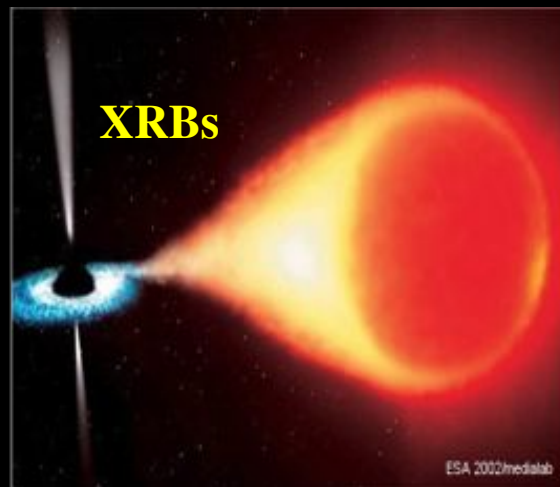


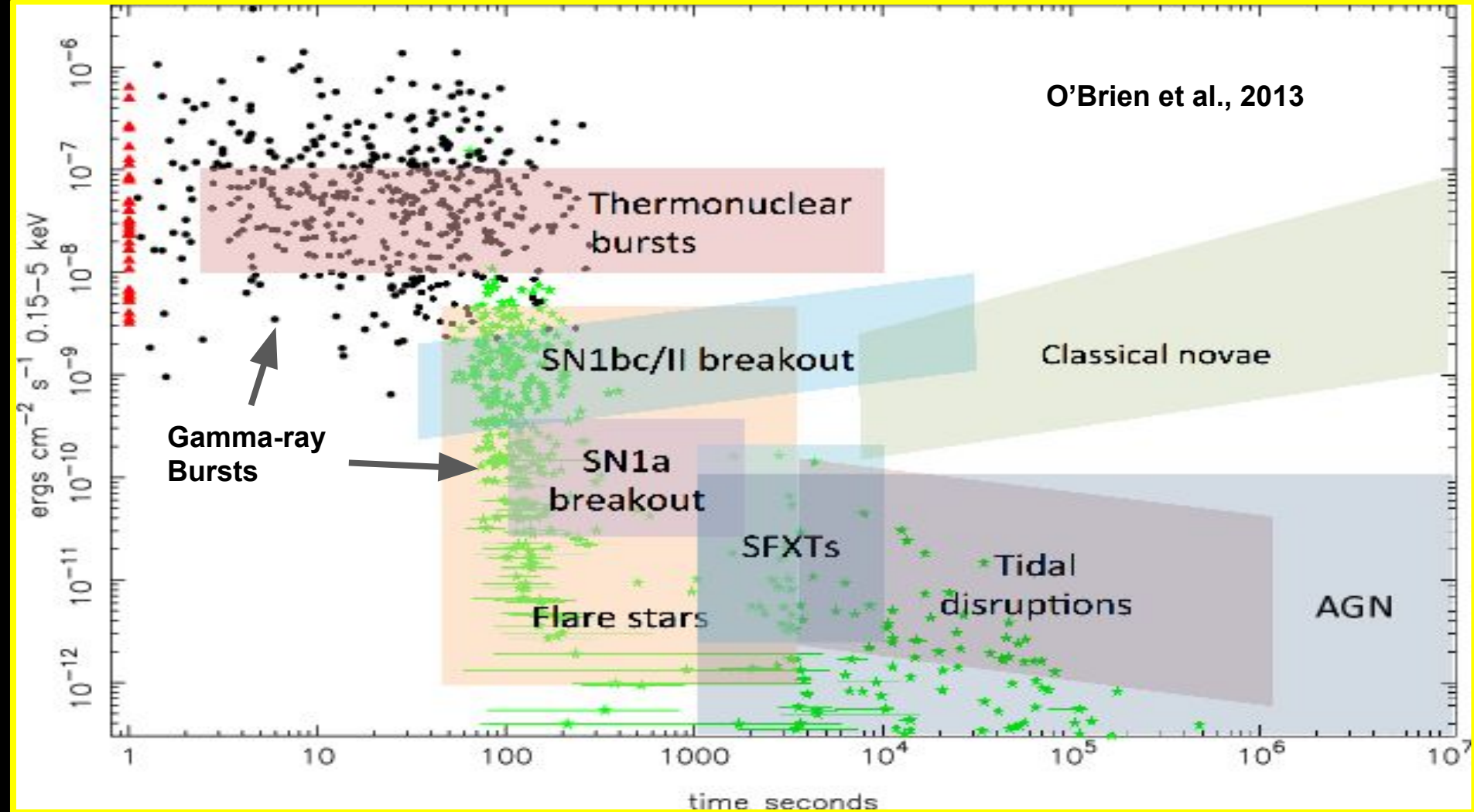
Astrophysical Timescales

The Universe is Transient





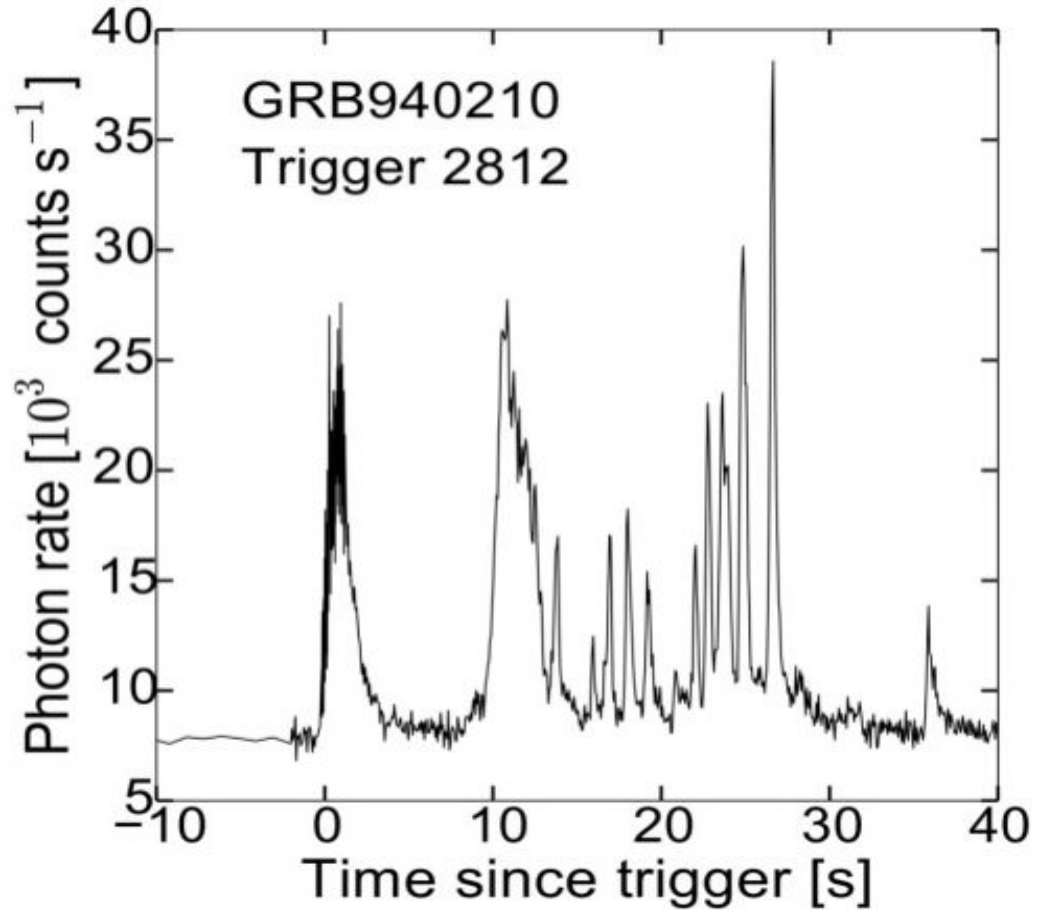
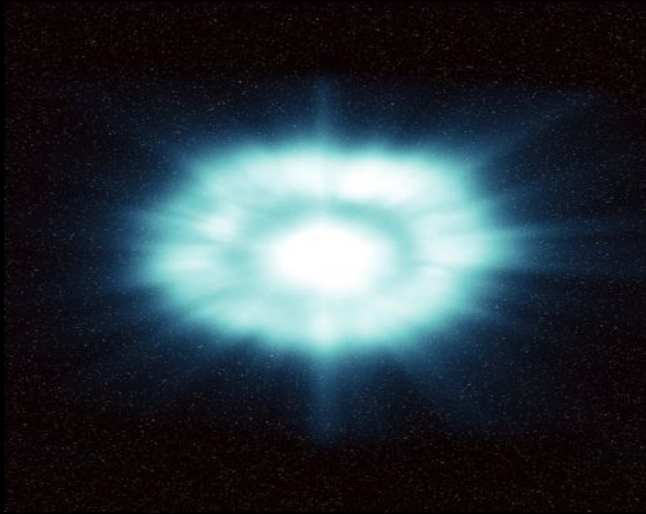
O'Brien et al., 2013



How do we use these timescales to learn what is going on with the physics of these objects?

How can we use their variability to learn what they are?

