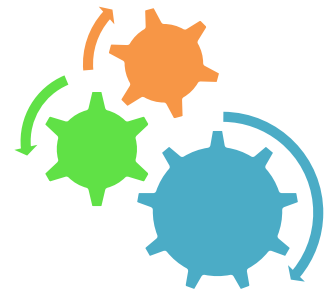


IT1311 Decision Analysis

Linear Programming 2



**NEXT, WE WILL USE LP TECHNIQUE IN THE
MARKETING AND FINANCE SCENARIO**

Linear Programming in Marketing

The Westchester Chamber of Commerce periodically sponsors public service seminars and programs. Currently, promotional plans are under way for this year's program. Advertising alternatives include television, radio and newspaper. Audience estimates, costs and maximum media usage limitations are listed below.

Constraint	Television (T)	Radio (R)	Newspaper (N)
Audience per advertisement	100,000	18,000	40,000
Cost per advertisement	\$2000	\$300	\$600
Maximum media usage	10	20	10

To ensure a balanced use of advertising media, radio advert must not exceed 50% of the total number of advertisements authorised. In addition, television should account for at least 10% of the total number of advert authorized.

If the promotion budget is limited to \$18,200, how many commercial messages should be run on each medium to maximize the total audience contact?

What is the allocation of the budget among the 3 media and what is the total audience reached?

Westchester Chamber of Commerce

Let T = no. of TV advertisements

R = no. of radio advertisements

N = no. of newspaper advertisements

Objective: Outreach = $100,000T + 18,000R + 40,000N$ (MAX)

Constraints:

$$2000*T + 300*R + 600*N \leq 18200$$

$$T \leq 10$$

$$R \leq 20$$

$$N \leq 10$$

$$R \leq 0.5*(T + R + N) \rightarrow 2R \leq T + R + N \rightarrow -T + R - N \leq 0$$

$$T \geq 0.1*(T + R + N) \rightarrow 10T \geq T + R + N \rightarrow 9T - R - N \geq 0$$

$$T, R, N \geq 0$$

	A	B	C	D	E	F	G	H	I	J
43	(Marketing)									
44										
45										
46										
47										
48										
49										
50										
51										
52										

Media	Audience Reached
T	100000
R	18000
N	40000

Constraints

Budget

Max TV

Max Radio

Max News

Max 50% Radio

Min 10% TV

Media		
T (Television)	R (Radio)	N (Newspaper)
\$2,000	\$300	\$600
1	0	0
0	1	0
0	0	1
-0.5	0.5	-0.5
0.9	-0.1	-0.1

not more than 10

not more than 20

not more than 10

not more than 50% for Radio

not less than 10% for TV

	A	B	C	D	E	F	G	H
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								

Decision Variables:

T

R

N

0

0

0

Objective Function Profit (Audience Reached):

100000

18000

40000

0.00

(MAX Audience reached)

Constraints:

