30

・積和、和積の公式（導出）

$$\sin\alpha\cos\beta = \frac{1}{2}\{\sin(\alpha+\beta)+\sin(\alpha-\beta)\}$$
$$\cos\alpha\sin\beta = \frac{1}{2}\{\sin(\alpha+\beta)-\sin(\alpha-\beta)\}$$
$$\cos\alpha\cos\beta = \frac{1}{2}\{\cos(\alpha+\beta)+\cos(\alpha-\beta)\}$$
$$\sin\alpha\sin\beta = -\frac{1}{2}\{\cos(\alpha+\beta)-\cos(\alpha-\beta)\}$$

$$\sin A+\sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$$
$$\sin A-\sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$$
$$\cos A+\cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$$
$$\cos A-\cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$$

（ざっくり証明）

$$\begin{cases}\sin(\alpha+\beta) = \sin\alpha\cos\beta+\cos\alpha\sin\beta & \cdots ① \\ \sin(\alpha-\beta) = \sin\alpha\cos\beta-\cos\alpha\sin\beta & \cdots ②\end{cases}$$

① + ② より

$$2\sin\underset{\frac{A+B}{2}}{\underset{\downarrow}{\alpha}}\cos\underset{\frac{A-B}{2}}{\underset{\downarrow}{\beta}} = \sin\underbrace{(\alpha+\beta)}_{A}+\sin\underbrace{(\alpha-\beta)}_{B}$$

$$\therefore \sin\alpha\cos\beta = \frac{1}{2}\{\sin(\alpha+\beta)+\sin(\alpha-\beta)\}$$
$$\sin A+\sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$$

① − ② より

$$2\cos\underset{\frac{A+B}{2}}{\underset{\downarrow}{\alpha}}\sin\underset{\frac{A-B}{2}}{\underset{\downarrow}{\beta}} = \sin\underbrace{(\alpha+\beta)}_{A}-\sin\underbrace{(\alpha-\beta)}_{B}$$

$$\therefore \cos\alpha\sin\beta = \frac{1}{2}\{\sin(\alpha+\beta)-\sin(\alpha-\beta)\}$$
$$\sin A-\sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$$

$$\begin{cases}\cos(\alpha+\beta) = \cos\alpha\cos\beta-\sin\alpha\sin\beta & \cdots ③ \\ \cos(\alpha-\beta) = \cos\alpha\cos\beta+\sin\alpha\sin\beta & \cdots ④\end{cases}$$

③ + ④ より

$$2\cos\underset{\frac{A+B}{2}}{\underset{\downarrow}{\alpha}}\cos\underset{\frac{A-B}{2}}{\underset{\downarrow}{\beta}} = \cos\underbrace{(\alpha+\beta)}_{A}+\cos\underbrace{(\alpha-\beta)}_{B}$$

$$\therefore \cos\alpha\cos\beta = \frac{1}{2}\{\cos(\alpha+\beta)+\cos(\alpha-\beta)\}$$
$$\cos A+\cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$$

③ − ④ より

$$-2\sin\underset{\frac{A+B}{2}}{\underset{\downarrow}{\alpha}}\sin\underset{\frac{A-B}{2}}{\underset{\downarrow}{\beta}} = \cos\underbrace{(\alpha+\beta)}_{A}-\cos\underbrace{(\alpha-\beta)}_{B}$$

$$\therefore \sin\alpha\sin\beta = -\frac{1}{2}\{\cos(\alpha+\beta)-\cos(\alpha-\beta)\}$$
$$\cos A-\cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$$

（例）次の式を三角関数の和または積の形に変形せよ

(1) $\sin 5\theta\cos 3\theta = \frac{1}{2}\{\sin(5\theta+3\theta)+\sin(5\theta-3\theta)\}$

$\sin 5\theta\cos 3\theta+\cos 3\theta\sin 5\theta$　　$\sin 5\theta\cos 3\theta-\cos 5\theta\sin 3\theta$

sin の加法定理

$= \frac{1}{2}(\sin 8\theta+\sin 2\theta)$ //

(2) $\sin 3\theta\cos 5\theta = \cos 5\theta\sin 3\theta$ ← $3\theta-5\theta=-2\theta$ と「−」がつくのを避けるため

sin の加法定理

$= \frac{1}{2}\{\sin(5\theta+3\theta)-\sin(5\theta-3\theta)\}$

$= \frac{1}{2}(\sin 8\theta-\sin 2\theta)$ //

(3) $\cos 5\theta\cos 3\theta = \frac{1}{2}\{\cos(5\theta+3\theta)+\cos(5\theta-3\theta)\}$

cos の加法定理

$= \frac{1}{2}(\cos 8\theta+\cos 2\theta)$ //

(4) $\sin 5\theta\sin 3\theta = -\frac{1}{2}\{\cos(5\theta+3\theta)-\cos(5\theta-3\theta)\}$

cos の加法定理

$= -\frac{1}{2}(\cos 8\theta-\cos 2\theta)$ //