APPLIED OPERATIONS RESEARCH

Assignment 2

Submitted by-

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Problem Statement

TABLE 117

Normal			Tumor			
1	2	3	1	2	3	Beam
16	12	8	20	12	6	1
12	10	6	18	15	8	2
9	8	13	13	10	17	3
4	12	12	6	18	16	4
9	4	11	13	5	14	5
8	7	7	10	10	10	6

Q) In treating a brain tumor with radiation, physicians want the maximum amount of radiation possible to bombard the tissue containing the tumors. The constraint is, however, that there is a maximum amount of radiation that normal tissue can handle without suffering tissue damage. Physicians must therefore decide how to aim the radiation so as to maximize the radiation that hits the tumor tissue subject to the constraint of not damaging the normal tissue. As a simple example of this situation, suppose six types of radiation beams (beams differ in for they are aimed and their intensity) can be aimed at a tumor. The region containing the tumor has been divided into six regions: three regions contain tumors and three contain normal tissue. The amount of radiation delivered to each region by each type of beam is shown in Table 117.

If each region of normal tissue can handle at most 40 units of radiation, then for beams should be used to maximize the total amount of radiation received by the tumors?

Verbal Formulation

Decision variables:

- Bi: 1 if Beam i is used for transmitting radiation, 0 otherwise ; for i=1,2,3,4,5,6
- RBi: Total amount of radiation radiated by beam i ;for i=1,2,3,4,5,6
- Ni: Amount of radiation falling on region i of normal tissue ;for i=1,2,3
- Ti: Amount of radiation falling on region i of Tumor ;for i=1,2,3
- Nbi: Amount of radiation falling on normal tissue from beam i ;for i=1,2,3,4,5,6
- Tbi: Amount of radiation falling on tumor from beam i ;for i= 1,2,3,4,5,6

Constraints:

- Normal tissue radiated by each beam constraint
- Normal tissue region constraint
- Radiation supply constraint by each beam

Objective function:

- Maximize the total radiation by beams

Mathematical Model

Decision variables:

- B_i: 1 if Beam i is used for transmitting radiation, 0 otherwise ; for i=1,2,3,4,5,6
- RB_i: Total amount of radiation radiated by beam i ;for i=1,2,3,4,5,6
- N_i: Amount of radiation falling on region i of normal tissue ;for i=1,2,3
- T_i: Amount of radiation falling on region i of Tumor ;for i=1,2,3
- Nb_i: Amount of radiation falling on normal tissue from beam i ;for i=1,2,3,4,5,6
- Tb_i: Amount of radiation falling on tumor from beam i ;for i= 1,2,3,4,5,6

Constraints:

- Normal tissue radiated by each beam constraint :

$N1= 16B1 + 12B2 + 9B3 + 4B4 + 9B5 + 8B6$
$N2= 12B1 + 10B2 + 8B3 + 12B4 + 4B5 + 7B6$
$N3= 8B1 + 6B2 + 13B3 + 12B4 + 11B5 + 7B6$

- Normal tissue region constraint: $N_i \le 40$ for i=1,2,3
- Radiation supply constraint by each beam : $RB1= 74B1$; $RB2= 69B2$; $RB3= 70B3$; $RB4= 68B4$; $RB5= 56B5$; $RB6= 52B6$;

Objective function:

- Maximize the total radiation by beams

Maximize $74B1+69B2+70B3+68B4+56B5+52B6$

LINGO Code

```
TITLE "Treating Brain Tumor";
!Decision Variables;
@BIN(B1); ! 1 if beam 1 is used;
@BIN(B2); ! 1 if beam 2 is used;
@BIN(B3); ! 1 if beam 3 is used;
@BIN(B4); ! 1 if beam 4 is used;
@BIN(B5); ! 1 if beam 5 is used;
@BIN(B6); ! 1 if beam 6 is used;
@FREE(RB1); ! Amount of Radiation from beam 1;
@FREE(RB2); ! Amount of Radiation from beam 2;
@FREE(RB3); ! Amount of Radiation from beam 3;
@FREE(RB4); ! Amount of Radiation from beam 4;
@FREE(RB5); ! Amount of Radiation from beam 5;
@FREE(RB6); ! Amount of Radiation from beam 6;
@FREE(N1);  ! Amount of radiation falling on region 1 of normal tissue;
@FREE(N2);  ! Amount of radiation falling on region 2 of normal tissue;
@FREE(N3);  ! Amount of radiation falling on region 3 of normal tissue;
@FREE(NB1);  ! Amount of radiation falling on normal tissue from beam 1;
@FREE(NB2);  ! Amount of radiation falling on normal tissue from beam 2;
@FREE(NB3);  ! Amount of radiation falling on normal tissue from beam 3;
@FREE(NB4);  ! Amount of radiation falling on normal tissue from beam 4;
@FREE(NB5);  ! Amount of radiation falling on normal tissue from beam 5;
@FREE(NB6);  ! Amount of radiation falling on normal tissue from beam 6;
@FREE(TB1);  ! Amount of radiation falling on tumor from beam 1;
@FREE(TB2);  ! Amount of radiation falling on tumor from beam 2;
@FREE(TB3);  ! Amount of radiation falling on tumor from beam 3;
@FREE(TB4);  ! Amount of radiation falling on tumor from beam 4;
@FREE(TB5);  ! Amount of radiation falling on tumor from beam 5;
@FREE(TB6);  ! Amount of radiation falling on tumor from beam 6;
@FREE(T1);  ! Amount of radiation falling on region 1 of tumor;
@FREE(T2);  ! Amount of radiation falling on region 2 of tumor;
@FREE(T3);  ! Amount of radiation falling on region 3 of tumor;


!OBJECTIVE FUNCTION;
max = 74*B1 + 69*B2 + 70*B3 + 68*B4 + 56*B5 + 52*B6;
```

```
!CONSTRAINTS;
N1= 16*B1 + 12*B2 + 9*B3 + 4*B4 + 9*B5 + 8*B6;
N2= 12*B1 + 10*B2 + 8*B3 + 12*B4 + 4*B5 + 7*B6;
N3= 8*B1 + 6*B2 + 13*B3 + 12*B4 + 11*B5 + 7*B6;
T1= 20*B1 + 18*B2 + 13*B3 + 6*B4 + 13*B5 + 10*B6;
T2= 12*B1 + 15*B2 + 10*B3 + 18*B4 + 5*B5 + 10*B6;
T3= 6*B1 + 8*B2 + 17*B3 + 16*B4 + 14*B5 + 10*B6;
N1 <= 40;
N2 <= 40;
N3 <= 40;
RB1= 74*B1;
NB1= 36*B1;
TB1= 38*B1;
RB2= 69*B2;
NB2= 28*B2;
TB2= 41*B2;
RB3= 70*B3;
NB3= 30*B3;
TB3= 40*B3;
RB4= 68*B4;
NB4= 28*B4;
TB4= 40*B4;
RB5= 56*B5;
NB5= 24*B5;
TB5= 32*B5;
RB6= 52*B6;
NB6= 22*B6;
TB6= 30*B6;
N1>=0;
N2>=0;
N3>=0;
T1>=0;
T2>=0;
T3>=0;
NB1>=0;
NB2>=0;
NB3>=0;
NB4>=0;
NB5>=0;
NB6>=0;
```

LINGO Output

```
Global optimal solution found.
Objective value:                              264.0000
Objective bound:                              264.0000
Infeasibilities:                              0.000000
Extended solver steps:                               0
Total solver iterations:                             0
Elapsed runtime seconds:                          0.10

Model Class:                                      MILP

Total variables:                     30
Nonlinear variables:                  0
Integer variables:                    6

Total constraints:                   52
Nonlinear constraints:                0

Total nonzeros:                     111
Nonlinear nonzeros:                   0
```

Variable	Value	Reduced Cost
B1	1.000000	-74.00000
B2	0.000000	-69.00000
B3	1.000000	-70.00000
B4	1.000000	-68.00000
B5	0.000000	-56.00000
B6	1.000000	-52.00000
RB1	74.00000	0.000000
RB2	0.000000	0.000000
RB3	70.00000	0.000000
RB4	68.00000	0.000000
RB5	0.000000	0.000000
RB6	52.00000	0.000000
N1	37.00000	0.000000
N2	39.00000	0.000000
N3	40.00000	0.000000
NB1	36.00000	0.000000
NB2	0.000000	0.000000
NB3	30.00000	0.000000
NB4	28.00000	0.000000
NB5	0.000000	0.000000
NB6	22.00000	0.000000
TB1	38.00000	0.000000
TB2	0.000000	0.000000
TB3	40.00000	0.000000
TB4	40.00000	0.000000
TB5	0.000000	0.000000
TB6	30.00000	0.000000
T1	49.00000	0.000000
T2	50.00000	0.000000
T3	49.00000	0.000000

Managerial Report

BEAM	NORMAL TISSUE				TUMOR				
	1	2	3	TOTAL NORMAL	1	2	3	TOTAL TUMOR	TOTAL RADIATION
1	16	12	8	36	20	12	6	38	74
2	0	0	0	0	0	0	0	0	0
3	9	8	13	30	13	10	17	40	70
4	4	12	12	28	6	18	16	40	68
5	0	0	0	0	0	0	0	0	0
6	8	7	7	22	10	10	10	30	52
TOTAL	**37**	**39**	**40**	**116**	**49**	**50**	**49**	**148**	**264**

According to the optimal solution obtained, total amount of radiation is **264** units out of which **116** units fall on Normal tissues and **148** units fall on Tumors.
Beams used to achieve this: Beam 1,Beam 3,Beam 4 and Beam 6

Thankyou