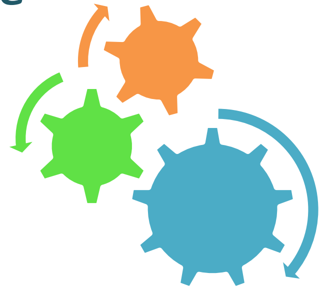
IT1311 Decision Analysis

Project Scheduling: PERT

Project Evaluation and Review Technique



Background

- We had been assuming that the project completion time for a task is fixed.
- But in the real-world, it’s very rare for any task to be fixed in duration e.g.
  – Construction meets bad weather
  – IT team has change in team members
  – Logistics meets pandemic
- To model uncertainty in a task’s duration, we can do some estimates

Project scheduling with uncertain activity times

- In the <u>three-time estimate approach</u>, the time to complete an activity is assumed to follow a Beta distribution.
- An activity's <u>mean completion time</u> is:

$$t = (a + 4m + b)/6$$

a = the <u>optimistic</u> completion time estimate

b = the <u>pessimistic</u> completion time estimate

m = the <u>most likely</u> completion time estimate

Project scheduling with uncertain activity times

- An activity's <u>completion time variance</u> is:

$$\sigma^2 = ((b-a)/6)^2$$

  a = the <u>optimistic</u> completion time estimate

  b = the <u>pessimistic</u> completion time estimate

  m = the <u>most likely</u> completion time estimate

- In the three-time estimate approach, the critical path is determined as if the mean times for the activities were fixed times.
- The <u>overall project completion time</u> is assumed to have a normal distribution with mean equal to the sum of the means along the critical path and variance equal to the sum of the variances along the critical path

An Example : The Daugherty Porta-Vac Project

The H.S. Daugherty Company is considering manufacturing a new product on cordless vacuum cleaner. The table below shows the activities involved in this project.

Activity	Description	Immediate Predecessor
A	Develop product design	-
B	Plan market research	-
C	Prepare routing	A
D	Build prototype model	A
E	Prepare marketing brochure	A
F	Prepare cost estimates	C
G	Do preliminary product testing	D
H	Complete market survey	B,E
I	Prepare pricing and forecast report	H
J	Prepare final report	F,G,I

The Daugherty Porta-Vac Project : Optimistic, Most Probable and Pessimistic Estimates

Note that these equations are based on the assumption that the activity time distribution can be described By ***Beta Probability Distribution***.

	Optimistic	Most Probable	Pessimistic
Activity	(a)	(m)	(b)
A	4	5	12
B	1	1.5	5
C	2	3	4
D	3	4	11
E	2	3	4
F	1.5	2	2.5
G	1.5	3	4.5
H	2.5	3.5	7.5
I	1.5	2	2.5
J	1	2	3

Using $t = (a + 4m + b)/6$

Activity A:

Mean completion time

$= (4+4(5) + 12)/6 = 6$

$\sigma^2 = ((b-a)/6)^2$

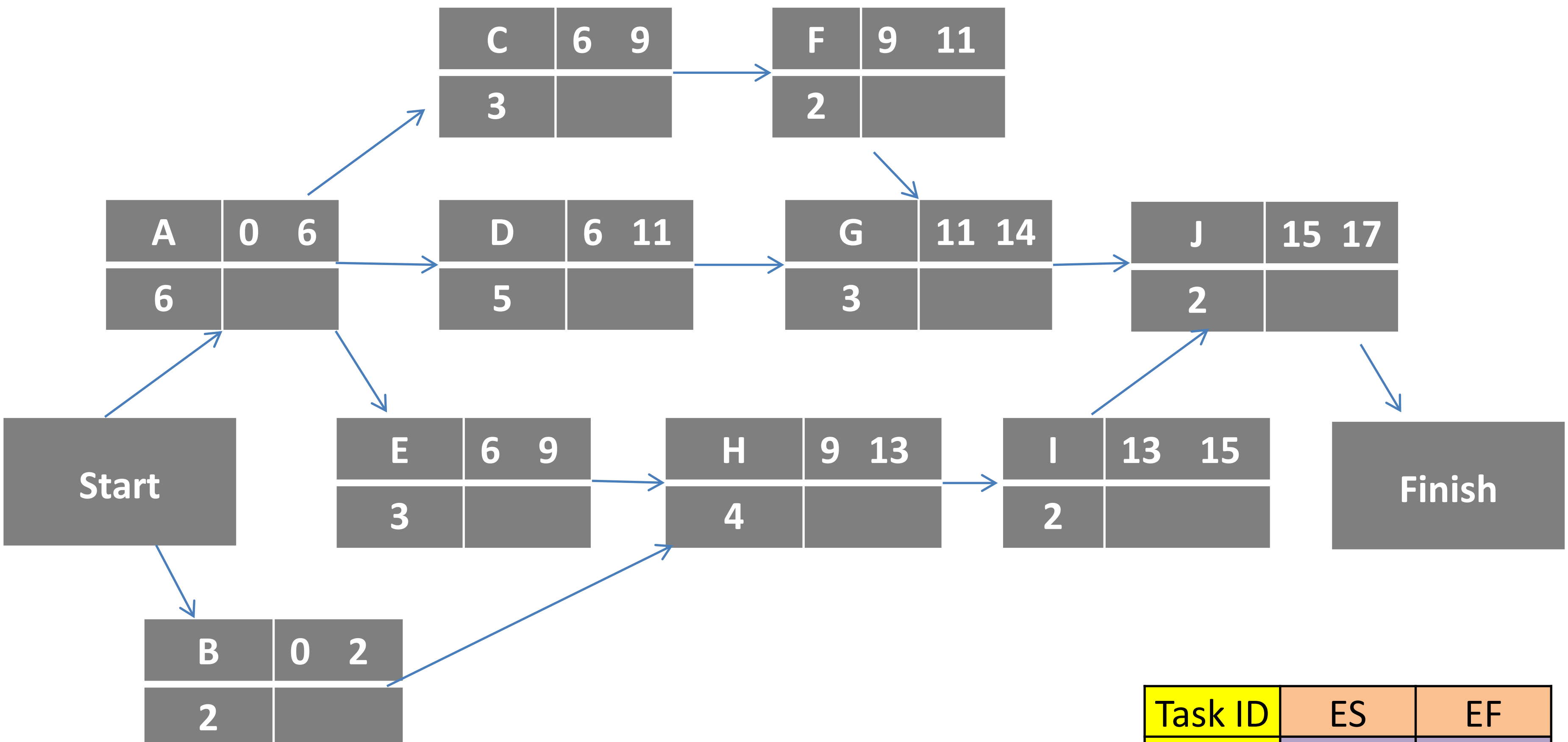
Variance $= ((12-4)/6)^2$

$= 1.78$

Expected times and Variances for the Porta-Vac Project Activities

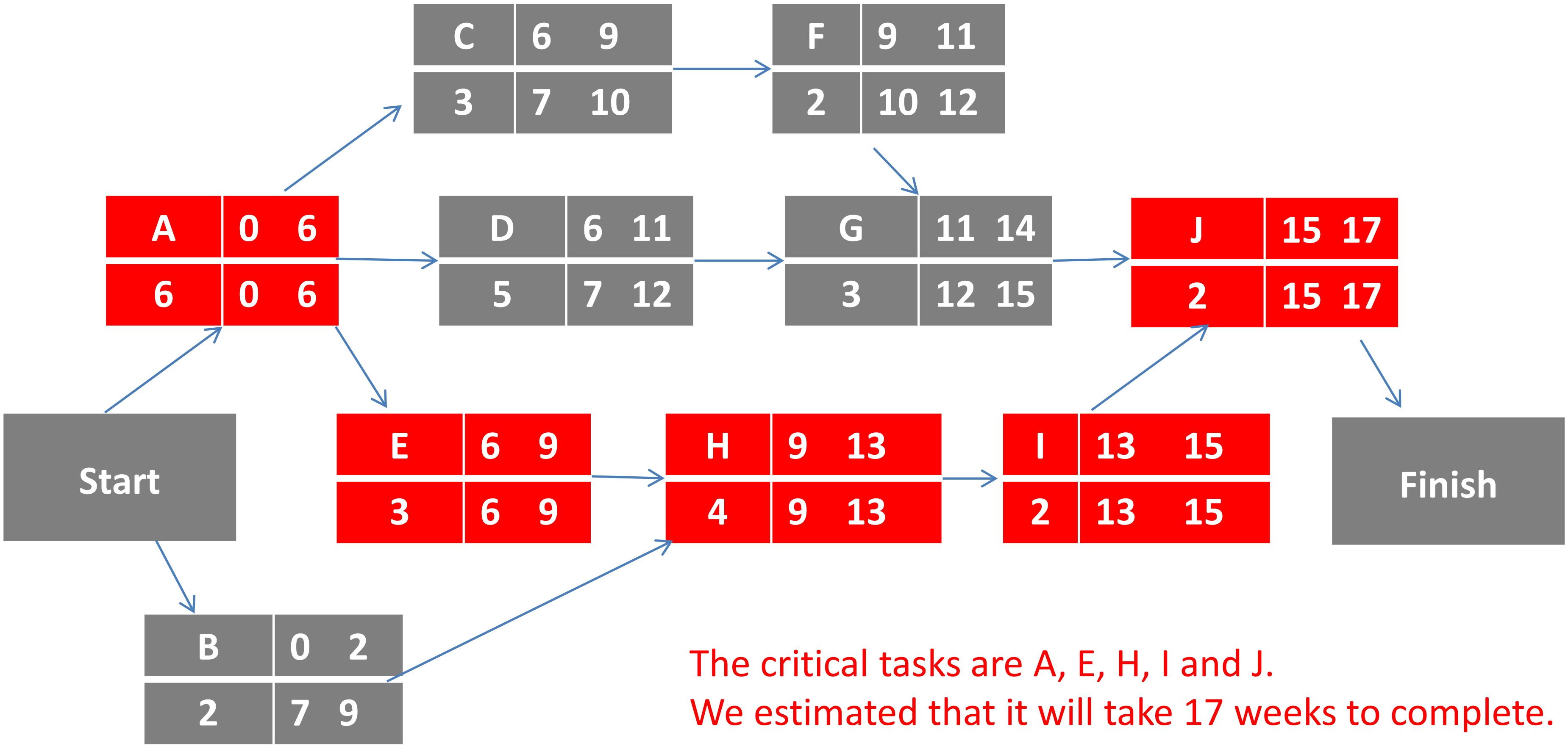
Activity	Expected Time (weeks)	Variance
A	6	1.78
B	2	0.44
C	3	0.11
D	5	1.78
E	3	0.11
F	2	0.03
G	3	0.25
H	4	0.69
I	2	0.03
J	2	0.11

Porta-Vac Project Network with Earliest Start and Earliest Finish Times



Task ID	ES	EF
Dur	LS	LF

Porta-Vac Project Network with Latest Start and Latest Finish Times



The critical tasks are A, E, H, I and J.
We estimated that it will take 17 weeks to complete.

Activity Schedule for Porta-Vac Project

Activity	Earliest Start (ES)	Latest Start (LS)	Earliest Finish (EF)	Latest Finish (LF)	Slack (LS – ES)	Critical?
A	0	0	6	6	0	yes
B	0	7	2	9	7	
C	6	10	9	13	4	
D	6	7	11	12	1	
E	6	6	9	9	0	yes
F	9	13	11	15	4	
G	11	12	14	15	1	
H	9	9	13	13	0	yes
I	13	13	15	15	0	yes
J	15	15	17	17	0	yes

Porta-Vac Project – Answer these questions

1. What is the total time required to complete the project?

   6 + 3 + 4 + 2 + 2 = 17 weeks

2. What is the variance in the project completion time?

   It is the sum of the variances of the critical activities

   (A, E, H, I , J)= 2.72

3. What is the probability of meeting the project completion within 20 weeks?

   Why do we ask this question?