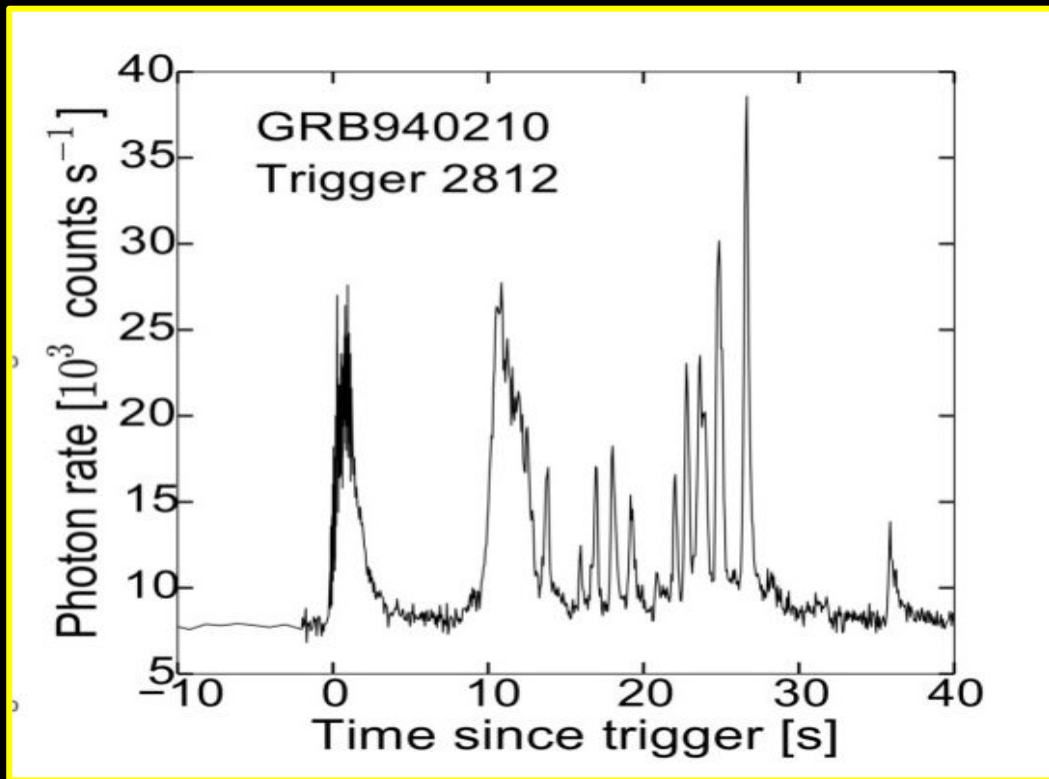
Gamma-ray Burst



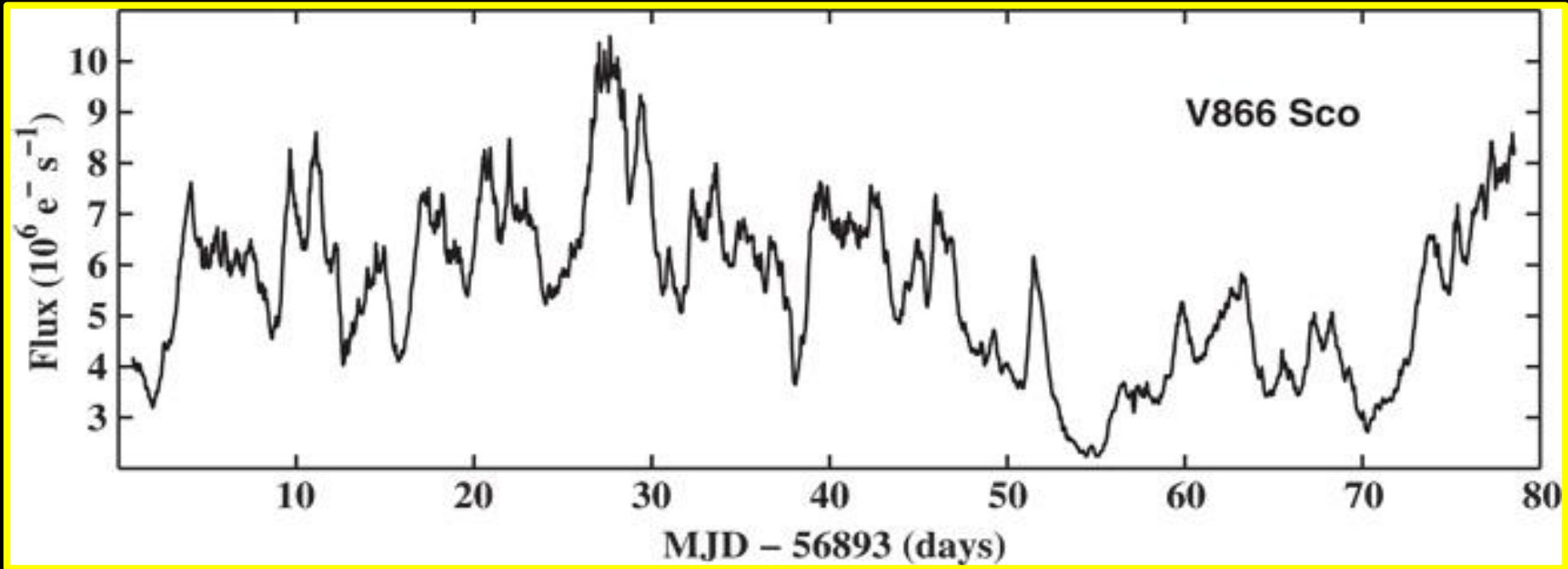
Light Crossing Timescale

$$t_{\text{LC}} \sim R/c$$

*Relativistic
corrections later.*



Compare to AGN variability \sim days



**What is the mass of a Schwarzschild black hole ($R=2GM/c^2$)
corresponding to $R = t_{\text{var}} c$?**

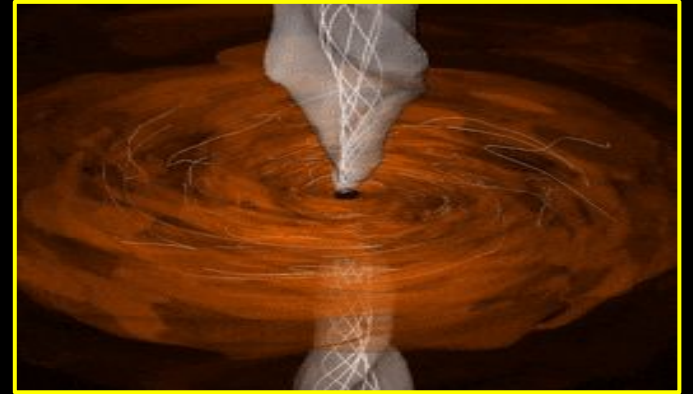
Free Fall (dynamical) Time

$$t_{\text{ff}} \sim (2R^3/GM)^{1/2}$$



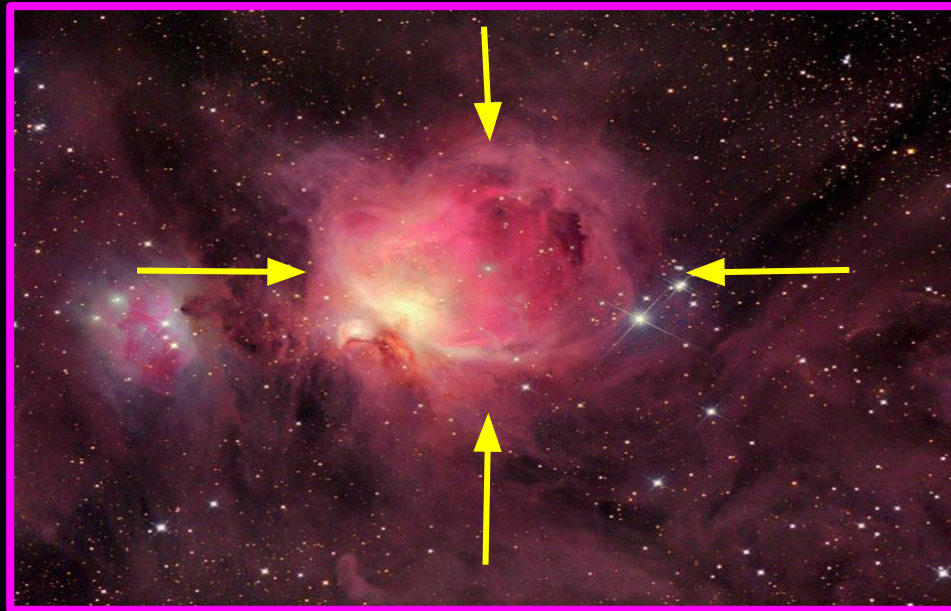
Keplerian Time

$$t_{\text{circ}} \sim (2\pi R^3/GM)^{1/2}$$



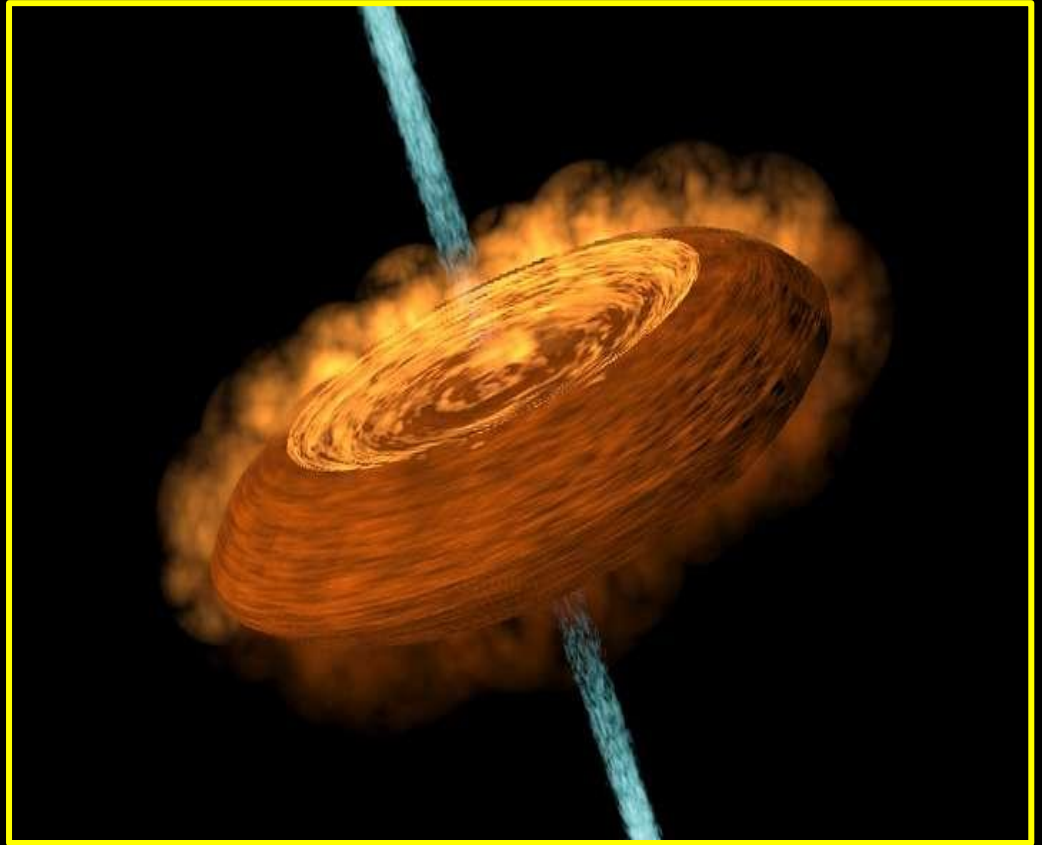
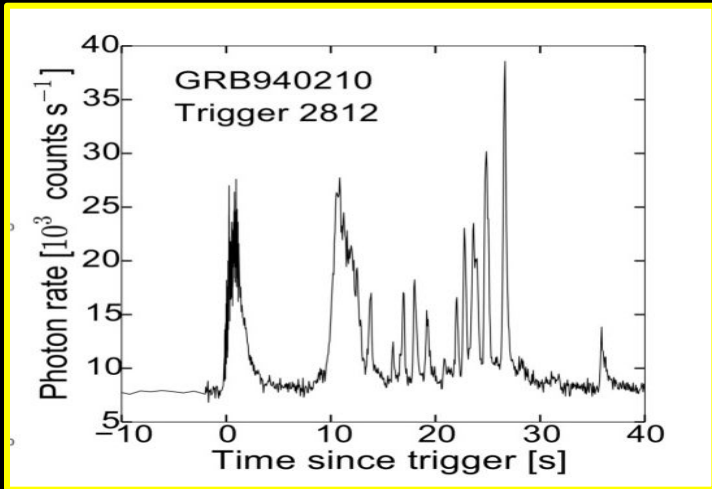
Exercise 1:

- What is the characteristic timescale for star formation if only gravity matters? Assume the density of a molecular cloud is $\sim 10^{-19} \text{g/cm}^3$? Is this consistent with observations?



Accretion Timescale - life of the inner engine

$$t_{\text{acc}} \sim (M/\dot{M})$$



Exercise 2:

- If a 20 solar mass star collapses and makes a 5 solar mass black hole, and we observe the central engine to be active for 20s, what is the accretion rate?
- How does this compare to the Eddington accretion rate?

