

# COLLATZ-HASSE-SYRACUSE-ULAM-KAKUTANI SEQUENCE : CONVERGENCE TO THE TRIVIAL CYCLE PROVED

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## ABSTRACT

The convergence of the *Collatz-Hasse-Syracuse-Ulam-Kakutani Sequence* is proved, thus proving the *Collatz Conjecture*, which has been an *unsolved problem*. The proof is based on the *isomorphism* established between the set of positive integers and a carefully designed *system* with a hierarchy (arborescence) of *binary exponential ladders* defined on the set of positive odd numbers.

Keywords: Collatz-Hasse-Syracuse-Ulam-Kakutani (CHSUK) Sequence;  
Collatz-Hasse-Syracuse-Ulam-Kakutani (CHSUK) Conjecture;  
Convergence, Isomorphism, Dedekind-Peano Axioms.

AMS MSC Mathematics Subject Classification: 11B50.

## 1. INTRODUCTION

The *Collatz-Hasse-Syracuse-Ulam-Kakutani Conjecture* (simply referred to as the *Collatz Conjecture*) states that the Collatz-Hasse-Syracuse-Ulam-Kakutani (CHSUK) Sequence (also referred to as the *Collatz Sequence*) converges to the trivial cycle  $\{(4 \rightarrow 2 \rightarrow 1)\}$ , starting from any positive integer.

This research report presents a proof for the same, reasoning with the most fundamental *Dedekind-Peano Axioms* and *Modulus Arithmetic* applied to a meticulously designed *Structured System Framework* of *Binary Exponential Ladders* defined on the set of positive odd numbers, and establishes an *isomorphism* between the *structured system framework* of *Binary Exponential Ladders* and the set of natural numbers.

## 2. PROBLEM DESCRIPTION

We define the *Collatz Function*  $C(n)$  with a positive integer  $n$  as its input argument, in terms of a 'pull-Down' operator  $D(n)$  and a 'push-Up' operator  $U(n)$  as follows:

$$\text{if } (n \text{ is even}) C(n) := D(n) = (n / 2); \text{ else } C(n) := U(n) = (3*n + 1); \quad [\text{Eqn.1}]$$

where the 'pull-Down'  $D(n)$  operator takes only an even number as its input argument whereas the 'push-Up' operator  $U(n)$  takes only an odd number as its input argument and gives an output that is an even number.

For convenience in our study of the Collatz Sequence, we define the *Compact Collatz Function*  $T(m)$  by the repeated application of the 'pull-down' operator  $D(m)$  wherever applicable, say,  $(p \geq 1)$  times, that is,  $D^*(m) := D^p(m)$  so as to get an output  $D^\#(m)$  that is an odd number:

$$\begin{aligned} \text{if } (m \text{ is even}) \quad T(m) &:= D^*(m) := D^p(m) = (m / 2^p) := D^\#(m); \\ \text{else } (m \text{ is odd}) \quad T(m) &:= U(m) = (3 * m + 1) := U^\#(m); \end{aligned} \quad [\text{Eqn.2}]$$

where  $D^\#(m)$  is called the "D-floor number" associated with the input argument  $m$ ; and  $U^\#(m)$  is called the "U-ceiling number" associated with the input argument  $m$ .

The *Compact Collatz Function*  $T(m)$  may as well be considered to have been redefined with the newly introduced two operators, the "D-floor operator"  $D^\#(m)$  and the "U-ceiling operator"  $U^\#(m)$  as given in [Eqn.2] above.

This new definition for the *Compact Collatz Function*  $T(m)$  facilitates our study of the corresponding *Compact Collatz Sequence*; which is no different from its equivalent Collatz Sequence, once we understand that the repeated application, say,  $(p \geq 1)$  times, of the 'pull-Down' operator  $D(m)$  has now been collapsed into an equivalent single "D-floor operator"  $D^\#(m)$  giving the D-floor number  $D^\#(m)$  as its output. The push-Up operator  $U$  has been simply redefined as the "U-ceiling operator"  $U^\#$  for uniformity and elegant completeness.

The *Compact Collatz Sequence* is obtained by the repeated sequential application of the *Compact Collatz Function*  $T(m)$  starting with the given initial input number  $m$  - represented by an alternating series of  $D^\#$  number and  $U^\#$  number - except possibly the starting initial 'seed' number  $m$  and the final terminating number, which as per the Collatz Conjecture, is anyway a  $D^\#$  number that is unity.

