Study of the M_{bl} kinematic endpoint in $t\bar{t}$ events

Konstantinos Avgerakis

National and Kapodistrian University of Athens

8 January 2025

Kinematics of the decay of a top quark (I)

$$t \rightarrow Wb \;\&\; W \rightarrow l\nu_l$$

$$m^2(bl) = (P_b + P_l)^{\mu}(P_b + P_l)_{\mu} = P_b^{\mu}P_{b\mu} + P_l^{\mu}P_{l\mu} + 2P_b^{\mu}P_{l\mu}$$

$$\Rightarrow m^2(bl) = 2E_bE_l(1 - \cos\theta)$$

Since cosθ is a bounded function, we expect the invariant mass of the b quark and lepton pair to have a minimum (θ=π) and a maximum (θ = 0). We also assume a limit of high energies and ignore the masses of the quark and lepton.

Konstantinos Avgerakis NKUA

Kinematics of the decay of a top quark (II)

Study of Extreme States θ = 0 & θ = π

Calculation of the Kinematic Endpoint

$$\theta = 0 : \quad b \leftarrow t \rightarrow W$$

$$l \leftarrow \circ \rightarrow \nu_l$$

$$m^2(bl)_{min} = 0$$

$$\theta = \pi : \quad b \leftarrow t \rightarrow W$$

$$\nu_l \leftarrow \circ \rightarrow l$$

$$m^2(bl)_{max} = 4E_b E_l$$

After carefully using the conservation of 4-momentum, we can calculate the energy of the b quark in t’s reference frame, as well as the energy of the lepton in W’s reference frame. Subsequently, we use a Lorentz boost to calculate the lepton’s energy in t’s reference frame. Finally, we derive the results:

$$E_b = \frac{m_t^2 - m_W^2}{2m_t} \qquad E_l = \frac{m_t}{2}$$

$$m(bl)_{max} = \sqrt{m_t^2 - m_W^2} = 152.6\ GeV$$

Where we used M_t = 172.5 GeV and M_W = 80.4 GeV [1]

Konstantinos Avgerakis NKUA

(1): Sirunyan, Albert M., et al. "Measurement of the top quark mass in the dileptonic $t\bar{t}$ decay channel using the mass observables M b ℓ, MT 2, and M b ℓ ν in pp collisio[illegible] *Physical Review D* 96.3 (2017): 032002.

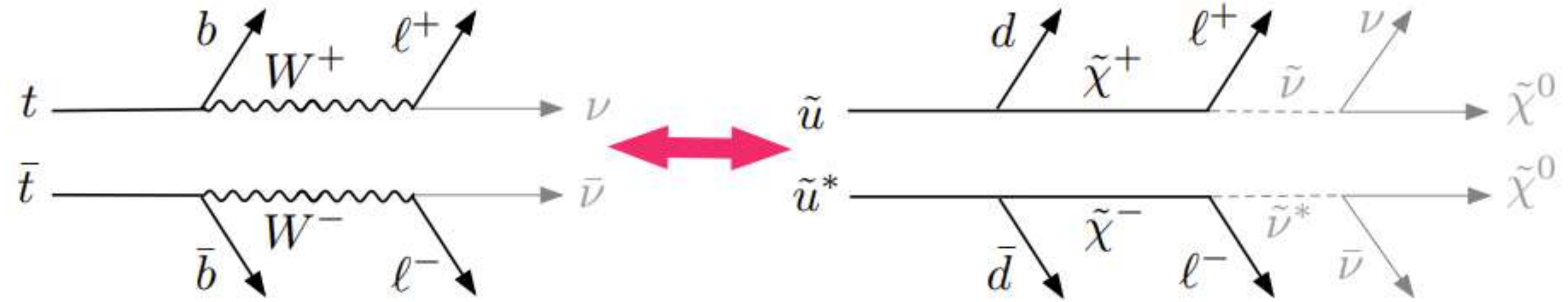
Why is this kinematic endpoint useful? (some examples)

Measurement of top quark's mass

Similarity with SUSY processes

In our case, the M_{bl} kinematic endpoint is explicitly related to the top quark's mass. The measurement of m_{top} affects various important theory predictions, such as particle production cross sections related to the exploration of the Higgs Boson's properties. Furthermore, determining m_{top} accurately can serve as a test of the consistency of the Standard Model, as well as a constraint to New Physics Models(2).

Observables such as M_{bl} in dilepton decays of $t\bar{t}$ events (which we study), and the methods that are used for such measurements, resemble the challenges that arise in similar New Physics scenarios, such as the following Supersymmetric decay chain(3):



(2): Giorgio Cortiana, Top-quark mass measurements: Review and perspectives, Reviews in Physics, Volume 1, 2016
(3): The CMS Collaboration., Chatrchyan, S., Khachatryan, V. *et al.* Measurement of masses in the tt¯ system by kinematic endpoints in pp collisions at s=7 TeV *Eur. Phys.*

Konstantinos Avgerakis NKUA

What will we do?

Debriefing all different final states

In our analysis we used CMS open data files from the CMS HEP Tutorial[4] in order to graph the distributions of the M_{bl} invariant mass for a variety of mutually exclusive final states originating from semi-leptonic and fully leptonic decays of $t\bar{t}$ pairs. Specifically, we used the final states:

- Two Muons & Two b Jets
- One Muon, One Electron & Two b Jets
- Two Muons & One b Jet
- One Muon, One Electron & One b Jet
- Two Muons & One b Jet

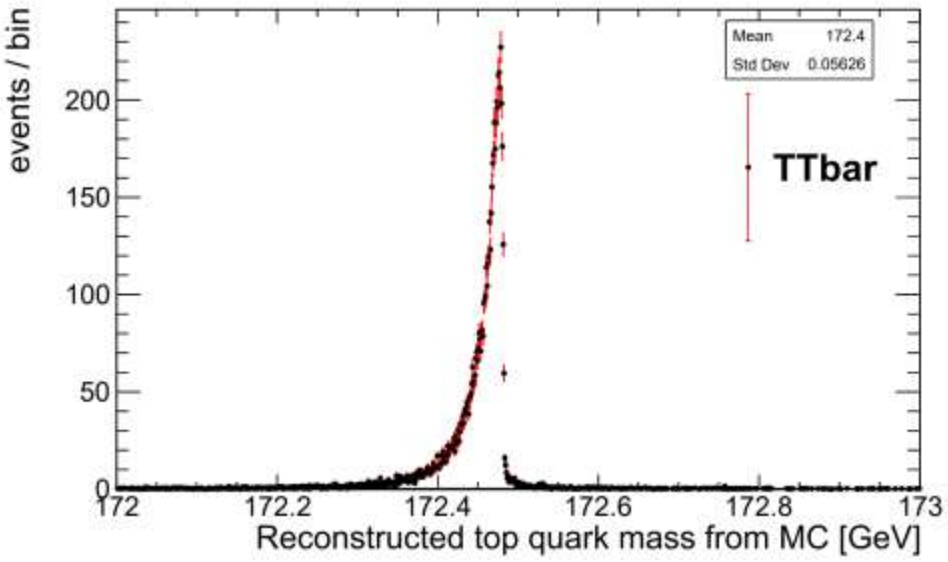
It is important to mention that the data we use has been preselected using a single muon trigger, hence the lack of states that only consist electrons regarding the leptonic part.

