

# Digit Recognition

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```
In [89]: 1 import pandas as pd
          2 import numpy as np
          3 import matplotlib.pyplot as plt
```

```
In [90]: 1 data = pd.read_csv("data.csv")
```

```
In [91]: 1 data.head()
```

Out[91]:

	label	pixel0	pixel1	pixel2	pixel3	pixel4	pixel5	pixel6	pixel7	pixel8	...	pixel774	pixel775
0	1	0	0	0	0	0	0	0	0	0	...	0	0
1	0	0	0	0	0	0	0	0	0	0	...	0	0
2	1	0	0	0	0	0	0	0	0	0	...	0	0
3	4	0	0	0	0	0	0	0	0	0	...	0	0
4	0	0	0	0	0	0	0	0	0	0	...	0	0

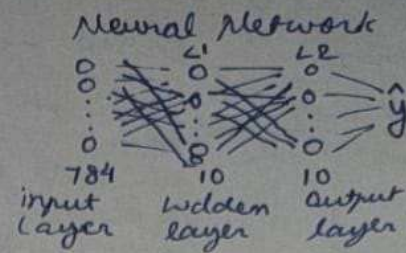
5 rows × 785 columns

```
In [92]: 1 data = np.array(data)
          2 m, n = data.shape
          3 np.random.shuffle(data)
          4
          5 data_test = data[0:1000].T
          6 Y_test = data_test[0]
          7 X_test = data_test[1:n]
          8 X_test = X_test / 255.
          9
          10 data_train = data[1000:m].T
          11 Y_train = data_train[0]
          12 X_train = data_train[1:n]
          13 X_train = X_train / 255.
          14 _, m_train = X_train.shape
```

$$X = \begin{bmatrix} x^1 \\ x^2 \\ \vdots \\ x^m \end{bmatrix}^T = [x^1 \ x^2 \ \dots \ x^m]$$

$$A^{(0)} = X \quad 784 \times m$$

$$L1: Z^{(1)} = W^{(1)} \cdot A^{(0)} + b^{(1)} \quad \begin{matrix} 10 \times m & 10 \times 784 & 784 \times m & 10 \times 1 \end{matrix}$$



Now, to introduce non-linearity in the neuron we will use  $\text{ReLU}(x)$  function (due to its simplicity)  
 $\text{ReLU}$ : Rectified linear unit.  $\text{ReLU}(x) = \begin{cases} x & x \geq 0 \\ 0 & x < 0 \end{cases}$

$$\therefore A^{(1)} = g(Z^{(1)}) = \text{ReLU}(Z^{(1)})$$

$$L2: Z^{(2)} = W^{(2)} A^{(1)} + b^{(2)} \quad \begin{matrix} 10 \times m & 10 \times 10 & 10 \times m & 10 \times 1 \end{matrix}$$

Here, activation function will be softmax fn.

$$df(x) = \frac{e^{x_i}}{\sum_{j=1}^n e^{x_j}} \quad \therefore A^{(2)} = \text{softmax}(Z^{(2)})$$

Backward propagation:

$$L2: dz^{(2)} = A^{(2)} - y \quad \begin{matrix} 10 \times m & 10 \times m & 10 \times m \end{matrix}$$

$$\rightarrow dW^{(2)} = \frac{1}{m} dz^{(2)} A^{(1)} \quad \text{(avg error wt)}$$

$$\rightarrow db^{(2)} = \frac{1}{m} \sum dz^{(2)} \quad \text{(avg error bias)}$$

$$L1: dz^{(1)} = W^{(2)} dz^{(2)}$$

$$\rightarrow dW^{(1)} = \frac{1}{m} dz^{(1)} x$$

$$\rightarrow db^{(1)} = \frac{1}{m} \sum dz^{(1)}$$

Update:

$$W^{(1)} = W^{(1)} - \alpha dW^{(1)} \quad \alpha \rightarrow \text{learning rate}$$

$$b^{(1)} = b^{(1)} - \alpha db^{(1)}$$

$$W^{(2)} = W^{(2)} - \alpha dW^{(2)}$$

$$b^{(2)} = b^{(2)} - \alpha db^{(2)}$$

forward prop  $\rightarrow$  Backward prop  $\rightarrow$  Update

