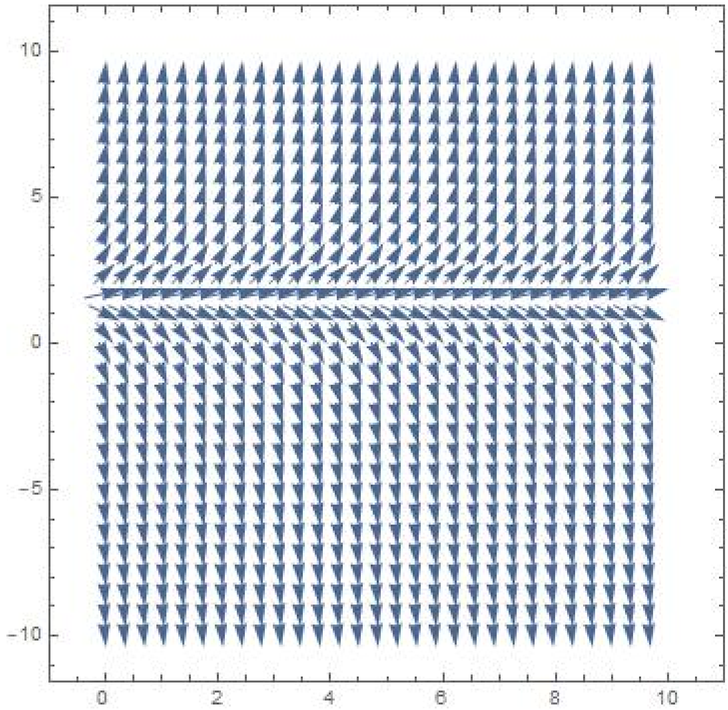
11612733 杜子豪

- **1.1——**
- 2.

  The direction field of the equation $y' = 2y - 3$ is

  as shown in the plot, for the initial point (t, y),

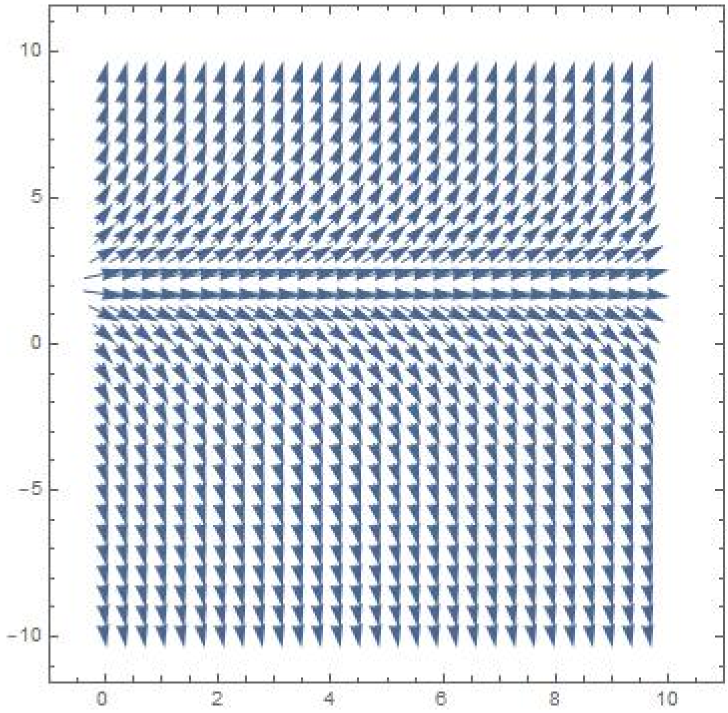
  if $y > \frac{3}{2},\ y \to +\infty \quad as\ t \to \infty$

  if $y < \frac{3}{2},\ y \to -\infty \quad as\ t \to \infty$

- 9.

