21 Permutations and Combinations

SYNOPSIS

- Introduction
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INTRODUCTION

The concept of permutation and combination is used to select or arrange some items out of a group according to certain predetermined conditions. This concept helps find the number of ways to arrange, select, or reject these items. The concepts of permutations and combinations derive from the principles of factorials the fundamental principle of counting.

FACTORIAL

The factorial of a natural number n, denoted by $n!$ And is the product of all natural numbers from 1 up to n.

$n! = 1 \times 2 \times 3 \times 4 \times \ldots \times (n-1) \times n$

OR

$n! = n \times \{(n-1) \times (n-2) \times (n-3) \times (n-4) \times \ldots \times 1\}$

The factorials of the first few natural numbers given as follows:

1!	1	1
2!	2 × 1	2
3!	3 × 2 × 1	6
4!	4 × 3 × 2 × 1	24
5!	5 × 4 × 3 × 2 × 1	120
6!	6 × 5 × 4 × 3 × 2 × 1	720
7!	7 × 6 × 5 × 4 × 3 × 2 × 1	5040
8!	8 × 7 × 6 × 5 × 4 × 3 × 2 × 1	40320
9!	9 × 8 × 7 × 6 × 5 × 4 × 3 × 2 × 1	362880
10!	10 × 9 × 8 × 7 × 6 × 5 × 4 × 3 × 2 × 1	3628800

Example: What is the value of 5! × 3!?

Solution: First expand both the factorials,

= (5 × 4 × 3 × 2 × 1) × (3 × 2 × 1)

= 120 × 6 = 720

Example: What is the value of $\frac{(12! \times 8!)}{(9! \times 10!)}$?

Solution: First of all, simplify the above given factorials,

$$\frac{12 \times 11 \times 10! \times 8!}{9 \times 8! \times 10!} = \frac{12 \times 11}{9}$$

$$= \frac{132}{9} = \frac{44}{3}$$

PERMUTATIONS

If there are n objects and one is supposed to arrange r ($r \le n$) objects out of these, then the number of ways in which this can be done is written as ${}^nP_r$ and is read as 'the number of permutations of n objects taken r at a time'.
The number of permutations is given as:

$${}^nP_r = \frac{n!}{(n-r)!}$$

Example: In how many ways can you arrange two books out of three?

Solution: ${}^3P_2 = \frac{3!}{(3-2)!}$

$= \frac{3!}{1!} = 3 \times 2 = 6$

Some specific cases related to permutations are:

If $r = 0$,

$${}^nP_0 = \frac{n!}{(n-0)!} = 1$$

If $r = 1$,

$${}^nP_1 = \frac{n!}{(n-1)!} = \frac{n \times (n-1)!}{(n-1)!} = n$$

If $r = n$,

$${}^nP_n = \frac{n!}{(n-n)!} = \frac{n!}{0!} = \frac{n!}{1} = n!$$

If $r = n - 1$,

$${}^nP_{n-1} = \frac{n!}{(n-n+1)!} = \frac{n!}{1!} = n!$$

Example: Find the value of ${}^5P_5$?

Solution: As we know, ${}^nP_n = n!$

So, ${}^5P_5 = 5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

Permutations of repeated objects

If n objects are to be arranged among themselves and these contain p identical objects of one kind, q identical objects of another kind, r identical objects of still another kind, and so on, the total number of ways in which they can be arranged is given by,

$$\frac{n!}{p!q!r!....}$$

Example: How many words can be formed by arranging the letters of the word 'weeded'?

Solution: Total number of letters in the word 'weeded' = 6.
D occurs 2 times and E occurs 3 times. This is a case where n objects (6 in this case) are to be arranged among themselves and they contain 2 identical objects of one kind (D) and 3 identical objects of another kind (E).

So, number of words = $\frac{6!}{2!3!}$ = 720/12 = 60.

Permutations with repetitions

The number of permutations of n different objects taken r at a time, when repetitions are allowed is n^r.

Example: How many four-digit numbers can be formed using the digits 4, 5, 6, 7, and 9 if repetition of digits is allowed?

Solution: Here, there are five different objects, i.e., the digits 4, 5, 6, 7, and 9; $n = 5$
Since repetition is allowed, the first digit can be taken in five different ways, the second can be taken in five different ways, and so on.
So, the number of four-digit numbers is 5 × 5 × 5 × 5 = 625.

COMBINATIONS

If there are n objects, and r out of them ($r \le n$) are to be selected, then the number of ways in which this can be done is ${}^nC_r$ and is read as 'the number of combinations of n objects taken r at a time'. The number of combinations is given as:

$${}^nC_r = \frac{n!}{(n-r)!r!}$$

Example: In how many different ways can you select 2 students out of 3?

Solution: It can be written as ${}^3C_2$.

$${}^3C_2 = \frac{3!}{(3-2)!2!} = 3.$$

The specific cases of combinations are:

$${}^nP_r = {}^nC_r \times r!$$

If $r = 0$,

$${}^nC_0 = \frac{n!}{(n-0)! \times 0!} = 1$$

If $r = 1$,

$${}^nC_1 = \frac{n!}{(n-1)! \times 1!} = n$$

$${}^nC_{n-1} = \frac{n!}{(n-n+1)! \times (n-1)!} = \frac{n!}{1! \times (n-1)!} = n$$

Example: Find the value of ${}^7C_7$?

If $r = n$,

$$^nC_n = \frac{n!}{(n-n)! \times n!} = 1$$

If $r = n - 1$,

Solution: $^7C_2 = \frac{7!}{(7-2)!2!}$

$$= \frac{7!}{5!2!} = \frac{5040}{120 \times 2} = 21.$$

Chapter Summary

$n! = 1 \times 2 \times 3 \times 4 \times \ldots \times (n-1) \times n$

Permutation $= {}^nP_r = \frac{n!}{(n-r)!}$

Some specific cases related to permutations are:

If $r = 0$,

$$^nP_0 = \frac{n!}{(n-0)!} = 1$$

If $r = 1$,

$$^nP_1 = \frac{n!}{(n-1)!} = \frac{n \times (n-1)!}{(n-1)!} = n$$

If $r = n$,

$$^nP_n = \frac{n!}{(n-n)!} = \frac{n!}{0!} = \frac{n!}{1} = n!$$

If $r = n - 1$,

$$^nP_{n-1} = \frac{n!}{(n-n+1)!} = \frac{n!}{1!} = n!$$

Combination $= {}^nC_r = \frac{n!}{(n-r)!r!}$

The specific cases of combinations are:

$$^nP_r = {}^nC_r \times r!$$

If $r = 0$,

$$^nC_0 = \frac{n!}{(n-0)! \times 0!} = 1$$

If $r = 1$,

$$^nC_1 = \frac{n!}{(n-1)! \times 1!} = n$$

If $r = n$,

$$^nC_n = \frac{n!}{(n-n)! \times n!} = 1$$

If $r = n - 1$,

$$^nC_{n-1} = \frac{n!}{(n-n+1)! \times (n-1)!} = \frac{n!}{1! \times (n-1)!} \; n$$

PRACTICE QUESTIONS

1. If the ratio between $\frac{n!}{2!(n-2)!}$ and $\frac{n!}{!(n-4)!}$ is 2:1, then find the value of n.

A. 5
B. 4
C. 3
D. 2

2. Find the value of n in $(n + 1)! = 12\ (n - 1)!$

A. 2

8. A gentleman has 5 friends to invite. In how many ways can he send invitation cards to them, if he has three servants to carry the cards?

A. 243
B. 247
C. 351
D. 498

9. In how many ways 3 prizes can be given away to 6 boys when each boy is eligible for any of the prizes?

B. 3
C. 4
D. 5

3. Find the value of n in $(n + 2)! = 60 (n - 1)!$
A. 1
B. 2
C. 3
D. 4

4. If ${}^{n}P_{3} = 120$, find the value of n.
A. 3
B. 4
C. 5
D. 6

5. If ${}^{10}P_{r} = 720$, then find the value of r.
A. 2
B. 3
C. 4
D. 5

6. How many numbers of four digits can be formed with digits 2, 4, 6, 7, and 8? (Repetition of digits is not allowed.)
A. 100
B. 110
C. 120
D. 130

7. How many numbers between 200 and 800 can be made with digits 0, 1, 2, 3, 4, and 5? (Repetition of digits not allowed.)
A. 50
B. 60
C. 70
D. 80

A. 195
B. 200
C. 216
D. 248

10. In how many ways can 7 boys be seated at a round table so that 3 particular boys are next to each other?
A. 12
B. 121
C. 11
D. 144

11. In how many ways can 7 boys be seated at a round table so that 2 particular boys are separated.
A. 480
B. 490
C. 500
D. 510

12. How many different letter arrangements can be made from the letters of the word ARISE?
A. 100
B. 120
C. 140
D. 160

13. How many different letter arrangements can be made from the letters of the word RECOVER?
A. 1,230
B. 1,240
C. 1,250
D. 1,260

14. How many permutations can be made out of the letters of the word TRIANGLE, which begin with *T* and end with *E*?
A. 720
B. 740
C. 760
D. 800

15. In how many ways can the letters of the word MOTHER be arranged so that all the vowels come together?
A. 100
B. 120
C. 140
D. 160

16. In how many ways can the letters of the word DIRECTOR be arranged so that the three vowels never come together?
A. 12,000
B. 15,000
C. 18,000
D. 20,000

21. In how many different ways can the letters of the word PEANUT be arranged?
A. 700
B. 710
C. 720
D. 730

22. A committee of five members is to be formed out of 4 students, 3 teachers, and 2 sports coaches. In how many ways can the committee be formed if the committee should consist of 2 students, 2 teachers, and 1 sports coach?
A. 6
B. 18
C. 36
D. 48

23. A committee of five members is to be formed out of 4 students, 3 teachers, and 2 sports coaches. In how many ways can the committee be formed if any five can be selected?

D. 20,000

17. There are 10 points in a plane out of which 4 are collinear. Find the number of straight lines formed by joining them.
A. 40
B. 45
C. 50
D. 55

18. In how many different ways can the letters of the word TRUST be arranged?
A. 60
B. 70
C. 80
D. 90

19. In how many different ways can the letters of the word ATTEND be arranged?
A. 320
B. 360
C. 400
D. 420

20. In how many different ways can the letters of the word BANKING be arranged?
A. 2,500
B. 2,520
C. 2,550
D. 3,000

selected?
A. 123
B. 124
C. 125
D. 126

24. In how many different ways can the letters of the word REPLACE be arranged?
A. 2,550
B. 2,540
C. 2,530
D. 2,520

25. In how many ways can a group of 5 men and 2 women be made out of a total of 7 men and 3 women?
A. 63
B. 64
C. 65
D. 66

26. How many different words can be formed with the letters of the word ALLAHABAD?
A. 7,560
B. 7,550
C. 7,540
D. 7,530

27. In how many different ways can the letters of the word SOFTWARE be arranged in

such a way that the vowels always come together?
A. 4,320
B. 4,330
C. 4,340
D. 4,350

28. In how many different ways can a group of 4 men and 4 women be formed out of 7 men and 8 women?
A. 2,400
B. 2,450
C. 2,500
D. 2,550

29. In how many different ways can the letters of the word MIRACLE be arranged?
A. 5,020
B. 5,040
C. 5,080
D. 5,160

30. A committee of 12 persons is to be formed from 9 women and 8 men. In how many ways can this be done if atleast 5 women have to be included in a committee?
A. 6,061
B. 6,062
C. 6,063
D. 6,064

31. A committee of 12 persons is to be formed from 9 women and 8 men. In how many

33. A committee of 5 members is to be formed out of 3 trainees, 4 professors, and 6 research associates. In how many different ways can this be done if the committee should have 2 trainees and 3 research associates?
A. 40
B. 50
C. 60
D. 70

34. In how many different ways can the number 25,69,774 be arranged, using each digit only once in each arrangement, such that the digits 6 and 5 are at the extreme ends in each arrangement?
A. 12
B. 24
C. 36
D. 48

35. Two girls and 4 boys are to be seated in a row in such a way that the girls do not sit together. In how many different ways can it be done?
A. 120
B. 240
C. 360
D. 480

36. In how many different ways can the letters of the word DRASTIC be arranged in such a way that the vowels always come together?

from 9 women and 8 men. In how many of these committees the women are in majority?

A. 2,700
B. 2,701
C. 2,702
D. 2,703

32. A committee of 5 members is to be formed out of 3 trainees, 4 professors, and 6 research associates. In how many different ways can this be done if the committee should have all 4 professors and 1 research associate or all 3 trainees and 2 professors?

A. 8
B. 10
C. 12
D. 14

A. 660
B. 680
C. 700
D. 720

37. In how many different ways can the letters of the word CASUAL be arranged?

A. 360
B. 180
C. 90
D. 45

38. A dinner party is to be fixed for a group of 100 persons. In this party 50 people do not prefer fish, 60 prefer chicken, and 10 do not prefer either chicken or fish. Find the number of people who prefer both fish and chicken?

A. 20
B. 30
C. 40
D. 50

39. In how many different ways the letters of the word RECTITUDE can be arranged so that vowels come together?

A. 4,320
B. 4,330
C. 4,340
D. 4,350

40. A person can go from Mumbai to Delhi by rail, road, or air. There are five routes to reach Manali from Delhi. Only one road goes from Manali to Ladakh. If Alina wants to go to Ladakh again but one of the roads between Delhi and Manali is closed, in how many different ways can she go from Mumbai to Ladakh?

A. 12
B. 24
C. 36
D. 48

41. Munim likes to have passwords of eight characters, the first four being different vowels and the last four being different digits. How many different passwords can he have?

A. 5,37,800
B. 6,04,800
C. 7,28,800
D. 8,94,500

42. A family of five goes in a five-seater car. In how many different ways can they sit?

A. 100
B. 120
C. 140
D. 160

A. 121
B. 144
C. 169
D. 181

45. Using all the letters of the word LINEAR, how many different words can be formed that start with a vowel but end with a consonant?

A. 214
B. 215
C. 216
D. 217

46. In how many different ways can the letters of the word READING be arranged in such a way that the vowels always come together?

A. 700
B. 710
C. 720
D. 730

47. Lakhan has 15 DVDs out of which 3 are of the movie Mother India, 5 are of the movie Sholay and the rest are all different. In how many ways can he arrange the DVDs on a shelf?

A. 1234567890
B. 1244985690
C. 1816214400
D. 1327528930

48. In how many ways can 5 people be seated around a circular table?

A. 20
B. 22
C. 24
D. 26

49. In how many ways can 6 different beads be arranged to form a necklace?

A. 30
B. 40

43. How many words can be formed by arranging the letters of the word HELPING?
A. 5,000
B. 5,040
C. 5,080
D. 5,120

44. Using all the letters of the word LINEAR, how many different words can be formed that start and end with a vowel?
C. 50
D. 60

50. In how many ways can four cards be selected from a pack of cards such that exactly one of them is an ace?
A. ${}^4C_2 \times {}^{52}C_4$
B. ${}^4C_3 \times {}^{52}C_3$
C. ${}^4C_4 \times {}^{48}C_4$
D. ${}^4C_1 \times {}^{48}C_3$

SOLUTIONS

1. (A) $\frac{n!/2!(n-2)!}{n!/4!(n-4)!} = \frac{2}{1}$

$\frac{n!}{2!(n-2)!} \times \frac{4!(n-4)!}{n!} = \frac{2}{1}$

$(n-2)(n-3) = 3 \times 2$

$n - 2 = 3$ and $n - 3 = 2$

So, $n = 5$

2. (B) $\frac{(n+1) \times n \times (n-1)!}{(n-1)!} = 12$

$(n + 1) \times n = 12$ or $(n + 1) \times n = 4 \times 3$

Hence, n = 3

3. (C) $\frac{(n+2) \times (n+1) \times n \times (n-1)!}{(n-1)!} = 60$

$(n + 2)(n + 1)\, n = 60$ or $(n + 2)(n + 1)\, n = 5 \times 4 \times 3$

Hence, $n = 3$

4. (D) $\frac{n!}{(n-3)!} = 120$

$\frac{n(n-1)(n-2)(n-3)!}{(n-3)!} = 120$

$n(n-1)(n-2) = 120 = 6 \times 5 \times 4$

Hence, $n = 6$

5. (B) $\frac{10!}{(10-r)!} = 720 = 10 \times 9 \times 8$

$\frac{10!}{(10-r)!} = \frac{10 \times 9 \times 8 \times 7!}{7!}$

$\frac{10!}{(10-r)!} = \frac{10!}{7!}$

$(10 - \boldsymbol{r})! = 7!$

So, $\boldsymbol{r} = 3$

6. (C) There are five numbers and number of places to be filled up = 4

So, required number of numbers is ${}^5P_4$

${}^5P_4 = \frac{5!}{(5-4)!} = 120$

7. (D) Any number between 200 and 800 will be of three digits and the first number must be fulfilled by 2, 3, 4, or 5 because if we start from 0 it will be lower than 200.

So, required number of ways to fill first place = 4

And, required number of ways to fill remaining two places = ${}^5P_2$

Required number of numbers = $4 \times {}^5P_2$

$= 4 \times \frac{5!}{(5-2)!} = 80$

8. (A) Invitation cards may be sent to each of 5 friends by any one of the three servants in 3 ways.

So, required ways = $3 \times 3 \times 3 \times 3 \times 3 = 3^5 = 243$.

9. (C) Each of the three prizes can be given away to any of the 6 boys in 6 ways.

So, the required number of ways = $6^3 = 216$ ways.

10. (D) Let the 3 particular boys be taken together as one unit, then the number of units will be 5.

They can sit around the table in 4! Ways, and for each of this arrangement, 3 can be interchanged in 3! Ways.

Hence, total number of arrangements = $4! \times 3! = 24 \times 6 = 144$

11. (A) Total ways in which 7 boys can be seated at a round table = $(7 - 1)! = 6!$

Total ways in which 2 particular boys sit together while arranging 7 boys = $5! \times 2!$

The arrangements that the two persons are separated = $6! - 5! \times 2! = 480$ ways.

12. (B) In word ARISE all 5 letters are different.

Hence, total number of permutations = ${}^5P_5$

$= \frac{5!}{(5-5)!} = 120$

13. (D) In word RECOVER, 2 letters E and R have come two times.

Total number of permutation = $\frac{7!}{2!2!} =$ 1260 ways.

14. (A) Total letters in word TRIANGLE = 8

Two letters T and E have fixed positions, so remaining letters = 6

Number of permutations = 6! = 720.

15. **(B)** Number of vowels in word MOTHER = (O, E) = 2
Total letters in MOTHER after leaving vowels = M, T, H, R = 4
As all vowels come together, so we will count them as 1, then we have only 4 + 1 = 5 letters.
Arrangements of these 5 letters = 5! = 120.

16. **(C)** Total number of letters = 8 and letter R occurring twice. Number of vowels = 3
Total number of arrangements when there is no restriction = $\frac{8!}{2!}$

When three vowels are together, taking them as one letter, we have only 5 + 1 = 6 letters.
These 6 letters can be arranged in $\frac{6!}{2!}$ ways, since R occurs twice.
Number of arrangements when 3 vowels are together = $\frac{6!}{2!} \times 3!$

Required number = $\frac{8!}{2!} - \frac{6!}{2!} \times 3! = 18000$.

17. **(A)** Let us suppose that the 10 points are such that no three of them are collinear. Now a straight line will be formed by any two of these 10 points. Thus forming a straight line amounts to selecting two of the 10 points.
Now out of 10 points 2 can be selected in ${}^{10}C_2$ ways.
Number of straight lines formed by 10 points when now let the four points become collinear, then ${}^4C_2$ straight lines formed by them will reduce to only one straight line.
Required number of lines formed = ${}^{10}C_2 - {}^4C_2 + 1 = 45 - 6 + 1 = 40$.

18. **(A)** The word TRUST consists of five letters in which T comes twice.
Number of arrangements = $\frac{5!}{2!} = 60$.

19. **(B)** The word ATTEND consists of six letters in which T comes twice.
Number of arrangements = $\frac{6!}{2!} = 360$.

20. **(B)** The word BANKING consists of seven letters in which N comes twice.
Number of arrangements = $\frac{7!}{2!} = 2{,}520$.

21. **(C)** The word PEANUT consists of six distinct letters.
Number of arrangements = 6! = 720.

22. **(C)** Required number of combinations = ${}^4C_2 \times {}^3C_2 \times {}^2C_1 = 36$.

23. **(D)** Required number of combinations = ${}^9C_5 = 126$.

24. **(D)** The word REPLACE consists of seven letters in which E comes twice.
Number of arrangements = $\frac{7!}{2!} = 2520$.

25. **(A)** There are 7 men and 3 women. We have to select 5 men out of 7 and 2 women out of 3. This can be done in ${}^7C_5 \times {}^3C_2$ ways.
The number of ways of making the selection = ${}^7C_5 \times {}^3C_2 = 63$.

26. **(A)** There are 9 letters in the word ALLAHABAD out of which 4 are As and 2 are *L*.
So, required number of words = $\frac{9!}{4!2!} = 7{,}560$.

27. **(A)** There are 8 letters in the word SOFTWARE, including 3 vowels (O, A, E) and 5 consonants (S, F, T, W, R).
Considering three vowels as one letter, we have six letters which can be arranged in ${}^6P_6 = 6!$ ways.
But corresponding to each way of these arrangements, the vowels can be put together in 3! ways.
So, the required number of words = 6! × 3! = 4320.

28. **(B)** 4 men out of 7 men and 4 women out of 8 women can be chosen in ${}^7C_4 \times {}^8C_4 = 2{,}450$.

29. **(B)** The word MIRACLE has 7 distinct letters.
So, the number of arrangements = 7! = 5,040.

30. **(B)** There are 9 women and 8 men. A committee of 12 consisting of at least 5 women can be formed by choosing.
5 women and 7 men
6 women and 6 men

7 women and 5 men
8 women and 4 men
9 women and 3 men
So, total number of ways of forming the committee = ${}^9C_5 \times {}^8C_7 + {}^9C_6 \times {}^8C_6 + {}^9C_7 \times {}^8C_5 + {}^9C_8 \times {}^8C_4 + {}^9C_9 \times {}^8C_3$
= 126 × 8 + 84 × 28 + 36 × 56 + 9 × 70 + 1 × 56 = 6,062.

31. (C) There are 9 women and 8 men. A committee of 12, consisting of at least 5 women, can be formed by choosing.
5 women and 7 men
6 women and 6 men
7 women and 5 men
8 women and 4 men
9 women and 3 men
As, women are in majority in 3rd, 4th, and 5th case.
So, total number of such committees = ${}^9C_7 \times {}^8C_5 + {}^9C_8 \times {}^8C_4 + {}^9C_9 \times {}^8C_3$
= 36 × 56 + 9 × 70 + 1 × 56 = 2,702.

32. (C) Number of combinations = ${}^4C_4 \times {}^6C_1 + {}^3C_3 \times {}^4C_2$
1 × 6 + 1 × 6 = 12.

33. (C) We have to select 2 trainees out of 3 and 3 research associates out of 6.
So, the number of combinations = ${}^3C_2 \times {}^6C_3$ = 60.

34. (D) Case 1 when arrangement is 6----5
Four empty places can be filled by 2, 9, 7 and 4 in 4! = 24 ways.
Case 1 when arrangement is 5----6
Four empty places can be filled by 2, 9, 7 and 4 in 4! = 24 ways.
Required number of arrangements = 24 + 24 = 48 ways.

35. (D) 4 boys can be seated in row in ${}^4P_4 = 4!$ ways.
Now in the 5 gaps 2 girls can be arranged in ${}^5P_2$ ways.
Hence, the number of ways in which no two girls sit together = 4! × ${}^5P_2$ = 480 ways.

36. (D) There are 7 letters in the word DRASTIC including 2 vowels (A, I) and 5 consonants (D, R, S, T, C), considering two vowels as one letter, we have 6 letters which can be arranged in 6! ways. But corresponding to each way of the arrangements, the vowels can be put together in 2! ways.
$= 6! \times \frac{2!}{2!} = 720$ ways.

37. (A) The word CASUAL has 6 letters in which letter A comes twice.
So, number of arrangements = $\frac{6!}{2!}$ = 360 ways.

38. (A) 50 out of 90 do not prefer fish.
So, the required answer = 60 – 40 = 20.

