MISSION ISRO CS & IT ENGINEERING

Theory of Computation

Lecture No.- 01

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Topics to be Covered

Topic

Regular Languages

- → Regular Expressions
  - $R_1 + R_2$
  - $R_1 \cdot R_2$
  - R^*
  - R^+
- → FA, Regular Grammars

$+$: OR

$a + b \Rightarrow a , b$

$a + \varepsilon \Rightarrow a , \varepsilon$

$a + \phi \Rightarrow a$

$\varepsilon + \phi \Rightarrow \varepsilon$

- (Concatenation) :

$$a . b \Rightarrow ab$$

$$a . \varepsilon \Rightarrow a$$

$$\varepsilon . \varepsilon \Rightarrow \varepsilon$$

$$\varepsilon . a \Rightarrow a$$

$$a . \phi \Rightarrow \phi$$

$$\phi . a \Rightarrow \phi$$

R^*

Kleene star of R

Kleene closure of R

$x^4 : xxxx$

$(ab)^4 : abababab$

$a^* \Rightarrow a^0, a^1, a^2, \ldots$

$\varepsilon, a, aa, \ldots$

$(ab)^* \Rightarrow \varepsilon, ab, abab, \underline{ab}\,\underline{ab}\,\underline{ab}, \ldots$

R^+

Kleene plus of R

Positive closure of R

$$\boxed{\begin{aligned} \varepsilon\varepsilon &= \varepsilon \\ \varepsilon^* &= \varepsilon \\ \varepsilon^+ &= \varepsilon \end{aligned}}$$

$a^+ \Rightarrow a^1, a^2, a^3, \ldots$

$a, aa, aaa, \ldots$

$(ab)^+ \Rightarrow ab, abab, ababab, \ldots$

$\varepsilon^+ \Rightarrow \varepsilon^1, \varepsilon^2, \varepsilon^3, \ldots$

$= \varepsilon$

$\boxed{R^0 = \varepsilon}$

$\phi^0 = \varepsilon$

$a^0 = \varepsilon$

$\varepsilon^0 = \varepsilon$

$b^0 = \varepsilon$

$(ab)^0 = \varepsilon$

$\phi^1 = \phi$

$a^1 = a$

$\varepsilon^1 = \varepsilon$

$(ab)^1 = ab$

$$\boxed{\begin{array}{l} R^1 = R \\ R^2 = R^1 R^1 = RR \end{array}}$$

$$\phi^* = \phi^0, \phi^1, \phi^2, \phi^3, \ldots = \varepsilon + \phi$$

$$= \varepsilon$$

$$\varepsilon, \phi$$

$$\boxed{\phi^* = \varepsilon}$$

$$\boxed{\phi^+ = \phi}$$

$L = \{w \in \{a, b\}^* : \#_a(w) \le 3\}$.

A. $b^* (a \cup \varepsilon) b^* (a \cup \varepsilon) b^* (a \cup \varepsilon) b^*$
B. $b^* (a) b^* (a \cup \varepsilon) b^* (a \cup \varepsilon) b^*$
C. $b^* (a) b^* (a) b^* (a \cup \varepsilon) b^*$
D. $b^* (a \cup \varepsilon) b^* (a) b^* (a) b^*$

$n_a = 0$
$= 1$
$= 2$
$= 3$

b^*
$b^* a b^*$
$b^* a b^* a b^*$
$b^* a b^* a b^* a b^*$

$b^*(a+\varepsilon)\, b^*(a+\varepsilon)\, b^*\, (a+\varepsilon) b^*$

$b^* \;\varepsilon\; b^* \;\varepsilon\; b^* \;\varepsilon\; b^*$

$b^* \;a\; b^* \;\varepsilon\; b^* \;\varepsilon\; b^*$

Slide 4

$L = \{w \in \{a, b\}^* : \#_a(w) \leq 3\}$.

A. $b^* (a \cup \varepsilon) b^* (a \cup \varepsilon) b^* (a \cup \varepsilon) b^*$

B. $b^* \, a \, b^* (a \cup \varepsilon) b^* (a \cup \varepsilon) b^*$ → $1 \leq n_a(w) \leq 3$

C. $b^* \, a \, b^* \, a \, b^* (a \cup \varepsilon) b^*$ → $2 \leq n_a(w) \leq 3$

D. $b^* (a \cup \varepsilon) b^* \, a \, b^* \, a \, b^*$ → $2 \leq n_a(w) \leq 3$

$\underbrace{a+\varepsilon}_{\text{zero or 1 } a}$

$b^* b^* = b^*$

$b^* \varepsilon = b^*$

$\underbrace{b^* a b^*}$ → $n_a(w) = 1$

$b^0 a b^0 \to a$

$b a \varepsilon \to ba$

$\varepsilon a b \to ab$

⋮

$L1 = a^*b^*$
$L2 = a^+b^+$

Find L2-L1.

A. a^*
B. b^*
C. $a^* + b^*$
D. None

$$L_1 = a^*b^* \rightarrow \varepsilon,\ a^+,\ b^+,\ a^+b^+ = L_2$$

$$L_1 - L_2 = \varepsilon + a^+ + b^+ = a^* + b^*$$

$$L_2 - L_1 = \phi$$

$$a^+b^+ - (\varepsilon, a^+, b^+, a^+b^+) = \phi$$

L1 = a*+b* and L2 = a*b*.
Which of the following is TRUE?

A. L1 = L2
B. L1 U L2 =(a+b)*
C. L1* = L2*
D. None

$L_1 \cup L_2 = L_2$

$L_1 = a^* + b^*$

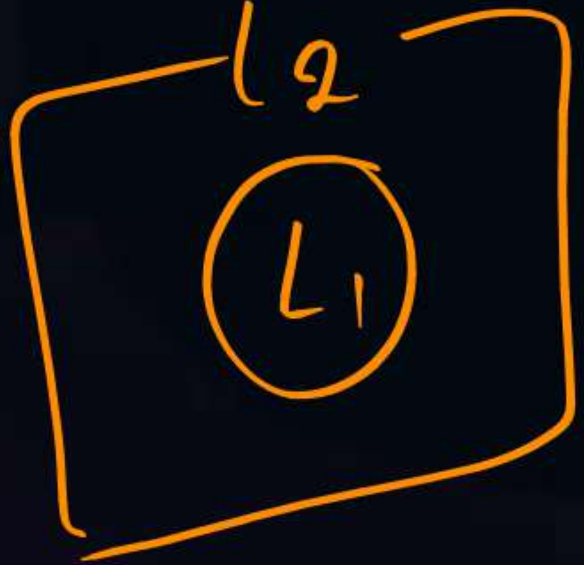
$L_2 = a^* b^* = \varepsilon + \overbrace{a^+ + b^+} + a^+ b^+$

$= \underbrace{a^*} + b^* + a^+ b^+$

$= L_1 + a^+ b^+$

$L_1^* = (a^* + b^*)^* = (a+b)^*$

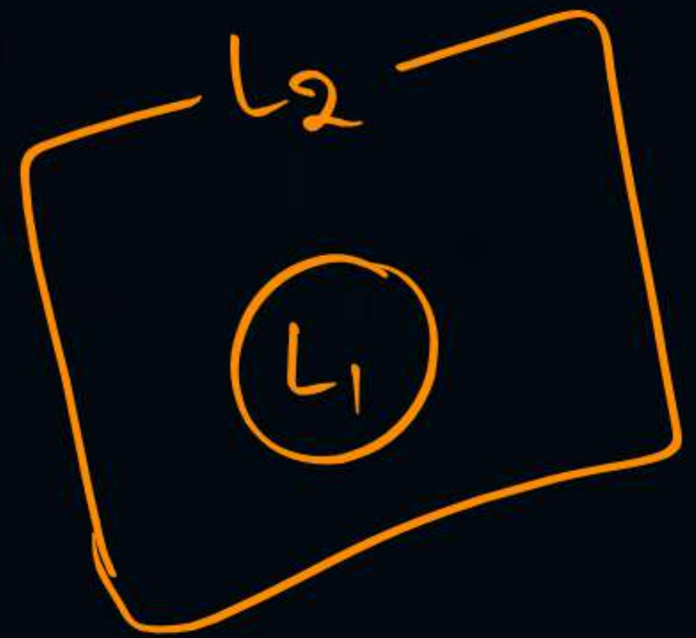
$L_2^* = (a^* b^*)^* = (a+b)^*$



Slide 6

$L1 = a^*+b^*$ and $L2 = a^*b^*$.
Which of the following is TRUE?

A. L1 is subset of L2
B. L2 is subset of L1
C. L1 U L2 = L1
D. None



$L1 = a^+b^+$ and $L2 = a^*b^*$.

Which of the following is FALSE?

A. L1 is subset of L2 $\rightarrow$ T

B. $L1^* = L2^*$ $\rightarrow$ FALSE

C. L1 U L2 = L2 $\rightarrow$ T

D. None

$$L_2 = a^* b^*$$

$$a^*b^* \rightarrow \varepsilon, \quad a^+, \quad b^+, \quad \underbrace{a^+b^+}_{L_1}$$

$$(a^+b^+)^* \neq \underbrace{(a^*b^*)^*}_{(a+b)^*}$$

$$a^+b^+ \rightarrow \varepsilon$$

$$\cancel{\phi}$$

$$\underbrace{a^+ b^+}_{\text{Atleast 1a followed by Atleast 1b}} \neq a^* b^*$$

$$(a^+ b^+)^* \neq (a^* b^*)^*$$

$L1 = a^+ + b^+$ and $L2 = a^* + b^*$.
Which of the following is TRUE?

A. $L1 = L2$
B. $L1^+ = L2^+$
C. $L1 \cup L2 = L2$ ✓
D. None

$L_1 = a^+ + b^+$

$L_2 = a^* + b^* = \varepsilon + L_1$

$L_1^+ = (a^+ + b^+)^+ = (a+b)^+$

$L_2^+ = (a^* + b^*)^+ = (a+b)^*$

L1 = a^+ and L2 = a^*

Which of the following is TRUE?

~~A.~~ $L1^+ = L2^*$ $\quad a^+ \neq a^*$

~~B.~~ $L1^+ = L2^+$ $\quad a^+ \neq a^*$

C. $L1^* = L2^+$ $\quad a^* = a^*$

D. None

$L_1 = a^+$

$L_2 = a^*$

$L_1^+ = (a^+)^+ = a^+$

$L_2^+ = (a^*)^+ = a^*$

$L_1^* = (a^+)^* = a^*$ | $L_2^* = (a^*)^* = a^*$

Slide 10

$$a^+ + b^+ = \{\underbrace{a, a^2, a^3, a^4, \dots}_{a^+}, \underbrace{b, b^2, b^3, \dots}_{b^+}\}$$

$$a^+ \cup b^+$$

$$a \quad a^2 \quad a^3 \dots \qquad b^1 \quad b^2 \quad b^3 \dots$$

$A = a^*$ and $B = b^*$

$AB = ?$

A. ~~{ $a^n b^n$ | n>=0}~~

B. { $a^m b^n$ | m, n>=0}

C. $(a+b)^*$

D. None

$A = a^* = \{a^n \mid n \geq 0\}$

$B = b^* = \{b^n \mid n \geq 0\}$

$A \cdot B = \{a^n \mid n \geq 0\} \cdot \{b^n \mid n \geq 0\}$

$= \{a^{n_1} b^{n_2} \mid n_1, n_2 \geq 0\}$

$= \{a^m b^n \mid m, n \geq 0\}$

$A = aa^*$ and $B = bb^*$

$(AUB)^* = ?$

A. $\{ a^n b^n \mid n>=0\}$

B. $\{ a^m b^n \mid m, n>=0\}$

C. $(a+b)^*$

D. None

$A = aa^* = a^+$

$B = bb^* = b^+$

$(A+B)^* = (a^+ + b^+)^*$

$= (a+b)^*$

Slide 12

Given the language L = {ab, aa, baa},
which of the following strings are not in L*?

1) abaabaaabaa $\in L.L.L.L.L$
2) aaaabaaaa $\in L.L.L.L.$
3) baaaaabaaaa$\boxed{b} \rightarrow \times$

A. 1 only
B. 2 only
C. 3 only
D. None

$L = \{ab, aa, baa\}$
$= ab + aa + baa$

$L^* = (ab + aa + baa)^*$
$= (x + y + z)^*$

The length of the shortest string NOT in the language (over $\Sigma = \{a, b\}$) of the following regular expression is ________________.

$a^*(ba)^*a^*$ → ε ✓, a ✓, $\boxed{b}$ ×

A. 1 ✓

B. 2

C. 3

D. 4

Shortest which is not possible

$\{\varepsilon, a, \boxed{b}, \ldots\}$

✓ ✓ ✗

L = a(a+b)* is equivalent to ________

A. $(ab^*)^+$

B. $(a^+b^*)^+$

C. $a^*(ab^*)^+$

~~D.~~ All of the above

$\{a, \underbrace{aa, ab}_{a\square}, \underbrace{\qquad}_{a\square\square}$

$\underbrace{a\,\Sigma^*}$

Slide 15

$L = (a+b)^*b$ is equivalent to ________

A. $(ab^*)^+ \longrightarrow a\Sigma^*$

B. $(a^+b^*)^+$

~~C.~~ $\boxed{b^*(ab^*)^*}\,b$

D. None

$= (a+b)^* b$

$= \{ b, \square b, \square\square b, \square\square\square b, \ldots \}$

a or b

.... $\boxed{b}$

$$(ab^*)^+ \Rightarrow a\Sigma^*$$

$$(ab^*)^* \Rightarrow a\Sigma^* + \varepsilon$$

$$b^*(ab^*)^* \Rightarrow \underbrace{a\Sigma^* + b\Sigma^* + \varepsilon}_{(a+b)^*}$$

$(b + ba)(b + a)^*(ab + b)$

A. $(a+b)^*$

B. $a(a+b)^*a$

C. $b(a+b)^*b$

D. None

$= b(\varepsilon + a)(a+b)^*(a+\varepsilon)b$

$= b(a+b)^*b$

every

$\{w \in \{a, b\}^* : \#_a(w) \equiv_3 0\}$.

A. $(b^*ab^*ab^*a)^*b^*$

B. $(b^*ab^*ab^*a)^*$ → b ×

C. $(ab^*ab^*a)^*$

D. $(ab^*ab^*a)^*b^*$ → baaa ×

Subset of answer

$\left(a \; a \; a \right)^*$

$(a \cup b)^* (a \cup \varepsilon) b^* =$

A. $(a+b)(a+b)^*$ $= (a+b)^+$

B. $(a+b)^*$ ✓

C. $(aa+b)^*$ $= (a^2+b)^*$

D. None

$$\underbrace{(a+b)^* (a+\varepsilon)}_{} b^*$$

$$\underbrace{(a+b)^* b^*}_{}$$

$$(a+b)^*$$

even

L={$w \in \{a, b\}^*$ | w has bba as a substring}
Which of the following describes L ?

A. $(a \cup b)^* \boxed{bba} (a \cup b)^*$
B. $(a \cup b)^+ \boxed{bba} (a \cup b)^*$
C. $(a \cup b)^+ \boxed{bba} (a \cup b)^+$
D. $(a \cup b)^* \boxed{bba} (a \cup b)^+$

Σ^* bba Σ^*

$L = \{w \in \{a, b\}^*\}$ $= (a+b)^*$

1. $(a + b)^*$
2. $(a + b + \text{epsilon})^+$
3. Epsilon + $(a + b)^+$
4. $(a^*b^*)^*$
5. $(b^*a^*)^*$
6. $(a^+b^+)^*$

How many of above are equivalent to given L ?

A. 4 B. 5 C. 6 D. 3

$\{w \mid w \in \{a,b\}^*\}$

$\{w \in \{a,b\}^*\}$

$\{w : w \in \{a,b\}^*\}$

every w in $(a+b)^*$

$a^*b^* \neq b^*a^*$

$(a^*b^*)^* = (b^*a^*)^*$

Slide 21

Which Two of the following four regular expressions are equivalent?

(i) $(00)^*(\varepsilon + 0)$ (ii) $(00)^*$ even (iii) 0^* All (iv) $0\,(00)^*$ odd

(a) (i) and (ii) (b) (ii) and (iii)

~~(c)~~ (i) and (iii) (d) (iii) and (iv)

(GATE - 96)

$$(00)^*(\varepsilon + 0)$$

$$\underbrace{(00)^*}_{\text{even}} + \underbrace{(00)^*0}_{\text{odd}} = \underbrace{0^*}_{\text{All}}$$

Slide 22

If the regular set A is represented by $A = (01+1)^*$ and the regular set 'B' is represented by $B = ((01)^*1^*)^*$, which of the following is true? **(GATE - 98)**

(a) $A \subset B$

(b) $B \subset A$

(c) A and B are incomparable

(d) $A = B$ ✓

$A = B$ ✓ $\quad A \supset B$ ✗

$A \subseteq B$ ✓

$A \supseteq B$ ✓

$A \subset B$ ✗

$A = (01+1)^* = (x+y)^*$, $x \rightarrow 01$, $y \rightarrow 1$

$B = ((01)^*1^*)^* = (x^*y^*)^*$

$\boxed{A = B}$

$\boxed{\text{The string 1101}}$ does not belong to the set represented by **(GATE - 98)**

1101 ∈ (a) $110^*(0+1)$ 1101 ∈ (b) $1(0+1)^*101$

1101 ∉ (c) $(10)^*(01)^*(00+11)^*$ (d) $(00+(11)^*0)^*$ 1101 ∉

c ✓

d ✓

Slide 24

Let S and T be languages over $\Sigma = \{a, b\}$ represented by the regular expressions $(a + b^*)^*$ and $(a + b)^*$, respectively.

Which of the following is true? **(GATE - 2000)**

(a) $S \subset T$

(b) $T \subset S$

(c) $S = T$

(d) $S \cap T = \phi$

Consider the set Σ^* of all strings over the alphabet $\Sigma = \{0, 1\}$.
Σ^* with the concatenation operator for strings **(GATE - 03)**

(a) Does not form a group

(b) Forms a non-commutative group

(c) Does not have a right identity element

(d) Forms a group if the empty string is removed from Σ^*

The regular expression $0^*(10^*)^*$ denotes the same set as **(GATE - 03)**

(a) $(1^*0)^*1^*$

(b) $0+(0+10)^*$

(c) $(0+1)^*10(0+1)^*$

(d) None of the above

Which one of the following languages over the alphabet $\{0, 1\}$ is described by the regular expression $(0+1)^*0(0+1)^*0(0+1)^*$ **(GATE - 09)**

(a) The set of all strings containing the substring 00

(b) The set of all strings containing at most two 0's

(c) The set of all strings containing at least two 0's

(d) The set of all strings that begin and end with either 0 or 1

Consider the languages $L_1 = \phi$ and $L_2 = \{a\}$. Which one of the following represents $L_1 L_2^* \cup L_1^*$? **(GATE - 13)**

(a) $\{\varepsilon\}$

(b) ϕ

(c) a^*

(d) $\{\varepsilon, a\}$

The length of the shortest string NOT in the language (over Σ = $\{a, b\}$) of the following regular expression is ________.

(GATE – 14-SET3)

$$a^*b^*(ba)^*a^*$$

Which one of the following regular expressions represents the language: the set of all binary strings having two consecutive 0's and two consecutive 1's? **(GATE – 16 – SET1)**

(a) $(0+1)^* 0011(0+1)^* + (0+1)^* 1100(0+1)^*$

(b) $(0+1)^* (00(0+1)^* 11 + 11(0+1)^* 00)(0+1)^*$

(c) $(0+1)^* 00(0+1)^* + (0+1)^* 11(0+1)^*$

(d) $00(0+1)^* 11 + 11(0+1)^* 00$

Which of the following regular expression identities are true?

(GATE - 92)

(a) $r(^*) = r^*$

(b) $(r^*s^*)^* = (r+s)^*$

(c) $(r+s)^* = r^* + s^*$

(d) $r^*s^* = r^* + s^*$

Which one of the following regular expressions represents the set of all binary strings with an odd number of $1'$s?

A. $((0+1)^*1(0+1)^*1)^*10^*$
B. $(0^*10^*10^*)^*0^*1$
C. $10^*(0^*10^*10^*)^*$
D. $(0^*10^*10^*)^*10^*$

Which one of the following regular expressions over $\{0, 1\}$ denotes the set of all strings **not** containing 100 as a substring?

