**Lab 6**
**Project Crashing**

Before this practical, you should have learned the Critical Path Method (CPM). CPM helps to answer the questions like:

- What is the total time to complete the project
- What are the scheduled start and finish dates for each task
- What tasks are critical and must complete in time in order not to miss the deadline

Another technique that we will be learning is the how to crash a project. For example, by looking at the critical path of the project and we realize that meeting the desired completion date is impossible unless we can shorten some selected activity time. This shortening of activity time which usually can be achieved by adding more resources is referred to as "crashing". However, the added resources associated with crashing activity time usually result in added project costs, so we will want to identify the activities that cost the least to crash and then crash those activities only the amount necessary to meet the desired project completion time. We will use the following scenario to learn all the above techniques mentioned.

There are 2 parts to be carried for this practical :
- Part 1 : Construct the project network diagram using the CPM approach
- Part 2 : Use Linear Programming technique to shorten the project schedule (also known as crashing the project schedule)

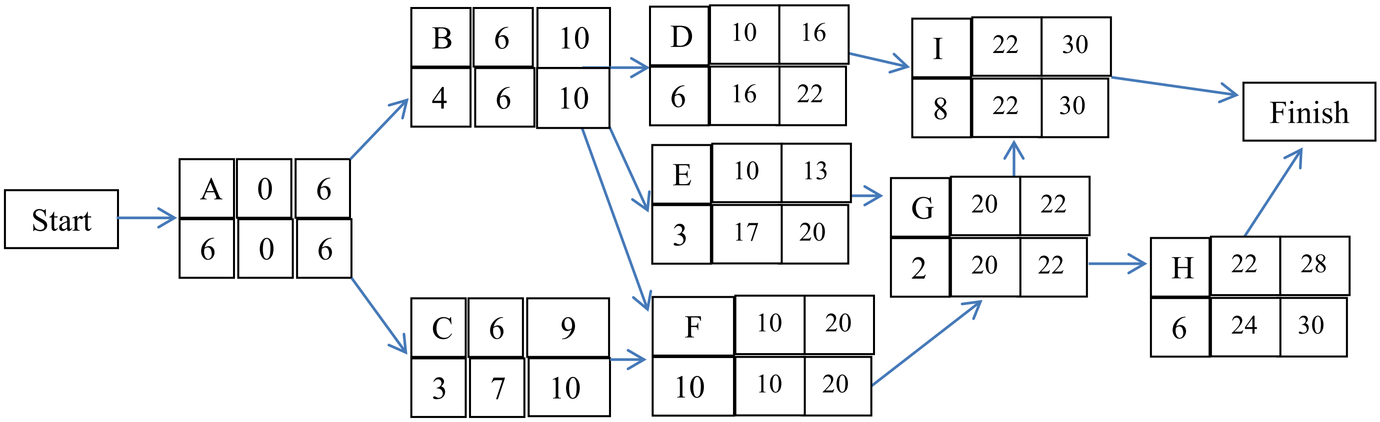
**Part 1: Construct the project network diagram using the CPM approach**

Frank's Fine Floats is the business of building elaborate parade floats. The company crew has a new float to build for the Christmas parade and they want to use PERT to help manage the project.

The activities identified for the project are tabulated below.

| Activity | Description | Immediate Predecessors | Completed Time (days) |
|---|---|---|---|
| A | Initial Paperwork | - | 6 |
| B | Build Body | A | 4 |
| C | Build Frame | A | 3 |
| D | Finish Body | B | 6 |
| E | Finish Frame | B | 3 |
| F | Final Paperwork | B,C | 10 |
| G | Mount Body to Frame | E,F | 2 |
| H | Install Skirt to Frame | G | 6 |
| I | Decorate at Site | D, G | 8 |

- Develop a network diagram for this project.
- Identify the critical tasks.




Critical Tasks : A, B, F, G, I

**Part 2: Use Linear Programming technique to shorten the project schedule (Project Crashing)**

In the above scenario, we derive that the project completion time will be 30 days. Which activities should be crashed in order for the project to be completed in 26 days?

To determine just where and how much to crash activity times, we need information on how much each activity can be crashed and how much the crashing process costs. Hence, we must ask for the following information:

- Activity cost under the normal or expected activity time
- Time to complete the activity under maximum crashing (i.e., the shortest possible activity time)
- Activity cost under maximum crashing

The following table provides the information needed:

| | | Normal | | Crash | |
|---|---|---|---|---|---|
| Activities | Description | Time | Cost ($) | Time | Cost ($) |
| A | Initial Paperwork | 6 | 800 | 5 | 10,000 |
| B* | Build Body | 4 | 1000 | 4 | 1000 |
| C | Build Frame | 3 | 5000 | 2 | 10,000 |
| D | Finish Body | 6 | 15000 | 3 | 30,000 |
| E | Finish Frame | 3 | 18,000 | 2 | 25,000 |
| F | Final Paperwork | 10 | 30,000 | 7 | 48,000 |
| G* | Mount Body to Frame | 2 | 10,000 | 2 | 10,000 |
| H | Install Skirt to Frame | 6 | 45,000 | 5 | 80,000 |
| I | Decorate at Site | 8 | 35,000 | 4 | 65,000 |

*indicates activity that cannot be crashed

We want to find out from the given information that which activities should be crashed – and by how much – in order **to meet the 26 days project completion deadline at minimum cost**! Note that activities B and G cannot be crashed at all!

