Exercise 3.7 Ross

Emil H. Andersen

April 21, 2016

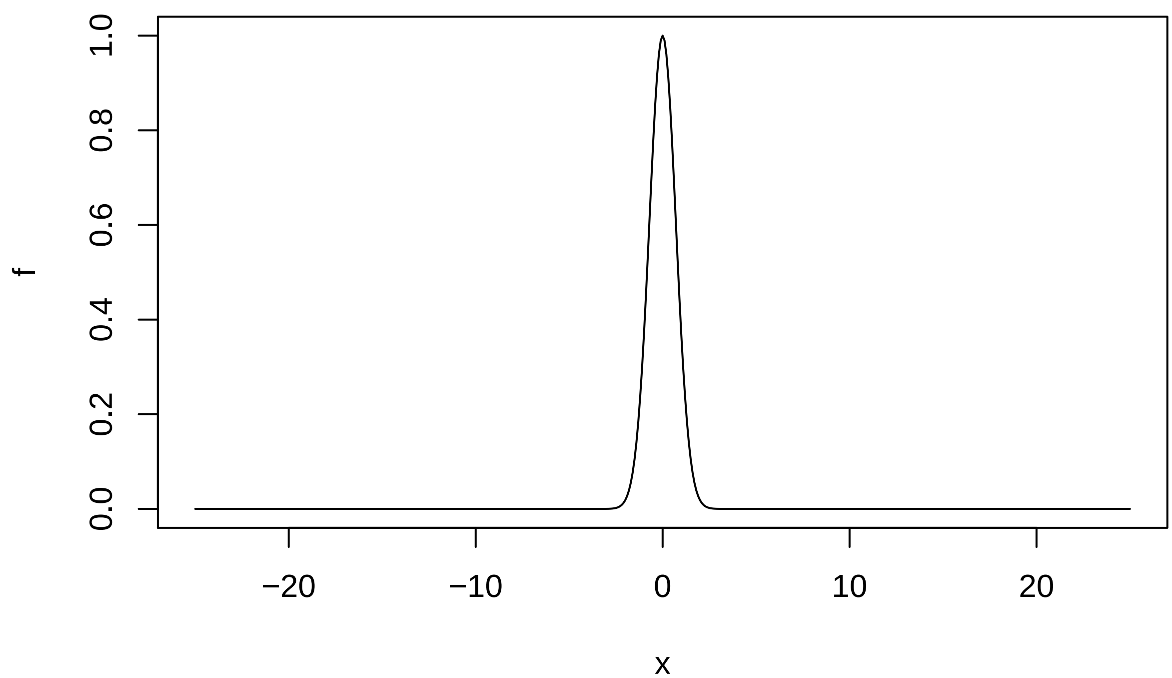
Exercise 3.7 in Ross book: Use simulation to approximate the following integral:

$$\int_{\infty}^{\infty} e^{-x^2} dx$$

Using the method described in chapter 3.2 of the Ross book. The result should be compared to the exact result if possible.

I first view the graph to see which area is relevant to estimate the integral

```
x = seq(-25,25,(1/10))
f = exp(-x^2)
plot(x,f,type="l")
```



As it is easily observed that the area is at least in the interval $x \in [-10, 10]$, I will estimate the integral within this interval. I first start by creating a vector to contain the 21 pieces that will be generated by estimating the integral in the previously mentioned interval. I do the simulation using a million random variables per run through to optimize the precision. Ux is the P is the vector of the part-results.

```
P<- rep(0,21)
for (i in -10:10){
  a = i
  b = i + 1
  Ux <- runif(1000000)
  Ux <- Ux * (b-a) + a
  f <- exp(-Ux^2)
  P[i+11] <- sum(f)/(1000000)
}
sum(P)
```

```
## [1] 1.772991
```

To get the exact result, a combination of calculus, polar coordinates and integration by substitution is necessary:

$$\left(\int_{-\infty}^{\infty} e^{-x^2} dx\right)^2 = \int_{-\infty}^{\infty} e^{-x^2} dx \cdot \int_{-\infty}^{\infty} e^{-y^2} dy = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(x^2+y^2)} dx dy$$

Now using polar coordinates, we get:

$$\int \int_{R^2} e^{-(x^2+y^2)} d(x,y) = \int_0^{2\pi} \int_0^{\infty} e^{-(-r^2)} r dr d\theta$$

Integrating first the θ part yields the result

$$2\pi \int_0^{\infty} e^{-(-r^2)} r dr$$

Now substituting $u = -r^2$, $-du \cdot \frac{1}{2r} = dr$ results in the following:

$$2\pi \int_0^{\infty} \frac{1}{2} e^u du = \pi [e^{-r^2}]_0^{\infty} = \pi$$

Now since

$$\left(\int_{-\infty}^{\infty} e^{-x^2} dx\right)^2 = \pi$$

then

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi} \approx 1.77245$$