Walking up Mount Everest without Oxygen is a bad idea for most of us. But how fast does the air pressure and density drop above sea level? And what happens if you climb ever higher?

1. Write down an expression for the variation of gas density ϱ (units of mass per volume) with height z above the Earth's surface, assuming the gas is at constant temperature T and in hydrostatic equilibrium. Take the gas to be made of a single kind of molecule of weight μ (units of mass). Express in terms of the density at ground level $\varrho_0 = \varrho(z = 0)$ and the density scale height $h = k_B T/(\mu g)$, where g is the gravitational acceleration. Neglect variations of g with z.

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2. Neglecting collisions with other molecules, what is the maximum height a molecule would attain if launched from $z = 0$ with the typical thermal velocity? Is this close to h?

   Note that this calculation is highly misleading insofar as it ignores collisions between molecules, which are crucial for describing air as a continuum fluid. Nevertheless, it provides a mnemonic for remembering the scale height, and it illustrates the sometimes surprisingly close connection between fluid mechanics and particle mechanics (a.k.a. kinetic theory).

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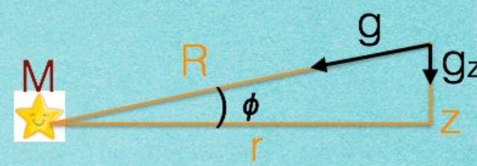
3. At what point (i.e,. height z) would you expect the equation in (1), which depends on the continuum hypothesis, to fail? This height marks the location of the base in planetary atmospheres. Just consider how intermolecular collisions validate the continuum approximation.

Think for later - come to us with an answer....

Now, lets change the geometry and have disks instead of spheres



4. Write down an expression for the hydrostatic variation of gas density ϱ with height z above the midplane of a circumstellar disk at radius r. Assume constant T and take the gravitational field to be that from the star alone (ignore the self-gravity of the disk). Work in the limit that z << r (known as the thin disk approximation). Express in terms of the density at the midplane ϱ_0 and the density scale height

$$h = \frac{\sqrt{k_B T/\mu}}{\Omega}$$

where Ω is the Keplerian orbital angular frequency.

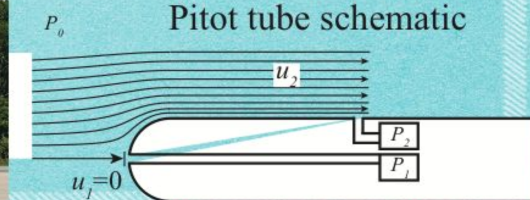
Does, $k_B T/\mu h$ seems familiar?

Is the density here has the same profile as the one for a sphere?

Have you ever wondered how a pilot knows the air speed of their plane? Or perhaps wondered what the tubes sticking out of the body of airplanes in various places are for? Let's investigate the Euler equations to find the answer.

Before having some fun with our friend Professor Euler, let's take a closer look at the various tubes sticking out of airplanes. These tubes are called Pitot tubes, and a diagram of the air flow past the Pitot tube is shown in the figure below, as seen in the frame of the airplane. Notice how the air must be at rest at the tip of the tube (we call this a stagnation point), while along the sides it is flowing past the tube with the velocity of the airplane. With this tube, measuring air speed (technically challenging) becomes a simple matter of measuring a pressure differential (technically straight forward).

Pitot tube schematic



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Some simplifications are in order:

- Let's consider an *incompressible* fluid, that is, one where the density does not change materially along streamlines. This often a good approximation for flows well below the sound speed, because in this case the pressure term is much larger than the ram pressure term and any compressions are quickly resisted by the pressure term.
- Suppose we are interested in flow past an airplane body, or along a fluid streamline. We can approximate such a flow as *1-dimensional*, with the velocity and pressure only changing along the direction of motion–let's say the x-direction, and with velocities perpendicular to the body of the airplane or the strealime small compared to the velocity along the streamline. Drop all $\partial/\partial y$ and $\partial/\partial z$ derivatives and set $\upsilon_y = \upsilon_z = 0$.
- We look for *stationary* solutions so that partial time derivatives vanish (our plane is flying through smooth air, the seatbelt sign is off). Drop all $\partial/\partial t$ derivatives.
- Finally, let's assume that all *external forces are negligible*, $f_{ext} = 0$.

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Using these assumptions, make the necessary simplifications to the incompressible Euler momentum equation

$$\rho \frac{\partial v}{\partial t} + \rho \left(\mathbf{v} \cdot \nabla \right) \mathbf{v} + \nabla P = \mathbf{f}_{\text{ext}}$$

Integrate the resulting ordinary differential equation to evaluate how the pressure depends on the velocity u.

Hint: remember the integration constant!