OVERALL ANALYSIS Solution Report

All Correct Answers Wrong Answers Not Attempted Questions

Q.1) Max Marks: 1

The binary operation defined on a,b∈z such that a*b= min(a,b) then (A,*) is

A Monoid

B Group

C Algebraic-structure

D Semi group — Correct Option

Solution: (D)

Explanation:

For Checking if $(Z,*)$ is a semigroup we need to check 2 conditions

1) Z is closed under * ------ True

"A set is closed under some operation if applying the operation on *any* elements of the set gives an element which is still in that set".

2) $*$ is an associative operation -------- True

Let us assume a<b<c | a,b,c ∈Z

$(a*b)*c=a*(b*c)$

$a*c=a*b$

$a=a$

Hence it is Semi-group

A monoid is a semigroup with an identity.

There exist no identity for operation * (min) in set of integers because there is no minimum element in Z to satisfy.

Hence it is not a monoid

Q.2) Max Marks: 1

Find the number of elements in the indicated cyclic group. The cyclic subgroup of

$\mathbb{Z}_{42}$

generated by 30.

A 6

B 7 — Correct Option

Solution: (B)

Consider the group $\mathbb{Z}_{42}$ and its subgroup $\langle 30\rangle$.

Observe that in $\mathbb{Z}_{42}$, $1(30) = 30$, $2(30) = 18$, $3(30) = 6$, $4(30) = 36$, $5(30) = 24$, $6(30) = 12$ and $7(30) = 0$. Thus $\langle 30\rangle = \{0, 6, 12, 18, 24, 30, 36\}$ and therefore $|\langle 30\rangle| = 7$.

C 8

D 9

Q.3) Max Marks: 1

The power set of the set S = {Phi, {Phi}} has ____ elements.

Correct Answer

Solution: (4)

Answer: 4

Explanation:

Explanation:

DEFINITIONS

The **power set** of S is the set of all subsets of S.
Notation: $P(S)$

X is a **subset** of Y if every element of X is also an element of Y.
Notation: $X \subseteq Y$

The emptyset $\emptyset$ and the set $\{\emptyset, \{\emptyset\}\}$ itself are subsets of $\{\emptyset, \{\emptyset\}\}$.

All other subsets need to contain some elements from $\{\emptyset, \{\emptyset\}\}$ but not all, which are thus $\{\emptyset\}$ and $\{\{\emptyset\}\}$.

$$P(\{\emptyset, \{\emptyset\}\}) = \{\emptyset, \{\emptyset, \}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$$

Q.4)

Max Marks: 1

Consider the poset ({3,5,9,15,24,45},|).
Which of the following is correct for the given poset ?

A **There exist a greatest element and a least element**

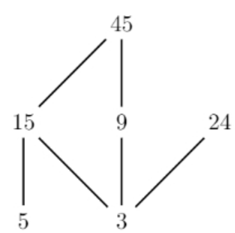
B **There exist a greatest element but not a least element**

C **There exist a least element but not a greatest element**

D **There does not exist a greatest element and a least element** Correct Option

Solution: (D)

Answer : D
Explanation:



There are two maximal elements 24 and 45.

There are two minimal elements 5 and 3.

So there is no greatest and least element.

∴ Option D is correct.

Q.5)

Max Marks: 1

A partially ordered set is said to be a lattice if every two elements in the set have

A **A unique least upper bound**

B **A unique greatest lower bound**

C **Both (A) and (B)** Correct Option

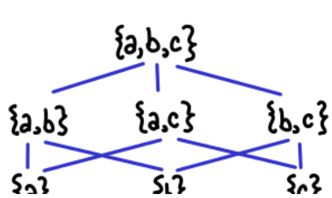
Solution: (C)

Answer: C

Explanation:

A partially ordered set is said to be a lattice if every two elements in the set have

1. A unique least upper bound
2. A unique greatest lower bound



D None of the above

Q.6) Max Marks: 1

The relation R = ∅ on the empty set S = ∅ :

A **reflexive, symmetric and transitive.** Correct Option

Solution: (A)

Answer: A

Explanation:

DEFINITIONS

A relation R on a set A is **reflexive** if $(a,a) \in R$ for every element $a \in A$.
A relation R on a set A is **symmetric** if $(b,a) \in R$ whenever $(a,b) \in R$
A relation R on a set A is **antisymmetric** if $(b,a) \in R$ and $(a,b) \in R$ implies $a = b$
A relation R on a set A is **transitive** if $(a,b) \in R$ and $(b,c) \in R$ implies $(a,c) \in R$

Given: The relation $R = \emptyset$ on the set $S = \emptyset$.

