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・正接の加法定理 (暗記+導出)

$$\begin{aligned}\tan(\alpha \oplus \beta) &= \frac{\tan \alpha \oplus \tan \beta}{1 \ominus \tan \alpha \tan \beta} \\ \tan(\alpha \ominus \beta) &= \frac{\tan \alpha \ominus \tan \beta}{1 \oplus \tan \alpha \tan \beta}\end{aligned}$$

(証明)

$$\begin{aligned}\tan(\alpha + \beta) &= \frac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)} \\ &= \frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\cos \alpha \cos \beta - \sin \alpha \sin \beta} \\ &= \frac{\frac{\sin \alpha}{\cos \alpha} + \frac{\sin \beta}{\cos \beta}}{1 - \frac{\sin \alpha \sin \beta}{\cos \alpha \cos \beta}} \quad \leftarrow \text{分母・分子を } \cos \alpha \cos \beta \\ &= \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}\end{aligned}$$

また、この式に  $\beta$  を  $-\beta$  にあてはめると

$$\begin{aligned}\tan(\alpha + (-\beta)) &= \frac{\tan \alpha + \tan(-\beta)}{1 - \tan \alpha \tan(-\beta)} \\ &= \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta} \quad \square \quad \leftarrow \tan(-\beta) = -\tan \beta\end{aligned}$$

(例)

$$\begin{aligned}\tan 75^\circ &= \tan(45^\circ + 30^\circ) \\ &= \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ} \\ &= \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1 \cdot \frac{1}{\sqrt{3}}} \\ &= \frac{\sqrt{3} + 1}{\sqrt{3} - 1} \\ &= \frac{\sqrt{3} + 1}{\sqrt{3} - 1} \cdot \frac{\sqrt{3} + 1}{\sqrt{3} + 1} \\ &= \frac{3 + 2\sqrt{3} + 1}{2} \\ &= 2 + \sqrt{3} \quad \text{,,}\end{aligned}$$