Fundamentos de Programação

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Summary

- Recursive functions
  - How recursion works
  - The rules for termination
  - Recursion vs. repetition
  - How to write recursive functions
- Examples
  - Operations on a list
  - Towers of Hanoi
  - A sorting algorithm (kind of quicksort)

A crazy idea

- What does this function do?

  `sumsq([1, 2, 3])   #-> 14`

  - Does it work on an empty list?
  - Can you write it with a generator expression? (Homework!)

```
def sumsq(lst):
    s = 0
    for x in lst:
        s += x**2
    return s
```

- Check out this weird version!
  - It squares <u>first</u> element;
  - Calls `sumsq` on the <u>rest</u>;
  - And adds.
  - For empty lst, just return zero.

```
def sumsq2(lst):
    s = 0
    if len(lst) > 0:
        sq0 = lst[0]**2
        s = sq0 + sumsq(lst[1:])
    return s
```

- It is <u>*equivalent*</u> to `sumsq` in every case, but still calls the original `sumsq`. Not very useful.
- But if `sumsq2` <=> `sumsq`, why not **call itself**?

Recursive functions

- This is what would result.

```
def sumsqR(lst):
    s = 0
    if len(lst) > 0:
        sq0 = lst[0]**2
        s = sq0 + sumsqR(lst[1:])
    return s
```

- This is a **recursive function**: a function that calls itself.
- Notice that there is no loop instruction, but code gets executed several times, anyway.
- How does it work?

How recursion works

- What happens when we call `sumsqR([1, 2, 3])` ?

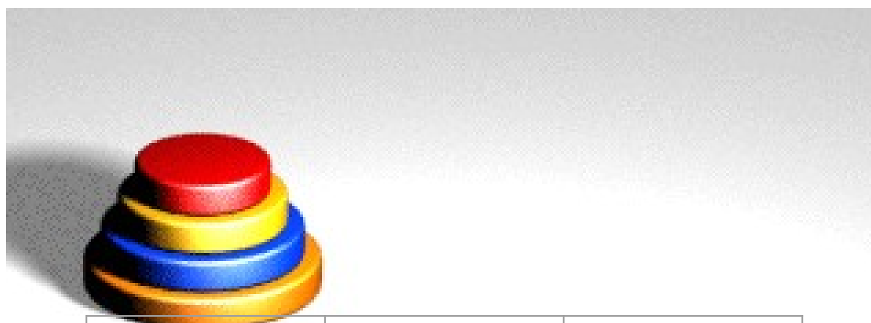
```
sumsqR([1, 2, 3])
|    sumsqR([2, 3])
|    |    sumsqR([3])
|    |    |    sumsqR([])
|    |    |    └>0
|    |    └>9   ( = 3**2 + 0 )
|    └>13   ( = 2**2 + 9 )
└>14   ( = 1**2 + 13 )
```

Play ▶

- Notice that at one point, there are 4 frames in memory.
  - 4 variables named `lst`, 4 named `s`, 3 named `sq0`, but all distinct!
- Each frame stores the *local context* of a single function call.
- The frames are stored in the **program stack**.

Example: Towers of Hanoi

- The Towers of Hanoi puzzle (Édouard Lucas, 1883).
- Move tower from A to C, using B temporarily.
  - Move only one disk at a time;
  - No disk may be put on top of a smaller disk.



A	B	C

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- Now solve it in 4 lines of code!