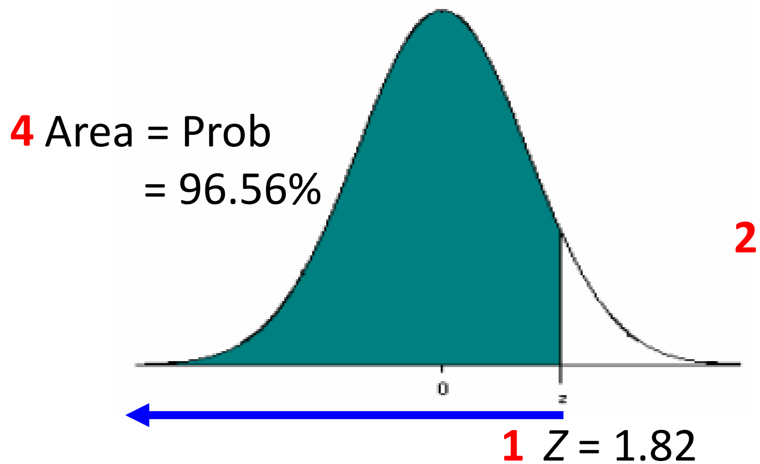
   Assuming that the project distribution follows the normal distribution, with x=20 and μ=17, σ = sqrt(2.72) = 1.65

   Z = (20 – 17) / 1.65 = 1.82

   Using Z = 1.82, p=0.9656 => 97% will complete within 20 weeks

Recall: Standardisation formula is $Z = \frac{X - \mu}{\sigma}$

Finding Prob from Normal Distribution Table

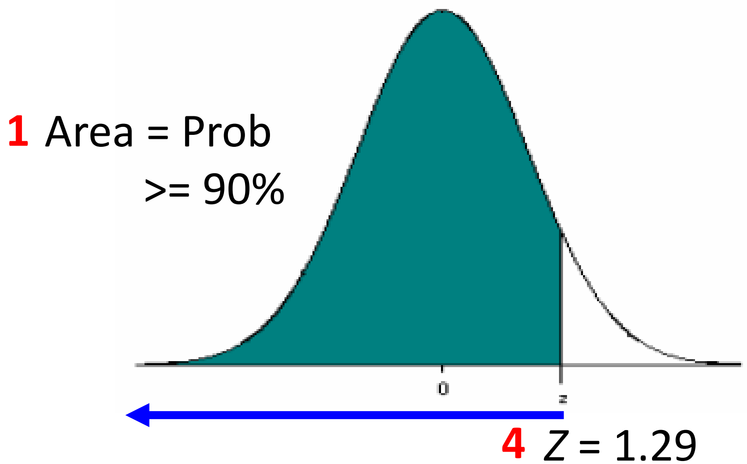


z	0.00	0.01	0.02	0.03
1.5	0.9332	0.9345	0.9357	0.9370
1.6	0.9452	0.9463	0.9474	0.9484
1.7	0.9554	0.9564	0.9573	0.9582
1.8	0.9641	0.9649	0.9656	0.9664
1.9	0.9713	0.9719	0.9726	0.9732
2.0	0.9772	0.9778	0.9783	0.9788

Steps

1. Calculate Z using $\frac{X-\mu}{\sigma}$ e.g Z = 1.82
2. Locate the first two digits of Z in the vertical column of the normal distribution table.
3. Locate the last digit of Z in the horizontal row of the normal distribution table.
4. The probability of any value less than Z happening is the intersection e.g. P(z <= Z) = 0.9656

Finding Value from Normal Distribution Table



z	0.07	0.08	0.09
1.0	0.8577	0.8599	0.8621
1.1	0.8790	0.8810	0.8830
1.2	0.8980	0.8997	0.9015
1.3	0.9147	0.9162	0.9177
1.4	0.9292	0.9306	0.9319

Steps

1. Given condition e.g. at least 90%, locate suitable probability in the middle section of the normal distribution table.
2. Locate the first two digits of Z horizontally.
3. Locate the last digit of Z vertically.
4. Combine the digits to get e.g. Z = 1.29

Summary

- PERT (Program Evaluation and review Technique) and CPM (Critical Path Method) can be used to plan, schedule and control a wide variety of projects
- The project schedule developed using this approach depicts the activities to be carried out in projects and their precedence relationships
- From there, the critical path can also be identified to help project managers closely monitor the progress
- We will also learn how to estimate uncertainty activity times and how to use this info to provide a probability statement about the chances of completing the project within the specified time
- If need be, crashing using linear optimization model may be used to reduce the activity time in order to meet the project completion deadlines