results

February 20, 2024

Solving boundary value problem:

$$\begin{cases} y''(x) + 4y(x) = -4x, \ x \in (0, 4) \\ y(0) - y'(0) = 2, \ y(4) = \cos(4) - 4 \end{cases}$$

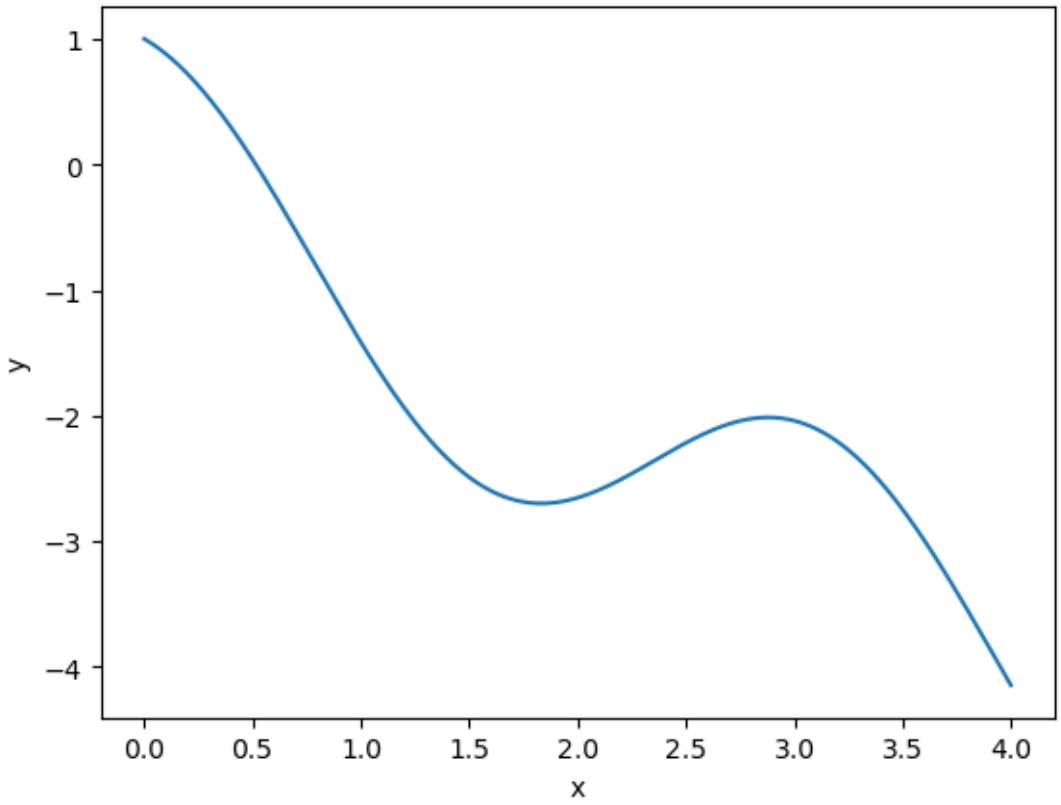
exaxct solution is y(x) = cos(2x) - x

1 Shooting method

testing nonlinear search of initial values

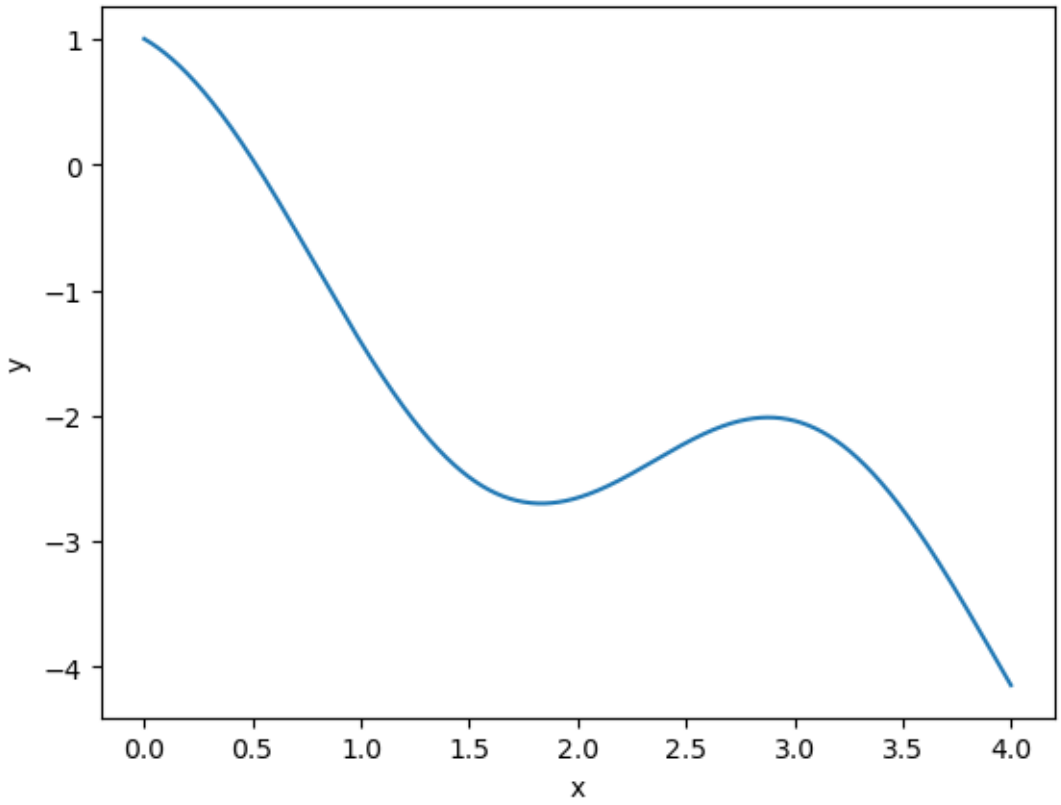
[ ]:
```
import matplotlib.pyplot as plt