Eddington Accretion:

- Radiation force balances gravity:

$$(\sigma_T/4\pi R^2)(L_{\text{Edd}}/c) = GMm_p/R^2$$

- $L_{\text{Edd}} = 4\pi GcMm_p/\sigma_T = \dot{M}_{\text{Edd}}c^2$

Exercise 3:

- What is the lifetime of a disk for a 10^9 solar mass black hole accreting at the Eddington rate (you'll need to make an assumption about the mass in the disk)?

Exercise 4:

- Compare the free fall time and light crossing time for a Schwarzschild black hole ($R = 2GM/c^2$) of mass M .

Radiative Timescale

$$t_{\text{rad}} = \text{Energy/Power}$$

$$\sim (\gamma m_e c^2) / (\sigma_T B^2 c \gamma^2) = m_e c / \sigma_T B^2 \gamma$$



Electron energy



Synchrotron
Radiation

Acceleration Timescale

$(\Delta p / \Delta t) r \sim E$ (work done on particle)

$$t_{\text{acc}} \sim \Delta p r / E \sim cR / (v_A)^2$$

↑
(Alfven Turbulence, e.g.)

Exercise 5:

- Electrons are accelerated in the presence of a magnetic field $B \sim 10 \text{ G}$, on a timescale 10^{-10} s . What is the maximum energy (or Lorentz factor) to which they can be accelerated?

But wait!

I want to simulate my astrophysical outflow. How do I describe my outflow? Is it a fluid? Is it a plasma?