To proof: R is symmetric, transitive and reflexive.

PROOF

Symmetric

Since $(a,b) \in R$ is always false (as R is the empty set), the conditional statement $((a,b) \in R) \to B$ is true for any statement B.

Let B be the statement "$(b,a) \in R$". The conditional statement "If $(a,b) \in R$, then $(b,a) \in R$" is then always true and thus the relation R is symmetric by the definition of symmetric.

Transitive

Since $(a,b) \in R$ is always false (as R is the empty set) and since $(b,a) \in R$ is always false, the statement "$(a,b) \in R$ and $(b,a) \in R$" is also always false.

Then the conditional statement $((a,b) \in R \text{ and } (b,a) \in R) \to B$ is true for any statement B.

Let B be the statement "$a = b$". The conditional statement "If $(a,b) \in R$ and $(b,a) \in R$, then $a = b$" is then always true and thus the relation R is transitive by the definition of transitive.

Reflexive

The definition of a reflexive relation is equivalent with: If $a \in A$, then $(a,a) \in R$

Since $a \in S$ is always false (as S is the empty set), the conditional statement $(a \in S) \to B$ is true for any statement B.

Let B be the statement "$(a,a) \in R$". The conditional statement "If $a \in S$, then $(a,a) \in R$" is then always true and thus the relation R is reflexive by the definition of reflexive .

$\square$

B **symmetric and transitive only.**

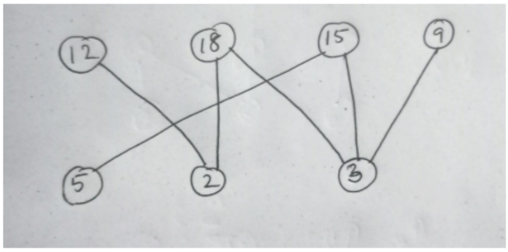
C **reflexive and transitive only.**

D **reflexive only.**

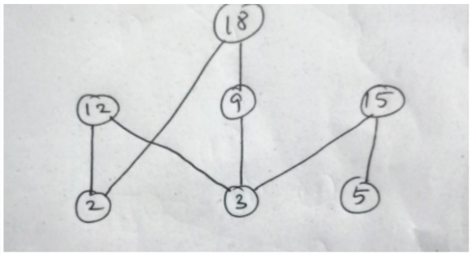
Q.7) Max Marks: 1

Which of the following is the correct Hasse diagrams of the divisibility relation on the set {2, 3, 5, 9, 12, 15, 18}.

A



B Correct Option



Solution: (B)

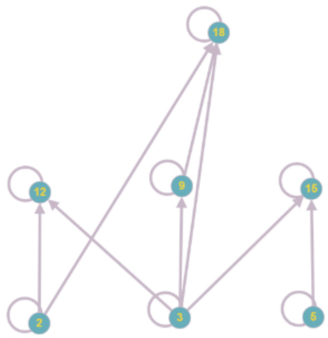
Answer: B

Explanation:

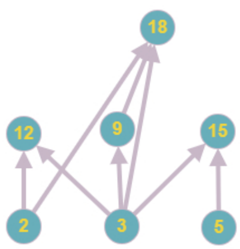
$$A = \{1, 2, 3, 4, 5, 6\}$$

$$R = \text{"Divisibility"}$$

We first draw the **directed graph** corresponding to the relation R.
Note that the directed graph R needs to contain loops at every vertex, because an element is always divisible by itself.
We need to draw an arrow from any integer a to another integer b, when a is divisible by b.

