

20

分母の有理化

(例1) 次の式の分母を有理化せよ。

$$(1) \frac{3}{\sqrt{2}} = \frac{3}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{2} //$$

$$(2) \frac{6}{\sqrt{3}} = \frac{2 \times (\sqrt{3})^2}{\sqrt{3}} \quad \leftarrow \begin{array}{l} \text{左側} \\ \text{右側} \end{array} = \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$$

としてもよいが、少し冗長

$$(3) \frac{\sqrt{3}}{\sqrt{2} + \sqrt{3}} = \frac{\sqrt{3}}{\sqrt{2} + \sqrt{3}} \times \frac{\sqrt{2} - \sqrt{3}}{\sqrt{2} - \sqrt{3}} \quad \leftarrow (a+b)(a-b) = a^2 - b^2$$

$$= \frac{\sqrt{3}(\sqrt{2} - \sqrt{3})}{(\sqrt{2})^2 - (\sqrt{3})^2}$$

$$= \frac{\sqrt{6} - 3}{-1}$$

= $3 - \sqrt{6}$ //

$$(4) \frac{\sqrt{5} + 2}{\sqrt{5} - 2} = \frac{\sqrt{5} + 2}{\sqrt{5} - 2} \times \frac{\sqrt{5} + 2}{\sqrt{5} + 2}$$

$$= \frac{(\sqrt{5} + 2)^2}{(\sqrt{5})^2 - 2^2}$$

$$= 5 + 4\sqrt{5} + 4$$

= $9 + 4\sqrt{5}$ //

(例2) 次の式の分母を有理化せよ。

$$(1) \frac{1}{1 + \sqrt{2} + \sqrt{3}} = \frac{1}{(1 + \sqrt{2}) + \sqrt{3}} \times \frac{(1 + \sqrt{2}) - \sqrt{3}}{(1 + \sqrt{2}) - \sqrt{3}} \quad \leftarrow \frac{(1^2 + (\sqrt{2})^2) - (\sqrt{3})^2}{(1 + \sqrt{2})^2 - (\sqrt{3})^2} = 0$$

$$= \frac{(1 + \sqrt{2}) - \sqrt{3}}{(1 + 2\sqrt{2} + 2) - 3}$$

$$= \frac{1 + \sqrt{2} - \sqrt{3}}{2\sqrt{2}}$$

$$= \frac{\sqrt{2} + 2 - \sqrt{6}}{4}$$

//

$$(2) \frac{\sqrt{5} + \sqrt{3} + \sqrt{2}}{\sqrt{5} + \sqrt{3} - \sqrt{2}} = \frac{\sqrt{5} + \sqrt{3} + \sqrt{2}}{\sqrt{5} + (\sqrt{3} - \sqrt{2})} \times \frac{\sqrt{5} - (\sqrt{3} - \sqrt{2})}{\sqrt{5} - (\sqrt{3} - \sqrt{2})} \quad \leftarrow \frac{(\sqrt{5})^2 - ((\sqrt{3})^2 + (\sqrt{2})^2)}{(\sqrt{5})^2 - (\sqrt{3} - \sqrt{2})^2} = 0$$

$$= \frac{(\sqrt{5} + \sqrt{3})^2 - (\sqrt{3})^2}{5 - (3 - 2\sqrt{6} + 2)}$$

$$= \frac{(\sqrt{5} + 2\sqrt{6} + 2) - 3}{2\sqrt{6}}$$

$$= \frac{4 + 2\sqrt{10}}{2\sqrt{6}}$$

$$= \frac{2 + \sqrt{10}}{\sqrt{6}}$$

$$= \frac{2\sqrt{6} + 2\sqrt{5}}{6}$$

= $\frac{\sqrt{6} + \sqrt{5}}{3}$ //