

ALGORITHM FOR THE CLASSICAL MONTY-HALL PROBLEM

Consider the standard classical Monty-Hall Problem as reported widely in the literature - with a prize hidden behind one of the three doors; a guest making a choice of the door (say, door-1) to pick the prize; the host who knows the location of the prize as well as the choice made by the guest, now *reveals a distinctly different yet a losing choice* (say, door-3). The host also offers the guest, an option to switch from the initial choice to the now available second choice, anticipating an enhanced chance of winning the prize, based on the knowledge obtained about a losing choice.

Let $x_r \in \{1,2,3\}$ be the door r behind which the prize x is hidden. Let $y_p \in \{1,2,3\}$ be the initial choice p of the guest y . Let $z \in \{1,2,3\}$ be the door q opened by the host z to show a losing choice. Also, x_r and y_p are mutually independent; but z_q is dependent on both y_p and x_r , that is, $z_q \neq (y_p, x_r)$. Let the symbol a_i denotes the event $[E\{a=i\}]$ for any 'agent' $a \in \{x,y,z\}$ and 'door' $i \in \{r,p,q\} = \{1,2,3\}$.

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There are exactly four *mutually-exclusive together-exhaustive* eventualities wherein Monty would open door-3 $[z3]$. They are listed below, with their probabilities:

(1) initial choice of door-1 by the guest $[y1]$ with the prize hidden behind door-1 $[x1]$; that is, $P_{113} = P[x1y1 | z3] = 1/6$;

(2) initial choice of door-2 by the guest $[y2]$ with the prize hidden behind door-1 $[x1]$; that is, $P_{123} = P[x1y2 | z3] = 1/3$;

(3) initial choice of door-1 by the guest $[y1]$ with the prize hidden behind door-2 $[x2]$; that is, $P_{213} = P[x2y1 | z3] = 1/3$;

(4) initial choice of door-2 by the guest $[y2]$ with the prize hidden behind door-2 $[x2]$; that is, $P_{223} = P[x2y2 | z3] = 1/6$;

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My approach is the following -

Adding (1) & (2) we get the probability of having the prize hidden behind door-1 $[x1]$ given that Monty had opened door-3 $[z3]$;

that is, $P[x1 | z3] = P_{113} + P_{123} = 1/2$;

Adding (3) & (4) we get the probability of having the prize hidden behind door-2 $[x2]$ given that Monty had opened door-3 $[z3]$;

that is, $P[x2 | z3] = P_{213} + P_{223} = 1/2$;

thus leading to the recommendation that there is neither any gain nor any loss in terms of the chances of winning the prize, irrespective of whether we opt to switch or not.

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INPUT DATA:

$P[x_r]; P[y_p]; P[z_q | \{x_r \& y_p\}];$

PROCESSING:

$P[\{x_r \& y_p\}] = P[x_r] * P[y_p];$

$P[\{z_q \& x_r \& y_p\}] = P[z_q | \{x_r \& y_p\}] * P[\{x_r \& y_p\}];$

$P[z_q | x_r] = \text{marginalized } P[\{z_q \& x_r \& y_p\}] \text{ over } y_p;$

$P[z_q] = \text{marginalized } P[\{z_q \& x_r \& y_p\}] \text{ over } [y_p \& x_r];$

$P[x_r | z_q] = [(P[z_q | x_r] * P[x_r])] / [(P[z_q])];$

OUTPUT:

$P[x_r | z_q];$