Folha 1

(7) $\neg \left[\forall y\, \exists x \left((q(x) \rightarrow p(y)) \vee (p(y) \wedge q(x)) \right)\right]$

$\equiv \exists y\, \forall x\, \neg \left((q(x) \rightarrow p(y)) \vee (p(y) \wedge q(x)) \right)$

$\equiv \exists y\, \forall x\, (q(x) \wedge \neg p(y)) \wedge (\neg p(y) \vee \neg q(x))$

$\equiv \exists y\, \forall x\; q(x) \wedge (\neg p(y) \wedge (\neg p(y) \vee \neg q(x)))$

$\equiv \exists y\, \forall x\; q(x) \wedge \underbrace{(\neg p(y) \wedge \neg p(y))}_{\neg p(y)} \vee (\neg p(y) \wedge \neg q(x))$

$\equiv \exists y\, \forall x\; q(x) \wedge (\neg p(y) \vee (\neg p(y) \wedge \neg q(x))$

$\equiv \exists y\, \forall x\, (q(x) \wedge \neg p(y)) \vee (q(x) \wedge (\neg p(y) \wedge \neg q(x))$

$\equiv \exists y\, \forall x\, (q(x) \wedge \neg p(y)) \vee \underbrace{(\boxed{q(x)} \boxed{\wedge} \neg p(y) \boxed{\wedge} \boxed{\neg q(x)})}_{\perp}$

$\equiv \exists y\, \forall x\, (q(x) \wedge \neg p(y))$

$\equiv \exists y\, \forall x\, \neg (\neg q(x) \vee p(y))$

$\equiv \exists y\, \forall x\, \neg (q(x) \rightarrow p(y))$

(11) a) $(\forall x\, S(x)) \rightarrow (\exists z\, P(z))$

$\equiv (\neg(\forall x\, S(x))) \vee (\exists z\, P(z))$

$\equiv (\exists x\, \neg S(x)) \vee (\exists z\, P(z))$

$\equiv \exists x,z\, \underbrace{(\neg S(x) \vee P(z))}_{1\ \vee\text{-cláusula}}$ FNP FNC FND

b) $\neg (\forall x\, (S(x) \rightarrow P(x)))$

$\equiv \neg\, \forall x\, (\neg (\neg S(x) \vee P(x))$

$\equiv \exists x\, (S(x) \boxed{\wedge} \neg P(x))$ FNP FNC (2 ∨ cláusulas) FND

c) $\forall x\, (P(x) \rightarrow (\exists y\, Q(x,y)))$

$\equiv \forall x\, (\neg P(x) \vee \exists y\, Q(x,y))$

$\equiv \forall x\, \exists y\, (\neg P(x) \vee Q(x,y))$

$\equiv \forall x\, \exists y\, \neg P(x) \vee Q(x,y)$ FNP FND FNC

d) $\exists x\, (\neg (\exists y\, P(x,y)) \rightarrow (\exists z\, (Q(z) \rightarrow R(x))))$

$\equiv \exists x\, ((\exists y\, P(x,y)) \vee (\exists z\, (\neg Q(z) \vee R(x)))$

$\equiv \exists x\, \exists y\, \exists z\, (P(x,y) \vee (\neg Q(z) \vee R(x)))$ FNP FNC FND

e) $\forall x\, \exists y\, \exists z\, ((\neg P(x,y) \wedge Q(x,z)) \vee R(x,y,z))$ FNP FND

$\equiv \forall x\, \exists y\, \exists z\, ((\neg P(x,y) \vee R(x,y,z)) \wedge (Q(x,z) \vee R(x,y,z)))$

Forma normal Skolem

FNS = FNP + FNC + não tem $\exists$

$\exists x\, \forall y \quad x + y = y + x = y$ → x é constante não depende de y

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$\forall x\, \exists y \quad x + y = 0$ Verdadeiro $y = f(x) = -x$

$\exists y\, \forall x \quad x + y = 0$ Falso

1.3.10

$\exists x\, \forall y\, \forall z\, \exists u\, \forall v\, \exists w\; P(x,y,z,u,v,w)$

$x = e$ → constante de Skolen

existe um elemento do domínio que satisfaz a condição

$\equiv \boxed{\forall y\, \forall z} \boxed{\exists u}\, \forall v\, \exists w\; P(e,y,z,u,v,w)$