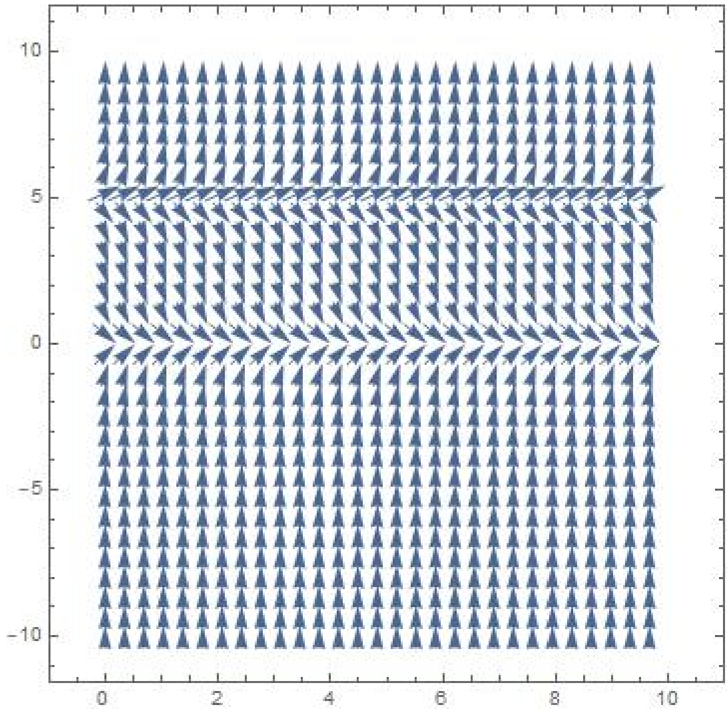
  Since all other solutions diverge from $y = 2$, we can get $y' = 0$ when $y = 2$, $y' > 0$ when $y > 2$ and $y' < 0$ when $y < 2$. Thus $0 = ay + b \Rightarrow 0 = 2a + b$ and $a > 0$

  Hence, let $a = 1,\ b = -2$, the equation will be $y' = y - 2$ and its direction field is



- 12.

  The direction field of equation $y' = -y(5-y)$ is

  as shown in the plot,

  if $y > 5,\ y \to +\infty \quad as\ t \to \infty;$

  if $y \in [-\infty, 5),\ y \to 0 \quad as\ t \to \infty$

- 15.

  All the solution converge to $y = 2$. Thus $y' = 0\ as\ y = 2$

We can know that the answer is (c) or (j)

Since $y' > 0 \; as \; y < 2 \quad and \quad y' < 0 \; as \; y > 2$

The equation is (j): $y' = 2 - y$

- 18.

  All the solution diverge from $y = 2$. Thus $y' = 0 \; as \; y = 2$

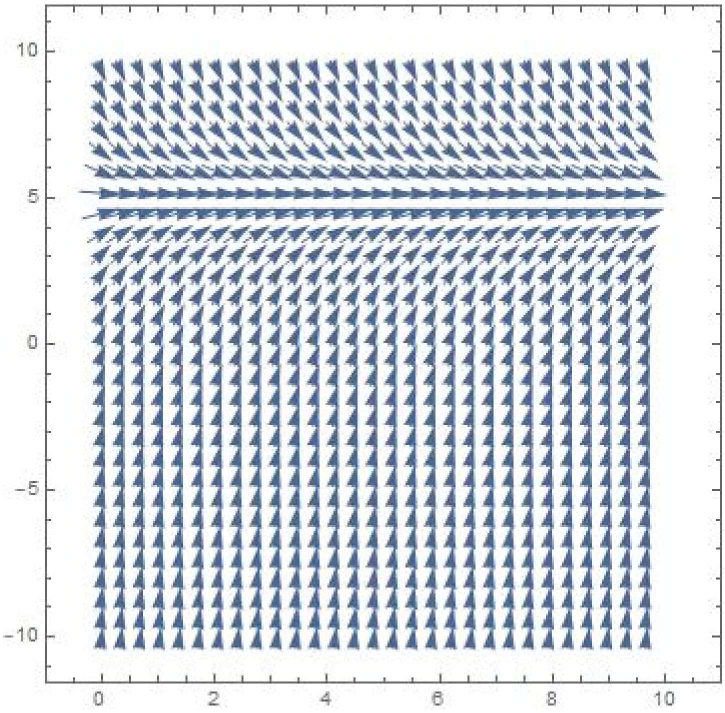
  We can know that the answer is (c) or (j), which is similar to question 15, however in the opposite direction when approach $y = 2$.

