Instrumental Variables

EC 425/525, Set 8

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Prologue

Schedule

Last time

Mathching and propensity-score methods

- Conditional independence
- Overlap

Today

Instrumental variables (and two-stage least squares)

Upcoming

- Admin: Assignment/project proposal this weekend
- Admin: Midterm very soon

Research designs

Research designs

Selection on observables and/or unobservables

We've been focusing on ***selection-on-observables*** **designs**, *i.e.*,

$$(\mathrm{Y}_{0i}, \mathrm{Y}_{1i}) \perp\!\!\!\perp \mathrm{D}_i | \mathrm{X}_i$$

for **observable** variables X_i.

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***Selection-on-unobservable* designs** replace this assumption with two new (but related) assumptions

1. $(\mathrm{Y}_{0i},\, \mathrm{Y}_{1i}) \perp \mathrm{Z}_i$

2. $\mathrm{Cov}(\mathrm{Z}_i,\, \mathrm{D}_i) \neq 0$

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Our main goal in causal-inference minded (applied) econometrics boils down to isolating **"good" variation** in D_i (exogenous/as-good-as-random) from **"bad" variation** (the part of D_i correlated with $\mathbf{Y}_{0i}$ and $\mathbf{Y}_{1i}$).

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   Seems more plausible. Possible to validate. May be underpowered.

Instrumental variables

Introduction

Instrumental variables (IV)[†] is the canonical selection-on-unobservables design—isolating *good variation* in $\mathbf{D}_i$ via some magical instrument $\mathbf{Z}_i$.

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Consider some model (structural equation)

$$\mathbf{Y}_i = \beta_0 + \beta_1 \mathbf{D}_i + \varepsilon_i \tag{1}$$

To guarantee consistent OLS estimates for β_1, want $\mathbf{Cov}(\mathbf{D}_i,\, \varepsilon_i) = \mathbf{0}$.
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Alternative: Estimate β_1 via instrumental variables.

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Definition

For our model

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   i.e., Z_i is excludable from equation (1). **(exclusion restriction)**

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Example

Back to the returns to a college degree,

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2. $\text{Cov}(\text{Lottery}_i, \varepsilon_i) = 0$ since the lottery is randomized.

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The IV estimator

The IV estimator for our model

$$\mathrm{Y}_i = \beta_0 + \beta_1 \mathrm{D}_i + \varepsilon_i \tag{1}$$

with (valid) instrument Z_i is

$$\hat{\beta}_{\mathrm{IV}} = \left(\mathrm{Z}'\mathrm{D}\right)^{-1}\left(\mathrm{Z}'\mathrm{Y}\right)$$

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If you have no covariates, then

$$\hat{\beta}_{\mathrm{IV}} = \frac{\mathrm{Cov}(\mathrm{Z}_i,\ \mathrm{Y}_i)}{\mathrm{Cov}(\mathrm{Z}_i,\ \mathrm{D}_i)}$$

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If you have additional (exogenous) covariates $\mathbf{X}_i$, then

$$\mathbf{Z} = \left[\, \text{Z}_i \quad \mathbf{X}_i \,\right]$$

$$\mathbf{D} = \left[\, \text{D}_i \quad \mathbf{X}_i \,\right]$$

Instrumental variables

Proof: Consistency

With a valid instrument $\mathbf{Z}_i$, $\hat{\beta}_{\text{IV}}$ is a consistent estiamtor for β_1 in

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$\mathbf{plim}\left(\hat{\beta}_{IV}\right)$

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&\quad = \beta \quad ✔
\end{aligned}$$

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First stage Estimate the effect of the instrument Z_i on our endogenous variable D_i and (predetermined) covariates X_i. Save $\widehat{\text{D}}_i$.

$$\text{D}_i = \gamma_1 \text{Z}_i + \gamma_2 \text{X}_i + u_i$$

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Second stage Estimate model we wanted—but only using the variation in $\mathbf{D}_i$ that correlates with Z_i, *i.e.*, $\widehat{\mathrm{D}}_i$.

$$\mathbf{Y}_i = \beta_1 \widehat{\mathrm{D}}_i + \beta_2 \mathbf{X}_i + \varepsilon_i$$

Note The controls $\mathbf{X}_i$ must match in the first and second stages.

Two-stage least squares

IV estimation

This two-step procedure, with a valid instrument, produces an estimator $\hat{\beta}_1$ that is consistent for β_1.

$$
\hat{\beta}_{\text{2SLS}} = \left(\mathrm{D}'\mathrm{P}_\mathrm{Z}\mathrm{D}\right)^{-1}\left(\mathrm{D}'\mathrm{P}_\mathrm{Z}\mathrm{Y}\right)
$$

$$
\mathrm{P}_\mathrm{Z} = \mathrm{Z}\left(\mathrm{Z}'\mathrm{Z}\right)^{-1}\mathrm{Z}'
$$

where $\mathbf{D}$ is a matrix of our treatment and predetermined covariates $(\mathbf{X}_i)$ and Z is a matrix of our instrument and our predetermined covariates.

Two-stage least squares

IV estimation

Important notes

- The controls (X_i) must match in the first and second stages.
- If you have exactly **one instrument** and exactly **one endogenous variable**, then 2SLS and IV are identical.
- Your second-stage standard errors are not correct.

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