Regression Discontinuity

EC 425/525, Set 9

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Prologue

Schedule

Last time

- Introduction to selection-on-unobservables designs
- Instrumental variables
- Two-stage least squares

Today

Regression discontinuity [†]

Upcoming

Midterm

† These notes largely follow notes from Michael Anderson and Imbens and Lemieux (2008).

Regression discontinuity

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Setup

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Regression discontinuity (**RD**) offers a particularly clear/clean research design based upon an arbitrary threshold (the *discontinuity*).

That said, most RDs boil down to an implementation of IV.

In addition, while RD is all the rage in modern applied econometrics, Thistlewaite and Campbell wrote about it back in 1960.

Our framework

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We will assume that $\mathbf{Y}_{0i}$ and $\mathbf{Y}_{1i}$ vary smoothly in $\mathbf{X}_i$.

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- An elector candidate wins if her vote share exceeds her competitors.
- Election runoffs are triggered if "winner" is below 50%.
- Antidiscrimination laws only apply to firms with >15 employees.
- Prisoners are eligible for early parole if some score exceeds a threshold.
- An individual is eligible for Medicare if her age is at least 65.
- You get a ticket if your speed exceeds the speed limit.
- Fifteen-percent discount at Sizzler if your age exceeds 60.
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In some cases, "treatment" is definite once we exceed the threshold.

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E.g., crossing some GRE threshold discontinuously increases your chances of getting into some grad schools (but doesn't guarantee admittance).

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To estimate the causal effect of $\mathbf{D}_i$ on $\mathbf{Y}_i$, we compare the mean of $\mathbf{Y}_i$ just *above* the threshold to the mean of $\mathbf{Y}_i$ just *below* the threshold.

More formally

We can write the comparison of means at the threshold as

$$\lim_{x \downarrow c} E[\mathrm{Y}_i \mid \mathrm{X}_i = x] - \lim_{x \uparrow c} E[\mathrm{Y}_i \mid \mathrm{X}_i = x]$$

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I.e., Because we don't observe Y_{0i} for treated individuals, we extrapolate $E[\mathrm{Y}_{0i} \mid \mathrm{X}_i = c - \varepsilon]$ to $E[\mathrm{Y}_{0i} \mid \mathrm{X}_i = x + \varepsilon]$ for small ε.

Sharp RDs

Estimation

Thus, we estimate

$$\tau_{\mathrm{SRD}} = \lim_{x \downarrow c} E[\mathrm{Y}_i - \mathrm{X}_i = x] - \lim_{x \uparrow c} E[\mathrm{Y}_i \mid \mathrm{X}_i = x]$$

as the diffrence between two regression functions estimated "near" c.

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as the diffrence between two regression functions estimated "near" c.

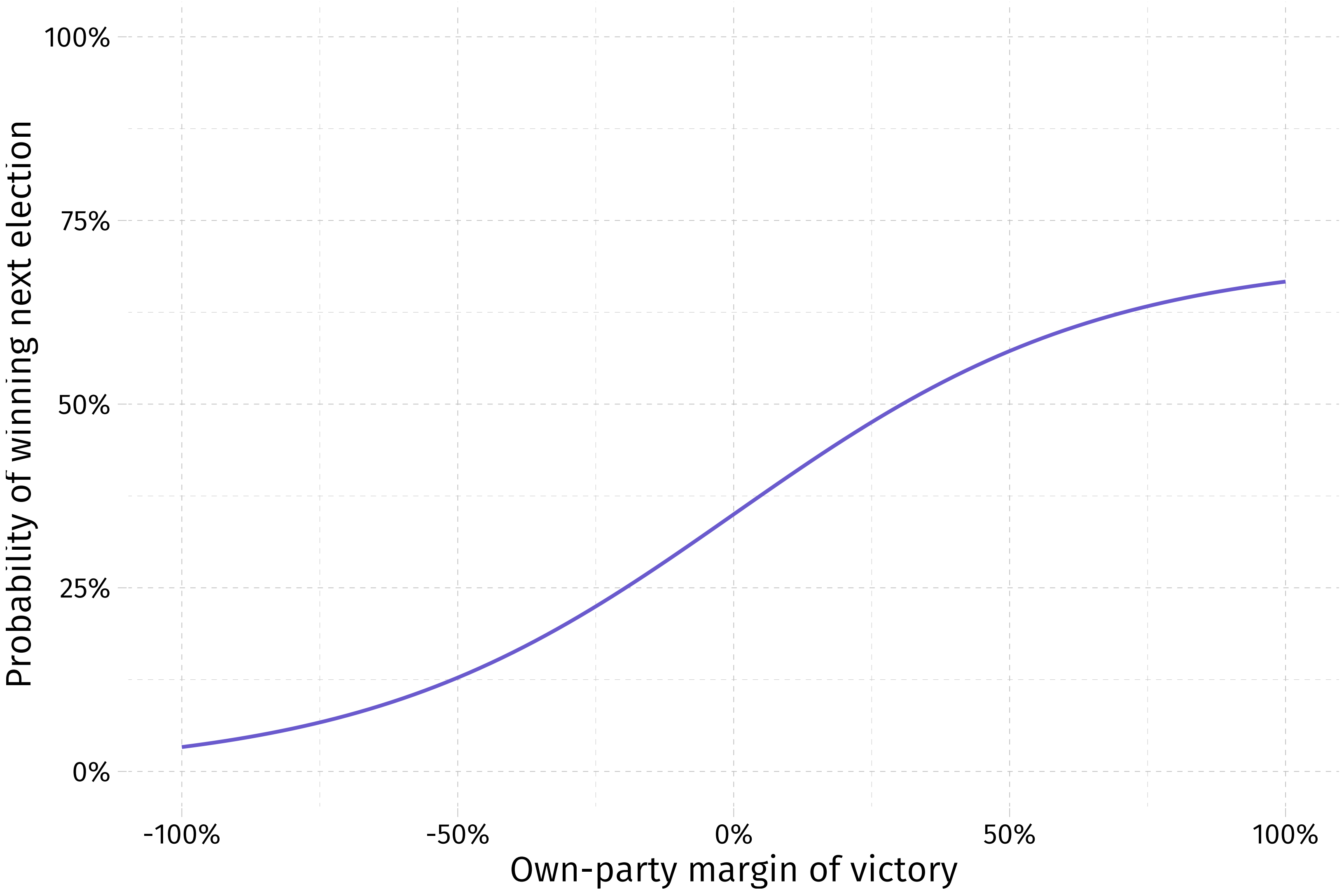
We must stay "near" to c to minimize the bias from extrapolating $E[\mathbf{Y}_{0i} \mid \mathbf{X}_i = c - \varepsilon]$ to $E[\mathbf{Y}_{0i} \mid \mathbf{X}_i = c + \varepsilon]$ (and assuming continuity).

Example Is there effect of a political party winning an election on that party's likelihood of winning the following election?

Is there a benefit of incumbency (at the party level)?[†]

† Lee (2008) addresses this question via RD. Caughey and Sekhon (2011) discuss RD in this setting.

