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## 対数の性質

$a > 0, a \neq 1, M > 0, N > 0$ ,  $\log_a$ が実数のとき

$$\log_a M + \log_a N = \log_a MN \quad +\text{足し算は掛け算に}$$

$$\log_a M - \log_a N = \log_a \frac{M}{N} \quad +\text{引き算はわり算に}$$

$$\log_a M^k = k \log_a M \quad +\text{底数は指数に}$$

(ざっくり証明)

$$P = \log_a M, \quad Q = \log_a N \text{ とおくと}$$

$$M = a^P, \quad N = a^Q$$

つまり

$$MN = a^{P+Q} \Leftrightarrow P+Q = \log_a MN$$

$$\frac{M}{N} = a^{P-Q} \Leftrightarrow P-Q = \log_a \frac{M}{N}$$

$$M^k = a^{kP} \Leftrightarrow kP = \log_a M^k$$

(例)

$$(1) \quad \log_{10} 2 + \log_{10} 5 = \log_{10} 2 \times 5 = \log_{10} 10 = 1,$$

$$(2) \quad \log_2 12 - \log_2 3 = \log_2 \frac{12}{3} = \log_2 4 = 2,$$

$$(3) \quad 2 \log_3 \sqrt{5} - \log_3 45 = \log_3 (\sqrt{5})^2 - \log_3 45$$

$$= \log_3 5 - \log_3 45$$

$$= \log_3 \frac{5}{45}$$

$$= \log_3 \frac{1}{9}$$

$$= -2,$$

(別解)

$$2 \log_3 \sqrt{5} - \log_3 45 = 2 \log_3 5^{\frac{1}{2}} - \log_3 (3^2 \cdot 5)$$

$$= 2 \cdot \frac{1}{2} \log_3 5 - (\log_3 3^2 + \log_3 5)$$

$$= -2,$$