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展開の公式

1	$(a+b)^2 = a^2 + 2ab + b^2$	$(a-b)^2 = a^2 - 2ab + b^2$
2	$(a+b)(a-b) = a^2 - b^2$	← 和と差の積は平方の差
3	$(x+a)(x+b) = x^2 + \underbrace{(a+b)}_{\text{和}}x + \underbrace{ab}_{\text{積}}$	
4	$(ax+b)(cx+d) = acx^2 + (ad+bc)x + bd$	
5	$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$	$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
6	$(a+b)(a^2-ab+b^2) = a^3+b^3$	$(a-b)(a^2+ab+b^2) = a^3-b^3$

(5)の証明

$$\begin{aligned}
 (a+b)^3 &= (a+b)(a+b)^2 \\
 &= (a+b)(a^2+2ab+b^2) \\
 &= a^3+2a^2b+ab^2+a^2b+2ab^2+b^3 \\
 &= a^3+3a^2b+3ab^2+b^3
 \end{aligned}
 \quad \leftarrow \begin{array}{l} (a+b)^2(a+b) \\ = (a^2+2ab+b^2)(a+b) \\ \vdots \\ \text{としてもよい。} \end{array}$$

よって

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

また、この式のbを-bにおき換えると

$$\begin{aligned}
 \{a+(-b)\}^3 &= a^3 + 3a^2(-b) + 3a(-b)^2 + (-b)^3 \\
 (a-b)^3 &= a^3 - 3a^2b + 3ab^2 - b^3 \quad \square
 \end{aligned}$$

(6)の証明

$$\begin{aligned}
 (a+b)(a^2-ab+b^2) &= a^3 - a^2b + ab^2 + a^2b - ab^2 + b^3 \\
 &= a^3 + b^3
 \end{aligned}$$

よって

$$(a+b)(a^2-ab+b^2) = a^3+b^3$$

また、この式のbを-bにおき換えると

$$\begin{aligned}
 \{a+(-b)\}(a^2-a(-b)+(-b)^2) &= a^3+(-b)^3 \\
 (a-b)(a^2+ab+b^2) &= a^3-b^3 \quad \square
 \end{aligned}$$

(例) 次の式を展開せよ。

$$\begin{aligned}
 (1) \quad (2x-3)^2 &= (2x)^2 + 2 \cdot 2x \cdot (-3) + (-3)^2 \quad \leftarrow \begin{array}{l} \{2x+(-3)\}^2 \text{と捉える} \\ (O+\Delta)^2 = O^2 + 2O\Delta + \Delta^2 \end{array} \\
 &= 4x^2 - 12x + 9 \quad \text{,,}
 \end{aligned}$$

$$\begin{aligned}
 (2) \quad (3a+2b)(3a-2b) &= (3a)^2 - (2b)^2 \quad \leftarrow (O+\Delta)(O-\Delta) = O^2 - \Delta^2 \\
 &= 9a^2 - 4b^2 \quad \text{,,}
 \end{aligned}$$

$$\begin{aligned}
 (3) \quad (x+1)(x+3) &= x^2 + (1+3)x + 1 \cdot 3 \quad \leftarrow (x+O)(x+\Delta) = x^2 + O\Delta + O\Delta \\
 &= x^2 + 4x + 3 \quad \text{,,}
 \end{aligned}$$

$$\begin{aligned}
 (4) \quad (x+2y)(x-3y) &= x^2 + \{2y+(-3y)\}x + 2y \cdot (-3y) \quad \leftarrow (x+O)(x+\Delta) = x^2 + O\Delta + O\Delta \\
 &= x^2 - xy - 6y^2 \quad \text{,,}
 \end{aligned}$$

$$\begin{aligned}
 (5) \quad (2x+1)(3x+4) &= 6x^2 + 8x + 3x + 4 \\
 &= 6x^2 + 11x + 4 \quad \text{,,}
 \end{aligned}$$

$$\begin{aligned}
 (6) \quad (x+2)^3 &= x^3 + 3x^2 \cdot 2 + 3x \cdot 2^2 + 2^3 \quad \leftarrow (O+\Delta)^3 = O^3 + 3O^2\Delta + 3O\Delta^2 + \Delta^3 \\
 &= x^3 + 6x^2 + 12x + 8 \quad \text{,,}
 \end{aligned}$$

$$\begin{aligned}
 (7) \quad (2a-b)^3 &= (2a)^3 + 3 \cdot (2a)^2 \cdot (-b) + 3 \cdot 2a \cdot (-b)^2 + (-b)^3 \quad \leftarrow \begin{array}{l} \{2a+(-b)\}^3 \text{と捉える} \\ (O+\Delta)^3 = O^3 + 3O^2\Delta + 3O\Delta^2 + \Delta^3 \end{array} \\
 &= 8a^3 - 12a^2b + 6ab^2 - b^3 \quad \text{,,}
 \end{aligned}$$

$$\begin{aligned}
 (8) \quad (x+2)(x^2-2x+4) &= (x+2)(x^2-2x+2^2) \\
 &= x^3 + 2^3 \\
 &= x^3 + 8 \quad \text{,,}
 \end{aligned}$$

$$\begin{aligned}
 (9) \quad (3a-2b)(9a^2+6ab+4b^2) &= (3a-2b)\{(3a)^2+3a \cdot 2b+(2b)^2\} \\
 &= (3a)^3 - (2b)^3 \\
 &= 27a^3 - 8b^3 \quad \text{,,}
 \end{aligned}$$