# accretion

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matter falling under gravity of a mass  $\,M$  from a large distance to a distance  $\,R$  from it releases potential energy

$$\sim \frac{GM}{R}$$

per unit mass

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$$R \lesssim 12 \frac{GM}{c^2}$$

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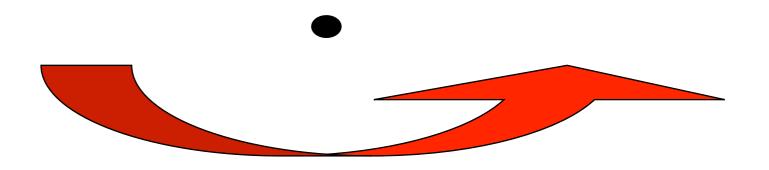
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accretion on to a supermassive black holes must power the brightest objects in the Universe

# accretion - e.g to a black hole

infalling matter does not hit black hole in general, but must *orbit* it:
— it is *never* `aimed' directly at the hole



— initial orbit is a rosette since potential is never exactly  $R^{-1}$ 

infalling gas: self—intersections  $\rightarrow$  dissipation  $\rightarrow$ 

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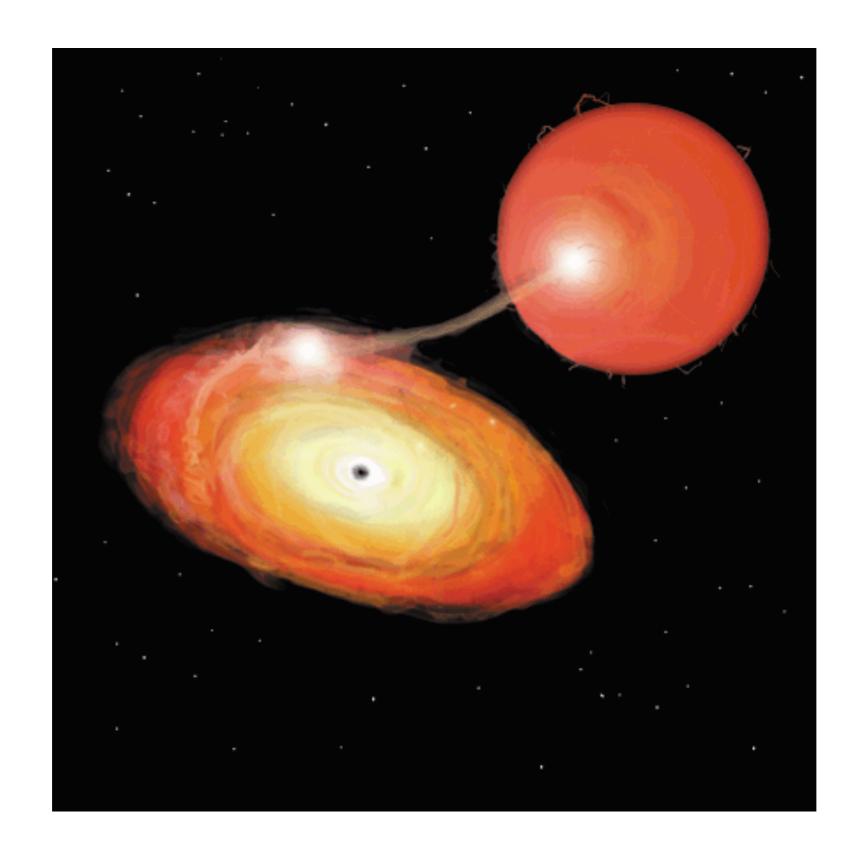
Kepler orbit with lowest energy for fixed a.m. is a circle

thus orbit circularizes, with radius such that it retains its orginal specific angular momentum

further energy loss only possible if angular momentum can be removed:

matter spirals inwards through a succession of circular orbits of decreasing angular momentum

#### accretion disc



close binary system with an accretion disc

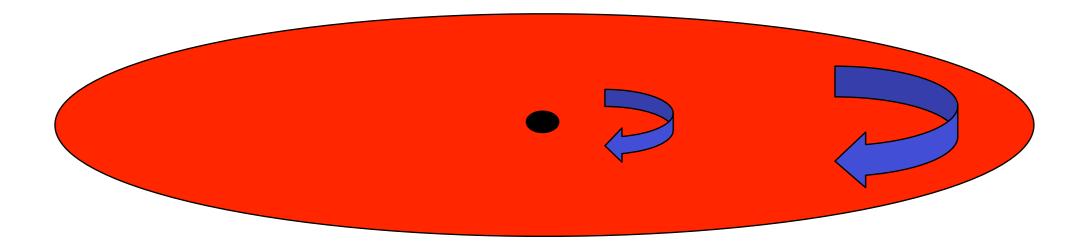
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all accreting gas has enough angular momentum to orbit the accretor, so a disc always forms

