Practical 5 Tutor's Guide
Linear Programming

Q1 Lecture Finance Example

A financial adviser at BMC bank needs to determine how to invest $100,000 in the following collection of bonds to maximize the annual return. The adviser wants to invest at least 50% of the money in short-term issues and no more than 50% in high-risk issues. At least 30% of the funds should go in tax-free investments and at least 40% of the total annual return should be tax free. Create a LP model for this scenario and find out the optimal solution.

Bond	Annual Return	Maturity	Risk	Tax-Free
A	9.5%	12 months	High	Yes
B	8.0%	6 months	Low	Yes
C	9.0%	12 months	Low	No
D	9.0%	12 months	High	Yes
E	9.0%	6 months	high	No

Objective Function (done in lecture):

Returns, R = 9.5%A + 8%B + 9%C + 9%D + 9%E (MAX)

Constraints (done in lecture):

1. A + B + C + D + E ≤ 100,000
2. B + E ≥ 0.5* (Total funds invested)
3. A + D + E ≤ 0.5* (Total funds invested)
4. A + B + D ≥ 0.3* (Total funds invested)
5. 9.5%A + 8%B + 9%D ≥ 0.4*(Returns)
6. A, B, C, D, E ≥ 0

Bond	Annual Return	Maturity	Risk	Tax-Free
A	9.50%	12 months	High	Yes
B	8.00%	6 months	Low	Yes
C	9.00%	12 months	Low	No
D	9.00%	12 months	High	Yes
E	9.00%	6 months	High	No

	Bond A	Bond B	Bond C	Bond D	Bond E			
Decision Variables	20339.0	20339.0	29661.0	0.0	29661.0	100000	(total amount invested)	
Objective Function:	9.50%	8.00%	9.00%	9.00%	9.00%	8898.305	(Annual return to be MAX)	
Constraints:								
Min 50% Short-term	0	1	0	0	1	50000	>=	$50,000
Max 50% high-risk	1	0	0	1	1	50000	<=	$50,000
Min 30% fund tax free	1	1	0	1	0	40677.97	>=	$30,000
Min 40% return tax free	1	1	0	1	0	3559.322	>=	$3,559
Total amount invested							=	$100,000

Q2 Winery

A winery has the following capacity to produce an exclusive dinner wine at either of its two vineyards at the indicated costs:

Vineyard	Capacity	Cost per Bottle
1	3,500 bottles	$23
2	3,100 bottles	$25

Four Italian restaurants around the country are interested in purchasing this wine. Because the wine is exclusive, they all want to buy as much as they need but will take whatever they can get. The maximum amounts required by the restaurants and the prices they are willing to pay per bottle are shown below.

Restaurant	Maximum Demand	Price
1	1,800 bottles	$69
2	2,300 bottles	$67
3	1,250 bottles	$70
4	1,750 bottles	$66

The costs of shipping a bottle from the vineyards to the restaurants are summarized in the following table.

	Restaurant			
Vineyard	1	2	3	4
1	$7	$8	$13	$9
2	$12	$6	$8	$7

The winery needs to determine the production and shipping plan that allows it to maximize its profits on this wine.

Let X_{ij} be the number of bottles produced at vineyard *i* sold to restaurant *j*
(Note that this value must be integer)

Hint – compute the profit of one bottle of wine from vineyard 1 that is sold to restaurant 1. Remember that Profit = Revenue – Cost.

For example:
The profit for selling one bottle of wine from vineyard 1 to restaurant 1 is represented by X_{11} and the profit = $69 – ($23+$7) = $69 - $30 = $39. So the same approach to compute for the rest of the combinations.

$$\text{MAX} \quad 39X_{11} + 36X_{12} + 34X_{13} + 34X_{14} + 32X_{21} + 36X_{22} + 37X_{23} + 34X_{24}$$

$$\text{ST} \quad X_{11} + X_{12} + X_{13} + X_{14} = 3{,}500$$

$$X_{21} + X_{22} + X_{23} + X_{24} = 3{,}100$$

$$X_{11} + X_{21} \leq 1800$$

$$X_{12} + X_{22} \leq 2300$$

$$X_{13} + X_{23} \leq 1250$$

$$X_{14} + X_{24} \leq 1750$$

$$X_{ij} \geq 0$$

X_{11} = 1,800, X_{12} = 1,700, X_{22} = 600, X_{23} = 1,250, X_{24} = 1,250, Maximum profit = $241,750