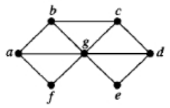
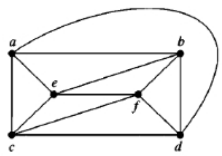
Q.1) Max Marks: 1

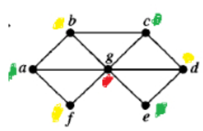
For the given graphs, their respective chromatic numbers are

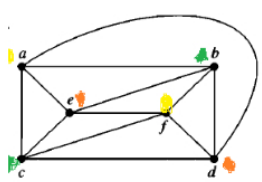




A 3 and 3 Correct Option

Solution: (A)
Answer: A
Explanation:
We can color both of them with 3 colors so that no two same color is adjacent. but how can we sure that it is not less than 3. As both the graph consist triangle it can't be color with less than 3. So we at least require 3 colors to color them.





B 6 and 5

C 3 and 4

D 4 and 4

Q.2) Max Marks: 1

$(\sim q \wedge (p \rightarrow q)) \rightarrow \sim p$ is

A Tautology Correct Option

Solution: (A)

Answer: A
Explanation:

$(\sim q \wedge (p \rightarrow q)) \rightarrow \sim p$

We know that $p \rightarrow q \equiv \sim p \vee q$

$(\sim q \wedge (\sim p \vee q)) \rightarrow \sim p$

Convert $\wedge \equiv \cdot, \vee \equiv +$

Now, $q'.(p' + q) \rightarrow p'$

$(q'.p' + q'.q) \rightarrow p'$

$(p'\cdot q' + 0) \rightarrow p'$

$p'.q' \rightarrow p'$

$(p.q') + p$

$p + q + p' = p + p' + q = 1 + q = 1$

Now, we can write $p \lor \sim p \equiv \sim q \lor p \equiv T$
So, this is a Tautology.

B Contradiction

C Contingency

D Unsatisfiable

Q.3) Max Marks: 1

Maximum no of edges in a triangle-free, simple planar graph with 10 vertices is ____

Correct Answer

Solution: (16)

Answer: 16

Explanation:

Minimum degree in any triangle free graph

$\delta = 4$

by using Euler's formula

$\boldsymbol{e\text{-}n\text{+}(k\text{+}1) = r}$ {k=#f connected components}

e-10+2=r

=> e-8=r

As, min degree of region is 4

$4r \leq 2e$

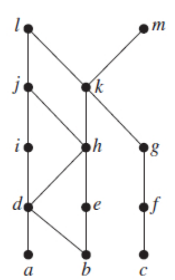
=> 4(e-8) $\leq$ 2e

=> 4e-32 $\leq$ 2e

=> $\boldsymbol{e \leq 16}$

Q.4) Max Marks: 1

Which of the given statements are TRUE for the partial order represented by the following Hasse diagram:



S1 : The maximal elements are l and m.

S2: There is no greatest element.

S3: The GLB of {f, g, h} does not exist.

A S1 and S2

B S2 and S3

C S1 and S3

D S1, S2 and S3 Correct Option

Solution: (D)

Answer: D

Explanation:

S1 : *l* and *m* are the maximal elements as they have no successor

S2 : There is no greatest elements as the 2 maximal elements are incomparable.
S3 : Greatest lower bound of {f,g,h} does not exist as they have no lower bounds in common

Q.5)

Max Marks: 1

Given the following conditional statements, which is/are tautology:

S1 : $[(p\to q)\wedge(q\to r)]\to(p\to r)$

S2 : $[(p\vee q)\wedge(p\to r)\wedge(q\to r)]\to r$

A **Only S1**

B **Only S2**

C **Both S1 and S2** — Correct Option

Solution: (C)

Answer: C

Explanation:

S1 : $[(p\to q)\wedge(q\to r)]\to(p\to r)$

If p, then q AND if q then r would mean if p, then r which is exactly what the RHS is. So, the implication is true. (But RHS implies LHS is not true here as RHS does not say that p should imply q or q should imply r)

S2 : $[(p\vee q)\wedge(p\to r)\wedge(q\to r)]\to r$

Either p or q is true; if p then r; if q then r. So, either way we have r and RHS is r only. So, again this implication is true.