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only exception — accretion on to an *extended* star — low accretion yield



flat, differentially rotating gas disc, thickness  $\mathcal{H}(R)$ 

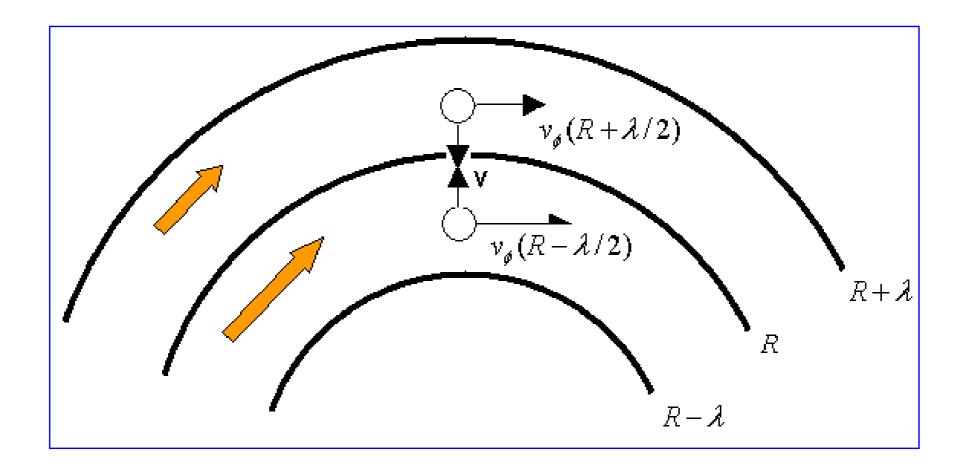
surface density (mass/area)  $\Sigma(R) = \rho H$ 

rotational angular velocity  $\Omega(R)$  increases towards centre

angular momentum  $R^2\Omega(R)$  decreases towards centre

disc is thin, 
$$\frac{H}{R} \sim \frac{c_s}{v_{\rm K}} << 1$$
, Keplerian  $R\Omega(R) = v_K = \left(\frac{GM}{R}\right)^{1/2}$ 

(pressure forces small) if and only if it can cool



torque of inner ring on outer one is  $G(R)=2\pi\nu\Sigma R^3\frac{\mathrm{d}\Omega}{\mathrm{d}R}$ , with  $\ \nu\sim\lambda v$ 

- driver of accretion is 'viscosity' some dissipative process which transports angular momentum outwards, against a.m. gradient
- currently unknown but may be magnetic
- characterized by a lengthscale  $\lambda$  and a speed v describing random motions around mean streaming (fluid) motion
- e.g. molecular viscosity has  $\lambda =$  mean free path, v = thermal speed of molecules (sound): other processes have larger  $\lambda$ , e.g. turbulence
- a viscosity transports fluid momentum and angular momentum within it
- gas spirals in, losing angular momentum and energy

disc surface density  $\Sigma(R,t)$  obeys a diffusion equation

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{R} \frac{\partial}{\partial R} \left( R^{1/2} \frac{\partial}{\partial R} \left[ \nu \Sigma R^{1/2} \right] \right)$$

where  $\nu$  is 'kinematic viscosity': parametrize as  $\nu = \alpha c_s H$ , with  $\alpha < 1$ ,  $\dot{M}(R,t) = 3\pi\nu\Sigma$ 