Collision Timescale

$$t_{\text{coll}} = \lambda/v =$$
$$\frac{(1/n\sigma_{\text{T}})}{v}$$

Mean free path



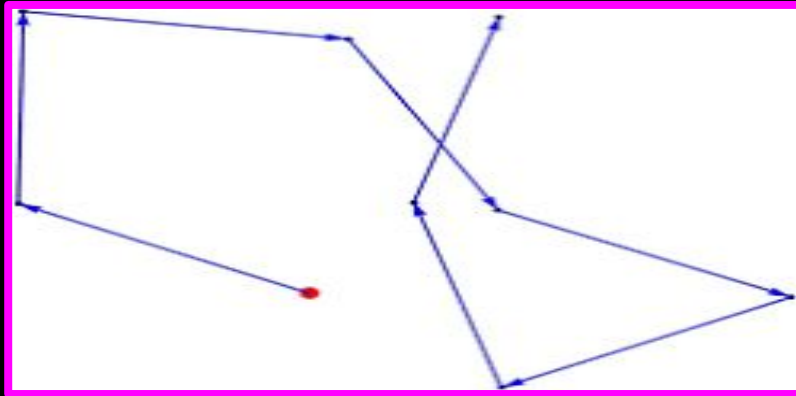
Diffusion Timescale

$$t_{\text{diff}} = N t_{\text{coll}} = (L/\lambda)^2 (\lambda/v) = \underline{L^2/\lambda v}$$

Random Walk

Size of system

Diffusion
Coefficient
(m²/s)



Exercise 6:

- Compute the time for photons to diffuse out of the sun.

Viscous Timescale

$$t_{\text{visc}} = L^2/\nu$$



Viscosity
(cm²/s)

Timescale for
evolution in a disk.
What is the
viscosity?

Exercise 7:

A typical (AGN) accretion disk has $R=10^{16}\text{cm}$, $T=10^4\text{K}$ and $n \sim 10^{16}\text{cm}^{-3}$. The kinematic viscosity is $\nu = v_T \lambda$, where v_T is the thermal velocity $v_T \sim (kT/m)^{1/2}$ and λ is the mean free path ($\sim 10^{-3}\text{ cm}$ for an ionized gas with this density and temperature - or you can make your own estimate!).

What is the viscous timescale for this accretion disk?

Is this a reasonable way to transport angular momentum and accrete?

Exercise 8:

Write down the viscous timescale for an alpha-disk:

- $\nu = \alpha c_s h,$
- $c_s = h\Omega, \Omega = (GM/R^3)^{1/2}$

Is this a better way to transport angular momentum? What is α ?

Plasma Timescales

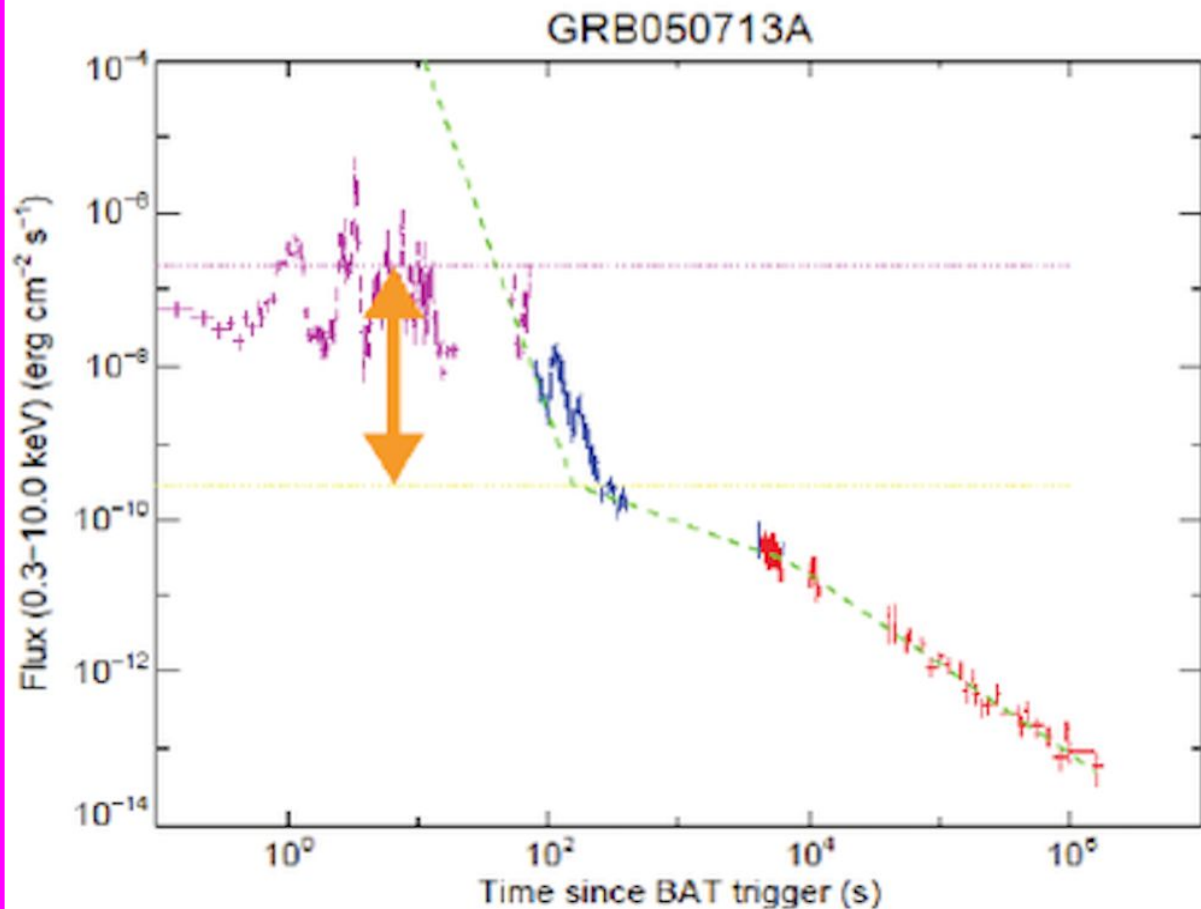
$$t_{\text{plas}} = \omega_p^{-1} = (m_*/4\pi n_* q^2)^{1/2}$$

