

$a^3+b^3+c^3-3abc$ の因数分解

$$\text{7 } a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$$

(証明)

$$\begin{aligned}
 & \underbrace{a^3+b^3+c^3-3abc} \\
 = & \underbrace{(a+b)^3-3ab(a+b)} + c^3 - 3abc \\
 = & (a+b)^3 + c^3 - 3ab(a+b) - 3abc \\
 = & \{(a+b)+c\} \{ (a+b)^2 - (a+b)c + c^2 \} - 3ab \{ (a+b)+c \} \\
 = & (a+b+c)(a^2+2ab+b^2-ca-bc+c^2-3ab) \\
 = & (a+b+c)(a^2+b^2+c^2-ab-bc-ca) \quad \square
 \end{aligned}$$

$$\begin{aligned}
 (a+b)^3 &= a^3+3a^2b+3ab^2+b^3 \\
 &= \underbrace{a^3+b^3} - 3ab(a+b) \\
 a^3+b^3 &= (a+b)^3 - 3ab(a+b)
 \end{aligned}$$