(GATE - 97)

(a) $0^*(1^+0)^*$

(b) $0^*\ 1010^*$

(c) $0^*1^*01^*$

(d) $0^*\ (10+1)^*$

THANK - YOU

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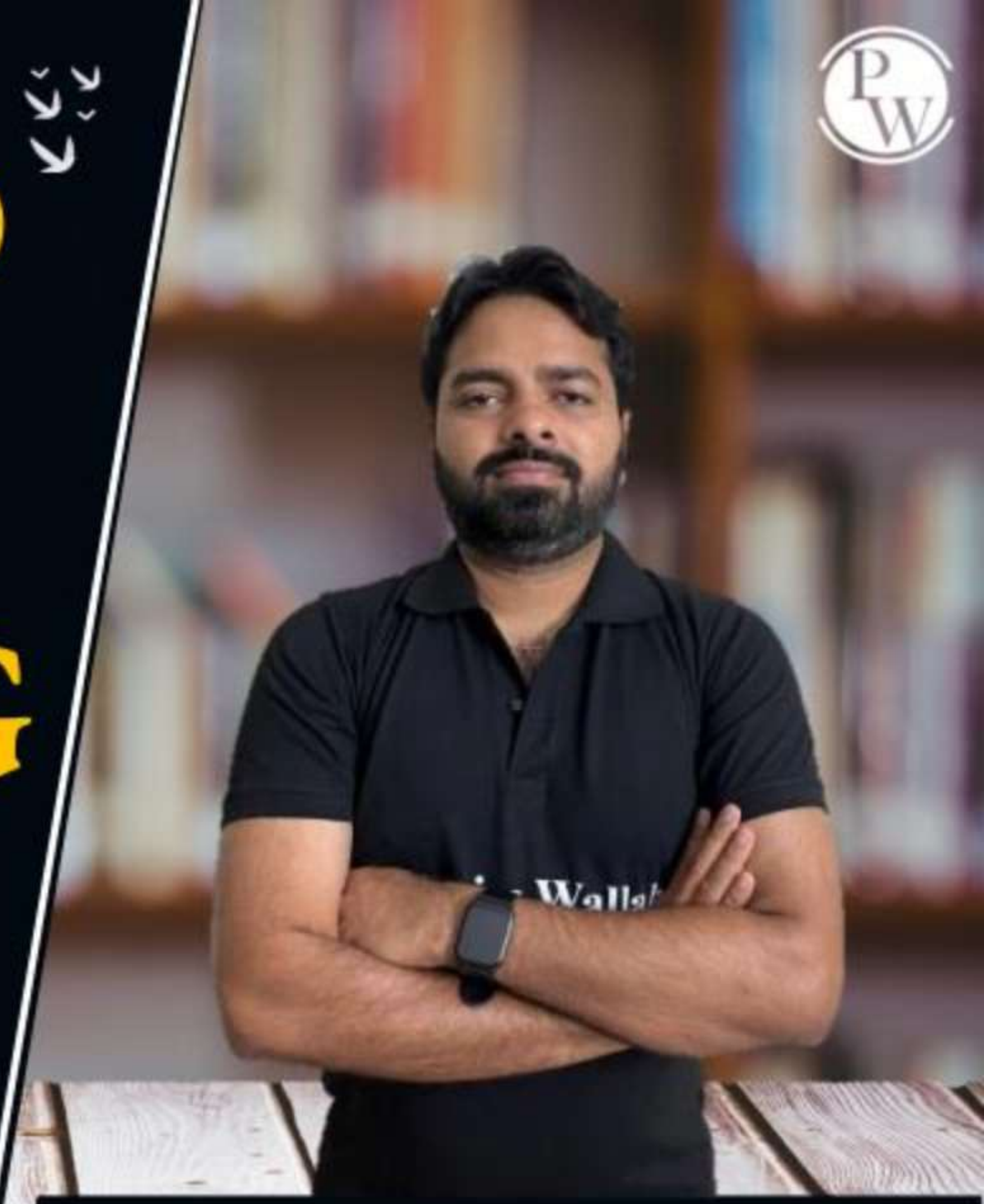
Theory of Computation

Lecture No.- 02



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Topics to be Covered

Topic

Regular Languages

- → Regular Expressions
- → FA
- → Regular Grammar
- → closure properties

Let S and T be languages over $\sum = \{a, b\}$ represented by the regular expressions $(a + b^*)^*$ and $(a + b)^*$, respectively. Which of the following is true? **(GATE - 2000)**

(a) $S \subset T$

(b) $T \subset S$

(c) $S = T$

(d) $S \cap T = \phi$

$S = (a+b^*)^* = T$

$T = (a+b)^*$

$S \cap T = S = T = (a+b)^*$

$S \subseteq T$

$T \subseteq S$

Slide 4

Consider the set Σ^* of all strings over the alphabet $\Sigma = \{0, 1\}$.
Σ^* with the concatenation operator for strings **(GATE - 03)**

(a) Does not form a group
(b) Forms a non-commutative group
(c) Does not have a right identity element
(d) Forms a group if the empty string is removed from Σ^*

(D, operation)

(Σ^*, Concatenation)

($\{\epsilon, 0, 1, 00, 01, 10, 11, \ldots\}$, $\cdot$)

Abelian group
group
Monoid

I) closed
II) Associative
III) Identity $= \epsilon$
IV) Inverse
V) Commutative

The regular expression $0^*(10^*)^*$ denotes the same set as **(GATE - 03)**

(a) $(1^*0)^*1^*$ (b) $0 + (0+10)^*$ — 1×

(c) $(0+1)^*10(0+1)^*$ — ε× (d) None of the above

$$0^*(10^*)^* = (0+1)^*$$
$$1^*(01^*)^* = \text{"}$$
$$(1^*0)^*1^* = \text{"}$$
$$(0^*1)^*0^* = \text{"}$$

Slide 6

Which one of the following languages over the alphabet $\{0, 1\}$ is described by the regular expression $(0+1)^*0(0+1)^*0(0+1)^*$ **(GATE - 09)**

(a) The set of all strings containing the substring 00

(b) The set of all strings containing at most two 0's

(c) The set of all strings containing at least two 0's

(d) The set of all strings that begin and end with either 0 or 1

$\underbrace{X\,0\,X\,0\,X}_{\text{Min 2 zero's}}$

Consider the languages $L_1 = \phi$ and $L_2 = \{a\}$. Which one of the following represents $L_1 L_2^* \cup L_1^*$? **(GATE - 13)**

(a) $\{\varepsilon\}$	(b) ϕ
(c) a^*	(d) $\{\varepsilon, a\}$

$L_1 . L_2^* \cup L_1^*$

$\underbrace{\phi . L_2^*}_{\phi} \cup \phi^*$

$\phi \cup \{\varepsilon\}$

$\{\varepsilon\}$

$\phi^* = \phi^0 \cup \phi^1 \cup \phi^2 \cup \ldots$

$\phi^0 \rightarrow \varepsilon$, $\phi^1 \rightarrow \phi$

$= \varepsilon + \phi$

$= \varepsilon$

Slide 8

The length of the shortest string NOT in the language (over Σ = {a, b}) of the following regular expression is __________.

a*b*(ba)*a* (GATE – 14-SET3)

$a^2 \varepsilon \varepsilon \varepsilon = a^2$

$a b \varepsilon \varepsilon = ab$

$\varepsilon \varepsilon ba \varepsilon = ba$

$\varepsilon b^2 \varepsilon \varepsilon = bb$

$a^3 \varepsilon \varepsilon \varepsilon = aaa$

$a^2 b \varepsilon \varepsilon = aab$

$a \varepsilon ba \varepsilon = aba$

$a b^2 \varepsilon \varepsilon = abb$

$\varepsilon \varepsilon ba\, a = baa$

bab

A) 2

~~B) 3~~

C) 4

D) 5

Which one of the following regular expressions represents the language: the set of all binary strings having two consecutive 0's and two consecutive 1's? **(GATE – 16 – SET1)**

(a) $(0+1)^* 0011(0+1)^* + (0+1)^* 1100(0+1)^*$

(b) $(0+1)^* (00(0+1)^* 11 + 11(0+1)^* 00)(0+1)^*$

(c) $(0+1)^* 00(0+1)^* + (0+1)^* 11(0+1)^*$

(d) $00(0+1)^* 11 + 11(0+1)^* 00$

$X\,0011\,X + X\,1100\,X \rightarrow$ contains 0011 or 1100

$X(00X11 + 11X00)X = \underbrace{X\,\underline{00}\,X\,\underline{11}\,X}_{} + \underbrace{X\,\underline{11}\,X\,\underline{00}\,X}_{}$

$X00X + X11X \rightarrow$ contains 00 or 11

$\underbrace{00X11}_{} + \underbrace{11X00}_{}$ OR

Starts with 00 and ends with 11

Starts with 11 and ends with 00

Which of the following regular expression identities are true?

(GATE - 92)

(a) $r(^*) = r^*$

(b) $(r^*s^*)^* = (r+s)^*$

(c) $(r+s)^* = r^* + s^*$

(d) $r^*s^* = r^* + s^*$

$\underbrace{(r+s)^*}_{\text{all}} \supseteq \underbrace{r^* + s^*}_{r^*, s^*}$

$\underbrace{r^{(*)}}_{\text{not wff}}$

$(r)^*$ ✓

(r^*) ✓

$r_1 + r_2$ ✓

$(r_1) + r_2$ ✓

$(r_1) + (r_2)$ ✓

$r_1 + (r_2)$ ✓

$(r_1 + r_2)$

$\cancel{r_1 (+) r_2}$ ✗

$(r_1 +) r_2$ ✗

Which one of the following regular expressions represents the set of all binary strings with an odd number of $1'$s?

~~A.~~ $((0+1)^*1(0+1)^*1)^*10^*$

~~B.~~ $(0^*10^*10^*)^*0^*1 \longrightarrow 10 \times$

~~C.~~ $10^*(0^*10^*10^*)^* \longrightarrow 01 \times$

~~D.~~ $(0^*10^*10^*)^*10^* \longrightarrow 01 \times$

No option is correct

$0^* 1 0^* (0^* 1 0^* 1 0^*)^* 0^*$ ✓

$0^* (0^* 1 0^* 1 0^*)^* 0^* 1 0^*$ ✓

Slide 12

Which one of the following regular expressions over {0, 1} denotes the set of all strings **not** containing 100 as a substring?

(GATE - 97)

(a) $0^*(1^+0)^*$ → 01 ×

(b) 0^*1010^* → ε ×

(c) $0^*1^*01^*$ → ε ×

(d) $0^*(10+1)^*$

***

L = { ε, 0, 1, 00, 01, 10, 11, 000, 001, 010, 011, 101, 110, 111, ... }

not having 100

not contain 100 but generates only some strings

$0^* 1$ some

101 some

$(11)^* 1$ some

$\boxed{(0+1)^* 1}$ all

ends with 1

Q

Consider the following grammar G. [MCQ]

$$\boxed{\begin{array}{l} S \rightarrow aA \mid \text{epsilon} \\ aA \rightarrow aaA \mid a \end{array}}$$

Unrestricted Grammar

What is the language generated by G?

$S \rightarrow \varepsilon$

$S \rightarrow aA \rightarrow a$

$S \rightarrow aA \rightarrow aaA \Rightarrow aa$

$S \rightarrow aA \rightarrow aaA \rightarrow aaaA \rightarrow aaa$

(A) (aa)*

(B) a* ✓

(C) a(aa)*

(D) None of these

$L = \{\varepsilon, a, a^2, a^3, \ldots\}$

Slide

Q

Find the Equivalent regular expressions for the set of odd number of 1's in binary string [MSQ]

A) 0*(0*10*10*)*1*

B) 1*(0*10*10*)* 0*

C) 0*1*(0*10*10*)* 0*

D) 0*(0*10*10*)* 1*0*

X

Slide

Q

How many states in minimized DFA that accepts set of all binary strings ending in 000 or 111? **[MCQ]**

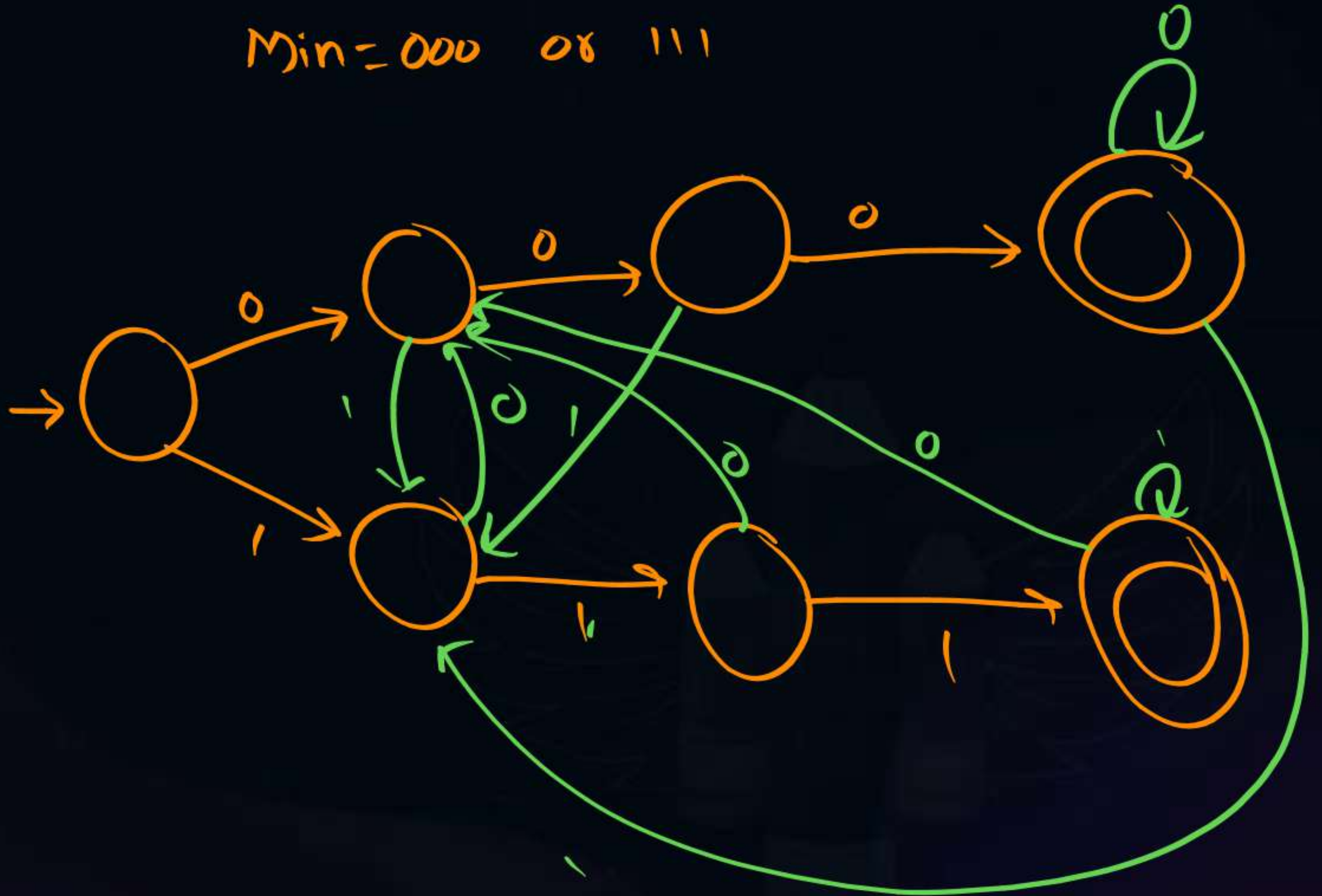
(A) 5

(B) 7

(C) 9

(D) 6

Min = 000 or 111



Slide

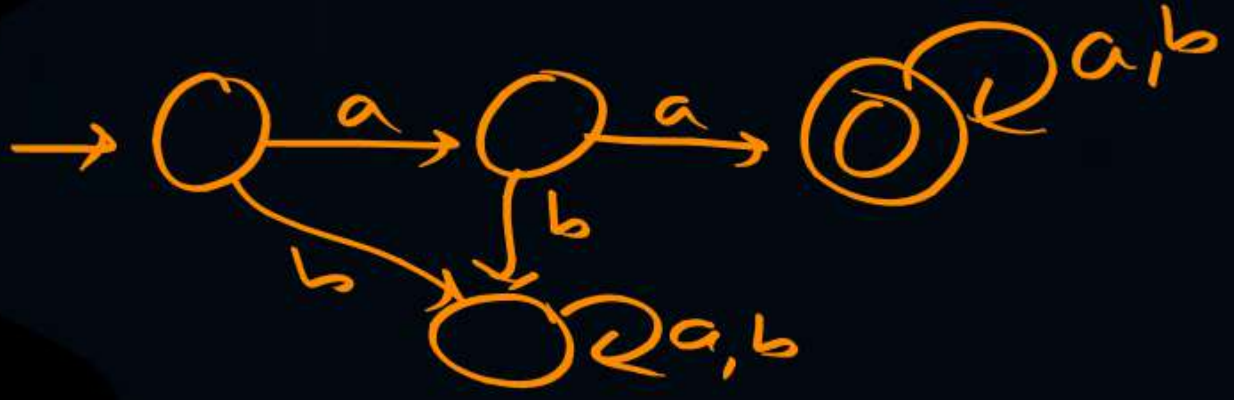
Q

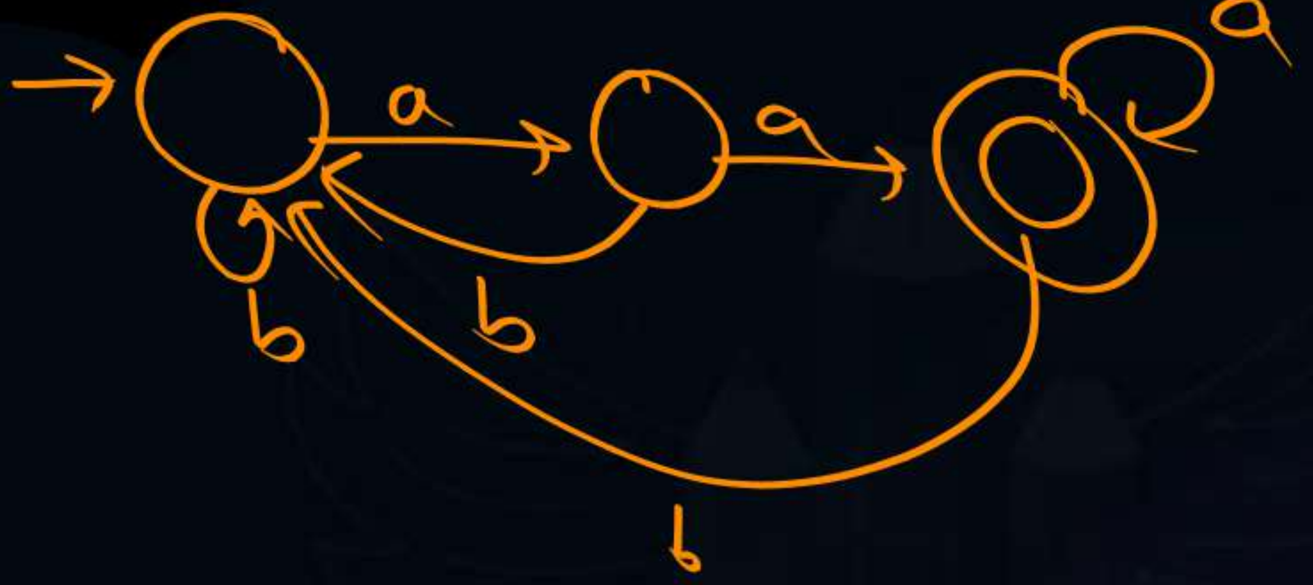
If x and y are number of equivalence classes for L1 and L2 respectively then $x+y=7$.

[MSQ]

$L1 = aa(a+b)^*$ $\longrightarrow$ 4 states

$L2 = (a+b)^*\,aa$ $\longrightarrow$ 3 states





(A) 6

(B) 7

(C) 8

(D) 9

Slide

Q.13

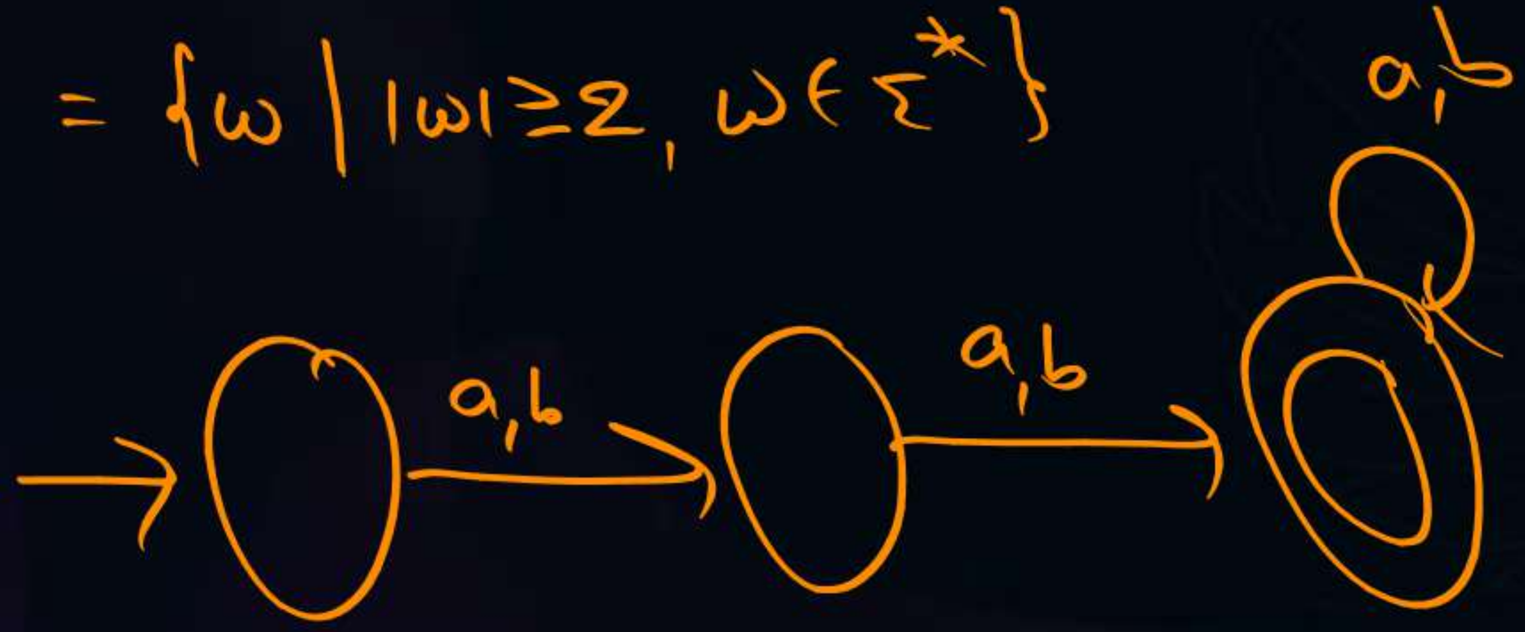
Let $L = (a+b)(a+b)(a+b)^*$ over alphabet $\Sigma = \{a, b\}$. **[NAT]**

If x is total number of final states and y is total number of non-final states, then $|x-y| = \underline{|1-2| = 1}$.

$$L = \Sigma.\Sigma.\Sigma^*$$
$$= \Sigma^2.\Sigma^*$$
$$= \{\omega \mid |\omega| \geq 2, \omega \in \Sigma^*\}$$

$x = 1$

$y = 2$



Slide

Q.14 Consider the following given languages: [MSQ]

L_1 = Regular Set

L_2 = Infinite Set

L_3 = Finite Set

Then which of the following is/are true?

(A) $L_1 - L_3$ is Regular   Reg - Fin ⇒ Reg - Reg ⇒ Reg

(B) $L_3 - L_1$ is Finite   Fin - L_1 ⇒ Fin

(C) $L_1 \cap L_2$ is Regular   Reg ∩ Inf ⇒ ?

(D) $L_1 \cup \boxed{L_2} \cup L_3$ is Infinite

A, B, D

Slide

Q.15

Consider the following two regular expressions r_1 and r_2:

[MCQ]

$r_1 = (0^*1^*)^* = (0+1)^*$

$r_2 = (1+0)^*$

if $L = r_1 / r_2$ then which of the following equivalent to L?