Let's start with $E[\mathrm{Y}_{0i} \mid \mathrm{X}_i]$



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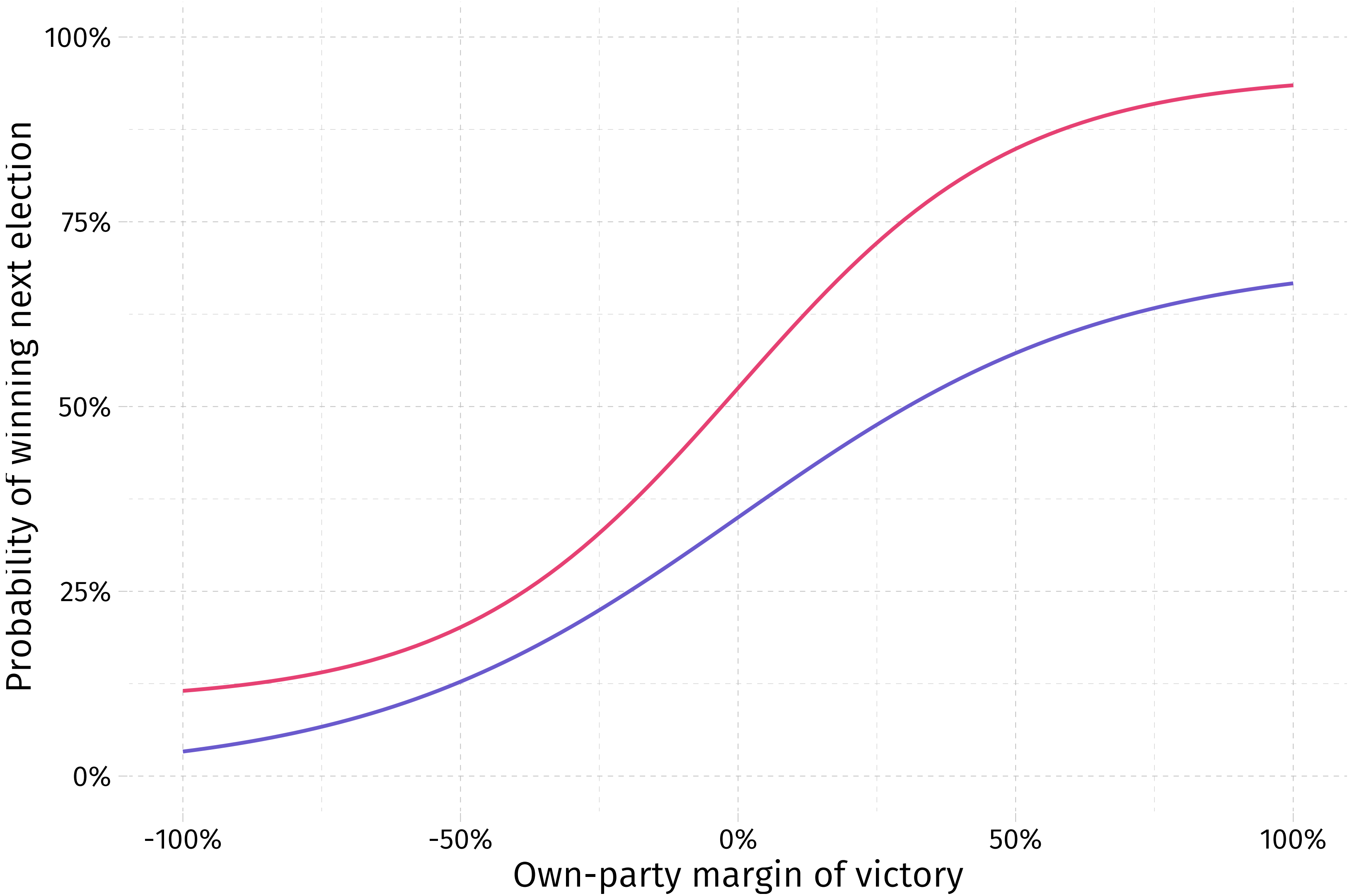
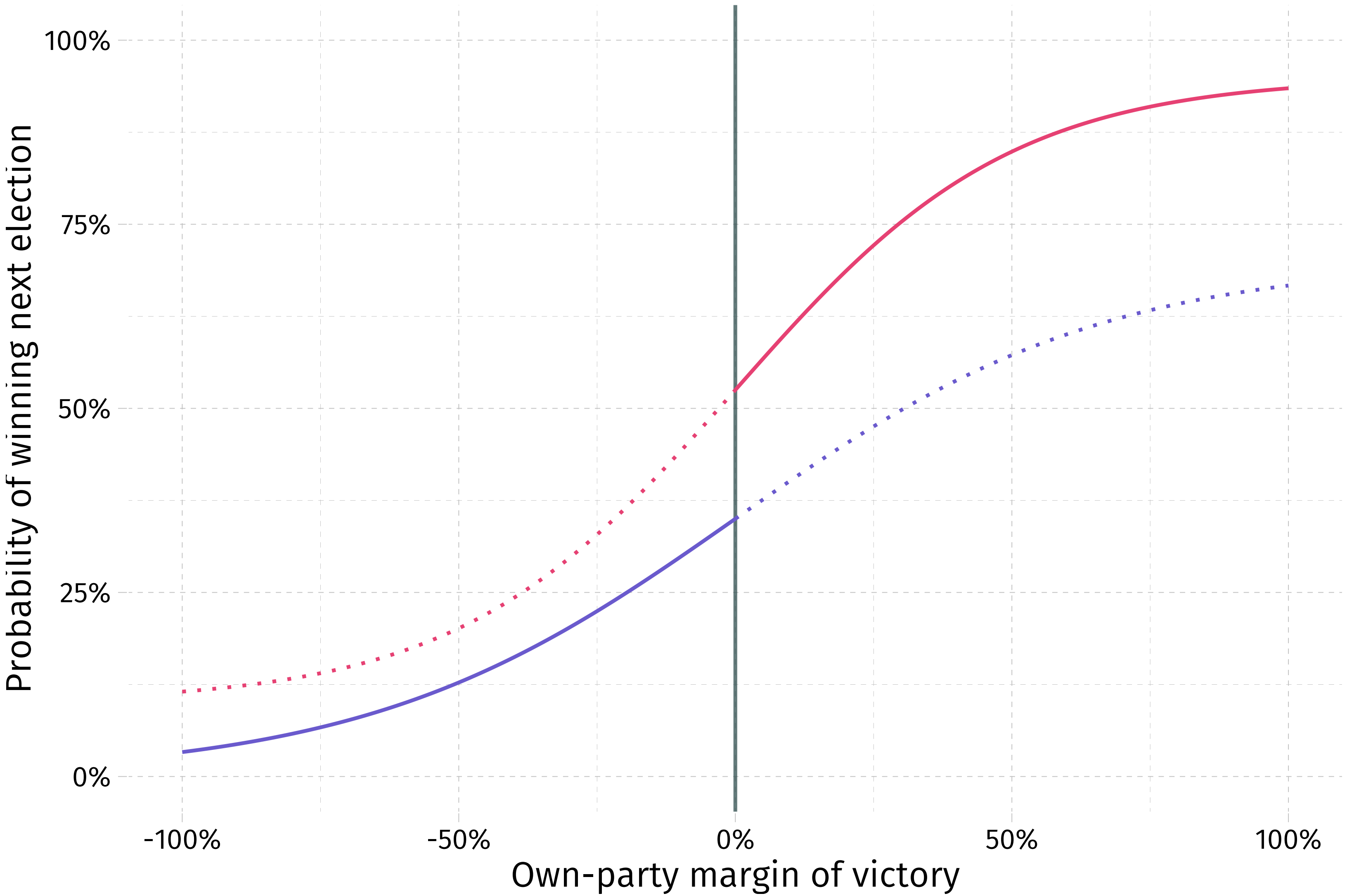
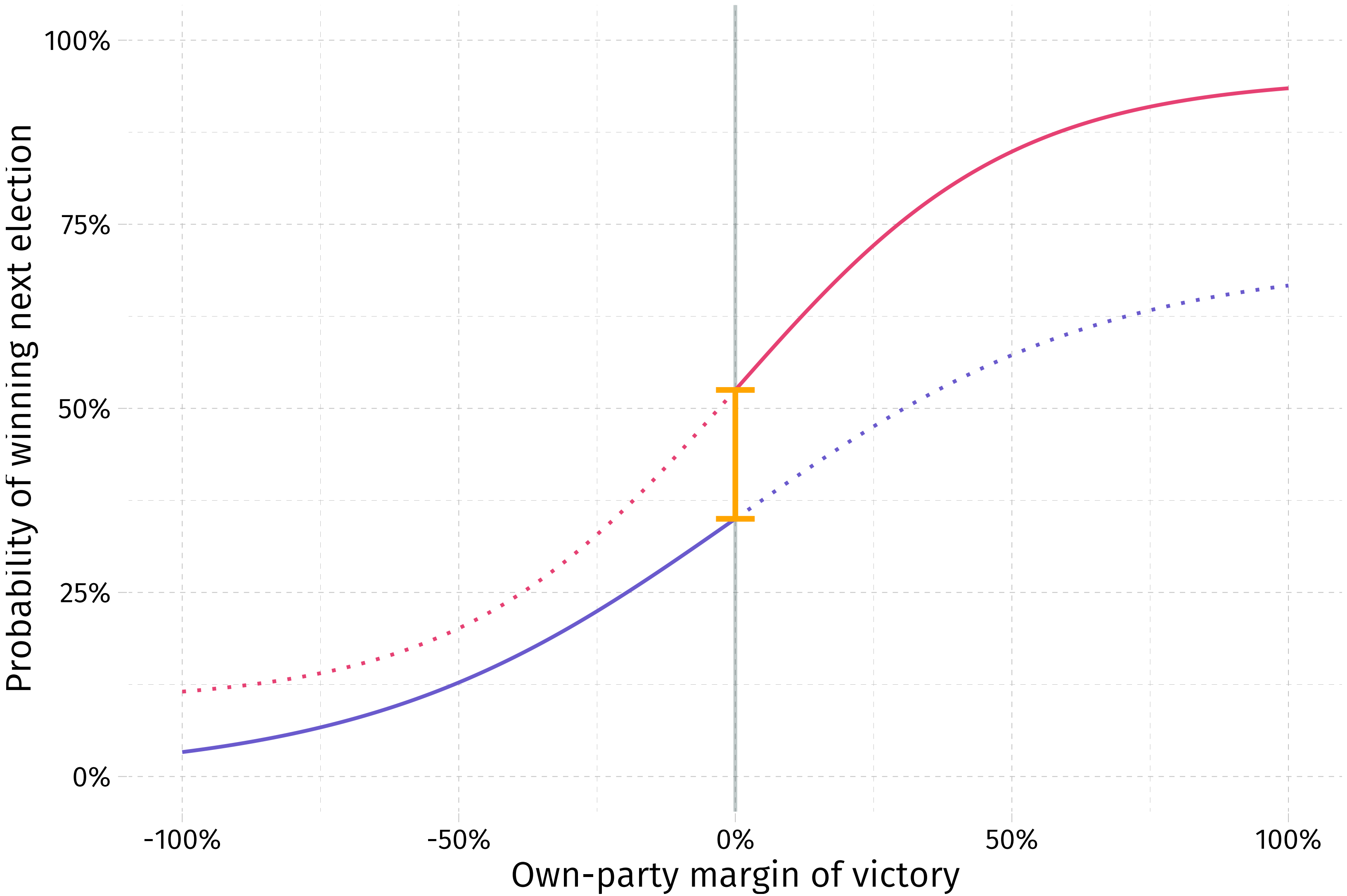


