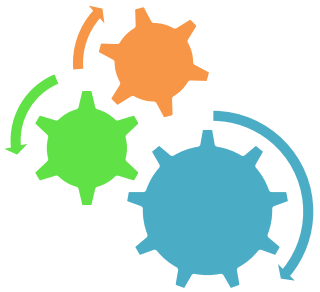
IT1311 Decision Analysis

Linear Programming



Introduction

- Linear Programming is concerned with the management of scarce resources
- For example
  - A marketing manager wants to determine how best to allocate a fixed advertising budget among alternative advertising media such as radio, television, newspaper and social media. The manager would like to determine the media mix that maximizes advertising effectiveness
  - A financial analyst must select an investment portfolio from a variety of stock and bond investment alternatives. The analyst would like to establish the portfolio that maximizes the return on investment
  - A manufacturer wants to develop a production schedule that will satisfy the market demand and at the same time minimize the total production and inventory costs

Learning Objectives

- To be able to formulate linear programming models for both maximizing and minimizing problems
- To know how to use Excel spreadsheet to solve linear programming problems
- To be able to carry out sensitivity analysis and interpretation of solutions

Reference

- Quantitative Methods for Business 12th edition, Chapter 7

Introduction to Linear Programming

- Linear programming has nothing to do with computer programming
- The use of the word “programming” here means “choosing a course of action”
- Linear programming involves choosing a course of action when the mathematical model of the problem contains only linear functions

Linear Programming Problem

- The maximization or minimization of some quantity is the objective in all linear programming problems.
- All LP problems have constraints that limit the degree to which the objective can be pursued.
- A feasible solution satisfies all the problem's constraints.
- An optimal solution is a feasible solution that results in the largest possible objective function value when maximizing (or smallest when minimizing).
- A graphical solution method can be used to solve a linear program with two variables. If both the objective function and the constraints are linear, the problem is referred to as a linear programming problem.
- Linear functions are functions in which each variable appears in a separate term raised to the first power and is multiplied by a constant (which could be 0).
- Linear constraints are linear functions that are restricted to be "less than or equal to", "equal to", or "greater than or equal to" a constant

Problem Formulation

- Problem formulation or modeling is the process of translating a verbal statement of a problem into a mathematical statement
- Formulating models is an art that can only be mastered with practice and experience
- Every LP problems has some unique features, but most problems also have common features
- General guidelines for LP model formulation are illustrated on the slides that follow

Guidelines for Model Formulation

- Understand the problem thoroughly
- Describe the objective
- Describe each constraint
- Define the decision variables
- Write the objective in terms of the decision variables
- Write the constraints in terms of the decision variables

An Example:

$$\text{Max} \quad 5x_1 + 7x_2 \qquad \text{Objective Function}$$

$$\text{s.t.} \quad x_1 \le 6$$
$$2x_1 + 3x_2 \le 19$$
$$x_1 + x_2 \le 8 \qquad \text{“Regular” Constraints}$$

$$x_1 \ge 0 \text{ and } x_2 \ge 0 \qquad \text{Non-negativity Constraints}$$

A Simple Maximization Problem

RMC is a small firm that produces a variety of chemical-based products. Three raw materials are used to produce two products: a fuel additive and a solvent base.

The three raw materials are blended to form the fuel additive and solvent base as shown below.

	Products	
	Fuel Additive (x)	Solvent Base (y)
Material 1	0.4	0.5
Material 2	-	0.2
Material 3	0.6	0.3

RMC's production is constrained by a limited availability of these three raw materials. For the current product period, the quantities availability for each raw material are shown on next slide.

	Amount available for production
Material 1	20 tons
Material 2	5 tons
Material 3	21 tons

In addition, the accounting department analyzed the production figures, assigned all relevant costs and arrived at prices for both products that will result in a profit contribution of $40 for every ton of fuel additive produced and $30 for every ton of solvent base produced.

