

Assignment-1

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1.Section-B : JEE Main/AIEEE

- 4) If $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfies $f(x+y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$ and $f(1) = 7$, then $\sum_{r=1}^n f(r)$ is (2003)
- $\frac{7n(n+1)}{2}$
 - $\frac{7n}{2}$
 - $\frac{7(n+1)}{2}$
 - $7n + (n+1)$
- 5) The function f from the set of natural numbers to integers is defined by (2003)
- $$f(n) = \begin{cases} \frac{n-1}{2} & \text{when } n \text{ is odd,} \\ -\frac{n}{2} & \text{when } n \text{ is even.} \end{cases} \text{ is}$$
- neither one-one nor onto
 - one-one but not onto
 - onto but not one-one
 - one-one and onto both.
- 6) The range of the function $f(x) = 7^{-x} P_{x-3}$ is (2004)
- $\{1, 2, 3, 4, 5\}$
 - $\{1, 2, 3, 4, 5, 6\}$
 - $\{1, 2, 3, 4\}$
 - $\{1, 2, 3\}$
- 7) If $f : \mathbb{R} \rightarrow \mathbb{S}$, defined by $f(x) = \sin x - \sqrt{3} \cos x + 1$, is onto, then the interval of \mathbb{S} is (2004)
- $[-1, 3]$
 - $[-1, 1]$
 - $[0, 1]$
 - $[0, 3]$
- 8) The graph of the function $y = f(x)$ is symmetrical about the line $x = 2$, then (2004)
- $f(x) = -f(-x)$
 - $f(2+x) = f(2-x)$
 - $f(x) = f(-x)$
 - $f(x+2) = f(x-2)$
- 9) The domain of the function $f(x) = \frac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ is (2004)
- $[1, 3]$
 - $[2, 3]$
 - $[1, 2]$
 - $[2, 3]$
- 10) Let $f : (-1, 1) \rightarrow B$, be a function defined by $f(x) = \tan^{-1} \frac{2x}{1-x^2}$, then f is both one-one and onto when B is the interval (2005)
- $(0, \frac{\pi}{2})$
 - $[0, \frac{\pi}{2})$
 - $[-\frac{\pi}{2}, \frac{\pi}{2}]$
 - $(-\frac{\pi}{2}, \frac{\pi}{2})$
- 11) A function is matched below against an interval where it is supposed to be increasing. Which of the following pairs is incorrectly matched? (2005)
- | Interval | Function |
|-----------------------------|------------------------|
| a) $(-\infty, \infty)$ | $x^3 - 3x^2 + 3x + 3$ |
| b) $[2, \infty)$ | $2x^3 - 3x^2 + 3x + 3$ |
| c) $(-\infty, \frac{1}{3}]$ | $3x^2 - 2x + 1$ |
| d) $(-\infty, -4)$ | $x^3 + 6x^2 + 6$ |
- 12) A real valued function $f(x)$ satisfies the functional equation
- $$f(x-y) = f(x)f(y) - f(a-x)f(a+y)$$
- where a is a given constant and $f(0) = 1, f(2a-x)$ is equal to (2005)
- $-f(x)$
 - $f(x)$
 - $f(a) + f(a-x)$
 - $f(-x)$
- 13) The Largest interval lying in $(-\frac{\pi}{2}, \frac{\pi}{2})$ for which the function, $f(x) = 4^{-x^2} + \cos^{-1}(\frac{x}{2} - 1) + \log(\cos x)$, is defined, is (2007)
- $[-\frac{\pi}{4}, \frac{\pi}{2})$

b) $[0, \frac{\pi}{2})$

c) $[0, \pi]$

d) $(-\frac{\pi}{2}, \frac{\pi}{2})$

- 14) Let $f : N \rightarrow Y$ be a function defined as $f(x) = 4x + 3$ where $Y = \{y \in \mathbb{N} : y = 4x + 3 \text{ for some } x \in \mathbb{N}\}$. Show that f is invertible and its inverse is (2008)

a) $g(y) = \frac{3y+4}{3}$

b) $g(y) = 4 + \frac{y+3}{4}$

c) $g(y) = \frac{y+3}{4}$

d) $g(y) = \frac{y-3}{4}$

- 15) Let $f(x) = (x+1)^2 - 1, x \leq -1$

Statement-1: The set

$$\{x : f(x) = f^{-1}(x) = \{0, -1\}\}$$

Statement-2: f is a bijection. (2009)

- a) Statement-1 is true, Statement-2 is true. Statement-2 is not a correct explanation for Statement-1.
 b) Statement-1 is true, Statement-2 is false.
 c) Statement-1 is false, Statement-2 is true.
 d) Statement-1 is true, Statement-2 is true. Statement-2 is not a correct explanation for Statement-1.

- 16) For real x , let $f(x) = x^3 + 5x + 1$, then (2009)

- a) f is onto \mathbb{R} but not one-one
 b) f is one-one and onto \mathbb{R}
 c) f is neither one-one nor onto \mathbb{R}
 d) f is one-one but not onto \mathbb{R}

- 17) The domain of the function $f(x) = \frac{1}{\sqrt{|x|-x}}$ is (2011)

- a) $(0, \infty)$
 b) $(-\infty, 0)$
 c) $(-\infty, \infty) - \{0\}$
 d) $(-\infty, \infty)$

- 18) For $x \in \mathbb{R} - \{0, 1\}$, let $f_1(x) = \frac{1}{x}, f_2(x) = 1 - x$ and $f_3(x) = \frac{1}{1-x}$ be the three given functions. If a function, $J(X)$ satisfies $(f_2 \circ J \circ f_1)(x) = f_3(x)$ then $J(x)$ is equal to: (JEE M 2019-9 Jan(M))

- a) $f_3(x)$
 b) $f_3(x)$
 c) $f_2(x)$
 d) $f_1(x)$