```
In [93]: 1 def init_params():
2         W1 = np.random.rand(10, 784) - 0.5
3         b1 = np.random.rand(10, 1) - 0.5
4         W2 = np.random.rand(10, 10) - 0.5
5         b2 = np.random.rand(10, 1) - 0.5
6         return W1, b1, W2, b2
7
8 def ReLU(Z):
9     return np.maximum(Z, 0)
10
11 def softmax(Z):
12     A = np.exp(Z) / sum(np.exp(Z))
13     return A
14
15 def forward_prop(W1, b1, W2, b2, X):
16     Z1 = W1.dot(X) + b1
17     A1 = ReLU(Z1)
18     Z2 = W2.dot(A1) + b2
19     A2 = softmax(Z2)
20     return Z1, A1, Z2, A2
21
22 def ReLU_deriv(Z):
23     return Z > 0
24
25 def one_hot(Y):
26     one_hot_Y = np.zeros((Y.size, Y.max() + 1))
27     one_hot_Y[np.arange(Y.size), Y] = 1
28     one_hot_Y = one_hot_Y.T
29     return one_hot_Y
30
31 def backward_prop(Z1, A1, Z2, A2, W1, W2, X, Y):
32     one_hot_Y = one_hot(Y)
33     dZ2 = A2 - one_hot_Y
34     dW2 = 1 / m * dZ2.dot(A1.T)
35     db2 = 1 / m * np.sum(dZ2)
36     dZ1 = W2.T.dot(dZ2) * ReLU_deriv(Z1)
37     dW1 = 1 / m * dZ1.dot(X.T)
38     db1 = 1 / m * np.sum(dZ1)
39     return dW1, db1, dW2, db2
40
41 def update_params(W1, b1, W2, b2, dW1, db1, dW2, db2, alpha):
42     W1 = W1 - alpha * dW1
43     b1 = b1 - alpha * db1
44     W2 = W2 - alpha * dW2
45     b2 = b2 - alpha * db2
46     return W1, b1, W2, b2
```

```
In [94]: ▶ 1 def get_predictions(A2):
2           return np.argmax(A2, 0)
3
4 def get_accuracy(predictions, Y):
5     #print(predictions, Y)
6     return np.sum(predictions == Y) / Y.size
7
8 def gradient_descent(X, Y, alpha, iterations):
9     W1, b1, W2, b2 = init_params()
10    for i in range(iterations):
11        Z1, A1, Z2, A2 = forward_prop(W1, b1, W2, b2, X)
12        dW1, db1, dW2, db2 = backward_prop(Z1, A1, Z2, A2, W1, W2, X, Y)
13        W1, b1, W2, b2 = update_params(W1, b1, W2, b2, dW1, db1, dW2, db
14        if i % 10 == 0:
15            print("Iteration: ", i)
16            predictions = get_predictions(A2)
17            print(get_accuracy(predictions, Y))
18    return W1, b1, W2, b2
```

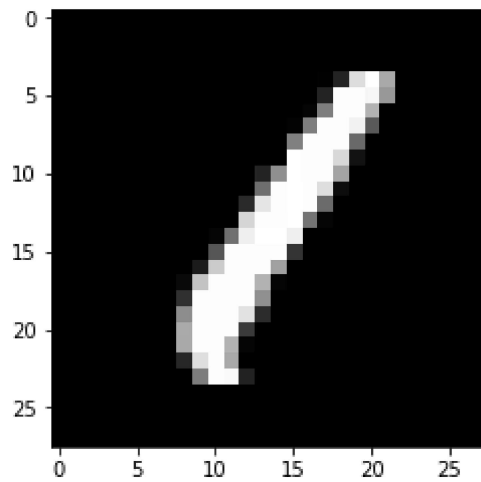
```
In [111]: ▶ 1 W1, b1, W2, b2 = gradient_descent(X_train, Y_train, 0.10, 1500)
0.8934634146341464
Iteration: 1360
0.8936829268292683
Iteration: 1370
0.8938780487804878
Iteration: 1380
0.8940487804878049
Iteration: 1390
0.8941219512195122
Iteration: 1400
0.8941707317073171
Iteration: 1410
0.8942926829268293
Iteration: 1420
0.894609756097561
Iteration: 1430
0.8948048780487805
Iteration: 1440
0.8949512195121951
Iteration: 1450
```

```
In [112]: ▶ 1 def make_predictions(X, W1, b1, W2, b2):
2             _, _, _, A2 = forward_prop(W1, b1, W2, b2, X)
3             predictions = get_predictions(A2)
4             return predictions
5
6 def test_prediction(index, W1, b1, W2, b2):
7     current_image = X_train[:, index, None]
8     prediction = make_predictions(X_train[:, index, None], W1, b1, W2, b2)
9     label = Y_train[index]
10    print("Prediction: ", prediction)
11    print("Label: ", label)
12
13    current_image = current_image.reshape((28, 28)) * 255
14    plt.gray()
15    plt.imshow(current_image, interpolation='nearest')
16    plt.show()
```

