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- **1.A——**
- 2.

  $(\frac{-1+\sqrt{3}i}{2})^3 = \frac{1}{8}(\sqrt{3}i-1)^2(\sqrt{3}i-1) = -\frac{1}{4}(\sqrt{3}i+1)(\sqrt{3}i-1) = -\frac{1}{4}[(\sqrt{3}i)^2-1] = 1$

  Hence, $\frac{-1+\sqrt{3}i}{2}$ is a cube root of 1

- 3.

  Suppose $x = a+bi$, where $a, b \in \mathbb{R}$

  $x^3 = i \Rightarrow (a+bi)^2 = i \Rightarrow (a^2-b^2)+2abi = i$

  It follows that $a^2 = b^2, 2ab = 1$

  Hence $b = \frac{\sqrt{2}}{2}$, $a = \pm b$

  Two square roots of i are $\frac{\sqrt{2}(1+i)}{2}$ and $-\frac{\sqrt{2}(1+i)}{2}$

- 7.

  Since $\alpha \in \mathbb{C}$, we can write α as $a+bi$ where $a, b \in \mathbb{R}$

  Suppose $\beta \in \mathbb{C}$, and $\beta = c+di$ where $c, d \in \mathbb{R}$

  Assume $\alpha+\beta = 0$, we can get $c = -a$ and $b = -d$ which implies that there exists the unique β s.t. $\alpha+\beta = 0$

- 8.

  Since $\alpha \in \mathbb{C}$, we can write α as $a+bi$ where $a, b \in \mathbb{R}$

  Suppose $\beta \in \mathbb{C}$, and $\beta = c+di$ where $c, d \in \mathbb{R}$

  $\alpha\beta = 1 \Leftrightarrow (ac-bd)+(ad+bc)i = 1 \Leftrightarrow ac-bd = 1 \; and \; ad+bc = 0 \Leftrightarrow c = \frac{a}{a^2+b^2} \; and \; d = -\frac{b}{a^2+b^2}$

- 9.

  Suppose $\lambda = m+ni$, $\alpha = a+bi$, $\beta = c+di$

  $\lambda(\alpha+\beta) = (m+ni)[(a+bi)+(c+di)] = (m+ni)(a+bi)+(m+ni)(c+di) = \lambda\alpha+\lambda\beta$

- 11.

  Suppose $\lambda = a+bi$

  Since $(a+bi)(2-3i) = 12-5i$

  Thus $2a+3b = 12$, $3a-2b = 5$. Hence $a = 3$, $b = 2$, $\lambda = 3+2i$

  However $(3+2i)(-6+7i) = -32+9i \neq -32+9i$

  Hence there does not exist such λ

- **1.B——**
- 1.

  $-(-v)$ is an additive inverse of $-v$

  v is also an additive inverse of $-v$

  Since additive inverse is unique, we can get $-(-v) = v$

- 2

  If $a \neq 0$, $\exists\ b \in \mathrm{F}$ s.t. $ab = 1$

  $v = 1 \cdot v = (ab)v = b(av) = b \cdot 0 = 0$

  If $a = 0$, which satisfies the condition as desire.

- 3.

  $v + 3x = w \Rightarrow x = \frac{v-w}{3}$ which proves the existance

  Suppose $y \in V$ and $y \neq x$ s.t. $v + 3y = w$

  $w - w = 0 = 3(x - y) \Rightarrow x = y.$

  Thus x is unique

- 5.

  $v + (-1)v = [1 + (-1)]v = 0v = 0$

  Thus they are equivalent

- 6.

  No. It is not a vector space, since $[+\infty + (-\infty)] + 1 = 1$, $-\infty + [(-\infty) + 1] = 0$

- **1.C——**
- 1.
  - a.

    Assume $x_{11} + 2x_{21} + 3x_{31} = 0$, $x_{12} + 2x_{22} + 3x_{32} = 0$

    $x_1 = (x_{11}, x_{21}, x_{31})$, $x_2 = (x_{12}, x_{22}, x_{32})$

    Addition:

    $x_1 + x_2 = (x_{11} + x_{12}, x_{21} + x_{22}, x_{31} + x_{32})$

    $(x_{11} + x_{12}) + 2(x_{21} + x_{22}) + 3(x_{31} + x_{32}) = 0$

    Hence, it is closed under addition

    Scalar Multiplication:

    $\lambda x_1 = (\lambda x_{11}, \lambda x_{21}, \lambda x_{31})$

$\lambda x_{11} + 2(\lambda x_{21}) + 3(\lambda x_{31}) = 0$

Hence, it is closed under scalar multiplication

Hence it is subspace of $\mathbf{F}^3$

- b.

