Q.1) Max Marks: 1

The time complexity to perform DFS on a graph for which the Adjacency Matrix is given and the graph is sparse in nature i.e. |E|<<|V|

A $O(|V|^2)$ Correct Option

Solution: (A)

Solution Option A

If the adjacency matrix is given then for each node we need to see its neighbours it takes O(|V|) time to check for each neighbour, therefore the time complexity is $O(|V|^2)$.

B O(|V|+|E|)

C O(|E|)

D None of the above.

Q.2) Max Marks: 1

Given G=(V,E) is an undirected graph where each edge has a unique cost or weight consider the following statements and choose which one of the following is TRUE.

I. For a given pair of vertices v_i and v_j there always exists a unique shortest path between them.

II. If the weights of the graph are multiplied by a positive constant the shortest paths remain unchanged.

A I and II are true.

B Only I is true.

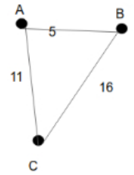
C Only II is true. Correct Option

Solution: (C)

Solution Option C.

Statement I

I need not be true consider the example of the following graph from c to b there are 2 paths of the same cost.



Statement II This is true because the relative cost of the edges remains unchanged if

$wt(e_1)<wt(e_2)$ then $wt(c^*e_1)<wt(c^*e_2)$.

D Neither I nor II are true.

Q.3) Max Marks: 1

Which if the following statements are true with respect to Dijkstra's single-source shortest path algorithm

1. The shortest path is updated only when a new path is discovered which is strictly less than the previous path computed so far.

2. It is also updated when it is less than or equal to the original path

3. All the paths which are equal are reported.

A 1 and 3

B 2 and 3

C 1 only. Correct Option

Solution: (C)
Solution. Option C
1. In the Dijkstra's algorithm the shortest path is updated only when a new path is discovered which is strictly less than the previous path computed so far.
2. Only one path is reported, multiple paths of equal cost exist are not reported.

D 2 only.

Q.4) Max Marks: 1

Which of the following is true about the Dijkstra's single-source shortest path algorithm with respect to a strongly connected graph.
Statement I: It is guaranteed to work if the graph does not have negative weight edges.
Statement II: It is guaranteed to work if the graph has negative weight edges but not negative weight cycles.

A I and II are true.

B Only I is true. Correct Option

Solution: (B)
It is guaranteed to work if the graph does not have negative weight edges, but it may fail even if the graph has a negative weight cycle and not a negative weight edge.

C Only II is true.

D Neither I nor II are true.

Q.5) Max Marks: 1

Consider a simple undirected weighted graph G(V, E) with 7 vertices and 21 edges, assume (u, v) are two vertices weight of each edge is given by the formula =lu- vl then the minimum cost of the spanning tree of G____

Correct Answer

Solution: (6)

In such a case the MST can be formed by connecting vertices in such a way that v_i is connected to v_{i+1}. and the weight of each edge is 1, Therefore the answer is 6*1=6.

Q.6) Max Marks: 1

Given a spanning tree S of a graph G(V, E) and we know that there are a few edges in the Graph which are not a part of the spanning tree S.
Statement I. Addition of an edge to S from E which is not part of S will result in a cycle for exactly one edge which belongs to E.
Statement II. Deletion of an edge from S will always result in more than one connected components.

A Statement I is true only.

B Statement II is true only. Correct Option

Solution: (B)
Statement I is false because the addition of any edge (not exactly one edge but for every edge) from E will make the MST or any spanning tree cyclic as a spanning tree is maximally acyclic.
Statement II is true because any spanning tree is minimally connected with |V|-1 edges and removal of an edge will make it disconnected but the resulting components will be connected.

C Both II and I are true.

D Neither I nor II is correct

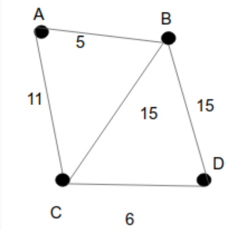
Q.7) Max Marks: 1

Given an undirected graph G(V,E) consider the following 2 statements
I. If G is a graph with unique and positive edge weights
II. It has only one unique minimal spanning tree

A Statement I implies statement II Correct Option

Solution: (A)
If a graph has a unique and positive edges then it has only one unique MST but if a graph has a unique MST it need not have unique and positive edges an example to prove the Statement II false is shown below

Statement II false is shown below:



In the above graph the MST consists of the edges AB, AC and CD and there is only one unique spanning tree for this graph but the edges are not unique.

B Statement I if and only if statement II

C Statement II implies statement I.

D None of the Above.

Q.8) Max Marks: 1

Total number of Minimal Spanning Trees of an unweighted complete graph K_n is given by

A $(n-1)!$

B C(n,2)

C $n^{(n-2)}$ Correct Option

Solution: (C)

Solution C

This is using krichoffs theorem we can conclude the number of minimum spanning trees for a complete graph K_n is $n^{(n-2)}$. One can also try by drawing for n=2, 3, 4 and substituting in the options.

D None of the above

Q.9) Max Marks: 1

Given G=(V, E) is an undirected graph where each edge has a unique cost or weight consider the following statements and choose which one of the following is TRUE.

I. MST(Minimal Spanning Tree) exists for the graph even if it contains negative weight cycles.

II. If the weights of the graph are multiplied by a non zero constant the MST of graph remains unchanged.

A I and II are true.

