Intro to AI and ML

Karthik Kanukollu, Praharsh Koppula

February 14, 2019

EE17BTECH11016, ME17BTECH11038

Original Question

"Let O be the vertex and Q be any point on the parabola $x^2 = 8y$. If the point P divides the line segment OQ internally in the ratio 1:3, then the locus of P is:"

Question in Matrix Form

Let O be the vertex and Q be any point on the parabola

$$\mathbf{x}^T \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \mathbf{x} + \mathbf{x}^T \begin{bmatrix} 0 \\ -8 \end{bmatrix} = 0.$$

If the point P divides the line segment OQ internally in the ratio 1:3, then find the locus of P.

Solution

Let $\mathbf{P} = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$ represent the point P. We need to find the locus of P.
Given P divides the line segment OQ internally in the ratio 1:3.
Thus, P = (1*Q + 3*O)/4.
That is, P=Q/4, or Q=4*P

Solution

But we know that Q lies on the given parabola. Thus,

$$16{*}\mathbf{P}^T \begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}\mathbf{P} + 4{*}\mathbf{P}^T\begin{bmatrix}0\\-8\end{bmatrix}=0.$$

Simplifying, we get:

$$\mathbf{P}^T \begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}\mathbf{P} + \mathbf{P}^T\begin{bmatrix}0\\-2\end{bmatrix}=0.$$

Solution

Replacing **P** with **x**, we get:

$$\mathbf{x}^T \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \mathbf{x} + \mathbf{x}^T \begin{bmatrix} 0 \\ -2 \end{bmatrix} = 0.$$

This is the locus of P.
Writing this in geometrical form, we get

$$x^2 - 2y = 0.$$

Plot

In the plot, we can see that for different values of Q on the original parabola, the corresponding values of P lie on the calculated locus, thus confirming that our answer is correct.