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正接の加法定理 (暗記+導出)

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta}$$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha \tan\beta}$$

(証明)

$$\begin{aligned}\tan(\alpha + \beta) &= \frac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)} \\ &= \frac{\sin\alpha \cos\beta + \cos\alpha \sin\beta}{\cos\alpha \cos\beta - \sin\alpha \sin\beta} \\ &= \frac{\frac{\sin\alpha}{\cos\alpha} + \frac{\sin\beta}{\cos\beta}}{1 - \frac{\sin\alpha \sin\beta}{\cos\alpha \cos\beta}} \quad \leftarrow \text{分子 分母を } \div \cos\alpha \cos\beta \\ &= \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta}\end{aligned}$$

また、この式の β を $-\beta$ に置き換えると

$$\begin{aligned}\tan(\alpha + (-\beta)) &= \frac{\tan\alpha + \tan(-\beta)}{1 - \tan\alpha \tan(-\beta)} \\ &= \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha \tan\beta} \quad \leftarrow \tan(-\beta) = -\tan\beta\end{aligned}$$

(例)

$$\begin{aligned}\tan 75^\circ &= \tan(45^\circ + 30^\circ) \\ &= \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ} \\ &= \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1 \cdot \frac{1}{\sqrt{3}}} \\ &= \frac{\sqrt{3} + 1}{\sqrt{3} - 1} \\ &= \frac{\sqrt{3} + 1}{\sqrt{3} - 1} \cdot \frac{\sqrt{3} + 1}{\sqrt{3} + 1} \\ &= \frac{3 + 2\sqrt{3} + 1}{2} \\ &= 2 + \sqrt{3},\end{aligned}$$