Contents

Unit ■ I

Discrete and Engineering Mathematics

■ Contents

UNIT I

Discrete and Engineering Mathematics

Syllabus :

Mathematical Logic: Propositional and first order logic.

Set Theory & Algebra: Sets, relations, functions, partial orders and lattices. Groups.

Combinatorics: Counting, recurrence relations, generating functions.

Graph Theory: Connectivity, matching, coloring.

Probability: Random variables. Uniform, normal, exponential, poisson and binomial distributions. Mean, median, mode and standard deviation. Conditional probability and Bayes theorem.

Linear Algebra: Matrices, determinants, system of linear equations, eigenvalues and eigenvectors, LU decomposition.

Calculus: Limits, continuity and differentiability. Maxima and minima. Mean value theorem. Integration.

Analysis of Previous GATE Papers

Exam Year	1 Mark Ques.	2 Marks Ques.	3 Marks Ques.	5 Marks Ques.	Total Marks
1990	1	2		–	**5**
1991	–	1		–	**2**
1992	3	1		3	**20**
1993	3	–		1	**8**
1994	4	5		–	**14**
1995	4	5		1	**19**
1996	7	7		–	**21**
1997	4	7		–	**18**
1998	7	7		1	**26**
1999	4	5		2	**24**
2000	2	5		2	**22**
2001	4	5		–	**14**
2002	6	4		3	**29**
2003	5	15		–	**35**
2004	5	11		–	**27**
2005	5	10		–	**25**
2006	3	10		–	**23**

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2007	4	9	**22**
2008	4	10	**24**
2009	4	6	**16**
2010	6	8	**24**
2011	–	5	**10**
2012	6	5	**16**
2013	6	3	**12**
2014 Set-1	6	9	**24**
2014 Set-2	5	8	**21**
2014 Set-3	7	8	**23**
2015 Set-1	5	8	**21**
2015 Set-2	5	8	**21**
2015 Set-3	7	6	**19**
2016 Set-1	5	4	**13**
2016 Set-2	6	3	**12**

1 Mathematical Logic

1.1 Indicate which of the following well-formed formula are valid:

(a) $((P \Rightarrow Q) \wedge (Q \Rightarrow R)) \Rightarrow (P \Rightarrow R)$

(b) $(P \Rightarrow Q) \Rightarrow (\neg P \Rightarrow \neg Q)$

(c) $(P \wedge (\neg P \vee \neg Q)) \Rightarrow Q$

(d) $((P \Rightarrow R) \vee (Q \Rightarrow R)) \Rightarrow ((P \vee Q)) \Rightarrow R)$.

[1990 : 2 Marks]

1.2 Which of the following predicate calculus statements is/are valid

(a) $(\forall x)\, P(x) \vee (\forall x)\, Q(x) \rightarrow (\forall x)\, \{P(x) \vee Q(x)\}$

(b) $(\exists x)\, P(x) \wedge (\exists x)\, Q(x) \rightarrow (\exists x)\, \{P(x) \wedge Q(x)\}$

(c) $(\exists x)\, \{P(x) \vee Q(x)\} \rightarrow (\forall x)\, P(x) \vee (\forall x)\, Q(x)$

(d) $(\exists x)\, \{P(x) \vee Q(x)\} \rightarrow \sim (\forall x)$

[1992 : 1 Mark]

1.3 Which of the following is/are tautology:

(a) $(a \vee b) \rightarrow (b \wedge c)$ (b) $(a \wedge b) \rightarrow (b \vee c)$

(c) $(a \vee b) \rightarrow (b \rightarrow c)$ (d) $(a \rightarrow b) \rightarrow (b \rightarrow c)$

[1992 : 1 Mark]

1.4 The proposition $p \wedge (\sim p \vee q)$ is

(a) a tautology (b) $\Leftrightarrow (p \wedge q)$

(c) $\Leftrightarrow (p \vee q)$ (d) a contradiction

[1993 : 1 Mark]

1.5 Let p and q be propositions. Using only the truth table decide whether $p \Leftrightarrow q$ does not imply $p \rightarrow \neg q$ is true or false.

[1994 : 2 Marks]

1.6 If the proposition $\neg p \Rightarrow q$ is true, then the truth value of the proposition $\neg p \vee (p \Rightarrow q)$, where $\neg$ is negation, '$\vee$' is inclusive or and '$\Rightarrow$' is implication, is

(a) true (b) multiple valued

(c) false (d) cannot be determined

[1995 : 2 Marks]

1.7 Which one of the following is false? Read $\wedge$ as AND, $\vee$ as OR, $\sim$ as NOT, $\rightarrow$ as one way implication and $\leftrightarrow$ as two way implication.

(a) $((x \rightarrow y) \wedge x) \rightarrow y$

(b) $((\sim x \rightarrow y) \wedge (\sim x \rightarrow \sim y)) \rightarrow x$

(c) $(x \rightarrow (x \vee y))$

(d) $((x \vee y) \leftrightarrow (\sim x \rightarrow \sim y))$

[1996 : 2 Marks]

1.8 Let a, b, c, d be propositions. Assume that the equivalence $a \leftrightarrow (b \vee \neg b)$ and $b \leftrightarrow c$ hold. Then the truth-value of the formula $(a \wedge b) \rightarrow (a \wedge c) \vee d$ is always

(a) True

(b) False

(c) Same as the truth-value of b

(d) Same as the truth-value of d

[2000 : 2 Marks]

1.9 What is the converse of the following assertion? I stay only if you go

(a) I stay if you go

(b) If I stay then you go

(c) If you do not go then I do not stay

(d) If I do not stay then you go

[2001 : 1 Mark]

1.10 Consider two well-formed formulas in propositional logic

F_1: $P \Rightarrow \neg P$

F_2: $(P \Rightarrow \neg P) \vee (\neg P \Rightarrow P)$

Which of the following statements is correct?

(a) F_1 is satisfiable, F_2 is valid

(b) F_1 is unsatisfiable, F_2 is satisfiable

(c) F_1 is unsatisfiable, F_2 is valid

(d) F_1 and F_2 are both satisfiable

[2001 : 1 Mark]

1.11 "If X then Y unless Z" is represented by which of the following formulas in propostional logic? ("$\neg$") is negation, "$\wedge$" is conjunction, and "$\rightarrow$" is implication)

(a) $(X \wedge \neg Z) \rightarrow Y$ (b) $(X \wedge Y) \rightarrow \neg Z$

(c) $X \rightarrow (Y \wedge \neg Z)$ (d) $(X \rightarrow Y) \wedge \neg Z$

[2002 : 1 Mark]

1.12 Which of the following is a valid first order formula? (Here α and β are first order formulae with x as their only free variable)

(a) $((\forall x)\, [\alpha] \Rightarrow (\forall x)[\beta]) \Rightarrow (\forall x)\, [\alpha \Rightarrow \beta]$

(b) $(\forall x)\, [\alpha] \Rightarrow (\exists x)\, [\alpha \wedge \beta]$

(c) $(\forall x)\, [\alpha \vee \beta\,] \Rightarrow (\exists x)\, [\alpha] \Rightarrow (\forall x)\, [\alpha]$

(d) $(\forall x)\, [\alpha \Rightarrow \beta\,] \Rightarrow ((\forall x)\, [\alpha] \Rightarrow (\forall x)\, [\beta])$

[2003 : 2 Marks]

1.13 Consider the following formula α and its two interpretations I_1 and I_2.

$\alpha : (\forall x)\ [P_x \Leftrightarrow (\forall y)\ [Q_{xy} \Leftrightarrow \neg\ Q_{yy}]]$
$\Rightarrow (\forall x)\ [\neg\ P_x]$

I_1: Domain : the set of natural numbers
$P_x \equiv$ 'x is a prime number'
$Q_{xy} \equiv$ 'y divides x'

I_2: Same as I_1 except that P_x = 'x is a composite number.'

Which of the following statements is true?

(a) I_1 satisfies α, I_2 does not
(b) I_2 satisfies α, I_1 does not
(c) Neither I_2 nor I_1 satisfies α
(d) Both I_1 and I_2 satisfy α

[2003 : 2 Marks]

1.14 The following resolution rule is used in logic programming: Derive clause $(P \vee Q)$ from clauses $(P \vee R)$, $(Q \vee \neg\ R)$

Which of the following statemnets related to this rule is FALSE?

(a) $(P \vee R) \wedge (Q \vee \neg R) \Rightarrow (P \vee Q)$ is logically valid
(b) $(P \vee Q) \Rightarrow (P \vee R) \wedge (Q \vee \neg R)$ is logically valid
(c) $(P \vee Q)$ is satisfiable if and only if $(P \vee R) \wedge (Q \vee \neg R)$ is satisfiable
(d) $(P \vee Q) \Rightarrow$ FALSE if and only if both P and Q are unsatisfiable

[2003 : 2 Marks]

1.15 Identify the correct translation into logical notation of the following assertion. Some boys in the class are taller than all the girls

Note: Taller (x, y) is true if x is taller than y.

(a) $(\exists x)$ (boy(x) $\rightarrow (\forall y)$ (girl(y) $\wedge$ taller (x, y)))
(b) $(\exists x)$ (boy(x) $\wedge (\forall y)$ (girl(y) $\wedge$ taller (x, y)))
(c) $(\exists x)$ (boy(x) $\rightarrow (\forall y)$ (girl(y) $\rightarrow$ taller (x, y)))
(d) $(\exists x)$ (boy(x) $\wedge (\forall y)$ (girl(y) $\rightarrow$ taller (x, y)))

[2004 : 1 Mark]

1.16 Let a(x, y), b(x, y,) and c(x, y) be three statements with variables x and y chosen from some universe. Consider the following statement:

$$(\exists x)(\forall y)[(a(x,y) \wedge b(x,y)) \wedge \neg c(x,y)]$$

Which one of the following is its equivalent?

(a) $(\forall x)(\exists y)[(a(x,y) \vee b(x,y)) \rightarrow c(x,y)]$
(b) $(\exists x)(\forall y)[(a(x,y) \vee b(x,y)) \wedge \neg\ c(x,y)]$
(c) $\neg(\forall x)(\exists y)[(a(x,y) \wedge b(x,y)) \rightarrow c(x,y)]$
(d) $\neg(\forall x)(\exists y)[(a(x,y) \vee b(x,y)) \rightarrow c(x,y)]$

[IT-2004 : 1 Mark]

1.17 Let p, q, r and s be four primitive statements. Consider the following arguments:

P: $[(\neg p \vee q) \wedge (r \rightarrow s) \wedge (p \vee r)] \rightarrow (\neg s \rightarrow q)$
Q: $[(\neg p \wedge q) \wedge [q \rightarrow (p \rightarrow r)]] \rightarrow \neg r$
R: $[[(q \wedge r) \rightarrow p] \wedge (\neg q \vee p)] \rightarrow r$
S: $[p \wedge (p \rightarrow r) \wedge (q \vee \neg r)] \rightarrow q$

Which of the above arguments are valid?

(a) P and Q only
(b) P and R only
(c) P and S only
(d) P, Q, R and S

[2004 : 2 Marks]

1.18 The following propositional statement is

$$(P \rightarrow (Q \vee R)) \rightarrow ((P \wedge Q) \rightarrow R)$$

(a) satisfiable but not valid
(b) valid
(c) a contraditiction
(d) None of the above

[2004 : 2 Marks]

1.19 Let P, Q and R be three atomic propositional assertions. Let X denote $(P \vee Q) \rightarrow R$ and Y denote $(P \rightarrow R) \vee (Q \rightarrow R)$. Which one of the following is a tautology?

(a) $X \equiv Y$
(b) $X \rightarrow Y$
(c) $Y \rightarrow X$
(d) $\neg\ Y \rightarrow X$

[2005 : 2 Marks]

1.20 What is the first order predicate calculus statement equivalent to the following? Every teacher is liked by some student

(a) $\forall$(x) [teacher (x) $\rightarrow \exists$ (y) [student (y) $\rightarrow$ likes (y, x)]]
(b) $\forall$(x) [teacher (x) $\rightarrow \exists$ (y) [student (y) $\wedge$ likes (y, x)]]
(c) $\exists$(y) $\forall$(x) [teacher (x) $\rightarrow$ [student (y) $\wedge$ likes (y, x)]]
(d) $\forall$(x) [teacher (x) $\wedge \exists$ (y) [student (y) $\rightarrow$ likes (y, x)]]

[2005 : 2 Marks]

1.21 Let P(x) and Q(x) be arbitrary predicates. Which of the following statements is always TRUE?

(a) $(\forall x(P(x) \vee Q(x))) \Rightarrow ((\forall x P(x)) \vee (\forall x Q(x)))$
(b) $(\forall x(P(x) \Rightarrow Q(x))) \Rightarrow ((\forall x P(x)) \Rightarrow (\forall x Q(x)))$
(c) $(\forall x(P(x) \Rightarrow (\forall x Q(x)))) \Rightarrow (\forall x(P(x) \Rightarrow Q(x)))$
(d) $((\forall x(P(x)) \Leftrightarrow (\forall x Q(x))) \Rightarrow (\forall x(P(x) \Leftrightarrow Q(x)))$

[IT-2005 : 2 Marks]

1.22 Consider the following first order logic formula in which R is a binary relation symbol.

$$\forall x\, \forall y\, (\, R(x,y) \Rightarrow R(y,x)\,)$$

The formula is

(a) satisfiable and valid
(b) satisfiable and so is its negation
(c) unsatisfiable but its negation is valid
(d) satisfiable but its negation is unsatisfiable

[IT-2006 : 2 Marks]

1.23 Which one of the first order predicate calculus statements given below correctly expresses the following English statement?
Tigers and lions attack if they are hungry or threatened.
(a) $\forall x\ [(\text{tiger}(x) \wedge \text{lion}(x)) \rightarrow \{(\text{hungry}(x) \vee \text{threatened}(x)) \rightarrow \text{attacks}(x)\}]$
(b) $\forall x\ [(\text{tiger}(x) \vee \text{lion}(x)) \rightarrow \{(\text{hungry}(x) \vee \text{threatened}(x)) \wedge \text{attacks}(x)\}]$
(c) $\forall x\ [(\text{tiger}(x) \vee \text{lion}(x)) \rightarrow \{(\text{attacks}(x) \rightarrow (\text{hungry}(x) \vee \text{Threatened}(x))\}]$
(d) $\forall x\ [(\text{tiger}(x) \vee \text{lion}(x)) \rightarrow \{(\text{hungry}(x) \vee \text{threatened}(x)) \rightarrow \text{attacks}(x)\}]$

[2006 : 2 Marks]

1.24 Consider the following propositional statements:
P1: $((A \wedge B) \rightarrow C)) \equiv ((A \rightarrow C) \wedge (B \rightarrow C))$
P2: $((A \vee B) \rightarrow C)) \equiv ((A \rightarrow C) \vee (B \rightarrow C))$
Which one of the following is true?
(a) P1 is a tautology, but not P2
(b) P2 is a tautology, but not P1
(c) P1 and P2 are both tautologies
(d) Both P1 and P2 are not tautologies

[2006 : 2 Marks]

1.25 A logical binary relation $\odot$, is defined as follows:

A	B	$A \odot B$
True	True	True
True	False	True
False	True	False
False	False	True

Let ~ be the unary negation (NOT) operator, with higher precedence, than $\odot$. Which one of the following is equivalent to $A \wedge B$?
(a) $(\sim A \odot B)$
(b) $\sim (A \odot \sim B)$
(c) $\sim (\sim A \odot \sim B)$
(d) $\sim (\sim A \odot B)$

[2006 : 2 Marks]

1.26 Let Graph (x) be a predicate which denotes that x is a graph. Let Connected (x) be a predicate which denotes that x is connected. Which of the following first order logic sentences DOES NOT represent the statement; "Not every graph is connected"?
(a) $\neg \forall x\ (\text{Graph}(x) \Rightarrow \text{Connected}(x))$
(b) $\exists x\ (\text{Graph}(x) \wedge \neg \text{Connected}(x))$
(c) $\neg \forall x\ (\neg \text{Graph}(x) \vee \text{Connected}(x))$
(d) $\forall x\ (\text{Graph}(x) \Rightarrow \neg \text{Connected}(x))$

[2007 : 2 Marks]

1.27 Which of the following is TRUE about formulae in Conjunctive Normal Form?
(a) For any formula, there is a truth assignment for which at least half the clauses evaluate to true.
(b) For any formula, there is a truth assignment for a which all the clauses evaluate to true.
(c) There is a formula such that for each truth assignment at most one-fourth of the clauses evaluate to true.
(d) None of the above

[2007 : 2 Marks]

1.28 Which one of these first-order logic formulae is valid?
(a) $\forall x(P(x) \Rightarrow Q(x)) \Rightarrow ((\forall xP(x)) \Rightarrow (\forall xQ(x)))$
(b) $\exists x(P(x) \vee Q(x)) \Rightarrow ((\exists x\ P(x)) \Rightarrow (\exists xQ(x)))$
(c) $\exists x(P(x) \wedge Q(x)) \Leftrightarrow ((\exists x\ P(x)) \wedge (\exists xQ(x)))$
(d) $\forall x \exists y\ P(x, y) \Rightarrow \exists y \forall x\ P(x, y)$

[IT-2007 : 2 Marks]

1.29 Let fsa and pda be two predicates such that fsa(x) means x is a finite state automaton, and pda(y) means, that y is a pushdown automaton. Let equivalent be another predicate such that equivalent (a, b) means a and b are equivalent. Which of the following first order logic statement represents the following:
Each finite state automaton has an equivalent pushdown automaton.
(a) $(\forall x\ \text{fsa}(x) \Rightarrow (\exists y\ \text{pda}(y) \wedge \text{equivalent}(x,y))$
(b) $\sim\forall y(\exists x\ \text{fsa}(x) \Rightarrow \text{pda}(y) \wedge \text{equivalent}(x,y))$
(c) $\forall x\ \exists y\ (\text{fsa}(x) \wedge \text{pda}(y) \wedge \text{equivalent}(x,y))$
(d) $\forall x\ \exists y\ (\text{fsa}(y) \wedge \text{pda}(x) \wedge \text{equivalent}(x,y))$

[2008 : 1 Mark]

1.30 Which of the following first order formulae is logically valid? Here $\alpha(x)$ is a first order formula with x as a free variable, and β is a first order formula with no free variable.