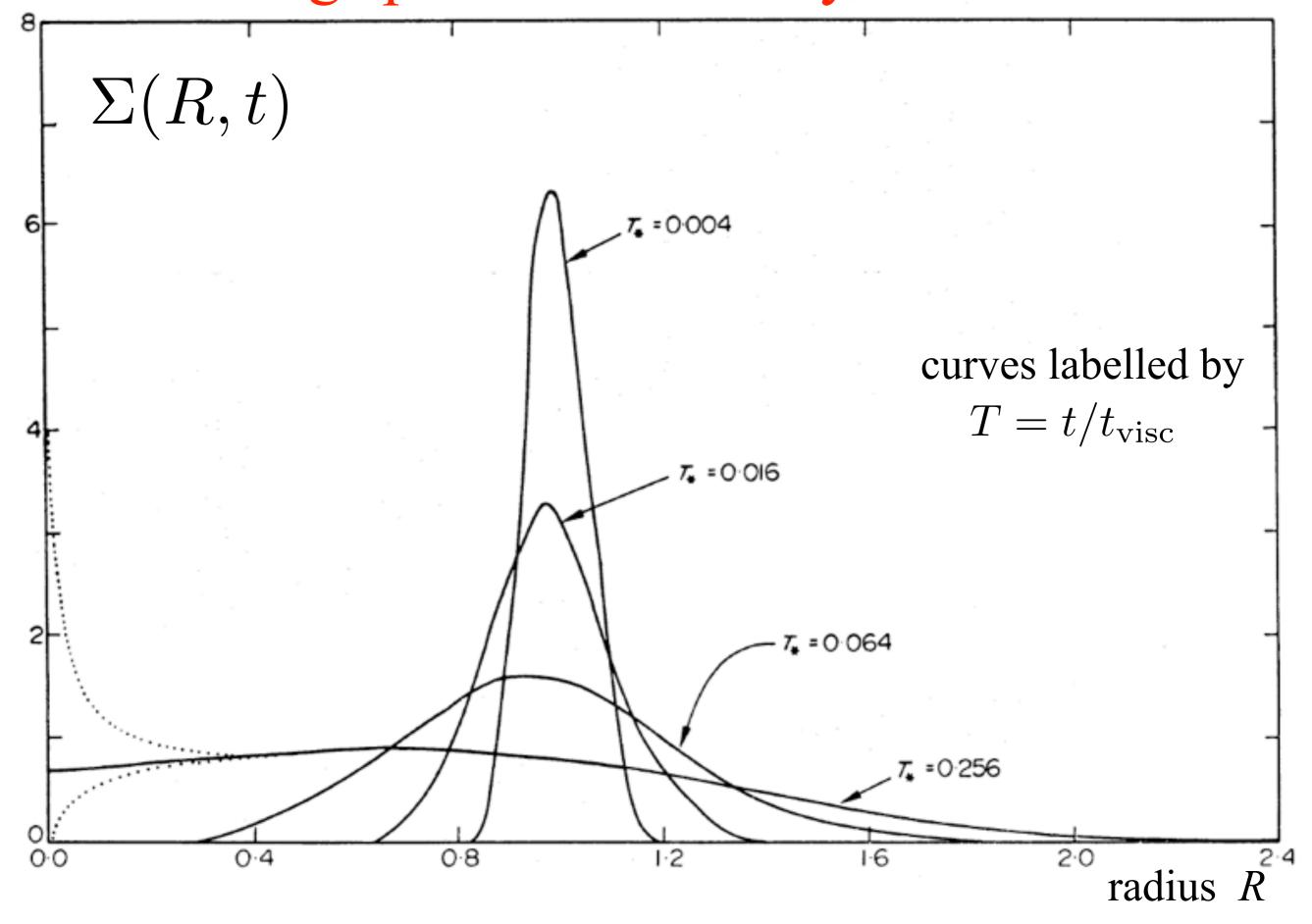
 $\Sigma$  spreads on viscous timescale

$$t_{\rm visc} = \frac{R^2}{\nu} = \frac{1}{\alpha} \left(\frac{R}{H}\right)^2 t_{\rm dyn}$$

where  $t_{\rm dyn}$  is the dynamical timescale  $R/v_K = (R^3/GM)^{1/2}$ 

this is long:  $t_{\rm visc} \simeq 10^{10} \ {\rm yr \ for} \ R \sim 1 \ {\rm pc} \ (H/R \lesssim 10^{-2})$ 

# initial ring spreads diffusively to make a disc



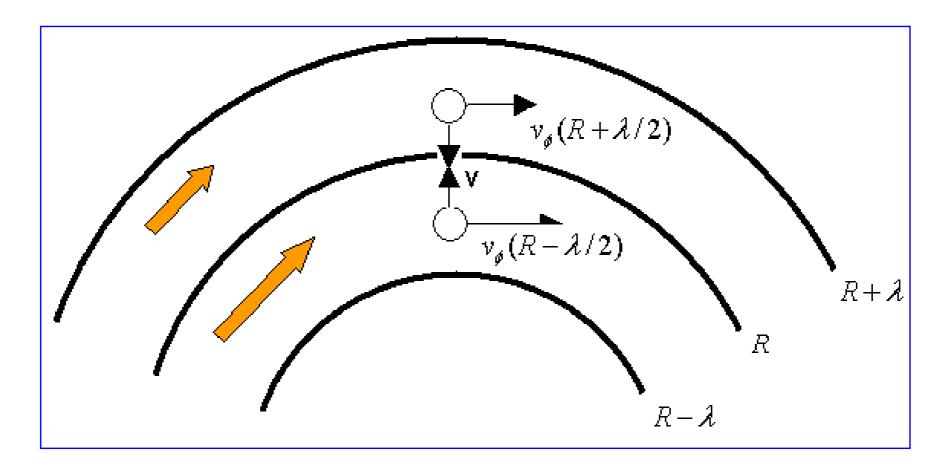
#### disc formation is unavoidable

all accreting gas has enough angular momentum to orbit the hole, so a disc always forms

