Universidade de Aveiro
Departamento de Matemática

Cálculo I - C **2024/2025**

Ficha de Exercícios 2 - Parte I

Primitivas. Integrais indefinidos

1. Determine os seguintes integrais indefinidos:

(a) $\int (3x^2+5x+7)\,dx$ (b) $\int \sqrt[3]{x}\,dx$ (c) $\int (x^3+1)^2\,dx$

(d) $\int \frac{\operatorname{arctg} x}{1+x^2}\,dx$ (e) $\int \frac{3x^2}{1+x^3}\,dx$ (f) $\int \frac{1}{x^7}\,dx$

(g) $\int \frac{x+1}{2+4x^2}\,dx$ (h) $\int 4x^3\cos x^4\,dx$ (i) $\int \frac{x}{\sqrt{1-x^2}}\,dx$

(j) $\int \operatorname{sen} x\cos^5 x\,dx$ (k) $\int \operatorname{tg} x\,dx$ (l) $\int \frac{\ln x}{x}\,dx$

(m) $\int e^{\operatorname{tg} x}\sec^2 x\,dx$ (n) $\int x7^{x^2}\,dx$ (o) $\int \operatorname{sen}(\sqrt{2}x)\,dx$

(p) $\int \frac{x^2+1}{x}\,dx$ (q) $\int \frac{x}{(7+5x^2)^{\frac{3}{2}}}\,dx$ (r) $\int \frac{x^3}{1+x^8}\,dx$

(s) $\int \frac{5x^2}{\sqrt{1-x^6}}\,dx$ (t) $\int \frac{1}{x^2+7}\,dx$ (u) $\int \frac{1}{x^2+2x+5}\,dx$

(v) $\int \frac{x}{1+x^4}\,dx$ (w) $\int \frac{x}{\sqrt{1-x^4}}\,dx$ (x) $\int \frac{x^3}{\sqrt{1-x^4}}\,dx$

2. Determine os seguintes integrais indefinidos:

(a) $\int \frac{e^{\operatorname{arcsen} x}}{\sqrt{1-x^2}}\,dx$ (b) $\int \operatorname{tg}^2 x\,dx$ (c) $\int \frac{1}{x}\cos(\ln x)\,dx$

(d) $\int \frac{6}{x\ln^3(4x)}\,dx$ (e) $\int \frac{e^{3x}}{(e^{3x}-2)^6}\,dx$ (f) $\int \operatorname{tg}^3 x\,dx$

(g) $\int \frac{1}{x\sqrt{1-\ln^2 x}}\,dx$ (h) $\int e^x\sqrt{1+e^x}\,dx$ (i) $\int \frac{1}{x\ln x}\,dx$

(j) $\int \frac{1}{\sqrt{x}}e^{\sqrt{x}}\,dx$ (k) $\int \frac{1+\cos x}{x+\operatorname{sen} x}\,dx$ (l) $\int \frac{e^{2x+1}}{e^{2x}+3}\,dx$

(m) $\int x^5\operatorname{sen}(x^6)\,dx$ (n) $\int \frac{\arccos x - x}{\sqrt{1-x^2}}\,dx$ (o) $\int \frac{\cos(\ln(x^2))}{x}\,dx$

(p) $\int \frac{1}{e^x+9e^{-x}}\,dx$ (q) $\int \frac{\operatorname{sen}(\operatorname{arctg} x)}{1+x^2}\,dx$

3. Considere a função g definida em $\mathbb{R}^+$ por $g(x)=\frac{(\ln x)^2}{x}$.

(a) Determine a família de todas as primitivas de g.

(b) Indique a primitiva da função g que se anula para $x=e$.

Resolução:

(a) $\displaystyle\int \frac{(\ln x)^2}{x}\,dx = \frac{(\ln x)^3}{3} + c, \quad c \in \mathbb{R}.$

(b) Para cada $c \in \mathbb{R}$, $G(x) = \frac{(\ln x)^3}{3} + c$ é uma primitiva de g.
Pretendemos então determinar $c \in \mathbb{R}$ tal que $G(e) = 0$.

$$G(e) = 0 \Leftrightarrow \frac{1}{3} + c = 0 \Leftrightarrow c = -\frac{1}{3}$$

Assim, $G(x) = \frac{(\ln x)^3}{3} - \frac{1}{3}$ é a primitiva de g que se anula para $x = e$.

4. Determine a primitiva F para a função $f(x) = \frac{2}{x} + \frac{3}{x^2}$ tal que $F(-1) = 1$.

5. Sabendo que a função f satisfaz a igualdade $\displaystyle\int f(x)\,dx = \operatorname{sen} x - x\cos x - \frac{1}{2}x^2 + c$, com $c \in \mathbb{R}$, determinar $f(\frac{\pi}{4})$.