  Since $y' > 0 \; as \; y > 2 \quad and \quad y' < 0 \; as \; y < 2$

  The equation is (j): $y' = 2 - y$

- **1.2——**
- 1.
  - a.

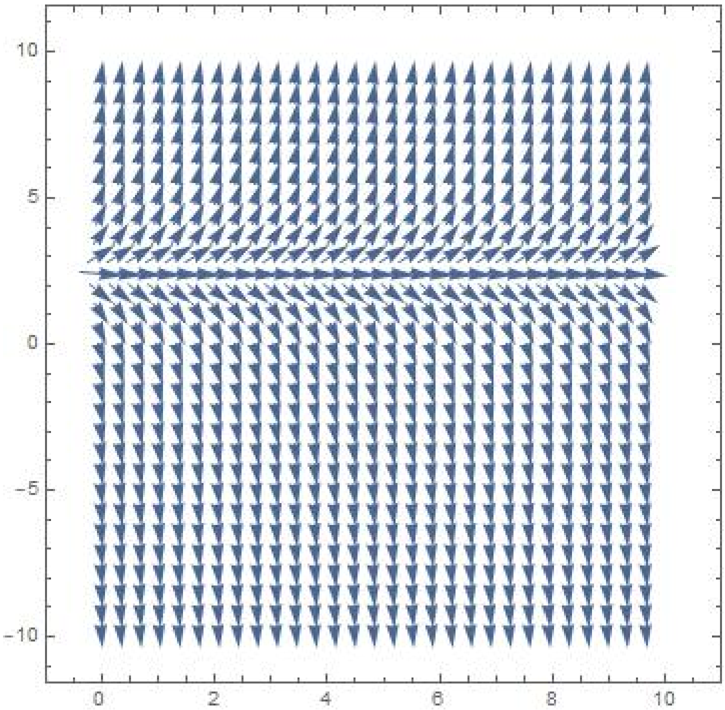
$$\begin{gathered} \mathrm{d}y/\mathrm{d}t = -y + 5 \\ \Rightarrow \frac{\mathrm{d}y/\mathrm{d}t}{y - 5} = -1 \\ \Rightarrow \mathrm{d}(\ln|y - 5|) = -\mathrm{d}t \\ \Rightarrow y = ce^{-t} + 5 \quad where \; c \; is \; an \; arbitrary \; constant \\ since \; y(0) = y_0 \\ c + 5 = y_0 \end{gathered}$$



all the solution will converge to $y = 5$

- 2.
  - b.

$$\frac{dy}{dt} = 2y - 5$$
$$\Rightarrow \frac{dy/dt}{y - 5/2} = 2$$
$$\Rightarrow d(\ln|y - 5/2|) = 2dt$$
$$\Rightarrow y = ce^{2t} + 5/2 \quad where\ c\ is\ an\ arbitrary\ constant$$
$$since\ y(0) = y_0$$
$$c + 5/2 = y_0$$



the solution diverge from $y = 5/2$

- 7.

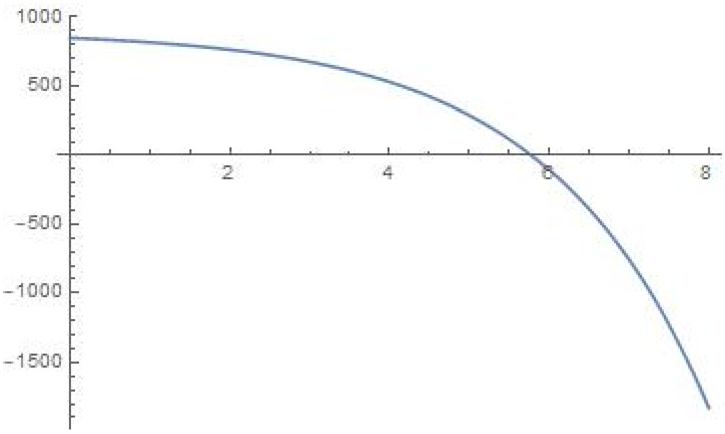
$$dp/dt = 0.5p - 450$$
$$\Rightarrow \frac{dp/dt}{p - 900} = \frac{1}{2}$$
$$\Rightarrow d(\ln|p - 900|) = \frac{1}{2}dt$$
$$\Rightarrow p = ce^{\frac{1}{2}t} + 900 \quad where\ c\ is\ an\ arbitrary\ constant$$

  - a.

