EC 425/525, Set 8

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Prologue

Schedule

Last time

Matching and propensity-score methods

- Conditional independence
- Overlap

Today

Instrumental variables (and two-stage least squares)

Upcoming

- Assignment due Sunday
- Proposal due Wednesday 5/22
- Midterm?

Selection on observables and/or unobservables

We've been focusing on **selection-on-observables designs**, i.e.,

$$(\mathbf{Y}_{0i},\,\mathbf{Y}_{1i}) \perp \!\!\! \perp \mathbf{D}_i | \mathbf{X}_i$$

for **observable** variables X_i .

Selection-on-unobservable designs replace this assumption with two new (but related) assumptions

- 1. $(Y_{0i}, Y_{1i}) \perp Z_i$
- 2. $Cov(\mathbf{Z}_i, \mathbf{D}_i) \neq 0$

Selection on observables and/or unobservables

Our main goal in causal-inference minded (applied) econometrics boils down to isolating **"good" variation** in D_i (exogenous/as-good-as-random) from **"bad" variation** (the part of D_i correlated with Y_{0i} and Y_{1i}).

(We want to avoid selection bias.)

- Selection-on-observables designs assume that we can control for all bad variation (selection) in D_i through a known (observed) X_i .
- Selection-on-unobservables designs assume that we can extract part of the good variation in D_i (generally using some Z_i) and then use this good part of D_i to estimate the effect of D_i on Y_i . We throw away the bad variation in D_i (it's bad).

Which route?

So set of research designs is more palatable?

- 1. There are plenty of bad applications of both sets. Violated assumptions, bad controls, *etc*.
- 1. **Selection on observables** assumes we know *everything* about selection into treatment—we can identify *all* of the good (or bad) variation in \mathbf{D}_i . Tough in non-experimental settings. Difficult to validate in practice.
- 1. Selection on unobservables assumes we can isolate some good/clean variation in D_i , which we then use to estimate the effect of D_i on Y_i . Seems more plausible. Possible to validate. May be underpowered.

Introduction

Instrumental variables (IV)[†] is the canonical selection-on-unobservables design—isolating good variation in D_i via some magical instrument Z_i .

Consider some model (structural equation)

$$\mathbf{Y}_i = \beta_0 + \beta_1 \mathbf{D}_i + \varepsilon_i \tag{1}$$

To guarantee consistent OLS estimates for β_1 , want $Cov(D_i, \varepsilon_i) = 0$. In general, this is a heroic assumption.

Alternative: Estimate β_1 via instrumental variables.

[†] For the moment, we're lumping together IV and two-stage least squares (2SLS) together—as many people do—even though they are technically different.

Definition

For our model

$$Y_i = \beta_0 + \beta_1 D_i + \varepsilon_i \tag{1}$$

A valid **instrument** is a variable \mathbb{Z}_i such that

- 1. $Cov(\mathbf{Z}_i, \mathbf{D}_i) \neq 0$ our instrument correlates with treatment (so we can keep part of \mathbf{D}_i)
- 1. $Cov(\mathbf{Z}_i, \varepsilon_i) = 0$ our instrument is uncorrelated with other (non- \mathbf{D}_i) determinants of \mathbf{Y}_i , i.e., \mathbf{Z}_i is excludable from equation (1). (exclusion restriction)

Example

Back to the returns to a college degree,

$$\mathrm{Income}_i = \beta_0 + \beta_1 \mathrm{Grad}_i + \varepsilon_i$$

OLS is likely biased.

What if that state conducts a (random) **lottery** for scholarships?

Let $Lottery_i$ denote an indicator for whether i won a lottery scholarship.

- 1. $Cov(Lottery_i, Grad_i) \neq 0 \ (> 0)$ if scholarships increase grad. rates.
- 1. $Cov(Lottery_i, \varepsilon_i) = 0$ since the lottery is randomized.

