Expressão geradora (*1)

Carta 1 $(1+x+x^2)$ | Carta 2 $(1+x+x^2+\dots)$

$x^0 \cdot 1 \cdot x^2 \cdot 1 + x^1 \cdot x^1 \cdot 1 + x^2 \cdot 1 \cdot x^0$
$= x^4(1+1+1) = 3x^4$

4 bolas ○○○○ — cores ← $x^4 \to n^{\circ}$ de bolas

3 possibilidades
$x^0 \cdot x^4$
$x^1 \cdot x^3$
$x^2 \cdot x^2$

Obs

$(1+x+x^2)(1+x+x^2+\dots)$

$\frac{1-x^3}{1-x} \cdot \frac{1}{1-x}$

$= \underbrace{(1-x^3)}_{\text{polinômio}} \cdot \underbrace{\frac{1}{(1-x)^2}}_{\text{Binomial Negativo}}$

$\frac{1}{(1-x)^2} = \sum_{n=0}^{\infty} \left(\binom{-2}{n}\right) x^n$

$\left(\binom{-2}{n}\right) = \binom{2+n-1}{n-1} = n+1$

$= (1-x^3) \cdot \sum_{n=0}^{\infty} \left(\binom{-2}{n}\right) x^n$

$= \sum_{n=0}^{\infty} 2(n+1)x^n - \sum_{n=0}^{\infty} (n+1)x^{n+3}$

$C_4 = \underbrace{[(n+1)]}_{n=4} - \underbrace{[(n+1)]}_{n=1}$

$= 5 - 2 = 3$

Folha 4

(24)

a) $x_1 + x_2 + x_3 + x_4 = n$

$0 \le x_1 \le 5 \quad 0 \le x_2 \le 3 \quad \dots$

pode ficar vazia
ou seja 0, 1, 2, 3, 4, 5 bolas (max)
(Binomial Neg.)

Sem restrição

$(1+x+x^2+\dots)^4 = \left(\frac{1}{1-x}\right)^4 = \frac{1}{(1-x)^4} = \sum_{n=0}^{\infty} \left(\binom{-4}{n}\right) x^n$

4 → n° de cores

Solução do problema

$(1+x+x^2+x^3+x^4+x^5) \cdot (1+x+x^2+x^3) \cdot (x^0+x^1+\dots+x^8) \cdot (1+x+x^2+x^3+x^4)$

$= x^0(1+x+\dots+x^5)$

$= \underbrace{\frac{1-x^6}{1-x}}_{x_1} \cdot \underbrace{\frac{1-x^4}{1-x}}_{x_2} \cdot \underbrace{x^0 + \frac{1-x^9}{1-x}}_{x_3} \cdot \underbrace{\frac{1-x^5}{1-x}}_{x_4} = (*)$

$(*) = \frac{x^0(1-x^6)(1-x^4)(1-x^9)(1-x^5)}{(1-x^4)}$

Série Gerador na forma racional

b) $x_1 + x_2 + x_3 + x_4 = n$

$(1+x^2+x^4+x^6+x^8) \cdot (x+x^3+x^5+x^7) \cdot (1+x+x^2+x^3+x^4)^2 = \frac{1-x^{10}}{1-x^8} \times \frac{1 \cdot x^8}{1+x^2} \left(\frac{1-x^4}{1-x}\right)^2$

$x_1 + x_2$ $x_3 + x_4$

$(x^2)^0$ $(x^2)^1$ $(x^2)^2$ $(x^2)^3$ $(x^2)^4$

$= x(1+x^2+x^4+x^6)$

$1-x^2 = (1-x)(1+x)$

$= \frac{(1-x^{10}) \times (1+x^8)(1-x^4)^2}{(1+x)^2(1-x)^2}$

(19) $x_1 + x_2 + x_3 + x_4 = 27 + n$

a) $(1+x+x^2+x^3+\dots)^4 = \left(\frac{1}{1-x}\right)^4 = \left(\frac{1}{(1-x)^4}\right) = \sum_{n=0}^{\infty} \left(\binom{-4}{n}\right) x^n$

4 → n° de condutores

$\left(\binom{-4}{n}\right) = \binom{-4-4-1}{n-4} = \binom{10}{3}$

b) $(x+x^2+x^3+x^4+\dots)^4 = \left(\frac{x}{1-x}\right)^4 = x^4 \cdot \frac{1}{(1-x)^4}$

polinômio — Binomial Negativo

$= x^4 \sum_{n=0}^{\infty} \left(\binom{-4}{n}\right) x^n = \sum_{n=0}^{\infty} \left(\binom{-4}{n}\right) x^{n+4}$

(x^{27})

$n+4 = 27 \Leftrightarrow n = 23$

$C_{23} = \left(\binom{-4}{23}\right) = \binom{4+23-1}{4-1} = \binom{26}{3}$

c) $(1+x+x^2+\dots+x^{10})^4 = \left(\frac{1-x^{11}}{1-x}\right)^4 = \underbrace{(1-x^{11})^4}_{\text{Sem Binomial}} \cdot \underbrace{\frac{1}{(1-x)^4}}_{\text{Binomial Negativo}}$