Total Budget

2000

300

600

0.00

<=

18200

Max TV

1

0

0

0

<=

10

Max Radio

0

1

0

0

<=

20

Max News

0

0

1

0

<=

10

Radio advert not 50% more than total advert

-1

1

-1

0

<=

0

Television advert no 10% less than total advert

9

-1

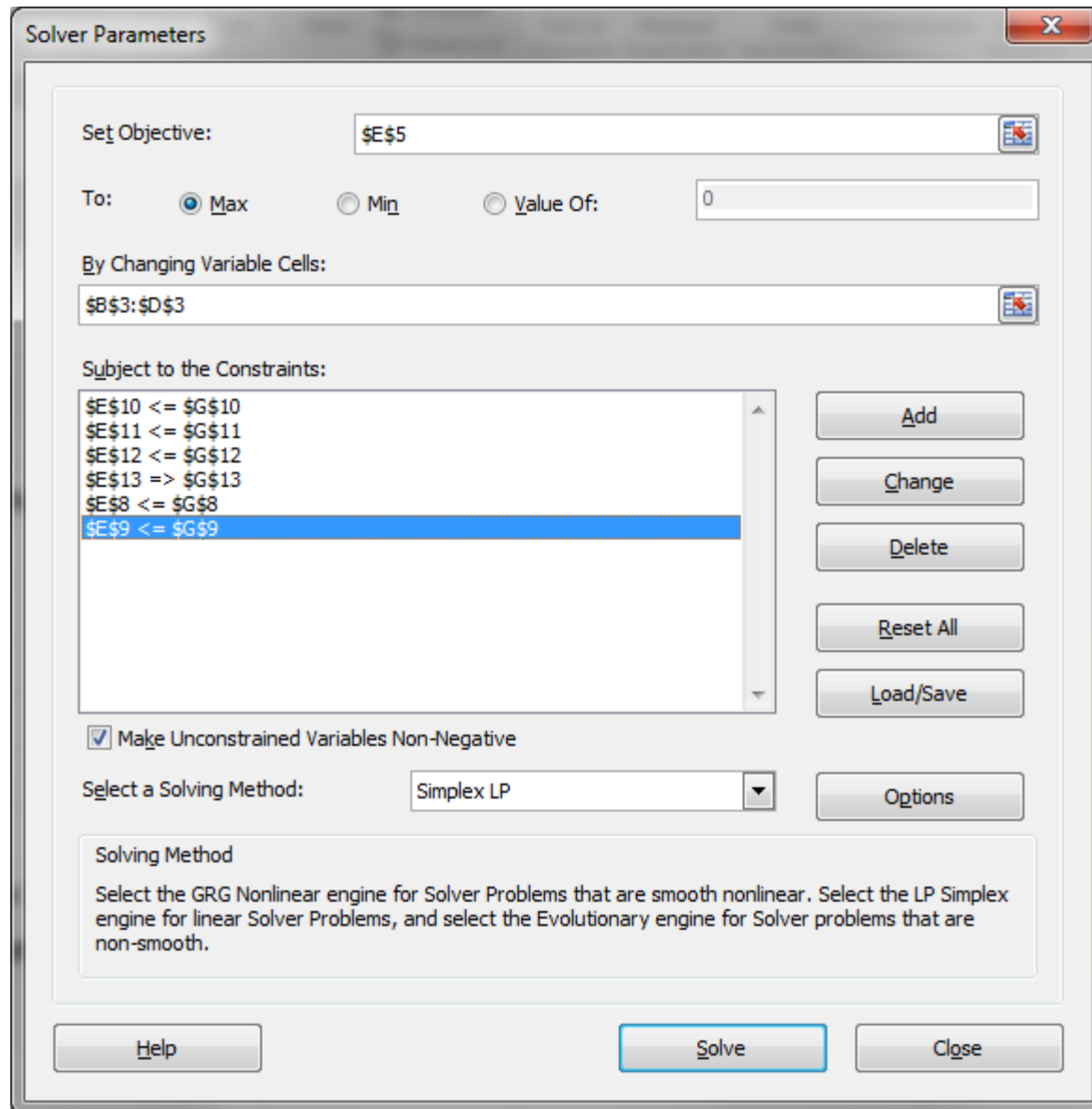
-1

0

>=

0

Excel Solver Setting



The image shows the 'Solver Parameters' dialog box in Microsoft Excel. The 'Set Objective' field is set to '\$E\$5'. The 'To' section has three radio buttons: 'Max' (selected), 'Min', and 'Value Of:'. The 'By Changing Variable Cells' field is set to '\$B\$3:\$D\$3'. The 'Subject to the Constraints' list contains five constraints: '\$E\$10 <= \$G\$10', '\$E\$11 <= \$G\$11', '\$E\$12 <= \$G\$12', '\$E\$13 => \$G\$13', and '\$E\$8 <= \$G\$8'. The constraint '\$E\$9 <= \$G\$9' is highlighted. To the right of the list are buttons for 'Add', 'Change', 'Delete', 'Reset All', and 'Load/Save'. Below the list is a checkbox labeled 'Make Unconstrained Variables Non-Negative' which is checked. The 'Select a Solving Method' dropdown is set to 'Simplex LP'. At the bottom, there is a text box explaining the solving methods: 'Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.' The bottom of the dialog has 'Help', 'Solve', and 'Close' buttons.

