CLASSICAL MONTY-HALL PROBLEM

Consider the standard classical Monty-Hall Problem as reported widely in the literature - with a prize hidden behind one of the three doors; a guest making a choice of the door (say, door-1) to pick the prize; the host who knows the location of the prize as well as the choice made by the guest, now *reveals a distinctly different yet a losing choice* (say, door-3). The host also offers the guest, an option to switch from the initial choice to the now available second choice, anticipating an enhanced chance of winning the prize, based on the knowledge obtained about a losing choice.

Let xr$\in$\{1,2,3\} be the door r behind which the prize x is hidden. Let yp$\in$\{1,2,3\} be the initial choice p of the guest y. Let z$\in$\{1,2,3\} be the door q opened by the host z to show a losing choice. Also, xr and yp are mutually independent; but zq is dependent on both yp and xr, that is, zq $\neq$ (yp, xr). Let the symbol ai denotes the event [E\{(a=i)\}] for any 'agent' a$\in$\{x,y,z\} and 'door' i$\in$\{r,p,q\}=\{1,2,3\}.

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There are exactly four *mutually-exclusive together-exhaustive* eventualities
wherein Monty would open door-3 [z3]. They are listed below, with their probabilities:
(1) initial choice of door-1 by the guest [y1] with the prize hidden behind door-1 [x1];
that is, P113 = P[x1y1 | z3] = 1/6;
(2) initial choice of door-2 by the guest [y2] with the prize hidden behind door-1 [x1];
that is, P123 = P[x1y2 | z3] = 1/3;
(3) initial choice of door-1 by the guest [y1] with the prize hidden behind door-2 [x2];
that is, P213 = P[x2y1 | z3] = 1/3;
(4) initial choice of door-2 by the guest [y2] with the prize hidden behind door-2 [x2];
that is, P223 = P[x2y2 | z3] = 1/6;

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My approach is the following -
Adding (1) & (2) we get the probability of having the prize hidden behind door-1 [x1]
given that Monty had opened door-3 [z3];
that is, P[x1 | z3] = P113 + P123 = 1/2;
Adding (3) & (4) we get the probability of having the prize hidden behind door-2 [x2]
given that Monty had opened door-3 [z3];
that is, P[x2 | z3] = P213 + P223 = 1/2;
thus leading to the recommendation that there is neither any gain nor any loss in terms
of the chances of winning the prize, irrespective of whether we opt to switch or not.

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I can explain to you if you need such explanations, as to why any other combinations among the above four eventualities, doesn't make any meaningful sense from the point of view of the guest who is faced with the decision-making question as to which of the now available two doors that he must choose, door-1 or door-2 (y1 xor y2) that is, whether to stay with the initial choice of door-1 (y1) or to switch to door-2 (y2).
For example, comparing P113 with P213 or comparing (P113+P223) with (P123+P213) etc. leads to serious error, as committed by the Leading Subject-Matter-Experts of those days – NOT YOU!
It is as if Savant had cast an enchantingly deep spell over the Frequentists who in turn pushed the Probabilists to mistake Bayes, leading to those errors of commission and/or errors of omission, also riddled with Logical Fallacy. We need to come out of that long drawn intellectual hibernation of over six decades, and wake up to reality.

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This is the central crux of my reasoning in the refutation of the logical fallacy (and the erroneous application of the Bayes Theorem) committed by the subject-matter-experts of those days on the Monty-Hall Problem.

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