$(0+1)^* / (0+1)^* = (0+1)^* / \{\varepsilon, 0, 1, \ldots\}$

$= (0+1)^*$

$u/\phi = u$

$0/\varepsilon = 0$

$L/\varepsilon = L$

$10/0 = 1$

$110/10 = 1$

$1010/1 = X$

$1010/0 = 101$

(A) 0^*

(B) 1^*

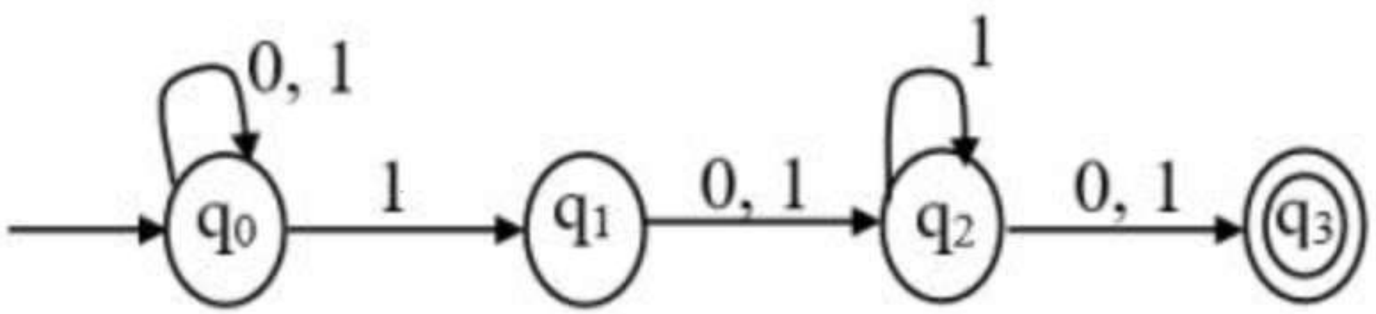
(C) $(1^*0^*)^* = (0+1)^*$

(D) None of these

(Hints: / is a quotient operation)

Slide

Consider the finite automaton in the following figure. **(GATE – 14-SET1)**



$q_0 \xrightarrow{0011} q_0$ | $q_0 \xrightarrow{0011} q_2$

$q_0 \xrightarrow{0011} q_1$ | $q_0 \xrightarrow{0011} q_3$ X

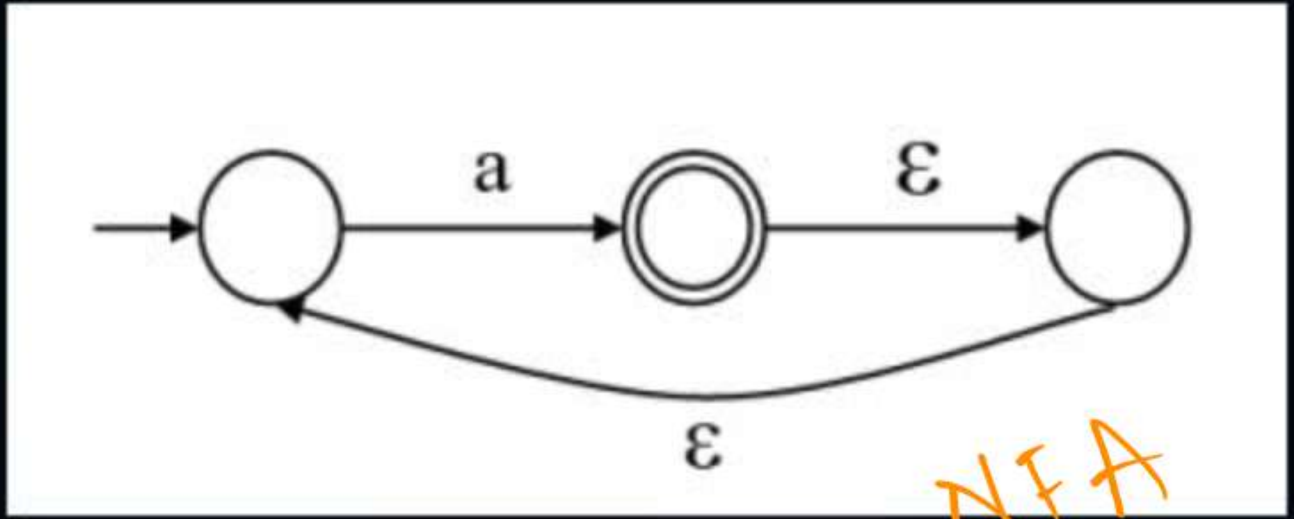
$\delta^*(q_0, 0011) = ?$

What is the set of reachable states for the input string 0011?

(a) $\{q_0, q_1, q_2\}$
(b) $\{q_0, q_1\}$
(c) $\{q_0, q_1, q_2, q_3\}$
(d) $\{q_3\}$

Slide

What is the complement of the language accepted by the $\boxed{\text{NFA}}$ shown below? Assume $\Sigma = \{a\}$ and ε is the empty string. **(GATE - 12)**



NFA

***

$L = \{a, a^2, a^3, \ldots\} = a^+$

$\bar{L} = \Sigma^* - a^+$

$= \{\varepsilon\}$

(a) ϕ (b) $\{\varepsilon\}$ (c) a^* (d) $\{a, \varepsilon\}$

Slide

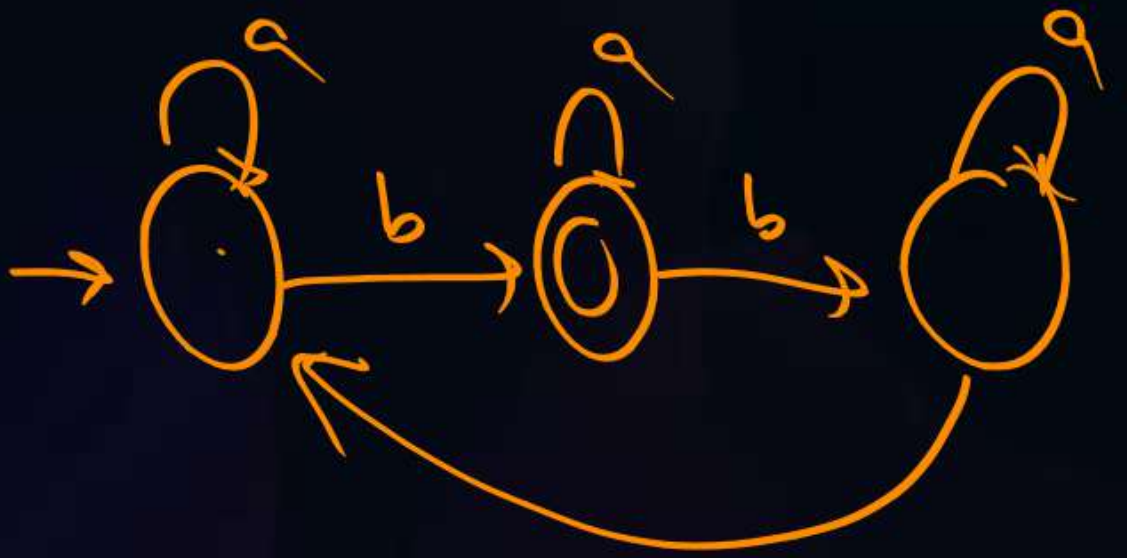
Which of the following is TRUE ?

(a) The language $L = \{a^n b^n \mid n \geq 0\}$ is regular.

(b) The language $L = \{a^n \mid n \text{ is prime}\}$ is regular.

(c) The language $L = \{w \mid w \text{ has } \underbrace{3k+1}_{1 \bmod 3} \text{ b's for some } k \in N \text{ with } \Sigma = \{a, b\}\}$ is regular.

(d) The language $L = \{ww \mid w \in \Sigma^* \text{ with } \Sigma = \{0, 1\}\}$ is regular



$$a^*(a^* b a^* b a^* b a^*)^* a^* b a^*$$

Slide

If $L_1 = \{a^n \mid n \geq 0\}$ and $L_2 = \{b^n \mid n \geq 0\}$, consider

$= a^*$ $= b^*$

~~I~~. $L_1 . L_2$ is a regular language

~~II~~. $L_1 . L_2 = \{a^n b^n \mid n \geq 0\}$

$= \{a^m b^n\}$

Which one of the following is CORRECT? **(GATE – 14-SET2)**

(a) Only (I)	(b) Only (II)
(c) Both (I) and (II)	(d) Neither (I) nor (II)

$L_1 L_2 = a^* b^*$

$= \{a^n\}\{b^n\}$

$= \{a^{n_1} b^{n_2}\}$

$= \{a^m b^n\}$

Slide

Let L be the language represented by the regular expression $\Sigma^*0011\Sigma^*$ where $\Sigma=\{0,1\}$. What is the minimum number of states in a DFA that recognizes $\overline{L}$ (complement of L)?

(GATE – 15 – SET3)

(a) 4 ~~(b)~~ 5

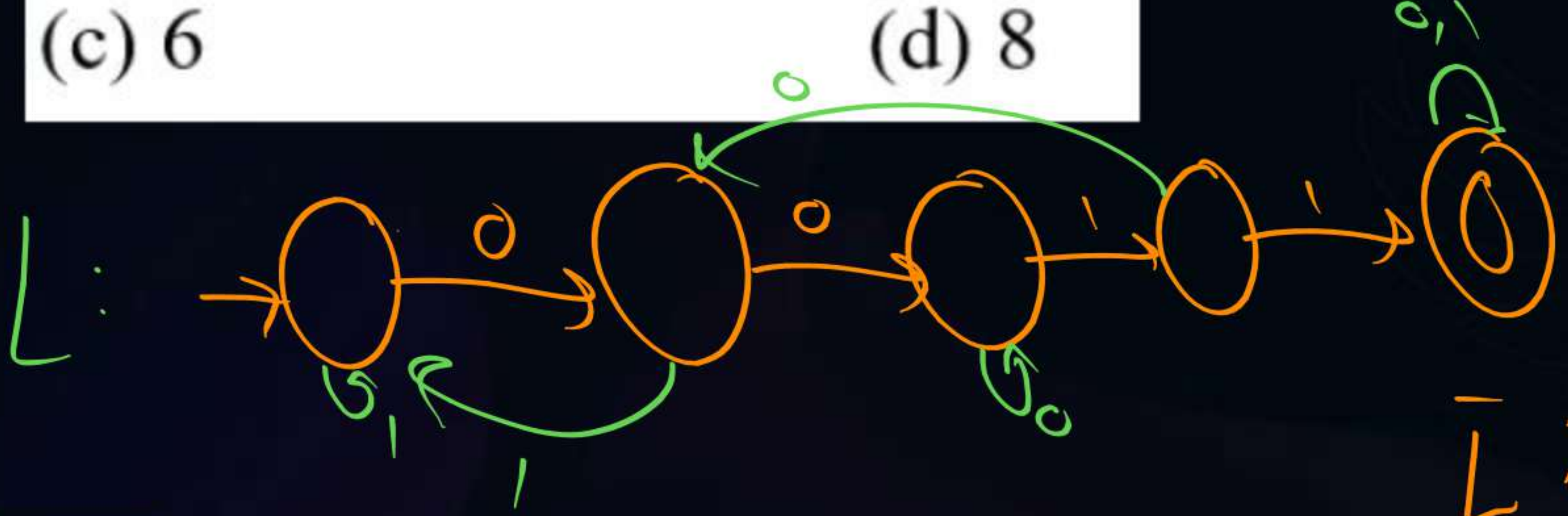
(c) 6 (d) 8

L : $\underbrace{X\,0011\,X}$

min = 0011

⇓

5 states



$\overline{L}$: 5 states

Slide

Which of the following languages is generated by the given grammar?

$$S \rightarrow aS \mid bS \mid \varepsilon$$ (GATE – 16 – SET1)

RLG

(a) $\{a^n b^m \mid n,m \geq 0\}$ – ba

(b) $\{w \in \{a,b\}^* \mid w$ has equal number of a's and b's$\}$ – a

(c) $\{a^n \mid n \geq 0\} \cup \{b^n \mid n \geq 0\} \cup \{a^n b^n \mid n \geq 0\}$ – ba

(d) $\{a, b\}^*$ ✓

$\boxed{S \rightarrow aS \mid bS \mid \varepsilon}$ OR $\boxed{S \rightarrow Sa \mid Sb \mid \varepsilon}$

Slide

Consider the language L given by the regular expression $(a+b)^*b(a+b)$ over the alphabet $\{a, b\}$. The smallest number of states needed in a deterministic finite-state automaton (DFA) accepting L is $2^2 = 4$ states.

(GATE – 17 – SET1)

$(a+b)^* \underset{3^{rd}}{b} (a+b)^2 \Rightarrow 8$ states

K^{th} symbol from end is 'a' over $\Sigma = \{a, b\}$

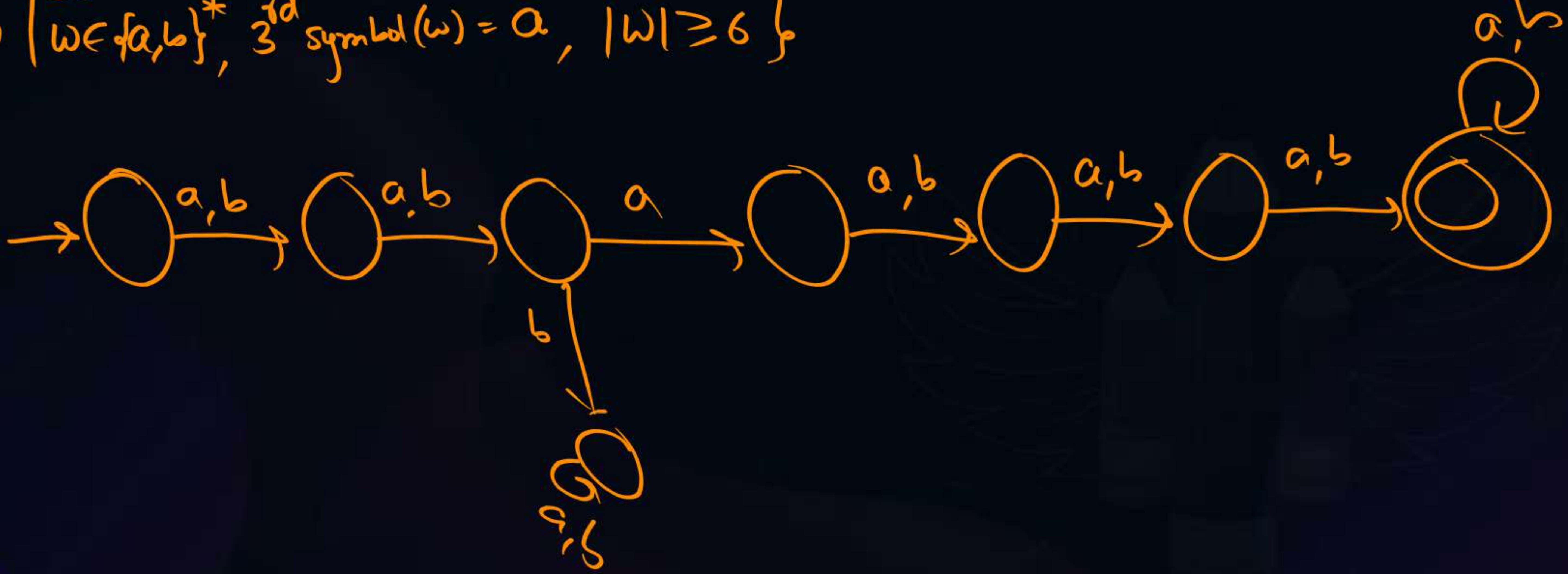
$= 2^K$ states

Slide

The minimum possible number of states of a deterministic finite automaton that accepts the regular language **(GATE – 17 – SET2)**

$L = \{\underbrace{\underbrace{w_1}_{2}\,\boxed{a}\,\underbrace{w_2}_{\geq 3}}_{\geq 6} \mid w_1, w_2 \in \{a, b\}^*, |w_1| = 2, |w_2| \geq 3\}$ is 8 //.

$= \{w \mid w \in \{a,b\}^*, 3^{rd}$ symbol$(w) = a, |w| \geq 6\}$



Slide

The lexical analysis for a modern computer language such as java needs the power of which one of the following machine models in a necessary and sufficient sense? **(GATE - 11)**

(a) Finite state automata

(b) Deterministic pushdown automata

(c) Non –deterministic pushdown automata

(d) Turing machine

Slide

The number of substrings (of all lengths inclusive) that can be formed from a character string of length n is **(GATE – 89 & 94)**

(a) ~~n~~

(b) ~~n^2~~

(c) ~~$\frac{n(n-1)}{2}$~~

(d) $\frac{n(n+1)}{2}+1$ ✓

$w = aaa$ $\quad$ $w = abc$

$$\underbrace{n+1}_{\text{min}} \le \#\text{substrings} \le \underbrace{\frac{n(n+1)}{2}+1}_{\text{max}}$$

$w = \underbrace{abc}$

$\to \varepsilon$
$\to a$
$\to b$
$\to c$
$\to ab$
$\to bc$
$\to abc$

substrings of w

$\frac{n(n+1)}{2}+1$

Slide

Let R_1 and R_2 be regular sets defined over the alphabet $\sum$ then:

(GATE - 90)

(a) $R_1 \cap R_2$ is not regular.

(b) $R_1 \cup R_2$ is regular.

(c) $\sum^* - R_1$ is regular.

(d) R_1^* is not regular.

Reg $\cap$ Reg $\Rightarrow$ Reg

Reg $\cup$ Reg $\Rightarrow$ Reg

$\Sigma^* - R_1 = \overline{R_1} \Rightarrow \overline{\text{Reg}} \Rightarrow$ Reg

$(\text{Reg})^* \Rightarrow$ Reg

Slide

Consider the following language.

$L = \{x \in \{a, b\}^* \mid \text{number of } a\text{'s in } x \text{ divisible by 2 but not divisible by 3}\}$

The minimum number of states in DFA that accepts L is ____________

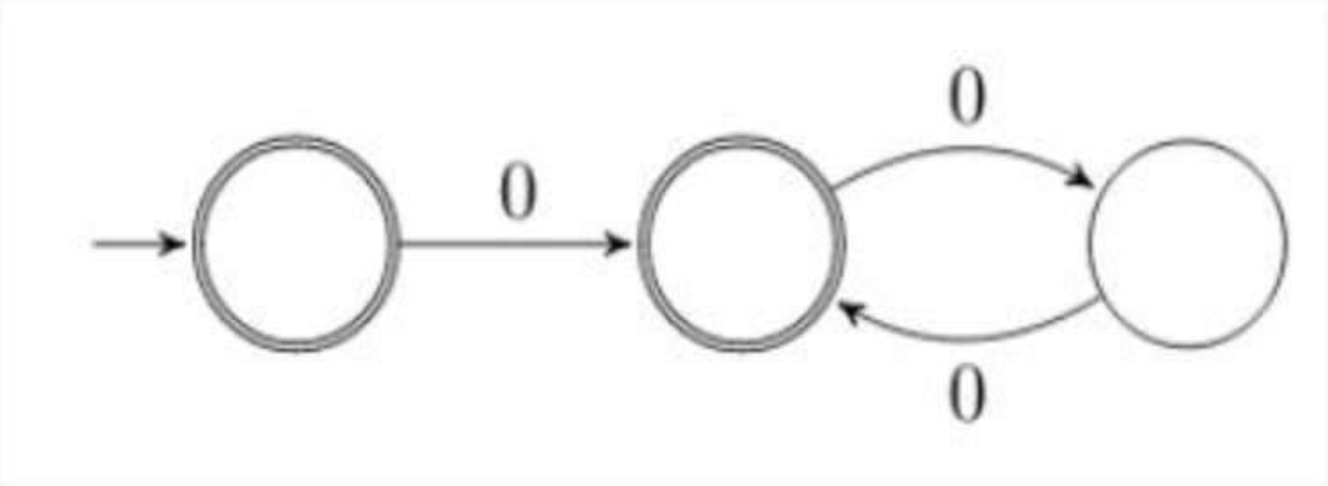
Slide

Given a language L, define L^i as follows:

$$L^0 = \{\varepsilon\}$$

$$L^i = L^{i-1} \bullet L \text{ for all } I > 0$$

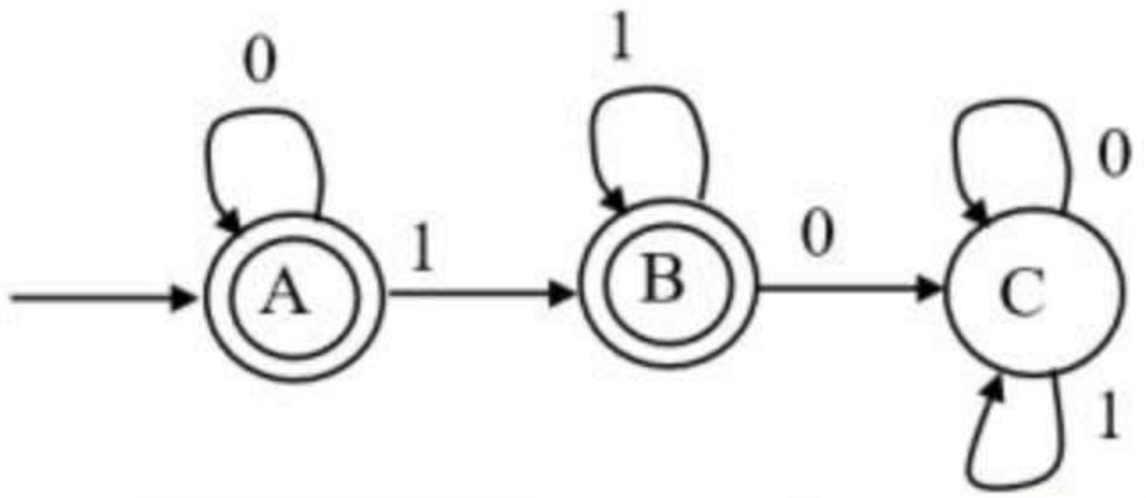
The order of a language L is defined as the smallest k such that $L^k = L^{k+1}$. Consider the language L_1 (over alphabet 0) accepted by the following automaton.



The order of L_1 is _____

Slide

The regular expression for the language recognized by the finite state automation of the below figure is _____ **(GATE - 94)**



Slide

A finite state machine with the following state table has a single input X and a single output Z.

Present state	Next state Z	
	X=1	X=0
A	D,0	B,0
B	B,1	C,1
C	B,0	D,1
D	B,1	C,0

If the initial state is unknown, then the shortest input sequence to reach the final state C is **(GATE - 95)**

(a) 01 (b) 10 (c) 101 (d) 110

Slide

Let L be the set of all binary strings whose last two symbols are the same .The number of states in the minimum state deterministic finite-state automaton accepting L is

(GATE - 98)

Slide

Consider a DFA over $\sum = \{a, b\}$ accepting all strings which have number of a's divisible by 6 and number of b's divisible by 8. What is the minimum number of states that the DFA will have?

(GATE - 01)

(a) 8 (b) 14

(c) 15 (d) 48

Slide

Consider the following languages:

$L_1 = \{w\,w \mid w \in \{a, b\}^*\}$

$L_2 = \{ww^R \mid w \in \{a,b\}^*, w^R \text{ is the reverse of } w\}$

$L_3 = \{0^{2i} \mid i \text{ is an integer}\}$

$L_4 = \{0^{i^2} \mid i \text{ is an integer}\}$

Which of the languages are regular?

(GATE - 01)

(a) Only L_1 and L_2

(b) Only L_2, L_3 and L_4

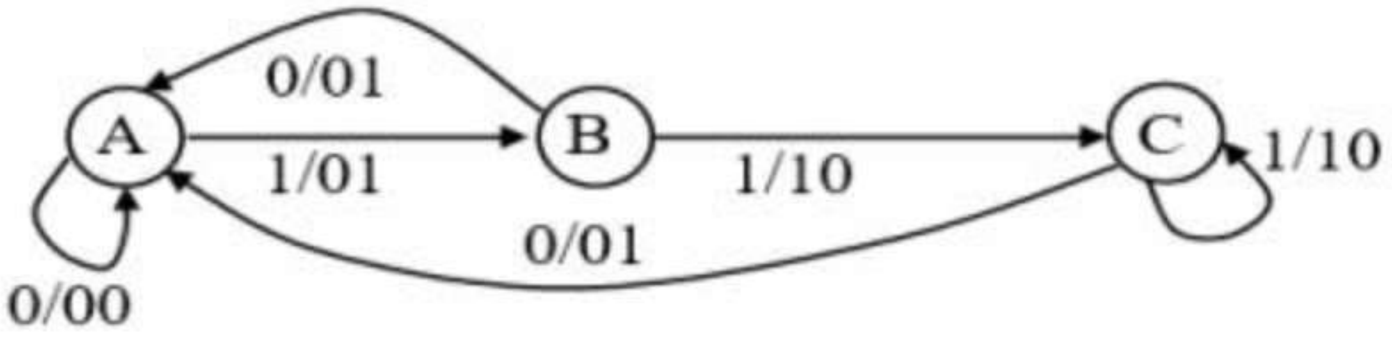
(c) Only L_3 and L_4

(d) Only L_3

Slide

The Finite state machine described by the following state diagram with A as starting state, where an arc label is x/y and x stands for 1-bit input and y stands for 2-bit output

(GATE - 02)



(a) Outputs the sum of the present and the previous bits of the input.