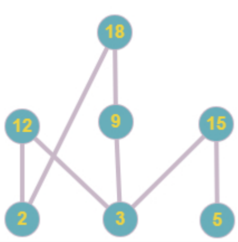


Next, we **remove all loops** from the diagram (because R is a partial order and since loops are always present in a partial order (reflexive), we do not have to shown them).

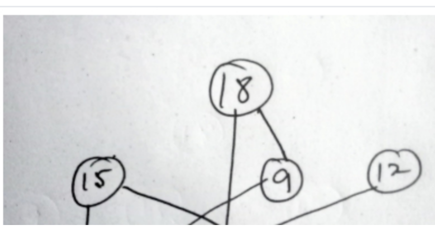


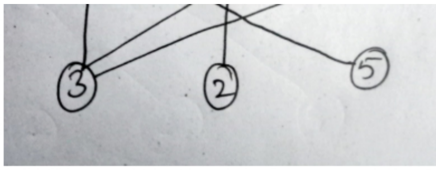
Finally, a partial order is also transitive and thus we do not have to show edges can be derived due to transitivity.
We make sure that the initial vertex is below the terminal vertex.
We remove any arrows on the directed edges (since all edges must point "upward").

The **Hassle diagram** is then:



c





D None of the above

Q.8)

Max Marks: 1

For the poset ({3, 5, 9, 15, 24, 45}, |), which of the following statement(s) is/are incorrect?

S1: The maximal elements are 24, 45

S2: The minimal elements are 3, 5

S3: The greatest element exists

S4: The least element doesn't exist

A S1 and S2

B S3 Correct Option

Solution: (B)

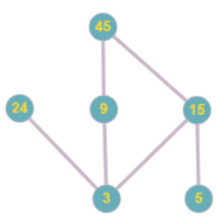
Answer: B

Explanation:

Given:

$$(\{3, 5, 9, 15, 24, 45\}, |)$$
$$S = \{3, 5, 9, 15, 24, 45\}$$
$$R = \{(a, b) | a \text{ divides } b\}$$

Let us first determine the **Hasse diagram**



(a) The **maximal elements** are all values in the Hasse diagram that do not have any elements above it.

$$\textbf{maximal elements} = 24, 45$$

(b) The **minimal elements** are all values in the Hasse diagram that do not have any elements below it.

$$\textbf{minimal elements} = 3, 5$$

(c) The **greatest element** only exists is there is exactly one maximal element and is then also equal to that maximal element.

$$\textbf{greatest element} = \text{Does not exist}$$

(d) The **least element** only exists is there is exactly one minimal element and is then also equal to that minimal element.

$$\textbf{least element} = \text{Does not exist}$$

C S4

D S2

Q.9)

Max Marks: 1

Consider the set {4,8,12,16}.
Which of the following is true?
S1: This set is a group under multiplication modulo 20.
S2: The identity element exists.
S3: The group is cyclic with generators as 8 and 13.

A S1, S2 and S3

B S1 and S2 Correct Option

Solution: (B)

Answer: B

Explanation:

Here is the Cayley table for set $S = \{4, 8, 12, 16\}$ under multiplication modulo 20:

	4	8	12	16
4	16	12	8	4
8	12	4	16	8
12	8	16	4	12
16	4	8	12	16

We observe that the identity is element 16. Let's list all its cyclic subgroups to check whether it is cyclic:

- $\langle 16 \rangle = \{16\}$
- $\langle 4 \rangle = \{16, 4\}$
- $\langle 8 \rangle = \{16, 8, 4, 12\} = S$
- $\langle 12 \rangle = \{16, 12, 4, 8\} = S$

We conclude that S is indeed cyclic, with generators 8 and 12.