  Assume $x_{11} + 2x_{21} + 3x_{31} = 0$, $x_{12} + 2x_{22} + 3x_{32} = 0$

  $x_1 = (x_{11}, x_{21}, x_{31})$, $x_2 = (x_{12}, x_{22}, x_{32})$

  Addition:

  $x_1 + x_2 = (x_{11} + x_{12}, x_{21} + x_{22}, x_{31} + x_{32})$

  $(x_{11} + x_{12}) + 2(x_{21} + x_{22}) + 3(x_{31} + x_{32}) = 8$

  Hence, it is not closed under addition

  Hence it is not a subspace of $\mathbf{F}^3$

- c.

  Assume $V = \{(x_1, x_2, x_3) \in \mathbf{F}^3 : x_1 x_2 x_3 = 0\}$

  $(1, 0, 0) \in V$ and $(0, 1, 1) \in V$

  $(1, 0, 0) + (0, 1, 1) = (1, 1, 1) \notin V$

  Hence it is not closed under addition.

  It is not a subspace of $\mathbf{F}^3$

- d.

  Assume $x_{11} = 5x_{31}$, $x_{12} = 5x_{32}$

  $x_1 = (x_{11}, x_{21}, x_{31})$, $x_2 = (x_{12}, x_{22}, x_{32})$

  Addition:

  $x_1 + x_2 = (x_{11} + x_{12}, x_{21} + x_{22}, x_{31} + x_{32})$

  $(x_{11} + x_{12}) = 5(x_{31} + x_{32})$

  Hence, it is closed under addition

  Scalar Multiplication:

  $\lambda x_1 = (\lambda x_{11}, \lambda x_{21}, \lambda x_{31})$

  $\lambda x_{11} = 5(\lambda x_{31})$

  Hence, it is closed under scalar multiplication

  Hence it is subspace of $\mathbf{F}^3$

- 2.
  - a.

    Assume $x_{31} = 5x_{41} + b$, $x_{32} = 5x_{42} + b$

    $x_1 = (x_{11}, x_{21}, x_{31}, x_{31})$, $x_2 = (x_{12}, x_{22}, x_{32}, x_{42})$

    Addition:

    $x_1 + x_2 = (x_{11} + x_{12}, x_{21} + x_{22}, x_{31} + x_{32}, x_{41} + x_{42})$

    $(x_{31} + x_{22}) = 5(x_{41} + x_{42}) + 2b$

Hence, it is closed under addition iff $2b = 0 \Leftrightarrow b = 0$

Scalar Multiplication:

$\lambda x_1 = (\lambda x_{11}, \lambda x_{21}, \lambda x_{31})$

$\lambda x_{11} = 5(\lambda x_{31}) + b$

Hence, it is closed under scalar multiplication iff $\lambda b \equiv 0 \Leftrightarrow b = 0$

Hence it is subspace of $\mathbf{F}^3$ iff $b = 0$

- b.

  Assume $f_1 : [0,1] \to \mathbb{R}$, $f_2 : [0,1] \to \mathbb{R}$

  Addition:

  $(f_1 + f_2)(x) = f_1(x) + f_2(x)$

  Thus $f_1 + f_2 : [0,1] \to \mathbb{R}$

  Hence it is closed under addition

  Scalar Multiplication:

  $(\lambda f_1)(x) = \lambda f_1(x)$

  Thus $\lambda f_1 : [0,1] \to \mathbb{R}$

  Hence it is close under scalar multiplication

  Hence it is a subspace of $\mathbb{R}^{[0,1]}$

- c.

  Assume $f_1 : \mathbb{R} \to \mathbb{R}$, $f_2 : \mathbb{R} \to \mathbb{R}$

  s.t. f_1' and f_2' exists

  Addition:

  $(f_1 + f_2)'(x) = f_1'(x) + f_2'(x)$

  Hence it is closed under addition

  Scalar Multiplication:

  $(\lambda f_1)'(x) = \lambda f_1'(x)$

  Thus $\lambda f_1 : [0,1] \to \mathbb{R}$

  Hence it is close under scalar multiplication

  Hence it is a subspace of $\mathbb{R}^{\mathbb{R}}$

- d.