Since $p(0) = 850$, $c + 900 = 850$. Thus $c = -50$

Hence, $p = -50e^{\frac{1}{2}t} + 900$

For $p = 0$, we get $e^{\frac{1}{2}t} = 18$, and the solution is $t = 2\ln 18$



- b.

  Since $p(0) = p_0$, $c + 900 = p_0$. Thus $c = p_0 - 900$

  Hence, $p = (p_0 - 900)e^{\frac{1}{2}t} + 900$

  For $0 < p < 900$, $p \to -\infty \; as \; t \to \infty$, and $p(0) > 0$

  It follows that $p = 0$ has a solution.

  let $p = 0$, we get $t = 2\ln(\frac{900}{900-p_0})$

- 8.

$$\mathrm{d}p/\mathrm{d}t = rp$$
$$\Rightarrow \frac{\mathrm{d}p/\mathrm{d}t}{p} = r$$
$$\Rightarrow \mathrm{d}(\ln|p|) = r\mathrm{d}t$$
$$\Rightarrow p = ce^{rt} \quad where\ c\ is\ an\ arbitrary\ constant$$
$$Here,\ we\ measure\ the\ time\ in\ day$$

- a.

  After 30 days, $p(30) = ce^{30r} = 2p(0) = 2c$

  Thus $e^{30r} = 2 \Rightarrow r = \frac{1}{30}\ln 2$

- b.

  After N days, $p(N) = ce^{Nr} = 2p(0) = 2c$

  Thus $e^{Nr} = 2 \Rightarrow r = \frac{1}{N}\ln 2$

- **1.3——**
- 1.

  Order: 2, Linear
- 2.

  Order: 2, Nonlinear
- 5.

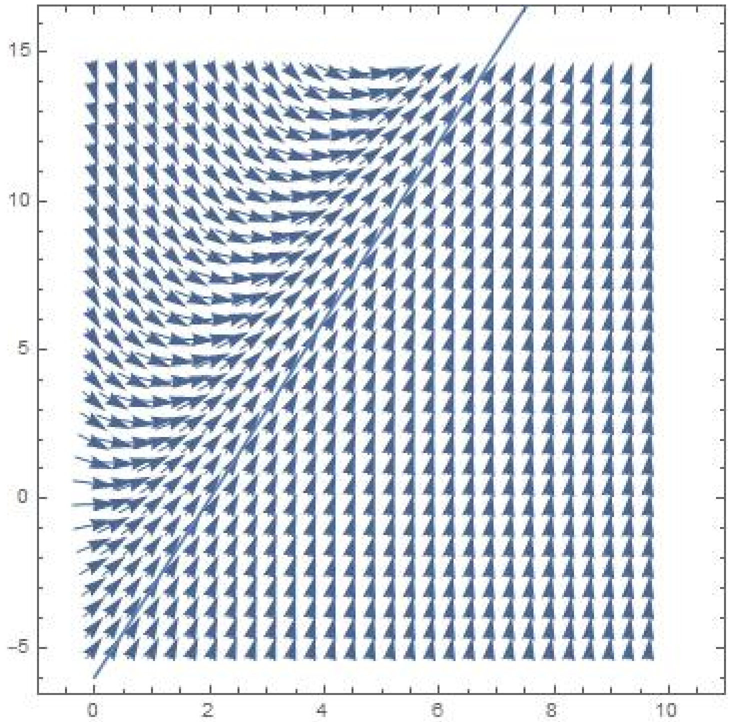
  Order: 2, Nonlinear

- 7.