## 3. OBSERVATIONS ON THE PULL-DOWN OPERATOR

The pull-Down operator  $D$  *always* takes only an even number  $n$  as its input argument. Every application of this pull-down operator results in an alternating

(toggling) effect on the  $n \text{MOD} 3$  property of the input argument number; that is, a  $1 \text{MOD} 3$  input gives a  $2 \text{MOD} 3$  output and a  $2 \text{MOD} 3$  input gives a  $1 \text{MOD} 3$  output; whereas a  $0 \text{MOD} 3$  input gives a  $0 \text{MOD} 3$  output. Repeated application of  $D$ , in case applicable, results in a final output that is an odd number and therefore becomes an input for the push-Up operator. In such a case, we call it a “D-floor operator”  $D^\#$  as defined in [Eqn.2] above, and its output a “D-floor number”  $D^\#(n)$  characterized by being a odd number;  $D^\#(n)$  may be in any one of the three possible types: (1) a  $1 \text{MOD} 6$  odd number, being a  $1 \text{MOD} 3$  odd number that is of the type  $(6m-5)$ ; (2) a  $5 \text{MOD} 6$  odd number, being a  $2 \text{MOD} 3$  odd number that is of the type  $(6m-1)$ ; (3) a  $3 \text{MOD} 6$  odd number, being a  $0 \text{MOD} 3$  odd number that is of the type  $(6m-3)$ .

#### 4. OBSERVATIONS ON THE PUSH-UP OPERATOR

The push-Up operator  $U$  *always* takes only an odd number  $m$  as its input argument, and *always* gives an output that is a  $4 \text{MOD} 6$  even number, being a  $1 \text{MOD} 3$  even number that is of the type  $(6m-2)$  - irrespective of whether the input is a  $1 \text{MOD} 6$  odd number or a  $3 \text{MOD} 6$  odd number or a  $5 \text{MOD} 6$  odd number. Note that one single application of the ‘push-Up’ operator  $U$  transforms any input odd number  $m$  into a  $4 \text{MOD} 6$  even number that becomes an input to the “D-floor operator  $D^\#$ ”. That is why we may as well call the push-Up operator  $U$  as the “U-ceiling operator  $U^\#$ ” as defined in [Eqn.2] above.

#### 5. OBSERVATIONS ON THE COMPACT COLLATZ FUNCTION

Start with any positive integer. (1) If the starting initial number  $n$  is even, then we apply the D-floor operator  $D^\#$  operator giving an output that is the D-floor number  $D^\#(n)$  which is given as input to the U-ceiling operator. Of course, if the starting number is a power of 2 we terminate at unity. So, now we have a D-floor number  $D^\#(n)$  that is an odd number greater than unity, in any non-trivial case, as the initial  $D^\#$  node in the Compact Collatz Sequence. (2) If on the other hand the starting initial number  $n$  is an odd number, we treat that itself as the initial  $D^\#$  node in the Compact Collatz Sequence.

Having thus obtained the initial  $D^\#$  node in the Compact Collatz Sequence, we apply the U-ceiling operator  $U^\#$  to get the U-ceiling number  $U^\#$  that is a  $4 \text{MOD} 6$  even number. That in turn is given as input to the D-floor operator  $D^\#$ . Now the process continues.

Note that the Compact Collatz Sequence can therefore be defined by a *trajectory* generated by an alternating sequence of a “D-floor number”  $D^\#$  and a “U-ceiling number”  $U^\#$ , with its starting initial node being a  $D^\#$  number. The Compact Collatz Function as presented in [Eqn.2] defines the unique link (directed arc) from any given D-floor number  $D^\#$  as the predecessor node to its corresponding unique U-ceiling number  $U^\#$  as the successor node and also the unique link (directed arc)

from any given U-ceiling number  $U^\#$  as the predecessor node to its corresponding unique D-floor number  $D^\#$  as the successor node. The unique link (directed arc) from a starting initial even “seed” number leading to the first node (D-floor number  $D^\#$ ) in the *trajectory* is similarly defined.

