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・平方根と次数下げ

(例)  $\alpha = 1 - \sqrt{2}$  のとき次の式の値を求めよ。

(1)  $\alpha^2 - 2\alpha - 1$       (2)  $\alpha^4 + \alpha^3 + \alpha^2 + \alpha + 1$

point

普通に代入しても良いが少し大変  
→ 次数下げを利用

(1)  $\alpha = 1 - \sqrt{2}$  より

$$\alpha - 1 = -\sqrt{2}$$

両辺を2乗して

$$(\alpha - 1)^2 = (-\sqrt{2})^2 \quad \leftarrow \Gamma \text{が消える}$$

$$\therefore \alpha^2 - 2\alpha - 1 = 0$$

(2) (1) より

$$\alpha^2 = 2\alpha + 1 \quad \leftarrow \begin{array}{l} \text{2次式(左辺)} \\ \rightarrow \text{1次式(右辺)にできる。} \end{array}$$

であるから

$$\alpha^3 = \alpha^2 \cdot \alpha = (2\alpha + 1)\alpha = 2\alpha^2 + \alpha = 2(2\alpha + 1) + \alpha = 5\alpha + 2$$

$$\alpha^4 = \alpha^3 \cdot \alpha = (5\alpha + 2)\alpha = 5\alpha^2 + 2\alpha = 5(2\alpha + 1) + 2\alpha = 12\alpha + 5$$

よって

$$\begin{aligned} & \alpha^4 + \alpha^3 + \alpha^2 + \alpha + 1 \\ &= (12\alpha + 5) + (5\alpha + 2) + (2\alpha + 1) + \alpha + 1 \\ &= 20\alpha + 9 \\ &= 20(1 - \sqrt{2}) + 9 \\ &= 29 - 20\sqrt{2} \end{aligned}$$