(a) $[\beta \rightarrow (\exists x, \alpha(x))] \rightarrow [\forall x, \beta \rightarrow \alpha(x)]$

(b) $[\exists x, \beta \rightarrow \alpha(x)] \rightarrow [\beta \rightarrow (\forall x, \alpha(x))]$

(c) $[(\exists x, \alpha(x)) \rightarrow \beta] \rightarrow [\forall x, \alpha(x) \rightarrow \beta]$

(d) $[(\forall x, \alpha(x)) \rightarrow \beta] \rightarrow [\forall x, \alpha(x) \rightarrow \beta]$

[IT-2008 : 2 Marks]

1.31 Which of the following is the negation of $[(\forall x, \alpha \rightarrow (\exists y, \beta \rightarrow (\forall u, \exists v, \gamma))]$

(a) $[\exists x, \alpha \rightarrow (\forall y, \beta \rightarrow (\exists u, \forall v, \gamma))]$

(b) $[\exists x, \alpha \rightarrow (\forall y, \beta \rightarrow (\exists u, \forall v, \neg\gamma))]$

(c) $[\forall x, \neg\alpha \rightarrow (\exists y, \neg\beta \rightarrow (\forall u, \exists v, \neg\gamma))]$

(d) $[\forall x, \alpha \wedge (\forall y, \beta \wedge (\exists u, \forall v, \neg\gamma))]$

[IT-2008 : 2 Marks]

1.32 P and Q are two propositions. which of the following logical expressions are equivalent?

1. $P \vee \sim Q$ **2.** $\sim(\sim P \wedge Q)$

3. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$

4. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge Q)$

(a) Only 1 and 2 (b) Only 1, 2 and 3

(c) Only 1, 2 and 4 (d) All of these

[2008 : 2 Marks]

1.33 Which one of the following is the most appropriate logical formula to represent the statement:

"Gold and silver ornaments are precious"

The following notations are used :

G(x): x is a gold ornament

S(x): x is a silver ornament

P(x): x is precious

(a) $\forall x(P(x) \rightarrow (G(x) \wedge S(x)))$

(b) $\forall x(G(x) \wedge (S(x) \rightarrow P(x)))$

(c) $\exists x((G(x) \wedge S(x)) \rightarrow P(x))$

(d) $\forall x((G(x) \vee S(x)) \rightarrow P(x))$

[2009 : 2 Marks]

1.34 The binary operation $\square$ is defined as follows:

P	Q	P$\square$Q
T	T	T
T	F	T
F	T	F
F	F	T

Which one of the following is equivalent to $P \vee Q$?

(a) $\neg Q \square \neg P$ (b) $P \square \neg Q$

(c) $\neg P \square Q$ (d) $\neg P \square \neg Q$

[2009 : 2 Marks]

1.35 Consider the following well-formed formulae:

I. $\neg\forall x(P(x))$ II. $\neg\exists x(P(x))$

III. $\neg\exists x(\neg P(x))$ IV. $\exists x(\neg P(x))$

Which of the above are equivalent?

(a) I and III (b) I and IV

(c) II and III (d) II and IV

[2009 : 2 Marks]

1.36 Suppose the predicate F(x, y, t) is used to represent the statement that person x can fool person y at time t. Which one of the statements below expresses best the meaning of the formula $\forall x \exists y \exists t (\neg F(x, y, t))$?

(a) Everyone can fool some person at some time

(b) No one can fool everyone all the time

(c) Everyone cannot fool some person all the time

(d) No one can fool some person at some time

[2010 : 2 Marks]

1.37 Which one of the following options is CORRECT given three positive integers x, y and z, and a predicate

$$P(x) = \neg(x = 1) \wedge \forall y(\exists z(x = y * z) \Rightarrow (y = x) \vee (y = 1))$$

(a) P(x) being true means that x is a prime number

(b) P(x) being true means that x is a number other than 1

(c) P(x) is always true irrespective of the value of x

(d) P(x) being true means that x has exactly two factors other than 1 and x

[2011 : 2 Marks]

1.38 Consider the following logical inferences.

I_1 : If it rains then the cricket match will not be played.
The cricket match was played.
Inference: There was no rain.

I_2 : If it rains then the cricket match will not be played.
It did not rain.
Inference: The cricket match was played.

Which of the following is TRUE?

(a) Both I_1 and I_2 are correct inferences

(b) I_1 is correct but I_2 is not a correct inference

(c) I_1 is not correct but I_2 is a correct inference

(d) Both I_1 and I_2 are not correct inferences

[2012 : 1 Mark]

1.39 What is the correct translation of the following statement into mathematical logic?

"Some real numbers are rational"

(a) $\exists x$ (real (x) $\vee$ rational (x))

(b) $\forall x$ (real (x) $\rightarrow$ rational (x))

(c) $\exists x$ (real (x) $\wedge$ rational (x))

(d) $\exists x$ (rational (x) $\rightarrow$ real (x))

[2012 : 1 Mark]

1.40 What is the logical translation of the following statements?

"None of my friends are perfect."

(a) $\exists x\,(F(x) \wedge \neg P(x))$

(b) $\exists x\,(\neg F(x) \wedge P(x))$

(c) $\exists x\,(\neg F(x) \wedge \neg P(x))$

(d) $\neg \exists x\,(F(x) \wedge P(x))$

[2013 : 2 Marks]

1.41 Which one of the following is NOT logically equivalent to $\neg\exists x(\forall y(\alpha)\wedge\forall z(\beta))$

(a) $\forall x(\exists z(\neg\beta) \rightarrow \forall y(\alpha))$

(b) $\forall x(\forall z(\beta) \rightarrow \exists y(\neg\alpha))$

(c) $\forall x(\forall y(\alpha) \rightarrow \exists z(\neg\beta))$

(d) $\forall x(\exists y(\neg\alpha) \vee \exists z(\neg\beta))$

[2013 : 2 Marks]

1.42 Consider the statement

"Not all that glitters is gold"

Predicate *glitters*(x) is true if x glitters and predicate gold(x) is true if x is gold. Which one of the following logical formulae represents the above statement?

(a) $\forall x : \text{glitters}(x) \Rightarrow \neg\text{gold}(x)$

(b) $\forall x : \text{gold}(x) \Rightarrow \text{glitters}(x)$

(c) $\exists x : \text{gold}(x) \wedge \neg\text{glitters}(x)$

(d) $\exists x : \text{glitters}(x) \wedge \neg\text{gold}(x)$

[2014 (Set-1) : 1 Mark]

1.43 Which one of the following propositional logic formulas is TRUE when exactly two of p, q, and r are **TRUE**?

(a) $((p \leftrightarrow q) \wedge r) \vee (p \wedge q \wedge \sim r)$

(b) $(\sim(p \leftrightarrow q) \wedge r) \vee (p \wedge q \wedge \sim r)$

(c) $((p \rightarrow q) \wedge r) \vee (p \wedge q \wedge \sim r)$

(d) $(\sim(p \leftrightarrow q) \wedge r) \wedge (p \wedge q \wedge \sim r)$

[2014 (Set-1) : 2 Marks]

1.44 Which one of the following Boolean expressions is NOT a tautology?

(a) $((a \rightarrow b) \wedge (b \rightarrow c)) \rightarrow (a \rightarrow c)$

(b) $(a \leftrightarrow c) \rightarrow (\sim b \rightarrow (a \wedge c))$

(c) $(a \wedge b \wedge c) \rightarrow (c \vee a)$

(d) $a \rightarrow (b \rightarrow a)$

[2014 (Set-2) : 2 Marks]

1.45 Consider the following statements:

P: Good mobile phones are not cheap

Q: Cheap mobile phones are not good

L: P implies Q

M: Q implies P

N: P is equivalent to Q

Which one of the following about L, M, and N is **CORRECT**?

(a) Only L is TRUE (b) Only M is TRUE

(c) Only N is TRUE (d) L, M and N are TRUE.

[2014 (Set-3) : 1 Mark]

1.46 The CORRECT formula for the sentence, "not all rainy days are cold" is

(a) $\forall d$(Rainy $(d) \wedge \sim$ Cold (d))

(b) $\forall d(\sim$Rainy $(d) \rightarrow$ Cold (d))

(c) $\exists d(\sim$Rainy $(d) \rightarrow$ Cold (d))

(d) $\exists d$(Rainy $(d) \wedge \sim$ Cold (d))

[2014 (Set-3) : 2 Marks]

1.47 Which one of the following is Not equivalent to $p \leftrightarrow q$?

(a) $(\neg p \vee q) \wedge (p \vee \neg q)$

(b) $(\neg p \vee q) \wedge (q \rightarrow p)$

(c) $(\neg p \wedge q) \vee (p \wedge \neg q)$

(d) $(\neg p \wedge \neg q) \vee (p \wedge q)$

[2015 (Set-1) : 1 Mark]

1.48 The binary operator $\neq$ is defined by the following truth table.

p	q	$p \neq q$
0	0	0
0	1	1
1	0	1
1	1	0

Which one of the following is true about the

(a) Both commutative and associative
(b) Commutative but not associative
(c) Not commutative but associative
(d) Neither commutative nor associative

[2015 (Set-1) : 2 Marks]

1.49 Consider the following two statements.

S1: If a candidate is known to be corrupt, then he will not be elected.

S2: If a candidate is kind , he will be elected.

Which one of the following statements follows from S1 and S2 as per sound inference rules of logic?

(a) If a person is known to be corrupt, he is kind
(b) If a person is not known to be corrupt, he is not kind
(c) If a person is kind, he is not known to be corrupt
(d) If a person is not kind, he is not known to be corrupt

[2015 (Set-2) : 1 Mark]

1.50 Which one of the following well formed formulae is a tautology?

(a) $x \quad y\, R(x, y) \qquad y \quad x\, R(x, y)$
(b) $(\quad x\, [\quad y\, R(x, y) \qquad S(x, y)]) \qquad x \quad y\, S(x, y)$
(c) $[\quad x \quad y\, (P(x, y) \qquad R(x, y))] \qquad [\quad x \quad y\, (\quad P(x, y) \qquad R(x, y))]$
(d) $x \quad y\, P(x, y) \qquad x \quad y\, P(y, x)$

[2015 (Set-2) : 2 Marks]

1.51 In a room there are only two types of people, namely Type 1 and Type 2. Type 1 people always tell the truth and Type 2 people always lie. You give a fair coin to a person in that room, without knowing which type he is from and tell him to toss it and hide the result from you till you ask for it. Upon asking, the person replies the following

"The result of the toss is head if and only if I am telling the truth."

Which of the following options is correct?

(a) The result is head
(b) The result is tail
(c) If the person is of Type 2, then the result is tail
(d) If the person is of Type 1, then the result is tail

[2015 (Set-3) : 1 Mark]

1.52 Let p, q, r, s represent the following propositions.

$p : x \quad \{8, 9, 10, 11, 12\}$
q : x is a composite number
r : x is a perfect square
s : x is a prime number

The integer $x \quad 2$ which satisfies $((p \quad q) \quad (\quad r \quad s))$ is _____.

[2016 (Set-1) : 1 Marks]

1.53 Consider the following expressions:

(i) false (ii) Q
(iii) true (iv) $P \quad Q$
(v) $-Q \quad P$

The number of expressions given above that are logically implied by $P \quad (P \quad Q)$ is _____.

[2016 (Set-2) : 1 Marks]

1.54 Which one of the following well-formed formulae in predicate calculus is **NOT** valid?

(a) $(\quad x\, p(x) \qquad x\, q(x)) \qquad (\quad x \quad p(x) \qquad x\, q(x))$
(b) $(\quad x\, p(x) \qquad x\, q(x)) \qquad x(p(x) \quad q(x))$
(c) $x\,(p(x) \quad q(x)) \qquad (\quad x\, p(x) \qquad x\, q(x))$
(d) $x\,(p(x) \quad q(x)) \qquad (\quad x\, p(x) \qquad x\, q(x))$

[2016 (Set-2) : 2 Marks]

Answers Mathematical Logic

1.1	(a)	1.2	(a)	1.3	(b)	1.4	(b)	1.6	(d)	1.7	(d)	1.8	(a)	1.9	(a)	1.10	(a)
1.11	(a)	1.12	(d)	1.13	(d)	1.14	(b)	1.15	(d)	1.16	(c)	1.17	(c)	1.18	(a)	1.19	(b)
1.20	(b)	1.21	(a)	1.22	(a)	1.23	(d)	1.24	(d)	1.25	(d)	1.26	(d)	1.27	(a)	1.28	(a)
1.29	(a)	1.30	(c)	1.31	(d)	1.32	(b)	1.33	(d)	1.34	(b)	1.35	(b)	1.36	(b)	1.37	(a)
1.38	(b)	1.39	(c)	1.40	(d)	1.41	(a,d)	1.42	(d)	1.43	(b)	1.44	(b)	1.45	(d)	1.46	(d)
1.47	(c)	1.48	(a)	1.49	(c)	1.50	(c)	1.51	(a)	154.	(d)						

Explanations Mathematical Logic

1.1 (a)

Option (a) is well known valid formula (tautology) because it is a rule of inference called hypothetical syllogism.

By applying boolean algebra and simplifying, we can show that (b), (c) and (d) are invalid.

For example,

Choice (b) $\equiv (P \Rightarrow Q) \Rightarrow (\neg P \Rightarrow \neg Q)$

$\equiv (P \Rightarrow Q) \Rightarrow (P' \Rightarrow Q')$

$\equiv (P' + Q) \Rightarrow (P + Q')$

$\equiv (P' + Q)' + (P + Q')$

$\equiv PQ' + P + Q'$

$\equiv P + Q'$

$\neq 1$

So, invalid.

1.2 (a)

According to distributive properties

$\forall x(P(x) \wedge Q(x)) \longleftrightarrow \forall xP(x) \wedge \forall xQ(x)$

$(\forall x P(x) \vee \forall xQ(x)) \longrightarrow \forall x(P(x) \vee Q(x))$

$\exists x(P(x) \vee Q(x)) \longleftrightarrow \exists xP(x) \vee \exists xQ(x)$

$\exists x(P(x) \wedge Q(x)) \longrightarrow \exists xP(x) \wedge \exists xQ(x)$

So option (a) is valid.

1.3 (b)

(a) $(a \vee b) \rightarrow (b \wedge c)$

$\equiv (a+b)' + bc$

$\equiv a'b' + bc$

Therefore, $((a \vee b) \rightarrow (b \wedge c))$ is contingency and not tautology.

