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- **Review Problem——**
- 13.
  Let $M(x.y) = e^{-x}\cos y - e^{2y}\cos x, N(x,y) = e^{-x}\sin y - 2e^{2y}\sin x$
  Hence $Mdx + Ndy = 0$
  $dM/dy = -e^{-x}\sin y - 2e^{2y}\cos x = dN/dx$, it follows that this equation is exact
  Thus define $\phi(x,y)$ s.t. $d\phi(x,y) = Mdx + Ndy = 0$
  Which implies $\phi(x,y) = \int(e^{-x}\cos y - e^{2y}\cos x)dx = -e^{-x}\cos y - e^{2y}\sin x + g(y)$
  $= \int(e^{-x}\sin y - 2e^{2y}\sin x)dy = -e^{-x}\cos y - e^{2y}\sin x + h(x)$
  Hence $\phi(x,y) = -e^{-x}\cos y - e^{2y}\sin xC =$ where C is an arbitrary constant
  We get $e^{-x}\cos y + e^{2y}\sin x = c$ where c is an arbitrary constant

- 14.
  Let $M(x.y) = e^{2x} + 3y, N(x,y) = -1$, thus $Mdx + Ndy = 0$
  Since this equation is not exact, we multiple both sides by $\mu(x)$ s.t. $(\mu M)_y = (\mu N)_x$
  which is $\mu M_y = \mu_x N + \mu N_x$ and we get $3\mu = -\mu_x$
  Thus let $\mu = e^{-3x}$ and $(e^{-x} + 3ye^{-3x})dx - e^{-3x}dy = 0$ is an exact euation
  $\int(e^{-x} + 3ye^{-3x})dx = -e^{-x} - ye^{-3x} + g(y) = \int(-e^{3x})dy = -ye^{-3x} + g(x)$
  Hence let $\phi(x,y) = -e^{-x} - ye^{-3x}, d\phi = 0$
  We get $e^{-x} + ye^{-3x} = c$ which is $y = ce^{3x} - e^{2x}$ where c is an arbitrary constant

- 15.
  Suppose $\mu(x) > 0$ s.t. $(\mu y)' = \mu y' + 2\mu y$.
  Thus $\mu' = 2\mu$, we can let $\mu = e^{2x}$
  Hence $(e^{2x}y)' = e^{-x^2}$, which implies $e^{2x}y = \int e^{-x^2}dx = \int_0^x e^{-s^2}ds + c$ where c is an arbitrary constant
  Since $y(0) = 3, c = 3$
  Hence $y = e^{-2x}\int_0^x e^{-s^2}dx + 3e^{-2x}$

- 16.
  Let $M(x.y) = y^3 + 2y - 3x^2, N(x,y) = 2x + 3xy^2$,hence $Mdx + Ndy = 0$
  Since $dM/dy = 3y^2 + 2 = dN/dx$, this equation is exact
  $\int(y^3 + 2y - 3x^2)dx = xy^3 + 2xy - x^3 + g(y) = \int(2x + 3xy^2)dy = 2xy + xy^3 + h(x)$
  Thus let $\phi(x,y) = xy^3 + 2xy - x^3, d\phi = 0$
  We get $xy^3 + 2xy - x^3 = c$ where c is an arbitrary constant

- 17.

  $dy/dx = e^x e^y \Rightarrow e^{-y}dy = e^x dx \Rightarrow -e^{-y} = e^x + C$ where C is an arbitrary constant

  Hence $e^x + e^{-y} = c$ where c is an arbitrary constant

- 18.

  Let $M(x,y) = 2y^2 + 6xy - 4, N(x,y) = 3x^2 + 4xy + 3y^2$, hence $Mdx + Ndy = 0$

  Since $dM/dy = 4y + 6x = dN/dx$, this equation is exact

  $\int(2y^2 + 6xy - 4)dx = 2xy^2 + 3x^2y - 4x + g(y) = \int(3x^2 + 4xy + 3y^2)dy = 3x^2y + 2xy^2 + y^3 + h(x)$

  Thus let $\phi(x,y) = 3x^2y + 2xy^2 - 4x + y^3, d\phi = 0$

  We get $3x^2y + 2xy^2 - 4x + y^3 = c$ where c is an arbitrary constant

- 19.

