1 Matrices

1. Find matrix A such that $2A - 3B + 5C = O$,where B $= \begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix}$ and C$= \begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$

2. If $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 3 & 1 & 1 \end{bmatrix}$, find $A{-}1$. Hence solve the following system of equations $x + y + z = 6, x + 2z = 7, 3x + y + z = 12.$

3. Find the inverse of the following matrix using elementary operations

$$\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 3 & 1 & 1 \end{bmatrix}$$

4. Find the value of x-y,if

$$\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}.$$

5. If A and B are square matrices of the same order 3, such that $\left|A\right| = 2$ and $AB = 2I$, write the value of$\left|B\right|$.

6. Using properties of determinants,prove that $\begin{vmatrix} a^2 + 2a & 2a + 1 & 1 \\ 2a + 1 & a + 2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a - 1)^3$

7. Using the properties of determinants,prove the following:

$$\begin{vmatrix} a + b + c & -c & -b \\ -c & a + b + c & -a \\ -b & -a & a + b + c \end{vmatrix} = 2(a + b)(b + a)(c + a)$$

2 Relations and functions

8. If $f(x) = x + 1$, find $\frac{d}{dx}\left(f \circ f\right)(x)$

9. Examine whether the operation $*$ defined on R by $a * b = ab + 1$ $(i)$is a binary operation.(ii) If a binary operation, is it associative or not?

10. prove that the function$f : N \to N$,defined by $f(x) = x^2 + x + 1$ is one-one but not onto.Find the inverse of $f : N \to S$,where S is range of f.

3 Algebra

11. Solve:$\tan^{-1} 4x + \tan^{-1} 6x = \frac{\pi}{4}$

12. If $\log\left(x^2+y^2\right) = 2\tan^{-1}\frac{y}{x}$,show that $\frac{dy}{dx} = \frac{x+y}{x-y}$

13. If $x^y - y^x = a^b$,find $\frac{dy}{dx}$

14. If $y = \left(\sin^{-1} x\right)^2$,prove that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$

15. Find:$\int \dfrac{\tan^2 x \sec^2 x}{1-\tan^6 x}, dx$

16. Solve for x:$\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$

4 Differentiation

17. Find the order and the degree of the differential equation $x^2\frac{d^2y}{dx^2} = \left\{1\left(\frac{dy}{dx}\right)^2\right\}^4$

18. Form the differential equation representing the family of curves $y = e^{2x}(a+bx)$, where 'a' and 'b' are arbitrary constants.

19. If $x = \cos t + log\tan\left(\frac{t}{2}\right), y = \sin t$,then find the values of $\frac{d^2y}{dt^2}$ and $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$

5 Vectors

20. If the sum of two unit vectors is a unit vector,prove that the magnitude of their difference is $\sqrt{3}$.

21. If $\overrightarrow{a} = 2\hat{i}+3\hat{j}+\hat{k}$, $\overrightarrow{b} = \hat{i}-2\hat{j}+\hat{k}$ and $\overrightarrow{c} = -3\hat{i}+\hat{j}+2\hat{k}$,Find $\begin{bmatrix}\overrightarrow{a} & \overrightarrow{b} & \overrightarrow{c}\end{bmatrix}$

22. If $\hat{i}+\hat{j}+k, 2\hat{i}+5\hat{j}, 3\hat{i}+2\hat{j}-3k, \hat{i}-6\hat{j}-k$ respectively are the position vectors of points A, B, C and D, then find the angle between the straight lines AB and CD. Find whether $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear or not.

23. Find the vector equation of the line which passes through the point $(3,4,5)$ and is parallel to the vector $2\hat{i}+2\hat{j}-3\hat{k}$

24. Find the vector and Cartesian equations of the plane passing through the points $(2,2\text{–}1), (3,4,2)$ and $(7,0,6)$. Also find the vector equation of a plane passing through $(4,3,1)$ and parallel to the plane obtained above.

