COLLATZ-HASSE-SYRACUSE-ULAM-KAKUTANI SEQUENCE
:
CONVERGENCE TO THE TRIVIAL CYCLE PROVED

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ABSTRACT

The convergence of the *Collatz-Hasse-Syracuse-Ulam-Kakutani Sequence* is proved by very simple reasoning, thus proving the *Collatz Conjecture*, which has been an *unsolved problem*.

Keywords: Collatz-Hasse-Syracuse-Ulam-Kakutani (CHSUK) Sequence; Collatz-Hasse-Syracuse-Ulam-Kakutani (CHSUK) Conjecture; Convergence.

AMS MSC Mathematics Subject Classification: 11B50.

1. INTRODUCTION

The *Collatz-Hasse-Syracuse-Ulam-Kakutani Conjecture* (simply referred to as the *Collatz Conjecture*) states that the Collatz-Hasse-Syracuse-Ulam-Kakutani (CHSUK) Sequence (also referred to as the *Collatz Sequence*) converges to the trivial cycle {(4->2->1)}, starting from any positive integer.

This research report presents a proof for the same, reasoning with the most fundamental *Peano Axioms* and *Modulus Arithmetic* applied to a meticulously designed *Structured System Framework* of *Binary-Exponential-Ladder*s that *partition*s the *set of positive integers* and establishes an isomorphism between the structured system framework of Binary-Exponential-Ladders and the set of natural numbers.

2. PROBLEM DESCRIPTION

We define the *Collatz Function* C(n) with a positive integer n as its input argument, in terms of a 'pull-Down' operator D(n) and a 'push-Up' operator U(n) as follows:

if (n is even) C(n) := D(n) = (n / 2); else C(n) := U(n) = (3*n + 1); [Eqn.1]

where the '*pull-Down*' D(n) operator takes only an even number as its input argument whereas the '*push-Up*' operator U(n) takes only an odd number as its input argument and gives an output that is an even number.

For convenience in our study of the Collatz Sequence, we define the *Compact Collatz Function* T(m) by the repeated application of the 'pull-down' operator D(m) wherever applicable, say, (p≥1) times, that is, $D^*(m) := D^p(m)$ so as to get an output $D^\#(m)$ that is an odd number:

if (m is even) $T(m) := D^*(m) := D^p(m) = (m / 2^p) := D^\#(m)$;
else (m is odd) $T(m) := U(m) = (3 * m + 1) := U^\#(m)$; [Eqn.2]

where $D^\#(m)$ is called the "D-floor number" associated with the input argument m; and $U^\#(m)$ is called the "U-ceiling number" associated with the input argument m.

The *Compact Collatz Function* T(m) may as well be considered to have been redefined with the newly introduced two operators, the "D-floor operator" $D^\#(m)$ and the "U-ceiling operator" $U^\#(m)$ as given in [Eqn.2] above.

This new definition for the *Compact Collatz Function* T(m) facilitates our study of the corresponding *Compact Collatz Sequence*; which is no different from its equivalent Collatz Sequence, once we understand that the repeated application, say, (p≥1) times, of the 'pull-Down' operator D(m) has now been collapsed into an equivalent single "D-floor operator" $D^\#(m)$ giving the D-floor number $D^\#(m)$ as its output. The push-Up operator U has been simply redefined as the "U-ceiling operator" $U^\#$ for uniformity and elegant completeness.

The *Compact Collatz Sequence* is obtained by the repeated sequential application of the *Compact Collatz Function* T(m) starting with the given initial input number m - represented by an alternating series of $D^\#$ number and $U^\#$ number - except possibly the starting initial 'seed' number m and the final terminating number, which as per the Collatz Conjecture, is anyway a $D^\#$ number that is unity.

3. OBSERVATIONS ON THE PULL-DOWN OPERATOR

The pull-Down operator D always takes only an even number n as its input argument. Every application of this pull-down operator results in an alternating (toggling) effect on the nMOD3 property of the input argument number; that is, a

1MOD3 input gives a 2MOD3 output and a 2MOD3 input gives a 1MOD3 output; whereas a 0MOD3 input gives a 0MOD3 output. Repeated application of D, in case applicable, results in a final output that is an odd number and therefore becomes an input for the push-Up operator. In such a case, we call it a "D-floor operator $D^{\#}$" as defined in [Eqn.2] above, and its output a "D-floor number" $D^{\#}(n)$ characterized by being a odd number; $D^{\#}(n)$ may be in any one of the three possible types: (1) a 1MOD6 odd number, being a 1MOD3 odd number that is of the type (6m-5); (2) a 5MOD6 odd number, being a 2MOD3 odd number that is of the type (6m-1); (3) a 3MOD6 odd number, being a 0MOD3 odd number that is of the type (6m-3).