The IV estimator

The IV estimator for our model

$$Y_i = \beta_0 + \beta_1 D_i + \varepsilon_i \tag{1}$$

with (valid) instrument Z_i is

$$\hat{eta}_{ ext{IV}} = \left(ext{Z'D}
ight)^{-1} \left(ext{Z'Y}
ight)$$

If you have no covariates, then

$$\hat{eta}_{ ext{IV}} = rac{ ext{Cov}(\mathbf{Z}_i,\,\mathbf{Y}_i)}{ ext{Cov}(\mathbf{Z}_i,\,\mathbf{D}_i)}$$

The IV estimator

The IV estimator for our model

$$Y_i = \beta_0 + \beta_1 D_i + \varepsilon_i \tag{1}$$

with (valid) instrument Z_i is

$$\hat{eta}_{ ext{IV}} = \left(ext{Z'D}
ight)^{-1} \left(ext{Z'Y}
ight)$$

If you have additional (exogenous) covariates X_i , then

$$\mathbf{Z} = [egin{array}{cc} \mathbf{Z}_i & \mathbf{X}_i \end{array}]$$

$$D = \begin{bmatrix} D_i & X_i \end{bmatrix}$$

Proof: Consistency

With a valid instrument \mathbf{Z}_i , $\hat{\beta}_{\mathrm{IV}}$ is a consistent estiamtor for β_1 in

$$\mathbf{Y}_i = \beta_0 + \beta_1 \mathbf{X}_i + \varepsilon_i \tag{1}$$

$$\operatorname{plim} \left(\hat{\beta}_{IV} \right)$$

$$=\operatorname{plim}\Bigl(\left(\operatorname{Z'D}
ight)^{-1} \left(\operatorname{Z'Y}
ight) \Bigr)$$

$$= ext{plim}\Big(ig(ext{Z'D}ig)^{-1}ig(ext{Z'D}eta+ ext{Z'}arepsilon\Big)\Big)$$

$$egin{aligned} &= \operatorname{plim}\!\left(\left(\operatorname{Z'D}
ight)^{-1} \left(\operatorname{Z'D}
ight) eta
ight) + \operatorname{plim}\left(rac{1}{N} \operatorname{Z'D}
ight)^{-1} \operatorname{plim}\!\left(rac{1}{N} \operatorname{Z'} arepsilon
ight) \end{aligned}$$

$$=\beta$$

Setup

You'll commonly see IV implemented as a two-stage process known as two-stage least squares (2SLS).

First stage Estimate the effect of the instrument Z_i on our endogenous variable D_i and (predetermined) covariates X_i . Save \widehat{D}_i .

$$\mathrm{D}_i = \gamma_1 \mathbf{Z}_i + \gamma_2 \mathbf{X}_i + u_i$$

Second stage Estimate model we wanted—but only using the variation in D_i that correlates with Z_i , *i.e.*, \widehat{D}_i .

$$\mathbf{Y}_i = eta_1 \widehat{\mathbf{D}}_i + eta_2 \mathbf{X}_i + arepsilon_i$$

Note The controls X_i must match in the first and second stages.

IV estimation

This two-step procedure, with a valid instrument, produces an estimator $\hat{\beta}_1$ that is consistent for β_1 .

$$\hat{eta}_{
m 2SLS} = \left({
m D}' {
m P}_{
m Z} {
m D}
ight)^{-1} \left({
m D}' {
m P}_{
m Z} {
m Y}
ight)$$
 ${
m P}_{
m Z} = {
m Z} \left({
m Z}' {
m Z}
ight)^{-1} {
m Z}'$

where \mathbf{D} is a matrix of our treatment and predetermined covariates (\mathbf{X}_i) and Z is a matrix of our instrument and our predetermined covariates.

IV estimation

Important notes

- The controls (X_i) must match in the first and second stages.
- If you have exactly **one instrument** and exactly **one endogenous variable**, then 2SLS and IV are identical.
- Your second-stage standard errors are not correct.

The reduced form

In addition to the regressions within the two stages of 2SLS

1.
$$\mathrm{D}_i = \gamma_1 \mathbf{Z}_i + \gamma_2 \mathbf{X}_i + u_i$$

2.
$$\mathbf{Y}_i = \beta_1 \widehat{\mathbf{D}}_i + \beta_2 \mathbf{X}_i + \varepsilon_i$$

there is a third important and related regression: the reduced form.

The **reduced form** regresses the outcome Y_i (LHS of the second stage) on our instrument Z_i and covariates X_i (RHS of the first stage).

$$\mathbf{Y}_i = \pi_1 \mathbf{Z}_i + \pi_2 \mathbf{X}_i + u_i$$

Thus, the reduced form provides a consistent estimate of the causal effect of our instrument on the outcome.

The reduced form, continued

While the reduced form estimates the causal effect of the instrument on our outcome, we're often actually interested in the effect of *treatment* (D_i) .

