$\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = F_0 \cos(\omega t)$, $\omega_0 > \gamma$, $\varphi_0 = 0$, $x(0) = 0$ $v(0) = 0$

Однор.

$$\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = 0$$

$$x = x_0 e^{i\Omega t} \qquad \hat{x}_0 = x_0 e^{i\varphi_0} = x_0$$

$$-\Omega^2 x_0 e^{i\Omega t} + 2\gamma i\Omega x_0 e^{i\Omega t} + \omega_0^2 x_0 e^{i\Omega t} = 0$$

$$\Omega^2 - 2\gamma i\Omega - \omega_0^2 = 0$$

$$\Omega = \gamma i \pm \sqrt{\omega_0^2 - \gamma^2}$$

$$i\Omega t = -\gamma t \mp i\sqrt{\omega_0^2 - \gamma^2}\, t$$

$$x = A e^{-\gamma t} e^{i\sqrt{\omega_0^2 - \gamma^2}\, t} + B e^{-\gamma t} e^{-i\sqrt{\omega_0^2 - \gamma^2}\, t} =$$

$$= A e^{-\gamma t} \cos\left(\sqrt{\omega_0^2 - \gamma^2}\, t\right) + B e^{-\gamma t} \sin\left(\sqrt{\omega_0^2 - \gamma^2}\, t\right)$$

Частн.

$$x = C e^{i\Omega t}$$

$$-C\Omega^2 e^{i\Omega t} + 2\gamma i C\Omega \cdot e^{i\Omega t} + \omega_0^2 C e^{i\Omega t} = F_0 e^{i\omega t}$$

$$(\omega_0^2 - \Omega^2 + 2i\gamma\Omega) C e^{i\Omega t} = F_0 e^{i\omega t}$$

$$C = \frac{F_0}{\omega_0^2 - \omega^2 + 2i\gamma\omega} = \frac{F_0}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\gamma^2\omega^2}} e^{i\varphi_0}$$

$$\omega_0^2 - \omega^2 + 2i\gamma\omega = \rho e^{-i\varphi_0}$$

$$\rho = \sqrt{(\omega_0^2 - \omega^2)^2 + 4\gamma^2\omega^2}$$

$$\boxed{\operatorname{tg}\varphi_0 = \frac{2\gamma\omega}{\omega^2 - \omega_0^2}} \Rightarrow \operatorname{tg}^2\varphi_0 + 1 = \frac{1}{\cos^2\varphi_0} \qquad \cos\varphi_0 = \sqrt{\frac{1}{\operatorname{tg}^2\varphi_0 + 1}}$$

$$\cos\varphi_0 = \sqrt{\frac{1}{\frac{2\gamma\omega}{\omega^2 - \omega_0^2} + 1}}$$

$$x = \frac{F_0}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\gamma^2\omega^2}} \cos(\omega t + \varphi_0)$$

Общее.

$$\boxed{x = \frac{F_0}{\sqrt{(\omega_0^2-\omega^2)^2+4\gamma^2\omega^2}}\cos(\omega t+\varphi_0) + A e^{-\gamma t}\cos\left(\sqrt{\omega_0^2-\gamma^2}\,t\right) + B e^{-\gamma t}\sin\left(\sqrt{\omega_0^2-\gamma^2}\,t\right)}$$

$$x(0) = 0 = \frac{F_0}{\sqrt{(\omega_0^2-\omega^2)^2+4\gamma^2\omega^2}}\cos\varphi_0 + A$$

$$A = \frac{-F_0\cos\varphi_0}{\sqrt{(\omega_0^2-\omega^2)^2+4\gamma^2\omega^2}}$$

$$v(0) = 0 = \frac{-F_0\,\omega}{\sqrt{(\omega_0^2-\omega^2)^2+4\gamma^2\omega^2}}\sin\varphi_0 - A\gamma + B\sqrt{\omega_0^2-\gamma^2}$$

$$\frac{-F_0}{\sqrt{(\omega_0^2-\omega^2)^2+4\gamma^2\omega^2}}(\omega\sin\varphi_0 - \cos\varphi_0) + B\sqrt{\omega_0^2-\gamma^2} = 0$$

$$B = \frac{F_0(\omega\sin\varphi_0 - \cos\varphi_0)}{\sqrt{(\omega_0^2-\omega^2)^2+4\gamma^2\omega^2}}\,\frac{1}{\sqrt{\omega_0^2-\gamma^2}}$$

$$x = \frac{F_0}{\sqrt{(\omega_0^2-\omega^2)^2+4\gamma^2\omega^2}}\left[\cos(\omega t+\varphi_0) - e^{-\gamma t}\cos\varphi_0\cos\left(\sqrt{\omega_0^2-\gamma^2}\,t\right) + \frac{\omega\sin\varphi_0-\cos\varphi_0}{\sqrt{\omega_0^2-\gamma^2}}e^{-\gamma t}\sin\left(\sqrt{\omega_0^2-\gamma^2}\,t\right)\right]$$