6. Determine a primitiva da função $f(x) = \dfrac{1}{x^2} + 1$ que se anula no ponto $x = 2$.

7. Determine a primitiva da função f definida por $f(x) = \frac{3\cos(\ln x)}{x}$ que toma o valor 2 em $x = 1$.

8. Determine a função g que verifica as seguintes condições:

$$g'(x) = \frac{1}{(1 + \operatorname{arctg}^2(x))(1 + x^2)} \quad \text{e} \quad \lim_{x \to +\infty} g(x) = 0.$$

9. Determine, usando a técnica de integração por partes, os seguintes integrais indefinidos:

(a) $\displaystyle\int (x+1)\operatorname{sen} x\,dx$

Resolução: Fazendo

$$\begin{array}{c} f'(x) = \operatorname{sen} x \quad \text{temos} \quad f(x) = -\cos x \\ g(x) = x + 1 \quad \text{temos} \quad g'(x) = 1 \end{array}$$

Assim,

$$\begin{aligned} \int (x+1)\operatorname{sen} x\,dx &= -(x+1)\cos x + \int \cos x\,dx \\ &= -(x+1)\cos x + \operatorname{sen} x + c, \quad c \in \mathbb{R} \end{aligned}$$

(b) $\displaystyle\int x\cos x\,dx$ (c) $\displaystyle\int x^2\cos x\,dx$ (d) $\displaystyle\int e^{-3x}(2x+3)\,dx$ (e) $\displaystyle\int \ln^2 x\,dx$

(f) $\displaystyle\int \ln x\,dx$ (g) $\displaystyle\int \ln(x^2+1)\,dx$ (h) $\displaystyle\int x\operatorname{arctg} x\,dx$ (i) $\displaystyle\int \cos(\ln x)\,dx$

(j) $\displaystyle\int e^{2x}\operatorname{sen}(x)\,dx$ (k) $\displaystyle\int \operatorname{sen}(\ln x)\,dx$ (l) $\displaystyle\int \operatorname{arcsen} x\,dx$ (m) $\displaystyle\int x\operatorname{arcsen} x^2\,dx$

(n) $\displaystyle\int x^3 e^{x^2}\,dx$ (o) $\displaystyle\int \operatorname{arctg} x\,dx$ (p) $\displaystyle\int \operatorname{arctg}\frac{1}{x}\,dx$ (q) $\displaystyle\int \sqrt{x}\ln x\,dx$

(r) $\displaystyle\int \operatorname{sen} x\cos(3x)\,dx$ (s) $\displaystyle\int \cos^2 x\,dx$ (t) $\displaystyle\int \sec^3 x\,dx$ (u) $\displaystyle\int \frac{x^2}{\sqrt{(1-x^2)^3}}\,dx$

10. Determine, usando a técnica de integração por substituição, os seguintes integrais indefinidos:

(a) $\int x\sqrt{x+1}\,dx$

Resolução:

Consideremos a substituição $x+1=t^2$, com $t\geq 0$. Definindo $\varphi(t)=t^2-1$, $t\geq 0$, temos que φ é invertível, diferenciável e $\varphi'(t)=2t$. Então

$$\begin{aligned}\int x\sqrt{x+1}\,dx &= \int (t^2-1)\cdot t\cdot 2t\,dt\\ &= \frac{2t^5}{5}-\frac{2t^3}{3}+c\,.\end{aligned}$$

Atendendo a que $x+1=t^2$, com $t\geq 0$, vem que $t=\sqrt{x+1}$. Assim,

$$\int x\sqrt{x+1}\,dx=\frac{2(x+1)^2\sqrt{x+1}}{5}-\frac{2(x+1)\sqrt{x+1}}{3}+c,\text{ com } c\in\mathbb{R}.$$

(b) $\int \frac{x}{1+\sqrt{x}}\,dx$ (c) $\int \frac{1}{x^2\sqrt{1-x^2}}\,dx$ (d) $\int \frac{1}{x^2\sqrt{x^2+4}}\,dx$ (e) $\int \frac{1}{x\sqrt{x^2-5}}\,dx$

(f) $\int x^2\sqrt{1-x}\,dx$ (g) $\int x^2\sqrt{4-x^2}\,dx$ (h) $\int \frac{1}{x\sqrt{x^2-1}}\,dx$ (i) $\int \frac{1}{x\sqrt{x^2+4}}\,dx$

(j) $\int \frac{1}{x^2\sqrt{9-x^2}}\,dx$ (k) $\int \frac{x^2}{\sqrt{1-2x-x^2}}\,dx$ (l) $\int \frac{1}{x^2\sqrt{x^2-7}}\,dx$ (m) $\int \frac{\sqrt{x}}{1+\sqrt[3]{x}}\,dx$