That said, the reduced form is still incredibly helpful/important:

- Clarifies your source of identifying variation.
- Does not suffer from weak instruments problems.
- Only requires $Cov(\mathbf{Z}_i, \, \varepsilon_i) = 0$.
- Offers insights into your estimates

$${\widehat eta}_1^{2{
m SLS}} = rac{{\widehat \pi}_1}{{\widehat \gamma}_1}$$

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The reduced form, intuition

This expression for the 2SLS (and IV) estimator can be very helpful.

$$\widehat{\beta}_1^{2\text{SLS}} = \frac{\widehat{\pi}_1}{\widehat{\gamma}_1} = \frac{\text{Reduced-form estimate}}{\text{First-stage estimate}}$$

What's the interpretation/intuition?

Back to our example: $\widehat{\beta}_1 = \text{est.}$ effect of college graduation on income.

 $\widehat{\pi}_1$ gives the estimated causal effect of the scholarship lottery on income, but what share of lottery winners graduate? We need to rescale if < 100%.

 $\hat{\gamma}_1$ estimates the effect of winning the scholarship lottery on graduation — the share of winners who graduated due to winning. We can scale with $\hat{\gamma}_1$!

The reduced form, example

To see why this scaling makes sense, imagine that 50% of lottery winners graduate from college due to the lottery, i.e., $\hat{\gamma}_1 = 0.50$.

Our reduced-form estimate of $\hat{\pi}_1 = \$5,000$ says that lottery winners make \$5,000 more than the control group, on average.

However, half of the winners did not graduate, so $\hat{\pi}_1$ "underestimates" the effect of college graduation by combining graduates by nongraduates.

Thus, we want to double $\hat{\pi}_1$, *i.e.*, divide by $\hat{\gamma}_1$: $\hat{\pi}_1/\hat{\gamma}_1$ = \$5,000/0.5 = \$10,000.

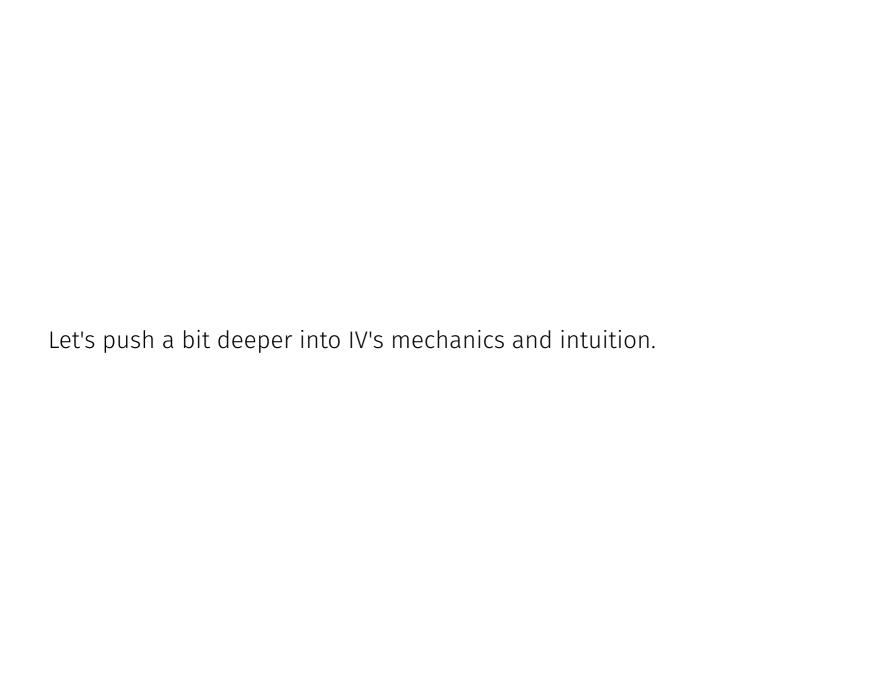
Q How do we get this magical expression? $\left(\widehat{\beta}_1^{\text{IV}} = \frac{\widehat{\pi}_1}{\widehat{\gamma}_1}\right)$