$$t_{\text{gyro}} = \omega_g^{-1} = (m_* c / q B)$$

$$t_{\text{Coulomb}} = \omega_c^{-1} = (T^{3/2} 10^6) / (n_* \ln \Lambda)$$

Onset of a
GRB
afterglow:

$$e = \Gamma^2 n m_p c^2$$



Deceleration Timescale

$$t_{\text{dec}} \sim (E/n)^{1/3} (\Gamma^8 c^5 m_p^2)^{-1/3}$$

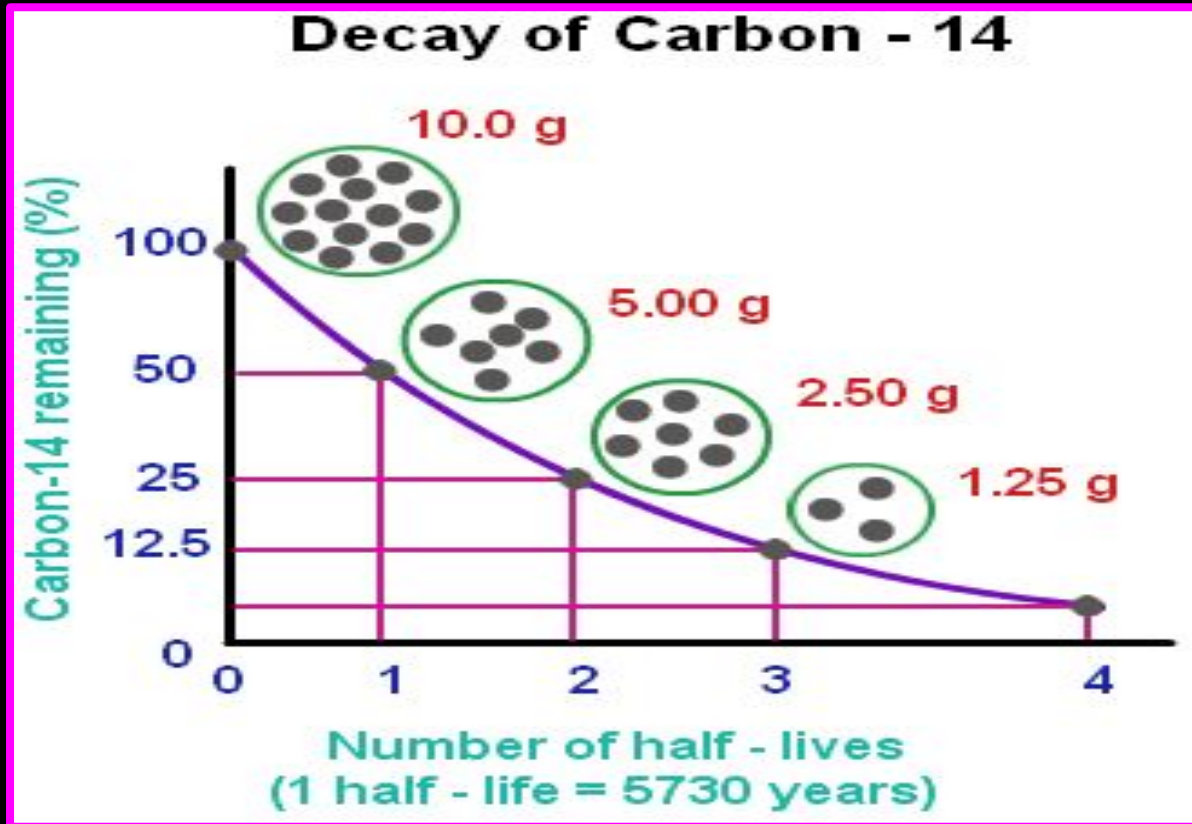


Radial and Angular Timescales (relativistic corrections):