25. Find the vector equation of the plane that contains the lines $r = \left(\hat{i} + \hat{j}\right) + \lambda \left(\hat{i} + 2\hat{j} – \hat{k}\right)$ and the point (–1, 3, –4). Also,find the length of the perpendicular drawn from the point (2, 1, 4) to the plane, thus obtained .

26. Find the value of λ, so that the lines $\frac{1-x}{3} = \frac{7y-14}{\lambda} = \frac{z-3}{2}$ and $\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles. Also, find whether the lines are intersecting or not.

27. If a line makes angles $90°, 135°, 45°$ with the x,y and z axes respectively,find its direction cosines.

6 Integration

28. Find: $\int \dfrac{\sec^2 x}{\sqrt{\tan^2 x + 4}}, dx$

29. Find: $\int \sin^{-1} 2x, dx$

30. Find: $\int \sqrt{1 - \sin 2x}, dx$, $\frac{\pi}{4} < x < \frac{\pi}{2}$

31. Find: $\int \sin^{-1} 2x, dx$

32. Find: $\int \dfrac{3x + 5}{x^2 + 3x - 18}, dx$

33. prove that $\int_0^a f(x), dx = \int_0^a f(a - x), dx$, hence evaluate $\int_0^\pi \frac{x \sin x}{1+\cos^2 x} dx$.

34. Solve the differential equation :$\left(1 + x^2\right) \frac{dy}{dx} + 2xy - 4x^2 = 0$,subject to the initial condition $y\,(0) = 0$

35. Solve the differential equation:$xdy - ydx = \sqrt{x^2 + y^2} dx$,given that $y = 0$ when $x = 1$.

7 Intersection of Conics

36. Using integration, find the area of triangle ABC, whose vertices are A$(2, 5)$, B$(4, 7)$ and C$(6, 2)$.

37. Find the area of the region lying above x-axis and included between the circle $x^2 + y^2 = 8x$ and inside of the parabola $y^2 = 4x$

38. Find the equation of tangent to the curve $y = \sqrt{3x – 2}$ which is parallel to the line $4x – 2y + 5 = 0$. Also, write the equation of normal to the curve at the point of contact.

8 Probability

39. A die marked $1, 2, 3$ in red and $4, 5, 6$ in green is tossed. Let A be the event "number is even" and B be the event "number is marked red. Find whether the events A and B are independent or not.

40. A die is thrown 6 times. If "getting an odd number" is a "success", what is the probability of (i) 5 successes ? (ii) atmost 5 successes ?

41. A manufacturer has three machine operators A, B and C. The first operator A produces 1% of defective items, whereas the other two operators B and C produces 5% and 7% defective items respectively. A is on the job for 50% of the time, B on the job 30% of the time and C on the job for 20% of the time. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by A ?

42. The random variable X has a probability distribution $P\left(X\right)$ of the following form, where k is some number:

$$P(X = x) = \begin{cases} k, & \text{if } x = 0 \\ 2k, & \text{if } x = 1 \\ 3k, & \text{if } x = 2 \\ 0, & \text{otherwise} \end{cases}$$

Determine the value of k.

9 Optimization

43. A manufacturer has employed 5 skilled men and 10 semi-skilled men and makes two models A and B of an article. The making of one item of model A requires 2 hours work by a skilled man and 2 hours work by a semi-skilled man. One item of model B requires 1 hour by a skilled man and 3 hours by a semi-skilled man. No man is expected to work more than 8 hours per day. The manufacturer's profit on an item of model A is ₹ 15 and on an item of model B is ₹ 10. How many of items of each model should be made per day in order to maximize daily profit ? Formulate the above LPP and solve it graphically and find the maximum profit.

44. A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is $8m^3$. If building of tank costs ₹ 70 per square metre for the base and ₹ 45 per square metre for the sides, what is the cost of least expensive tank ?