Derivation

$$\widehat{eta}_1^{\mathrm{IV}} = \left(\mathbf{Z}'\mathbf{D}\right)^{-1} \left(\mathbf{Z}'\mathbf{Y}\right)$$
 $= \left(\widetilde{\mathbf{Z}}'\widetilde{\mathbf{D}}\right)^{-1} \left(\widetilde{\mathbf{Z}}'\mathbf{Y}\right)$ applying FWL to reduce \mathbf{D} and \mathbf{Z} to vectors. $\mathbf{Cov}\left(\widetilde{\mathbf{Z}},\mathbf{Y}\right)$ $\mathbf{Cov}\left(\widetilde{\mathbf{Z}},\mathbf{Y}\right)/\mathbf{Vov}\left(\widetilde{\mathbf{Z}}\right)$

$$=\frac{\operatorname{Cov}\!\left(\widetilde{\boldsymbol{\mathbf{Z}}}_{i},\,\boldsymbol{\mathbf{Y}}_{i}\right)}{\operatorname{Cov}\!\left(\widetilde{\boldsymbol{\mathbf{Z}}}_{i},\,\widetilde{\boldsymbol{\mathbf{D}}}_{i}\right)}=\frac{\operatorname{Cov}\!\left(\widetilde{\boldsymbol{\mathbf{Z}}}_{i},\,\boldsymbol{\mathbf{Y}}_{i}\right)/\operatorname{Var}\!\left(\widetilde{\boldsymbol{\mathbf{Z}}}_{i}\right)}{\operatorname{Cov}\!\left(\widetilde{\boldsymbol{\mathbf{Z}}}_{i},\,\widetilde{\boldsymbol{\mathbf{D}}}_{i}\right)/\operatorname{Var}\!\left(\widetilde{\boldsymbol{\mathbf{Z}}}_{i}\right)}$$

$$=rac{\widehat{\pi}_1}{\widehat{\gamma}_1}$$



Setup

In this section, we'll use medical trials as a working example.[†]

We are interested in the regression model for the effect of some treatment (e.g., blood-pressure medication) on medical outcome Y_i

$$\mathbf{Y}_i = \beta_0 + \beta_1 \mathbf{D}_i + \varepsilon_i$$

 D_i indicates whether *i takes* the treatment (medication). ε_i captures all other factors that affect Y_i . Or in potential-outcomes framework:

$$egin{aligned} \mathbf{Y}_i &= \mathbf{Y}_{1i} \mathbf{D}_i + \mathbf{Y}_{0i} (1 - \mathbf{D}_i) \ \mathbf{Y}_{0i} &= eta_0 + arepsilon_i \ \mathbf{Y}_{1i} &= \mathbf{Y}_{0i} + eta_1 \end{aligned}$$

Research design

Goal Estimate the effect of blood-pressure medication on blood pressure.

Challenge **Selection bias:** Even if treatment reduces blood pressure, selection bias will fights against the estimated effect.

Solution Randomized medical trial: Ask randomly chosen individuals in treatment group to take the pill. Control individual get placebo (or nothing).

Analysis 1 Intention to treat (ITT):
$$\widehat{eta}_1^{ ext{ITT}} = \overline{Y}_{ ext{Trt}} - \overline{Y}_{ ext{Ctrl}}$$

ITT problem Bias from noncompliance: People don't always follow rules. E.g., treated folks who don't take pills; control folks who take pills.

Analysis 2 **IV!** Instrument medication D_i with intention to treat Z_i .

The IV solution

First question: Is \mathbf{Z}_i a valid instrument for \mathbf{D}_i ?

- 1. $Cov(\mathbf{Z}_i, \, \varepsilon_i) = 0$ as \mathbf{Z}_i was randomly assigned (exclusion restriction).
- 1. $Cov(\mathbf{Z}_i, \mathbf{D}_i) \neq \mathbf{0}$ if assignment to treatment changes the likelihood you take the pills (first stage).
- \therefore \mathbf{Z}_i is a valid instrument for \mathbf{D}_i and IV consistently estimates β_1 .

Noncompliance

Noncompliant individuals do not abide by their treatment assignment.

Let's see how IV "solves" this problems.

First, assume noncompliance only affects treated individuals—*i.e.*, treated folks sometimes don't take their pills; control folks never take pills.

Noncompliance, continued

The **first stage** recovers the share of treatment individuals who take the pill

$$D_i = \gamma_1 Z_i + u_i$$

i.e., if 50% of treated individuals take the medication, $\widehat{\gamma}=0.50$.

The **reduced form** estimates the *ITT*

$$\mathbf{Y}_i = \pi_1 \mathbf{Z}_i + v_i$$

which we know IV rescales using the first stage

$$\widehat{eta}_1^{ ext{IV}} = rac{\widehat{\pi}_1}{\widehat{\gamma}_1} = rac{\widehat{\pi}_1}{0.50} = 2 imes \widehat{\pi}_1$$

Noncompliance, continued

IV solves the noncompliance issue by rescaling by the rate of compliance.

