① $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$= 1\times 0 + 2\times 1 = 2$

$1\times -1 + 2\times 0 = -1$

$3\times 0 + 4\times 1 = 4$

$3\times -1 + 4\times 0 = -3$

$$AB = \begin{bmatrix} 2 & -1 \\ 4 & -3 \end{bmatrix}$$

$B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ } → Represents a 90° degree. Counterclockwise rotation about axis.

In AB, the transformation and rotation are applied to A.

② $C = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

$$|C - \lambda I| = \det \begin{bmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{bmatrix} = 0$$

$= (2-\lambda)(2-\lambda) - 1\times 1$

$= (2-\lambda)^2 - 1 \implies \lambda^2 - 4\lambda + 3 = 0$

$= (\lambda - 3)(\lambda - 1) = 0$

$\lambda_1 = 3, \ \lambda_2 = 1$

$\lambda_1 = 3$

$(C - 3I)X = 0$

$$C - 3I = \begin{bmatrix} 2-3 & 1 \\ 1 & 2-3 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$= -x + y = 0 \Rightarrow y = x.$

$X_1 = k \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \ k \in R.$

$\lambda_2 = 1$

$(C - I)X = 0$

$$C - I = \begin{bmatrix} 2-1 & 1 \\ 1 & 2-1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$x + y = 0 \Rightarrow y = -x.$

$X_2 = k \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \ k \in R.$

For $\lambda_1 = 3$ — $X_1 = k \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

for $\lambda_2 = 1$ — $X_2 = k \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

③ Given initial position $p = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

Robot Arm rotates 90° Counterclockwise about origin. let us take it as R.

$$R = \begin{bmatrix} \cos(90^\circ) & -\sin(90^\circ) \\ \sin(90^\circ) & \cos(90^\circ) \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

To find P after rotation, multiply P and R.

let new position be X

$$X = PR = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$X = \begin{bmatrix} 0 \times 2 + -1 \times 1 \\ 1 \times 2 + 0 \times 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 2 & 0 \end{bmatrix}$$

$X = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \rightarrow$ X is robot arm in 2D space.

④ For Bayesian inference.

A = Where object is there (present)

B = where object is actually detected.

$P(B|A) = 80\% = 0.8$ $\quad$ $P(A) = 30\% = 0.3$

$P(B/\bar{A}) = 0.1$ $\quad$ $P(\bar{A}) = 0.7$.

$$P(A/B) = \frac{P(A)\ P(B/A)}{P(A)\ P(B/A) + P(\bar{A})\ P(B/\bar{A})}$$

$$= \frac{0.3 \times 0.8}{0.3 \times 0.8 + 0.7 \times 0.1} = \frac{0.24}{0.31}$$

$$= \frac{24}{31}$$

⑤ $\mu = 5$ (Mean)

$\sigma = 2$ (standard deviation)

$X = 7$ (probability)

$$Z = \frac{X - \mu}{\sigma} = Z = \frac{7-5}{2} = \frac{2}{2} = 1$$

$P(X > 7) \rightarrow$ To find. where $P(Z > 1)$.

By using standard normal distribution table,

$P(Z \leq 1) = 0.8413$

$P(Z > 1) = 1 - P(Z \leq 1) = 1 - 0.8413 = 0.1587$

Probability of reading being greater than 7 is 0.1587.

⑥ $\mu = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $\Sigma = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ $x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$f(x) = \frac{1}{(2\pi)^{d/2} |\Sigma|^{1/2}} \operatorname{Exp}\left(-\frac{1}{2}(x-\mu)^T \Sigma^{-1} (x-\mu)\right)$$

$d = 2.$

$$|\Sigma| = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = 4 - 1 = 3.$$

$$\Sigma^{-1} = \frac{1}{|\Sigma|} = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$$x - \mu = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$$(x-\mu)^T = [0 \quad -1]$$

$$f(x) = \frac{1}{(2\pi)(\sqrt{3})} \operatorname{Exp}\left(-\frac{1}{2}[0 \quad -1] \frac{1}{3}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\right.$$

$$= \frac{1}{(2\pi)(\sqrt{3})} \operatorname{Exp}\left(-\frac{1}{2} \cdot \frac{2}{3}\right) \Rightarrow \frac{1}{2\sqrt{3}\pi} \operatorname{Exp}\left(\frac{-1}{3}\right)$$

$$= \frac{1 \times 0.7165}{6.2832 \times 1.732} = 0.063.$$