1. Powers and Roots

(i) $a^0 = 1;\ a \neq 0$

(ii) $a^m a^n = a^{m+n}$

(iii) $\dfrac{a^m}{a^n} = a^{m-n}$

(iv) $(ab)^m = a^m b^m$

(v) $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$

(vi) $\left(a^m\right)^n = a^{mn}$

(vii) $a^{-m} = \dfrac{1}{a^m}$

(viii) $\sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}$

(ix) $\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$

(x) $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

2. Logarithms

$$e = \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = 2.71828182846\ldots$$

(i) $\log_a 1 = 0$

(ii) $\log_a a = 1$

(iii) $\log_a(mn) = \log_a m + \log_a n$

(iv) $\log_a\left(\dfrac{m}{n}\right) = \log_a m - \log_a n$

(v) $\log_a\left(m^n\right) = n\log_a m$

(vi) $\log_b a = \dfrac{1}{\log_a b}$

(vii) $\log_{(a^k)}(m) = \dfrac{1}{k}\log_a m$

(viii) $\log_a m = \log_b m \cdot \log_a b$ where $b > 0$ and $b \neq 1$

(ix) $\log_a m = \dfrac{\log_b m}{\log_b a}$

(x) $x^{\log_a y} = y^{\log_a x}$

(xi) $x = x^{\log_a a} = a^{\log_a x}$

(xii) $x = e^{\ln x} = \ln e^x$

Product Formulas

(a) $(a+b)^2 = a^2 + 2ab + b^2$

(b) $(a-b)^2 = a^2 - 2ab + b^2$

(c) $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

(d) $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

Factoring Formulas

(a) $a^2 - b^2 = (a-b)(a+b)$

(b) $a^3 - b^3 = (a-b)\left(a^2 + ab + b^2\right)$

(c) $a^3 + b^3 = (a+b)\left(a^2 - ab + b^2\right)$

(d) $a^{2n} - b^{2n} = \left(a^n - b^n\right)\left(a^n + b^n\right)$

(e) $a^n - b^n = (a-b)\left(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \ldots + ab^{n-2} + b^{n-1}\right)$

Formulas for summation

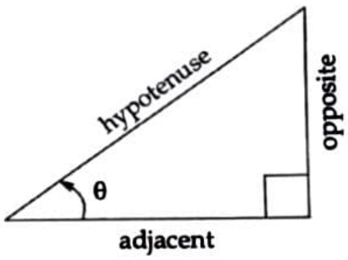
1. $1 + 2 + \ldots + n = \dfrac{n(n+1)}{2}$
2. $1 + 3 + \ldots + (2n-1) = n^2$
3. $2 + 4 + \ldots + (2n) = n(n+1)$
4. $1^2 + 2^2 + \ldots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$
5. $1^3 + 2^3 + \ldots + n^3 = \dfrac{n^2(n+1)^2}{4}$

Definition of Trigonometric Functions

$\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$ $\quad \csc\theta = \dfrac{\text{hyp}}{\text{opp}}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$ $\quad \sec\theta = \dfrac{\text{hyp}}{\text{adj}}$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}}$ $\quad \cot\theta = \dfrac{\text{adj}}{\text{opp}}$

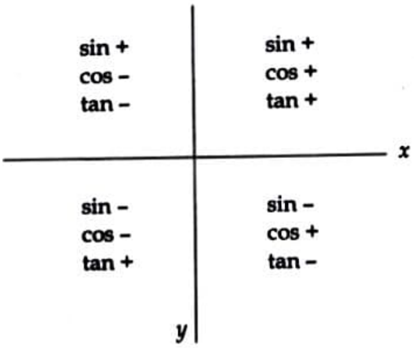


Trigonometric Functions of common angles

θ	0	30	45	60	90
sinθ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cosθ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tanθ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

Signs of Trig Functions by Quadrant (Quadrant rule)

