Exercise 6

Exercise 6, chapter 3

Use simulation to approximate the following integral. Compare your estimate with the exact answer if known.

$$\int_0^\infty x(1+x^2)^{-2}$$

I use chapter 3.2, I get that it can be calculated by:

$$\int_0^1 \frac{\left(\frac{1}{y}-1\right)\left(1+\left(\frac{1}{y}-1\right)^2\right)^{-2}}{y^2}dy$$

Then I approximate its solution in R.

```
n <- 50000
RV <- c()
for(i in 1:n){
  U <- runif(1)
  h <- (((1/U)-1)*(1+((1/U)-1)^2)^(-2))/(U^2)
  q <- mean(h)
  RV <- append(RV,q)
}
result <- sum(RV)/n
```

This give me the estimate ≈ 0.498.
To find the exact answer then I calculated it in maple and got the following.

$$\int_0^1 \frac{\left(\frac{1}{y}-1\right)\left(1+\left(\frac{1}{y}-1\right)^2\right)^{-2}}{y^2}dy = \frac{1}{2}$$