NCERT - 9.6.19

EE224BTECH11044 - Muthyala koushik

I. DIFFERENTIAL EQUATIONS

Question: $\left(1-y^2\right)\frac{dx}{dy}+yx=ay$ $(-1<y<1)$; $y=0$ when $x=1$

Solution: The given equation is a linear differential equation of type $\frac{dx}{dy}+Px=Q$

$$\frac{dx}{dy}+x\frac{y}{1-y^2}=\frac{ay}{1-y^2} \tag{1}$$

where $P=\frac{y}{1-y^2}$ and $Q=\frac{ay}{1-y^2}$. Therefore

$$I.F=e^{\int\frac{y}{1-y^2}dy} \tag{2}$$

Let $u=1-y^2$, so $du=-2ydy$. then:

$$\int\frac{y}{1-y^2}dy=-\frac{1}{2}\int\frac{1}{u}du=-\frac{1}{2}\ln u=-\frac{1}{2}\ln\left|1-y^2\right| \tag{3}$$

$$I.F=e^{-\frac{1}{2}\ln\left|1-y^2\right|}=\frac{1}{\sqrt{\left|1-y^2\right|}} \tag{4}$$

Hence, the solution of the differential equation is given by

$$x.\frac{1}{\sqrt{\left|1-y^2\right|}}=\int\left(\frac{ay}{1-y^2}\right)\frac{1}{\sqrt{\left|1-y^2\right|}}dx+C \tag{5}$$

$$x.\frac{1}{\sqrt{\left|1-y^2\right|}}=\int\frac{ay}{(1-y^2)^{\frac{3}{2}}} \tag{6}$$

Let $u=1-y^2$, so $du=-2ydy$. the integral becomes:

$$\int\frac{ay}{(1-y^2)^{\frac{3}{2}}}dy=-\frac{a}{2}\int u^{-\frac{3}{2}}du \tag{7}$$

$$\int u^{-\frac{3}{2}}=-2u^{-\frac{1}{2}} \tag{8}$$

$$\int\frac{ay}{(1-y^2)^{\frac{3}{2}}}=-\frac{a}{2}(-2)\,u^{-\frac{1}{2}} \tag{9}$$

$$\int\frac{ay}{(1-y^2)^{\frac{3}{2}}}=\frac{a}{\sqrt{1-y^2}} \tag{10}$$

Substitute back into the equation:

$$\frac{x}{\sqrt{\left|1-y^2\right|}} = \frac{a}{\sqrt{1-y^2}} + C \tag{11}$$

$$x = a + C\sqrt{\left|1-y^2\right|} \tag{12}$$

Final Solution:

$$x = a + C\sqrt{\left|1-y^2\right|} \tag{13}$$

Solution by the method of finite differences: Let make a=0;C=1;

$$\frac{dx}{dy} = -\frac{xy}{1-y^2} \tag{14}$$

Using the method of finite differences, we approximate the derivative as

$$\frac{dx}{dy} \approx \frac{x_{n+1}-x_n}{h} \tag{15}$$

Substitute equation(15) in equation(14)

$$\frac{x_{n+1}-x_n}{h} = -\frac{x_n y_n}{1-{y_n}^2} \tag{16}$$

$$x_{n+1}-x_n = -h\left(\frac{x_n y_n}{1-{y_n}^2}\right) \tag{17}$$

$$x_{n+1} = x_n - h\left(\frac{x_n y_n}{1-{y_n}^2}\right) \tag{18}$$

The initial conditions are given as: $x_0 = 1$, $y_0 = 0$, $h = 0.003$. Using the recurrence relation (17), we compute values of x_n and y_n. These values can be used to approximate the solution numerically for a given range of x.

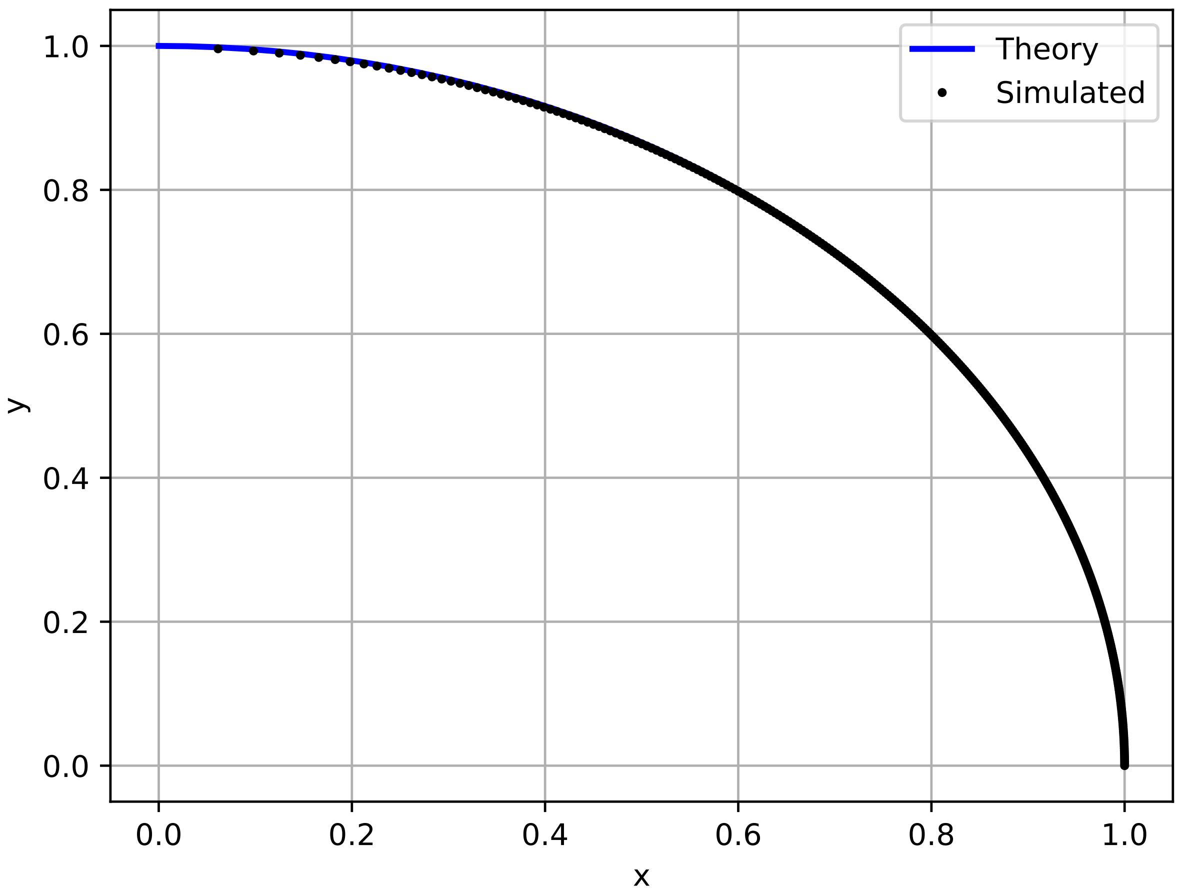

Fig. 0. Solution of given DE