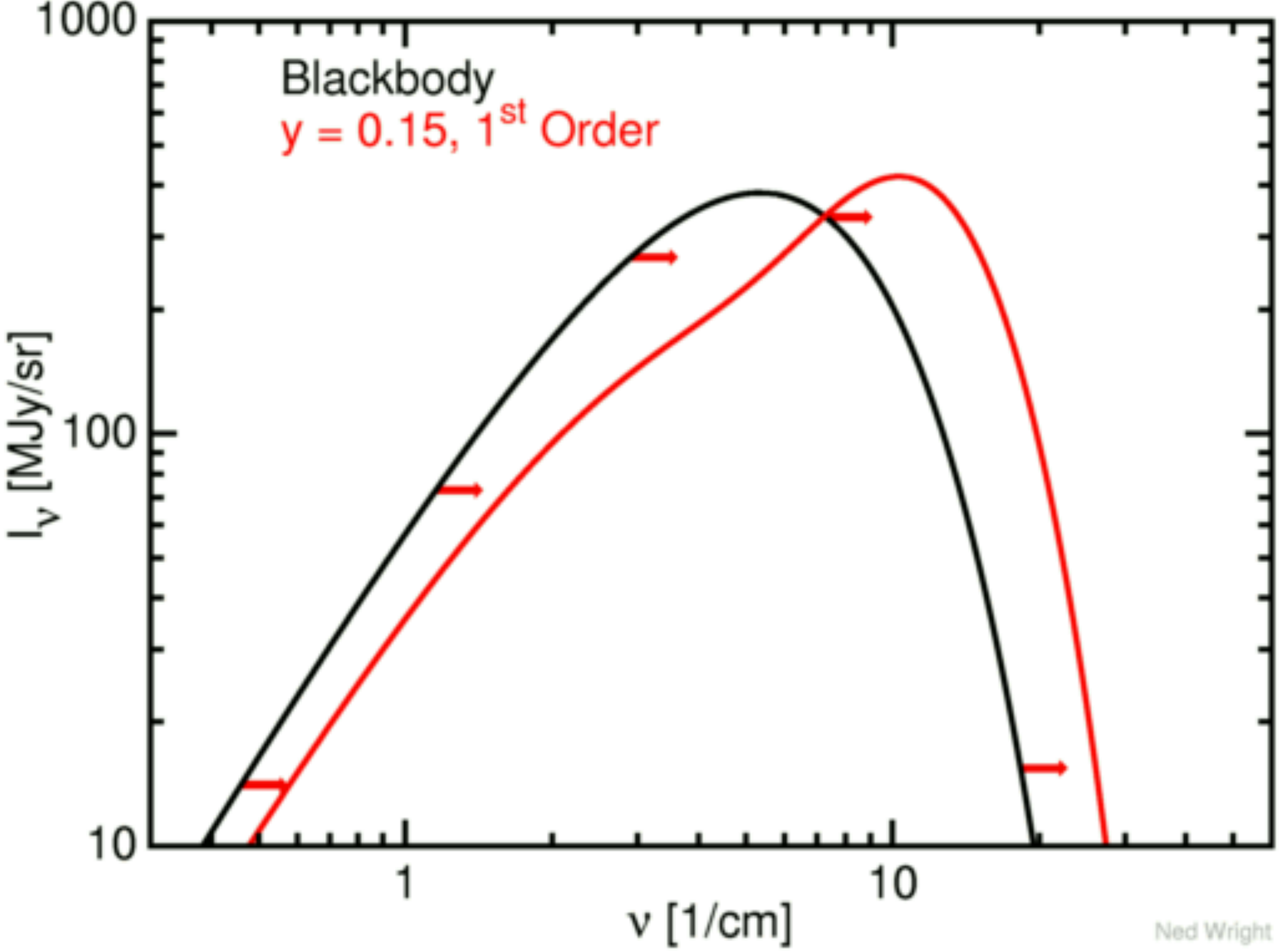
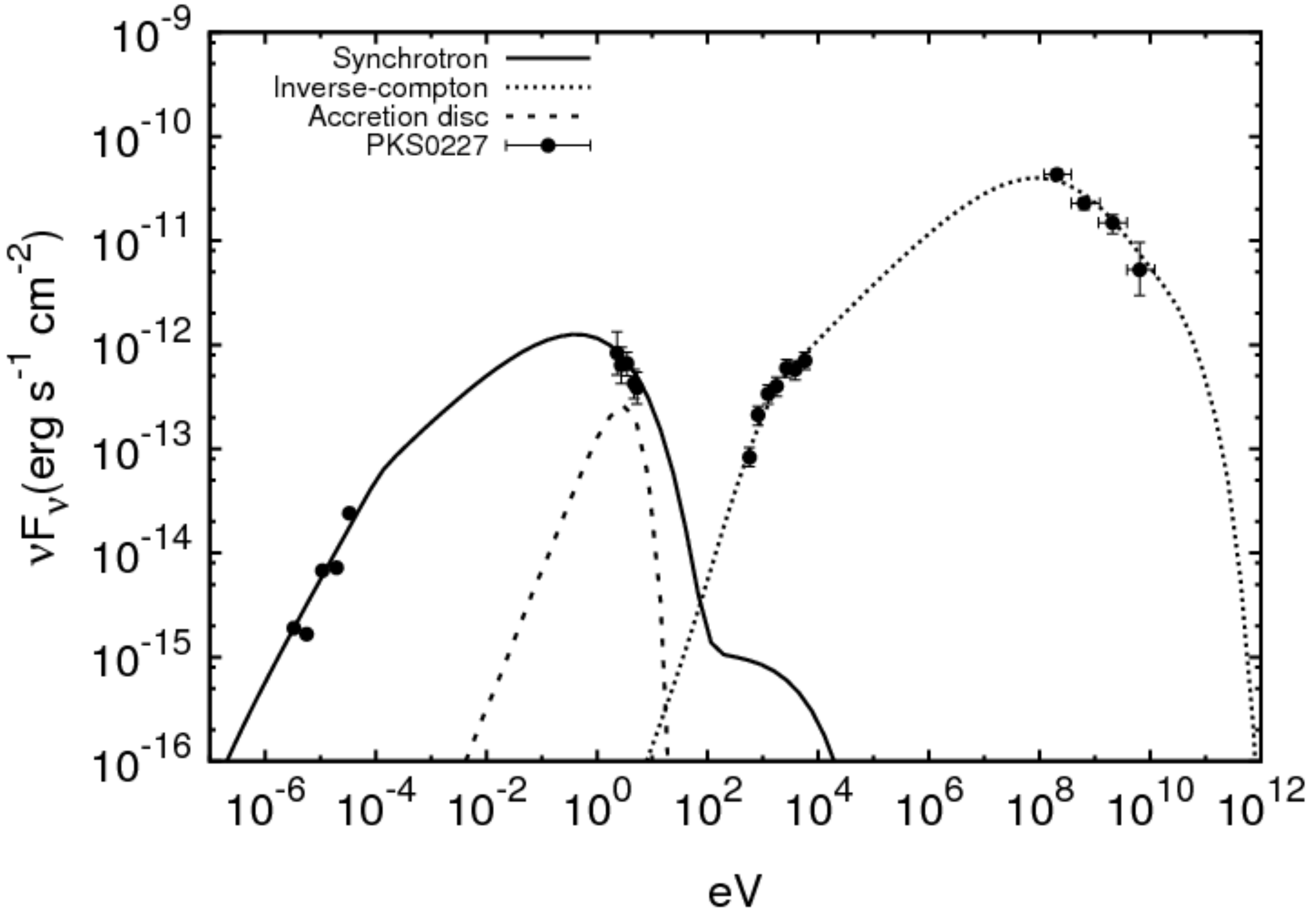
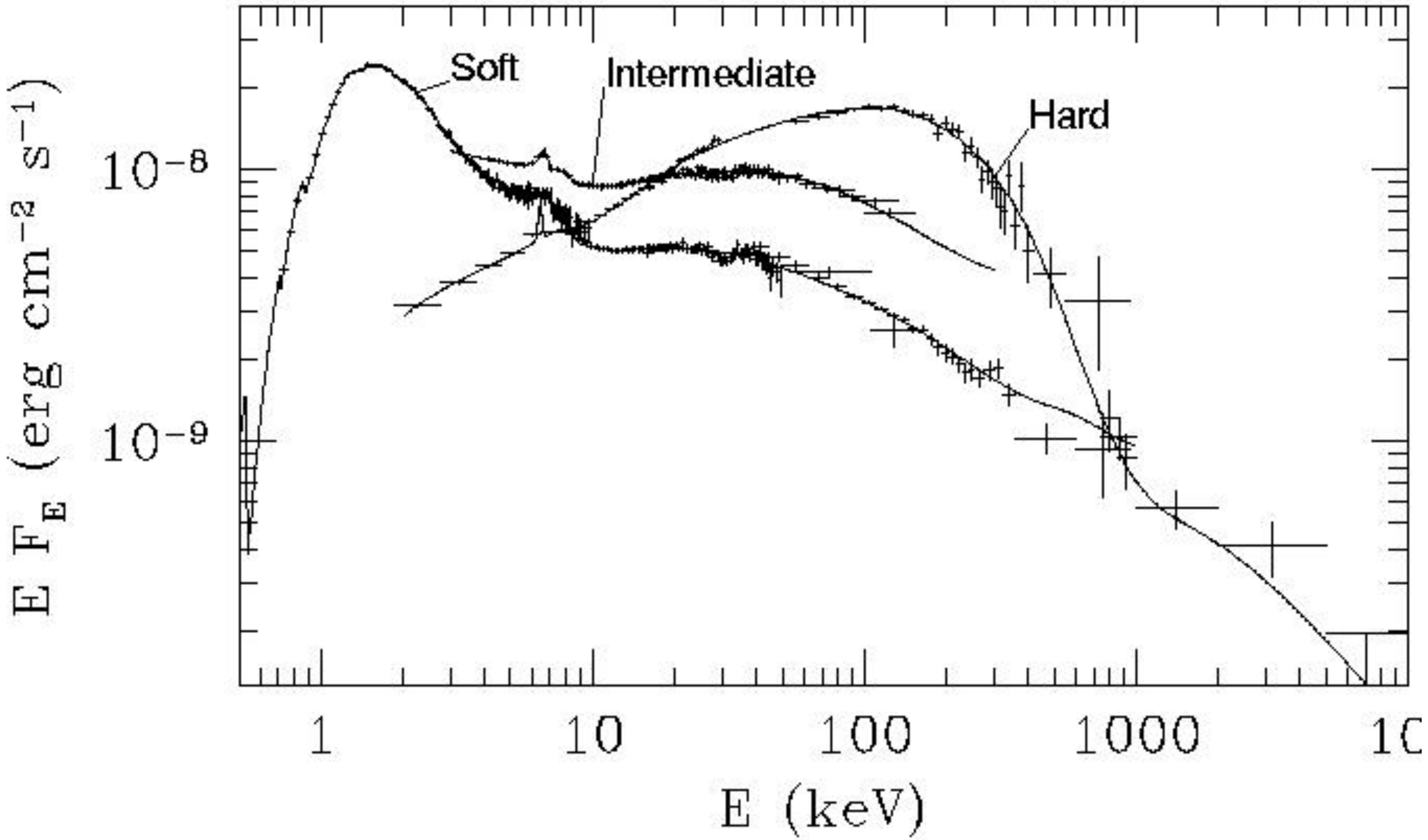
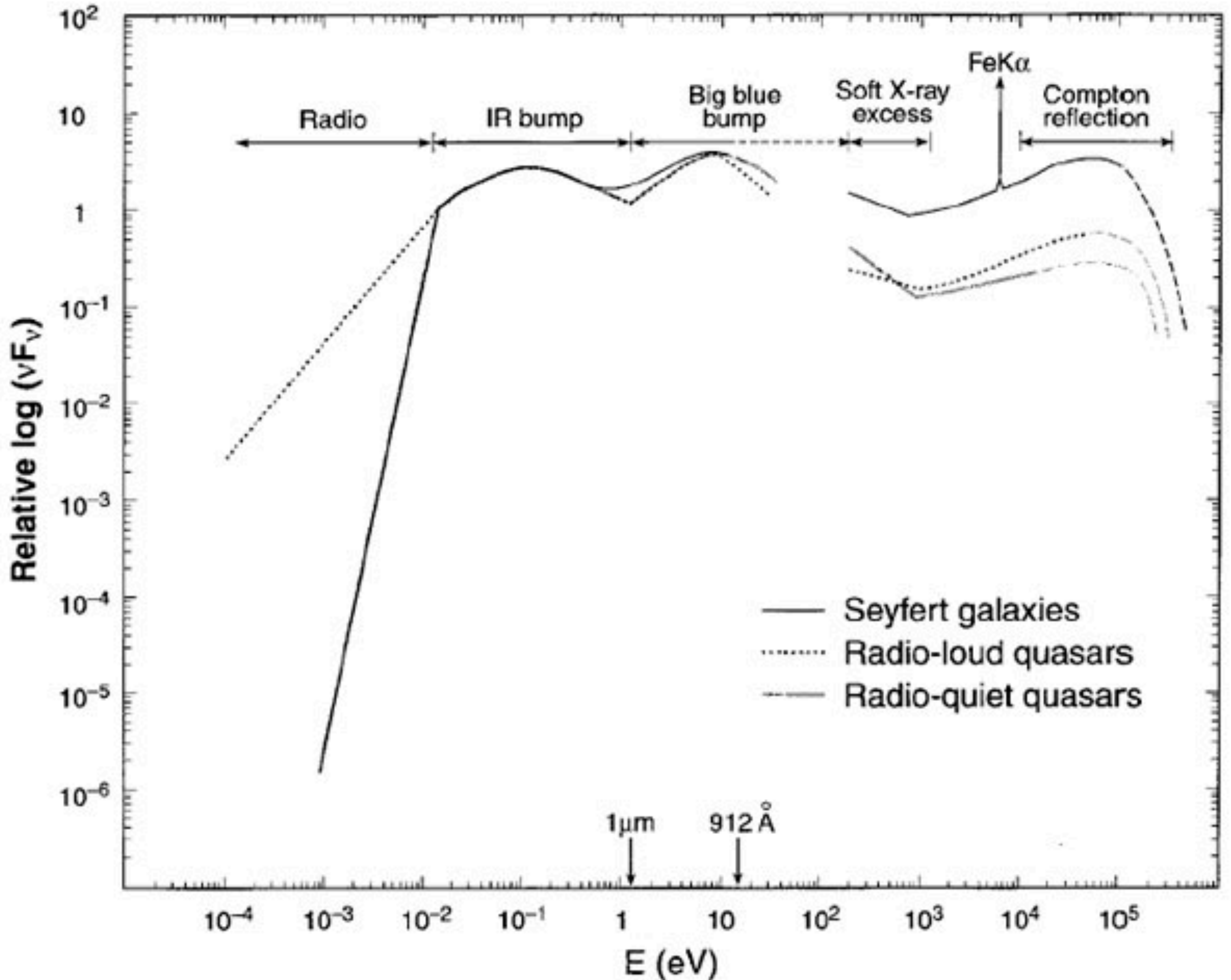
Inverse Compton
Lecture and Tutorial