39. (A) There are 9 letters in the word RECTITUDE including 4 vowels (E, I, U, E) and 5 consonants (R, C, T, T, D), and letters T and E come twice.
So, number of arrangements = $\frac{6! \times 4!}{2! \times 2!}$ = 4,320 ways.

40. (A) Mumbai to Delhi = 3 ways (rail, road, air)
Delhi to Manali = 5 ways (5 routes)
Manali to Ladakh = 1 way (one road)
So, total ways to reach Ladakh from Mumbai are 3 × 5 × 1 = 15 ways.
Since one road between Delhi and Manali is closed, there are four possible roads.
Hence, total ways = 3 × 4 × 1 = 12 ways.

41. (B) The first 4 characters of Munim's password are to be chosen from the 5 vowels. Since there are 5 vowels available, the first vowel can be chosen in 5 different ways. Now, as all four vowels need to be different, the second vowel can be chosen only in 4 ways, the third in 3 ways, and the last in 2 ways. Similarly, the four digits have to be different. The first digit can be chosen from the 10 digits in 10 ways, the second can be chosen from the remaining 9 digits in 9 ways. Similarly, the third and fourth digit can be chosen in 8 and 7 ways, respectively. Hence, the four numbers can be chosen from the 10 digits from 0 to 9 in 10, 9, 8, and 7 ways, respectively.

Thus the number of different passwords that Munim can have = 5 × 4 × 3 × 2 × 10 × 9 × 8 × 7 = 604800.

42. (B) Number of seats = 5 and number of people = 5

46. (C) The word READING has 7 different letters.
The vowels (E, A, I) can be arranged among themselves in 3! = 6 ways
Since E, A, and I are considered to be

people = 5.
So, required arrangement = ${}^5P_5$ = 5! = 120 ways.

43. **(B)** The word HELPING has 7 distinct letters.
So, the number of arrangements = 7! = 5040.

44. **(B)** The word LINEAR has 3 vowels = I, E and A.
If a word starts and ends with a vowel, the two letters to occupy the first and the last positions can be selected and arranged in ${}^3P_2$ = 6 ways.
The remaining 4 letters can be arranged among themselves in ${}^4P_4$ = 4! = 24 ways.
Hence, the number of words that start and end with a vowel = 24 × 6 = 144.

45. **(C)** The word LINEAR has 3 vowels (I, E, A) and 3 consonants (L, N, R).
If a word starts with a vowel but ends with a consonant, its first letter can be selected from I, E, and A in 3 ways. Its last letter can be selected from L, N, and R in 3 ways. The remaining four letters can be arranged in 4! ways.
Hence, the number of words that start with a vowel but end with a consonant = 3 × 3 × 4! = 9 × 24 = 216.

together, consider these as one letter.
Hence, letters to be arranged are R, D, N, G, vowels. These can be arranged in 5! = 120 ways
So, the required number of ways = 120 × 6 = 720 ways.

47. **(C)** There are 15 DVDs out of which 3 are of one kind and 5 are of another kind.
So, the total number of arrangements possible = $\frac{15!}{3! \times 5!}$ = 1816214400.

48. **(C)** n objects can be arranged in a circle in $(n - 1)!$ ways.
Similarly, 5 people can be seated around a circular table in (5 – 1)! = 4! = 24 ways.

49. **(D)** n objects can be arranged in a necklace in ½ × $(n - 1)!$ ways
Thus, 6 beads can form a necklace in $1/2 \times (6 - 1)! = \frac{5!}{2}$ = 60 ways.

50. **(D)** An ace can be selected from 4 aces in ${}^4C_1$ ways.
Since exactly one card is an ace, the remaining three cards can be selected from the 52 – 4 = 48 non-ace cards in ${}^{48}C_3$ ways.
So, total number of selections = ${}^4C_1 \times {}^{48}C_3$.

22 Probability

SYNOPSIS

- Introduction
- General terms of Probability
- Types of Events
- Probability
- Addition Theorem of Probability
- Conditional Probability
- Odds

Probability is a method of expressing the event of something occurring or not happening. The higher the probability, the greater the odds of an event occurring and vice versa.

SOME GENERAL TERMS OF PROBABILITY

Deterministic Experiment: The experiment that

TYPES OF EVENT

Simple Event: If an event contains only one sample point, then it is called a simple event. Consider the sample space of the experiment of rolling an unbiased die,

S = {1, 2, 3, 4, 5, 6}

Any subset of S is called an *event*.

The event *E* of getting *a* perfect cube when an unbiased die is rolled once is an example of a simple event, i.e., *E = {1}*.

Impossible Event: An impossible event is one in which the sample set is empty, i.e., it contains no sample points. For example, getting 7 by throwing a die is an impossible event.

Certain Event: If the event is sure to happen, it is a certain event. For example, getting a value less than 7 on throwing an unbiased die.

Deterministic Experiment: The experiment that gives a definite result is called a deterministic experiment. For example, rolling a fair die: each number on a six-sided die has the same odds (1/6) of coming up.

Random Experiment: A random experiment is an action that gives one or more results. For example, we toss a coin three times and watch the heads/tails sequence. The sample space in this case could be described as follows: S = {(H, H, H), (H, H, T), (H, T, H), (T, H, H), (H, T, T), (T, H, T),(T, T, H), (T, T, T)}.

Outcome: The result of a random experiment is called an outcome.

Sample Space: The sample space (S) is the set of all possible outcome of an event. The number of elements in the sample space is denoted by *n*(S). For example, when a single die is thrown, it has 6 outcomes since it has 6 faces. Therefore, the sample is given as *S* = {1, 2, 3, 4, 5, and 6}.

Unbiased Experiment: A random experiment having equally likely outcomes is called an unbiased experiment.

Complementary Event: Let A be an event in the sample space S. Then A is a subset of S. The complement of A is (S - A) It is represented as A'.

Combination of Events: The *union* of the events A and B of a sample space (A U B) is the event that either A or B or both take place.

The *intersection* of the events A and B of a sample space (A ∩ B) is the event that both A and B take place.

Exhaustive Events: If two events A and B of a sample space S are such that (A U B) = S, then A and B are called exhaustive events.

If A and B are exhaustive, (A U B) = S

So, *n*(A U B) = *n*(S)

Independent Events: If the happening of one event, A has no effect on the other event B, then A and B are said to be independent of each other.

Example: When three unbiased dice are rolled, what type of events are these —'getting a total greater than 2' and 'getting a total greater than 20'?

Solution: When three dice are rolled, the minimum total is 1 + 1 + 1 = 3 and the maximum total is 6 + 6 + 6 = 18.

Hence, the event getting a total greater than 3" degrees will always happen and is hence a 'certain event'. On the other hand, the event 'getting a total greater than 18' will never happen and hence, an 'impossible event'.

PROBABILITY

Probability refers to the occurrence of an event occurring. If S is an unbiased experiment's sample space and E is an event, then the probability that it occurs is,

$$P(E) = \frac{n(E)}{n(S)}$$

Example: Consider tossing an unbiased coin. Let E be the event that the result is heads. Let E' be an event that the result is tails.

Then, *n*(S) = 2; *n*(E) = 1 and *n*(E') = 1

So, $P(E) = \frac{1}{2}$ and $P(E') = \frac{1}{2}$

So when a coin is tossed, the probability of occurrence of heads is ½. Similarly, the probability of occurrence of tails is ½.

Example: Two unbiased coins are tossed. What is the probability that both the tosses give different results?

Solution: When a coin is tossed the result is either heads (H) or tails (T).

Tossing two coins gives 4 outcomes, i.e., S = {HH, HT, TH, TT}

Hence, P(A U B) = P(A) + P(B)

Example: A bag contains 20 coins numbered 1 to 20. At random, a coin is chosen. What is the probability that the coin will come up with a number that is a multiple of 3 or 5?

Solution: Multiples of 3 from 1 to 20 = 3, 6, 9, 12, 15, 18, i.e., 6 values.

Multiples of 5 from 1 to 20 = 5, 10, 15, 20, i.e., 4 values.

Multiples of both 3 and 5 = 15, i.e., only 1 value.

So, number of multiples of 3 or 5 = 6 + 4 - 1 = 9

There are 20 numbers in all.

Hence, required probability = $\frac{9}{20}$

CONDITIONAL PROBABILITY

Conditional probability is the probability of an event or outcome that depends on the occurrence of a preceding event or outcome.

Let A and B be two events defined on a sample space S.

Let P(B) > *0*. Then the conditional probability of A given B, is denoted by P(A/B) and is defined by,

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Example: There is a 0.03 probability that it is Friday and a student is absent. As only school is open only for 5 days, there is a 0.2 probability that it is Friday. Given that it is Friday, what is the probability that a student will be absent?

Hence, $n(S) = 4$

Now, let A be the event that both the tosses give different results. The first coin shows heads and the second shows tails or the first coin shows tails and the second shows heads.

So, A = {HT, TH} and $n(A) = 2$

Hence, $P(A) = \frac{n(A)}{n(S)} = \frac{2}{4} = \frac{1}{2}$.

ADDITION THEOREM OF PROBABILITY

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

If A and B are mutually exclusive events, $n(A \cap B) = 0$.

So, $P(A \cap D) = 0$

Solution:

$P(\text{Absent} \mid \text{Friday}) = \frac{P(\text{Absent} \cap \text{Friday})}{P(\text{Friday})} = \frac{0.03}{0.2}$,

i.e., 0.15 = 15%

ODDS

$$\text{Odds in Favour} = \frac{\text{Number of favourable cases}}{\text{Number of unfavourable cases}}$$

$$\text{Odds Against} = \frac{\text{Number of unfavourable cases}}{\text{Number of favourable cases}}$$

Example: A boy randomly picks a marble from his bag containing 10 yellow, 18 orange, 6 red, and 20 blue marbles. What are the odds that his marble is blue or orange?

Solution: Total marbles 10 + 18 + 6 + 20 = 54

So, total events = ways of picking 1 marble out of 54 = 54

Number of favourable events = ways of picking a blue or orange marble = 20 + 18 = 38.

And, the number of unfavourable events = 54 – 38 = 16.

Hence, odds of getting a blue or orange marble = 38:16 = 19:8.

PRACTICE QUESTIONS

1. A bag contains seven balls, four of which are red and three of which are blue. How probable is it that a blue ball will be chosen?
A. 4/7
B. 3/7
C. 2/7
D. 1/7

2. A dice is tossed. Calculate the probability of getting a multiple of three.
A. 1/12
B. 1/9
C. 1/6
D. 1/3

3. What is the probability of throwing a number larger than 4 using regular dice with numbered faces ranging from 1 to 6?
A. 1/3
B. 1/4
C. 1/5
D. 1/6

4. Three balls are picked at random from a bag containing four white and five black balls. Calculate the probability that all three are black.
A. 1/14
B. 2/21
C. 5/42
D. 1/42

6. Tickets with numbers ranging from 1 to 20 are mixed together and a ticket is selected at random. What is the probability that the ticket will contain a number that is a multiple of 3 or 7?
A. 1/5
B. 2/5
C. 3/5
D. 4/5

7. A word is made up of nine letters, five consonants, and four vowels. At random, three letters are picked. What is the probability that more than one vowel will be chosen?
A. $({}^3C_2 \times {}^4C_2 + {}^4C_3)/{}^9C_3$
B. $({}^2C_2 \times {}^3C_2 + {}^1C_2)/{}^9C_3$
C. $({}^4C_2 \times {}^5C_1 + {}^4C_3)/{}^9C_3$
D. $({}^1C_3 \times {}^3C_1 + {}^4C_3)/{}^9C_3$

8. From a group of three men, two women, and four children, four people will be picked at random. What is the probability of choosing one man, one woman, and two children?
A. 1/7
B. 3/7
C. 5/7
D. 2/7

9. From a group of three men, two women, and four children, four people will be

5. Two dice are thrown at the same time. What is the probability of having an even-numbered doublet?

A. 1/12
B. 1/3
C. 1/4
D. 1/6

picked at random. What is the probability of selecting exactly two children?

A. 8/21
B. 3/7
C. 10/21
D. 11/21

10. From a group of three men, two women, and four children, four people will be picked at random. What is the probability of choosing two women?

A. 1/6
B. 1/3
C. 1/2
D. 2/3

11. What are the chances that a leap year with 53 Sundays is chosen at random?

A. 1/7
B. 4/7
C. 6/7
D. 2/7

12. If the year chosen is not a leap year, what are the chances of 53 Sundays?

A. 2/7
B. 1/7
C. 4/7
D. 3/7

13. From a typical deck of 52 playing cards, a single card is chosen at random. What are the chances of picking a king or a club?

A. 1/13
B. 2/13
C. 3/13
D. 4/13

14. There are 17 boys and 13 girls in a maths class of 30 students. Four boys and five girls received an A on a unit test. What are the chances of picking a female or an 'A grade student' at random from the class?

A. 7/15
B. 8/15
C. 17/30
D. 19/30

15. From a class of 50 males, a teacher selects one student at random. What is the probability that the chosen student is a boy?

A. 1/2
B. 1
C. 1/4
D. 1/3

16. There are 5 white and 8 black balls in each bag. Two consecutive draws of three balls are made, with the balls not being replaced before the second draw. What is the probability that the first draw will provide three white balls and the second draw will yield three black balls?

A. 7/429
B. 5/666
C. 8/671
D. 13/456

17. A has a 1/3 probability of hitting the target, but B has a 2/5 probability of hitting it. If each of A and B shoots at the target, what is the probability that the target will be hit?

A. 1/5
B. 2/5
C. 3/5
D. 4/5

18. Six red, five green, and eight blue balls are contained in a basket. What is the probability that all four balls are red or that any two of the four are green if four balls are chosen at random?

A. 437/3474
B. 675/3214
C. 925/3876
D. 435/3475

19. Six blue, two red, four green, and three yellow balls are contained in a basket. What is the probability that two balls are taken at random and both are green or both are yellow?

A. 1/35
B. 2/35
C. 3/35
D. 4/35

20. Six blue, two red, four green, and three yellow balls are contained in a basket. What is the probability that at least one of the five balls chosen at random is blue?

A. 137/141
B. 37/41
C. 137/143
D. 37/43

21. Six blue, two red, four green, and three yellow balls are contained in a basket. What is the probability that two balls are drawn at random and both turn out to be blue?
A. 1/7
B. 1/9
C. 1/11
D. 1/13

22. Six blue, two red, four green, and three yellow balls are contained in a basket. What is the probability that two balls are red and two are green if four balls are chosen at random?
A. 1/455
B. 2/455
C. 3/455
D. 4/455

23. There are six blue, two red, four green, and three yellow balls in a basket. What is the probability that none of the three balls chosen at random are yellow?
A. 6/13
B. 43/91
C. 44/91
D. 3/13

24. Three blue and four red balls are contained in a basket. What is the probability that three balls are picked at random from the basket and all three are blue or red?
A. 1/3
B. 1/5
C. 1/6
D. 1/7

25. Four green, five blue, two red, and three yellow marbles are contained in an urn. What is the probability that both or at least one marble will be red if two marbles are picked at random?
A. 23/91
B. 25/91
C. 27/91
D. 29/91

26. Four green, five blue, two red, and three yellow marbles are contained in an urn. What is the probability that at least one marble will be yellow if three marbles are picked at random?
A. 197/364
B. 199/364
C. 201/364
D. 50/91

27. Four green, five blue, two red, and three yellow marbles are contained in an urn. What is the probability that there are equal numbers of marbles of each colour if 8 marbles are picked at random?
A. 50/1001
B. 60/1001
C. 70/1001
D. 80/1001

28. Four green, five blue, two red, and three yellow marbles are contained in an urn. What is the probability that none of the three marbles picked at random are green?
A. 20/91
B. 30/91
C. 40/91
D. 50/91

29. Four green, five blue, two red, and three yellow marbles are contained in an urn. What is the probability that two blue marbles and two red marbles are picked at random?
A. 10/1001
B. 20/1001
C. 30/1001
D. 40/1001

30. There are 13 white and 7 black balls in each bag. At random, two balls are picked. What's the probability they're the same colour?
A. 1/2
B. 99/190
C. 99/199
D. 95/199

31. What is the probability that one card picked at random from a well-shuffled pack of 52 playing cards will be a black king?
A. 1/13
B. 2/13
C. 3/13
D. 4/13

32. What is the probability of rolling two dice and obtaining the number 4 consecutively?
A. 1/9
B. 1/18
C. 1/27
D. 1/36

33. Three red balls, two blue balls, and one black ball are contained in a bag. What are the chances of getting a blue ball out of the bag?
A. 1/6
B. 1/2
C. 1/3
D. 1/4

34. What is the probability of getting an even number when a fair die is thrown? What's the probability you'll obtain 4 or a greater number?
A. 1/3
B. 1/4
C. 1/2
D. 1/6

35. Two dice are thrown at the same time. What is the probability that the sum of the thrown numbers is less than 2?
A. 1/9
B. 1/36
C. 0
D. 1/18

36. Two dice are thrown at the same time. What is the probability that the sum of the thrown numbers is 9?
A. 1/18
B. 1/3
C. 1/9
D. 1/12

37. Two dice are thrown at the same time. What is the probability that the sum of the thrown numbers is even?
A. 1/2
B. 1/3
C. 1/4
D. 1/5

38. What is the probability of receiving the same number on both dice if two dice are thrown at the same time?
A. 1/3
B. 1/6
C. 1/9
D. 1/12

39. What is the probability of receiving the same face on a coin tossed twice?
A. 1
B. 1/4
C. 3/4
D. 1/2

40. What is the probability of receiving the same face if three coins are tossed simultaneously?
A. 1/2
B. 1/3
C. 1/4
D. 1/5

41. In a class of 20, 13 students passed the statistics exam. What is the probability of a student passing statistics if they are chosen at random from this group?
A. 7/20
B. 11/20
C. 12/20
D. 13/20

42. What is the probability of getting two consecutive numbers on a pair of dice if both are rolled at the same time?
A. 1/9
B. 1/6
C. 2/9
D. 5/18

43. What is the probability of getting the same face on a dice if you roll it three times?
A. 1/9
B. 1/18
C. 1/27
D. 1/36

47. What are the chances of getting a sum of 10 if a pair of dice is rolled?
A. 1/12
B. 1/3
C. 1/6
D. 1/9

44. Two dice are rolled at the same time. What is the probability that the sum of the faces will be more than three?
A. 4/9
B. 5/9
C. 7/12
D. 11/12

45. What is the probability of getting at least one 6 if a fair die is thrown twice?
A. 11/27
B. 11/36
C. 1/18
D. 7/36

46. Ram tosses a die into the air. What are the chances that it won't land on the same side two times in a row?
A. 1/6
B. 1/2
C. 5/7
D. 1/3

48. Ravi will win the game if he gets a 4 on the die roll. What is the probability?
A. 1/3
B. 1/4
C. 1/5
D. 1/6

49. Calculate the probability of getting a multiple of two when you roll a die.
A. 2/3
B. 1/2
C. 1/6
D. 1/3

50. Three dice are thrown at the same time. What are the chances of all three faces being the same on the top?
A. 1/36
B. 1/27
C. 1/18
D. 1/9

SOLUTIONS

1. **(B)** Number of favourable outcomes = 3
Total number of outcomes = 7
Probability of one ball = 3/7

2. **(D)** S = {1, 2, 3, 4, 5, 6}
Total number of outcomes = 6
Total number of favourable event = {3, 6} = 2
Probability of getting a multiple of 3 = 2/6 = 1/3

3. **(A)** Total number of outcomes = 6
Total number of favourable event = {5, 6} = 2
Probability of getting a number greater than 4 = 2/6 = 1/3

4. **(C)** 3 balls can be selected from 9 balls in $n(S) = {}^9C_3$ ways
If A be the event of getting 3 black balls in $n(A) = {}^5C_3$ ways
Required probability $P(A) = \frac{n(A)}{n(S)} = \frac{5}{42}$

5. **(A)** Here, $n(S) = 36$
And doublet of even number, $n(A)$ = {(2, 2) (4, 4) (6, 6)} = 3
So, probability = 3/36 = 1/12

6. **(B)** Let A be the required event then, A = {3, 6, 7, 9, 12, 14, 15, 18}
and, $n(A) = 8$; $n(S) = 20$
Required probability = 8/20 = 2/5

7. **(C)** Three letters can be chosen out of 9 letters in ${}^9C_3$ ways

More than one vowel can be chosen in the following way. 2 vowels and one consonant or 3 vowels.
If A be the required event, then, $n(A) = ({}^4C_2 \times {}^5C_1 + {}^4C_3)/{}^9C_3$
Hence, this is the required probability.

8. **(D)** Out of 9 persons, 4 can be selected in ${}^9C_4 = 126$ ways.
Hence, $n(S) = 126$
Let A be the required event then, $n(A) = {}^3C_1 \times {}^2C_1 \times {}^4C_2 = 36$
Hence required probability = 36/126 = 2/7

9. **(C)** Out of 9 persons, 4 can be selected in ${}^9C_4 = 126$ ways.