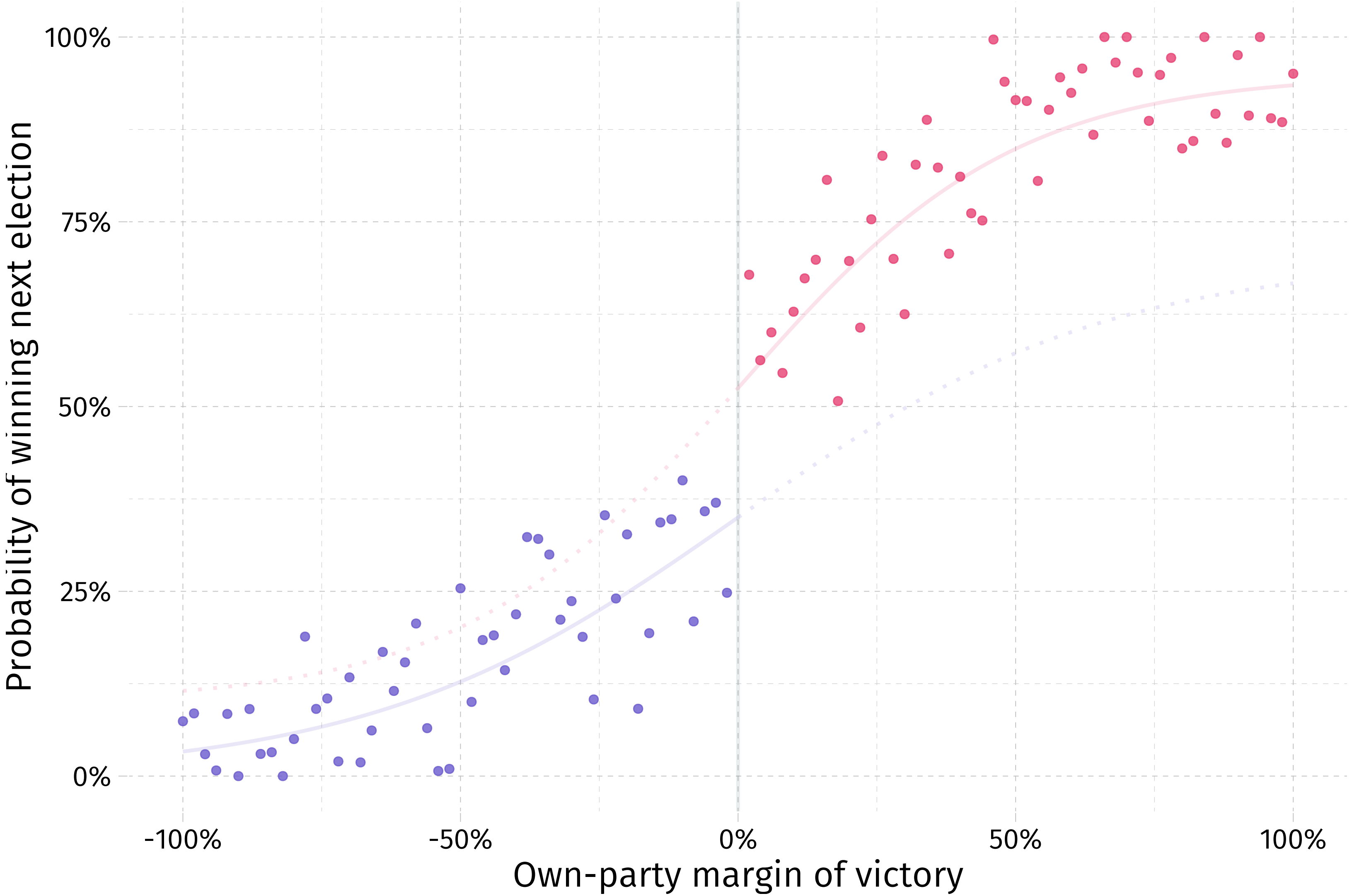
You only win an election if your **margin of victory exceeds zero**.



$E[\mathrm{Y}_{1i} \mid \mathrm{X}_i] - E[\mathrm{Y}_{0i} \mid \mathrm{X}_i]$ **at the discontinuity** gives τ_{SRD}.



Real data are a bit trickier. We must estimate $E[\mathrm{Y}_{1i} \mid \mathrm{X}_i]$ and $E[\mathrm{Y}_{0i} \mid \mathrm{X}_i]$.

