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- **2.3——**
- 2.

  Suppose that the salt in tank is y g

  Thus $y' = 2\gamma - 2\frac{y}{120}$, $y(0) = 0$

  $y' = 2\gamma - 2\frac{y}{120} \Rightarrow y' = 2\gamma - y/60 \Rightarrow$

  $60y' = -y + 120\gamma \Rightarrow 60\frac{y'}{y-120\gamma} = -1 \Rightarrow d(\ln|y - 120\gamma|) = -\frac{1}{60}dt \Rightarrow$

  $y = \pm e^{-\frac{1}{60}t+C} + 120\gamma = ce^{-\frac{1}{60}t} + 120\gamma$ where c is an arbitrary constant

  Substituting with $y(0) = 0$, we get $c = -120\gamma$

  Hence $y = -120\gamma e^{-\frac{1}{60}t} + 120\gamma$

- 6.
  - a.

    $S' = rS + k \Rightarrow$

    $dS/(S + \frac{k}{r}) = rdt \Rightarrow d(\ln|S + \frac{k}{r}|) = rt \Rightarrow S = ce^{rt} - k/r$ where c is an arbitrary constant

    Substitute with $S(0) = 0$, and we get $c = k/r$

    Hence $S(t) = k(e^{rt} - 1)/r$
  - b.

    Substitute with $r = 7.5\%$ and $t = 40$, $S(40) = k(e^3 - 1)/0.075$

    Solving $k(e^{30} - 1) = 75000$, we get $k \approx \$3929.68$
  - c.

    Substitute with $k = \$2000$ and $t = 40$, $S(40) = 2000(e^{40r} - 1)/r$

    Solving $(e^{40r} - 1)/r = 500$, we get $r \approx 9.8\%$

- 12.

  Suppose the temperature of coffee is T, and cooling rate is r

  Thus $T' = r(T - 70) \Rightarrow d\ln(T - 70) = rdt \Rightarrow T = e^{rt+c} + 70$ where c is an arbitrary constant

  Substitute $T(0) = 200$, and we get $e^c = 130$ thus $T = 130e^{rt} + 70$

  Substitute $T(1) = 190$, and we get $r = \ln(12/13)$ thus $T = 130e^{tln(12/13)} + 70$

  Solving T(t)=150, we get $t \approx 6.1\text{s}$

- 20.
  - a.

    $v' = -g - kv/m \Rightarrow d\ln(v + mg/k) = -\frac{k}{m}dt \Rightarrow v(t) = ce^{-kt/m} - mg/k$

Substitute $v(0) = v_0$, and we get $c = v_0 + mg/k$

Hence $v(t) = (v_0 + mg/k)e^{-kt/m} - mg/k$

- b.

  if $k \to 0$, $v(t) \to v_0 - gt$

  which satisfies the case in a vacuum

- c.

  if $m \to 0$, $v(t) \to 0$

- 21.
  - a.

    The volumn of the spherical body is $V = \frac{4}{3}\pi a^3$

    Thus $B = \frac{4}{3}\pi\rho' a^3 g$, $m = \frac{4}{3}\pi\rho a^3 g$, $$

  - b.
  -

- **2.6——**
- 1.

  $\partial(2x+3)/\partial y = 0, \partial(2y-2)/\partial x = 0$

  Hence this equation is exact

  $\int(2x+3)dx = x^2 + 3x + g(y) = \int(2y-2)dy = y^2 - 2y + h(x)$

  Hence, let $\phi(x,y) = x^2 + 3x + y^2 - 2y$, $d\phi = 0$

  We get $x^2 + 3x + y^2 - 2y = c$ where c is an arbitrary constant

- 3.

  $\partial(3x^2 - 2xy + 2)/\partial y = 2x, \partial(6y^2 - x^2 + 3)/\partial x = 2x$

  Hence this equation is exact

  $\int(3x^2 - 2xy + 2)dx = x^3 - x^2 y + 2x + g(y) = \int(6y^2 - x^2 + 3)dy = 2y^3 - x^2 y + 3y + h(x)$

  Hence, let $\phi(x,y) = x^3 + 2x - x^2 y + 2y^3 + 3y,\ d\phi = 0$

  We get $x^3 + 2x - x^2 y + 2y^3 + 3y = c$ where c is an arbitrary constant

- 5.

  $\partial(ax - by)/\partial y = -b,\ \partial(bx - cy)/\partial x = b$

  Hence this equation is exact iff $b = 0$

  When $b = 0$, $cydy = axdx$

  $\frac{1}{2}cy^2 - \frac{1}{2}ax^2 = c$ where c is an arbitrary constant

- 7.

