Complex No. & PDE

Unit → 1

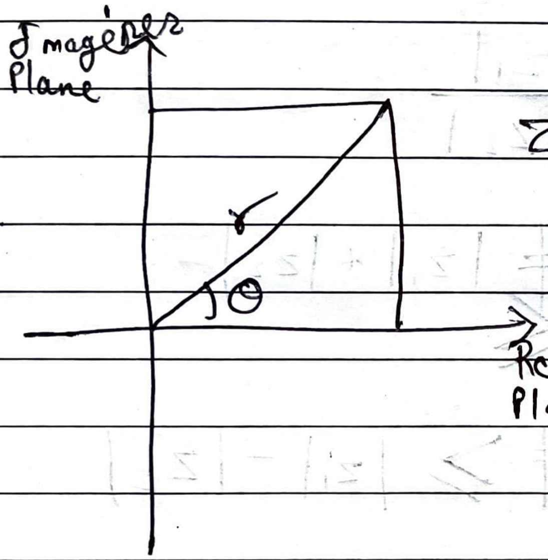
→ **Parts of complex :–** $x + iy$ ($i \to \sqrt{-1}$)

x ↓ Real Part

y ↘ imaginary

→ **Form of complex :–** $a + ib$

$r(\cos\theta + i\sin\theta)$



$Z_0 = x + iy$

$Z = r\cos\theta + ir\sin\theta$

$Z = re^{i\theta}$

$Z = r\,\text{cis}\,\theta$

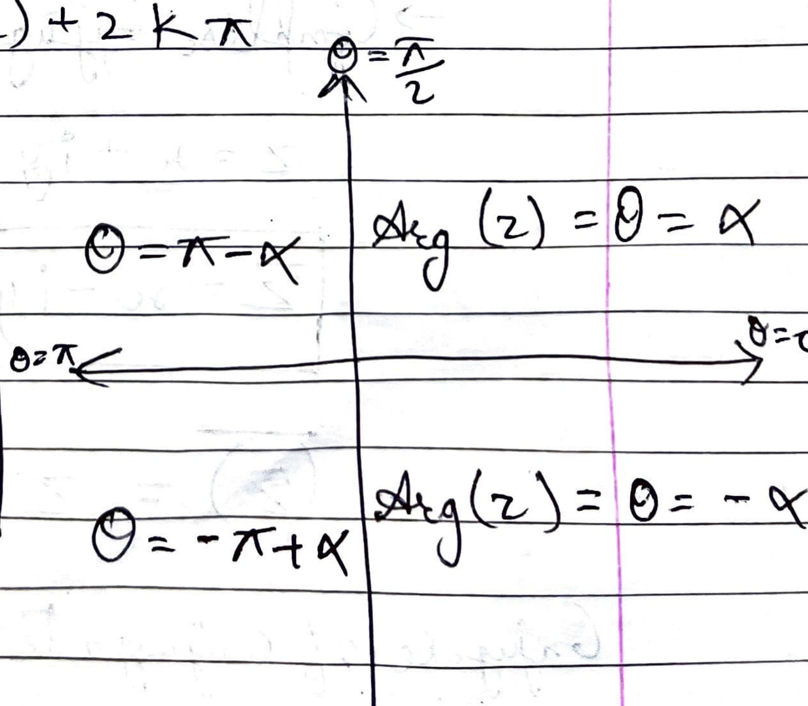
→ **Argument of complex :–**

$\arg(z) = \text{Arg}(z) + 2k\pi$

$Z = x + iy$

$r = \sqrt{x^2 + y^2}$

$\alpha = \tan^{-1}\left|\dfrac{y}{x}\right|$



→ Operation :- ($+$, $-$, $\times$, $\div$)

→ Absolute value :-

$$|z| = \sqrt{x^2 + y^2}$$

$$z_1 = z_2 \rightarrow |z_1| = |z_2|$$

For ($\times$) $|z_1 \cdot z_2| = |z_1|\,|z_2|$

For ($\div$) $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$

For ($+$) $|z_1 + z_2| \neq |z_1| + |z_2|$
$\leqslant$

For ($-$) $|z_1 - z_2| \neq \geqslant |z_1| - |z_2|$

$$|z| = 0 \Leftrightarrow z = 0$$

→ Complex Conjugate :-

$$z = x + iy$$

$$\boxed{\bar{z} = x - iy}$$

$$\overline{(\bar{z})} = z$$

Conjugate of conjugate is number (z).