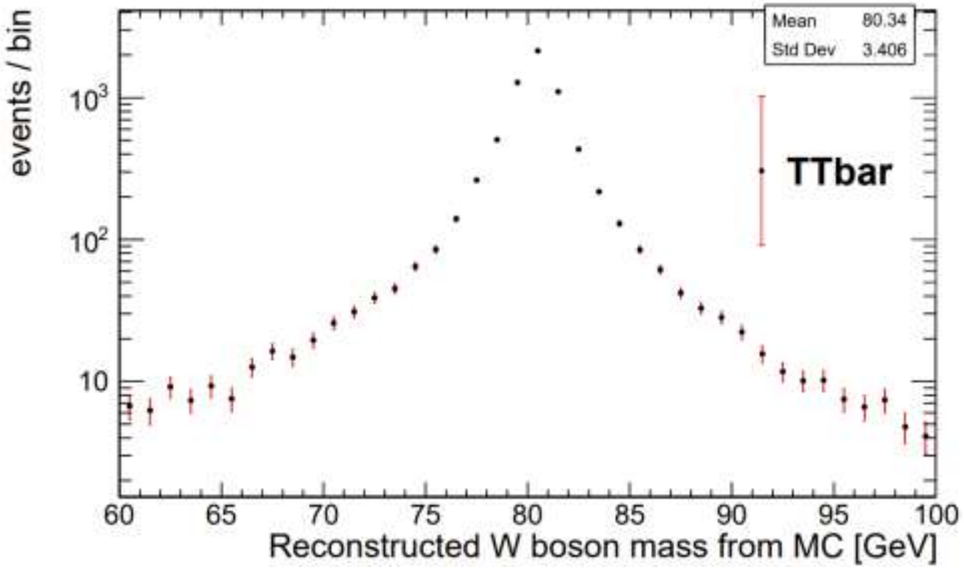
Konstantinos Avgerakis NKUA

(4): Sander, C., & Schmidt, A. (2012). CMS HEP Tutorial (No. IPPOG-RDB-2022-049).

Some sanity checks before we start... (I)

Before we start our analysis, it is instructive to test that our code works according to what we expect. In our case, we will be testing the Monte Carlo simulation of $t\bar{t}$ events. Initially, we will be reconstructing the mass of the W boson and the top quark that originate from the decay: t $\rightarrow$ Wb $\rightarrow q\bar{q}$ b. Our results are portrayed in the following graphs:





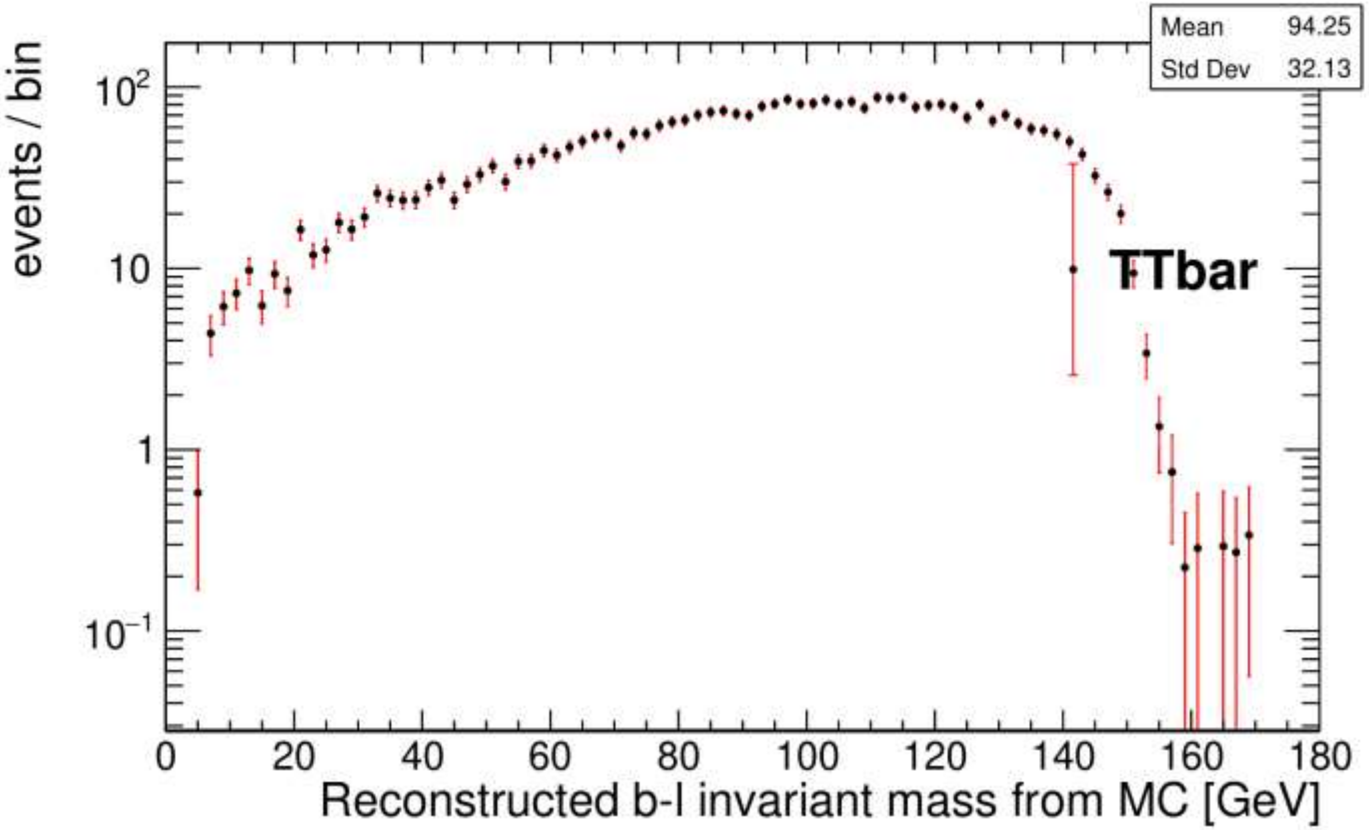
Konstantinos Avgerakis
NKUA

Some sanity checks before we start… (II)