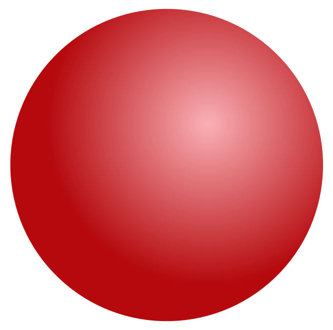

Blackbody
y = 0.15, 1st Order
I_ν [MJy/sr]
ν [1/cm]
Ned Wright


Synchrotron
Inverse-compton
Accretion disc
PKS0227
νF_ν(erg s^{-1} cm^{-2})
eV


Soft
Intermediate
Hard
E F_E (erg cm^{-2} s^{-1})
E (keV)

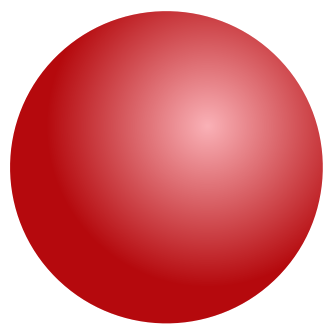

Radio
IR bump
Big blue bump
Soft X-ray excess
FeKα
Compton reflection
Relative log (νFν)
Seyfert galaxies
Radio-loud quasars
Radio-quiet quasars
1μm
912 Å
E (eV)

Four-Vectors

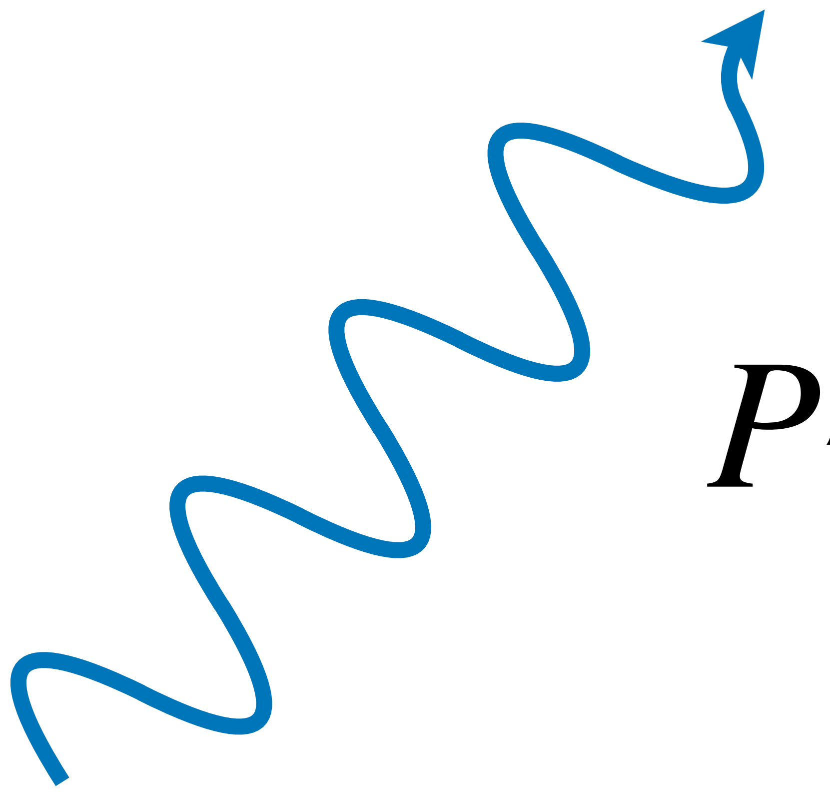


$$x^{\mu} = \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix}$$

Four-Vectors



$$P^{\mu}{}_{\mathrm{e}} = \Gamma m_{\mathrm{e}} c \begin{pmatrix} 1 \\ \beta_{\mathrm{x}} \\ \beta_{\mathrm{y}} \\ \beta_{\mathrm{z}} \end{pmatrix}$$



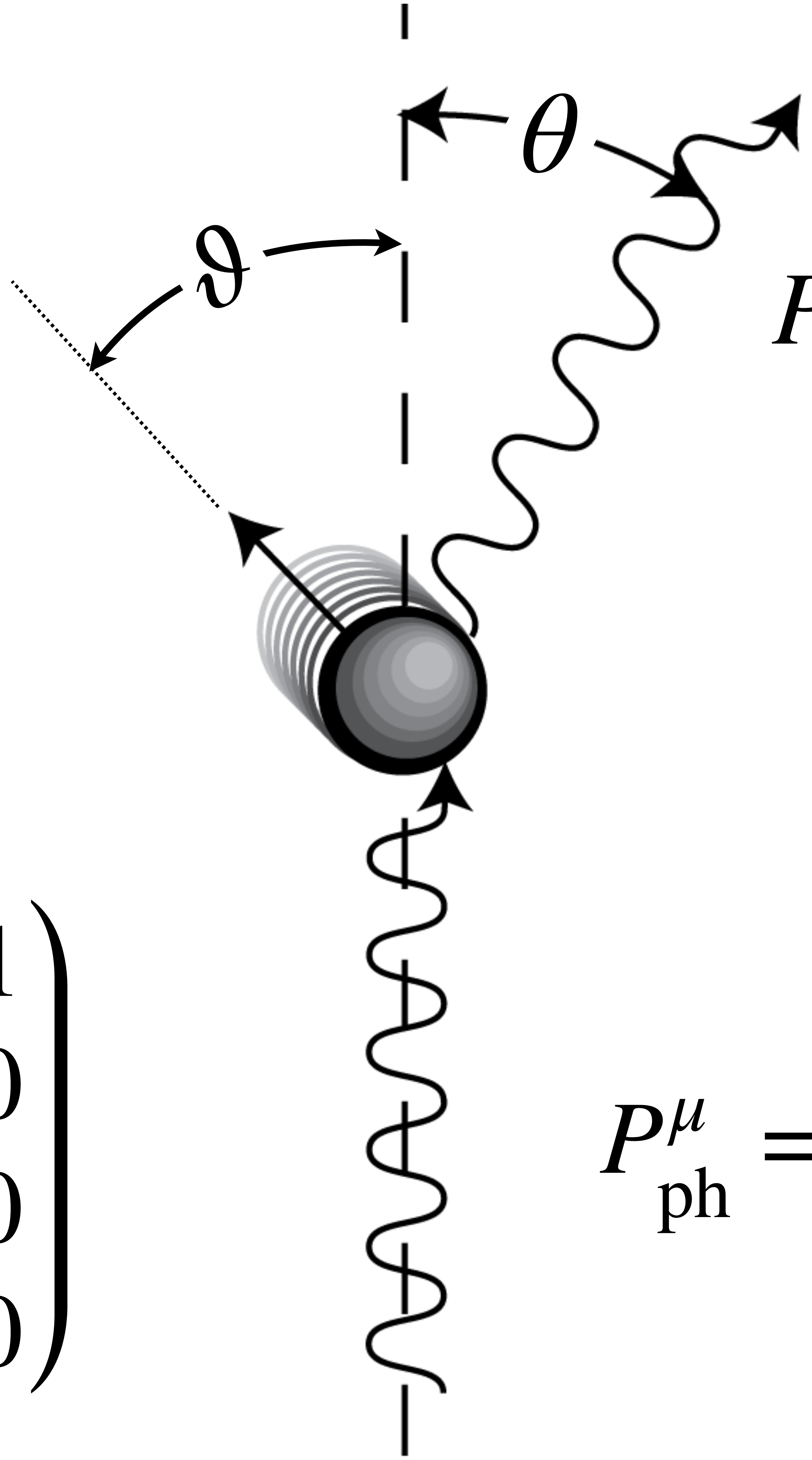
$$P^{\mu}{}_{\mathrm{photon}} = \frac{h\nu}{c} \begin{pmatrix} 1 \\ n_{\mathrm{x}} \\ n_{\mathrm{y}} \\ n_{\mathrm{z}} \end{pmatrix}$$

$$\vec{\beta} \equiv \frac{\vec{v}}{c}$$

$$\Gamma \equiv \sqrt{\frac{1}{1 - \beta^2}}$$

After

$$P^{\mu}_{\mathrm{e},1} = \Gamma m_{\mathrm{e}} c \begin{pmatrix} 1 \\ \beta \cos\vartheta \\ \beta \sin\vartheta \\ 0 \end{pmatrix}$$



$$P^{\mu}_{\mathrm{ph}} = \frac{h\nu_1}{c} \begin{pmatrix} 1 \\ \cos\theta \\ \sin\theta \\ 0 \end{pmatrix}$$

$$P^{\mu}_{\mathrm{e},0} = m_{\mathrm{e}} c \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$P^{\mu}_{\mathrm{ph}} = \frac{h\nu_0}{c} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

Before

Compton Scattering:

- $\lambda_1 = \lambda_0 + \lambda_C \left[1 - \cos(\theta)\right]$ with $\lambda_C = \dfrac{h}{m_e c}$

- Photon ***loses*** energy

- When is recoil loss in energy important?