(b) $(a \wedge b) \rightarrow (b \vee c)$

$\equiv ab \rightarrow b + c$

$\equiv (ab)' + b + c$

$\equiv a' + b' + b + c$

$\equiv a' + 1 + c$

$\equiv 1$

So $((a \wedge b) \rightarrow (b \vee c))$ is tautology.

(c) $(a \vee b) \rightarrow (b \rightarrow c)$

$\equiv (a + b) \rightarrow (b' + c)$

$\equiv (a + b)' + b' + c$

$\equiv a'b' + b' + c$

$\equiv b' + c$

So $((a \vee b) \rightarrow (b \rightarrow c))$ is contingency but not tautology.

(d) $(a \rightarrow b) \rightarrow (b \rightarrow c)$

$\equiv (a' + b) \rightarrow (b' + c)$

$\equiv (a' + b)' + b' + c$

$\equiv ab' + b' + c$

$\equiv b' + c$

Therefore, $((a \rightarrow b) \rightarrow (b \rightarrow c))$ is contingency but not tautology.

1.4 (b)

The proposition $p \wedge (\sim p \vee q)$

$\equiv p(p' + q) \equiv pq$

$\equiv p \wedge q$

1.5 Sol.

TRUE

p	q	$p \leftrightarrow q$	$p \rightarrow \sim q$	$(p \leftrightarrow q) \rightarrow (p \rightarrow \sim q)$
T	T	T	F	F
T	F	F	T	T
F	T	F	T	T
F	F	T	T	T

From the truth table, $(p \leftrightarrow q) \rightarrow (p \rightarrow \sim q)$ is not tautology, hence it is true that $p \leftrightarrow q$ doesn't imply $p \rightarrow \neg q$.

1.6 (d)

p	q	$\neg p \rightarrow q$
0	0	0
0	1	1
1	0	1
1	1	1

Now since $\neg p \rightarrow q$ is given true, we reduce the truth table as follows

p	q	$\neg p \rightarrow q$
0	1	1
1	0	1
1	1	1

In the reduced truth table we need to find the truth value of $\neg p \vee (p \rightarrow q) \equiv p' + (p \rightarrow q)$

$\equiv p' + p' + q \equiv p' + q$

The truth value of $p' + q$ in the reduced truth table is given below.

p	q	$p' + q$
0	1	1
1	0	0
1	1	1

Since in the reduced truth table also, the given expression is sometimes true and sometimes false, therefore the truth value of proposition $\neg p \vee (p \rightarrow q)$ can not be determined.

1.7 (d)

(a) $((x \rightarrow y) \wedge x) \rightarrow y$
$= \sim((\sim x \vee y) \wedge x) \vee y$
$= \sim((\sim x \wedge x) \vee (y \wedge x)) \vee y$
$= \sim(F \vee (y \wedge x) \vee y$
$= \sim(y \wedge x) \vee y$
$= \sim y \vee \sim x \vee y$
$= (\sim y \vee y) \vee \sim x$
$= T \vee \sim x$
$= T$

(b) $((\sim x \rightarrow y) \wedge (\sim x \rightarrow \sim y)) \rightarrow x$
$= \sim(\sim(\sim x) \vee y) \wedge (\sim(\sim x) \vee \sim y)) \vee x$
$= \sim((x \vee y) \wedge (x \vee \sim y)) \vee x$
$= \sim(x \vee (y \wedge \sim y)) \vee x$
$= \sim(x \vee F) \vee x$
$= \sim x \vee x$
$= T$

(c) $(x \rightarrow (x \vee y))$
$= (\sim x \vee (x \vee y))$
$= ((\sim x \vee x) \vee y)$
$= (T \vee y)$
$= T$

(d) $((x \vee y) \leftrightarrow (\sim x \rightarrow \sim y))$
$= (x \vee y) \leftrightarrow (\sim (\sim x) \vee \sim y)$
$= (x \vee y) \leftrightarrow (x \vee \sim y)$
$= ((x \vee y) \wedge (x \vee \sim y)) \vee (\sim(x \vee y) \wedge \sim(x \vee \sim y))$
$= ((x \vee (y \wedge \sim y)) \vee ((\sim x \wedge \sim y) \wedge (\sim x \wedge y))$
$= (x \vee F) \vee ((\sim x \wedge \sim y) \wedge (\sim x \wedge y))$
$= x \vee (\sim x \wedge (y \wedge \sim y)) = x \vee (\sim x \wedge F)$
$= x \vee F = x$

1.8 (a)

- $a \leftrightarrow (b \vee \neg b)$
  $a \leftrightarrow$ True
  So a is true, i.e. $a = 1$
- $b \leftrightarrow c$ holds. So $b = c$
  Now the given expression is
  $(a \wedge b) \rightarrow ((a \wedge c) \vee d) \equiv (a \cdot b) \rightarrow ((a \cdot c) + d)$
  Putting $a = 1$ in above expression we get
  $1 \cdot b \rightarrow ((1 \cdot c) + d)$
  $\equiv b \rightarrow c + d$
  $\equiv b' + c + d$

Now putting $b = c$ in above expression we get
$\equiv c' + c + d \equiv 1 + d \equiv 1$
So the expression is always true.

1.9 (a)

Let p : I stay
q : you go
I stay only if you go
$p \rightarrow q$
Converse of $p \rightarrow q$ is $q \rightarrow p$
Now convert the answers one-by-one into boolean form. Only option (a) i.e. "*I stay if you go*" converts to $q \rightarrow p$.

1.10 (a)

F_1: $P \rightarrow \sim P \equiv p \rightarrow p' \equiv p' + p' \equiv p'$
So F_1 is contingency. Hence, F_1 is satisfiable but not valid.
F_2: $(P \rightarrow \sim P) \vee (\sim P \rightarrow P)$
$\equiv (p \rightarrow p') + (p' \rightarrow p)$
$\equiv (p' + p') + (p + p)$
$\equiv p' + p \equiv 1$
So F_2 is tautology and therefore valid.

1.11 (a)

If X then Y unless Z is represented by
$X \rightarrow Y$ unless $Z \equiv (X \rightarrow Y) + Z$
$\equiv X' + Y + Z$
Now convert the answers one-by-one into boolean form only choice (a) converts to $X' + Y + Z$ as can be seen below
$(X \wedge \neg Z) \rightarrow Y \equiv XZ' \rightarrow Y$
$\equiv (XZ')' + Y$
$\equiv X' + Y + Z$

1.12 (d)

$(\forall x)\ [\alpha \Rightarrow \beta] \Rightarrow ((\forall x)\ [\alpha] \Rightarrow \forall(x)\ [\beta])$ is a logical equivalence and therefore, a valid first order formula.

1.13 (d)

$Q_{yy} \equiv$ "y divides y" is always true
$\therefore\ Q_{xy} \Leftrightarrow \neg Q_{yy}$ is same as $Q_{xy} \Leftrightarrow$ False
Now α becomes
$(\forall x)[P(x) \Leftrightarrow (\forall y)(Q_{xy} \Leftrightarrow \text{false})]$
$\Rightarrow (\forall x)\ [\neg P(x)]$
Now consider I_1 : $P(x) \equiv$ "x is a prime number".
α becomes
($\forall$x x is a prime number if and only if $\forall$y (y does not divide x)) $\Rightarrow$ ($\forall$x) x is not prime.
which means that x is a prime number if and only if no number divides x implies that no number is prime.

Since x always divides x, the above sentence is true.

Now consider $I_2 : P(x) \equiv$ "x is a composite number".

Now α becomes

($\forall$x x is a composite number if and only if $\forall$y (y does not divide x)) $\Rightarrow$ ($\forall$x) x is not composite.

Which means that x is a composite number if and only if no number divides x implies that no number is composite.

Since x always divides x, the above sentence is true.

$\therefore$ Both I_1 and I_2 satisfy α.

1.14 (b)

Derive clause $P \vee Q$ from clauses $P \vee R$, $Q \vee \neg R$ means that $(P \vee R) \wedge (Q \vee \neg R) \Rightarrow P \vee Q$

$\therefore$ (a) is true

Since, $x \Rightarrow y$ does not imply that $y \Rightarrow x$

$\therefore$ $P \vee Q \Rightarrow (P \vee R) \wedge (Q \vee \neg R)$

$\therefore$ may or may not be true.

Hence (b) is false.

1.15 (d)

The statement is "some boys in the class are taller than all the girls".

So the notation for the given statement is

$(\exists x)\ (\text{boy}(x) \wedge (\forall y)\ (\text{girl}(y) \rightarrow \text{taller}\ (x, y)))$

1.16 (c)

Choice (c) is

$\neg(\forall x)(\exists y)[(a(x,y) \wedge b(x,y)) \rightarrow c(x,y)]$

$\equiv \neg(\forall x)(\exists y)[a \wedge b \rightarrow c]$

$\equiv \neg(\forall x)(\exists y)[(ab)' + c]$

$\equiv \exists x\, \forall y[(ab)' + c]'$

$\equiv \exists x\, \forall y[abc'] \equiv \exists x\, \forall y[a \wedge b \wedge \neg c]$

which is same as the given expression.

$(\exists x)(\forall y)[(a(x,y) \wedge b(x,y)) \wedge \neg c(x,y)]$

1.17 (c)

P: $[(\neg p \vee q) \wedge (r \rightarrow s) \wedge (p \vee r)] \rightarrow (\neg s \rightarrow q)$

$\equiv [(p \rightarrow q) \wedge (r \rightarrow s) \wedge (p \vee r)] \rightarrow (q \vee s)$

which is a rule of inference called constructive dilemma and therefore valid.

S: $[p \wedge (p \rightarrow r) \wedge (q \vee \neg r)] \rightarrow q$

$\equiv p(p' + r)(q + r') \rightarrow q$

$\equiv pr(q + r') \rightarrow q$

$\equiv prq \rightarrow q$

$\equiv (prq)' + q$

$\equiv p' + r' + q' + q$

$\equiv p' + r' + 1 \equiv 1$

Therefore S is valid.

Q and R can be similarly simplified in boolean algebra to show that they are both not equivalent to 1.

So only P and S are valid.

1.18 (a)

$(P \rightarrow (Q \vee R)) \rightarrow ((P \wedge Q) \rightarrow R)$

$\equiv (P \rightarrow Q + R) \rightarrow (PQ \rightarrow R)$

$\equiv [P' + Q + R] \rightarrow [(PQ)' + R]$

$\equiv [P' + Q + R] \rightarrow [P' + Q' + R]$

$\equiv (P' + Q + R)' + P' + Q' + R$

$\equiv P\,Q'\,R' + P' + Q' + R$

$\equiv Q' + Q'\,P\,R' + P' + R$

$\equiv Q' + P' + R$ (by absorption law)

Which is a contingency (i.e. satisfiable but not valid).

1.19 (b)

X: $(P \vee Q) \rightarrow R$

Y: $(P \rightarrow R) \vee (Q \rightarrow R)$

X: $P + Q \rightarrow R \equiv (P + Q)' + R \equiv P'\,Q' + R$

Y: $(P' + R) + (Q' + R) \equiv P' + Q' + R$

Clearly $X \neq Y$

Consider $X \rightarrow Y$

$\equiv (P'Q' + R) \rightarrow (P' + Q' + R)$

$\equiv (P'Q' + R)' + P' + Q' + R$

$\equiv (P'Q')' \cdot R' + P' + Q' + R$

$\equiv (P + Q) \cdot R' + P' + Q' + R$

$\equiv PR' + QR' + P' + Q' + R$

$\equiv (PR' + R) + (QR' + Q') + P'$

$\equiv (P + R)\,(R' + R) + (Q + Q') \times (R' + Q') + P'$

$\equiv (P + R) + (R' + Q') + P'$

$\equiv P + P' + R + R' + Q'$

$\equiv 1 + 1 + Q' \equiv 1$

$\therefore$ $X \rightarrow Y$ is a tautology.

1.20 (b)

"Every teacher is liked by some student: then the logical expression is $\forall\ (x)\ [\text{teacher}(x) \rightarrow \exists(y)\ [\text{student}\ (y) \wedge \text{likes}\ (y, x)]]$

Where likes (y, x) means y likes x, such that y represent the student and x represents the teacher.

1.21 (b)

Consider choice (b)

$(\forall x(P(x) \Rightarrow Q(x))) \Rightarrow ((\forall xP(x)) \Rightarrow (\forall xQ(x)))$

Let the LHS of this implication be true

This means that

$P_1 \rightarrow Q_1$
$P_2 \rightarrow Q_2$
$\vdots$
$P_n \rightarrow Q_n$

Now we need to check if the RHS is also true.

The RHS is $((\forall xP(x)) \Rightarrow (\forall xQ(x)))$

To check this let us take the LHS of this as true i.e. take $\forall x\, P(x)$ to be true. This means that $(P_1, P_2, \ldots P_n)$ is taken to be true. Now P_1 along with $P_1 \rightarrow Q_1$ will imply that Q_1 is true. Similarly P_2 along with $P_2 \rightarrow Q_2$ will imply that Q_2 is true. And so on...

Therefore $(Q_1, Q_2, \ldots Q_n)$ all true.

i.e. $\forall x\, Q(x)$ is true. Therefore the statement (b) is a valid predicate statement.

1.22 (b)

Since a relation may or may not be symmetric, the given predicate is satisfiable but not valid. So (a) is clearly false.

Whenever a predicate is satisfiable its negation also is satisfiable. So option (b) is the correct answer.

1.23 (d)

The given statement should be read as

"If an animal is a tiger or a lion, then (if the animal is hungry or threatened, then it will attack). Therefore the correct translation is $\forall x$ [(tiger (x) $\vee$ lion (x)) $\rightarrow$ {(hungry (x) $\vee$ threatened (x)) $\rightarrow$ attacks (x)}]

which is choice (d).

1.24 (d)

P_1: $((A \wedge B) \rightarrow C)) \equiv ((A \rightarrow C) \wedge (B \rightarrow C))$

LHS:

$(A \wedge B) \rightarrow C$

$\equiv AB \rightarrow C$

$\equiv (A\,B)' + C$

$\equiv A' + B' + C$

RHS:

$(A \rightarrow C) \wedge (B \rightarrow C)$

$\equiv (A' + C)\,(B' + C)$

$\equiv A'B' + C$

Clearly, LHS $\neq$ RHS

P_1 is not a tautology

P_2: $((A \vee B \rightarrow C)) \equiv ((A \rightarrow C) \vee (B \rightarrow C))$

LHS $\equiv (A + B \rightarrow C)$

$\equiv (A + B)' + C$

$\equiv A'B' + C$

RHS $\equiv (A \rightarrow C) \vee (B \rightarrow C)$

$\equiv (A' + C) + (B' + C)$

$\equiv A' + B' + C$

Clearly, LHS $\neq$ RHS $\Rightarrow P_2$ is also not a tautology.

Therefore, both P_1 and P_2 are not tautologies.

Correct choice is (d).

1.25 (d)

By using min terms we can define

$A \odot B = AB + AB' + A'B'$

$= A + A'B'$

$= (A + A') \cdot (A + B') = A + B'$

(a) $\sim A \odot B = A' \odot B = A' + B'$

(b) $\sim (A \odot \sim B) = (A \odot B')' = (A + (B')')' = (A + B)' = A'B'$

(c) $\sim (\sim A \odot \sim B) = (A' \odot B')' = (A' + (B')')' = (A' + B)' = AB'$

(d) $\sim (\sim A \odot B) = (A' \odot B)' = (A' + B')' = A \cdot B = A \wedge B$

$\therefore$ Only, choice (d) $\equiv A \wedge B$

Note: This problem can also be done by constructing truth table for each choice and comparing with truth table for $A \wedge B$.

1.26 (d)

The statement "Not every graph is connected" is same as "There exists some graph which is not connected" which is same as

$\exists x$ {graph (x) $\wedge \neg$ connected (x)}

Which is choice (b)

By boolean algebra we can see that option (a) and (c) are same as (b). Only option (d) is not the same as (b).

Infact option (d) means that "all graphs are not connected".

Alternate solution

We can translate the given statement "NOT (every graph is connected)" as $\neg(\forall x \text{ graph}(x) \rightarrow \text{connected}(x))$

$\equiv \exists x \neg(\text{graph}(x) \rightarrow \text{connected}(x))$

$\equiv \exists x \neg(\neg\text{graph}(x) \vee \text{connected}(x))$

$\equiv \exists x (\text{graph}(x) \wedge \neg\text{connected}(x))$

By boolean algebra we can see that option (a) and (c) are same as (b). Only option (d) is not the same as (b).