$(1-x^{11})^4 = \sum_{j=0}^{4} \binom{4}{j} (-1)^j x^{11j}$

$\frac{1}{(1-x)^4} = \sum_{k=0}^{\infty} \left(\binom{-4}{k}\right) x^k$

$x^{11j} \cdot x^k = x^{\overbrace{11j+k}^{n}}$

$11j + k = 27$

$C_{27} = \left(\binom{4}{0}\right)(-1)^0 \cdot \left(\binom{-4}{27}\right) + \left(\binom{4}{1}\right)(-1)^1 \cdot \left(\binom{-4}{16}\right) = \binom{30}{3} - 4 \cdot \binom{19}{3}$

$(j=0 \wedge k=27) \quad (j=1 \wedge k=16)$

coeficiente

Objetivo

Dois fragmentos

$\sum_{n=0}^{\infty} \frac{a_n}{n!} x^n$ — coeficiente — [Série exponencial]

$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

Cartão 1 $(1+x+\frac{x^2}{2})$ | Cartão 2 e^x

Série Taylor $= \frac{1}{0!} + \frac{1}{1!}x + \frac{1}{2!}x^2$

$(1+x+\frac{x^2}{2}) \cdot e^x = (1+x+\frac{x^2}{2}) \cdot \sum_{n=0}^{\infty} \frac{x^n}{n!} = \sum_{n=0}^{\infty} \frac{1x^n}{n!} + \sum_{n=0}^{\infty} \frac{1x^{n+1}}{n!} + \sum_{n=0}^{\infty} \frac{1}{2} \frac{x^{n+2}}{n!}$

$= \sum_{n=0}^{\infty} \frac{1x^n}{n!} + \sum_{n=0}^{\infty} \frac{1(n+1)x^n}{(n+1)\,n!} + \sum_{n=0}^{\infty} \frac{1}{2} \frac{x^{n+2}}{(n+2)(n+1)\,n!}$

$(n+1)! \qquad (n+2)!$

Coeficiente 4 $\quad C_4 = 1 + \underbrace{[(n+1)]}_{n=3} + \underbrace{\left[\frac{1}{2}(n+1)(n+2)\right]}_{n=2} = 1+4+6 = 11$

Soma de 4

Folha 4

(19) a)

$(2+x)^4 = \sum_{n=0}^{4} \underbrace{\left(\binom{4}{n} 2^{4-n}\right)}_{\text{coeficiente}} x^n + 0x^5 + 0x^6 + \dots$

$a_n = \begin{cases} \binom{4}{n} 2^{4-n} & \text{se } 0 \le n \le 4 \\ 0 & \text{se } n \ge 5 \end{cases}$

b) $\frac{6x}{(1+2x)^2} + 2 - x^2$ — Obs:

$2 - x^2 = 2 + 0 + 0x + (-1)x^2 + 0x^3 + 0x^4 + \dots$

$\frac{6x}{(1+2x)^2} = 6x \cdot \frac{1}{(1-(-2x))^2}$ $\qquad \frac{1}{(1-x)^2} = \sum_{n=0}^{\infty} \underbrace{\left(\binom{-2}{n}\right)}_{n+1} x^n$

$= 6x \cdot \sum_{n=0}^{\infty} \left(\binom{-2}{n}\right) (-2x)^n$

$= 6x \cdot \sum_{n=0}^{\infty} (n+1) \cdot (-2x)^n$

$= 6 \cdot \sum_{n=0}^{\infty} (n+1) \cdot (-2)^n \cdot x^{n+1}$

$= \sum_{n=0}^{\infty} 6(n+1)(-2)^n x^{n+1}$

$= \sum_{n=1}^{\infty} 6n(-2)^{n-1} x^n$

Coeficientes

$a_n \begin{cases} 0, & n = 0 \\ -25, & n = 2 \\ 6n(-2)^{n-1}, & \text{caso contrário} \end{cases}$

Série Geradora vs Eq. Recorrência

$\begin{cases} a_n = 2a_{n-1} + 1, & n \ge 2 \\ a_1 = 1 \end{cases}$

$A = \sum_{n=1}^{\infty} a_n x^n$

Encontrar a Recorrência

$A = a_1 x + \sum_{n=2}^{\infty} a_n x^n$

$= x + \sum_{n=2}^{\infty} a_n x^n$

$= x + \sum_{n=2}^{\infty} (2a_{n-1} + 1) x^n$

$= x + \sum_{n=2}^{\infty} x^n + 2x \sum_{n=2}^{\infty} a_{n-1} x^{n-1} = \frac{x}{1-x} + 2xA$

Fechando

$A = \frac{x}{1-x} + 2xA$

$\Leftrightarrow A = \frac{1}{(1-2x)(1-x)}$ (Função racional)

Conferindo a Série

$\frac{1}{(1-2x)(1-x)} = \frac{A}{(1-2x)} + \frac{B}{(1-x)}$ $\left[\begin{array}{l} \text{Verifica } A = 2 \\ \text{e } B = -1 \end{array}\right]$

$= \sum_{n=0}^{\infty} (2x)^n - \sum_{n=0}^{\infty} x^n$

$= \sum_{n=1}^{\infty} \underbrace{(2^n - 1)}_{a_n} x^n$

se for $n=0$
dá $0 \neq 1$
não deu ok