Play

Example: quicksort

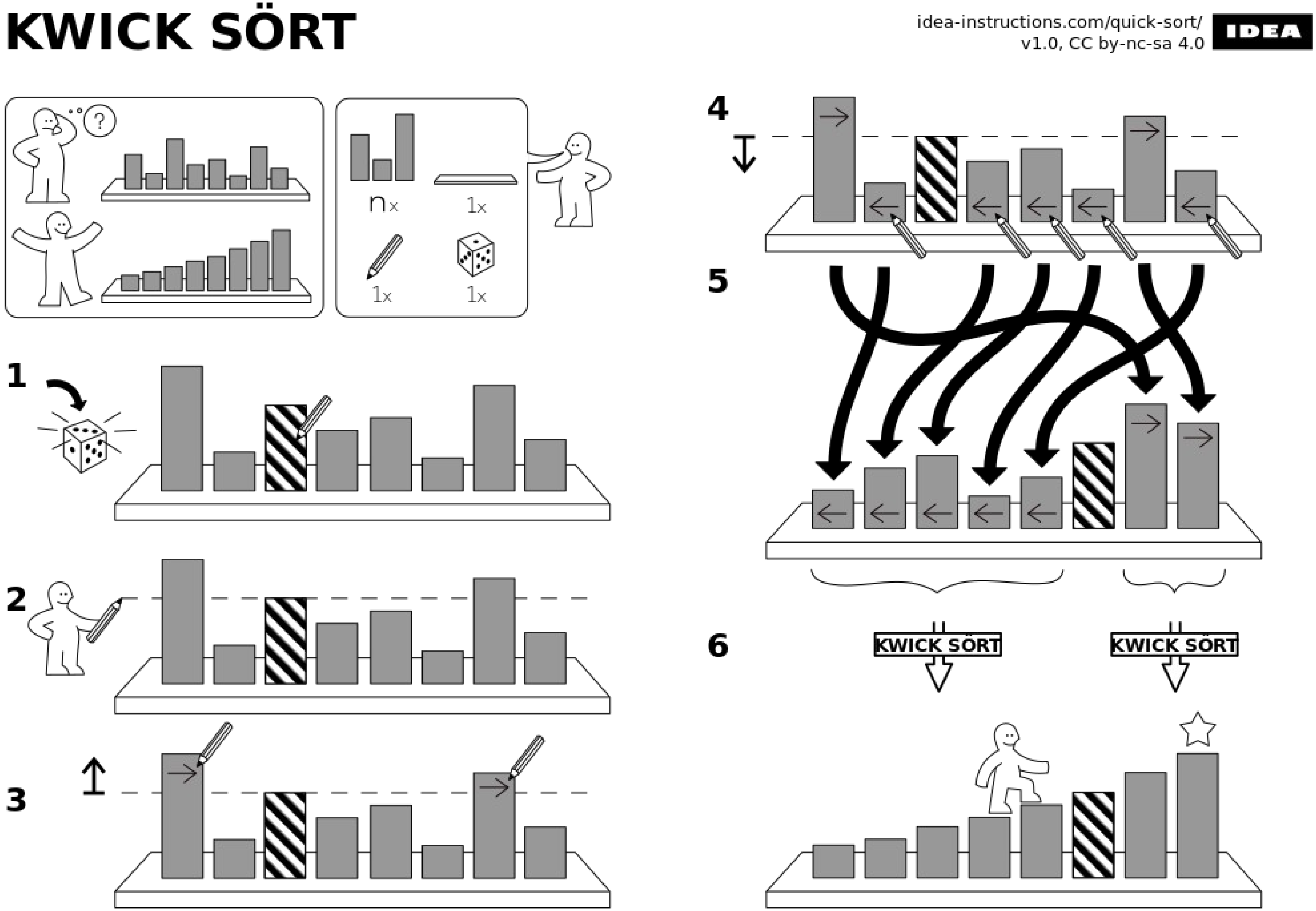
- The quicksort algorithm (by C.A.R. Hoare) goes like this:
  1. Pick one of the values in the list (e.g. the first) and store in T.
  2. Put values smaller than T into a list L1, the others into a list L2.
  3. Sort L1 and L2 (using same algorithm, by the way)
  4. Result is L1 + [T] + L2.
- Of course, there's a few more details (the base case).

```python
def qsorted(lst):
    if len(lst)<=1:      # no need to sort
        return lst[:]    # just return a copy
    T = lst[0]
    L1 = [x for x in lst[1:] if x<T]
    L2 = [x for x in lst[1:] if x>=T]
    return qsorted(L1) + [T] + qsorted(L2)
```

- This is simple to understand and quite efficient!

The actual quicksort modifies the list in-place, and is slightly harder to write.

Example: quicksort



The rules for recursion termination

To guarantee that a recursive function **terminates**, it must respect some **rules**:

1. There must be cases that can be solved *without* recursive calls. These are called the **base cases**.
   - In `sumsqR`, the base case is `len(lst)==0`. In that case, return 0.
2. In the other cases, the *context passed* to recursive calls **must always differ** from the *context received*.*
   - In `sumsqR`, the argument `lst[1:] != lst` (always)
3. In successive recursive calls, the context must **converge** towards the base cases.
   - In `sumsqR`, the `lst` is shortened on each call, until it's empty.

* The ***context*** is the set of arguments (and global values) that have an impact on the base case / recursive case selection.

Recursion vs repetition

- Any problem that can be solved by repetition may be solved by recursion, and vice-versa.
- For certain complex problems, recursive solutions are usually more concise and easier to understand.
- Recursive implementations may incur extra time and memory cost because of function calls and stack usage.
- If the problem has a simple iterative solution, that is usually the most efficient, too.

Writing recursive functions

- To develop a recursive function to solve some problem, follow these guidelines:
  1. First, **define** the **arguments** you need, what they **mean**, and the **result** you **expect**, as *rigorously* as possible.
  2. Now, **assume** the function will work. Describe how the solution to a problem can be obtained by **modifying** the solutions to **smaller** versions of the problem. This will be the <u>recursive part</u> of the algorithm.
  3. Finally, **determine** the **base cases**: which conditions have a trivial solution? This will be the <u>non-recursive part</u> of the algorithm. (Hint: base cases are usually *outside the domain* of the recursive part.)
- While in step 2, you may realize that you need *extra arguments*. Just add them and go back to step 1.