4. OBSERVATIONS ON THE PUSH-UP OPERATOR

The push-Up operator U always takes only an odd number m as its input argument, and always gives an output that is a 4MOD6 even number, being a 1MOD3 even number that is of the type (6m-2) - irrespective of whether the input is a 1MOD6 odd number or a 3MOD6 odd number or a 5MOD6 odd number. Note that one single application of the 'push-Up' operator U transforms any input odd number m into a 4MOD6 even number that becomes an input to the "D-floor operator $D^{\#}$". That is why we may as well call the push-Up operator U as the "U-ceiling operator U#" as defined in [Eqn.2] above.

5. OBSERVATIONS ON THE COMPACT COLLATZ FUNCTION

Start with any positive integer. (1) If the starting initial number n is even, then we apply the D-floor operator $D^{\#}$ operator giving an output that is the D-floor number $D^{\#}(n)$ which is given as input to the U-ceiling operator. Of course, if the starting number is a power of 2 we terminate at unity. So, now we have a D-floor number $D^{\#}(n)$ that is an odd number greater than unity, in any non-trivial case, as the initial $D^{\#}$ node in the Compact Collatz Sequence. (2) If on the other hand the starting initial number n is an odd number, we treat that itself as the initial $D^{\#}$ node in the Compact Collatz Sequence.

Having thus obtained the initial $D^{\#}$ node in the Compact Collatz Sequence, we apply the U-ceiling operator $U^{\#}$ to get the U-ceiling number $U^{\#}$ that is a 4MOD6 even number. That in turn is given as input to the D-floor operator $D^{\#}$. Now the process continues.

Note that the Compact Collatz Sequence can therefore be defined by a *trajectory* generated by an alternating sequence of a "D-floor number" $D^{\#}$ and a "U-ceiling number" $U^{\#}$, with its starting initial node being a $D^{\#}$ number. The Compact Collatz Function as presented in [Eqn.2] defines the unique link (directed arc) from any given D-floor number $D^{\#}$ as the predecessor node to its corresponding unique U-ceiling number $U^{\#}$ as the successor node and also the unique link (directed arc) from any given U-ceiling number $U^{\#}$ as the predecessor node to its corresponding unique D-floor number $D^{\#}$ as the successor node. The unique link (directed arc) from a starting initial even “seed” number leading to the first node (D-floor number $D^{\#}$) in the *trajectory* is similarly defined.

As mentioned earlier, the application of the D-floor operator $D^{\#}$ on a U-ceiling number $U^{\#}$ that is a 4MOD6 even number of the form (6m-2) can lead to a D-floor number $D^{\#}$ that is an odd number that can be either: (1) a 1MOD6 odd number, being a 1MOD3 odd number that is of the type (6m-5); (2) a 5MOD6 odd number, being a 2MOD3 odd number that is of the type (6m-1); but can never be (3) a 3MOD6 odd number, that is a 0MOD3 odd number of the type (6m-3). Note that the only situation when the D-floor operator $D^{\#}$ gives an output D-floor number $D^{\#}$ that is a 3MOD6 odd number of the type (6m-3) is when its input is a 0MOD6 even number, which is impossible for any U-ceiling number $U^{\#}$, although such an input may come in those special cases wherein the starting initial ‘seed’ number itself is a 0MOD6 even number that is of the form $(6m-3).2^p$ leading to an output $D^{\#}$ that is again a 3MOD6 odd number of the form (6m-3).