Monte Carlo Truth Level

Reconstruction of M_{bl} distribution

Our next test to certify the code works smoothly, is the reconstruction of M_{bl} invariant mass distribution. We want to ensure that the kinematic endpoint we predicted theoretically is apparent ***at parton level*** in our simulations as well. We give the graph:



Konstantinos Avgerakis
NKUA

Combinatorics

Why some combinations are incorrect

In the context of a simulation, it is easy to distinguish the decay products that originate from the top quark and the anti-top quark. Hence, creating the M_{bl} distribution proves to be an easy task. In real data, finding the correct lepton - b jet pair is challenging. Ergo, our problem lies in combinatorics. The initial thought is to include every possible b-l pair in our graphs (even though half of them will not correspond to the invariant mass we study).

$t_1 \to W\, b_1 \to l_1\, v_1$

$t_2 \to W\, b_2 \to l_2\, v_2$

Correct pairs:

- $b_1\, l_1$
- $b_2\, l_2$

Incorrect Pairs:

- $b_1\, l_2$
- $b_2\, l_1$

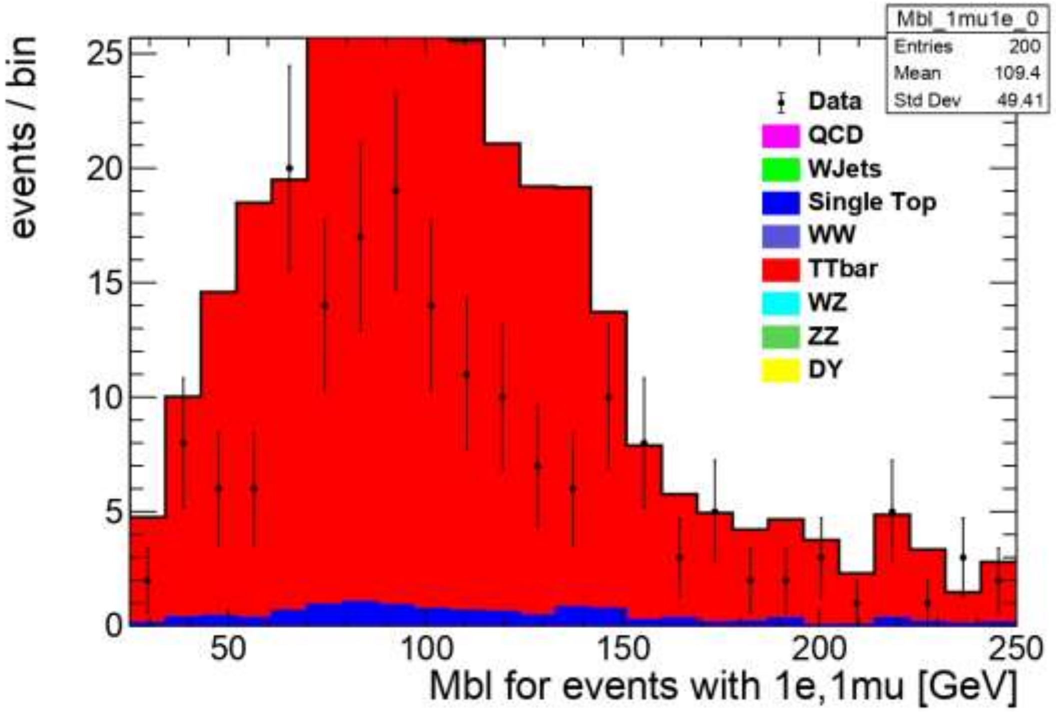
While pairing algorithms do exist[(1)] in our analysis we will persist and keep all possible combinations. We will later see that the effects of this "mistake" will not have a severe effect to our final results.

(1): pages 5-6.

Konstantinos Avgerakis NKUA

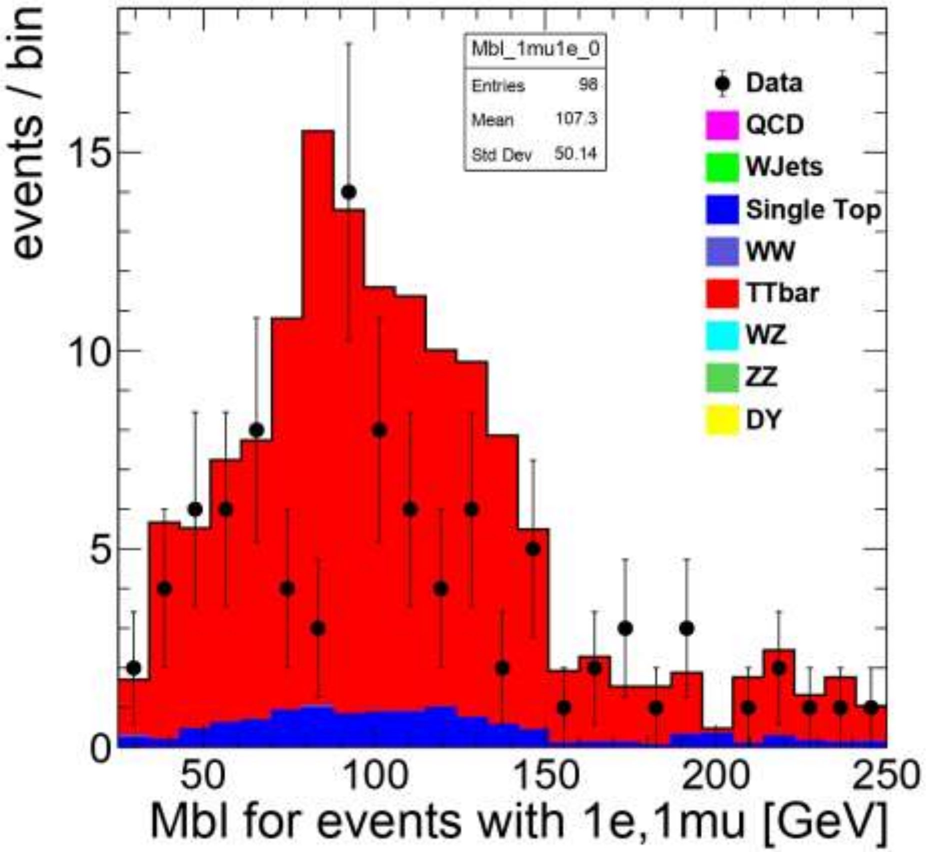
One Muon, One Electron & 2 (b) Jets

We start off with the fully leptonic processes that consist of different flavored leptons. These are quite easy to work with, since they are very clear, $t\bar{t}$ originated events and the background is minimal. The important part of our analysis is that we could use the b-tagging algorithm only for the first jet we use; this would almost guarantee the next jet would be b flavored. This is very useful since the b-tagging algorithm is not perfect and is bound to either miss events or mistag jets.



