INTRODUCTION TO EARTHQUAKE ENGINEERING

CE0440R

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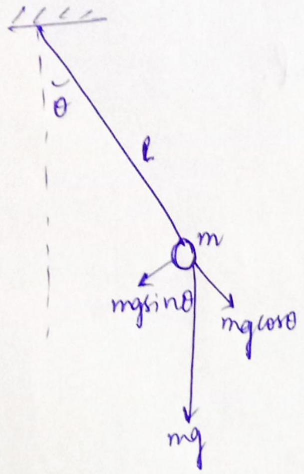
1)(a) Oscillation of simple pendulum

mass of bob is m and string length 'l'



By applying Newton's second law for rotational system, the equation of motion for the pendulum may be obtained.

$$\tau = I\alpha$$

$$\Rightarrow -mg\sin\theta\, L = mL^2 \frac{d^2\theta}{dt^2}$$

and rearranged as

$$\boxed{\frac{d^2\theta}{dt^2} + \frac{g}{L}\sin\theta = 0}$$

If the amplitude of angular displacement is small enough, so the small angle approximation holds true, then the equation of motion reduces to the equation of simple harmonic motion.

$$\boxed{\frac{d^2\theta}{dt^2} + \frac{g}{L}\theta = 0}$$

The simple harmonic motion is

$$\theta(t) = \theta_0 \cos(\omega t)$$

where θ_0 is the initial angular displacement and $\omega = \sqrt{g/L}$ the natural frequency of the motion. The time period of this system is

$$\boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{L/g}}$$

Pramod

1(b)

Damping ratio is 5%

so $\xi = 0.05$

~~we have~~ to reduce displacement amplitude to 48% of the initial value.

$x_n = 48\%$ of x_0

$x_n = 0.48\, x_0$

Let n be the no. of cycles required.

So the amplitude will always reduce.

$$\frac{x_0}{x_n} = \left(e^{\frac{2\pi \xi}{\sqrt{1-\xi^2}}}\right)^n$$

taking log both sides.

$$\log\left(\frac{x_0}{0.48\, x_0}\right) = n \times \frac{2\pi \xi}{\sqrt{1-\xi^2}}$$

$$\Rightarrow \log\left(\frac{1}{0.48}\right) = \frac{n \times 2\pi\ 0.05}{\sqrt{1-(0.05)^2}}$$

$$\Rightarrow 0.319 = \frac{0.314 \times n}{\sqrt{0.998}}$$

$$\Rightarrow 0.319 \times 0.9987 = 0.314 \times n$$

$$\Rightarrow n = \frac{0.319 \times 0.9987}{0.314} = \boxed{1.015}$$ (rounding to 3 decimal places)

So n is a decimal value.

The system needs a complete one cycle and some parts of the second cycle. So so talk about full cycle it will be ~~red~~ reduced to 48% of initial after 2 cycles.

3) (a) so $m = 48$ kg

frequency $\omega = 2200 \times \frac{2\pi}{60} = 230.3$ rad/s

Isolation to be done is 80%

Hence transmissibility $= 1 - 0.8 = 0.2 = T_r$

$$T_r = \sqrt{\frac{1 + (2\xi_e r)^2}{(1-r^2)^2 + (2\xi_e r)^2}}$$

Given to ~~go~~ neglect damping of the system so $\xi_e = 0$

$$T_r = \frac{1}{r^2 - 1}$$

$$T_r = \frac{1}{\left(\frac{\omega}{\omega_n}\right)^2 - 1}$$

$$\left(\frac{\omega}{\omega_n}\right)^2 - 1 = \frac{1}{T_r}$$

$$\left(\frac{\omega}{\omega_n}\right)^2 = \frac{1 + T_r}{T_r}$$

$$\left(\frac{\omega}{\omega_n}\right)^2 = \frac{1 + 0.2}{0.2}$$

$$\left(\frac{\omega}{\omega_n}\right)^2 = 6$$

$\frac{\omega}{\omega_n} = 2.45$ (approx to 2 decimal place.)

