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・正接の加法定理（暗記＋導出）

$$\boxed{\begin{aligned} \tan(\alpha+\beta) &= \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta} \\ \tan(\alpha-\beta) &= \frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta} \end{aligned}}$$

(証明)

$$\tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}$$

$$= \frac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\cos\alpha\cos\beta-\sin\alpha\sin\beta}$$

$$= \frac{\frac{\sin\alpha}{\cos\alpha}+\frac{\sin\beta}{\cos\beta}}{1-\frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}$$ ← 分母・分子を÷$\cos\alpha\cos\beta$

$$= \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

また、この式のβを−βにおきかえると

$$\tan\{\alpha+(-\beta)\} = \frac{\tan\alpha+\tan(-\beta)}{1-\tan\alpha\tan(-\beta)}$$

$$= \frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta} \quad \square$$ ← $\tan(-\beta) = -\tan\beta$

(例)

$$\tan 75^\circ = \tan(45^\circ+30^\circ)$$

$$= \frac{\tan 45^\circ+\tan 30^\circ}{1-\tan 45^\circ\tan 30^\circ}$$

$$= \frac{1+\frac{1}{\sqrt{3}}}{1-1\cdot\frac{1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$= \frac{\sqrt{3}+1}{\sqrt{3}-1}\cdot\frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{3+2\sqrt{3}+1}{2}$$

$$= 2+\sqrt{3}$$ //