So, both are tautologies.

D **None of the above**

Q.6)

Max Marks: 1

Which of the following is True

S1. Every satisfiable is a contingency

S2. Every satisfiable is a tautology

S3. Every invalid is a contradiction

A **S1 and S2**

B **S2 and S3**

C **S1 and S3**

D **None of the above** — Correct Option

Solution: (D)

Explanation:

Tautology: Compound proposition which is always True.

Contradiction: Compound proposition which is always False.

Contingency: Compound proposition which is sometimes True and sometimes False.

Satisfiability: A Compound proposition is satisfiable if there is at least one True in its truth table.

So, Tautology is always satisfiable.

Unsatisfiability: Not even a single True result in its truth table.

So, Contradiction is always unsatisfiable.

Valid: A Compound preposition is valid when it is a tautology.

Invalid: A Compound preposition is invalid when it is either contradiction or contingency.

Important Results:

Tautology is always satisfiable but satisfiable is not always the tautology.

Invalid does not mean that a compound preposition is always False. If a compound preposition is sometimes True and False, then also is said to be invalid.

Summary:

Tautology	Contradiction	Contingency
always TRUE	always FALSE	Sometimes TRUE or FALSE
Satisfiable	Unsatisfiable	Satisfiable
Valid	Invalid	Invalid

Therefore, S1 , S2 and S3 are all False. Thus, Option D is correct.

Q.7)

Max Marks: 1

Number of relations S over set $\{0,1,2,3\}$ such that $(x, y) \in S \Rightarrow x = y$ is ___

Correct Answer

Solution: (16)

Answer: 16

Explanation:
Only four pairs that satisfy the condition are: (0,0), (1,1), (2,2), (3,3).

Now the relation can include or leave each of these 4 pairs. Hence 2^4 ways.

For all other 12 possible pairs, the relation must leave them. Hence 1 way.

So no of relations = 2^4 × 1 = 16.

Note that the empty relation is also valid. In that case the condition is vacuously valid.

Q.8)

Max Marks: 1

What is the largest no of maximal independent set of complete bipartite graph K(4,2)?

A 2

B 3

C 4 Correct Option

Solution: (C)

Answer: C

Explanation:

We know that for bipartite complete graph $K_{m,n}$, we can partition the vertex set into 2 sets such that one contains m and other contains n elements(vertices) and within each of this sets there does not exist any edge between any pair of vertices.So it is nothing but independent set.

So for bipartite complete graph $K_{m,n}$, we have 2 independent sets.

So here max(4,2) = 4.Hence 4 should be the size of the largest maximal independent set.

D 5

Q.9)

Max Marks: 1

Consider a set S {2,3,4,.....,23,24} and R is a relation on S such that aRb if a divides b, then find the number of minimal elements in its hasse diagram

A 9 Correct Option

Solution: (A)

Answer: A

Explanation:
Minimal Element : Let (A, R) be a poset. Then "a" in A is a minimal element if there does not exist an element b in A such that bRa.

So, for the given poset (S, /)

Minimal elements : {2,3,5,7,11,13,17,19,23} = 9 elements

B 10

C 12

D 16

Q.10)

Max Marks: 1

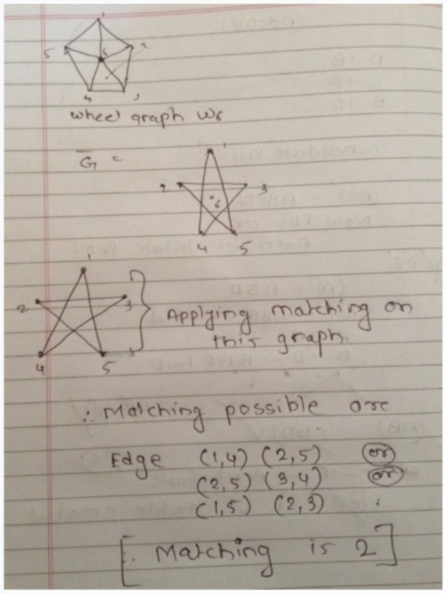
If G is a wheel graph with 6 vertices, then matching number of $\overline{G}$ = ______.

Correct Answer

Solution: (2)

Answer: 2

Explanation:



Q.11)

Max Marks: 2

Which of the following statement(s) is/are True?

