spline_interpolations

April 18, 2024

1 Spline Interpolations of a given analytic function

1.1 0. Definitions of Plotting parameters

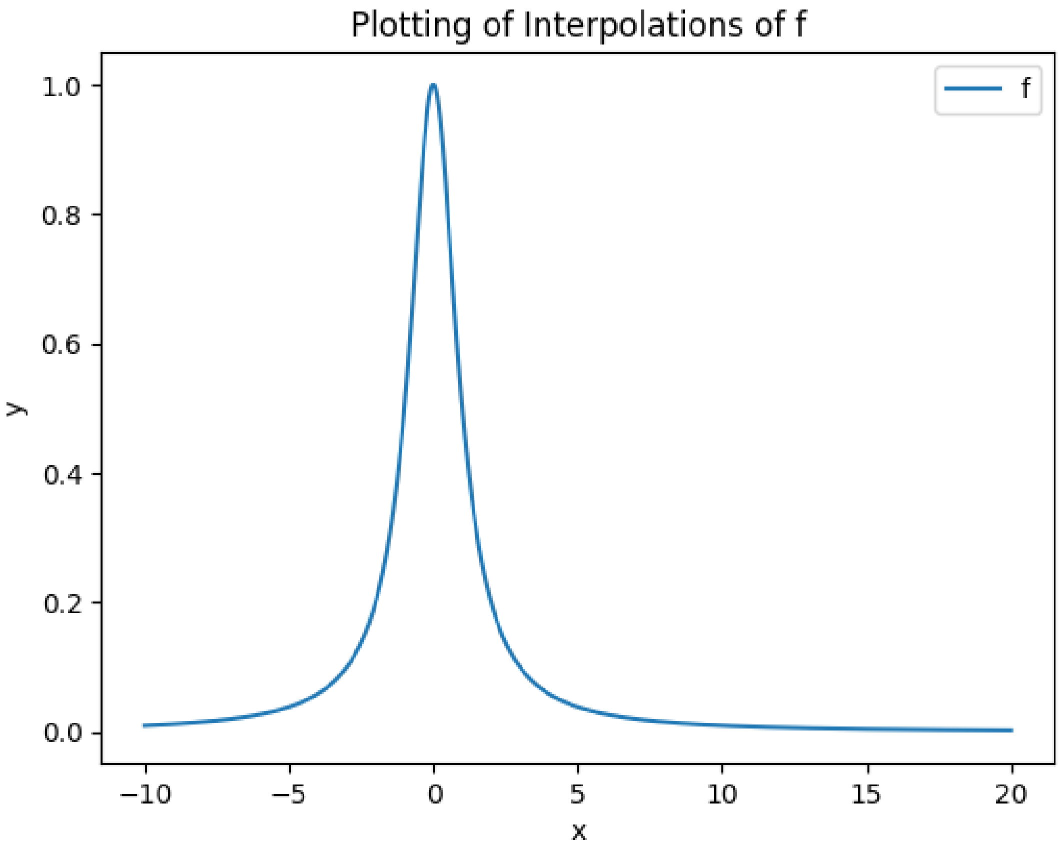
[ ]:

```python
import numpy as np

a, b, n_plot = -10, 20, 1000
x_plot = np.linspace(a, b, n_plot)
# print("x_plot =", x_plot)
```

1.2 1. Definition of f

[ ]:

```python
from utils import *

# f_exp = "cos(x)" # "1/(1+x**2)"
# def f(x):
    # return eval(f_exp, {"x": x})

f = lambda x: 1/(1+x**2)

fig, ax = set_fig()
plot_f(ax, f, x_plot)
```