Signs of Trig Function by Quadrant



Important Identities

(i) Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

(ii) Even-Odd Identities

$$\sin(-x) = -\sin x$$

$$\cos(-x) = \cos x$$

$$\tan(-x) = -\tan x$$

(iii) Sum-Difference Formulas

$$\sin(x+y)=\sin x\cos y+\cos x\sin y$$

$$\sin(x-y)=\sin x\cos y-\cos x\sin y$$

$$\cos(x+y)=\cos x\cos y-\sin x\sin y$$

$$\cos(x-y)=\cos x\cos y+\sin x\sin y$$

$$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$$

$$\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$$

Double Angle Formulas

$$\sin 2x = 2\sin x\cos y$$

$$\begin{aligned}\cos 2x &= \cos^2 x-\sin^2 x\\ &=2\cos^2 x-1\\ &=1-2\sin^2 x\end{aligned}$$

$$\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$$

$$\sin(2x)=\frac{2\tan(x)}{1+\tan^2(x)}$$

$$\cos 2x=\frac{1-\tan^2 x}{1+\tan^2 x}$$

Power-Reducing/Half Angle Formulas

$$\sin^2(x)=\frac{1-\cos(2x)}{2}$$

$$\cos^2(x)=\frac{1+\cos(2x)}{2}$$

$$\tan^2(x)=\frac{1-\cos(2x)}{1+\cos(2x)}$$

Product-to-Sum Formulas

$$\sin(x)\sin(y)=\frac{1}{2}[\cos(x-y)-\cos(x+y)]$$

$$\cos(x)\cos(y)=\frac{1}{2}[\cos(x-y)+\cos(x+y)]$$

$$\sin(x)\cos(y)=\frac{1}{2}[\sin(x+y)+\sin(x-y)]$$

$$\cos(x)\sin(y)=\frac{1}{2}[\sin(x+y)-\sin(x-y)]$$

Sum-to-Product Formulas

$$\sin(x)+\sin(y)=2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin(x)-\sin(y)=2\sin\left(\frac{x-y}{2}\right)\cos\left(\frac{x+y}{2}\right)$$

$$\cos(x)+\cos(y)=2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\cos(x)-\cos(y)=-2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

Multiple Angle Formulas

$$\sin 3x = 3\sin x - 4\sin^3 x$$

$$\cos 3x = 4\cos^3 x - 3\cos x$$

$$\tan 3x = \frac{3\tan x - \tan^3 x}{1-3\tan^2 x}$$

Multiple Angle Formulas

$$\sin 3x = 3\sin x - 4\sin^3 x$$

$$\cos 3x = 4\cos^3 x - 3\cos x$$

$$\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$$

Relations to Hyperbolic functions

$$\sin ix = i\sin hx$$

$$\cos ix = \cos hx$$

$$\sec ix = \sec hx$$

$$\csc ix - i\csc hx$$

$$\tan ix = i\tan hx$$

$$\cot ix = -i\cot hx$$

Linear Equation in One Variable

$$ax + b = 0$$

$$x = -\frac{b}{a}$$

Quadratic Equation

$$ax^2 + bx + c = 0$$

Roots: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Discriminant : $D = b^2 - 4ac$