14. **(C)** $n(S) = 30$
P(Girl or A) = P(Girl) + P(A) – P (Girl and A)
= 13/30 + 9/30 – 5/30 = 17/30

15. **(B)** As all the students are boys, so required probability is,
= 50/50 = 1

16. **(A)** Let A be the event of drawing 3 white balls in the first draw and B be the event of drawing 3 black balls.
Hence, required probability = $P(A \cap B) = P(A) \times P(B/A)$
P(A) = 5/143
After drawing 3 white balls in the first draw 10 balls are left in the bag, out of which 8

Hence, $n(S) = 126$
Let A be the required event then, $n(A) = {}^4C_2 \times {}^5C_2 = 60$
Hence required probability = 60/126 = 10/21

10. (A) Out of 9 persons, 4 can be selected in ${}^9C_4 = 126$ ways.
Hence, $n(S) = 126$
Let A be the required event then, $n(A) = {}^2C_2 \times {}^7C_2 = 21$
Hence required probability = 21/126 = 1/6

11. (D) A leap year has 366 days so it has 52 complete weeks and 2 more days. The two days can be Sunday and Monday, Monday and Tuesday, Tuesday and Wednesday, Wednesday and Thursday, Thursday and Friday, Friday and Saturday, Saturday and Sunday, i.e., $n(S) = 7$.
Out of these 7 cases, cases favourable for more Sundays are (Sunday and Monday, Saturday and Sunday), i.e., $n(E) = 2$
Hence, required probability = 2/7

12. (B) When the year is not a leap year, it has 52 complete weeks and 1 more day that can be Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, $n(S) = 7$
Out of these 7 cases, cases favourable for one more Sunday is $n(E) = 1$.
Hence, required probability = 1/7

13. (D) Here, $n(S) = 52$
So, P(King or Club) = P(King) + P(Club) – P(King and Club) = 4/52 + 13/52 – 1/52
= 16/52 = 4/13

are black balls.
So, P(B/A) = 7/15
Hence, $P(A \cap B) = P(A) \times P(B/A) = 5/143 \times 7/15 = 7/429$

17. (C) Let A and B are the events of hitting the target and A' and B' are not hitting the target.
So, P(A) = 1/3; P(A') = 2/3; P(B) = 2/5; P(B') = 3/5
Required probability = A hits the target and B does not hit the target
Or
B hits the target and A does not hit the target
Or
A hits the target and B hits the target
= (1/3 × 3/5) + (2/5 × 2/3) + (1/3 × 2/5) = 3/5

18. (C) Number of balls = 19
Ways of selecting 4 balls out of 19 = ${}^{19}C_4$ = 3,876
Selecting 4 red balls or any two green balls out of the four = 15 + 910 = 925
Hence, required probability = 925/3,876

19. (C) Total numbers of the balls in the basket = 15
Number of ways of selecting 2 balls out of 15 balls = ${}^{15}C_2 = 105$
Favourable number of cases = ${}^4C_2 + {}^3C_2 = 9$
Hence, required probability = 9/105 = 3/35

20. (C) Total numbers of the balls in the basket = 15
Number of ways of selecting 5 balls out of 15 balls = ${}^{15}C_5 = 105$

Let no blue ball be selected
So, number of ways of selecting 5 balls out of 9 balls without blue balls = ${}^9C_5$
Hence, required probability = $1 - {}^9C_5 / {}^{15}C_5$
= 1 – 6/143 = 137/143

21. (A) Total numbers of the balls in the basket = 15
Number of ways of selecting 2 balls out of 15 balls = ${}^{15}C_2 = 105$
Favourable number of cases = ${}^6C_2 = 15$
Hence, required probability = 15/105 = 1/7

22. (B) Number of ways of selecting 4 balls out of 15 balls = ${}^{15}C_4 = 1365$
Favourable number of cases = ${}^2C_2 + {}^4C_2 = 6$
Hence, required probability = 6/1365 = 2/455

23. (C) Number of ways of selecting 3 balls out of 15 balls = ${}^{15}C_3 = 455$
Favourable number of cases = ${}^{12}C_3 = 220$
Hence, required probability = 220/455 = 44/91

24. (D) Number of possible outcomes = ${}^7C_3 = 35$

No ball is green. So, selection of 3 marbles out of 5 blue, 2 red and 3 yellow marbles = ${}^{10}C_3 = 120$
Hence, required probability = 120/364 = 30/91

29. (A) Total possible outcome = ${}^{10}C_3 = 1{,}001$
Favourable outcome = ${}^5C_2 \times {}^2C_2 = 10$
Hence, required probability = 10/1,001

30. (B) Total possible outcome = ${}^{20}C_2 = 190$
Total favourable outcome = ${}^{13}C_2 + {}^7C_2 = 78 + 21 = 99$
Hence, required probability = 99/190

31. (C) Total possible outcome = ${}^{52}C_1 = 52$
Favourable number of cases = ${}^{12}C_1 = 12$
Required probability = 12/52 = 3/13

32. (D) Total outcome for a pair of dice = 36
Favourable outcome = 1
Hence, required probability = 1/36

33. (C) Total number of balls = 6
Favourable outcome = 2 blue balls
Probability of blue ball = 2/6 = 1/3

Favourable number of cases = $^4C_3 + {}^3C_3$ = 4 + 1 = 5
Hence, required probability = 5/35 = 1/7

25. **(B)** Total number of marbles in the urn = 4 + 5 + 2 + 3 = 14
Total possible outcome = Selection of 2 marbles out of 14 marbles = $^{14}C_2 = 91$
Favourable number of cases = $^2C_2 + {}^2C_1 \times {}^{12}C_1 = 1 + 2 \times 12 = 25$
Hence, required probability = 25/91

26. **(B)** Total possible outcome = $^{14}C_3 = 364$
When no marble is yellow, favourable number of cases = $^{11}C_3 = 165$
So, probability that no marble is yellow = 165/364
Hence, required probability = 1 – 165/364 = 199/364

27. **(B)** Total possible outcome = $^{14}C_6 = 3003$
Favourable number of cases = $^4C_2 \times {}^5C_2 \times {}^2C_2 \times {}^3C_2 = 180$
Hence, required probability = 180/3003 = 60/1001

28. **(B)** Total possible outcome = $^{14}C_3 = 364$

34. **(C)** For even number,
Total outcome = 6
Favourable outcome (even numbers) = 3
Hence, required probability = 3/6 = 1/2
For 4 or higher number,
Total outcome = 6
Favourable outcome (4, 5, and 6) = 3
Hence, required probability = 3/6 = 1/2

35. **(C)** Total outcome = 36
Favourable outcome (sum is less than 2) = 0
Hence, required probability = 0/6 = 0

36. **(C)** Total outcome = 36
Favourable outcome, sum is 9 {(4, 5) (5, 4) (3, 6) (6, 3)} = 4
Hence, required probability = 4/36 = 1/9

37. **(A)** Total outcome = 36
Favourable outcome (even sum) = 18
Hence, required probability = 18/36 = 1/2

38. **(B)** Total outcome = 36
Favourable outcome (same number) = {(1, 1) (2, 2) (3, 3) (4, 4) (5, 5) (6, 6)} = 6
Hence, required probability = 6/36 = 1/6

39. **(D)** Total outcome $(2^2) = 4$
Favourable outcome (same face appearing) = {(H, H) (T, T)} = 2
Hence, required probability = 2/4 = 1/2

40. **(C)** Total outcome $(2^3) = 8$
Favourable outcome (same face appearing) = {(H, H, H) (T, T, T)} = 2
Hence, required probability = 2/8 = 1/4

41. **(D)** Total outcome (number of students) = 20
Favourable outcome (passed in statistics) = 13
Hence, required probability = 13/20

42. **(D)** Total outcome = 36
Favourable outcome (two consecutive numbers) = {(1, 2) (2, 3) (3, 4) (4, 5) (5, 6) (2, 1) (3, 2) (4, 3) (5, 4) (6, 5)} = 10 ways
Hence, required probability = 10/36 = 5/18

43. **(D)** Total outcome $(6^3) = 216$
Favourable outcome (having same face) = {(1, 1, 1) (2, 2, 2) (3, 3, 3) (4, 4, 4) (5, 5, 5) (6, 6, 6) = 6 ways
Hence, required probability = 6/216 = 1/36

44. **(D)** Total outcome = 36
Favourable outcome (sum less than or equal to 3) = {(1, 1) (2, 1) (1, 2)} = 3
Hence, required probability = 33/36 = 11/12

45. **(B)** Total outcome = 36
Favourable outcome (at least one 6) = {(1, 6) (2, 6) (3, 6) (4, 6) (5, 6) (6, 6) (6, 1) (6, 2) (6, 3) (6, 4) (6, 5)} = 11
Hence, required probability = 11/36

46. **(C)** Total outcome = 36
Possible outcome with same side = {(1, 1) (2, 2) (3, 3) (4, 4) (5, 5) (6, 6)} = 6 outcome
So, probability of same side = 6/36 = 1/6
Hence, probability of not landing on same side = 1 – 1/6 = 5/6

47. **(A)** Total outcome = 36
Favourable outcome (getting a sum of 10) = {(4, 6) (5, 5) (6, 4)} = 3
Hence, required probability = 3/36 = 1/12

48. **(D)** Total outcome = 6
Favourable outcome (rolling a 4) = 1
Hence, required probability = 1/6

49. **(B)** Total outcome = 6
Favourable outcome (multiple of 2) = {2, 4, 6} = 3
Hence, required probability = 3/6 = 1/2

50. **(A)** Total outcome $(6^3) = 216$
Favourable outcome (having all the three same faces on the top) = {(1, 1, 1) (2, 2, 2) (3, 3, 3) (4, 4, 4) (5, 5, 5) (6, 6, 6) = 6 outcome
Hence, required probability = 6/216 = 1/36

23 AREA AND PERIMETER

SYNOPSIS

- Introduction
- 2D Shapes
- Quadrilaterals
- Area
- Perimeter
- Formulae Related to Two-Dimensional Figures
- Circles
- Triangle
- Polygons
- Regular Polygon
- Regular Hexagon

INTRODUCTION

This chapter deals with the concepts related to all the two-dimensional (2D) shapes that also includes the area and perimeter of all 2D figures. In this chapter, you will revise how to calculate the perimeter and area of squares, rectangles, triangles, and circles. The perimeter of a shape is the distance all the way around the sides of the shape. The area of a shape is the flat space inside the shape. You will also learn how to calculate the areas of different 2D figures, as well as investigate the effect on the perimeter and area of a shape when its dimensions are changed.

2D SHAPES

The 2D shapes are also known as flat shapes. These are the shapes having two dimensions only. It has length and breadth. It does not have thickness. The two different measures

QUADRILATERALS

Any four-sided polygon is called as a quadrilateral in general.

Different quadrilaterals and their properties

1. **Rectangle**
   a. Adjacent sides are at right angles to each other.
   b. Opposite sides are equal and parallel.
   c. Diagonals are equal.
2. **Square**
   a. All four sides are equal.
   b. All four angles are equal to 90°.
   c. The diagonals are equal.
   d. Diagonals are perpendicular bisector of each other.
3. **Rhombus**
   a. All sides are equal.
   b. All angles are not necessarily equal to 90°.
   c. All diagonals are perpendicular bisectors of each other but are not equal.
4. **Kite**
   a. The adjacent sides are equal.
   b. The diagonals are perpendicular to each other.
   c. The longer diagonal bisects the shorter one.
5. **Trapezium**
   A quadrilateral in which one pair of lines is parallel.
6. **Isosceles Trapezium**
   The non-parallel lines are equal.
7. **Parallelogram**
   a. Opposite sides are equal and parallel.
   b. Opposite angles are equal

used for measuring the flat shapes are area and the perimeter. Two-dimensional shapes are the shapes that can be drawn on the piece of paper. Some of the examples of 2D shapes are square, rectangle, circle, and triangle.

b. Opposite angles are equal.
c. Diagonals bisect each other.
d. Sum of adjacent angles is 180°.

*Rectangle, Square, and Rhombus are special cases of parallelogram.

AREA

An area is a quantity that expresses the extent of a 2D figure or shape or planar lamina in the plane. Lamina shapes include 2D figures that can be drawn on a plane, e.g., circle, square, triangle, rectangle, trapezium, rhombus, and parallelogram. Area of shapes such as circle, triangle, square, rectangle, parallelogram, etc., are the region occupied by them in space.

Polygon shape: A polygon is a 2D shape that is formed by straight lines. The examples of polygons are triangles, hexagons, and pentagons. The names of shapes describe how many sides exist in the shape. For instance, a triangle consists of three sides and a rectangle has four sides. Hence, any shape that can be formed using three straight lines is known as a triangle and any shape that can be drawn by linking four lines is known as a quadrilateral. The area is the region inside the boundary/perimeter of the shapes which is to be considered.

PERIMETER

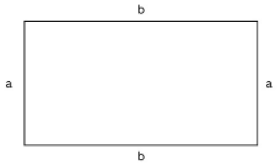
A perimeter is a closed path that surrounds a 2D shape. The Perimeter of a shape is defined as the total distance around the shape, it is the length of the outline or boundary of any 2D geometric shape. The word perimeter has been derived from the Greek word 'peri' meaning around, and 'metron' which means measure. Perimeter is the total length of the sides of a two-dimensional shape.

We often find perimeter when we have to put fencing around our field or when have to calculate the dimensions of a park. We use ruler to measure the length of the sides of a small regular shape. The perimeter is determined by adding the lengths of the sides/edges of the shape. For small irregular shapes, we can use a string of thread and place it exactly along the boundary of the shape, once. The total length of the string used along the boundary is the perimeter of the shape. The perimeter of all polygons can be determined by adding the lengths of their sides/edges.

FORMULAE RELATED TO TWO-DIMENSIONAL FIGURES

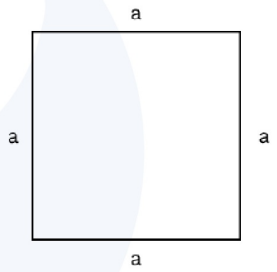
1. Rectangle

a. Area of rectangle = $a \times b$
b. Perimeter of a rectangle = $2(a + b)$
c. Length of diagonal = $\sqrt{a^2 + b^2}$



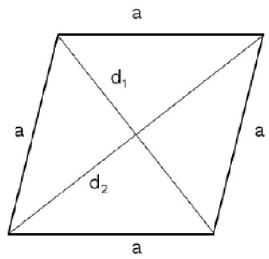
2. Square

a. Area of square = a^2
b. Perimeter of square = $4a$
c. Length of diagonal = $a\sqrt{2}$



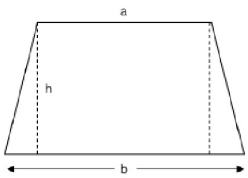
3. Rhombus

a. Area = $\frac{1}{2} \times d_1 \times d_2$
b. Side = $\frac{1}{2} \sqrt{d1^2 + d2^2}$



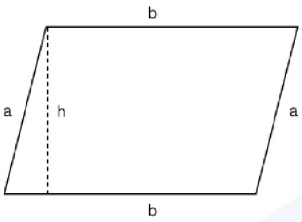
4. Trapezium

Area of trapezium = $\frac{1}{2} \times (a + b) \times h$



5. Parallelogram

Area of parallelogram = $b \times h$

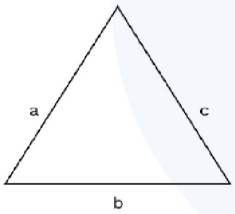


6. Triangle

a. Area of triangle = $\frac{1}{2} \times b \times h$

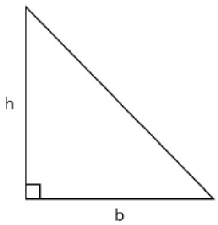
b. Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$;

where $s = \frac{a+b+c}{2}$



7. Right triangle

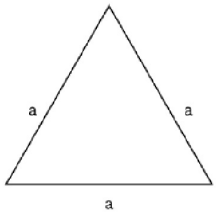
Area = $\frac{1}{2} \times b \times h$



8. Equilateral Triangle

a. Area = $\frac{\sqrt{3}}{4} a^2$

b. Perimeter = $3a$

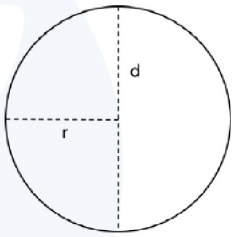


CIRCLES

A circle is a set of points in a plane, which are at a constant distance from a fixed point in the plane. The fixed point is then known as the centre and the fixed distance is called the radius of the circle.

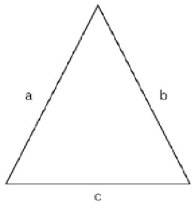
Circumference (perimeter) of a circle = $2\pi r = \pi d$, where r is the radius and d the diameter.

Area of circle = πr^2



TRIANGLE

A triangle is a plane figure bounded by three straight lines, or it is defined as a polygon of three sides. A vertex of a triangle is a point where two straight lines meet. Thus, in a triangle there are three vertices. The symbol DABC is used to denote the triangle with vertices A, B, and C. The three straight lines AB, BC, and CA are called the sides and three angles $\angle$BAC, $\angle$ABC and $\angle$BCA are called the angles of the D*ABC*.



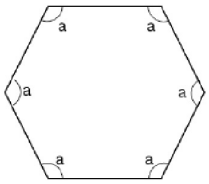
Area of triangle = $\frac{1}{2} \times$ base $\times$ height

POLYGONS

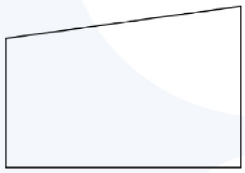
A figure bounded by three or more sides is called a polygon. There are different polygons with different number of sides which are as follows:

NUMBER OF SIDES	NAME
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon

A polygon is regular, if all it's sides as well as angles are equal, else it is an irregular polygon.



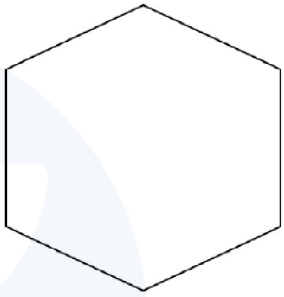
Regular Polygon


Irregular Polygon

REGULAR POLYGON

- Interior angle + exterior angle = 180°
- Perimeter = number of sides × length of side(s)
- Each interior angle = $\frac{(n-2)\times 180}{n}$; where n is the number of sides
- Each exterior angle = $\frac{360°}{n}$
- Sum of exterior angles = 360°

REGULAR HEXAGON



- Perimeter = 6 × Side
- Area = $\frac{3\sqrt{3}}{2}$ × (side)^2

PRACTICE QUESTIONS

1. Find the area of rhombus whose diagonals are 8 cm and 10 cm.
A. 40 cm^2
B. 36 cm^2
C. 44 cm^2
D. 38 cm^2

2. Find the area of an equilateral triangle whose side is 6 cm.
A. $6\sqrt{3}$ cm^2
B. $9\sqrt{3}$ cm^2
C. $9\sqrt{6}$ cm^2
D. $8\sqrt{3}$ cm^2

3. In a four-sided field, the longer diagonal is 220 m. The perpendicular that is drawn from the opposite vertices to the longer diagonal are 40.4 m and 50.8 cm. Find the area of the field.
A. 10,032 m^2
B. 10,080 m^2
C. 10,400 m^2
D. 10,060 m^2

4. If the perimeter of the square is 16 cm, find its area.
A. 10 cm^2
B. 16 cm^2
C. 14 cm^2
D. 18 cm^2

5. If the area of a circle is 314 cm^2, find the circumference of the circle.
A. 60 cm
B. 58.4 cm
C. 61.4 cm
D. 62.8 cm

6. The perimeter of a rectangle is 326 cm and

A. 72,000
B. 68,000
C. 70,000
D. 75,000

12. Find the area of a square if the length of the diagonal of the square is 10 m.
A. 40 m^2

its length is 98 cm. Find the area of the rectangle.
A. 6,370
B. 6,820
C. 7,420
D. 6,680

7. The breadth of a rectangle is 12 cm and its perimeter is 50 cm. Find the length of the rectangle.
A. 15 cm
B. 18 cm
C. 14 cm
D. 13 cm

8. The area of a rectangle is 1,029 cm^2 and if the sides of the rectangle are in the ratio 3 : 7, then find its perimeter.
A. 120
B. 180
C. 140
D. 160

9. The length and breadth of a garden is 10 m and 7 m, respectively. If the cost of fencing is Rs. 40/m, then what will be the cost for fencing the garden?
A. 1,400
B. 1,360
C. 1,340
D. 1,260

10. The perimeter of a rectangle is 50 m and the sides of the rectangle are in the ratio 3 : 2. Find the area of the rectangle.
A. 600 m^2
B. 580 m^2
C. 640 m^2
D. 560 m^2

11. The perimeter of the square hall is 200 ft. If the cost of flooring is Rs. 50 per m^2, then find the total cost of flooring the hall.
B. 50 m^2
C. 54 m^2
D. 46 m^2

13. Find the area of the right-angled triangle whose hypotenuse is 15 cm and one of the sides is 12 cm.
A. 60 cm^2
B. 48 cm^2
C. 50 cm^2
D. 54 cm^2

14. Find the area of a circle whose diameter is 20 cm.
A. 300
B. 310
C. 314
D. 306

15. The area of the triangle is 3,125 cm^2, and the base and height are in the ratio 5 : 2. What is the height of the triangle?
A. 40
B. 48
C. 50
D. 46

16. The perimeter of a triangle is 96 cm. If two sides of the triangle are 40 cm and 24 cm, then find the area of the triangle.
A. 384
B. 380
C. 428
D. 364

17. The base and height of a parallelogram field is 54 m and 24 m, respectively. Find the cost of levelling the field at the rate of Rs. 5 per m^2.
A. 6,110
B. 6,850
C. 7,420
D. 6,480

18. The perimeter of one square is 24 m and that of another is 32 m. Perimeter of a square whose area is equal to the sum of the areas of the two squares will be:
A. 40
B. 38
C. 44
D. 36

19. Find the area of a triangle whose sides are 3 cm, 4 cm, and 5 cm, respectively.
A. 8 cm^2
B. 4 cm^2
C. 5 cm^2
D. 6 cm^2

20. The area of a parallelogram is 192 cm^2, if the height of the parallelogram is one-third of the distance of 3 m apart. The number of required poles will be?
A. 40
B. 48
C. 38
D. 36

25. If the length of the diagonal of the square is 5.2 cm, then find the area of the square.
A. 14 cm^2
B. 13.50 cm^2
C. 14.42 cm^2
D. 13.52 cm^2

26. The perpendicular and area of a right-angled triangle is 20 cm and 10 cm^2. Find the base of the triangle.
A. 1 cm

base, then find the height of the parallelogram.
A. 6
B. 12
C. 8
D. 10

21. The perimeter and area of rhombus are 180 cm and 315 cm^2, respectively. Find the altitude of the rhombus.
A. 10 cm
B. 7 cm
C. 8 cm
D. 6 cm

22. The side of a square shaped pool is 10 m and if the cost of flooring is Rs. 100 per m^2 then, find the total cost of flooring the pool.
A. 12,000
B. 10,000
C. 8,000
D. 6,000

23. The area of a square is increased by 60 cm^2 when its side is increased by 4 cm. Find the side of the square.
A. 5.5
B. 8
C. 4.5
D. 5

24. The length and breadth of a field are 36 m and 21 m, respectively. Poles are required to be fixed all along the boundary at a
B. 3 cm
C. 4 cm
D. 5 cm

27. The perimeter of the rectangle is 50 cm and its breadth is 10 cm. Find its area.
A. 100 cm^2
B. 150 cm^2
C. 140 cm^2
D. 160 cm^2

28. The perimeter of a square garden is 400 m and the cost of gardening is Rs. 3 per m^2, then find the total cost for gardening the garden.
A. Rs. 1,000
B. Rs. 2,800
C. Rs. 4,000
D. Rs. 3,000

29. Find the area of the rhombus whose length of the diagonal is 10 cm.
A. 40 cm^2
B. 38 cm^2
C. 50 cm^2
D. 46 cm^2

30. Find the height of the triangle whose area is 100 cm^2 and length of the base is 20 cm.
A. 10
B. 8
C. 14
D. 6

SOLUTIONS

1. **(A)** Area of rhombus = $\frac{1}{2} \times d1 \times d2$
Area = $\frac{1}{2} \times 8 \times 10$
= 40 cm^2, hence, the option (A) is correct.

2. **(B)** Area of equilateral triangle = $\frac{\sqrt{3}}{4}a^2$
= $\frac{\sqrt{3}}{4} \times 6 \times 6 = 9\sqrt{3}$
The area of equilateral triangle is $9\sqrt{3}$ cm^2, thus, option (B) is correct.

3. **(A)** Area of the field = $\frac{1}{2}$ × diagonal × sum of the perpendicular from the opposite vertices on to this diagonal
= $\frac{1}{2} \times 220 \times (40.4 + 50.8)$
= 10,032 m^2, thus, option (A) is correct.

4. **(B)** Perimeter of square = $4a$
$4a = 16$, $a = 4$
Area of square = a^2
= $4 \times 4 = 16$ cm^2

5. **(D)** Area of a circle = πr^2
= $3.14 \times r^2 = 314$
= $r^2 = 100$, $r = 10$ cm

8. **(C)** Given: Area = 1,029 cm^2
Area = $l \times b$
Let the length and breadth be $7x$ and $3x$
$1029 = 3x \times 7x$
$1029 = 21x^2$
$x^2 = 49$, $x = 7$
Length = 7(7) = 49
Breadth = 3(7) = 21
Perimeter = $2(l + b)$
= 2(49 + 21)
= 140 cm, hence, option (C) is correct.

9. **(B)** Given: Length = 10 m, Breadth = 7 m
Perimeter = $2(l + b)$
= 2(10 + 7)
= 34 m
Cost of fencing = 34 × 40 = Rs. 1,360, hence, option (B) is correct.

10. **(A)** Let the length and breadth be $3x$, and $2x$ respectively
Perimeter = $2(l + b)$
$50 = 2(3x + 2x)$
$50 = 6x + 4x$

$r^2 = 100$, $r = 10$ cm
Circumference of a circle = $2\pi r$
$= 2 \times 3.14 \times 10 = 62.8$ cm
The circumference of the circle is 62.8 cm; hence, option D is correct.

6. **(A)** Given: Perimeter = 326 cm, Length = 98 cm
Perimeter = $2(l + b)$
$326 = 2(98 + b)$
$326 = 196 + 2b$
$2b = 130$
Breadth = 65 cm
Area = Length × Breadth
Area = $98 \times 65 = 6370$ cm^2, hence, option (A) is correct.

7. **(D)** Given: Breadth = 12 cm, Perimeter = 50 cm
Perimeter = $2(l + b)$
$50 = 2(l + 12)$
$50 = 2l + 24$
$2l = 26$, $l = 13$
The length of the rectangle is 13 cm, hence, option (D) is correct.

$50 = 10x$, $x = 10$
Length = $10(3) = 30$ m, Breadth = $10(2) = 20$ m
Area = $l \times b$
$= 30 \times 20 = 600$ m^2, hence, option (A) is correct.

11. **(D)** Given: Perimeter = 200 ft
Perimeter = $4a$
$200 = 4a$, $a = 50$ ft
Area = side × side
$= 50 \times 50 = 2{,}500$ ft^2
Cost of flooring = $2{,}500 \times 50$ = Rs. 75,000, hence, option (D) is correct.

12. **(B)** Given: Diagonal = 10 m
Diagonal = $a\sqrt{2}$
$10 = a\sqrt{2}$
$100 = 2a^2$
$a^2 = 50$
The area of the square is 50 m^2, hence, option (B) is correct.