S1 : Set of Integers on addition operation is Monoid but not group.

S2 : Set of Integers on subtraction operation is Monoid but not group.

S3 : Set of Integers on multiplication operation is group but not Abelian group.

A) S1 only

B) S2 only

C) S1 and S3

D) None of the above — Correct Option

Solution: (D)

Answer: D

Explanation:

S1: It is False

Addition of Integer is closed

Addition of integer is associative ((1+2)+3)=(1+(2+3))

Identity property also satisfied 0+1=1, here identity element is 0

Inverse of 1 is (-1), 1+(-1)=0, So, inverse also satisfied

So, Integer addition will be a group

S2: It is also false

Associativity also not satisfied

((7-2)-3) not equal with (7-(2-3))

i.e.(5-3) not equal with (7+1)

but in monoid we have to satisfy associativity and identity property

S3: Also False

As, Integer multiplication does not satisfy inverse property

inverse of 2 will be 1/2 , which is not an integer

So, Integer multiplication cannot form a group

Q.12)

Max Marks: 2

Consider an undirected graph with n vertices, vertex 1 has degree 1, while each vertex 2,3......, n – 1 has degree 4. The degree of vertex n is unknown. Which of the following statements must be TRUE?

A) Vertex n has degree 1.

B) Graph is connected.

C) There is a path from vertex 1 to vertex n. — Correct Option

Solution: (C)

Answer: C

Explanation:

A) false.

By handshaking lemma you can argue that degree of n must be odd but it doesn't mean it will be 1 always.

For example 1,4,4,4,4,3 is a valid degree sequence.

B) false.

Suppose the graph has 2 components as degree sequence G1 = 1,4,4,4,4,3 and G2= 4,4,4,4,4. Which means, it is not always true that graph is connected.

C) True

Vertex 1 and vertex n should be a vertex of same component of graph otherwise it will not follow handshaking lemma. Graph is undirected and vertex 1 and n are connected vertex 1 and vertex n should be a vertex of same component of graph otherwise it will not follow handshaking lemma. Graph is undirected and vertex 1 and n are connected

So, there must be a path from 1 to n.

D) false

Not every spanning tree will contain the edge joining 1 to n.

D **Spanning tree will include the edge connecting vertex 1 and n.**

Q.13) Max Marks: 2

Consider the following two statements:

S_1 : All clear explanations are satisfactory.

S_2 : Some excuses are unsatisfactory.

Which one of the following statement(s) follows from S_1 and S_2 as per inference rules of logic?

A **Every excuses are not clear explanations**

B **Some excuses are clear explanations**

C **Some excuses are not clear explanations** Correct Option

Solution: (c)

Answer: C

Explanation:

Let's define some predicates :

$CE(x)$: x is a Clear Explanation.

$Ex(x)$: x is an Excuse.

$S(x)$: x is Satisfactory.

Now, given statements can be written as follows :

$S_1 = \forall x(CE(x) \rightarrow S(x))$ (Interpretation : For all x, if x is Clear Explanation then x is Satisfactory).

$S_2 = \exists x(Ex(x) \land \sim S(x))$ (Interpretation : There is some x such that x is an Excuse and x is Not Satisfactory)

So, We are given S_1 and S_2. And considering these two statements True, we need to show which of the Options is always True.

Using Statement S_2, you can say that, in the domain, there is at least one x for which $Ex(x)$ is True and $S(x)$ is False. So, Let this $x=a$ and now from S_2 we have $Ex(a)=T$ and $S(a)=F$

Let's see S_1 for this element a in the domain.
We will find that,

$CE(a) \rightarrow S(a)$ must be True. But we know that $S(a)=F$, So, We have $CE(a)=F$ (Because $CE(a) \rightarrow S(a)$ is True)

So, From S_1 and S_2, we have $Ex(a)=T$ and $CE(a)=F$

So, Option C is Correct because it says $\exists x(Ex(x) \land \sim CE(x))$, which is Always True or we can say that Option C can never be False when S_1 and S_2 are True. Because, for $x=a$, $\exists x(Ex(x) \land \sim CE(x))$ becomes True.

So, Option C is implied from S_1 and S_2.