1.3 2. Definition of Interpolation parameters

[ ]:

```
n = 40

# Defintion of Uniforms points
x_uniform = np.linspace(a, b, n)
y_uniform = [f(x) for x in x_uniform]
print("Uniforms points")
print("x_uniform =", x_uniform)
print("\ny_uniform =", y_uniform)
```

```
Uniforms points
x_uniform = [-10.          -9.23076923  -8.46153846  -7.69230769  -6.92307692
  -6.15384615  -5.38461538  -4.61538462  -3.84615385  -3.07692308
  -2.30769231  -1.53846154  -0.76923077   0.           0.76923077
   1.53846154   2.30769231   3.07692308   3.84615385   4.61538462
   5.38461538   6.15384615   6.92307692   7.69230769   8.46153846
   9.23076923  10.          10.76923077  11.53846154  12.30769231
  13.07692308  13.84615385  14.61538462  15.38461538  16.15384615
  16.92307692  17.69230769  18.46153846  19.23076923  20.        ]
```

```
y_uniform = [0.009900990099009901, 0.011599972544443685, 0.013774553753362133,
0.01661913659160193, 0.020437779658967224, 0.025726899071395953,
0.033339909252318015, 0.044839479968161323, 0.0633195953540652,
0.09553420011305824, 0.15809167446211417, 0.29701230228471004,
0.6282527881040898, 1.0, 0.6282527881040887, 0.29701230228471004,
0.15809167446211406, 0.09553420011305815, 0.06331959535406517,
0.04483947996816129, 0.03333990925231801, 0.025726899071395953,
0.020437779658967224, 0.016619136591601923, 0.013774553753362128,
0.011599972544443674, 0.009900990099009901, 0.008548737923010773,
0.007455114914641137, 0.006558267685979275, 0.005813753483091954,
0.005188983389112346, 0.004659626678430615, 0.004207224476586421,
0.003817569857010548, 0.0034795857439930823, 0.0031845333433831425,
0.0029254444425210746, 0.0026967081012941, 0.0024937655860349127]
```

1.4 3. Test of Linear Spline Interpolation

[ ]:

```python
from polynomial.newton_poly import  Spline1Poly

x = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
y = [1, 4, 9, 16, 25, 36, 49, 64, 81, 100]
print("x =", x)
print("y =", y)
polynomial = Spline1Poly(x, y)
print(polynomial)

x = 1.5
value = polynomial.horner_eval(x)
print(f"P({x}) = {value}")
```

```
x = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
y = [1, 4, 9, 16, 25, 36, 49, 64, 81, 100]
P(x) =   1.0 + 3.0 * (x - 1.0)   if x in [1.0, 2.0]
         4.0 + 5.0 * (x - 2.0)   if x in [2.0, 3.0]
         9.0 + 7.0 * (x - 3.0)   if x in [3.0, 4.0]
         16.0 + 9.0 * (x - 4.0)   if x in [4.0, 5.0]
         25.0 + 11.0 * (x - 5.0)   if x in [5.0, 6.0]
         36.0 + 13.0 * (x - 6.0)   if x in [6.0, 7.0]
         49.0 + 15.0 * (x - 7.0)   if x in [7.0, 8.0]
         64.0 + 17.0 * (x - 8.0)   if x in [8.0, 9.0]
         81.0 + 19.0 * (x - 9.0)   if x in [9.0, 10.0]

P(1.5) = 2.5
```