13. **(D)** $AB^2 = AC^2 - BC^2$
$= (15)^2 - (12)^2$
$= 225 - 144$

$AB^2 = 81$
AB = 9 cm
Area = $\frac{1}{2}$ × base × height
$= \frac{1}{2} \times 12 \times 9$
$= 54$ cm^2, hence, option (D) is correct.

14. **(C)** Given: Diameter = 20 cm, radius = 10 cm
Area of circle = πr^2
$= 3.14 \times 10 \times 10$
$= 314$ cm^2, hence, option (C) is correct.

15. **(C)** Let the base and height be $5x$ and $2x$, respectively.
Area of triangle = $\frac{1}{2}$ × base × height
$3125 = \frac{1}{2} \times 5x \times 2x$
$3125 = 5x^2$
$625 = x^2$
$x = 25$
Height of triangle = $2(25) = 50$ cm, thus, option (C) is correct.

16. **(A)** Given: Perimeter = 96 cm, a = 40 cm, b = 24 cm
$96 = 40 + 24 + c$
$c = 32$ cm
$S = \frac{(a + b + c)}{2}$
$= \frac{(32 + 24 + 40)}{2} = \frac{96}{2}$
$= 48$ cm
Area of triangle = $\sqrt{[s(s-a)(s-b)(s-c)]}$
$= \sqrt{[48(48-40)(48-24)(48-32)]}$
$= \sqrt{(48 \times 8 \times 24 \times 16)}$
$= 384$ cm^2, hence, option (A) is correct.

17. **(D)** Area of parallelogram = base × height

19. **(D)** $a = 3$ cm, $b = 4$ cm, $c = 5$ cm
$S = \frac{(a + b + c)}{2} = \frac{(3 + 4 + 5)}{2} = 6$ cm
Area of triangle = $\sqrt{[s(s-a)(s-b)(s-c)]}$
$= \sqrt{[6(6-3)(6-4)(6-5)]}$
$= \sqrt{36} = 6$ cm^2, hence, option (D) is correct.

20. **(C)** Let the height and base of parallelogram be x cm and $3x$ cm respectively.
Area = 192 cm^2
Area of parallelogram = base × height
$192 = 3x(x)$
$192 = 3x^2$
$x^2 = 64$, $x = 8$
The height of the parallelogram is 8 cm; hence, option (C) is correct.

21. **(B)** Perimeter = 180 cm
Side = 180/4 = 45 cm
Area = $b \times h$
$315 = 45 \times h$
$h = 7$ cm, hence, option (B) is correct.

22. **(B)** Area = side × side
$= 10 \times 10 = 100$ m^2
Total cost of flooring = 100×100 = Rs. 10,000
Total cost of flooring is Rs. 10,000, hence option (B) is correct.

23. **(A)** Let the side be x cm
Therefore, $(x + 4)^2 - x^2 = 60$
$= x^2 + 8x + 16 - x^2 = 60$
$x = 5.5$ cm, hence, option (A) is correct.

24. **(C)** Perimeter = $2(l \times b)$
$= 2(36 + 21) = 144$ m
Number of poles required = 144/3 = 38,

= 54 × 24 = 1296 m^2
Cost of levelling = 1,296 × 5 = 6,480
The total cost of levelling is Rs. 6,480, hence, option (D) is correct.

18. **(A)** Perimeter of first square = 24 m
Side of first square = 24/4 = 6
Perimeter of second square = 32 m
Side of second square = 32/4 = 8
Area of first square = 6 × 6 = 36 m^2
Area of second square = 8 × 8 = 64 m^2
Sum of areas = 36 + 64 = 100 m^2
Side of square will be = $\sqrt{100}$ = 10 m
Perimeter = 4 × 10 = 40 m, hence, option (A) is correct.

hence, option (C) is correct.

25. **(D)** Area = $\frac{1}{2}$ × diagonal × diagonal
= $\frac{1}{2}$ × 5.2 × 5.2
= 13.52 cm^2, hence, option (*D*) is correct.

26. **(A)** Given: Perpendicular = 20 cm, area = 10 cm^2
Area = $\frac{1}{2}$ × base × perpendicular
10 = $\frac{1}{2}$ × base × 20
Base = 1 cm, hence, option (A) is correct.

27. **(B)** Perimeter = 2(*l* + *b*)
Given: Perimeter = 50 cm, Breadth = 10 cm

50 = 2(*l* + 10)
50 = 2*l* + 20
l = 15
Area = *l* × *b*
= 15 × 10 = 150 cm^2, hence, option (B) is correct.

28. **(D)** Perimeter = 4 × side
400 = 4 × side
Side = 100 m
Area = 100 × 100 = 10,000 m^2

Total cost for gardening = 10,000 × 3 = Rs. 3,000, hence, option (D) is correct.

29. **(C)** Area = $\frac{1}{2}$ × $diagonal^2$
= $\frac{1}{2}$ × 10 × 10
= 50 cm^2, hence, option (C) is correct.

30. **(A)** Area = $\frac{1}{2}$ × base × height
Given: Area = 100 cm^2, Base = 20 cm
100 = $\frac{1}{2}$ × 20 × h
200 = 20 h
h = 10 cm, hence, option (A) is correct.

24 Surface Area and Volume

SYNOPSIS

- Introduction
- Surface Area
- Total Surface Area
- Curved Surface Area
- Volume
- Formulae Related to Three-Dimensional Figures
- Summary

INTRODUCTION

This topic deals with the mensuration which means measurement of lengths in a geometric figure that deals with the relation between lengths, area and volume of two-dimensional (2D) and three-dimensional (3D) figures. Mensuration deals with relating lengths of 2D figures with their area and perimeter. For 3D figures or solids, how volume and surface area is related to the measure of its sides:

Area = Product of Sides; Volume = Base area × side length (Height)

SURFACE AREA

The surface area and volume can be calculated for any 3D geometrical shape. The surface of any area is the region occupied by the surface of an object. The volume is the amount of space available in an object. We have different types of shapes such as a hemisphere, sphere, cube, cuboid, cylinder, etc. All 3D shapes have area and volume. But in 2D shapes such as square, rectangle, triangle, circle, etc., 2D, we can only measure the area. The area occupied by a 3D object by its outer surface is called the surface area. It is measured in square units.

The area is of two types:

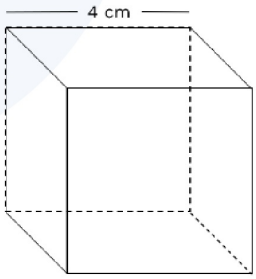
- Total surface area
- Curved surface area/lateral surface area

TOTAL SURFACE AREA

The area including the base(s) and the curved portion corresponds to the overall surface area. It is the amount of the area enclosed by the object's surface. If the form has a curved base and surface, so the sum of the two regions would be the total area. The total surface area can be defined as 'the total area covered by an object including its base as well as the curved part. If an object has both the base and curved area then the total surface area will be equal to the sum of a base and curved area'.

The total surface area is the total area occupied by an object. For example, take cuboid as an example the cuboid has 6 faces, 12 edges, and 8 vertices. The sum of all those 6 areas will be our total surface area of the particular shape.

Example: Given below figure is a cube whose side is 4 cm. Find the total surface area of the cube.



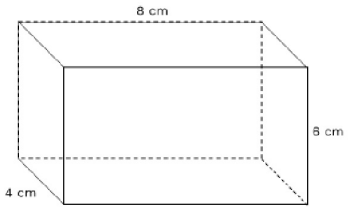
Solution: Given side = 4 cm

Total surface area of cube = $6a^2$ sq. units

$= 6\ (4)^2\ cm^2$

$= 96\ cm^2$

Example: Given below figure is a cuboid having its dimensions: Length = 4 cm, Height = 6 cm and Breadth = 8 cm. Find the total surface of the cuboid.



Solution: Given L = 4 cm, B = 8 cm, H = 6 cm

Total surface area = $2[(l \times b) + (l \times h) + (b \times h)]$

$= 2[(4 \times 8) + (4 \times 6) + (8 \times 6)]$

$= 2[(32) + (24) + (48)]$

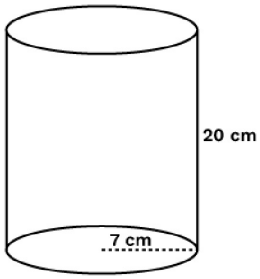
$= 2(104)$

$= 208\ cm^2$

CURVED SURFACE AREA

Curved surface area, except its centre, corresponds to the area of only the curved portion of the shape (s). For shapes such as a cone, it is often called the lateral surface area. The lateral surface area can be defined as 'the area which includes only the curved surface area of an object or lateral surface area of an object by excluding the base area of an object'. The lateral surface area is also known as the curved surface area. Most of the shapes or objects refer to the curved surface area, the shape or object-like cylinder refers to it as a lateral surface area. In simple, 'The area which is visible to us is called a lateral surface area'.

Example: Given below figure is a cylinder and height and radius of the cylinder is 7 cm and 20 cm, respectively. Find the curved surface area of the cylinder.



Solution: Given, $r = 7$ cm, $H = 20$ cm

Curved surface area = $2\pi rh$

$= 2 \times \frac{22}{7} \times 7 \times 20$

$= 880$

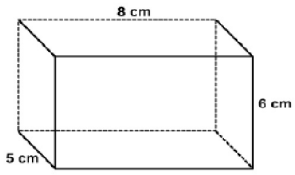
VOLUME

The volume is the amount of space in a certain 3D object. The total amount of space that an object or substance occupies is called volume. It is measured in cubic units. We have already learnt about volumes of certain figures (objects) in earlier classes. Recall that solid objects occupy space. The measure of this occupied space is called the volume of the object.

If an object is solid, then the space occupied by such an object is measured and is termed the volume of the object. On the other hand, if the object is hollow, then interior is empty, and can be filled with air, or some liquid that will take the shape of its container. In this case, the volume of the substance that can fill the interior is called the capacity of the container. In short, the volume of an object is the measure of the space it occupies, and the capacity of an object is the volume of substance its interior can accommodate. Hence, the unit of measurement of either of the two is cubic unit.

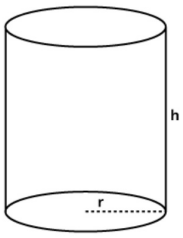
So, if we were to talk of the volume of a cuboid, we would be considering the measure of the space occupied by the cuboid. Further, the area or the volume is measured as the magnitude of a region. So, correctly speaking, we should be finding the area of a circular region, or volume of a cuboidal region, or volume of a spherical region, etc. But for the sake of simplicity, we say, find the area of a circle, volume of a cuboid or a sphere even though these mean only their boundaries.

Example: Given below figure is a cuboid, if length, breadth, and height of the cuboid is 8cm, 5 cm, 6 cm, respectively, then find the volume of the cuboid.



Solution: Given, $l = 8$ cm, $h = 6$ cm, $b = 4$ cm

Volume = $l \times b \times h$



$= 8 \times 4 \times 6$

$= 192 \text{ cm}^3$

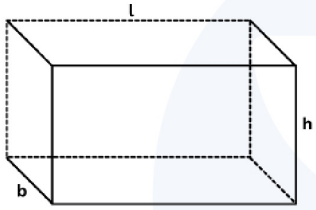
Formulae related to three-dimensional figures

1. Cuboid

a. Volume = $(l \times b \times h)$ *cu.* units

b. Surface area = $2(lb + bh + lh)$ sq. units

c. Longest diagonal = $\sqrt{l^2+b^2+h^2}$



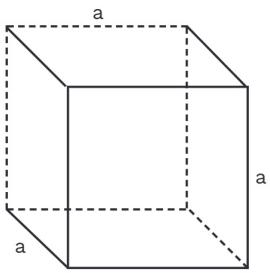
2. Cube

Each edge is 'a' unit.

a. Volume = a^3 cu. units

b. Total surface area = $6a^2$ sq. units

c. Longest diçgonal = $\sqrt{3a^2} = \sqrt{3}a$



3. Right circular cylinder

a. Curved surface area = $2\pi rh$

b. Area of each circular face = πr^2

c. Total surface area = $2\pi r\,(r + h)$

d. Volume = $\pi r^2 h$

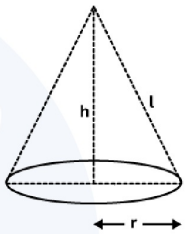
4. Right circular cone

a. Curved surface area = πrl;

where $l = \sqrt{h^2+r^2}$;

b. Total surface area = $\pi r^2 + \pi rl = \pi r(r + l)$

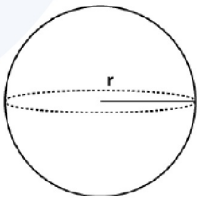
c. Volume = $\frac{1}{3}\pi r^2 h$



5. Sphere

a. Volume = $\frac{4}{3}\pi r^3$

b. Total surface area = $4\pi r^2$

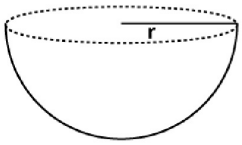


6. Hemisphere

a. Volume = $\frac{2}{3}\pi r^3$

b. Curved surface area = $2\pi r^2$

c. Total surface area = $2\pi r^2 + \pi r^2 = 3\pi r^2$



PRACTICE QUESTIONS

1. What is the total surface area of a cuboid whose dimensions are 25 cm, 10 cm, and 2 cm?

A. 250 cm^2

B. 400 cm^2

C. 500 cm^2

D. 1,000 cm^2

2. The diameter of the base of a right circular cylinder is 56 cm and its height is 20 cm. What is the curved surface of the cylinder?

A. 3,520 cm^2

B. 1,120 cm^2

C. 1,760 cm^2

D. 3,000 cm^2

3. A conical tent has floor area of 616 m^2 and

A. 3.74 m^2

B. 7.48 m^2

C. 74.8 m^2

D. 37.4 m^2

8. Two cubes each of volume 125 cm^3 are joined from end to end then, what is the total surface area of the resulting cuboid?

A. 125 cm^2

B. 200 cm^2

C. 250 cm^2

D. 300 cm^2

9. Find the total surface area of a cylinder whose base radius is 7 cm and height is 14 cm.

A. 564 cm^2

B. 231 cm^2

a straight height of 48 m. If canvas costs Rs. 42/m^2, find the cost of making the tent.
A. Rs. 46,200
B. Rs. 92,400
C. Rs. 88,704
D. Rs. 69,300

4. A solid sphere of radius 21 mm is melted to form 27 equal spherical balls. What is the surface area of each of the smaller spheres?
A. 204 mm^2
B. 308 mm^2
C. 412 mm^2
D. 616 mm^2

5. How much water can be stored in a cone with its base radius 7 m and height 12 m?
A. 204 m^3
B. 308 m^3
C. 412 m^3
D. 616 m^3

6. The volume of a cube is 343 cm^3, find the edge of the cube.
A. 7 cm
B. 14 cm
C. 21 cm
D. 3.5 cm

7. Find the area of the cloth required to cover a cylindrical vessel of height 1 m and diameter 140 cm.
C. 462 cm^2
D. 924 cm^2

10. If the radius of a dome of a building is 7 m, then find the cost of whitewashing the dome from outside if cost of whitewashing is Rs. 5/m^2.
A. Rs. 770
B. Rs. 1,540
C. Rs. 1,410
D. Rs. 1,630

11. How many spherical balls of 2 cm radius can be made out of a solid cube whose edge measures 44 cm?
A. 2,542
B. 5,084
C. 1,271
D. 7,626

12. Find the volume of a right circular cone whose diameter of base is 6 cm and height is 14 cm.
A. 33 cm^3
B. 66 cm^3
C. 132 cm^3
D. 44 cm^3

13. If the volume of a metallic cube is 125 m^3, then find the cost of painting the cube if cost of painting is Rs. 10/m^2.
A. Rs. 3000
B. Rs. 2250
C. Rs. 1800
D. Rs. 1500

14. What is the curved surface area of a hemisphere of radius 14 cm?
A. 616 cm^2
B. 924 cm^2
C. 1,232 cm^2
D. 1,848 cm^2

15. The volume and radius of base of a right circular cylinder is 770 cm^3 and 7 cm respectively. Find the height of the cylinder.
A. 3 cm
B. 4 cm
C. 5 cm
D. 6 cm

16. If the surface area of a sphere is equal to its volume, then find the surface area of the sphere.
A. 113
B. 85
C. 97
D. 109

17. The dimensions of a cuboid are 10 m, 12 m, and 8 m. Find the total cost of covering the cuboid with a sheet, if the cost of sheet is Rs. 12/m^2.
A. Rs. 11,520
B. Rs. 10,840
C. Rs. 10,080
D. Rs. 12,960

18. The dimensions of a box are 10 cm, 10 cm, and 5 cm. Calculate the length of the longest stick that can be put into the box.
A. 14 cm
B. 15 cm
C. 16 cm
D. 18 cm

19. Find the water holding capacity of a conical flask having height 21 cm and radius of base 7 cm.
A. 1,078 cm^3
B. 1,144 cm^3
C. 796 cm^3
D. 1,012 cm^3

20. The radius of a hemisphere is 14 m. Find the cost of painting it if the cost of painting is Rs. 10/m^2.
A. Rs. 9,240
B. Rs. 12,320
C. Rs. 16,240
D. Rs. 18,480

SOLUTIONS

1. **(C)** Total surface area of cuboid = $l \times b \times h$
$= 25 \times 10 \times 2$
$= 500\ cm^2$, hence, option (C) is correct.

2. **(A)** Curved surface area of cylinder = $2\pi rh$
Given: Base = 56 cm, Height = 20 cm
$= 2 \times \pi \times 28 \times 20$
$= 3520\ cm^2$, hence, option (A) is correct.

3. **(B)** Floor area = $\pi r^2 = 616$
$= r^2 = \frac{616 \times 7}{22} = 196$ m
$r = 14$ m
Slant height (l) = $\sqrt{r^2 + h^2} = \sqrt{14^2 + 48^2}$
$= 50$ m
Surface area = $\pi rl = \frac{22}{7} \times 14 \times 50 = 2{,}200\ m^2$
Cost of tent making = $2{,}200 \times 42 = 92{,}400$
Hence, option (B) is correct.

4. **(D)** Volume of the larger sphere = $\frac{4}{3} \times \pi \times (21)^3$
Volume of each smaller sphere = $\frac{4 \times \pi \times (21)^3}{3 \times 27}$
Let the radius of each small sphere be r then
$= \frac{4}{3}\pi (r)^3 = \frac{4}{3}\pi (\frac{21}{3})^3$
$r = 7$ mm
Surface area of each smaller sphere = $4\pi r^2$
$= 4 \times \frac{22}{7} \times 7 \times 7 = 616\ mm^2$, hence, option (D) is correct.

5. **(D)** Given: Radius = 7 m, Height = 12 m
Volume of a cone = $\frac{1}{3}\pi r^2 h$
$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 12 = 616\ m^3$, hence, option (D) is correct.

6. **(A)** Volume of cube = side^3
Side^3 = 343 cm
Side = $(7 \times 7 \times 7)^{\frac{1}{3}}$
Side = 7 cm, hence, option (A) is correct.

7. **(B)** Required cloth = Total surface area of cylinder
Total surface area of cylinder = $2\pi r\ (h + r)$
$= 2 \times \frac{22}{7} \times \frac{70}{100}(1 + \frac{70}{100})$
$= 2 \times 22 \times 0.1 \times 1.7 = 7.48\ m^2$
Hence, option (B) is correct.

8. **(C)** Volume = side^3
125 = side^3
Side = 5 cm
After joining 2 cubes, its length will be 5 + 5 = 10 cm
Total surface area of a cube = $2(lb + bh + lh)$
$= 2(10 \times 5 + 5 \times 5 + 10 \times 5)$
$= 2(50 + 25 + 50)$
$= 250\ cm^2$, hence, option (C) is correct.

9. **(D)** Total surface area of cylinder = $2\pi r\ (r + h)$
Given: Radius = 7 cm, Height = 14 cm
$= 2 \times \frac{22}{7} \times 7(7 + 14)$
$= 44 \times 21 = 924\ cm^2$
Hence, option (D) is correct.

$= \frac{4}{3} \times \frac{22}{7} \times 2 \times 2 \times 2 = 33.5\ cm^3$
Number of spherical balls = $\frac{85{,}184}{33.5}$ = 2,542.8 ≅ 2,542, hence, option (A) is correct.

12. **(C)** Volume of cone = $\frac{1}{3}\pi r^2 h$
Given: Diameter = 6 cm, radius = 3 cm, Height = 14 cm
$= \frac{1}{3} \times \frac{22}{7} \times 3 \times 3 \times 14$
$= 132\ cm^3$, hence, option (C) is correct.

13. **(D)** Volume = side^3
125 = side^3
Side = 5 m
Surface area = 6 (side^2)
$= 6 \times 25 = 150\ m^2$
Cost of painting = 150×10 = Rs. 1500, hence, option (D) is correct.

14. **(C)** Curved surface area = $2\pi r^2$
Given: Radius = 14 cm
$= 2 \times \frac{22}{7} \times 14 \times 14$
$= 1{,}232\ cm^2$, hence, option (C) is correct.

15. **(C)** Given: Volume = 770 cm^3, Radius = 7 cm
Volume = $\pi r^2 h$
$770 = \frac{22}{7} \times 7 \times 7 \times h$
$h = \frac{770}{154} = 5$ cm
The height of the cylinder is 5 cm, hence, option (C) is correct.

Hence, option (D) is correct.

10. **(B)** Given: Radius = 7 m
Surface area of dome = $2\pi r^2$
$= 2 \times \frac{22}{7} \times 7 \times 7 = 308\ m^2$
Cost of whitewashing = 308 × 5 = Rs. 1,540, hence, option (B) is correct.

11. **(A)** Volume of cube = $side^3$
$= (44)^3 = 85{,}184\ cm^3$
Volume of sphere = $\frac{4}{3}\ \pi r^3$

option (C) is correct.

16. **(A)** Let 'r' be the radius of the sphere
Surface area of sphere = volume of sphere
$4\pi r^2 = \frac{4}{3}\pi r^3$
r = 3 cm
Surface area = $4 \times \frac{22}{7} \times 3 \times 3$
= 113.04 ≅ 113, hence, option (A) is correct.

17. **(A)** Given: Dimensions = 10 m, 12 m and 8 m
Total surface area = $l \times b \times h$

= 10 × 12 × 8
= 960
Total cost of covering = 960 × 12 = Rs. 11,520
Hence, option (A) is correct.

18. **(B)** Given: Dimensions = 10 cm, 10 cm, 5 cm
Length of longest stick = Diagonal of cuboid
Required length = $\sqrt{10^2 + 10^2 + 5^2}$
= $\sqrt{225}$ = 15 cm
The longest stick that can be put into box is 15 cm, hence, option (B) is correct.

19. **(A)** Given: Height = 21 cm, Radius = 7 cm
Volume = $\frac{1}{3}\ \pi r^2 h$
$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 21$
= 1,078 cm^3, hence, option (A) is correct.

20. **(D)** Surface area = $3\pi r^2$
$= 3 \times \frac{22}{7} \times 14 \times 14$
= 1,848 m^2
Total cost of painting = 1,848 × 10 = Rs. 18,480, hence, option (D) is correct.

25 Angles and Triangles

SYNOPSIS

- Definition of triangle
- Properties of a triangle
- Types of triangles
  - On the basis of length
    - Equilateral triangle
    - Isosceles triangle
    - Scalene triangle
  - On the basis of angle
    - Acute angle triangle
    - Right angle triangle
    - Obtuse angle triangle
- Area and perimeter of triangle and its types
- Some Important Theorems Related to Triangles

DEFINITION OF A TRIANGLE

A triangle can be defined as a polygon with three vertices and three edges.

In a triangle, the sum of all the three internal angles must be equal to 180°. This is called the *angle sum property of a triangle*.

A triangle is a two-dimensional closed shape. It's a polygon having three sides. Straight lines run along all three sides. The vertex is the intersection of two straight lines. As a result, there are three vertices in the triangle. An angle is formed by each vertex.

