

NCERT - 10.4.ex.4

EE224BTECH11044 - Muthyala koushik

I. QUADRATIC EQUATIONS

Question: Find the roots of the quadratic equation $3x^2 - 2\sqrt{6}x + 2 = 0$

Solution: The given equation:

$$3x^2 - 2\sqrt{6}x + 2 = 3x^2 - \sqrt{6}x - \sqrt{6}x + 2 \quad (1)$$

$$= \sqrt{3}x(\sqrt{3}x - \sqrt{2}) - \sqrt{2}(\sqrt{3}x - \sqrt{2}) \quad (2)$$

$$= (\sqrt{3}x - \sqrt{2})(\sqrt{3}x - \sqrt{2}) \quad (3)$$

So, the roots of the equation are the values of x for which

$$(\sqrt{3}x - \sqrt{2})(\sqrt{3}x - \sqrt{2}) = 0 \quad (4)$$

$$\sqrt{3}x - \sqrt{2} = 0 \quad (5)$$

$$x = \sqrt{\frac{2}{3}} \quad (6)$$

Therefore, the roots of $3x^2 - 2\sqrt{6}x + 2 = 0$ are $\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}$

Solution by the method of Fixed point Iteration

Rearrange the equation to $x = g(x)$

$$x = \frac{1}{3} \left(2\sqrt{6} - \frac{2}{x} \right) \quad (7)$$

This gives the iteration function:

$$g(x) = \frac{1}{3} \left(2\sqrt{6} - \frac{2}{x} \right) \quad (8)$$

Iteration: Use the formula repeatedly:

$$x_{n+1} = \frac{1}{3} \left(2\sqrt{6} - \frac{2}{x_n} \right) \quad (9)$$

Stop when $|x_{n+1} - x_n| < \epsilon$

Root: 0.8173993362392574, Iterations: 900

Actual Root: 0.81649658092

Solution by the Newton-Raphson method: we have;

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad (10)$$

$$x_{n+1} = x_n - \frac{3x^2 - 2\sqrt{6}x + 2}{6x - 2\sqrt{6}} \quad (11)$$

Iterating and updating the value of x_n , we can obtain the roots of the quadratic equation.

Newton-Raphson Root: 0.8164972809158475, Iterations: 18

Actual Root: 0.81649658092

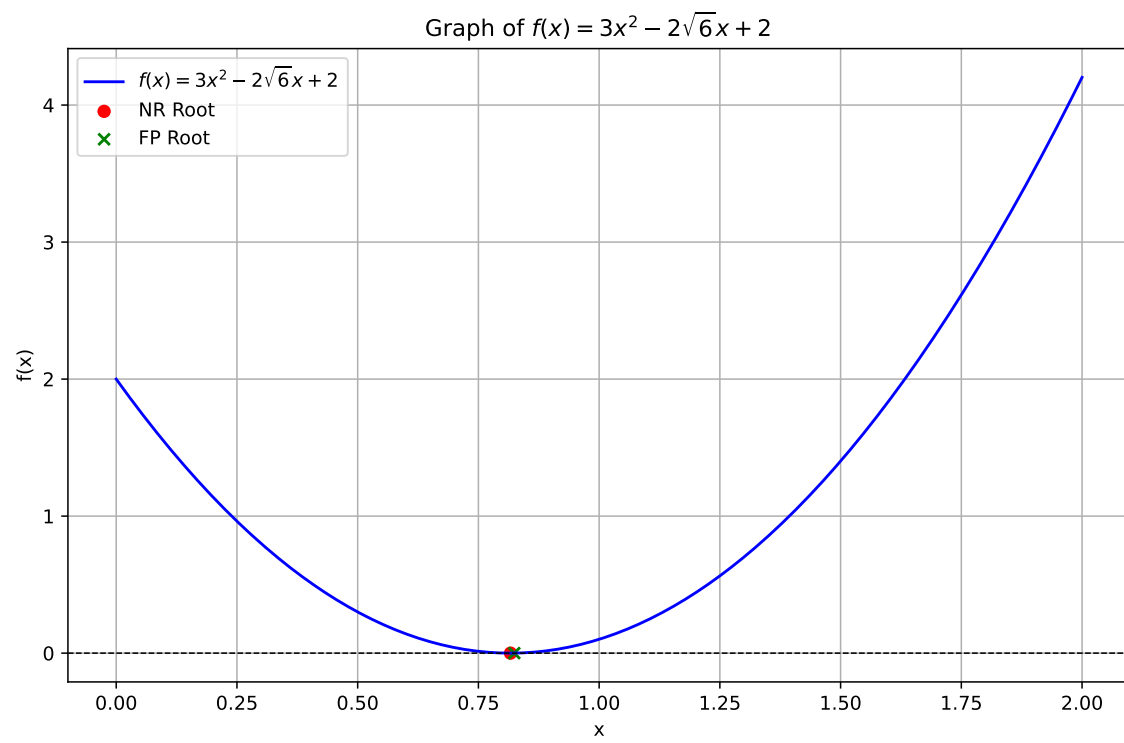


Fig. 0. Solution of given DE