B Only I is true. Correct Option

Solution: (B)

Statement I is true MST exists even if there is a negative weight cycle in the graph, as irrespective of the cycle and the weight of the edges the MST is calculated by adding one edge after another until is cycle is formed or all the vertices are covered.

Statement II can be true if the constant is positive otherwise the statement II is false the MST will change in case the constant is negative.

C Only II is true.

D Neither I nor II are true.

Q.10) Max Marks: 1

What is the maximum number of edges in a directed graph which contains 7 vertices such that it has no cycles?

Correct Answer

Solution: (21)

Self-loops and cycles are not allowed we can construct a graph, by adding edges from vertices v_i to v_j if and only if i<j or i>j but not both,

No of edges such edges=E=v(v-1)/2, substituting v=7, we get 21.

Q.11)

Max Marks: 2

For a given undirected graph G (V, E) which is sparse, the adjacency matrix representation is given however the Dijkstra's algorithm works better on the adjacency list representation a modified procedure is written to get the adjacency list of the graph and then Dijkstra's algorithm is applied on the adjacency list of the graph G, what is the time complexity of this complete process.

A $O(|V|^2)$ — Correct Option

Solution: (A)

Time taken to convert Adj Matrix to Adj list $=O(|V|^2)$
Time taken for Dijkstra's algorithm O(VlogV+ElogE)
Total Time taken is $O(|V|^2)$.

B $O(|V|+|E|)$

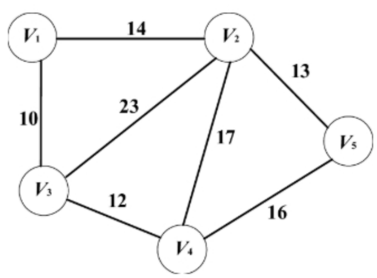
C $O(|E|)$

D None Of the Above

Q.12)

Max Marks: 2

Given the following graph, what is the cost of the Minimal spanning tree of the graph is_____



Correct Answer

Solution: (49)
Solution 49
Applying Kruskal's algorithm sort the edges in ascending order
V1-V3, V3-V4, V2-V5, V1-V2, V4-V5, V4-V2, V2-V3.
we get
First, add V1-V3
Then add V3-V4
Then V2-V5
Then V1-V2
All the vertices are covered we can stop.
cost=10+12+13+14=49

Q.13)

Max Marks: 2

Match the following

List -1	List-2
1. Bellman-Ford	A. Minimal Spanning Tree
2. Floyd Warshal	B. All pairs shortest path
3. Dijkstra's Algorithm	C. Single source shortest path
4. Prims Algorithm	D. Graph Traversal

A 1-C, 2-B, 3-C, 4-D

B 1-C, 2-B, 3-C, 4-A — Correct Option

Solution: (B)
Solution Option B
Bellman-Ford and Dijkstra's algorithm is a single-source shortest path algorithm.
Prim's algorithm is to calculate the minimal spanning tree.
Floyd Warshall algorithm is to calculate the all-pairs shortest path algorithm.

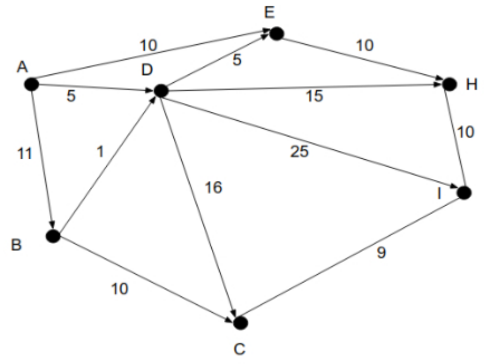
C 1-C, 2-B, 3-A, 4-A

D	1-D, 2-B, 3-C, 4-A

Q.14)

Max Marks: 2

Consider the following Graph the shortest path from A to I reported by the Dijkstra's Algorithm is



The edges without directions are bi-directional for example edge (C, I).

A	ADI	Correct Option

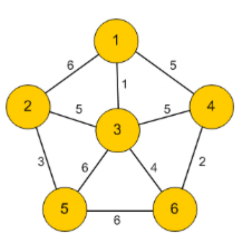
Solution: (A)
Solution Option A
On Running Dijkstra's Algorithm
1. E.d=10
D.d=5
B.d=11
Source=A
2. D is removed from the priority Q
H.d=20
I.d=30
C.d=21
Source=D
We have seen the distance of I=30
Also by observing the figure we cannot find a better distance so this will not be updated by the Dijkstra's Algorithm.
ADI is the path reported by Dijkstra's algorithm.

B	AEHI
C	ADHI
D	ABCI

Q.15)

Max Marks: 2

Which of the following is not the BFS traversal of the given graph



A	1, 2, 3, 4, 5, 6
B	1, 4, 3, 2, 6, 5
C	5, 2, 6, 3, 1, 4

D	6, 5, 3, 1, 4, 2	Correct Option

Solution: (D)
Solution D
If we apply DFS from vertex 1 we get options A and B
If we apply BFS form vertex 5 we get option C
If we apply BFS from vertex 6 we cannot explore vertex 1 before vertex 4 because it is at distance 2 and vertex 4 is at distance 1 from the source. In BFS the vertices are

If we apply **BFS** from vertex 6 we cannot explore vertex 1 before vertex 4 because it is at distance 2 and vertex 4 is at distance 1 from the source. In **BFS** the vertices are explored in order of edge distance.

close