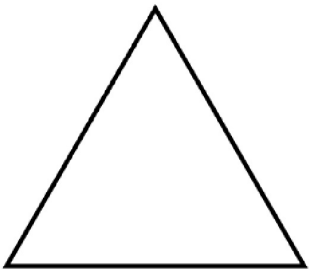


Fig. 5.1

PROPERTIES OF A TRIANGLE

- There are three edges and three sides of a triangle
- Sum of interior angles of a triangle is equal to 180°.
- Sum of exterior angles of a triangle is equal to 360°.
- The sum of two sides of a triangle will always be greater than the third side.
- The difference between any two sides of a triangle is always less than the length of the third side.
- The shortest side of a triangle will always be opposite to the smallest interior angle.
- The longest side of a triangle will always be opposite to the largest interior angle.

TYPES OF TRIANGLE

Triangles can be categories on two bases, i.e., on the basis of length of sides and on the basis of angle.

On the basis of length of sides

There are three types of triangles based on the length of its sides. They are as:

1. **Equilateral triangle:** The triangle whose all the three sides are equal is called an equilateral triangle. Due to equal sides, it also has equal interior angles, each measuring 60°.

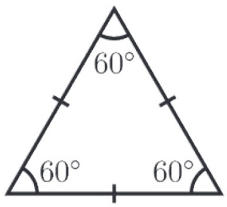


Fig. 5.2

2. **Isosceles Triangle:** The triangle that has two equal sides are called isosceles triangles.

The interior angles opposite to the two equal sides are also equal

equal sides are also equal.

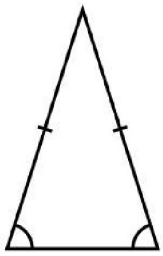


Fig. 5.3

3. **Scalene Triangle:** A triangle that has all sides of different lengths and thus no equal interior angles, is called a scalene triangle.

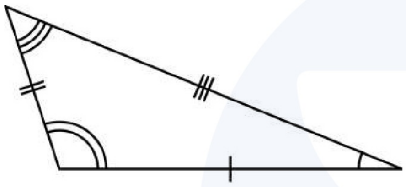


Fig. 5.4

On the basis of angles

There are three types of triangles on the basis of their angles. They are as follows:

1. **Acute Angle Triangle:** The triangle which has all of its angle smaller than 90° is called an acute angle triangle.

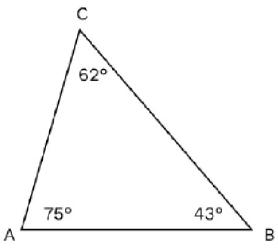


Fig. 5.5

2. **Obtuse Angle Triangle:** The triangle that has one of its angles more than 90° is called an obtuse angle triangle.

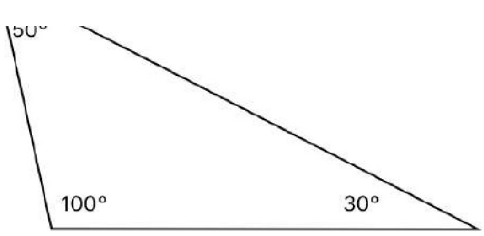


Fig. 5.6

3. **Right-Angle Triangle:** The triangle whose one angle is 90° is called a right-angle triangle.

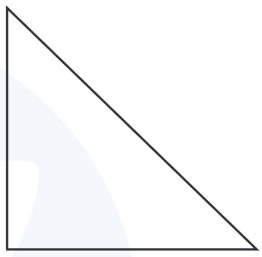


Fig. 5.7

AREA AND PERIMETER OF TRIANGLES AND ITS TYPES

Perimeter of a triangle = Sum of all sides of a triangles

Area of a triangle = Half of product of base and height
= 1/2 × Base × Height

Area of triangle using Heron's formula

If the height of a triangle is not given, we use Heron's formula to calculate the area of the given triangle.

For this we need the semi perimeter of the triangle.

$$S = \frac{\text{Perimeter of the triangle}}{2}$$

Consider a triangle with sides a, b, and c. Then,

$$S = \frac{a+b+c}{2}$$

Then the area of the triangle is given by:

$$A = \left[(ss - - as)(bs)(-c) \right]$$

4. If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.
5. If in two triangles, sides of one triangle are proportional to (i.e., in the same ratio of) the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similar.
6. If one angle of a triangle is equal to one angle of the other triangle and the sides

Area of isosceles triangle using only sides

$$\frac{1}{2}\sqrt{\frac{a^2}{4}-b^2}\times b$$

Here a = length of the equal side
b = base of the triangle
h = height of the triangle

PERIMETER OF ISOSCELES TRIANGLE

The perimeter of the isosceles triangle;
$P = 2a + b$
Here a = length of the equal side
b = length of the unequal side

AREA OF ISOSCELES RIGHT-ANGLE TRIANGLE

$$A = \frac{1}{2} \times a^2$$

SOME IMPORTANT THEOREMS RELATED TO TRIANGLE

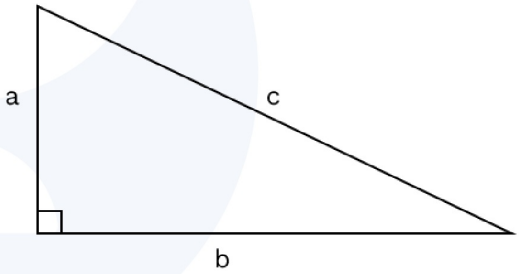
1. Two triangles are similar, if
   a. Their corresponding angles are equal and
   b. Their corresponding sides are in the same ratio (or proportion).
2. The ratio of any two corresponding sides in two equiangular triangles is always the same.
3. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

including these angles are proportional, then the two triangles are similar.

7. The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

PYTHAGORAS THEOREM

In a right-angle triangle, sum of the square of the hypotenuse is equal to the sum of the square of the other two sides.



$$a^2 + b^2 = c^2$$

Fig. 5.8

Here, c is the hypotenuse and a and b are the other two sides of the triangle.

- If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.
- In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.

PRACTICE QUESTIONS

1. Which of the three sides cannot be used to construct a triangle?
   A. 2, 4, 7
   B. 3, 4, 5
   C. 5, 2, 5
   D. 8, 6, 10

2. Calculate the perimeter of a triangle with 13 cm on each side.
   A. 38 cm
   B. 39 cm
   C. 40 cm
   D. 41 cm

3. In a triangle with a perimeter of 55 cm and two sides of 17 cm each, what is the missing side length?
   A. 23 cm

Duggu's. What is the perimeter of the triangle formed by their houses?
   A. 35 feet
   B. 36 feet
   C. 37 feet
   D. 38 feet

8. What is the perimeter of a right triangle with a height 56 and a 33 cm base?
   A. 151 cm
   B. 152 cm
   C. 153 cm
   D. 154 cm

9. The triangle's three sides are 12 cm, 15 cm, and 18 cm. Calculate the area of the triangle.
   A. 75.9 cm^2
   B. 85.9 cm^2

B. 22 cm
C. 21 cm
D. 20 cm

4. What is the area of a triangle with a 26 cm base and a 12 cm height?
A. 153 cm^2
B. 154 cm^2
C. 155 cm^2
D. 156 cm^2

5. One side of an equilateral triangle is 15 cm long. What is the triangle's perimeter?
A. 45 cm
B. 47 cm
C. 43 cm
D. 41 cm

6. The lengths of the two short sides of a right triangle are 6 and 8. What is the triangle's perimeter?
A. 23 cm
B. 24 cm
C. 25 cm
D. 26 cm

7. The residences of Tuku, Bobbie, and Duggu form a triangle. Bobbie's residence is 9 feet distant from Tuku's. Bobbie's residence is 13 feet away from Duggu's. Tuku's residence is 15 feet distant from

C. 95.9 cm^2
D. 105.9 cm^2

10. Find the base of the triangle with an area of 102 m^2 and a height of 12 m.
A. 14 cm
B. 15 cm
C. 16 cm
D. 17 cm

11. Find the height of the triangle with a 14-unit base and a 147-square-unit area.
A. 20 units
B. 21 units
C. 22 units
D. 23 units

12. A triangle's base and height are in the ratio 13:12 and its area is 312 m^2. What is the sum of triangle's height and base?
A. 50
B. 54
C. 56
D. 58

13. Find the area of a triangle whose sides are 10 cm, 24 cm, and 26 cm.
A. 110 cm^2
B. 120 cm^2
C. 130 cm^2
D. 140 cm^2

14. The area of triangle whose base is 8 cm is equal to the area of a circle of radius 7 cm. What is the height of the triangle?
A. 18.5 cm
B. 28.5 cm
C. 38.5 cm
D. 48.5 cm

15. A right angled triangle has its area equal to that of a square whose side is 6 cm. What is the altitude of the triangle, if it is twice its base?
A. 42 cm
B. 32 cm
C. 22 cm
D. 12 cm

16. What is the area of a triangle having sides 4 cm, 7 cm, and 9 cm?
A. 13.41 cm^2
B. 15.43 cm^2
C. 17.45 cm^2
D. 19.47 cm^2

17. A triangular field's base is three times its height. Find the base and height of the field if the cost of cultivating the field at Rs. 28 per hectare be Rs. 364.
A. 249:90
B. 259:91

20. A triangle having the base *AB* is *ABC*. *D* is a point on *AB* where *AB* equals 8 cm and *DB* equals 6 cm. What is the ratio of area of triangle *ADC* to that of the area of triangle *ABC*?
A. 5 cm
B. 6 cm
C. 7 cm
D. 8 cm

21. The area of right angled triangle is 20 times its base. What is the height?
A. 40 cm
B. 41 cm
C. 42 cm
D. 43 cm

22. If a triangle's area is 1176 cm^2 and the base-to-height ratio is 3:4, what is the triangle's altitude?
A. 55 cm
B. 56 cm
C. 57 cm
D. 58 cm

23. A triangle's three sides are 9 cm, 40 cm, and 41 cm, respectively. Then its area is?
A. 160 cm^2
B. 170 cm^2
C. 180 cm^2
D. 190 cm^2

C. 269:92
D. 279:93

18. The ratio of the areas of two triangles is 4:3, while the ratio of their heights is 3:4. Calculate their base ratio.
A. 18:6
B. 17:7
C. 16:9
D. 15:8

19. The base of a triangle is 17 cm and height is 14 cm. The height of another triangle of double the area having the base 20 cm is?
A. 23.8 cm
B. 33.8 cm
C. 43.8 cm
D. 53.8 cm

24. The sides of a triangle are in the ratio of 1/4 : 1/6 : 1/8. If the perimeter is 52 cm, then the length of the small side is?
A. 15 cm
B. 14 cm
C. 13 cm
D. 12 cm

25. A triangle has a surface area of 216 cm^2 and sides in the ratio 3:4:5. What is the triangle's perimeter?
A. 71 cm
B. 72 cm
C. 73 cm
D. 74 cm

26. The sides of a triangle are 6 cm, 8 cm, and 10 cm. The area of the triangle formed by joining the mid-points of the sides of this triangle is?

A. 4 cm^2
B. 5 cm^2
C. 6 cm^2
D. 7 cm^2

27. One side of a right angle triangle is two times the other, and the hypotenuse is 10 cm. The area of the triangle is?
A. 50 cm^2
B. 40 cm^2
C. 30 cm^2
D. 20 cm^2

28. A right angled triangle has a perimeter of 60 cm. It has a 26-cm hypotenuse. What is the triangle's area?
A. 120 cm^2
B. 122 cm^2
C. 124 cm^2
D. 126 cm^2

29. If an isosceles right triangle's perimeter is $(6+3\sqrt{2})$ m, what is the triangle's area?
A. 2.5 m^2
B. 3.5 m^2
C. 4.5 m^2
D. 5.5 m^2

30. A triangle's perimeter is 30 cm and its area is 30 cm^2. What is the length of the shortest side of the triangle if the greatest side is 13 cm?
A. 2 cm
B. 3 cm
C. 4 cm
D. 5 cm

SOLUTIONS

1. **(A)** We already know that the total of any two triangle sides must be bigger than the third side. Option (A) does not meet this requirement.

2. **(B)** The triangle is an equilateral triangle because all three sides are of the same length.
Perimeter = 3 × side
= 3 × 13 = 39 cm.

3. **(C)** The triangle is isosceles triangle as only two sides are equal here.
So, perimeter = $2L + B$
$55 = 2 \times 17 + B$
$B = 21$ cm

4. **(D)** Area of triangle = ½ × b × h
= ½ × 26 × 12

7. **(C)** The formula for calculating the perimeter of a triangle is to add the lengths of the three sides together.
The three sides of this triangle are 9ft, 13ft, and 15ft in length. When you combine these three sides together, you get a perimeter of 37 feet.

8. **(D)** Using the Pythagoras theorem,
$(56)^2 + (33)^2 = C^2$
$3136 + 1089 = C^2$
So, third side = 65 cm
Perimeter = 56 + 33 + 65 = 154 cm.

9. **(C)** Perimeter = 13 + 15 + 18 = 46 cm
Semi perimeter = $\frac{46}{2}$ = 23 cm
Area = $\sqrt{23(23-13)(23-15)(23-18)}$

= ½ × 26 × 12
= 156 cm²

5. **(A)** Perimeter of triangle = 3 × 15 = 45 cm.
6. **(B)** For perimeter, first we have to find the length of the third side.
Using Pythagoras theorem to find the third side,
$a^2 + b^2 = c^2$
$6^2 + 8^2 = c^2$
Hence, (C) or third side = 10
Now, perimeter = 6 + 8 + 10 = 24 cm.

$= \sqrt{23 \times 10 \times 8 \times 5}$
$= \sqrt{9200} = 95.9$ cm²

10. **(D)** Area = ½ × b × h
102 = ½ × b × 12
Base = $\frac{102}{6}$ = 17 cm.
11. **(B)** Area = ½ × b × h
147 = ½ × 14 × h
Height = $\frac{147}{7}$ = 21 units.
12. **(A)** Let's take height and base is $12x$ and $13x$

Area = ½ × b × h
312 = ½ × $13x$ × $12x$
312 = $78x^2$
x = 2
So, height = 12 × 2 = 24
and, base = 13 × 2 = 26

13. **(B)** Firstly we have to find the semiperimeter of this triangle = $\frac{10+24+26}{2}$ = 30 cm
Now, area of triangle
$= \sqrt{30(30-10)(30-24)(30-26)}$
$= \sqrt{30 \times 20 \times 6 \times 4}$ = 120 cm²
14. **(C)** Area of circle = πr^2
= 22/7 × 7 × 7 = 154 cm²
Area of triangle = ½ × b × h
154 = ½ × 8 × h
Height = 38.5 cm.
15. **(D)** Area of square = $(6)^2$ = 36 cm²
Since the altitude of the triangle is twice the base, let the base and altitude is b and $2b$, respectively.
Area of triangle = ½ × b × h
36 = ½ × b × $2b$
b = 6
So, altitude = $2b$ = 12 cm.
16. **(A)** Firstly we have to find the semi-perimeter of this triangle = $\frac{4+7+9}{2}$ = 10 cm
Now, area of triangle
$= \sqrt{10(10-4)(10-7)(10-9)}$
$= \sqrt{10 \times 6 \times 3 \times 1}$ = 13.41 cm²
17. **(D)** Area of the field = Total cost/Rate = 364/28 = 13 hectare = 1,30,000 m²
Let altitude be x and base be $3x$
Then, ½ × $3x$ × x = 1,30,000
x = 93
Hence, base = 279 and altitude = 93
18. **(C)** Let the bases of two triangles be x and y and their heights be $3h$ and $4h$, respectively.
Then, (½ × x × $3h$)/(½ × y × $4h$) = 4/3
x/y = 4/3 × 4/3 = 16/9
Hence, required ratio = 16:9

Hence, 238 = ½ × 20 × h
Height = 23.8 cm.

20. **(B)** Here, semi-perimeter = ½ (8 + 6 + 10) = 12 cm
Area = $\sqrt{12 \times 4 \times 6 \times 2}$ = 24 cm²
Now, ½ × 8 × h = 24
So, height = 6 cm.
21. **(A)** Area = ½ × b × h
20 × b = ½ × b × h
So, height = 40 cm.
22. **(B)** Let base be $3x$ and height is $4x$
Then, ½ × $3x$ × $4x$ = 1176
$12x^2$ = 2,352
x = 14 cm
So, altitude = 4 × 14 = 56 cm
23. **(C)** It is a right angle triangle. So,
Area = ½ × 40 × 9 = 180 cm²
24. **(D)** Ratio of sides = $\frac{1}{4} : \frac{1}{6} : \frac{1}{8}$ = 6:4:3
Perimeter = 52 cm. So, sides are 24 cm, 16 cm, and 12 cm
Hence, length of the smallest side = 12 cm
25. **(B)** Let sides be $3x$ cm, $4x$ cm, and $5x$ cm.
So, semiperimeter = $6x$ cm
Area = $\sqrt{6x \times 3x \times 2x \times x}$ = $6x^2$ cm²
216 = $6x^2$
So, x = 6
Hence, sides are 18 cm, 24 cm, and 30 cm.
So, perimeter = 18 + 24 + 30 = 72 cm.
26. **(C)** It is a right angle triangle with base 6 cm and height 8 cm.
So, area = ½ × 6 × 8 = 24 cm²
Hence, area of required triangle = ¼ × 24 = 6 cm²
27. **(D)** Let the sides be x and $2x$
Then, $(x)^2 + (2x)^2 = (10)^2$
x^2 = 20
Area = ½ × x × $2x$ = x^2 = 20 cm²
28. **(A)** Perimeter = $b + h$ + 26
60 − 26 = $b + h$ or $(b + h)^2 = 34^2$
Also, $b^2 + h^2 = 26^2$
So, $(b + h)^2 - (b^2 + h^2) = 34^2 - 26^2$

Hence, required ratio = 16:9

19. **(A)** Area of first triangle = ½ × 17 × 14 = 119 cm^2
So the area of second triangle = 238 cm^2

$2bh = (34 + 26)(34 - 26)$
$bh = 240$ or ½ $bh = 120$
Hence, area = 120 cm^2

29. **(C)** Let the sides be a, a and b meters.
Then, $2a + b = 6 + 3\sqrt{2}$ and $b^2 = a^2 + a^2 = 2a^2$
$b = \sqrt{2}a$
So, $2a + \sqrt{2}a = 6 + 3\sqrt{2}$
$a = 3$
Hence, area = ½ × 3 × 3 = 4.5 m^2

30. **(D)** Let the smallest side be x

Then, the other sides are 13 cm. and $(17 - x)$ cm
Here, $a = 13$, $b = x$ and $c = (17 - x)$ with semi-perimeter = 15
So, Area = $\sqrt{15 \times 2 \times (15 - x) \times (x - 2)}$

$30^2 = 30 \times (15 - x)(x - 2)$
$x = 5$
Hence, smallest side = 5 cm.

26 Circles

SYNOPSIS

- Definition of Circle
- Parts of Circle
  - Radius
  - Diameter
  - Sector
  - Segment
  - Arc
  - Secant
  - Tangent
  - Chord
  - Centre
- Circle Formulas
  - Areas and perimeter
- Properties of Circle
- Some Important Theorems

DEFINITION OF CIRCLE

Circle may be defined as a collection of points in a plane which are equidistant from a fixed point in the same plane.

A circle divides the plane it exists on into 3 parts. They are: (i) inside the circle, also known as the circle's interior; (ii) the circle; and (iii) outside the circle, also known as the circle's exterior. The circular region is made up of the circle and its inside.

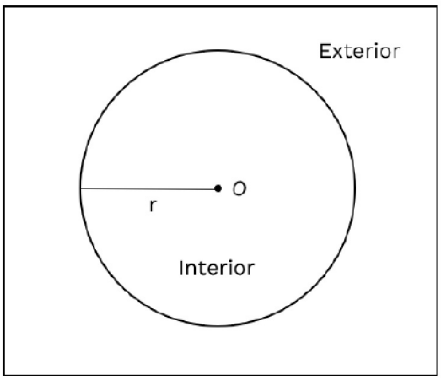


Fig. 6.1

Fig. 9.1 shows the interior of a circle, the exterior of a circle and the circle.

PARTS OF A CIRCLE

- **Centre:** The fixed point as shown in Fig. 6.1 (represented by O) is called as the centre of the circle.
- **Radius:** The fixed distance between the center of the circle and all the points forming the circle is defined as the radius of the circle. It is generally represented by *r* (see Fig. 6.1).
- **Chord:** A line segment whose both the endpoints lie on the circle is called a chord.
- **Diameter:** A line segment which has both its endpoints on the circle and passes through the center is called the diameter of the circle. It is generally denoted by *d* and is twice the size of the radius of the circle.

Diameter is the longest chord of the circle.

$$d = 2r$$
$$r = d/2$$

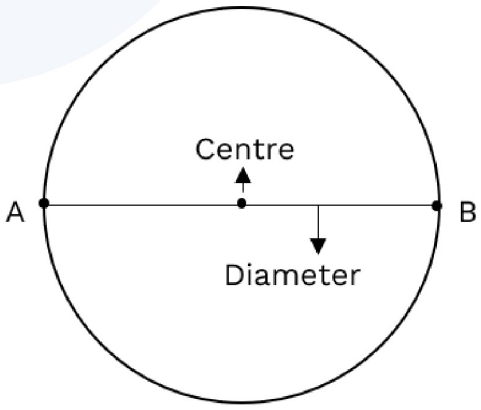
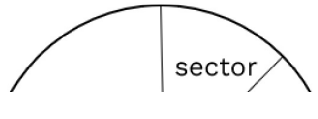


Fig. 6.2

- **Arc:** Arc is basically a part of the circumference of the circle. It is denoted by '⌒' or '⌓'.



- **For example:** In the Fig. 6.3, AB is the arc and is read as 'arc AB'.

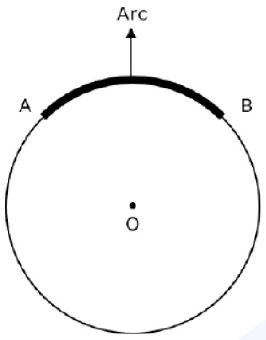


Fig. 6.3

- **Secant:** Secant can be defined as an extended chord which cuts the circle at two points.
- **Segment:** The region bounded by a chord and an arc between the two endpoints of the chord. It may be both in interior as well as exterior of the circle.
- **Sector:** The region in the interior of the circle bound by two radii and an arc formed between the two end points of the radii on the circle.
- **Tangent:** A line segment touching the circle at one and only point on the exterior is called a tangent.

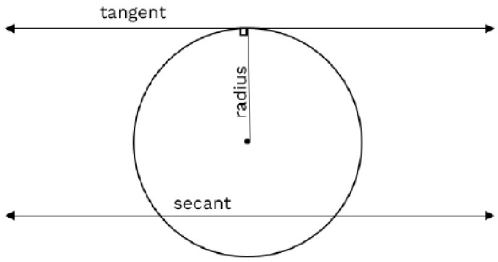


Fig. 6.4

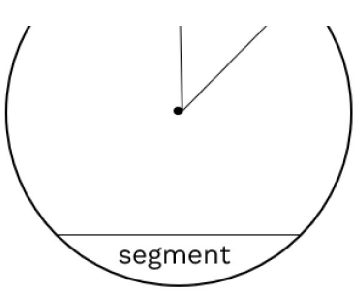
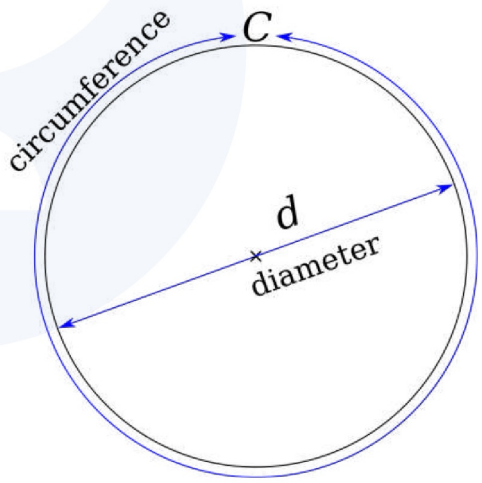


Fig. 6.5

Fig. 6.4 shows the sector and segment of a circle. Tangent and secant can be clearly seen in Fig. 6.5.

CIRCLE FORMULAS

- **Circumference of a circle**: Circumference of a circle is defined as the distance around the circle.



