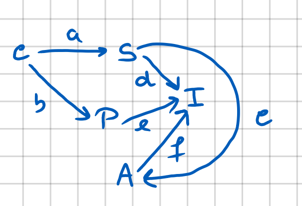
# Folha 5

④

a)




rancho → catsa

| | |
|---|---|
| Cauda | 1 |
| Cabeça | -1 |

b) Matriz incidência (vértice - aresta)

Ordem = 5 = nº de vértices = linhas

Dimensão = 6 = nº de arcos / arestas = colunas

$$\begin{array}{c} \\ e \\ S \\ P \\ A \\ I \end{array} \begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & -1 & -1 \end{bmatrix} \end{array}$$

Matriz adjacência (vértice - vértice)

$$\begin{array}{c} \\ C \\ S \\ P \\ A \\ I \end{array} \begin{array}{c} \begin{array}{ccccc} C & S & P & A & I \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

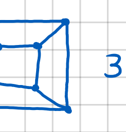
⑤ Num grafo regular todos os vértices têm o mesmo grau

$\Delta(G) = \delta(G)$

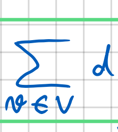
$\Delta(G)$ → grau máx de G

$\delta(G)$ → grau min de G

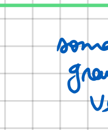
Exemplos



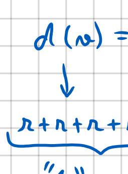
0 regular (nenhum vértice tem arestas adjacentes)



1 regular



2 regular



3 regular (Grafo cúbico)

$$\boxed{\sum_{v \in V} d(v) = 2\varepsilon}$$

$\sum_{v \in V} d(v)$ → somatório dos graus de cada vértice

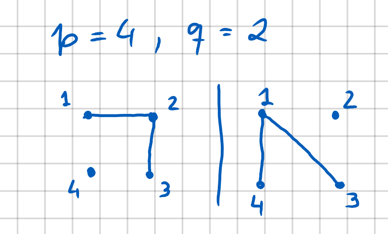
$\varepsilon$ → arestas

$d(v) = r$

$$\underbrace{r+r+r+r+\dots}_{\text{"}p\text{" vezes}} = 2\varepsilon$$

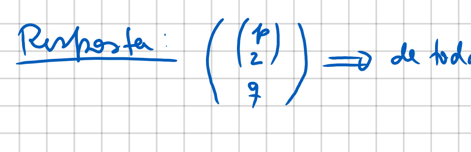
$\Leftrightarrow pr = 2\varepsilon \Leftrightarrow \varepsilon = \dfrac{pr}{2}$

b) $\boxed{\varepsilon(K_p) = \binom{p}{2}} \Longrightarrow \varepsilon(K_p)$ = subconjunto de 2 elementos de um conjunto de cardinalidade $p$



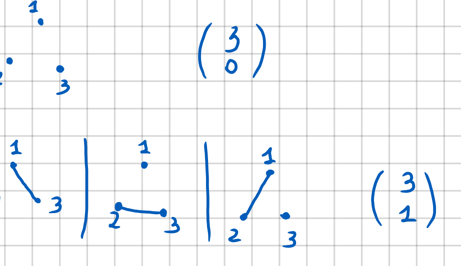
c)

i) $p = 3$, $q = 2$




$\binom{3}{2}$

$p = 4$, $q = 2$




(...) $\binom{\binom{4}{2}}{2}$

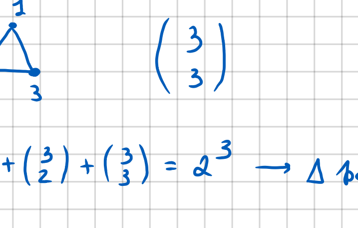
Resposta: $\binom{\binom{p}{2}}{q} \Longrightarrow$ de todas as arestas possíveis (grafo completo — arestas do grafo completo) escolher $q$ arestas

d) $p = 3$

0 arestas

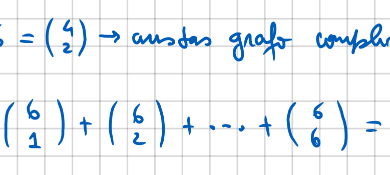

$\binom{3}{0}$

1 aresta



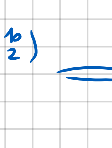

$\binom{3}{1}$

2 arestas




$\binom{3}{2}$

3 arestas




$\binom{3}{3}$

$\binom{3}{0} + \binom{3}{1} + \binom{3}{2} + \binom{3}{3} = 2^3$ → Δ pascal

$p = 4$  $6 = \binom{4}{2}$ → arestas grafo completo

$\binom{6}{0} + \binom{6}{1} + \binom{6}{2} + \dots + \binom{6}{6} = 2^6$

$2^{\binom{p}{2}} \Longrightarrow$ nº de grafos numerados com 3 vértices

⑥ $\boxed{\delta(G) \le d(v) \le \Delta(G)}$ , $v \in V$

$$n\,\underbrace{\delta(G)}_{\text{grau}} \le \underbrace{\sum_{v \in V} d(v)}_{2m} \le n\,\Delta(G)$$

