Assignment-1

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1.Section-B : JEE Main/AIEEE

4) If $f : \mathbb{R} \to \mathbb{R}$ satisfies $f(x+y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$ and $f(1) = 7$, then $\sum_{r=1}^{n} f(r)$ is (2003)
   a) $\frac{7n(n+1)}{2}$
   b) $\frac{7n}{2}$
   c) $\frac{7(n+1)}{2}$
   d) $7n + (n+1)$
5) The function f from the set of natural numbers to integers is defined by (2003)
   $f(n) = \begin{cases} \frac{n-1}{2} & \text{when } n \text{ is odd,} \\ -\frac{n}{2} & \text{when } n \text{ is even.} \end{cases}$ is
   a) neither one-one nor onto
   b) one-one but not onto
   c) onto but not one-one
   d) one-one and onto both.
6) The range of the function $f(x) = P(7-x, x-3)$ is (2004)
   a) $\{1,2,3,4,5\}$
   b) $\{1,2,3,4,5,6\}$
   c) $\{1,2,3,4\}$
   d) $\{1,2,3\}$
7) If $f : R \to S$, defined by $f(x) = \sin x - \sqrt{3}\cos x + 1$,is onto,then the interval of S is (2004)
   a) $[-1,3]$
   b) $[-1,1]$
   c) $[0,1]$
   d) $[0,3]$
8) The graph of the function $y = f(x)$ is symmertrical about the line x=2,then (2004)
   a) $f(x) = -f(-x)$
   b) $f(2+x) = f(2-x)$
   c) $f(x) = f(-x)$
   d) $f(x+2) = f(x-2)$
9) The domain of the function $f(x) = \frac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ is (2004)
   a) $[1,3]$
   b) $[2,3)$
   c) $[1,2]$
   d) $[2,3]$
10) Let $f : (-1,1) \to B$,be a function defined by $f(x) = tan^{-1}\frac{2x}{1-x^2}$,then f is both one-one and onto when B is the interval (2005)
   a) $\left(0, \frac{\pi}{2}\right)$
   b) $\left[0, \frac{\pi}{2}\right)$
   c) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
   d) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
11) A function is macthed below against an interval where it is supposed to increasing.Which of the following pairs is incorrectly matcher? (2005)

Interval	**Function**
a) $(-\infty, \infty)$	$x^3 - 3x^2 + 3x + 3$
b) $[2, \infty)$	$2x^3 - 3x^2 + 3x + 3$
c) $\left(-\infty, \frac{1}{3}\right]$	$3x^2 - 2x + 1$
d) $(-\infty, -4)$	$x^3 + 6x^2 + 6$

12) A real valued function $f(x)$ satisfies the functional equation
   $$f(x-y) = f(x)f(y) - f(a-x)f(a+y)$$
   where a given constant and $f(0)$=1,$f(2a-x)$ is equal to (2005)
   a) $-f(x)$
   b) $f(x)$
   c) $f(a) + f(a-x)$
   d) $f(-x)$
13) The Largest interval lying in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$for which the function, $f(x) = 4^{-x^2} + \cos^{-1}\left(\frac{x}{2} - 1\right) + \log(\cos x)$,is defined,is (2007)
   a) $\left[-\frac{\pi}{4}, \frac{\pi}{2}\right)$

b) $\left[0, \frac{\pi}{2}\right)$

c) $[0, \pi]$

d) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

14) Let $f : N \rightarrow Y$ be a function defined as $f(x) = 4x+3$ where Y=$\{y \in \mathbb{N} : y = 4x+3 for some x \in \mathbb{N}\}$.Show that f is invertible and its inverse is (2008)

a) $g(y) = \frac{3y+4}{3}$

b) $g(y) = 4 + \frac{y+3}{4}$

c) $g(y) = \frac{y+3}{4}$

d) $g(y) = \frac{y-3}{4}$

15) Let $f(x) = (x+1)^2 - 1, x \leq -1$
Statement-1: The set $\{x$:$f(x)$=$f^{-1}(x)$=$\{0,-1\}$
Statement-2:f is a bijection. (2009)

a) Statement-1 is true,Statement-2 is true.Statement-2 is not a correct explanation for Statement-1.
b) Statement-1 is true,Statement-2 is flase.
c) Statement-1 is false,Statement-2 is true.
d) Statement-1 is true,Statement-2 is true.Statement-2 is not a correct explanation for Statement-1.

16) For real x,let $f(x) = x^3 + 5x + 1$,then (2009)

a) f is onto $\mathbb{R}$ but not one-one
b) f is one-one and onto $\mathbb{R}$
c) f is neither one-one nor onto $\mathbb{R}$
d) f is one-one but not onto $\mathbb{R}$

17) The domain of the function $f(x) = \frac{1}{\sqrt{|x|-x}}$ is (2011)

a) $(0, \infty)$
b) $(-\infty, 0)$
c) $(-\infty, \infty) - \{0\}$
d) $(-\infty, \infty)$

18) $x \in \mathbb{R} - \{0, 1\}$,let $f_1(x) = \frac{1}{x}$,$f_2(x) = 1 - x$ and $f_3(x) = \frac{1}{1-x}$ be the three given functions.If a function,$J(X)$ satisfies $(f_2 o J o f_1)(x)$=$f_3(x)$ then J(x) is equal to: (JEE M 2019-9 Jan(M))

a) $f_3(x)$
b) $f_3(x)$
c) $f_2(x)$
d) $f_1(x)$