Lab session week 17 : Exercises Chapter 3

Exercise 1 & 2

- $x_0 = 5$ and $x_n = 3x_{n-1} \mod 150$.

```
x <- 5
for(i in 1:10){
  x[i + 1] <- (3*x[i]) %% 150
}
```

- $x_0 = 3$ and $x_n = (5x_{n-1} + 7) \mod 200$.

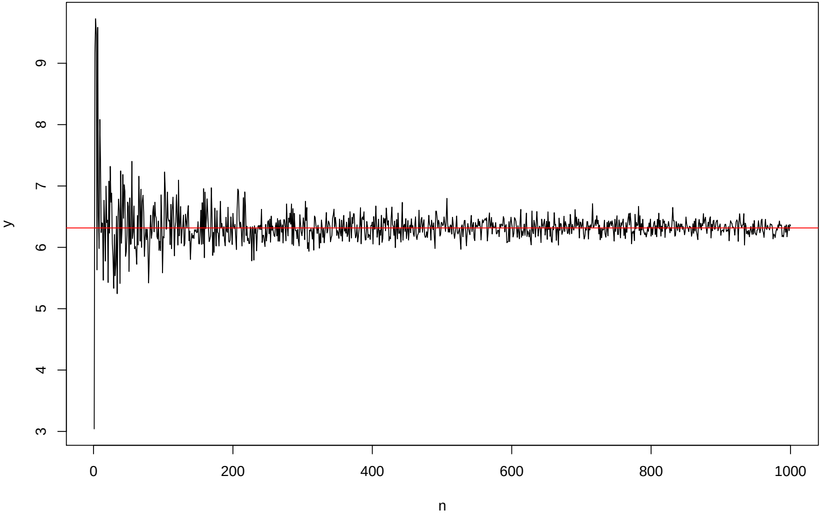
```
x <- 3
for(i in 1:10){
  x[i + 1] <- (5*x[i] + 7) %% 200
}
```

Exercise 3

- Estimation of $\int_0^1 e^{e^x}\,dx$.

```
simba <- function(n){
  gem <- c()
  for (i in 1:n) {
    x <- runif(1)
    evalsimba <- exp(exp(x))
    gem <- c(evalsimba,gem)
  }
  estimat <- sum(gem)/n
  return(estimat)
}
```

Exercise 3

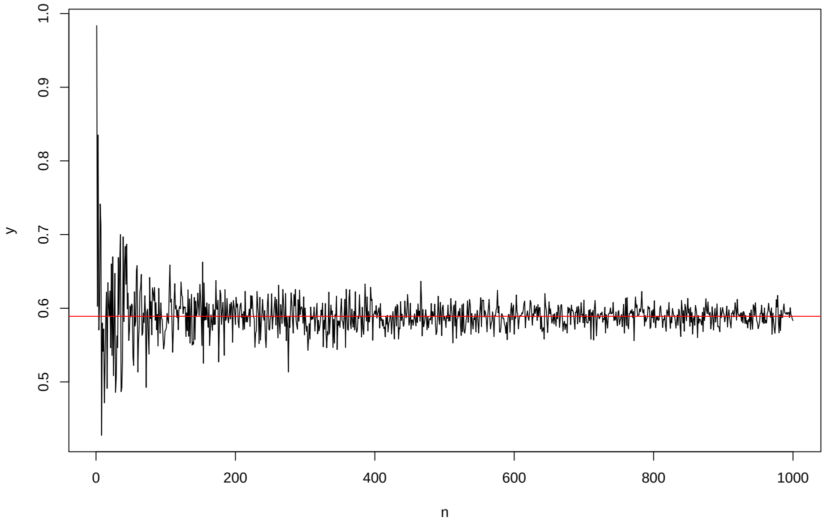


Exercise 4

- Estimation of $\int_0^1 (1-x^2)^{3/2} dx = \frac{3\pi}{16}$.

```
n = 10^3
U = runif(n,0,1)
V = sapply(U, function(u) ((1-u^2)^(3/2)))/n
Integral = sum(V)
```

Exercise 4



Exercice 5-7

Assuming we want to estimate $\int_a^b f(x)dx$ for any real integrable function f and $b > a$

- for $|a| < +\infty$ and $|b| < +\infty$

$$\int_a^b f(x)dx = (b-a)\int_0^1 f((b-a)*x+a)dx$$

- for $|a| < +\infty$ and $b = +\infty$ or $a = -\infty$ and $|b| < +\infty$

$$\int_a^{+\infty} f(x)dx = \int_0^1 f\left(\frac{1}{x} - 1 + a\right) x^{-2}dx$$