As mentioned earlier, the application of the D-floor operator  $D^\#$  on a U-ceiling number  $U^\#$  that is a 4MOD6 even number of the form  $(6m-2)$  can lead to a D-floor number  $D^\#$  that is an odd number that can be either: (1) a 1MOD6 odd number, being a 1MOD3 odd number that is of the type  $(6m-5)$ ; (2) a 5MOD6 odd number, being a 2MOD3 odd number that is of the type  $(6m-1)$ ; but can never be (3) a 3MOD6 odd number, that is a 0MOD3 odd number of the type  $(6m-3)$ . Note that the only situation when the D-floor operator  $D^\#$  gives an output D-floor number  $D^\#$  that is a 3MOD6 odd number of the type  $(6m-3)$  is when its input is a 0MOD6 even number, which is impossible for any U-ceiling number  $U^\#$ , although such an input may come in those special cases wherein the starting initial ‘seed’ number itself is a 0MOD6 even number that is of the form  $(6m-3).2^p$  leading to an output  $D^\#$  that is again a 3MOD6 odd number of the form  $(6m-3)$ .

## 6. ANALYSIS OF THE COMPACT COLLATZ SEQUENCE

From the above observations, it is clear that corresponding to every positive integer  $n$  as the starting initial ‘seed’ number, there is a starting initial node in the trajectory representing the *Compact Collatz Sequence*, that is a  $D^\#$  number in exactly one of the three possible forms as mentioned above - that can be an input argument to the U-ceiling operator  $U^\#$  giving exactly one unique output  $U^\#$  which itself can be an input to the D-floor operator  $D^\#$  so that the process continues. Successive application of each of these two operators ( $U^\#$  and  $D^\#$ ) wherever applicable, traces a unique *trajectory*, wherein each node represents a number that is the unique output number of the appropriate operation applied to the input number represented by the preceding node in the trajectory.

The anticipated terminating trivial cycle  $\{(4 \rightarrow 2 \rightarrow 1)\}$  can be obtained only through a final application of the D-floor operator  $D^\#$  on a 4MOD6 even number that is of the form  $(6m-2)$ .

## 7. BINARY-EXPONENTIAL-LADDER WITH ITS DEFINING-BASE-RUNG $D^\#$

Here, we present a meticulously designed *Structured System Framework* that *partitions* the *set of positive integers* to facilitate a *general systems analysis* of the *Compact Collatz Sequence*.

Let every positive odd number be associated with a *Binary-Exponential-Ladder*, denoted by  $BEL(2m-1)$  and defined as a sequence  $\{(2m-1).2^u\}$  with  $(u \geq 0)$ ; its *defining-base-rung* given by the odd number  $(2m-1)$ . Thus, we establish an exact

one-to-one mapping between the *set of positive odd numbers* that form the  $D^\#$  value for the *defining-base-rung* and the corresponding *Binary-Exponential-Ladder*  $BEL(D^\#)$ .

Every positive even number in the form  $(2m-1).2^u$  with  $(u>0)$ ; for which there exists its corresponding  $D^\#$  value,  $D^\#((2m-1).2^u) = (2m-1)$ ; for which there exists exactly one corresponding *Binary-Exponential-Ladder*  $BEL(2m-1)$  that contains the given even number  $(2m-1).2^u$  as one of its higher rungs in that  $BEL(2m-1)$  ladder.

Thus, we establish that *the set of all Binary-Exponential-Ladders form a partition of the set of all positive integers*; with an exact one-to-one correspondence between each positive odd number  $D^\#$  and the corresponding *Binary-Exponential-Ladder* for which it is the *defining-base-rung*  $D^\#$ ; whereas each of the positive even numbers correspond to exactly one of the higher rungs of a specific *Binary-Exponential-Ladder* identified by the  $D^\#$ -floor number  $D^\#$  associated with that given positive even number.

This partitioned framework of positive integers goes another step deeper because of the fact that the *defining-base-rung*  $D^\#$  of a *Binary-Exponential-Ladder*  $BEL(D^\#)$  can itself be in one of the three possible forms  $1 \text{MOD} 6$  or  $5 \text{MOD} 6$  or  $3 \text{MOD} 6$  whereas all the upper rungs of the *Binary-Exponential-Ladder* are either (1) alternately  $2 \text{MOD} 6$  and  $4 \text{MOD} 6$  or (2) all being  $0 \text{MOD} 6$  numbers.