6. ANALYSIS OF THE COMPACT COLLATZ SEQUENCE

From the above observations, it is clear that corresponding to every positive integer n as the starting initial ‘seed’ number, there is a starting initial node in the trajectory representing the *Compact Collatz Sequence*, that is a $D^{\#}$ number in exactly one of the three possible forms as mentioned above - that can be an input argument to the U-ceiling operator $U^{\#}$ giving exactly one unique output $U^{\#}$ which itself can be an input to the D-floor operator $D^{\#}$ so that the process continues. Successive application of each of these two operators ($U^{\#}$ and $D^{\#}$) wherever applicable, traces a unique *trajectory*, wherein each node represents a number that is the unique output number of the appropriate operation applied to the input number represented by the preceding node in the trajectory.

The anticipated terminating trivial cycle {(4->2->1)} can be obtained only through a final application of the D-floor operator $D^{\#}$ on a 4MOD6 even number of the form (6m-2).

7. BINARY-EXPONENTIAL-LADDER WITH ITS DEFINING-BASE-RUNG

Here, we present a meticulously designed *Structured System Framework* that *partition*s the *set of positive integers* to facilitate a *general systems analysis* of the *Compact Collatz Sequence*.

Let every positive odd number be associated with a *Binary-Exponential-Ladder*, denoted by BEL(2m-1) and defined as a sequence $\{(2m-1).2^u\}$ with ($u \geq 0$); its *defining-base-rung* given by the odd number (2m-1). Thus, we establish an exact one-to-one mapping between the *set of positive odd numbers* that form the $D^{\#}$

value for the *defining-base-rung* and the corresponding *Binary-Exponential-Ladder* BEL($D^{\#}$).

Every positive even number in the form $(2m-1).2^u$ with (u>0); for which there exists its corresponding $D^{\#}$ value, $D^{\#}((2m-1).2^U) = (2m-1)$; for which there exists exactly one corresponding *Binary-Exponential-Ladder* BEL(2m-1) that contains the given even number $(2m-1).2^u$ as one of its higher rungs in that BEL(2m-1) ladder.

Thus, we establish that *the set of all Binary-Exponential-Ladders form a partition of the set of all positive integers*; with an exact one-to-one correspondence between each positive odd number and the corresponding *Binary-Exponential-Ladder* for which it is the *defining-base-rung* $D^{\#}$; whereas each of the given positive even numbers correspond to exactly one of the higher rungs of some specific Binary-Exponential-Ladder identified by the D-floor number associated with that given positive even number.

This partitioned framework of positive integers goes another step deeper because of the fact that the *defining-base-rung* $D^{\#}$ of a Binary-Exponential-Ladder BEL($D^{\#}$) can itself be in one of the three possible forms 1MOD6 or 5MOD6 or 3MOD6 whereas all the upper rungs of the Binary-Exponential-Ladder are either (1) alternatingly 2MOD6 and 4MOD6 or (2) all being 0MOD6 numbers.

The Collatz Conjecture states that every Collatz Sequence, starting from any positive integer, converges to the trivial cycle {(4->2->1)} which is in the Binary-Exponential-Ladder BEL(1) that is uniquely identified by its defining-base-rung $D^{\#}$ value that is unity. Therefore, our focus will be the set of all Binary-Exponential-Ladders centered around BEL(1) and its relationship with every other Binary-Exponential-Ladder BEL($D^{\#}$).

As seen above, $D^{\#}$ can be (1) either of the form (6m-5) that is a 1MOD6 number; (2) or of the form (6m-1) that is a 5MOD6 number; (3) or of the form (6m-3) that is a 3MOD6 number. BEL(6m-5) contains the output of $U^{\#}$ at $(6m-5)2^W$ with w being an even exponent of the form (2k) wherein the input of $U^{\#}$ is given by $[\{(6m-5).2^W -1\}/3]$. BEL(6m-1) contains the output of $U^{\#}$ at $(6m-1)2^V$ with v being an odd exponent of the form (2k-1) wherein the input of $U^{\#}$ is given by $[\{(6m-1).2^V -1\}/3]$. However, BEL(6m-3) cannot contain any such output of the U-ceiling operator $U^{\#}$ irrespective of any input argument.

8. IMMEDIATE NEIGHBORHOOD OF A BINARY-EXPONENTIAL-LADDER

The relationship between a pair of Binary-Exponential-Ladders BEL(m) and BEL(n) can be considered to be defined and characterized by the relationship between the corresponding pair of the *defining-base-rung* $D^{\#}$ values m and n along with the corresponding pair $U^{\#}(m)$ and $U^{\#}(n)$.

