Q.1) Max Marks: 1

The value of

$$\lim_{x\to 0}\frac{x^2 sin\frac{1}{x}}{sinx} =$$

A 1

B 0 Correct Option

Solution: (B)

Solution B

$$= \lim_{x\to 0}\frac{x^2 sin\frac{1}{x}}{sinx}$$

$$= \lim_{x\to 0}\frac{x}{sinx}\times x sin\frac{1}{x}$$

$$= 1\times 0$$

$$\lim_{x\to 0} x sin\frac{1}{x} = 0\ as \mid x sin\frac{1}{x}\mid \le \mid x\mid$$

C 1/2

D None of these

Q.2) Max Marks: 1

The function

$$f(x) = \frac{x}{1+\mid x\mid}$$

A **Is differentiable on R** Correct Option

Solution: (A)

Solution A.

f(x) can be rewritten as

$$f(x) = \begin{cases} \frac{x}{1+x} & if x \ge 0 \\ \frac{x}{1+x} & if x < 0 \end{cases}$$

Since x/(1+x) and x/(1-x) and x<0 have non zero polynomials in their denominators, they are differentiable in the respective domains. For x=0 we can check directly

$$f'(0+) = \lim_{h\to 0+}\frac{f(h)-f(0)}{h}$$

$$= \lim_{h\to 0+}\frac{\frac{h}{1+h}-0}{h-0}$$

$$= \lim_{h\to 0+}\frac{1}{1+h} = 1$$

$$f'(0-) = \lim_{h\to 0-}\frac{f(h)-f(0)}{h}$$

$$= \lim_{h\to 0-}\frac{\frac{h}{1-h}-0}{h-0}$$

$$= \lim_{h\to 0-}\frac{1}{1-h} = 1$$

Thus f is derivable at x=0 and also f is derivable.

B Is differentiable only on R^+

C Is differentiable only on R^-

D **None of the above.**

Q.3)

Max Marks: 1

$$\int_8^{15} \frac{dx}{(x-3)\sqrt{x+1}} =$$

A $\frac{1}{2}log\frac{5}{3}$

Correct Option

Solution: (A)

Put $\sqrt{(x+1)} = t$

$$= \int_3^4 \frac{2t}{(t^2-4)t}dt = \frac{2}{(2)(2)}log\left|\frac{t-2}{t+2}\right|_2^4$$
$$= \frac{1}{2}\left[log\frac{1}{3} - log\frac{1}{5}\right]$$
$$= \frac{1}{2}log\ \frac{5}{3}$$

B $2log\frac{1}{3}$

C $\frac{1}{2}log\frac{1}{5}$

D $2log\frac{3}{5}$

Q.4)

Max Marks: 1

Given the following function

$$f(x) = \frac{x^2-1}{x^2+1}$$

The minimum value of f.

A **Does not exist**

B **is not attained even though f is bounded**

C **is equal to 1**

D **is equal to -1**

Correct Option

Solution: (D)

Solution: D option

f(x) can we rewritten as 1- $(2/(x^2+1))$

f(x) will be minimum when x^2+1 is least i.e. when x=0, thus the minimum value of f(x)=f(0)=-1.

Q.5)

Max Marks: 1

If $f(x)=x^2+8x+14$, defined over the interval (-6,-2) then what can be said about f(x).

I. f(x) is continuous over its domain.
II. There exists a x∈ (-6,-2) such that f′(x)=0

A **I and II are true**

Correct Option

Solution: (A)

Solution A.

Since a is a polynomial in x it is continuous on R

f(-6)=2

f(-2)=2
According to Rolle's Theorem, we have a point in (-6,-2) such that f'(x)=0. Therefore II is also true.

B I is only true

C II is true

D Neither I nor II is true

Q.6) Max Marks: 1

The number of points at which the function is discontinuous in the interval [0, π] is______.

$$f(x) = \frac{1 + cos5x}{1 - cos4x}$$

Correct Answer

Solution: (3)

Solution
Clearly this f(x) is only discontinuous when 1-cos4x=0
Cos 4x=1
4x=0 or 2π or 4π
x=0,π/2,π.
The no of values is 3.

