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MSc Basismodul

The Adaptive Radix Tree

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1 Introduction

Main-Memory Databases increasingly become a viable option for many applications. Whilst main memory is a considerably faster medium than a secondary disk, utilizing caches more efficiently would lead to even better access times.

Leis et al. [2] propose the Adaptive Radix Tree (ART), an in-memory data structure which efficiently stores and retrieves data, even outperforming red-black trees. As we will see later, ART achieves its exceptional performance, and space efficiency, by compressing the tree both vertically and horizontally. Higher space effiency allows ART to utilize caches more optimal.

The goal of this project is to implement ART, as proposed by [2] in C++ and compare its transactional throughput against other in-memory data structures, i.e. red-black trees and hash tables. In Section 2 we describe how ART is constructed by applying vertical and horizontal compression to a trie. Next, we describe the point and range query procedure, as well as value insertion and removal in **??**. Finally, a benchmark of ART, a red-black tree and a hashtable is presented in **??**.

2 Adaptive Radix Tree (ART)

The WAPI is a hierarchical index and indexes the properties of nodes. It takes into account if an index node is volatile before performing structural index modifications. If a node is considered volatile, we do not remove it from the index. In the following section, we will see how to add, query and remove nodes from the index.

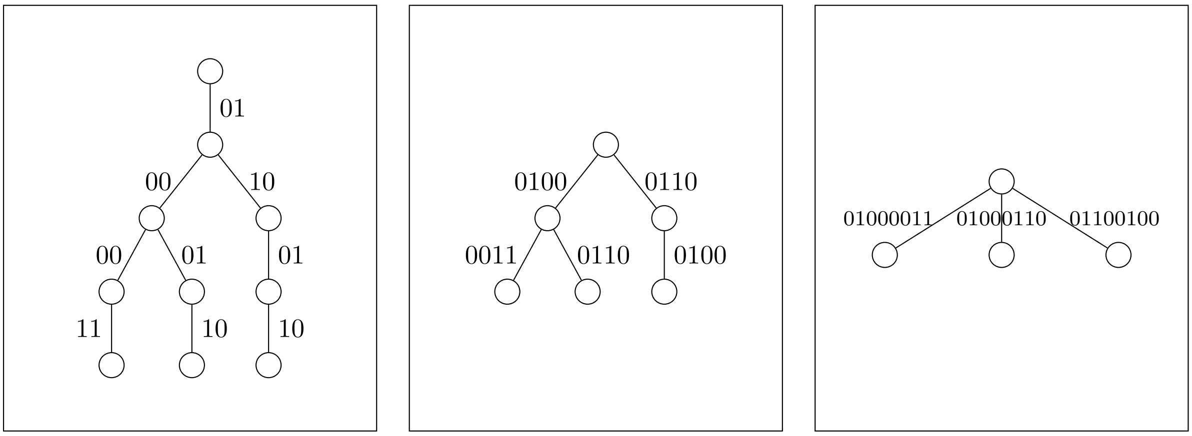
2.1 Trie

The trie [1] is a hierarchical data structure which stores key-value pairs. The trie can answer both point and range queries efficiently since keys are stored sorted according to their lexicographic order. A node's path represents the node's key. This is done by splitting a key into chunks of s bits, where s is called *span*.

Each inner node has 2^s child nodes, one for each possible s-bit sequence. During tree traversal, we propagate down to the child node identified by the d-th s-bit chunk of the key, where d is the depth of the current node. Using an array of 2^s pointers, this lookup can be done without any additional comparison.

Figure 1 depicts tries storing the 8-bit keys “01000011”, “01000110” and “01100100” with $s = 2, 4, 8$. Span s is critical for the performance of the trie because s determines the height of the trie. We observe that by increasing the span, we decrease the tree height. A trie storing k bit keys has $\lceil \frac{k}{s} \rceil$ levels of inner nodes. As a consequence, point queries, insertions and deletions have $O(k)$ complexity. Having a data structure with time complexity not dependend on n, makes it very attractive for large data sets.

Span s also determines the space consumption of the tree. A node with span s requires 2^s pointers. The tries in Figure 1 require a total of $240, 224, 384$ and 2048 bytes respectively, for the bookkeeping of child nodes, assuming 64-bit pointers. An apparent trade off exists between tree height versus space efficiency that depends on s.



$s = 2$ $s = 4$ $s = 8$

Figure 1: Tries with span $s = 2, 4, 8$ storing keys "01000011", "01000110" and "01100100".