Circumference of a circle (C) = $2\pi r = \pi d$

- **Area of a circle**: The total amount of space occupied by a circle is defined as the area of the circle.
  Area of the circle = πr^2
- **Circumference of a semicircle:** Circumference of a semicircle is half the area of a circle.
  Area of a semicircle = $\frac{1}{2}\pi r$

- **Area of sector:** $\frac{\theta}{360°} \times \pi r^2$ (when θ is in degrees)
- **Perimeter of a sector**: 2 Radius + $(\frac{\theta}{360°} \times 2\pi r)$
- **Area of a segment**: Area of a segment in degrees = $\frac{1}{2} \times r^2 \times [(\frac{\pi}{180°})\ \theta - sin\ \theta]$
  Area of segment in radians = $\frac{1}{2} \times r^2\ (\theta - Sin\ \theta)$
- **Perimeter of a segment:** The perimeter of a segment is the sum of chord length and arch length.

- Diameter of a circle divides it into two equal parts.
- Two circles with equal radii are similar circles.

SOME IMPORTANT THEOREMS

- Angle made by equal chords at the centre are equal
- If two angles made by chords at the centre are equal the chords are equal.
- A perpendicular from the centre of a circle bisects the chord in two equal halves.

Perimeter of segment in degrees = $(\frac{\theta}{360°} \times 2\pi r) + (2r \sin \frac{\theta}{2})$

Perimeter of segment in radians = $r\theta + (2r \sin \frac{\theta}{2})$

PROPERTIES OF A CIRCLE

- Two circles with equal radii are congruent to each other.
- Diameter is the longest chord of a circle and is double the radius in length.
- There can be one and only one circle passing through three non-collinear points.
- Two equal chords of the circle are at equal distance from the centre.
- Two chords which are at equal distance from the centre are equal in length.
- The angle subtended by an arc at the center is double the angle subtended by it at any point on the remaining part of the circle.
- Angles in the same segment of a circle are equal.

PRACTICE QUESTIONS

1. Calculate the area of a circle with a radius of 6.
A. 28π
B. 36π
C. 42π
D. 46π

2. Calculate the circumference of a circle with a radius of 49.
A. 306
B. 307
C. 308
D. 309

3. Calculate the diameter of a circle with a circumference of 17π.
A. 14
B. 15
C. 16
D. 17

4. Find the area of a sector of a 28-cm-radius circle with a 90-degree central angle.
A. 616 cm^2
B. 626 cm^2
C. 636 cm^2
D. 646 cm^2

5. What is the area of the segment if the area of a sector is 128 sq. ft. and the area of the enclosed triangle is 67 square feet?
A. 61 sq. ft.
B. 71 sq. ft.
C. 81 sq. ft.
D. 91 sq. ft.

6. AB subtends a 72-degree angle to the centre of a circle with a radius of 21 cm, given that arc. Determine the length of the arc AB.
A. 26 cm
B. 26.4 cm
C. 26.8 cm
D. 26.5 cm

7. If the area of the appropriate minor segment is 37 sq. units and the radius is 7 units, find the area of the major segment of a circle.
A. 115 sq. units
B. 116 sq. units
C. 117 sq. units
D. 118 sq. units

8. If the central angle of a pizza slice is 60 degrees and the radius is 6 units, determine the area of the segment generated by removing the triangle part of the pizza slice.
A. 3.19 sq. units
B. 4.27 sq. units
C. 3.67 sq. units
D. 3.27 sq. units

9. Calculate the length of an arc of a circle that forms a 150-degree angle with the centre of a circle with a radius of 27 cm

13. Calculate the length of an arc, having a radius of 17 m and an angle of 33.67 radians, in radians.
A. 575.39 m
B. 574.39 m
C. 573.39 m
D. 572.39 m

14. If a wheel revolves 49 times to cover 343 meters, what is its radius?
A. 3.83 m
B. 2.83 m
C. 0.83 m
D. 1.83 m

15. A ring's circumference and diameter differ by 28 cm. Calculate the ring's radius.
A. 5.53 cm
B. 6.53 cm
C. 7.53 cm
D. 8.53 cm

centre of a circle with a radius of 27 cm.

A. 70.45 cm
B. 70.55 cm
C. 70.65 cm
D. 70.75 cm

10. An arc is 70 meters long. Find the angle subtended by the arc if the radius of the circle is 21 m.

A. 191 degrees
B. 191.08 degrees
C. 191.80 degrees
D. 192 degrees

11. Find the radius of an arc with a length of 127 cm and a 120-degree angle to the circle's centre.

A. 6.44 cm
B. 6.54 cm
C. 6.64 cm
D. 6.74 cm

12. An equilateral triangle is the form of a wire. The triangle's sides are 14 cm long. A circle is formed by bending the wire. Calculate the circumference of the resulting circle.

A. 44.63 square cm
B. 45.63 square cm
C. 46.63 square cm
D. 47.63 square cm

16. 5:00 p.m. is displayed on a circular clock. The minute hand measures 14 units in length. When the time is 5:30 p.m., find the distance travelled by the tip of the minute hand.

A. 44 units
B. 46 units
C. 48 units
D. 50 units

17. A circle's circumference is 576 yards. Calculate the circle's diameter.

A. 183.43 yards
B. 185.43 yards
C. 187.43 yards
D. 189.43 yards

18. The shape of a race track is that of a circular ring. The track has a 47-yard inner radius and a 49-yard outside radius. Find the area of the race track.

A. 601.42 sq. yards
B. 603.42 sq. yards
C. 605.42 sq. yards
D. 607.42 sq. yards

19. The area ratio of two circles is 16:25. Find the ratio of their radii using the area of circle formula.

A. 1:5
B. 2:5
C. 4:5
D. 8:5

20. If the radius is 28 cm and the angle is 108°, calculate the area of the sector.

A. 43.2 cm^2
B. 44.2 cm^2
C. 45.2 cm^2
D. 46.2 cm^2

21. A man walks twice a day on a walking track that is shaped like a sector with an angle of 111° and a radius of 97 m. In a single day, determine the area traversed by the guy of the walking track.

A. 17642.68 m
B. 17742.68 m
C. 17852.68 m
D. 17967.68 m

22. In a field, a horse is grazing. It's attached to a pole via a 7-m rope. The horse moves from point A to point B, forming a 72-degree arch. Find the horse's grazing place in the sector.

A. 26.8 sq. meter
B. 28.8 sq. meter
C. 30.8 sq. meter
D. 32.8 sq. meter

25. A bicycle's wheels have a diameter of 77 cm. To cover a distance of 147 meters, how many revolutions will each wheel make?

A. 41 revolutions
B. 51 revolutions
C. 61 revolutions
D. 71 revolutions

26. Each wheel of a motorcycle has a radius of 0.280 m. If each wheel rotates 100 times, how far will the motorcycle travel? Assume that the motorcycle is on a straight path.

A. 146 m
B. 156 m
C. 166 m
D. 176 m

27. To form a circle, a piece of wire in the shape of a rectangle with a length of 80 cm and a width of 35 cm is cut and folded. Calculate the radius and circumference of the completed circle.

A. 36.62 cm
B. 38.62 cm
C. 40.62 cm
D. 42.62 cm

28. Calculate the perimeter of a 0.14-meter-radius circular flower garden.

A. 0.088 m
B. 0.88 m

23. If the area of the sector is 57 m and the part of a triangle in the sector is 23 m, find the area of the segment.
A. 31 sq. meter
B. 32 sq. meter
C. 33 sq. meter
D. 34 sq. meter

24. If the angle subtended by a sector's arc at its centre is 180 degrees, the sector's area in square units is?
A. $\frac{\pi r^2}{2}$
B. $\frac{\pi r^2}{4}$
C. $\frac{3\pi r^2}{2}$
D. πr^2
C. 1.88 m
D. 2.88 m

29. What is the area of a sector of a circle with a radius of 46 cm that is cut off by a 12.5 cm long arc?
A. 277.5 cm^2
B. 287.5 cm^2
C. 297.5 cm^2
D. 267.5 cm^2

30. When a circular disc with a radius of 10 cm is partitioned into sectors with angles of 120 and 150, the ratio of the two sectors' areas is?
A. 1:5
B. 2:5
C. 3:5
D. 4:5

SOLUTIONS

1. **(B)** Area = πr^2
$= \pi (6)^2$
$= 36\pi$

2. **(C)** Circumference = $2\pi r$
$= 2 \times \frac{22}{7} \times 49$
$= 2 \times 22 \times 7 = 308.$

3. **(D)** Circumference = πd
$17\ \pi = \pi \times d$
Hence, diameter = 17.

4. **(A)** Here, r = 28 cm and θ = 90
$l = \frac{\theta}{360} \times 2\pi r = \frac{90}{360} \times 2 \times \frac{22}{7} \times 28 = 44$ cm
Perimeter = $l + 2r = 44 + 2 \times 28 = 100$ cm
Area = ½ × 44 × 28 = 616 cm^2

5. **(A)** Area of the segment = area of the sector- area of the triangle
= 128 sq. ft. – 67 sq. ft.
= 61 sq. ft.
Therefore, the area of the segment is 61 sq. ft.

6. **(B)** Here r = 21 cm, θ = 72 degrees.
By substitution,
The length of an arc = $2\pi r\ (\frac{\theta}{360})$
Length of the arc = $2 \times \frac{22}{7} \times 21 \times \frac{72}{360}$
= 26.4 cm.

7. **(C)** Area of the major segment = area of the circle - area of the minor Segment
$= \pi r^2 - 62$
$= (\frac{22}{7}) \times 7 \times 7 - 37$
= 117 sq. units
Therefore, the area of the major segment = 117 sq. units.

8. **(D)** The radius of pizza is, r = 6 units.
The central angle is, θ = 60 degrees.

10. **(B)** The length of an arc = $2\pi r\ (\frac{\theta}{360})$
$70 = 2 \times 3.14 \times 21 \times (\frac{\theta}{360})$
$70 = 131.88 \times \frac{\theta}{360}$
Multiply both sides by 360 to remove the fraction.
$25200 = 131.88\theta$
Divide both sides by 131.88
θ = 191.08 degrees.

11. **(D)** The length of an arc = $2\pi r\ (\frac{\theta}{360})$
$127 = 2 \times 3.14 \times r \times \frac{120}{360}$
$127 = 18.84\ r$
Divide both sides by 18.84
r = 6.74 cm.
So, the radius of the arc is 6.74 cm.

12. **(A)** Perimeter of the triangle = 3 × side = 3 × 14 = 42 cm.
Perimeter of the triangle = circumference of the circle.
Thus, the perimeter of the triangle is also 42 cm.
Circumference of a circle = $2\pi r = 2 \times \frac{22}{7} \times r = 42$.
$r = \frac{(42 \times 7)}{44} = 6.68$.
Therefore, the radius of the circle is 6.68 cm.
Area of a circle = $\pi r^2 = \frac{22}{7} \times (6.68)^2 = 44.63$ square cm.
Therefore, the area of a circle is 44.63 square cm.

13. **(D)** Arc length = $r\ \theta$
= 17 × 33.67 = 572.39 m.

14. **(C)** One rotation of the wheel = circumference of the wheel

The area of the segment is,
$r^2 [\pi \frac{\theta}{360} - \sin \frac{\theta}{2}]$
$= 6^2 [3.142 \times \frac{60}{360} - \sin \frac{60}{2}]$
≈ 3.27 square units.
Therefore, the area of the segment of the pizza = 3.27 square units.

9. **(C)** The length of an arc = $2\pi r (\frac{\theta}{360})$
$= 2 \times 3.14 \times 27 \times 150/360$
$= 70.65$ cm.

49 rotations = 343 m
1 rotation = 7 m
So, circumference = 7 m
$2\pi r = 7$ m
$r = 7 \times \frac{7}{22} \times \frac{1}{2}$
$r = 0.83$ m

15. **(B)** Circumference – Diameter = 28 cm
$2\pi r - 2r = 28$ cm
$2r (\pi - 1) = 28$ cm
$2r (\frac{22}{7} - 1) = 28$ cm

$2r (\frac{15}{7}) = 28$ cm
$r = 28 \times \frac{7}{15} \times \frac{1}{2}$
$r = 6.53$ cm

16. **(A)** At 5:30 p.m., the minute hand covers half of the circle. As a result, the minute hand's travel distance is actually half of the circumference.
Distance = πr (where r is the length of the minute hand).
As a result, the distance travelled is $\frac{22}{7} \times 14$
$= 22 \times 2 = 44$ units.
As a result, the total trip distance is 44 units.

17. **(A)** Circumference = $2\pi R$
$576 = 2 \times 3.14 \times R$
$576 = 6.28R$
Divide both sides by 6.28 to get,
$R = 91.71$
Therefore, the radius of the circle is 91.71 yards. But, since the diameter is twice the radius of a circle, the diameter is equal to 183.43 yards.

18. **(B)** $R = 49$ yd, $r = 47$ yd.
Let the area of outer circle be A_1 and the area of inner circle be A_2
Area of race track = $A_1 - A_2 = \pi R^2 - \pi r^2 = \pi (49^2 - 47^2) = \frac{22}{7} \times 192 = 603.42$ square yards.
Therefore, the area of the race track is 603.42 square yards.

19. **(C)** It is given that $A_1 : A^2 = 16 : 25$
Area of a Circle = πr^2
$\pi R_1^2 : \pi R_2^2 = 16 : 25$
Taking square roots of both sides,
$R1 : R2 = 4 : 5$
Therefore, the ratio of the radii = 4:5

20. **(D)** Area of sector = $\frac{108}{360} \times \frac{22}{7} \times 28 \times 28 = 46.2$ cm^2

21. **(B)** Area of the sector = $\frac{111}{360} \times \frac{22}{7} \times 97 \times 97$
$= 8871.34$ square meter
Area covered by the man of the walking track in a day = 8871.34 + 8871.34 = 17742.68 m.

22. **(C)** Area of sector = $\frac{72}{360} \times \frac{22}{7} \times 7 \times 7$

23. **(D)** Area of segment = Area of sector – aea of triangle
$= 57 - 23 = 34$ square meter.

24. **(A)** It's a semicircle, since the central angle is 180 degrees.
As a result, the area of this sector is $\frac{1}{2}$ the area of the circle = $\frac{\pi r^2}{2}$

25. **(C)** Circumference = $\frac{22}{7} \times 77 = 242$ cm
To get the number of revolutions of the wheel, divide the distance covered by the circumference of the wheel.
We need to convert 147 meters to cm before dividing, so we multiply 147 by 100 to get 14,700 cm. Therefore,
Number of revolutions = $\frac{14{,}700 \text{ cm}}{242 \text{ cm}}$
= 61 revolutions (approx).

26. **(D)** Circumference = $2 \times \frac{22}{7} \times 0.280 = 1.76$ m.
To find the distance travelled, multiply the circumference of the wheel by the number of revolutions taken.
Distance = $1.76 \times 100 = 176$ m
Therefore, the distance travelled is equal to 176 meters.

27. **(A)** The circumference of the circle formed = the perimeter of the rectangular wire.
Perimeter of a rectangle = 2(80 + 35) cm = 230 cm.
Therefore, the circumference of the circle will be 230 cm.
Circumference = $2 \pi R$
$230 \text{ cm} = 2 \times \pi \times R$
$230 \text{ cm} = 2 \times 3.14 \times R$
$R = 36.62$ cm
So, the radius of the circle will be 36.62 cm.

28. **(B)** Circumference = $2 \times \frac{22}{7} \times 0.14 = 0.88$ m.

29. **(B)** Area = $½ \times IR = ½ \times 12.5 \times 46 = 287.5$ cm^2

30. **(D)** Here, $\frac{120}{360} = \frac{1}{3}$
and, $\frac{150}{360} = \frac{5}{12}$
So, sector with angle 120 and 150 is part $\frac{1}{3}$ and $\frac{5}{12}$.
Hence, Ratio of area of two sectors = Ratio of central angle = 120:150 = 4:5.

= 30.8 square meter

27 Trignometry

SYNOPSIS

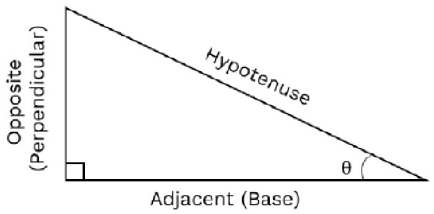
- Some Important Definitions
- Pythagoras Theorem
- Trigonometric Ratios
- Reciprocal Relationships
- Trigonometric Table
- Relation between Degree and Radian
- Table for Degree and Radian Relation
- Trigonometric Identities
- Pythagoras Identities
- Heights and Distances

INTRODUCTION

Trigonometry can be defined as the branch of mathematics which deals with the relation between the side lengths and angles of a right-angled triangle.

SOME IMPRORTANT DEFINITIONS

- **Hypotenuse:** Hypotenuse is the largest side of the triangle and is opposite to the right angle of the triangle.
- **Adjacent:** The side of the triangle on which the right angle lies is called as the adjacent of the triangle. It is also referred as base of the triangle.
- **Opposite:** Side perpendicular to the base of the right-angled triangle. It is also referred as perpendicular for trigonometric calculations.



PHTYAGORAS THEOREM

Pythagoras theorem states that the square of the hypotenuse in a right-angled triangle is equal to the sum of squares of the other two sides.

$\text{Hypotenuse}^2 = \text{Adjacent side}^2 + \text{Opposite side}^2$

TRIGONOMETRIC RATIOS

$\sin\theta$	$\frac{\text{Opposite side}}{\text{Hypotenuse}}$
$\tan\theta$	$\frac{\text{Opposite side}}{\text{Adjacent side}}$
$\cos\theta$	$\frac{\text{Adjacent side}}{\text{Hypotenuse}}$
$\text{Cosec}\theta$	$\frac{\text{Hypotenuse}}{\text{Opposite side}}$
$\sec\theta$	$\frac{\text{Hypotenuse}}{\text{Adjacent side}}$
$\cot\theta$	$\frac{\text{Adjacent side}}{\text{Opposite side}}$

RECIPROCAL RELATIONSHIP

$\text{Cosec}\theta = \frac{1}{\sin\theta}$

$\sec\theta = \frac{1}{\cos\theta}$

$\cot\theta = \frac{1}{\tan\theta}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\text{Cosec}\theta \times \sin\theta = 1$

$\sec\theta \times \cos\theta = 1$

$\cot\theta \times \tan\theta = 1$

TRIGONOMETRIC TABLE

Angles	0°	30°	45°	60°	90°	180°	270°	360°

Sinθ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
Cosθ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1
Tanθ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0	∞	0
Cosecθ	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	∞	-1	∞
Secθ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞	-1	∞	1
Cotθ	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	∞	0	∞

RELATION BETWEEN DEGREE AND RADIAN

2π radian = 360°

TABLE FOR DEGREE AND RADIAN RELATION

DEGREE	30°	45°	60°	90°	180°	270°	360°
RADIAN	π/6	π/4	π/3	π/2	π	3π/2	2π

TRIGONOMETRIC IDENTITIES

If an equation involves trigonometric ratios of an angle and is true for all the values of an angle, it is called trigonometric identity.

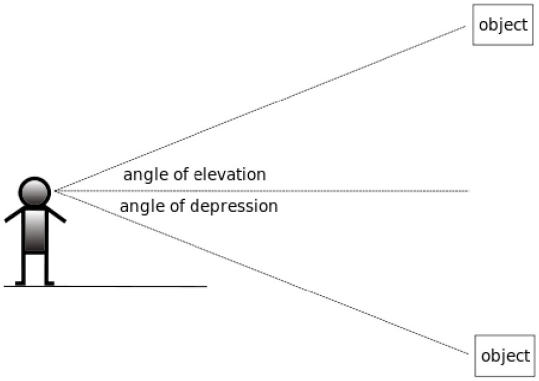
PYTHAGORAS IDENTITIES

$\sin^2\theta + \cos^2\theta = 1;$
$\tan^2\theta + 1 = \sec^2\theta;$
$\cot^2\theta + 1 = \text{cosec}^2\theta;$

HEIGHT AND DISTANCE

Trigonometry is best used in real-life for finding height and distance.

- **Angle of Elevation:** The angle formed by the line of sight with the horizontal, when the point being observed is above the horizontal level is called as the angle of elevation.
- **Angle of Depression:** The angle formed by the line of sight with the horizontal when the point being observed is below the horizontal level is called as the angle of depression.



Points to Remember

- In a right angled triangle, if the angles are in the ratio 45°: 45°:90°, then the sides are in the ratio 1:1:2.
- In a right angled triangle, if the angles are in the ratio 30°:60°:90°, then the sides are in the ratio 1:3:2.

PRACTICE QUESTIONS

1. The length of the shadow of a tree is equal to its height. What is the angle of the elevation of the light?

A. 30° B. 60°
C. 90° D. 45°

2. During a cyclone, a tree in the coastal area broke in such a way that its top touched the ground making an angle of 60°. If the initial height of the tree is 15 m, at what height is the tree broken?

A. $15\left(1+\sqrt{3}\right)$ m

B. $\frac{15\sqrt{3}}{2+\sqrt{3}}$ m

C. $\frac{30\sqrt{3}}{\left(2+\sqrt{3}\right)}$ m

D. 5 m

3. The angle of elevation of a building from a point P is 45°. If P is 20 m away from the foot of the pole, then what is the height of the pole?

A. $20\sqrt{2}$ m
B. 10 m
C. 20 m
D. 15 m

4. There are two points P and Q situated 18m and 32m away from the foot of a tower. The angles of elevations of the top of the tower from these two points are complementary. What is the height of the tower?

A. 20 m
B. 24 m
C. 36 m
D. 32 m

5. The angle of elevation of the top of a tower is 30° from the top of a 5 m high platform. If the height of the tower is 45 m, what is the distance between the tower and the platform?

A. $13\sqrt{3}$ m
B. $40\sqrt{3}$ m
C. $15\sqrt{4}$ m
D. $24\sqrt{5}$ m

6. There are 2 cars 200 m apart, the angles of depression of the cars from the top of a building are 45° and 30° towards east. What is the height of the building?

A. 100 m
B. 173 m
C. 200 m
D. 273 m

7. The angle of elevation of an under construction building at a certain point 150 m from its base is 30°. If, as per the actual construction plan, the angle of elevation at the same point of the fully constructed building is to be 45°, then the building has to be raised by how many metres?

A. 59.4 m
B. 61.4 m
C. 62.4 m
D. 63.4 m

8. What is the height of the pillar when from the top of a large statue, which has the height of 90 metres, the angles of depression of the top and bottom of a pillar are 30° and 60°?

A. 30 m
B. 45 m
C. 60 m
D. 75 m

9. What is the height of a large plastic structure when the plastic structure got broken at a point and its top touches the ground at a distance 20 m from the base of the structure making an angle 30° with the ground?

A. $\frac{40}{\sqrt{3}\,\text{m}}$

B. $20\sqrt{3}\,\text{m}$

C. $40\sqrt{3}\,\text{m}$

D. 30 m

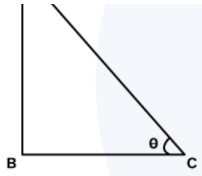
10. When the flood light's elevation is 30°, the shadow of a house is 15 m. How long must have been the shadow if the flood light's elevation would have been 60°?