Use linear programming technique to determine the number of tons of fuel additive and the number of tons of solvent base to produce in order to maximize the total profit contribution.

Formulating the model

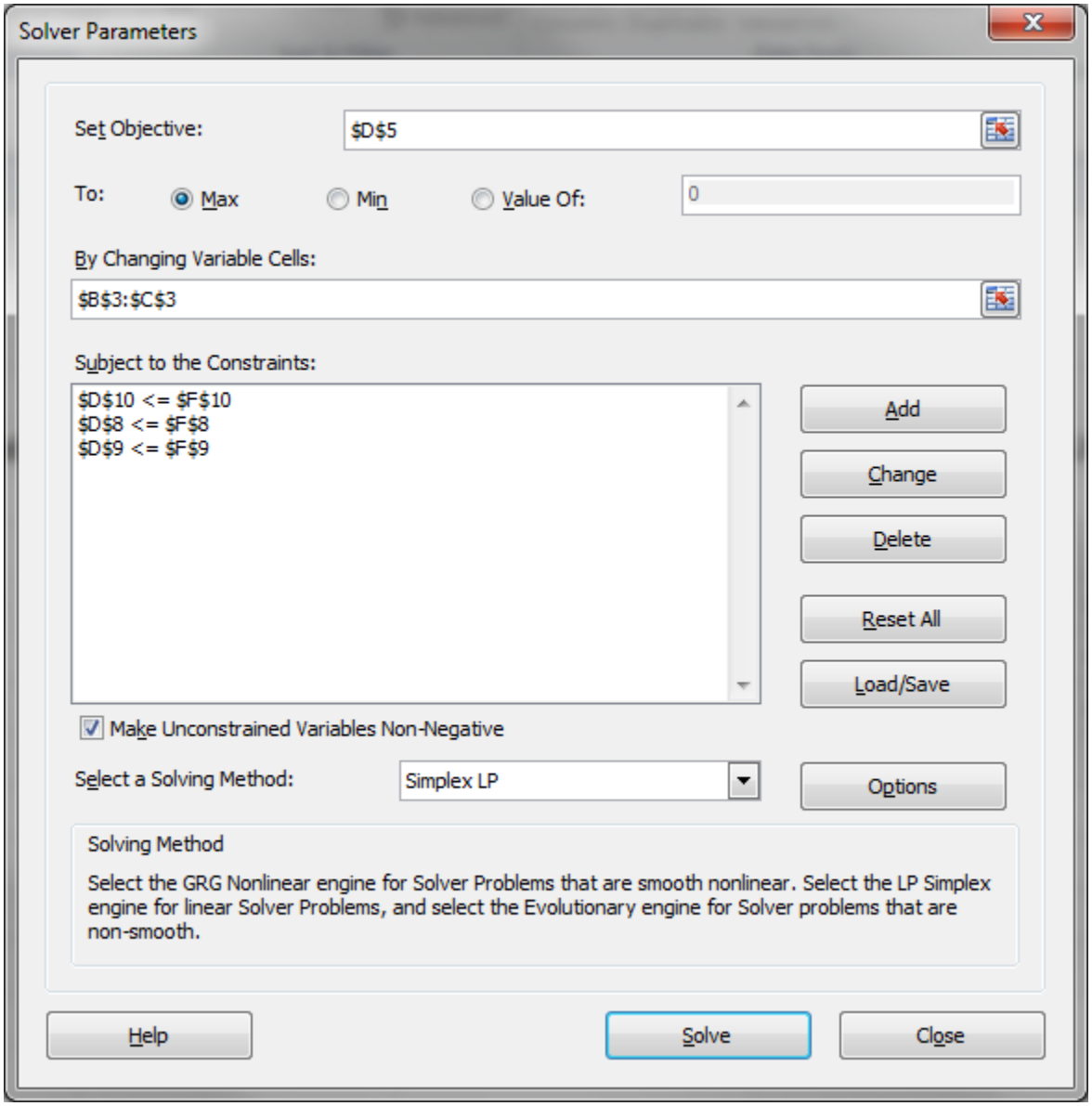
(maximization)