Solver Parameters

Set Objective:

To: ☒ Max ☐ Min ☐ Value Of:

By Changing Variable Cells:

Subject to the Constraints:

- \$E\$10 <= \$G\$10
- \$E\$11 <= \$G\$11
- \$E\$12 <= \$G\$12
- \$E\$13 => \$G\$13
- \$E\$8 <= \$G\$8
- \$E\$9 <= \$G\$9

Buttons: Add, Change, Delete, Reset All, Load/Save

☒ Make Unconstrained Variables Non-Negative

Select a Solving Method:

Options

Solving Method

Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Buttons: Help, Solve, Close

Recommendation

	A	B	C	D	E	F	G
1							
2		T	R	N			
3	Decision Variables:	4	14	10			
4							
5	Objective Function Profit (Audience Reached):	100000	18000	40000	1052000.00	(MAX Audience reach	
6							
7	Constraints:						
8	Total Budget	2000	300	600	18200.00	<=	18200
9	Max TV	1	0	0	4	<=	10
10	Max Radio	0	1	0	14	<=	20
11	Max News	0	0	1	10	<=	10
12	Radio advert not 50% more than total advert	-1	1	-1	0	<=	0
13	Television advert no 10% less than total advert	9	-1	-1	12	>=	0

Recommendation:

4 runs for Television broadcast

14 runs for Radio broadcast

10 runs for Newspaper advertisement

to reach out to 1,052,000 audiences

Linear Programming in Finance

A financial adviser at BMC bank needs to determine how to invest \$100,000 in the following collection of bonds to maximize the annual return.

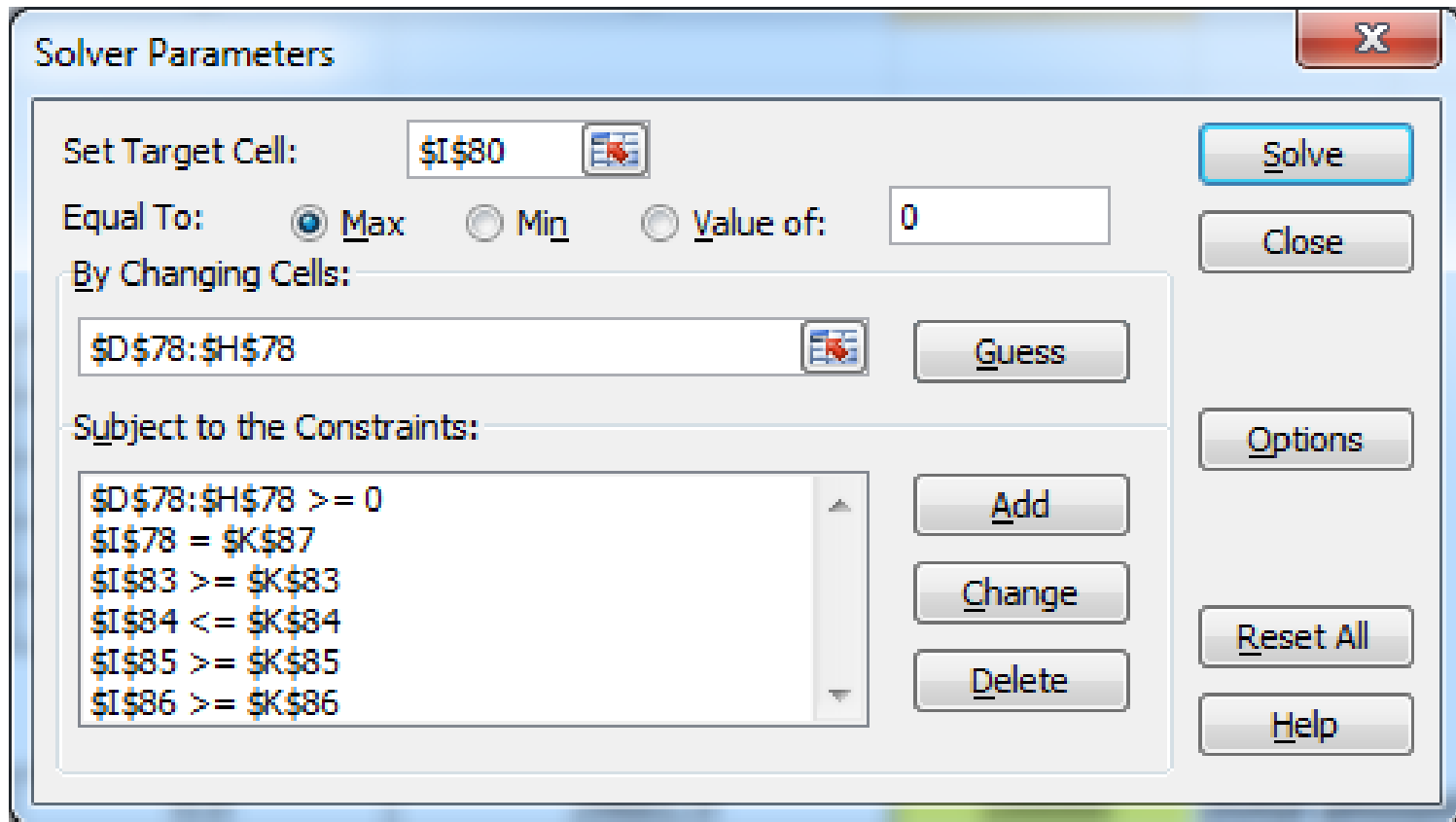
The adviser wants to invest at least 50% of the money in short-term issues and no more than 50% in high-risk issues. At least 30% of the funds should go in tax-free investments and at least 40% of the total annual return should be tax free. Create a LP model for this scenario and find out the optimal solution.