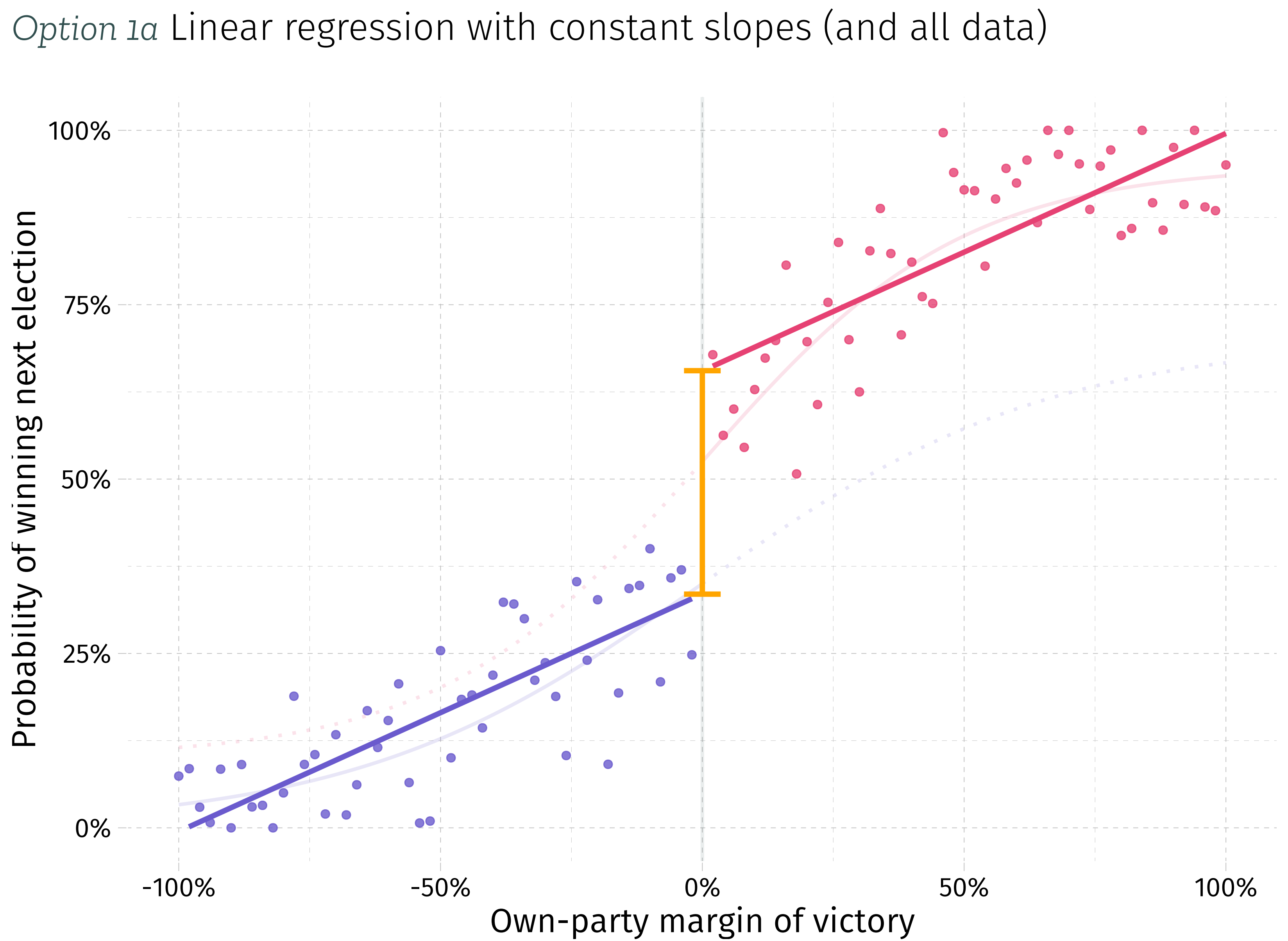


Questions

1. How should we estimate $E[\mathbf{Y}_{1i} \mid \mathbf{X}_i]$ and $E[\mathbf{Y}_{0i} \mid \mathbf{X}_i]$?

2. How much data should we use—*i.e.*, what is the right **bandwidth** size?

Option 1a Linear regression with constant slopes (and all data)


Option 1a Linear regression with constant slopes (and all data)
Probability of winning next election
100%
75%
50%
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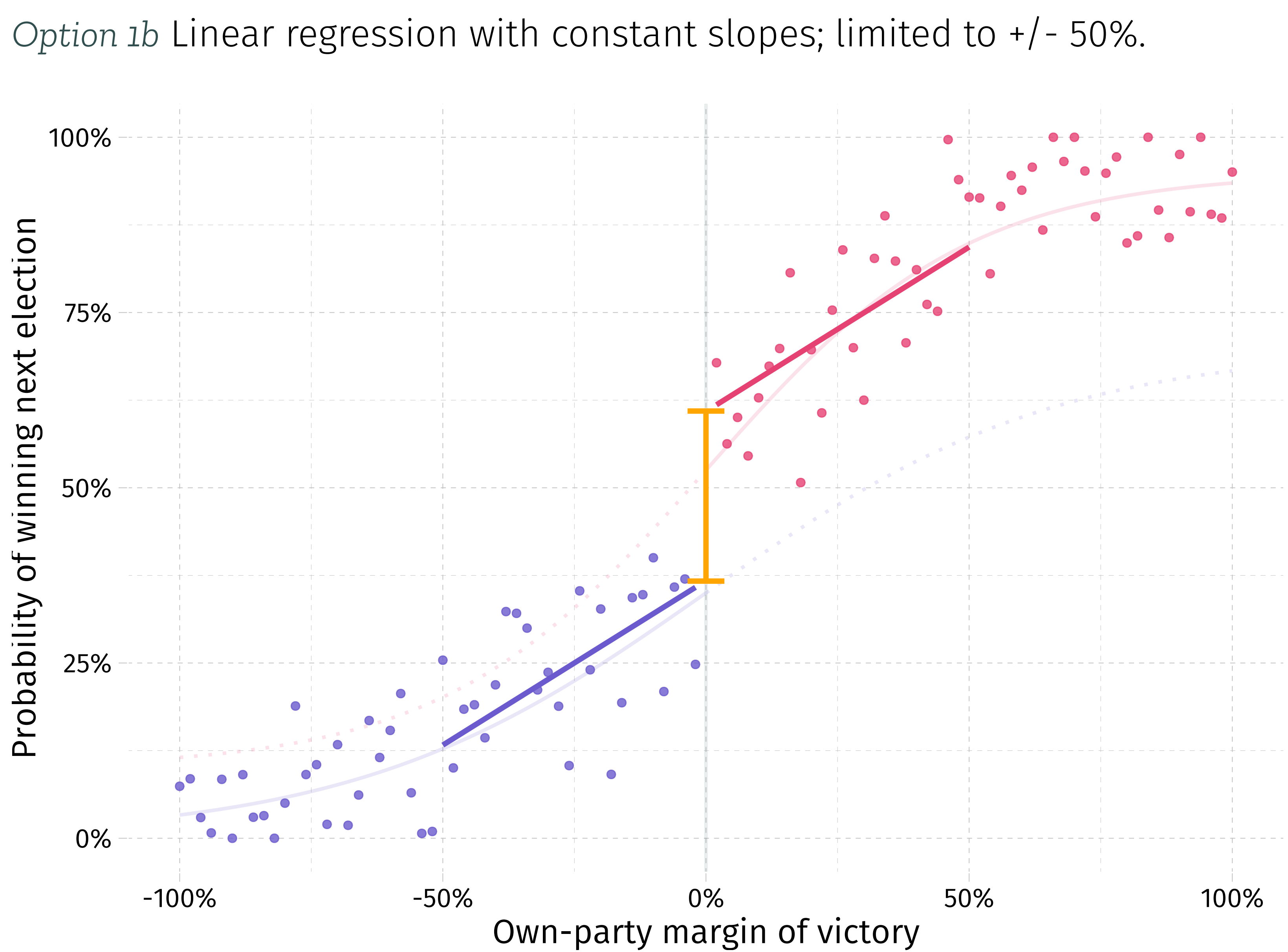
0%
-100%
-50%
0%
50%
100%
Own-party margin of victory