  $y_1' = e^t = y_1''$, Hence $y_1'' - y_1 = e^t - e^t = 0$, which is a solution

  $y_2 = \cosh t = \frac{e^t+e^{-t}}{2}$, Thus $y_2'' = (\frac{e^t-e^{-t}}{2})' = \frac{e^t+e^{-t}}{2} = y_2$, Hence $y_2'' - y_2 = 0$, which is a solution

- **2.1——**
- 9.
  - a.

  - b.

    For large t, the solution converge to $y = 3t - 6$
  - c.

    Multiple both sides of the equation by $\mu(t)$, we get

$$2\mu y' + \mu y = 3\mu t \Rightarrow \mu y' + \frac{1}{2}\mu y = \frac{3}{2}\mu t > 0$$

$$\exists\ \mu(t),\ s.t.\,(\mu y)' = \mu y' + \frac{1}{2}\mu y$$

$$\Rightarrow \mu' = \frac{1}{2}\mu$$

$$\Rightarrow \mathrm{d}\mu/\mu = \frac{1}{2}\mathrm{d}t$$

$$\Rightarrow \ln\mu = \frac{1}{2}t + C,\ where\ C\ is\ an\ arbitrary\ constant$$

$$\Rightarrow \mu = e^{t/2+C}$$

$$Let\ C = 0,\ \mu = e^{t/2}$$

Hence $\mu y' + \frac{1}{2}\mu y = (\mu y)' = (e^{t/2}y)' = \frac{3}{2}e^{t/2}t$

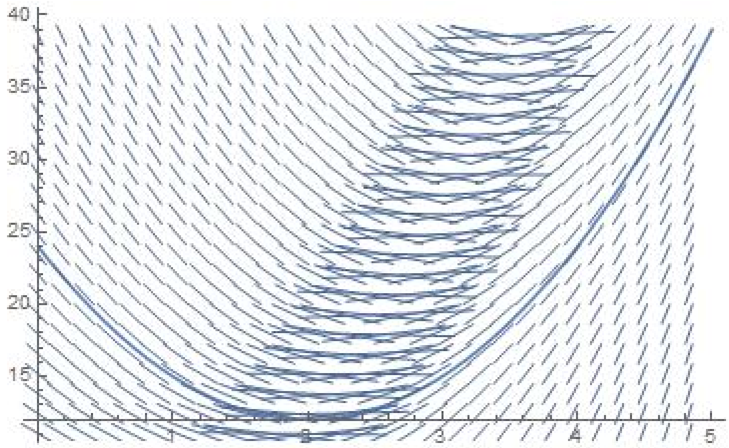
We get $y = \frac{3}{2}e^{-t/2}[\int_{t_0}^{t} se^{s/2}ds + c] =$

$$\int se^{s/2}ds = 2\int s(e^{s/2}d(\frac{s}{2})) = 2\int sd(e^{s/2})$$

$$= 2\int d(se^{s/2}) - 2\int ds(e^{s/2}) = 2se^{s/2} - 4e^{s/2}$$

Hence $y = \frac{3}{2}e^{-t/2}(2te^{t/2} - 4e^{t/2} + c) = 3t - 6 + \frac{3}{2}ce^{-t/2}$

- 12.
  - a.



  - b.

    For large t, the solution converge to $y = 3t^2 - 12t + 24$

  - c.

    Multiple both sides by $\mu(t)$, we get

$$2\mu y' + \mu y = 3\mu t^2 \Rightarrow \mu y' + \frac{1}{2}\mu y = \frac{3}{2}\mu t^2$$

$$\exists\ \mu(t) > 0,\ s.t. (\mu y)' = \mu y' + \frac{1}{2}\mu y$$

$$\Rightarrow \mu' = \frac{1}{2}\mu$$

$$\Rightarrow \mathrm{d}\mu/\mu = \frac{1}{2}\mathrm{d}t$$

$$\Rightarrow \ln\mu = \frac{1}{2}t + C,\ where\ C\ is\ an\ arbitrary\ constant$$

$$\Rightarrow \mu = e^{t/2+C}$$

$$Let\ C = 0,\ \mu = e^{t/2}$$

Hence $\mu y' + \frac{1}{2}\mu y = (\mu y)' = (e^{t/2}y)' = \frac{3}{2}e^{t/2}t^2$

