NCERT -11.16.1.7

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I. PROBABILITY

Question: One die of red colour, one of white colour and one of blue colour are placed in a bag. One die is selected at random and rolled, its colour and the number on its uppermost face is noted. Describe the sample space.

Solution:

1) **Total Number of Possible Outcomes**
   Let X be the random variable representing the outcome of the experiment. The sample space is:

   $$S = \{(C, n) \mid C \in \{R, W, B\}, \ n \in \{1, 2, 3, 4, 5, 6\}\}. \tag{1}$$

   Since there are 3 colours and 6 numbers per die,

   $$|S| = 3 \times 6 = 18. \tag{2}$$

2) **Probability Mass Function (PMF)**
   Because each die is equally likely to be chosen and every face is equally likely to occur, each outcome in S has the same probability. Thus, the PMF of X is:

   $$P_X(C, n) = \begin{cases} \dfrac{1}{18}, & \text{if } (C, n) \in S, \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

3) **Cumulative Distribution Function (CDF)**
   The CDF for the discrete random variable X is defined as

   $$F_X(C, n) = P(X \leq (C, n))$$

   If the outcomes are ordered lexicographically (for example, with $R < W < B$ and for each colour, $1 < 2 < \cdots < 6$), and if (C, n) is the k-th outcome in this ordering, then:

   $$F_X(C, n) = \frac{k}{18}. \tag{4}$$

4) **Numerical Solution (Monte Carlo Simulation)**
   An alternative approach is to verify these probabilities via a Monte Carlo simulation. By repeatedly simulating the process of randomly selecting a die, rolling it, and recording the outcome, one can estimate the PMF and CDF empirically.

Probability Mass Function (PMF) of Combined Outcomes

Cumulative Distribution Function (CDF) of Combined Outcomes