Or You can say that $(\forall x(CE(x) \rightarrow S(x))) \land (\exists x(Ex(x) \land \sim S(x))) \rightarrow (\exists x(Ex(x) \land \sim CE(x)))$ is a Tautology.

Now, the above method is a Formal way of proving answer. But intuitively, just by looking at the Question, We could say that C is Correct. But, in this direct way, we might make mistakes for complicated questions, So, Knowing the above formal approach gives you assurity of the answer.

D **Some explanations are clear excuses.**

Q.14) Max Marks: 2

From the given statements, which among the following statement(s) is/are TRUE ?

S1 : ({ 0,1,2....(m-1) } , $+_m$) where $+_m$ stands for "addition-modulo-m"

S2 : ({0,1,2....m} , $+_m$) where $+_m$ stands for "addition-modulo-m".

A ONLY S1 is a group. Correct Option

Solution: (A)

Answer: A

Explanation:

The answer will be A. Only $S1$ is the group.

Because in the second case, Identity element does not exist.

In the first case identity element is 0. That means for all $a \in S1$, $a +_m 0 = a$. because $a < m$

In the second case, 0 can not be the identity element. For example: for one of the member m of the set we have $m +_m 0 = 0$, It should come m. That's why $S2$ is not a group.

However, both $S1$ and $S2$ are Semigroup as they satisfy closure and associativity property.

B ONLY S2 is a group.

C BOTH S1 AND S2 are groups.

D NEITHER S1 NOT S2 is a group.

Q.15) Max Marks: 2

Let $G = \{a_1, a_2, \ldots, a_{12}\}$ be an Abelian group of order 12 . Then the order of the element $(\prod_{i=1}^{12} a_i)$ is

A 1

B 2 Correct Option

Solution: (B)

Answer: B

Explanation:

Every element is multiplied with its inverse. The identity element is its own inverse. That leaves us with 11 elements other than identity. So, one element (except the identity element) is its own inverse. The other 10 elements get cancelled out as they are multiplied with their respective inverse elements. So, the order of the element $\prod_{i=1}^{12} a_i$ is 2.

C 8

D 12

Q.16) Max Marks: 2

A={1,2,3,4,5,6}
Given that set A is a group with respect to multiplication mod 7.
G is also a cyclic group, what are the generators of G?

A 4 and 6

B 2 and 4

C 3 and 6

D 3 and 5 Correct Option

Solution: (D)

Answer: D

Explanation:
From the given question, we know G is cyclic.

Once we figured out that G is cyclic with $\mathrm{O(G)} = 6$
Then $\phi(\mathbf{6})$ will give no of generators in G
$\Rightarrow \phi(\mathbf{6}) = \phi(\mathbf{2^1 * 3^1})$
$\Rightarrow \phi(\mathbf{6}) = \phi(\mathbf{2^1}) * \phi(\mathbf{3^1})$
$\Rightarrow \phi(\mathbf{6}) = [2^1 - 2^0] * [3^1 - 3^0]$
$\Rightarrow \phi(\mathbf{6}) = 2$

⇒ And these two gen are 3 and 5

Q.17)

Max Marks: 2

Which of the following statement(s) is/are true?

S1: If R1 and R2 are the equivalence relation on X then R1 ∩ $R2^{-1}$ (Inverse of R2) is also an equivalence relation.

S2: If R is reflexive and transitive relation on X then R ∩ R^{-1} is an equivalence relation.

A **Only S1**

B **Only S2**

C **Both S1 and S2** — Correct Option

Solution: (c)

Answer: C

Explanation:

Inverse of a relation: If (a,b) ε R, then (b,a) ε R^{-1} for all a,b ε X.

<u>Statement 1:</u>

We know '∩' of two equivalence relations is equivalence. So, we just need to prove R_2^{-1} is an equivalence relation.

1. If (a,a) ε R_2, then (a,a) ε R_2^{-1} also for all a ε X. So R_2^{-1} is reflexive.

2. If symmetric pairs (a,b) & (b,a) ε R_2, then corresponding elements (b,a) & (a,b) also ε R_2^{-1}. So R_2^{-1} is symmetric.