$$\lambda_0 < \lambda_C \quad \text{or} \quad h\nu > m_e c^2$$

- $\sigma_T = \dfrac{8\pi r_e^2}{3}$ and $\dfrac{d\sigma_T}{d\Omega} = r_e^2 \left[1 + \cos^2(\theta)\right]$ for $\boxed{h\nu \ll m_e c^2}$

Tomson Scattering:

- $\lambda_f = \lambda_0 + \lambda_C\left[1 - \cos(\theta)\right]$ with $\lambda_C = \dfrac{h}{m_e c}$
- Photon ***loses*** energy
- When is recoil loss in energy important?

$$\lambda_0 < \lambda_C \quad \text{or} \quad h\nu > m_e c^2$$

We will take the low energy limit: $h\nu \ll m_e c^2$

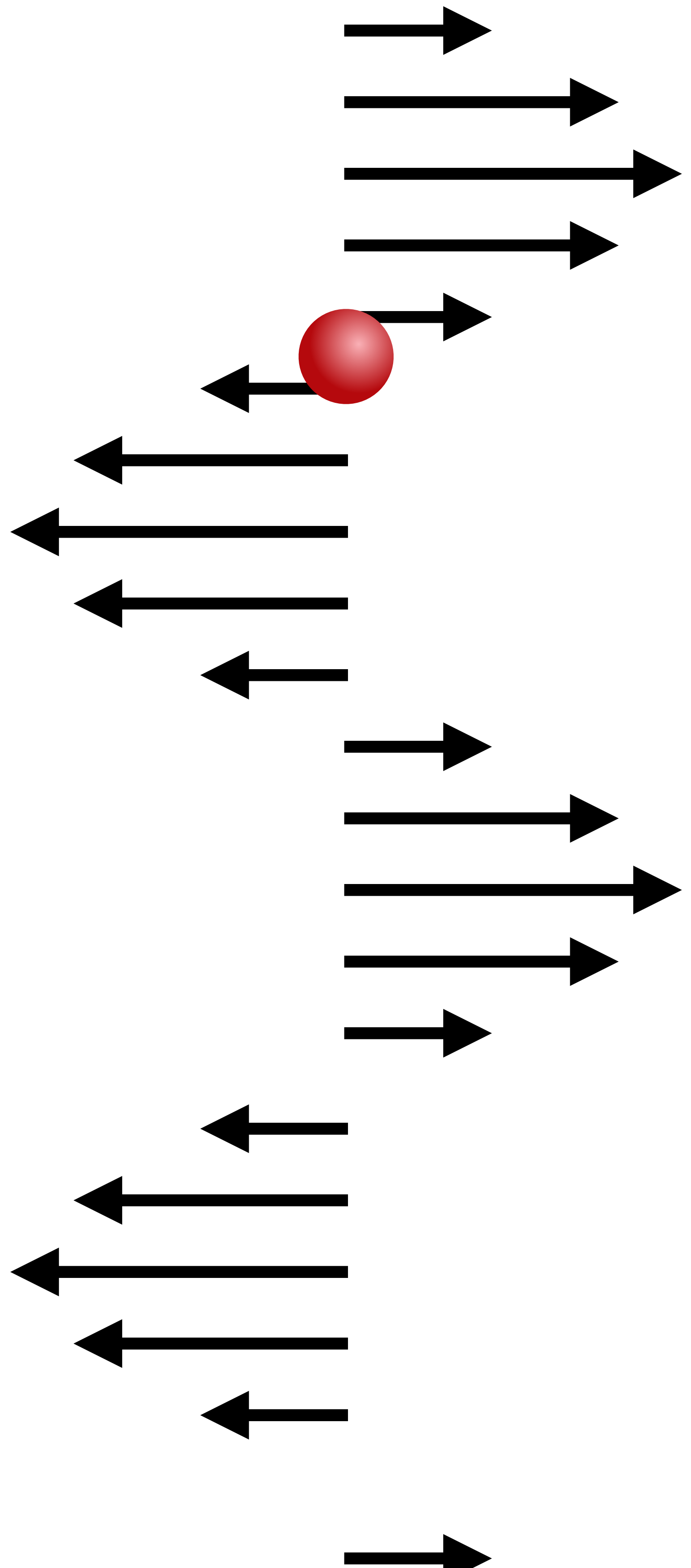
Then:

$$h\nu_f = h\nu_i$$

$$\sigma = \sigma_T$$

- $\sigma_T = \dfrac{8\pi r_e^2}{3}$ and $\dfrac{d\sigma_T}{d\Omega} = r_e^2\left[1 + \cos^2(\theta)\right]$ for $\boxed{h\nu \ll m_e c^2}$

A Photon Meets a Moving Electron...



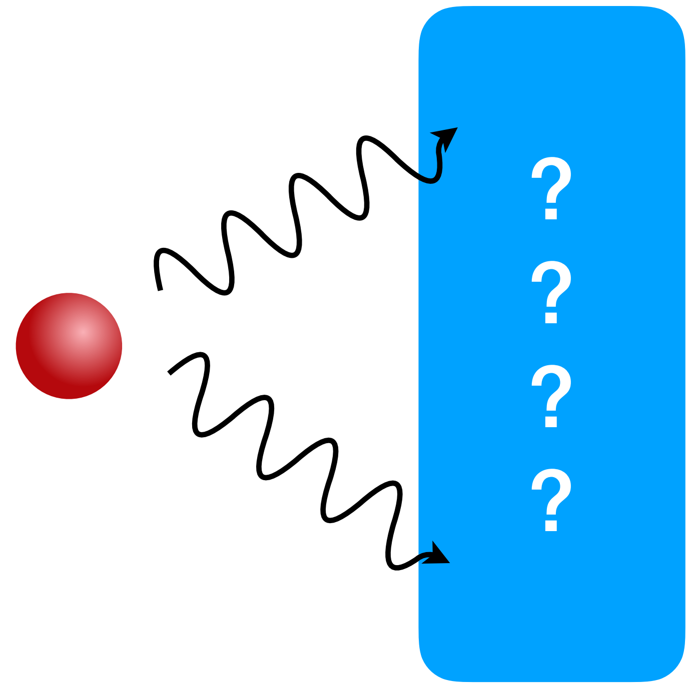
A Photon Meets a Moving Electron...

Step-by-step guide to **inverse Compton scattering:**

1. Transform to electron rest frame

2. Thomson scatter

3. Transform back to observer frame



Compton scattering

Inverse Compton Scattering

Lorentz Transforms

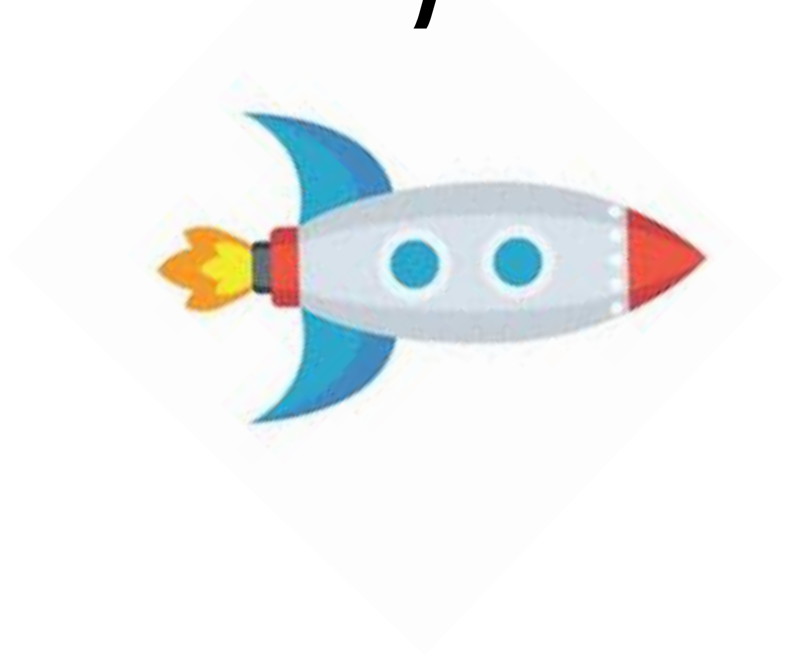
$$\Lambda^{\mu}{}_{\nu}(\overrightarrow{\beta}) = \begin{pmatrix} \Gamma & -\Gamma\beta_{\mathrm{x}} & -\Gamma\beta_{\mathrm{y}} & -\Gamma\beta_{\mathrm{z}} \\ -\Gamma\beta_{\mathrm{x}} & 1 + (\Gamma - 1)\,\beta_{\mathrm{x}}^2/\beta^2 & (\Gamma - 1)\,\beta_{\mathrm{x}}\beta_{\mathrm{y}}/\beta^2 & (\Gamma - 1)\,\beta_{\mathrm{x}}\beta_{\mathrm{z}}/\beta^2 \\ -\Gamma\beta_{\mathrm{y}} & (\Gamma - 1)\,\beta_{\mathrm{x}}\beta_{\mathrm{y}}/\beta^2 & 1 + (\Gamma - 1)\,\beta_{\mathrm{y}}^2/\beta^2 & (\Gamma - 1)\,\beta_{\mathrm{y}}\beta_{\mathrm{z}}/\beta^2 \\ -\Gamma\beta_{\mathrm{z}} & (\Gamma - 1)\,\beta_{\mathrm{x}}\beta_{\mathrm{z}}/\beta^2 & (\Gamma - 1)\,\beta_{\mathrm{y}}\beta_{\mathrm{z}}/\beta^2 & 1 + (\Gamma - 1)\,\beta_{\mathrm{z}}^2/\beta^2 \end{pmatrix}$$

$$P^{\mu'} = \Sigma_{\nu=0}^{3}\Lambda^{\mu}{}_{\nu}P^{\nu} \equiv \Lambda^{\mu}{}_{\nu}P^{\nu}$$

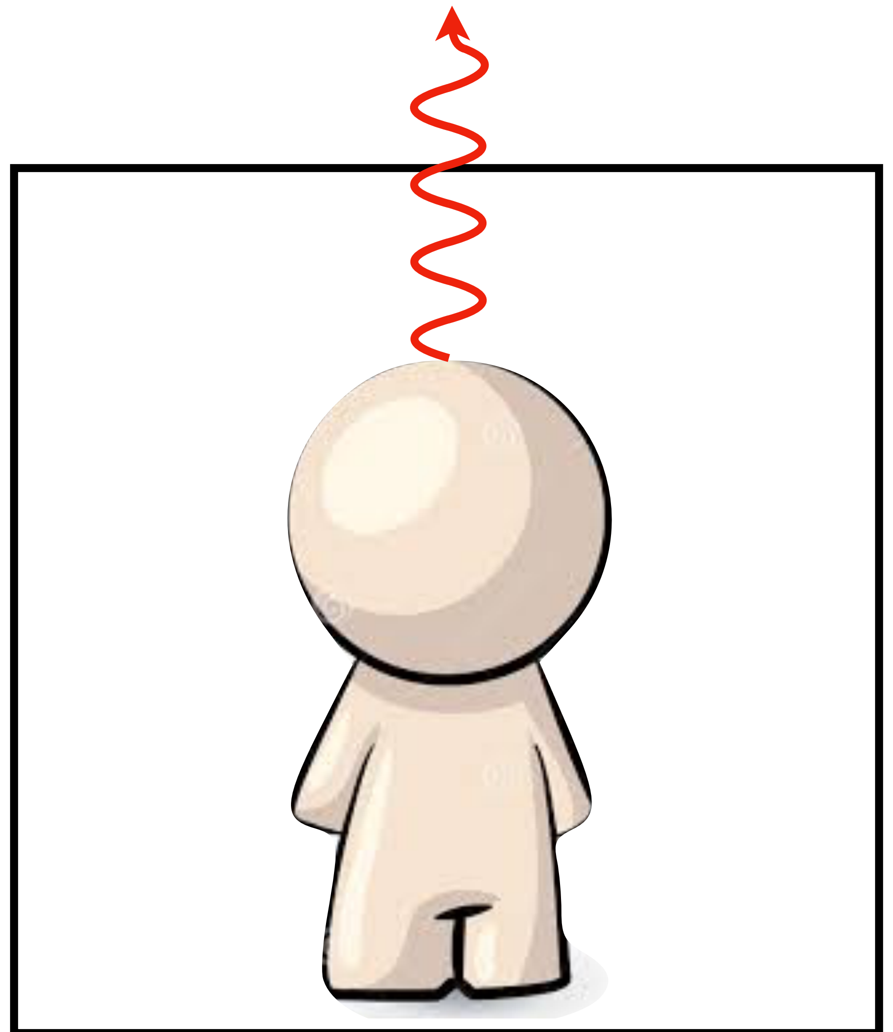
Lorentz Transforms

$$\Lambda^{\mu}{}_{\nu}(\overrightarrow{\beta}) = \begin{pmatrix} \Gamma & -\Gamma\beta & 0 & 0 \\ -\Gamma\beta & \Gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\Gamma\beta$



$$P^{\mu'} = \frac{h\nu_0}{c}\begin{pmatrix} \Gamma \\ -\Gamma\beta \\ 1 \\ 0 \end{pmatrix}$$