import pandas as pd
import numpy as np

res = pd.read_csv('out/shoot.csv')
plt.plot(res['x'], res['y'])
plt.xlabel("x")
plt.ylabel("y")
plt.show()
```



testing linear search of initial values

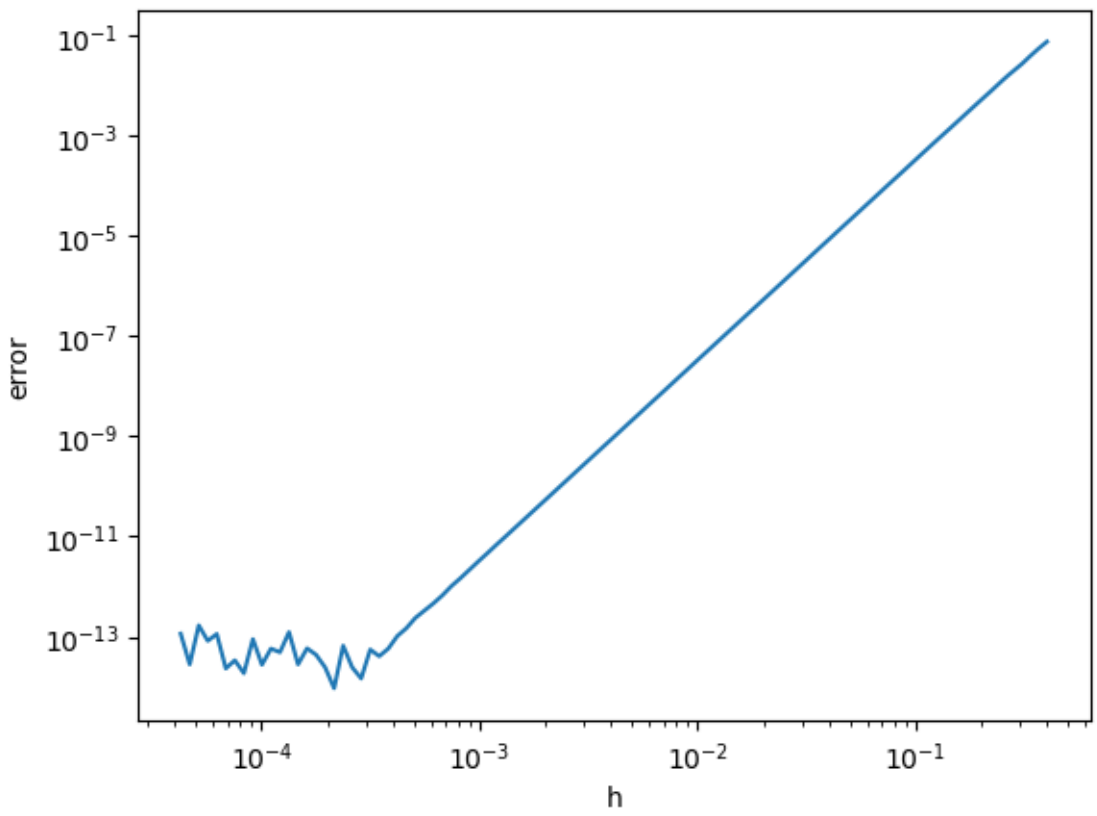
[ ]:
```
res = pd.read_csv('out/shoot_linear.csv')
plt.plot(res['x'], res['y'])
plt.xlabel("x")
plt.ylabel("y")
plt.show()
```