disc must be small enough for matter to accrete on reasonable timescales, i.e. ~ 0.1 pc for an AGN

this requires any feeding mechanism to produce an accurate 'shot' towards the black hole

feeding SMBH efficiently is difficult

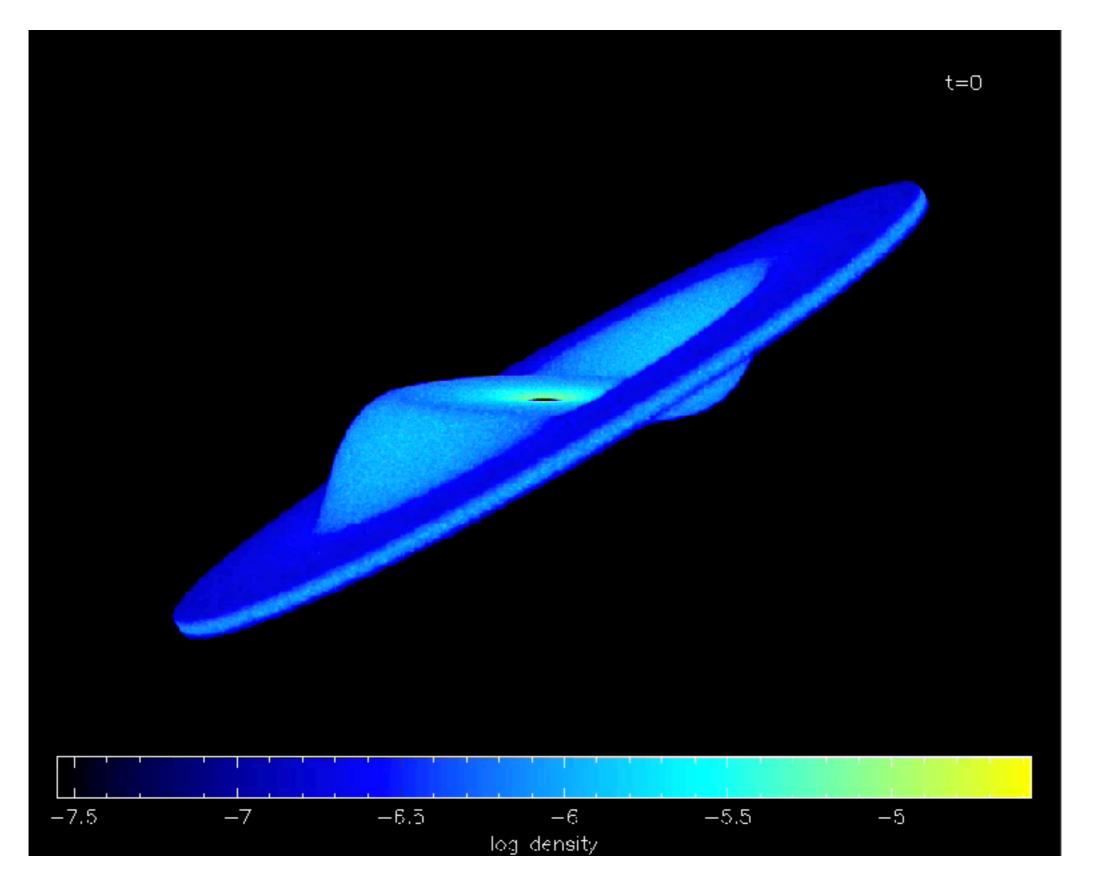


torque of inner ring on outer one is  $G(R) = 2\pi\nu\Sigma R^3 \frac{\mathrm{d}\Omega}{\mathrm{d}R}$ , with  $\nu \sim \lambda v$ 