One Muon, One Electron & 1 b Jet

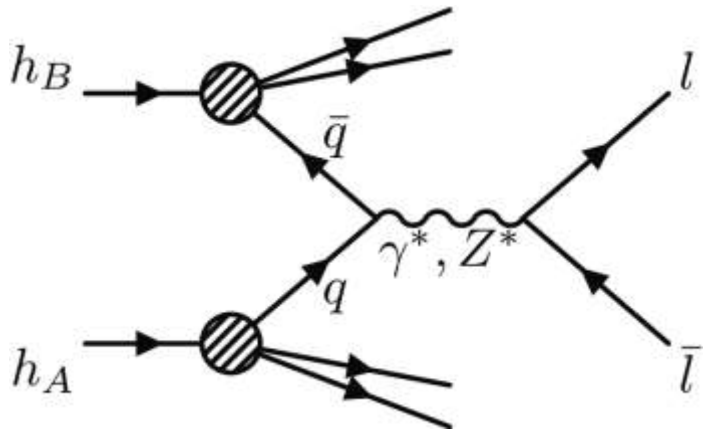
This case is extremely similar to the one with 2 jets and provides the same ease in its analysis. We provide the graph:



Konstantinos
Avgerakis
NKUA

Drell-Yan Process Background

The other cases we cover in our analysis consist of 2 muons in the final state. These cases have the challenge of a very significant Drell-Yan background. The Drell-Yan process describes high mass lepton pair production in hadron-hadron collisions, which closely resemble the final states originating from $t\bar{t}$ events. The process is described by the diagram[5]:



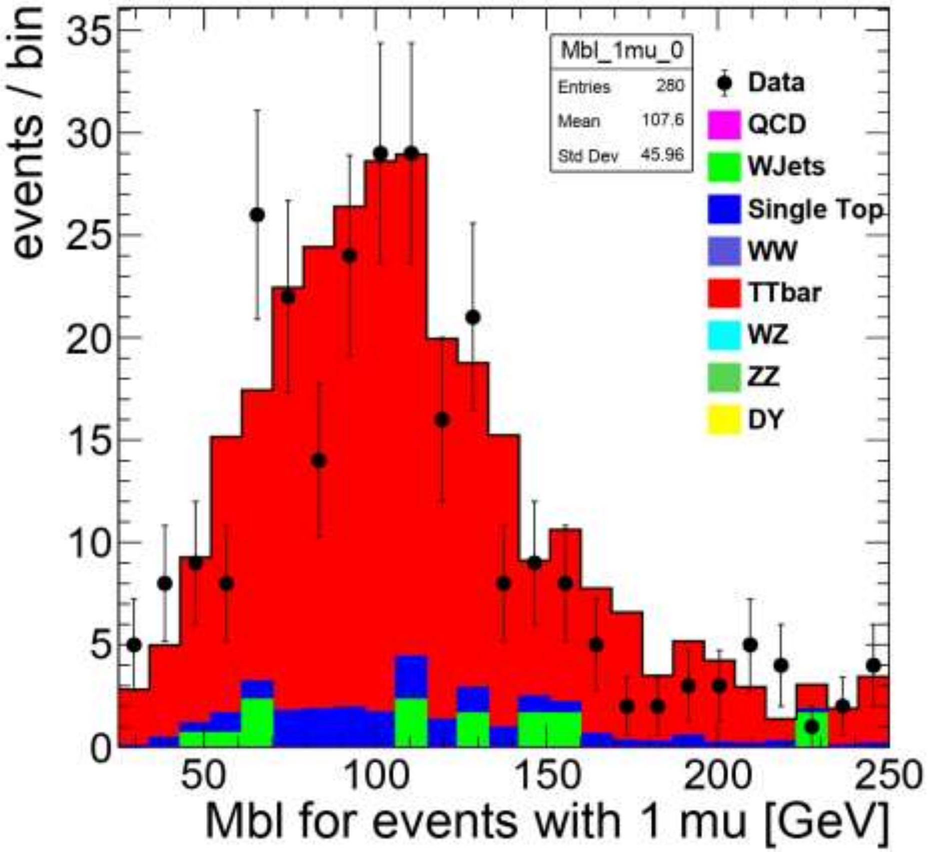
To suppress this background, we introduce a cut in the lepton pair invariant mass M_{ll} around the Z boson's mass; specifically, the cut we use (and will use henceforth) excludes events with 75 GeV < M_{ll} < 105 GeV. This will not be enough on its own, but we (will) analyze this later.

(5): Bechtel, Florian. "The underlying event in proton-proton collisions." (2009).

One Muon & 2 (b) Jets

1 μ 2 Jets
Final state

This process is the only Semi-Leptonic decay we study in this analysis, and consists of only one Muon and two b Jets. For this particular process, we require two b Jets or more along with a demand of MET > 50 GeV.