We get $y = \frac{3}{2}e^{-t/2}[\int_{t_0}^{t} s^2e^{s/2}ds + c] =$

$$\int s^2 e^{s/2} ds = 2\int s^2 (e^{s/2} d(\frac{s}{2})) = 2\int s^2 d(e^{s/2})$$
$$= 2\int d(s^2 e^{s/2}) - 2\times 2\int e^{s/2} s ds$$

From 9.(a) we know that $\int se^{s/2}ds = 2se^{s/2} - 4e^{s/2}$

Hence $y = \frac{3}{2}e^{-t/2}(2t^2e^{t/2} - 8te^{t/2} + 16e^{t/2} + c) = 3t^2 - 12t + 24 + \frac{3}{2}ce^{-t/2}$

- 16.

  Multiple both sides by $\mu(t)$

$$\mu y' + \mu(2/t)y = \mu(\cos t)/t^2$$
$$\exists\ \mu(t) > 0,\ s.t. (\mu y)' = \mu y' + \frac{2}{t}\mu y$$
$$\Rightarrow \mu' = \frac{2}{t}\mu$$
$$\Rightarrow \mathrm{d}\mu/\mu = \frac{2}{t}\mathrm{d}t$$
$$\Rightarrow \ln\mu = 2\ln t + C,\ where\ C\ is\ an\ arbitrary\ constant$$
$$\Rightarrow \mu = e^{2\ln t + C}$$
$$Let\ C = 0,\ \mu = t^2$$

  Hence $\mu y' + \mu(2/t)y = (\mu y)' = (t^2 y)' = \cos t$

  We get $y = t^{-2}[\int_{t_0}^{t}\cos s ds + c] = \frac{\sin t}{t^2} + c/t^2$

  Since $y(\pi) = 0$, $c = 0$. Hence $y = \sin t/t^2$

- 20.

  Multiple both sides by $\mu(t)$

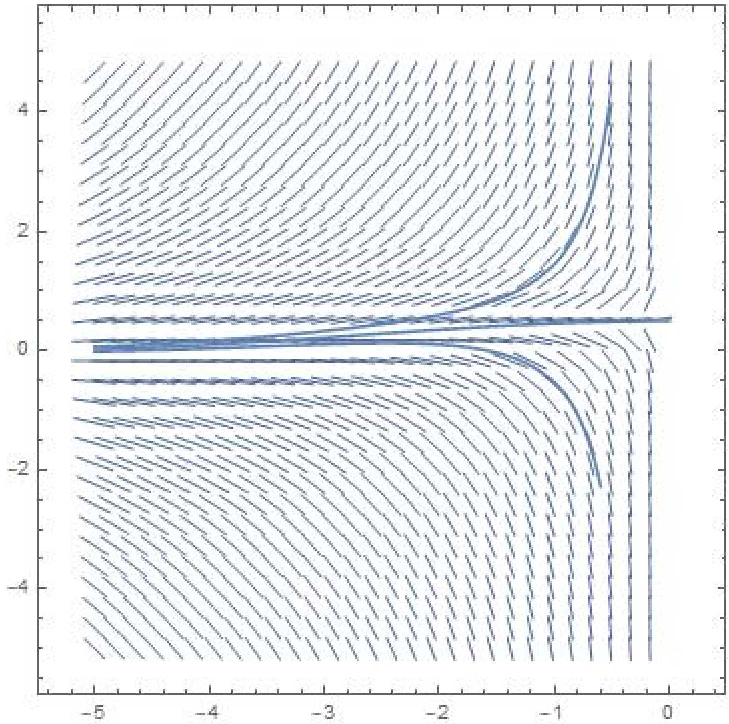
$$\mu t y' + \mu(t+1)y = \mu t \Rightarrow \mu y' + \mu\frac{t+1}{t}y = \mu$$
$$\exists\ \mu(t) > 0,\ s.t. (\mu y)' = \mu y' + \frac{t+1}{t}\mu y$$
$$\Rightarrow \mu' = \frac{t+1}{t}\mu$$
$$\Rightarrow \mathrm{d}\mu/\mu = \frac{t+1}{t}\mathrm{d}t$$
$$\Rightarrow \ln\mu = t + \ln t + C,\ where\ C\ is\ an\ arbitrary\ constant$$
$$\Rightarrow \mu = e^{t + \ln t + C}$$
$$Let\ C = 0,\ \mu = te^t$$

