11612733 杜子豪

- **3.5——**
- 2.

  Solving $y'' - y' - 2y = 0$ we get $r^2 - r - 2 = 0$ thus $r_1 = 2, r_2 = -1$

  Hence the solution is $y(t) = c_1 e^{2t} + c_2 e^{-t}$ where c_1, c_2 are arbitrary constants

  Assume $Y(t) = At^2 + Bt + C$, $Y' = 2At + B$, $Y'' = 2A$

  Thus $2A - 2At - B - 2At^2 - 2Bt - 2C = -2t + 4t^2$, we get $A = -2, B = 3, C = -\frac{7}{2}$

  Hence $y = c_1 e^{2t} + c_2 e^{-t} - 2t^2 + 3t - \frac{7}{2}$ where c_1, c_2 are arbitrary constants

- 5.

  Solving $y'' + 2y' = 0$ we get $r^2 + 2r = 0$ thus $r_1 = -2, r_2 = 0$

  Hence the solution is $y(t) = c_1 e^{-2t} + c_2$

  Assume $Y_1(t) = A \sin 2t + B \cos 2t$

  $Y' = 2A \cos 2t - 2B \sin 2t$, $Y'' = -4A \sin 2t - 4B \cos 2t$

  Thus $-4(A + B) \sin 2t + 4(A - B) \cos 2t = 4 \sin 2t$, we get $A = B = -\frac{1}{2}$

  Assume $Y_2(t) = Kt$, thus $(Kt)'' + 2(Kt)' = 2K = 3$, we get $K = \frac{3}{2}$

  Hence $y = c_1 e^{-2t} + c_2 + \frac{3}{2}t - \frac{1}{2}(\sin 2t + \cos 2t)$ where c_1, c_2 are arbitrary constants

- 11.

  Solving $y'' + y' - 2y = 0$ we get $r^2 + r - 2 = 0$ thus $r_1 = -2, r_2 = 1$

  Hence the solution is $y(t) = c_1 e^{-2t} + c_2 e^t$ where c_1, c_2 are arbitrary constants

  Assume $Y(t) = At + B$, $Y' = A$, $Y'' = 0$

  Thus $A - 2At - 2B = 2t$, we get $A = -1, B = -\frac{1}{2}$

  Hence $y = c_1 e^{-2t} + c_2 e^t - t - \frac{1}{2}$ where c_1, c_2 are arbitrary constants

  Substitute $y(0) = 0, y'(0) = 1$, we get $c_1 + c_2 = \frac{1}{2}, c_2 - 2c_1 = 2$

  Thus $c_1 = -\frac{1}{2}, c_2 = 1$ the solution is $y = -\frac{1}{2}e^{-2t} + r^t - t - \frac{1}{2}$

- 16.
  - a.

    $Y(t) = (A_1 t + A_2 t^2 + A_3 t^3 + A_4 t^4 + A_5 t^5) + (B_1 t + B_2 t^2 + B_3 t^3)e^{-3t} + C \cos 3t + D \sin 3t$

- 28.

  $y'' - 3y' - 4y = (D - 4)(D + 1)y = 3e^{2t}$

  Solving $u' - 4u = 3e^{2t}$, we get $u = ce^{4t} - \frac{3}{2}e^{-2t}$ where c is an arbitrary constant

Solving $y' + y = ce^{4t} - \frac{3}{2}e^{-2t}$, we get $y = c_1 e^{-t} + c_2 e^{4t} - \frac{1}{2}e^{2t}$

where c_1, c_2 are arbitrary constants

- **3.6——**
- 2.

  Solving $y'' - y' - 2y = 0$ we get $r^2 - r - 2 = 0$ thus $r_1 = 2, r_2 = -1$

  Hence two solutions are $y_1(t) = e^{2t}, y_2 = e^{-t}$

  $W[y_1, y_2] = y_1 y_2' - y_2 y_1' = -3e^t, g = 2e^{-t}$

  $-y_1 \int_0^t \frac{y_2 g}{W[y_1,y_2]} ds + y_2 \int_0^t \frac{y_1 g}{W[y_1,y_2]} ds$

  $= -e^{2t} \int_0^t -\frac{2}{3}e^{-3s} ds + e^{-t} \int_0^t -\frac{2}{3} ds = \frac{2}{9}y_1 - \frac{2}{9}y_2 - \frac{2}{3}te^{-t}$

  Hence $Y(t) = -\frac{2}{3}te^{-t}$

- 4.

  Sloving $y'' + y = 0$ we get $r^2 + 1 = 0$ thus $r_1 = i, r_2 = -i$

  Hence two solutions are $y_1(t) = \cos t, y_2(t) = \sin t$

  $W[y_1, y_2] = y_1 y_2' - y_2 y_1' = 1, g = \tan t$

  $-y_1 \int_0^t \frac{y_2 g}{W[y_1,y_2]} ds + y_2 \int_0^t \frac{y_1 g}{W[y_1,y_2]} ds$

  $= -\cos t \int_0^t \sin^2 s \cos^{-1} s ds + \sin t \int_0^t \sin s ds = -\cos t \ln(\sec t + \tan t) - \sin t \cos t + \sin t$

  Hence $Y(t) = -\cos t \ln(\sec t + \tan t) - \sin t \cos t$

- 18.
  - a.