Infact option (d) means that "all graphs are not connected".

1.27 (a)

In conjunction normal form, for any particular assignment of truth values, all except one clause, will always evaluate to true. So, the proportion of clauses which evaluate to true to the total number of clauses is equal to $\frac{2^n - 1}{2^n}$.

Now putting n = 1, 2, . . ., we get 1/2, 3/4, 7/8 ...

All of these proportions are $\geq 1/2$ and so choice (a) atleast half of the clauses evaluate to true, is the correct answer.

1.28 (a)

Option (a) is a standard one way distributive property of predicates.

1.29 (a)

"For x which is an fsa, there exists a y which is a pda and which is equivalent to x."

$(\forall x\, \text{fsa}(x) \Rightarrow (\exists y\, \text{pda}(y) \wedge \text{equivalent}(x, y))$ is the logical representation.

1.30 (c)

Option (c) is

$[(\exists x, \alpha(x)) \rightarrow \beta] \rightarrow [\forall x, \alpha(x) \rightarrow \beta]$

Let us check the validity of this predicate.

Let the LHS of this predicate be true.

This means that some $\alpha \rightarrow \beta$.

Let $\alpha_5 \rightarrow \beta$

Now we will check if the RHS is true. The RHS is $[\forall x, \alpha(x) \rightarrow \beta]$ to check this implication let us take $\forall x, \alpha(x)$ to we true.

This means that all the α are true. It means that α_5 is also true.

But $\alpha_5 \rightarrow \beta$. Therefore β is true.

So the RHS $[\forall x, \alpha(x) \rightarrow \beta]$ is true.

Whenever the LHS $[(\exists x, \alpha(x)) \rightarrow \beta]$ is true. So option (c) is valid.

1.31 (d)

The given predicate is

$[\forall x, \alpha \rightarrow (\exists y, \beta \rightarrow (\forall u, \exists v, \gamma))]$

The negation is of this predicate is

$\neg[\forall x\ \alpha \rightarrow (\exists y\ \beta \rightarrow \forall u\ \exists v\ \gamma)]$

$\neg[\forall x\ \alpha \rightarrow (\forall y\ \neg\beta \vee \forall u\ \exists v\ \gamma)]$

$\neg[\exists x\ \neg\alpha \vee (\forall y\ \neg\beta \vee \forall u\ \exists v\ \gamma)]$

$\forall x\ \alpha \wedge \exists y\ \beta \wedge \exists u \forall v\ \neg\gamma$

Which is option (d).

1.32 (b)

(i) $P \vee \sim Q \equiv p + q'$

(ii) $\sim(\sim P \wedge Q) \equiv (p'q)' \equiv p + q'$

(iii) $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$

$\equiv pq + pq' + p'q'$

$\equiv p(q + q') + p'q'$

$\equiv p + p'q'$

$\equiv (p + p')(p + q')$

$\equiv p + q'$

(iv) $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge Q)$

$\equiv pq + pq' + p'q$

$\equiv p(q + q') + p'q$

$\equiv p + p'q$

$\equiv (p + p')(p + q)$

$\equiv p + q$

Clearly (i), (ii) and (iii) are equivalent. Correct choice is (b).

1.33 (d)

The correct translation of "Gold and silver ornaments are precious" is choice (d)

$$\forall x((G(x) \vee S(x)) \rightarrow P(x))$$

which is read as "if an ornament is gold or silver, then it is precious".

Now since a given ornament cannot be both gold and silver at the same time.

Choice (b) $\forall x((G(x) \wedge S(x)) \rightarrow P(x))$ is incorrect.

1.34 (b)

The given table can be converted into boolean function by adding minterms corresponding to true rows.

$\therefore$

P	Q	P□Q
T	T	T
T	F	T
F	T	F
F	F	T

Since there is only one false in the above truth table, we can represent the function P □ Q more efficientely, in conjunctive normal form.

Translates P □ Q = P + Q′ (the max-term corresponding to the third row, where the function is false).

Now, we can easily translate the choices into boolean algebra as follows:

Choice (a) $\neg Q \,\square\, \neg P \equiv Q' \,\square\, P' \equiv Q' + P$

Choice (b) $P \,\square\, \neg Q \equiv P \,\square\, Q' \equiv P + Q$

Choice (c) $\neg P \,\square\, Q \equiv P' \,\square\, Q \equiv P' + Q'$

Choice (d) $\neg P \,\square\, \neg Q \equiv P' \,\square\, Q' \equiv P' + Q$

As we can clearly see only choice (b) $P \,\square\, \neg Q$ is equivalent to P + Q.

1.35 (b)

I $\neg\forall x P(x) \equiv \exists x \neg P(x)$

and IV $\exists x \neg P(x)$

Clearly, choices I and IV are equivalent.

II $\neg \exists x P(x) \equiv \forall x \neg P(x)$

and III $\neg \exists x[\neg P(x)] \equiv \forall x P(x)$

Clearly II and III are not equivalent to each other or to I and IV.

1.36 (b)

$\forall x\, \exists y\, \exists t\, \neg F(x, y, t)$

$\equiv \neg\{\exists x\, \forall y\, \forall t\, F(x, y, t)\}$

≡ it is not true that (someone can fool all people at all time)

≡ no one can fool everyone all the time

1.37 (a)

If P(x) is true, then

$x \neq 1$ and also

x is broken into two factors, only if, one of the factors is x itself and the other factor is 1, which is exactly the definition of a prime number.

So P(x) is true means x is a prime number.

1.38 (b)

Let p : It rains

q : cricket match will not be played.

I_1 : $p \Rightarrow q$

$\underline{\sim q}$

$\therefore \sim p$

Clearly I_1 is correct since it is in the form of Modus Tollens (rule of contrapositive)

I_2 : $p \Rightarrow q$

$\underline{\sim p}$

$\therefore \sim q$

which corresponds $[p \Rightarrow q \wedge \sim p] \Rightarrow \sim q$

$\equiv [(p' + q)\, p'] \Rightarrow q'$

$\equiv [p' + qp'] \Rightarrow q'$

$\equiv p' \Rightarrow q'$

$\equiv (p')' + q' \equiv p + q'$

which is not a tautology.

So I_2 is incorrect inference.

1.39 (c)

Some real nos are rational

$\equiv \exists x$ [real (x) ^ rational (x)]

1.40 (d)

None of my friends are perfect

i.e. all of my friends are not perfect

$\forall x((F(x) \rightarrow \neg P(x))$

$\forall x(\neg F(x) \vee \neg P(x))$

$\neg \exists x(F(x) \wedge P(x))$

Alternatively

$\exists x\, (F(x) \wedge P(x))$ gives

there exist some of my friends who are perfect.

$\neg \exists x\, (F(x) \wedge P(x))$

there does not exits any friend who is perfect

i.e. none of my friends are perfect.

So (d) is correct option.

1.41 (a)

Let $\forall y(\alpha) = P,\ \forall z(\beta) = Q$

Then $\exists y(\neg\alpha) = \neg P$ and $\exists z(\neg\beta) = \neg Q$

Given $\neg\exists x(\forall y(\alpha) \wedge \forall z(\beta))$

$= \neg\exists x(P \wedge Q)$

$= \neg\forall x(\neg P \vee \neg Q)$

$= \forall x(P \rightarrow \neg Q)$

(a) $\forall x(\exists z(\neg\beta) \rightarrow \forall y(\alpha)) = \forall x(\neg Q \rightarrow P)$

(b) $\forall x(\forall z(\beta) \rightarrow \exists y(\neg\alpha)) = \forall x(Q \rightarrow \neg P)$

$= \forall x(P \rightarrow \neg Q)$

(c) $\forall x(\forall y(\alpha) \rightarrow \exists z(\neg\beta)) = \forall x(P \rightarrow \neg Q)$

(d) $\forall x(\exists y(\neg\alpha) \vee \exists z(\neg\beta)) = \forall x(\neg P \vee \neg Q)$

$= \forall x(P \rightarrow \neg Q)$

$\therefore$ Only (a) is not logically equivalent to $\forall x(P \rightarrow \neg Q)$.

1.42 (d)

(a) $\forall x$ glitters$(x) \Rightarrow \neg$gold(x)
All glitters are not gold

(b) $\forall x$ gold$(x) \Rightarrow$ glitters(x)
All golds are glitters

(c) $\exists x$ gold$(x) \wedge \neg$ glitters(x)
There exist gold which is not glitter i.e. not all golds are glitters.

(d) $\exists x$ glitters$(x) \wedge \neg$ gold(x)
Not all that glitters is gold i.e., there exist some which glitters and which is not gold.

1.43 (b)

Option (b) is $(\sim(p \leftrightarrow q) \wedge r) \vee (p \wedge q \wedge \sim r)$
$\equiv ((p \oplus q)\ r) + pqr'$
$\equiv (pq' + p'q)\ r + pqr'$
$\equiv pq'r + p'qr + pqr'$
$\equiv pqr' + pq'r + p'qr$
This is exactly the min-term form of a logical formula which is true when exactly two variables are true (only p, q true or only p, r true or only q, r true).

1.44 (b)

$(a \leftrightarrow c) \rightarrow (\sim b \rightarrow (a \wedge c))$
$\equiv (a \leftrightarrow c)' + (b' \rightarrow ac)$
$\equiv (a \oplus c)' + (b' \rightarrow ac)$
$\equiv ac' + a'c + b + ac$
$\equiv a(c' + c) + a'c + b$
$\equiv a + a'c + b \equiv a + c + b$
which is not a tautology.

1.45 (d)

g : mobile is good
c : mobile is cheap
P: Good mobile phones are not cheap

$$g \rightarrow c' \equiv (g' + c')$$

Q: Cheap mobile phones are not good

$$c \rightarrow g' \equiv (c' + g')$$

$\therefore$ Both P and Q are equivalent.

L: $P \rightarrow Q$
M: $Q \rightarrow P$
N: $P \equiv Q$
Since both P and Q are equivalent, all three of L, M, N are true.
So correct option is (d).

1.46 (d)

Not (all rainy days are cold):
$\sim (\forall d$ (Rainy $(d) \rightarrow$ Cold $(d)))$
$\equiv \sim (\forall d$ ($\sim$ Rainy $(d) \vee$ Cold $(d)))$
$\equiv \exists d$ (Rainy $(d) \wedge \sim$ Cold (d))

Alternate Method
Not all rainy days are cold is same as some rainy days are not cold which is same as
$\exists d$ (Rainy $(d) \wedge \sim$ Cold (d))

1.47 (c)

Here, option (a) and (b) can be reduced to $(p \rightarrow q) \wedge (q \rightarrow p)$ and hence $\equiv p \leftrightarrow q$
Option (d) is $p'q' + pq \equiv p \leftrightarrow q$.
Option (c) is $p'q + pq' = p \oplus q$ which is not equivalent to $p \leftrightarrow q$.

1.48 (a)

The given truth table corresponds to $p'q + pq' = p \oplus q$.
$\oplus$ is known to be both commutative and associative.

1.49 (c)

C : Person is corrupt
K : Person is kind.
E : Person is elected
$S_1 : C \rightarrow \bar{E}$
$S_2 : K \rightarrow E$
$S_2 \equiv \bar{E} \rightarrow \bar{K}$.

So from S_1 and S_2 : $(C \rightarrow \bar{E}) \wedge (\bar{E} \rightarrow \bar{K}) \equiv C \rightarrow \bar{K}$
We can conclude $C \rightarrow \bar{K}$ which is same as $K \rightarrow \bar{C}$, which is same as option (c).

1.50 (c)

Since $P \rightarrow R \equiv \neg P \vee R$, and the quantifiers on both the sides are same $(\forall x \exists y)$.
Option (c) is clearly a tautology.

1.51 (a)

Type 1 always tells truth.
Type 2 always tells lies.
The statement is "toss is head if and only if I am telling the truth".
There are two cases possible.
Case-1: Let the person be type 1.
Case-2: Let the person be type 2.
In each case we shall prove that result is head.
Case-1: Let the person be type 1. Type 1 always tells truth.
So the statement "toss is head if and only if I am telling the truth" is true.
So toss head $\Leftrightarrow$ telling truth. Since type 1 is telling truth so toss head is also true. So in case 1 result is that toss is head.
Case-2: Let the person be type 2. Type 2 always tells lies.
So the statement "toss is head if and only if I am telling the truth" is false.
So toss head $\Leftrightarrow$ telling truth is false. So toss head $\oplus$ telling truth. So toss head and telling truth have opposite truth values. Now, since type 2 telling truth is false, so toss head has to have opposite truth value which is true.
So toss head is true. So in case 2 also, result is head.
So in both cases we have proved that the result is head.
So option (a) is correct.

1.52 Sol.

We wish to make

$\neg((p \Rightarrow q) \wedge (\neg r \vee \neg s)) = 1$

$\Rightarrow (p \Rightarrow q) \wedge (\neg r \vee \neg s) = 0$

$\Rightarrow (p \Rightarrow q) = 0$ ...(1)

or $\neg r \vee \neg s = 0$ ...(2)

Now (1) is satisfies only when $p = 1$ and $q = 0$.
Equation (2) $\neg r \vee \neg s = 0$, iff $r \wedge s = 1$
i.e. $r = 1$ and $s = 1$
i.e. x is a perfect square and x is a prime number. Which is not possible so condition (2) cannot be satisfied by any x.
So condition (1) must be satisfies which is $p = 1$ and $q = 0$ i.e. $x \in \{8, 9, 10, 11, 12,\}$ and x is not a composite.
Now the only value of x which satisfies this is $x = 11$. So correct answer is $x = 11$.

1.53 Sol.

$p \wedge (p \Rightarrow q) = p(p' + q) \equiv pq$

Take (i) false

$$\begin{aligned} pq \Rightarrow \text{false} &\equiv pq \Rightarrow 0 \\ &\equiv (pq)' + 0 \\ &\equiv p' + q' + 0 \\ &\equiv \text{not valid} \end{aligned}$$

Take (ii)

$$\begin{aligned} pq \Rightarrow q &\equiv (pq)' + q \\ &\equiv (pq)' + q \\ &\equiv p' + 1 \equiv 1 \equiv \text{valid} \end{aligned}$$

Take(iii)

$$\begin{aligned} pq \Rightarrow \text{true} &\equiv pq \Rightarrow 1 \\ &\equiv (pq)' + 1 \equiv 1 \equiv \text{valid} \end{aligned}$$

Take(iv)

$$\begin{aligned} pq \Rightarrow p + q &\equiv (pq)' + p + q \\ &\equiv p' + q' + p + q \\ &\equiv 1 + 1 \equiv 1 \equiv \text{valid} \end{aligned}$$

Take(v)

$$\begin{aligned} pq \Rightarrow q' + p &\equiv (pq)' + q' + p \\ &\equiv p' + q' + q' + p \\ &\equiv 1 \equiv \text{valid} \end{aligned}$$

So the number of expressions that are logically implies by $p \wedge (p \Rightarrow q)$ is 4.

1.54 (d)

$\forall x$ is only one way distribution over "$\vee$" so let us check (d)

(d) $\forall x\, (p(x) \vee q(x)) \Rightarrow (\forall x\, p(x) \vee \forall x\, q(x))$

Let LHS be true, so we have,

$p_1 \vee q_1$ (true)

$p_2 \vee q_2$ (true)

$p_3 \vee q_3$(true)

$\vdots$

Now take p_1 is true, q_1 is false and p_2 is false, q_2 is true.

Now LHS is true, but RHS, $\forall x\, p(x)$ is false (since p_2 is false) and $\forall x\, q(x)$ is also false (since q_1 is false)

So, LHS $\not\Rightarrow$ RHS.

■■■■

2 Set Theory and Algebra

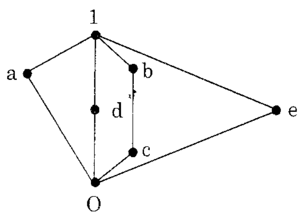
2.1 State whether the following statement are TRUE or FALSE:
The union of two equivalence relations is also an equivalence relation.

[1987 : 1 Mark]

2.2 (a) How many binary relations are there on a set A with n elements?
(b) How many one–to–one functions are there from a set A with n elements onto itself.