  Hence $\mu y' + \mu\frac{t+1}{t}y = (\mu y)' = (te^t y)' = te^t$

  We get $y = \frac{1}{t}e^{-t}[\int_{t_0}^{t} se^s ds + c] = \frac{1}{t}e^{-t}[(t-1)e^t + c] = \frac{t-1+ce^{-t}}{t}$

Since $y(\ln 2) = 1$, $c = 2$, Hence $y = \frac{t-1+2e^{-t}}{t}$.

- 25.
  - a.



Thus the solution will diverge from the curve with initial value of $a_0 \approx 0.5$

  - b.

Multiple both sides by $\mu(t)$

$$\mu t y' + 2\mu y = \mu(\sin t)/t \Rightarrow \mu y' + \frac{2}{t}\mu y = \mu(\sin t)/t^2$$
$$\exists\ \mu(t) > 0,\ s.t. (\mu y)' = \mu y' + \frac{2}{t}\mu y$$
$$\Rightarrow \mu' = \frac{2}{t}\mu$$
$$\Rightarrow \mathrm{d}\mu/\mu = \frac{2}{t}\mathrm{d}t$$
$$\Rightarrow \ln \mu = 2\ln t + C,\ where\ C\ is\ an\ arbitrary\ constant$$
$$\Rightarrow \mu = e^{2\ln t + C}$$
$$Let\ C = 0,\ \mu = t^2$$

Hence $\mu y' + \frac{2}{t}\mu y = (\mu y)' = (t^2 y)' = \mu(\sin t)/t^2 = \sin t$

We get $y = t^{-2}[\int_t^0 \sin s ds + c] = \frac{c - \cos t}{t^2}$

For $t \to 0$, $\cos t \to 1$. Thus $y \to -\infty \quad as\ c < 1;\ y \to +\infty \quad as\ c > 1$

When $c = 0$, $y(-\pi/2) = (1-0)/(-\pi/2)^2 = 4/\pi^2$

Hence the value of a_0 is $4/\pi^2$

- c.

  For $a_0 = 4/\pi^2$, \$y\to1\quad as\;t\to0

  For $a_0 > 4/\pi^2$, $y \to +\infty \quad as\; t \to 0$

  For $a_0 < 4/\pi^2$, $y \to -\infty \quad as\; t \to 0$\$

- 29.
  - a.

    Multiple both sides by $\mu(t)$

$$\mu y' + \frac{1}{4}\mu y = \mu(3 + 2\cos 2t)$$
$$\exists\ \mu(t) > 0,\ s.t.\, (\mu y)' = \mu y' + \frac{1}{4}\mu y$$
$$\Rightarrow \mu' = \frac{1}{4}\mu$$
$$\Rightarrow \mathrm{d}\mu/\mu = \frac{1}{4}\mathrm{d}t$$
$$\Rightarrow \ln\mu = \frac{1}{4}t + C,\ where\ C\ is\ an\ arbitrary\ constant$$
$$Let\ C = 0,\ \mu = e^{\frac{1}{4}t}$$

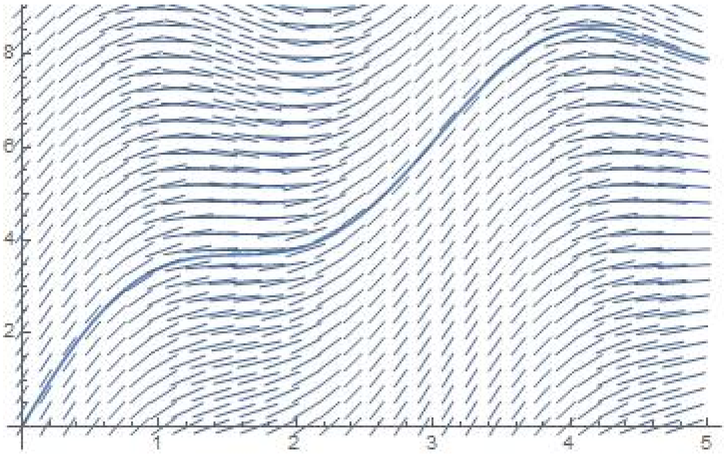
Hence $(\mu y)' = (e^{\frac{1}{4}t}y)' = 3e^{t/4} + 2e^{t/4}\cos 2t$