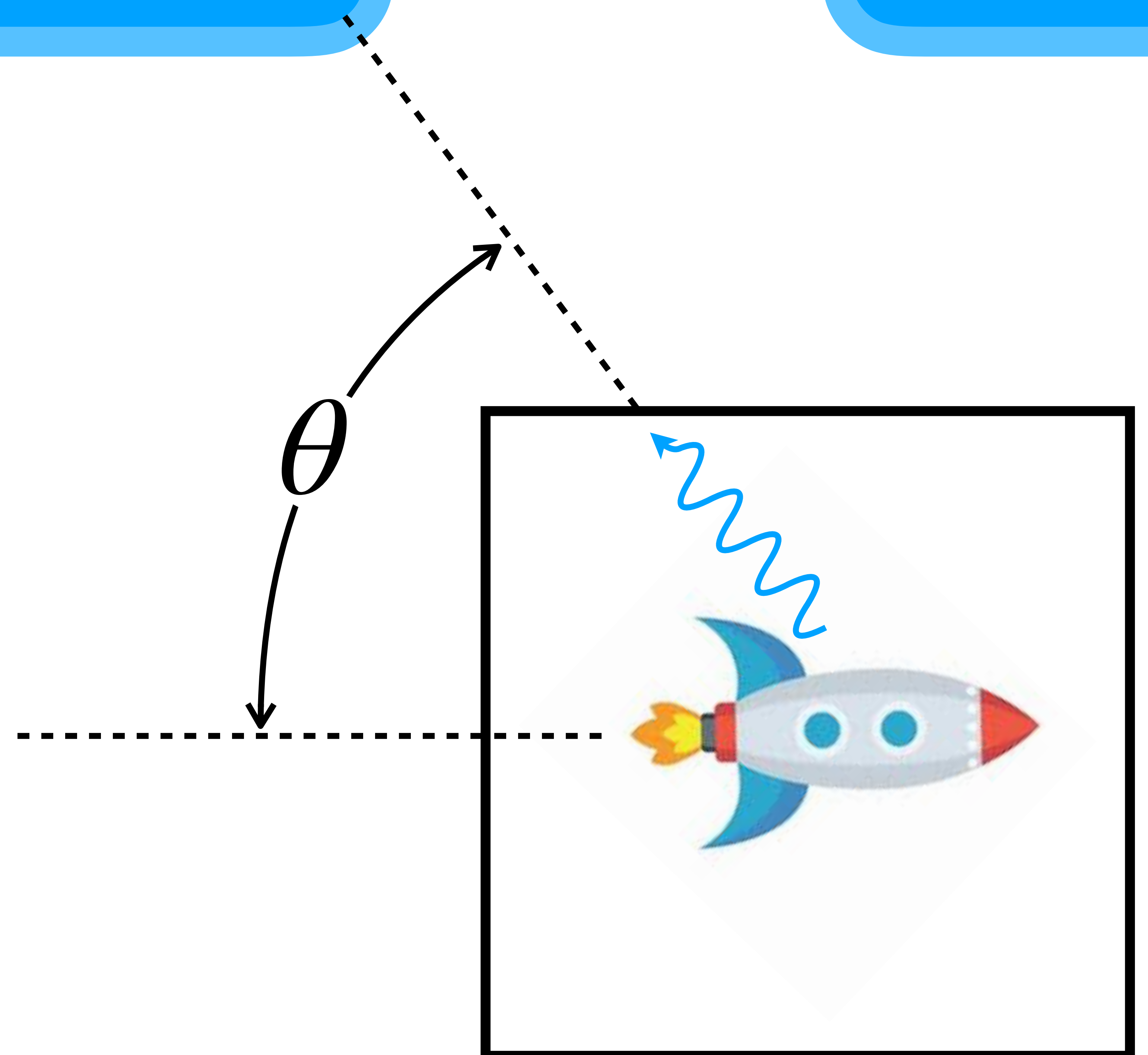


$$P^{\mu} = \frac{h\nu_0}{c}\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

Lorentz Transforms

$$\boxed{h\nu' = \Gamma h\nu_0}$$

$$\boxed{\tan(\theta') = -\frac{1}{\Gamma\beta}}$$



$$P^{\mu'} = \frac{h\nu_0}{c}\begin{pmatrix}\Gamma \\ -\Gamma\beta \\ 1 \\ 0\end{pmatrix}$$

A Photon Meets a Moving Electron…

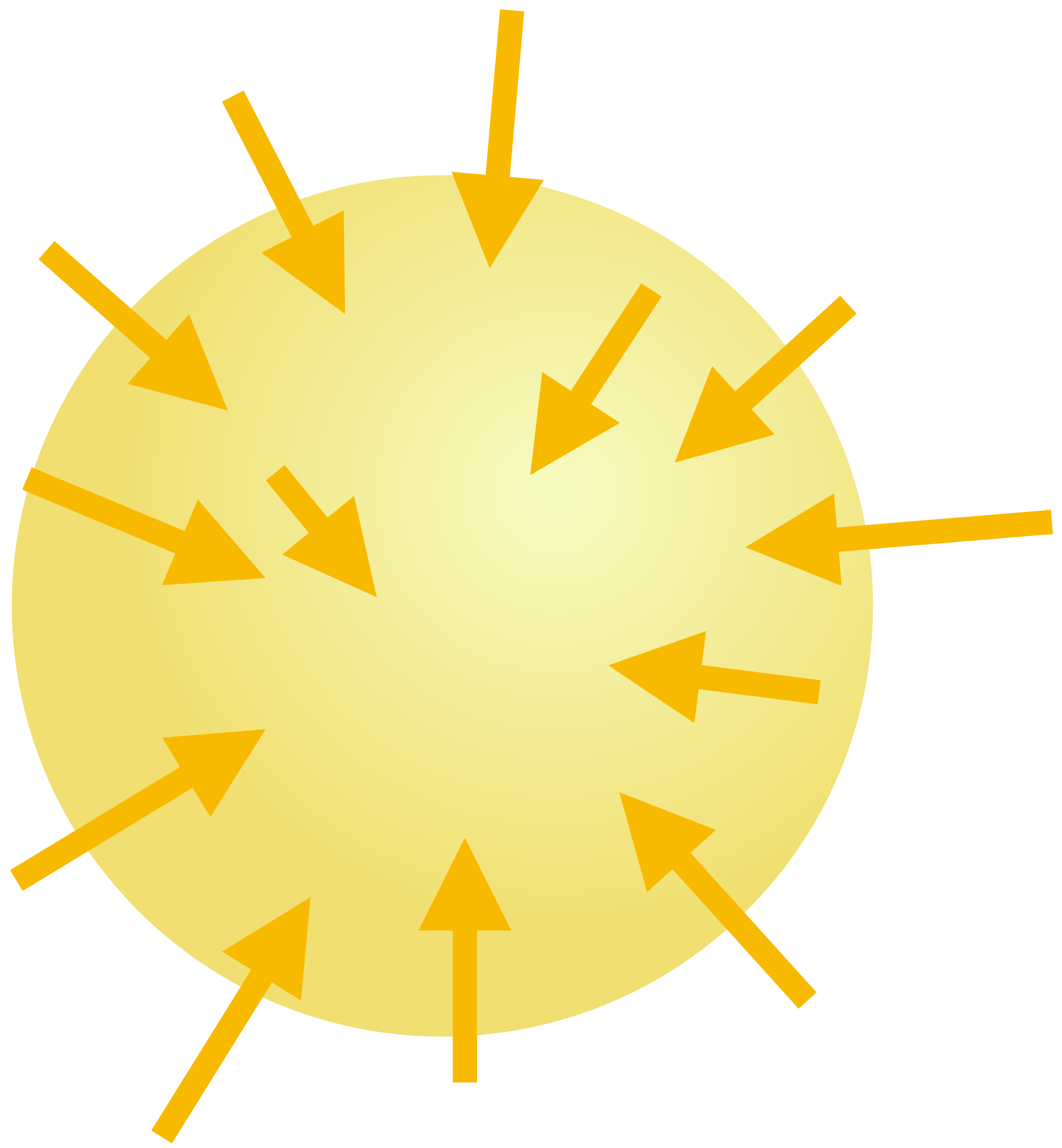
Step-by-step guide to **inverse Compton scattering:**

1. Transform to electron rest frame

2. Thomson scatter

3. Transform back to observer frame

$$h\nu_1 = \Gamma^2\left(1-\beta\cos\vartheta\right)\left(1+\beta\cos\vartheta'\right)h\nu_0$$



$$\boxed{\langle\Delta h\nu\rangle = \langle h\nu_1 - h\nu_0\rangle = \frac{4}{3}\Gamma^2\beta^2 h\nu_0}$$

$$h\nu_1 = \Gamma^2 \left(1 - \beta \cos\vartheta\right) \left(1 + \beta \cos\vartheta'\right) h\nu_0$$

Caution:
There are some subtleties in this step…

$$\langle \Delta h\nu \rangle = \langle h\nu_1 - h\nu_0 \rangle = \frac{4}{3}\Gamma^2\beta^2 h\nu_0$$

Power per Particle

$$P = \underbrace{\left(\frac{4}{3}\langle h\nu_0\rangle\Gamma^2\beta^2\right)}_{\text{Average energy gain}} \times \left(\sigma_{\mathrm{T}} \times \underbrace{n_{\mathrm{photons}}c}_{\text{Photon flux density}}\right)$$

Power per Particle

$$P = \underbrace{\left(\frac{4}{3}\langle h\nu_0\rangle\Gamma^2\beta^2\right)}_{\text{Average energy gain}} \times \left(\overbrace{\sigma_{\mathrm{T}} \times n_{\mathrm{photons}} c}^{\text{Rate of scattering}}\right)$$