(n) $\int x(2x+5)^{10}\,dx$ (o) $\int e^{\sqrt{x}}\,dx$ (p) $\int \frac{\ln x}{x\cdot\sqrt{1+\ln x}}\,dx$ (q) $\int \frac{1+\operatorname{tg}^2 x}{\sqrt{\operatorname{tg} x-1}}\,dx$

11. Determine os seguintes integrais indefinidos:

(a) $\int \frac{x+2}{(x-1)^2(x^2+4)}\,dx$

Resolução:

A determinação deste integral indefinido passa por decompor em frações simples a fração

$$\frac{x+2}{(x-1)^2(x^2+4)}.$$

Isto é, passa por escrever a dita fração na seguinte forma

$$\frac{x+2}{(x-1)^2(x^2+4)}=\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{Cx+D}{x^2+4}\quad(*)$$

com $A,\ B,\ C$ e D constantes reais a determinar.

Temos então que

$$\frac{x+2}{(x-1)^2(x^2+4)}=\frac{(A+C)x^3+(-A+B-2C+D)x^2+(4A+C-2D)x-4A+4B+D}{(x-1)^2(x^2+4)}$$

donde resulta a igualdade de polinómios

$$x+2=(A+C)x^3+(-A+B-2C+D)x^2+(4A+C-2D)x-4A+4B+D.$$

Atendendo à condição de igualdade de polinómios resulta que

$$\begin{cases} A+C=0 \\ -A+B-2C+D=0 \\ 4A+C-2D=1 \\ -4A+4B+D=2 \end{cases} \Leftrightarrow \begin{cases} A=-\frac{1}{25} \\ B=\frac{15}{25} \\ C=\frac{1}{25} \\ D=-\frac{14}{25} \end{cases}$$

Voltando a $(*)$, podemos escrever

$$\frac{x+2}{(x-1)^2(x^2+4)} = \frac{-\frac{1}{25}}{x-1} + \frac{\frac{15}{25}}{(x-1)^2} + \frac{\frac{1}{25}x-\frac{14}{25}}{x^2+4}.$$

Assim

$$\begin{aligned} \int \frac{x+2}{(x-1)^2(x^2+4)}\,dx &= -\tfrac{1}{25}\int \frac{1}{x-1}\,dx + \frac{15}{25}\int (x-1)^{-2}\,dx + \frac{1}{25}\int \frac{x-14}{x^2+4}\,dx \\ &= -\tfrac{1}{25}\ln|x-1| - \tfrac{3}{5(x-1)} + \tfrac{1}{25}\int \frac{x}{x^2+4}\,dx - \frac{14}{25}\int \frac{1}{x^2+4}\,dx \\ &= -\tfrac{1}{25}\ln|x-1| - \tfrac{3}{5(x-1)} + \tfrac{1}{50}\ln(x^2+4) - \tfrac{7}{25}\text{arctg}\,\tfrac{x}{2} + c, \quad c\in\mathbb{R} \end{aligned}$$

(b) $\displaystyle\int \frac{2x-1}{(x-2)(x-3)(x+1)}\,dx$ (c) $\displaystyle\int \frac{1}{(x-1)(x+1)^3}\,dx$ (d) $\displaystyle\int \frac{1}{x^3+8}\,dx$

(e) $\displaystyle\int \frac{x^8}{1+x^2}\,dx$ (f) $\displaystyle\int \frac{1}{x^3(1+x^2)}\,dx$ (g) $\displaystyle\int \frac{8}{x^4+4x^2}\,dx$

(h) $\displaystyle\int \frac{x^5+x^4-8}{x^3-4x}\,dx$ (i) $\displaystyle\int \frac{x^2}{(x-1)^3}\,dx$ (j) $\displaystyle\int \frac{x^3+3x-1}{x^4-4x^2}\,dx$

(k) $\displaystyle\int \frac{x+1}{x^3-1}\,dx$ (l) $\displaystyle\int \frac{x^4}{x^4-1}\,dx$ (m) $\displaystyle\int \frac{1}{x(x^2+1)^2}\,dx$

(n) $\displaystyle\int \frac{x+1}{x^2+4x+5}\,dx$

12. Determine

(a) $\displaystyle\int \text{sen}^2\theta\,d\theta$ (b) $\displaystyle\int \text{sen}^4 x\,dx$ (c) $\displaystyle\int \text{sen}\,x\cos^2 x\,dx$

(d) $\displaystyle\int \text{sen}^3 x\,dx$ (e) $\displaystyle\int \text{sen}^5 x\cos^2 x\,dx$ (f) $\displaystyle\int \cos^3 x\,dx$