Plotting errors depending on step size h

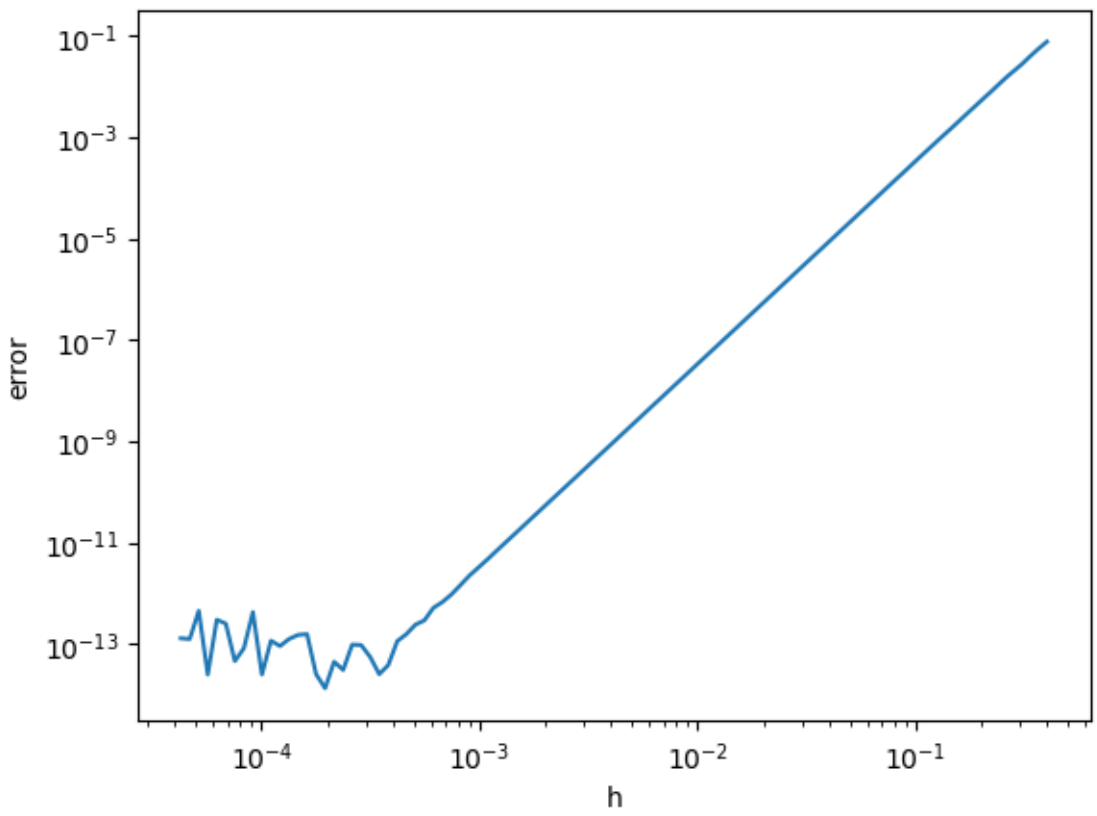
[ ]:
```
res = pd.read_csv('out/shoot_error.csv')
h, err = res['h'], res["error"]
h = h[h > 10e-3]
err = err[0:h.size]
plt.loglog(res['h'], res["error"])
plt.xlabel("h")
plt.ylabel("error")
m, b = np.polyfit(np.log(h), np.log(err), 1)
print("Order of convergence: ", m)
plt.show()
```

```
Order of convergence:  3.984927519676901
```