Power per Particle

$$P = \left(\frac{4}{3} \langle h\nu_0 \rangle \Gamma^2 \beta^2 \right) \times \left(\sigma_{\mathrm{T}} \times n_{\mathrm{photons}} c \right)$$

$$P = \frac{4}{3} \sigma_{\mathrm{T}} \Gamma^2 \beta^2 U_{\mathrm{rad}}$$

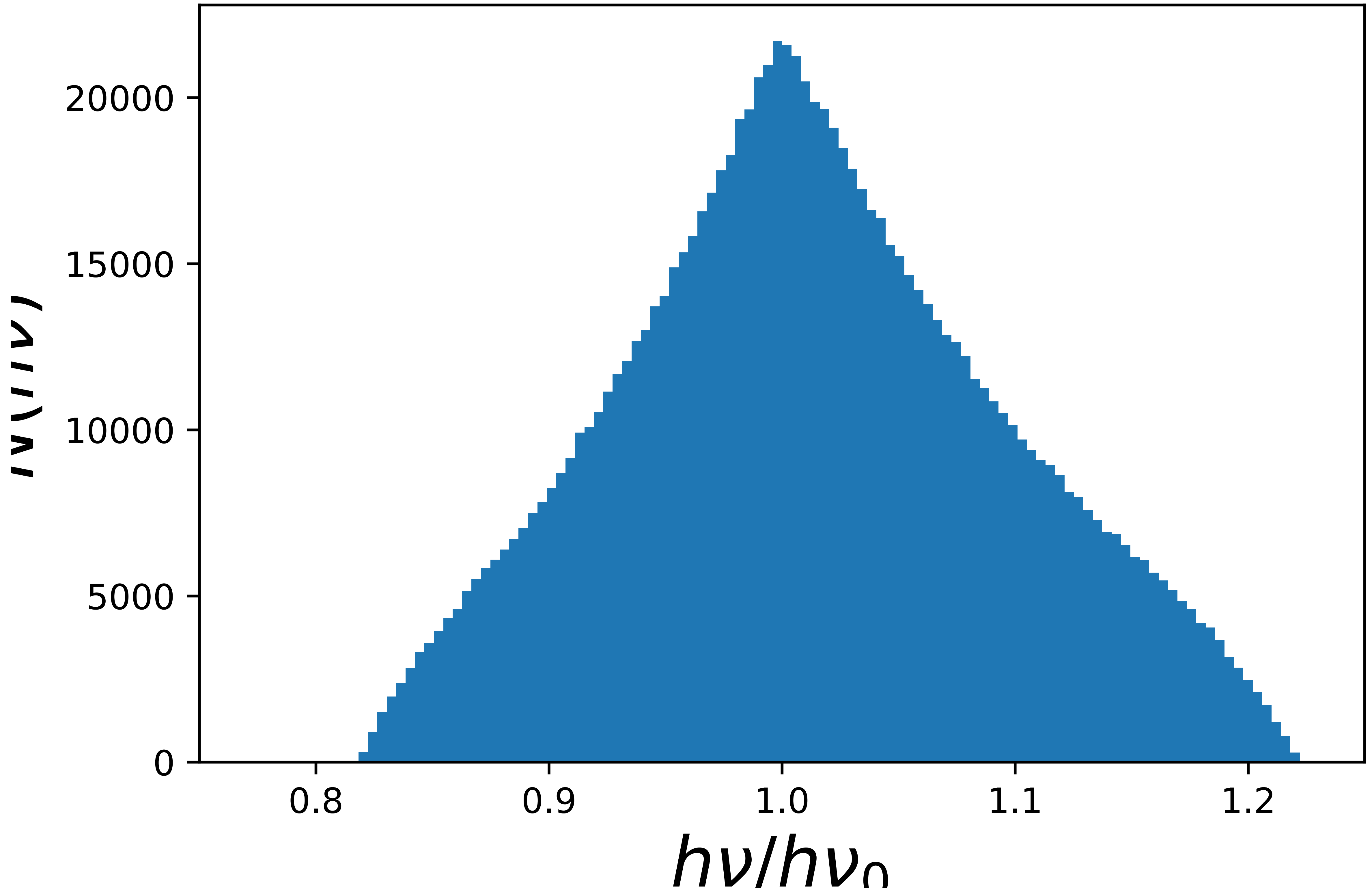
Power per Particle

$$P = \left(\frac{4}{3} \langle h\nu_0 \rangle \Gamma^2 \beta^2 \right) \times \left(\sigma_{\mathrm{T}} \times n_{\mathrm{photons}} c \right)$$

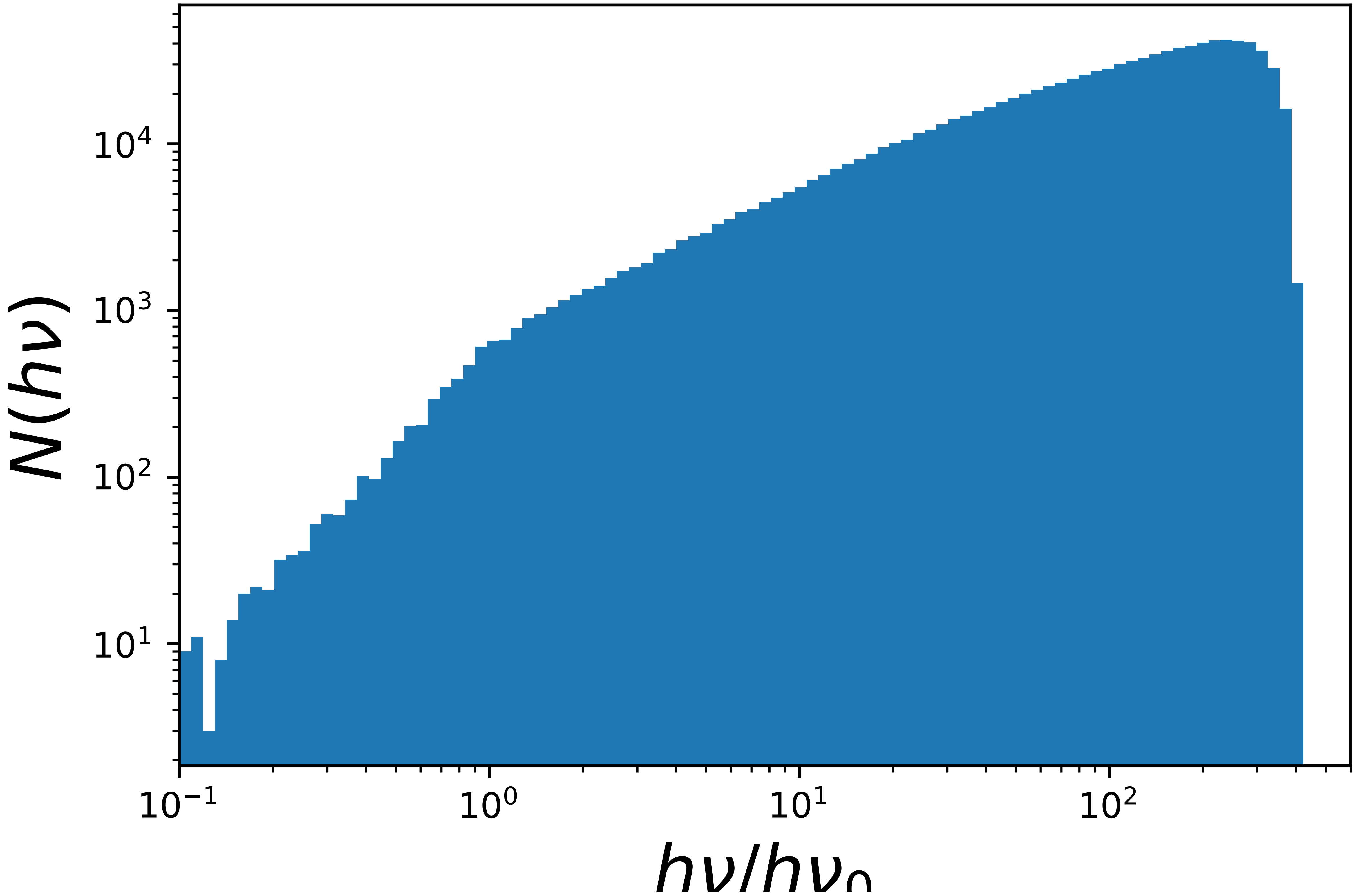
$$\boxed{P = \frac{4}{3} \sigma_{\mathrm{T}} \Gamma^2 \beta^2 \left(U_{\mathrm{rad}} + U_{\mathrm{B}} \right)}$$