Among the set of all Binary-Exponential-Ladders, the immediate-neighborhood of a given Binary-Exponential-Ladder BEL($D^{\#}$) is defined by the immediate-predecessors and immediate-successors, w.r.t U the push-Up operator; *since the* *pull-Down operator is applicable only within a given Binary-exponential-Ladder and not between a pair of them*.

It turns out that the only *one single unique immediate successor* of BEL(m) is BEL($D^{\#}(U^{\#}$(m))) that contains $U^{\#}$(m) as one of its higher rungs, with its identifying characteristic D-floor number $D^{\#}(U^{\#}$(m)) as its defining-base-rung. However, there exists a *set of immediate-predecessors* for each BEL($D^{\#}$) of the form BEL(6m-5) and BEL(6m-1) although none for BEL(6m-3).

BEL(1MOD6) or equivalently BEL(6m-5) has, as its set of immediate-predecessors, {BEL([(1MOD6).2^W - 1]/3)} or equivalently {BEL([(6m-5).2^W - 1]/3)} with w being an positive even exponent of the form (2k), wherein the input of $U^{\#}$ is given by [{(1MOD6).2^W -1}/3] or equivalently [{(6m-5).2^W -1}/3] and the output of $U^{\#}$ being {(1MOD6).2^W} or equivalently {(6m-5).2^W} that is contained in BEL(1MOD6) or equivalently BEL(6m-5).

BEL(5MOD6) or equivalently BEL(6m-1) has, its set of immediate-predecessors, {BEL([(5MOD6).2^V - 1]/3)} or equivalently {BEL([(6m-1).2^V - 1]/3)} with v being a positive odd exponent of the form (2k-1), wherein the input of $U^{\#}$ is given by [{(5MOD6).2^V -1}/3] or equivalently [{(6m-1).2^V -1}/3] and the output of $U^{\#}$ being {(5MOD6).2^V} or equivalently {(6m-1).2^V} that is contained in BEL(5MOD6) or equivalently BEL(6m-1).

The above observed property, that *only* the alternating rungs, defined by (1MOD6).4^u or (5MOD6).2.4^u, of the *Binary-Exponential-Ladder* BEL(1MOD6) or BEL(5MOD6), being the 'active' nodes in the CHSUK-Sequence; naturally makes it convenient to define a system of *Quarternary-Exponential-Ladder*s (QEL) wherein every rung of QEL becomes an 'active' node in the CHSUK-Sequence. This concept is not directly needed for proving the convergence of the Collatz Sequence, and therefore we will leave it at this point.

Considering BEL(1) as our central focus of interest, which itself belongs to the type BEL(1MOD6) or equivalently BEL(6m-5); it is interesting to note that it has its single unique immediate-successor as BEL($D^{\#}(U^{\#}$(1))) that is BEL(1) itself because of the trivial cycle {(4->2->1)} being contained within BEL(1).

9. STRUCTURED SYSTEM FRAMEWORK OF BINARY-EXPONENTIAL-LADDERS

From the above discussion we find that it is convenient for our study to consider a *Structured System Framework* H = {H_0, H_1, H_2, . . .} consisting of a countably infinite hierarchy of countably infinite set of *Binary Exponential Ladder*s; the 'root' being the singleton set H_0 = {BEL(1)} at tier-0 level of the hierarchy. The set of k^{th} immediate predecessors of BEL(1) form the set H_k at tier-k in the hierarchy, etc.

The uniqueness characteristic of the *immediate-successor* relationship among the Binary-Exponential-Ladders can be considered to be a *strict-ordering* relation among these sets H_k except for the first element H_0 = BEL(1) which is its own successor, implying that H_0 = {BEL(1)} acts as a final sink node in the corresponding sequence.

However, the multiplicity of the *immediate-predecessor* relationship among the Binary-Exponential-Ladders requires that the set of all immediate-predecessors of every element of H_{k-1} form the elements of the set H_k as indicated in the above definition of the Structured System Framework.

This Structured System Framework H of Binary Exponential Ladders has a direct one-to-one correspondence (mapping) with the set of positive integers, considering the distinctly specific rungs of each of the Binary Exponential Ladders; the lowest rung in each BEL being the *defining-base-rung* that is mapped to the corresponding odd number and each of the higher rungs being mapped to the corresponding even number.