Product	Profit contributions
Fuel Additive	$40
Solvent Base	$30

Materials	Product Composition: Fuel Additive	Product Composition: Solvent Base	
Material 1	0.4	0.5	20 tons per week
Material 2	0	0.2	5 tons per week
Material 3	0.6	0.3	21 tons per week

	A	B	C	D	E	F
1						
2		Fuel Additive	Solvent Base			
3	Decision Variables:	25	20			
4						
5	Objective Function Profit ($ per unit):	40	30	1600.00	(MAX Profit)	
6						
7	Constraints:					
8	Material 1 (20 tons per week)	0.4	0.5	20.00	<=	20
9	Material 2 (5 tons per week)	0	0.2	4.00	<=	5
10	Material 3 (21 tons per week)	0.6	0.3	21.00	<=	21

Excel Solver Setting



Recommendation

	Fuel Additive	Solvent Base	
Decision Variables	25	20	(this gives the qty of Fuel Additive, Solvent Base to max profits)
Objective Function:	$40	$30	$1,600.00 (Profit to be MAX)

Constraints:

Material 1	0.4	0.5	20	<=	20	tons per week
Material 2	0	0.2	4	<=	5	tons per week
Material 3	0.6	0.3	21	<=	21	tons per week

Recommendation:

The optimal production quantities for RMC is at 25 tons of fuel additive and 20 tons of solvent base and this yields a profit contribution of 40(25) + 30(20) = $1600

A Simple Minimization Problem

M&D Chemicals produces two products that are sold as raw materials to companies manufacturing bath soaps and laundry detergents. Based on an analysis of current inventory levels and potential demand for the coming month, M&D management has specified that the combined production for products A and B must total at least 350 gallons. Separately, a major customer order for 125 gallons of product A must also be satisfied.

Product A requires 2 hours of processing time per gallon while product B requires 1 hour of processing time per gallon. For the coming month, 600hours of processing time are available.

M&D objective is to satisfy these requirements at a minimum production cost. Product costs are $2 per gallon for product A and $3 per gallon for product B.

Formulating the model

(Minimization)

Product	Production Costs
Product A	$2
Product B	$3

	Product Composition		
Constraints	Product A	Product B	
Major Customer	1	0	at least 125 gallons
Combined Production	1	1	at least 350 gallons
Processing time	2	1	not more than 600 hours

	Product A	Product B	
Decision Variables	0	0	(this gives the qty of Product A & B to min production costs)
Objective Function:	$2	$3	$0.00 (Production cost to be MIN)

Cost=2A+3B

Constraints:

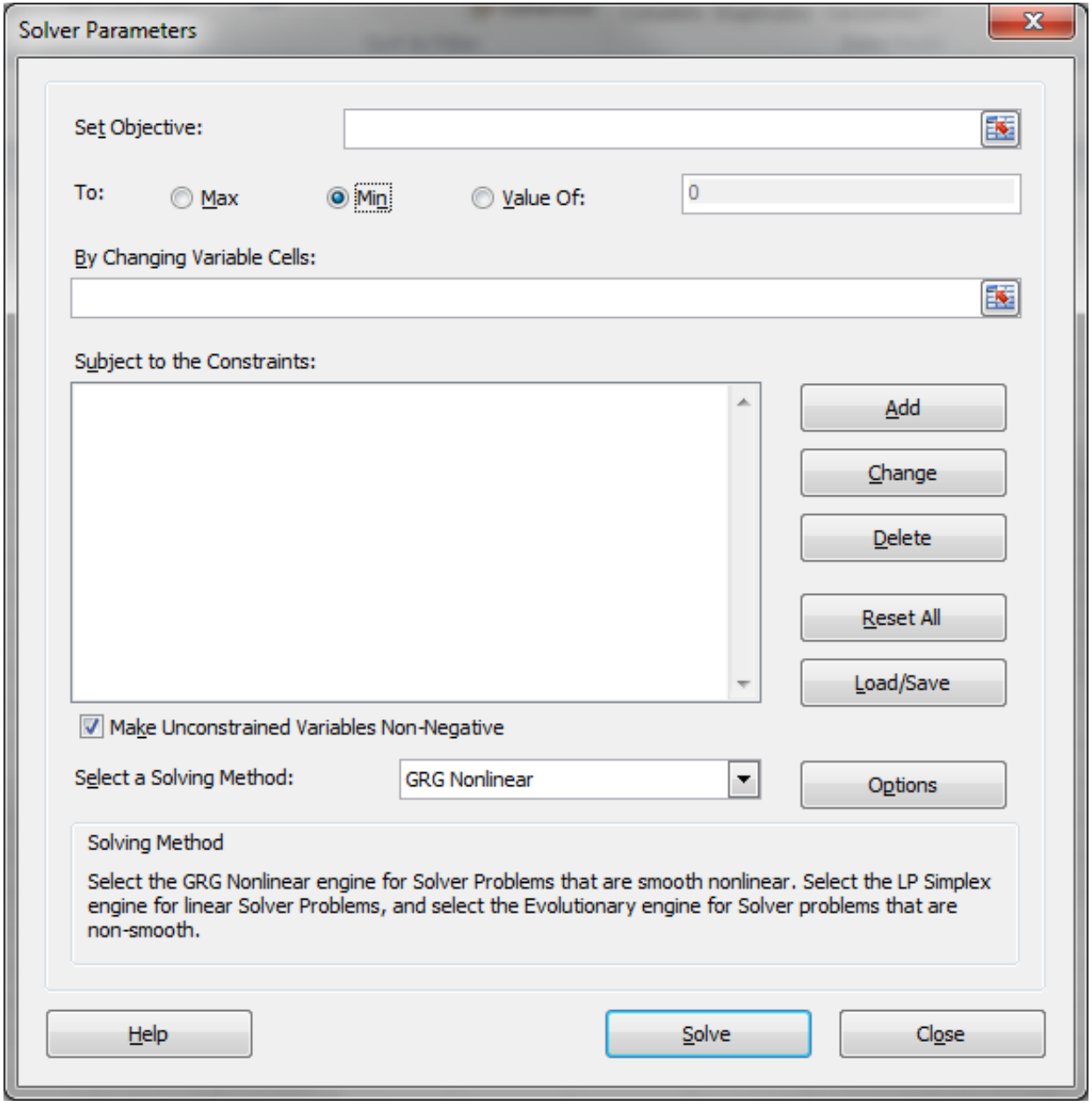
Major Customer	1	0	0	>=	125	gallons
Combined Production	1	1	0	>=	350	gallons
Processing time	2	1	0	<=	600	hours

And non negativity constraints

A >= 125

A + B >= 350, 2A + B <= 600

Excel Solver Setting



Recommendation

Product	Production Costs
Product A	$2
Product B	$3

	Product Composition		
Constraints	Product A	Product B	
Major Customer	1	0	at least 125 gallons
Combined Production	1	1	at least 350 gallons
Processing time	2	1	not more than 600 hours

Set these cells to 0 decimal place

	Product A	Product B			
Decision Variables	250.000001	99.999999	(this gives the qty of Product A & B to min production costs)		
Objective Function:	$2	$3	$800.00	(Production cost to be MIN)	
Constraints:					
Major Customer	1	0	250.000001	>=	125 gallons
Combined Production	1	1	350	>=	350 gallons
Processing time	2	1	600.000001	<=	600 hours

Recommendation:

The minimum cost solution to M&D Chemicals is to produce 250 units of product A and 100 units of product B with minimum production cost of 250($2) + 100($3) = $800