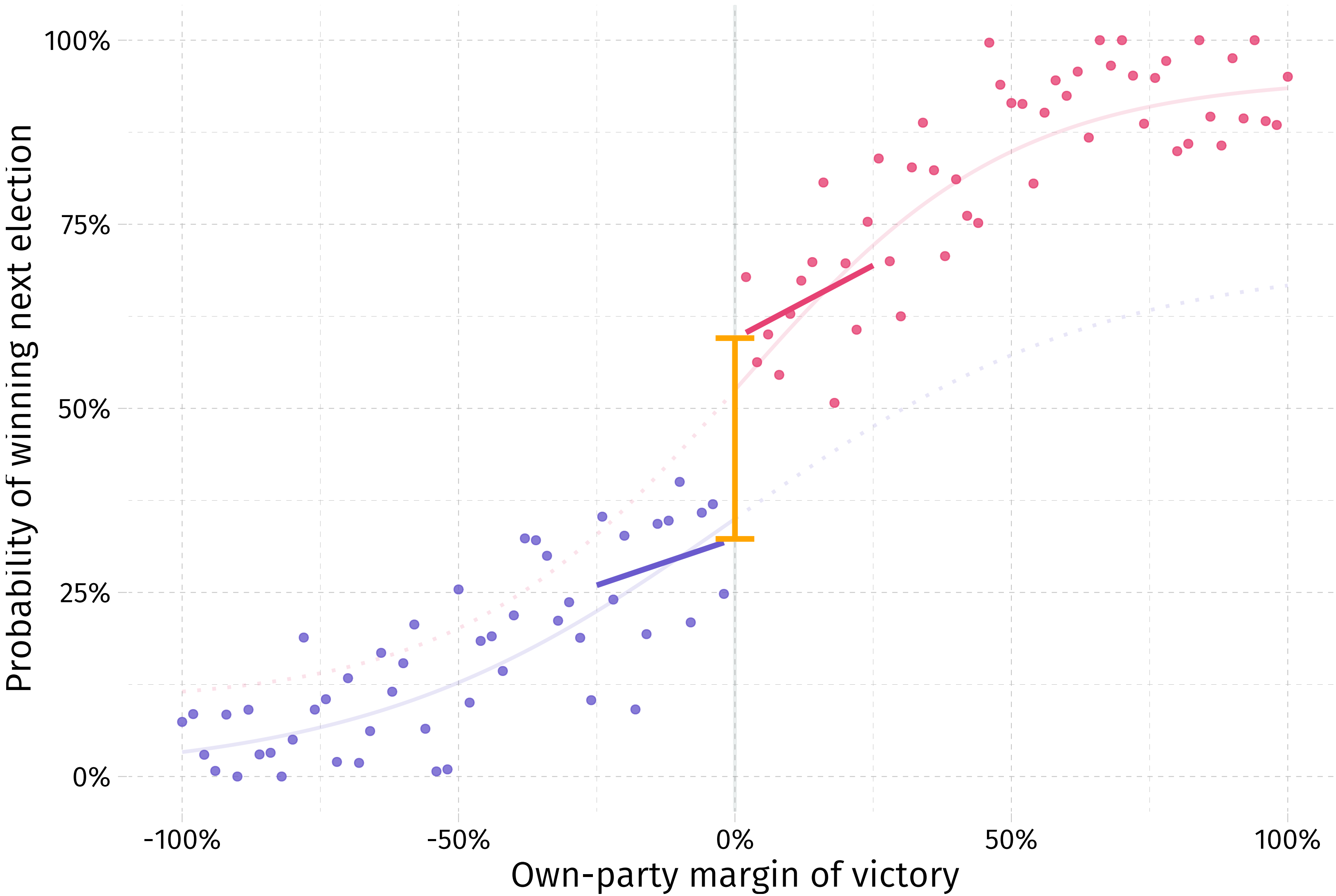


Option 1b Linear regression with constant slopes; limited to +/- 50%.
Probability of winning next election
100%
75%
50%
25%
0%
-100%
-50%
0%
50%
100%
Own-party margin of victory

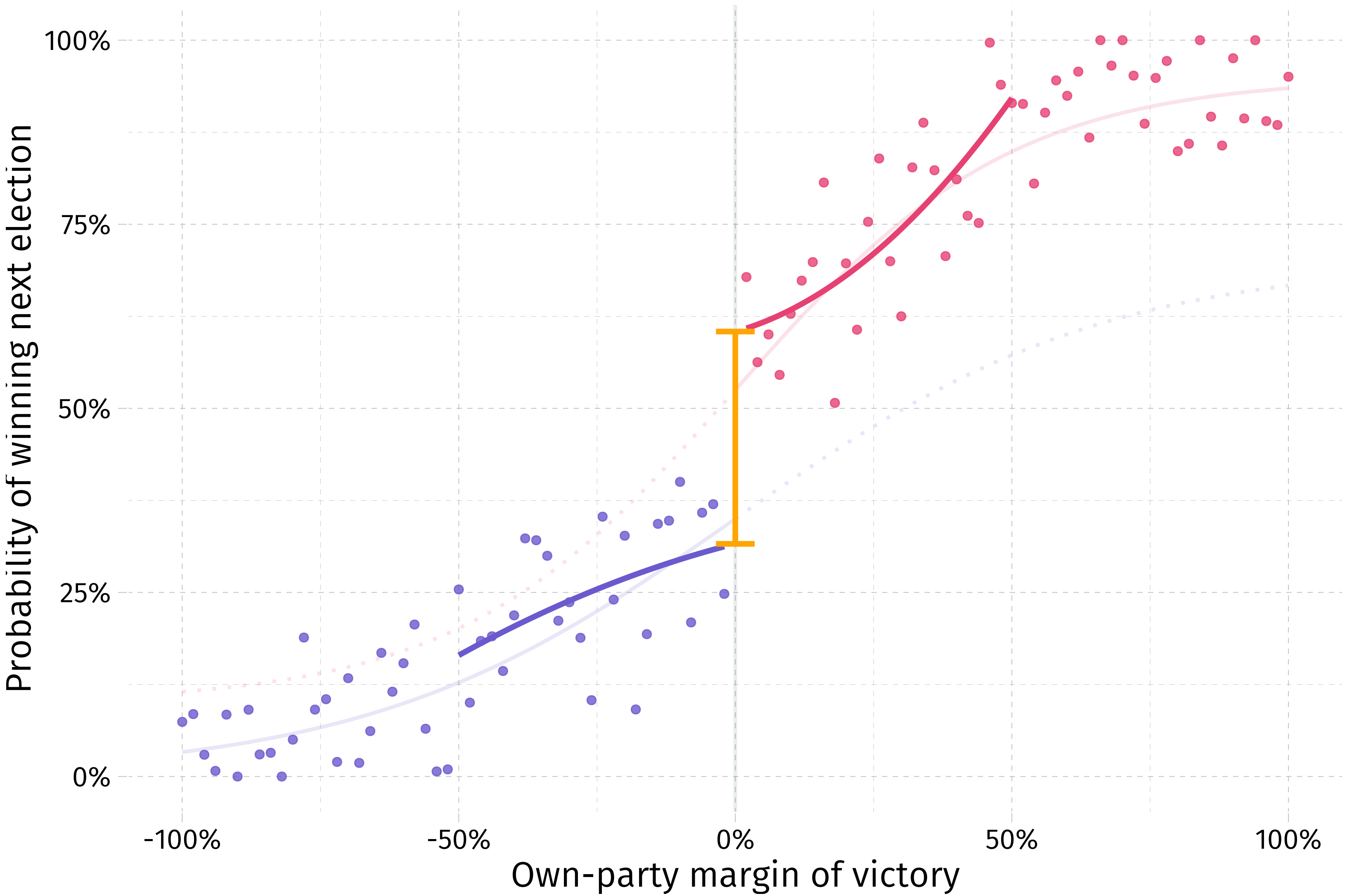
Option 2a Linear regression with differing slopes (and all data)

Option 2b Linear regression with differing slopes; limited to +/- 50%.

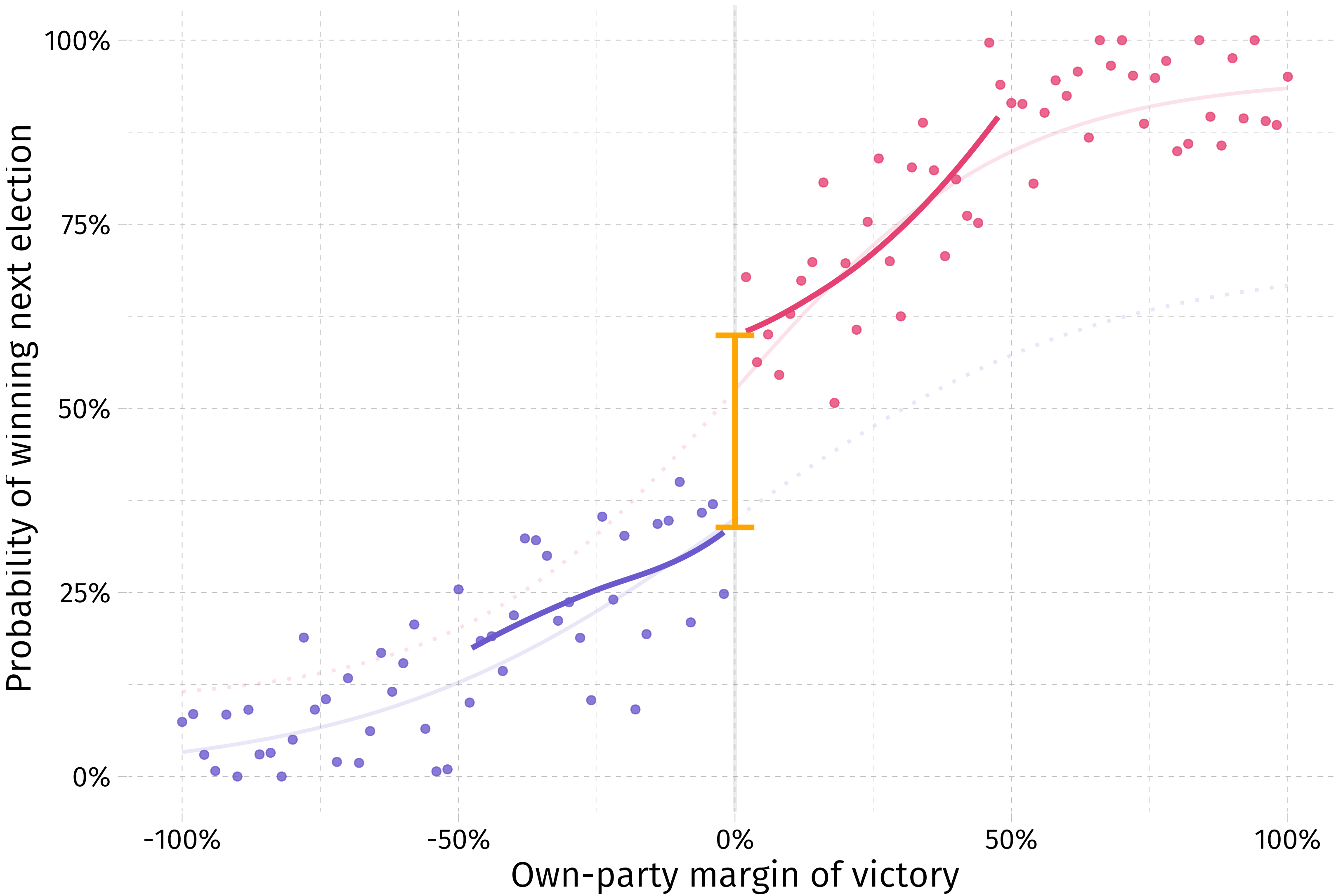
Option 2c Linear regression with differing slopes; limited to +/- 25%.