The Collatz Conjecture states that every Collatz Sequence, starting from any positive integer, converges to the trivial cycle  $\{(4 \rightarrow 2 \rightarrow 1)\}$  which is in  $BEL(1)$  that is uniquely identified by its *defining-base-rung*  $D^\#$  value that is unity. Therefore, our focus will be the *Binary-Exponential-Ladders*  $BEL(1)$  and its relationship with every other *Binary-Exponential-Ladder*  $BEL(D^\#)$ .

As seen above,  $D^\#$  can be (1) either a  $1 \text{MOD} 6$  number of the form  $(6m-5)$ ; (2) or a  $5 \text{MOD} 6$  number of the form  $(6m-1)$ ; (3) or a  $3 \text{MOD} 6$  number of the form  $(6m-3)$ .  $BEL(6m-5)$  contains the output of  $U^\#$  at  $(6m-5)2^w$  with  $w$  being an even exponent that is of the form  $(2k)$  wherein the input of  $U^\#$  is given by  $\lceil \{(6m-5).2^w - 1\} / 3 \rceil$ .  $BEL(6m-1)$  contains the output of  $U^\#$  at  $(6m-1)2^v$  with  $v$  being an odd exponent that is of the form  $(2k-1)$  wherein the input of  $U^\#$  is given by  $\lceil \{(6m-1).2^v - 1\} / 3 \rceil$ . However,  $BEL(6m-3)$  cannot contain any such output of the  $U^\#$ -ceiling operator  $U^\#$  irrespective of any input argument.

## 8. IMMEDIATE NEIGHBORHOOD OF A BINARY-EXPONENTIAL-LADDER

The relationship between a pair of *Binary-Exponential-Ladders*  $BEL(m)$  and  $BEL(n)$  can be considered to be defined and characterized by the relationship between the corresponding pair of the *defining-base-rung*  $D^\#$  values  $m$  and  $n$  along with the corresponding pair  $U^\#(m)$  and  $U^\#(n)$ .

The immediate-neighborhood of a given *Binary-Exponential-Ladder*  $BEL(D^\#)$  is defined by the immediate-predecessors and immediate-successors, considering

the U-ceiling operator  $U^\#$ ; since the D-floor operator  $D^\#$  is applicable only within a given Binary-exponential-Ladder and not between a pair of them.

It turns out that the *only one single unique immediate successor* of  $BEL(m)$  is  $S(BEL(m)) = BEL(D^\#(U^\#(m)))$  that contains  $U^\#(m)$  as one of its higher rungs, with its identifying characteristic D-floor number  $D^\#(U^\#(m))$  as its defining-base-rung. However, there exists a *set of immediate-predecessors* for each  $BEL(D^\#)$  of the form  $BEL(6m-5)$  and  $BEL(6m-1)$  although none for  $BEL(6m-3)$ .

$BEL(1 \text{ MOD } 6)$  or equivalently  $BEL(6m-5)$  has, as its *set of immediate-predecessors*,  $\{BEL([(1 \text{ MOD } 6).2^w - 1]/3)\}$  or equivalently  $\{BEL([(6m-5).2^w - 1]/3)\}$  with  $w$  being an positive even exponent of the form  $(2k)$ , wherein the input of  $U^\#$  is given by  $\{[(1 \text{ MOD } 6).2^w - 1]/3\}$  or equivalently  $\{[(6m-5).2^w - 1]/3\}$  and the output of  $U^\#$  being  $\{(1 \text{ MOD } 6).2^w\}$  or equivalently  $\{(6m-5).2^w\}$  that is contained in  $BEL(1 \text{ MOD } 6)$  or equivalently  $BEL(6m-5)$ .

$BEL(5 \text{ MOD } 6)$  or equivalently  $BEL(6m-1)$  has, its *set of immediate-predecessors*,  $\{BEL([(5 \text{ MOD } 6).2^v - 1]/3)\}$  or equivalently  $\{BEL([(6m-1).2^v - 1]/3)\}$  with  $v$  being a positive odd exponent of the form  $(2k-1)$ , wherein the input of  $U^\#$  is given by  $\{[(5 \text{ MOD } 6).2^v - 1]/3\}$  or equivalently  $\{[(6m-1).2^v - 1]/3\}$  and the output of  $U^\#$  being  $\{(5 \text{ MOD } 6).2^v\}$  or equivalently  $\{(6m-1).2^v\}$  that is contained in  $BEL(5 \text{ MOD } 6)$  or equivalently  $BEL(6m-1)$ .

