ODEs Lecture 1

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Textbook:
Elementary Differential Equations and Boundary Value Problems, 10th edition, William E. Boyce and Richard C. DiPrima, Wiley, 2012.

Reference:
《常微分方程教程》, 第二版, 丁同仁, 李承治, 高等教育出版社, 2004年.

Grade: homework (20%) + quizzes (20%) + mid-term test (25%) + final exam (35%).

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Office hours: Wednesday 16:30-18:00, Thursday 16:30-18:00 or by appointment

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- Important applications to other fields.

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Many processes in the natural world, for instance, the motion of fluids, the dissipation of heat in solid objects, the increase or decrease of populations, etc, could be modeled and simulated by some differential equations.

This is a kind of **mathematical modeling with differential equations**.

To gain a deep insight into these processes, we should know something about differential equations.

A differential equation that describes some physical process is often called a **mathematical model** of the process.

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How to study the differential equations?

This is our main task of the course.

Here, we mainly focus on ODEs.

The main purpose of this course is to discuss some of the **properties of solutions of ODEs**, and to present some of the **methods** that have proved effective in **finding solutions**, or, in some cases, approximating them.

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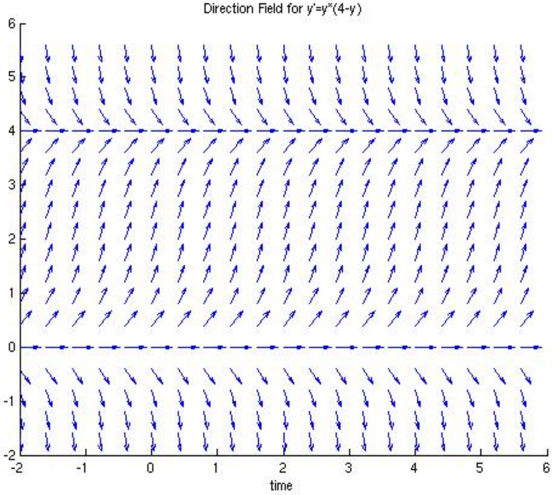
Newton's second law

The mass of an object m times its acceleration a is equal to the net force F on the object. In mathematics,

$$F = ma.$$


Direction Field for y'=y*(4-y)
6
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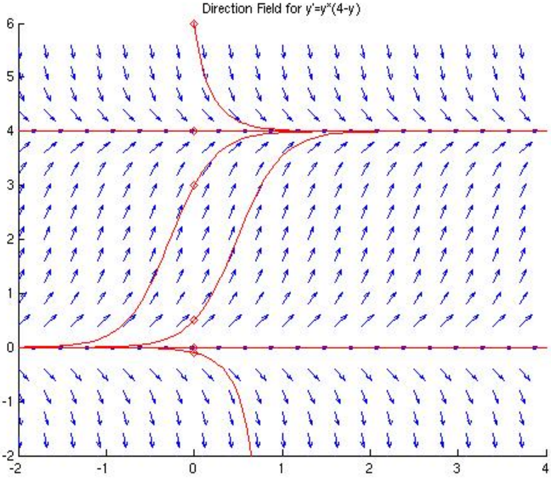
3
2
1
0
-1
-2
-2
-1
0
1
2
3
4
5
6
time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4

How to construct mathematical models with differential equations?

See page 7 in the textbook.

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1. Identify the independent and dependent variables and assign letters to represent them. Often the independent variable is time.
2. Choose the units of measurement for each variable. In a sense the choice of units is arbitrary, but some choices may be much more convenient than others. For example, we chose to measure time in seconds for the falling-object problem and in months for the population problem.
3. Articulate the basic principle that underlies or governs the problem you are investigating. This may be a widely recognized physical law, such as Newton's law of motion, or it may be a more speculative assumption that may be based on your own experience or observations. In any case, this step is likely not to be a purely mathematical one, but will require you to be familiar with the field in which the problem originates.
4. Express the principle or law in step 3 in terms of the variables you chose in step 1. This may be easier said than done. It may require the introduction of physical constants or parameters (such as the drag coefficient in Example 1) and the determination of appropriate values for them. Or it may involve the use of auxiliary or intermediate variables that must then be related to the primary variables.
5. Make sure that all terms in your equation have the same physical units. If this is not the case, then your equation is wrong and you should seek to repair it. If the units agree, then your equation at least is dimensionally consistent, although it may have other shortcomings that this test does not reveal.
6. In the problems considered here, the result of step 4 is a single differential equation, which constitutes the desired mathematical model. Keep in mind, though, that in more complex problems the resulting mathematical model may be much more complicated, perhaps involving a system of several differential equations, for example.