Option 3 Differing quadratic regressions (limited to +/- 50%).



Option 4a Differing local (LOESS) regressions (limited to +/- 50%).



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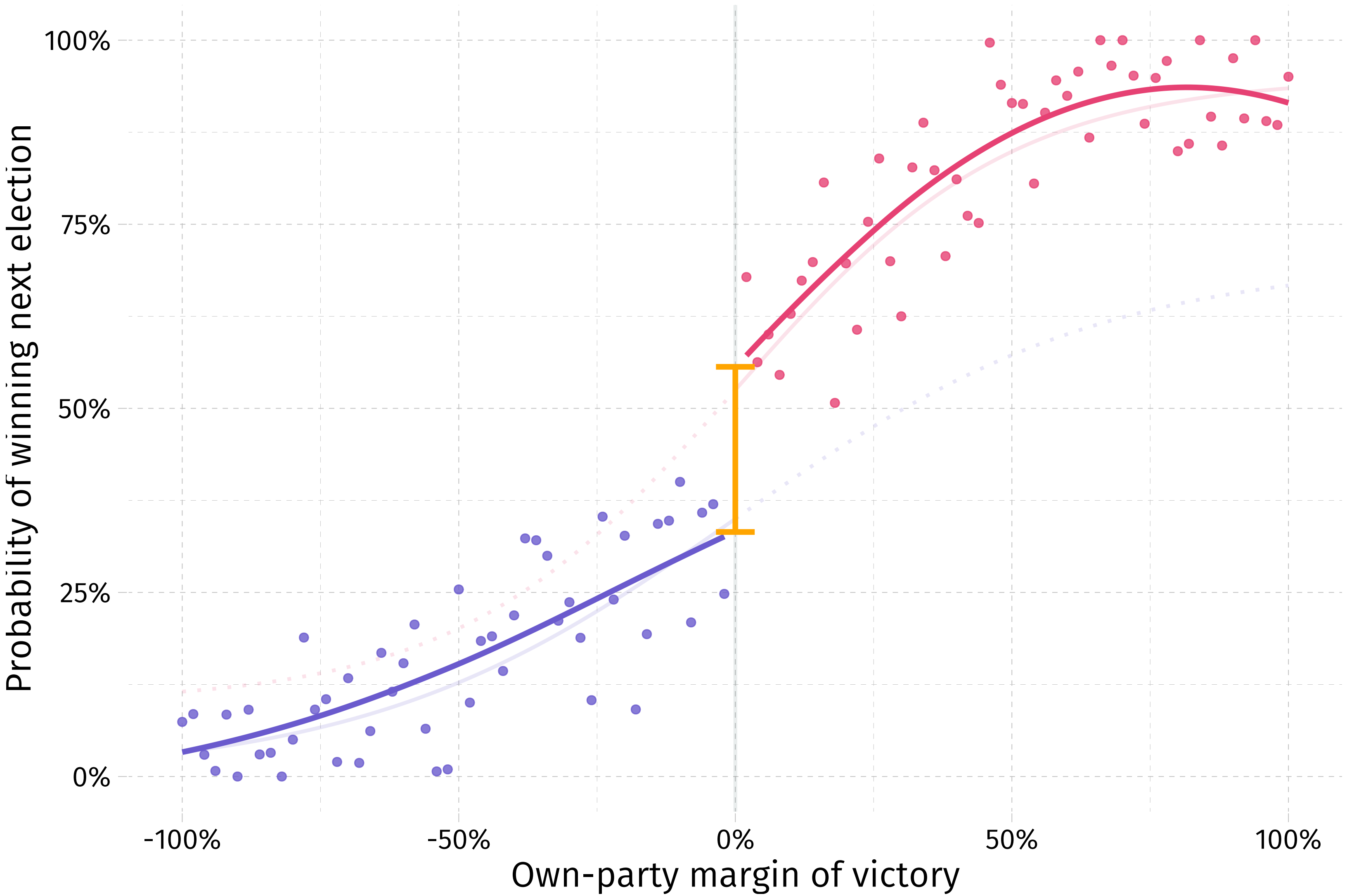
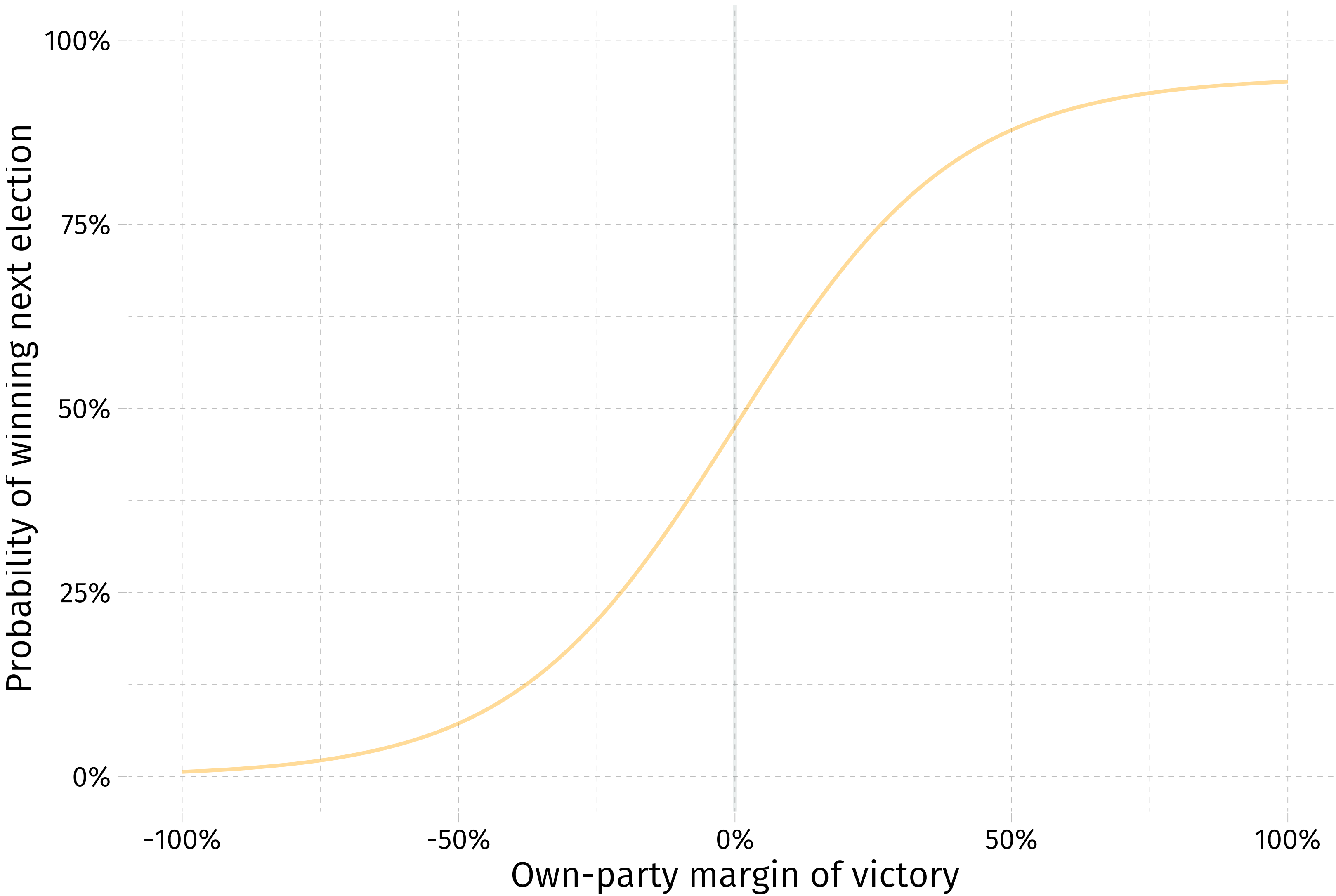


Note Functional form can be very important.