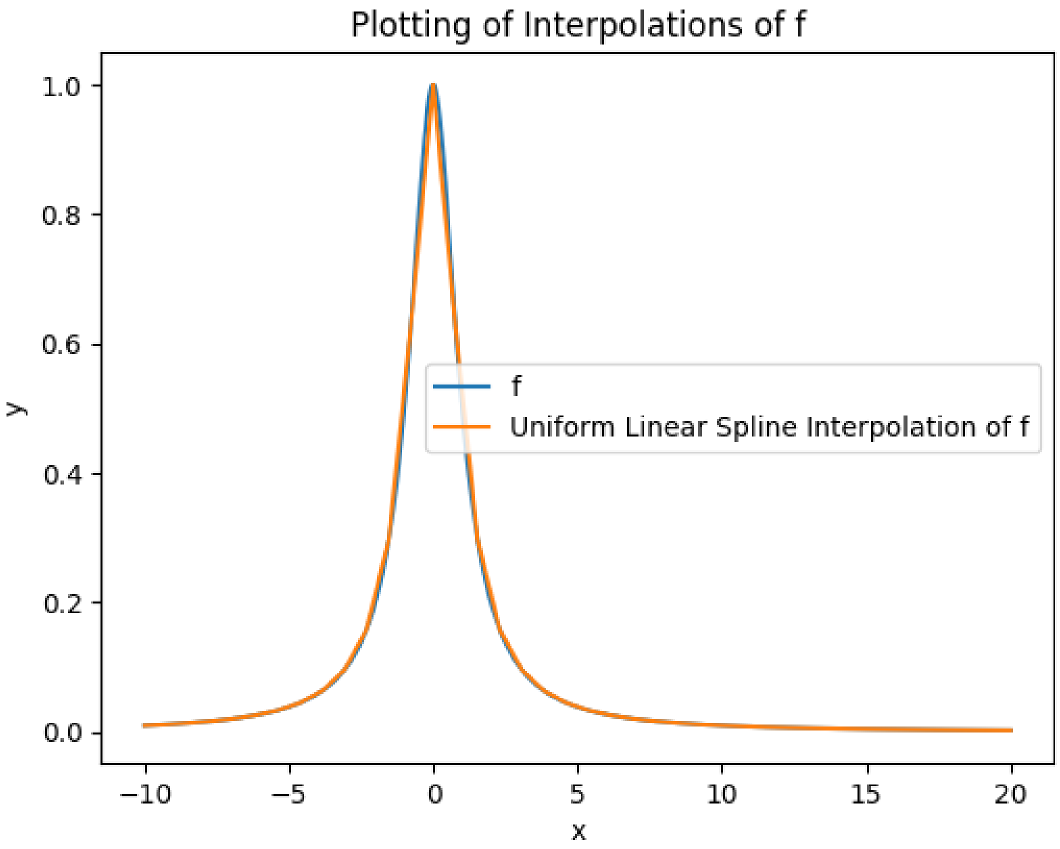
1.5 4. Uniform Linear Spline Interpolation of f

```
[ ]: uni_linear_spline_poly = Spline1Poly(x_uniform, y_uniform,␣
      ↪"Uni_linear_spline_poly")

     print(uni_linear_spline_poly)

     x0 = 1
     print(f"\nUni_linear_spline_poly({x0}) =", uni_linear_spline_poly.
      ↪horner_eval(x0))

     # print("\nx_uniform =", x_uniform)
     # print("\ny_uniform =", y_uniform)

     fig, ax = set_fig()
     plot_f(ax, f, x_plot)

     uni_linear_spline_poly.plot(ax, "Uniform Linear Spline Interpolation of f")
```

```
Uni_linear_spline_poly(x) =      0.009901 + 0.002209 * (x + 10.0)   if x in
[-10.0, -9.23076923076923]
         0.0116 + 0.002827 * (x + 9.230769)   if x in [-9.23076923076923,
-8.461538461538462]
         0.013775 + 0.003698 * (x + 8.461538)   if x in [-8.461538461538462,
-7.692307692307692]
         0.016619 + 0.004964 * (x + 7.692308)   if x in [-7.692307692307692,
-6.923076923076923]
         0.020438 + 0.006876 * (x + 6.923077)   if x in [-6.923076923076923,
-6.153846153846153]
         0.025727 + 0.009897 * (x + 6.153846)   if x in [-6.153846153846153,
-5.384615384615384]
         0.03334 + 0.014949 * (x + 5.384615)   if x in [-5.384615384615384,
-4.615384615384615]
         0.044839 + 0.024024 * (x + 4.615385)   if x in [-4.615384615384615,
-3.846153846153846]
         0.06332 + 0.041879 * (x + 3.846154)   if x in [-3.846153846153846,
-3.0769230769230766]
         0.095534 + 0.081325 * (x + 3.076923)   if x in [-3.0769230769230766,
-2.3076923076923075]
         0.158092 + 0.180597 * (x + 2.307692)   if x in [-2.3076923076923075,
-1.5384615384615383]
         0.297012 + 0.430613 * (x + 1.538462)   if x in [-1.5384615384615383,
-0.7692307692307683]
         0.628253 + 0.483271 * (x + 0.769231)   if x in [-0.7692307692307683,
0.0]
         1.0 + (-0.483271) * x   if x in [0.0, 0.76923076923077]
         0.628253 + (-0.430613) * (x - 0.769231)   if x in [0.76923076923077,
1.5384615384615383]
```

```
         0.297012 + (-0.180597) * (x - 1.538462)   if x in [1.5384615384615383,
2.3076923076923084]
         0.158092 + (-0.081325) * (x - 2.307692)   if x in [2.3076923076923084,
3.0769230769230784]