3. If (a,b), (b,c) & (a,c) ε R_2, then (b,a), (c,b) & (c,a) ε R_2^{-1} satisfying transitivity property, ((c,b) & (b,a) --> (c,a)). So R_2^{-1} is transitive.

Thus R_2^{-1} is an equivalence relation.

So *$R_1 \cap R_2^{-1}$ is also equivalence.*

<u>Statement 2:</u>

1. Since R is reflexive, so R^{-1} will also contain reflexive elements. So R ∩ R^{-1} is reflexive.

2. Since R is not symmetric, then only some but not every symmetric pairs (a,b) & (b,a) are present in R. R contains only (a,b) but not (b,a) for some a,b ε X, then R^{-1} will contain only (b,a) but not (a,b). So R ∩ R^{-1} contains neither (a,b) nor (b,a) thus ensuring no asymmetric elements. R contains both (a,b) & (b,a) for some a,b ε X, then R^{-1} will also contain both (b,a) & (a,b). So R ∩ R^{-1} contains both (a,b) & (b,a) thus ensuring both symmetric pairs are present. So R ∩ R^{-1} will either contain both symmetric pairs or none at all making it symmetric.

3. Since R is transitive, so if (a,b), (b,c) & (a,c) ε R for some a,b,c ε X, then (b,a), (c,b) & (c,a) ε R^{-1}. This means R ∩ R^{-1} doesn't contain any of (a,b), (b,c) & (a,c) or (b,a), (c,b) & (c,a) elements, so no need to check transitivity property here.

Finally *$R \cap R^{-1}$ is equivalence relation* as it is reflexive, symmetric and transitive!

D **None of the above.**

Q.18)

Max Marks: 2

The proposition $(P \Rightarrow Q) \wedge (Q \Rightarrow P)$ is a

A **Tautology**

B **Contradiction**

C **Contingency** — Correct Option

Solution: (c)

Answer: C

Explanation:

P	Q	P->Q	Q-> P	(P->Q)^(Q->P)
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

The given proposition is bidirectional, which is neither tautology nor contradiction.

A proposition which is neither tautology nor contradiction is contingency

A proposition which is neither tautology nor contradiction is contingency.

Hence, option C is correct.

D **Absurdity**

Q.19)

Max Marks: 2

Let f : A —> B and g : B —> C be functions.
Which of the following statement(s) is/are true?

a: If g o f is injective, then f is injective.
b: If g o f is surjective, then g is surjective.

A **a only**

B **b only**

C **Both a and b** Correct Option

Solution: (C)
Part a)

Suppose $g \circ f$ is injective. Let $x, y \in A$ such that

$$\begin{aligned} \underline{f(x) = f(y)} &\Rightarrow g(f(x)) = g(f(y)) \\ &\Rightarrow (g \circ f)(x) = (g \circ f)(y) \\ &\Rightarrow \underline{x = y} \qquad\qquad (g \circ f \text{ is injective}) \end{aligned}$$

Therefore, f is injective.
Part b)

Suppose $g \circ f$ is surjective. Then, for any $z \in C$ there exists $x \in A$ such that

$$z = (g \circ f)(x) \iff z = g(f(x))$$

Therefore, if we define $y = f(x)$, for any $z \in C$ there exists $y \in B$ such that $g(y) = z$, that is, g is surjective.

D **None of the above**

Q.20)

Max Marks: 2

Which of the following is not a boolean algebra?

I: [D_{110}; /]
II: [D_{91}; /]
III: [D_{45}; /]
IV: [D_{64}; /]

A **I only**

B **II only**

C **II and III**

D **III and IV** Correct Option

Solution: (D)
Answer: D

Explanation: Let n = p1p2 ... pk, where the pi are distinct primes. Then Dn is a Boolean algebra.

• A) $\{D_{110}, 1\} = \{\underline{2 \times 5 \times 11}\}$ ✓

• All are distinct prime

B) $\{D_{91}, 1\} = \{\underline{7, 13}\}$ ✓

• All are distinct prime

c) $\{D_{45}, 1\} = \{3 \times 3 \times 5\}$ ×

• All are Not Distinct

d) $\{D_{64}, 1\} = \{2 \times 2 \times 2 \times 2 \times 2 \times 2\}$ ×

- All are Not Distinct prime.

close