The above observed property, that *only* the alternating rungs, defined by  $(1 \text{ MOD } 6).4^u$  of  $BEL(1 \text{ MOD } 6)$  or  $(5 \text{ MOD } 6).2.4^u$  of  $BEL(5 \text{ MOD } 6)$  being the 'active' nodes in the CHSUK-Sequence; naturally makes it convenient to define a system of *Quarternary-Exponential-Ladders* QEL wherein every rung of QEL becomes an 'active' node in the CHSUK-Sequence. This concept is not directly needed for proving the convergence of the Collatz Sequence, and therefore we will not take up this line of study in this research report.

Considering  $BEL(1)$  as our central focus of interest, which itself belongs to the type  $BEL(1 \text{ MOD } 6)$  or equivalently  $BEL(6m-5)$ ; it is interesting to note that it has its *single unique immediate-successor* as  $S(BEL(1)) = BEL(D^\#(U^\#(1))) = S(BEL(1))$  that is  $BEL(1)$  itself because of the trivial cycle  $\{(4 \rightarrow 2 \rightarrow 1)\}$  contained within  $BEL(1)$ .

## 9. STRUCTURED SYSTEM FRAMEWORK OF BINARY-EXPONENTIAL-LADDERS

From the above discussion we find that it is convenient for our study to consider a *Structured System Framework*  $H$  as a sequence of sets  $\{ \dots H_{k-1} < H_k < H_{k+1} \dots \}$  wherein the *unique ordering relationship* between the adjacent terms of the sequence is derived from the uniqueness characteristic of the *immediate-successor* relationship among the BELs that form these adjacent terms in the sequence. Specifically,  $H_{k-1}$  is defined as the set formed by the unique immediate-successor of each BEL belonging to  $H_k$  and  $H_k$  is defined as the set formed by the unique immediate-successor for each BEL belonging to  $H_{k+1}$  etc.



$$H_{k-1} := \{S(BEL(m)) \mid BEL(m) \in H_k\} \text{ and } H_k := \{S(BEL(m)) \mid BEL(m) \in H_{k+1}\} \quad [\text{Eqn.3}]$$

Now, we may as well define the predecessor relationship as the inverse of the above defined successor relationship, as –

$$H_{k-1} := S(H_k) \quad \text{and} \quad H_k := S(H_{k+1}) \quad [\text{Eqn.4}]$$

and

$$P(H_{k-1}) := H_k \quad \text{and} \quad P(H_k) := H_{k+1} \quad [\text{Eqn.5}]$$

However, the multiplicity of the *immediate-predecessor* relationship among the BELs requires that the set of all immediate-predecessors of every element of  $H_{k-1}$  form the elements of the set  $H_k$  so as to guarantee the strict and complete ordering relation  $H_{k-1} < H_k < H_{k+1}$  among these sets, in spite of only a partial ordering relationship among the BELs.

Note that this *structured system framework*  $H = \{ \dots H_{k-1} < H_k < H_{k+1} \dots \}$  is a *connected digraph* that is like a linear directed path (chain) with no forking but merging only at the deeper BEL level, because of the above-mentioned successor predecessor relationships between the BEL elements that are members of the adjacent terms in the sequence, and that H is merely a well-organized system structure framework meticulously designed to contain the entire set of all the Binary-Exponential-Ladders.

As observed earlier in the previous section,  $BEL(1)$  itself is its single unique immediate-successor and does not have any immediate-successor distinct from itself, although it has multiple immediate-predecessors. Therefore, the above sequence H must necessarily have a term  $H_0$  with  $BEL(1)$  being its singleton element, thus forcing the sequence H to be truncated from below at  $H_0$  - the modified and updated sequence becomes  $H = \{H_0, H_1, H_2, \dots\}$  which we consider as the defining structure for the *Structured System Framework* H. To be precise,  $H_{k-1}$  contains the unique immediate-successor of each of the BEL in  $H_k$  but none else, whereas  $H_k$  contains all the immediate-predecessors of each of the BEL in  $H_{k-1}$  but none else, for every  $k > 0$ .

From the above discussion we observe that the *Structured System Framework*  $H = \{H_0, H_1, H_2, \dots\}$  is an infinite sequence of terms, each term being a countably infinite set of *Binary Exponential Ladders* except its 'root'  $H_0 = \{BEL(1)\}$  being a singleton set. The set of  $k^{\text{th}}$  immediate predecessors of  $BEL(1)$  form the set  $H_k$  at tier-k level in the hierarchy, if one wishes to consider it as a hierarchy.

