Name: _____

MATH1107 Linear Algebra and Geometry	Winter 2024

Week 6 – Lab 6 _____/32

Please show your work and explain your answers for each question.

1) Suppose **A**, **B**, **C**, **D and E** are matrices with the following sizes: **/8**

$\mathbf{A}$ (4×5), $\mathbf{B}$ (4×5), $\mathbf{C}$ (5×2), $\mathbf{D}$ (4×2), $\mathbf{E}$ (5×4)

Determine which of the following matrix expressions are defined. For those which are defined, give the size of the resulting matrix

Expression	Defined?	Size?
BA	No	nothing
AC + **D**	Yes	4*2
AE + **B**	No	Nothing
AB + **B**	No	Nothing
E(**A** + **B**)	Yes	5*5
E(**AC**)	No	nothing
$\mathbf{E}^T\mathbf{A}$	NO	Nothing
$(\mathbf{A}^T + \mathbf{E})\mathbf{D}$	Yes	5*2

t

Name: _____

$15 + \frac{2}{3} \rightarrow \frac{\quad}{3}$

2) Compute the following products **/4**

a. $\begin{bmatrix} 2 & -4 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} \frac{3}{4} & 5 \\ -2 & \frac{-2}{3} \end{bmatrix}$

2×2 $\quad 2$

$= \begin{bmatrix} \frac{3 \cdot 8}{4} & \frac{38}{3} \\ \frac{17}{4} & \frac{47}{3} \end{bmatrix}$

b. $\begin{bmatrix} 4 & -2 & 5 \end{bmatrix} \begin{bmatrix} -3 & 0 & -2 \\ 4 & 3 & -6 \\ 1 & -3 & 2 \end{bmatrix}$

$1 \times$

$\begin{bmatrix} -15 & -21 & 14 \end{bmatrix}$ 1×3

Name: _____

3) Consider the matrices: **/18**

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix} \quad E = \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix} \quad F = \begin{bmatrix} 6 \\ 2 \\ 4 \end{bmatrix}$$

Compute the following. If the computations are not possible, explain why.

a. AB

$$\begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 12 & -3 \\ -4 & 5 \\ 4 & 1 \end{bmatrix}$$

b. BF **Cause B is 2*2 and the E is 3*3 therefore we have problem with computing them**

Undefined

c. BA **B is 2*2 and a is 3*2 so we cannot do the calculation**

undefined

Name: _____

d. $(2E)D$

$$2\begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 5 \end{bmatrix} \times \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix} =$$

$$\begin{bmatrix} 12 & 2 & 6 \\ -2 & 2 & 4 \\ 8 & 2 & 6 \end{bmatrix} \times$$

$$\begin{bmatrix} 28 & 72 & 50 \\ 8 & -2 & 14 \\ 24 & 52 & 42 \end{bmatrix}$$

e. $(AB)C$

$$\begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 12 & -3 \\ -4 & 5 \\ 4 & 1 \end{bmatrix}_{3\times2} \times \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}_{2\times3} =$$

$$\begin{bmatrix} 3 & 45 & 9 \\ 11 & -11 & 17 \\ 7 & 17 & 9 \end{bmatrix}_{3\times3}$$

f. AF

Undefined because the a is 3*2 and f is 3*1 therefore we cannot perform this expression.

Name: _____

g. $\boldsymbol{CF}$

$$\begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}_{2\times 3} \times \begin{bmatrix} 6 \\ 2 \\ 4 \end{bmatrix}_{3\times 1} = \begin{bmatrix} 22 \\ 40 \end{bmatrix}_{2\times 1}$$

h. $\boldsymbol{FA^T}$ **undefined due to difference of size we cannot perform**

$$\begin{bmatrix} 6 \\ 2 \\ 4 \end{bmatrix}_{3\times 1} \times \begin{bmatrix} 3 & -1 & 1 \\ 0 & 2 & 1 \end{bmatrix}_{2\times 3}$$

$1 \neq 2$ ✓

i. $\boldsymbol{A(BC)}$

$$\begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}_{3\times 2} \times \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}_{2\times 2} \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}_{2\times 3} = {}_{2\times 3} \neq {}_{2\times 3}$$

$$\begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}_{3\times 2} \times \begin{bmatrix} 1 & 15 & 3 \\ 6 & 2 & 10 \end{bmatrix}_{2\times 3} = \begin{bmatrix} 3 & 45 & 9 \\ 11 & -11 & 17 \\ 7 & 17 & 13 \end{bmatrix}_{3\times 3}$$