If $D > 0$ then Roots are real and distinct

If $D = 0$ then Roots are real and equal

If $D < 0$ then Roots are complex conjugates

1) FORMULAE

(a) $\lim_{x\to 0}\frac{\sin x}{x}=1$

(b) $\lim_{x\to 0}\frac{\tan x}{x}=1$

(c) $\lim_{x\to 0}\frac{e^x-1}{x}=1$

(d) $\lim_{x\to 0}\frac{\ln(1+x)}{x}=1$

(e) $\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}$

(f) $\lim_{x\to \infty}\frac{\sin x}{x}=0$

(g) $\lim_{x\to 0}(1+x)^{1/x}=e$

(h) $\lim_{x\to 0}(1+ax)^{1/x}=e^a$

(i) $\lim_{x\to \infty}\left(1+\frac{1}{x}\right)^x=e$

(j) $\lim_{x\to \infty}\left(1+\frac{a}{x}\right)^x=e^a$

(k) $\lim_{x\to \infty}(x)^{1/x}=1$

(l) $\lim_{x\to 0}(x)^x=1$

2) BASIC

(i) $\lim_{x\to a}[cf(x)]=c\lim_{x\to a}f(x)$

(ii) $\lim_{x\to a}[f(x)\pm g(x)]=\lim_{x\to a}f(x)\pm\lim_{x\to a}g(x)$

(iii) $\lim_{x\to a}[f(x)g(x)]=\lim_{x\to a}f(x)\lim_{x\to a}g(x)$

(iv) $\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right]=\frac{\lim_{x\to a}f(x)}{\lim_{x\to a}g(x)}$ provided $\lim_{x\to a}g(x)\neq 0$

(v) $\lim_{x\to a}[f(x)]^n=\left[\lim_{x\to a}f(x)\right]^n$

(vi) $\lim_{x\to a}\left[\sqrt[n]{f(x)}\right]=\sqrt[n]{\lim_{x\to a}f(x)}$

3) DERIVATIVE

(i) $\frac{d}{dx}\left[x^n\right]=nx^{n-1}$

(ii) $\frac{d}{dx}\left[a^x\right]=a^x\ln(a)$

(iii) $\frac{d}{dx}\left[e^x\right]=e^x$

(iv) $\frac{d}{dx}[\log_a x]=\frac{1}{x\ln(a)}$

(v) $\frac{d}{dx}[\ln x]=\frac{1}{x}$

(vi) $\frac{d}{dx}[\sin x]=\cos x$

(vii) $\frac{d}{dx}[\cos x]=-\sin x$

(viii) $\frac{d}{dx}[\tan x]=\sec^2 x$

(ix) $\frac{d}{dx}[\cot x]=-\csc^2 x$

(x) $\frac{d}{dx}[\sec x]=\sec x\tan x$

(xi) $\frac{d}{dx}[\csc x]=-\csc x\cot x$

(xii) $\frac{d}{dx}\left[\sin^{-1}x\right]=\frac{1}{\sqrt{1-x^2}}$

(xiii) $\frac{d}{dx}\left[\cos^{-1}x\right]=\frac{-1}{\sqrt{1-x^2}}$

(xiv) $\frac{d}{dx}\left[\tan^{-1}x\right]=\frac{1}{1+x^2}$

(xv) $\frac{d}{dx}\left[\sec^{-1}x\right]=\frac{1}{x\sqrt{x^2-1}}$

(xvi) $\frac{d}{dx}\left[\csc^{-1}x\right]=\frac{-1}{x\sqrt{x^2-1}}$

(xvii) $\frac{d}{dx}\left[\cot^{-1}x\right]=\frac{-1}{1+x^2}$

(xviii) $\frac{d}{dx}[\sin hx]=\cos hx$

(xix) $\frac{d}{dx}[\cos hx]=\sin hx$

(xx) **Product rule:** $\frac{d}{dx}[f(x)g(x)]=f(x)g'(x)+g(x)f'(x)$

(xxi) **Quotient rule:** $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{f'(x)g(x)-f(x)g'(x)}{[g(x)]^2}$

(xxii) **Chain rule:** $\frac{d}{dx}[f(g(x))]=f'(g(x))g'(x)$

Definite integral

1. $\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$

2. $\int_a^b f(x)dx = \int_a^b f(t)dt$

3. $\int_a^b f(x)dx = -\int_b^a f(x)dx$

4. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ where $a < c < b$

5. $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

6. $$\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_0^a f(x)\,dx, & \text{if } f(-x) = f(x); \quad \text{Even function} \\ 0, & \text{if } f(-x) = -f(x); \quad \text{Odd function} \end{cases}$$

