Assignment-7

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I. Section-A:JEE Main 2022-29 June-shift-2

1) The set of all α, for which the vectors $\mathbf{a} = \alpha t\hat{i} + 6\hat{j} - 3\hat{k}$ and $\mathbf{b} = t\hat{i} - 2\hat{j} - 2\alpha t\hat{k}$ are inclined at an obtuse angle for all $t \in \mathbb{R}$, is
   a) $\left(-\frac{4}{3}, 1\right)$
   b) $\left(-\frac{4}{3}\ ,\ 0\right]$
   c) $[0\ ,\ 1)$
   d) $(-2\ ,\ 0]$

2) Let $\mathbf{P}(x, y, z)$ be a point in the first octant, whose projection in the xy-plane is the point $\mathbf{Q}$. Let $OP = \gamma$; the angle between OQ and the positive x-axis be θ; and the angle between OP and the positive z-axis be ϕ, where $\mathbf{O}$ is the origin. Then the distance of $\mathbf{P}$ from x-axis is
   a) $\gamma\sqrt{1 + \cos^2\phi \sin^2\theta}$
   b) $\gamma\sqrt{1 - \sin^2\phi \cos^2\theta}$
   c) $\gamma\sqrt{1 + \cos^2\theta \sin^2\phi}$
   d) $\gamma\sqrt{1 - \sin^2\theta \cos^2\phi}$

3) Let the circles $C_1 : (x - \alpha)^2 + (y - \beta)^2 = {r_1}^2$ and $C_2 : (x - 8)^2 + \left(y - \frac{15}{2}\right)^2 = {r_2}^2$ touch each other externally at the point $(6, 6)$. If the point $(6, 6)$ divides the line segment joining the centers of the circles C_1 and C_2 internally in the ratio $2 : 1$, then $(\alpha + \beta) + 4\left({r_1}^2 + {r_2}^2\right)$ equals
   a) 110
   b) 125
   c) 145
   d) 130

4) Let $I(x) = \int \frac{6}{\sin^2 x(1-\cot x)^2}dx$. If $I(0) = 3$, then $I\left(\frac{\pi}{12}\right)$ is equal to
   a) $2\sqrt{3}$
   b) $6\sqrt{3}$

c) $\sqrt{3}$

d) $3\sqrt{3}$

5) If $\sin x = -\frac{3}{5}$, where $\pi < x < \frac{3\pi}{2}$, then $80\left(\tan^2 x - \cos x\right)$ is equal to

a) 19

b) 18

c) 109

d) 108

6) Let z be a complex number such that $|z+2| = 1$ and $\mathrm{Im}\left(\frac{z+1}{z+2}\right) = \frac{1}{5}$. Then the value of $\left|\mathrm{Re}\left(\overline{z+2}\right)\right|$ is

a) $\frac{2\sqrt{6}}{5}$

b) $\frac{24}{5}$

c) $\frac{\sqrt{6}}{5}$

d) $\frac{1+\sqrt{6}}{5}$

7) Let $f(x) = 4\cos^3 x + 3\sqrt{3}\cos^2 x - 10$. The number of points of local maxima of f in interval $(0, 2\pi)$ is

a) 4

b) 1

c) 2

d) 3

8) For the function $f(x) = (\cos x) - x + 1$, $x \in \mathbb{R}$, between the following two statements
(S_1) $f(x) = 0$ for only one value of x in $[0, \pi]$.
(S_2) $f(x)$ is decreasing in $\left[0, \frac{\pi}{2}\right]$ and increasing in $\left[\frac{\pi}{2}, \pi\right]$.

a) Both (S_1) and (S_2) are incorrect.

b) Only (S_1) is correct.

c) Both (S_1) and (S_2) are correct.

d) Only (S_2) is correct.

9) Let $A = \begin{bmatrix} 2 & a & 0 \\ 1 & 3 & 1 \\ 0 & 5 & b \end{bmatrix}$. If $A^3 = 4A^2 - A - 21I$, where I is the identity matrix of order 3×3, then $2a + 3b$ is equal to

a) -10

b) -9

c) -12

d) -13

10) Let $[t]$ be the greatest integer less than or equal to t. Let A be the set of all prime factors of 2310 and $f : A \to \mathbb{Z}$ be the function $f(x) = \left[\log_2\left(x^2 + \left[\frac{x^3}{5}\right]\right)\right]$. The number of one-to-one functions from A to the range of f is

a) 25

b) 120

c) 20

d) 24

11) Let $y = y(x)$ be the solution of the differential equation $\left(1 + y^2\right) e^{\tan x} dx + \cos^2 x \left(1 + e^{2\tan x}\right) dy = 0, y(0) = 1$. Then $y\left(\frac{\pi}{4}\right)$ is equal to

a) $\frac{1}{e^2}$

b) $\frac{1}{e}$

c) $\frac{2}{e}$

d) $\frac{2}{e^2}$

12) The equation of two sides AB and AC of a triangle ABC are $4x+y = 14$ and $3x-2y = 5$, respectively. The point $\left(2, -\frac{4}{3}\right)$ divides the third side BC internally in the ratio $2 : 1$. The equation of the side BC is

a) $x - 3y - 6 = 0$

b) $x - 6y - 10 = 0$

c) $x + 3y + 2 = 0$

d) $x + 6y + 6 = 0$

13) If the shortest distance between the lines

$$L_1 : \mathbf{r} = (2 + \lambda)\hat{i} + (1 - 3\lambda)\hat{j} + (3 + 4\lambda)\hat{k}, \lambda \in \mathbb{R}$$

$$L_2 : \mathbf{r} = 2(1 + \mu)\hat{i} + 3(1 + \mu)\hat{j} + (5 + \mu)\hat{k}, \in \mathbb{R}$$

is $\frac{m}{\sqrt{n}}$, where $\gcd(m, n) = 1$, then the value of $m + n$ equals

a) 377

b) 390

c) 387

d) 384

14) If the set $R = \{(a, b) : a + 5b = 42, a, b \in \mathbb{R}\}$ has m elements and $\sum_{n=1}^{m} \left(1 - i^{n!}\right) = x+iy$, where $i = \sqrt{-1}$, then the value of $m + x + y$ is

a) 8

b) 5

c) 4

d) 12

15) The value of $k \in \mathbb{N}$ for which the integral $I_n = \int_0^1 \left(1 - x^k\right)^n dx$, $n \in \mathbb{N}$, satisfies $147 I_{20} = 148 I_{21}$ is

a) 8

b) 10

c) 7

d) 14