ASSIGNMENT 1

Applied Probability and Statistics (MG221)

Application of CLT

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MEAN

Distribution	Parameters	Observed Outcomes
Uniform	(0,1)	Approaches normality for n =6
Binomial		
	(12,0.01)	Slowly approaches normality for n= 300
	(20,0.5)	Rapidly approaches normality for n= 40
	(18,0.95)	Slowly approaches normality for n = 120
Poisson		
	(0.001)	Approaches normality for n= 6000
	(1)	Rapidly approaches normality for n = 50
	(25)	Rapidly approaches normality for n = 10
Exponential		
	(0.001)	Approaches normality for n = 50
	(1)	Rapidly approaches normality for n = 25
	(25)	Rapidly approaches normality for n= 30
Gamma		
	(0.001,1)	Slowly Approaches normality for n = 15000
	(2,1)	Rapidly approaches normality for n= 30
	(30,1)	Rapidly Approaches normality for n= 20
Beta		
	(2,2)	Rapidly Approaches normality for n= 10
	(20,2)	Rapidly Approaches normality for n= 25
	(2,8)	Rapidly Approaches normality for n = 20
	(0.5,0.5)	Rapidly Approaches normality for n = 10
	(0.5,0.2)	Rapidly Approaches normality for n = 20
	(0.2,5)	Rapidly Approaches normality for n = 50
Cauchy	(0,1)	Does not approach normality
Normal	(0,1)	Approaches normality for n= 1

Inferences:

1. Mean approaches normality for all distributions except Cauchy distribution.
2. The rate at which the mean approaches normality varies from one parent distribution to other.
3. The mean approaches normality for increasing value of n, as per the central limit theorem.

MEDIAN

Distribution	Parameters	Observed Outcomes
Uniform	(0,1)	Approaches normality for n =10
Binomial		
	(12,0.01)	Does not approach normality
	(20,0.5)	Does not approach normality
	(18,0.95)	Does not approach normality
Poisson		
	(0.001)	Does not approach normality
	(1)	Does not approach normality
	(25)	Does not approach normality
Exponential		
	(0.001)	Approaches quickly to normal as n approaches 40
	(1)	Gradually approaches to normal as n approaches 50
	(25)	Gradually approaches to normal as n approaches 50
Gamma		
	(0.001,1)	Does not approach normality
	(2,1)	Approaches quickly to normal as n approaches 25
	(30,1)	Rapidly approaches to normal as n approaches 2
Beta		
	(2,2)	Rapidly approaches to normal as n approaches 4
	(20,2)	Approaches quickly to normal as n approaches 25
	(20,8)	Approaches quickly to normal as n approaches 30
	(0.5,0.5)	Approaches quickly to normal as n approaches 28
	(0.5,0.2)	Gradually approaches to normal as n approaches 600
	(0.2,5)	Gradually approaches to normal as n approaches 700
Cauchy	(0,1)	Gradually approaches to normal as n approaches 50
Normal	(0,1)	Rapidly approaches to normal as n approaches 10

Inferences:

4. Median approaches normality for Uniform, Normal, Exponential, Cauchy and Beta.
5. The median does not approach normality for Gamma (0.001,1) since the value of alpha is very small. For other values of Gamma parameters, median approaches normality.

STANDARD DEVIATION

Distribution	Model	Observed outcomes
Uniform	(0,1)	Rapidly approaches to normal as n approaches 9
Binomial		
	(12,0.01)	Gradually approaches to normal as n approaches 220
	(20,0.5)	Approaches quickly to normal as n approaches 35
	(18,0.95)	Approaches quickly to normal as n approaches 40
Poisson		
	(0.001)	Approaches quickly to normal as n approaches 9000
	(1)	Gradually approaches to normal as n approaches 40
	(25)	Rapidly approaches to normal as n approaches 20
Exponential		
	(0.001)	Gradually approaches to normal as n approaches 50
	(1)	Gradually approaches to normal as n approaches 100
	(25)	Gradually approaches to normal as n approaches 80
Gamma		
	(0.001,1)	Very slowly approaches to normal as n approaches 30000
	(2,1)	Approaches quickly to normal as n approaches 60
	(30,1)	Rapidly approaches to normal as n approaches 40
Beta		
	(2,2)	Rapidly approaches to normal as n approaches 4
	(20,2)	Approaches quickly to normal as n approaches 30
	(20,8)	Approaches quickly to normal as n approaches 20
	(0.5,0.5)	Approaches quickly to normal as n approaches 32
	(0.5,0.2)	Approaches quickly to normal as n approaches 70
	(0.2,5)	Approaches quickly to normal as n approaches 35
Cauchy	(0,1)	Does not approach normality
Normal	(0,1)	Rapidly approaches to normal as n approaches 6

Inferences:

1. Distribution of Standard Deviation achieve normality for Uniform, Binomial, Poisson, Exponential, Normal, Beta.
2. Standard Deviation does not achieve normality for Cauchy Distribution.
3. For lower shape factor in Gamma Distribution, S.D. approaches normal very slowly while for higher shape factor it normalizes rapidly.