  The equation can be transformed into $y' + \frac{t+1}{t}y = \frac{e^{2t}}{t}$

  Suppose $\mu(t) > 0$ s.t. $(\mu y)' = \mu y' + \frac{t+1}{t}\mu y.$

  Thus $\mu' = \frac{t+1}{t}\mu$, we can let $\mu = te^t$

  Hence $(te^t y)' = e^{3t}$ which implies that $te^t y = \int e^{3t}dt = \frac{1}{3}e^{3t} + c$

  Hence $y = \frac{e^{2t} + 3ce^{-t}}{3t}$

- 20.

  Suppose $v(x) = y(x)/x$, thus $xdv + v = dy$

  It follows that the equation can be transformed into $xv' + v = v + e^v$ which is $e^{-v}dv = dx/x$

  Thus $\ln|x| + e^{-v} = c$ where c is an arbitrary constant

  We get $\ln|x| + e^{\frac{y}{x}} = c$ where c is an arbitrary constant

- 21.

  Let $u = x^2, dy/dx = (dy/du)(du/dx) = 2xdy/du$

  Thus the equation can be transformed into $2dy/du = 1/(uy + y^3)$ which is $(2uy + 2y^3)dy - du = 0$

  Let $M(y,u) = 2uy + 2y^3, N(y,u) = -1$

  This equation is not exact, thus suppose $\mu(y)$ s.t. $(\mu M)_u = (\mu N)_y$ which is $\mu M_u = \mu_y N + \mu N_y$

  We get $2y\mu = -\mu_y$, thus we can let $\mu = e^{-y^2}$

  Thus $(2uye^{-y^2} + 2y^3e^{-y^2})dy - e^{-y^2}du = 0$ which is exact

  $\int(2uye^{-y^2} + 2y^3e^{-y^2})dy = -ue^{-y^2} - (y^2+1)e^{-y^2} + g(u) = \int(-e^{-y^2})du = -ue^{-y^2} + h(y)$

  Thus let $\phi(y,u) = -ue^{-y^2} - (y^2+1)e^{-y^2}, d\phi = 0$

  Hence $-ue^{-y^2} - (y^2+1)e^{-y^2} = c$ which is $-x^2e^{-y^2} - (y^2+1)e^{-y^2} = C$ where C is an arbitrary constant

We get $x^2+y^2+1=ce^{y^2}$ where c is an arbitrary constant

- 22.

  Suppose $xv(x)=y(x)$, thus $y'=xv'+v$

  Thus the equation can be transformed into $xv'+v=\frac{1+v}{1-v}$ which is $xv'=\frac{1+v^2}{1-v}$

  Hence $(\frac{1}{1+v^2}-\frac{1}{2}\frac{2v}{1+v^2})dv=dx/x$

  we get $\arctan(v)-\ln\sqrt{|1+v^2|}=\ln|x|+c$ where c is an arbitrary constant

  Hence $\arctan(\frac{y}{x})-\ln\sqrt{x^2+y^2}=c$ where c is an arbitrary constant

- 23.

  Suppose $xv(x)=y(x)$, thus $y'=xv'+v$

  Thus the equation can be transformed into $xv'+v=\frac{3v^2+2v}{2v+1}$ which is $xv'=\frac{v^2+v}{2v+1}$

  Hence $(\frac{1}{v}+\frac{1}{1+v})dv=dx/x$

  we get $v(v+1)=cx$ where c is an arbitrary constant

  Hence $y^2+xy=cx^3$ where c is an arbitrary constant

- 24.

  Suppose $v(x)=[y(x)]^{-1}$, thus $y=v^{-1}, y'=-v^{-2}v'$

  Thus the equation can be transformed into $xv'-v=-e^{2x}$ which is $v'-\frac{1}{x}v=-\frac{e^{2x}}{x}$

  Multiple both sides by some $\mu(x)>0$ s.t. $(\mu v)'=\mu v'-\frac{1}{x}\mu v$ thus $\mu'=-\frac{1}{x}\mu$

  we can suppose $\mu=\frac{1}{x}$, and $(\frac{v}{x})'=-\frac{e^{2x}}{x^2}$

  Hence $\frac{v}{x}=-\int\frac{e^{2x}}{x^2}dx=-\int_1^x\frac{e^{2s}}{s^2}ds+c$

  We get $v=-x\int_1^x\frac{e^{2s}}{s^2}ds+cx$, $y^{-1}=-x\int_1^x\frac{e^{2s}}{s^2}ds+cx$

  Since $y(1)=2$, $c=\frac{1}{2}$

  Hence $y^{-1}=-x\int_0^x\frac{e^{2s}}{s^2}ds+\frac{2}{x}$