Applied Operation Research

ASSIGNMENT 4 – DYNAMIC PROGRAMMING

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7.2.3 Equipment Replacement Problem using Dynamic Programming

Q) This DP formulation is illustrated by solving a numerical problem for a 4-year planning horizon. The age of the machine at the beginning of the first year is 2. The cost of a new machine in year 1 is $58,000 and increases by $2,000 every year. Annual operating costs along with trade-in and salvage values are time independent.

Ans) DP Formulation:

Stage n: Each year count(index) i.e., 1,2,3,4,5

State S_n: Age of the machine in service at the beginning of a year(stage)

Decision X_n: “buy” a new machine or “keep” the old one at the beginning of Stage n

$f_n(S_n, X_n)$: Total net cost from Stage n to the end of year N(4), starting with an S_n year old machine in stage n and decision is X_n.

Optimal value function ($f_n^*(S_n)$): minimum cost of owning a machine from the beginning of Stage n to the end of year N i.e., 4^{th} year (or beginning of year 5^{th}), starting Stage n with a machine that just turned age S_n.

X_n^* : Optimal Action: keep or buy for getting $f_n^*(S_n)$ at that stage n

$C_n(S_n, X_n)$: Cost in the stage n depending on the age of the machine(S_n) and decision X_n

Cost in Stage n:

At the beginning of a stage, two possible decisions can be made: keep the current machine or replace the machine. If we decide to keep the machine, then the only cost in the current stage will be the cost of operating the machine. If the decision is to replace the machine, the annual cost will include the price of a new machine, minus the trade-in value received for the current machine, plus the operating cost of a new machine. The cost in stage n, $C_n(S_n, X_n)$, depends on the age of the current machine and the decision:

$C_n(S_n, X_n) = a(n) - t(S_n) + r(0)$, if X_n = buy a new machine

$C_n(S_n, X_n) = r(S_n)$, if X_n = keep the current machine

State for the next stage based on the current state, stage, and decision:

$S_{n+1} = 1$, if X_n = “buy”

$S_{n+1} = S_n + 1$, if X_n = “keep”

Stage 5:

Let us begin with the specification of the boundary condition. For this purpose, it is convenient to view the end of year 4 as the beginning of a final stage 5, **where the only available action is to salvage the machine in service**. In this stage the revenue received from salvaging a machine can be interpreted as a negative cost.

Age of Machine at the beginning of that stage = S_5	Total net cost incurred At Stage S_5: $f_5(S_5, X_5)$	Minimum net cost incurred at Stage S_5 : $f_{5*}(S_5)$
1	-30	-30
2	-20	-20
3	-10	-10
4	-5	-5
6	0	0

Stage 4:

Age of Machine at the beginning of that stage = S_4	Total net cost incurred from Stage S_4 to the end by taking decision=X_4 :$f_4(S_4, X_4)= C_4(S_4,X_4) + f_{5*}(S_5)$ Buy	Keep	Minimum cost from Stage S_4 to the end : $f_{4*}(S_4)$	Optimal Action (X_4*)
1	(58+6)−35+12+(−30)= 11	15+(-20)= -5	-5	Keep
2	(58+6)−25+12+(−30)= 21	25+(-10)= 15	15	keep
3	(58+6)−15+12+(−30) =31	35+(-5)= 30	30	buy
5	(58+6)−5+12+(−30) =41	80+(0)= 80	41	buy

Stage 3:

Age of Machine at the beginning of that stage = S_3	Total net cost incurred from Stage S_3 to the end by taking decision=X_3: $f_3(S_3, X_3)= C_3(S_3,X_3) + f_{4*}(S_4)$ Buy	Keep	Minimum cost from Stage S_3 to the end : $f_{3*}(S_3)$	Optimal Action (X_3*)
1	(58+4)−35+12+(−5)= 34	15+15= 30	30	keep
2	(58+4)−25+12+(−5)= 44	25+30= 55	44	buy
4	(58+4)−15+12+(−5) =54	60+30= 90	54	buy

Stage 2:

Age of Machine at the beginning of that stage = S_2	Total net cost incurred from Stage S_2 to the end by taking decision=X_2: $f_2(S_2, X_2)= C_2(S_2,X_2) + f_{3*}(S_3)$ Buy	Keep	Minimum cost from Stage S_2 to the end :$f_{2*}(S_2)$	Optimal Action (X_2*)
1	(58+2)−35+12+30= 67	15+44= 59	59	Keep
3	(58+2)−15+12+30= 87	35+54= 89	87	buy

Stage 1:

Age of Machine at the beginning of that stage = S_1	Total net cost incurred from Stage S_1 to the end by taking decision=X_1: $f_1(S_1, X_1)= C_1(S_1,X_1) + f_{2*}(S_2)$ Buy	Keep	Minimum cost from Stage S_1 to the end :$f_{1*}(S_1)$	Optimal Action (X_1*)
2	58−25+12+59= 104	25+87= 112	104	buy

Optimal solution:

The total minimum cost for the 4-year planning horizon is **$104,000** based on an optimal replacement policy that includes **buying a new machine at the beginning of years 1 and 3**.

Recursive formulation:

If (X_n*= “buy”), Then

$f_n(S_n, X_n)= a(n)-t(S_n)-r(0)+ f_{n+1}^*(1)$ for S_n =1 ,2,...,i−1,i+1

Else

$f_n(S_n, X_n)= r(S_n)+f_{n+1}^*(S_n+1)$ for S_n =1 ,2,...,i−1,i+1

$f_n^*(S_n)= \min(f_n(S_n, X_n))$