7. $$\int_0^{2a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx, & \text{if } f(2a-x) = f(x) \\ 0, & \text{if } f(2a-x) = -f(x) \end{cases}$$

c formulae

$\int k\,dx = kx + C$	4.	$\int \frac{1}{x}\,dx = \ln(x) + C$ {for positive values of x only}
$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$	5.	$\int \frac{c}{ax+b}\,dx = \frac{c}{a}\ln(ax+b) + C$
$\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$ (for $n \neq -1$)		

ponential and Log

1.	$\int e^x\,dx = e^x + C$	2.	$\int a^x\,dx = \frac{a^x}{\ln a} + C$

1.	$\int \ln x\,dx = x\ln x - x + C$	2.	$\int \log_a x\,dx = x\log_a x - \frac{x}{\log a} + C$

igonometry

1.	$\int \sin x\,dx = -\cos x + C$	2.	$\int \cos x\,dx = \sin x + C$
3.	$\int \tan x\,dx = -\ln(\cos x) + C$ $= \ln(\sec x) + C$	4.	$\int \cot x\,dx = \ln(\sin x) + C$
5.	$\int \sec x\,dx = \ln(\sec x + \tan x) + C$ $= \log\tan\left(\frac{\pi}{4} + \frac{x}{2}\right) + C$	6.	$\int \operatorname{cosec} x\,dx = -\ln(\operatorname{cosec} x + \cot x) + C$ $= \log\tan\frac{x}{2}$

igonometry

1.	$\int \sin x\,dx = -\cos x + C$	2.	$\int \cos x\,dx = \sin x + C$
3.	$\int \tan x\,dx = -\ln(\cos x) + C$ $= \ln(\sec x) + C$	4.	$\int \cot x\,dx = \ln(\sin x) + C$
5.	$\int \sec x\,dx = \ln(\sec x + \tan x) + C$ $= \log\tan\left(\frac{\pi}{4} + \frac{x}{2}\right) + C$	6.	$\int \operatorname{cosec} x\,dx = -\ln(\operatorname{cosec} x + \cot x) + C$ $= \log\tan\frac{x}{2}$
7.	$\int \sec^2 x\,dx = \tan x + C$	8.	$\int \operatorname{cosec}^2 x\,dx = -\cot x + C$
9.	$\int \sec x \tan x\,dx = \sec x + C$	10.	$\int \operatorname{cosec} x \cot x\,dx = -\operatorname{cosec} x + C$

lgebraic

.	$\int \frac{1}{\sqrt{a^2 - x^2}}dx = \sin^{-1}\frac{x}{a} + C$	2.	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) = \frac{-1}{a}\cot^{-1}\left(\frac{x}{a}\right)$
.	$\int \frac{1}{x\sqrt{x^2 - a^2}}dx = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C$	4.	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\log\left(\frac{a+x}{a-x}\right) = \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right)$
5.	$\int \sqrt{x^2 - a^2}\,dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\cosh^{-1}\left(\frac{x}{a}\right)$	6.	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\log\left(\frac{x-a}{x+a}\right)$
7.	$\int \frac{1}{\sqrt{x^2 + a^2}}dx = \ln(x + \sqrt{x^2 + a^2}) + C$	8.	$\int \frac{1}{\sqrt{x^2 - a^2}}dx = \ln(x + \sqrt{x^2 - a^2}) + C$
9.	$\int \sqrt{a^2 + x^2}\,dx = \frac{x}{2}\sqrt{(a^2 + x^2)} + \frac{a^2}{2}\sinh^{-1}\left(\frac{x}{a}\right)$	10.	$\int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{(a^2 - x^2)} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)$

Hyperbolic

1.	$\int \sinh x\,dx = \cosh x + C$	4.	$\int \operatorname{cosech} x\,dx = \ln\left(\tanh\frac{x}{2}\right) + C$
2.	$\int \cosh x\,dx = \sinh x + C$	5.	$\int \operatorname{sech} x\,dx = \tan^{-1}(\sinh x) + C$
3.	$\int \tanh x\,dx = \ln(\cosh x) + C$	6.	$\int \coth x\,dx = \ln(\sinh x) + C$