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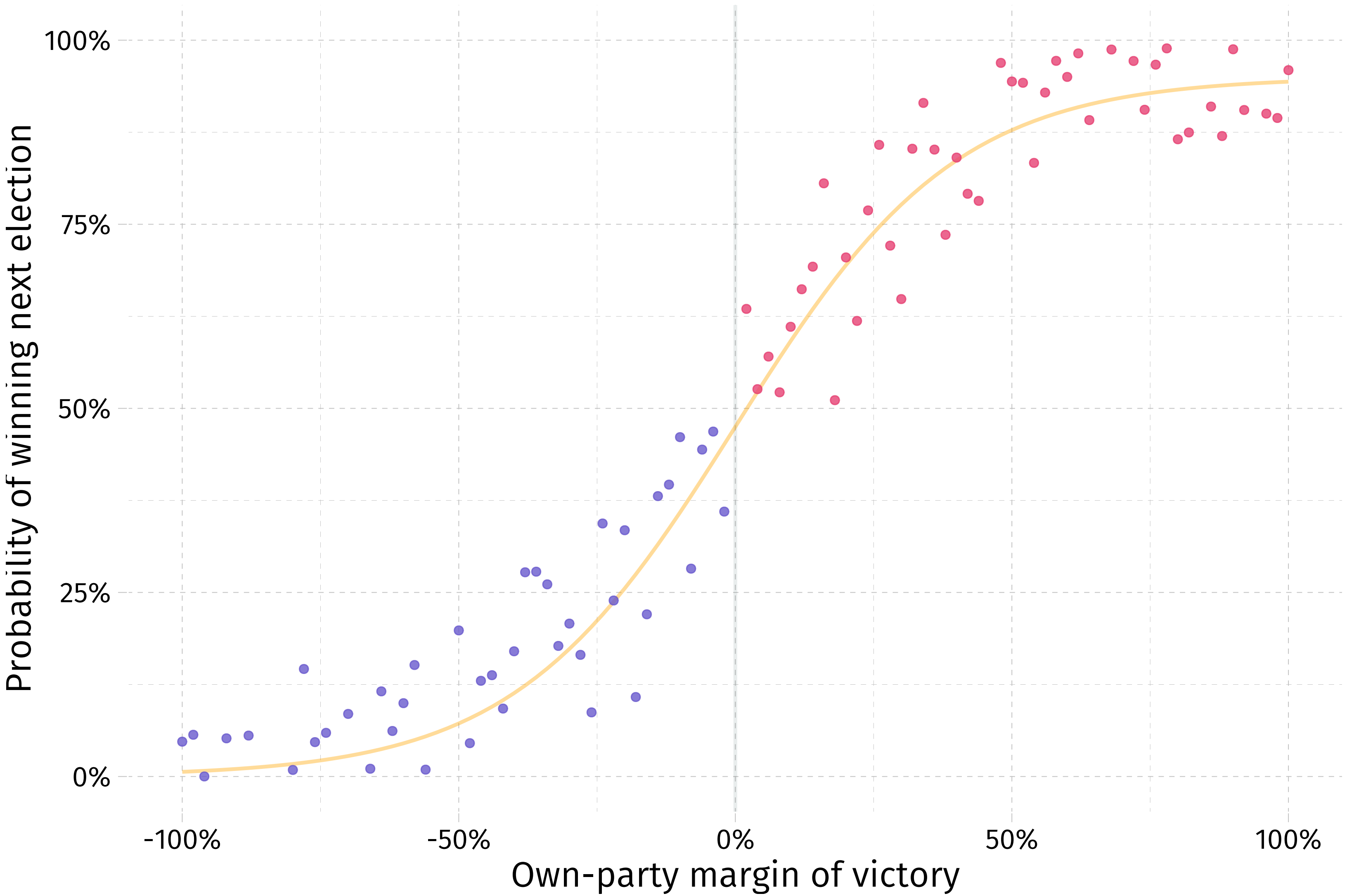

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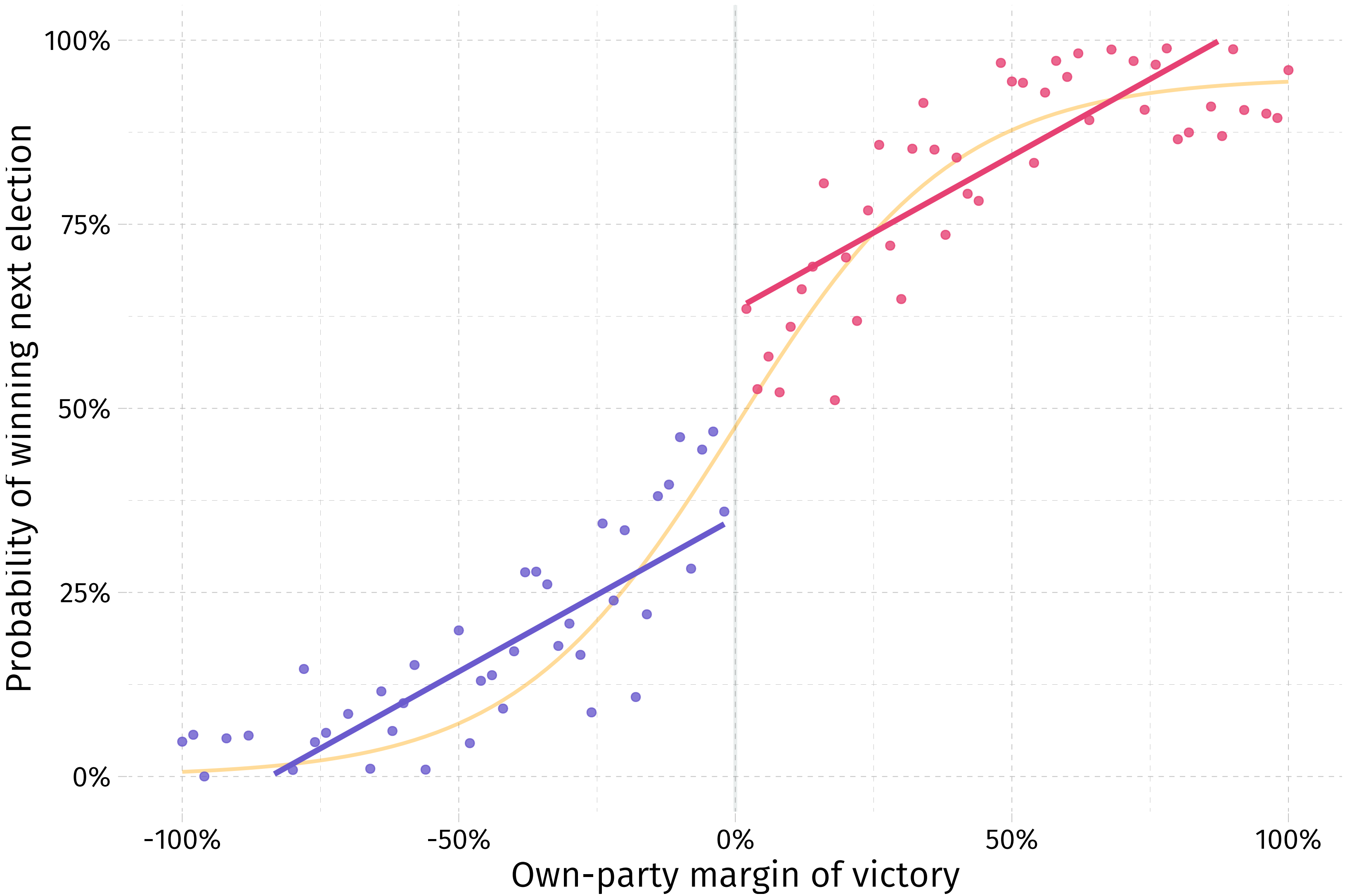