dissipation per unit disc face area of a steady thin disc is

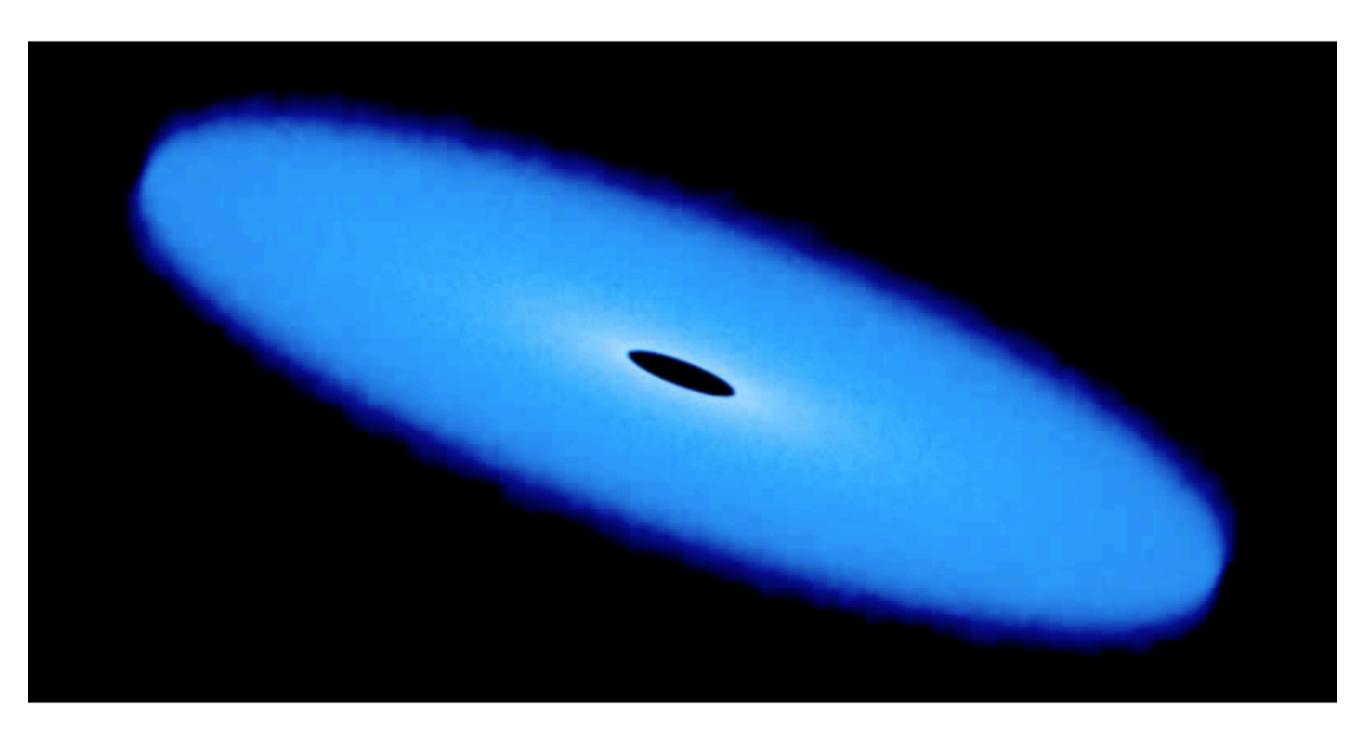
$$D(R) = \frac{3GM\dot{M}}{8\pi R^3} \left[ 1 - \left(\frac{R_{\rm in}}{R}\right)^{1/2} \right]$$

