



K. N. Toosi University of Technology

Faculty of Physics
Educational Group of
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Special Topics I Problem Sets (Problem Set 3 - Part 1 - Linear Algebra)

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Notes

- These questions require thought, but do not require long answers. Please be as concise as possible.
- If you are writing your solutions out by hand, please write clearly and in a reasonably large font using a dark pen to improve legibility.

Gradients and Hessians

Recall that a matrix $A \in \mathbb{R}^{n \times n}$ is *symmetric* if $A^T = A$, that is, $A_{ij} = A_{ji}$ for all i, j . Also recall the gradient $\nabla f(x)$ of a function $f : \mathbb{R}^n \rightarrow \mathbb{R}$, which is the n -vector of partial derivatives

$$\nabla f(x) = \begin{bmatrix} \frac{\partial}{\partial x_1} f(x) \\ \vdots \\ \frac{\partial}{\partial x_n} f(x) \end{bmatrix} \quad \text{where} \quad x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \quad (1)$$

The hessian $\nabla^2 f(x)$ of a function $f : \mathbb{R}^n \rightarrow \mathbb{R}$ is the $n \times n$ symmetric matrix of twice partial derivatives,

$$\nabla^2 f(x) = \begin{bmatrix} \frac{\partial^2}{\partial x_1^2} f(x) & \frac{\partial^2}{\partial x_1 \partial x_2} f(x) & \cdots & \frac{\partial^2}{\partial x_1 \partial x_n} f(x) \\ \frac{\partial^2}{\partial x_2 \partial x_1} f(x) & \frac{\partial^2}{\partial x_2^2} f(x) & \cdots & \frac{\partial^2}{\partial x_2 \partial x_n} f(x) \\ \vdots & \vdots & \cdots & \vdots \\ \frac{\partial^2}{\partial x_n \partial x_1} f(x) & \frac{\partial^2}{\partial x_n \partial x_2} f(x) & \cdots & \frac{\partial^2}{\partial x_n^2} f(x) \end{bmatrix} \quad (2)$$

1. Let $f(x) = \frac{1}{2}x^T A x + b^T x$, where A is a symmetric matrix and $b \in \mathbb{R}^n$ is a vector. What is $\nabla f(x)$?
2. Let $f(x) = g(h(x))$, where $g : \mathbb{R} \rightarrow \mathbb{R}$ is differentiable and $h : \mathbb{R}^n \rightarrow \mathbb{R}$ is differentiable. What is $\nabla f(x)$?
3. Let $f(x) = \frac{1}{2}x^T A x + b^T x$, where A is symmetric and $b \in \mathbb{R}^n$ is a vector. What is $\nabla^2 f(x)$?
4. Let $f(x) = g(a^T x)$, where $g : \mathbb{R} \rightarrow \mathbb{R}$ is continuously differentiable and $a \in \mathbb{R}^n$ is a vector. What are $\nabla f(x)$ and $\nabla^2 f(x)$?