This Structured System Framework H of Binary Exponential Ladders has a direct one-to-one correspondence (mapping) with the set of positive integers, considering the distinctly specific rungs of each of the Binary Exponential Ladders; the lowest rung in each  $BEL(D^{\#})$  being the *defining-base-rung*  $D^{\#}$  that is mapped

to the corresponding odd number and each of the higher rungs being mapped to the corresponding even number.

Note that there is a strict and complete ordering relation  $H_{k-1} < H_k < H_{k+1}$  between the terms in the sequence or the different levels of the hierarchy or the tier levels, because of the above-mentioned successor predecessor relationships between them, and a clear idempotent element  $H_0$  which is its own successor.

The (1) strict and complete ordering in hierarchy of  $H$  with a single unique root node  $H_0$ ; with the above mentioned (2) successor predecessor relationships between the Binary-Exponential-Ladders that form the elements of  $H_k$  at tier level  $k$  and those of  $H_{k-1}$  at tier level  $k-1$  in the hierarchy; and the fact that (3) there is an exact one-to-one correspondence between the set of positive integers and the set of all rungs in all the Binary-Exponential-Ladders; imply that - the *Structured System Framework*  $H$  has been meticulously designed to represent a hierarchy (an *arborescence*) of Binary-Exponential-Ladders, wherein there is *exactly one single unique directed path (chain)* with no forking but merging only at the deeper BEL level, from *every positive integer to the trivial cycle that is at the defining base rung of BEL(1)*.

## 10. COLLATZ-HASSE-SYRACUSE-ULAM-KAKUTANI (CHSUK) THEOREM

### STATEMENT OF THE CHSUK THEOREM

The CHSUK Sequence converges to the trivial cycle  $\{(4 \rightarrow 2 \rightarrow 1)\}$ .

### PROOF

We show that the *Structured System Framework*  $H$  by its very design, satisfies the Dedekind-Peano's axioms (replacing the 'successor' by the 'predecessor') and therefore  $H$  is isomorphic with the set of natural numbers; and satisfies the above stated convergence statement.

#### DEDEKIND-PEANO AXIOM : Existence of 0.

$H_0 \in H$ .

The trivial cycle  $\{(4 \rightarrow 2 \rightarrow 1)\}$  is contained in  $H_0 \in H$ .

#### DEDEKIND-PEANO AXIOM : Existence of a *successor function*.

By the very design of  $F$ , for every positive integer  $k$ ,

$H_k \in H$  is the *predecessor* of  $H_{k+1} \in H$ .

Application of the Collatz Function with the input from numbers in  $H_k$  yield the output contained in  $H_{k+1}$ .

#### DEDEKIND-PEANO AXIOM : 0 is not a successor.



$H_0$  is its own predecessor. However, there *does not exist any*  $H_k \in H$ ,  $k \neq 0$ ; that is distinct from  $H_0$ ; with  $H_k \neq H_0$ ; such that  $H_0$  is the predecessor of  $H_k$ . Once the Collatz Sequence reaches the trivial cycle (sink) there is no exit from it.

**DEDEKIND-PEANO AXIOM : Successor function is a unique one-to-one mapping.**

If  $H_u$  is the predecessor of  $H_v$  and also  $H_u$  is the predecessor of  $H_w$ ;  
then it necessarily implies  $H_v = H_w$  by the very design of  $H$ ;

and,

If  $H_v$  is the predecessor of  $H_u$  and also  $H_w$  is the predecessor of  $H_u$ ;  
then it necessarily implies  $H_v = H_w$  by the very design of  $H$ ;

This is because the predecessor relation in  $H$  is a unique one-to-one mapping (bijection).

Also, note that for each positive integer  $k$  there corresponds a unique set  $H_k \in H$ , and for each  $H_k \in H$  there corresponds a unique positive integer  $k$ ; thus, establishing a *one-to-one mapping (bijection) between the set of positive integers and the Structured System Framework  $H$ .*

This guarantees the Compact Collatz Sequence to be a linear directed path (chain) with no forking or merging in  $H$  (although merging is observed deeper at the level of the BELs) and the path traces through  $\dots H_{k+1}$  onto  $H_k$  onto  $H_{k-1} \dots$  etc in that order, each of these terms in  $H$  forming a hypernode in the CHSUK-Sequence.

