

対数の性質

$a > 0, a \neq 1, M > 0, N > 0, k$ が実数のとき

$$\log_a M + \log_a N = \log_a MN \quad \leftarrow \text{足し算はかけ算に}$$

$$\log_a M - \log_a N = \log_a \frac{M}{N} \quad \leftarrow \text{引き算はわり算に}$$

$$k \log_a M = \log_a M^k \quad \leftarrow \text{係数は指数に}$$

(ざっくり証明)

$$p = \log_a M, \quad q = \log_a N \text{ とおくと}$$

$$M = a^p, \quad N = a^q$$

つまり

$$MN = a^{p+q} \Leftrightarrow p+q = \log_a MN$$

$$\frac{M}{N} = a^{p-q} \Leftrightarrow p-q = \log_a \frac{M}{N}$$

$$M^k = a^{kp} \Leftrightarrow kp = \log_a M^k$$

(例)

$$(1) \quad \log_{10} 2 + \log_{10} 5 = \log_{10} 2 \times 5 = \log_{10} 10 = 1 //$$

$$(2) \quad \log_2 12 - \log_2 3 = \log_2 \frac{12}{3} = \log_2 4 = 2 //$$

$$\begin{aligned} (3) \quad 2 \log_3 \sqrt{5} - \log_3 45 &= \log_3 (\sqrt{5})^2 - \log_3 45 \\ &= \log_3 5 - \log_3 45 \\ &= \log_3 \frac{5}{45} \\ &= \log_3 \frac{1}{9} \\ &= -2 // \end{aligned}$$

(別解)

$$\begin{aligned} 2 \log_3 \sqrt{5} - \log_3 45 &= 2 \log_3 5^{\frac{1}{2}} - \log_3 (3^2 \cdot 5) \\ &= 2 \cdot \frac{1}{2} \log_3 5 - (\log_3 3^2 + \log_3 5) \\ &= -2 // \end{aligned}$$