$$t_r \sim R / (2c\Gamma^2)$$



Radioactive Decay Time



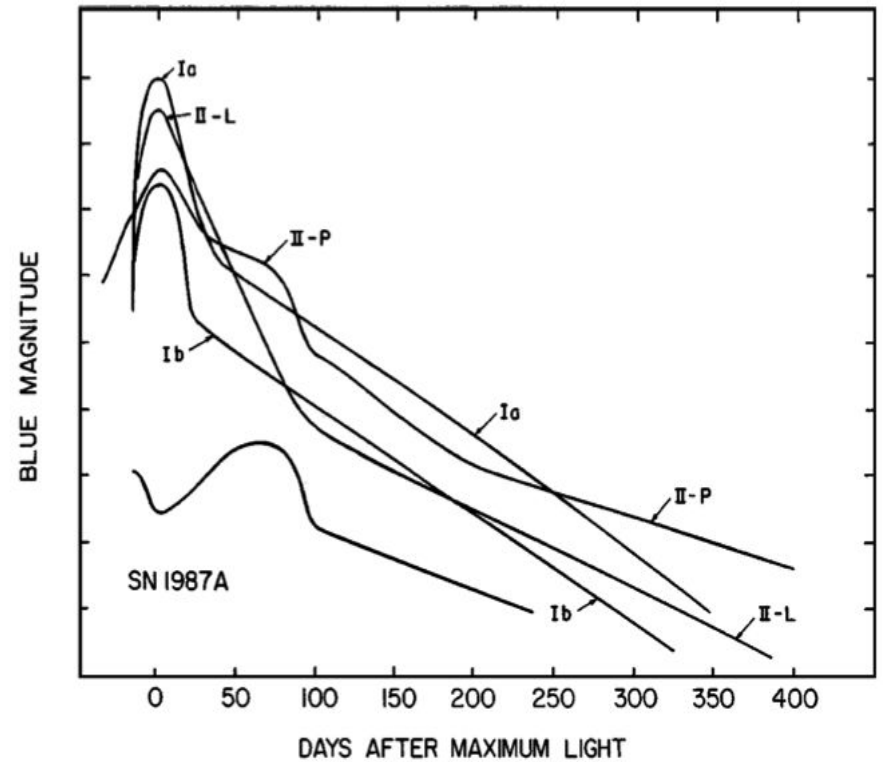
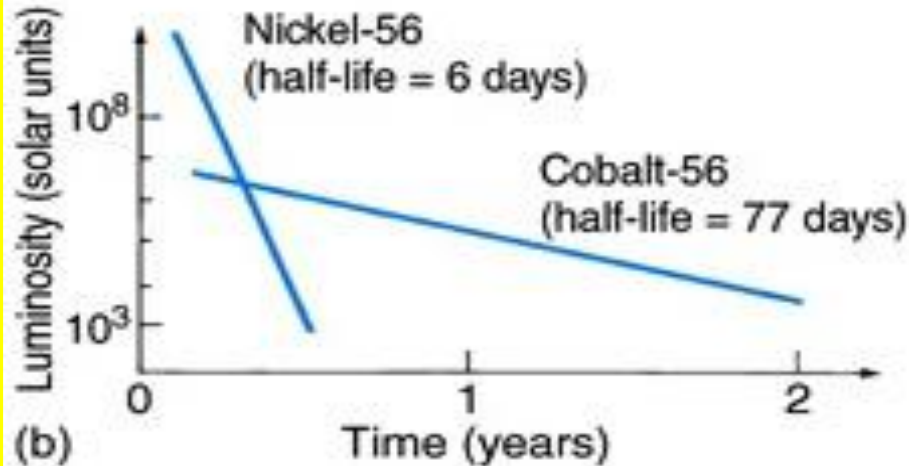
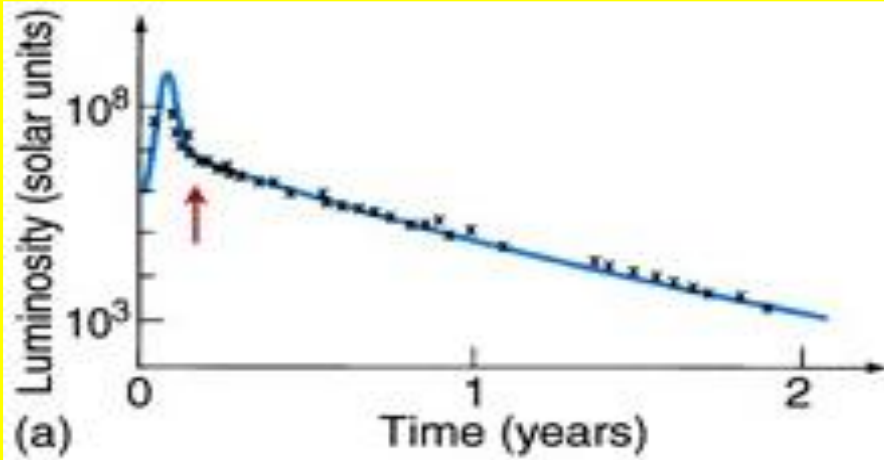
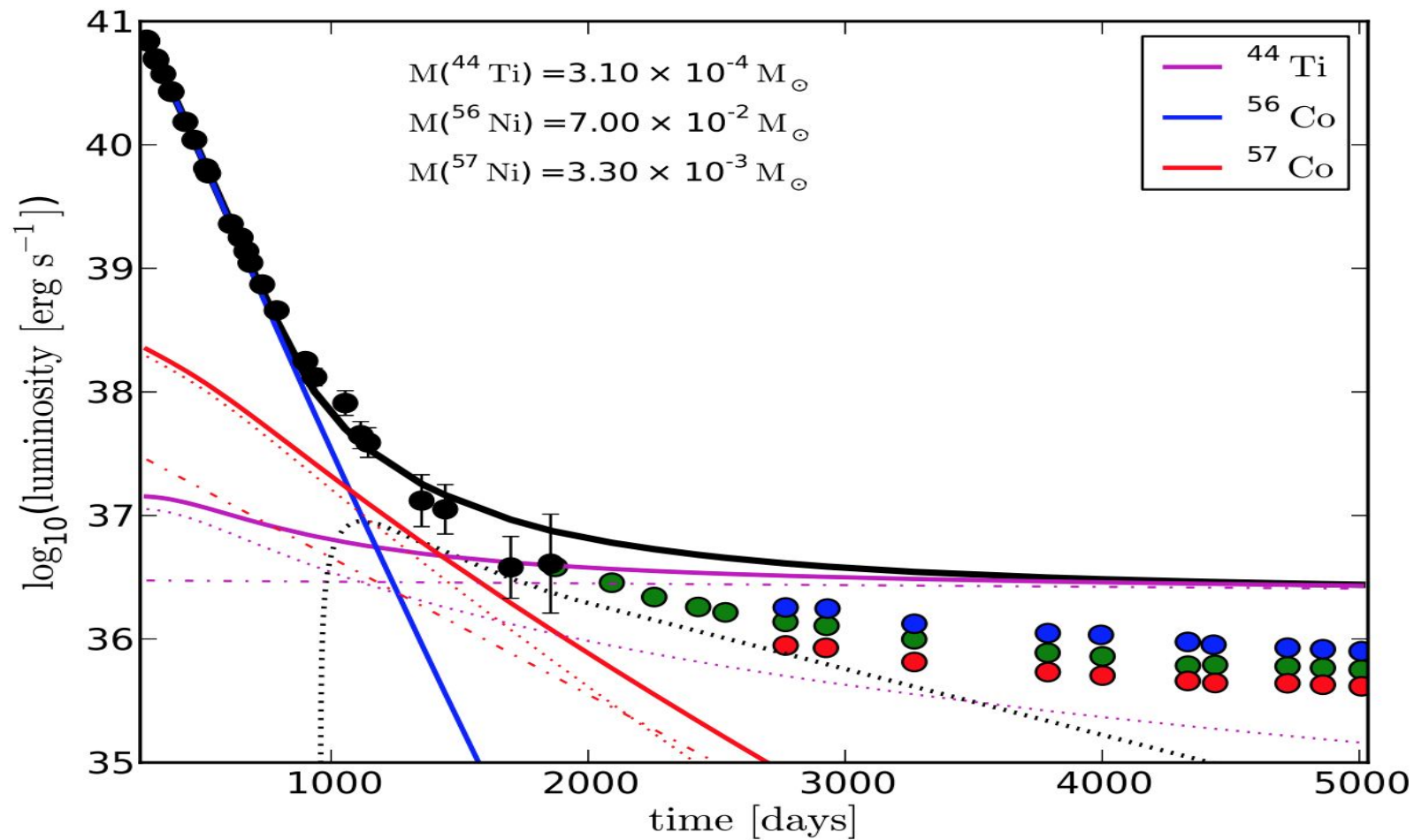


Figure 3 Schematic light curves for SNe of Types Ia, Ib, II-L, II-P, and SN 1987A. The curve for SNe Ib includes SNe Ic as well, and represents an average. For SNe II-L, SNe 1979C and 1980K are used, but these might be unusually luminous.

