LOW ANGULAR MOMENTUM ACCRETION PROBLEMS

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Contents

1. Version history

Just to keep track of what changed when in this document.

Version	Date	Changes
1.2	2019-07-19	Taking into account comments from Sebastiaan
1.1	2019-07-02	More edits
1.0	2019-06-25	Initial version

2. Motivation

In many astrophysical scenarios, the accretor is moving through an ambient medium. A typical example is the flow in high-mass X-ray binaries where the donor star is a massive O-, B- or supergiant star. In this case the accretion does not proceed through a Roch-lobe overflow (with subsequent formation of a disk) but via accretion of the wind from the massive star. The Bondi-Hoyle-Lyttleton theory (BHL) gives a good indication for the flow and provides with a simple estimate of the accretion rate.

Here we investigate the BHL problem in the general relativisitic regime with numerical simulations and examine the validity of the simplified theory.

3. Theoretical background

A comprehensive review of BHL accretion is given by [1]. We are interested in particular in the relations from [2], know as the ballistic approximation. This means that the motion of fluid follows that of particle trajectories, a valid assumption when pressure gradients are neglected (highly supersonic regime). The equations of motion for a particle coming from far away with impact parameter ξ and velocity v_∞ read

$$\ddot{r} - r\dot{\theta}^2 = -\frac{GM}{r^2} \tag{1}$$

$$r^2\dot{\theta} = \xi v_\infty \tag{2}$$

where the first equation follows from projecting the force-balance on the radial direction and the second equation describes conservation of the angular momentum of the flow. See Figure 1 for the overall geometry of the problem.

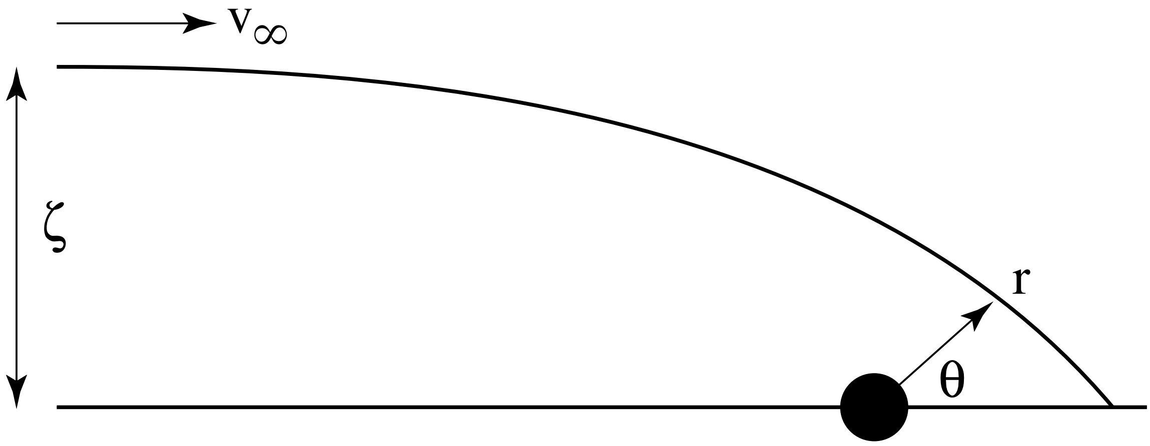

FIGURE 1. Geometry of the Bondi-Hoyle-Lyttleton accretion. Adopted from [1].

The analytic stationary solutions were first given by [3] and read

$$r = \frac{\xi^2 v_\infty^2}{GM(1+\cos\theta) + \xi v_\infty^2 \sin\theta} \tag{3}$$

$$v_r = -\sqrt{v_\infty^2 + \frac{2GM}{r} - \frac{\xi^2 v_\infty^2}{r^2}} \tag{4}$$

$$v_\theta = \frac{\xi v_\infty}{r} \tag{5}$$

$$\rho = \frac{\rho_\infty \xi^2}{r\sin\theta(2\xi - r\sin\theta)} \tag{6}$$

Here, the density follows from the requirement that $\rho\mathbf{v}$ is divergence-free for a stationary solution.