C S3

D None of the above

Q.10)

Max Marks: 1

Let $f : A \to B$ be a function and S and T be subsets of B. Consider the following statements about image (range) :

$$S1: \quad f^{-1}(S \cup T) = f^{-1}(S) \cup f^{-1}(T)$$
$$S2: \quad f^{-1}(S \cap T) = f^{-1}(S) \cap f^{-1}(T)$$

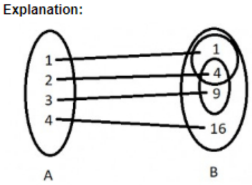
A Only S1 is true

B Only S2 is true

C Both S1 and S2 is true — Correct Option

Solution: (C)

Explanation:



A = {1,2,3,4}
B = {1,4,9,16}
S = {1,4}
T = {4,9}

for S1: $f^{-1}(S \cup T) = f^{-1}(S) \cup f^{-1}(T)$

$\Rightarrow f^{-1}(1,4,9) = f^{-1}(1,4) \cup f^{-1}(4,9)$
$\Rightarrow \{1,2,3\} = \{1,2\} \cup \{2,3\}$

$\Rightarrow \{1,2,3\} = \{1,2,3\}$

hence S1 is true

for S2: $f^{-1}(S \cap T) = f^{-1}(S) \cap f^{-1}(T)$

$\Rightarrow f^{-1}(4) = f^{-1}(1,4) \cap f^{-1}(4,9)$

$\Rightarrow \{2\} = \{1,2\} \cap \{2,3\}$
$\Rightarrow \{2\} = \{2\}$
hence S2 is true

D Neither S1 nor S2 is true

Q.11)

Max Marks: 2

Which of these functions is a bijection from R to R.

A f (x) = −3x + 4 — Correct Option

Solution: (A)

Answer: A

Explanation:

Part a -
$f(x) = -3x + 4$ is a bijection.
It is one-to-one as $f(x) = f(y) \Rightarrow -3x + 4 = -3y + 4 \Rightarrow x = y$
It is onto as $f(\frac{4-x}{3}) = x$

Part b -
$f(-x) = f(x)$. Hence the function is not a bijection.

Part c -
There is no real number x such that $f(x) = \frac{x+1}{x+2} = 1$. Hence the function is not a bijection.

Part d -

It is incorrect as the correct answer is Option A

B f (x) = $-3x^2 + 7$

C f (x) = (x + 1)/(x + 2)

D None of the above

Q.12)

Max Marks: 2

Given f : Z × Z → Z.

Determine which of the following is not onto

A f (m, n) = 2m − n.

B f (m, n) = $m^2 - n^2$.

Correct Option

Solution: (B)

Answer: B

Explanation:

DEFINITIONS

A **function** f from A to B has the property that each element of A has been assigned to exactly one element of B.

The function f is **onto** if and only if for every element $b \in B$ there exist an element $a \in A$ such that $f(a) = b$.

$$A = \mathbf{Z} \times \mathbf{Z}$$
$$B = \mathbf{Z}$$

(a) Given: $f(m, n) = 2m - n$

The function f is onto, because for every integer $x \in \mathbf{Z}$, the pair $(0, -x) \in \mathbf{Z} \times \mathbf{Z}$ has image x.

$$f(0, -x) = 0 - (-x) = 0 + x = x$$

(b) Given: $f(m, n) = m^2 - n^2$

Perfect squares x^2: $0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, \ldots$

The function f is not onto, because you cannot write 2 as the difference of perfect squares (we note that the difference between two perfect squares in the above list is either 1 or larger than 2).

(c) Given: $f(m, n) = m + n + 1$

The function f is onto, because for every integer $x \in \mathbf{Z}$, the pair $(0, x - 1) \in \mathbf{Z} \times \mathbf{Z}$ has image x.

$$f(0, x - 1) = 0 + (x - 1) + 1 = x - 1 + 1 = x$$

(d) Given: $f(m, n) = |m| - |n|$

For every integer $x \in \mathbf{Z}$ with x nonnegative, the pair $(x, 0) \in \mathbf{Z} \times \mathbf{Z}$ has image x.

$$f(x, 0) = |x| - |0| = x - 0 = x$$

and for every integer $x \in \mathbf{Z}$ with x negative, the pair $(0, x) \in \mathbf{Z} \times \mathbf{Z}$ has image x.

$$f(0, x) = |0| - |x| = 0 - (-x) = 0 + x = x$$

Thus for all integers $x \in \mathbf{Z}$, there exists an element $(a, b) \in \mathbf{Z} \times \mathbf{Z}$ such that $f(a, b) = x$ and thus the function f is onto.