Single-Scattering Kernel

$v = 0.1c$

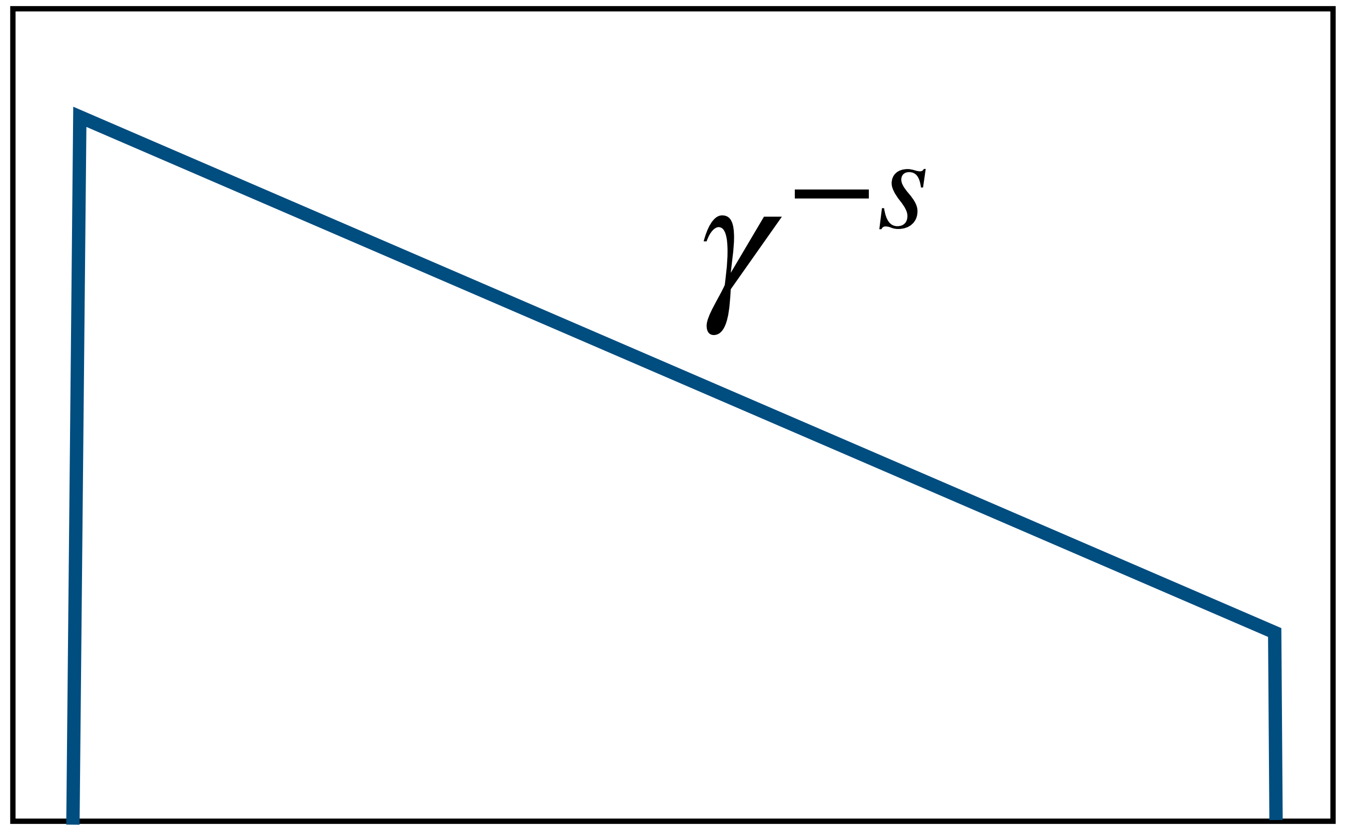


$\Gamma = 10$

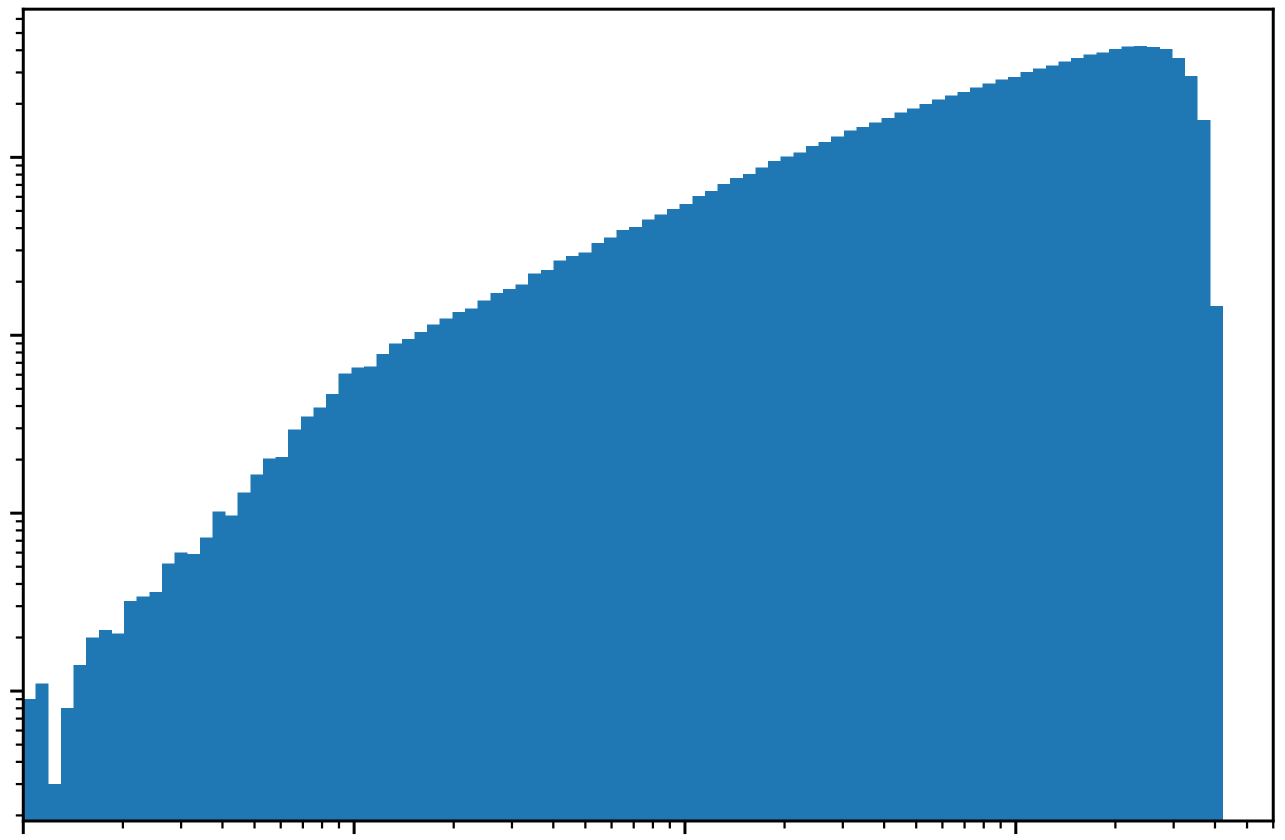


Powerlaw Electron Distribution

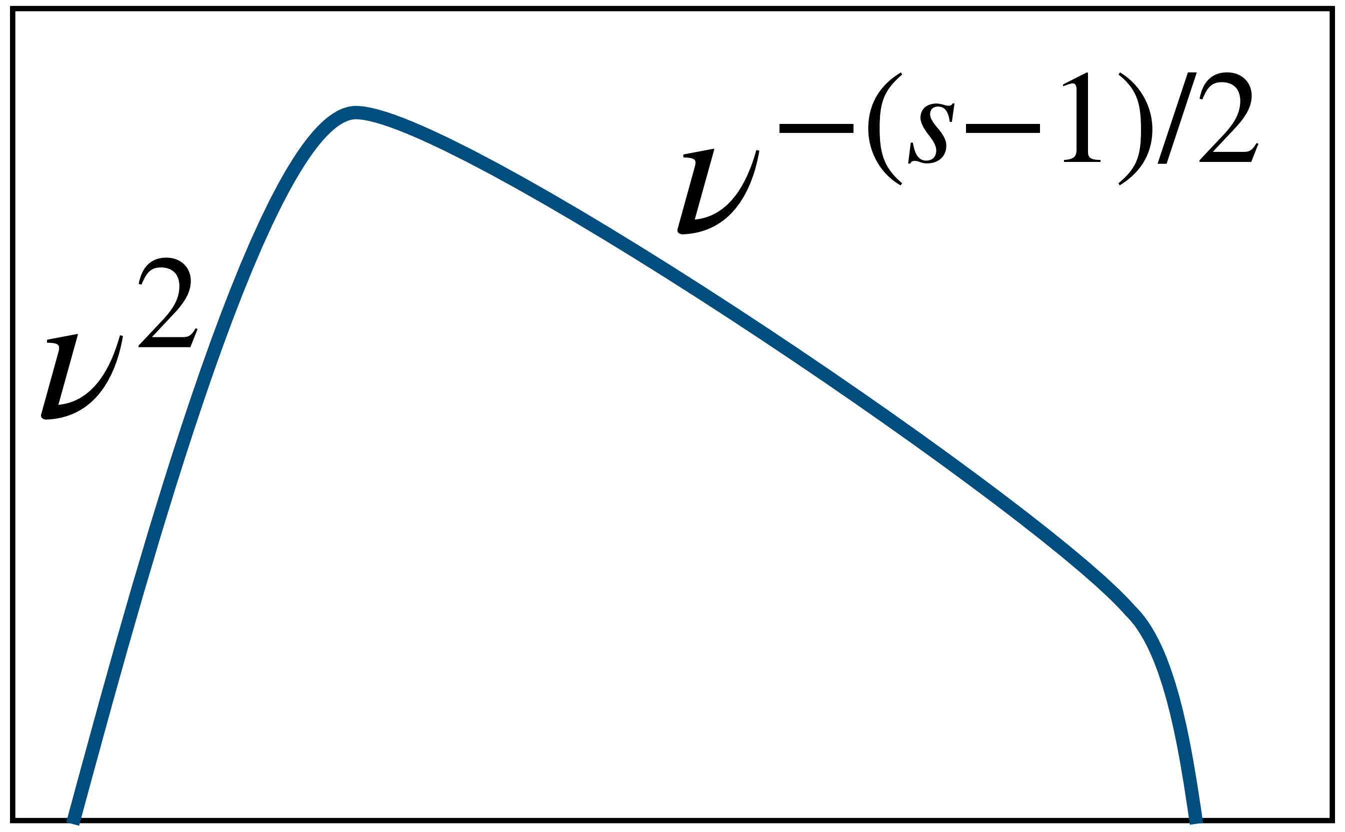
Electron distribution



Compton kernel