(g) $\displaystyle\int \frac{1}{\sqrt{2+x^2}}\,dx$ (h) $\displaystyle\int \frac{\text{sen}\sqrt{x}}{\sqrt{x}}\,dx$ (i) $\displaystyle\int \frac{x}{x^2-5x+6}\,dx$

(j) $\displaystyle\int \frac{1}{\sqrt{2x-x^2}}\,dx$ (k) $\displaystyle\int x\sqrt{(1+x^2)^3}\,dx$ (l) $\displaystyle\int \frac{\sqrt{x}}{1+\sqrt{x}}\,dx$

(m) $\displaystyle\int x\ln x\,dx$ (n) $\displaystyle\int \frac{1+e^x}{e^{2x}+4}\,dx$ (o) $\displaystyle\int \frac{x}{\cos^2 x}\,dx$

(p) $\displaystyle\int \frac{\text{sen}\,x}{(1-\cos x)^3}\,dx$ (q) $\displaystyle\int (2x^2+3)\text{arctg}\,x\,dx$ (r) $\displaystyle\int \frac{1}{\sqrt{x^2+2x-3}}\,dx$

(s) $\displaystyle\int \sqrt{1+e^x}\,dx$ (t) $\displaystyle\int \frac{1}{\sqrt{e^x-1}}\,dx$ (u) $\displaystyle\int \cos x\cos(5x)\,dx$

(v) $\displaystyle\int \frac{\sin^3 x}{\sqrt{\cos x}}\,dx$ (w) $\displaystyle\int \text{sen}^5 x\,dx$ (x) $\displaystyle\int \frac{\ln x}{x(\ln^2 x+1)}\,dx$

Exercícios de revisão

13. Determine a primitiva da função $f(x) = \operatorname{tg} x$ cujo gráfico passa pelo ponto de coordenadas $(\pi, 3)$.

14. Determine a função $f : \mathbb{R} \to \mathbb{R}$ tal que

$$f'(x) = \frac{2e^x}{3+e^x} \quad \text{e} \quad f(0) = \ln 4.$$

15. Determine a função $f : \mathbb{R} \to \mathbb{R}$ tal que $f(0) = 1, f'(0) = 2$ e $f''(x) = 12x$, para todo o $x \in \mathbb{R}$.

16. Determine os seguintes integrais indefinidos:

   (a) $\displaystyle\int \operatorname{sen}(2x) e^{\cos(2x)}\, dx$

   (b) $\displaystyle\int \frac{1}{\sqrt{x} - \sqrt[4]{x}}\, dx$

   (c) $\displaystyle\int \frac{1}{x^2\sqrt{x^2-9}}\, dx$

   (d) $\displaystyle\int x^2 \operatorname{arctg} x\, dx$

   (e) $\displaystyle\int \frac{x+2}{x(x^2+4)}\, dx$

   (f) $\displaystyle\int \frac{x^2}{\sqrt{1+x^3}}\, dx$

   (g) $\displaystyle\int \frac{1}{x^2\sqrt{1+x^2}}\, dx$

   (h) $\displaystyle\int \frac{3x-1}{x^3+x}\, dx$

   (i) $\displaystyle\int \frac{-\cos x}{(1+\operatorname{sen} x)^2}\, dx$

   (j) $\displaystyle\int x \cdot \ln(1+x^2)\, dx$

   (k) $\displaystyle\int \cos x \cdot \ln(\operatorname{sen} x)\, dx$

   (l) $\displaystyle\int \frac{1}{x\sqrt{x^2-4}}\, dx$

   (m) $\displaystyle\int x^3 \cos(x^2)\, dx$

   (n) $\displaystyle\int x(\ln x)^2\, dx$

   (o) $\displaystyle\int \cos(\sqrt{x})\, dx$

   (p) $\displaystyle\int \operatorname{tg}^3 x\, dx$

17. Determine a função f tal que

$$f'(x) = \frac{5x-4}{x(x^2-2x+2)} \quad \text{e} \quad \lim_{x\to+\infty} f(x) = 0.$$

Soluções:

1. (a) $x^3 + \frac{5}{2}x^2 + 7x + c, \quad c \in \mathbb{R}$
   (b) $\frac{3}{4}\sqrt[3]{x^4} + c, \quad c \in \mathbb{R}$
   (c) $\frac{x^7}{7} + \frac{x^4}{2} + x + c, \quad c \in \mathbb{R}$
   (d) $\frac{(\operatorname{arctg} x)^2}{2} + c, \quad c \in \mathbb{R}$
   (e) $\ln|1 + x^3| + c, \quad c \in \mathbb{R}$
   (f) $-\frac{1}{6x^6} + c, \quad c \in \mathbb{R}$
   (g) $\frac{1}{8}\ln(2 + 4x^2) + \frac{\sqrt{2}}{4}\operatorname{arctg}(\sqrt{2}x) + c, \quad c \in \mathbb{R}$
   (h) $\operatorname{sen} x^4 + c, \quad c \in \mathbb{R}$
   (i) $-\sqrt{1 - x^2} + c, \quad c \in \mathbb{R}$
   (j) $-\frac{\cos^6 x}{6} + c, \quad c \in \mathbb{R}$
   (k) $-\ln|\cos x| + c, \quad c \in \mathbb{R}$
   (l) $\frac{(\ln x)^2}{2} + c, \quad c \in \mathbb{R}$
   (m) $e^{\operatorname{tg} x} + c, \quad c \in \mathbb{R}$
   (n) $\frac{1}{2\ln 7}7^{x^2} + c, \quad c \in \mathbb{R}$
   (o) $-\frac{\sqrt{2}}{2}\cos(\sqrt{2}x) + c, \quad c \in \mathbb{R}$
   (p) $\frac{x^2}{2} + \ln|x| + c, \quad c \in \mathbb{R}$
   (q) $-\frac{1}{5\sqrt{7+5x^2}} + c, \quad c \in \mathbb{R}$
   (r) $\frac{1}{4}\operatorname{arctg}(x^4) + c, \quad c \in \mathbb{R}$
   (s) $\frac{5}{3}\operatorname{arcsen}(x^3) + c, \quad c \in \mathbb{R}$
   (t) $\frac{\sqrt{7}}{7}\operatorname{arctg}\left(\frac{x}{\sqrt{7}}\right) + c, \quad c \in \mathbb{R}$
   (u) $\frac{1}{2}\operatorname{arctg}\left(\frac{x+1}{2}\right) + c, \quad c \in \mathbb{R}$
   (v) $\frac{1}{2}\operatorname{arctg}\left(x^2\right) + c, \quad c \in \mathbb{R}$
   (w) $\frac{1}{2}\operatorname{arcsen}\left(x^2\right) + c, \quad c \in \mathbb{R}$
   (x) $-\frac{1}{2}\sqrt{1 - x^4} + c, \quad c \in \mathbb{R}$

2. (a) $e^{\operatorname{arcsen} x} + c, \quad c \in \mathbb{R}$
   (b) $\operatorname{tg} x - x + c, \quad c \in \mathbb{R}$
   (c) $\operatorname{sen}(\ln x) + c, \quad c \in \mathbb{R}$
   (d) $-\frac{3}{\ln^2(4x)} + c, \quad c \in \mathbb{R}$
   (e) $-\frac{1}{15(e^{3x}-2)^5} + c, \quad c \in \mathbb{R}$
   (f) $\frac{\operatorname{tg}^2 x}{2} + \ln|\cos x| + c, \quad c \in \mathbb{R}$
   (g) $\operatorname{arcsen}(\ln x) + c, \quad c \in \mathbb{R}$
   (h) $\frac{2}{3}\sqrt{(1 + e^x)^3} + c, \quad c \in \mathbb{R}$
   (i) $\ln|\ln x| + c, \quad c \in \mathbb{R}$
   (j) $2e^{\sqrt{x}} + c, \quad c \in \mathbb{R}$
   (k) $\ln|x + \operatorname{sen} x| + c, \quad c \in \mathbb{R}$
   (l) $\frac{e}{2}\ln(e^{2x} + 3) + c, \quad c \in \mathbb{R}$
   (m) $-\frac{\cos(x^6)}{6} + c, \quad c \in \mathbb{R}$

(n) $-\frac{1}{2}(\arccos x)^2 + \sqrt{1-x^2} + c, \quad c \in \mathbb{R}$
(o) $\frac{1}{2}\text{sen}\,(\ln(x^2)) + c, \quad c \in \mathbb{R}$
(p) $\frac{1}{3}\text{arctg}\,\left(\frac{e^x}{3}\right) + c, \quad c \in \mathbb{R}$
(q) $-\cos(\text{arctg}\,x) + c, \quad c \in \mathbb{R}$