C f (m, n) = m + n + 1.

D f (m, n) = |m| − |n|.

Q.13) Max Marks: 2

Which of the following is not a POSET?

Is (S, R) a poset if S is the set of all people in the world and (a, b) ∈ R, where a and b are people,

A **if a is taller than b?** Correct Option

Solution: (A)

Answer: A

Explanation:

DEFINITIONS

A relation R on a set S is a **partial ordering** if the relation R is reflexive, antisymmetric and transitive.
(S, R) is then called a **poset**.

A relation R on a set A is **reflexive** if $(a, a) \in R$ for every element $a \in A$.
A relation R on a set A is **antisymmetric** if $(b, a) \in R$ and $(a, b) \in R$ implies $a = b$
A relation R on a set A is **transitive** if $(a, b) \in R$ and $(b, c) \in R$ implies $(a, c) \in R$

$$S = \text{Set of all people in the world}$$

(a)

$$R = \{(a, b) | a \text{ is taller than } b\}$$

R is **not reflexive**, because an individual is not taller than themself.

R is **antisymmetric**, because $(a, b) \in R$ and $(b, a) \in R$ cannot both be true at the same time (if a is taller than b, then b cannot be taller than a).

R is **transitive**, because if $(a, b) \in R$ and $(b, c) \in R$, then a is taller than b and b is taller than c, which implies that a is taller than c and thus $(a, c) \in R$.

(R, S) is **not a poset**, because R is not reflexive.

(b)

$$R = \{(a, b) | a \text{ is not taller than } b\}$$

R is **reflexive**, because an individual is not taller than themself.

R is **antisymmetric**, because $(a, b) \in R$ and $(b, a) \in R$ implies a is not taller than b and b is not taller than a. Both statements can only be true if $a = b$.

R is **transitive**, because if $(a, b) \in R$ and $(b, c) \in R$, then a is not taller than b and b is not taller than c, which implies that a is not taller than c and thus $(a, c) \in R$

(R, S) is **a poset**, because R is reflexive, antisymmetric and transitive.

(c)

$$R = \{(a, b) | a = b \text{ or } a \text{ is an ancestor of } b\}$$

R is **reflexive**, because all elements with $a = b$ are included in the relation R.

R is **antisymmetric**, because $(a, b) \in R$ and $(b, a) \in R$ implies (a is an ancestor of b or $a = b$) and (b is an ancestor of a or $a = b$). a cannot be an ancestor of b when b is an ancestor of a, thus the statements can only be true if $a = b$.

R is **transitive**, because if $(a, b) \in R$ and $(b, c) \in R$, then(a is an ancestor of b or $a = b$) and (b is an ancestor of c or $b = c$), which implies that (a is an ancestor of c or $a = c$) and thus $(a, c) \in R$

(R, S) is **a poset**, because R is reflexive, antisymmetric and transitive.

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d) False as option a is the right answer.

B **a is not taller than b?**

C **a = b or a is an ancestor of b?**

D **None of the above**

Q.14)

Let A and B be the multisets {3 • a, 2 • b, 1 • c} and {2 • a, 3 • b, 4 • d}, respectively.
Which of the following is incorrect?

Max Marks: 2

A. A ∪ B = {3.a, 3.b, 1.c, 4.d}

B. A ∩ B = {2.a, 2.b}

C. **A − B = {1.a, 1.c}**

D. **B − A = {1.b, 3.d}** Correct Option

Solution: (D)

Answer: D

Explanation:

$a)\ A \cup B = \{3.a, 3.b, 1.c, 4.d\}$

In $A \cup B$, the element with the highest multiplicity is chosen from A and B. For example, a has multiplicities 2 and 3, so $A \cup B$ contains $3.a$.

