18

・三角関数を含む方程式、不等式 ②

(例) $0 \leqq \theta < 2\pi$ のとき、次の方程式、不等式を解け。

(1) $\sin\left(\theta+\frac{\pi}{4}\right)=\frac{1}{2}$　(2) $\cos\left(\theta-\frac{\pi}{3}\right)>-\frac{1}{2}$

(3) $\tan\left(2\theta+\frac{\pi}{3}\right)=1$

point

おきかえ → おきかえた文字の範囲に注意

$\sin\left(\underbrace{\theta+\frac{\pi}{4}}_{t\text{とおく}}\right)$

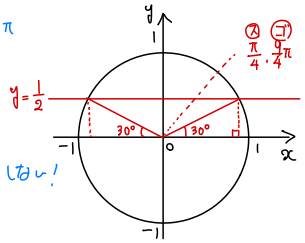
$0 \leqq \theta < 2\pi$

$\frac{\pi}{4} \leqq \theta+\frac{\pi}{4} \leqq \frac{9}{4}\pi$

$\frac{\pi}{4} \leqq t \leqq \frac{9}{4}\pi$

(1) $t=\theta+\frac{\pi}{4}$ とおくと、$0 \leqq \theta < 2\pi$ のとき

$\frac{\pi}{4} \leqq \theta+\frac{\pi}{4} < \frac{9}{4}\pi$ つまり $\frac{\pi}{4} \leqq t < \frac{9}{4}\pi$



このとき

$\sin t=\frac{1}{2}$　$\therefore t=\frac{\pi}{6}, \frac{13}{6}\pi$

↑ $\frac{\pi}{6}, \frac{5}{6}\pi$ としない!

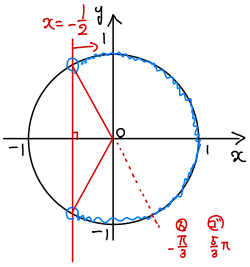
よって

$\theta+\frac{\pi}{4}=\frac{\pi}{6}, \frac{13}{6}\pi$

$\therefore \theta=\frac{7}{12}\pi, \frac{23}{12}\pi$

(2) $t=\theta-\frac{\pi}{3}$ とおくと、$0 \leqq \theta < 2\pi$ のとき

$-\frac{\pi}{3} \leqq \theta-\frac{\pi}{3} < \frac{5}{3}\pi$ つまり $-\frac{\pi}{3} \leqq t < \frac{5}{3}\pi$



このとき

$\cos t > -\frac{1}{2}$

$\therefore -\frac{\pi}{3} \leqq t < \frac{2}{3}\pi, \frac{4}{3}\pi < t < \frac{5}{3}\pi$

よって

$-\frac{\pi}{3} \leqq \theta-\frac{\pi}{3} < \frac{2}{3}\pi, \frac{4}{3}\pi < \theta-\frac{\pi}{3} < \frac{5}{3}\pi$

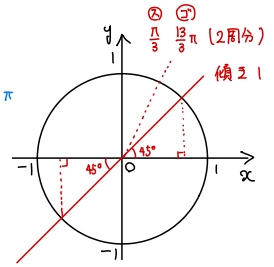
つまり

$0 \leqq \theta < \pi, \frac{5}{3}\pi < \theta < 2\pi$

(3) $t=2\theta+\frac{\pi}{3}$ とおくと、$0 \leqq \theta < 2\pi$ のとき

$0 \leqq 2\theta < 4\pi$

$\frac{\pi}{3} \leqq 2\theta+\frac{\pi}{3} < \frac{13}{3}\pi$ つまり $\frac{\pi}{3} \leqq t < \frac{13}{3}\pi$



このとき

$\tan t=1$

$\therefore t=\frac{5}{4}\pi, \frac{9}{4}\pi, \frac{13}{4}\pi, \frac{17}{4}\pi$

よって

$2\theta+\frac{\pi}{3}=\frac{5}{4}\pi, \frac{9}{4}\pi, \frac{13}{4}\pi, \frac{17}{4}\pi$

$2\theta=\frac{11}{12}\pi, \frac{23}{12}\pi, \frac{33}{12}\pi, \frac{47}{12}\pi$

$\therefore \theta=\frac{11}{24}\pi, \frac{23}{24}\pi, \frac{33}{24}\pi, \frac{47}{24}\pi$