Name: _____

j. CC^T

$$\begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}_{2\times3} \begin{bmatrix} 1 & 3 \\ 4 & 1 \\ 2 & 5 \end{bmatrix}_{3\times2} = \begin{bmatrix} 21 & 17 \\ 17 & 35 \end{bmatrix}_{2\times2}$$

k. $(DA)^T$

$$A^T D^T \rightarrow \begin{bmatrix} 3 & -1 & 1 \\ 0 & 2 & 1 \end{bmatrix}_{2\times3} \times \begin{bmatrix} 1 & -1 & 3 \\ 5 & 0 & 2 \\ 2 & 1 & 4 \end{bmatrix}_{3\times3} =$$

$$\begin{bmatrix} 0 & -2 & 11 \\ 12 & 1 & 8 \end{bmatrix}_{2\times3}$$

l. $(C^T B)A^T$

$$\begin{bmatrix} 1 & 3 \\ 4 & 1 \\ 2 & 5 \end{bmatrix}_{3\times2} \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}_{2\times2}$$

$$\begin{bmatrix} 4 & 5 \\ 16 & -2 \\ 8 & 8 \end{bmatrix}_{2\times2} \times \begin{bmatrix} 3 & -1 & 1 \\ 9 & 2 & 1 \end{bmatrix}_{2\times3} = \begin{bmatrix} 12 & 6 & 9 \\ 48 & -20 & 14 \\ 24 & 8 & 16 \end{bmatrix}_{3\times3}$$

Name: _____

m. $(DD^T)^T$

$D \quad D^T \rightarrow \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix} \times \begin{bmatrix} 1 & -1 & 3 \\ 5 & 0 & 2 \\ 2 & 1 & 4 \end{bmatrix} =$

$\begin{bmatrix} 29 & 1 & 21 \\ 1 & 2 & 1 \\ 21 & 1 & 21 \end{bmatrix}_{3\times3}$

n. $(3D^T - 2E)A$

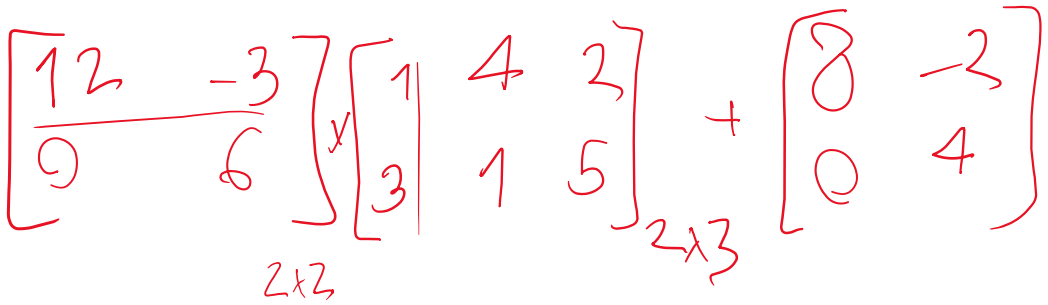
$\left(3\begin{bmatrix} 1 & -1 & 3 \\ 5 & 0 & 2 \\ 2 & 1 & 4 \end{bmatrix} - 2\begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix} \right) \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}$

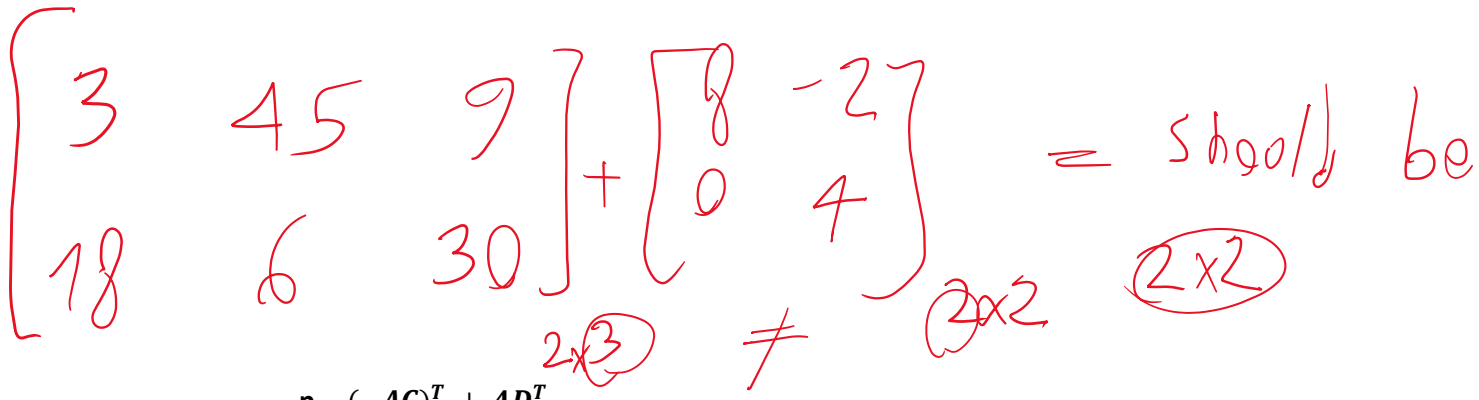
$\left(\begin{bmatrix} 3 & -3 & 9 \\ 15 & 0 & 6 \\ 6 & 3 & 12 \end{bmatrix} - \begin{bmatrix} 12 & 2 & 6 \\ -2 & 2 & 4 \\ 8 & 2 & 6 \end{bmatrix} \right) \times \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} =$