         0.095534 + (-0.041879) * (x - 3.076923)   if x in [3.0769230769230784,
3.8461538461538467]
         0.06332 + (-0.024024) * (x - 3.846154)   if x in [3.8461538461538467,
4.615384615384617]
         0.044839 + (-0.014949) * (x - 4.615385)   if x in [4.615384615384617,
5.384615384615385]
         0.03334 + (-0.009897) * (x - 5.384615)   if x in [5.384615384615385,
6.153846153846153]
         0.025727 + (-0.006876) * (x - 6.153846)   if x in [6.153846153846153,
6.923076923076923]
         0.020438 + (-0.004964) * (x - 6.923077)   if x in [6.923076923076923,
7.692307692307693]
         0.016619 + (-0.003698) * (x - 7.692308)   if x in [7.692307692307693,
8.461538461538463]
         0.013775 + (-0.002827) * (x - 8.461538)   if x in [8.461538461538463,
9.230769230769234]
         0.0116 + (-0.002209) * (x - 9.230769)   if x in [9.230769230769234,
10.0]
         0.009901 + (-0.001758) * (x - 10.0)   if x in [10.0, 10.76923076923077]
         0.008549 + (-0.001422) * (x - 10.769231)   if x in [10.76923076923077,
11.53846153846154]
         0.007455 + (-0.001166) * (x - 11.538462)   if x in [11.53846153846154,
12.30769230769231]
         0.006558 + (-0.000968) * (x - 12.307692)   if x in [12.30769230769231,
13.076923076923077]
         0.005814 + (-0.000812) * (x - 13.076923)   if x in [13.076923076923077,
13.846153846153847]
         0.005189 + (-0.000688) * (x - 13.846154)   if x in [13.846153846153847,
14.615384615384617]
         0.00466 + (-0.000588) * (x - 14.615385)   if x in [14.615384615384617,
15.384615384615387]
         0.004207 + (-0.000507) * (x - 15.384615)   if x in [15.384615384615387,
16.153846153846157]
         0.003818 + (-0.000439) * (x - 16.153846)   if x in [16.153846153846157,
16.923076923076923]
         0.00348 + (-0.000384) * (x - 16.923077)   if x in [16.923076923076923,
17.692307692307693]
         0.003185 + (-0.000337) * (x - 17.692308)   if x in [17.692307692307693,
18.461538461538463]
         0.002925 + (-0.000297) * (x - 18.461538)   if x in [18.461538461538463,
19.230769230769234]
         0.002697 + (-0.000264) * (x - 19.230769)   if x in [19.230769230769234,
20.0]
```

```
Uni_linear_spline_poly(1) = 0.5288806423582754
```