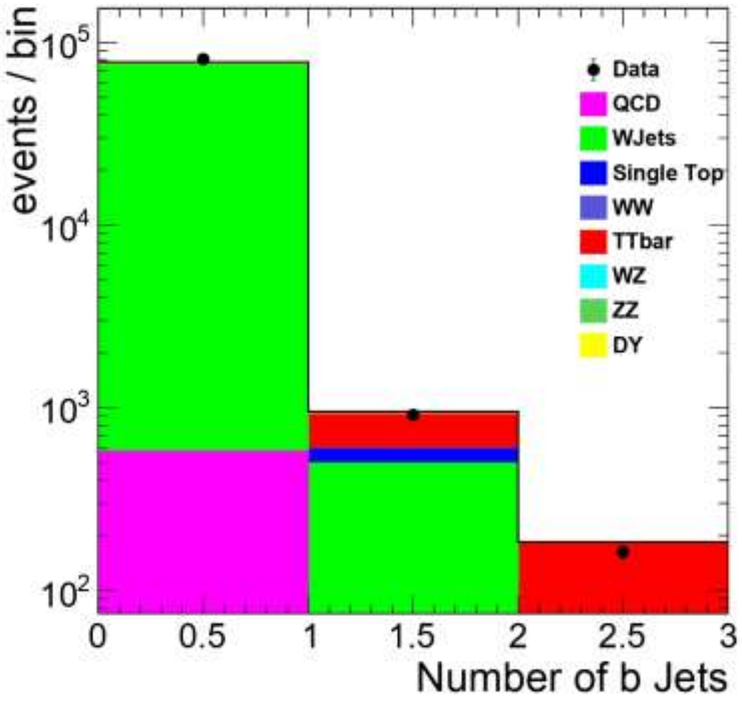


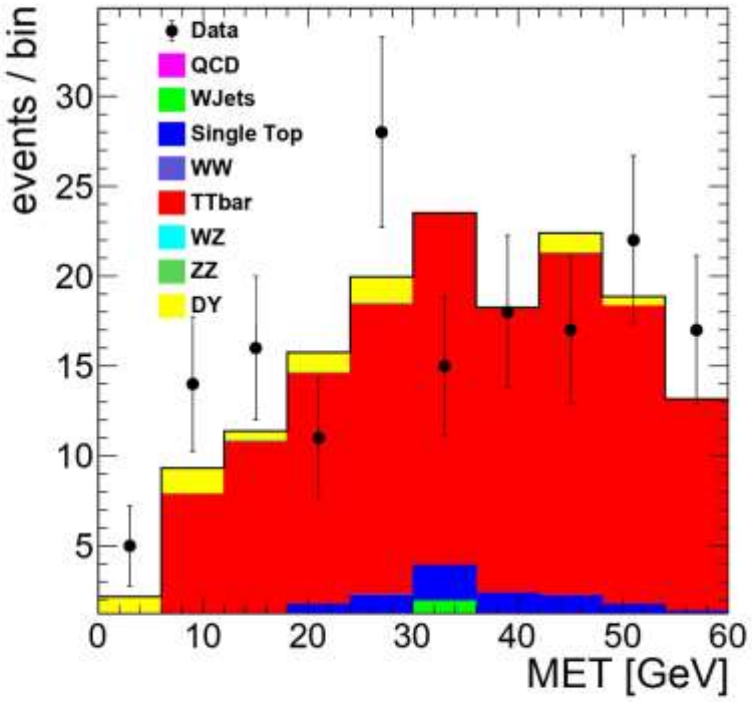
How do we decide cuts? N-1 plots (I)

1 μ 2 Jets
Necessary cuts

Not exactly rigorous but works

In the last process we introduced cuts that might seem arbitrary; specifically, we demanded at least two b jets, and MET>50 GeV. In reality, the way we decide how to introduce these cuts is through plotting each variable we use for the cuts, with respect to all the others. We will repeat this for the rest of the processes that required cuts. In this case, the graphs that are important are the following:

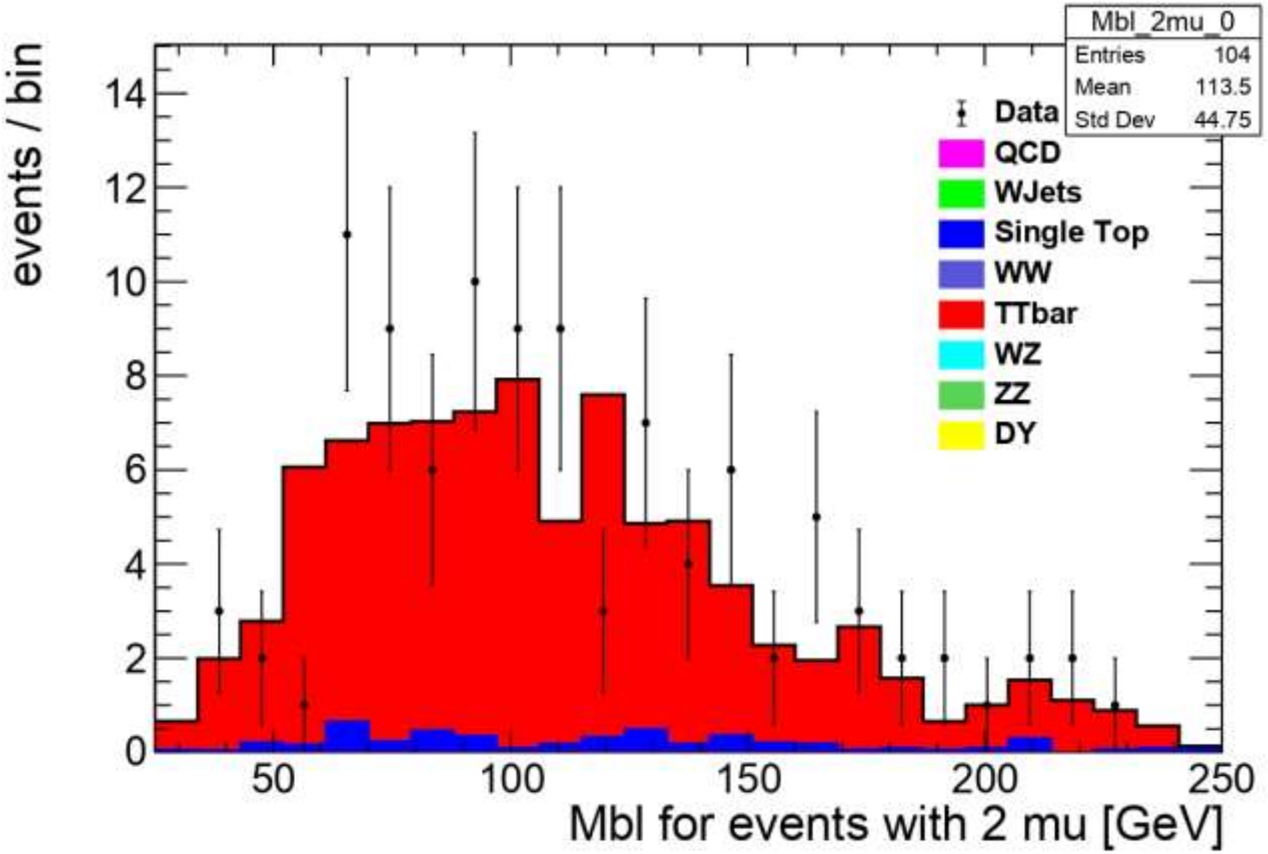




Konstantinos Avgerakis NKUA

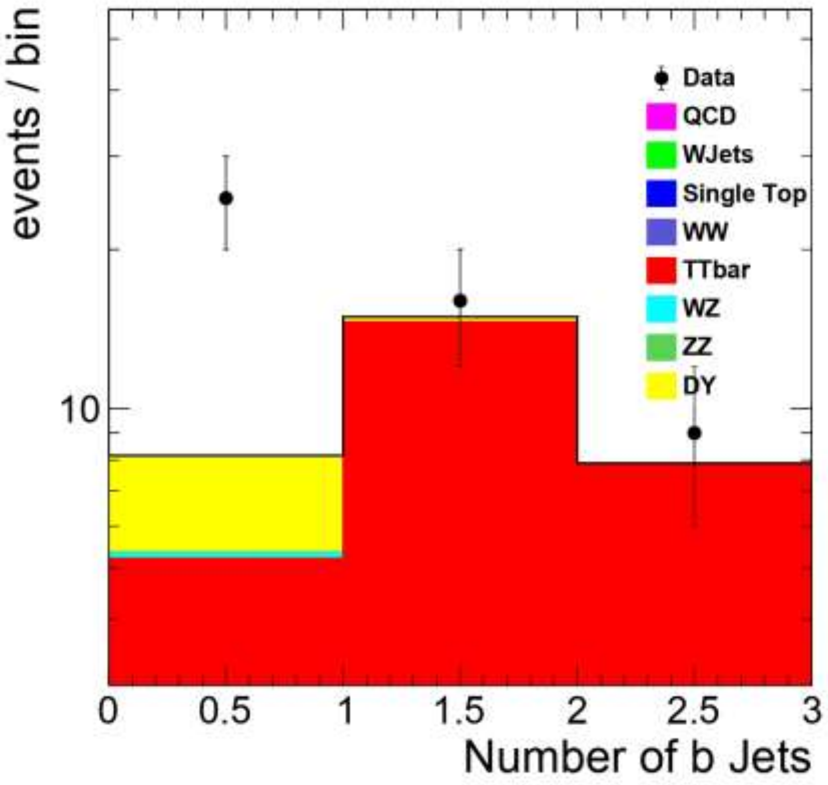
Two Muons & 2 (b) Jets

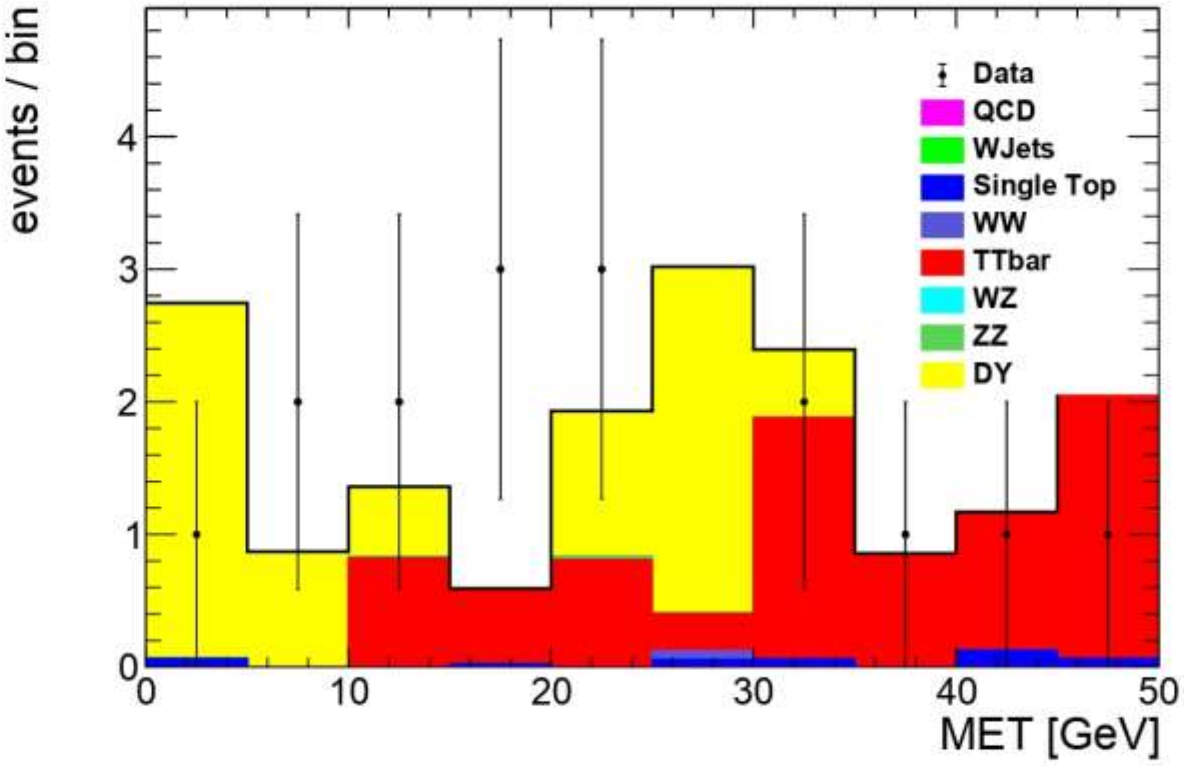
This process has similarities with the corresponding semi-leptonic one, but the cuts we introduced differ. In this case, we required the number of b jets to be at least one (similar logic with the one electron – one muon processes), and we also demanded MET > 35 GeV. The resulting plot is thus:



How do we decide cuts? N-1 plots (II)

