NCERT - 8.3.17

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I. APPLICATION OF INTEGRALS

Question: The area bounded by the curve $y = x\,|x|$, x-axis and the ordinates $x = -1$ and $x = 1$ is given by

Solution: $y = x^2$ if $x > 0$ and $y = -x^2$ if $x < 0$

$$\text{Area}\,(A) = \int_{-1}^{1} |y|\,dx \tag{1}$$

$$\tag{2}$$

Split the integral at $x = 0$, as the function changes;

$$A = \int_{-1}^{0} \left(-x^2\right) dx + \int_{0}^{1} x^2 dx \tag{3}$$

$$A = -\int_{-1}^{0} \left(-x^2\right) dx + \int_{0}^{1} x^2 dx \tag{4}$$

$$A = -\left[\frac{x^3}{3}\right]_{-1}^{0} + \left[\frac{x^3}{3}\right]_{0}^{1} \tag{5}$$

$$A = -\left(0 - \left(-\frac{(-1)^3}{3}\right)\right) + \left(\frac{1}{3} - 0\right) \tag{6}$$

$$A = \frac{1}{3} + \frac{1}{3} \tag{7}$$

$$A = \frac{2}{3} \tag{8}$$

$$A = 0.66666666667 \tag{9}$$

Computational Solution:
Using the trapezoidal rule,

$$J = \int_{a}^{b} f(x)\ dx \approx h\left(\frac{1}{2}f(a) + f(x_1) + f(x_2) \cdots + f(x_{n-1}) + \frac{1}{2}f(b)\right) \tag{10}$$

$$h = \frac{b-a}{n} \tag{11}$$

$$A = A_n,\ \text{where, } A_{i+1} = A_i + h\frac{f(x_{n+1}) + f(x_n)}{2} \tag{12}$$

$$A_{i+1} = A_i + \frac{h}{2}\left(f(x_{n+1}) + f(x_n)\right) \tag{13}$$

$$A_{i+1} = A_i + \frac{h}{2}\left({x_{n+1}}^2 + {x_n}^2\right) \tag{14}$$

$$x_{n+1} = x_n + h \tag{15}$$

Initial Conditions:

- $a = -1$
- $b = 1$
- $A_0 = 0$
- $h = \frac{2}{n}$ (depending on the chosen number of subintervals n)
- Here we assume $n = 1000$.

$\implies$ The theoritical value of Area is 0.66666666667.

$\implies$ The computational value of Area is 0.6666679999999998.