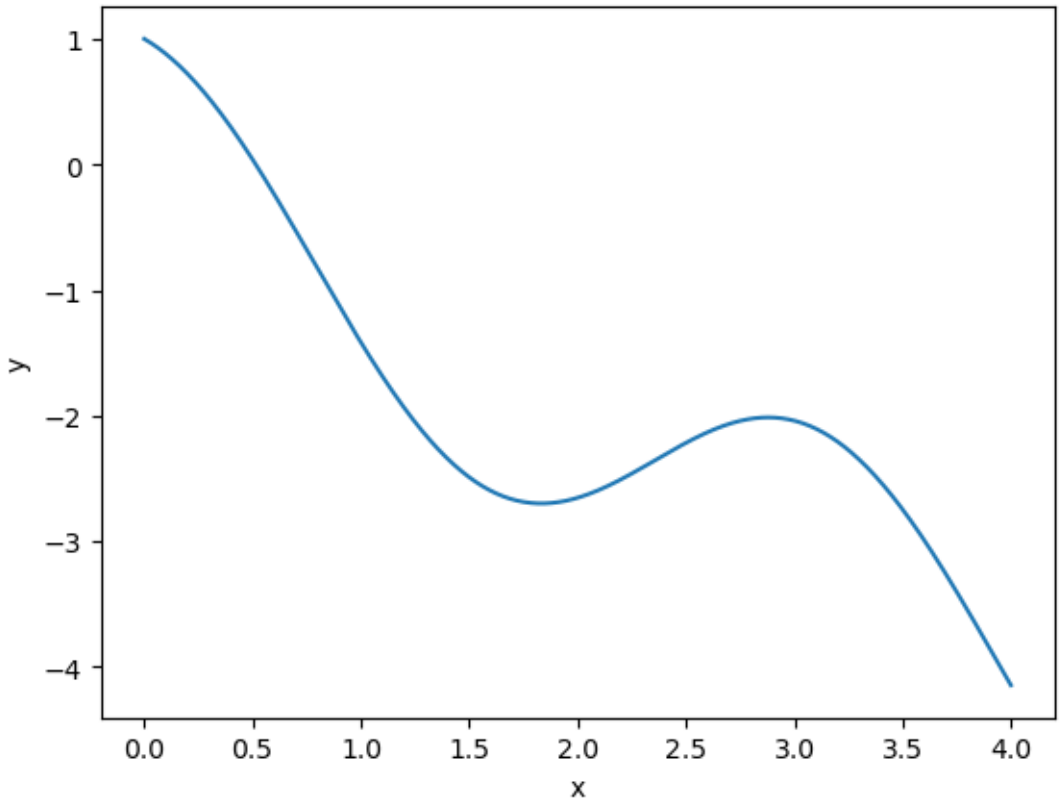
[ ]:
```
res = pd.read_csv('out/shoot_linear_error.csv')
h, err = res['h'], res["error"]
h = h[h > 10e-3]
err = err[0:h.size]
plt.loglog(res['h'], res["error"])
plt.xlabel("h")
plt.ylabel("error")
m, b = np.polyfit(np.log(h), np.log(err), 1)
print("Order of convergence: ", m)
plt.show()
```

```
Order of convergence:  3.9849275770276478
```