$b)\ A \cap B = \{2.a, 2.b\}$

In $A \cap B$, the element with the minimum multiplicity is chosen from A and B. In case an element does not belong to both the sets A and B, its minimum multiplicity is considered to be zero. So, it does not appear in the intersection. For example, a has multiplicities 2 and 3, so $A \cap B$ contains $2.a$. Also, c does not belong to B, so it does not appear in $A \cap B$.

$c)\ A - B = \{1.a, 1.c\}.$

Elements in $A - B$ are the elements of A with multiplicities equal to its multiplicity in A minus its multiplicity in B, provided it is positive, otherwise it is taken to be zero. For example, with this method, multiplicity of b came out to be -1 so we took it as zero.

$d)\ B - A = \{1.b, 4.d\}.$

Therefore, the answer is Option D

Q.15)

Which of these relations on the set of all functions from Z to Z are equivalence relations?

Max Marks: 2

a) {(f, g) | f(1) = g(1)}

b) {(f, g) | f(0) = g(0) or f(1) = g(1)}

c) {(f, g) | f(x) − g(x) = 1 for all x ∈ Z}

d) {(f, g) | for some C ∈ Z, for all x ∈ Z, f(x) − g(x) = C}

e) {(f, g) | f(0) = g(1) and f(1) = g(0)}

A. **a and b**

B. **a, b and c**

C. **a and d** Correct Option

Solution: (C)

Answer: C

Explanation:

In order to determine if a relation is an equivalent relation, we need to see if said relation is reflexive, symmetric and transitive. If it is, then it's an equivalent relation; otherwise, it isn't.

The relation is:

Reflexive if every element is related to itself. $(\forall x \in A : x \rightarrow x)$

Symmetric if for all x and y elements in a set, if x is related to y then y is related to x. $(\forall x, y \in A :$ if $x \rightarrow y$ then $y \rightarrow x)$

Transitive if whenever an element x is related to an element y, and y is related to an element z, then x is also related to z. $(\forall x, y, z \in A :$ if $x \rightarrow y$ and $y \rightarrow z$ then $x \rightarrow z)$

(a) is clearly reflexive, symmetric and transitive (since everything is equal to everything).

(d) is reflexive, because $f(x) - f(x) = 0 = C, C = 0 \in Z$. Is symmetric, because if $f(x)$ *and* $g(x) \in Z$ *then* $f(x) - g(x) = C_1$ *and* $g(x) - f(x) = C_2, C_1$ *and* $C_2 \in Z$. Is also transitive because $f(x) - g(x) = C_1,\ g(x) - h(x) = C_2$ *and* $f(x) - h(x) = C_3; C_1,\ C_2$ *and* $C_3 \in Z$.

Thus, only (a) and (d) are equivalence relations.

(b) is not transitive. Note that $f(0) = g(0)$ OR $f(1) = g(1)$ (the OR means that meeting one of the equalities is enough). So, we can have that $f(0) = g(0)$ and $g(1) = h(1)$ but $f(0) \neq h(0)$ and $f(1) \neq h(1)$.

(c) R is not reflexive, not symmetric and not transitive. See that $f(x) - f(x) = 0 \neq 1$, thus, R is not reflexive. Also, if $f(x) - g(x) = 1$ then $g(x) - f(x)$ cannot be equal to 1, thus, R is not symmetric. Furthermore, if $f(x) - g(x) = 1$ and $g(x) - h(x) = 1$ then $f(x) - h(x) = 2 \neq 1$, thus, R is not transitive.

(e) Is clearly neither reflexive nor transitive. See that $f(0)$ might not be equal to $f(1)$, thus, R is not reflexive. Also see that if fRg *and* gRh, then $f(0) = g(1) = h(0)$ and $f(1) = g(0) = h(1)$; looking at this we know that $f(0) = h(0)$ *and* $f(1) = h(1)$, which does not necessarily mean that fRh, that should be $f(0) = h(1)$ *and* $f(1) = h(0)$; thus, R is not transitive.

D

a only

close