If everyone perfectly complies, then $\widehat{\gamma}_1=1$ and $\widehat{eta}_1^{ ext{IV}}=\widehat{\pi}_1/1=\widehat{eta}_1^{ ext{ITT}}$.

Further example N_{Trt} = 10; trt. compliance = 50%; ctrl. compliance = 100%.

$$\overline{Y}_{\mathrm{Trt}} = rac{5(eta_0 + eta_1) + 5(eta_0)}{10} = eta_0 + rac{eta_1}{2} ext{ and } \overline{Y}_{\mathrm{Ctrl}} = eta_0.$$

So our reduced-form estimate (the ITT) is $\widehat{\gamma}_1=rac{eta_1}{2}$ (half the true effect).

IV consistently estimates β_1 via rescaling the ITT by the rate of compliance

$$\widehat{eta}_1^{ ext{IV}} = rac{\pi}{\gamma} = rac{eta_1/2}{1/2} = eta_1$$

Takeaways

Main points

- 1. IV **rescales** the causal effect of \mathbf{Z}_i on \mathbf{Y}_i by the causal effect of \mathbf{Z}_i on \mathbf{D}_i .
- 1. IV **does not** compare treated compliers to untreated compliers. Such a comparison/estimator would re-introduce selection bias.

Thus far, we assumed homogeneous treatment effects. **Q** What happens when treatment effects are heterogeneous?