2.2 Vertical (Prefix) Compression

When storing long keys, chains start to form where each node only has a single child. As a consequence, we waste a lot of space on structural information. Morrison introduced *Patricia* [3]. Patricia is a space-optimized trie in which each node with no siblings is merged with its parent, i.e. inner nodes are only created if they are required to distinguish at least two leaf nodes. Doing so, we eliminate chains caused by long keys which make tries space-inefficient. Although Morrison's Patricia tree is a bitwise trie, i.e. has a span $s = 1$, the principle can be applied to tries with any span. We refer to this heuristic as *vertical compression*.

Vertical compression is implemented by storing an additional variable, called *prefix*, inside each node. This variable stores the concatenation of partial keys of descendants that were eliminated because they had no siblings. Figure 2 depicts two tries, one with and one without vertical compression. We observe that nodes with no siblings, color coded red, are eliminated and their partial key is appended to the parent's prefix. With even longer keys, the results of vertical compression are even more astonishing.

Tall trees may become very compact, and so not only do we save space that was otherwise wasted on structural inforation, we also are now able to traverse the data structure even faster since the height decreased.

2.3 Horizontal Compression (Adaptive Nodes)

With large values of span s, we sacrifice an excessive amount of space for a smaller tree height. Space is allocated for pointers which keep references of child nodes. In order

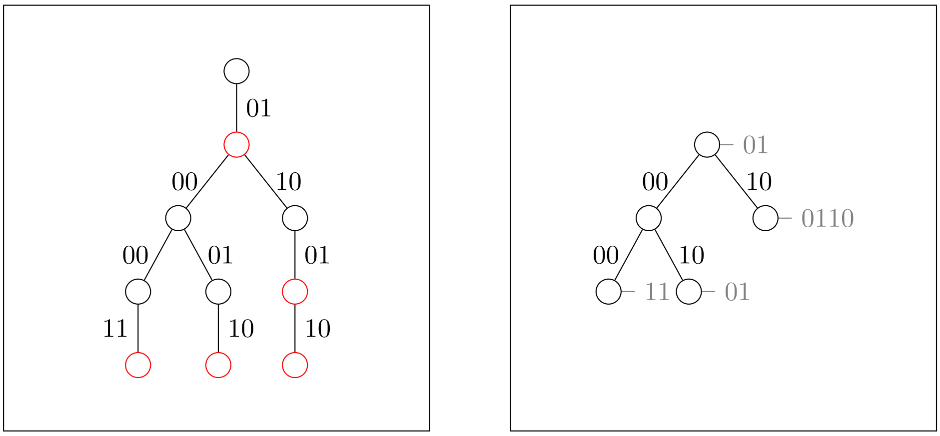


Figure 2: Tries with span $s = 2$ storing keys "01000011", "01000110" and "01100100". The trie on the right incorporates vertical compression. Red nodes indicate nodes which get eliminated under vertical compression. Gray strings represent the value of the "prefix" property.

to reduce the space needed to keep such references, Leis et al. propose *Adaptive Nodes* [2], which make use of dynamic data structures instead of static arrays for child node bookkeeping. Doing so, we allocate a minimal amount of space when the number of children is small and add more space if required, i.e. more children are added. We refer to this heuristic as *horizontal compression*. Leis et al. also fix the span $s = 8$, i.e. partial keys are 1 byte long and therefore each node can have up to $2^8 = 256$ children.

An adaptive node is in one of four configurations, depending on the number of children. Each of the four configurations is optimized for a different number of children. The most compact configuration is called *Node4* which can carry up to four children. In the same manner, we also have *Node16*, *Node48* and *Node256*.

We now describe the structure of each of the four configurations. The node types are also illustrated in Figure 3. We store the partial keys 65, 82, 84 and their corresponding child nodes α, β, γ in each node type with the purposes of explaining their structures. Note that $\emptyset$ is a `null` pointer.

A node of type Node4 contains two static arrays, each of which can hold up to four values. The first array, called the "partial keys" array, holds partial keys which identify children of that node. The second array, called the "children" array, holds pointers to the child nodes. Partial keys and pointers are stored at corresponding positions and the partial keys are sorted. A node of type Node16 is structured similar to Node4, the only difference being the lengths of the two static arrays, which are now 16 each.

An instance of Node48 contains a 256-element array named "indexes" and a 48-element array called "children". Partial keys are stored implicitly in "indexes", i.e. can be indexed with partial key bytes directly. As the name suggests, "indexes" stores the index of a child node inside the "children" array. This node can be compared to virtual memory, since the address space (256 addresses) is wider than the actual available storage space (48 slots).