There is a strict ordering relation $\{H_{j-1}\} < \{H_j\}$ between the different levels of the hierarchy or the tier levels, because of the predecessor successor relationship between them, and a clear idempotent element $\{H_0\}$ which is its own successor.

The (1) strict ordering in hierarchy of H; with the above mentioned (2) predecessor-successor relationship among the Binary-Exponential-Ladders that form the elements of H in each level of the hierarchy; and the fact that (3) there is an exact one-to-one correspondence between the set of positive integers and the set of all rungs in all the Binary-Exponential-Ladders BEL(2m-1); imply that - the *Structured System Framework* H has been designed to represent an *arborescence*, wherein there is *exactly one single unique directed path from every positive integer to the trivial cycle* {(4->2->1)} *at the defining-base-rung of* BEL(1).

10. COLLATZ-HASSE-SYRACUSE-ULAM-KAKUTANI (CHSUK) THEOREM

STATEMENT OF THE CHSUK THEOREM

The CHSUK Sequence converges to the trivial cycle {(4->2->1)}.

PROOF

We show that the *Structured System Framework* H by its very design, satisfies the Peano's axioms (replacing the 'successor' by the 'predecessor') and therefore H is isomorphic with the set of natural numbers; and satisfies the above stated convergence statement.

PEANO'S AXIOM : Existence of 0.
$\{H_0\} \in$ H.
The trivial cycle {(4->2->1)} is contained in $\{H_0\} \in$ H.

PEANO'S AXIOM : Existence of a *successor function*.
By the very design of F, for every positive integer k,

$\{H_k\} \in F$ is the *predecessor* of $\{H_{k-1}\} \in H$.
Application of the Collatz Function with the input from numbers in $\{H_k\}$ yield the output contained in $\{H_{k-1}\}$.

PEANO'S AXIOM : 0 is not a successor.
$\{H_0\}$ is its own predecessor. However, there *does not exist any* $\{H_k\} \in H$, $k \neq 0$; that is distinct from $\{H_0\}$; with $\{H_k\} \neq \{H_0\}$; such that $\{H_0\}$ is the predecessor of $\{H_k\}$. Once the Collatz Sequence reaches the trivial cycle (sink) there is no exit from it.

PEANO'S AXIOM : Successor function is a unique one-to-one mapping.
If $\{H_u\}$ is the predecessor of $\{H_v\}$ and also $\{H_u\}$ is the predecessor of $\{H_w\}$;
then it necessarily implies $\{H_v\} = \{H_w\}$ by the very design of H;
and,
If $\{H_v\}$ is the predecessor of $\{H_u\}$ and also $\{H_w\}$ is the predecessor of $\{H_u\}$;
then it necessarily implies $\{H_v\} = \{H_w\}$ by the very design of H;
This is because the predecessor relation is a unique one-to-one mapping (bijection). The Compact Collatz Sequence is a linear directed path (chain) with no forking or merging.

PEANO'S AXIOM : Principle of induction.
Collatz Sequence starting with numbers from $\{H_0\}$ converge in the trivial cycle that is contained in $\{H_0\}$.
Collatz Sequence starting with a positive number from $\{H_k\}$ passes through $\{H_{k-1}\}$.
Therefore, the Collatz Sequence starting with any positive integer being contained in some $\{H_k\} \in F$, $k \geq 0$; must necessarily reach $\{H_0\}$ and therefore converge in the trivial cycle.

Thus, we establish a direct isomorphism between the *Structured System Framework* H and the set of Natural Numbers N; and the proof of convergence of the Collatz Sequence is an immediate consequence of this isomorphism according to the property of induction as mentioned above.

END OF PROOF

11. SOME EXPLICIT FORMS FOR THE BEL-NEIGHBORHOOD

We can perform some simple algebraic manipulation to get the parametric relation [Eqn.4] that gives a generic form for the set B_k that is the set of k^{th} predecessors of H_0 = BEL(1) wherein the set H_k corresponding to the set of tier-k Binary-Exponential-Ladders {BEL(n)} each of which contains the set of positive integers n that form its rungs - the lowest defining-base-rung characterized by the corresponding D-floor number $D^{\#}(n)$ with the parameter $z_{k-1} = 0$ and the higher rungs defined with the parameter z_{k-1} taking values from the set of positive integers.

