

# Assignment-1

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## 1.Section-B : JEE Main/AIEEE

- 4) If  $f : \mathbb{R} \rightarrow \mathbb{R}$  satisfies  $f(x+y) = f(x) + f(y)$  for all  $x, y \in \mathbb{R}$  and  $f(1) = 7$ , then  $\sum_{r=1}^n f(r)$  is (2003)
- $\frac{7n(n+1)}{2}$
  - $\frac{7n}{2}$
  - $\frac{7(n+1)}{2}$
  - $7n + (n+1)$
- 5) The function  $f$  from the set of natural numbers to integers is defined by (2003)
- $$f(n) = \begin{cases} \frac{n-1}{2} & \text{when } n \text{ is odd,} \\ -\frac{n}{2} & \text{when } n \text{ is even.} \end{cases} \text{ is}$$
- neither one-one nor onto
  - one-one but not onto
  - onto but not one-one
  - one-one and onto both.
- 6) The range of the function  $f(x) = {}^{7-x}P_{x-3}$  is (2004)
- $\{1, 2, 3, 4, 5\}$
  - $\{1, 2, 3, 4, 5, 6\}$
  - $\{1, 2, 3, 4\}$
  - $\{1, 2, 3\}$
- 7) If  $f : R \rightarrow S$ , defined by  $f(x) = \sin x - \sqrt{3} \cos x + 1$ , is onto, then the interval of  $S$  is (2004)
- $[-1, 3]$
  - $[-1, 1]$
  - $[0, 1]$
  - $[0, 3]$
- 8) The graph of the function  $y = f(x)$  is symmetrical about the line  $x = 2$ , then (2004)
- $f(x) = -f(-x)$
  - $f(2+x) = f(2-x)$
  - $f(x) = f(-x)$
  - $f(x+2) = f(x-2)$
- 9) The domain of the function  $f(x) = \frac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$  is (2004)
- $[1, 3]$
  - $[2, 3]$
  - $[1, 2]$
  - $[2, 3]$
- 10) Let  $f : (-1, 1) \rightarrow B$ , be a function defined by  $f(x) = \tan^{-1} \frac{2x}{1-x^2}$ , then  $f$  is both one-one and onto when  $B$  is the interval (2005)
- $(0, \frac{\pi}{2})$
  - $[0, \frac{\pi}{2})$
  - $[-\frac{\pi}{2}, \frac{\pi}{2}]$
  - $(-\frac{\pi}{2}, \frac{\pi}{2})$
- 11) A function is matched below against an interval where it is supposed to be increasing. Which of the following pairs is incorrectly matched? (2005)
- | Interval                    | Function               |
|-----------------------------|------------------------|
| a) $(-\infty, \infty)$      | $x^3 - 3x^2 + 3x + 3$  |
| b) $[2, \infty)$            | $2x^3 - 3x^2 + 3x + 3$ |
| c) $(-\infty, \frac{1}{3}]$ | $3x^2 - 2x + 1$        |
| d) $(-\infty, -4)$          | $x^3 + 6x^2 + 6$       |
- 12) A real valued function  $f(x)$  satisfies the functional equation
- $$f(x-y) = f(x)f(y) - f(a-x)f(a+y)$$
- where  $a$  is a given constant and  $f(0) = 1, f(2a-x)$  is equal to (2005)
- $-f(x)$
  - $f(x)$
  - $f(a) + f(a-x)$
  - $f(-x)$
- 13) The Largest interval lying in  $(-\frac{\pi}{2}, \frac{\pi}{2})$  for which the function,  $f(x) = 4^{-x^2} + \cos^{-1}(\frac{x}{2} - 1) + \log(\cos x)$ , is defined, is (2007)
- $[-\frac{\pi}{4}, \frac{\pi}{2})$

b)  $[0, \frac{\pi}{2})$

c)  $[0, \pi]$

d)  $(-\frac{\pi}{2}, \frac{\pi}{2})$

- 14) Let  $f : N \rightarrow Y$  be a function defined as  $f(x) = 4x + 3$  where  $Y = \{y \in \mathbb{N} : y = 4x + 3 \text{ for some } x \in \mathbb{N}\}$ . Show that  $f$  is invertible and its inverse is (2008)

a)  $g(y) = \frac{3y+4}{3}$

b)  $g(y) = 4 + \frac{y+3}{4}$

c)  $g(y) = \frac{y+3}{4}$

d)  $g(y) = \frac{y-3}{4}$

- 15) Let  $f(x) = (x+1)^2 - 1, x \leq -1$

**Statement-1:** The set

$$\{x : f(x) = f^{-1}(x) = \{0, -1\}\}$$

**Statement-2:**  $f$  is a bijection. (2009)

- a) Statement-1 is true, Statement-2 is true. Statement-2 is not a correct explanation for Statement-1.  
 b) Statement-1 is true, Statement-2 is false.  
 c) Statement-1 is false, Statement-2 is true.  
 d) Statement-1 is true, Statement-2 is true. Statement-2 is not a correct explanation for Statement-1.

- 16) For real  $x$ , let  $f(x) = x^3 + 5x + 1$ , then (2009)

- a)  $f$  is onto  $\mathbb{R}$  but not one-one  
 b)  $f$  is one-one and onto  $\mathbb{R}$   
 c)  $f$  is neither one-one nor onto  $\mathbb{R}$   
 d)  $f$  is one-one but not onto  $\mathbb{R}$

- 17) The domain of the function  $f(x) = \frac{1}{\sqrt{|x|-x}}$  is (2011)

- a)  $(0, \infty)$   
 b)  $(-\infty, 0)$   
 c)  $(-\infty, \infty) - \{0\}$   
 d)  $(-\infty, \infty)$

- 18) For  $x \in \mathbb{R} - \{0, 1\}$ , let  $f_1(x) = \frac{1}{x}, f_2(x) = 1 - x$  and  $f_3(x) = \frac{1}{1-x}$  be the three given functions. If a function,  $J(X)$  satisfies  $(f_2 \circ J \circ f_1)(x) = f_3(x)$  then  $J(x)$  is equal to: (JEE M 2019-9 Jan(M))

- a)  $f_3(x)$   
 b)  $f_3(x)$   
 c)  $f_2(x)$   
 d)  $f_1(x)$