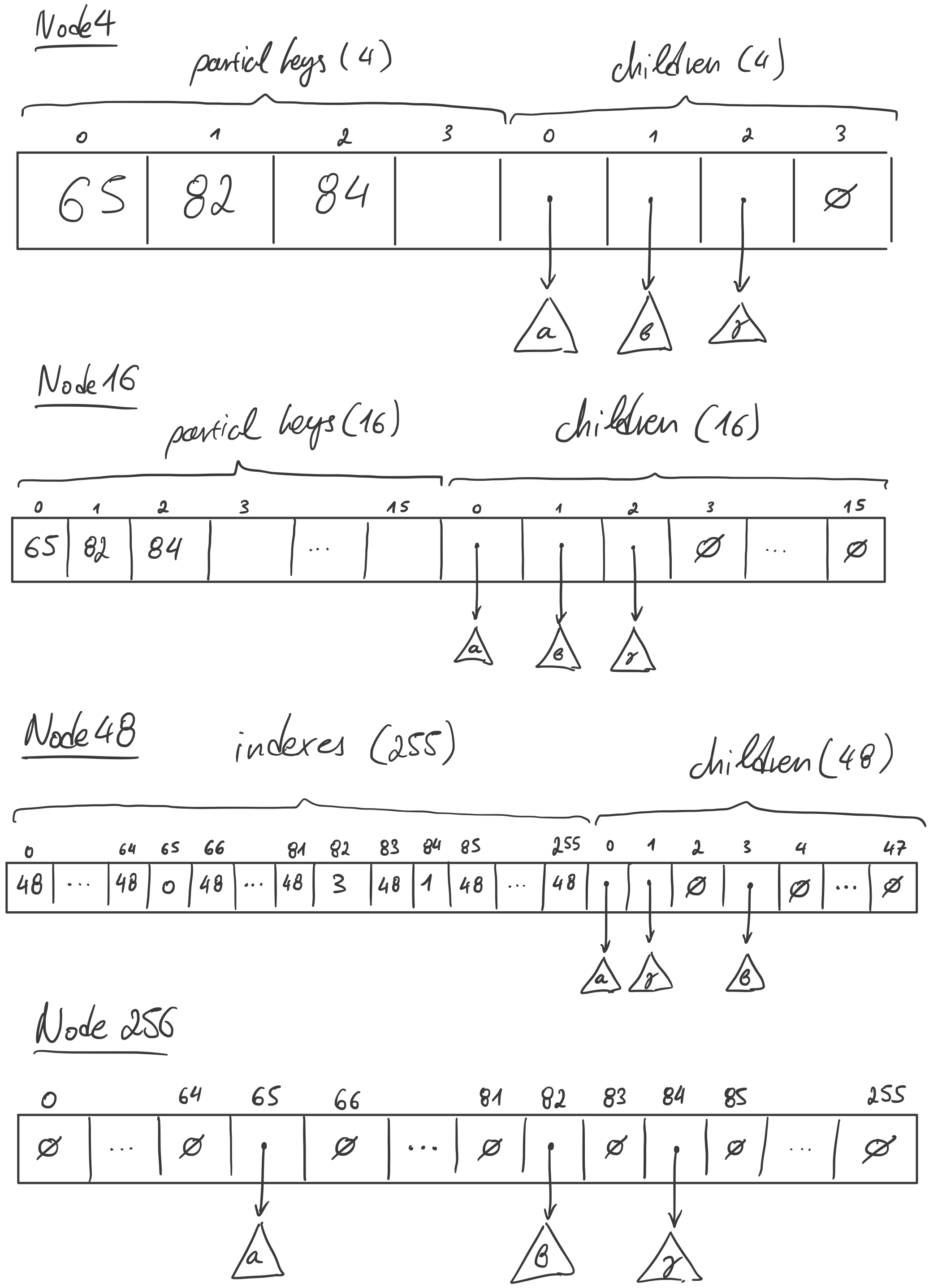


Figure 3: Tries with span $s = 2$ storing keys “01000011”, “01000110” and “01100100”. The trie on the right incorporates vertical compression. Red nodes indicate nodes which get eliminated under vertical compression. Gray strings represent the value of the “prefix” property.

References

[1] E. Fredkin. Trie memory. Communications of the ACM, 3(9):490–499, 1960.

[2] V. Leis, A. Kemper, and T. Neumann. The adaptive radix tree: Artful indexing for main-memory databases. In 2013 IEEE 29th International Conference on Data Engineering (ICDE), pages 38–49. IEEE, 2013.

[3] D. R. Morrison. Patricia—practical algorithm to retrieve information coded in alphanumeric. Journal of the ACM (JACM), 15(4):514–534, 1968.