(b) Outputs 01 whenever the input sequence contains 11

(c) Outputs 00 whenever the input sequence contains 10

(d) None of the above

Slide

The smallest finite automation which accepts the language

$L=\{x \mid \text{length of } x \text{ is divisible by } 3\}$ has

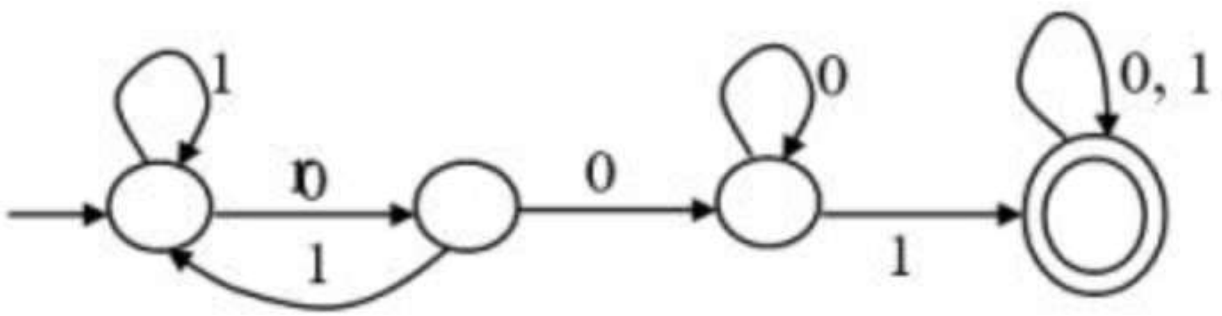
(GATE - 02)

(a) 2 states (b) 3 states

(c) 4 states (d) 5states

Slide

Consider the following deterministic finite state automation M.



Let S denote the set of seven bit binary strings in which the first, the fourth, and the last bits are 1. The number of strings in S that are accepted by M is **(GATE - 03)**

(a) 1 (b) 5 (c) 7 (d) 8

Which of the following sets can be recognized by a Deterministic Finite-state Automaton? **(GATE - 98)**

(a) The numbers 1, 2, 4, 8,..... 2^n,....... written in binary.

(b) The numbers 1, 2, 4,...2^n, ... written in unary.

(c) The set of binary strings in which the number of zeros is the same as the number of ones.

(d) The set {1, 101, 11011, 1110111, ...}

Slide

Consider the regular expression (0+1), (0+1)……n times. The minimum state finite automaton that recognizes the language represented by this regular expression contains: **(GATE - 99)**

Slide

Given an arbitrary non-deterministic finite automaton (NFA) with N states, the maximum number of states in an equivalent minimized DFA is at least **(GATE - 01)**

(a) N^2

(b) 2^N

(c) $2N$

(d) $N!$

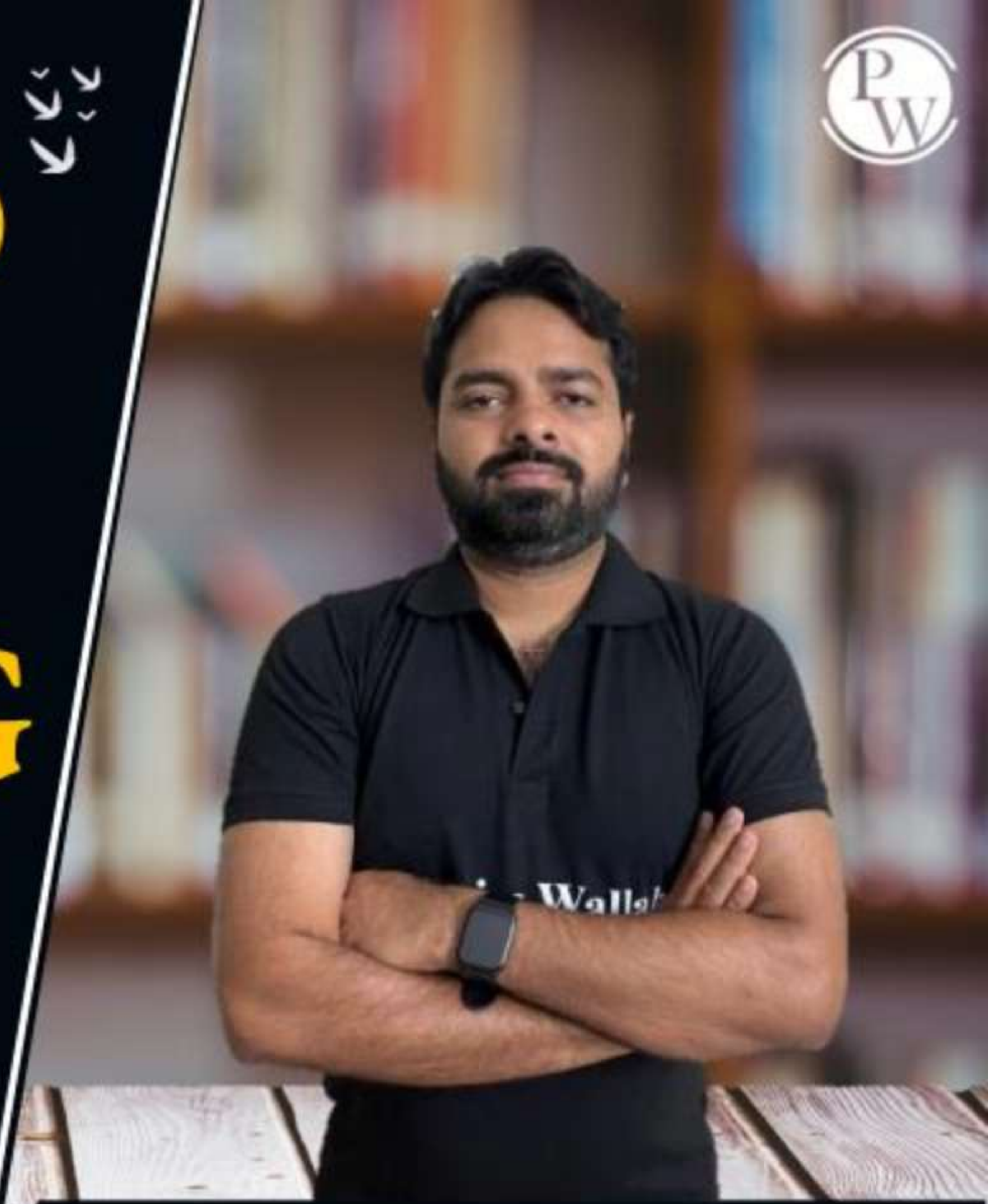
Slide

THANK - YOU

MISSION ISRO CS & IT ENGINEERING

Theory of Computation

Lecture No.- 03



By- Mallesham Devasane Sir

Topics to be Covered

Topic

Regulars

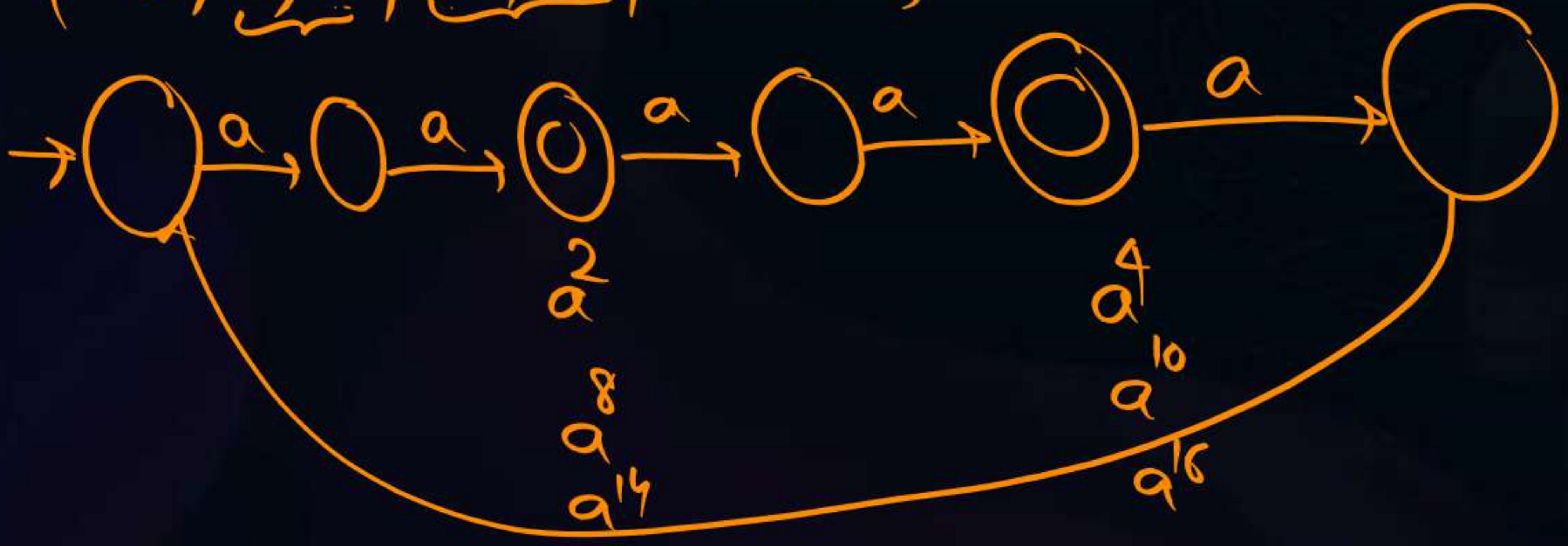
CFLs

Consider the following language.

$$L = \{x \in \{a, b\}^* \mid \text{number of } a\text{'s in } x\ \underbrace{\text{divisible by } 2}_{\varepsilon,\, a^2,\, a^4,\, a^6,\, \dots} \text{ but } \boxed{\text{not}}\ \underbrace{\text{divisible by } 3}_{\varepsilon,\, a^3,\, a^6,\, a^9,\, \dots}\}$$

The minimum number of states in DFA that accepts L is __________ $= 6$

$$L = \{ a^2, a^4, a^8, a^{10}, a^{14}, \dots \}$$



Slide

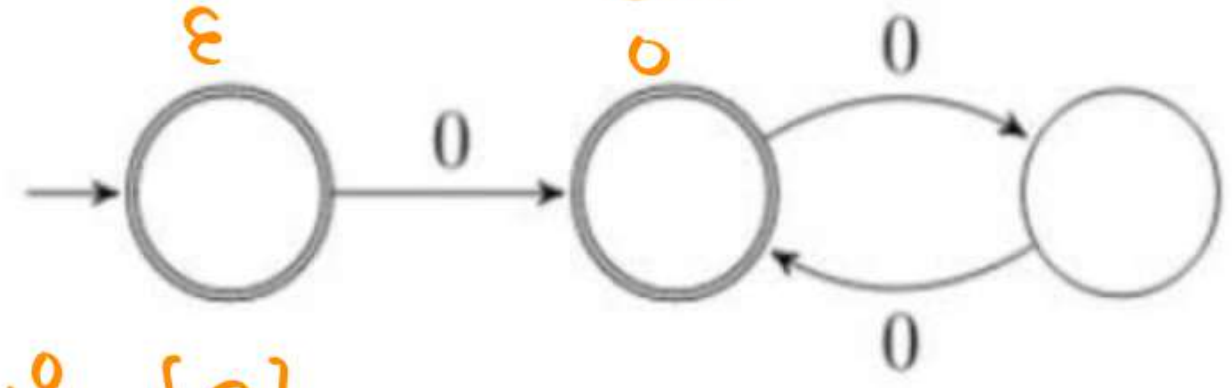
Given a language L, define L^i as follows:

$$L^0 = \{\varepsilon\}$$

$$L^i = L^{i-1} \bullet L \text{ for all } i > 0$$

The order of a language L is defined as the smallest k such that $\boxed{L^k = L^{k+1}}$. Consider the language L_1 (over alphabet 0) accepted by the following automaton.

$L = \varepsilon + 0(00)^*$



The order of L_1 is $\underline{2}$

$L^0 = \{\varepsilon\}$

$L^1 = L^0 \cdot L = \varepsilon \cdot L = L = \varepsilon + 0(00)^*$

$L^2 = L^1 \cdot L = (\varepsilon + 0(00)^*) \cdot (\varepsilon + 0(00)^*)$

$= \{\varepsilon, 0, 00, 000, 0 \ldots\} = 0^*$

$L^3 = L^2 \cdot L = 0^* \cdot L = 0^*$

$L^0 = \{\varepsilon\}$

$L^i = L^{i-1} \cdot L$

$O(L) = K$

$L^K = L^{K+1}$ for smallest K

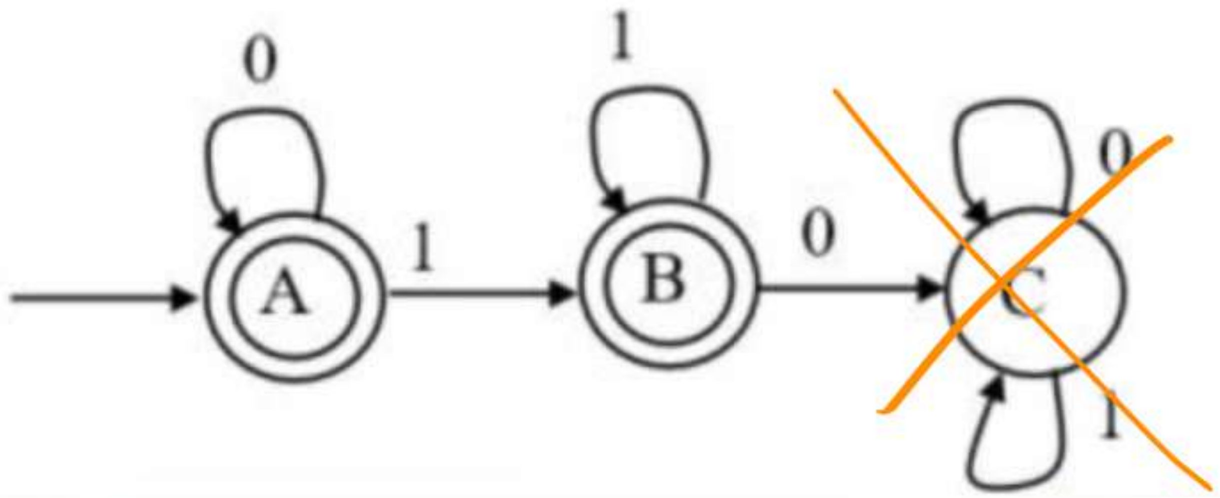
$K = 0 \Rightarrow L^0 = L^1$ ✗

$K = 1 \Rightarrow L^1 = L^2$ ✗

$K = 2 \Rightarrow L^2 = L^3$ ✓

Slide

The regular expression for the language recognized by the finite state automation of the below figure is _____ **(GATE - 94)**



$L = 0^* 1^*$

A finite state machine with the following state table has a single input X and a single output Z.

Present state	Next state Z	
	X=1	X=0
A	D,0	B,0
B	B,1	C,1
C	B,0	D,1
D	B,1	C,0

$B \xrightarrow{1} B \xrightarrow{0} C$

01/10/101/110

$A \longrightarrow C$

$B \xrightarrow{10} C$ ✓

$C \longrightarrow C$

$D \longrightarrow C$

If the initial state is $\boxed{\text{unknown}}$, then the shortest input sequence to reach the final state C is **(GATE - 95)**

(a) 01 (b) 10 (c) 101 (d) 110

Slide

Let L be the set of all binary strings whose last two symbols are the same .The number of states in the minimum state deterministic finite-state automaton accepting L is $= 5$

(GATE - 98)

$L = (0+1)^*(00+11)$

Slide

Consider a DFA over $\sum = \{a, b\}$ accepting all strings which have number of a's divisible by $\boxed{6}_{m}$ and number of b's divisible by $\boxed{8}_{n}$. What is the minimum number of states that the DFA will have?

(GATE - 01)

(a) 8 (b) 14

(c) 15 (d) 48

$6 \times 8 = 48$

$m \times n$

Slide

Consider the following languages:

$L_1 = \{\underline{w}\ \underline{w} \mid w \in \{a, b\}^*\}$ $\Rightarrow$ CSL $\Rightarrow$ Not Regular

$L_2 = \{ww^R \mid w \in \{a,b\}^*, w^R \text{ is the reverse of } w\}$ $\Rightarrow$ CFL $\Rightarrow$ Not Regular

$L_3 = \{0^{2i} \mid i \text{ is an integer}\}$ $= (00)^* \Rightarrow$ Regular

$L_4 = \{0^{i^2} \mid i \text{ is an integer}\}$ $= \{\varepsilon, 0^1, 0^4, 0^9, \ldots\} \Rightarrow$ Not Regular $\Rightarrow$ CSL

Which of the languages are regular?

(GATE - 01)

(a) Only L_1 and L_2

(b) Only L_2, L_3 and L_4

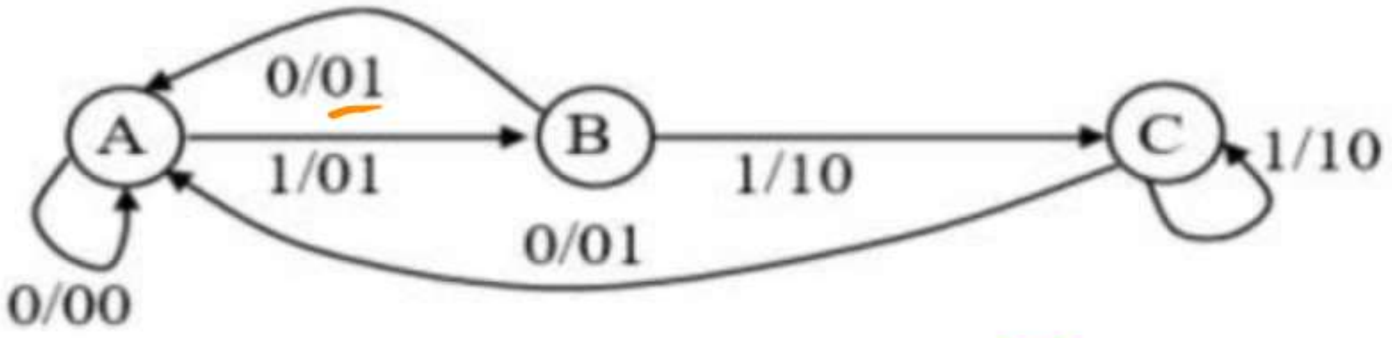
(c) Only L_3 and L_4

(d) Only L_3 ✓

Slide

The Finite state machine described by the following state diagram with A as starting state, where an arc label is x/y and x stands for 1-bit input and y stands for 2-bit output

(GATE - 02)



$A \xrightarrow[00]{0} A \xrightarrow[00]{0} A \xrightarrow[01]{1} B \xrightarrow[10]{1} C \xrightarrow[01]{0} A \xrightarrow[01]{1} B$

(a) Outputs the sum of the present and the previous bits of the input.

(b) Outputs 01 whenever the input sequence contains 11

(c) Outputs 00 whenever the input sequence contains 10

(d) None of the above

0011010

01011011000

Slide

The smallest finite automation which accepts the language

$L = \{x \mid \text{length of } x \text{ is divisible by } 3\}$ has

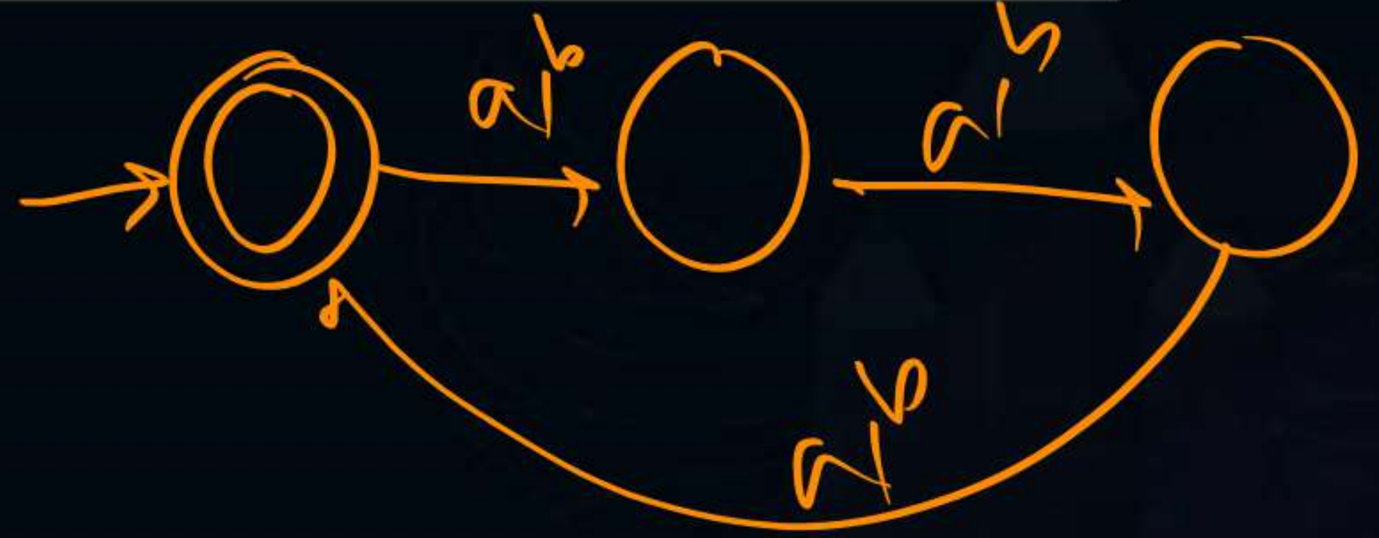
0 1 2

3 remainders

(GATE - 02)

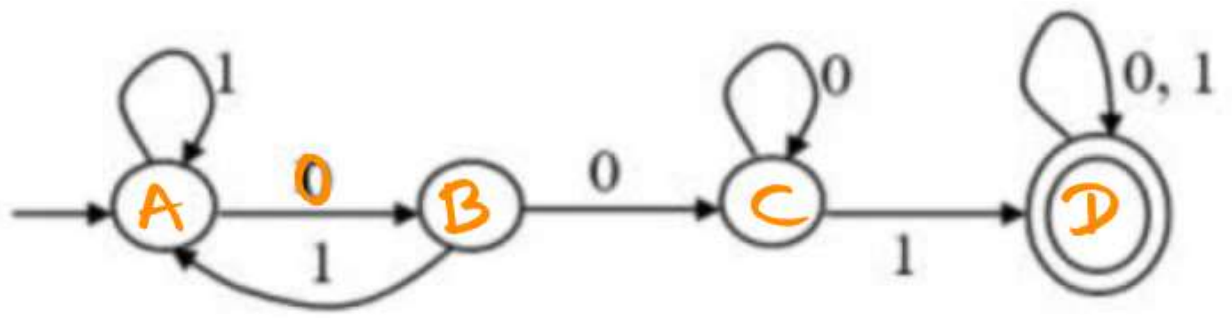
(a) 2 states
(b) 3 states
(c) 4 states
(d) 5states

$\Sigma = \{a, b\}$



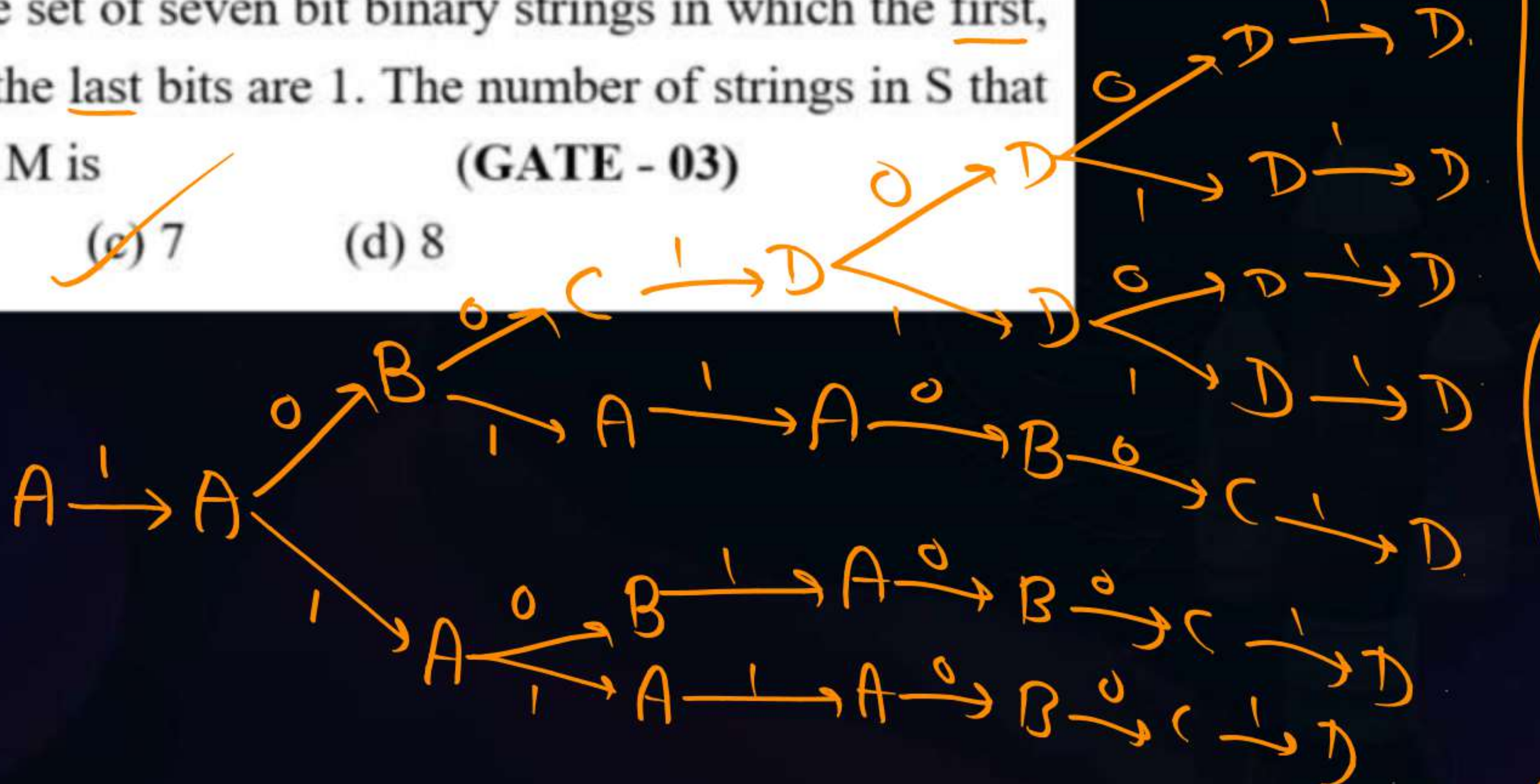
Slide

Consider the following deterministic finite state automation M.