What happens if the flow meets the axis $\theta = 0$? In first approximation, the flow will collide with stream-lines from the other side such that v_θ will cancel out. The remaining radial velocity component becomes $v_r = -v_\infty$ (from Eqs. 4,3) and the radial position where the flow hits the axis (from Eq. 3) is $r = (\xi v_\infty)^2/2GM$. If we consider that bound material at the axis will eventually be accreted, we have

$$\frac{1}{2}v_\infty^2 - \frac{GM}{r} < 0 \tag{7}$$

and after eliminating the radius we find the critical impact parameter

$$\xi < \xi_{\mathrm{HL}} := \frac{2GM}{v_\infty^2} \tag{8}$$

To find the amount of material being accreted, we just need to consider the cross-section $\pi\zeta_{\mathrm{HL}}^2$ sweeping through the medium:

$$\dot{M}_{\mathrm{HL}} = \pi\zeta_{\mathrm{HL}}^2 v_\infty \rho_\infty = \frac{4\pi G^2 M^2 \rho_\infty}{v_\infty^3} \tag{9}$$

which we denote as Hoyle-Lyttleton accretion rate.

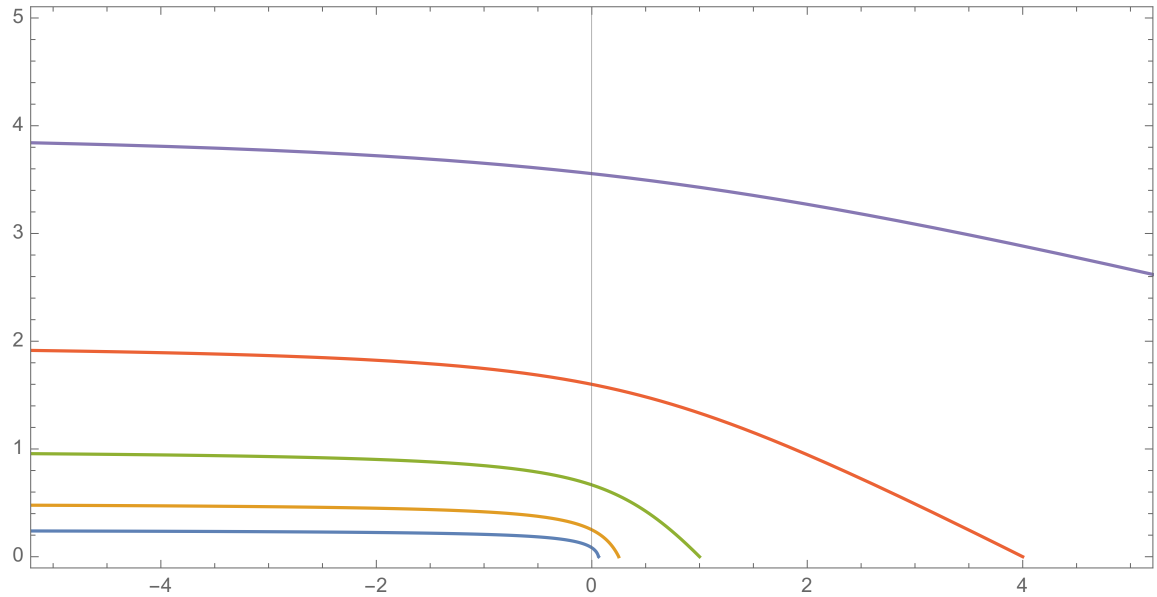

FIGURE 2. Selected flow lines in ballistic approximation with impact parameters $\xi/\xi_{\mathrm{HL}} \in \{0.25, 0.5, 1, 2, 4\}$. All distances are given in terms of the critical impact parameter $\xi_{\mathrm{HL}} = 2GM/v_\infty^2$.

4. INITIAL CONDITIONS

To simulate BHL accretion, we choose spherical polar coordinates and simulate only the $r-\theta$ plane as the problem is axi-symmetric.

The initial state is determined by choosing values "at infinity", the density ρ_∞, the velocity v_∞ and the Mach-number M_∞.

$$v^r = -\frac{1}{\sqrt{\gamma_{rr}}} v_\infty \cos\theta \tag{10}$$

$$v^\theta = +\frac{1}{\sqrt{\gamma_{\theta\theta}}} v_\infty \sin\theta \tag{11}$$

$$v^\phi = 0 \tag{12}$$

$$\rho = \rho_\infty \tag{13}$$

$$p = p_\infty \tag{14}$$

where the pressure follows from the chosen relativistic Mach number

$$\mathcal{M} = \frac{\Gamma v}{\Gamma_s c_s} \tag{15}$$

and the sound speed for an ideal gas with adiabatic index $\hat{\gamma}$ is

$$c_s^2 = \hat{\gamma}\frac{p}{\rho + \frac{\hat{\gamma}}{\hat{\gamma}-1}p}. \tag{16}$$

Without loss of generality, we can set the initial density $\rho_\infty = 1$ always (Why is this the case)?