- $z \cdot \bar{z} = |z|^2 = |\bar{z}|^2$
- If $z = x + iy$ than $\bar{z} = x - iy$ $\rightarrow x = \frac{z+\bar{z}}{2}$, $y = \frac{z-\bar{z}}{2i}$

$z = \frac{(1+i)}{1-i} \times \frac{(1+i)}{1+i}$

$= \frac{1+i+i+i^2}{2}$

$z = \frac{2i}{2}$

$\boxed{\bar{z} = -\frac{2i}{2}}$

$z = (1+i) + (2+i)^2$

$= (1+i) + 3 + 4i$

$= 4 + 5i$

$\bar{z} = 4 - 5i$

$z = \frac{2+6i}{3i} = \frac{-6i+18}{9}$

$z = \frac{18}{9} - \frac{6}{9}i = 2 - \frac{2}{3}i$

$\bar{z} = 2 + \frac{2}{3}i$

* Properties :-

1. $|\bar{z}| = |z|$
2. $Arg(\bar{z}) = -Arg(z)$
3. $z = re^{i\theta} = \bar{z} = re^{-i\theta}$
4. $z = r(\cos\theta + i\sin\theta)$

   $\bar{z} = r(\cos\theta - i\sin\theta)$
5. $\overline{(z_1+z_2)} = \bar{z}_1 + \bar{z}_2$
6. $\overline{z_1 - z_2} = \bar{z}_1 - \bar{z}_2$
7. $\overline{z_1 \cdot z_2} = \bar{z}_1 \cdot \bar{z}_2$
8. $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$

Pro. 1 Find complex conjugate of following - ① $(2-3i)(1-2i)$ ② $\frac{(2i-3)(3-i)}{2i}$ ③ $\frac{2-3i}{2+3i}$ ④ $z = \frac{1+i}{1-i}$

$\frac{6i+2-9+3i}{2i}$

$\frac{-7+9i}{2i}$

$\frac{14i+18}{4} = \frac{18}{4} + \frac{14}{4}i$

$\bar{z} = \frac{9}{2} - \frac{7}{2}i$ $\quad z = \frac{9}{2} + \frac{7}{2}i$

$\frac{(2-3i)(2-3i)}{(2)^2-(3i)^2}$

$\frac{4-6i-6i-9}{13}$

$\frac{-5-12i}{13}$

$z = \frac{-5}{13} - \frac{12}{13}i$

$z = -\frac{5}{13} - \frac{12}{13}i$

$\bar{z} = \frac{-5}{13} + \frac{12}{13}i$

* Circular Trigonometry fun :-

$$\cos\theta = \frac{z+\bar{z}}{2} = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

$$\sin\theta = \frac{z-\bar{z}}{2i} = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

$$\tan\theta = \frac{e^{i\theta} - e^{-i\theta}}{i(e^{i\theta} - e^{i\theta})}$$

* Hyperbolic Trigonometry fun^n :-

$$\cosh\theta = \frac{e^{\theta} + e^{-\theta}}{2}$$

$$\sinh\theta = \frac{e^{\theta} - e^{-\theta}}{2}$$

$$\tanh\theta = \frac{e^{\theta} - e^{-\theta}}{e^{\theta} + e^{-\theta}}$$

* Relations b/w above two fun^n :-

$$\cosh(i\theta) = \cos\theta$$

$$\sinh(i\theta) = i\sin\theta$$

$$\tanh(i\theta) = i\tan\theta$$