$$n = [2^z - \{3^0.2^{z0} + 3^1.2^{z1} + 3^2.2^{z2} + \ldots + 3^{k-1}.2^{zk-1}\}] / 3^k \qquad \text{[Eqn.4]}$$

wherein k, z, z_0, z_1, z_2, . . . z_k, are the parameters that take specific values corresponding to each positive integer n. Or equivalently, each positive integer can be considered to be defined by the corresponding set of these parameters. Here k, z are positive integers; z_0, z_1, z_2, . . . z_{k-1}, z_k are non-negative integers of decreasing values all less than z; ($z_k := 0$ and $z_{k-1} = 0$ for positive odd number n).

Now, define $p_0 := (z - z_0)$; $p_j := (z_{j-1} - z_j)$; where p_j corresponds to the number of rungs in BEL{H_j} above the *defining-base-rung* of BEL{H_j} for the node located in BEL{H_j} that the Collatz sequence/trajectory passes through; BEL{H_j} being the *Binary-Exponential-Ladder* at tier-j with j=0,1,2, . . . k. Thus, we may as well redefine the set of (k+1) parameters as {P_k} = {p_0, p_1, p_2, . . . p_k} that is, a set of (k+1) CHSUK(generative)parameters that generate each positive integer n as per the parametric relation [Eqn.4] given above ($p_k = 0$ for positive odd number n).

For any positive integer value of k, the above set of exponents z, z_0, z_1, z_2, z_3, . . . z_k, can be redefined in terms of the newly defined CHSUK(generative)parameters, by rewriting the above definition as $z := (z_0 + p_0)$; $z_{j-1} := (z_j + p_j)$; with $p_k = 0$ for positive odd number n and $z_k := 0$.

Table-1 gives some of the possible set of valid CHSUK(generative)parameter and therefore the corresponding valid values of the exponents in [Eqn.4] above along with their corresponding n values. Note that the set of valid values of the exponents in [Eqn.4] above are governed by certain rules as can be seen from the earlier observations above, regarding the matching relationship between their uMOD3 value with the nMOD3 of its predecessor.

Table-1 : CHSUK(generative)parameters

k	z	z0	z1	z2	z3	p0	p1	p2	p3	n
1	2	0				2	0			1
1	4	0				4	0			5
1	6	0				6	0			21
1	8	0				8	0			85
1	10	0				10	0			341
1	12	0				12	0			1365
2	5	1	0			4	1	0		3
2	7	3	0			4	3	0		13
2	9	5	0			4	5	0		53
2	11	7	0			4	7	0		213
2	13	9	0			4	9	0		853
2	15	11	0			4	11	0		3413
2	10	2	0			8	2	0		113
2	12	4	0			8	4	0		453
2	14	6	0			8	6	0		1813
2	16	8	0			8	8	0		7253

2	18	10	0			8	10	0		29013
2	20	12	0			8	12	0		116053
2	11	1	0			10	1	0		227
2	13	3	0			10	3	0		909
2	15	5	0			10	5	0		3637
2	17	7	0			10	7	0		14549
2	19	9	0			10	9	0		58197
2	21	11	0			10	11	0		232789
3	9	5	2	0		4	3	2	0	17
3	11	7	4	0		4	3	4	0	69
3	13	9	6	0		4	3	6	0	277
3	15	11	8	0		4	3	8	0	1109
3	17	13	10	0		4	3	10	0	4437
3	19	15	12	0		4	3	12	0	17749
3	10	6	1	0		4	5	1	0	35
3	12	8	3	0		4	5	3	0	141
3	14	10	5	0		4	5	5	0	565
3	16	12	7	0		4	5	7	0	2261
3	18	14	9	0		4	5	9	0	9045
3	20	16	11	0		4	5	11	0	36181
3	15	11	2	0		4	9	2	0	1137
3	17	13	4	0		4	9	4	0	4549
3	19	15	6	0		4	9	6	0	18197
3	21	17	8	0		4	9	8	0	72789
3	23	19	10	0		4	9	10	0	291157
3	25	21	12	0		4	9	12	0	1164629
3	16	12	1	0		4	11	1	0	2275
3	18	14	3	0		4	11	3	0	9101
3	20	16	5	0		4	11	5	0	36405
3	22	18	7	0		4	11	7	0	145621
3	24	20	9	0		4	11	9	0	582485
3	26	22	11	0		4	11	11	0	2329941
3	12	2	1	0		10	1	1	0	151
3	14	4	3	0		10	1	3	0	605
3	16	6	5	0		10	1	5	0	2421
3	18	8	7	0		10	1	7	0	9685
3	20	10	9	0		10	1	9	0	38741
3	22	12	11	0		10	1	11	0	154965
3	17	7	2	0		10	5	2	0	4849
3	19	9	4	0		10	5	4	0	19397
3	21	11	6	0		10	5	6	0	77589
3	23	13	8	0		10	5	8	0	310357
3	25	15	10	0		10	5	10	0	1241429
3	27	17	12	0		10	5	12	0	4965717
3	18	8	1	0		10	7	1	0	9699