5. Assignments

First set up the initial conditions as given in the previous section. A resolution of $N_r = 256$ and $N_\theta = 128$ should be sufficient for the beginning. Note that the length scale is given in units of GM/c^2 and velocities are measured in units of the speed of light c.

5.1. **Bondi-Hoyle-Lyttleton.**

(1) For your first run, choose a Schwarzschild metric $a = 0$, $M_\infty = 1.28$, $\hat{\gamma} = 5/3$, $v_\infty = 0.2c$.
  (a) Visualize the density field and try to reproduce Figure 3.
  (b) What is ξ_{HL} for this configuration, does the simulation match your expectation? For this, try to visualize the velocity vectors. There is already a function prepared for you in
  `harm_script.py`
  called
  `int_movie(dumpstart,dumpend, xmax1=20, skip1=16, skip2=16)`
  What is different from the ballistic approximation?
  (c) The code measures the accretion rate. Plot the accretion rate against time. There is already a function prepared for you in
  `harm_script.py`
  It is called
  `accretion_rate(dumpstart,dumpend, v_inf)`
  It normalizes the rate in terms of $\dot{M}_{HL}$ and time in terms of the crossing time $t_{\mathrm{HL}} = \xi_{HL}/v_\infty$.
  (d) Double the resolution and make a new run, are your results converged?
  (e) Optional: if you prepare a run with quadruple resolution, you can measure the "empirical order of convergence" index p:

$$p = \frac{\log\left(|\dot{M}_N - \dot{M}_{2N}|/|\dot{M}_{2N} - \dot{M}_{4N}|\right)}{\log 2} \tag{17}$$

  Plot p against time, which convergence rate do you measure?
(2) Now change the Mach-number of your setup and repeat the analysis of (1). Run also a case with $\mathcal{M}_\infty < 1$. Does this match your expectations? How long does it take to reach a stationary state?
(3) Scale your simulation to physical units, assume for example the black hole has a mass of $10M$ and the density corresponds to typical ISM values $\rho_\infty = 10^{-24}\,\mathrm{g cm^{-3}}$.
  (a) What is the length- and time-scale of your simulation in cgs units (reminder: in the code we set $r_g = GM/c^2 = 1$ and $c = 1$)?
  (b) Compute the accretion rate in physical units and also the accretion luminosity $L = \dot{M}c^2$ (assuming 100% efficiency). Could we observe such an object? How far out?

   (c) Use Eq. 9 to extrapolate to a somewhat more realistic value of $v_\infty = 100 \mathrm{kms}^{-1}$. What about now?
(4) For spherically symmetric accretion with sub-sonic velocity at infinity, it is common to define the "accretion radius" $r_{\mathrm{acc}} = 2GM/c_{s,\infty}^2$ where the gravitational pull dominates over the thermal energy of the gas [4]. Use this to motivate the interpolation formula due to [4, 5]:

$$\dot{M}_{\mathrm{BH}} = \frac{4\pi G^2 M^2 \rho_\infty}{(c_{s,\infty}^2 + v_\infty^2)^{3/2}} \tag{18}$$

and compare your results.
(5) Find a set of parameters where the shock becomes attached to the black hole.
(6) If you look at a movie of your simulation, you can see waves propagating from the bow-shock. In which of the cases and in which variables are they most pronounced? Are they physical or numerical?

5.2. **Black-Hole Disk interaction.** Let's beef up the BHL scenario a little bit. In black-hole binary systems (the Quasar OJ287 is speculated to be such an object comprised of two supermassive black holes), an accretion disk forms around the primary black hole (the heavier one, in OJ287 it likely has $10^{10} M_\odot$). Depending on the orbit, the secondary black hole will run through the accretion disk which could give rise to a periodic observational signature. This problem was first studied numerically by [6]. To simulate this, we let the black hole impact on a dense slab of gas (the accretion disk) and study the fluid-dynamics (e.g. evolution of the bow-shock) and accretion rate.

5.2.1. *Initial conditions.* Modify the initial conditions from the BHL setup to include a Gaussian slab of gas (the accretion-disk around the primary black hole in cross-section) which moves with v_∞ towards the secondary black hole. Use the following parameters:

(1) The disk thickness Δ_d
(2) The initial vertical position of the disk mid-plane z_d
(3) The peak-density of the disk ρ_d

5.2.2. *Assignments.*

(1) For your first run, choose $\Delta_d = 25$, $\rho_d = 100$ and $z_d = 500$. This shall leave enough time to establish a BHL flow before the disk impacts on the system.
   (a) Make a movie of the density evolution, what happens to the bow-shock during traversal?
   (b) Try also to visuzlize the flow-lines in the reference frame of the disk.
   (c) Plot the accretion rate, compare time-scale and magnitude of the "flare" with a simple estimate.
(2) In which case do you expect the gas motion during the BH-disk interaction to follow the BHL theory? Write down an an explicit condition by comparing the relevant length-scales.
(3) Start varying parameters, e.g. choose a denser disk or a more narrow disk. Make movies and compare the accretion rates.