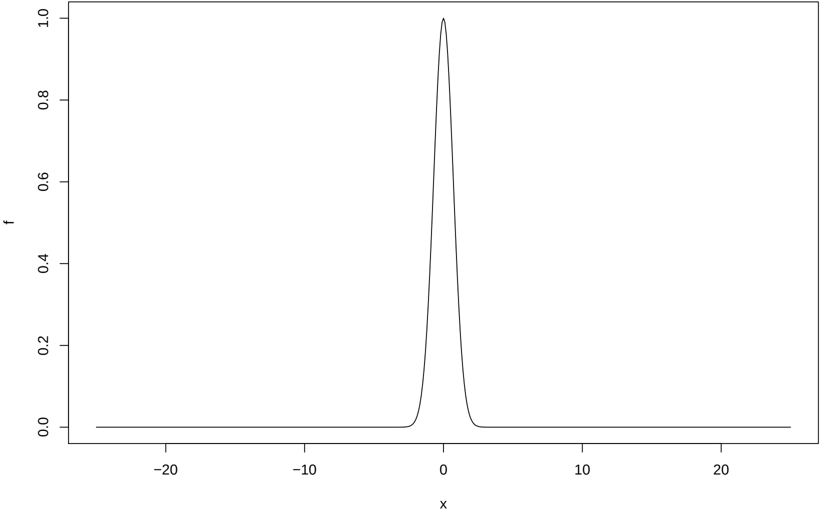
$$\int_{-\infty}^b f(x)dx = \int_0^1 f\left(1 - \frac{1}{x} + b\right) x^{-2}dx$$

- for $a = -\infty$ and $b = +\infty$

$$\int_{-\infty}^{+\infty} f(x)dx = \int_0^{+\infty} f(x) + f(-x)dx$$

Exercise 7

- Estimation of $\int_{-\infty}^{+\infty} e^{-x^2} dx = \sqrt{\pi}$.

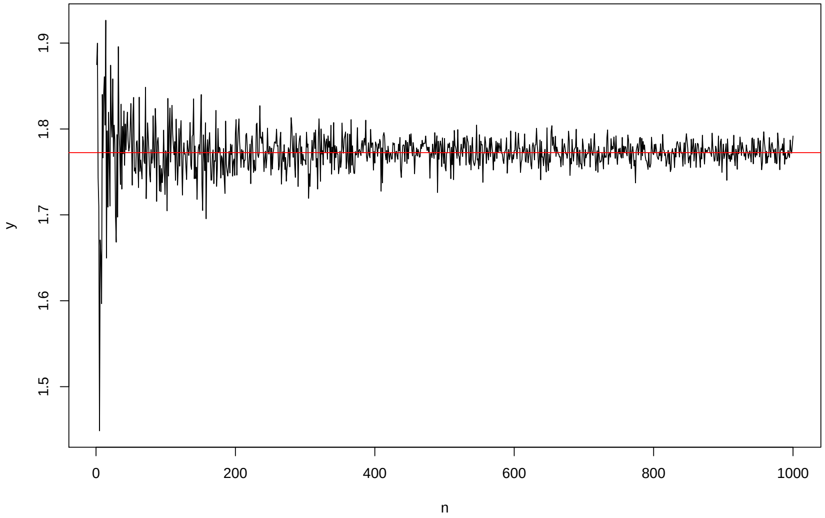


Exercise 7

```
P<- rep(0,21)
for (i in -10:10){
  a = i
  b = i + 1
  Ux <- runif(1000000)
  Ux <- Ux * (b-a) + a
  f <- exp(-Ux^2)
  P[i+11] <- sum(f)/(1000000)
}
sum(P)
```

```
## [1] 1.772276
```

Exercise 7

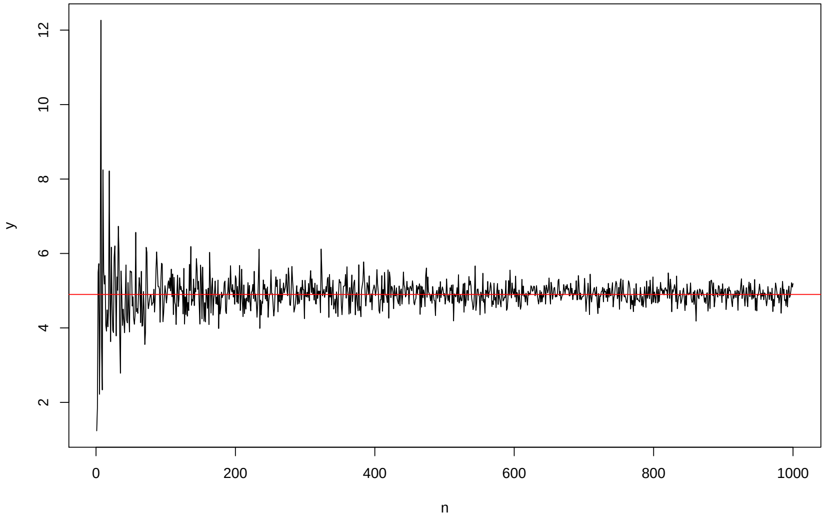


Exercise 8

- Estimation of $\int_0^1 \int_0^1 e^{(x+y)^2} dxdy$.

```r
estInt1 <- function(n) {
  storeVal1 <- c()
  for(i in 1:n){
    U1 <- runif(1)
    U2 <- runif(1)
    evalFun <- exp((U1 + U2)^2)
    storeVal1 <- c(evalFun, storeVal1)
  }
  est <- sum(storeVal1)/n
  return(est)
}
```