$$\omega_n = \frac{\omega}{2.45}$$

$$= \frac{230.3}{2.45}$$

$\omega_n = 94$ rad/s

$$\omega_n = \sqrt{k/m}$$

$$m\omega_n^2 = k$$

Putting $m = 48$

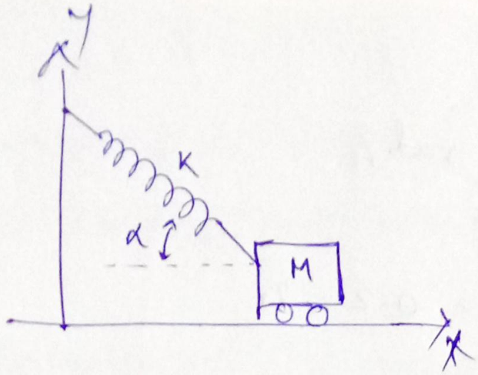
$$k = 48 \times (94)^2$$

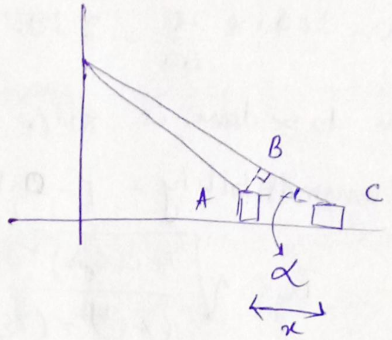
$$= 424{,}128 \text{ N/m}$$

$$\boxed{k = 4.24 \times 10^5 \text{ N/m}}$$

Ans.

3) (b)





Suppose we move the mass to the right by x distance which is very small.

As x is very small.

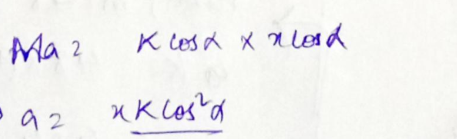
$\angle ACB$ is also α.

So the length of $BC = x \cos\alpha$

the length of BC is the amount of extention in the spring.

~~So, By energy equation.~~

FBD in X-plane



Equating both sides we get

$$Ma = K\cos\alpha \times x\cos\alpha$$

$$\Rightarrow a = \frac{xK\cos^2\alpha}{M}$$

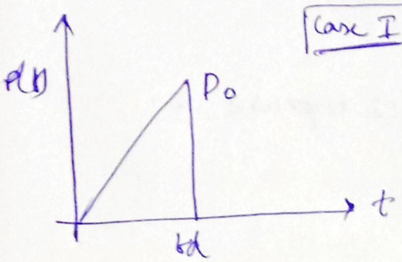
we can replace a by $\omega^2 x$

$$\Rightarrow \omega^2 \cancel{x} = \frac{\cancel{x}K\cos^2\alpha}{M}$$

$$\Rightarrow \boxed{\omega = \sqrt{\frac{K\cos^2\alpha}{M}}}$$ (Answer)

4) Undamped ~~SDO~~ SDOF system is subjected to a ramp impulse as shown

Steady state response in the forced and free vibration states using Duhamel integral.



$\boxed{\text{Case I}}$ for $0 < t \leq td$

$P(t) = \frac{P_o t}{td}$

The steady state response in forced vibration.

$$x = x_o \cos \omega t + \frac{\dot{x}_o}{\omega} \sin \omega t + \frac{1}{m\omega_n} \int_0^t \frac{P_o \tau}{td} \sin (t-\tau) \, d\tau$$

Assuming initial condition at $t=0$ $x_o = 0$, $\dot{x}_o = 0$

$$x = \frac{1}{m\omega_n} \int_0^{td} \frac{P_o \tau}{td} \sin \omega_n (t-\tau) \, d\tau$$

$$x = \frac{P_o}{m\omega_n td} \int_0^{td} \tau \sin \omega_n (t-\tau) \, d\tau$$

$$x = \frac{P_o}{m\omega_n td} \left[\tau \frac{1}{\omega_n} \cos(\omega_n (t-\tau)) - \int 1 \cdot \frac{1}{\omega_n} \cos(\omega_n (t-\tau)) d\tau \right]_0^t$$

$$x = \frac{P_o}{m\omega_n^2 td} \left[\tau \cos(\omega_n (t-\tau)) + \frac{1}{\omega_n} \sin(\omega_n (t-\tau)) \right]_0^t$$

$$x = \frac{P_o}{m\omega_n^2 td} \left[t - \frac{1}{\omega_n} \sin \omega_n t \right]$$

$$x = \frac{P_o}{m\omega_n^2} \left[\frac{t}{td} - \frac{1}{\omega_n td} \sin \omega_n t \right]$$

$$x = \frac{P}{k} \left[\frac{t}{td} - \frac{1}{\omega_n td} \sin \omega_n t \right] \quad \boxed{k = \omega_n^2 m}$$