Q.7) Max Marks: 1

The function

$$f(x) = \frac{cosx - sinx}{cos\ 2x}$$

Is not defined at x= π/4. The value of f(π/4) so that f(x) is continuous everywhere is

A 1

B -1

C √2

D 1/√2 Correct Option

Solution: (D)

Solution option d

For the function to be continous the function should be defined and the limit should exist

$$= \lim_{x \to \frac{\pi}{4}} \frac{cosx - sinx}{cos\ 2x}$$

Applying L'Hopitals rule

$$= \lim_{x \to \frac{\pi}{4}} \frac{-sinx - cosx}{-2sin\ 2x}$$

$$= \frac{1}{\sqrt{2}}$$

Q.8) Max Marks: 1

The value of

$$\lim_{x \to 1} \frac{x + x^2 + x^3 ... + x^n - n}{x - 1} =$$

A n(n-1)/2

B n

C n(n+1)/2 — Correct Option

Solution: (C)
Solution:

$$\lim_{x\to 1}\frac{x+x^2+x^3...+x^n-n}{x-1}$$

$$\frac{0}{0} form Applying L'Hopital Rule$$

$$=\lim_{x\to 1}\frac{x+2x+3x^2+4x^3...+nx^{n-1}}{1}$$

$$=1+2+3...+n$$

$$\frac{n(n+1)}{2}$$

D (n+1)/2

Q.9) Max Marks: 1

$\int x\sqrt{x+3}=$

A (⅔)x(x+3)$^{3/2}$-(4/15)(x+3)$^{5/2}$+C — Correct Option

Solution: (A)

Solution A.

Performing integration by parts

$$u=x \quad \text{and} \quad dv=\sqrt{x+3}\,dx=(x+3)^{1/2}\,dx$$

$$du=(1)dx=dx \quad \text{and} \quad v=\frac{(x+3)^{3/2}}{(3/2)}=(2/3)(x+3)^{3/2}\ .$$

$$\int x\sqrt{x+3}\,dx=x(2/3)(x+3)^{3/2}-\int(2/3)(x+3)^{3/2}\,dx$$

$$=(2/3)x(x+3)^{3/2}-(2/3)\int(x+3)^{3/2}\,dx$$

$$=(2/3)x(x+3)^{3/2}-(2/3)\frac{(x+3)^{5/2}}{(5/2)}+C$$

$$=(2/3)x(x+3)^{3/2}-(4/15)(x+3)^{5/2}+C\ .$$

B (⅔)x(x+3)$^{3/2}$+(4/15)(x+3)$^{5/2}$+C

C (⅔)x(x-3)$^{3/2}$-(4/15)(x+3)$^{5/2}$+C

D None of the above.

Q.10) Max Marks: 1

Let f and g be defined by $f(x)=\sqrt{x}$ and $g(x)=x-1$ Then which of the following are true.

I. $f(g(x))$ is continuous on $[0,\infty)$
II. $g(f(x))$ is continuous on $[0,\infty)$
III. $f(g(x))$ is continuous on $[1,\infty)$

A I and II

B II and III — Correct Option

Solution: (B)

Solution B Option

f(g(x))=√(x-1) is defined only for x>=1 therefore I is false
III is correct as the f is defined in [1,∞) and it is a polynomial in x, therefore, it is continuous in the given range.

g(f(x))=(√x)-1 it is defined for x>=0 i.e. in the range [0,∞) and it is also continuous in the given range. III is true.

C **I and III**

D **None of the above.**

Q.11)

Max Marks: 2

If the displacement of a particle as a function of time is given by $f(t)=7t^3+3t^2-2$ the rate of change of the speed at time t=10 is ______

Correct Answer

Solution: (426)

Solution 426

The speed of the particle is the rate of change of displacement
speed(t)=f'(t)=$21t^2+6t$
The rate of change of speed is acceleration is given by the derivative of speed.
f''(t)=42t+6
At t=10
f''(100)=426.