A. 3 m
B. 4 m
C. 5 m
D. 6 m

SOLUTIONS

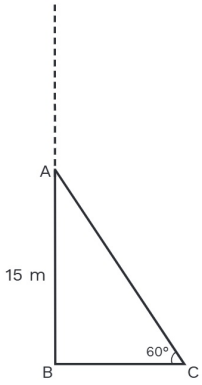
1. (D)

Let the tree be BAC after falling. Total length BAC = 15 m
BA + AC = 15



Let AB is the tree and BC be the shadow of the tree.
Given, AB = BC
tan θ = AB/BC = 1
θ = 45°

2. (B)



Let the height at which tree is broken is x.
BA = x; AC = 15 – x
Sin 60° = AB/AC

$$\frac{\sqrt{3}}{2} = \frac{x}{15-x}$$

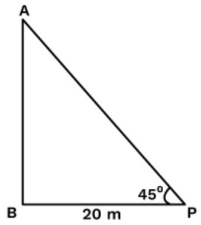
$$\sqrt{3}(15-x) = 2x$$

$$15\sqrt{3} - \sqrt{3}x = 2x$$

$$15\sqrt{3} = 2x + \sqrt{3}x$$

$$x = \frac{15\sqrt{3}}{2+\sqrt{3}}$$

3. (C)

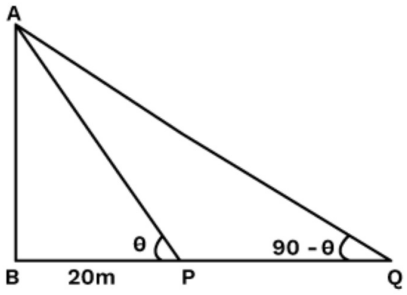


tan 45° = AB/BP

$$1 = \frac{AB}{20}$$

AB = 20 m

4. (B)



Let angle at P = θ and height of the tower be h.
Then angle at P = (90 – θ)

$$\tan\theta = \frac{AB}{BP} = \frac{h}{18}$$

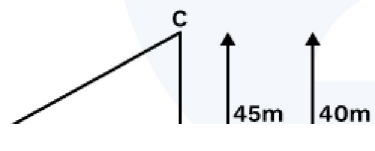
$$\tan(90-\theta) = \frac{h}{32};\ \cot\theta = \frac{h}{32}$$

$$\tan\theta \times \cot\theta = 1 = \frac{h}{18} \times \frac{h}{32}$$

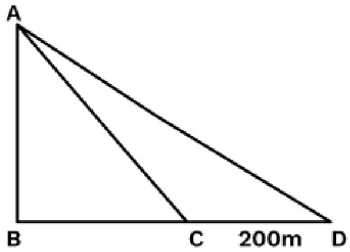
$$18 \times 32 = h^2$$

h = 24 m

5. (B)



6. (B)



Given,
∠ACB = 45°
∠ADB = 30°
Distance between two cars, i.e.,
CD = 200 m
Then, AB = ?
Let BC = x m
In ΔABC,
tan 45º = AB/BC
(∵ tan 45° = 1)

$$1 = \frac{AB}{x}$$

AB = x m ....(i)

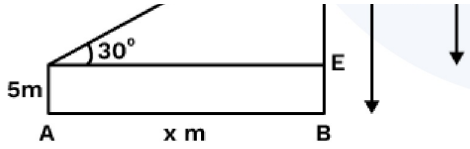
In ΔABD, tan 30º = AB/BD

$$\frac{1}{\sqrt{3}} = \frac{AB}{x+200}$$

$$(\because \tan 30° = \frac{1}{\sqrt{3}})$$

x = $\sqrt{3}$, AB – 200 ....(ii)

From equations (i) and (ii),
AB = $\sqrt{3}$ AB – 200



In ΔDEC,

$\tan 30° = \frac{CE}{DE}$

$\tan 30° = \frac{40}{x}$

$\frac{1}{\sqrt{3}} = \frac{40}{x}$

$x = 40\sqrt{3}$ m

$\sqrt{3}$ AB – AB = 200

0.732 AB = 200

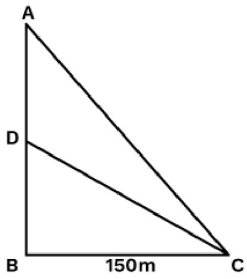
($\because \sqrt{3} = 1.732$)

$AB = \frac{200}{0.732} = 273.22$

= 273 m

Hence, option (D) is correct.

7. (D)



Given, BC = 150 m

∠ACB = 30°

and, ∠DCB = 45°

Then, AD = ?

In ΔABC, tan 30° = AB/BC

$\frac{1}{\sqrt{3}} = \frac{AB}{150}$

$AB = \frac{150}{\sqrt{3}} = 86.6$ m

In ΔDBC, tan 45° = DB/BC

$1 = \frac{DB}{150}$

DB = 150

AD + AB = 150

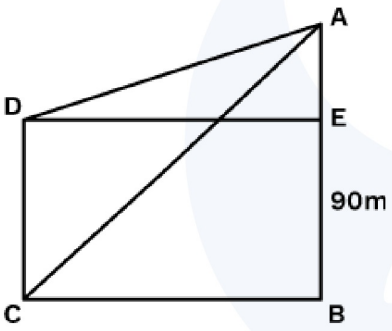
[$\because$ DB = AD + AB]

AD = 150 – AB

= 150 – 86.6 = 63.4 m

Hence, option (D) is correct.

8. (C)



Given, AB = 90 m

∠ADE = 30°

And ∠ACB = 60°

Then, DC = ?

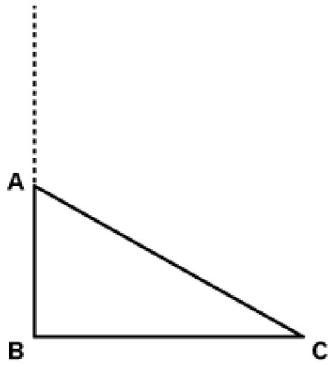
Ratio of angles,

tan 30º/tan 60º = (AE/ED)/(AB/BC)

[$\because$ ED = BC]

$\frac{1}{\sqrt{3}} = \frac{AE}{}$

9. (B)



Given, BC = 20 m

∠ACB = 30°

Total height of the large plastic structure is (AB + CA) = ?

In Δ ABC, tan30° = AB/BC

$\frac{1}{\sqrt{3}} = \frac{AB}{20}$

$AB = \frac{20}{\sqrt{3}}$ m

Now, cos30º = BC/AC

$\frac{\sqrt{3}}{2} = \frac{20}{AC}$

$AC = \frac{40}{\sqrt{3}}$ m

So, $AB + CA = \frac{20}{\sqrt{3}} + \frac{40}{\sqrt{3}} = \frac{60}{\sqrt{3}}$

$= 20\sqrt{3}$ m

Hence, option (B) is correct.

10. (C)

$\frac{\sqrt{3}}{\sqrt{3}} = \frac{AE}{90}$

$\frac{1}{3} = \frac{AE}{90}$

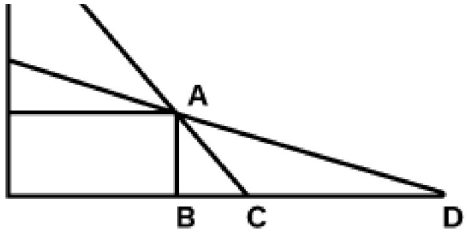
AE = 30 m

Now, DC = EB

= AB – AE

= 90 – 30 = 60 m

Hence, option (C) is correct.



Given, ∠ADB = 30° and ∠ACB = 60°

When the flood light's elevation is 30°, the shadow of house is 'BD = 15 m' and when the flood light's elevation is 60°, the shadow of house is 'BC = ?'

Let, BC = x m

In ΔABD, $\tan 30° = \frac{AB}{BD}$

$\frac{1}{\sqrt{3}} = \frac{AB}{15}$

$AB = \frac{15}{\sqrt{3}}$ ....(i)

In ΔABC, $\tan 60° = AB/BC$

$\sqrt{3} = AB/x$

$AB = x\sqrt{3}$ ...(ii)

From equations (i) and (ii), we get

$x\sqrt{3} = \frac{15}{\sqrt{3}}$

x = 5 m

Hence, option (C) is correct.

28 Data Interpretation

SYNOPSIS

- Table
- Pie chart
- Line graph
- Bar chart
- Mixed graph
- Caselet
- Most common types of questions and important formulae
- Some important tricks useful in data interpretation
- Summary

INTRODUCTION

Data interpretation is analysing the provided data and calculating the required value asked based upon that data. Data in competitive questions can be in various forms like tables, pie charts, bar graphs, line graphs, etc.

TABLES

In tabular data interpretations (DIs), data are divided systematically into horizontal rows and vertical columns followed by certain questions based on that data that are to be answered by students. For example, the table given below represents the loss of three companies in the period of 3 years from 2018 to 2020.

YEAR COMPANY	2018 (LOSS %)	2019 (LOSS %)	2020 (LOSS %)
A	18	11	7
B	13	17	11
C	7	14	15

Comprehending the Data Table

If a question asks to calculate the increase/decrease in loss of B's revenue from 2019 to 2020 when total revenue for the years is 2 crores:

1. Identify loss in 2019, i.e., the intersection of B row and 2019 column, 17%
2. Similarly, loss in 2020, 11%
3. Finally, calculate the change in loss = 17% of 2 crores – 11% of 2 crores

Note: Pay attention to the units.

Illustration:

Directions (Questions 1–4): Study the given table carefully and answer the questions that follow:

STATES	TOTAL NUMBER OF PEOPLE WHO VOTED	PERCENTAGE OF PEOPLE WHO VOTED FOR DIFFERENT PARTIES FROM RESPECTIVE STATES

	PEOPLE WHO VOTED	PARTIES FROM RESPECTIVE STATES			
		A	B	C	D
P	80	25	20	15	40
Q	100	24	33	21	22
R	200	32	20	17	31
S	250	30	10	20	50

1. In state Q, the total number of people who voted for parties A and C together is what per cent less than the total number of people who voted for B and D?

A. $16\frac{2}{11}$ B. $15\frac{2}{11}$

C. $18\frac{2}{11}$ D. $17\frac{3}{11}$

Solution: Total number of people who voted for parties A and C together in Q

$$= 100 \times \frac{24}{100} + 100 \times \frac{21}{100}$$

$$= 24 + 21 = 45$$

Total number of people who voted for parties B and D together in Q

$$= 100 \times \frac{33}{100} + 100 \times \frac{22}{100}$$

$$= 33 + 22 = 55$$

Required percentage $= \frac{(55 - 45)}{55} \times 100 = 18\frac{2}{11}$

2. What is the average number of people who voted in states P, Q, and S for party C?

A. 23 B. 27

C. 29 D. 32

Solution: Number of people who voted in state P for party C $= 80 \times \frac{15}{100} = 12$

Number of people who voted in state R for party C $= 200 \times \frac{17}{100} = 34$

Number of people who voted in state S for party C $= 250 \times \frac{20}{100} = 50$

Required average $= \frac{(12 + 34 + 50)}{3} = 32$

3. What is the difference between the total number of people who voted in state P for parties A and D together and the total number of people who voted in state R for the same party together?

A. 63 B. 74

C. 85 D. 96

Solution: Total number of people who have voted in P for parties A and D together

$$= 80 \times \frac{25}{100} + 80 \times \frac{40}{100}$$

$$= 20 + 32 = 52$$

Total number of people who have voted in R for parties A and D together

$$= 200 \times \frac{32}{100} + 200 \times \frac{31}{100}$$

$$= 64 + 62 = 126$$

Required difference = 126 − 52 = 74

4. It is given that in S, 42% are females. And if 20% of these total females voted for party B, what is the number of male people who voted for party B from state S?

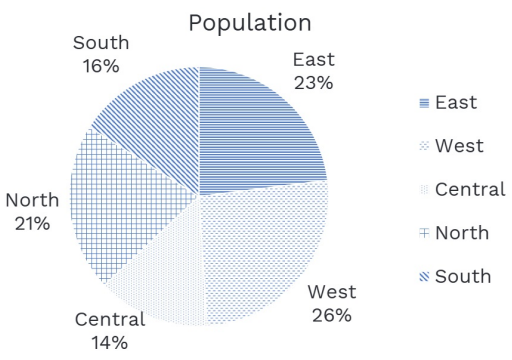
A. 145 B. 132

C. 129 D. 117

Solution: In S, females = 42%

Males = 58%

Total number of male people in S who voted for party B $= \frac{58}{100} \times 250 = 145$



PIE CHART

COMPREHENDING PIE CHARTS

1. If data are represented in degrees:

In pie charts, data are represented as sectors of circular charts, which are proportional to the quantities they represent. And the total quantity over the whole circle, i.e., 360° or 100%. For example, the following pie chart represents the distribution of population in various districts of Town X.

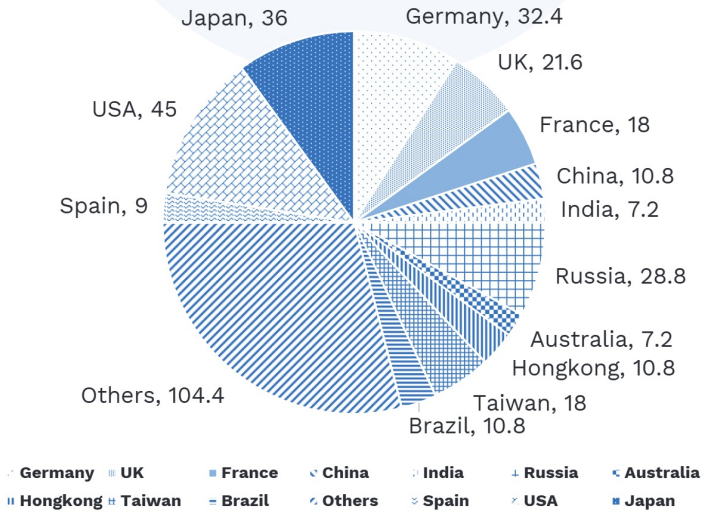
1. **If data are represented in degrees:**
   Value of any sector =
   $$\frac{\text{Angle of any sector}}{360°} \times \text{Total value}$$
2. **If data are represented in percentage:**
   Value of any sector
   $$= \frac{\text{Per cent of the sector}}{100} \times \text{Total value}$$

Thus, from the above chart the population of any zone can be calculated using the later formula.

Illustration:

Directions (Questions 1–5): Study the given table carefully and answer the questions that follow:

Country-wise Exports (in degree) Total = 72000 billion



1. By how much does the value of the imports of the USA exceed that of Germany?
A. 2326 billion B. 2789 billion
C. 2900 billion D. 2520 billion

Solution: Difference in the angles subtended by the USA and Germany = 45° – 32.4° = 12.6°
The difference in the imports of the USA and Germany = $\frac{12.6°}{360°} \times 72000$ billion = 2520 billion

2. The difference in the values of the imports of Japan and France is how many times that of the UK and Taiwan?
A. 2 times B. 3 times
C. 4 times D. 5 times

Solution: The difference in the angles of the import of Japan and France
= 36° – 18° = 18°
The difference in the angles subtended by the UK and Taiwan
= 21.6° – 18° = 3.6°
Hence, the difference in the value of imports
Required difference = 4176 – 2880 = 1296 billion

4. If imports of developing countries accounted for 36% of the total worldwide imports, then what is the value of the imports of Japan as a percentage of the imports of the developing countries?
A. 23.3% B. 27.7%
C. 29.9% D. 33.3%

Solution: Imports of developing countries = 36% of total
Imports of Japan = $\frac{36°}{360°} \times 100 = 10\%$ of total
Required per cent = $\frac{10}{36} \times 100 = 27.7\%$

5. What is the number of countries whose imports are more than the average imports per country?
A. 6138.5 billion B. 5276.4 billion
C. 4829.3 billion D. 5142.8 billion

Solution: Average imports
Total imports 72000

of Japan and France is 5 times that of the UK and Taiwan.

3. The value of the imports of the Organization of the Petroleum Exporting Countries (OPECs) organisation is how much more than the value of the imports of India and Australia put together, given that OPEC has a 20% share in the value of the imports of others?

A. 2673 billion B. 2569 billion
C. 1296 billion D. 1325 billion

Solution: Value of imports of India and Australia

$$= \frac{(7.2° + 7.2°)}{360°} \times 72000 = 2880 \text{ billion}$$

Value of imports of OPEC countries

$$= \frac{20}{100} \times \left(\frac{104.4°}{360°} \times 72000 \right) = 4176 \text{ billion}$$

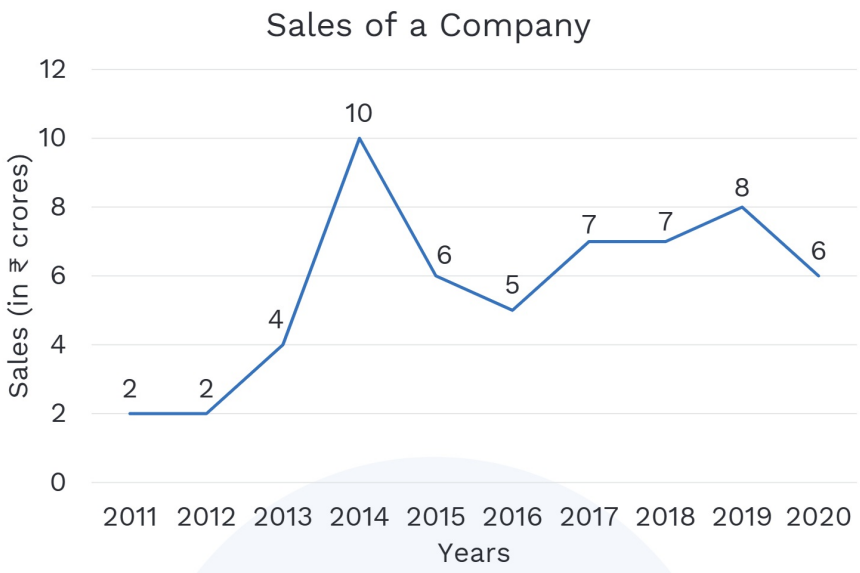
$$= \frac{\text{Total imports}}{\text{Number of countries}} = \frac{72000}{14} = 5142.8 \text{ billion}$$

After calculating imports of each country individually, only USA, Japan, Germany, Russia, and Others (i.e., 5) have imports greater than average.

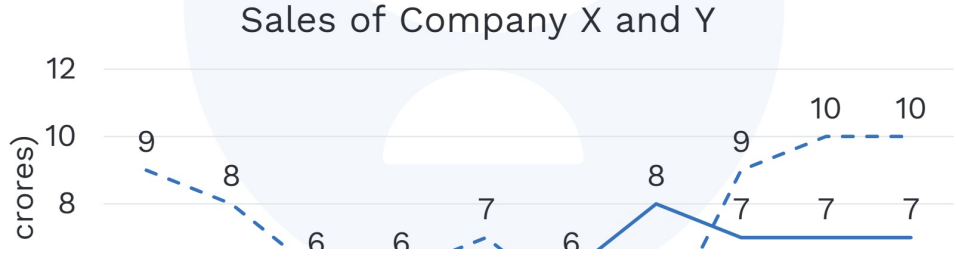
LINE GRAPHS

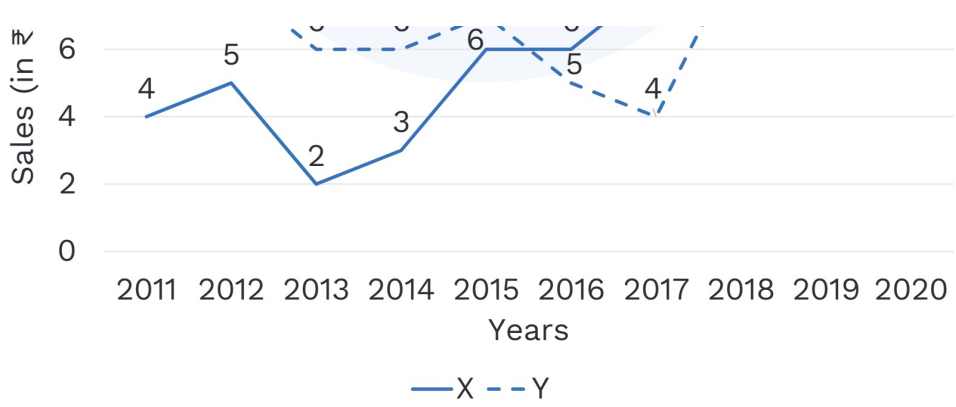
In a line graph, data are distributed on X and Y-axes, which represent a variation of quantity with respect to these perimeters over axes.
Types of line graph

1. Single-line graph: Used for a single variable. For example, the following graph represents the sales of a company in the years 2011–2020.

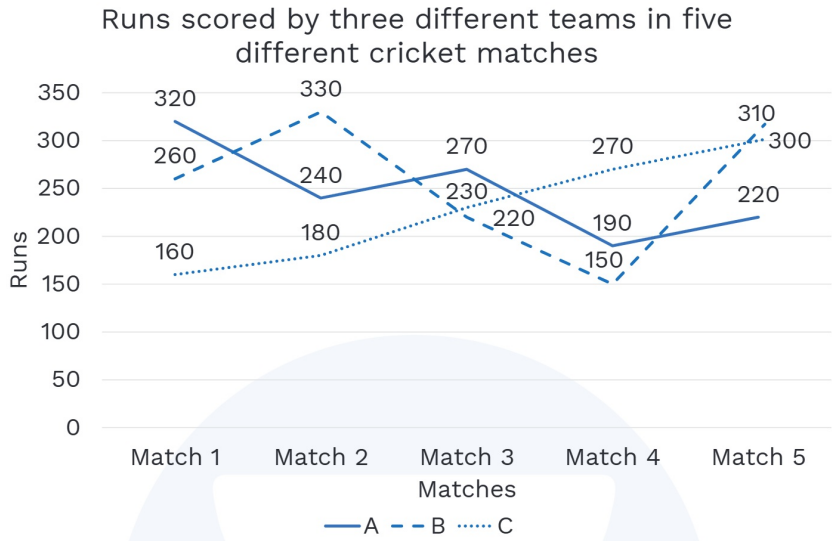


2. Two-variable graph:
For example, the following graph represents the sales of two companies X and Y in the years 2011–2020.





3. **Three-variable graph:**
For example, the following graph represents the runs scored by three different countries in five different cricket matches.



COMPREHENDING LINE GRAPH

If a question asks, the sales of company X in 2015 is by how much per cent more than the sales of company Y in 2016:

1. Identify sales of company X in the year 2015 from the blue line
2. Similarly, sales of company Y in the year 2016 from the orange line
3. Finally, calculate increase = 6 crores – 5 crores

Illustration:

Directions (Questions 1–5): Study the given line graph carefully and answer the questions that follow:

1. What is the difference between the total number of articles sold by both the shops together on the 2nd day and that by both the shops together on the 4th day?

A. 230 B. 270
C. 290 D. 320

Solution: Total number of articles sold on 2nd day $= 120 + 90 = 210$

And the total number of articles sold on the 4th day $= 240 + 200 = 440$

Required difference $= 440 - 210 = 230$

2. The number of articles sold by shop Y on the 1st day is what per cent of the number of articles sold by the same shop on the 5th day?

A. $25\frac{1}{4}$ B. $22\frac{3}{4}$
C. $22\frac{1}{4}$ D. $25\frac{3}{4}$

Solution: Required percentage $= \frac{60}{270} \times 100 = \frac{600}{27}$

$= 22.25$ or $22\frac{1}{4}$

3. What is the average number of articles sold by shop Y on the 1st and 4th day?

A. 130 B. 170
C. 190 D. 220

Solution: Average number of articles sold by shop Y on 1st and 4th day $= \frac{60 + 200}{2} = \frac{260}{2} = 130$

4. What is the respective ratio between the total number of articles sold by shop X on the 4th and 5th day together and by the same shop on the 2nd and 3rd day?

A. 3:2 B. 5:1
C. 7:3 D. 9:5

Solution: Total articles sold on 4th and 5th day $= 240 + 300 = 540$

And total articles sold on 2nd and 3rd day $= 120 + 180 = 300$

Required ratio $= 540 : 300 = 9 : 5$

5. The number of articles sold by shop X on the 3rd day is what per cent more than that sold by the same Y on the same day?

