26

・2倍角の公式（暗記＋導出）

$$\boxed{\begin{aligned} \sin 2\alpha &= 2\sin\alpha\cos\alpha \\ \cos 2\alpha &= \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha \\ \tan 2\alpha &= \frac{2\tan\alpha}{1-\tan^2\alpha} \end{aligned}}$$

(ざっくり証明)

$$\sin(\alpha+\underset{\alpha}{\beta}) = \sin\alpha\cos\underset{\alpha}{\beta} + \cos\alpha\sin\underset{\alpha}{\beta}$$

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$\cos(\alpha+\underset{\alpha}{\beta}) = \cos\alpha\cos\underset{\alpha}{\beta} - \sin\alpha\sin\underset{\alpha}{\beta}$$

$$\begin{aligned} \cos 2\alpha &= \underbrace{\cos^2\alpha}_{1-\sin^2\alpha} - \underbrace{\sin^2\alpha}_{1-\cos^2\alpha} \\ &= 2\cos^2\alpha - 1 \\ &= 1 - 2\sin^2\alpha \end{aligned}$$

$$\tan(\alpha+\underset{\alpha}{\beta}) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\underset{\alpha}{\beta}} \ \alpha$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$$

(例) $\frac{\pi}{2} < \alpha < \pi$, $\sin\alpha = \frac{4}{5}$ のとき, $\sin 2\alpha$, $\cos 2\alpha$, $\tan 2\alpha$ の値を求めよ。

$\frac{\pi}{2} < \alpha < \pi$ より

$$\cos\alpha < 0$$

であるから

$$\begin{aligned} \cos\alpha &= -\sqrt{1-\sin^2\alpha} \\ &= -\sqrt{1-\left(\frac{4}{5}\right)^2} \\ &= -\frac{3}{5} \end{aligned}$$

よって

$$\sin 2\alpha = 2\sin\alpha\cos\alpha = 2\cdot\frac{4}{5}\cdot\left(-\frac{3}{5}\right) = -\frac{24}{25}$$

$$\cos 2\alpha = 1 - 2\sin^2\alpha = 1 - 2\cdot\left(\frac{4}{5}\right)^2 = -\frac{7}{25}$$

$$\tan 2\alpha = \frac{\sin 2\alpha}{\cos 2\alpha} = \frac{-\frac{24}{25}}{-\frac{7}{25}} = \frac{24}{7} \,_{//}$$

(別解)

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{\frac{4}{5}}{-\frac{3}{5}} = -\frac{4}{3}$$

であるから

$$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{2\cdot\left(-\frac{4}{3}\right)}{1-\left(-\frac{4}{3}\right)^2} = \frac{24}{7} \,_{//}$$