$n$ → nº de vértices (ordem)

$m$ → nº de arestas (dimensão)

$\Leftrightarrow \delta(G) \le \dfrac{2m}{n} \le \Delta(G)$

⑦ $\nu(G) \ge 2$ → nº de vértices

Grafo G simples (s/ lacets ou arestas paralelas)

| |
|---|
| Grau → nº de arestas incidentes ao vértice |

$\Delta(K_\nu) = \delta(K_\nu) = \nu - 1$

O grafo completo $K_\nu$ é $(\nu-1)$ regular

$d(v) \in \{0, \dots, \nu-1\}$

Caso 1: Se G não tem vértices isolados (de grau 0)

$d(v) \in \underbrace{\{1, \dots, \nu-1\}}_{D}$

$D$ → graus

$f: V \longrightarrow D$

$v \mapsto d(v)$

$|V| > |D|$, pelo princípio da gaiola de pombos, dois grafos têm o mesmo grau

Caso 2: G tem vértices isolados

$d(v) \in D = \{0, \dots, \nu-2\}$

$f: V \longrightarrow D$

$v \mapsto d(v)$

$|V| > |D|$

A conclusão é a mesma do caso anterior

⑨ $\nu(G) = 56$ → vértices

$\varepsilon(G^c) = 80$ → arestas do complementar

$\varepsilon(G) + \varepsilon(G^c) = \varepsilon(K_\nu)$ → arestas do completo (com $\nu$ vértices)

$K_\nu$ → grafo completo

$\Leftrightarrow \varepsilon(G) + 80 = \binom{56}{2}$

$\Leftrightarrow \varepsilon(G) = \binom{56}{2} - 80$

⑩ Grafo simples regular com 24 arestas

$\nu = ?$ → vértices

24 arestas → $\varepsilon = 24$

$$\boxed{\sum_{v \in V} d(v) = 2\varepsilon}$$

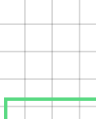
$\sum_{v \in V} d(v) = 2 \times 24$ , $d(v) = K$

$\Leftrightarrow K \cdot \nu = 48$

$K$ → grau

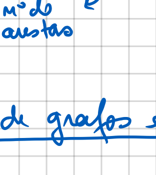
$\nu$ → nº de vértices

$K = 1$, $\nu = 48$




$K = 2$, $\nu = 24$

$\varepsilon = 24$




2 regular

$K = 3$, $\nu = 16$

$K = 4$, $\nu = 12$

$K = 6$, $\nu = 8$

~~$K = 8$, $\nu = 6$~~

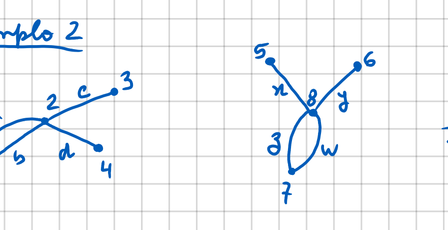
$\boxed{24 \le \binom{\nu}{2}}$

24 → nº de arestas

$\binom{\nu}{2}$ → nº de arestas completo

## 4. Isomorfismos de grafos e subgrafos

Exemplo 1



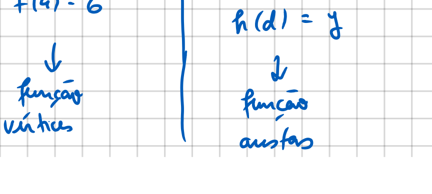

$f: \{1,2,3\} \longrightarrow \{1,2,3\}$

$f(1) = 3$

$f(2) = 2$

$f(3) = 1$

Exemplo 2




$\Longrightarrow$ Grafos ≠ mas Isomorfos

Bijeção → objetos ≠ correspondem imagens ≠

| Função vértices | Função arestas |
|---|---|
| $f(1) = 7$ | $h(a) = z$ |
| $f(2) = 8$ | $h(b) = w$ |
| $f(3) = 5$ | $h(c) = x$ |
| $f(4) = 6$ | $h(d) = y$ |