

Cap. 3 - Arranjos e combinações

3.1. Permutações e Arranjos

Um arranjo com repetição de n elementos é uma sequência

em que os elementos pertencem a um universo de n elementos

Nesta sequência interessa a ordem e podem existir elementos repetidos

De outro modo é o n.º de funções na forma $f: \{1, \dots, k\} \rightarrow \{1, \dots, m\}$

$$\text{n.º de arranjos c/ repetição} = A^n(n, k) = \underbrace{m \times m \times \dots \times m}_k \text{ fatores} = m^k$$

ex 3.1.3

$$\begin{array}{cccccc} \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 7 & 7 & 7 & 7 & 7 & 7 \end{array}$$

$$A^6(7, 6) = 7^6$$

ex 3.1.4

$$\begin{array}{cccc} \downarrow & \downarrow & \downarrow & \downarrow \\ 3 & 3 & 3 & 3 \end{array}$$

$$A^4(3, 5) = 3^5$$

Um arranjo sem repetição de n elementos K a K é uma sequência de comprimento K com repetição, em que os elementos pertencem a um universo de tamanho n

OU uma função injetiva na forma

$$f: \{1, \dots, k\} \rightarrow \{1, \dots, n\}$$

$$\text{n.º de arranjos s/ repetição} = A^S(n, k) = \underbrace{n(n-1) \dots (n-k+1)}_k \text{ fatores} = \frac{n!}{(n-k)!}$$

ex 3.1.7

$A^0(n, k)$ é o n.º de maneiras de reunir k pessoas em k cadeiras seguidas quando o universo de pessoas tem tamanho n

ex 3.1.8

2 arranjos com 2 elementos
 $2 \times A^2(2, 2) = 2 \times 2!$
permutações de 2

ex 3.1.9

$$A^8(8, 8) = 8!$$

3.2. Combinações

Uma combinação de n elementos K a K

é um subconjunto de K elementos de

um conjunto de tamanho n (não interessa a ordem dos elementos)

$$\text{n.º de subconjuntos de } K \text{ a } K = \binom{n}{k} = \frac{A^S(n, k)}{k!} = \frac{n!}{(n-k)! \times k!}$$

ex 3.2.4

$$\binom{6}{3} = \frac{6!}{(6-3)! \times 3!} = \frac{6 \times 5 \times 4 \times 3!}{3! \times 3!} = \frac{6 \times 5 \times 4}{3 \times 2} = \frac{3 \times 2 \times 5 \times 4}{3 \times 2} = 20$$

ex 3.2.5

16 rapazes

15 rapazes

$$\begin{array}{c} \begin{array}{c} \text{+} \\ \text{OU} \end{array} \\ \begin{array}{c} 3 \text{ rapazes} \times 2 \text{ rapazes} \\ \left(\binom{15}{3} \right) \times \left(\binom{16}{2} \right) \end{array} \quad \begin{array}{c} \text{+} \\ \text{OU} \end{array} \\ \begin{array}{c} 4 \text{ rapazes} \times 1 \text{ rapaz} \\ \left(\binom{15}{4} \right) \times \left(\binom{16}{1} \right) \end{array} \quad \begin{array}{c} \text{+} \\ \text{OU} \end{array} \\ \begin{array}{c} 5 \text{ rapazes} \\ \left(\binom{15}{5} \right) \end{array} \end{array}$$

$$R: \left(\binom{15}{3} \right) \times \left(\binom{16}{2} \right) + \left(\binom{15}{4} \right) \times 16 + \left(\binom{15}{5} \right) = 15 \times 16 \times 3 = 720$$

ERRADO

Exemplo + pequeno

Grupo c/ 2 rapazes e 2 raparigas
N.º de subconjuntos de dois em que pelo menos um é rapaz

$$\begin{array}{l} 2 \times 2 + 1 = 5 \\ \frac{\binom{5}{2} - \binom{2}{2}}{1} = 5 \\ \frac{2 \times 3 + 6}{2} = 6 \end{array}$$

$$\frac{\binom{5}{2} - \binom{2}{2}}{1} = 5$$

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