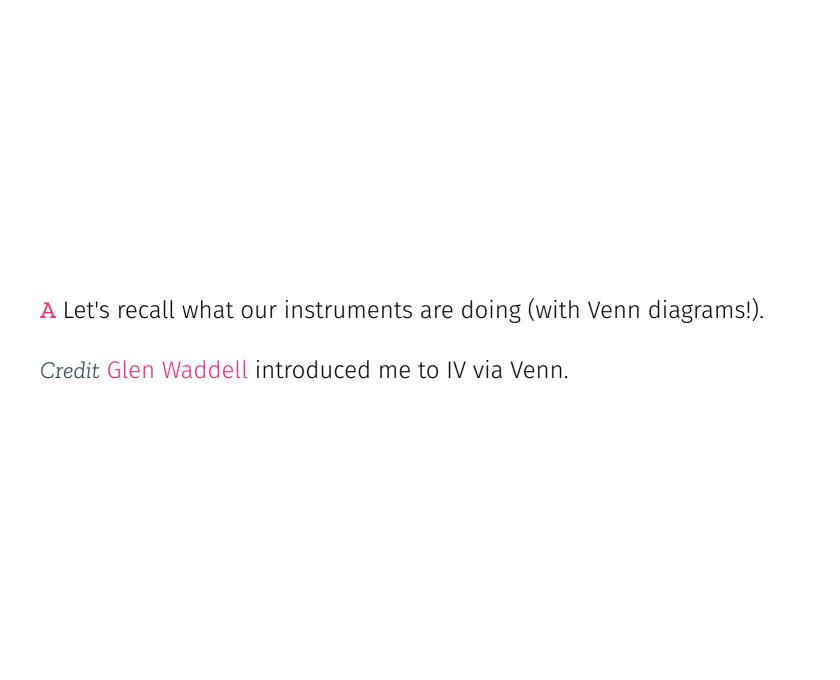


Figure 1

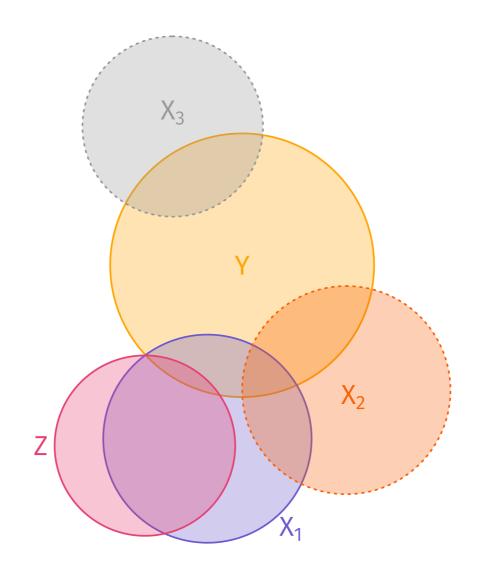
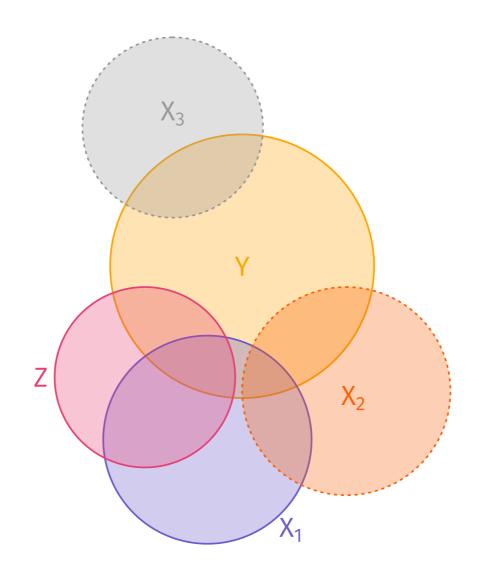


Figure 2



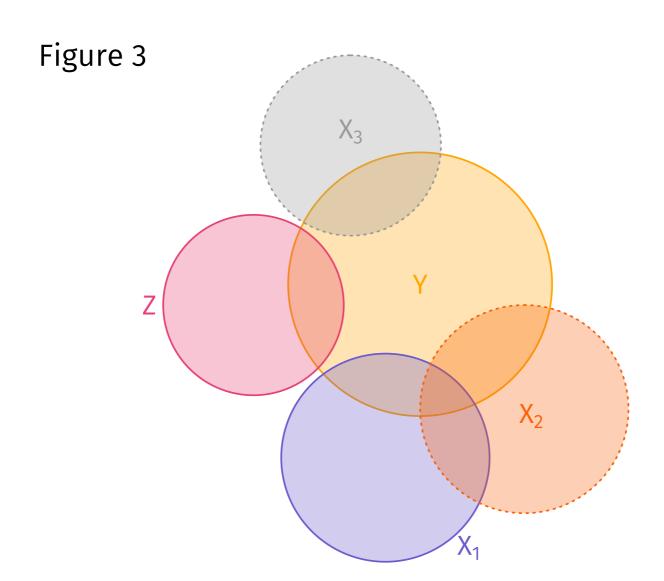


Figure 4

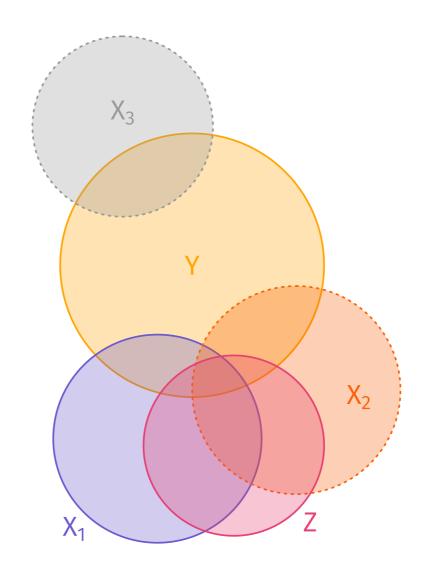
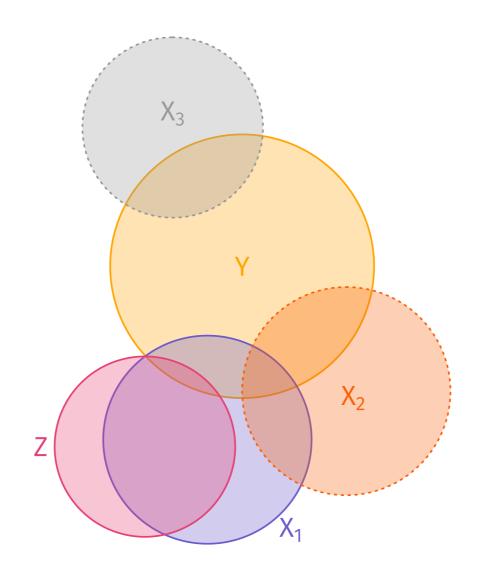


Figure 1



Recap

Throughout the course, we've discussed two concepts of treatment effects.

- 1. Average treatment effect (ATE) The average treatment effect for an individual randomly drawn from our sample.
- 1. **Treatment on the treated (TOT)** The average treatment effect for a **treated** individual randomly drawn from our sample.