  Assume $f_1 : (0,3) \to \mathbb{R}$, $f_2 : (0,3) \to \mathbb{R}$

  s.t. $f_1'(2) = f_2'(2) = b$

  Addition:

  $(f_1 + f_2)'(2) = f_1'(2) + f_2'(2) = 2b$

  Hence it is closed under addition iff $b = 0$

  Scalar Multiplication:

  $(\lambda f_1)'(2) = \lambda f_1'(2) = \lambda b$

Hence it is close under scalar multiplication iff $\lambda b \equiv 0 \Leftrightarrow b = 0$

Hence it is a subspace of $\mathbb{R}^{(0,3)}$ iff $b = 0$

- 6.
  - a.

    In $\mathbb{R}$, $a^3 = b^3 \Leftrightarrow a = b$

    Thus $\{(a, b, c) \in \mathbb{R}^3 : a^3 = b^3\} = \{(a, b, c) \in \mathbb{R}^3 : a = b\}$

    Which is a subspace of $\mathbb{R}^3$
  - b.

    Let $x^3 = 1$, we can get $x_1 = 1$, $x_2 = \frac{-1+\sqrt{3}i}{2}$, $x_3 = \frac{-1-\sqrt{3}i}{2}$

    Assume $\alpha = (x_1, x_2, 0)$, $\beta = (x_1, x_3, 0)$

    $\alpha + \beta = (2, -1, 0)$ where $2^3 \neq (-1)^3$

    Hence it is not a subspace of $\mathbb{R}^3$

- 7.

  Let $U = \{(x, y) : x, y \in \mathbb{Q}\}$

  Since addition in $\mathbb{Q}$ is closed and $\{0\} \in \mathbb{Q}$ Hence, $-u \in U$ whenver $u \in U$

  But if $\lambda = \pi$, $(1, 1) \in U$, $\pi(1, 1) \notin U$

  Hence U is not a subspace of $\mathbb{R}^2$

- 8.

  Let $U = \{(x, y) : x = 0 \ or \ y = 0\}$

  Thus $\lambda(x, y) \in U$

  However, $(1, 0) \in U$ and $(0, 1) \in U$, $(1, 0) + (0, 1) = (1, 1) \notin U$

  Hence U is not a subspace of $\mathbb{R}^2$

- 10.

  Suppose $\forall x, y \in U_1 \cap U_2$

  Thus $x + y \in U_1$ and $x + y \in U_2$, $x + y \in U_1 \cap U_2$

  $\lambda x \in U_1$ and $\lambda x \in U_2$, Thus $\lambda x \in U_1 \cap U_2$

  Hence $U_1 \cap U_2$ is a subspace of V

- 19.

  Counterexample:

  Suppose $U_1 = \{(x, y) \in \mathbb{R}^2\}$, $U_2 = \{(x, 0) \in \mathbb{R}^2\}$, $W = \{(0, y) \in \mathbb{R}^2\}$

  $U_1 + W = R^2 = U_2 + W$ but $U_1 \neq U_2$

- 21.

  $W=\{(0,0,\alpha,\beta,\gamma)\in \mathbf{F}^5\}$

- 22.

  $W_1=\{(0,0,\alpha,0,0)\in \mathbf{F}^5\}$

  $W_2=\{(0,0,0,\beta,0)\in \mathbf{F}^5\}$

  $W_3=\{(0,0,0,0,\gamma)\in \mathbf{F}^5\}$

- 23.

  Counterexample:

  Suppose $U_1=\{(x,0)\in \mathbb{R}^2\}, U_2=\{(0,y)\in \mathbb{R}^2\}, W=\{(\alpha,\alpha)\in \mathbb{R}^2\}$

  $U_1+W=R^2=U_2+W$ and $U_1\cap W=(0,0)=U_2\cap W$

  However, $U_1\neq U_2$

- 24.

  If a function is odd and even, $f(x)=-f(x)\Rightarrow f(x)\equiv 0$

  Thus $U_e\cap U_o=\{f:f\equiv 0\}$

  For $\forall f\in \mathbb{R}$, constract $g(x)=\frac{f(x)+f(-x)}{2}$ and $h(x)=\frac{f(x)-f(-x)}{2}$

  It is easy to know that $g(x)$ is even and $h(x)$ is odd

  And $f(x)=g(x)+h(x)$

  Thus $U_e+U_o=\mathbb{R}^{\mathbb{R}}$

  Hence $\mathbb{R}^{\mathbb{R}}=U_e\oplus U_o$