Assignment-6

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I. Section-A:JEE Main 2022-29 June-shift-2

1) Let α be a root of the equation $1 + x^2 + x^4 = 0$. Then the value of $\alpha^{1011} + \alpha^{2022} - \alpha^{3033}$ is equal to:
   a) 1

   b) α

   c) $1 + \alpha$

   d) $1 + 2\alpha$

2) Let $\arg(z)$ represent the principal argument of the complex number z. Then, $|z| = 3$ and $\arg(z-1) - \arg(z+1) = \frac{\pi}{4}$ intersect:
   a) Exactly at one point

   b) Exactly at two points

   c) Nowhere

   d) At infinitely many points.

3) Let $A = \begin{pmatrix} 2 & -1 \\ 0 & 2 \end{pmatrix}$. If $B = I - {}^5C_1\,(\operatorname{adj} A) + {}^5C_2(\operatorname{adj} A)^2 - \cdots - {}^5C_5(\operatorname{adj} A)^5$, then the sum of all elements of the matrix B is:
   a) -5

   b) -6

   c) -7

   d) -8

4) The sum of the infinite series $1 + \frac{5}{6} + \frac{12}{6^2} + \frac{22}{6^3} + \frac{35}{6^4} + \frac{51}{6^5} + \frac{70}{6^6} + \ldots$ is equal to:
   a) $\frac{425}{216}$

   b) $\frac{429}{216}$

   c) $\frac{288}{125}$

   d) $\frac{280}{125}$

5) The value of $\lim_{x\to 1} \frac{(x^2-1)\sin^2 \pi x}{x^4-2x^3+2x-1}$ is equal to:
   a) $\frac{\pi^2}{6}$

   b) $\frac{\pi^2}{3}$

   c) $\frac{\pi^2}{2}$

   d) π^2

6) Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = (x-3)^{n_1} + (x-5)^{n_2}$, $n_1, n_2 \in \mathbb{N}$. Then, which of the following is NOT true?
   a) For $n_1 = 3$, $n_2 = 4$, there exists $\alpha \in (3,5)$ where f attains local maxima.

   b) For $n_1 = 4$, $n_2 = 3$, there exists $\alpha \in (3,5)$ where f attains local maxima.

   c) For $n_1 = 3$, $n_2 = 5$, there exists $\alpha \in (3,5)$ where f attains local maxima.

   d) For $n_1 = 4$, $n_2 = 6$, there exists $\alpha \in (3,5)$ where f attains local maxima.

7) Let f be a real valued continuous function on $[0,1]$ and $f(x) = x + \int_0^1 (x-t)\, f(t)\, dt$. Then, which of the following points (x,y) lies on the curve $y = f(x)$?
   a) $(2,4)$

   b) $(1,2)$

   c) $(4,17)$

   d) $(6,8)$

8) If $\int_0^2 \left(\sqrt{2x} - \sqrt{2x - x^2}\right) dx = \int_0^1 \left(1 - \sqrt{1-y^2} - \frac{y^2}{2}\right) dy + \int_1^2 \left(2 - \frac{y^2}{2}\right) dy + I$ then I equals to:
   a) $\int_0^1 \left(1 + \sqrt{1-y^2}\right) dy$

   b) $\int_0^1 \left(\frac{y^2}{2} - \sqrt{1-y^2} + 1\right) dy$

   c) $\int_0^1 \left(1 - \sqrt{1-y^2}\right) dy$

   d) $\int_0^1 \left(\frac{y^2}{2} + \sqrt{1-y^2} + 1\right) dy$

9) If $y = y(x)$ is the solution of the differential equation $\left(1 + e^{2x}\right) \frac{dy}{dx} + 2\left(1 + y^2\right) e^x = 0$ and $y(0) = 0$, then $6\left(y'(0) + \left(y\left(\log_e \sqrt{3}\right)\right)^2\right)$ is equal to:
   a) 2

   b) -2

   c) -4

   d) -1

10) Let $P : y^2 = 4ax$, $a > 0$ be a parabola with focus S. Let the tangents to the parabola P make an angle of $\frac{\pi}{4}$ with the line $y = 3x + 5$ touch the parabola at A and B. Then the value of a for which A, B and S are collinear is:

a) 8 only

b) 2 only

c) $\frac{1}{4}$ only

d) any $a > 0$

11) Let a triangle ABC be inscribed in the circle $x^2 - \sqrt{2}(x + y) + y^2 = 0$ such that $\angle BAC = \frac{\pi}{2}$. If the length of side AB is $\sqrt{2}$, then the area of the $\triangle ABC$ is equal to:

a) $\frac{(\sqrt{2}+\sqrt{6})}{3}$

b) $\frac{(\sqrt{6}+\sqrt{3})}{2}$

c) $\frac{(3+\sqrt{3})}{4}$

d) $\frac{(\sqrt{6}+2\sqrt{3})}{4}$

12) Let $\frac{x-2}{3} = \frac{y+1}{-2} = \frac{z+3}{-1}$ lie on the plane $px - qy + z = 5$, for some $p, q \in \mathbb{R}$. The shortest distance of the plane from the origin is:

a) $\sqrt{\frac{3}{109}}$

b) $\sqrt{\frac{5}{142}}$

c) $\sqrt{\frac{5}{71}}$

d) $\sqrt{\frac{1}{142}}$

13) The distance of the origin from the centroid of the triangle whose two sides have the equations $x - 2y + 1 = 0$ and $2x - y - 1 = 0$ and whose orthocenter is $\left(\frac{7}{3}, \frac{7}{3}\right)$ is:

a) $\sqrt{2}$

b) 2

c) $2\sqrt{2}$

d) 4

14) Let $\mathbf{Q}$ be the mirror image of the point $\mathbf{P}(1, 2, 1)$ with respect to the plane $x + 2y + 2z = 16$. Let T be a plane passing through the point $\mathbf{Q}$ and contains the line $\mathbf{r} = -\hat{k} + \lambda\left(\hat{i} + \hat{j} + 2\hat{k}\right), \lambda \in \mathbb{R}$. Then, which of the following points lies on T?

a) $(2, 1, 0)$

b) $(1,2,1)$

c) $(1,2,2)$

d) $(1,3,2)$

15) Let $\mathbf{A},\mathbf{B},\mathbf{C}$ be three points whose position vectors respectively are:
$\mathbf{a} = \hat{i} + 4\hat{j} + 3\hat{k}$
$\mathbf{b} = 2\hat{i} + \alpha\hat{j} + 4\hat{k}, \alpha \in \mathbb{R}$
$\mathbf{c} = 3\hat{i} - 2\hat{j} + 5\hat{k}$
If α is the smallest positive integer for which $\mathbf{a},\mathbf{b},\mathbf{c}$ are non-collinear, then the length of the median, in $\triangle\mathbf{ABC}$, through $\mathbf{A}$ is:

a) $\frac{\sqrt{82}}{2}$

b) $\frac{\sqrt{62}}{2}$

c) $\frac{\sqrt{69}}{2}$

d) $\frac{\sqrt{66}}{2}$