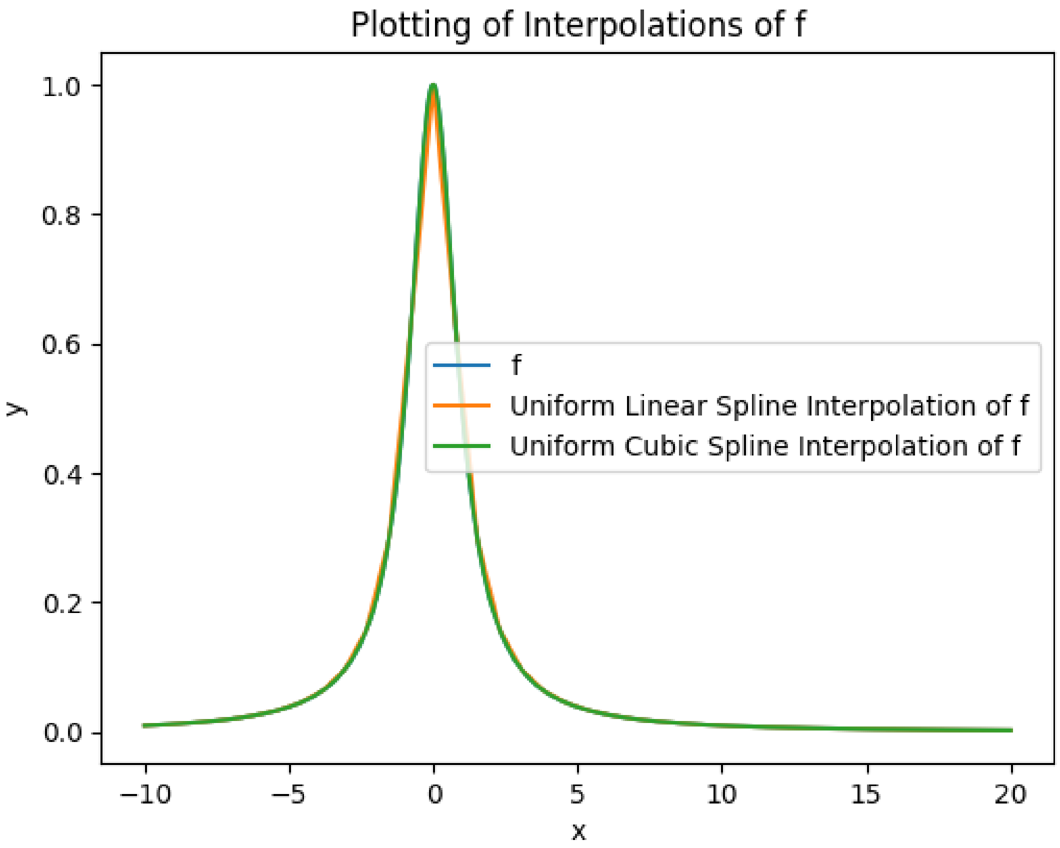
1.6 5. Uniform Cubic Spline Interpolation of f