Exercise 8



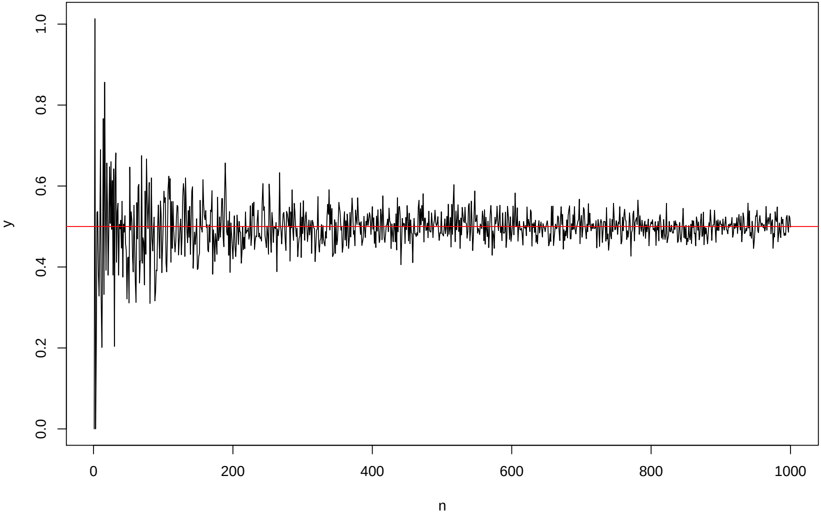
Exercise 9

- Estimation of $\int_0^{+\infty} \int_0^x e^{-(x+y)} dxdy = \frac{1}{2}$.

```
f <- function(x){
    return(ifelse(x[1]<x[2], exp(-(x[1]+x[2])), 0))
  }

simul9<-function(n){
  u <- sapply(1:n,function(x) runif(2))
  r <- apply(u,2,function(x) f(1/x-1)/(x[1]*x[2])^2 )
  return(mean(r))
}
```

Exercise 9



Exercise 10

- Estimation of $\mathrm{Cov}(U, e^U) = 1 - \frac{e-1}{2}$

```
n=10^3
U = runif(n)
X = sapply(U, function(u) exp(u))
cov(U,X)
```

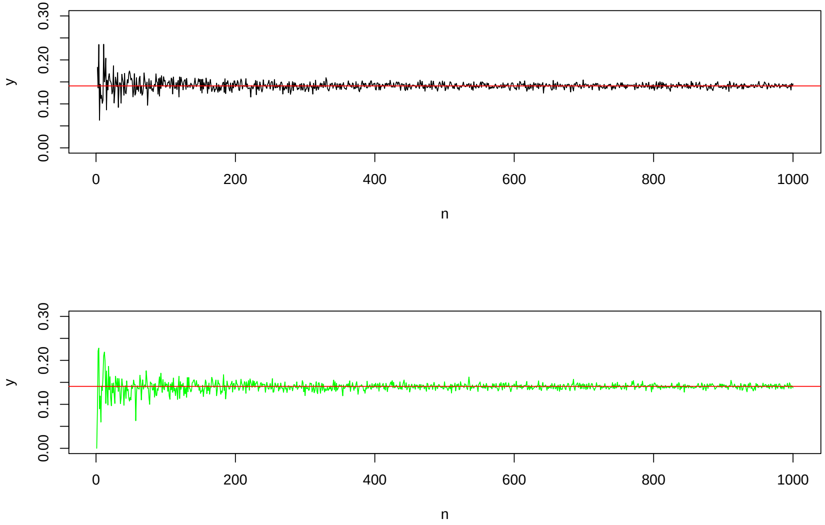
```
## [1] 0.1314415
```

or

```
mean(U*X)-mean(U)*mean(X)
```

```
## [1] 0.1313101
```

Exercise 10



y
0.00
0.10
0.20
0.30
0
200
400
600
800
1000
n
y
0.00
0.10
0.20
0.30
0
200
400
600
800
1000
n

Exercise 11

- Estimation of $\mathrm{Cor}(U, \sqrt{1-U^2})$ and $\mathrm{Cor}(U^2, \sqrt{1-U^2})$.

```
n<-1000
u<-runif(n)
w<-sqrt(1-u^2)
cor(u,w)
```

```
## [1] -0.9183402
```

Exercise 12

- Estimation of $\mathbb{E}[N]$ where $N = \min\{n : \sum_{i=1}^{n} U_i > 1\}$.

```
RN12<-function(n){
  N2<-rep(0,n)
  for(i in 1:n){
    N=0
    S=0
    while(S<=1){
      U<-runif(1)
      S=S+U
      N=N+1
    }
  N2[i]=N
  }
return(mean(N2))
}
```

Exercise 13

- Estimation of $\mathbb{E}[N]$ where $N = \max\{n : \prod_{i=1}^{n} U_i > e^{-3}\}$.

```
RN13<-function(n){
  N2<-rep(0,n)
  for(i in 1:n){
    P=1
    N=0
    while(P>exp(-3)){
      U<-runif(1)
      P=P*U
      N=N+1
    }
    N2[i]=N
  }
  return(mean(N2))
}
```