Let S denote the set of seven bit binary strings in which the first, the fourth, and the last bits are 1. The number of strings in S that are accepted by M is **(GATE - 03)**

(a) 1 (b) 5 (c) 7 (d) 8



7

Slide

Which of the following sets can be recognized by a Deterministic Finite-state Automaton? **(GATE - 98)**

(a) The numbers 1, 2, 4, 8,..... 2^n,....... written in binary.

$\to$ 1, 01, 001, ... $\to$ 10 $\to$ 100 $\to$ 1000

$\to 0^* 1 0^*$

(b) The numbers 1, 2, 4,...2^n, ... written in unary.

$\to$ a, aa, aaaa, ...

$\{a^{2^n}\}$ $\to$ not regular $\to$ CSL

(c) The set of binary strings in which the number of zeros is the same as the number of ones. $\{w \mid w \in \{0,1\}^*, n_0(w) = n_1(w)\}$ $\to$ Not reg $\to$ DCFL

(d) The set {1, 101, 11011, 1110111, ...}

$\{1\} \cup \{1^n 0 1^n \mid n \geq 1\}$ $\to$ not reg $\to$ DCFL

Slide

Consider the regular expression $\overbrace{(0+1).(0+1)\ldots\ldots n}^{\text{n length string}}$ times. The minimum state finite automaton that recognizes the language represented by this regular expression contains: **(GATE - 99)**

$|w| = n \Rightarrow n+2$ states in DFA

$n+1$ states in NFA

Slide

Given an arbitrary non-deterministic finite automaton (NFA) with N states, the maximum number of states in an equivalent minimized DFA is at least (GATE - 01)

(a) N^2

(b) 2^N

(c) $2N$

(d) $N!$

NFA $\Longrightarrow$ DFA

N state $\leq \boxed{2^N}$

Slide

Regular Languages

- Regular Exp.
- FA
  - DFA
  - NFA
- Regular Grammar

Q

Consider the language

$L1 = \{0^i1^j \mid i \neq j\}$, $= \{0^i1^j \mid \underbrace{i<j}_{\text{1's left on i/p}} \text{ or } \underbrace{i>j}_{\text{0's left on stack}}\} \longrightarrow DCFL \to CFL$

$L2 = \{0^i1^j \mid i = j\}$, $\longrightarrow DCFL \to CFL$

$L3 = \{0^i1^j \mid i = 2j + 1\}$, $= \{0\, 0^{2j}\, 1^j\}$

$L4 = \{0^i1^j \mid i \neq 2j\}$. $= \{0^i 1^j \mid i<2j \text{ or } i>2j\}$

Which one of the following statements is true? [2010: 2 Marks]

(A) Only L2 is context free

(B) Only L2 and L3 are context free

(C) Only L1 and L2 are context free

(D) All are context free

Q

Consider the languages L1, L2 and L3 as given below:

$L1 = \{0^p 1^q \mid p, q \in N\}$, $= 0^* 1^* \rightarrow$ Reg

$L2 = \{0^p 1^q \mid p, q \in N \text{ and } p = q\}$ and $\rightarrow$ DCFL

$L3 = \{0^p 1^q 0^r \mid p, q, r \in N \text{ and } p = q = r\}$ $= 0^n 1^n 0^n \rightarrow$ CSL

Which of the following statements is NOT TRUE? [2011: 2 Marks]

(A) Push Down Automata (PDA) can be used to recognize L1 and L2. T

(B) L1 is a regular language. T

(C) All the three languages are context free. F

(D) Turing machines can be used to recognize all the languages. T

Q

Consider the following languages:

I. $\{a^m b^n c^p d^q \mid m + p = n + q, \text{ where } m, n, p, q \geq 0\}$

II. $\{a^m b^n c^p d^q \mid m = n \text{ and } p = q, \text{ where } m, n, p, q \geq 0\}$

III. $\{a^m b^n c^p d^q \mid m = n = p \text{ and } p \neq q, \text{ where } m, n, p, q \geq 0\}$

IV. $\{a^m b^n c^p d^q \mid mn = p + q, \text{ where } m, n, p, q \geq 0\}$

Which of the language above are context-free?

[2012: 2 Marks]

(A) I and IV only

(B) I and II only

(C) II and III only

(D) II and IV only

Q

Consider the following languages

$L1 = \{0^p 1^q 0^r \mid p, q, r \geq 0\}$

$L2 = \{0^p 1^q 0^r \mid p, q, r \geq 0, p \neq r\}$

Which one of the following statements is FALSE?

[2013: 2 Marks]

(A) L2 is context-free

(B) $L1 \cap L2$ is context-free

(C) Complement of L2 is recursive

(D) Complement of L1 is context-free but not regular

Q

Consider the following languages over the alphabet $\Sigma = \{0, 1, c\}$:

$L_1 = \{0^n 1^n \mid n \geq 0\}$

$L_2 = \{wcw^r \mid w \in \{0, 1\}^*\}$

$L_3 = \{ww^r \mid w \in \{0, 1\}^*\}$

Here, w^r is the reverse of the string w. Which of these languages are deterministic Context-free languages?

[2014-Set3: 2 Marks]

(A) None of the languages

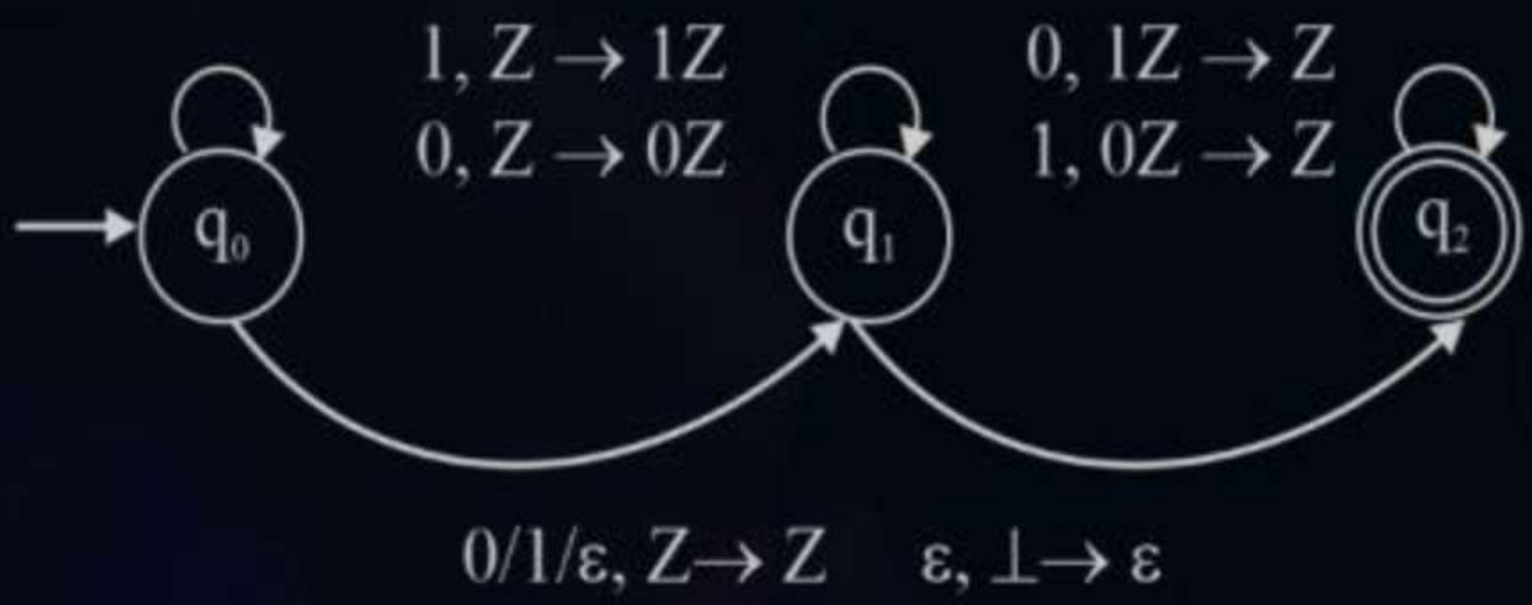
(B) Only L_1

(C) Only L_1 and L_2

(D) All the three languages

Q

Consider the NPDA $\langle Q = \{q_0, q_1, q_2\}, \Sigma = \{0, 1\}, \Gamma = \{0, 1, \perp\}, \delta, q_0, \perp, F = \{q_2\} \rangle$, where (as per usual convention) Q is the set of states, Σ is the input alphabet, Γ is stack alphabet, δ is the state transition function, q_0 is the initial state, $\perp$ is the initial stack symbol, and F is the set of accepting states, The state transition is as follows:



Which of the following sequences must follow the string 101100 so that the overall string is accepted by the automaton?

[2015-Set1: 2 Marks]

(A) 10110

(B) 10010

(C) 01010

(D) 01001

Q

Which of the following languages are context-free?

$L_1 = \{a^m\, b^n\, a^n\, b^m \mid m, n \geq 1\}$

$L_2 = \{a^m\, b^n\, a^m\, b^n \mid m, n \geq 1\}$

$L_3 = \{a^m\, b^n \mid m = 2n + 1\}$

[2015(Set-3): 1 Marks]

(A) L_1 and L_2 only

(B) L_1 and L_3 only

(C) L_2 and L_3 only

(D) L_3 only

Q

Consider the following context-free grammars:

G_1: $S \rightarrow aS \mid B$, $B \rightarrow b \mid bB$

G_2: $S \rightarrow aA \mid bB$, $A \rightarrow aA \mid B \mid \varepsilon$, $B \rightarrow bB \mid \varepsilon$

Which one of the following pairs of languages is generated by G_1 and G_2, respectively?

[2016(Set-1): 2 Marks]

(A) $\{a^m b^n \mid m > 0 \text{ or } n > 0\}$ and $\{a^m\, b^n \mid m > 0 \text{ and } n > 0\}$.

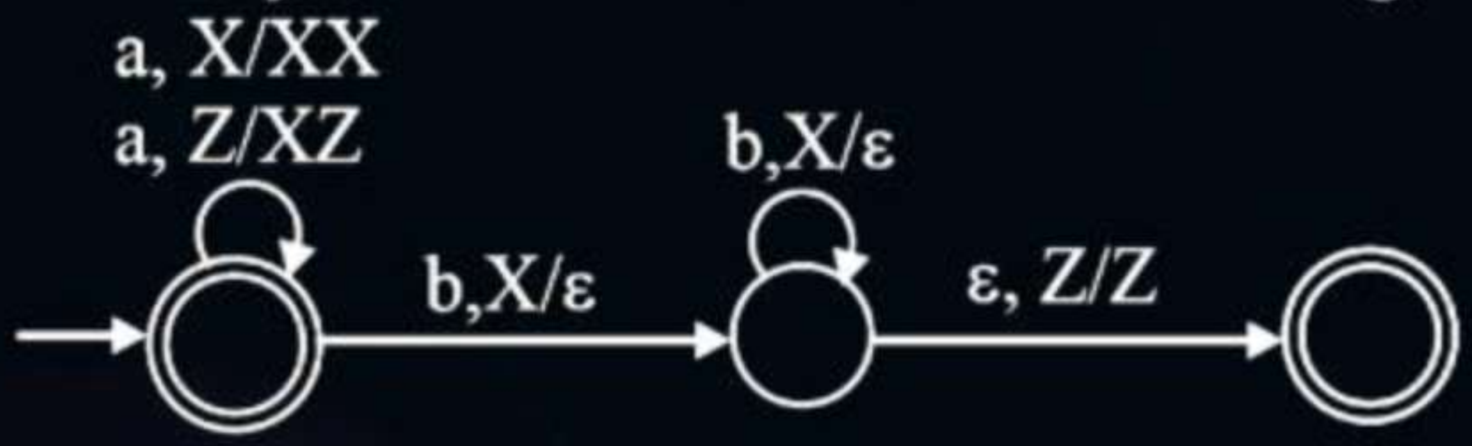
(B) $\{a^m b^n \mid m > 0 \text{ and } n > 0\}$ and $\{a^m\, b^n \mid m > 0 \text{ or } n \geq 0\}$.

(C) $\{a^m b^n \mid m \geq 0 \text{ or } n > 0\}$ and $\{a^m\, b^n \mid m > 0 \text{ and } n > 0\}$.

(D) $\{a^m b^n \mid m \geq 0 \text{ and } n > 0\}$ and $\{a^m\, b^n \mid m > 0 \text{ or } n > 0\}$.

Q

Consider the transition diagram of a PDA given below with input alphabet $\Sigma = \{a, b\}$ and stack alphabet $\Gamma = \{X, Z\}$. Z is the initial stack symbol. Let L denote the language accepted by the PDA.



Which one of the following is TRUE?

[2016(Set-1): 2 Marks]

(A) $L = \{a^n b^n \mid n \geq 0\}$ and is not accepted by any finite automata.

(B) $L = \{a^n \mid n \geq 0\} \cup \{a^n b^n \mid n \geq 0\}$ and is not accepted by any deterministic PDA.

(C) L is not accepted by any Turing machine that halts on every input.

(D) $L = \{a^n \mid n \geq 0\} \cup \{a^n b^n \mid n \geq 0\}$ and is deterministic context-free.

Q

Consider the following languages:

$L_1 = \{a^n b^m c^{n+m} : m, n \geq 1\}$

$L_2 = \{a^n b^n c^{2n} : n \geq 1\}$

Which one of the following is TRUE?

[2016(Set-2): 2 Marks]

(A) Both L_1 and L_2 are context-free.

(B) L_1 is context-free while L_2 is not context-free

(C) L_2 is context-free while L_1 is not context-free

(D) Neither L_1 nor L_2 is context-free

Q.

Language L_1 is defined by the grammar: $S_1 \rightarrow aS_1b | \varepsilon$
Language L_2 is defined by the grammar: $S_2 \rightarrow abS_2 | \varepsilon$
Consider the following statements:
P: L_1 is regular
Q: L_2 is regular
Which one of the following is TRUE?

[2016(Set-2): 1 Marks]

(A) Both P and Q are true

(B) P is true and Q is false

(C) P is false and Q is true

(D) Both P and Q are false

Q

Consider the following context-free grammar over the alphabet $\Sigma = \{a, b, c\}$ with S as the start symbol

$$S \rightarrow abScT \mid abcT$$

$$T \rightarrow bT \mid b$$

Which one of the following represents the language generated by the above grammar?

[2017(Set-1): 1 Marks]

(A) $\{(ab)^n(cb)^n \mid n \geq 1\}$

(B) $\{(ab)^n cb^{m_1} cb^{m_2} \ldots cb^{m_n} \mid n, m_1, m_2, \ldots, m_n \geq 1\}$

(C) $\{(ab)^n(cb^m)^n \mid m, n \geq 1\}$

(D) $\{(ab)^n(cb^n)^m \mid m, n \geq 1\}$

Q

Consider the following language over the alphabet $\Sigma = \{a, b, c\}$.
Let $L_1 = \{a^n b^n c^m \mid m, n \geq 0\}$ and
$L_2 = \{a^m b^n c^n \mid m, n \geq 0\}$.
Which of the following are context-free languages?

I. $L_1 \cup L_2$

II. $L_1 \cap L_2$

[2017(Set-1): 2 Marks]

(A) I only

(B) II only

(C) I and II

(D) Neither I nor II

Q

Consider the context-free grammars over the alphabet $\{a, b, c\}$ given below. S and T are non-terminals.

$G_1: S \rightarrow aSb \mid T, T \rightarrow cT \mid \in$

$G_2: S \rightarrow bSa \mid T, T \rightarrow cT \mid \in$

The language $L(G_1) \cap L(G_2)$ is [2017-Set1: 1 Mark]

(A) Finite

(B) Not finite but regular

(C) Context-free but not regular

(D) Recursive but not context-free

Q

Identify the language generated by the following grammar, where S is the start variable.

$$S \rightarrow XY$$
$$X \rightarrow aX \mid a$$
$$Y \rightarrow aYb \mid \in$$

[2017(Set-2): 1 Marks]

A $\{a^m b^n \mid m \geq n, n > 0\}$

B $\{a^m b^n \mid m \geq n, n \geq 0\}$

C $\{a^m b^n \mid m > n, n \geq 0\}$

D $\{a^m b^n \mid m > n, n > 0\}$

Q

Let L_1, L_2 be any two context-free languages and R be any regular language. Then which of the following is/are CORRECT?

I. $L_1 \cup L_2$ is context-free

II. $\overline{L_1}$ is context-free

III. $L_1 - R$ is context-free

IV. $L_1 \cap L_2$ is context-free

[2017(Set-2): 1 Marks]

(A) I, II and IV only

(B) I and III only

(C) II and IV only

(D) I only

Q

Consider the following languages:

$L_1 = \{a^p \mid p \text{ is a prime number}\}$

$L_2 = \{a^n b^m c^{2m} \mid n \geq 0, m \geq 0\}$

$L_3 = \{a^n b^n c^{2n} \mid n \geq 0\}$

$L_4 = \{a^n b^n \mid n \geq 1\}$

Which of the following are CORRECT?

I. L_1 is context-free but not regular.

II. L_2 is not context-free.

III. L_3 is not context-free but recursive.

IV. L_4 is deterministic context-free.

[2017(Set-2): 2 Marks]

(A) I, II and IV only

(B) II and III only

(C) I and IV only

(D) III and IV only

Q

Which one of the following languages over $\Sigma = \{a, b\}$ is NOT context-free?

[2019: 2 Marks]

(A) $\{a^n b^i \mid i \in \{n, 3n, 5n\}, n \geq 0\}$

(B) $\{w a^n w^R b^n \mid w \in \{a, b\}^*, n \geq 0\}$

(C) $\{w w^R \mid w \in \{a, b\}^*\}$

(D) $\{w a^n b^n w^R \mid w \in \{a, b\}^*, n \geq 0\}$

Q

Consider the following languages:

$L_1 = \{wxyx \mid w, x, y \in (0 + 1)^+\}$

$L_2 = \{xy \mid x, y \in (a + b)^*, |x| = |y|, x \neq y\}$

Which of the following is TRUE

[2020: 2 Marks]

(A) L_1 is regular and L_2 is context-free.

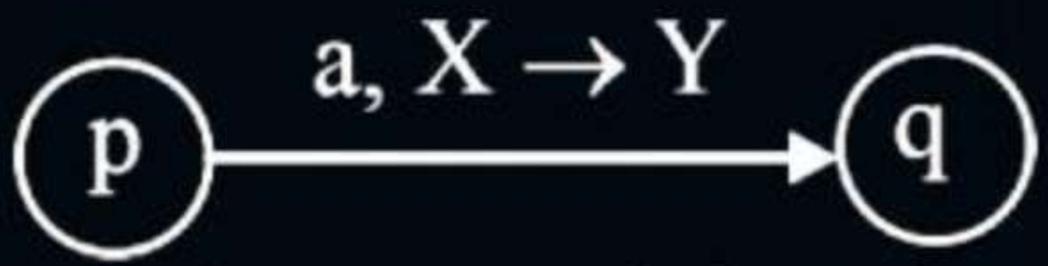
(B) L_1 is context-free but L_2 is not context-free.

(C) Neither L_1 nor L_2 is context-free.

(D) L_1 is context-free but not regular and L_2 context-free.