Special function

1.	$\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx) + C$
2.	$\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx) + C$
3.	$\int \cos ax\cosh bx\,dx = \frac{1}{a^2+b^2}(a\sin ax\cosh bx + b\cos ax\sinh bx) + C$
4.	$\int \sin ax\cosh bx\,dx = \frac{1}{a^2+b^2}(b\sin ax\sinh bx - a\cos ax\cosh bx) + C$

BASICS

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	−1	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	−1	0	−1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0	$-\infty$	0

1. $\sin(-x) = -\sin x$
2. $\cos(-x) = \cos x$
3. $\sin(x + y) = \sin x \cos y + \cos x \sin y$
4. $\sin(x - y) = \sin x \cos y - \cos x \sin y$
5. $\cos(x + y) = \cos x \cos y - \sin x \sin y$
6. $\cos(x - y) = \cos x \cos y + \sin x \sin y$
7. $\cos\left(\frac{\pi}{2} - x\right) = \sin x$
8. $\sin\left(\frac{\pi}{2} - x\right) = \cos x$

Special function

1.	$\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)+C$
2.	$\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)+C$
3.	$\int \cos ax\cosh bx\,dx = \frac{1}{a^2+b^2}(a\sin ax\cosh bx + b\cos ax\sinh bx)+C$
4.	$\int \sin ax\cosh bx\,dx = \frac{1}{a^2+b^2}(b\sin ax\sinh bx - a\cos ax\cosh bx)+C$

BASICS

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	–1	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	–1	0	–1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0	$-\infty$	0

1. $\sin(-x) = -\sin x$
2. $\cos(-x) = \cos x$
3. $\sin(x + y) = \sin x \cos y + \cos x \sin y$
4. $\sin(x - y) = \sin x \cos y - \cos x \sin y$
5. $\cos(x + y) = \cos x \cos y - \sin x \sin y$
6. $\cos(x - y) = \cos x \cos y + \sin x \sin y$
7. $\cos\left(\frac{\pi}{2} - x\right) = \sin x$
8. $\sin\left(\frac{\pi}{2} - x\right) = \cos x$

9. (i) $\sin\left(\frac{\pi}{2}+x\right) = \cos x$ (ii) $\cos\left(\frac{\pi}{2}+x\right) = -\sin x$

(iii) $\sin(\pi - x) = \sin x$ (iv) $\cos(\pi - x) = -\cos x$

(v) $\sin(\pi + x) = -\sin x$ (vi) $\cos(\pi + x) = -\cos x$

(vii) $\sin(2\pi - x) = -\sin x$ (viii) $\cos(2\pi - x) = \cos x$

10. $\tan(x + y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$

11. $\tan(x - y) = \dfrac{\tan x - \tan y}{1 + \tan x \tan y}$

12. $\tan\left(\frac{\pi}{4} + x\right) = \dfrac{1 + \tan x}{1 - \tan x}$

13. $\tan\left(\frac{\pi}{4} - x\right) = \dfrac{1 - \tan x}{1 + \tan x}$

14. $\cot(x + y) = \dfrac{\cot x \cot y + 1}{\cot y + \cot x}$

15. $\cot(x - y) = \dfrac{\cot x \cot y + 1}{\cot y - \cot x}$

16. $\sin 2x = 2\sin x \cos x = \dfrac{2\tan x}{1 + \tan^2 x}$

17. $\cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x = \dfrac{1 - \tan^2 x}{1 + \tan^2 x}$

18. $\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$

19. $\sin^2 x = 1 - \cos^2 x$

20. $\cos^2 x = 1 - \sin^2 x$

21. $e^{it} = \cos t + i \sin t$