3. Resolvido

4. $F(x) = 2\ln|x| - \frac{3}{x} - 2$

5. $\frac{\pi}{8}(\sqrt{2} - 2)$

6. $F(x) = -\frac{1}{x} + x - \frac{3}{2}$

7. $F(x) = 3\text{sen}\,(\ln x) + 2$

8. $g(x) = \text{arctg}\,(\text{arctg}\,x) - \text{arctg}\,(\pi/2)$

9. (a) Resolvido
(b) $x\text{sen}\,x + \cos x + c, \quad c \in \mathbb{R}$
(c) $x^2\text{sen}\,x + 2x\cos x - 2\text{sen}\,x + c, \quad c \in \mathbb{R}$
(d) $-\frac{2x+3}{3}e^{-3x} - \frac{2}{9}e^{-3x} + c, \quad c \in \mathbb{R}$
(e) $x(\ln^2 x - 2\ln x + 2) + c, \quad c \in \mathbb{R}$
(f) $x\ln x - x + c, \quad c \in \mathbb{R}$
(g) $x\ln(x^2+1) - 2(x - \text{arctg}\,x) + c, \quad c \in \mathbb{R}$
(h) $\frac{x^2}{2}\text{arctg}\,x - \frac{1}{2}(x - \text{arctg}\,x) + c, \quad c \in \mathbb{R}$
(i) $\frac{x}{2}\cos(\ln x) + \frac{x}{2}\text{sen}\,(\ln x) + c, \quad c \in \mathbb{R}$
(j) $\frac{-e^{2x}\cos x + 2e^{2x}\text{sen}\,x}{5} + c, \quad c \in \mathbb{R}$
(k) $\frac{x\text{sen}\,(\ln x) - x\cos(\ln x)}{2} + c, \quad c \in \mathbb{R}$
(l) $x\text{arcsen}\,x + \sqrt{1-x^2} + c, \quad c \in \mathbb{R}$
(m) $\frac{x^2}{2}\text{arcsen}\,(x^2) + \frac{1}{2}\sqrt{1-x^4} + c, \quad c \in \mathbb{R}$
(n) $\frac{1}{2}e^{x^2}(x^2-1) + c, \quad c \in \mathbb{R}$
(o) $x\text{arctg}\,x - \frac{1}{2}\ln(1+x^2) + c, \quad c \in \mathbb{R}$
(p) $x\text{arctg}\,\frac{1}{x} + \frac{1}{2}\ln(1+x^2) + c, \quad c \in \mathbb{R}$
(q) $\frac{2}{3}\sqrt{x^3}\ln x - \frac{4}{9}\sqrt{x^3} + c, \quad c \in \mathbb{R}$
(r) $\frac{\cos x\cos(3x) + 3\text{sen}\,x\text{sen}\,(3x)}{8} + c, \quad c \in \mathbb{R}$
(s) $\frac{\cos x\text{sen}\,x + x}{2} + c, \quad c \in \mathbb{R}$
(t) $\frac{\sec x tg x + \ln|\sec x + \text{tg}\,x|}{2} + c, \quad c \in \mathbb{R}$
(u) $\frac{x}{\sqrt{1-x^2}} - \text{arcsen}\,x + c, \quad c \in \mathbb{R}$

10. (a) Resolvido
(b) $\frac{2}{3}\sqrt{x^3} - x + 2\sqrt{x} - 2\ln|\sqrt{x}+1| + c, \quad c \in \mathbb{R}$
(c) $-\frac{\sqrt{1-x^2}}{x} + c, \quad c \in \mathbb{R}$
(d) $-\frac{\sqrt{4+x^2}}{4x} + c, \quad c \in \mathbb{R}$
(e) $\frac{1}{\sqrt{5}}\arccos\left(\frac{\sqrt{5}}{x}\right) + c, \quad c \in \mathbb{R}$
(f) $-\frac{2}{3}(1-x)\sqrt{1-x} - \frac{2}{7}(1-x)^3\sqrt{1-x} + \frac{4}{5}(1-x)^2\sqrt{1-x} + c, \quad c \in \mathbb{R}$

(g) $2\text{arcsen}\,\frac{x}{2} - \frac{x(2-x^2)\sqrt{4-x^2}}{4} + c, \quad c \in \mathbb{R}$

(h) $\arccos\frac{1}{x} + c, \quad c \in \mathbb{R}$

(i) $-\frac{1}{2}\ln\left|\frac{\sqrt{x^2+4}}{x} + \frac{2}{x}\right| + c, \quad c \in \mathbb{R}$

(j) $-\frac{\sqrt{9-x^2}}{9x} + c, \quad c \in \mathbb{R}$

(k) $2\text{arcsen}\,\frac{x+1}{\sqrt{2}} - \frac{(x+1)\sqrt{2-(x+1)^2}}{2} + 2\sqrt{2-(x+1)^2} + c, \quad c \in \mathbb{R}$

(l) $\frac{\sqrt{x^2-7}}{7x} + c, \quad c \in \mathbb{R}$

(m) $\frac{6}{7}x\sqrt[6]{x} - \frac{6}{5}\sqrt[6]{x^5} + 2\sqrt{x} - 6\sqrt[6]{x} + 6\text{arctg}\,\sqrt[6]{x} + c, \quad c \in \mathbb{R}$

