Assignment-4

EE224BTECH11044 - Muthyala Koushik

I. Intersection of Conics

Question: Using integration, find the area of the region bounded by the line $y = 3x + 2$, the $x - axis$ and the ordinates $x = -2$ and $x = 1$
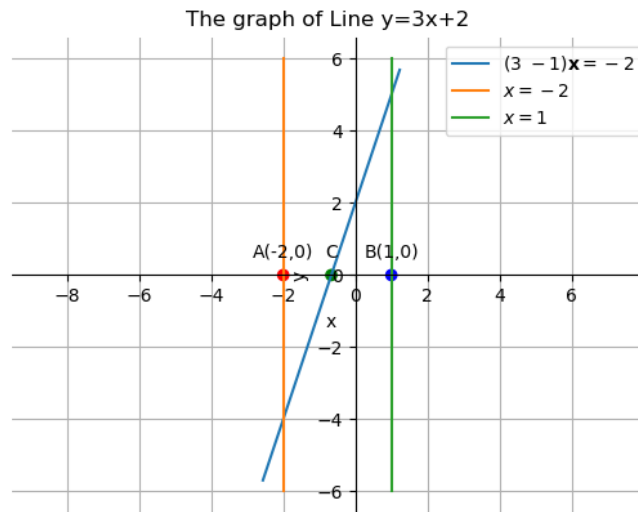
Solution:

Fig. 1: The plot of the Line $y = 3x + 2$

The line $y = 3x + 2$ can be represented as $(3 - 1)$x=-2

Let $\mathbf{C}\begin{pmatrix} a \\ 0 \end{pmatrix}$ be the point on line $y = 3x + 2$ which intersect on x-axis:

$$(3 - 1)\begin{pmatrix} a \\ 0 \end{pmatrix} = -2 \tag{1}$$

$$3a = -2 \iff a = -\frac{2}{3} \tag{2}$$

From the Fig. 1 the area of the line between lines x=-2 and x=1 is given by $A = A_1 + A_2$
A_1:Area between $x = -2$ to $x = -2/3$, A_2:Area between $x = -2/3$ to $x = 1$

$$A = \int_{-2}^{-\frac{2}{3}} |3x + 2| \, dx + \int_{-\frac{2}{3}}^{1} |3x + 2| \, dx \tag{3}$$

$$A = \int_{-2}^{-\frac{2}{3}} -(3x + 2) \, dx + \int_{-\frac{2}{3}}^{1} (3x + 2) \, dx \tag{4}$$

$$A = \left[-\left(\frac{3}{2}x^2 + 2x \right) \right]_{-2}^{-\frac{2}{3}} + \left[\left(\frac{3}{2}x^2 + 2x \right) \right]_{-\frac{2}{3}}^{1} \tag{5}$$

$$A = \left(\frac{2}{3} - (-2) \right) + \left(\frac{7}{2} - \left(-\frac{2}{3} \right) \right) \tag{6}$$

$$A = \left(\frac{2}{3} + 2 \right) + \left(\frac{7}{2} + \frac{2}{3} \right) \tag{7}$$

$$A = \frac{8}{3} + \frac{25}{6} \tag{8}$$

$$A = \frac{41}{6} \tag{9}$$

The area of the region bounded by the line $y = 3x + 2$, the $x - axis$ and the ordinates $x = -2$ and $x = 1$ is $\boxed{\dfrac{41}{6}}$