Module - propriétés -

① $|z| \in \mathbb{R}_+$; ② $|z| = |\bar{z}| = |-z|$

③ $|z| = \sqrt{z\bar{z}}$; ④ $|z| = 1 \Leftrightarrow \bar{z} = \frac{1}{z}$

⑤ $|zz'| = |z| \cdot |z'|$; ⑥ $\left|\frac{z}{z'}\right| = \frac{|z|}{|z'|}$ $(z' \neq 0)$

⑦ $|z^n| = |z|^n$ $(n \in \mathbb{N}^*)$

Si $z = a + ib$ $(a \in \mathbb{R}$ et $b \in \mathbb{R})$ alors :

$$z\bar{z} = a^2 + b^2 = |z|^2$$

Exemples : Soit $z = 1 + i\sqrt{3}$ et $z' = 1 + i$

$|z| = |1 + i\sqrt{3}| = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{1+3} = 2 \in \mathbb{R}_+$

$|\bar{z}| = |1 - i\sqrt{3}| = \sqrt{1 + (-\sqrt{3})^2} = \sqrt{1+3} = 2 = |z|$

$|-z| = |-1 - i\sqrt{3}| = 2$

Ainsi $|z| = |\bar{z}| = |-z|$

$|zz'| = |z| \cdot |z'| = 2 \cdot \sqrt{1+1} = 2\sqrt{2}$

$\left|\frac{z'}{z}\right| = \frac{|z'|}{|z|} = \frac{\sqrt{2}}{2}$

$|z^2| = |z|^2 = 2^2 = 4$

$z\bar{z} = (1 + i\sqrt{3})(1 - i\sqrt{3}) = 1 + (\sqrt{3})^2 = 4 = |z|^2$

Ainsi $z\bar{z} = |z|^2$