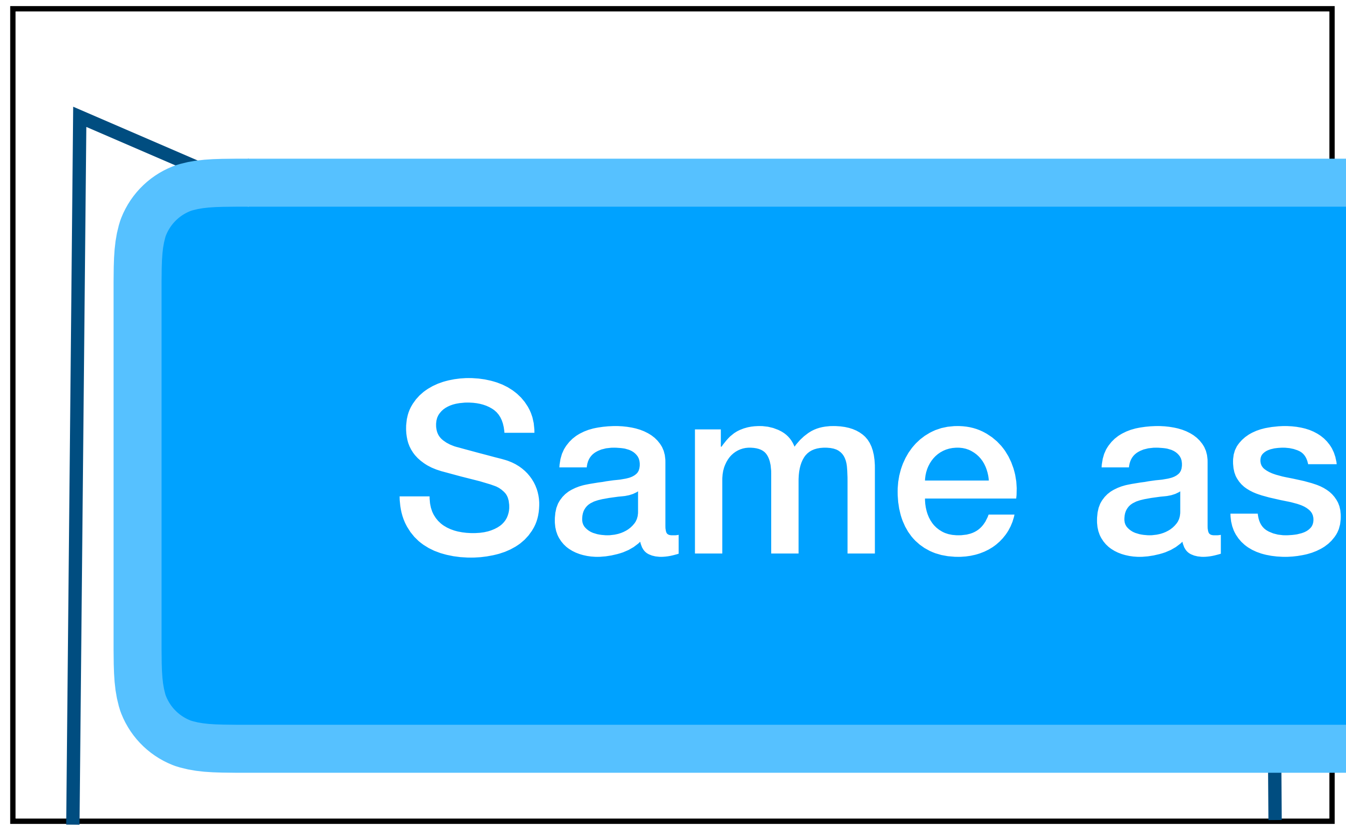


Inverse Compton Spectrum



Powerlaw Electron Distribution

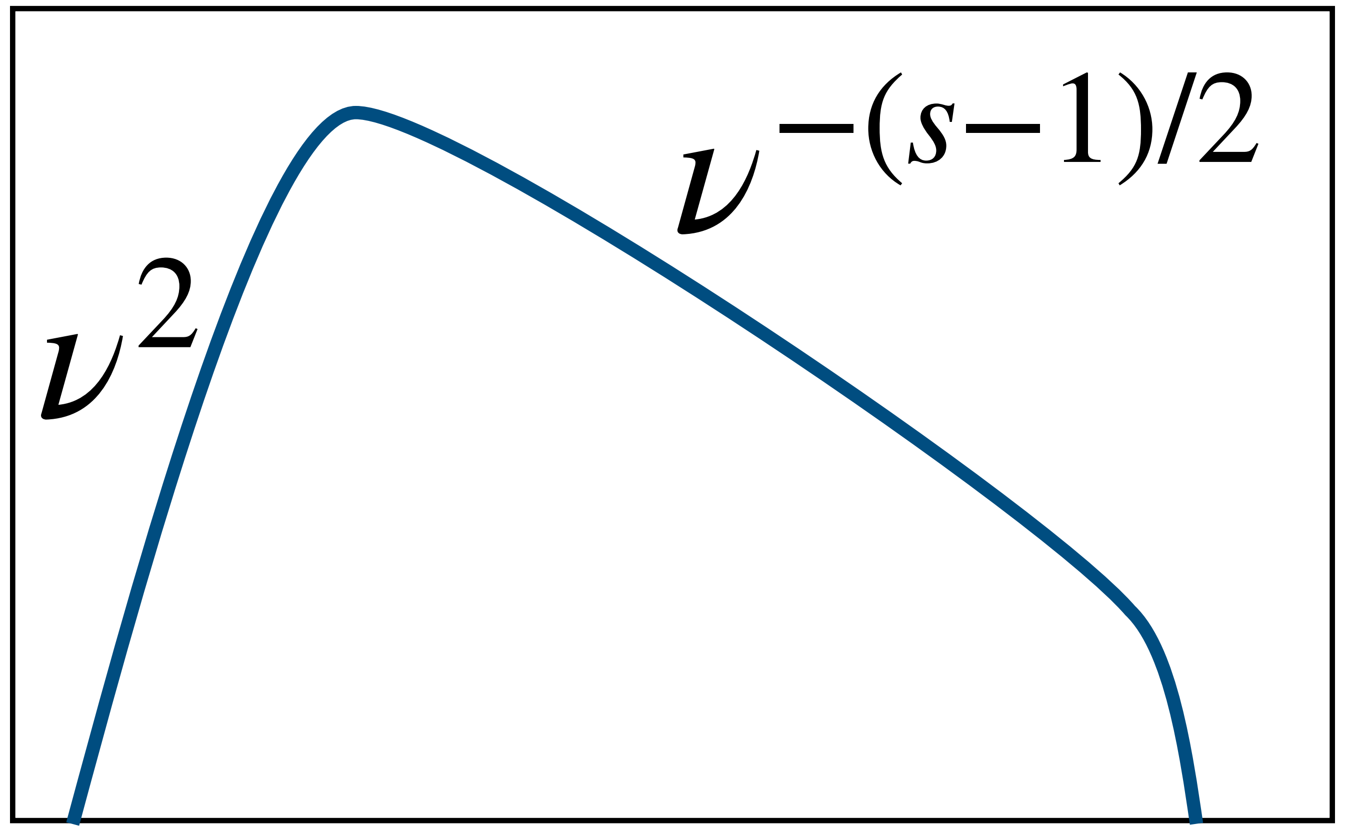
Electron distribution



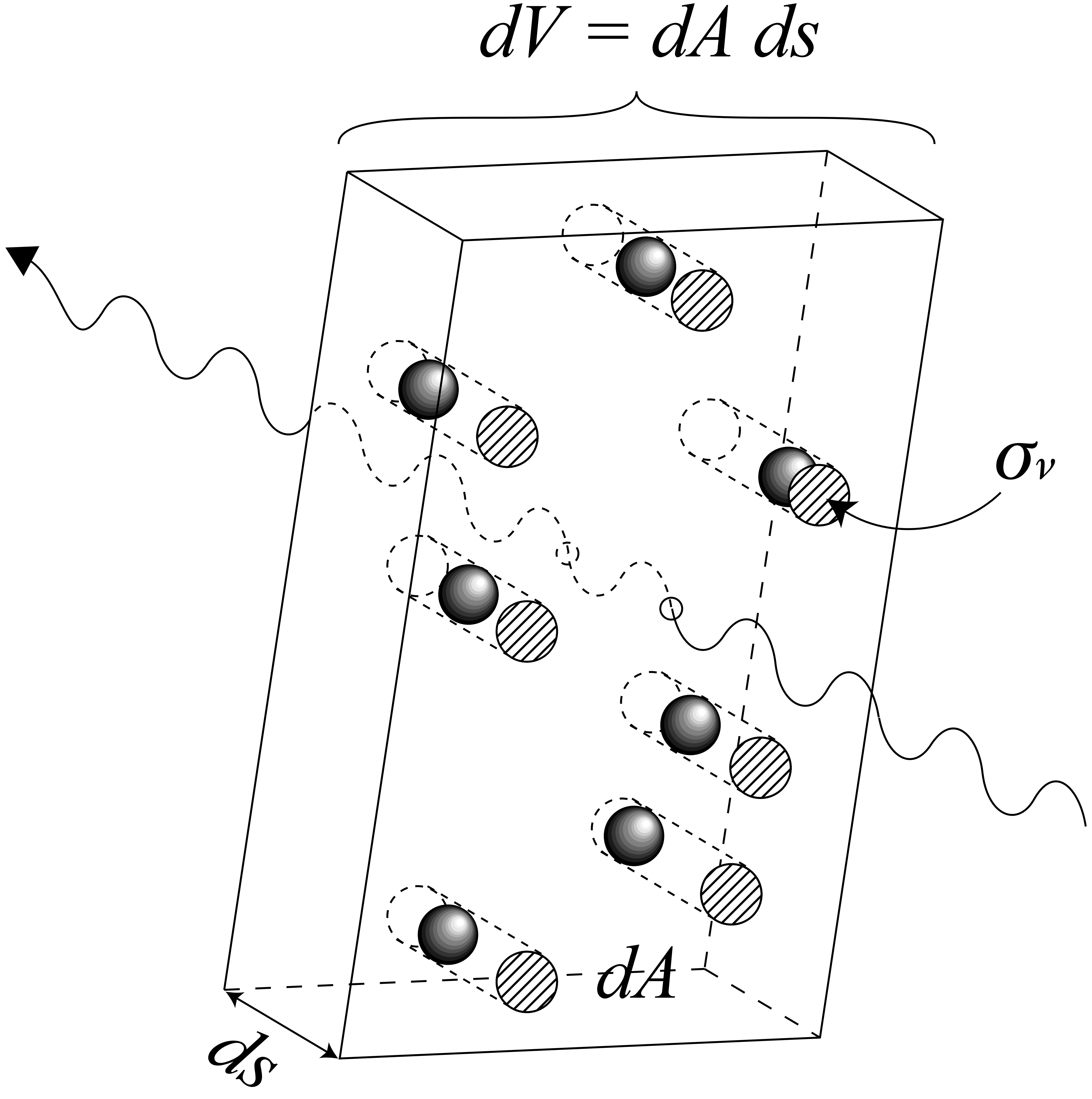
Compton kernel

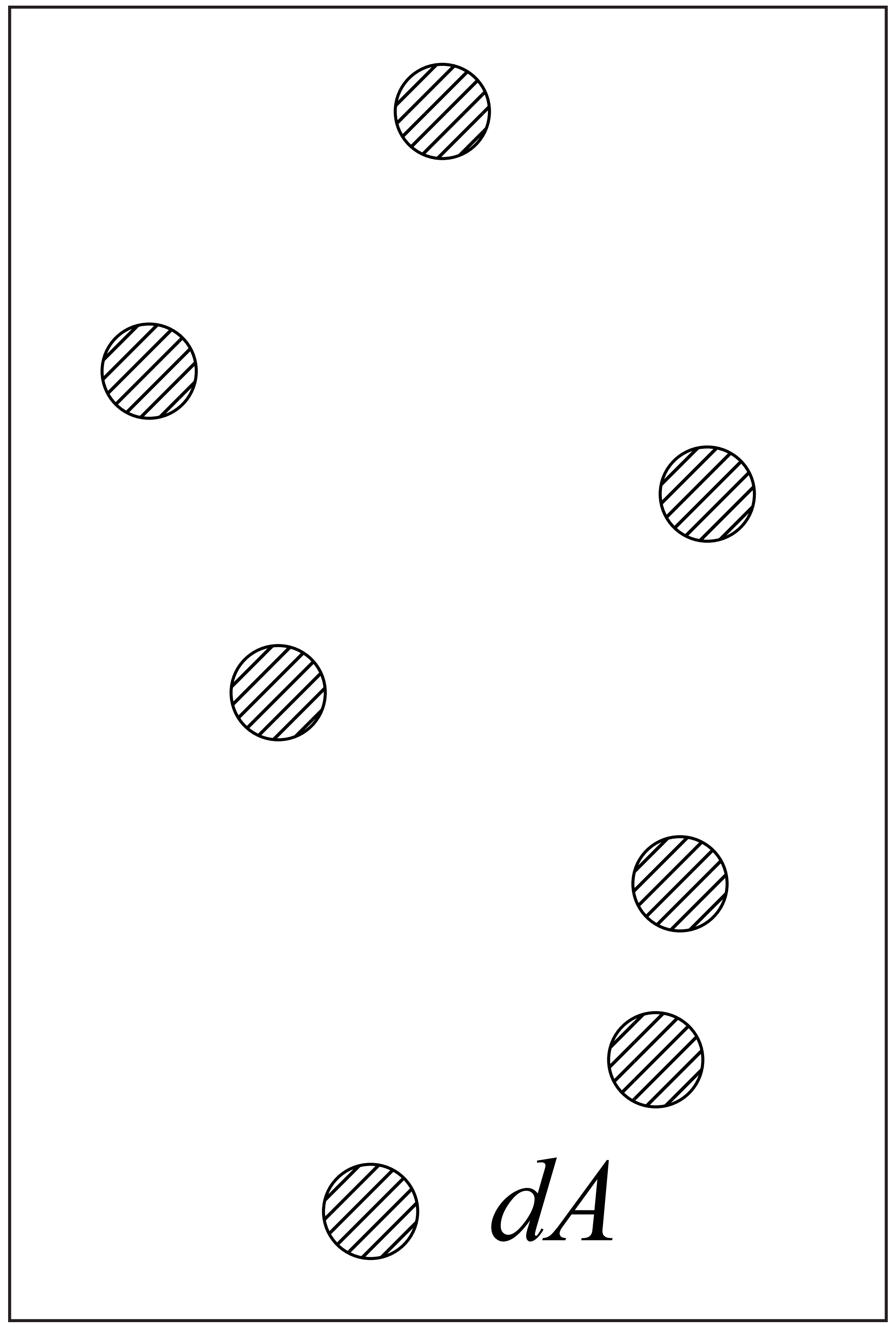
Same as synchrotron!