#### assumed warp (Lodato & Price 2010)

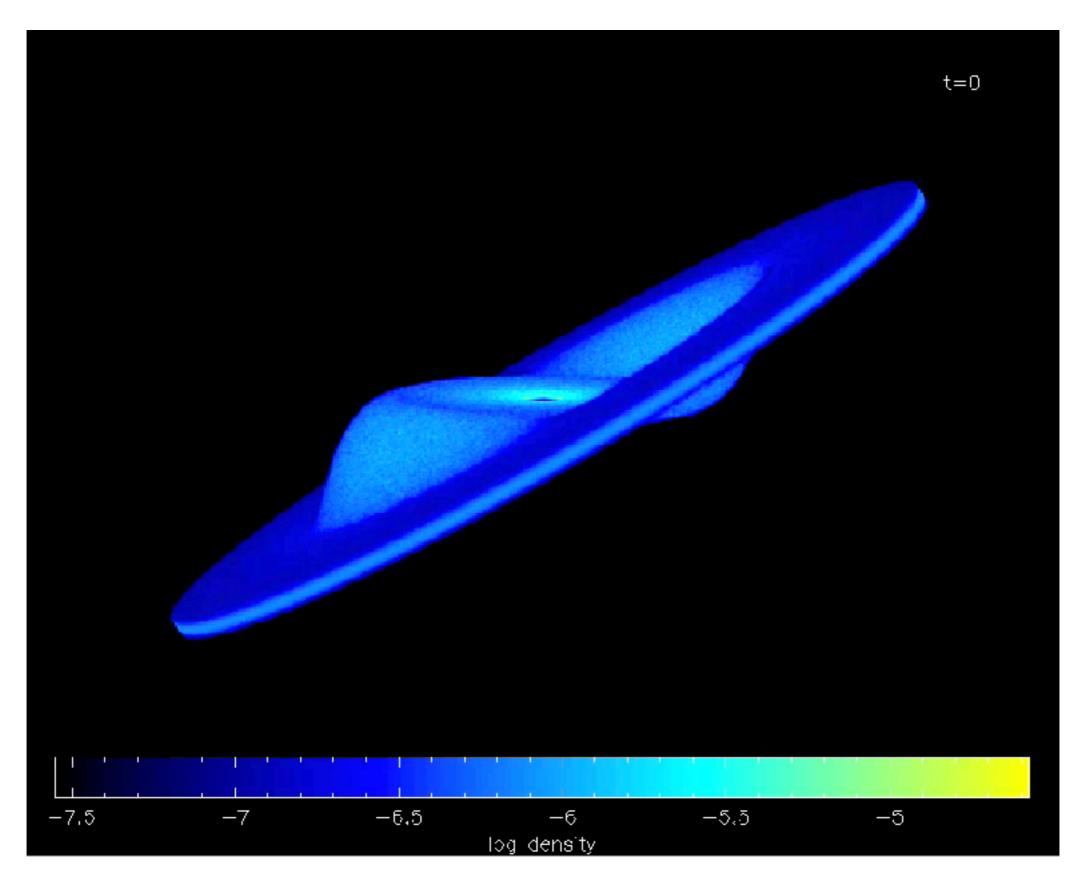


strong warp, significant viscosity

induced warp: Lense-Thirring with small tilt (Nixon & King, 2011)