[1987 : 1 Mark]

2.3 The complement(s) of the element 'a' in the lattice shown in figure is (are) ......



[1988 : 2 Marks]

2.4 The transitive closure of the relation {(1, 2) (2, 3) (3, 4) (5, 4)} on the set A = {1, 2, 3, 4, 5} is ____

[1989 : 2 Marks]

2.5 Let S be an infinite set and $S_1, S_2, \ldots S_n$ be sets such that $S_1 \cup S_2 \cup \ldots\ldots \cup S_n = S$. Then
(a) At least one of the sets S_i is a finite set
(b) Not more than one of the sets S_i can be finite
(c) At least one of sets S_i is infinite
(d) Not more than one the sets S_i is an a infinite set.

[1993 : 1 Mark]

2.6 Let A be a finite set of size n. The number of elements in the power set of A × A is
(a) 2^{2^n} (b) 2^{n^2}
(c) 2^n (d) 2^n

[1993 : 1 Mark]

2.7 Some group (G, o) is known to be abelian. Then, which one of the following is true for G?
(a) $g = g^{-1}$ for every $g \in G$.
(b) $g = g^2$ for every $g \in G$
(c) $(goh)^2 = g^2 \text{ o } h^2$ for every $g, h \in G$.
(d) G is of finite order

[1994 : 2 Marks]

2.8 Let R be a symmetric and transitive relation on a set A. Then
(a) R is reflexive and hence an equivalence relation
(b) R is reflexive and hence a partial order
(c) R is reflexive and hence not an equivalence relation
(d) None of the above

[1995 : 1 Mark]

2.9 The number of elements in the power set P(S) of the set S = {(ϕ), 1, (2, 3)} is:
(a) 2 (b) 4
(c) 8 (d) None of these

[1995 : 1 Mark]

2.10 Let A be the set of all nonsingular matrices over real numbers and let * be the matrix multiplication operator. Then
(a) A is closed under * but ⟨A, *⟩ is not a semigroup.
(b) ⟨A, *⟩ is a semigroup but not a monoid.
(c) ⟨A, *⟩ is a monoid but not a group.
(d) ⟨A, *⟩ is a group but not an abelian group

[1995 : 2 Marks]

2.11 Let A and B be sets and let A^c and B^c denote the complements of the sets A and B. The set $(A - B) \cup (B - A) \cup (A \cap B)$ is equal to
(a) $A \cup B$ (b) $A^c \cup B^c$
(c) $A \cap B$ (d) $A^c \cap B^c$

[1996 : 1 Mark]

2.12 Let X = {2, 3, 6, 12, 24}. Let ≤ be the partial order defined by x ≤ y if x divides y. The number of edges in the Hasse diagram of (X, ≤) is
(a) 3 (b) 4
(c) 9 (d) None of these

[1996 : 1 Mark]

2.13 Suppose X and Y are sets and $|X|$ and $|Y|$ are their respective cardinalities. It is given that there

are exactly 97 functions from X to Y. From this one can conclude that
(a) $|X| = 1, |Y| = 97$
(b) $|X| = 97, |Y| = 1$
(c) $|X| = 97, |Y| = 97$
(d) None of the above

[1996 : 1 Mark]

2.14 Which of the following statements is false?
(a) The set of rational numbers is an abelian group under addition
(b) The set of integers in an abelian group under addition
(c) The set of rational numbers form an abelian group under multiplication
(d) The set of real numbers excluding zero in an abelian group under multiplication

[1996 : 1 Mark]

2.15 Let R denote the set of real numbers. Let $f: R \times R \to R \times R$ be a bijective function defined by $f(x, y) = (x + y, x - y)$. The inverse function of f is given by

(a) $f^{-1}(x,y) = \left(\frac{1}{x+y}, \frac{1}{x-y}\right)$
(b) $f^{-1}(x, y) = (x - y, x + y)$
(c) $f^{-1}(x,y) = \left(\frac{x+y}{2}, \frac{x-y}{2}\right)$
(d) $f^{-1}(x, y) = (2(x - y), 2(x + y))$

[1996 : 2 Marks]

2.16 Let R be a non-empty relation on a collection of sets defined by ARB if and only if $A \cap B = \phi$. Then, (pick the true statement)
(a) R is reflexive and transitive
(b) R is symmetric and not transitive
(c) R is an equivalence relation
(d) R is not reflexive and not symmetric

[1996 : 2 Marks]

2.17 Which one of the following is false?
(a) The set of all bijective functions on a finite set forms a group under function composition.
(b) The set $\{1, 2, ..., p - 1\}$ forms a group under multiplication mod p where p is a prime number.
(c) The set of all strings over a finite alphabet Σ forms a group under concatenation.
(d) A subset $s \neq \phi$ of G is a subgroup of the group $\langle G, *\rangle$ if and only if for any pair of elements $a, b \in s$, $a * b^{-1} \in s$.

[1996 : 2 Marks]

2.18 The number of equivalence relations on the set $\{1, 2, 3, 4\}$ is
(a) 15
(b) 16
(c) 24
(d) 4

[1997 : 1 Mark]

2.19 Suppose A is a finite set with n elements. The number of elements in the Largest equivalence relation of A is
(a) n
(b) n^2
(c) 1
(d) n + 1

[1998 : 1 Mark]

2.20 Let R_1 and R_2 be two equivalence relations on a set. Consider the following assertions:
(i) $R_1 \cup R_2$ is an equivalence relation
(ii) $R_1 \cap R_2$ is an equivalence relation

Which of the following is correct?
(a) both assertions are true
(b) assertion (i) is true but assertion (ii) is not true
(c) assertion (ii) is true but assertion (i) is not true
(d) neither (i) nor (ii) is true

[1998 : 1 Mark]

2.21 The number of functions from an m element set to an n element set is
(a) m + n
(b) m^n
(c) n^m
(d) m * n

[1998 : 1 Mark]

2.22 The binary relation R = {(1, 1), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4)} on the set A = {1, 2, 3, 4} is
(a) Reflexive, symmetric and transitive
(b) Neither reflexive, nor irreflexive but transitive
(c) Irreflexive, symmetric and transitive
(d) Irreflexive and antisymmetric

[1998 : 2 Marks]

2.23 Suppose A = {a, b, c, d} and Π_1 is the following partition of A

$$\Pi_1 = \{\{a, b, c\}, \{d\}\}$$

(a) List the ordered pairs of the equivalence relations induced by Π_1
(b) Draw the graph of the above equivalence relation

[1998 : 2 Marks]

2.24 Let (A, $*$) be a semigroup. Furthermore, for every a and b in A, if $a \neq b$, then $a * b \neq b * a$.
(a) Show that for every a in A
$a * a = a$
(b) Show that for every a, b in A
$a * b * a = a$
(c) Show that for every a, b, c in A
$a * b * c = a * c$
[1998 : 2 Marks]

2.25 The number of binary relations on a set with n elements is
(a) n^2 (b) 2n
(c) 2^{n^2} (d) None of these
[1999 : 1 Mark]

2.26 (a) Mr. X claims the following:
If a relation R is both symmetric and transitive, then R is reflexive. For this, Mr. X offers the following proof.
"From xRy, using symmetry we get yRx. Now because R is transitive, xRy and yRx together imply xRx. Therefore, R is reflexive."
Briefly point out the flaw in Mr. X's proof
(b) Give an example of a relation R which is symmetric and transitive but not reflexive.
[1999 : 2 Marks]

2.27 A relation R is defined on the set of integers as xRy iff (x + y) is even. Which of the following statements is true?
(a) R is not an equivalence relation
(b) R is an equivalence relation having 1 equivalence class
(c) R is an equivalence relation having 2 equivalence classes
(d) R is an equivalence relation having 3 equivalence classes
[2000 : 2 Marks]

2.28 Let P(S) denote the power set of a set S. Which of the following is always true?
(a) $P(P(S)) = P(S)$ (b) $P(S) \cap P(P(S)) = \{\varnothing\}$
(c) $P(S) \cap S = P(S)$ (d) $S \notin P(S)$
[2000 : 2 Marks]

2.29 Consider the following relations
R_1(a, b) iff (a + b) is even over the set of integers
R_2(a, b) iff (a + b) is odd over the set of integers
R_3(a, b) iff a.b > 0 over the set of non-zero rational numbers.
R_4(a, b) iff $|a - b| \leq 2$ over the set of natural numbers
Which of the following statements is correct?
(a) R_1 and R_2 are equivalence relations, R_3 and R_4 are not
(b) R_1 and R_3 are equivalence relations, R_2 and R_4 are not
(c) R_1 and R_4 are equivalence relations, R_2 and R_3 are not
(d) R_1, R_2 R_3 and R_4 are all equivalence relations
[2001 : 1 Mark]

2.30 Consider the following statements:
S_1: There exist infinite sets A, B, C such that $A \cap (B \cap C)$ is finite.
S_2: There exist two irrational numbers x and y such that (x + y) is rational.
Which of the following is true about S_1 and S_2?
(a) Only S_1 is correct
(b) Only S_2 is correct
(c) Both S_1 and S_2 are correct
(d) None of S_1 and S_2 is correct
[2001 : 2 Marks]

2.31 Let $f : A \rightarrow B$ be a function, and let E and F be subsets of A. Consider the following statements about images.
S1: $f(E \cup F) = f(E) \cup f(F)$
S2: $f(E \cap F) = f(E) \cap f(F)$
(a) Only S1 is correct
(b) Only S2 is correct
(c) Both S1 and S2 are correct
(d) None of S1 and S2 is correct
[2001 : 2 Marks]

2.32 The binary relation S = f(empty set) on set A = {1, 2, 3} is
(a) Neither reflexive nor symmetric
(b) Symmetric and reflexive
(c) Transitive and reflexive
(d) Transitive and symmetric
[2002 : 2 Marks]

2.33 Consider the set Σ^* of all strings over the alphabet $\Sigma = \{0, 1\}$. Σ^* with the concatenation operator for strings
(a) does not form a group
(b) forms a non-commutative group
(c) does not have a right identity element
(d) forms a group if the empty string is removed from Σ^*
[2003 : 1 Mark]

2.34 Let $(S, \leq)$ be a partial order with two minimal elements a and b, and a maximum element c. Let $P : S \rightarrow \{True, False\}$ be a predicate defined on S. Suppose that P(a) = True, P(b) = False and $P(x) \Rightarrow P(y)$ for all $x, y \in S$ satisfying $x \leq y$, where $\Rightarrow$ stands for logical implication. Which of the following statements CANNOT be true?
(a) P (x) = True for all $x \in S$ such that $x \neq b$
(b) P (x) = False for all $x \in S$ such that $x \neq a$ and $x \neq c$
(c) P (x) = False for all $x \in S$ such that $b \leq x$ such that $x \neq c$
(d) P (x) = False for all $x \in S$ such that $a \leq x$ and $b \leq x$

[2003 : 2 Marks]

2.35 Consider the set {a, b, c} with binary operators + and × defined as follows :

+	a	b	c
a	b	a	c
b	a	b	c
c	a	c	b

×	a	b	c
a	a	b	c
b	b	c	a
c	c	c	b

For example, a + c = c, c + a = a, c × b = c and b × c = a.
Given the following set of equations:
$(a \times x) + (a \times y) = c$
$(b \times x) + (c \times y) = c$
The number of solution (s) (i.e., pair (s) (x, y) that satisfies the equations) is
(a) 0
(b) 1
(c) 2
(d) 3

[2003 : 2 Marks]

2.36 Consider the binary relation:
$S = \{(x, y) \mid y = x + 1 \text{ and } x, y \in \{0, 1, 2, ...\}\}$
The reflexive transitive closure of S is
(a) $\{(x, y) \mid y > x \text{ and } x, y \in \{0, 1, 2, ...\}\}$
(b) $\{(x, y) \mid y \geq x \text{ and } x, y \in \{0, 1, 2, ...\}\}$
(c) $\{(x, y) \mid y < x \text{ and } x, y \in \{0, 1, 2, ...\}\}$
(d) $\{(x, y) \mid y \leq x \text{ and } x, y \in \{0, 1, 2, ...\}\}$

[2004 : 1 Mark]

2.37 The number of different n × n symmetric matrices with each element being either 0 or 1 is: (Note : power (2, x) is same as 2^x)
(a) power(2, n)
(b) power(2, n^2)
(c) power(2, $(n^2 + n)/2$)
(d) power (2, $(n^2 - n)/2$)

[2004 : 1 Mark]

2.38 In a class of 200 students, 125 students have taken Programming Language course, 85 students have taken Data Structures course, 65 students have taken Computer Organization course; 50 students have taken both Programming Language and Data Structures, 35 students have taken both Data Structures and Computer Organization; 30 students have taken both Data Structures and Computer Organization, 15 students have taken all the three courses. How many students have not taken any of the three courses?
(a) 15
(b) 20
(c) 25
(d) 30

[IT-2004 : 1 Mark]

2.39 Let R_1 be a relation from A = {1, 3, 5, 7} to B= {2, 4, 6, 8} and R_2 be another relation from B to C = {1, 2, 3, 4} as defined below:
(i) An element x in A is related to an element y in B (under R_1) if x + y is divisible by 3.
(ii) An element x in B is related to an element y in C (under R_2) if x + y is even but not divisible by 3.
Which is the composite relation R_1R_2 from A to C?
(a) R_1R_2={(1, 2), (1, 4), (3, 3), (5, 4), (7, 3)}
(b) R_1R_2={(1, 2), (1, 3), (3, 2), (5, 2), (7, 3)}
(c) R_1R_2={(1, 2), (3, 2), (3, 4), (5, 4), (7, 2)}
(d) R_1R_2={(3, 2), (3, 4), (5, 1), (5, 3), (7, 1)}

[IT-2004 : 1 Mark]

2.40 The following is the incomplete operation table of a 4-element group.

*	e	a	b	c
e	e	a	b	c
a	a	b	c	e
b				
c				

The last row of the table is
(a) c a e b
(b) c b a e
(c) c b e a
(d) c e a b

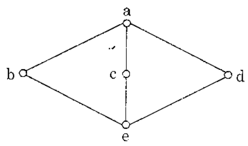
[2004 : 2 Marks]

2.41 The inclusion of which of the following sets into S = {{1, 2}, {1,2,3}, {1, 3, 5}, {1, 2, 4}, {1, 2, 3, 4, 5}} is necessary and sufficient to make S a complete lattice under the partial order defined by set containment?

(a) {1}
(b) {1}, {2, 3}
(c) {1}, {1, 3}
(d) {1}, {1, 3}, {1, 2, 3, 4}, {1, 2, 3, 5}
[2004 : 2 Marks]

2.42 Let A, B and C be non-empty sets and let X = (A – B) – C and Y = (A – C) – (B – C)
Which one of the following is TRUE?
(a) X = Y (b) $X \subset Y$
(c) $Y \subset X$ (d) None of these
[2005 : 1 Mark]

2.43 The following is the Hasse diagram of the poset [{a, b, c, d,e}, ≤]



The poset is
(a) not a lattice
(b) a lattice but not a distributive lattice
(c) a distributive lattice but not a Boolean algebra
(d) a Boolean algebra
[2005 : 1 Mark]

2.44 The set {1, 2, 4, 7, 8, 11, 13, 14} is a group under multiplication modulo 15. The inverses of 4 and 7 are respectively
(a) 3 and 13 (b) 2 and 11
(c) 4 and 13 (d) 8 and 14
[2005 : 1 Mark]

2.45 Let R and S be any two equivalence relations on a non-empty set A. Which one of the following statements is TRUE?
(a) $R \cap S$, $R \cup S$ are both equivalence relations
(b) $R \cup S$ is an equivalence relation
(c) $R \cap S$ is an equivalence relation
(d) Neither $R \cup S$ nor $R \cap S$ is an equivalence relation
[2005 : 2 Marks]

2.46 Let f: B → C and g: A → B be two functions and let h = f o g. Given that h is an onto function. Which one of the following is TRUE?
(a) f and g should both be onto functions
(b) f should be onto but g need not be onto
(c) g should be onto but f not be onto
(d) both f and g need not be onto
[2005 : 2 Marks]

2.47 Consider the set H of all 3 × 3 matrices of the type

$$\begin{bmatrix} a & f & e \\ 0 & b & d \\ 0 & 0 & c \end{bmatrix}$$

where a, b, c, d, e and f are real numbers and abc ≠ 0. Under the matrix multiplication operation, the set H is
(a) a group
(b) a monoid but not group
(c) a semigroup but not a monoid
(d) neither a group nor a semigroup
[2005 : 2 Marks]

2.48 Let f be a function from a set A to a set B, g a function from B to C, and h a function from A to C, such that h(a) = g(f(a)) for all $a \in A$. Which of the following statements is always true for all such functions f and g?
(a) g is onto ⇒ h is onto
(b) h is onto ⇒ f is onto
(c) h is onto ⇒ g is onto
(d) h is onto ⇒ f and g are onto
[IT-2005 : 2 Marks]

2.49 Let A be a set with n elements. Let C be a collection of distinct subsets of A such that for any two subsets S_1 and S_2 in C, either $S_1 \subset S_2$ or $S_2 \subset S_1$. What is the maximum cardinality of C?
(a) n (b) n+1
(c) $2^{n-1}+1$ (d) n!
[IT-2005: 2 Marks]

2.50 For the set N of natural numbers and a binary operation f : N × N → N, an element $z \in N$ is called an identity for f, if f(a, z) = a = f(z, a), for all $a \in N$. Which of the following binary operations have a identity?
I. f(x, y) = x+y – 3
II. f(x, y) = max(x, y)
III. f(x, y) = x^y
(a) I and II only (b) II and III only
(c) I and III only (d) None of these
[IT-2006 : 1 Mark]

2.51 Let, X, Y, Z be sets of sizes x, y and z respectively. Let W = X × Y and E be the set of all subsets of W. The number of functions from Z to E is
(a) z (b) $z \times 2^{xy}$
(c) 2^z (d) 2^{xyz}
[2006 : 1 Mark]

2.52 The set {1, 2, 3, 5, 7, 8, 9} under multiplication modulo 10 is not a group. Given below are four possible reasons. Which one of them is false?