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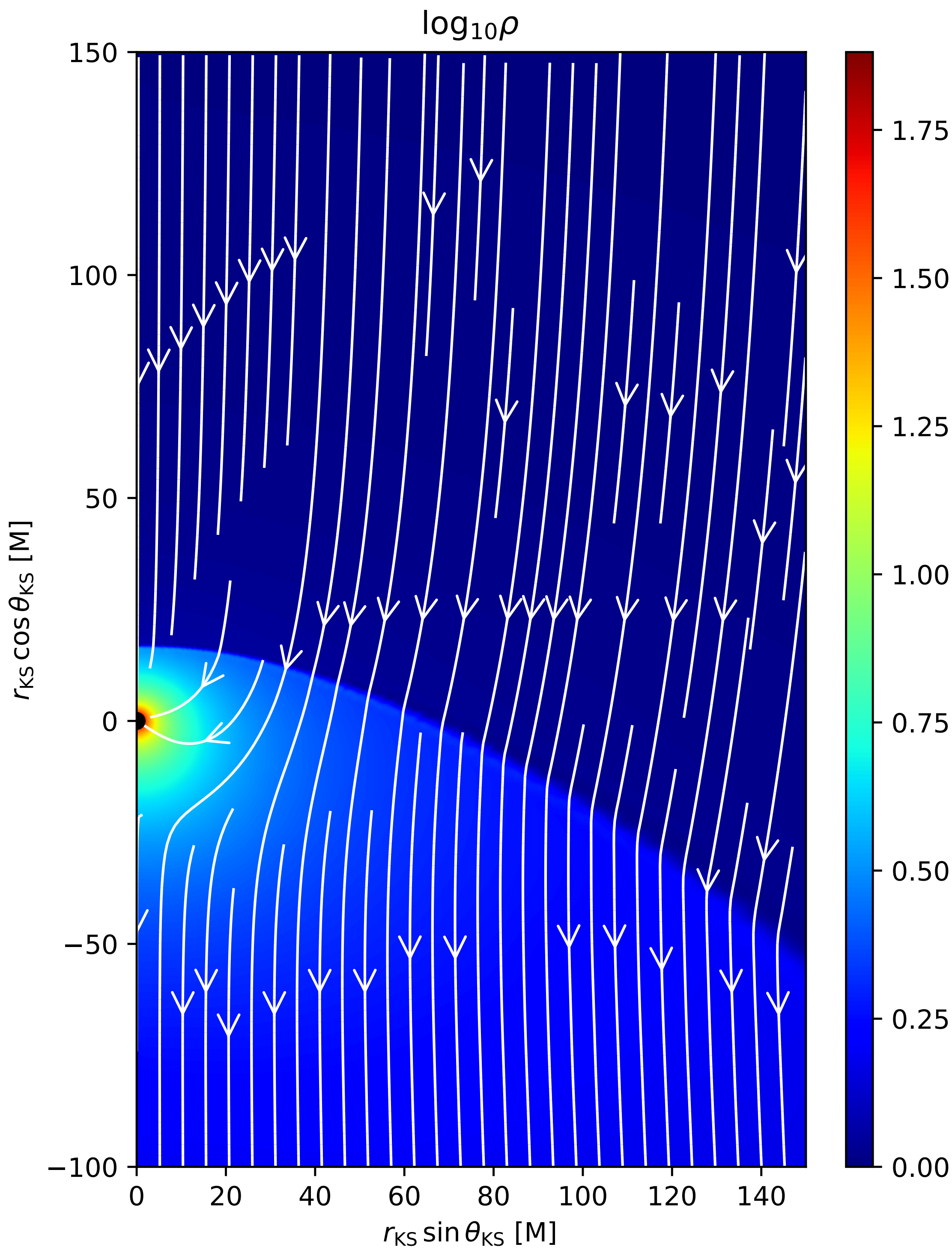

FIGURE 3. Simulation output with Mach number $M_\infty = 1.28$, $\hat{\gamma} = 5/3$, and velocity $v_\infty = 0.2c$. Logarithm of rest-frame density and streamlines of the four velocity u^μ.

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