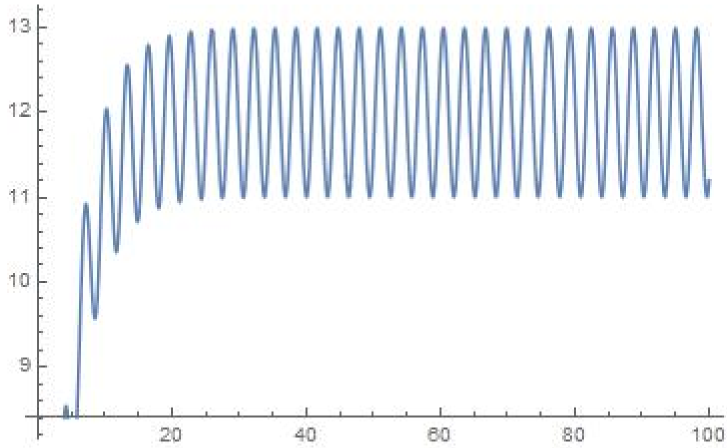
We get $y = e^{-t/4}[3\int_{t_0}^{t} e^{s/4}ds + 2\int_{t_0}^{t} e^{s/4}\cos 2sds + c]$

$$A = \int e^{s/4}\cos 2sds = 4\int d(e^{s/4})\cos 2s = 4[\int d(e^{s/4}\cos 2s) + 2\int e^{s/4}\sin 2sds]$$
$$= 4e^{s/4}\cos 2s + 8\times 4[\int d(e^{s/4}\sin 2s) - 2\int e^{s/4}\cos 2sds]$$
$$= 4e^{s/4}\cos 2s + 32e^{s/4}\sin 2s - 64A$$
$$A = (4e^{s/4}\cos 2s + 32e^{s/4}\sin 2s)/65$$

Hence $y = 12 + \frac{8\cos 2t + 64\sin 2t}{65} + \frac{c}{e^{t/4}}$

Since $y(0) = 0$,$c = -12 - 8/65$. Thus $y = 12 + \frac{8}{65}(\cos 2t + 8\sin 2t) - \frac{788}{65}e^{-t/4}$





Thus, when $t \to \infty$, y will oscillate between $12 \pm \frac{8}{\sqrt{65}}$

- b.

  If $y = 12$, we can get $\frac{8}{\sqrt{65}} \sin(2t + \arctan\frac{1}{8}) = \frac{788}{65}e^{-t/4}$

  Thus ……(不知道到这里是不是做错了，卡住)

- **2.2——**
- 1.

$$\frac{dy}{dx} = \frac{x^2}{y} \Rightarrow ydy = x^2dx \Rightarrow \int ydy = \int x^2dx$$
$$\Rightarrow \frac{1}{2}y^2 = \frac{1}{3}x^3 + c \quad where\ c\ is\ an\ arbitrary\ constant$$
$$y = \pm\sqrt{\frac{2}{3}x^2 + 2c}$$

- 10.
  - a.

$$\frac{dy}{dx} = \frac{1-2x}{y} \Rightarrow ydy = (1-2x)dx \Rightarrow \int ydy = \int (1-2x)dx$$
$$\Rightarrow \frac{1}{2}y^2 = x - x^2 + c$$
$$substitute\ x = 0\ and\ y = 1$$
$$\frac{1}{2} = c$$
$$y = \pm\sqrt{-2x^2 + 2x + 1}$$

  - c.

    The solution is defined only when $-2x^2 + 2x + 1 \geq 0$.

    That is $x \in [\frac{1-\sqrt{3}}{2}, \frac{1+\sqrt{3}}{2}]$