Q

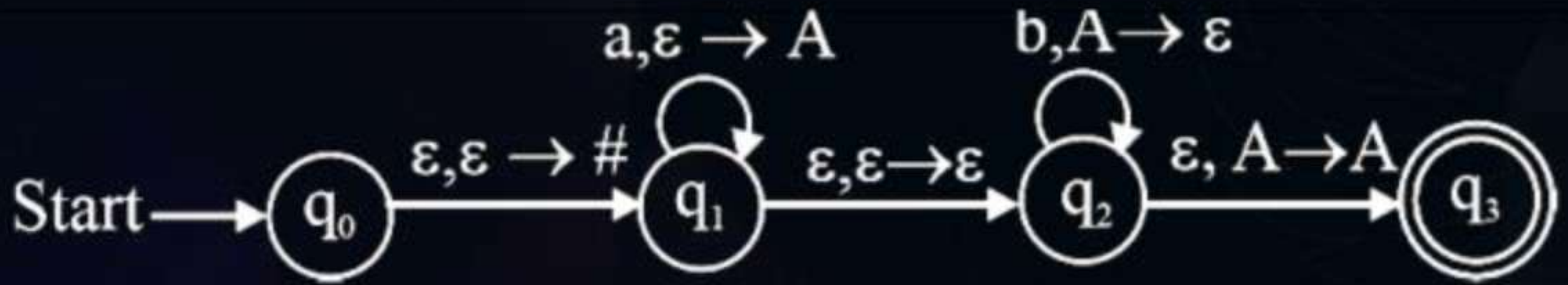
In a pushdown automaton $P = (Q, \Sigma, \Gamma, \delta, q_0, F)$, a transition of the form,



Where $p, q, \in Q$, $a \in \Sigma \cup \{\epsilon\}$, and $X, Y \in \Gamma \cup \{\epsilon\}$ represents

$$(q, Y) \in \delta(p, a, X)$$

Consider the following pushdown automaton over the input alphabet $\Sigma = \{a, b\}$ and stack alphabet $\Gamma = \{\#, A\}$.



The number of strings of length 100 accepted by the above pushdown automaton is ______.

[2021(Set-1): 2 Marks]

Q

Suppose that L_1 is a regular language and L_2 is a context-free language. Which one of the following languages is NOT necessarily context-free?

[2021(Set-1): 2 Marks]

(A) $L_1 \cdot L_2$

(B) $L_1 \cup L_2$

(C) $L_1 - L_2$

(D) $L_1 \cap L_2$

Q

For a string w, we define w^R to be the reverse of w. For example, if w = 01101 then w^R = 10110. Which of the following languages is/are context-free?

[2021(Set-2): 2 Marks]

(A) $\{wxw^Rx^R \mid w, x \in \{0, 1\}^*\}$

(B) $\{wxw^R \mid w, x \in \{0, 1\}^*\}$

(C) $\{ww^Rxx^R \mid w, x \in \{0, 1\}^*\}$

(D) $\{wxx^Rw^R \mid w, x \in \{0, 1\}^*\}$

Q

Let L_1 be a regular language and L_2 be a context-free language. Which of the following languages is/are context-free?

[2021(Set-2)MSQ: 1 Marks]

(A) $L_1 \cap \overline{L_2}$

(B) $\overline{\overline{L_1} \cup \overline{L_2}}$

(C) $L_1 \cup (L_2 \cup \overline{L_2})$

(D) $(L_1 \cap L_2) \cup (\overline{L}_1 \cap L_2)$

Q

Consider the following languages:

$L_1 = \{a^n w a^n \mid w \in \{a, b\}^*\}$

$L_2 = \{wxw^R \mid w, x \in \{a, b\}^*\}, |w|, |x| > 0\}$

Note that w^R is the reversal of the string w. Which of the following is/are TRUE?

[2022: MSQ: 2 Marks]

(A) L_1 and L_2 are regular.

(B) L_1 and L_2 are context-free.

(C) L_1 is regular and L_2 is context-free.

(D) L_1 and L_2 are context-free but not regular.

Q

Consider the following languages:

$L_1 = \{ww \mid w \in \{a, b\}^*\}$

$L_2 = \{a^n b^n c^m \mid m, n \geq 0\}$

$L_3 = \{a^m b^n c^n \mid m, n \geq 0\}$

Which of the following statements is/are FALSE?

[2022: 2 Marks]

(A) L_1 is not context-free but L_2 and L_3 are deterministic context-free.

(B) Neither L_1 nor L_2 is context-free.

(C) L_2, L_3 and $L_2 \cap L_3$ all are context-free.

(D) Neither L_1 nor its complement is context-free

Q

Which of the following are decidable?

1. Whether the intersection of two regular languages is infinite
2. Whether a given context-free language is regular
3. Whether two push-down automata accept the same language
4. Whether a given grammar is context-free

[2008: 1 Marks]

(A) 1 and 2

(B) 1 and 4

(C) 2 and 3

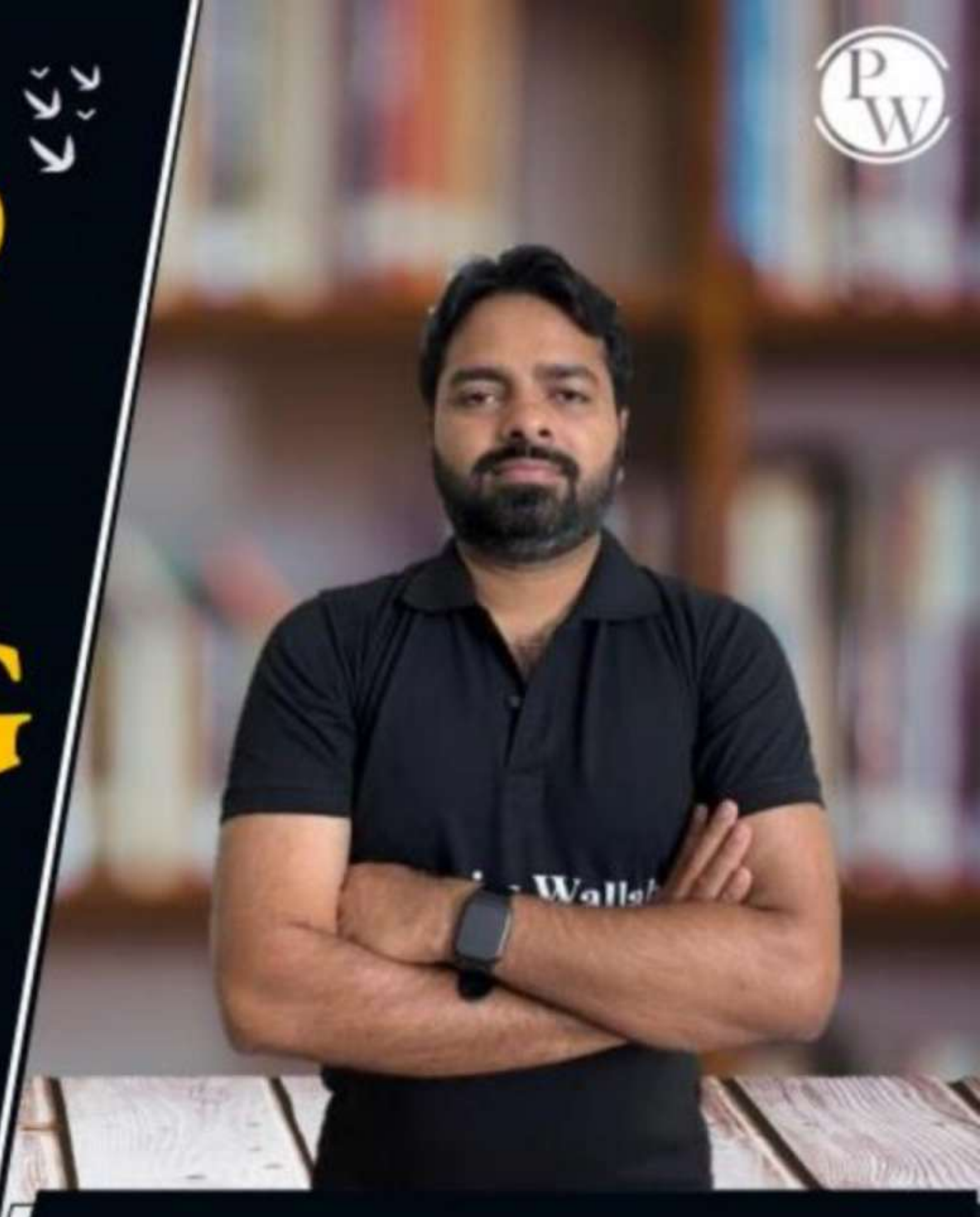
(D) 2 and 4

THANK - YOU

MISSION ISRO CS & IT ENGINEERING

Theory of Computation

Lecture No.- 04



By- Mallesham Devasane Sir

Topics to be Covered

Topic CFLs

Practice on CFLs

Q

Consider the following languages:

I. $\{a^m b^n c^p d^q \mid m + p = n + q, \text{ where } m, n, p, q \geq 0\}$

II. $\{a^m b^n c^p d^q \mid m = n \text{ and } p = q, \text{ where } m, n, p, q \geq 0\}$

III. $\{a^m b^n c^p d^q \mid m = n = p \text{ and } p \neq q, \text{ where } m, n, p, q \geq 0\}$

IV. $\{a^m b^n c^p d^q \mid \boxed{mn} = p + q, \text{ where } m, n, p, q \geq 0\}$

Which of the language above are context-free?

[2012: 2 Marks]

A) I and IV only

B) I and II only

C) II and III only

D) II and IV only

push a's, pop a's

CFL

Not CFL

$a^m \boxed{b^n}$

a's = b's, Push c's for c's, Pop c's for d's

a's > b's, push c's for c's, Pop c's Pop a's for d's for d's

a's < b's, Push b's for rem b's, Pop b's for c's

c's = b's on stack

c's > b's on stack

Q

Consider the following languages

$L1 = \{0^p 1^q 0^r \mid p, q, r \geq 0\}$ $= 0^* 1^* 0^* \rightarrow$ Regular

$L2 = \{0^p 1^q 0^r \mid p, q, r \geq 0, p \neq r\} \rightarrow$ DCFL

Which one of the following statements is $\boxed{\text{FALSE?}}$

[2013: 2 Marks]

(A) L2 is context-free T

(B) $\underbrace{L1 \cap L2}_{= L_2}$ is context-free T

(C) Complement of L2 is recursive T

(D) Complement of L1 is context-free but $\boxed{\text{not regular}}$ FALSE

Q

Consider the following languages over the alphabet $\Sigma = \{0, 1, c\}$:

$L_1 = \{0^n 1^n \mid n \geq 0\}$ ⟶ DCFL

$w\#w^R$ $\quad L_2 = \{wcw^r \mid w \in \{0, 1\}^*\}$

$L_3 = \{ww^r \mid w \in \{0, 1\}^*\}$ ⟶ CFL but not DCFL

Here, w^r is the reverse of the string w. Which of these languages are deterministic Context-free languages? [2014-Set3: 2 Marks]

(A) None of the languages

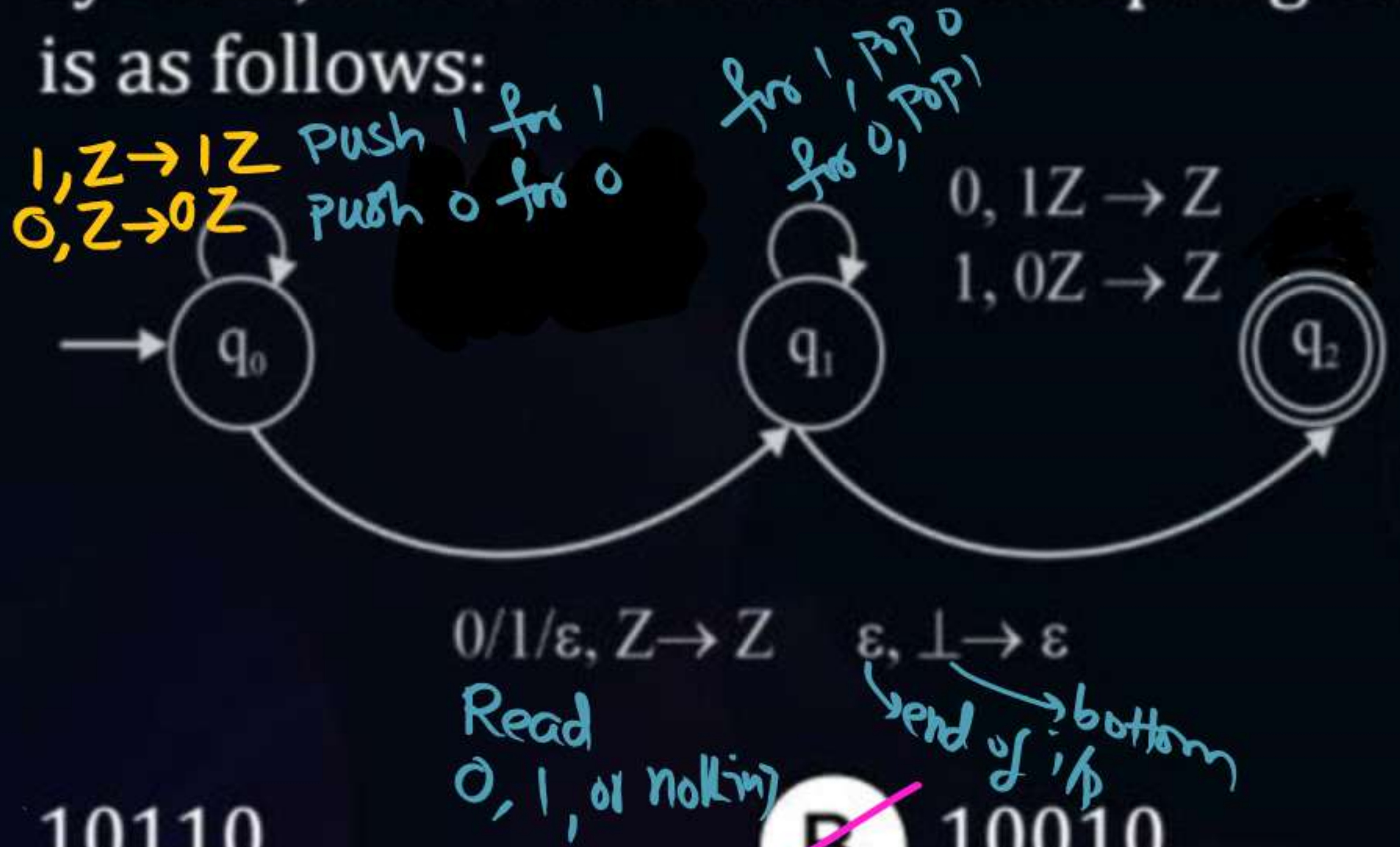
(B) Only L_1

(C) Only L_1 and L_2

(D) All the three languages

Q

Consider the NPDA $\langle Q = \{q_0, q_1, q_2\}, \Sigma = \{0, 1\}, \Gamma = \{0, 1, \perp\}, \delta, q_0, \perp, F = \{q_2\} \rangle$, where (as per usual convention) Q is the set of states, Σ is the input alphabet, Γ is stack alphabet, δ is the state transition function, q_0 is the initial state, $\perp$ is the initial stack symbol, and F is the set of accepting states, The state transition is as follows:



Which of the following sequences must follow the string 101100 so that the overall string is accepted by the automaton?

[2015-Set1: 2 Marks]

$\underbrace{101100}_{\text{push}}\underbrace{\boxed{0}}_{\text{skip}}10010 \in L$

(A) 10110

(B) 10010 ✓

(C) 01010

(D) 01001

Practice on CFLs

Q

Which of the following languages are context-free?

push a's, push b's, pop b's, pop a's

$L_1 = \{a^m b^n a^n b^m \mid m, n \geq 1\}$ ⟶ DCFL → CFL

$L_2 = \{a^m b^n a^m b^n \mid m, n \geq 1\}$ ⟶ Not CFL

$L_3 = \{a^m b^n \mid m = 2n + 1\}$ ⟶ DCFL → CFL

$a^{2n+1} b^n = a \cdot a^{2n} b^n$

skip, push a for 2a's, pop a for b

[2015(Set-3): 1 Marks]

(A) L_1 and L_2 only

(B) L_1 and L_3 only

(C) L_2 and L_3 only

(D) L_3 only

Q

Consider the following context-free grammars:

G_1: $S \rightarrow aS \mid B$, $B \rightarrow b \mid bB$

G_2: $S \rightarrow aA \mid bB$, $A \rightarrow aA \mid B \mid \varepsilon$, $B \rightarrow bB \mid \varepsilon$

Which one of the following pairs of languages is generated by G_1 and G_2, respectively?

[2016(Set-1): 2 Marks]

$L_1 = a^* b^+$

G_1: $S \rightarrow aS \mid B$

$B \rightarrow b \mid bB \Rightarrow b^+$

b^* b^* $a^* b^*$ $a^+ b^*$ b^+

$L_2 = b^+ + a^+ b^*$

$\cancel{a^+ b^*} + a^* b^+$

A) $\{a^m b^n \mid m > 0 \text{ or } n > 0\}$ and $\{a^m b^n \mid m > 0 \text{ and } n > 0\}$.

$a^+ b^+$

B) $\{a^m b^n \mid m > 0 \text{ and } n > 0\}$ and $\{a^m b^n \mid m > 0 \text{ or } n \geq 0\}$.

$a^* b^* + a^* b^+$

C) $\{a^m b^n \mid m \geq 0 \text{ or } n > 0\}$ and $\{a^m b^n \mid m > 0 \text{ and } n > 0\}$.

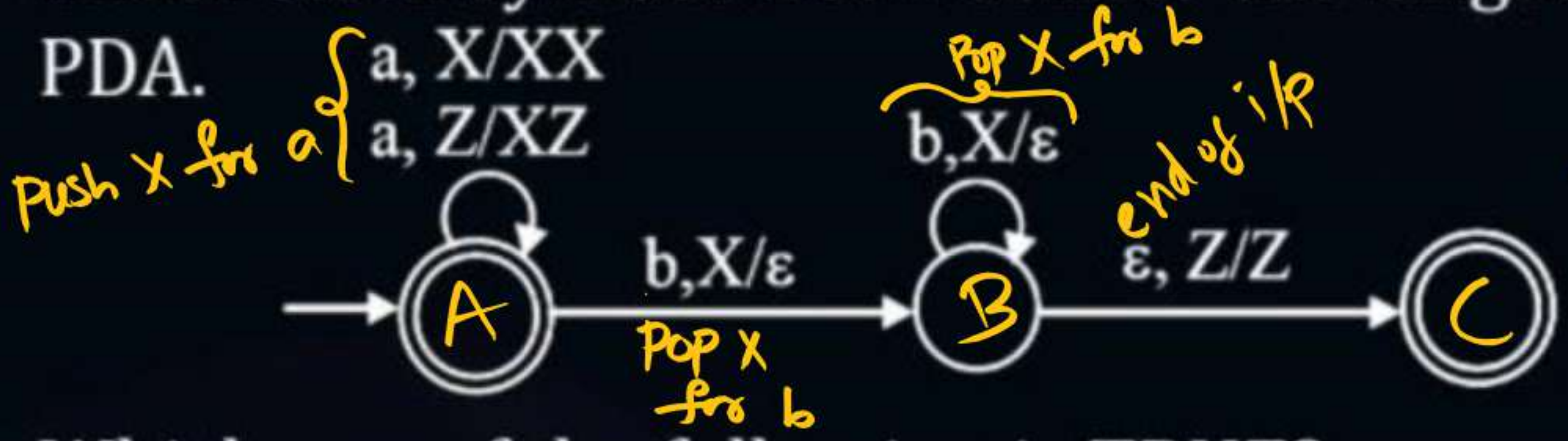
D) $\{a^m b^n \mid m \geq 0 \text{ and } n > 0\}$ and $\{a^m b^n \mid m > 0 \text{ or } n > 0\}$.

$a^* b^+$

$a^+ b^* + a^* b^+$ b^+ $a^+ b$

Q

Consider the transition diagram of a PDA given below with input alphabet $\Sigma = \{a, b\}$ and stack alphabet $\Gamma = \{X, Z\}$. Z is the initial stack symbol. Let L denote the language accepted by the PDA.



$A = a^*$

$C = \{a^n b^n \mid n \geq 1\}$

$A + C = a^* \cup \{a^n b^n \mid n \geq 1\}$

$= a^* \cup \{a^n b^n\} \rightarrow$ DCFL

Which one of the following is TRUE?

[2016(Set-1): 2 Marks]

(A) $L = \{a^n b^n \mid n \geq 0\}$ and is not accepted by any finite automata.

(B) $L = \{a^n \mid n \geq 0\} \cup \{a^n b^n \mid n \geq 0\}$ and is not accepted by any deterministic PDA.

(C) L is not accepted by any Turing machine that halts on every input.

(D) $L = \{a^n \mid n \geq 0\} \cup \{a^n b^n \mid n \geq 0\}$ and is deterministic context-free.

Q

Consider the following languages:

$L_1 = \{a^n b^m c^{n+m} : m, n \geq 1\}$ → DCFL → CFL

$L_2 = \{a^n b^n c^{2n} : n \geq 1\}$ → Not CFL

Which one of the following is TRUE?

[2016(Set-2): 2 Marks]

(A) Both L_1 and L_2 are context-free.

(B) L_1 is context-free while L_2 is not context-free

(C) L_2 is context-free while L_1 is not context-free

(D) Neither L_1 nor L_2 is context-free

Q.

Language L_1 is defined by the grammar: $S_1 \rightarrow aS_1b|\varepsilon$

Language L_2 is defined by the grammar: $S_2 \rightarrow abS_2|\ \varepsilon$

Consider the following statements:

P: L_1 is regular

Q: L_2 is regular

Which one of the following is TRUE?

$L(S_1) = a^n b^n = L_1 \rightarrow$ Not reg languag

$L(S_2) = L_2 = (ab)^* \rightarrow$ Reg languag

[2016(Set-2): 1 Marks]

(A) Both P and Q are true

(B) P is true and Q is false

(C) P is false and Q is true

(D) Both P and Q are false

Q

Consider the following context-free grammar over the alphabet $\Sigma = \{a, b, c\}$ with S as the start symbol

$S \to abScT \mid abcT$

$\boxed{T \to bT \mid b}$ $T = b^+$

$S \to abScb^+ \mid abcb^+$

$S \to xSy \mid z \Rightarrow x^n z y^n$

Which one of the following represents the language generated by the above grammar?

[2017(Set-1): 1 Marks]

$L = (ab)^n \underline{ab}c\underline{b^+}(cb^+)^n \mid n \geq 0$

$= \{(ab)^n (cb^+)^n \mid n \geq 1\} = \{(ab)^n \underbrace{cb^+ cb^+ \ldots}_{n \text{ times}}\}$

$= \{(ab)^n cb^{m_1} cb^{m_2} \ldots cb^{m_n} \mid n \geq 1, m_1, m_2, \ldots, m_n \geq 1\}$

(A) $\{(ab)^n(cb)^n \mid n \geq 1\}$

(B) $\{(ab)^n cb^{m_1} cb^{m_2} \ldots cb^{m_n} \mid n, m_1, m_2, \ldots, m_n \geq 1\}$

(C) $\{(ab)^n(cb^{\boxed{m}})^n \mid m, n \geq 1\}$

(D) $\{(ab)^n(cb^n)^m \mid m, n \geq 1\}$

Q

Consider the following language over the alphabet $\Sigma = \{a, b, c\}$.

Let $L_1 = \{a^n b^n c^m \mid m, n \geq 0\}$ and $= a^n b^n c^*$

$L_2 = \{a^m b^n c^n \mid m, n \geq 0\}$. $= a^* b^n c^n$

$L_1 \cup L_2 = \{a^i b^j c^k \mid i=j \text{ OR } j=k\}$ → CFL → IAL

$L_1 \cap L_2 = \{a^n b^n c^n\}$ → CSL

Which of the following are context-free languages?

I. $L_1 \cup L_2$

II. $L_1 \cap L_2$

[2017(Set-1): 2 Marks]

(A) I only

(B) II only

(C) I and II

(D) Neither I nor II

Q

Consider the context-free grammars over the alphabet $\{a, b, c\}$ given below. S and T are non-terminals.