Bond	Annual Return	Maturity	Risk	Tax-Free
A	9.5%	12 months	High	Yes
B	8.0%	6 months	Low	Yes
C	9.0%	12 months	Low	No
D	9.0%	12 months	High	Yes
E	9.0%	6 months	high	No

Formulating the model

	A	B	C	D	E	F	G	H	I	J	K	L
66												
67												
68												
69	(Finance)											
70			<u>Bond</u>	<u>Annual Return</u>	<u>Maturity</u>	<u>Risk</u>	<u>Tax-Free</u>					
71			A	9.5%	12 months	High	Yes					
72			B	8.0%	6 months	Low	Yes					
73			C	9.0%	12 months	Low	No					
74			D	9.0%	12 months	High	Yes					
75			E	9.0%	6 months	High	No					
76												
77			Bond A	Bond B	Bond C	Bond D	Bond E					
78	Decision Variables		0.0	0.0	0.0	0.0	0.0	0	(total amount invested)			
79												
80	Objective Function:		9.5%	8.0%	9.0%	9.0%	9.0%	0	(Annual return to be MAX)			
81												
82	Constraints:											
83	Min 50% Short-term		0	1	0	0	1	0	>=	\$50,000		
84	Max 50% high-risk		1	0	0	1	1	0	<=	\$50,000		
85	Min 30% fund tax free		1	1	0	1	0	0	>=	\$30,000		
86	Min 40% return tax free		1	1	0	1	0	0	>=	\$0		
87	Total amount invested								equals	\$100,000		
88												

Excel Solver Setting



The image shows the 'Solver Parameters' dialog box in Microsoft Excel. The dialog is titled 'Solver Parameters' and has a standard Windows window border with a close button (X) in the top right corner.

Set Target Cell: The text '\$I\$80' is entered in the input field, followed by a small icon of a spreadsheet with a red arrow.

Equal To: There are three radio buttons: 'Max' (selected), 'Min', and 'Value of:'. To the right of 'Value of:' is a text box containing the number '0'.

By Changing Cells: The text '\$D\$78:\$H\$78' is entered in the input field, followed by a small icon of a spreadsheet with a red arrow. To the right of this field is a 'Guess' button.

Subject to the Constraints: A list box contains the following constraints:

- \$D\$78:\$H\$78 >= 0
- \$I\$78 = \$K\$87
- \$I\$83 >= \$K\$83
- \$I\$84 <= \$K\$84
- \$I\$85 >= \$K\$85
- \$I\$86 >= \$K\$86

To the right of the list box are three buttons: 'Add', 'Change', and 'Delete'.

Buttons: On the right side of the dialog, there are five buttons stacked vertically: 'Solve', 'Close', 'Options', 'Reset All', and 'Help'.

Recommendation

	Bond A	Bond B	Bond C	Bond D	Bond E		
Decision Variables	20339.0	20339.0	29661.0	0.0	29661.0	100000	(total amount invested)
Objective Function:	9.5%	8.0%	9.0%	9.0%	9.0%	8898.305085	(Annual return to be MAX)
Constraints:							
Min 50% Short-term	0	1	0	0	1	50000	>= \$50,000
Max 50% high-risk	1	0	0	1	1	50000	<= \$50,000
Min 30% fund tax free	1	1	0	1	0	40677.9661	>= \$30,000
Min 40% return tax free	1	1	0	1	0	3559.322034	>= \$3,559
Total amount invested							equals \$100,000

With total return of \$8,898

Summary

- A linear program is a mathematical model with the following qualities:
 - A linear objective function that is to be maximized or minimized
 - A set of linear constraints
 - Variables restricted to non-negative values
- We have also seen how this technique is applied in the marketing and finance scenario