**DEDEKIND-PEANO AXIOM : Principle of induction.**

Collatz Sequence starting with numbers from  $H_0$  converge in the trivial cycle that is contained in  $H_0$ .

Collatz Sequence starting with any positive number that passes through  $H_k$  must necessarily pass through  $H_{k-1}$  because by design  $H_{k-1} := S(H_k)$ .

Therefore, the Collatz Sequence starting with any positive integer being contained in some  $H_k \in H$ ,  $k \geq 0$ ; must necessarily reach  $H_0$  and therefore converge in the trivial cycle.

Thus, we establish a direct isomorphism between the *Structured System Framework  $H$*  and the set of Natural Numbers  $N$ ; and the proof of convergence of the Collatz Sequence is an immediate consequence of this isomorphism according to the property of induction as mentioned above.

END OF PROOF

## 11. SOME EXPLICIT FORMS FOR THE BEL-NEIGHBORHOOD

We can perform some simple algebraic manipulation to get the parametric relation [Eqn.6] that gives a generic form for the set  $H_k$  that is the set of  $k^{\text{th}}$  predecessors of  $H_0 = \{\text{BEL}(1)\}$ ; that is, the set  $H_k$  corresponds to the set of tier- $k$  level of the

hierarchy with the set of Binary-Exponential-Ladders  $\{BEL(m)\}$  each with its defining-base-rung  $m$  being a positive odd number  $m > 1$ .

$$m = [2^z - \{3^0 \cdot 2^{z_0} + 3^1 \cdot 2^{z_1} + 3^2 \cdot 2^{z_2} + \dots + 3^{k-1} \cdot 2^{z_{k-1}}\}] / 3^k \quad [\text{Eqn.6}]$$

wherein  $k > 0$  is the tier-level whereas  $z > 0$  and the  $k$ -tuple  $(z_0, z_1, z_2, \dots, z_{k-1})$  form the set of non-negative integer exponents in [Eqn.6] each of which takes a unique value corresponding to each **positive odd number  $m > 1$** . That is, each positive odd number  $m > 1$  can be considered to be defined by the corresponding unique set of these parameters. Here the set of values for the  $k$ -tuple  $(z_0, z_1, z_2, \dots, z_{k-1})$  are of decreasing values all less than  $z$ ; ( $z_k := 0$ ;  $z_{k-1} = 0$  for positive odd number  $m > 1$ ).

Now, define  $p_0 := (z - z_0)$ ;  $p_j := (z_{j-1} - z_j)$ ; where  $p_j$  corresponds to the number of rungs in  $BEL\{H_j\}$  above the *defining-base-rung* of  $BEL\{H_j\}$  for the node located in  $BEL\{H_j\}$  that the Collatz sequence/trajectory passes through;  $BEL\{H_j\}$  being the *Binary-Exponential-Ladder* at tier- $j$  with  $j=0,1,2, \dots, k$ . Thus, we may as well redefine the set of  $(k+1)$  parameters as a **tuple  $(P_k) := (p_0, p_1, p_2, \dots, p_k)$**  the set of  $(k+1)$  **CHSUK generative parameters** that generate each positive integer  $n$  as per the parametric relation [Eqn.6] given above ( $p_k = 0$  for positive odd number  $m$ ).

For any  $k > 0$ , the above set of exponents  $z, z_0, z_1, z_2, z_3, \dots, z_k$ , can be redefined in terms of the newly defined **CHSUK generative parameters**, by rewriting the above definition as  **$z := (z_0 + p_0)$ ;  $z_{j-1} := (z_j + p_j)$ ;  $z_k := 0$ ;  $p_k = 0$  for positive odd number  $m$** .

Table-1 gives some of the possible set of valid **CHSUK generative parameters** and therefore the corresponding valid values of the exponents in [Eqn.6] above along with the resultant  **$n(P_k) := n(p_0, p_1, p_2, \dots, p_k)$**  values. Note that the set of valid values for the CHSUK generative parameters and therefore for the exponents in [Eqn.6] above are governed by certain rules as can be seen from the earlier observations above, regarding the matching relationship between the  $((D^\#) \text{MOD} 3)$  of the predecessor and the  $((U^\#) \text{MOD} 3)$  of the successor in the CHSUK Sequence.