  $\partial(\frac{y}{x}+6x)/\partial y=\frac{1}{x},\ \partial(\ln x-2)/\partial x=\frac{1}{x}$

  Hence this equation is exact

  $\int(\frac{y}{x}+6x)dx=y\ln x+3x^2+g(y)=\int(\ln x-2)dy=y\ln x-2y+h(x)$

  Hence, let $\phi(x,y)=y\ln x+3x^2-2y,\ d\phi=0$

  $y\ln x+3x^2-2y=c$ where c is an arbitrary constant

- 11.

  $\partial(xy^2+bx^2y)/\partial y=2xy+bx^2,\ \partial(x^3+x^2y)/\partial x=3x^2+2xy$

  Hence this equation is exact iff $b=3$

  When $b=3$

  $\int(xy^2+3x^2y)dx=\frac{1}{2}x^2y^2+x^3y+g(y)=\int(x^3+x^2y)dy=x^3y+\frac{1}{2}x^2y^2+h(x)$

  Hence, let $\phi(x,y)=x^3y+\frac{1}{2}x^2y^2,\ d\phi=0$

  $x^3y+\frac{1}{2}x^2y^2=c$ where c is an arbitrary constant

- 14.

  $dM(x)/dy=0=dN(y)/dx$

  Hence this equation is exact

- 15.

  $\partial(x^2y^3)/\partial y=3x^2y^2,\ \partial(x+xy^2)/\partial x=1+2xy$

  Hence it is not exact

  When multiplied by $\frac{1}{xy^3}$

  $\partial x/\partial y=0,\ \partial(\frac{1}{y^3}+\frac{1}{y})/\partial x=0$

  This time it is exact

  $\int xdx=\frac{1}{2}x^2+g(y)=\int(\frac{1}{y^3}+\frac{1}{y})dy=-\frac{1}{2}y^{-2}+\ln|y|+h(x)$

  Hence, let $\phi(x,y)=\frac{1}{2}x^2-\frac{1}{2}y^{-2}+\ln|y|,\ d\phi=0$

  We get $\frac{1}{2}x^2-\frac{1}{2}y^{-2}+\ln|y|=c$ where c is an arbitrary constant

- 18.

  Let $M(x,y)=3x^2y+2xy+y^3,\ N(x,y)=x^2+y^2$

  Suppose $\exists\mu(x)$ s.t. $(\mu M)_y=(\mu N)_x\Rightarrow\mu M_y=\mu_x N+\mu N_x\Rightarrow\mu(M_y-N_x)/N=\mu'$

  $(M_y-N_x)/N=(3x^2+2x+3y^2-2x)/(x^2+y^2)=3\Rightarrow 3dx=d\mu/\mu$

  Thus we can let $\mu(x)=e^{3x}$

  $\int\mu(x)M(x,y)dx=x^2ye^{3x}+\frac{1}{3}y^3e^{3x}+g(x)=\int\mu(x)N(x,y)dy=x^2ye^{3x}=\frac{1}{3}y^3e^{3x}+h(x)$

  Hence, let $\phi(x,y)=e^{3x}(x^2y+\frac{1}{3}y^3),\ d\phi=0$

We get $e^{3x}(x^2y+\frac{1}{3}y^3)=c$ where c is an arbitrary constant

- 19.

  Let $M(x,y)=e^{2x}+y-1,\ N(x,y)=-1$

  From problem 18, we get $\mu(M_y-N_x)/N=\mu'$

  Thus $(M_y-N_x)/N=(1-0)/-1=-1 \Rightarrow -dx=d\mu/\mu$

  We can let $\mu(x)=e^{-x}$

  $\int \mu(x)M(x,y)dx=e^x+(1-y)e^{-x}+g(y)=\int \mu(x)N(x,y)=-ye^{-x}+h(x)$

  Hence, let $\phi(x,y)=e^x+(1-y)e^{-x},\ d\phi=0$

  We get $e^x+(1-y)e^{-x}=c \Rightarrow y=e^{2x}-ce^x+1$ where c is an arbitrary constant

- 20.

  Let $M(x,y)=1,\ N(x,y)=\frac{x}{y}-\sin y$

  Suppose $\exists\mu(y)$ s.t. $(\mu M)_y=(\mu N)_x \Rightarrow \mu_y M+\mu M_y=\mu N_x \Rightarrow \mu(N_x-M_y)/M=\mu'$$

  Thus $(N_x-M_y)/M=(\frac{1}{y}-0)/1=\frac{1}{y} \Rightarrow dy/y=d\mu/\mu$

  We can let $\mu(y)=y$

  $\int \mu(y)M(x,y)dx=xy+g(y)=\int \mu(y)N(x,y)dy=xy+y\cos y-\sin y+h(x)$

  Hence, let $\phi(x,y)=xy+y\cos y-\sin y,\ d\phi=0$

  We get $xy+y\cos y-\sin y=c$ where c is an arbitrary constant