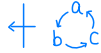


7

・ おき換えを利用した展開の工夫

$$\text{7} \quad (a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$$

(証明)

$$\begin{aligned} (a+b+c)^2 &= (a+b+c)(a+b+c) \\ &= a^2+ab+ac+ab+b^2+bc+ac+bc+c^2 \\ &= (a^2+b^2+c^2+2ab+2bc+2ac) \\ &= a^2+b^2+c^2+2ab+2bc+2ca \quad \square \end{aligned}$$


(別の証明)

 $a+b=A$ とおく \leftarrow おき換えの工夫

$$\begin{aligned} (\text{与式}) &= (A+c)^2 \\ &= A^2+2Ac+c^2 \\ &= (a+b)^2+2(a+b)c+c^2 \\ &= a^2+2ab+b^2+2ca+2bc+c^2 \\ &= a^2+b^2+c^2+2ab+2bc+2ca \quad \square \end{aligned}$$

(例1) 次の式を展開せよ。

$$\begin{aligned} (a-2b+3c)^2 &= \{a+(-2b)+(3c)\}^2 \\ &= a^2+(-2b)^2+(3c)^2+2a(-2b)+2(-2b)(3c)+2(3c) \cdot a \\ &= a^2+4b^2+9c^2-4ab-12bc+6ca \quad \square \end{aligned}$$

(例2) 次の式を展開せよ。

$$(1) \quad (a^2+ab+b^2)(a^2-ab+b^2)$$

$$a^2+b^2=A \text{ とおく}$$

$$\begin{aligned} (\text{与式}) &= (A+ab)(A-ab) \\ &= A^2-(ab)^2 \\ &= (a^2+b^2)^2-a^2b^2 \\ &= a^4+2a^2b^2+b^4-a^2b^2 \\ &= a^4+a^2b^2+b^4 \quad \square \end{aligned}$$

$$(2) \quad (a+b-c)(a-b+c)$$

$$b-c=A \text{ とおく}$$

$$\begin{aligned} (\text{与式}) &= \{a+(b-c)\}\{a-(b-c)\} \\ &= (a+A)(a-A) \\ &= a^2-A^2 \\ &= a^2-(b-c)^2 \\ &= a^2-(b^2-2bc+c^2) \\ &= a^2-b^2+2bc-c^2 \quad \square \end{aligned}$$