A. 30% B. 20%
C. 40% D. 10%

Solution: Required percentage

$= \frac{180 - 150}{150} \times 100 = 20\%$

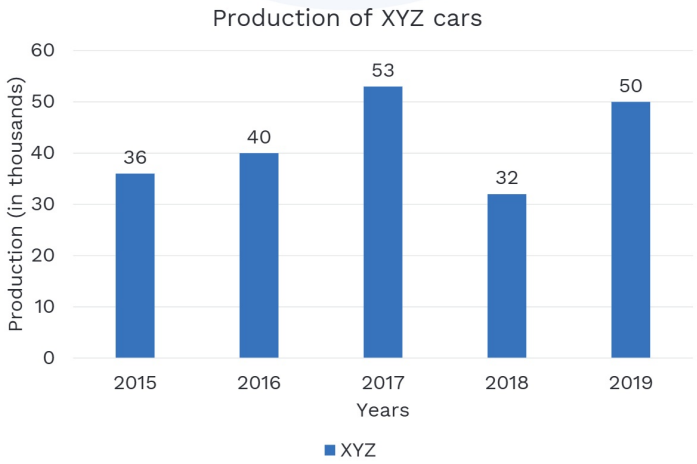
BAR GRAPH

A bar chart represents data in the form of rectangles whose lengths are proportional to the data they represent. And one axis represents variables while the other represents a parameter that is changing in the graph

Types of bar charts

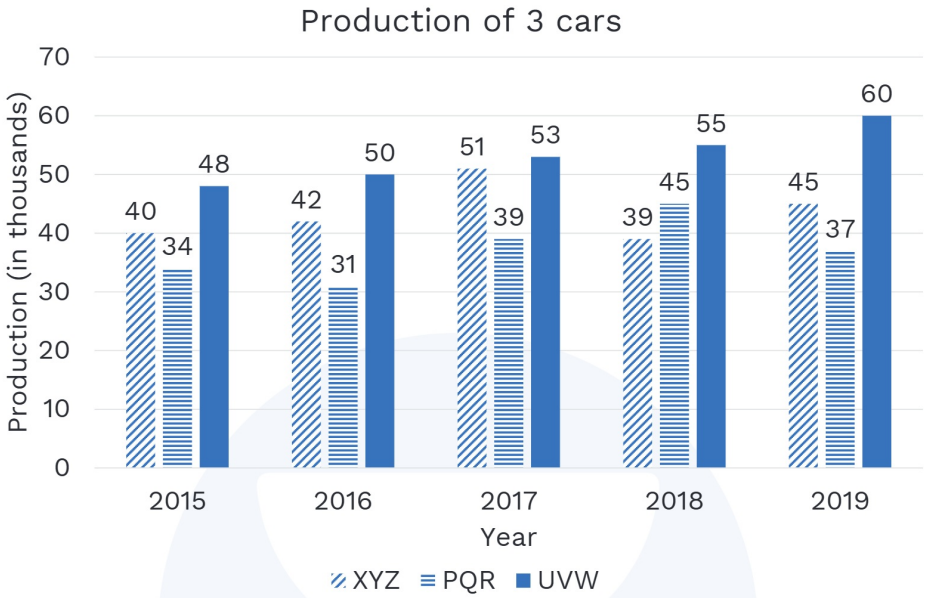
1. Simple bar graph

For example, the following bar graph represents the production of XYZ cars in five different years.



2. Grouped bar graph

For example, the following bar graph represents the production of XYZ, PQR, and UVW cars in five different years.



3. Stacked bar graph

For example, the following bar graph represents the production of various models of XYZ cars over three years 2019, 2020 and 2021.

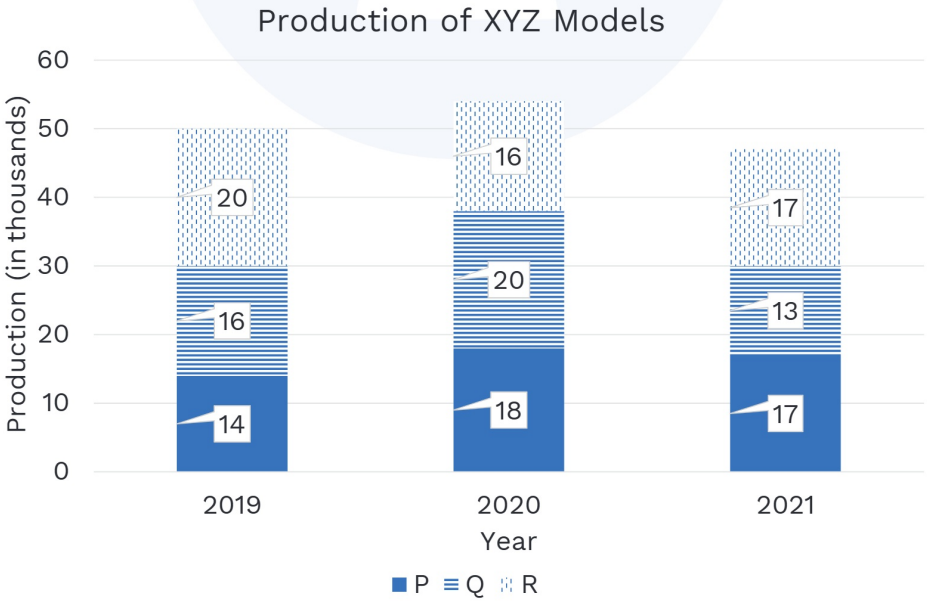
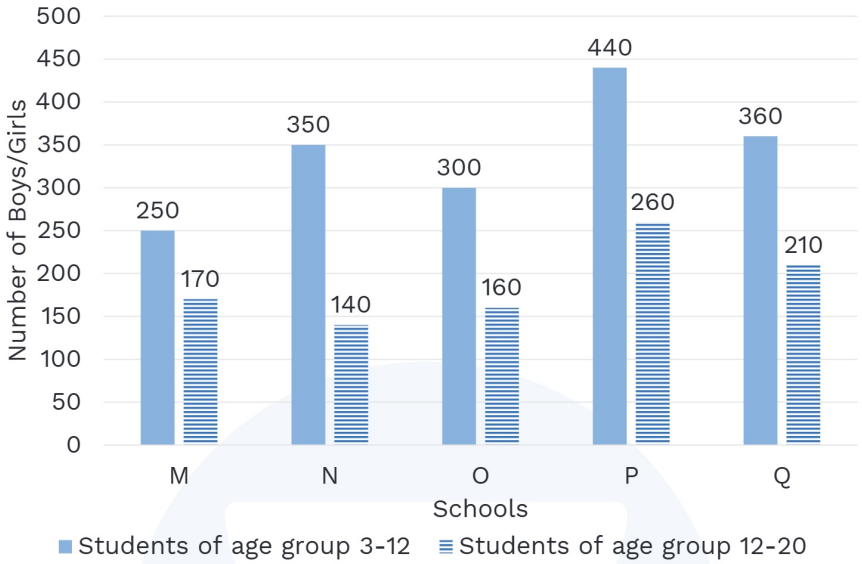


Illustration:

Directions (Questions 1–5): Study the given table carefully and answer the questions that follow:



1. What is the difference between the total number of students of age group 3-12 studying in N and O together and the total number of students of age group 12-20 in the same schools together?

A. 210 B. 350
C. 490 D. 570

Solution: Total students of age group 3-12 $= 350 + 300 = 650$

Total students of age group 12-20 $= 140 + 160 = 300$

Required difference $= 650 - 300 = 350$

2. The number of students of age group 3-12 and students of age group 12-20 together in school N is what per cent less than that in school P?

A. 30% B. 70%
C. 50% D. 20%

Solution: Total students in N $= 140 + 150 = 290$

Total students in P $= 260 + 440 = 700$

Required per cent $= \frac{700 - 490}{700} \times 100 = 30\%$

3. The number of students of age group 3-12 and students of age group 12-20 studying in class X of Q school is 20% less than those in class XII of the same school. How many students study in class X then?

A. 456 B. 472
C. 489 D. 423

Solution: Total students of Q school in class XII $= 210 + 380 = 590$

Number of students in class X $= 590 - 590 \times \frac{20}{100} = 472$

4. The number of students of age group 3-12 studying in Q school is what per cent more than the number of students of age group 3-12 studying in M?

A. 48% B. 52%
C. 54% D. 56%

Solution: Students of age group 3-12 in Q $= 380$

Students of age group 3-12 in M $= 250$

Required percentage $= \frac{380 - 250}{250} \times 100 = 52\%$

5. What is the respective ratio between the number of students of age group 3-12 and students of age group 12-20 together studying in M and that in P?

A. 21 : 23 B. 23 : 27
C. 27 : 29 D. 29 : 31

types of Data interpretation questions require the questions to be solved using both the interdependent data.

Types of mixed graphs

1. Any two graphs/charts together

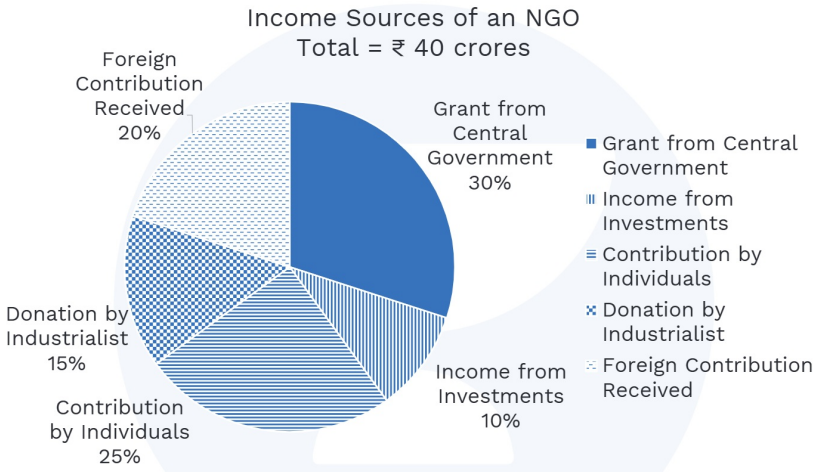
Solution: Required ratio = (170 + 250) : (160 + 300)
= 420 : 460 = 21 : 23

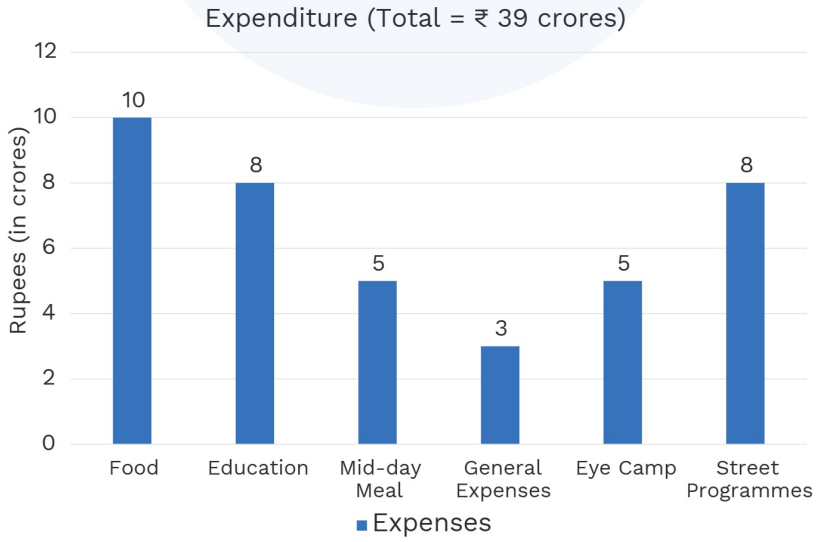
MIXED GRAPH

Mixed graphs present data in a combination of two or more forms of data presentation. These

Illustration:

Directions (Questions 1–5): The following pie chart represents the different income sources of an NGO and the bar graph represents the expenditure of this NGO over different heads. Answer the questions that follow:





1. What percentage of money is saved?
A. 2.3% B. 2.5%
C. 2.7% D. 2.9%

Solution: Saving = Total income – total expenditure
= 40 – 39 crore = 1 crores

Per cent of money saved $= \frac{1}{40} \times 100 = 2.5\%$

2. If the industries stop donating and the expenditure pattern remains the same, then what will be the decrease in money spent on the mid-day meal programme?
A. 0.69 crores B. 0.73 crores
C. 0.76 crores D. 0.63 crores

Solution: Decreased income = 40 crores –

Solution: Income from investment = $\frac{10}{100} \times 40 = 4$ crores

4 crores is 0.75 times of 3 crores (i.e., general expense)

5. If in the next year, a grant from the central government increases by 10%, foreign contribution decreases by 10% and other income amounts and expense patterns remain the same. What is the per cent increase in expenditure on Food?
A. 1% B. 2%
C. 3% D. 4%

Solution: Initially, amount of grant from central government $= \frac{30}{}$ 40 crores 12 crores

$\frac{15}{100} \times 40$ crores

$= 34$ crores

Percentage of expenditure on mid-day meal $= \frac{5}{39} \times 100 = 12.8\%$

New expenditure after a gradual decrease in expenditure with respect to income $= \frac{39}{40} \times 34$

$= 33.15$ crores

New expenditure on mid-day meal $= \frac{12.8}{100} \times 33.15 =$ ₹ 4.24 crores

Required decrease in expenditure on mid-day meal $= 5 - 4.24$ crores $= 0.76$ crores

3. What is the ratio of expenditure on food and mid-day meal programmes together to that of a grant from the central government?

A. 2 : 1 B. 3 : 2

C. 4 : 3 D. 5 : 4

Solution: Total expense on food and mid-day meal = (10 + 5) = 15 crores

Grant from central government

$= \frac{30}{100} \times 40 = 12$ crores

Required ratio = 15 : 12 = 5 : 4

4. The general expenses are how many times income from the investment?

A. 0.25 times B. 0.50 times

C. 0.75 times D. 1 time

government $= \frac{}{100} \times 40$ crores $= 12$ crores

After 10% increase $= \frac{110}{100} \times 12 = 13.2$ crores

Similarly new foreign contribution $= \frac{8}{100} \times 90 = 7.2$ crores

New income

$= 40 + (13.2 - 12) - (8 - 7.2) = 40.4$ crores

Gradual increase in expenditure

$= \frac{39}{40} \times 40.4 = 39.9$ crores

Gradual increase in Food expense

$= \frac{10}{39} \times 39.9 = 10.1$ crores

Required per cent increase $\frac{10.1 - 10}{10} \times 100 = 1\%$

CASELET

Caselets DIs present information in the form of long paragraphs, instead of tables or graphs usually. And are followed by questions that are to be solved using that data. For example:

In data for 2020-2021 total minerals extracted in India were 10 lakh tonnes. The data recorded extraction of three major minerals: iron, aluminium, and sulphur. Of which Iron accounted for 70% of the total, while Aluminum and Sulphur were in the ratio of 2:1. In the 2018-2019 annual year these same were extracted 10% less than the total extraction

in 2019-2020 but were 10% more than the total for 2017-2018. While the total extraction in 2019-2020 was also 10% more than that in 2020-2021. And the data hypothesis is that the extraction of iron increases at a rate of 5% per annum.

Comprehending caselet

1. Construct the data into a simple table, if it can be converted for reference:

YEAR	EXTRACTION (IN LAKH TONNES)
2020–2021	10
2019–2020	11
2018–2019	9.9
2017–2018	9.91

2. Use data accordingly for each question.

Illustration:

interested in the same. The total number of students interested in research is 204.

1. The difference in the number of boys and girls who are interested in research is what percentage of the total number of students who are interested in debating?

A. 51% B. 53%

C. 57% D. 59%

2. Find the percentage of students of the university who are interested in mooting.

A. 16.25% B. 17.5%

C. 18.75% D. 19%

3. Find the ratio of the number of boys interested in dancing to the number of girls interested in the same.

A. 67 : 70 B. 83 : 80

C. 67 : 90 D. 83 : 67

4. The total number of girls in arbitration is what percentage of the total number of students in Arbitration?

A. 23% B. 27%

C. 29% D. 32%

Directions (Questions 1–5): Study the given table carefully and answer the questions that follow:

There are 1400 students in ABC University in the academic year 2021. The ratio of the boys to the girls in the University is 4 : 3. All the students are interested in different extracurricular activities: mooting, debating, dancing, research and arbitration and one student is interested in only one extra-curricular activity. The number of boys interested in research is 153. The ratio of the number of boys who are interested in mooting to the number of boys who are interested in dancing is 72 : 89. 32% of the students are interested in arbitration. The number of boys interested in debating is 12.5% to the total number of boys. The number of girls interested in arbitration is 142 which is 37 less than the number of girls enrolled in dancing. The number of girls interested in mooting is 46 more than the number of boys

5. Find the number of girls who are enrolled in Debating.

A. 68 B. 72
C. 76 D. 80

Solution 1: Total number of students = 1400

Total number of boys = $\frac{4}{7} \times 1400 = 800$

Total number of girls = 1400 – 800 = 600

Number of boys interested in research = 153

Number of students interested in arbitration = $\frac{32}{100}$ of 1400 = 448

Number of girls interested in arbitration = 144

Number of boys interested in arbitration = 448 – 144 = 304

Number of boys interested in debating = $\frac{12.5}{100}$ of 1400 = 100

Number of girls interested in dancing = 720

Number of students interested in research = 204

Number of girls interested in research = 204 – 153 = 51

Number of boys interested in mooting and dancing together = 800 – (100 + 153 + 304) = 242

Number of boys interested in Mooting = $= \frac{72}{(72+89)} \times 242 = 108$

Number of girls interested in mooting = 154

Number of boys interested in dancing = 242 – 108 = 134

Number of girls interested in debating = 600 – (154 + 180 + 51 + 144) = 72

ACTIVITY	BOYS	GIRLS	TOTAL
Mooting	108	154	262
Dancing	134	180	314
Debating	100	72	172
Research	153	51	204
Arbitration	304	144	448

Difference in boys and girls interested in Research = (153 – 51) = 102

Required percentage $= \frac{102}{172} \times 100 = 59\%$

Solution 2: Required percentage

$= \frac{262}{1400} \times 100 = 18.75\%$

Solution 3: Required ratio = 134 : 180

= 67 : 90

Illustration:

Following is the table of sales of total sales of almonds in different years.

YEAR	2010	2011	2012	2013
Sale (In Tonne)	90	95	94	101

1. By what per cent did sales grow over the years 2010–2013?

Solution: If quantity changes from A to B,

$$\text{Growth Rate} = \frac{B - A}{A} \times 100$$

Thus, here,

$$\text{Growth Rate} = \frac{101 - 90}{90} \times 10 = \frac{11}{9} \times 10 = 12.2\%$$

Note

- If B is greater the A, then growth is positive.
- If B is smaller than A, the growth rate is negative.

2. What was the average annual growth rate of sales over the years 2010–2013?

Solution: average annual/monthly growth rate is an average growth rate over the years/months.

$$\text{Average growth rate} = \frac{\text{Total growth rate}}{}$$

Solution 4: Required percentage

$= \frac{144}{448} \times 100 = 32\%$

Solution 5: B

COMMON TYPE OF QUESTIONS AND IMPORTANT FORMULAE

1. Growth rate

Finding the growth rate of quantities is one of the most asked questions in DI. It is the percentage increase/decrease in a quantity over a period of time.

$$\text{Average annual growth rate} = \frac{\text{Total growth}}{\text{No. of years or months elapsed}}$$

Hence, here,

average annual growth rate of sales

$= \frac{12.2\%}{3} = 4.07\%$ per year

Note

Though the number of years given is 4 (2010, 2011, 2012, 2013) the growth happens over a year, i.e., as in 2002–2003. Thus, the number of years elapsed in the time period will be 3 (2010–2011, 2011–2012, 2012–2013)

2. Quantity X is how much of Quantity Y

This type of questions include calculating, how much is quantity 1 in respect of quantity 2 or quantity 1 in respect of total quantity or ratio of two quantities, etc.

Illustration:

Following is the table of the number of students appearing for an entrance exam in different cities.

CITY	A	B	C	D	E	F
Number of students (in thousand)	9.4	13	11	14.7	15.6	12

1. The number of students appearing from city E is what per cent of the number of students appearing from city A?

Solution:

$\text{Required percentage} = \frac{15600}{9400} \times 100 = 165.9\%$

2. If the number of students appearing from city G is 70% more than students appearing from city E, what is the number of students appearing for the entrance from city G?

Solution:

$\text{Number of students} = 15600 + \frac{70}{100} \times 15600$

Or 170% of 15600 = 26520

3. Average of all or some units

Illustration:

Following is the table units sold (in hundred) by five companies over different years

YEAR	A	B	C	D	E
2001	1.3	2.2	1.7	2.2	1.4
2002	2.0	1.6	1.5	1.9	1.7
2003	0.9	1.6	1.0	1.5	1.1
2004	0.4	1.3	1.4	1.2	2.5
2005	1.5	1.2	2.1	1.1	2.6

1. What is the average number of units sold by company D over all the years?

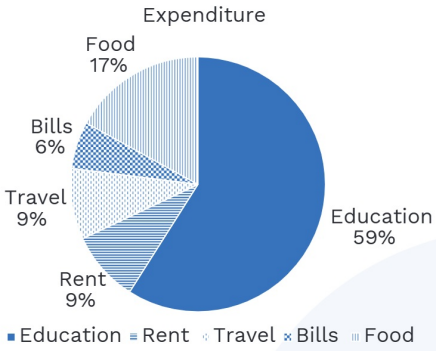
Solution: $\text{Average} = \frac{\text{Sum of units}}{\text{Number of years}}$

$$= \frac{220 + 190 + 150 + 120 + 110}{5} = 158$$

4. Calculating X if the total is given or vice versa

Illustration:

Following is the pie chart representing the percentage distribution of budget expenditure of a household



1. If the house has a total expenditure of ₹ 30 thousand, then how much did they spend on Travel?

Solution:

$$\text{Expenditure on travel} = \frac{9}{100} \times 30000 = ₹\ 2700$$

2. If 9000 were spent on bills, then what would have been the total expenditure of the house?

Solution:

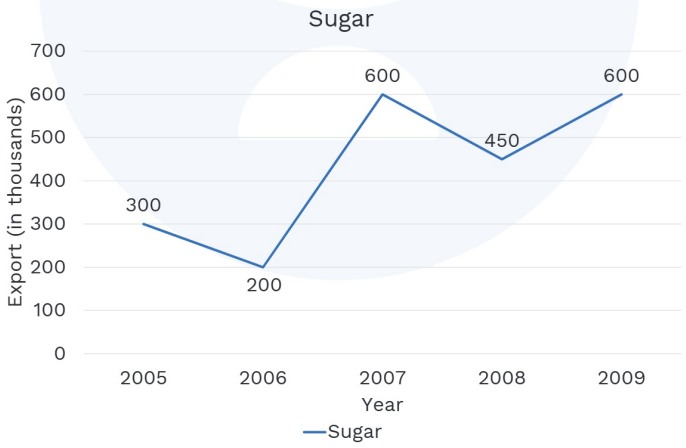
$$\text{Amount spent on bills} = \frac{6}{100} \times \text{total}$$

$$\text{Total} = \frac{9000}{6} \times 100 = 150000$$

Which year has the highest quantity

Illustration:

Following is the line graph of export of sugar in different years



1. Which year has the highest per cent increase in exports as compared to the preceding year?

Solution: $\text{Per cent change} = \frac{B - A}{A} \times 100$

$$2006 = \frac{200 - 300}{300} \times 100 = 33.33\% \text{ decrease}$$

$$2007 = \frac{600 - 200}{200} \times 100 = 200\% \text{ increase}$$

$$2008 = \frac{450 - 600}{600} \times 100 = 25\% \text{ decrease}$$

$$2009 = \frac{600}{450} \times 100 = 33.33\% \text{ increase}$$

Thus, the highest per cent increase is in the year 2007, i.e., 200%

SOME IMPORTANT TRICKS USEFUL IN DATA INTERPRETATION

- If the data are given in text form, converting the data first into a simple table saves a lot of time and is easy to refer to in a shortage of time.
- If the quantity of the item is not mentioned in bar graphs or line graphs, etc., label the quantity beforehand so that you don't look back again and again at both axes.
- Use approximation if options are not close. ex: in question, you do not need to calculate $\frac{11}{9}$ first and then multiply it by 100 or in any other longer way. You can take an approximation of $\frac{100}{9}$ as 11.1 or 11, and thus the answer can be calculated in one step as 11^2.

  Or,

  In question $\frac{15600}{9400}$ could be approximated as $\frac{15000}{9000} \times 100 = \frac{10}{6} \times 100$

  Next, you know $\frac{1}{6}$ is 0.166%. Thus, $\frac{1000}{6}$ = 166.6%, and $\frac{15600}{9400}$ will be near to 166.6%; hence, one coluld ultimatly choose 165.9% from the options.
- Write the calculated values at a separate place so that if any question refers to them again, you do not have to spend time again in calculating.

Chapter Summary

- If data are represented in degrees:
  $$\text{Value of any sector} = \frac{\text{Angle of any sector}}{360^\circ} \times \text{Total Value}$$
- If data are represented in percentage:
  $$\text{Value of any sector} = \frac{\text{Per cent of any sector}}{360^\circ} \times \text{Total Value}$$
- $$\text{Growth Rate} = \frac{B - A}{A} \times 100$$
- $$\text{Average} = \frac{\text{Sum of units}}{\text{Number of years}}$$

PRACTICE QUESTIONS

Data: The table lists the share of students (in per cent) per total population who had at least one smartphone available at home and the share of students (in per cent) who bought a new smartphone after 2020 in the ASER survey of the year 2020 and 2021, categorised according to the level of education of their parents. Given that the total population of India is 1.5 billion.