Inverse Compton Spectrum



Radiative Transfer



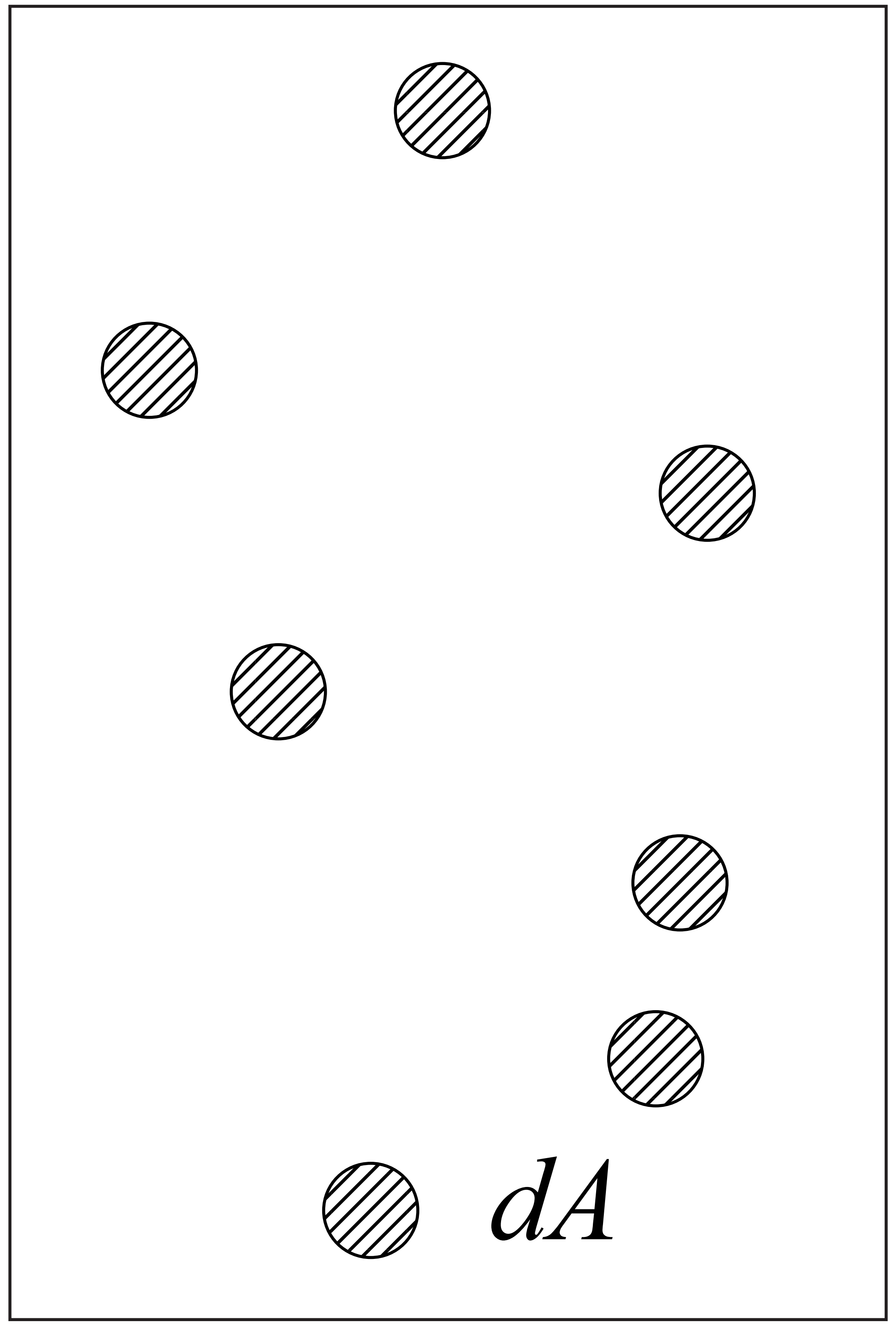


area dA as seen by I_ν

$$dA = N\sigma ds$$

$$N = ndV = nAds$$

$$f_{\mathrm{blocked}} = \frac{dA}{A} = \frac{n\sigma Ads}{A} = n\sigma ds$$



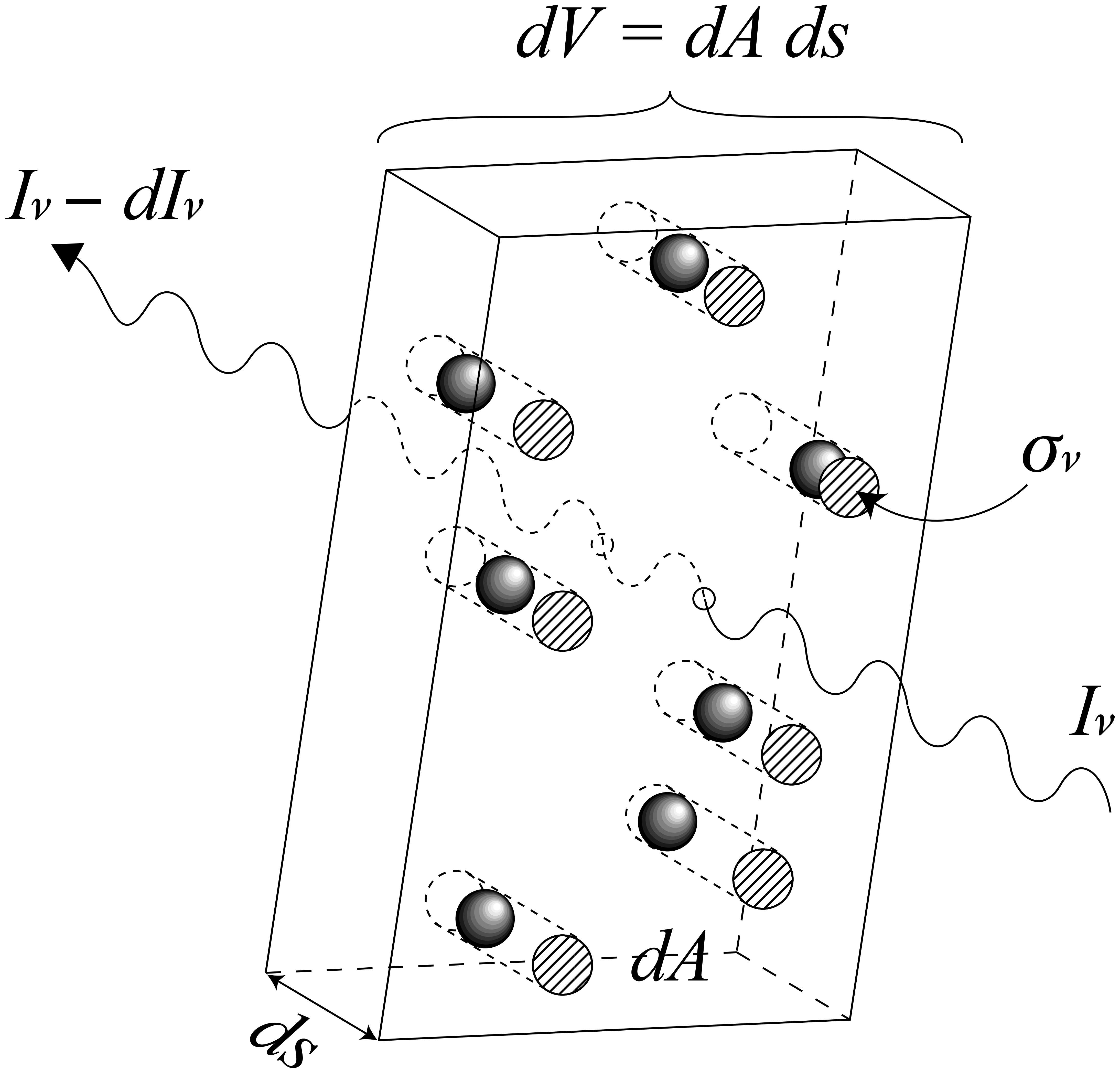
$$dI_{\mathrm{blocked}} = -If_{\mathrm{blocked}} = -I\,n\sigma ds = -Id\tau$$

$$d\tau \equiv n\sigma ds$$

$$\frac{dI}{d\tau} = -I$$

$$I = I_0 e^{-\int d\tau} = I_0 e^{-\tau}$$

$$\Delta I = I_0(1 - e^{-\tau})$$



Quantifying Energy Gain

- How much energy gain per photon?
- Compton y-parameter tells you how important inverse Compton scattering is for a spectrum
- Depends on the electron velocity distribution and optical depth

$$y \equiv \frac{\langle h\nu_f - h\nu_0 \rangle}{\langle h\nu_0 \rangle}$$

Small optical depth

a) Energy gain proportion to energy

b) Fixed probability P of repeating

ALWAYS produces power law

Small optical depth

- What fraction of particles scatter at least once?

$$1 - e^{-\tau} \approx 1 - (1 - \tau) = \tau$$

- What fraction of particles scatter at least k times?

$$f(> k) = \tau^k$$

- How much energy does a particle have after k scatterings?

$$h\nu_{\mathrm{k}} = h\nu_0 \left(\frac{4}{3}\gamma^2\beta^2\right)^k = h\nu_0 \xi^k$$

Small optical depth

- How many scatterings has a photon of energy h\nu undergone?

$$k = \ln\left(\frac{\nu}{\nu_0\xi}\right)$$

- Fraction of electrons with energy > hnu:

$$f(>\nu) = \tau^k = e^{\ln\tau\cdot\ln(\nu/\nu_0\xi)} = \left(\frac{\nu}{\nu_0}\right)^{\ln(\tau)/\ln\xi} = \left(\frac{\nu}{\nu_0}\right)^{\alpha}$$

- How much energy does a particle have after k scatterings?

$$P_\nu = \dot{n}_0 h \frac{d\left[f(>\nu)h\nu\right]}{d\nu} = \dot{n}_0 h\,(1+\alpha)\left(\frac{\nu}{\nu_0}\right)^{\alpha}$$

Small optical depth

- How many scatterings has a photon of energy h\nu undergone?

$k =$

- Fraction of

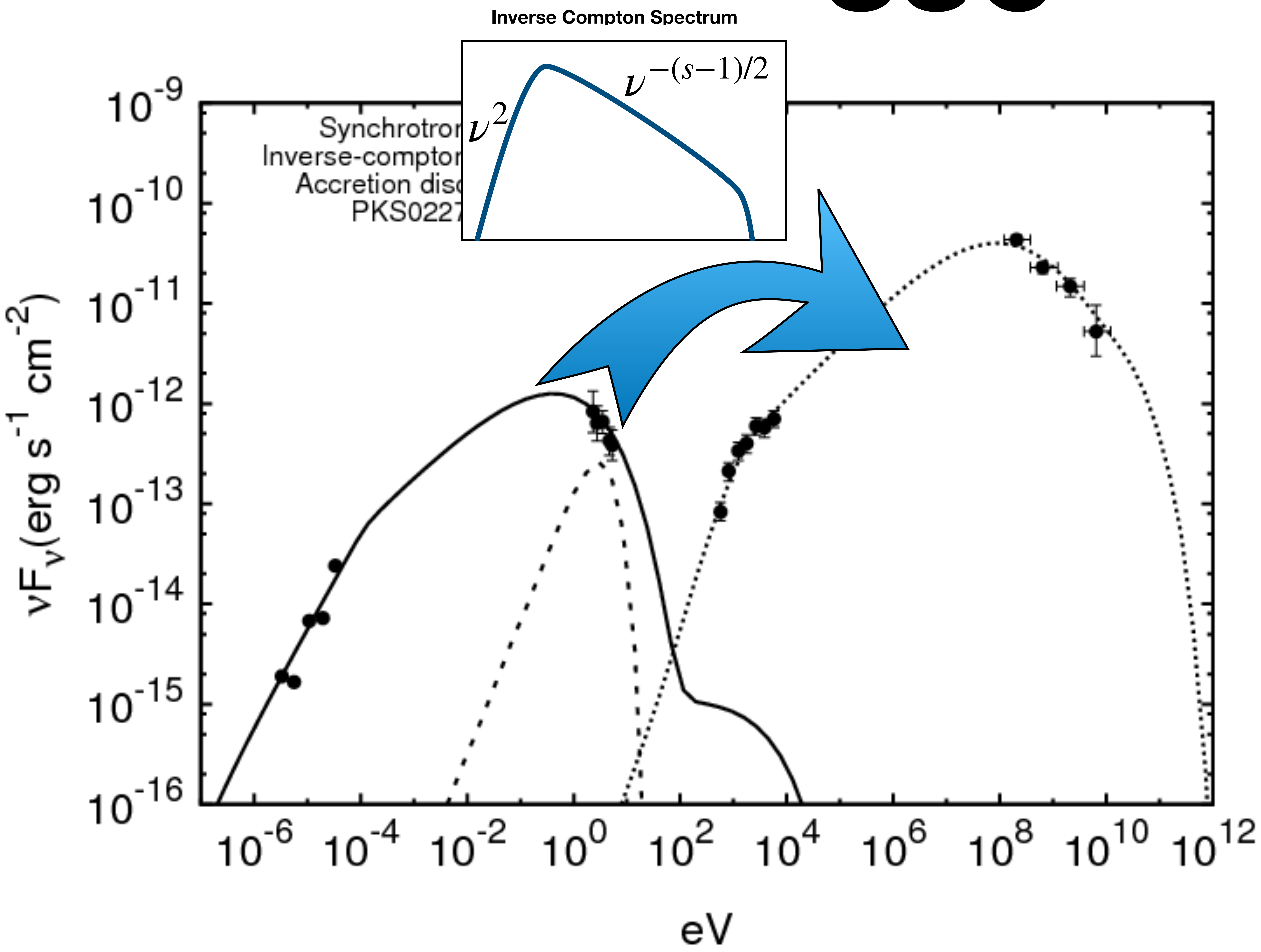
$f(>$

- How much

$$P_\nu = \dot{n}_0 h \frac{d\left[f(> \nu) h\nu\right]}{d\nu} = \dot{n}_0 h (1+\alpha)\left(\frac{\nu}{\nu_0}\right)^\alpha$$

$$\alpha = \frac{\ln(\tau)}{\ln(4/3\langle\gamma\rangle^2)}$$

SSC



Inverse Compton Spectrum
ν^2
$\nu^{-(s-1)/2}$
Synchrotron
Inverse-compton
Accretion disc
PKS0227
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
10^{-15}
10^{-16}
νF_ν(erg s^{-1} cm^{-2})
10^{-6} 10^{-4} 10^{-2} 10^{0} 10^{2} 10^{4} 10^{6} 10^{8} 10^{10} 10^{12}
eV

Inverse Compton Catastrophe

What happens when $U_{\mathrm{rad}} > U_{\mathrm{B}}$?

1. Self-Compton amplification leads to diverging P

2. What will happen in real life?

$$P = \frac{4}{3}\sigma_{\mathrm{T}}\Gamma^2\beta^2\left(U_{\mathrm{rad}} + U_{\mathrm{B}}\right)$$

Inverse Compton

- Ubiquitous radiation process in HE
- Can self-generate power law spectra
- Responsible for hard emission in AGN and XRBs
- Energy gain is proportional to energy
- Compton y-parameter tells you about importance of IC and spectrum

From Posterior to Monte-Carlo