This is not a simple question to solve if we try to do it manually. Instead, we are going to use the technique of Linear Programming for minimum cost crashing to help us find out the answer. The following guide shows how the Linear Programming Model can be constructed using the Microsoft Excel spreadsheet.

(You must have completed the PERT chart in the earlier section to do this part of the practical.)

**Step 1: Define the objective to be achieved in this case**
Let $x_A, x_B, x_C, x_D, x_E, x_F, x_G, x_H$ and $x_I$ be the decision variables for crashing the activity A, B, C, D, E, F, G, H and I respectively.

We want to achieve minimum cost in crashing the schedule. Therefore, the objective function is defined as:

*Minimize the crashing cost :* $9200x_A + 5000x_C + 5000x_D + 7000x_E + 6000x_F + 35000x_H + 7500x_I$

**Step 2: Identify the constraints**
There are 3 categories of constraints to be defined in this case.

a) The maximum number of days can be crashed for each activity
b) The predecessor relations between activities and the start time and reduction.
c) The total duration of the **critical tasks** must be less than 26 days

Define these 3 categories in the model. (Refer to LP model in P8.xlsx)

***Note:*** A predecessor relation of $A \to B$ can be modelled by the following inequality:

$$\boxed{y_B \geq T_A - x_A + y_A}$$

where $y_B$ is the start time of activity $B$,
$T_A$ is the normal activity time of activity $A$,
$x_A$ is the time reduction of activity $A$, and
$y_A$ is the start time of activity $A$.

a) $x_A \leq 1$
$x_C \leq 1$
$x_D \leq 3$
$x_E \leq 1$
$x_F \leq 3$
$x_H \leq 1$
$x_I \leq 4$

b) $y_B \geq T_A - x_A + y_A$
$y_C \geq T_A - x_A + y_A$
$y_D \geq T_B - x_B + y_B$
$y_E \geq T_B - x_B + y_B$
$y_F \geq T_B - x_B + y_B$
$y_F \geq T_C - x_C + y_C$
$y_G \geq T_E - x_E + y_E$
$y_G \geq T_F - x_F + y_F$
$y_I \geq T_D - x_D + y_D$
$y_I \geq T_G - x_G + y_G$
$y_H \geq T_G - x_G + y_G$
$y_J \geq T_H - x_H + y_H$
$y_J \geq T_I - x_I + y_I$ (Note: The node J is actually the end node.)

Note: 1 inequality per arrow / relation in the network diagram, except the arrows that originate from the start node.

c) $y_J \leq 26$

**Step 3 : Run Excel** Solver to find the optimal solution

From the result, the additional cost incurred at minimum will be $25500 and the recommended days to crash for each activity are:

| Activity | Normal Time | Normal Cost | Crash Time | Crash Cost | Predecessor | Y (Start) | X (Red.) | Cost / Time |
|---|---|---|---|---|---|---|---|---|
| A | 6.0 | 800 | 5.0 | 10000 | | 0 | 0 | 9200 |
| B | 4.0 | 1000 | 4.0 | 1000 | A | 6 | 0 | 0 |
| C | 3.0 | 5000 | 2.0 | 10000 | A | 7 | 0 | 5000 |
| D | 6.0 | 15000 | 3.0 | 30000 | B | 13 | 0 | 5000 |
| E | 3.0 | 18000 | 2.0 | 25000 | B | 10 | 0 | 7000 |
| F | 10.0 | 30000 | 7.0 | 48000 | B,C | 10 | 3 | 6000 |
| G | 2.0 | 10000 | 2.0 | 10000 | E,F | 17 | 0 | 0 |
| H | 6.0 | 45000 | 5.0 | 80000 | G | 20 | 0 | 35000 |
| I | 8.0 | 35000 | 4.0 | 65000 | D,G | 19 | 1 | 7500 |
| J | 0 | 0 | 0 | 0 | H,I | 26 | 0 | 0 |

***Conclusion:***

Crashing the project schedule refers to shortening the activity times by adding **resources** and hence usually increases **project cost**.

***Extra Exercise:***

Recall that we drew the project network diagram to find the project completion time of 30 days. Could we have found out what this original project completion time is without drawing the network diagram? *(Hint: Use the spreadsheet we just created.)*

**Answer:**

- Set maximum completion time to a large number like 100 days to make this constraint non-binding.
- Run the Solver without changing any parameter.
- Since the maximum completion time is no longer binding, the only binding constraints are the maximum crashing times and the predecessor relationships.
- The value of Yj generated by Solver will be the original project completion time of 30 days.