(a) It is not closed
(b) 2 does not have an inverse
(c) 3 does not have an inverse
(d) 8 does not have an inverse

[2006 : 1 Mark]

2.53 A relation R is defined on ordered pairs of integers as follows: $(x, y)R(u,v)$ if $x < u$ and $y > v$. Then R is

(a) Neither a Partial Order nor an Equivalence Relation
(b) A Partial Order but not a Total Order
(c) A Total Order
(d) An Equivalence Relation

[2006 : 1 Mark]

2.54 Let E, F and G be finite sets.
Let $X = (E \cap F) - (F \cap G)$ and
$Y = (E - (E \cap G)) - (E - F)$.

Which one of the following is true?

(a) $X \subset Y$
(b) $X \supset Y$
(c) $X = Y$
(d) $X - Y \neq \varnothing$ and $Y - X \neq \varnothing$

[2006 : 2 Marks]

2.55 Let $S = \{1, 2, 3, \ldots, m\}$, $m > 3$. Let $X_1, X_2, \ldots, X_n$ be subsets of S each of size 3. Define a function f from S to the set of natural numbers as, f(i) is the number of sets X_j that contains the element i. That is $f(i) = |\{j \mid i \in X_j\}|$.

Then $\sum_{i=1}^{m} f(i)$ is

(a) 3m (b) 3n
(c) 2m + 1 (d) 2n + 1

[2006 : 2 Marks]

2.56 Let P, Q and R be sets let Δ denote the symmetric difference operator defined as $P \Delta Q = (P \cup Q) - (P \cap Q)$. Using Venn diagrams, determine which of the following is/are TRUE?

I. $P \Delta (Q \cap R) = (P \Delta Q) \cap (P \Delta R)$
II. $P \cap (Q \Delta R) = (P \cap Q) \Delta (P \cap R)$

(a) I only (b) II only
(c) Neither I nor II (d) Both I and II

[IT-2006 : 2 Marks]

2.57 What is the cardinality of the set of integers X defined below? $X = \{n \mid 1 \leq n \leq 123$, n is not divisible by 2, 3 or 5}

(a) 28 (b) 33
(c) 37 (d) 44

[IT-2006 : 2 Marks]

2.58 Let S be a set of n elements. The number of ordered pairs in the largest and the smallest equivalence relations on S are

(a) n and n (b) n^2 and n
(c) n^2 and 0 (d) n and 1

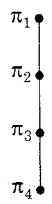
[2007 : 1 Mark]

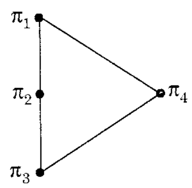
2.59 Consider the set S = {a, b, c, d}. Consider the following 4 partitions $\pi_1, \pi_2, \pi_3, \pi_4$ on

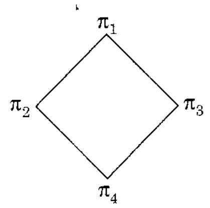
S: $\pi_1 = \{\overline{abcd}\}, \pi_2 \{\overline{ab}, \overline{cd}\}, \pi_3 = \{\overline{abc}, \overline{d}\},$

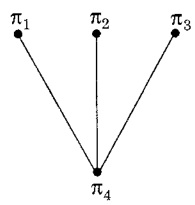
$\pi_4 = \{\overline{a}, \overline{b}, \overline{c}, \overline{d}\}$

Let $\prec$ be the partial order on the set of partitions $S' = (\pi_1, \pi_2, \pi_3, \pi_4)$ defined as follows: $\pi_i \prec \pi_j$ if and only if π_i refines π_j. The poset diagram for $(S', \prec)$ is

(a) 

(b) 

(c) 

(d) 

[2007 : 2 Marks]

2.60 A partial order P is defined on the set of natural numbers as follows. Here x/y denotes integer division.

(i) $(0, 0) \in P$.
(ii) $(a, b) \in P$ if and only if $a \% 10 \leq b \% 10$ and $(a/10, b/10) \in P$.

Consider the following ordered pairs:

(i) (101, 22) (ii) (22, 101)
(iii) (145, 265) (iv) (0, 153)

Which of these ordered pairs of natural numbers are contained in P?

(a) (i) and (iii) (b) (ii) and (iv)
(c) (i) and (iv) (d) (iii) and (iv)

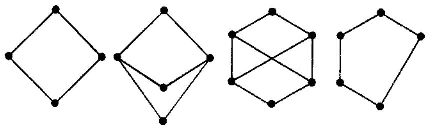
[IT-2007 : 2 Marks]

2.61 If P, Q, R are subsets of the universal set U, then $(P \cap Q \cap R) \cup (P^c \cap Q \cap R) \cup Q^c \cup R^c$ is

(a) $Q^c \cup R^c$ (b) $P \cup Q^c \cup R^c$
(c) $P^c \cup Q^c \cup R^c$ (d) U

[2008 : 1 Mark]

2.62 Consider the following Hasse diagrams.



(i) (ii) (iii) (iv)

Which all of the above represent a lattice?

(a) (i) and (iv) only (b) (ii) and (iii) only
(c) (iii) only (d) (i), (ii) and (iv) only

[IT-2008 : 2 Marks]

2.63 Which one of the following is NOT necessarily a property of a Group?

(a) Commutativity
(b) Associativity
(c) Existence of inverse for every element
(d) Existence of identity

[2009 : 1 Mark]

2.64 Consider the binary relation: R = {(x, y), (x, z), (z, x), (z, y)} on the set {x, y, z}. Which one of the following is TRUE?

(a) R is symmetric but NOT antisymmetric
(b) R is NOT symmetric but antisymmetric
(c) R is both symmetric and antisymmetric
(d) R is neither symmetric nor antisymmetric

[2009 : 1 Mark]

2.65 For the composition table of a cyclic group shown below:

*	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

Which one of the following choices is correct?

(a) a, b are generators
(b) b, c are generators
(c) c, d are generators
(d) d, a are generators

[2009 : 2 Marks]

2.66 What is the possible number of reflexive relations on a set of 5 elements?

(a) 2^{10} (b) 2^{15}
(c) 2^{20} (d) 2^{25}

[2010 : 1 Mark]

2.67 Consider the set $S = \{1, \omega, \omega^2\}$, where ω and ω^2 are cube roots of unity. If $*$ denotes the multiplication operation, the structure $\{S, *\}$ forms

(a) a group (b) a ring
(c) an integral domain (d) a field

[2010 : 1 Mark]

2.68 How many onto (or surjective) functions are there from an n-element $(n \geq 2)$ set to a 2-element set?

(a) 2^n (b) $2^n - 1$
(c) $2^n - 2$ (d) $2(2^n - 2)$

[2012 : 2 Marks]

2.69 A binary operation $\oplus$ on a set of integers is defined as $x \oplus y = x^2 + y^2$. Which one of the following statements is TRUE about $\oplus$?

(a) Commutative but not associative
(b) Both commutative and associative
(c) Associative but not commutative
(d) Neither commutative nor associative

[2013 : 1 Mark]

2.70 Let S denote the set of all functions $f : \{0,1\}^4 \to \{0,1\}$. Denote by N the number of functions from S to the set {0, 1}. The value of $\log_2 \log_2 N$ is ________.

[2014 (Set-1) : 2 Marks]

2.71 Consider the following relation on subsets of the set S of integers between 1 and 2014. For two distinct subsets U and V of S we say $U < V$ if the minimum element in the symmetric difference of the two sets is in U.

Consider the following two statements:

S1: There is a subset of S that is larger than every other subset.

S2: There is a subset of S that is smaller than every other subset.

Which one of the following is CORRECT?

(a) Both S1 and S2 are true
(b) S1 is true and S2 is false
(c) S2 is true and S1 is false
(d) Neither S1 nor S2 is true

[2014 (Set-2) : 2 Marks]

2.72 Let X and Y be finite sets and $f: X \rightarrow Y$ be a function. Which one of the following statements is TRUE?

(a) For any subsets A and B of X,
$|f(A \cup B)| = |f(A)| + |f(B)|$

(b) For any subsets A and B of X,
$f(A \cap B) = f(A) \cap f(B)$

(c) For any subsets A and B of X,
$|f(A \cap B)| = \min\{|f(A)|, |f(B)|\}$

(d) For any subsets S and T of Y,
$f^{-1}(S \cap T) = f^{-1}(S) \cap f^{-1}(T)$

[2014 (Set-3) : 1 Mark]

2.73 Let G be a group with 15 elements. Let L be a subgroup of G. It is known that $L \neq G$ and that the size of L is at least 4. The size of L is ________.

[2014 (Set-3) : 1 Mark]

2.74 Consider the set of all functions f : {0,1, ..., 2014} → {0,1, ...,2014} such that $f(f(i)) = i$, for all $0 \le i \le 2014$. Consider the following statements:

P: For each such function it must be the case that for every i, $f(i) = i$.

Q: For each such function it must be the case that for some i, $f(i) = i$.

R : Each such function must be onto.

Which one of the following is CORRECT?

(a) P, Q and R are true
(b) Only Q and R are true
(c) Only P and Q are true
(d) Only R is true

[2014 (Set-3) : 2 Marks]

2.75 There are two elements x, y in a group $(G, *)$ such that every element in the group can be written as a product of some number of x's and y's in some order. It is known that

$$x*x = y*y = x*y*x*y = y*x*y*x = e$$

where e is the identity element. The maximum number of elements in such a group is ____.

[2014 (Set-3) : 2 Marks]

2.76 If $g(x) = 1 - x$ and $h(x) = \dfrac{x}{x-1}$, then $\dfrac{g(h(x))}{h(g(x))}$ is

(a) $\dfrac{h(x)}{g(x)}$ (b) $\dfrac{-1}{x}$

(c) $\dfrac{g(x)}{h(x)}$ (d) $\dfrac{x}{(1-x)^2}$

[2015 (Set-1) : 1 Mark]

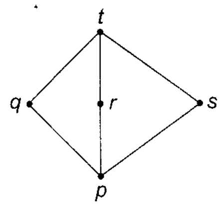
2.77 For a set A, the power set of A is denoted by 2^A. If A = {5, {6}, {7}}, which of the following options are True?

1. $\varnothing \in 2^A$ 2. $\varnothing \subseteq 2^A$
3. $\{5,\{6\}\} \in 2^A$ 4. $\{5,\{6\}\} \subseteq 2^A$

(a) 1 and 3 only (b) 2 and 3 only
(c) 1, 2 and 3 only (d) 1, 2 and 4 only

[2015 (Set-1) : 1 Mark]

2.78 Suppose $\mathcal{L} = \{p, q, r, s, t\}$ is a lattice represented by the following Hasse diagram:



For any $x, y \in \mathcal{L}$, not necessarily distinct, $x \vee y$ and $x \wedge y$ are join and meet of x, y respectively. Let $\mathcal{L}^3 = \{(x,y,z): x, y, z \in \mathcal{L}\}$ be the set of all ordered triplets of the elements of $\mathcal{L}$. Let p_r be the probability that an element $(x,y,z) \in \mathcal{L}^3$ chosen equiprobably satisfies $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$. Then

(a) $p_r = 0$ (b) $p_r = 1$
(c) $0 < p_r \le 1/5$ (d) $1/5 < p_r < 1$

[2015 (Set-1) : 2 Marks]

2.79 Let R be the relation on the set of positive integers such that aRb if and only if a and b are distinct and have a common divisor other than 1. Which one of the following statements about R is True?

(a) R is symmetric and reflexive but not transitive
(b) R is reflexive but not symmetric and not transitive
(c) R is transitive but not reflexive and not symmetric
(d) R is symmetric but not reflexive and not transitive

[2015 (Set-2) : 1 Mark]

2.80 The cardinality of the power set of {0, 1, 2..., 10} is ________.

[2015 (Set-2) : 1 Mark]

2.81 The number of onto functions (surjective functions) from set X = {1, 2, 3, 4} to set Y = {a, b, c} is ________.

[2015 (Set-2) : 2 Marks]

2.82 Let X and Y denote the sets containing 2 and 20 distinct objects respectively and F denote the set of all possible functions defined from X and Y. Let f be randomly chosen from F. The probability of f being one-to-one is ________.

2.83 Let # be a binary operator defined as X # Y = X′ + Y′ where X and Y are Boolean variables. Consider the following two statements.

S1: (P # Q) # R = P # (Q # R)
S2: Q # R = R # Q

Which of the following is/are true for the Boolean variables P, Q and R?

(a) Only S1 is True
(b) Only S2 is True
(c) Both S1 and S2 are True
(d) Neither S1 nor S2 are True

[2015 (Set-3) : 1 Mark]

2.84 Suppose U is the power set of the set S = {1,2,3,4,5,6}. For any $T \in U$, let $|T|$ denote the number of elements in T and T' denote the complement of T. For any $T, R \in U$, let $T \setminus R$ be the set of all elements in T which are not in R. Which one of the following is true?

(a) $\forall X \in U (|X| = |X'|)$
(b) $\exists X \in U \exists Y \in U (|X| = 5, |Y| = 5 \text{ and } X \cap Y = \varnothing)$
(c) $\forall X \in U \forall Y \in U (|X| = 2, |Y| = 3 \text{ and } X \setminus Y = \varnothing)$
(d) $\forall X \in U \forall Y \in U (X \setminus Y = Y' \setminus X')$

[2015 (Set-3) : 1 Mark]

2.85 Let R be a relation on the set of ordered pairs of positive integers such that $((p, q), (r, s)) \in R$ if and only if $p-s = q-r$. Which one of the following is true about R?

(a) Both reflexive and symmetric
(b) Reflexive but not symmetric
(c) Not reflexive but symmetric
(d) Neither reflexive nor symmetric

[2015 (Set-3) : 2 Marks]

2.86 A function $f : \mathbb{N}^+ \rightarrow \mathbb{N}^+$, defined on the set of positive integers $\mathbb{N}^+$, satisfies the following properties:

$f(n) = f(n/2)$ if n is even
$f(n) = f(n + 5)$ if n is odd

Let $R = \{i \mid \exists j : f(j) = i\}$ be the set of distinct values that f takes. The maximum possible size of R is ______.

[2016 (Set-1) : 2 Marks]

2.87 A binary relation R on $N \times N$ is defined as follows: $(a, b)\ R\ (c, d)$ if $a \le c$ or $b \le d$. Consider the following propositions

P: R is reflexive
Q: R is transitive

Which one of the following statements is TRUE?

(a) Both P and Q are true
(b) P is true and Q are false
(c) P is false and Q are true
(a) Both P and Q are false

[2016 (Set-2) : 2 Marks]

2.88 Consider a set U of 23 different compoundin a Chemistry lab. There is a subset S of U of 9 compounds, each of which reacts with exactly 3 compounds of U. Consider the following statements:

I. Each compound in $U \setminus S$ reacts with an odd number of compounds.
II. At least one compound in $U \setminus S$ reacts with an odd number of compounds.
III. Each compound in $U \setminus S$ reacts with an even number of compounds

Which one of the above statements is ALWAYS TRUE?

