NCERT - 9.4.15

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I. DIFFERENTIAL EQUATIONS

Question: $2xy + y^2 - 2x^2\frac{dx}{dy} = 0$; $y = 2$ when $x = 1$

Solution: (Theoretical Solution) The given differential equation can be written as

$$\frac{dy}{dx} = \frac{y}{x} + \frac{y^2}{2x^2} \tag{1}$$

To solve it, we make the substitution

$$y = xt \tag{2}$$

Differentiating equation (2) with respect to x, we get

$$\frac{dy}{dx} = t + x\frac{dt}{dx} \tag{3}$$

Substituting the value of y and $\frac{dy}{dx}$ in equation (1), we get

$$t + \frac{t^2}{2} = t + x\frac{dt}{dx} \tag{4}$$

$$\frac{t^2}{2} = x\frac{dt}{dx} \tag{5}$$

$$\frac{dx}{2x} = \frac{dt}{t^2} \tag{6}$$

$$\int \frac{dx}{2x} = \int \frac{dt}{t^2} \tag{7}$$

$$\frac{\ln x}{2} = -\frac{1}{t} + c \tag{8}$$

Replacing t, we have

$$\frac{\ln x}{2} = -\frac{x}{y} + c \tag{9}$$

Substituting $x = 1$, $y = 2$ to find c, we get

$$\frac{\ln 1}{2} = -\frac{1}{2} + c \tag{10}$$

$$c = \frac{1}{2} \tag{11}$$

By solving, we obtain y as

$$y = \frac{2x}{1 - \ln x} \tag{12}$$

Solution by the method of finite differences:

$$\frac{dy}{dx} = \frac{y}{x} + \frac{y^2}{2x^2} \tag{13}$$

Using the method of finite differences, we approximate the derivative as

$$\frac{dy}{dx} \approx \frac{y_{n+1} - y_n}{h} \tag{14}$$

Substitute equation(14) in equation(13)

$$\frac{y_{n+1} - y_n}{h} = \frac{y_n}{x_n} + \frac{{y_n}^2}{2{x_n}^2} \tag{15}$$

$$y_{n+1} - y_n = h(\frac{y_n}{x_n} + \frac{{y_n}^2}{2{x_n}^2}) \tag{16}$$

$$y_{n+1} = y_n + h(\frac{y_n}{x_n} + \frac{{y_n}^2}{2{x_n}^2}) \tag{17}$$

The initial conditions are given as: $x_0 = 1$, $y_0 = 2$, $h = 0.005$. Using the recurrence relation (17), we compute values of x_n and y_n. These values can be used to approximate the solution numerically for a given range of x.

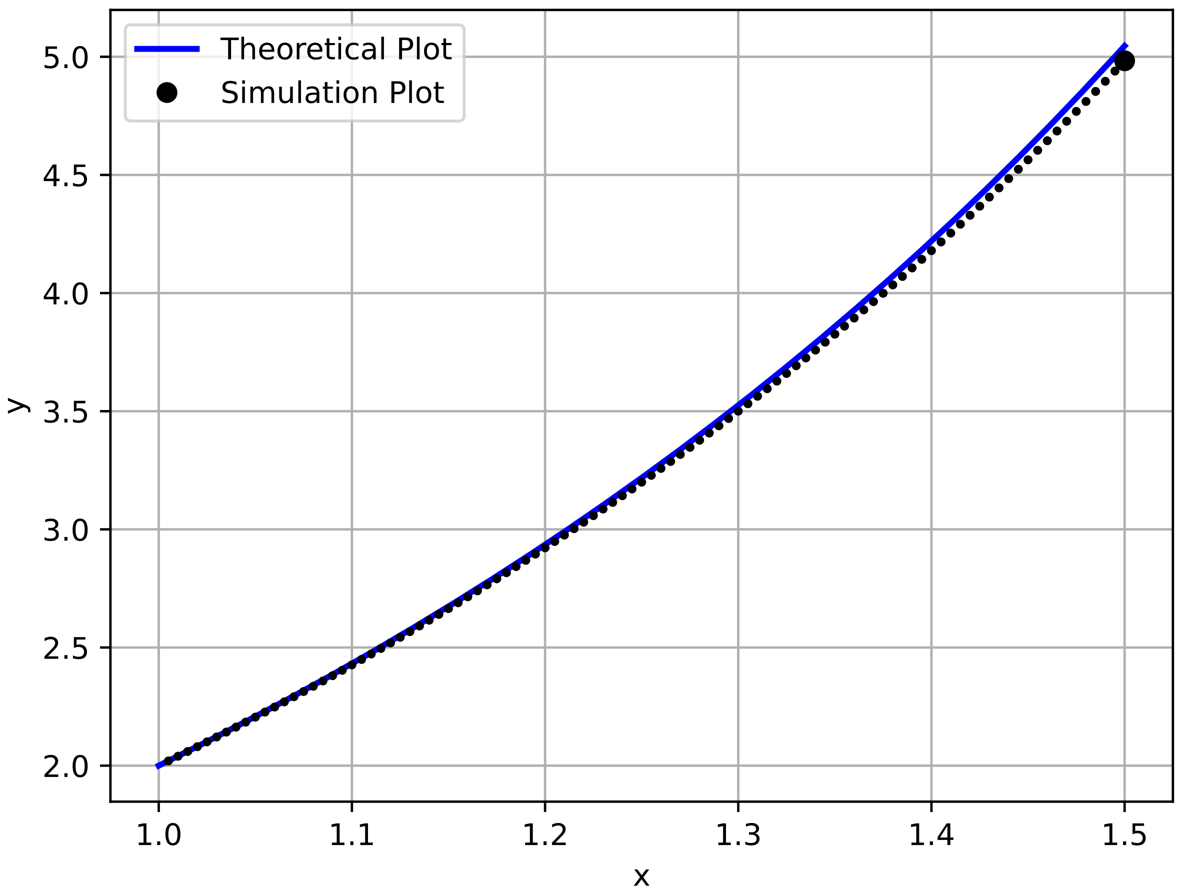

Fig. 0. Solution of given DE