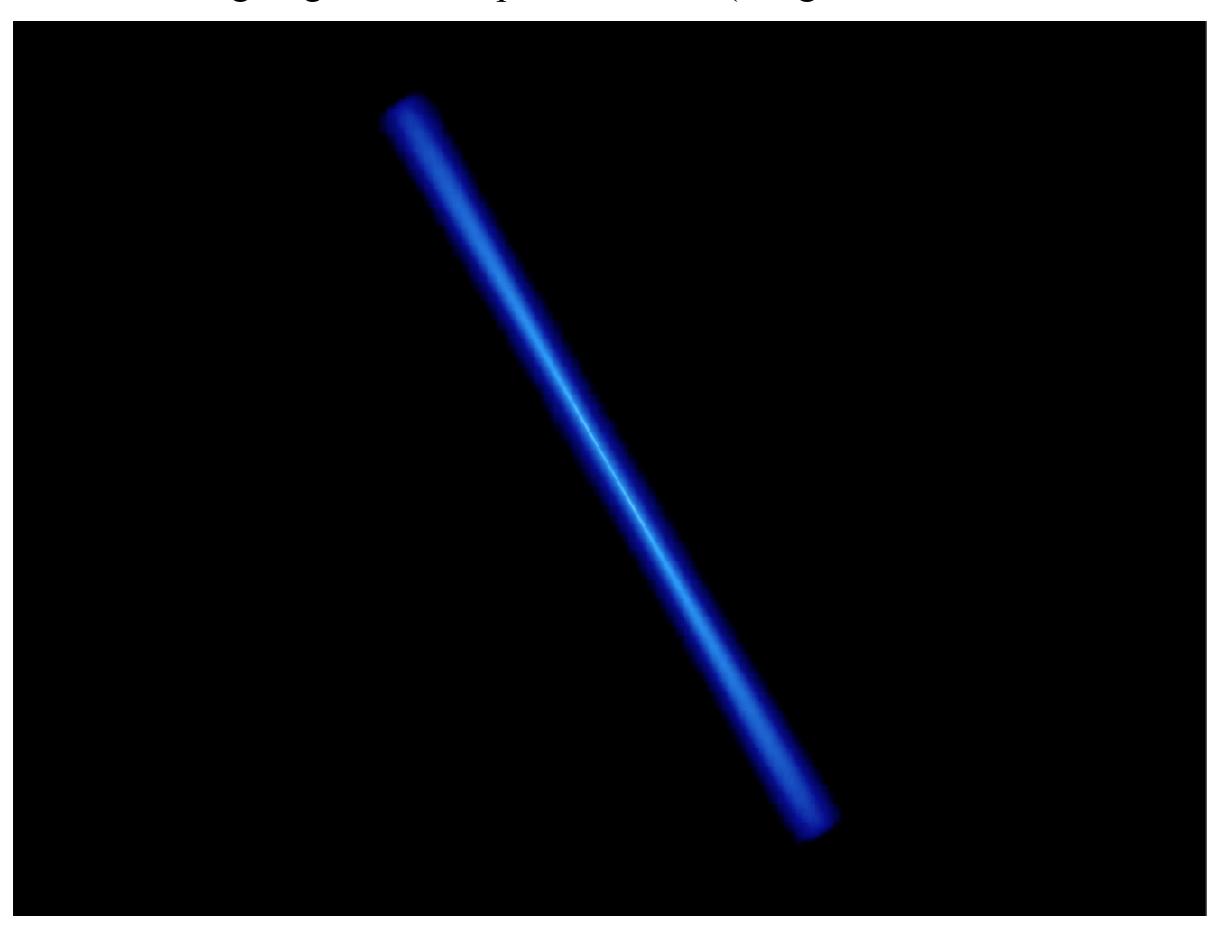


larger assumed warp (Lodato & Price, 2010)