(a) Only I (b) Only II
(c) Only III (d) None

[2016 (Set-2) : 2 Marks]

■■■■

Answers: Set Theory and Algebra

2.5 (c)	2.6 (b)	2.7 (c)	2.8 (d)	2.9 (c)	2.10 (d)	2.11 (a)	2.12 (b)	2.13 (a)
2.14 (c)	2.15 (c)	2.16 (b)	2.17 (c)	2.18 (a)	2.19 (b)	2.20 (c)	2.21 (c)	2.22 (b)
2.25 (c)	2.27 (c)	2.28 (b)	2.29 (b)	2.31 (a)	2.32 (d)	2.33 (a)	2.34 (d)	2.35 (c)
2.36 (b)	2.37 (c)	2.38 (c)	2.39 (c)	2.40 (d)	2.41 (a)	2.42 (a)	2.43 (b)	2.44 (c)
2.45 (c)	2.46 (b)	2.47 (a)	2.48 (c)	2.49 (b)	2.50 (a)	2.51 (d)	2.52 (c)	2.53 (a)
2.54 (c)	2.55 (b)	2.56 (b)	2.57 (b)	2.58 (b)	2.59 (c)	2.60 (d)	2.61 (d)	2.62 (a)
2.63 (a)	2.64 (d)	2.65 (c)	2.66 (c)	2.67 (a)	2.68 (c)	2.69 (a)	2.71 (a)	2.72 (d)
2.74 (b)	2.76 (a)	2.77 (c)	2.78 (d)	2.79 (d)	2.83 (b)	2.84 (d)	2.85 (c)	2.87 (b)
2.88 (b)								

Explanations Set Theory and Algebra

2.1 Sol.

A relation is said to be equivalence relation is

(i) Reflexive

(ii) Symmetric, and

(iii) Transitive

Union of two reflexive relations and two symmetric relations are reflexive and symmetric respectively. However, union of two transitive relations need not to be transitive. Therefore, union of two equivalence relations need not be an equivalence relation.

Example:

Let R_1 and R_2 on set A = {1, 2, 3}

R_1 = {(1, 1), (2, 2,), (3, 3) (1, 2), (2, 1)} is an equivalence relation

R_2 = {(1, 1),(2, 2), (3, 3), (2, 3), (3, 2)} is an equivalence relation

$R_1 \cup R_2$ = {(1, 1), (2, 2), (3, 3,), (1, 2), (2, 1), (2, 3), (3, 2)} is not an equivalence relation,

because (1, 2) & (2, 3) needs (1, 3) element to be in transitive relation.

2.2 Sol.

(a) Set A contain n elements. Every subset of A × A is a binary relation on the set A.

∴ Number of binary relations on a set A with n elements = 2^{n^2}.

(b) Number of one-to-one functions from a set A with m-elements to a set B with n-elements is nP_m.

So the number of one-to-one functions from a set A with n-elements to itself is $nP_n = n!$.

2.3 Sol.

The complement of an element x is x' iff LUB of x and x' is 1 (greatest element) and GLB of x and x' is 0 (least element).

∴ The complement of the element 'a' in the lattice = {b, c, d, e}.

2.4 Sol.

The relation have (1, 2) (2, 3) then add (1, 3) to relation. Since relation have (2, 3) (3, 4) then add (2, 4) to relation.

So the resultant relation is

= {(1, 2), (2, 3), (3, 4), (5, 4), (1, 3), (2, 4)}

Now the resultant relation have (1, 2) (2, 4) then add (1, 4) to the relation

∴ {(1,2), (2,3), (3,4), (5,4), (1,3), (2,4), (1,4)}

So the transitive closure of the relation is

= {(1,2), (2,3), (3,4), (5,4), (1,3), (2,4), (1,4)}

2.5 (c)

$S = S_1 \cup S_2 \cup S_3 \cup ... \cup S_n$.

For S to be infinite set, atleast one of sets S_i must be infinite, since if all S_i were finite, then S will also be finite.

2.6 (b)

Number of elements in A × A = n^2

∴ Number of elements in the power set of A × A

= 2^{n^2}.

2.7 (c)

$(g\circ h)^2 = (g\circ h)\circ(g\circ h)$

Since group is abelian so it is commutative as well as associative.

$= g\circ(hog)h$

$= g\circ(goh)\circ h$

$= (g\circ g)\circ(hh) = g^2\circ h^2$

So $(g\circ h)^2 = g^2\circ h^2$ for every $g, h \in G$

2.8 (d)

A relation which is symmetric and transitive, need not be reflexive relation.

(i) $R = \{\}$; on the set $A = \{a, b\}$. The relation R is symmetric and transitive but not reflexive.

(ii) $R = \{(a, a), (b, b)\}$; on the set $A = \{a, b\}$. The relation R is symmetric, transitive and also reflexive.

$\therefore$ A relation is transitive and symmetric relation but need not be reflexive relation.

2.9 (c)

If a set has n elements then its powers set has 2^n elements.

Given set $S = \{(\phi), 1, (2, 3)\}$

Number of elements in S = 3

$\therefore$ The number of elements in powerset (S) $= 2^3 = 8$.

2.10 (d)

(i) **Closure Property:** Multiplication of two non-singular matrices is also non-singular matrix. Matrix multiplication over non-singular matrices follows closure properties.

(ii) **Associative Property:** Multi-plication over any set of matrices is associative.

$$(AB)C = A(BC)$$

Where A, B and C are non-singular matrices

(iii) **Identity Element:** Identity matrix I is the identity element for matrix multiplication over matrices and I is non-singular.

(iv) **Inverse Element:** For every non-singular matrix its inverse exists. So, for non-singular matrices inverse element exist.

(v) **Commutative:** Matrix multi-plication is not commutative

$$AB \neq BA$$

Where A, B are non-singular matrices.

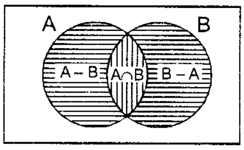
Matrices multiplication is not commutative.

So <A, *> is a group but not an abelian group.

2.11 (a)

$$(A - B) \cup (B - A) \cup (A \cap B)$$

Representing above set using Venn diagram as follows.



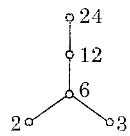
$\therefore$ By Venn diagram,

$$(A - B) \cup (B - A) \cup (A \cap B) \equiv A \cup B$$

2.12 (b)

$$x = \{2, 3, 6, 12, 24\}$$

The Hasse diagram of $(x, \leq)$ is



Therefore, the number of edges in the Hasse diagram = 4

2.13 (a)

X and Y are sets. The cardinalities of X and Y are $|X|$ and $|Y|$ respectively.

The number of functions from X to Y $= (|Y|)^{|X|}$

Given that number of functions from X to Y = 97

$\therefore \quad 97 = (|Y|)^{|X|}$

So above implies that $|X| = 1$ and $|Y| = 97$

2.14 (c)

0 is also a rational number and for 0, inverse doesn't exist under multiplication.

Therefore, the set of rational numbers doesn't form an abelian group under multiplication.

2.15 (c)

$$f(x, y) = ((x + y), (x - y))$$

So let $z_1 = x + y \quad \ldots(i)$

$z_2 = x - y \quad \ldots(ii)$

$$f(x, y) = (z_1, z_2)$$

So $f^{-1}(z_1, z_2) = (x, y)$

Adding (i) and (ii)

$$z_1 + z_2 = 2x$$

$$x = \frac{z_1 + z_2}{2}$$

Subtracting (i) and (ii)

$$z_1 - z_2 = 2y$$

$$y = \frac{z_1 - z_2}{2}$$

So the inverse function of F is given by

$$f^{-1}(z_1, z_2) = (x, y) = \left(\frac{z_1 + z_2}{2}, \frac{z_1 - z_2}{2}\right)$$

Since z_1 and z_2 are just dummy variables so replacing z_1 and z_2 by x and y respectively

$$f^{-1}(x, y) = \left(\frac{x + y}{2}, \frac{x - y}{2}\right)$$

Alternate Method

Given $f(x, y) = ((x + y), (x - y))$

Take some random ordered pair say (2, 3)

$f(2, 3) = ((2 + 3), (2 - 3)) = (5, -1)$

Now the correct inverse must map (5, –1) back to (2, 3).

Trying the options one-by-one we find that only option (c) maps (5, –1) to (2, 3).

2.16 (b)

(i) **Reflexive**

$A \cap A = A \neq \phi$

So; (A, A) doesn't belongs to relation R.

$\therefore$ Relation R is not reflexive.

(ii) **Symmetric**

If $A \cap B = \phi$ then $B \cap A = \phi$ is also true

$\therefore$ relation R is symmetric relation.

(iii) **Transitive**

If $A \cap B = \phi$ and $B \cap C = \phi$, it need not be true that $A \cap C = \phi$.

For example:

$A = \{1, 2\}$, $B = \{3, 4\}$, $C = \{1, 5, 6\}$

$A \cap B = \phi$ and $B \cap C = \phi$ but $A \cap C = \{1\} \neq \phi$

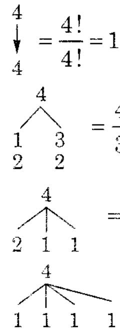
$\therefore$ Relation R is not transitive relation.

2.17 (c)

In option (c), the set of all strings over a finite alphabet Σ doesn't forms a group under concatenation because the inverse of a string doesn't exist with respect to concatenation.

2.18 (a)

Corresponding to every partition of the set {1, 2, 3, 4}, there exists a unique equivalence relation. So we count every type of unordered partitions of the set of 4 elements into one block, two block, three block and four block partitions, as shown below.



$$= \frac{4!}{4!} = 1$$

$$= \frac{4!}{3!} + \frac{4!}{2!\,2!\,2!} = 4 + 3 = 7$$

$$= \frac{4!}{2!\,2!} = 6$$

$$= \frac{4!}{4!} = 1$$

Total = 1 + 7 + 6 + 1 = 15

So the number of equivalence relations on the set {1, 2, 3, 4} is 15.

2.19 (b)

A is a finite set with n elements. The largest equivalence relation of A is the cross-product $A \times A$ and the number of elements in the cross product $A \times A$ is n^2.

2.20 (c)

A relation is said to be equivalence relation if relation is

(i) Reflexive

(ii) Symmetric

(iii) Transitive

Reflexive and symmetric properties are both closed under $\cup$ & $\cap$.

Transitive property is closed under $\cap$ but not $\cup$.

So equivalence relations are closed under $\cap$ but not $\cup$.

Therefore $R_1 \cap R_2$ is an equivalence relation but $R_1 \cup R_2$ is not necessarily an equivalence relation.

2.21 (c)

The number of functions from an m element set to an n element set is n^m.

2.22 (b)

The relation R doesn't contain (4, 4), so R is not reflexive relation.

Since relation R contains (1, 1), (2, 2) and (3, 3).

Therefore, relation R is also not irreflexive.

That R is transitive, can be checked by systematically checking for all (a, b) and (b, c) in R, whether (a, c) also exists in R.

So option (b) is correct.

2.23 Sol.

Given partition of A is

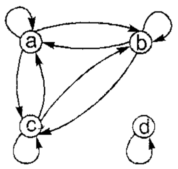
$\pi_1 = \{(a, b, c), (d)\}$

(a) The ordered pairs of the equivalence relations induced by π_1 is

$R = (a, b, c) \times (a, b, c) \cup (d) \times (d)$

$R = \{(a, a), (a, b), (a, c), (b, a), (b, b), (b, c), (c, a), (c, b), (c, c), (d, d)\}$

(b) The diagraph of the above equivalence relation is as follows



Diagraph of Relation R

2.24 Sol.

Given $a \neq b \Rightarrow a*b \neq b*a$

The contrapositive version of this is

$a*b = b*a \Rightarrow a = b$

(a) Now since (A, $*$) is given to be a semigroup, $*$ is associative

i.e. $a * (a * a) = (a * a) * a$

Hence $a = a * a$

(b) $(a * b * a) * a = a * b * (a * a)$

$= a * b * a$

$\Rightarrow a * (a * b * a) = (a * a) * b * a$

$= a * b * a$

as $a * a = a$ [proved in part a]

So $(a * b * a) * a = a * (a * b * a)$

Hence $a * b * a = a$

(c) $(a * b * c) * (a * c) = a * b * (c * a * c)$

$= a * b * c$

$(a * c) * (a * b * c) = (a * c * a) * b * c$

$= a * b * c$

So $(a*b*c) * (a * c) = (a * c) * (a * b * c)$

Hence $a * b * c = a * c$

2.25 (c)

The maximum number of elements in a binary relation on a set A with n elements = Number of elements in $A \times A = n^2$

Each element has two choices, either to appear on a binary relation or doesn't appear on a binary relation.

$\therefore$ Number of binary relations $= 2^{n^2}$.

2.26 Sol.

(a) According to Mr. X if xRy is in relation then yRx also in relation because of symmetricity. Now because R is transitive, xRy and yRx implies xRx.

Flaw in Mr. X claim is that if xRy present then only it implies yRx and hence xRx.

However, if xRy is not present then according to symmetricity yRx is also not present. So xRx need not be present in relation. So it need not be reflexive.

For Example: Empty relation ϕ is both symmetric and transitive. However, ϕ is not reflexive relation.

(b) Examples of a relations which are symmetric and transitive but not reflexive.

(i) Empty relation ϕ is symmetric and transitive but not reflexive.

(ii) Relation $R = \{(a, b), (b, a), (a, a), (b, b)\}$ over $A = \{a, b, c\}$ is both symmetric and transitive but not reflexive.

2.27 (c)

A relation R is defined as xRy iff $(x + y)$ is even over set of integers.

$(x + y)$ is even iff

(i) both x and y are even

(ii) both x and y are odd

Therefore, relation R is equivalence relation because relation is

(i) Reflexive

$x + x = 2x =$ even

So (x, x) belongs to R. So relation is reflexive.

(ii) Symmetric

If $x + y =$ even then $y + x$ is also even

So relation is symmetric.

(iii) Transitive

If $x + y =$ even and $y + z =$ even

Then $x + y + y + z =$ even + even

$\Rightarrow x + z + 2y =$ even

$\Rightarrow x + z =$ even $- 2y$

$\Rightarrow x + z =$ even

$\therefore$ Relation R is transitive.

So relation R is an equivalence relation which divides the set of integer into two equivalence classes: One is of all even and other is of odd integer.

Equivalence classes of R are
$[0] = \{..., -6, -4, -2, 0, 2, 4, 6, ...\}$
$[1] = \{..., -7, -5, -3, -1, 1, 3, 5, 7, ...\}$

2.28 (b)

ϕ always present in any powerset of a set and ϕ is the only common element between P(S) and P(P(S))
$\therefore P(S) \cap P(P(S)) = \{\phi\}$

2.29 (b)

(I) Relation $R_1(a, b)$ iff $(a + b)$ is even over the set of integers.
 (i) $a + a = 2a$ which is even
 So (a, a) belongs to R_1
 $\therefore$ R_1 is reflexive relation
(ii) If $(a + b)$ is even, then $(b + a)$ is also even
 $\therefore$ R_1 is symmetric relation
(iii) If $(a + b)$ and $(b + c)$ are even then
 $a + c = (a + b) + (b + c) - 2b$
 = even + even – even
 = even
 $\therefore$ R_1 is transitive relation.
 Since R_1 is reflexive, symmetric and transitive so R_1 is an equivalence relation.

(II) $R_2(a, b)$ iff $(a + b)$ is odd over set of integers.
 (i) $a + a = 2a$ which is not odd
 So (a, a) doesn't belong to R_2
 $\therefore$ R_2 is not reflexive relation
 Since R_2 is not reflexive, it is not an equivalence relation.

(III) $R_3(a, b)$ iff $a.b > 0$ over set of non-zero relational numbers.
 (i) $a. a. > 0$ for every non-zero rational number.
 $\therefore$ R_3 is reflexive relation.
 (ii) If $a.b > 0$ then $b.a > 0$
 $\therefore$ R_3 is symmetric relation.
 (iii) $a.b > 0$ and $b.c > 0 \Rightarrow$ All a,b,c are positive or all a,b,c are negative.
 So $a.c > 0$
 $\therefore$ R_3 is transitive relation.
 So R_3 is an equivalence relation.