(n) $\frac{1}{48}(2x+5)^{12} - \frac{5}{44}(2x+5)^{11} + c, \quad c \in \mathbb{R}$

(o) $2e^{\sqrt{x}}(\sqrt{x}-1) + c, \quad c \in \mathbb{R}$

(p) $\frac{2}{3}\left(\sqrt{1+\ln x}\right)^3 - 2\sqrt{1+\ln x} + c, \quad c \in \mathbb{R}$

(q) $2\sqrt{\text{tg}\,x - 1} + c, \quad c \in \mathbb{R}$

11. (a) Resolvido

(b) $-\ln\mid x-2\mid + \frac{5}{4}\ln\mid x-3\mid - \frac{1}{4}\ln\mid x+1\mid + c, \quad c \in \mathbb{R}$

(c) $\frac{1}{8}\ln\mid x-1\mid - \frac{1}{8}\ln\mid x+1\mid + \frac{1}{4(x+1)} + \frac{1}{4(x+1)^2} + c, \quad c \in \mathbb{R}$

(d) $\frac{1}{12}\ln|x+2| - \frac{1}{24}\ln(x^2-2x+4) + \frac{\sqrt{3}}{12}\text{arctg}\left(\frac{x-1}{\sqrt{3}}\right) + c, \quad c \in \mathbb{R}$

(e) $\frac{x^7}{7} - \frac{x^5}{5} + \frac{x^3}{3} - x + \text{arctg}\,x + c, \quad c \in \mathbb{R}$

(f) $-\ln|x| - \frac{1}{2x^2} + \frac{1}{2}\ln(1+x^2) + c, \quad c \in \mathbb{R}$

(g) $-\frac{2}{x} - \text{arctg}\left(\frac{x}{2}\right) + c, \quad c \in \mathbb{R}$

(h) $\frac{x^3}{3} + \frac{x^2}{2} + 4x + 2\ln\mid x\mid + 5\ln\mid x-2\mid - 3\ln\mid x+2\mid + c, \quad c \in \mathbb{R}$

(i) $\ln\mid x-1\mid - \frac{2}{x-1} - \frac{1}{2(x-1)^2} + c, \quad c \in \mathbb{R}$

(j) $-\frac{3}{4}\ln\mid x\mid - \frac{1}{4x} + \frac{13}{16}\ln\mid x-2\mid + \frac{15}{16}\ln|x+2| + c, \quad c \in \mathbb{R}$

(k) $\frac{1}{3}(2\ln|x-1| - \ln(x^2+x+1)) + c, \quad c \in \mathbb{R}$

(l) $\frac{1}{4}(4x + \ln|x-1| - \ln|x+1| - 2\text{arctg}\,x) + c, \quad c \in \mathbb{R}$

(m) $\ln|x| - \frac{1}{2}\ln(1+x^2) + \frac{1}{2(x^2+1)} + c, \quad c \in \mathbb{R}$

(n) $\frac{1}{2}\ln(x^2+4x+5) - \text{arctg}\,(x+2) + c, \quad c \in \mathbb{R}$

12. (a) $\frac{1}{2}\theta - \frac{1}{4}\text{sen}\,(2\theta) + c, \quad c \in \mathbb{R}$

(b) $\frac{3}{8}x - \frac{1}{4}\text{sen}\,(2x) + \frac{1}{32}\text{sen}\,(4x) + c, \quad c \in \mathbb{R}$

(c) $-\frac{\cos^3 x}{3} + c, \quad c \in \mathbb{R}$

(d) $-\cos x + \frac{1}{3}\cos^3 x + c, \quad c \in \mathbb{R}$

(e) $-\frac{\cos^3 x}{3} + \frac{2}{5}\cos^5 x - \frac{\cos^7 x}{7} + c, \quad c \in \mathbb{R}$

(f) $\text{sen}\,x - \frac{\text{sen}^3 x}{3} + c, \quad c \in \mathbb{R}$

(g) $\ln\mid\sqrt{\frac{2+x^2}{2}} + \frac{x}{\sqrt{2}}\mid + c, \quad c \in \mathbb{R}$