$u = f(y,z)$ → função Skolen

u pode depender de y e z por causa do alcance

$\equiv \forall y\, \forall z\, \forall v\, \exists w\; P(e,y,z,f(y,z),v,w)$

$w = g(y,z,v)$ → função Skolen

$\equiv \forall y\, \forall z\, \forall v\; P(e,y,z,f(y,z),v,g(y,z,v))$ FNS

1.3.11

$\forall x\, \exists y\, \exists z\, ((\neg P(x,y) \wedge Q(x,z)) \vee R(x,y,z))$

$y = f(x)$ → função Skolen

$z = g(x)$

y desaparece

$\equiv ((\neg P(x,f(x)) \vee R(x,f(x),g(x)) \wedge (Q(x,g(x)) \vee R(x,f(x),g(x)))$ FNS

(12)

a) $\neg ((\forall x\, P(x)) \rightarrow (\exists y\, P(y)))$

$\equiv \neg (\neg \forall x\, P(x) \vee \exists y\, P(y))$

$\equiv (\forall x\, P(x)) \wedge (\neg \exists y\, P(y))$

$\equiv (\forall x\, P(x)) \wedge (\forall y\, \neg P(y))$

$\equiv \forall x\, \forall y\; P(x) \wedge \neg P(y)$

$\equiv \forall x\; \underbrace{P(x) \wedge \neg P(x)}_{\emptyset}$

$\equiv \perp$

b) $\neg ((\forall x\, P(x)) \rightarrow (\exists y\, \forall z\, Q(y,z)))$

$\equiv (\forall x\, P(x)) \wedge (\neg (\exists y\, \forall z\, Q(y,z)))$

$\equiv (\forall x\, P(x)) \wedge (\forall y\, \exists z\, \neg Q(y,z))$ y não depende de x, logo pode trocar

$\equiv \forall x\, \forall y\; P(x) \wedge \exists z\, (\neg Q(y,z))$

$\equiv \forall x\, \forall y\, \exists z\; P(x) \wedge \neg Q(y,z)$

$z = f(x,y)$

$\equiv \forall x\, \forall y\; P(x) \wedge \neg Q(y, f(x,y))$

$\equiv (\forall x\, P(x)) \wedge (\forall x\, \exists z\, \neg Q(x,z))$

$\equiv \forall x\, (P(x) \wedge \exists z\, \neg Q(x,z))$

$\equiv \forall x\, \exists z\, (P(x) \wedge \neg Q(x,z))$ FNP

$z = f(x)$ → função Skolen

$\equiv \forall x\, (P(x) \wedge \neg Q(x, f(x)))$

Unificação

1.4.6

$\theta = \{ f(x,z)/x,\ g(y, f(x,y))/y,\ h(x,y)/z,\ v/u \}$

substitui x por f(x,z)

$\hat\theta(t) = s(\underbrace{f(x,z)}_{x}, f(\underbrace{g(y,f(x,y))}_{y}, \underbrace{v}_{u}), h(\underbrace{f(x,z)}_{x}, \underbrace{h(x,y)}_{z})$

$t\hat\theta$

1.4.7

$\hat\theta(E_1) = f(a, f(b), g(c))$

$\hat\theta(E_2) = P(h(a), e, f(f(b)))$

$\theta = \{ x/x,\ f(x)/y \} = \{ f(x)/y \}$

substituição identidade (pode ser retirada)

Var $s = \{x, y, z\}$

substituição identidade ⇐ $\varepsilon = \{ x/x,\ y/y,\ z/z \} = \{\}$

Folha 1

(14) a) $E\hat\theta = P(h(a), g(x), f(g(x)))$

b) $E\hat\theta = F(a, h(a), f(y), h(a))$

1.4.10

$\theta \Delta \delta = \{ a/x,\ g(f(y))/y,\ z/z,\ x/u \}$

θ "após" δ

$y \xrightarrow{\delta} g(x) \xrightarrow{\hat\theta} g(f(y))$ entre termos | $z \xrightarrow{\delta} y \xrightarrow{\theta} z$ | $u \xrightarrow{\delta} u \xrightarrow{\theta} x$

$\delta \Delta \theta = \{ f(g(x))/x,\ y/z,\ a/u \}$

$x \xrightarrow{\theta} f(y) \xrightarrow{\hat\delta} f(g(x))$