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・三角関数を含む方程式、不等式⑤

(例) $0 \leqq \theta < 2\pi$ のとき、次の方程式、不等式を解け。

(1) $\sqrt{3}\sin\theta - \cos\theta = \sqrt{3}$　　(2) $\sqrt{3}\cos\theta - \sin\theta + 1 < 0$

point

合成を用いて、三角関数を統一する

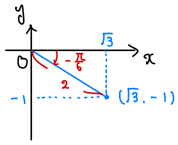
おきかえ → おきかえた文字の範囲に注意

(1)

$\sqrt{3}\sin\theta - 1\cos\theta = \sqrt{3}$

$2\sin\left(\theta - \frac{\pi}{6}\right) = \sqrt{3}$

$\sin\left(\theta - \frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$



$t = \theta - \frac{\pi}{6}$ とおくと、$0 \leqq \theta < 2\pi$ のとき

$-\frac{\pi}{6} \leqq \theta - \frac{\pi}{6} < \frac{11}{6}\pi$　つまり　$-\frac{\pi}{6} \leqq t < \frac{11}{6}\pi$

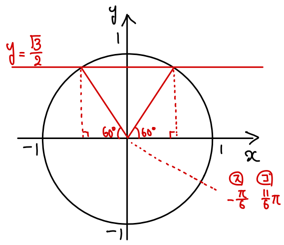
このとき

$\sin t = \frac{\sqrt{3}}{2}$　　$\therefore\ t = \frac{\pi}{3}, \frac{2}{3}\pi$

よって

$\theta - \frac{\pi}{6} = \frac{\pi}{3}, \frac{2}{3}\pi$

$\therefore\ \theta = \frac{\pi}{2}, \frac{5}{6}\pi$

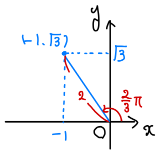


(2)

$\sqrt{3}\cos\theta - 1\sin\theta + 1 < 0$

$2\sin\left(\theta + \frac{2}{3}\pi\right) < -1$

$\sin\left(\theta + \frac{2}{3}\pi\right) < -\frac{1}{2}$



$t = \theta + \frac{2}{3}\pi$ とおくと、$0 \leqq \theta < 2\pi$ のとき

$\frac{2}{3}\pi \leqq \theta + \frac{2}{3}\pi < \frac{8}{3}\pi$　つまり　$\frac{2}{3}\pi \leqq t < \frac{8}{3}\pi$

このとき

$\sin t < -\frac{1}{2}$

$\therefore\ \frac{7}{6}\pi < t < \frac{11}{6}\pi$

よって

$\frac{7}{6}\pi < \theta + \frac{2}{3}\pi < \frac{11}{6}\pi$

つまり

$\frac{\pi}{2} < \theta < \frac{7}{6}\pi$ //