2 Sweep (Thomas) method

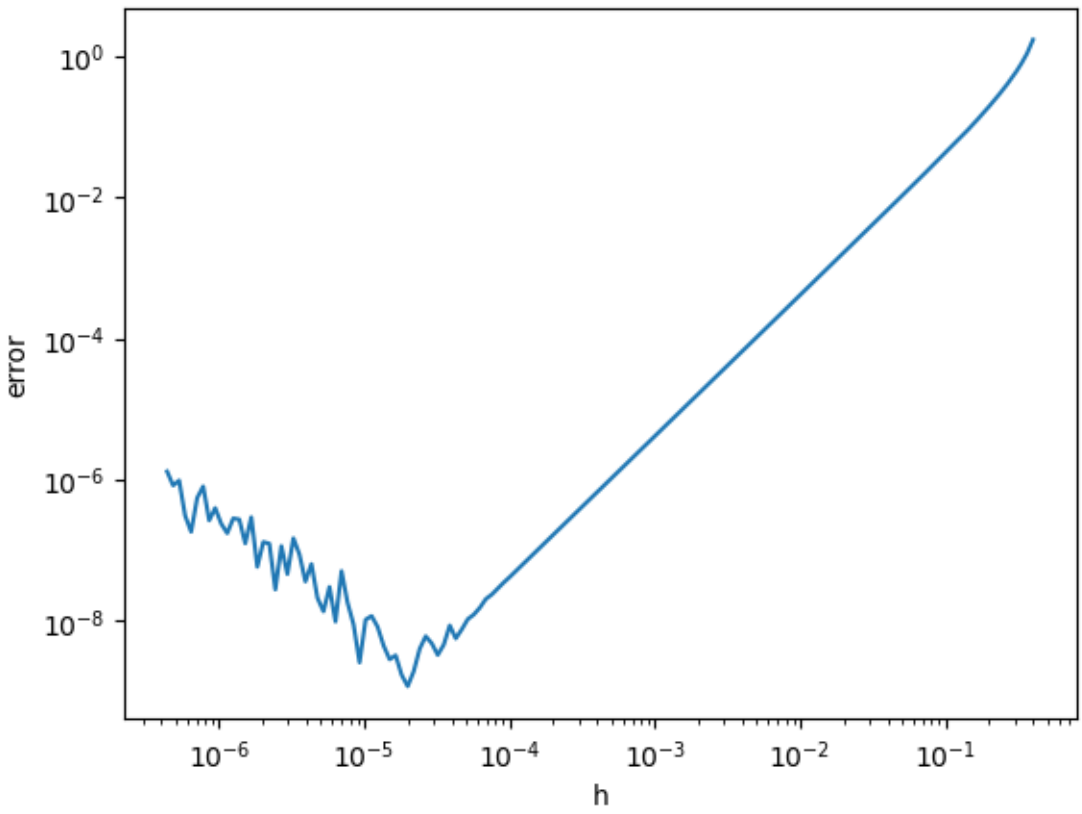
[ ]:
```
res = pd.read_csv('out/thomas.csv')
plt.plot(res['x'], res['y'])
plt.xlabel("x")
plt.ylabel("y")
plt.show()
```



Plotting errors depending on step size h

[ ]:
```
res = pd.read_csv('out/thomas_error.csv')
h, err = res['h'], res["error"]
h = h[h > 10e-4]
err = err[0:h.size]
plt.loglog(res['h'], res["error"])
plt.xlabel("h")
plt.ylabel("error")
m, b = np.polyfit(np.log(h), np.log(err), 1)
print("Order of convergence: ", m)
plt.show()
```

```
Order of convergence:  2.0596438787726656
```


10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
error
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
h