    Solving $y'' + y = 0$ we get $r_1 = i, r_2 = -i$ thus $y_1 = \cos t, y_2 = \sin t$

    $W[y_1, y_2] = 1$

    $Y(t) = \int_{t_0}^t (\cos s \sin t - \cos t \sin s) g(s) ds = \int_{t_0}^t \sin(t-s) g(s) ds$

    $Y(t_0) = 0, Y'(t_0) = 0$
  - b.

    Solving $L(u) = 0, u(0) = y_0, u'(0) = y_0'$ we get $r_1 = i, r_2 = -i$

    Thus $u(t) = y_0 \cos t + y_0' \sin t$ where c_1, c_2 are arbitrary constants

    Solving $L(v) = g(t), v(0) = 0, v'(0) = 0$ we get $v(t) = \int_{t_0}^t \sin(t-s) g(s) ds$

    Hence the solution is $y = \int_{t_0}^t \sin(t-s) g(s) ds + y_0 \cos t + y_0' \sin t$

- 24.

  The equation can be transformed into $y'' - \frac{2}{t}y' + \frac{2}{t^2}y = 4$

  Let $y = tv(t)$ we get $tv'' = 4$

Hence $v = 4t \ln t + c_1 t + c_2$, weget $y = 4t^2 \ln t + c_1 t^2 + c_2 t$

- **3.7——**
- 3.

  Since there is no damping, $mu'' + ku(t) = 0$

  $mg = kL$ and we get $k = mg/L = 1/0.05 = 20N/m$

  Hence $0.1u'' + 20u = 0$ and solving this we get $u = c_1 \cos 10\sqrt{2}t + c_2 \sin 10\sqrt{2}t$

  where c_1, c_2 are arbitrary constants

  Substitute $y(0) = 0, y'(0) = 0.1$ we get $c_1 = 0, c_2 = \frac{1}{100\sqrt{2}}$

  Hence $u = \frac{\sqrt{2}}{2} \sin 10\sqrt{2}t$ with u in cm and t in s

  When $u = 0$, we get $t = k\pi/(10\sqrt{2})$ for $k = 0, 1, \ldots$

  Thus the first returning to its equilibrium, $t = \frac{\sqrt{2}}{20}\pi$

- 8.

  $T_d/T = (1 - \frac{\gamma^2}{4km})^{-\frac{1}{2}} = 1/\sqrt{1 - \frac{1}{4}\gamma^2} = 1.5$

  Thus $\gamma = \sqrt{\frac{20}{9}}$

- 17.

  Solving $\frac{3}{2}u'' + ku = 0$ we get $u = c_1 \cos \omega t + c_2 \sin \omega t$

  where c_1, c_2 are arbitrary constants and $\omega^2 = 2k/3$

  Substitute $u(0) = 2$, we get $c_1 = 2$

  Since $3 = \sqrt{c_1^2 + c_2^2} = \sqrt{4 + c_2^2}$, we get $c_2 = \pm\sqrt{5}$

  Since $T = \pi = 2\pi/\omega$, we get $k = 6$

  Thus $u = 2\cos 2t \pm \sqrt{5} \sin 2t$ and $u'(0) = v = \pm 2\sqrt{5}$

- 18.
  - a.

    Solving $mu'' + \gamma u' + ku = 0$ since $\gamma^2 - 4km < 0$ we get $u = c_1 e^{\lambda t} \cos \mu t + c_2 e^{\lambda t} \sin \mu t$

    where c_1, c_2 are arbitrary constants and $\mu = \frac{1}{2m}(4km - \gamma^2)^{\frac{1}{2}}, \lambda = -\gamma/(2m)$

    Substitute $u(0) = u_0, u'(0) = v_0$ we get $c_1 = u_0, c_2 = (v_0 - u_0\lambda)/\mu$

    Hence $u = u_0 e^{\lambda t} \cos \mu t + \frac{v_0 - \lambda u_0}{\mu} e^{\lambda t} \sin \mu t$ where $\mu = \frac{1}{2m}(4km - \gamma^2)^{\frac{1}{2}}, \lambda = -\gamma t/(2m)$
  - b.

    $R^2 = c_1^2 + c_2^2 = 4m(mv_0^2 + \gamma v_0 u_0 + ku_0^2)/(4km - \gamma^2)$
  - c.

Let $r(\gamma) = \frac{A+B\gamma}{C-\gamma^2}$ where $A, B, C > 0$

$r'(\gamma) = \frac{B(C-\gamma^2)+2(A+B\gamma)\gamma}{(C-\gamma^2)^2} = \frac{BC+2A\gamma+B\gamma^2}{(C-\gamma^2)^2} > 0$

Hence R increases as γ increases

- 23.

  $ma = mu'' = F_{total} = -ku - \gamma u'$ which is equivalent to equation (21)

  In this case the equilibrium position is just the position that

  makes the spring unstretched or uncompressed

  However, in the text, the equilibrium position balance the gravity