MINIMUM

Distribution	Parameter	Observed outcomes
Uniform	(0,1)	Does not approach normality
Binomial		
	(12,0.01)	Does not approach normality
	(20,0.5)	Does not approach normality
	(18,0.95)	Does not approach normality
Poisson		
	(0.001)	Does not approach normality
	(1)	Does not approach normality
	(25)	Does not approach normality
Exponential		
	(0.001)	Does not approach normality
	(1)	Does not approach normality
	(25)	Does not approach normality
Gamma		
	(0.001,1)	Does not approach normality
	(2,1)	Does not approach normality
	(30,1)	Does not approach normality
Beta		
	(2,2)	Does not approach normality
	(20,2)	Does not approach normality
	(20,8)	Does not approach normality
	(0.5,0.5)	Does not approach normality
	(0.5,0.2)	Does not approach normality
	(0.2,5)	Does not approach normality
Cauchy	(0,1)	Does not approach normality
Normal	(0,1)	Rapidly approaches to normal as n approaches 1

Inferences:

1. Minimum distributions do not approach to normality for any of the distributions.

MAXIMUM

Distribution	parameter	Observed outcomes
Uniform	(0,1)	Does not approach normality
Binomial		
	(12,0.01)	Does not approach normality
	(20,0.5)	Does not approach normality
	(18,0.95)	Does not approach normality
Poisson		
	(0.001)	Does not approach normality
	(1)	Does not approach normality
	(25)	Does not approach normality
Exponential		
	(0.001)	Does not approach normality
	(1)	Does not approach normality
	(25)	Does not approach normality
Gamma		
	(0.001,1)	Does not approach normality
	(2,1)	Does not approach normality
	(30,1)	Does not approach normality
Beta		
	(2,2)	Does not approach normality
	(20,2)	Does not approach normality
	(20,8)	Does not approach normality
	(0.5,0.5)	Does not approach normality
	(0.5,0.2)	Does not approach normality
	(0.2,5)	Does not approach normality
Cauchy	(0,1)	Does not approach normality
Normal	(0,1)	Does not approach normality

Inferences:

1. Maximum distribution does not approach normality for any of the distributions as it generally has skewed distribution.

Inter Quartile Range

Distribution	Parameters	Observed Outcome
Uniform	(0,1)	Rapidly approaches to normal as n approaches 12
Binomial		
	(12,0.01)	Does not approach normality
	(20,0.5)	Does not approach normality
	(18,0.95)	Does not approach normality
Poisson		
	(0.001)	Does not approach normality
	(1)	Does not approach normality
	(25)	Does not approach normality
Exponential		
	(0.001)	Slowly approaches to normal as n approaches 85
	(1)	Slowly approaches to normal as n approaches 80
	(25)	Slowly approaches to normal as n approaches 120
Gamma		
	(0.001,1)	Does not approach normality
	(2,1)	Rapidly approaches to normal as n approaches 80
	(30,1)	Rapidly approaches to normal as n approaches 50
Beta		
	(2,2)	Rapidly approaches to normal as n approaches 25
	(20,2)	Rapidly approaches to normal as n approaches 45
	(2,8)	Rapidly approaches to normal as n approaches 50
	(0.5,0.5)	Rapidly approaches to normal as n approaches 60
	(0.5,0.2)	Rapidly approaches to normal as n approaches 240
	(0.2,5)	Slowly approaches to normal as n approaches 400
Cauchy	(0,1)	Slowly approaches to normal as n approaches 105
Normal	(0,1)	Rapidly approaches to normal as n approaches 15

Inferences:

1. The IQR achieves normality for all the parameters of Uniform, Exponential, Beta, Cauchy, Normal while Does not approach normality for any of the parameters for Poisson and Binomial. The IQR Does not approach normality for Gamma (0.001,1) while approaches normality for other parameters.
2. Rate of approaching to normal varies from one parent distribution to other.

Conclusions

1. There was no significant pattern of various statistics for different distributions was observed. It varies with each distribution.

2. It was observed that not all statistics/distributions achieve Normality, for example mean of Cauchy (0,1) distribution.

3. The range of sample sizes to attain normality differ for different statistics across distributions.

4. The rate at which various statistics Normalize is not unique; some normalize rapidly ($n<10$), some quickly ($10<n<100$), some gradually ($100<n<1000$) and some very slowly ($n>1000$).

5. The speed of distribution approaching normality depends of symmetry of the parent distribution irrespective of the shape e.g. Beta (0.5,0.5).

6. The distribution approaching top normality depends on the statistic being considered on the way it's considered e.g. Minimum and maximum are unlikely to approach normal for any of the distribution as the graph for the extreme values being equally likely to occur tend to be highly skewed.

7. The distribution approaching top normality depends on the shape of the parent distribution being close to the nor mal distribution e.g. Beta (2,2).

8. The averages of IIDs are more likely to occur than other statistics.