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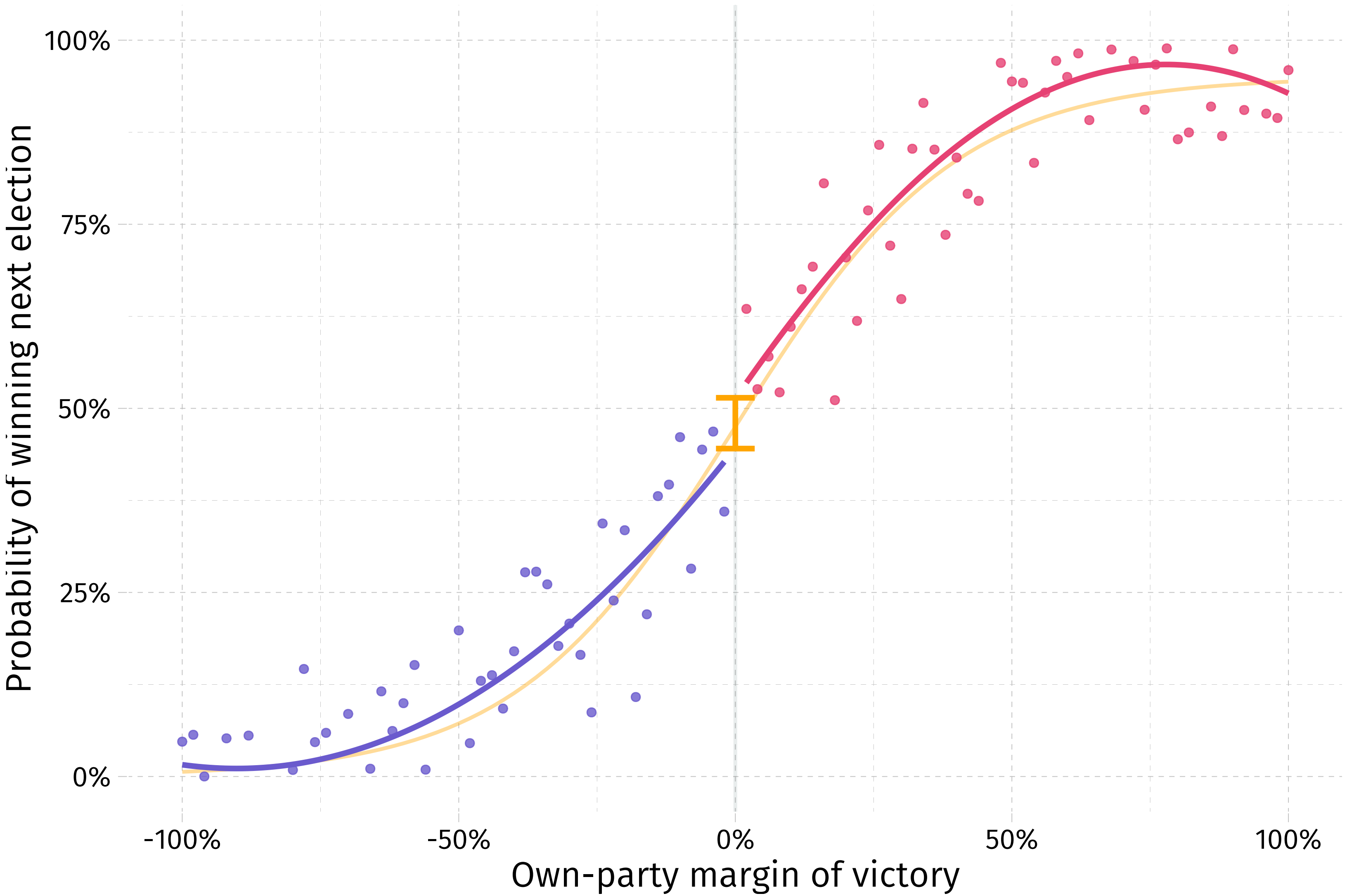
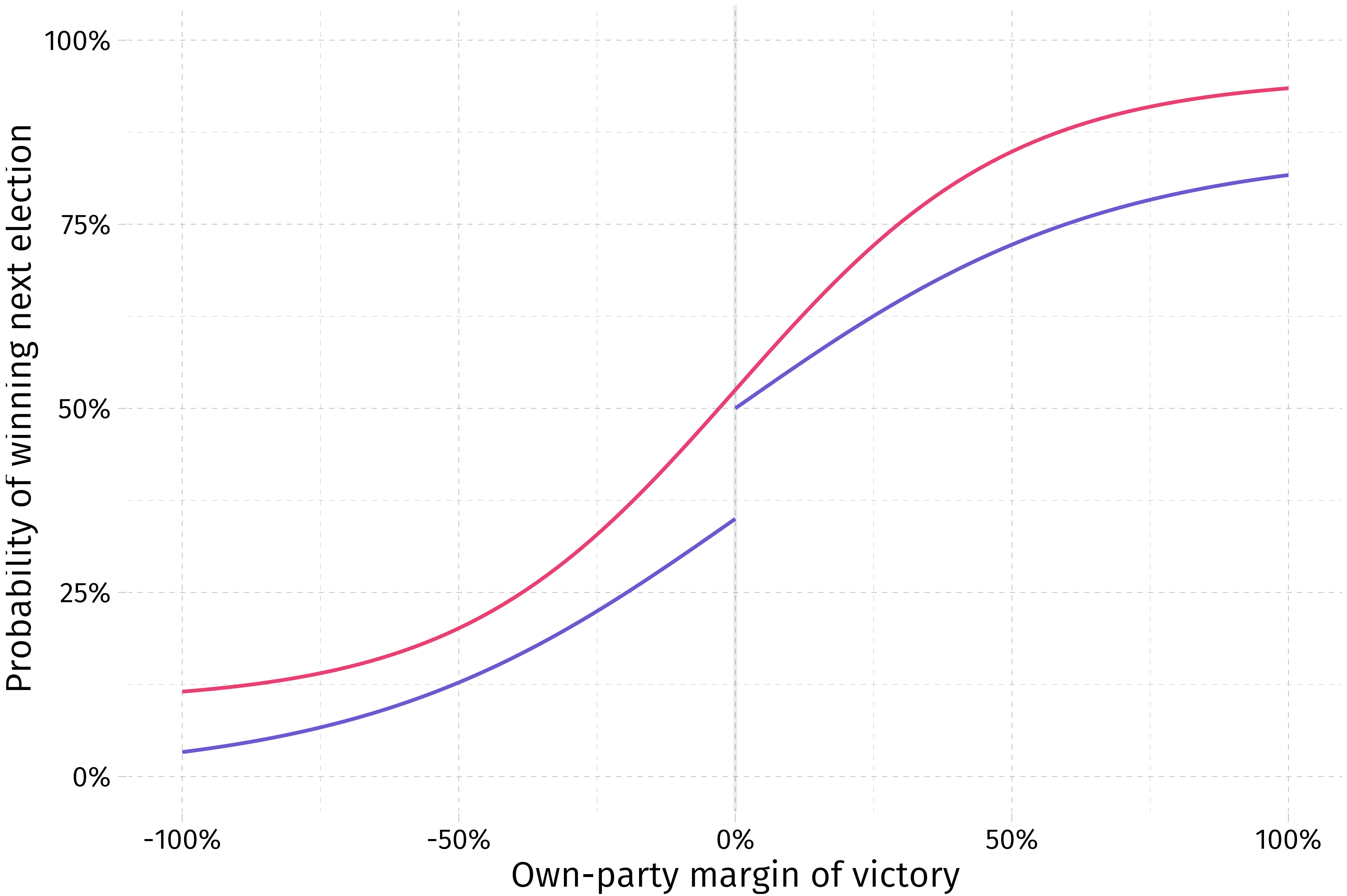


The continuity of $E[\text{Y}_{0i} \mid \text{X}_i = x]$ (in x) is also very important. No sorting.



Sharp RDs

In practice

Gelman and Imbens (2018) on functional form:

> We argue that controlling for global high-orderpolynomials in regression discontinuity analysis is a flawed approach with three major problems: it leads to noisy estimates, sensitivity to the degree of the polynomial, and poor coverage of confidence intervals. We recommend researchers instead use estimators based on local linear or quadratic polynomials or othersmooth functions.

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See Imbens and Kalyanaraman (2012) for optimal bandwidth selection.

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Setup

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Admin

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