3	20	10	3	0		10	7	3	0	38797
3	22	12	5	0		10	7	5	0	155189
3	24	14	7	0		10	7	7	0	620757
3	26	16	9	0		10	7	9	0	2483029
3	28	18	11	0		10	7	11	0	9932117
k	z	z0	z1	z2	z3	p0	p1	p2	p3	n
Table-1 : CHSUK(generative)parameters										

12. A CHALLENGE TO MY COOL-HEADED BRAVE-HEARTS

If you can prove that for every positive integer n there exists a unique set of CHSUK generative parameters, then you can directly prove the convergence of the CHSUK Sequence to the trivial cycle {(4->2->1)}.

13. CONCLUSION

We have presented a meticulously designed *structured system framework* of *Binary-Exponential-Ladders* and established its isomorphism with the set of positive integers, that directly leads to a simple and elegant proof of the convergence of the CHSUK Sequence.

14. RECOMMENDED READING

[1]. Wikipedia Page – https://en.wikipedia.org/wiki/Collatz_conjecture

[2]. Jeffrey C Lagarias;
"The 3x+1 problem: An annotated bibliography (1963--1999) (sorted by author)";
https://arxiv.org/abs/math/0309224

[3]. Jeffrey C Lagarias;
"The 3x+1 Problem: An Annotated Bibliography, II (2000-2009)";
https://arxiv.org/abs/math/0608208

[4]. Jeffrey C Lagarias;
"The 3x + 1 Problem : An Overview"
https://arxiv.org/abs/2111.02635

[5]. Halemane, K. P. (2014);
"Unbelievable O($L^{1.5}$) worst case computational complexity
achieved by *spdspds* algorithm for linear programming problem";
https://arxiv.org/abs/1405.6902 (2025).

[6]. Halemane, K. P. (2025);
"Refutation of the Logical Fallacy Committed by the Subject Matter Experts on the Monty-Hall Problem";
https://engrxiv.org/preprint/view/5102

[7]. Halemane, K. P. (2025);
"Monty-Hall Theorem";
https://engrxiv.org/preprint/view/5594

15. ACKNOWLEDGEMENT

I acknowledge the fact that the most revered Number-Theory Expert Paul Erdos once said about the Collatz Conjecture - "Mathematics is not yet ready for such problems" as quoted by Jeffrey Lagarias [4].

I must necessarily confess here that the *core idea behind this analysis is so stunningly & elusively simple*, that one may simply be taken aback in a profound wonder-struck jaw-drop-silence, maybe with an after-thought: "*oh my goodness, how could it be that it never flashed on me any time earlier*"! as was also the case in earlier research reports [5][6]&[7].

16. DEDICATION

To my ಅಜ್ಜ(ajja) Karinja Halemane Keshava Bhat & ಅಜ್ಜಿ(ajji) Thirumaleshwari, ಅಪ್ಪ(appa) Shama Bhat & ಅಮ್ಮ(amma) Thirumaleshwari, for their *teachings through love*, that *quality matters more than quantity*; to my wife Vijayalakshmi for her *ever consistent love & support*; to my daughter Sriwidya.Bharati and my twin sons Sriwidya.Ramana & Sriwidya.Prawina for their *love & affection*.

To all the *cool-headed brave-hearts*, eagerly awaited but probably yet to be visible among the world professionals, especially the *Subject-Matter-Experts*, who would be attracted to and certainly capable of effectively understanding without any prejudice and appreciating the deeper insights enshrined in this research report.

ॐतत्सत्