[ ]:

```
from polynomial.taylor_poly import Spline3Polys

uni_spline3_poly = Spline3Polys(x_uniform, y_uniform, "Uni_spline3_poly")

print(uni_spline3_poly)

x0 = 1
print(f"\nUni_spline3_poly({x0}) =", uni_spline3_poly.horner_eval(x0))

fig, ax = set_fig()
plot_f(ax, f, x_plot)

# print("\nx_uniform =", x_uniform)
# print("\ny_uniform =", y_uniform)
```

```python
uni_linear_spline_poly.plot(ax, "Uniform Linear Spline Interpolation of f")
uni_spline3_poly.plot(ax, n_plot, "Uniform Cubic Spline Interpolation of f")
```

```
Uni_spline3_poly(x) =    0.009901 + 0.002088 * (x + 10.0) + 0.000204 * (x +
10.0)^3   if x in [-10.0, -9.23076923076923]
         0.0116 + 0.00245 * (x + 9.230769) + 0.00047 * (x + 9.230769)^2 +
2.6e-05 * (x + 9.230769)^3   if x in [-9.23076923076923, -8.461538461538462]
         0.013775 + 0.00322 * (x + 8.461538) + 0.000531 * (x + 8.461538)^2 +
0.000118 * (x + 8.461538)^3   if x in [-8.461538461538462, -7.692307692307692]
         0.016619 + 0.004246 * (x + 7.692308) + 0.000804 * (x + 7.692308)^2 +
0.000169 * (x + 7.692308)^3   if x in [-7.692307692307692, -6.923076923076923]
         0.020438 + 0.005782 * (x + 6.923077) + 0.001193 * (x + 6.923077)^2 +
0.000297 * (x + 6.923077)^3   if x in [-6.923076923076923, -6.153846153846153]
         0.025727 + 0.008145 * (x + 6.153846) + 0.001878 * (x + 6.153846)^2 +
0.000519 * (x + 6.153846)^3   if x in [-6.153846153846153, -5.384615384615384]
         0.03334 + 0.011956 * (x + 5.384615) + 0.003076 * (x + 5.384615)^2 +
0.001059 * (x + 5.384615)^3   if x in [-5.384615384615384, -4.615384615384615]
         0.044839 + 0.018569 * (x + 4.615385) + 0.005521 * (x + 4.615385)^2 +
0.002041 * (x + 4.615385)^3   if x in [-4.615384615384615, -3.846153846153846]
         0.06332 + 0.030687 * (x + 3.846154) + 0.010232 * (x + 3.846154)^2 +
0.005614 * (x + 3.846154)^3   if x in [-3.846153846153846, -3.0769230769230766]
         0.095534 + 0.056393 * (x + 3.076923) + 0.023187 * (x + 3.076923)^2 +
0.011992 * (x + 3.076923)^3   if x in [-3.0769230769230766, -2.3076923076923075]
         0.158092 + 0.113352 * (x + 2.307692) + 0.05086 * (x + 2.307692)^2 +
0.047526 * (x + 2.307692)^3   if x in [-2.3076923076923075, -1.5384615384615383]
         0.297012 + 0.275964 * (x + 1.538462) + 0.160535 * (x + 1.538462)^2 +
0.052661 * (x + 1.538462)^3   if x in [-1.5384615384615383, -0.7692307692307683]
         0.628253 + 0.616422 * (x + 0.769231) + 0.282061 * (x + 0.769231)^2 +
(-0.591704) * (x + 0.769231)^3   if x in [-0.7692307692307683, 0.0]
         1.0 + (-1.08341) * x^2 + 0.591704 * x^3   if x in [0.0,
0.7692307692307692307677]
         0.628253 + (-0.616422) * (x - 0.769231) + 0.282061 * (x - 0.769231)^2 +
(-0.052661) * (x - 0.769231)^3   if x in [0.76923076923077, 1.5384615384615383]
         0.297012 + (-0.275964) * (x - 1.538462) + 0.160535 * (x - 1.538462)^2 +
(-0.047526) * (x - 1.538462)^3   if x in [1.5384615384615383,
2.3076923076923084]
         0.158092 + (-0.113352) * (x - 2.307692) + 0.05086 * (x - 2.307692)^2 +
(-0.011992) * (x - 2.307692)^3   if x in [2.3076923076923084,
3.0769230769230784]
         0.095534 + (-0.056393) * (x - 3.076923) + 0.023187 * (x - 3.076923)^2 +
(-0.005614) * (x - 3.076923)^3   if x in [3.0769230769230784,
3.8461538461538467]
         0.06332 + (-0.030687) * (x - 3.846154) + 0.010232 * (x - 3.846154)^2 +
