NCERT -10.3.4.2.2

EE224BTECH11044 - Muthyala koushik

I. DIFFERENTIAL EQUATIONS

Question: Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?

Solution: Let Nuri's current age is x and Sonu's current age is y:

- Five years ago, Nuri was thrice as old as Sonu.

$$x - 5 = 3(y - 5) \implies x - 3y = -10 \tag{1}$$

- Ten years later, Nuri will be twice as old as Sonu.

$$x + 10 = 2(y + 10) \implies x - 2y = 10 \tag{2}$$

The system of equations can be written in matrix form as:

$$\begin{pmatrix} 1 & -3 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -10 \\ 10 \end{pmatrix}. \tag{3}$$

Let:

$$A = \begin{pmatrix} 1 & -3 \\ 1 & -2 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} -10 \\ 10 \end{pmatrix}. \tag{4}$$

Doolittle's Algorithm:
The LU decomposition splits A into a lower triangular matrix L and an upper triangular matrix U such that:

$$A = LU \tag{5}$$

Where:

$$L = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ L_{21} & 1 & 0 & \cdots & 0 \\ L_{31} & L_{32} & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ L_{n1} & L_{n2} & L_{n3} & \cdots & 1 \end{pmatrix}, \quad U = \begin{pmatrix} U_{11} & U_{12} & U_{13} & \cdots & U_{1n} \\ 0 & U_{22} & U_{23} & \cdots & U_{2n} \\ 0 & 0 & U_{33} & \cdots & U_{3n} \\ \vdots & \vdots & \vdots & \ddots & U_{n-1,n} \\ 0 & 0 & 0 & \cdots & U_{nn} \end{pmatrix}. \tag{6}$$

The Doolittle algorithm is computed as follows:
Elements of the U Matrix: For each column j:

$$U_{ij} = A_{ij} \quad \text{if } i = 0, \tag{7}$$

$$U_{ij} = A_{ij} - \sum_{k=0}^{i-1} L_{ik} U_{kj} \quad \text{if } i > 0. \tag{8}$$

Elements of the L Matrix: For each row i:

$$L_{ij} = \frac{A_{ij}}{U_{jj}} \quad \text{if } j = 0, \tag{9}$$

$$L_{ij} = \frac{A_{ij} - \sum_{k=0}^{j-1} L_{ik} U_{kj}}{U_{jj}} \quad \text{if } j > 0. \tag{10}$$

We decompose the matrix A into the product of a lower triangular matrix L and an upper triangular matrix U, i.e., $A = LU$.

Let:

$$L = \begin{pmatrix} 1 & 0 \\ l_{21} & 1 \end{pmatrix}, \quad U = \begin{pmatrix} u_{11} & u_{12} \\ 0 & u_{22} \end{pmatrix}. \tag{11}$$

We compute the elements of L and U:

$$u_{11} = a_{11} = 1, \tag{12}$$

$$u_{12} = a_{12} = -3, \tag{13}$$

$$l_{21} = \frac{a_{21}}{u_{11}} = \frac{1}{1} = 1, \tag{14}$$

$$u_{22} = a_{22} - l_{21} u_{12} = -2 - 1 \cdot (-3) = 1. \tag{15}$$

Thus, the LU decomposition is:

$$L = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \quad U = \begin{pmatrix} 1 & -3 \\ 0 & 1 \end{pmatrix}. \tag{16}$$

First, solve $L\mathbf{y} = \mathbf{b}$ for $\mathbf{y}$:

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -10 \\ 10 \end{pmatrix}. \tag{17}$$

This gives:

$$y_1 = -10, \tag{18}$$

$$y_1 + y_2 = 10 \implies -10 + y_2 = 10 \implies y_2 = 20. \tag{19}$$

Thus:

$$\mathbf{y} = \begin{pmatrix} -10 \\ 20 \end{pmatrix}. \tag{20}$$

Next, solve $U\mathbf{x} = \mathbf{y}$ for $\mathbf{x}$:

$$\begin{pmatrix} 1 & -3 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -10 \\ 20 \end{pmatrix}. \tag{21}$$

This gives:

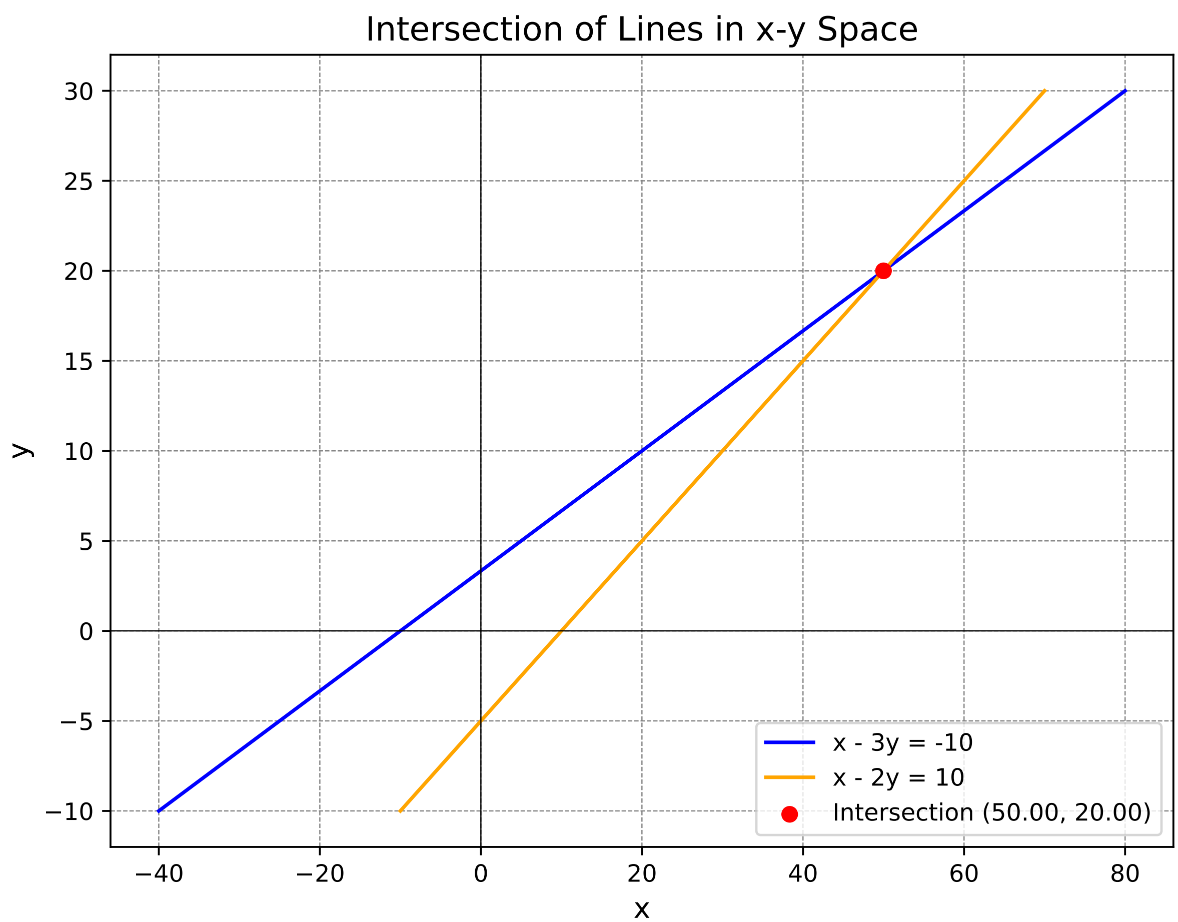
$$y = 20, \tag{22}$$

$$x - 3y = -10 \implies x - 3(20) = -10 \implies x = 50. \tag{23}$$

Final Answer:

$$x = 50, \quad y = 20. \tag{24}$$

So, Nuri is 50 years old and Sonu is 20 years old.



Intersection of Lines in x-y Space
30
25
20
15
10
5
0
−5
−10
y
−40
−20
0
20
40
60
80
x
x - 3y = -10
x - 2y = 10
Intersection (50.00, 20.00)