$G_1: S \rightarrow aSb \mid T,\ T \rightarrow cT \mid \epsilon$ $L_1 = a^n c^* b^n$

$G_2: S \rightarrow bSa \mid T,\ T \rightarrow cT \mid \epsilon$ $L_2 = b^n c^* a^n$

The language $L(G_1) \cap L(G_2)$ is c^* [2017-Set1: 1 Mark]

(A) Finite

(B) Not finite but regular

(C) Context-free but not regular

(D) Recursive but not context-free

Q

Identify the language generated by the following grammar, where S is the start variable.

$$S \rightarrow XY$$

$$\boxed{X \rightarrow aX \mid a} \quad X = a^+$$

$$\boxed{Y \rightarrow aYb \mid \epsilon} \quad Y = a^n b^n$$

[2017(Set-2): 1 Marks]

$$L = \underbrace{a^+ a^n}_{\#a's} > \underbrace{b^n}_{\#b's} = \{a^i b^j \mid i > j \geq 0\}$$

$$= \{a^i b^j \mid i > j,\ i, j \geq 0\}$$

$$= \{a^i b^j \mid i > j,\ j \geq 0,\ i \geq 1\}$$

(A) $\{a^m b^n \mid m \geq n, n > 0\}$

(B) $\{a^m b^n \mid m \geq n, n \geq 0\}$

(C) $\{a^m b^n \mid m > n, n \geq 0\}$

(D) $\{a^m b^n \mid m > n, n > 0\}$

Q

Let L_1, L_2 be any two context-free languages and R be any regular language. Then which of the following is/are $\boxed{\text{CORRECT?}}$

I. $L_1 \cup L_2$ is context-free

II. $\overline{L_1}$ is context-free

III. $L_1 - R$ is context-free

IV. $L_1 \cap L_2$ is context-free

$CFL_1 \cup CFL_2 \Rightarrow CFL$

$\overline{CFL_1} \Rightarrow CSL$

$CFL - Reg \Rightarrow CFL \cap \overline{Reg} \Rightarrow CFL \cap Reg \Rightarrow CFL$

$CFL_1 \cap CFL_2 \Rightarrow CSL$ (need not be CFL)

[2017(Set-2): 1 Marks]

(A) I, II and IV only

(B) I and III only

(C) II and IV only

(D) I only

Q

Consider the following languages:

$L_1 = \{a^p \mid p \text{ is a prime number}\} \rightarrow$ CSL

$L_2 = \{a^n b^m c^{2m} \mid n \geq 0, m \geq 0\} \rightarrow$ DCFL

$L_3 = \{a^n b^n c^{2n} \mid n \geq 0\} \rightarrow$ CSL

$L_4 = \{a^n b^n \mid n \geq 1\} \rightarrow$ DCFL

Which of the following are CORRECT?

I. L_1 is context-free but not regular.

II. L_2 is not context-free.

III. L_3 is not context-free but recursive.

IV. L_4 is deterministic context-free.

[2017(Set-2): 2 Marks]

(A) I, II and IV only

(B) II and III only

(C) I and IV only

(D) III and IV only

Q

Which one of the following languages over $\Sigma = \{a, b\}$ is NOT context-free?

[2019: 2 Marks]

(A) $\{a^n b^i \mid i \in \{n, 3n, 5n\}, n \geq 0\}$ $= \{a^n b^n\} \cup \{a^n b^{3n}\} \cup \{a^n b^{5n}\} \rightarrow$ CFL

(B) $\{w a^n w^R b^n \mid w \in \{a, b\}^*, n \geq 0\}$ $\rightarrow$ Not CFL

(C) $\{\underbrace{w}_{\text{Push}}\underbrace{w^R}_{\text{Pop}} \mid w \in \{a, b\}^*\}$ $\rightarrow$ CFL

(D) $\{\underbrace{w}_{\text{Push}}\underbrace{a^n}_{\text{Push}}\underbrace{b^n}_{\text{Pop } a\text{'s}}\underbrace{w^R}_{\text{Pop}} \mid w \in \{a, b\}^*, n \geq 0\}$ $\rightarrow$ CFL

Q

Consider the following languages:

$L_1 = \{wxyx \mid w, x, y \in (0 + 1)^+\}$ → $w0y0 + w1y1$ → Regular

$L_2 = \{xy \mid x, y \in (a + b)^*, |x| = |y|, x \neq y\}$ → CFL but not DCFL

Which of the following is TRUE ?

[2020: 2 Marks]

(A) L_1 is regular and L_2 is context-free.

(B) L_1 is context-free but L_2 is not context-free.

(C) Neither L_1 nor L_2 is context-free.

(D) L_1 is context-free but not regular and L_2 context-free.

Q

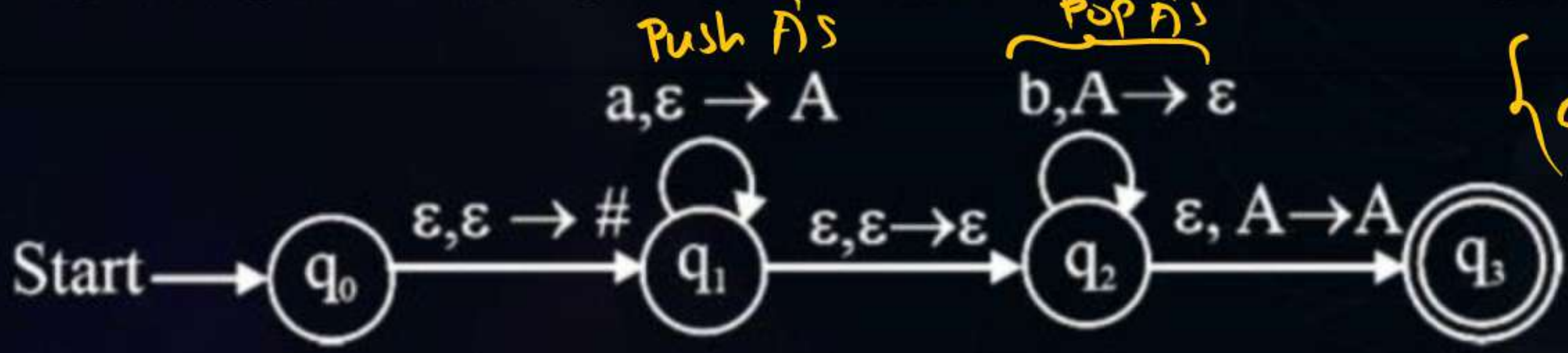
In a pushdown automaton $P = (Q, \Sigma, \Gamma, \delta, q_0, F)$, a transition of the form,

$$p \xrightarrow{a,\ X \to Y} q$$

Where $p, q, \in Q$, $a \in \Sigma \cup \{\epsilon\}$, and $X, Y \in \Gamma \cup \{\epsilon\}$ represents

$$(q, Y) \in \delta(p, a, X)$$

Consider the following pushdown automaton over the input alphabet $\Sigma = \{a, b\}$ and stack alphabet $\Gamma = \{\#, A\}$.



The number of strings of length 100 accepted by the above pushdown automaton is $= 50$.

[2021(Set-1): 2 Marks]

$a^m b^n \mid m+n=100$

$\{a^{100}, a^{99}b, a^{98}bb, \ldots, a^{51}b^{49}\}$

$m>n$

100	0
99	1
98	2
⋮	⋮
51	49

50 string

$\{a^m b^n \mid m>n\geq 0\}$

Q

Suppose that L_1 is a regular language and L_2 is a context-free language. Which one of the following languages is NOT necessarily context-free?

[2021(Set-1): 2 Marks]

Reg.CFL → CFL

(A) $L_1 \cdot L_2$

Reg ∪ CFL → CFL

(B) $L_1 \cup L_2$

(C) $L_1 - L_2$

(D) $L_1 \cap L_2$

Reg ∩ CFL ⇒ CFL

Reg − CFL = Reg ∩ $\overline{\text{CFL}}$ ⇒ Need not CFL

Q

For a string w, we define w^R to be the reverse of w. For example, if w = 01101 then w^R = 10110. Which of the following languages is/are context-free?

[2021(Set-2): 2 Marks]

(A) $\{wxw^Rx^R \mid w, x \in \{0, 1\}^*\}$

(B) $\{wxw^R \mid w, x \in \{0, 1\}^*\} = (0+1)^* \rightarrow$ Reg $\rightarrow$ CFL

(C) $\{ww^Rxx^R \mid w, x \in \{0, 1\}^*\} \rightarrow$ CFL

(D) $\{wxx^Rw^R \mid w, x \in \{0, 1\}^*\}$

Not DCFLs

Q

Let L_1 be a regular language and L_2 be a context-free language. Which of the following languages is/are context-free?

[2021(Set-2)MSQ: 1 Marks]

(A) $L_1 \cap \overline{L_2}$ $= Reg \cap \overline{CFL} \Rightarrow CSL$

(B) $\overline{\overline{L_1} \cup \overline{L_2}}$ $\Rightarrow L_1 \cap L_2 \Rightarrow Reg \cap CFL \Rightarrow CFL$

(C) $L_1 \cup \underbrace{(L_2 \cup \overline{L_2})}$ $= Reg \cup \Sigma^* = \Sigma^* \Rightarrow Reg \Rightarrow CFL$

(D) $(L_1 \cap L_2) \cup (\overline{L}_1 \cap L_2)$ $\Rightarrow \underbrace{(Reg \cap CFL)}_{CFL} \cup \underbrace{(\overline{Reg} \cap CFL)}_{CFL} \Rightarrow CFL$

Q

Consider the following languages:

$L_1 = \{a^n w a^n \mid w \in \{a, b\}^*\}$ $= w = (a+b)^*$

$L_2 = \{w x w^R \mid w, x \in \{a, b\}^*\}, |w|, |x| > 0\}$ $= axa + bxb \mapsto$ Regular

Note that w^R is the reversal of the string w. Which of the following is/are TRUE?

Note: Every Regular is CFL

[2022: MSQ: 2 Marks]

(A) L_1 and L_2 are regular.

(B) L_1 and L_2 are context-free.

(C) L_1 is regular and L_2 is context-free.

(D) L_1 and L_2 are context-free but not regular.

Q

Consider the following languages: $\rightarrow \overline{L_1}$ is CFL

$L_1 = \{ww \mid w \in \{a, b\}^*\}$ $\rightarrow$ Not CFL

$L_2 = \{a^n b^n c^m \mid m, n \geq 0\}$ $\rightarrow a^n b^n c^* \rightarrow$ CFL $\quad L_2 \cap L_3 = a^n b^n c^n \rightarrow$ CSL

$L_3 = \{a^m b^n c^n \mid m, n \geq 0\}$ $\rightarrow a^* b^n c^n \rightarrow$ CFL

Which of the following statements is/are FALSE?

[2022: 2 Marks]

(A) L_1 is not context-free but L_2 and L_3 are deterministic context-free. T

(B) Neither L_1 nor L_2 is context-free. F

(C) L_2, L_3 and $L_2 \cap L_3$ all are context-free. F

(D) Neither L_1 nor its complement is context-free F

Q

Which of the following are decidable?

1. Whether the intersection of two regular languages is infinite
2. Whether a given context-free language is regular
3. Whether two push-down automata accept the same language
4. Whether a given grammar is context-free

[2008: 1 Marks]

(A) 1 and 2

(B) 1 and 4

(C) 2 and 3

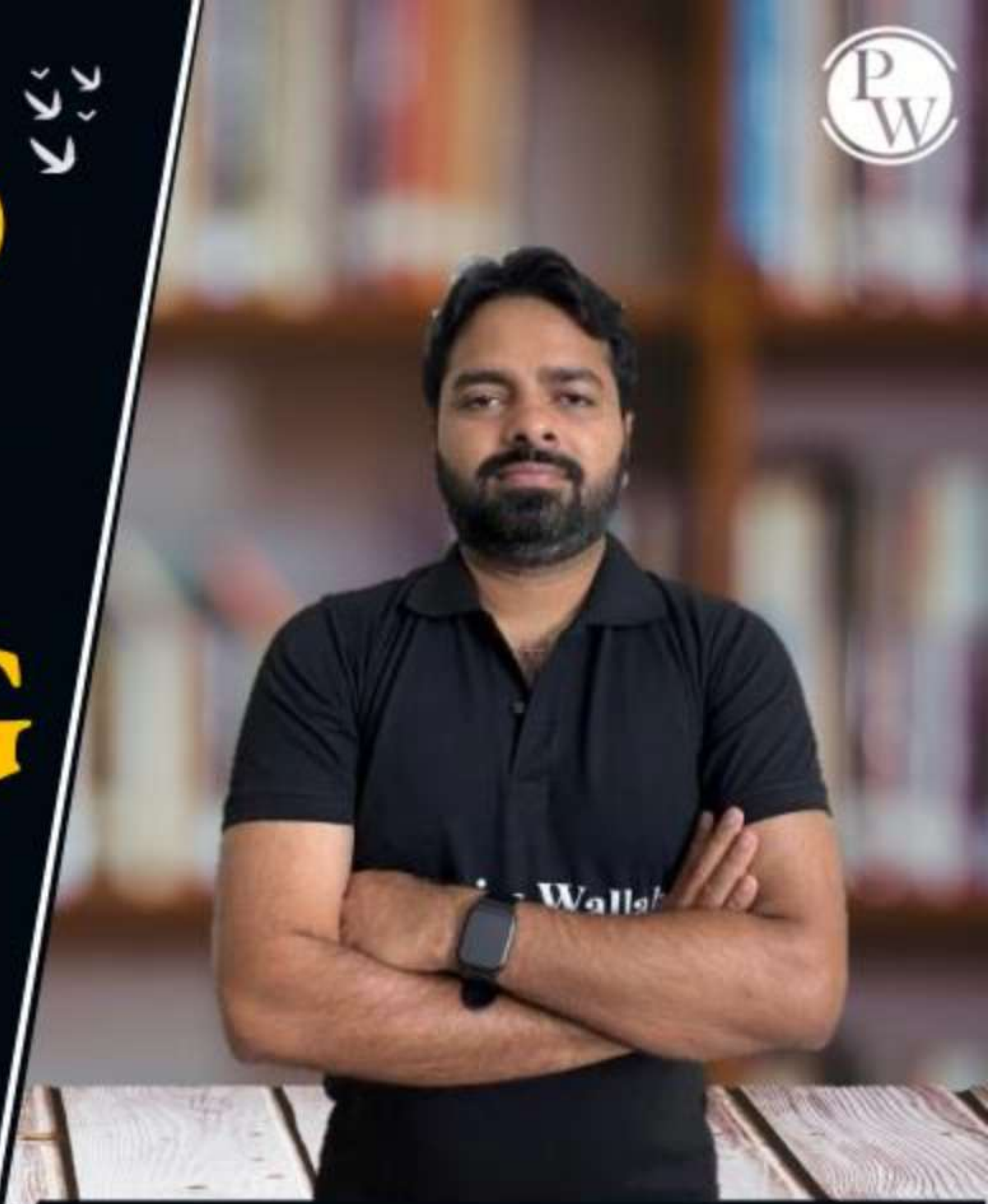
(D) 2 and 4

THANK - YOU

MISSION ISRO CS & IT ENGINEERING

Theory of Computation

Lecture No.- 05



By- Mallesham Devasane Sir

Topics to be Covered

Topic TM & Undecidability

TM & Undecidability

TM $\Rightarrow$ REL

$\Uparrow$

HTM $\Rightarrow$ Decidable Language

$\Uparrow$

LBA $\Rightarrow$ CSL

$\Uparrow$

PDA $\Rightarrow$ CFL

$\Uparrow$

DPDA $\Rightarrow$ DCFL

$\Uparrow$

FA $\Rightarrow$ Regular

$w \longrightarrow \boxed{\text{TM}}$

L

If $w \in L$, always halts at final

There are Two types of TM

(a) DTM (Deterministic TM)

$$\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \left\{ \underset{\text{Left}}{\overset{L}{\downarrow}}, \underset{\text{Right}}{\overset{R}{\downarrow}} \right\}$$

(b) NTM (Non- Deterministic TM)

$$\delta : Q \times \Gamma \rightarrow 2^{Q \times \Gamma \times \{L, R\}}$$

Equivalence of various TMs

$TM \cong$ Single tape TM

$TM \cong$ One-way infinite tape TM

$TM \cong$ Two-way infinite tape TM

$TM \cong$ Multi tape and multi head TM

$TM \cong$ Universal TM

$TM \cong$ Two stack PDA

$TM \cong$ Multi stack PDA

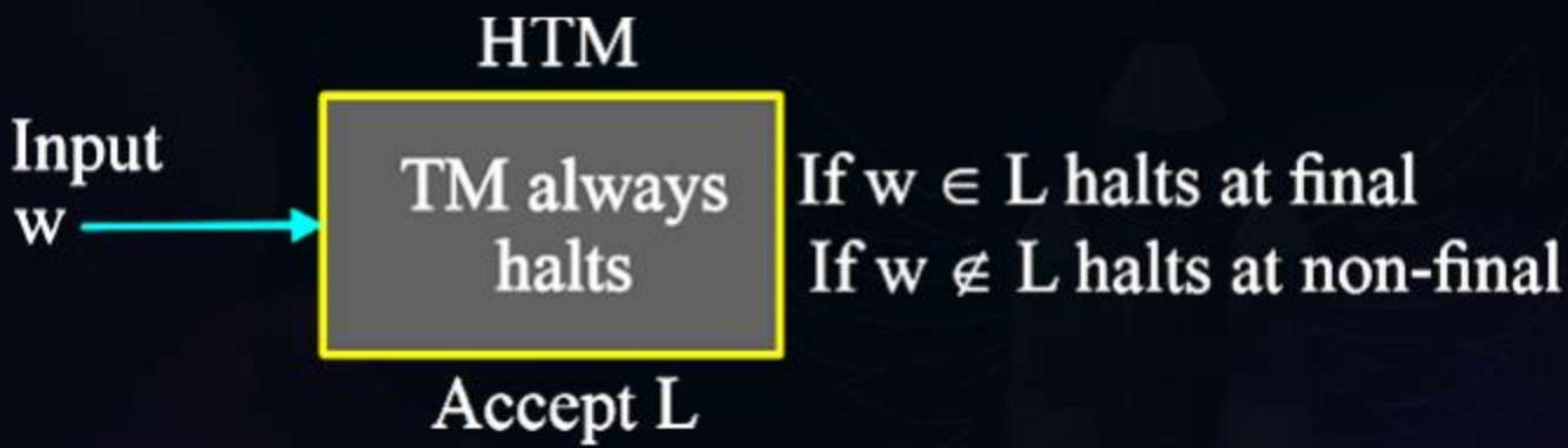
$TM \cong$ FA with two stacks

$TM \cong$ FA + R/W tape + Bidirectional head

Recursive Language

- Recursive language is acceptable by HTM, and hence acceptable by TM.
- Recursive also called as decidable language.
- Recursive also called as Turing decidable language.

If TM always halts, then TM is called as HTM.



Note :

1. Union:

$$\text{REL} \cup \text{Finite} \Rightarrow \text{REL}$$
$$\text{REL} \cup \text{Regular} \Rightarrow \text{REL}$$
$$\text{REL} \cup \text{CFL} \Rightarrow \text{REL}$$
$$\text{REL} \cup \text{Recursive} \Rightarrow \text{REL}$$
$$\text{REL}_1 \cup \text{REL}_2 \Rightarrow \text{REL}$$

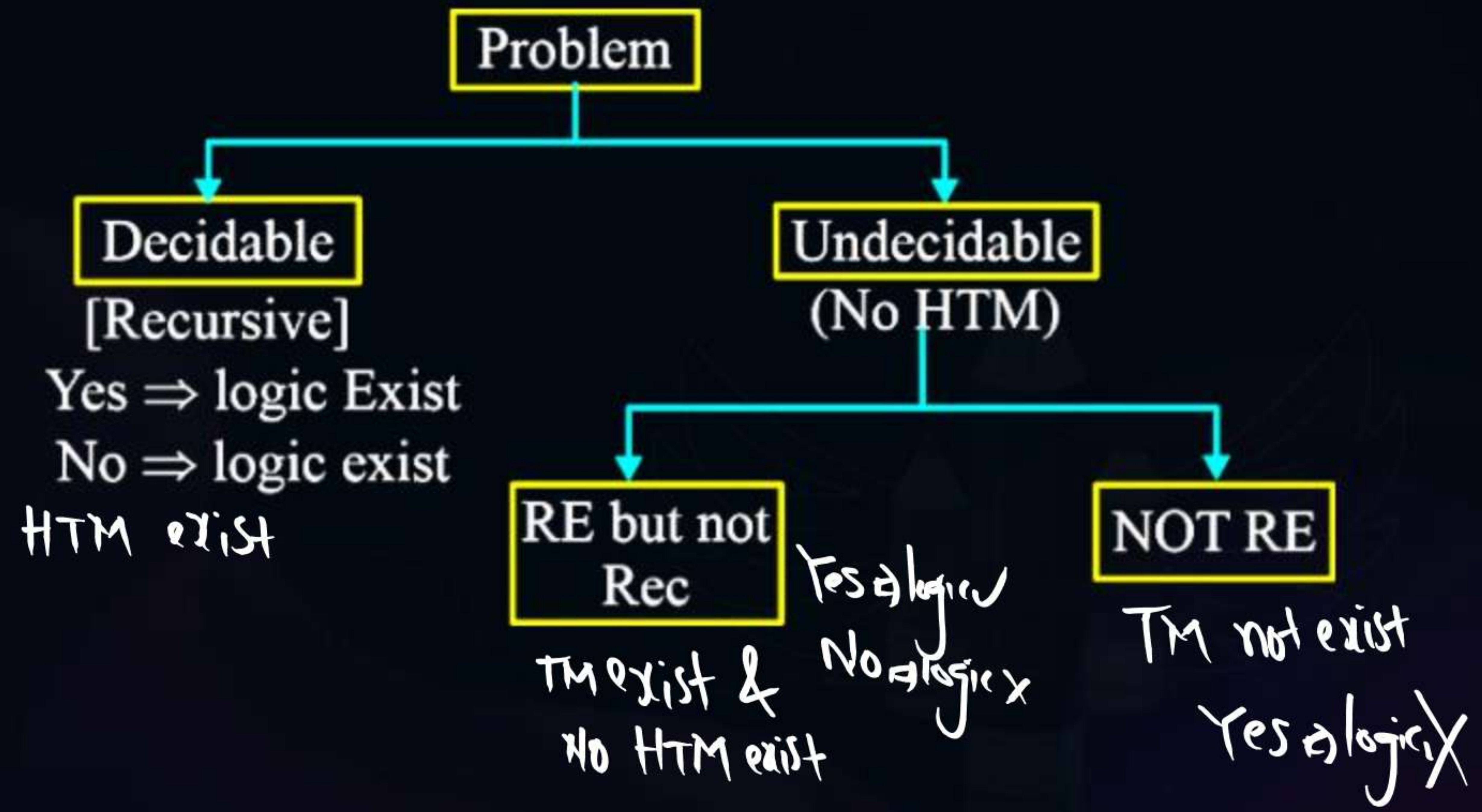
$$\text{REL} \cup \begin{array}{c} \text{Fin} \\ \hline \text{Reg} \\ \hline \text{CFL} \\ \hline \text{CSL} \\ \hline \text{Rec} \\ \hline \text{REL} \end{array} = \text{REL}$$

