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- **2.2——**
- 27.
  - a.

    $\frac{x^2+3y^2}{2xy} = \frac{1+3(\frac{y}{x})^2}{2(\frac{y}{x})}$

    Thus this equation is homogeneous
  - b.

    Let $y = xv(x)$

    Hence $\frac{dy}{dx} = v + xv' = \frac{1+3v^2}{2v} \Rightarrow \frac{dv}{dx} = \frac{1+v^2}{2xv} \Rightarrow \frac{2v}{1+v^2}dv = \frac{dx}{x}$

    Thus $d(\ln|1+v^2|) = d(\ln x) \Rightarrow 1+v^2 = cx$ where c is an arbitrary constant

    Since $v^2 = y^2/x^2,\ x^2+y^2 = cx^3$

- 29.
  - a.

    $\frac{4x+3y}{2x+y} = \frac{4+3\frac{y}{x}}{2+\frac{y}{x}}$

    Thus this equation is homogeneous
  - b.

    Let $y = xv(x)$

    Hence $\frac{dy}{dx} = v + xv' = -\frac{4+3v}{2+v} \Rightarrow \frac{xdv}{dx} = -\frac{v^2+5v+4}{2+v} \Rightarrow \frac{v+2}{(v+1)(v+4)}dv = -\frac{dx}{x}$

    Thus $\frac{1}{3}[d(\ln|v+1| + 2d(\ln|v+4|)] = -d\ln|x| \Rightarrow (v+1)(v+4)^2x^3 = c$ where c is an arbitrary constant

    Since $v = y/x,\ (x+y)(4x+y) = c$

- **2.4——**
- 1.

  Change the form of equation, we get $y' + \frac{\ln t}{t-3}y = \frac{2t}{t-3}$

  Hence, $t \in (0,3) \cup (3,+\infty)$

  Since $y(1) = 2$, $0 < t < 3$

- 3.

  Change the form of equation, we get $y' + \frac{2t}{4-t^2}y = \frac{3t^2}{4-t^2}$

  Hence, $(-\infty,-2) \cup (-2,2) \cup (2,+\infty)$

  Since $y(-3) = 1,\ t < -2$

- 5.

  Denote $f(t,y)=\sqrt{1-t^2-y^2}$

  $\partial f/\partial y=-\frac{2}{2\sqrt{1-t^2-y^2}}$

  Hence, $1-t^2-y^2>0$ which is $y^2+t^2<1$

- 7.

  Denote $f(t,y)=(t^2+y^2)^{\frac{3}{2}}$

  $\partial f/\partial y=6y\sqrt{t^2+y^2}$

  Hence, $y^2+t^2\geq 0$, which is everywhere in the ty-plane

- 9.

  $y'=-4t/y\Rightarrow ydy=-4tdt\Rightarrow \frac{1}{2}y^2=-2t^2+c\Rightarrow y^2+4t^2=c$ where c is an arbitrary constant

  Since $y(0)=y_0,\ c=y_0^2,\ y=\pm\sqrt{y_0^2-4t^2}$ where $y_0\neq 0$ and $4t^2<y_0^2$ which is $-\frac{|y_0|}{2}<t<\frac{|y_0|}{2}$

- 11.

  $y'+y^3=0\Rightarrow \frac{dy}{y^3}=-dt\Rightarrow \frac{1}{2}y^{-2}=t+\frac{1}{2}c\Rightarrow y^{-2}=2t+c$ where c is an arbitrary constant

  Since $y(0)=y_0,\ c=y_0^{-2},\ y=\frac{y_0}{\sqrt{2ty_0^2+1}}$ where $t>-\frac{1}{2y_0^2}$ for $y_0\neq 0$; $t\in\mathbb{R}$ for $y_0=0$

- 18.
  - a.

    $(1-t)'=-1,\ \frac{-t+\sqrt{t^2+4(1-t)}}{2}=\frac{-t+|t-2|}{2}$

    Thus $y_1(t)=1-t$ is a solution which valid when $t\geq 2$

    $(-t^2/4)'=-t/2,\ \frac{-t+\sqrt{t^2-t^2}}{2}=-t/2$

    Thus $y_2(t)=-t^2/4$ is a solution and valid for $\forall t\in\mathbb{R}$

  - b.

    Denote $f(t,y)=\frac{-t+\sqrt{t^2+4y}}{2}$

    $\partial f/\partial y=\frac{1}{\sqrt{t^2+4y}}$, thus when $t^2+4y=0,\ f_y$ is not continuous

    However $4y_1$

- 24.

  Let $v=y^{-1}, dv/dt=-y^{-2}dy/dt,\ y'y^{-2}=ry^{-1}-k\Rightarrow -v'=rv-k\Rightarrow \frac{dv}{v-\frac{k}{r}}=-rdt$

  Thus $v=ce^{-rt}+\frac{k}{r}$ where c is an arbitrary constant

  Hence $y=\frac{r}{cre^{-rt}+k}$ where c is an arbitrary constant

- 25.

  Let $v = y^{-2},\ dv/dt = -2y^{-3}dy/dt,\ y'y^{-3} = \epsilon y^{-2} - \sigma \Rightarrow -\frac{1}{2}v' = \epsilon v - \sigma \Rightarrow \frac{dv}{v-\frac{\sigma}{\epsilon}} = -2\epsilon dt$

  Thus $v = ce^{-2\epsilon t} + \frac{\sigma}{\epsilon}$ where c is an arbitrary constant

  Hence $y = \pm(\frac{\epsilon}{c\epsilon e^{-2\epsilon t}+\sigma})^{\frac{1}{2}}$ where c is an arbitrary constant

- **Riccati Equations——**
- 26.
  - a.

    $y' = 1 + t^2 - 2ty + y^2 = q_1(t) + q_2(t)y + q_3(t)y^2$

    Thus $q_1(t) = 1 + t^2,\ q_2(t) = -2t,\ q_3(t) = 1$

    $dv/dt = -(-2t + 2t)v - 1 = -1$ which implies $v = -t + c$ where c is an arbitrary constant

    Hence $y = t + \frac{1}{c-t}$ where c is an arbitrary constant
  - b.

    $y' = -\frac{1}{t^2} - \frac{y}{t} + y^2 = q_1(t) + q_2(t)y + q_3(t)y^2$

    Thus $q_1(t) = -\frac{1}{t^2},\ q_2(t) = -\frac{1}{t},\ q_3(t) = 1$

    $dv/dt = -(-\frac{1}{t} + \frac{2}{t})v - 1 = -\frac{v}{t} - 1$ which implies $v = \frac{c-t^2}{2t}$ where c is an arbitrary constant

    Hence $y = \frac{1}{t} + \frac{2t}{c-t^2}$ where c is an arbitrary constant