Table-1 : Some typical CHSUK generative parameter tuples																	
p0	p1	p2	p3	p4	p5	p6		n		z	z0	z1	z2	z3	z4	z5	z6
0								1		2	0						
1								2									
2								4									
3								8									
4								16									
4	0							5		4	0	0					
4	1	0						3		5	1	0	0				
4	3	0						13		7	3	0	0				
4	5	0						85		9	5	0	0				
4	3	2	0					17		9	5	2	0	0			
4	3	2	1	0				11		10	6	3	1	0	0		
4	3	2	1	1	0			7		11	7	4	2	1	0	0	
4	3	2	1	1	2	0		9		13	9	6	4	3	2	0	0
Table-1 : Some typical CHSUK generative parameter tuples																	

## 12. A CHALLENGE TO MY COOL-HEADED BRAVE-HEARTS

If you can prove that corresponding to every positive odd number  $m > 1$  there exists a unique valid set of CHSUK generative parameters  $\{p_0, p_1, p_2, \dots, p_k\}$  and therefore the corresponding valid set of exponents  $\{z, z_0, z_1, z_2, z_3, \dots, z_{k-1}, z_k\}$  in the parametric equation [Eqn.6] given above that generates every positive odd number  $m > 1$ , then you can directly prove the CHSUK Conjecture establishing the convergence of the CHSUK Sequence to the trivial cycle  $\{(4 \rightarrow 2 \rightarrow 1)\}$ .

## 13. CONCLUSION

We have presented a meticulously designed *structured system framework* of *Binary-Exponential-Ladders* and established its isomorphism with the set of positive integers, that directly leads to a simple and elegant proof of the convergence of the CHSUK Sequence.

## 14. RECOMMENDED READING

- [1]. Wikipedia Page – [https://en.wikipedia.org/wiki/Collatz\\_conjecture](https://en.wikipedia.org/wiki/Collatz_conjecture)
- [2]. Jeffrey C Lagarias;  
“The  $3x+1$  problem: An annotated bibliography (1963--1999) (sorted by author)”;  
<https://arxiv.org/abs/math/0309224>
- [3]. Jeffrey C Lagarias;  
“The  $3x+1$  Problem: An Annotated Bibliography, II (2000-2009)”;  
<https://arxiv.org/abs/math/0608208>
- [4]. Jeffrey C Lagarias;  
“The  $3x + 1$  Problem : An Overview”  
<https://arxiv.org/abs/2111.02635>
- [5]. Halemane, K. P. (2014);  
“Unbelievable  $O(L^{1.5})$  worst case computational complexity achieved by *spdspds* algorithm for linear programming problem”;  
<https://arxiv.org/abs/1405.6902> (2025).
- [6]. Halemane, K. P. (2025);  
“Monty-Hall Theorem”;  
<https://engrxiv.org/preprint/view/5594>

## 15. ACKNOWLEDGEMENT

I acknowledge the fact that the most revered Number-Theory Expert Paul Erdos once said about the Collatz Conjecture - "Mathematics is not yet ready for such problems" as quoted by Jeffrey Lagarias [4].

I must necessarily confess here that the *core idea behind this analysis is so stunningly & elusively simple*, that one may simply be taken aback in a profound wonder-struck jaw-drop-silence, maybe with an after-thought: "*oh my goodness, how could it be that it never flashed on me any time earlier*"! as was also the case in earlier research reports [5]&[6].

## 16. DEDICATION

To my ಅಜ್ಜಿ (ajja) Karinja Halemane Keshava Bhat & ಅಜ್ಜಿ (ajji) Thirumaleshwari, ಅಪ್ಪ (appa) Shama Bhat & ಅಮ್ಮ (amma) Thirumaleshwari, for their *teachings through love*, that *quality matters more than quantity*; to my wife Vijayalakshmi for her *ever consistent love & support*; to my daughter [Sriwidya.Bharati](#) and my twin sons [Sriwidya.Ramana](#) & [Sriwidya.Prawina](#) for their *love & affection*.

Whereas [this Original Author-Creator](#) holds the (PIPR:©:) Perpetual Intellectual Property Rights, his legal heirs (three children mentioned above) may avail the same for perpetuity.

To all the *cool-headed brave-hearts*, eagerly awaited but probably yet to be visible among the world professionals, especially the *Subject-Matter-Experts*, who would be attracted to and certainly capable of effectively understanding without any prejudice and appreciating the deeper insights enshrined in this research report.

ॐ तत्सत्