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展開の公式

1 $(a+b)^2 = a^2 + 2ab + b^2$	$(a-b)^2 = a^2 - 2ab + b^2$
2 $(a+b)(a-b) = a^2 - b^2$	← 和と差の積が平方の差
3 $(x+a)(x+b) = x^2 + \underbrace{(a+b)x}_{\text{和}} + ab$	
△ 4 $(ax+b)(cx+d) = acx^2 + (ad+bc)x + bd$	
5 $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$	$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
6 $(a+b)(a^2 - ab + b^2) = a^3 + b^3$	$(a-b)(a^2 + ab + b^2) = a^3 - b^3$

(5の証明)

$$\begin{aligned} (a+b)^3 &= (a+b)(a+b)^2 \\ &= (a+b)(a^2 + 2ab + b^2) \\ &= a^3 + 2a^2b + ab^2 + a^2b + 2ab^2 + b^3 \\ &= a^3 + 3a^2b + 3ab^2 + b^3 \end{aligned}$$

$$\begin{array}{c} (a+b)^2(a+b) \\ =(a^2+2ab+b^2)(a+b) \\ \vdots \\ \text{としてもよい。} \end{array}$$

よって

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

また、この式のbを-bに替えると

$$\begin{aligned} \{a+(-b)\}^3 &= a^3 + 3a^2(-b) + 3a(-b)^2 + (-b)^3 \\ (a-b)^3 &= a^3 - 3a^2b + 3ab^2 - b^3 \quad \square \end{aligned}$$

(6の証明)

$$\begin{aligned} (a+b)(a^2 - ab + b^2) &= a^3 - a^2b + ab^2 + a^2b - ab^2 + b^3 \\ &= a^3 + b^3 \end{aligned}$$

よって

$$(a+b)(a^2 - ab + b^2) = a^3 + b^3$$

また、この式のbを-bに替えると

$$\begin{aligned} \{a+(-b)\}\{a^2 - a \cdot (-b) + (-b)^2\} &= a^3 + (-b)^3 \\ (a-b)(a^2 + ab + b^2) &= a^3 - b^3 \quad \square \end{aligned}$$

(例) 次の式を展開せよ。

$$\begin{aligned} (1) \quad (2x-3)^2 &= (2x)^2 + 2 \cdot 2x \cdot (-3) + (-3)^2 \quad \leftarrow \begin{array}{l} \{2x+(-3)\}^2 \text{と見える} \\ (O+\Delta)^2 = O^2 + 2O\Delta + \Delta^2 \end{array} \\ &= 4x^2 - 12x + 9 \quad \square \end{aligned}$$

$$\begin{aligned} (2) \quad (3a+2b)(3a-2b) &= (3a)^2 - (2b)^2 \quad \leftarrow (O+\Delta)(O-\Delta) = O^2 - \Delta^2 \\ &= 9a^2 - 4b^2 \quad \square \end{aligned}$$

$$\begin{aligned} (3) \quad (x+1)(x+3) &= x^2 + (1+3)x + 1 \cdot 3 \quad \leftarrow (x+a)(x+b) = x^2 + \textcircled{+}ab + \textcircled{\times} \end{aligned}$$

$$\begin{aligned} &= x^2 + 4x + 3 \quad \square \end{aligned}$$

$$\begin{aligned} (4) \quad (x+2y)(x-3y) &= x^2 + \{2y+(-3y)\}x + 2y \cdot (-3y) \quad \leftarrow (x+a)(x+b) = x^2 + \textcircled{+}ab + \textcircled{\times} \end{aligned}$$

$$= x^2 - 2y^2 - 6y^2 \quad \square$$

$$\begin{aligned} (5) \quad (2x+1)(3x+4) &= 6x^2 + 8x + 3x + 4 \\ &= 6x^2 + 11x + 4 \quad \square \end{aligned}$$

$$\begin{aligned} (6) \quad (x+2)^3 &= x^3 + 3x^2 \cdot 2 + 3x \cdot 2^2 + 2^3 \quad \leftarrow (O+\Delta)^3 = O^3 + 3O^2\Delta + 3O\Delta^2 + \Delta^3 \\ &= x^3 + 6x^2 + 12x + 8 \quad \square \end{aligned}$$

$$\begin{aligned} (7) \quad (2a-b)^3 &= (2a)^3 + 3(2a)^2(-b) + 3 \cdot 2a \cdot (-b)^2 + (-b)^3 \quad \leftarrow \begin{array}{l} \{2a+(-b)\}^3 \text{と見える} \\ (O+\Delta)^3 = O^3 + 3O^2\Delta + 3O\Delta^2 + \Delta^3 \end{array} \\ &= 8a^3 - 12a^2b + 6ab^2 - b^3 \end{aligned}$$

$$\begin{aligned} (8) \quad (x+2)(x^2 - 2x + 4) &= (x+2)(x^2 - 2x + 2^2) \\ &= x^3 + 2^3 \\ &= x^3 + 8 \quad \square \end{aligned}$$

$$\begin{aligned} (9) \quad (3a-2b)(9a^2 + 6ab + 4b^2) &= (3a-2b)\{(3a)^2 + 3a \cdot 2b + (2b)^2\} \\ &= (3a)^3 - (2b)^3 \\ &= 27a^3 - 8b^3 \quad \square \end{aligned}$$