(-0.002041) * (x - 3.846154)^3   if x in [3.8461538461538467, 4.615384615384617]
         0.044839 + (-0.018569) * (x - 4.615385) + 0.005521 * (x - 4.615385)^2 +
(-0.001059) * (x - 4.615385)^3   if x in [4.615384615384617, 5.384615384615385]
         0.03334 + (-0.011956) * (x - 5.384615) + 0.003077 * (x - 5.384615)^2 +
(-0.00052) * (x - 5.384615)^3   if x in [5.384615384615385, 6.153846153846153]
        0.025727 + (-0.008145) * (x - 6.153846) + 0.001878 * (x - 6.153846)^2 +
(-0.000296) * (x - 6.153846)^3   if x in [6.153846153846153, 6.923076923076923]
        0.020438 + (-0.005782) * (x - 6.923077) + 0.001195 * (x - 6.923077)^2 +
(-0.000172) * (x - 6.923077)^3   if x in [6.923076923076923, 7.692307692307693]
        0.016619 + (-0.004249) * (x - 7.692308) + 0.000798 * (x - 7.692308)^2 +
(-0.000107) * (x - 7.692308)^3   if x in [7.692307692307693, 8.461538461538463]
        0.013775 + (-0.003211) * (x - 8.461538) + 0.000551 * (x - 8.461538)^2 +
(-6.9e-05) * (x - 8.461538)^3   if x in [8.461538461538463, 9.230769230769234]
        0.0116 + (-0.002484) * (x - 9.230769) + 0.000393 * (x - 9.230769)^2 +
(-4.6e-05) * (x - 9.230769)^3   if x in [9.230769230769234, 10.0]
        0.009901 + (-0.00196) * (x - 10.0) + 0.000287 * (x - 10.0)^2 +
(-3.1e-05) * (x - 10.0)^3   if x in [10.0, 10.76923076923077]
        0.008549 + (-0.001574) * (x - 10.769231) + 0.000215 * (x - 10.769231)^2
+ (-2.2e-05) * (x - 10.769231)^3   if x in [10.76923076923077,
11.53846153846154]
        0.007455 + (-0.001283) * (x - 11.538462) + 0.000164 * (x - 11.538462)^2
+ (-1.6e-05) * (x - 11.538462)^3   if x in [11.53846153846154,
12.30769230769231]
        0.006558 + (-0.001059) * (x - 12.307692) + 0.000127 * (x - 12.307692)^2
+ (-1.2e-05) * (x - 12.307692)^3   if x in [12.30769230769231,
13.076923076923077]
        0.005814 + (-0.000884) * (x - 13.076923) + 0.0001 * (x - 13.076923)^2 +
(-9e-06) * (x - 13.076923)^3   if x in [13.076923076923077, 13.846153846153847]
        0.005189 + (-0.000746) * (x - 13.846154) + 8e-05 * (x - 13.846154)^2 +
(-7e-06) * (x - 13.846154)^3   if x in [13.846153846153847, 14.615384615384617]
        0.00466 + (-0.000635) * (x - 14.615385) + 6.4e-05 * (x - 14.615385)^2 +
(-5e-06) * (x - 14.615385)^3   if x in [14.615384615384617, 15.384615384615387]
        0.004207 + (-0.000545) * (x - 15.384615) + 5.3e-05 * (x - 15.384615)^2
+ (-4e-06) * (x - 15.384615)^3   if x in [15.384615384615387,
16.153846153846157]
        0.003818 + (-0.000471) * (x - 16.153846) + 4.3e-05 * (x - 16.153846)^2
+ (-3e-06) * (x - 16.153846)^3   if x in [16.153846153846157,
16.923076923076923]
        0.00348 + (-0.00041) * (x - 16.923077) + 3.6e-05 * (x - 16.923077)^2 +
(-2e-06) * (x - 16.923077)^3   if x in [16.923076923076923, 17.692307692307693]
        0.003185 + (-0.000359) * (x - 17.692308) + 3.1e-05 * (x - 17.692308)^2
+ (-3e-06) * (x - 17.692308)^3   if x in [17.692307692307693,
18.461538461538463]
        0.002925 + (-0.000317) * (x - 18.461538) + 2.4e-05 * (x - 18.461538)^2
+ 1e-06 * (x - 18.461538)^3   if x in [18.461538461538463, 19.230769230769234]
        0.002697 + (-0.000277) * (x - 19.230769) + 2.7e-05 * (x - 19.230769)^2
+ (-1.2e-05) * (x - 19.230769)^3   if x in [19.230769230769234, 20.0]


Uni_spline3_poly(1) = 0.5003753458364024
```



