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・半角の公式（理 暗記＋導出）

$$\boxed{\begin{aligned}\sin^2\frac{\alpha}{2} &= \frac{1-\cos\alpha}{2}\\ \cos^2\frac{\alpha}{2} &= \frac{1+\cos\alpha}{2}\\ \tan^2\frac{\alpha}{2} &= \frac{1-\cos\alpha}{1+\cos\alpha}\end{aligned}}$$

（ざっくり証明）

$$\cos 2\alpha = 1-2\sin^2\alpha = 2\cos^2\alpha - 1$$

$$\sin^2\alpha = \frac{1-\cos 2\alpha}{2} \qquad \cos^2\alpha = \frac{1+\cos 2\alpha}{2}$$

αを$\frac{\alpha}{2}$でおきかえ

$$\sin^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{2} \qquad \cos^2\frac{\alpha}{2} = \frac{1+\cos\alpha}{2}$$

$$\tan^2\frac{\alpha}{2} = \frac{\sin^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}} = \frac{\frac{1-\cos\alpha}{2}}{\frac{1+\cos\alpha}{2}} = \frac{1-\cos\alpha}{1+\cos\alpha}$$

(例1) $\sin\frac{\pi}{8}$, $\cos\frac{\pi}{8}$, $\tan\frac{\pi}{8}$ の値を求めよ。

$0<\frac{\pi}{8}<\frac{\pi}{2}$ であるから

$$\sin\frac{\pi}{8}>0,\quad \cos\frac{\pi}{8}>0$$

よって.

$$\sin\frac{\pi}{8} = \sqrt{\frac{1-\cos\frac{\pi}{4}}{2}} = \sqrt{\frac{1-\frac{1}{\sqrt{2}}}{2}} = \sqrt{\frac{2-\sqrt{2}}{4}} = \frac{\sqrt{2-\sqrt{2}}}{2}$$

$$\cos\frac{\pi}{8} = \sqrt{\frac{1+\cos\frac{\pi}{4}}{2}} = \sqrt{\frac{1+\frac{1}{\sqrt{2}}}{2}} = \sqrt{\frac{2+\sqrt{2}}{4}} = \frac{\sqrt{2+\sqrt{2}}}{2}$$

$$\tan\frac{\pi}{8} = \frac{\sin\frac{\pi}{8}}{\cos\frac{\pi}{8}} = \frac{\frac{\sqrt{2-\sqrt{2}}}{2}}{\frac{\sqrt{2+\sqrt{2}}}{2}} = \sqrt{\frac{2-\sqrt{2}}{2+\sqrt{2}}} = \sqrt{3-2\sqrt{2}} = \sqrt{2}-1 \,/\!/$$

(例2) $\frac{\pi}{2}<\alpha<\pi$, $\sin\alpha=\frac{3}{5}$ のとき, $\sin\frac{\alpha}{2}$, $\cos\frac{\alpha}{2}$, $\tan\frac{\alpha}{2}$ の値を求めよ。

$\frac{\pi}{2}<\alpha<\pi$ より

$$\cos\alpha<0$$

であるから

$$\cos\alpha = -\sqrt{1-\sin^2\alpha} = -\sqrt{1-\left(\frac{3}{5}\right)^2} = -\frac{4}{5}$$

また, $\frac{\pi}{2}<\alpha<\pi$ より $\frac{\pi}{4}<\frac{\alpha}{2}<\frac{\pi}{2}$ であるから

$$\sin\frac{\alpha}{2}>0,\quad \cos\frac{\alpha}{2}>0$$

よって

$$\sin\frac{\alpha}{2} = \sqrt{\frac{1-\cos\alpha}{2}} = \sqrt{\frac{1-\left(-\frac{4}{5}\right)}{2}} = \frac{3}{\sqrt{10}} \,/\!/$$

$$\cos\frac{\alpha}{2} = \sqrt{\frac{1+\cos\alpha}{2}} = \sqrt{\frac{1+\left(-\frac{4}{5}\right)}{2}} = \frac{1}{\sqrt{10}} \,/\!/$$

$$\tan\frac{\alpha}{2} = \frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} = \frac{\frac{3}{\sqrt{10}}}{\frac{1}{\sqrt{10}}} = 3 \,/\!/$$