

NCERT - 8.3.17

EE224BTECH11044 - Muthyala koushik

I. APPLICATION OF INTEGRALS

Question: The area bounded by the curve $y = x|x|$, x -axis and the ordinates $x = -1$ and $x = 1$ is given by

Solution: $y = x^2$ if $x > 0$ and $y = -x^2$ if $x < 0$

$$\text{Area (A)} = \int_{-1}^1 |y| dx \quad (1)$$

(2)

Split the integral at $x = 0$, as the function changes;

$$A = \int_{-1}^0 (-x^2) dx + \int_0^1 x^2 dx \quad (3)$$

$$A = - \int_{-1}^0 (-x^2) dx + \int_0^1 x^2 dx \quad (4)$$

$$A = - \left[\frac{x^3}{3} \right]_{-1}^0 + \left[\frac{x^3}{3} \right]_0^1 \quad (5)$$

$$A = - \left(0 - \left(-\frac{(-1)^3}{3} \right) \right) + \left(\frac{1}{3} - 0 \right) \quad (6)$$

$$A = \frac{1}{3} + \frac{1}{3} \quad (7)$$

$$A = \frac{2}{3} \quad (8)$$

$$A = 0.666666666667 \quad (9)$$

Computational Solution:

Using the trapezoidal rule,

$$J = \int_a^b f(x) dx \approx h \left(\frac{1}{2} f(a) + f(x_1) + f(x_2) \cdots + f(x_{n-1}) + \frac{1}{2} f(b) \right) \quad (10)$$

$$h = \frac{b-a}{n} \quad (11)$$

$$A = A_n, \text{ where, } A_{i+1} = A_i + h \frac{f(x_{n+1}) + f(x_n)}{2} \quad (12)$$

$$A_{i+1} = A_i + \frac{h}{2} (f(x_{n+1}) + f(x_n)) \quad (13)$$

$$x_{n+1} = x_n + h \quad (14)$$

$$h = \frac{2}{n} \quad (15)$$

(16)

Area calculated from c code is 0.6666679999999998