

## ・三角関数の相互関係

1. $\tan \theta = \frac{\sin \theta}{\cos \theta}$	△ 三角比のとこと 同じ
2. $\sin^2 \theta + \cos^2 \theta = 1$	
3. $1 + \tan^2 \theta = \frac{1}{\cos^2 \theta}$	

(例1)  $\frac{3}{2}\pi < \theta < 2\pi$ ,  $\cos \theta = \frac{12}{13}$  のとき,  $\sin \theta$  と  $\tan \theta$  は? $\frac{3}{2}\pi < \theta < 2\pi$  より

$\sin \theta < 0$

よって,  $\sin^2 \theta + \cos^2 \theta = 1$  より

$$\begin{aligned} \sin \theta &= -\sqrt{1 - \cos^2 \theta} \\ &= -\sqrt{1 - \left(\frac{12}{13}\right)^2} \\ &= -\frac{5}{13} \end{aligned}$$

また

$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{-\frac{5}{13}}{\frac{12}{13}} = -\frac{5}{12}$

(例2)  $\tan \theta = 3$  のとき,  $\sin \theta$  と  $\cos \theta$  は?

+ θに制限なし

$1 + \tan^2 \theta = \frac{1}{\cos^2 \theta}$  より

$$\begin{aligned} \frac{1}{\cos^2 \theta} &= 1 + \tan^2 \theta \\ &= 1 + 3^2 \\ &= 10 \end{aligned}$$

よって

$\cos^2 \theta = \frac{1}{10} \quad \therefore \cos \theta = \pm \frac{1}{\sqrt{10}}$

また,  $\tan \theta = \frac{\sin \theta}{\cos \theta}$  より

$\cos \theta = \frac{1}{\sqrt{10}}$  のとき

$\sin \theta = \tan \theta \cos \theta = 3 \cdot \frac{1}{\sqrt{10}} = \frac{3}{\sqrt{10}}$

$\cos \theta = -\frac{1}{\sqrt{10}}$  のとき

$\sin \theta = \tan \theta \cos \theta = 3 \cdot \left(-\frac{1}{\sqrt{10}}\right) = -\frac{3}{\sqrt{10}}$

以上より

$$\begin{aligned} \cos \theta &= \frac{1}{\sqrt{10}}, \sin \theta = \frac{3}{\sqrt{10}} \quad \text{または} \quad \cos \theta = -\frac{1}{\sqrt{10}}, \sin \theta = -\frac{3}{\sqrt{10}}, \quad + \cos \theta = \pm \frac{1}{\sqrt{10}}, \sin \theta = \pm \frac{3}{\sqrt{10}} \text{ となる}, \\ \cos \theta &= \pm \frac{1}{\sqrt{10}}, \sin \theta = \pm \frac{3}{\sqrt{10}} \quad (\text{符号同値}) \end{aligned}$$

(例3) 等式  $\tan \theta + \frac{1}{\tan \theta} = \frac{1}{\sin \theta \cos \theta}$  を証明せよ。

$$\begin{aligned} (\text{左辺}) &= \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \quad + \tan \theta = \frac{\sin \theta}{\cos \theta} \\ &= \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} \\ &= \frac{1}{\sin \theta \cos \theta} \quad + \sin^2 \theta + \cos^2 \theta = 1 \\ &= (\text{右辺}) \quad \square \end{aligned}$$