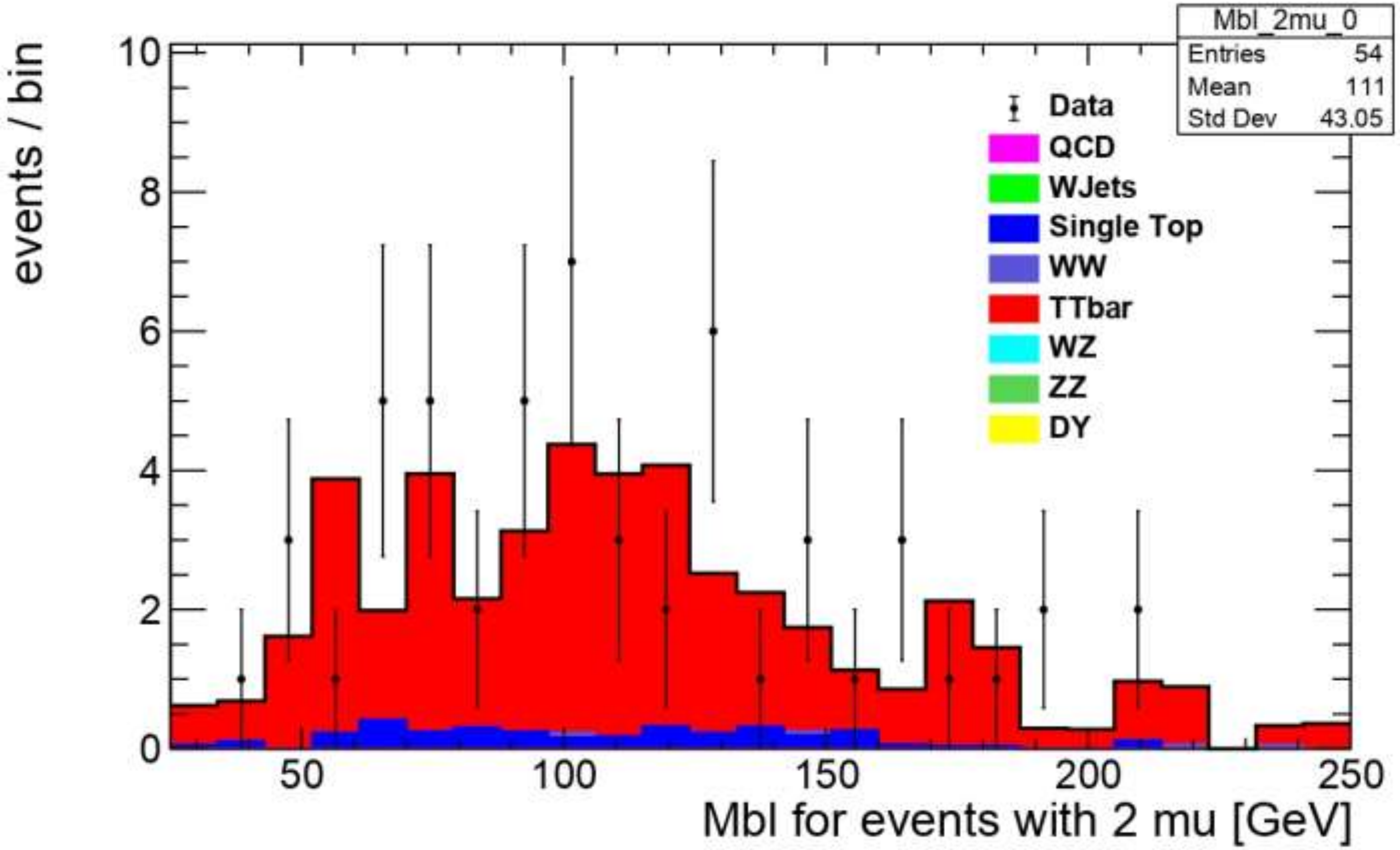
Once again, we will provide the N-1 (in our case N=2) plots that describe the cuts we introduced in the study of this process. The resulting graphs are shown below:





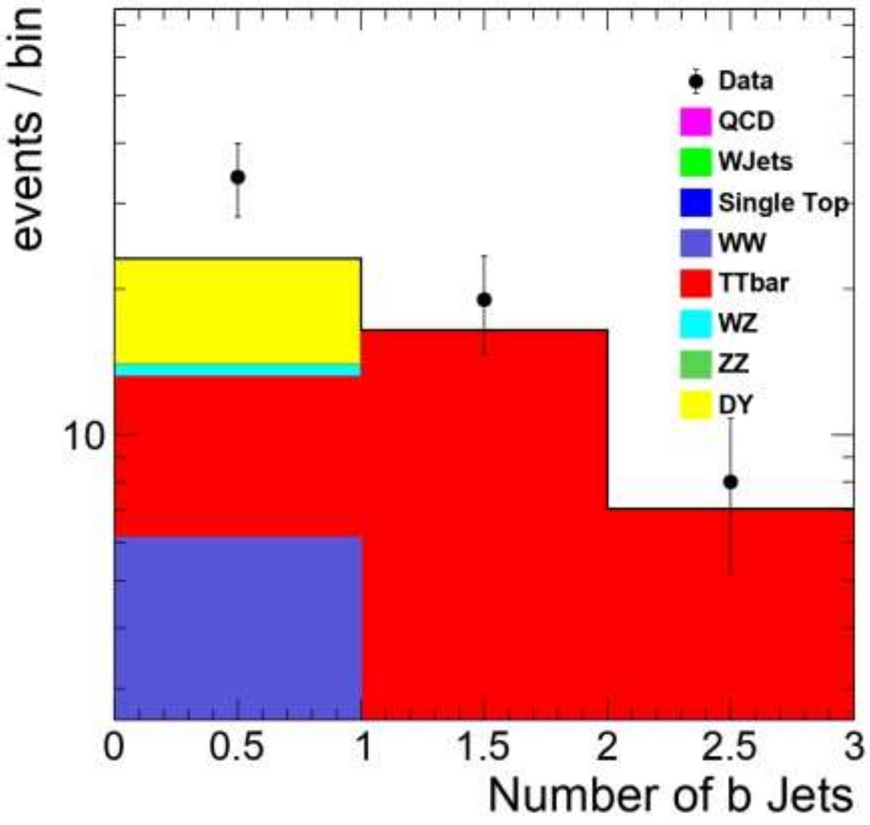
Two Muons & 1 b Jet

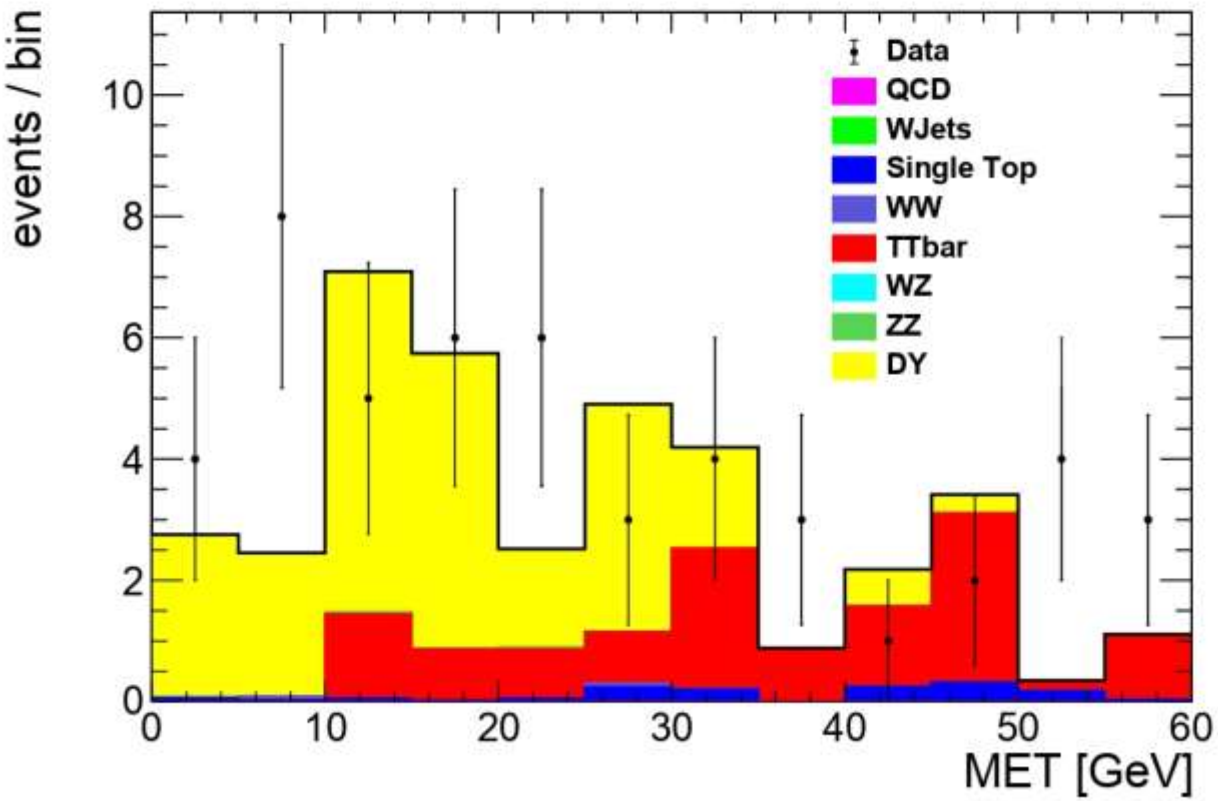
The analysis for this process closely resembles the others. Regarding cuts, we demanded at least one b jet and MET > 50 GeV. The resulting plot is as follows:



How do we decide cuts? N-1 plots (III)

We repeat the process we followed before. The resulting graphs are shown below:

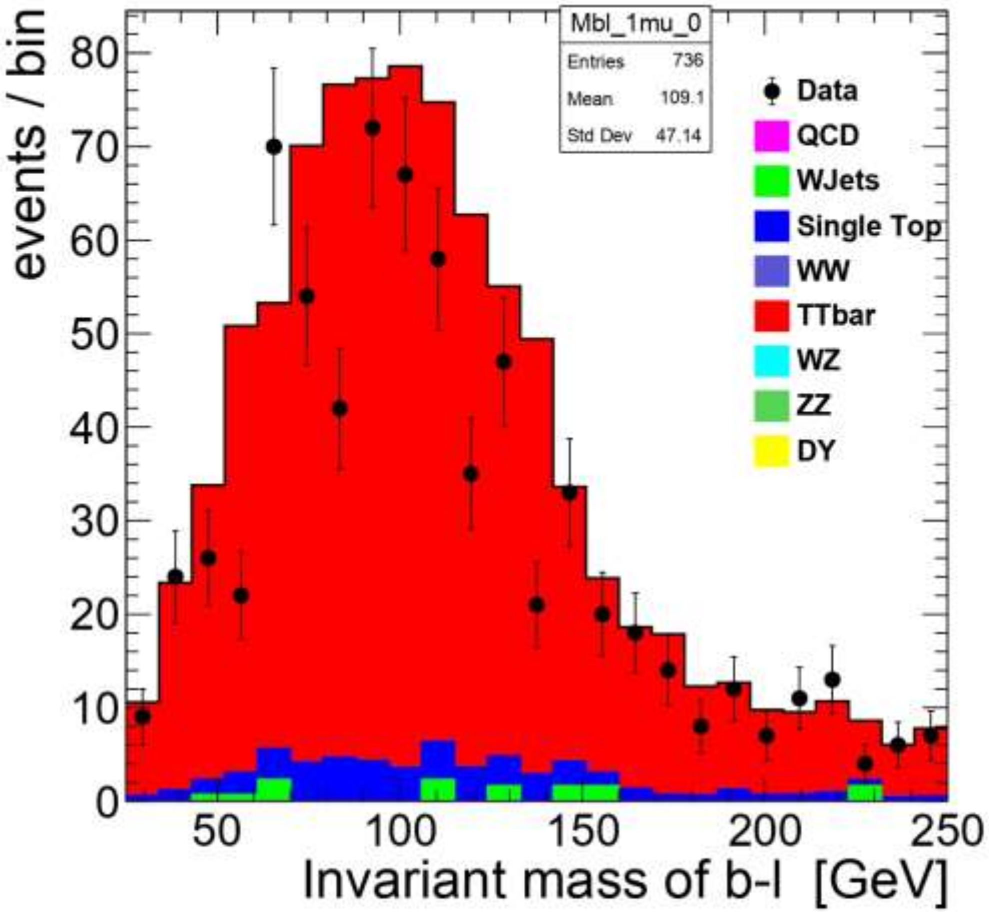




Konstantinos
Avgerakis
NKUA

Combined Results, Can we see the Endpoint?

After our analysis, we are finally in a spot where we can combine all our results in one graph regarding the M_{bl} invariant mass. The endpoint can be spotted close to the theoretical prediction!



Bibliography and References

- (1): Sirunyan, Albert M., et al. "Measurement of the top quark mass in the dileptonic tt¯ decay channel using the mass observables M b ℓ, MT 2, and M b ℓ v in pp collisions at s= 8 TeV *Physical Review D* 96.3 (2017): 032002.

- (2): Giorgio Cortiana, Top-quark mass measurements: Review and perspectives, Reviews in Physics, Volume 1, 2016

- (3): The CMS Collaboration., Chatrchyan, S., Khachatryan, V. *et al.* Measurement of masses in the tt¯ system by kinematic endpoints in pp collisions at s=7 TeV . *Eur. Phys. J. C* **73**, 2494 (2013).

- (4): Sander, C., & Schmidt, A. (2012). CMS HEP Tutorial (No. IPPOG-RDB-2022-049)

- (5): Bechtel, Florian. "The underlying event in proton-proton collisions." (2009).

- (6): Theofilatos, Konstantinos. "The terascale tutorial." arXiv preprint arXiv:2403.08907 (2024).

In memory of Luc Pape (1939 – 2021)