Figure Credit: Wheeler, J. C., & Harkness, R. P. 1990, RPPH, 53, 1467



Answers to Exercises:

Exercise 1:

- What is the characteristic timescale for star formation if only gravity matters? Assume the density of a molecular cloud is $\sim 10^{-19} \text{g/cm}^3$? Is this consistent with observations?
- Answer is free fall time $\sim 2,700 \text{s}/\sqrt{\rho} = 10 \text{ Myr}$. But the peak of star formation is around 1 Gyr.
- \Rightarrow Star formation inefficient!

Exercise 2:

- If a 20 solar mass star collapses and makes a 5 solar mass black hole, and we observe the central engine to be active for 20s, what is the accretion rate?
 - $20\text{s} \sim \text{mass of the disk}/\text{accretion rate} \Rightarrow \text{accretion rate} \sim 0.5 \text{ solar masses/second}$
- How does this compare to the Eddington accretion rate?
 - $\dot{M}_{\text{Eddington}} = (4 * \pi * G * M * m_p) / (\sigma T * c) = 10^{-17} (\text{s}^{-1}) * (M)$. If $M=5$ solar masses, $\dot{M}_{\text{Eddington}} = 10^{-16}$ solar masses/second!
 - The accretion rate above (not unreasonable for gamma-ray bursts is very very highly super Eddington).

Exercise 3:

- What is the lifetime of a disk for a 10^9 solar mass black hole accreting at the Eddington rate (you'll need to make an assumption about the mass in the disk)?
 - $\dot{M}_{\text{Eddington}} = (4 * \pi * G * M * m_p) / (\sigma T * c)$. I assumed the mass in the disk is roughly 10^8 solar masses. For a 10^9 solar mass black hole, the Eddington accretion rate is 10^{-8} solar masses/second. The lifetime of the disk is then $\dot{M} / \dot{M}_{\text{Eddington}} \sim 10^{16} \text{s} \sim 10^9 \text{yr}$!

Exercise 4:

- Compare the free fall time and light crossing time for a Schwarzschild black hole ($R = 2GM/c^2$) of mass M .
 - Remember the free fall time is $t_{\text{ff}} \sim (2R^3/GM)^{1/2}$ and the light crossing time is $t_{\text{LC}} \sim R/c$. If we plug in the relationship between mass and radius for a Schwarzschild black hole (given above), we find $t_{\text{ff}} \sim t_{\text{LC}} \sim 2GM/c^3$. That is, they are the same.

Exercise 5:

- Electrons are accelerated in the presence of a magnetic field $B \sim 10$ G, on a timescale 10^{-10} s. What is the maximum energy (or Lorentz factor) to which they can be accelerated?
 - To solve this, we want to consider the energy at which the radiative time is equal to the acceleration time - it is at this energy that particles lose their energy to radiation as fast as they can be accelerated, and so this sets the maximum energy scale to which they may be accelerated:
 - $t_{\text{rad}} = (m_e \cdot c) / (B^2 \cdot \sigma \cdot \gamma) = t_{\text{acc}}$. Plugging in the numbers given above and solving for γ , we find $\gamma \sim 10^5$!
 - Bonus: If the magnetic field went up by an order of magnitude (to 100 G), how does this energy/Lorentz factor change?

Exercise 6:

- Compute the time for photons to diffuse out of the sun.
 - $t_{\text{diffusion}} \sim L^2/\lambda v$, where λ is the mean free path $\sim 1/n\sigma_T$. The particle density $n \sim \rho_{\text{sun}}/m_H$ and σ_T is the Thomson cross section. Here, $v=c$ and $L=\text{radius of the sun}$. Plugging in the numbers, we find the mean free path is roughly 1 cm, so $t_{\text{diffusion}} \sim 10^{11}$ s.

Exercise 7:

A typical (AGN) accretion disk has $R=10^{16}\text{cm}$, $T=10^4\text{K}$ and $n \sim 10^{16}\text{cm}^{-3}$. The kinematic viscosity is $\nu = v_T \lambda$, where v_T is the thermal velocity $v_T \sim (kT/m)^{1/2}$ and λ is the mean free path ($\sim 10^{-3}\text{ cm}$ for an ionized gas with this density and temperature - or you can make your own estimate!).

- What is the viscous timescale for this accretion disk?
 - The mean free path is given as 10^{-3} cm . Plugging in the temperature to compute the thermal velocity, we can compute the viscosity ν .
 - The viscous timescale is then $R^2/\nu \sim 10^{29}\text{s} \sim 10^{22}\text{ year!!}$ (so this simple kinematic viscosity cannot be responsible for the loss of angular momentum that allows material to accrete onto the central black hole - another mechanism must be at play to transport angular momentum, allowing material to accrete!

Exercise 8:

Write down the viscous timescale for an alpha-disk:

- $v = \alpha c_s h$,
- $c_s = h\Omega$, $\Omega = (GM/R^3)^{1/2}$

Is this a better way to transport angular momentum? What is α ?

Bonus Exercise (only if time):

Relativistic electrons are scattered off Alfven turbulence. B-field is = 10G

Density ρ is = 10^{-18} g/cm³

$$(v_A = B/(4\pi\rho)^{1/2})$$

To what energy can the particles be accelerated?

Extra Time? MADs. MAD definition,
MAD timescales.