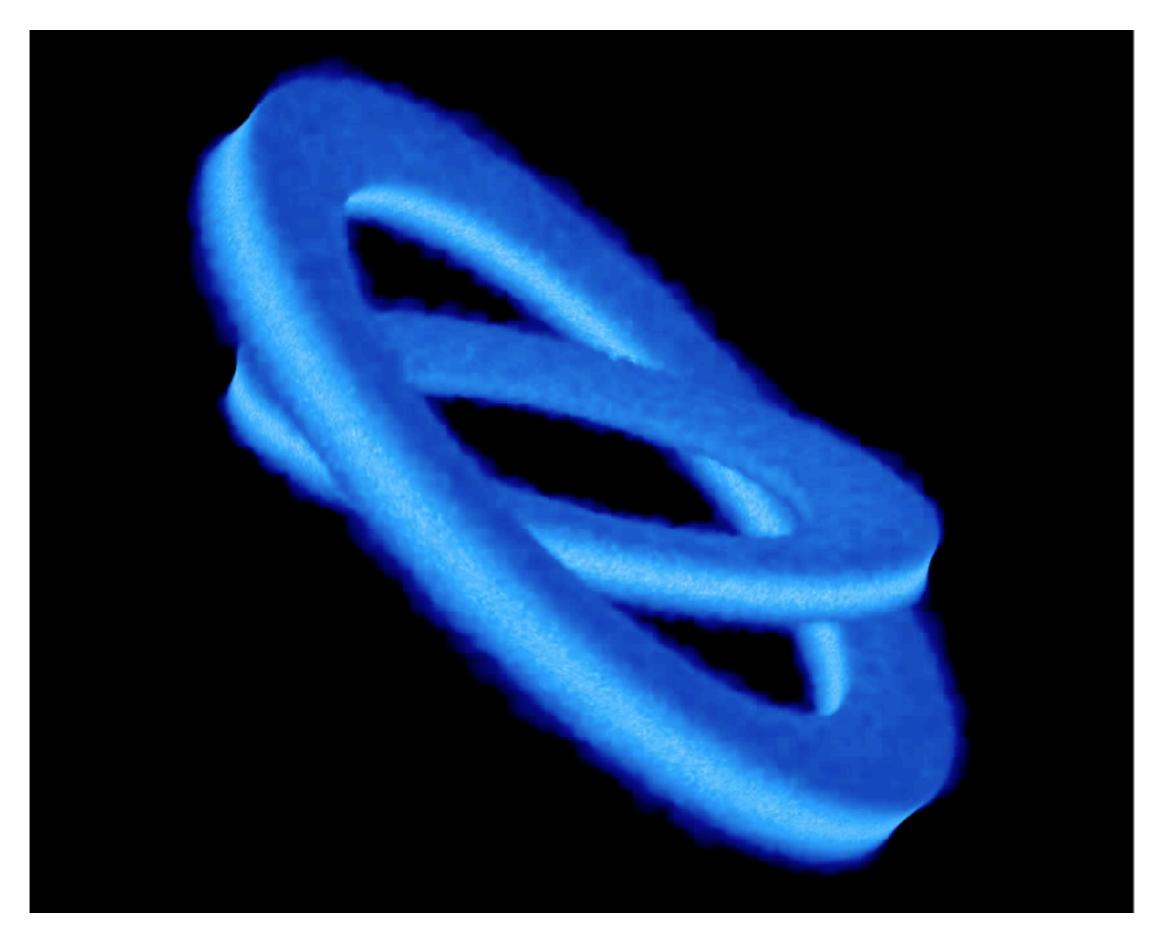


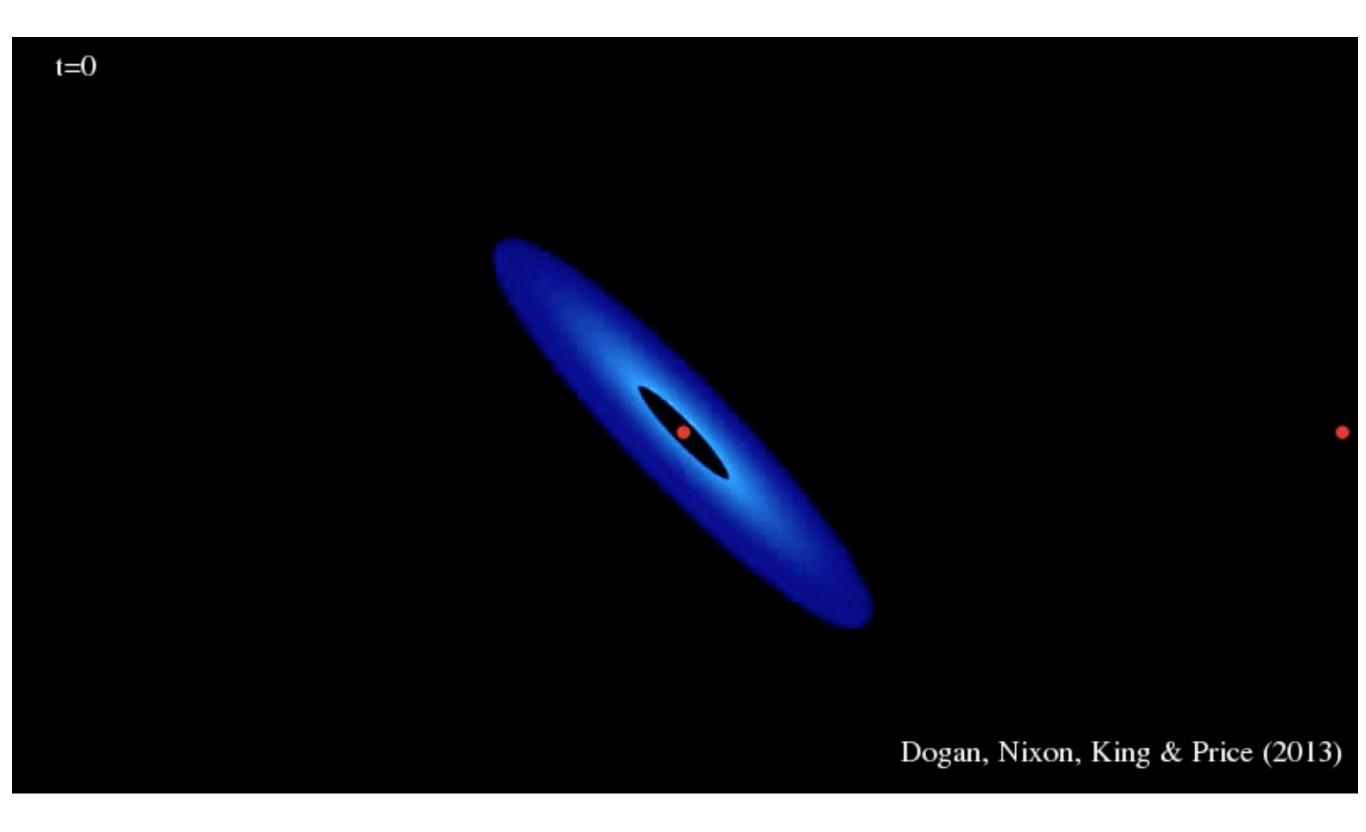
strong warp, viscosity relatively weaker: disc breaks!

Lense-Thirring: big tilt ==> rapid accretion (King & Nixon 2012



rapid accretion: counterrotating discs (Nixon & King, 2012)





# work problem: accretion disc spectrum

$$D(R) = \frac{3GM\dot{M}}{8\pi R^3} \left[ 1 - \left(\frac{R_{\rm in}}{R}\right)^{1/2} \right]$$

we can take

$$D(R) = \sigma T_{\text{eff}}^4$$

as defining the disc surface temperature

simplest case: disc radiates locally as a blackbody:

$$I_{\nu}(R) = \frac{2h\nu^3}{e^{h\nu/kT_{\text{eff}}} - 1}$$

plot the continuum spectrum of a blackbody disc