$\begin{bmatrix} -9 & -5 & 3 \\ 17 & 42 & 2 \\ -2 & & 6 \end{bmatrix}_{3\times3} \times \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}_{3\times2} = \begin{bmatrix} -19 & -7 \\ 55 & -2 \\ -1 & 8 \end{bmatrix}_{3\times2}$

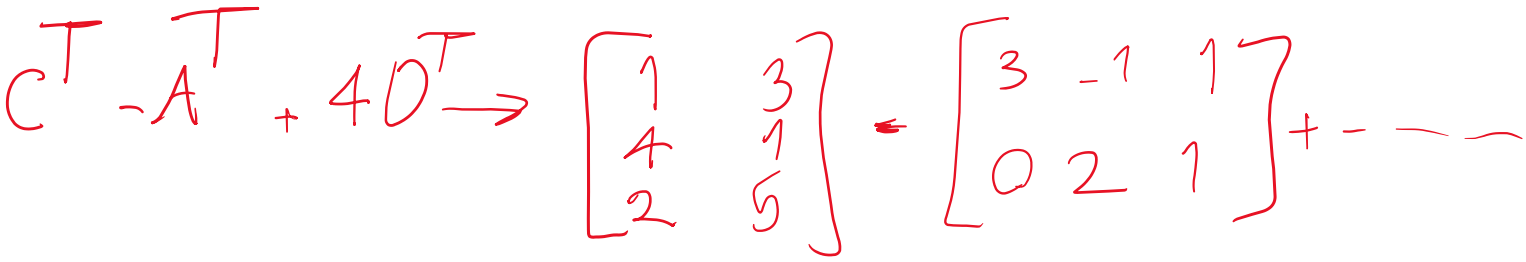
Name: _____

o. $(3B)C + 2B$ **undefined because of (shown in answer)./**



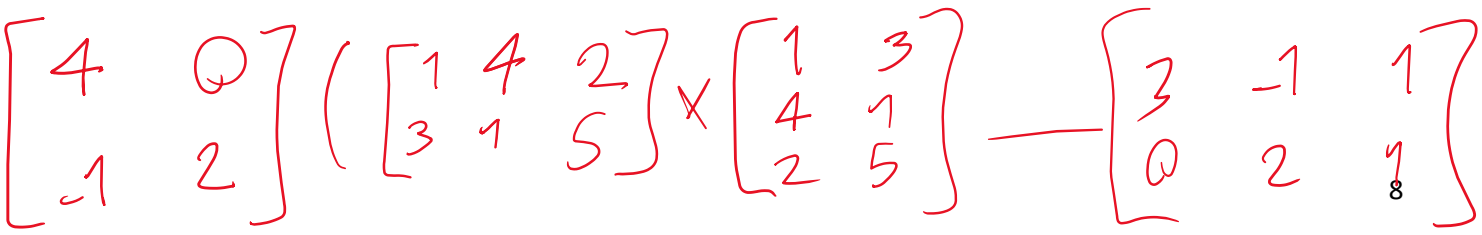


p. $(-AC)^T + 4D^T$



Undefined because the subtraction is not possible cause they don't have the same size.

q. $B^T(CC^T - A^TA)$



Name: _____

$$\left(\begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}\right)^T \begin{bmatrix} 4 & 9 \\ -1 & 2 \end{bmatrix} \left(\begin{bmatrix} 21 & 17 \\ 1 & 35 \end{bmatrix} - \begin{bmatrix} 11 & -1 \\ -1 & 5 \end{bmatrix} \right)$$

$$\begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix} \times \begin{bmatrix} 10 & 18 \\ 2 & 30 \end{bmatrix} = \begin{bmatrix} 40 & 72 \\ -6 & 42 \end{bmatrix}$$

r. $\boldsymbol{D^T E^T - (ED)^T}$

$$D^T E^T - (D^T E^T) = 0$$

Is zero.

4) What type of transformation is represented by the following 2D matrix? **/2**

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

Clockwise by 90 degree or mathematically –90 degree

Name: _____