Ordinary Differential Equation

Catalog

Chapter 1

1.2 Solutions of Some Differential Equations

General Form——

- To solve

$$\frac{\mathrm{d}y}{\mathrm{d}t} = ay - b \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}t} = a(y - \frac{b}{a}) \tag{1}$$

- If $y \neq \frac{b}{a}$

$$\begin{aligned} \frac{\mathrm{d}y/\mathrm{d}t}{y - \frac{b}{a}} &= a \\ \Rightarrow \frac{\mathrm{d}}{\mathrm{d}t}\ln|y - \frac{b}{a}| &= a \\ \Rightarrow \ln|y - \frac{b}{a}| &= at + C \end{aligned} \tag{2}$$

- Hence

$$y = \frac{b}{a} + ce^{at}, \quad where\ c = \pm e^C \tag{*}$$

Initial Value Problem(IVP)——

- If for some x_0, we have additional condition s.t. $y(x_0) = y_0$
- By eq(*) in General Form.

$$\begin{aligned} \frac{b}{a} + ce^{ax_0} &= y_0 \\ \Rightarrow c &= \frac{y_0 - b/a}{e^{ax_0}} \end{aligned} \tag{1}$$

1.3 Classification of Differential Equations

Ordinary and Partial Differential Equation——

- One important classification is based on whether the unknown function depends on a single independent variable or on several independent variables.
- For example, ordinary——

$$L\frac{\mathrm{d}^2Q(t)}{\mathrm{d}t^2}+R\frac{\mathrm{d}Q(t)}{\mathrm{d}t}+\frac{1}{C}Q(t)=E(t) \tag{1}$$

- for the charge Q(t) on a capacitor(电容器) in a circuit with capacitance C, resistance R, and inductance(电感) L.
- For example, partial——

$$\alpha^2\frac{\partial^2u(x,t)}{\partial x^2}=\frac{\partial^2u(x,t)}{\partial t} \tag{2}$$

System of Differential Equation

Chapter 2

Solving the 1st Order Equation

Method of Integral Fraction——

- For the equation

$$y'+p(t)y=q(t)$$

- Multiple $\mu(t)$ on both sides, we get

$$\mu y'+\mu py=\mu q \tag{1}$$

- If we can find a $\mu>0$, s.t.

$$(\mu y)'=\mu y'+\mu py \text{i.e.} \quad \mu'=\mu p \tag{2}$$

- Thus

$$\mu'=\mathrm{d}\mu/\mathrm{d}t=\mu p\Rightarrow \mathrm{d}\mu/\mu=p\mathrm{d}t \tag{3}$$
$$\Rightarrow \ln\mu=\int p\mathrm{d}t$$

- Hence

$$\mu=e^{\int p\mathrm{d}t}$$

- It follows that

$$(\mu y)'=p\mu \text{i.e.} \quad y=\frac{\int p\mu\mathrm{d}t}{e^{\int p\mathrm{d}t}} \tag{*}$$

HW——due on Week 2 Wed. In class question

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{x^2+y^2}{xy}$$