When we assume homogeneous/constant treatment effects, ATE = TOT.

- Q If treatment effects vary, then what do IV and 2SLS estimate?
- A Not ATE. And not TOT. They estimate the LATE.[†]

[†] See Angrist, Imbens, and Rubin (1996).

The LATE

IV generally estimates the LATE—the Local Average Treatment Effect.

Recall IV "works" by isolating variation in D_i induced by our instrument Z_i .

In other words: IV focuses on the individuals whose D_i changes due to Z_i .

Angrist, Imbens, and Rubin (1996) call these folks compliers.

However, compliers are only one of four possible groups.

- 1. Compliers $D_i = 1$ iff $Z_i = 1$.
- 2. Always-takers $D_i = 1 \ \forall Z_i$.
- 3. Never-takers $D_i = 0 \ \forall Z_i$.
- 4. Defiers $D_i = 1$ iff $Z_i = 0$.

Only take pills when treated.

Always take pills.

Never take pills.

Only take pills when untreated.

The LATE

Because IV only uses variation in D_i that correlates with Z_i , IV mechanically drops always-takers and never-takers.

Most IV derivations/applications assume away the existence of defiers.

Thus, IV estimates a treatment effect using only compliers.

Hence the "local" in local average treatment effect.

The LATE: Medical-trial example

Imagine treatment works for some $(\beta_{1,i} < 0)$ and not for others $(\beta_{1,j} = 0)$.

Suppose individuals know their response to blood-pressure medication.

- $\beta_{1,i} < 0$ individuals always take the pill.
- $\beta_{1,j} = 0$ individuals only take the pill when treated.

Then our compliers will be individuals for whom $\beta_{1,j} = 0$.

Thus, IV's LATE will indicate no treatment effect $\left(\widehat{eta}_1^{\mathrm{IV}}=0\right)$.

The LATE

Q So is IV actually inconsistent?

A It depends what you are trying to estimate (and how you interpret it).

IV doesn't estimate the ATE or TOT, so it would be inconsistent for them.[†]

IV estimates the *local* average treatment effect.

Takeaway Because IV identifies off of compliers, it estimates an average treatment effect for these individuals (who comply with the instrument).

Takeaway₂ Different instruments have different LATEs.

[†] Just as the TOT is not consistent for the ATE.

Monotonicity

We've already written down the two classical IV/2SLS assumptions

- First stage: $Cov(Z_i, D_i) > 0$
- Exclusion restriction: $\operatorname{Cov}(\mathbf{Z}_i,\, \varepsilon_i) = 0$

but we need a third assumption to get ensure IV's complier-based LATE interpretation.

• Monotonicity (Uniformity): $D_i(z) \ge D_i(z')$ or $D_i(z) \le D_i(z') \ \forall i$ Heckman: Uniformity of responses across persons. Imbens and Angrist (1994): Instrument has monotone effect on D_i .

Monotonicity

If "defiers" exist, then monotonicity/uniformity is violated.

In this case, the IV estimand is

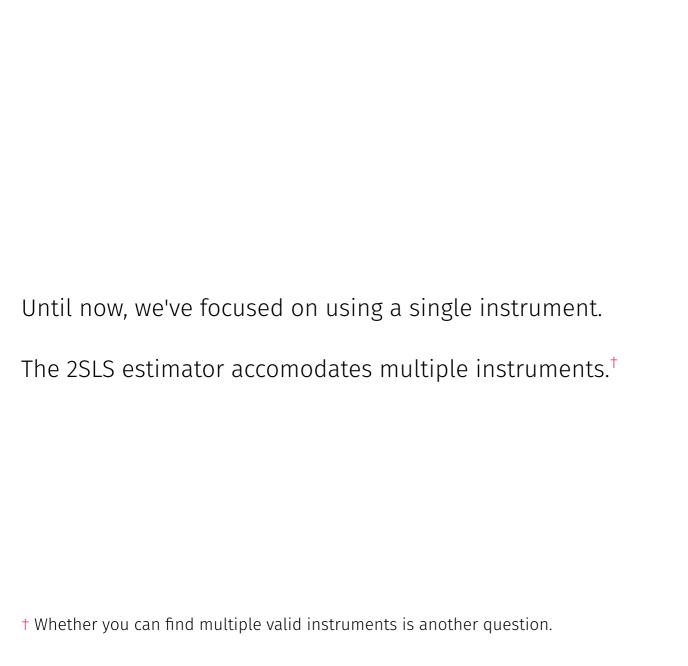
$$rac{ au_c \operatorname{Pr}(\operatorname{complier}) - au_d \operatorname{Pr}(\operatorname{defier})}{\operatorname{Pr}(\operatorname{complier}) - \operatorname{Pr}(\operatorname{defier})}$$

which is not bound between τ_c and τ_d .

