APPLIED OPERATIONS RESEARCH

Linear Programming Assignment

Submitted by-

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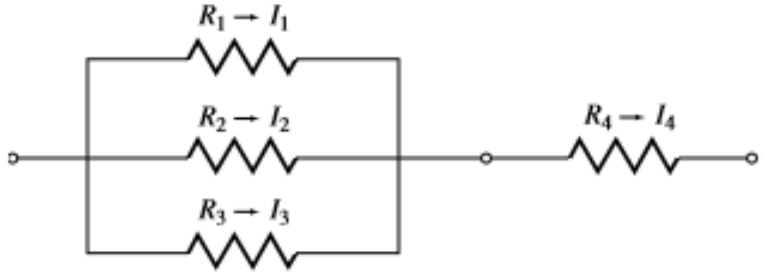
Problem Statement



‡Based on Lanzenauer et al. (1987).

Q) In the electrical circuit of Figure 11, It = current (in amperes) flowing through resistor t, Vt = voltage drop (in volts) across resistor t, and Rt = resistance (in ohms) of resistor t. Kirchoff's Voltage and Current Laws imply that V1 = V2 = V3 and I1 + I2 + I3 = I4. The power dissipated by the current flowing through resistor t is It 2Rt. Ohm's Law implies that Vt = ItRt. The two parts of this problem should be solved independently.

- a) Suppose you are told that I1 = 4, I2 = 6, I3 = 8, and I4 = 18 are required. Also, the voltage drop across each resistor must be between 2 and 10 volts. Choose the Rt's to minimize the total dissipated power. Formulate an LP whose solution will solve your problem.
- b)Suppose you are told that V1 = 6, V2 = 6, V3 = 6, and V4 = 4 are required. Also, the current flowing through each resistor must be between 2 and 6 amperes. Choose the Rt's to minimize the total dissipated power. Formulate an LP whose solution will solve your problem.

a) Verbal Formulation

Decision variables:

- Ri: Resistance across resistor i where i=1,2,3,4
- Vi: Voltage drop across resistor i where i=1,2,3,4
- Ii: Current passing through each resistor i where i=1,2,3,4
- Pi: Power dissipated across each resistor i where i=1,2,3,4

Constraints:

- Kirchoff's voltage law
- Kirchoff's current law
- Ohm's Law
- Voltage drop constraint
- Current constraint

Objective function:

- Minimize the total power dissipated

Mathematical Model

Decision variables:

- Ri: Resistance across resistor i where i=1,2,3,4
- Vi: Voltage drop across resistor i where i=1,2,3,4
- Ii: Current passing through each resistor i where i=1,2,3,4
- Pi: Power dissipated across each resistor i where i=1,2,3,4

Constraints:

- Kirchoff's voltage law: V1=V2=V3
- Kirchoff's current law: I1+I2+I3=I4
- Ohm's Law : Vi=Ii*Ri for i=1,2,3,4
- Voltage drop constraint: 2<=Vi<=10 for i=1,2,3,4
- Current Constraint: I1=4, I2=6, I3=8, I4=18

Objective function:

- Minimize the total power dissipated:

  Minimize P1+P2+P3+P4

Where Pi = Ii^2*Ri for i=1,2,3,4

LINDO Code

```
MINIMIZE  P1+P2+P3+P4
SUBJECT TO

CUR1) I1 = 4
CUR2) I2 = 6
CUR3) I3 = 8
CUR4) I4 = 18

OHM1) V1-4R1=0
OHM2) V2-6R2=0
OHM3) V3-8R3=0
OHM4) V4-18R4=0

KV1) V1-V2=0
KV2) V2-V3=0

KC1) I1+I2+I3-I4=0

V1A) V1<=10
V1B) V1>=2
V2A) V2<=10
V2B) V2>=2
V3A) V3<=10
V3B) V3>=2
V4A) V4<=10
V4B) V4>=2

POW1) P1-16R1=0
POW2) P2-36R2=0
POW3) P3-64R3=0
POW4) P4-324R4=0
```

LINDO Output

```
 LP OPTIMUM FOUND AT STEP      6

        OBJECTIVE FUNCTION VALUE

        1)       72.00000

 VARIABLE         VALUE          REDUCED COST
        P1         8.000000          0.000000
        P2        12.000000          0.000000
        P3        16.000000          0.000000
        P4        36.000000          0.000000
        I1         4.000000          0.000000
        I2         6.000000          0.000000
        I3         8.000000          0.000000
        I4        18.000000          0.000000
        V1         2.000000          0.000000
        R1         0.500000          0.000000
        V2         2.000000          0.000000
        R2         0.333333          0.000000
        V3         2.000000          0.000000
        R3         0.250000          0.000000
        V4         2.000000          0.000000
        R4         0.111111          0.000000

      ROW    SLACK OR SURPLUS      DUAL PRICES
     CUR1)         0.000000          0.000000
     CUR2)         0.000000          0.000000
     CUR3)         0.000000          0.000000
     CUR4)         0.000000          0.000000
     OHM1)         0.000000          4.000000
     OHM2)         0.000000          6.000000
     OHM3)         0.000000          8.000000
     OHM4)         0.000000         18.000000
      KV1)         0.000000         -4.000000
      KV2)         0.000000        -10.000000
      KC1)         0.000000          0.000000
      V1A)         8.000000          0.000000
      V1B)         0.000000          0.000000
      V2A)         8.000000          0.000000
      V2B)         0.000000          0.000000
      V3A)         8.000000          0.000000
      V3B)         0.000000        -18.000000
      V4A)         8.000000          0.000000
      V4B)         0.000000        -18.000000
     POW1)         0.000000         -1.000000
     POW2)         0.000000         -1.000000
     POW3)         0.000000         -1.000000
     POW4)         0.000000         -1.000000
```

b) Verbal Formulation

Decision variables:

- Ci: Inverse of Resistance across resistor i where i=1,2,3,4
- Vi: Voltage drop across resistor i where i=1,2,3,4
- Ii: Current passing through each resistor i where i=1,2,3,4
- Pi: Power dissipated across each resistor i where i=1,2,3,4

Constraints:

- Kirchoff's voltage law
- Kirchoff's current law
- Ohm's Law
- Voltage constraint
- Current constraint

Objective function:

- Minimize the total power dissipated

Mathematical Model

Decision variables:

- Ci: Inverse of Resistance (1/Ri) across resistor i where i=1,2,3,4
- Vi: Voltage drop across resistor i where i=1,2,3,4
- Ii: Current passing through each resistor i where i=1,2,3,4
- Pi: Power dissipated across each resistor i where i=1,2,3,4

Constraints:

- Kirchoff's voltage law: V1=V2=V3
- Kirchoff's current law: I1+I2+I3=I4
- Ohm's Law : Ii=Vi*Ci for i=1,2,3,4
- Voltage constraint: V1=6,V2=6,V3=6,V4=4
- Current constraint: 2<=Ii<=6 for i=1,2,3,4

Objective function:

- Minimize the total power dissipated:

  Minimize P1+P2+P3+P4

Where Pi = Vi^2*Ci for i=1,2,3,4

LINDO Code

```
MINIMIZE  P1+P2+P3+P4
SUBJECT TO

Vol1) V1 = 6
Vol2) V2 = 6
Vol3) V3 = 6
CVol4) V4 = 4

OHM1) I1 - 6C1=0
OHM2) I2 - 6C2=0
OHM3) I3 - 6C3=0
OHM4) I4 - 4C4=0

KC1) I1+I2+I3-I4=0

I1A) I1<=6
I1B) I1>=2
I2A) I2<=6
I2B) I2>=2
I3A) I3<=6
I3B) I3>=2
I4A) I4<=6
I4B) I4>=2

POW1) P1-36C1=0
POW2) P2-36C2=0
POW3) P3-36C3=0
POW4) P4-16C4=0
```

LINDO Output

```
LP OPTIMUM FOUND AT STEP      5

        OBJECTIVE FUNCTION VALUE

        1)       60.00000

 VARIABLE        VALUE          REDUCED COST
       P1        12.000000          0.000000
       P2        12.000000          0.000000
       P3        12.000000          0.000000
       P4        24.000000          0.000000
       V1         6.000000          0.000000
       V2         6.000000          0.000000
       V3         6.000000          0.000000
       V4         4.000000          0.000000
       I1         2.000000          0.000000
       C1         0.333333          0.000000
       I2         2.000000          0.000000
       C2         0.333333          0.000000
       I3         2.000000          0.000000
       C3         0.333333          0.000000
       I4         6.000000          0.000000
       C4         1.500000          0.000000

      ROW    SLACK OR SURPLUS      DUAL PRICES
     VOL1)         0.000000          0.000000
     VOL2)         0.000000          0.000000
     VOL3)         0.000000          0.000000
    CVOL4)         0.000000          0.000000
     OHM1)         0.000000          6.000000
     OHM2)         0.000000          6.000000
     OHM3)         0.000000          6.000000
     OHM4)         0.000000          4.000000
      KC1)         0.000000          4.000000
      I1A)         4.000000          0.000000
      I1B)         0.000000        -10.000000
      I2A)         4.000000          0.000000
      I2B)         0.000000        -10.000000
      I3A)         4.000000          0.000000
      I3B)         0.000000        -10.000000
      I4A)         0.000000          0.000000
      I4B)         4.000000          0.000000
     POW1)         0.000000         -1.000000
     POW2)         0.000000         -1.000000
     POW3)         0.000000         -1.000000
     POW4)         0.000000         -1.000000
```

Report for (a) part:

- According to the optimal solution that has been obtained, minimum Power dissipated is 72 Watts.
- Resistors: R1= 0.5 ohms, R2=0.33ohms, R3= 0.25 ohms, R4=0.111 ohms
- Current: I1=4A, I2=6A, I3=8A, I4=18A
- Voltage drop: V1=V2=V3=V4=2 Volts
- Power dissipated across each resistor: P1=8W, P2=12W, P3=16W, P4=36W
- Total Power dissipated= P1+P2+P3+P4= 72W

Report for (b) part:

- According to the optimal solution that has been obtained, minimum Power dissipated is 60 Watts.
- Resistors:

R1= 1/C1= 1/0.333= 3ohms

R2= 1/C2= 1/0.333= 3ohms

R3= 1/C3= 1/0.333= 3ohms

R4= 1/C4= 1/1.5= 0.66667 ohms

- Current: I1=2A, I2=2A, I3=2A, I4=6A
- Voltage drop: V1=6v ,V2=6v ,V3=6v ,V4=4v
- Power dissipated across each resistor: P1=12W, P2=12W, P3=12W, P4=24W
- Total Power dissipated= P1+P2+P3+P4= 60W

Thankyou