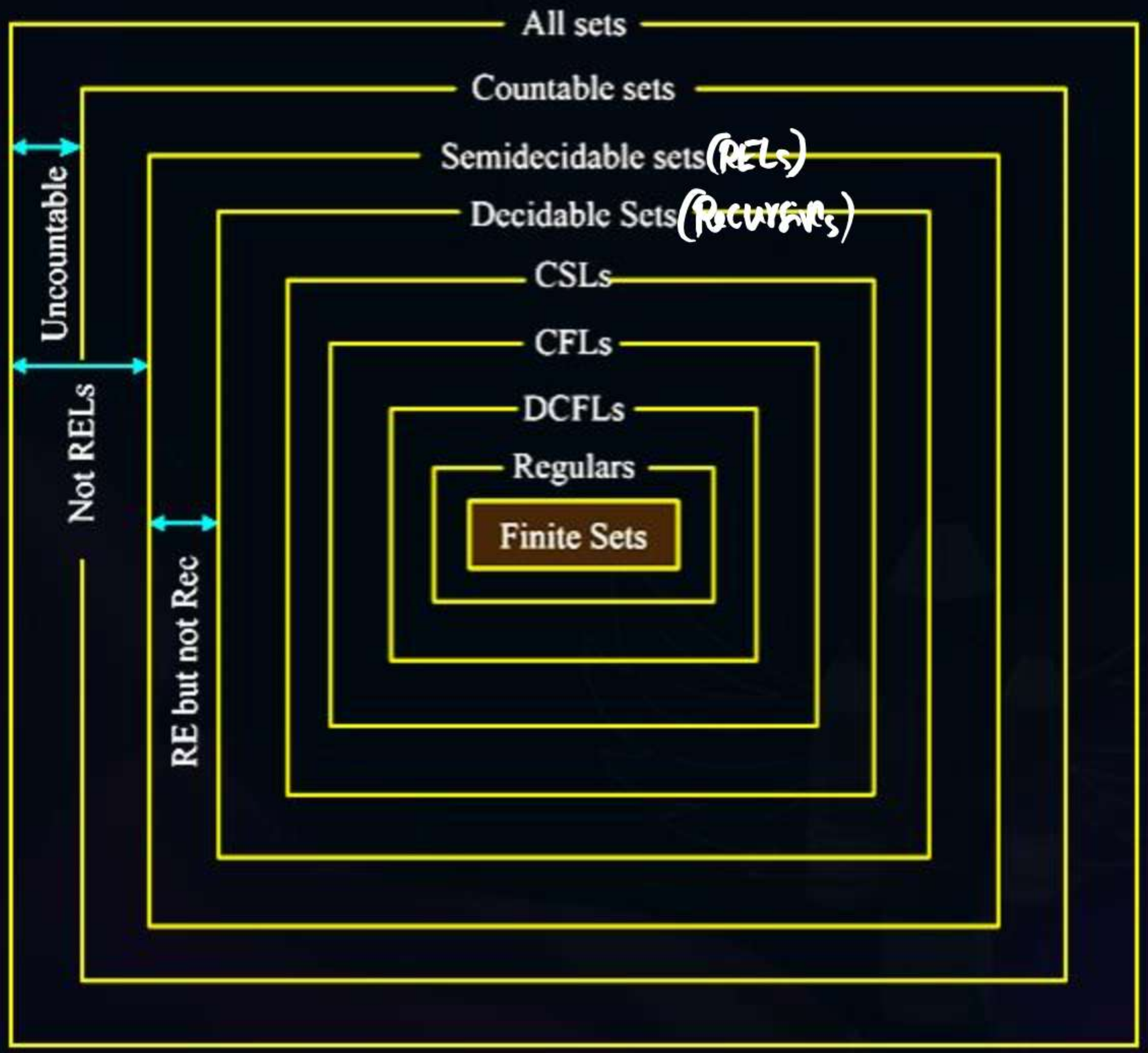
2. Intersection:

$$\text{REL} \cap \text{Finite} \Rightarrow \text{REL (Finite)}$$
$$\text{REL} \cap \text{CFL} \Rightarrow \text{REL}$$
$$\text{REL} \cap \text{Rec} \Rightarrow \text{REL}$$
$$\text{REL}_1 \cap \text{REL}_2 \Rightarrow \text{REL}$$

$$\text{REL} \cap \text{"} = \text{REL}$$

Decidable and Undecidable




All sets
Countable sets
Semidecidable sets(RELs)
Decidable Sets(Recursives)
CSLs
CFLs
DCFLs
Regulars
Finite Sets
Uncountable
Not RELs
RE but not Rec

Decision Properties Table

- D: Decidable
- UD: Undecidable

	FA	DPDA	PDA	LBA/HTM	TM
H (Halting)	D	D	D	D	UD
M (Membership)	D	D	D	D	UD
E_m (Emptiness)	D	D	D	UD	UD
F (Finiteness)	D	D	D	UD	UD
T (Totality)	D	D	UD	UD	UD
E_q (Equivalence)	D	D	UD	UD	UD
D (Disjoint)	D	UD	UD	UD	UD
S (Set Containment)	D	UD	UD	UD	UD

Classification of Languages based on Decidability

All RE languages can be classified into 3 important classes.

1 Regarding the power of recognition of languages, which of the following statement is false?

(GATE - 98)

(a) The non-deterministic finite-state automata are equivalent to deterministic finite-state automata. T

(b) Non-deterministic Push-down automata are equivalent to deterministic Push-down automata. F

(c) Non-deterministic Turing machines are equivalent to deterministic Turing machines. T

(d) Multi-tape Turing machines are equivalent to Single-tape Turing machines. T

NFA $\cong$ DFA

PDA $\ncong$ DPDA

NTM $\cong$ DTM

Multitape TM $\cong$ Single Tape TM

3

If the strings of a language L can be effectively enumerated in lexicographic (i.e., alphabetic) order, which of the following statements is true? (GATE - 03)

(a) L is necessarily finite

(b) L is regular but not necessarily finite

(c) L is context free but not necessarily regular

(d) L is recursive but not necessarily context free

L is enumerable effectively

iff

L is Recursive Language (Decidable)

L is enumerable

iff

L is REL

4

Which of the following is true for the language $\{a^p \mid p \text{ is prime}\}$? **(GATE - 08)**

(a) It is not accepted by a Turing machine ✗

(b) It is regular but not context-free ✗

(c) It is context-free but not regular ✗

(d) It is neither regular nor context-free, but accepted by a Turing machine ✓

$\{a^1, a^2, a^3, a^5, a^7, a^{11}, \ldots\}$

→ Not regular

→ Not CFL

→ CSL

→ Recursive

→ REL

5 Which of the following statements is/are **FALSE**?

1. For every non–deterministic Turing machine, there exists an equivalent deterministic Turning machine T
2. Turing recognizable languages are closed under union and complementation F
3. Turing decidable languages are closed under intersection and complementation T
4. Turing recognizable languages are closed under union and intersection T **(GATE - 13)**

(a) 1 and 4 only (b) 1 and 3 only

(c) 2 only (d) 3 only

NTM $\Rightarrow$ DTM

RELs closed under $\cup, \cap$
not closed under complement

Recursives closed under
$\cup, \cap$, complement

6 For any two languages L_1 and L_2 such that L_1 is context-free and L_2 is recursively enumerable but not recursive, which of the following is/are necessarily true?

I. $\overline{L}_1$ (complement of L_1) is recursive ✓

II. $\overline{L}_2$ (complement of L_2) is recursive ✗

III. $\overline{L}_1$ is context – free ✗

IV. $\overline{L}_1 \cup L_2$ is recursively enumerable ✓

(GATE – 15 – SET1)

(a) I only
(b) III only
(c) III and IV only
(d) I and IV only ✓

L_1 : CFL
L_2 : RE but not Rec

$\overline{\text{CFL}}$ is CSL
$\overline{\text{RE but not Rec}}$ is Not REL

CSL ∪ RE but not Rec ⟹ RE

7

Consider the following types of languages:

L_1: Regular, L_2: Context-free, L_3: Recursive, L_4: Recursively enumerable.

Which of the following is/are TRUE?

I. $\overline{L_3} \cup L_4$ is recursively enumerable $\overline{Rec} \cup RE \Rightarrow Rec \cup RE \Rightarrow RE$

II. $\overline{L_2} \cup L_3$ is recursive $\overline{CFL} \cup Rec \Rightarrow CSL \cup Rec \Rightarrow Rec$

III. $L_1^* \cap L_2$ is context-free $Reg^* \cap CFL \Rightarrow Reg \cap CFL \Rightarrow CFL$

IV. $L_1 \cup \overline{L_2}$ is context-free $Reg \cup \overline{CFL} \Rightarrow Reg \cup CSL \Rightarrow CSL$

(GATE – 16 – SET2)

(a) I only

(b) I and III only

(c) I and IV only

(d) I, II and III only

8

Let L_1 be a recursive language. Let L_2 and L_3 be languages that are "recursively enumerable but not recursive". Which of the following statement is not necessarily true?

(GATE - 10)

T (a) $L_2 - L_1$ is recursively enumerable.

False (b) $L_1 - L_3$ is recursively enumerable.

T (c) $L_2 \cap L_3$ is recursively enumerable.

T (d) $L_2 \cup L_3$ is recursively enumerable.

$L_1 - L_2 = L_1 \cap \overline{L_2}$

RE but not Rec − Rec ⇒ RE but not Rec $\cap$ $\overline{\text{Rec}}$ ⇒ RE

Rec − RE but not Rec ⇒ Rec $\cap$ $\overline{\text{RE but not Rec}}$ ⇒ Rec $\cap$ Not RE ⇒ Any

9

Recursive languages are: (GATE - 90)

(a) A proper superset of context free languages. X

(b) Always recognizable by pushdown automata. X

(c) Also called Type (0) languages. ?

(d) Recognizable by Turing machines.

Set of Recursive languages

Recursive languages are

Set of Recursive language is Superset of Set of CFLs

10

In which of the cases stated below is the following statement true?

"For every nondeterministic machine M_1 there exists an equivalent deterministic machine M_2 recognizing the same language".

(GATE - 92)

(a) M_1 is nondeterministic finite automation NFA ⇒ DFA

(b) M_1 is a nondeterministic PDA NPDA ⇒ DPDA X

(c) M_1 is a nondeterministic Turing machine NTM ⇒ DTM

(d) For no machine M_1 use the above statement true

11

Which of the following conversions is not possible (algorithmically)?

(GATE - 94)

(a) Regular grammar to context-free grammar Algo exist

(b) Non-deterministic FSA to deterministic FSA

NFA ⇒ DFA Subset construction

(c) Non-deterministic PDA to deterministic PDA

$\boxed{\text{PDA} \Rightarrow \text{DPDA}}$ No algo

(d) Non-deterministic Turing machine to deterministic Turing machine.

NTM ⇒ DTM Algo exist

12

Which of the following is true? **(GATE - 02)**

(a) The complement of a recursive language is recursive.

(b) The complement of a recursively enumerable language is recursively enumerable.

(c) The complement of a recursive language is either recursive or recursively enumerable.

(d) The complement of a context-free language is context-free.

$\overline{Rec} \Rightarrow Rec \Rightarrow RE$

13

The C language is (GATE - 02)

(a) A context free language

(b) A context sensitive language

(c) A regular language

(d) Parsable fully only by a Turing machine

14

ε: Halts
0
1
00
01
10
11

A single tape Turing Machine M has two states q_0 and q_1 of which q_0 is the starting state. The tape alphabet of M is {0, 1, B} and its input alphabet is {0, 1}. The symbol B is the blank symbol used to indicate end of an input string. The transition function of M is described in the following table.

	0	1	B
→ q_0	q_1, 1, R	q_1, 1, R	Halt
q_1	q_1, 1, R	q_0, 1, L	q_0, B, L

(a) M does not halt on any string in $(0 + 1)^+$

(b) M does not halt on any string in $(00 + 1)^*$

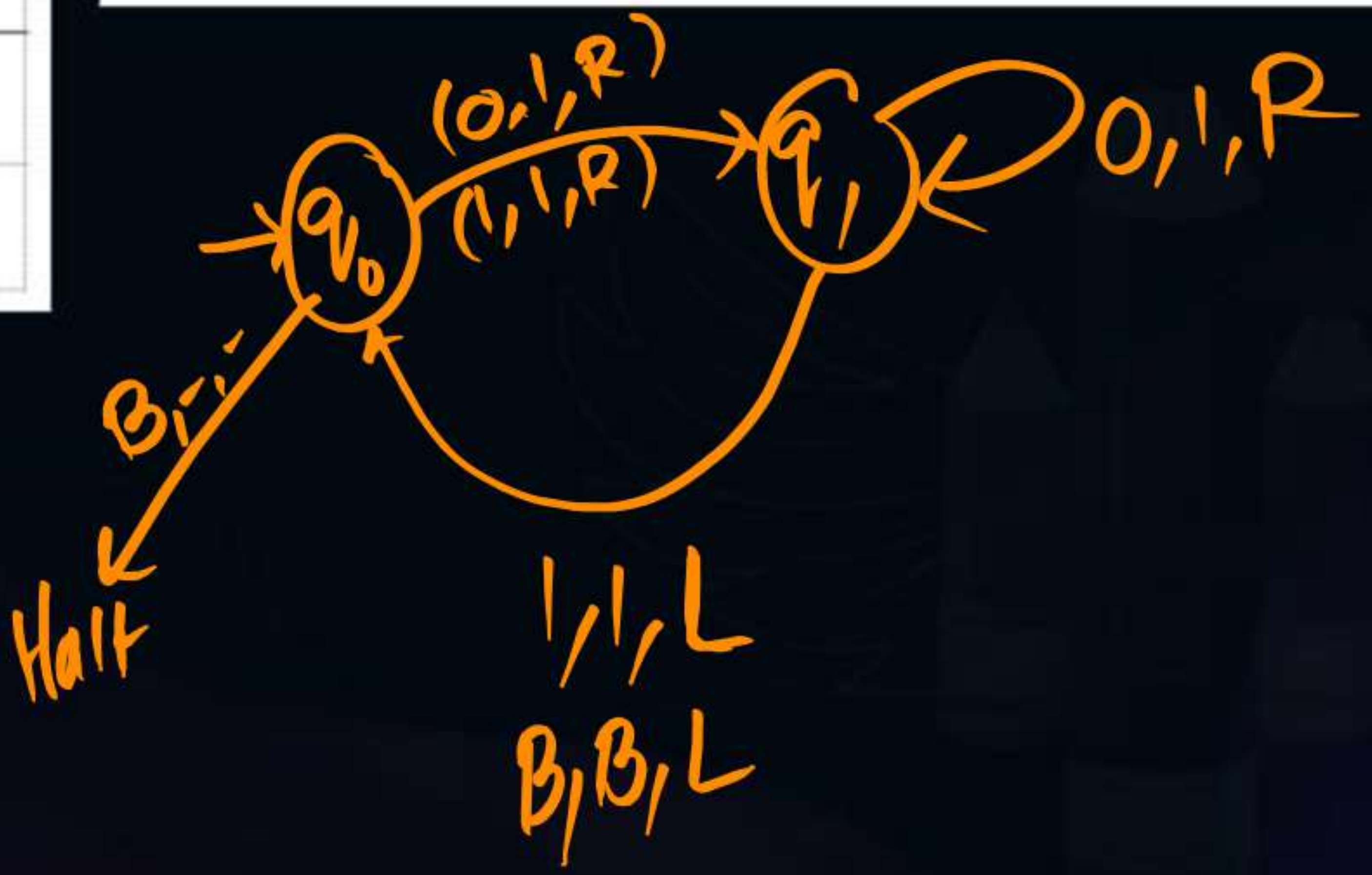
(c) M halts on all string ending in a 0

(d) M halts on all string ending in a 1

The table is interpreted as illustrated below.

The entry (q_1, 1, R) in row q_0 and column 1 signifies that if M is in state q_0 and reads 1 on the current tape square, then it writes 1 on the same tape square, moves its tape head one position to the right and transitions to state q_1.

Which of the following statements is true about M? **(GATE - 03)**



15

Define languages L_0 and L_1 as follows

$L_0 = \{<M, w, 0> \mid M \text{ halts on } w\}$

$L_1 = \{<M, w, 1> \mid M \text{ does not halts on } w\}$

Here $<M, w, i>$ is a triplet, whose first component, M is an encoding of a Turing Machine, second component, w, is a string, and third component, i, is a bit.

Let $L = L_0 \cup L_1$. Which of the following is true? **(GATE - 03)**

(a) L is recursively enumerable, but $\overline{L}$ is not

(b) $\overline{L}$ is recursively enumerable, but L is not

(c) Both L and $\overline{L}$ are recursive

(d) Neither L nor $\overline{L}$ is recursively enumerable

16 L_1 is a recursively enumerable language over Σ. An algorithm A effectively enumerates its words as $w_1, w_2, w_3, \ldots$.

Define another language L_2 over $\Sigma \cup \{\#\}$ as $\{w_i\#w_j : w_i, w_j \in L_1, i<j\}$. Here # is a new symbol. Consider the following assertions:

S_1: L_1 is recursive implies L_2 is recursive

S_2: L_2 is recursive implies L_1 is recursive

Which of the following statements is true?

(GATE – 04)

(a) Both S_1 and S_2 are true.

(b) S_1 is true but S_2 is not necessarily true

(c) S_2 is true but S_1 is not necessarily true

(d) Neither is necessarily true

17 Let L_1 be a recursive language and let L_2 be a recursively enumerable but not a recursive language. Which one of the following is TRUE? **(GATE - 05)**

(a) $\overline{L_1}$ is recursive and $\overline{L_2}$ is recursively enumerable

(b) $\overline{L_1}$ is recursive and $\overline{L_2}$ is not recursively enumerable

(c) $\overline{L_1}$ and $\overline{L_2}$ are recursively enumerable

(d) $\overline{L_1}$ is recursively enumerable and $\overline{L_2}$ is recursive

18 For $s \in (0+1)^*$, let $d(s)$ denote the decimal value of s (e. g. $d(101) = 5$)

Let $L = \{s \in (0+1)^* \mid d(s) \bmod 5 = 2$ and $d(s) \bmod 7 \neq 4\}$

Which one of the following statement is true?

(GATE - 06)

(a) L is recursively enumerable, but not recursive

(b) L is recursive, but not context-free

(c) L is context-free, but not regular

(d) L is regular

19

Let L_1 be a regular language, L_2 be a deterministic context-free language and L_3 be a recursively enumerable, but not recursive language. Which one of the following statement is false? **(GATE - 06)**

(a) $L_1 \cap L_2$ is a deterministic CFL

(b) $L_3 \cap L_1$ is recursive

(c) $L_1 \cup L_2$ is context-free

(d) $L_1 \cap L_2 \cap L_3$ is recursively enumerable

20

If L and $\overline{L}$ are recursively enumerable then L is (GATE - 08)

(a) Regular
(b) Context-free
(c) Context-sensitive
(d) Recursive.

21

Let L be a language and $\overline{L}$ be its complement. Which one of the following is NOT a viable possibility?

(GATE – 14-SET1)

(a) Neither L nor $\overline{L}$ is recursively enumerable (r.e).

(b) One of L and $\overline{L}$ is r.e. but not recursive; the other is not r.e.

(c) Both L and $\overline{L}$ are r.e. but not recursive.

(d) Both L and $\overline{L}$ are recursive.

22

Let X be a recursive language and Y be a recursively enumerable but not recursive language. Let W and Z be two languages such that $\overline{Y}$ reduces to W, and Z reduces to $\overline{X}$ (reduction means the standard many-one reduction). Which one of the following statements is TRUE? **(GATE – 16 – SET1)**

(a) W can be recursively enumerable and Z is recursive.

(b) W can be recursive and Z is recursively enumerable.

(c) W is not recursively enumerable and Z is recursive.

(d) W is not recursively enumerable and Z is not recursive.

23 Let A and B be finite alphabets and let # be a symbol outside both A and B. Let f be a total function from A^* to B^*. We say f is computable if there exists a Turing machine M which given an input x in A^*, always halts with f(x) on its tape. Let L_f denote the language $\{x\#f(x) | x \in A^*\}$.

Which of the following statements is true:

(GATE – 17 – SET1)

(a) f is computable if and only if L_f is recursive

(b) f is computable if and only if L_f is recursively enumerable

(c) If f is computable then L_f is recursive, but not conversely

(d) If f is computable then L_f is recursively enumerable, but not conversely

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Which of the following statements are true?

(GATE - 92)

(a) Union of two recursive languages is recursive

(b) The language $\{0^n | n \text{ is prime}\}$ is not regular

(c) Regular languages are closed under infinite union

25 Let $A \leq_m B$ denotes that language A is mapping reducible (also known as many-to-one reducible) to language B. Which one of the following is FALSE?

(GATE – 14-SET2)

(a) If $A \leq_m B$ and B is recursive then A is recursive.

(b) If $A \leq_m B$ and A is undecidable then B is un-decidable.

(c) If $A \leq_m B$ and B is recursively enumerable then A is recursively enumerable.

(d) If $A \leq_m B$ and B is not recursively enumerable then A is not recursively enumerable.

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GATE2020

Which of the following languages are undecidable? Note that $\langle M \rangle \langle M \rangle$ indicates encoding of the Turing machine M.

$L1=\{\langle M \rangle | L(M)=\emptyset\}$

$L2=\{\langle M,w,q \rangle |$M on input w reaches state q in exactly 100 steps

$L3=\{\langle M \rangle | L(M)$ is not recursive

$L4=\{\langle M \rangle | L(M)$contains at least 21 members

A. L1, L3, and L4 only
B. L1 and L3 only
C. L2 and L3 only
D. L2, L3, and L4 only

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GATE2018

The set of all recursively enumerable languages is:

A. closed under complementation
B. closed under intersection
C. a subset of the set of all recursive languages
D. an uncountable set

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GATE2019

Consider the following sets:

S1: Set of all recursively enumerable languages over the alphabet $\{0,1\}$

S2: Set of all syntactically valid C programs

S3: Set of all languages over the alphabet $\{0,1\}$

S4: Set of all non-regular languages over the alphabet $\{0,1\}$

Which of the above sets are uncountable?

A. S1 and S2
B. S3 and S4
C. S2 and S3
D. S1 and S4

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GATE2018

Consider the following problems. L(G) denotes the language generated by a grammar G. L(M) denotes the language accepted by a machine M.

I. For an unrestricted grammar G and a string w, whether $w \in L(G)$
II. Given a Turing machine M, whether L(M) is regular
III. Given two grammarG1 and G2, whether L(G1)=L(G2)
IV. Given an NFA N, whether there is a deterministic PDA P such that N and P accept the same language

Which one of the following statement is correct?

A. Only I and II are undecidable
B. Only II is undecidable
C. Only II and IV are undecidable
D. Only I, II and III are undecidable

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GATE2017

Let L(R) be the language represented by regular expression R. Let L(G) be the language generated by a context free grammar G. Let L(M) be the language accepted by a Turing machine M. Which of the following decision problems are undecidable?

I. Given a regular expression R and a string w, is $w \in L(R)$?
II. Given a context-free grammar G, is $L(G) = \emptyset$
III. Given a context-free grammar G, is $L(G) = \Sigma*$ for some alphabet Σ?
IV. Given a Turing machine M and a string w, is $w \in L(M)$?

A. I and IV only
B. II and III only
C. II, III and IV only
D. III and IV only

THANK - YOU