(h) $-2\cos\sqrt{x} + c, \quad c \in \mathbb{R}$

(i) $3\ln|x-3| - 2\ln|x-2| + c, \quad c \in \mathbb{R}$

(j) $\text{arcsen}\,(x-1) + c, \quad c \in \mathbb{R}$

(k) $\frac{(1+x^2)^2\sqrt{1+x^2}}{5} + c, \quad c \in \mathbb{R}$

(l) $x - 2\sqrt{x} + 2\ln(1+\sqrt{x}) + c, \quad c \in \mathbb{R}$

(m) $\frac{x^2}{2}\ln x-\frac{x^2}{4}+c, \quad c\in\mathbb{R}$

(n) $\frac{1}{4}x-\frac{1}{8}\ln(e^{2x}+4)+\frac{1}{2}\text{arctg}\,\frac{e^x}{2}+c, \quad c\in\mathbb{R}$

(o) $x\text{tg}\,x+\ln|\cos x|+c, \quad c\in\mathbb{R}$

(p) $-\frac{1}{2(1-\cos x)^2}+c, \quad c\in\mathbb{R}$

(q) $(\frac{2}{3}x^3+3x)\text{arctg}\,x-\frac{1}{3}x^2-\frac{7}{6}\ln(1+x^2)+c, \quad c\in\mathbb{R}$

(r) $\ln\left|\frac{x+1+\sqrt{(x+1)^2-4}}{2}\right|+c, \quad c\in\mathbb{R}$

(s) $2\sqrt{1+e^x}+\ln|\sqrt{1+e^x}-1|-\ln(\sqrt{1+e^x}+1)+c, \quad c\in\mathbb{R}$

(t) $2\text{arctg}\,\sqrt{e^x-1}+c, \quad c\in\mathbb{R}$

(u) $\frac{1}{12}\text{sen}\,(6x)+\frac{1}{8}\text{sen}\,(4x)+c, \quad c\in\mathbb{R}$

(v) $-2\sqrt{\cos x}+\frac{2}{5}\sqrt{\cos^5 x}+c, \quad c\in\mathbb{R}$

(w) $-\cos x+\frac{2}{3}\cos^3 x-\frac{1}{5}\cos^5 x+c, \quad c\in\mathbb{R}$

(x) $\frac{1}{2}\ln(\ln^2 x+1)+c, \quad c\in\mathbb{R}$

13. $F(x)=-\ln|\cos x|+3$

14. $f(x)=2\ln(e^x+3)-\ln 4$

15. $f(x)=2x^3+2x+1$

16. (a) $-\frac{1}{2}e^{\cos(2x)}+c, \quad c\in\mathbb{R}$

(b) $4\left(\frac{\sqrt{x}}{2}+\sqrt[4]{x}+\ln(\sqrt[4]{x}-1)\right)+c, \quad c\in\mathbb{R}$

(c) $\frac{\sqrt{x^2-9}}{9x}+c, \quad c\in\mathbb{R}$

(d) $\frac{x^3}{3}\text{arctg}\,x-\frac{x^2}{6}-\frac{1}{6}\ln(1+x^2)+c, \quad c\in\mathbb{R}$

(e) $\frac{1}{2}\ln|\,x\,|-\frac{1}{4}\ln(4+x^2)+\frac{1}{2}\text{arctg}\,(\frac{x}{2})+c, \quad c\in\mathbb{R}$

(f) $\frac{2}{3}\sqrt{1+x^3}+c, \quad c\in\mathbb{R}$

(g) $-\frac{\sqrt{1+x^2}}{x}+c, \quad c\in\mathbb{R}$

(h) $\frac{1}{2}(\ln(x^2+1)-2\ln|x|+6\text{arctg}\,x)+c, \quad c\in\mathbb{R}$

(i) $\frac{1}{1+\text{sen}\,x}+c, \quad c\in\mathbb{R}$

(j) $\frac{x^2+1}{2}\ln(1+x^2)-\frac{x^2}{2}+c, \quad c\in\mathbb{R}$

(k) $\text{sen}\,x\cdot\ln(\text{sen}\,x)-\text{sen}\,x+c, \quad c\in\mathbb{R}$

(l) $\frac{1}{2}\arccos\left(\frac{2}{x}\right)+c, \quad c\in\mathbb{R}$

(m) $\frac{x^2}{2}\text{sen}\,(x^2)+\frac{1}{2}\cos(x^2)+c, \quad c\in\mathbb{R}$

(n) $\frac{x^2}{2}\left((\ln x)^2-\ln x+\frac{1}{2}\right)+c, \quad c\in\mathbb{R}$

(o) $2\,(\sqrt{x}\cdot\text{sen}\,(\sqrt{x})+\cos(\sqrt{x}))+c, \quad c\in\mathbb{R}$

(p) $\frac{\text{tg}^2 x}{2}+\ln|\cos x|+c, \quad c\in\mathbb{R}$

17. $f(x)=\ln\left(\frac{x^2-2x+2}{x^2}\right)+3\text{arctg}\,(x-1)-\frac{3\pi}{2}$