1.7 6. Errors of SPline Interpolations of f

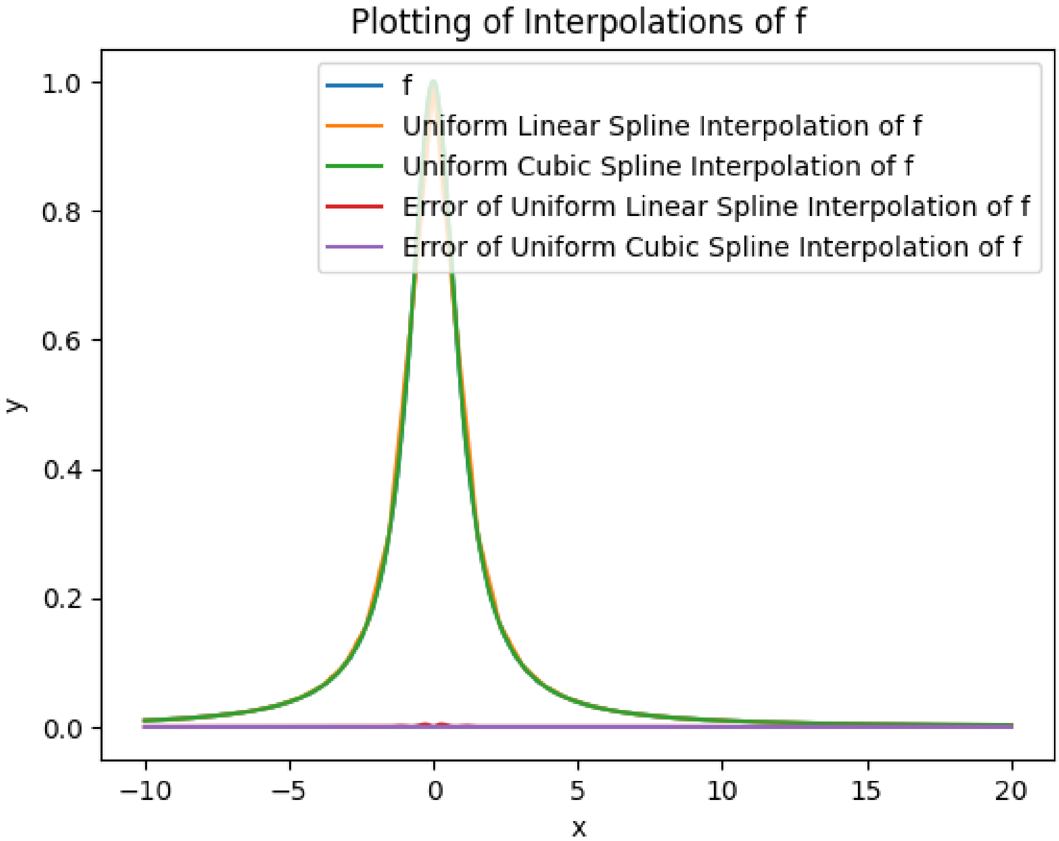
[ ]:

```python
from integration import gauss_integration

func_err_spline1 = lambda x: (f(x) - uni_linear_spline_poly.horner_eval(x))**2
func_err_spline3 = lambda x: (f(x) - uni_spline3_poly.horner_eval(x))**2
err_spline1 = sqrt(gauss_integration(func_err_spline1, a, b))
err_spline3 = sqrt(gauss_integration(func_err_spline3, a, b))
print("err_spline1 =", err_spline1)
print("err_spline3 =", err_spline3)

fig, ax = set_fig()

plot_f(ax, f, x_plot)
uni_linear_spline_poly.plot(ax, "Uniform Linear Spline Interpolation of f")
uni_spline3_poly.plot(ax, n_plot, "Uniform Cubic Spline Interpolation of f")

y_uni_plot = [func_err_spline1(x) for x  in x_plot]
```

```
ax.plot(x_plot, y_uni_plot, label="Error of Uniform Linear Spline Interpolation␣
 ↪of f")

y_tche_plot = [func_err_spline3(x) for x  in x_plot]
ax.plot(x_plot, y_tche_plot, label="Error of Uniform Cubic Spline Interpolation␣
 ↪of f")

ax.legend()
```

```
err_spline1 = 0.07612371912240237
err_spline3 = 0.0010423319658657137
```

```
[ ]: <matplotlib.legend.Legend at 0x7e7ec7355c90>
```


Plotting of Interpolations of f
f
Uniform Linear Spline Interpolation of f
Uniform Cubic Spline Interpolation of f
Error of Uniform Linear Spline Interpolation of f
Error of Uniform Cubic Spline Interpolation of f
y
x