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- **Review Problems——**
- 7.

  Multiple both sides by $\mu(x) > 0$

  Thus $\mu x y' + 2\mu y = \mu \sin x / x \Rightarrow \mu y' + (2\mu/x)y = \mu \sin x / x^2$

  Suppose $\mu y' + (2\mu/x)y = (\mu y)'$ we can let $\mu = x^2$

  It follows that $(x^2 y)' = \sin x \Rightarrow y = \frac{c - \cos x}{x^2}$ where $c$ is an arbitrary constant

  Substitute $y(2) = 1$, we get $c = 4 + \cos 2$

  Hence $y = \frac{4 + \cos 2 - \cos x}{x^2}$

- 8.

  Let $M(x, y) = 2xy + 1,\ N(x, y) = x^2 + 2y$, thus $Mdx + Ndy = 0$

  Since $dM/dy = 2x,\ dN/dx = 2x$, this equation is exact

  $\int (2xy + 1)dx = x^2 y + x + g(y) = \int (x^2 + 2y)dy = x^2 y + y^2 + h(x)$

  Thus let $\phi'(x, y) = Mdx + Ndy = 0$, $\phi(x, y) = x^2 y + x + y^2 = c$ where $c$ is an arbitrary constant

- 9.

- 10.

  Let $M(x, y) = x^2 + y,\ N(x, y) = x + e^y$, thus $Mdx + Ndy = 0$

  Since $dM/dy = 1,\ dN/dx = 1$, this equation is exact

  $\int (x^2 + y)dx = \frac{1}{3}x^3 + xy + g(y) = \int (x + e^y)dy = xy + e^y + h(x)$

  Thus let $\phi'(x, y) = Mdx + Ndy = 0$, $\phi(x, y) = xy + \frac{1}{3}x^3 + e^y = c$ where $c$ is an arbitrary constant

- 11.

  Let $M(x, y) = x + y,\ N(x, y) = x + 2y$, thus $Mdx + Ndy = 0$

  Since $dM/dy = 1,\ dN/dx = 1$, this equation is exact

  $\int (x + y)dx = \frac{1}{2}x^2 + xy + g(y) = \int (x + 2y)dy = xy + y^2 + h(x)$

  Thus let $\phi'(x, y) = Mdx + Ndy = 0$, $\phi(x, y) = \frac{1}{2}x^2 + xy + y^2 = c$ where $c$ is an arbitrary constant

  Substitute $y(2) = 3$, we get $c = 17$

  Hence $\frac{1}{2}x^2 + xy + y^2 = 17$

- 12.

- **Part 2——**
- 1.

  Since $f(x, y)$ is continues on R, $\forall\epsilon > 0, \exists\delta > 0, \forall y \in R, |y - y_0| < \delta, |f(x, y) - f(x, y_0| < \epsilon$

  Since $\phi_n(x)$ converges uniformly to $\phi(x)$,

  $\forall\epsilon > 0, \exists N \in \mathbb{N}^*, for\ n > N, \forall x \in R, |\phi_n(x) - \phi(x)| < \epsilon$

  Thus $\forall\epsilon > 0, \forall\delta > 0, \exists N(\delta) \in \mathbb{N}^*, for\ n > N(\delta), \forall x \in R, |\phi_n(x) - \phi(x)| < \delta$

  It follows that $\exists\delta_0 > 0$, since $|\phi_n(x) - \phi(x)| < \delta_0$ for all $n > some\ N, |f(x, \phi_n(x)) - f(x, \phi(x))| < \epsilon$

  Hence $f(x, \phi_n(x))$ converges uniformly to $\phi(x)$

- 2.

  Can't.

  Since by step 1 construction of Euler's polugonal line, they are not equivalent

  And the lack of Lipschitz condition makes the iterator fail, since each iterator cannot ensure the region being smaller or equal

- 3.

  $L = \max_{(x,y \in R)} |2y| = 2, M = \max_{(x,y) \in R} |x^2 + y^2| = 2, h = 1/2$

  Thus solving $ML^n h^{n+1}/(n+1)! = 1/(n+1)! < 0.05$, we get $n \geq 3$

  Thus $\phi_1(x) = \frac{1}{3}x^3$, $\phi_2(x) = \frac{1}{63}x^7 + \frac{1}{3}x^3$, $\phi_3(x) = \frac{1}{59535}x^{15} + \frac{1}{63}x^7 + \frac{1}{3}x^3$

- 4.
  - 1.

    f(x,y) is continuous on $(x, y) \in \mathbb{R}^2$

    $dy/(y(y-1)) = dx \Rightarrow (\frac{1}{y-1} - \frac{1}{y})dy = dx \Rightarrow d\ln|\frac{y-1}{y}| = dx$

    Thus $y = \frac{1}{1-ce^x}$ where $c$ is an arbitrary constant or $y = 0$

    Suppose $y(x_0) = y_0$ is a initial point

    If $y_0 \neq 0, c = e^{-x_0}(1 - \frac{1}{y_0}), y(x) = \frac{1}{1-e^{x-x_0}(1-\frac{1}{y_0})}$

    Let $1 - e^{x-x_0}(1 - \frac{1}{y_0}) = 0$

    We get $x = \ln(\frac{y_0}{y_0-1}) + x_0$, if $y_0 \in (-\infty, 0) \cup (1, +\infty)$

    or $1 - e^{x-x_0}(1 - \frac{1}{y_0}) > 0$, if $y_0 \in (0, 1]$

    Hence

    $y_0 \in (-\infty, 0) \cup (1, +\infty)$, the maximum existance interval is $(-\infty, \ln(\frac{y_0}{y_0-1}) + x_0)$ or

    $(\ln(\frac{y_0}{y_0-1}) + x_0, +\infty)$

    $y_0 \in [0, 1]$, the maximum existance interval is $\mathbb{R}$

  - 2.