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計算の順序を意識した展開の工夫

(例) 次の式を展開せよ。

$$(1) \quad (x+1)^2(x-1)^2 = [(x+1)(x-1)]^2 \quad \leftarrow a^2 b^2 = (ab)^2 \\ = (x^2 - 1)^2 \\ = x^4 - 2x^2 + 1$$

$$(2) \quad (a^2 + b^2)(a+b)(a-b) = (a^2 + b^2)(a^2 - b^2) \\ = a^4 - b^4$$

$$(3) \quad (x+1)(x+2)(x+3)(x+4) = [(x+1)(x+4)][(x+2)(x+3)] \\ = (x^2 + 5x + 4)(x^2 + 5x + 6)$$

ここで、 $x^2 + 5x = X$ とおくと

$$\begin{aligned} (\text{与式}) &= (x+4)(x+6) \\ &= X^2 + 10X + 24 \\ &= (X^2 + 5X)^2 + 10(X^2 + 5X) + 24 \\ &= X^4 + 10X^3 + 25X^2 + 10X^2 + 50X + 24 \\ &= X^4 + 10X^3 + 35X^2 + 50X + 24 \end{aligned}$$

$$\begin{aligned} (4) \quad (a+2b)(a-2b)(a^2+2ab+4b^2)(a^2-2ab+4b^2) \\ &= [(a+2b)(a^2-2ab+4b^2)][(a-2b)(a^2+2ab+4b^2)] \\ &= (a^3 + 8b^3)(a^3 - 8b^3) \\ &= a^6 - 64b^6 \end{aligned}$$