Case II For $t > t_d$

The steady response in free vibration state

$$x = x_0 \cos \omega_n t + \frac{\dot{x}_0}{\omega_n} \sin \omega_n t + \frac{1}{m\omega_n} \int_{t_d}^{t} 0 \cdot \sin \omega_n (t-\tau) \, d\tau \rightarrow 0$$

$$x = x_0 \cos \omega_n t + \frac{\dot{x}_0}{\omega_n} \sin \omega_n t$$

For x_0, Putting $t = t_d$ in eq. of steady state response in forced vibration.

$$x_0 = \frac{P}{K}\left[\frac{t_d}{t_d} - \frac{1}{\omega_n t_d} \sin \omega_n t_d\right]$$

$$x_0 = \frac{P}{k}\left[1 - \frac{1}{\omega_n t_d} \sin \omega_n t_d\right]$$

For $\dot{x}_0$, differentiate the eq. of steady state response in forced vibration.

$$\dot{x} = \frac{dx}{dt} = \frac{P}{k}\left[\frac{1}{t_d} - \frac{1}{\omega_n t_d} \cos \omega_n t \cdot \omega_n\right]$$

$$= \frac{P}{k}\left[\frac{1}{t_d} - \frac{\cos \omega_n t}{t_d}\right]$$

Putting $t = t_d$ for $\dot{x}_0$

$$\dot{x}_0 = \frac{P}{k}\left[\frac{1}{t_d} - \frac{\cos \omega_n t_d}{t_d}\right]$$

So steady state response in free vibration state.

$$\boxed{x = \frac{P}{K}\left[1 - \frac{1}{\omega_n t_d} \sin \omega_n t_d\right] \cos \omega_n t + \frac{P}{\omega_n K}\left[\frac{1}{t_d} - \frac{\cos \omega_n t_d}{t_d}\right] \sin \omega_n t}$$

5) a) Importance factor is determined from design loads for Building and other structure based on occupancy category. It is utilized in calculating flood, wind, snow, seismic and ice design loads.
It is different for different buildings.

(i) 1.5 for critical and lifeline structures.

(ii) 1.2 for business continuing structures

(iii) 1.0 for the rest.

(b) ~~h=15m~~ height(h)=15m

~~N=2.~~ seismic weight (W) = 2000kN

total stiffness of column (K) = 1000 kN/m

$g = 10\ m/s^2$

$$\text{mass } (m) = \frac{W}{g} = \frac{2000}{10} \times 10^3\ kg = 200 \times 10^3\ kg$$

$$T = 2\pi\sqrt{\frac{m}{K}} = 2\pi\sqrt{\frac{200\times10^3}{1000\times10^3}} = 2.809\ sec.$$

$T = 2.809$ sec.

$$S_a/g = 1.36/T \text{ for } T = 0.55-4$$

$$= \frac{1.36}{2.809}$$

$$S_a/g = 0.484$$

Lateral forces at the top : $F = mA_h$

$$A_h = \frac{Z}{2} \cdot \frac{I}{R} \cdot \frac{S_a}{g}.$$

My city is in low (II) seismic zone.

$Z = 0.1$

$I = 1.5$

$R = 5$

$A_h = \frac{0.1}{2} \cdot \frac{1.5}{5} \cdot (0.484)$

$A_h = 0.00726 \ m/s^2$

$F = m A_h$

$F = 200 \times 10^3 \times 0.00726 = 1452 N$

lateral forces at the top is $\boxed{1452 N}$ (Ans)

So Bending moment at the base level is $= F \cdot h$

$= (1452) \times 15$

$\boxed{M_b = 21780 \ N\text{-}m}$

(Ans).