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・三角関数を含む方程式、不等式⑤

(例) $0 \leq \theta < 2\pi$ のとき、次の方程式、不等式を解け。

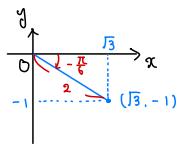
$$(1) \sqrt{3}\sin\theta - \cos\theta = \sqrt{3} \quad (2) \sqrt{3}\cos\theta - \sin\theta + 1 < 0$$

point

合成を用いて、三角関数を統一する

おまかせ → おまかえた文字の範囲に注意

$$(1) \begin{aligned} \sqrt{3}\sin\theta - \cos\theta &= \sqrt{3} \\ 2\sin(\theta - \frac{\pi}{6}) &= \sqrt{3} \\ \sin(\theta - \frac{\pi}{6}) &= \frac{\sqrt{3}}{2} \end{aligned}$$



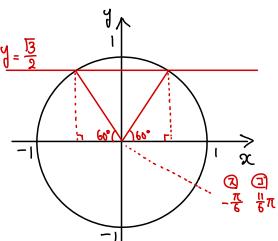
$$\begin{aligned} t &= \theta - \frac{\pi}{6} \text{ とおくと, } 0 \leq \theta < 2\pi \text{ のとき} \\ -\frac{\pi}{6} \leq \theta - \frac{\pi}{6} &< \frac{11}{6}\pi \text{ つまり } -\frac{\pi}{6} \leq t < \frac{11}{6}\pi \end{aligned}$$

このとき

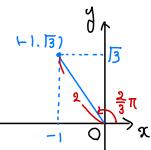
$$\sin t = \frac{\sqrt{3}}{2} \quad \therefore t = \frac{\pi}{3}, \frac{2}{3}\pi$$

よって

$$\begin{aligned} \theta - \frac{\pi}{6} &= \frac{\pi}{3}, \frac{2}{3}\pi \\ \therefore \theta &= \frac{\pi}{2}, \frac{5}{6}\pi \end{aligned}$$



$$(2) \begin{aligned} \sqrt{3}\cos\theta - \sin\theta + 1 &< 0 \\ 2\sin(\theta + \frac{2}{3}\pi) &< -1 \\ \sin(\theta + \frac{2}{3}\pi) &< -\frac{1}{2} \end{aligned}$$



$$\begin{aligned} t &= \theta + \frac{2}{3}\pi \text{ とおくと, } 0 \leq \theta < 2\pi \text{ のとき} \\ \frac{2}{3}\pi \leq \theta + \frac{2}{3}\pi &< \frac{11}{3}\pi \text{ つまり } \frac{2}{3}\pi \leq t < \frac{8}{3}\pi \end{aligned}$$

このとき

$$\begin{aligned} \sin t &< -\frac{1}{2} \\ \therefore \frac{7}{6}\pi &< t < \frac{11}{6}\pi \end{aligned}$$

よって

$$\frac{7}{6}\pi < \theta + \frac{2}{3}\pi < \frac{11}{6}\pi$$

つまり

$$\frac{\pi}{2} < \theta < \frac{7}{6}\pi$$