Example
$$\tau_c = 1$$
 and $\tau_d = 2$. $\Pr(\text{complier}) = 2/3$ and $\Pr(\text{defier}) = 1/3$.

Then the "LATE" is 0.[†]

[†] Some people would instead say that there is no LATE when you violate monotonicity.



Motivation

Q Why include multiple instruments?

A Multiple instruments can capture more variation in D_i (efficiency).

Using terminology from the system-of-equations literature,

- one instrument for one endogenous variable: just identified
- multiple instruments for one endogenous variable: over identified

In practice

With (valid) instruments \mathbf{Z}_{1i} and \mathbf{Z}_{2i} , or first stage becomes

$$\mathrm{D}_i = \gamma_0 + \gamma_1 \mathrm{Z}_{1i} + \gamma_2 \mathrm{Z}_{2i} + \gamma_3 \mathrm{X}_i + u_i$$

while our second stage is still

$$\mathbf{Y}_i = eta_0 + eta_1 \widehat{\mathbf{D}}_i + eta_2 \mathbf{X}_i + v_i$$

Example: Quarter of birth

Back to our quest to estimate the returns to education.

Angrist and Krueger (1991) proposed *quarter of birth* as a set of instruments for years of schooling.

Accordingly, their first stage looks something like[†]

$$egin{aligned} ext{Schooling}_i &= \gamma_0 + \gamma_1 \mathbb{I}(ext{Born Q1})_i + \gamma_2 \mathbb{I}(ext{Born Q2})_i \ &+ \gamma_3 \mathbb{I}(ext{Born Q3})_i + \gamma_4 \mathbb{I}(ext{Born Q4})_i \ &+ \gamma_5 ext{X}_i + u_i \end{aligned}$$

[†] We need to drop one of the quarter-of-birth indicators to avoid perfect collinearity.

Example: Quarter of birth

Q Is quarter of birth a valid instrument?

Q1 Why would quarter of birth affect schooling? (First stage)

A1 Students cannot drop out of school until a certain age, and quarter of birth affects your age at the time you begin school.

Example Some states require students to stay in school until they are 16.

- Students who start school at age 6 drop out after 10 years of schooling.
- Students who start school at age 5 drop out after 11 years of schooling.

Example: Quarter of birth

If students must begin school in calendar year in which they turn 6

- December birthdates: begin school at 5.75; drop out with 10.25 yrs.
- January birthdates: begin school at 6.75; drop out with 9.25 yrs.

For some group, quarter of birth may affect the number of years in school.

Example: Quarter of birth

It turns out that the first stage is also pretty weak in this setting.

Weak instruments can cause several problems for 2SLS/IV:

- 1. Our estimator is a ratio of the reduced form and the first stage, so a weak first stage can blow up reduced-form estimates (amplifying reduced-form noise/bias).
- 1. Many weak instruments lead to a finite-sample issue in which 2SLS is biased toward OLS—our first stage is essentially overfitting.

What about our other requirements for a valid instrument?

Example: Quarter of birth

Q2 Is quarter of birth uncorrelated with ε_i (excludable)?

A2 While quarter of birth may be fairly arbitrary for some families, other families might time births.

If these birth timers differ from other couples along other dimensions (e.g., income or education), then quarter of birth may correlate with ε_i .

Example: Quarter of birth

Q3 Is the effect monotone?

A3 Some[†] argue that monotonicity may be violated in this setting.

Consider December births.

- Original idea: December birthdates will start school at age 5.7, inducing more years of education before 16.
- *Redshirting* idea: Parents hold back December kids so they can be older (*i.e.*, 6.7), inducing fewer years of education before 16.

IV and 2SLS

Conclusions

- 1. IV/2SLS focus on **isolating some "good" variation** in D_i via Z_i .
- 2. Important **requirements**: strong first stage, excludability, monotonicity.
- 3. IV and 2SLS **rescale the reduced form** with the first stage.
- 4. Estimates are **LATE from compliers**.
- 5. Different instruments can produce **different LATEs**.
- 6. A **weak first stage** can lead to problems.

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