Q.12)

Max Marks: 2

$\int \frac{5x^8+7x^6}{(x^2+1+2x^7)^2} dx =$

A $\frac{x^7}{2x^7+1+x^2}$ +C

Correct Option

Solution: (A)

$$\int \frac{5x^8+7x^6}{(x^2+1+2x^7)^2} dx$$

$$= \int \frac{5x^8+7x^6}{x^{14}(\frac{1}{x^5}+\frac{1}{x^7}+2)^2} dx$$

$$= \int \frac{x^{14}(\frac{5}{x^6}+\frac{7}{x^8})}{x^{14}(\frac{1}{x^5}+\frac{1}{x^7}+2)^2} dx$$

$$= put\ t = \frac{1}{x^5} + \frac{1}{x^7} + c$$

$$= \int \frac{-dt}{t^2}$$

$$= \frac{1}{t} + c$$

$$= \frac{x^7}{x^2+1+2x^7} + c$$

B $\frac{x^5}{2x^7+1+x^2}$ +C

C $\frac{-1}{2x^7+1+x^2}$ +C

D **None of the above.**

Q.13)

Max Marks: 2

For x∈R 1 Marks

$$\lim_{x\to\infty}\left(\frac{x-3}{x+2}\right)^x =$$

A e

B e^{-1}

C e^{-5}

Correct Option

Solution: (C)

Solution:

$$\lim_{x\to\infty}\left(\frac{x-3}{x+2}\right)^x$$

$$=\lim_{x\to\infty}\left(1-\frac{5}{x+2}\right)^x$$

$$=\lim_{x\to\infty}\left(1-\frac{5}{x+2}\right)^{-\frac{x+2}{5}\times\frac{-5}{x+2}\times x}$$

$$=\lim_{x\to\infty}\left(1-\frac{5}{x+2}\right)^{-\frac{x+2}{5}\times\frac{-5}{x+2}\times x}$$

$$=\lim_{x\to\infty}\left(1-\frac{5}{x+2}\right)^{-\frac{x+2}{5}\times 5\times\frac{1}{1+\frac{2}{x}}}$$

$$=e^{-5\times 1}$$

$$=e^{-5}$$

D e^{5}

Q.14)

Max Marks: 2

Compute the area of the region enclosed by the graphs of the equations

$x=y^3$ and $x=y^2+2y$ in the positive quadrant.________ (upto 3 decimal places)

Correct Answer

Solution: (2.667)

Solution: 2.667

Begin by finding the points of intersection of the two graphs.

$x=y^3$

$x=y^2+2y$

$y^3=y^2+2y$

$y(y+1)(y-2)=0$

$y=0,-1,2$

For $y=0$ $x=0$

For $y=-1$ $x=1$

For $y=2$ $x=8$

Now as we are concerned about the +ve quadrant we need from x=0 to 2

The area enclosed can be given by (area of the higher curve - area of the lower curve)

In the interval [0,2]

As both the equation are of the form x= some polynomial in y we can compute the area using y as the independent variable.

$$\int_0^2 (Right\ -\ Left)\,dy$$

$$\int_0^2 \left((y^2+2y)-y^3\right)dy$$

$$\left(\frac{y^3}{3}+y^2-\frac{y^4}{4}\right)\Big|_0^2$$

$$\left(\frac{8}{3}+4-4\right)-\left(0\right)$$

$\left(\frac{}{3} + 4 - 4\right) - (0)$

=2.667

Q.15)

Max Marks: 2

The following integral can be given by

$$\int \frac{1}{x^2-4}\,dx$$

A $\frac{1}{4} ln \frac{|x+2|}{|x-2|} + C$

B $\frac{1}{4} ln \frac{|x-2|}{|x+2|} + C$

Correct Option

Solution: (B)

Solution B

$$= \int \frac{1}{(x+2)(x-2)}\,dx$$

Dividing into partial fractions.

$$= \int \left(\frac{A}{x+2} + \frac{B}{x-2}\right) dx$$

$$\text{let } x = -2: \; A(-4) + B(0) = 1 \longrightarrow A = -\frac{1}{4}$$

$$\text{let } x = 2: \; A(0) + B(4) = 1 \longrightarrow B = \frac{1}{4}\ .)$$

$$= \int \left(\frac{-1/4}{x+2} + \frac{1/4}{x-2}\right) dx$$

$$= \int \left(-(1/4)\frac{1}{x+2} + (1/4)\frac{1}{x-2}\right) dx$$

$$= -\frac{1}{4}\ln|x+2| + \frac{1}{4}\ln|x-2| + C$$

$$= \frac{1}{4}\Big(\ln|x-2| - \ln|x+2|\Big) + C$$

$$= \frac{1}{4}\ln\frac{|x-2|}{|x+2|} + C\ .$$

C $\frac{1}{2} ln \frac{|x-2|}{|x+2|} + C$

D None of the above

close