```
In [118]: 1 for i in range(5,10):  
          2     test_prediction(i, W1, b1, W2, b2)
```

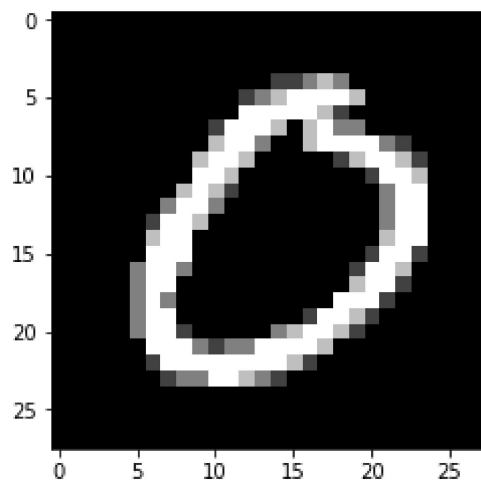
Prediction: [1]

Label: 1



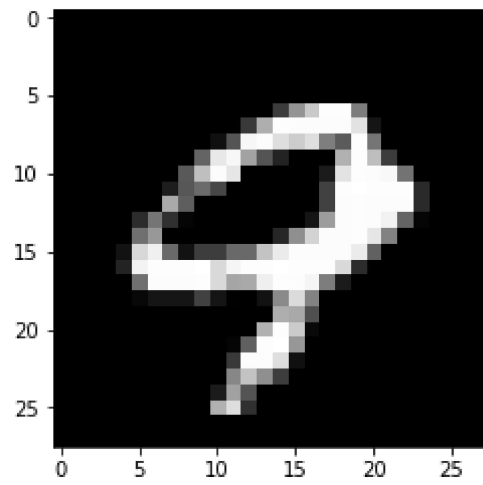
Prediction: [0]

Label: 0



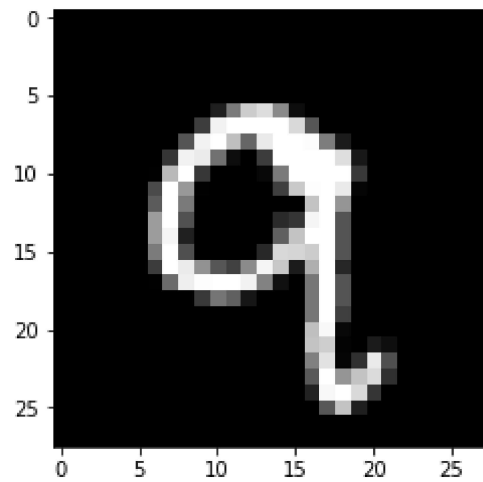
Prediction: [9]

Label: 9



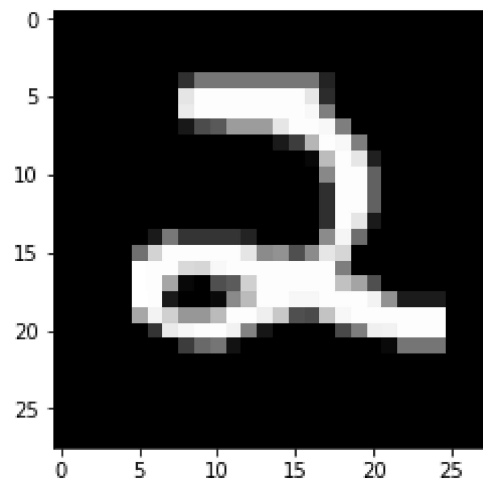
Prediction: [9]

Label: 9



Prediction: [2]

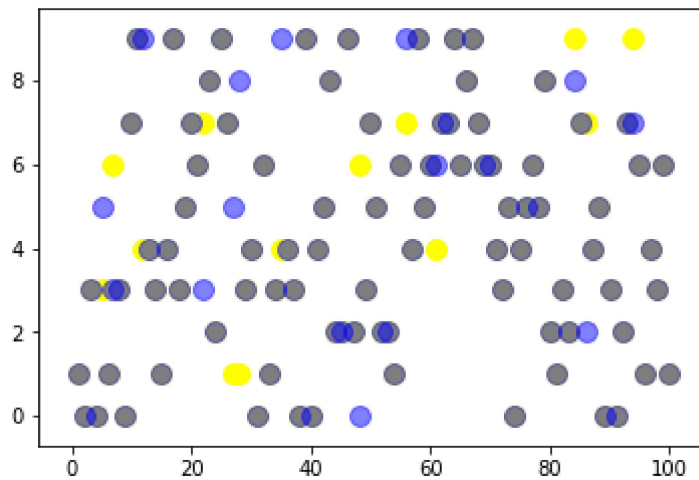
Label: 2



```
In [114]: 1 dev_predictions = make_predictions(X_test, W1, b1, W2, b2)
          2 get_accuracy(dev_predictions, Y_test) * 100
```

Out[114]: 88.6

```
In [115]: 1 x_axis = range(1, 101)
          2 plt.scatter(x_axis, dev_predictions[0:100], s=100, color="yellow")
          3 plt.scatter(x_axis, Y_test[0:100], s=100, color="blue", alpha=0.5)
          4 plt.show()
```



```
In [ ]: 1
```