(IV) R_4 (a, b) iff $|a - b| \le 2$ over the set of natural number
 (i) $|a - a| \le 2$
 $0 \le 2$
 $\therefore$ R_4 is reflexive relation.
 (ii) If $|a - b| \le 2$ then also $|b - a| \le 2$
 $\therefore$ R_4 is symmetric relation
 (iii) If $|a - b| \le 2$ and $|b - c| \le 2$ then it is not necessary that $|a - c| \le 2$
 Ex. $|3 - 5| \le 2$ and $|5 - 7| \le 2$ but
 $|3 - 7| \nleq 2$
 $\therefore$ R_4 is not transitive.
 Since R_4 is reflexive and symmetric not transitive, so R_4 is not an equivalence relation.

2.30 (c)

S_1: Let A = set of integers
 B = set of odd integers
 C = set of even integers
$A \cap (B \cap C) = \phi$ and ϕ is finite set.
Therefore, S_1 is true.
S_2: Let two irrational number x and y are respectively $(1+\sqrt{2})$ and $(1-\sqrt{2})$.
So $x + y = 1+\sqrt{2}+1-\sqrt{2}$
 $= 2$ which is rational number
Therefore,. S_2 is true. Since both S_1 and S_2 are true, option (c) is true.

2.31 (a)

Given a function f: $x \to y$ and subsets E and F of A then we have
 $f(E \cup F) = f(E) \cup f(F)$ and
 $f(E \cap F) \subseteq f(E) \cap f(F)$
Therefore S_1 is correct and S_2 is false.

2.32 (d)

The empty relation on any set is always transitive and symmetric but not reflexive.

2.33 (a)

$$\Sigma = \{0, 1\}$$
$$\Sigma^* = \{0, 1\}^*$$
$$= \{\varepsilon, 0, 1, 01, 10, 11, 000, ...\}$$

So $(\Sigma^*, \cdot)$ is an algebraic system, where $\cdot$ (concatenation) is a binary operation.
So $(\Sigma^*, \cdot)$ is a group if and only if the following conditions are satisfied.

1. $\cdot$ (Concatenation) is a closed operation.
2. $\cdot$ is an associative operation.
3. There is an identity.
4. Every element of Σ^* has a inverse

Condition 1: * is a closed operation because for any $\omega_1 \in \Sigma^*$ and
$\omega_2 \in \Sigma^*$, $\omega_1 \cdot \omega_2 \in \Sigma^*$

Condition 2: For any string x, y, $z \in \Sigma^*$,
$x \cdot (y \cdot z) = (x \cdot y) \cdot z$

So it is associative for example let

$x = 01, y = 11, z = 00$ then

$$\begin{aligned} \text{L.H.S.} &= x\cdot(y\cdot z) \\ &= 01\cdot(11\cdot00) = 01\cdot(1100) \\ &= 011100 \\ \text{R.H.S.} &= (x\cdot y)\cdot z \\ &= (01\cdot11)\cdot00 = (0111)\cdot00 \\ &= 011100 \end{aligned}$$

Condition 3: The Identity is ε or empty string because for any string $\omega \in \Sigma^*$,

$\varepsilon\omega = \omega\varepsilon = \omega$.

Now, since $\varepsilon \in \Sigma^*$, identity exists.

Condition 4: There is no inverse exist for Σ^* because any string $\omega \in \Sigma^*$, there is no string ω^{-1} such that $\omega \cdot \omega^{-1} = \varepsilon = \omega^{-1}\omega$.

So Σ^* with the concatenation operator for strings doesn't form a group but it does form a monoid.

2.34 (d)

If $a \le x$, since $p(x) \Rightarrow p(y)$ whenever $x \le y$

$\therefore$ $p(a) \Rightarrow p(x)$

Now since p(a) = True, p(x) = cannot be false.

$\therefore$ (d) cannot be true.

2.35 (c)

The possible solution pairs are (a,a), (a, b), (a, c), (b, a), (b,b), (b,c), (c, a,), (c, b) and (c,c). Substitute them one by one in both equations and see which of them satisfies both the equations.

The given equations are:

$(a \times x) + (a \times y) = c$ ...(i)

$(b \times x) + (c \times y) = c$ ...(ii)

Substitute first (x, y) = (a, a)

LHS of equation (i) becomes $(a \times a) + (a \times a) = a + a = b$

Now RHS of equation (i) = c

Therefore LHS $\neq$ RHS. This means that (a, a) is not a solution pair.

Similarly try each of the remaining seven possible solution pairs.

It will be found that only two pairs (b, c) and (c, b) will satisfy both equation (i) & (ii) simultaneously.

Therefore choice (c) is correct.

2.36 (b)

$S = \{(x, y) \mid y = x + 1, \text{ and } x, y \in \{0, 1, 2, ...\}\}$
$= \{(0, 1), (1, 2), (2, 3), (3, 4), ...\}$

Now let T_1 be the reflexive closure of S.

$$\begin{aligned} T_1 &= \{(0, 0), (1, 1), (2, 2), (3, 3) ...\} \cup \\ &\quad \{(0, 1), (1, 2), (2, 3), (3, 4) ...\} \\ &= \{(0, 0), (0, 1), (1, 1), (1, 2), (2, 2), \\ &\quad (2, 3), (3, 3), (3, 4) ...\} \end{aligned}$$

Let T_2 be the transitive closure of S.

$(0, 1), (1, 2) \in S \Rightarrow (0, 2) \in T_2$
$(0, 2), (2, 3) \in S \Rightarrow (0, 3) \in T_2$
$(0, 3), (3, 4) \in S \Rightarrow (0, 4) \in T_2$
and so on ...

Also $(1, 2), (2, 3) \in S \Rightarrow (1, 3) \in T_2$
$(1, 3), (3, 4) \in S \Rightarrow (1, 4) \in T_2$
$(1, 4), (4, 5) \in S \Rightarrow (1, 5) \in T_2$
and so on ...

$\therefore$ $T_2 = \{(0, 1), (0, 2), (0, 3), ..., (1, 2), (1, 3), (1, 4), ...\}$

Now the reflexive, transitive closure of S will be $T_3 = T_1 \cup T_2 = \{(0, 0), (0, 1), (0, 2), ..., (1, 1), (1, 2), (1, 3), ..., (2, 2)\ (2, 3), (2, 4), ...\}$.

Option (b) is correct.

2.37 (c)

In a symmetric matrix, the lower triangle must be the mirror image of upper triangle using the diagonal as mirror. Diagonal elements may be anything. Therefore, when we are counting symmetric matrices we count how many ways are there to fill the upper triangle and diagonal elements. Since the first row has n elements, second (n – 1) elements, third row (n – 2) elements and so on upto last row, one element.

Total number of elements in diagonal + upper triangle

$$= n + (n-1) + (n-2) + ... + 1$$
$$= n(n+1)/2$$

Now, each one of these elements can be either 0 or 1. So total number of ways we can fill these elements is

$2^{\frac{n(n+1)}{2}}$ = power $(2, (n^2 + n)/2)$

Since there is no choice for lower triangle elements the answer is power $(2, (n^2 + n)/2)$ which is choice (c).

2.38 (c)

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A - C| - |B \cap C| + |A \cap B \cap C|$

A ≡ students who have taken programming.

B ≡ Students who have taken Data structures.

C ≡ Students who have taken Computer Organization.

So, the number of students who have taken any of the 3 courses is given by:

= 125 + 85 + 65 – 0 – 35 – 30 + 15 = 175

Therefore, the number of students who haven't taken any of the 3 courses is:

= 200 – 175 = 25

2.39 (c)

$R_1 = \{(1, 2),(1, 8),(3, 6),(5, 4),(7, 2),(7, 8)\}$

$R_2 = \{(2, 2),(4, 4),(6, 2),(6, 4),(8, 2)\}$

R_1R_2 goes from A to C. So it is actually $R_2.R_1$.

$R_1R_2 = \{(1, 2),(3, 2),(3, 4),(5, 4),(7, 2)\}$

2.40 (d)

Step 1:

By looking at the row for e, we see that it is a copy of the column headers.

So e must be the identity element. Since right identity and left identity element must both be same. The column corresponding to e must be a copy of the row headers.

We can now say that the operation table is

*	e	a	b	c
e	e	a	b	c
a	a	b	c	e
b	b	–	–	–
c	c	–	–	–

Step 2:

From table above we see that a*c = e

∴ c*a must also be = e (if a is the inverse of c, then c is the inverse of a)

Now the operation table looks like

*	e	a	b	c
e	e	a	b	c
a	a	b	c	e
b	b	–	–	–
c	c	e	–	–

Step 3:

The blank in second column must be c (since in a cayley table, every row and every column is a unique permutation of the row and column headers).

Now the operation table looks like

*	e	a	b	c
e	e	a	b	c
a	a	b	c	e
b	b	c	–	–
c	c	e	–	–

Step 4:

Now the blanks in third row can be filled as a, e or e, a. Let us try each one in turn.

If we fill a, e in third row the operation table will look like

*	e	a	b	c
e	e	a	b	c
a	a	b	c	e
b	b	c	a	e
c	c	e	–	–

Now the blank in fourth row and third column must be filled by e.

However this is not possible since e is already entered in fourth row and second column. Therefore filling a, e in third row blanks is wrong. So let us try filling the third row blanks with e, a

Now the operation table looks like

*	e	a	b	c
e	e	a	b	c
a	a	b	c	e
b	b	c	e	a
c	c	e	–	–

Now the blanks in fourth row has to be filled with a, b

The final operation table looks like

*	e	a	b	c
e	e	a	b	c
a	a	b	c	e
b	b	c	e	a
c	c	e	a	b

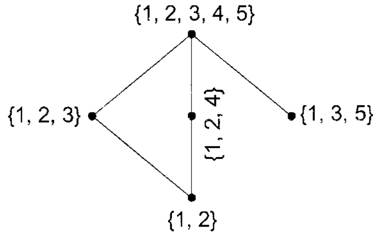
which is consistent with all the rules of a Cayley Table.

The last row of this table is c, e, a, b

Therefore the correct answer is (d).

2.41 (a)

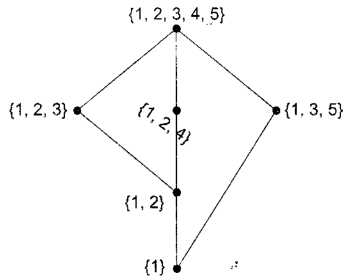
The hasse diagram of the given poset is



In a complete lattice L, every non empty subset of L, has both LUB and GLB.

Now it is necessary to add {1} since GLB of {1, 2} and {1, 3, 5} is {1}.

The hasse diagram now becomes



Now the above hasse diagram represents a complete lattice, since every non empty subset has both LUB and GLB. Therefore adding {1} is not only necessary but it is also sufficient to make the given lattice a complete lattice.

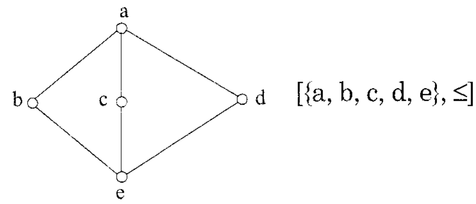
Therefore the correct choice is (a).

2.42 (a)

$$
\begin{aligned}
X &= (A - B) - C \\
&= (A \cap B') - C \\
&= (A \cap B') \cap C' \\
&= AB'C' \\
Y &= (A - C) - (B - C) \\
&= (A \cap C') - (B \cap C') \\
&= (AC') - (BC') \\
&= (AC') \cap (BC')' \\
&= (AC') \cap (B' + C) \\
&= (AC') \cdot (B' + C) \\
&= AC'B' + AC'C \\
&= AC'B' \qquad \text{(since } C'C = 0) \\
&= AB'C' \qquad \text{(commutative property)}
\end{aligned}
$$

$\therefore \quad X = Y$

2.43 (b)



The poset [{a, b, c, d, e}, ≤] is a lattice (since every pair of elements has LUB and GLB) but it is not a distributive lattice. Because distributive lattice satisfy the following conditions. For any x, y, z,

$x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$

$x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$

Where $\wedge$ and $\vee$ are meet and join operations but for given poset [{a, b, c, d, e} ≤]

$b \wedge (c \vee d) = b \wedge a = b$

$(b \wedge c) \vee (b \wedge d) = e \vee e = e$

So it is not distributive. (Also, element b has 2 complements c and d, which is not possible in a distributive lattice, since, in a distributive lattice, complement if it exists, is always unique).

2.44 (c)

The set S = {1, 2, 4, 7, 8, 11, 13, 14} is a group under multiplication modulo 15.

The identity element for this group is e = 1 since, $\forall x \in S, 1 \cdot x \bmod 15 = x$

Now let the inverse of 4 be 4^{-1}.

Now$(4 \cdot 4^{-1}) \bmod 15 = e = 1$

Since$(4 \cdot 4) \bmod 15 = 1$

$\therefore \quad 4^{-1} = 4$ (This inverse is unique)

Similarly let the inverse of 7 be 7^{-1}.

$(7 \cdot 7^{-1}) \bmod 15 = 1$

putting each element of set as 7^{-1} by trial and error we get

$(7 \cdot 13) \bmod 15 = 91 \bmod 15 = 1$

$\therefore \quad 7^{-1} = 13$

So 4^{-1} and 7^{-1} are respectively 4 and 13.

Correct choice is (c).

2.45 (c)

$R \cap S$ is an equivalence relation as can be seen from proof given below.

Let $\forall x \in A$ $(x, x) \in R$ and $(x, x) \in S$ (since R and S are reflexive)

$\therefore$ $(x, x) \in R \cap S$ also $\therefore$ $R \cap S$ is reflexive.

Now, $(x, y) \in R \cap S$

$\Rightarrow (x, y) \in R$ and $(x, y) \in S$

$\Rightarrow (y, x) \in R$ and $(y, x) \in S$

(Since R and S are symmetric)

$\Rightarrow (y, x) \in R \cap S$

$\therefore (x, y) \in R \cap S$

$\Rightarrow (y, x) \in R \cap S$

$R \cap S$ is therefore symmetric

Now consider

(x, y) and $(y, z) \in R \cap S$

$\Rightarrow (x, y)$ and $(y, z) \in R$

and (x, y) and $(y, z) \in S$

$\Rightarrow (x, z) \in R$ and $(x, z) \in S$

(Since R and S are transitive)

$\Rightarrow (x, z) \in R \cap S$

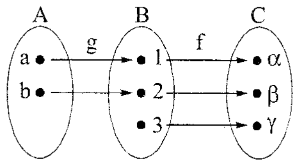
$\therefore$ $R \cap S$ is transitive also. Since $R \cap S$ is reflexive, symmetric and transitive.

$\therefore$ $R \cap S$ is equivalence relation.

Note: A similar argument cannot be made from $R \cup S$.

2.46 (b)

Consider the arrow diagram shown below



$h(a) = f{\cdot}g(a) = \alpha$

$h(b) = f{\cdot}g(b) = \beta$

Here f is onto but g is not onto, yet h is onto.

As can be seen from diagram if f is not onto, h cannot be onto.

$\therefore$ f should be onto, but g need not be onto.

$\therefore$ Answer is (b).

2.47 (a)

(i) The set H is closed, since multiplication of upper triangular matrices will result only in upper triangular matrix.

(ii) Matrix multiplication is associative, i.e. $A * (B * C) = (A * B) * C$.

(iii) Identity element is

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and this belongs to H as I is an upper triangular as will as lower triangular matrix.

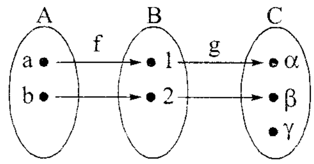
(iv) If $A \in H$, then $|A| = abc$. Since it is given that $abc \neq 0$, this means that $|A| \neq 0$ i.e. every matrix belonging to H is non-singular and has a unique inverse.

$\therefore$ the set H along with matrix multiplication is a group.

2.48 (c)

Given $h = g(f(x)) = g.f$

Consider the following arrow diagram



From above diagram it is clear that g is not onto $\Rightarrow h = g.f$ is also not onto, since the co-domain of g is same as the co-domain of g.f.

The contrapositive version of the above implication is

h is onto $\Rightarrow$ g is onto

which also has to be true since direct $\equiv$ contrapositive.

So option (c) is true.

2.49 (b)

The way C is defined in the question, it contains only comparable subsets of A.

i.e. the set C is the set of all comparable subsets of the set A. Such a set is called a chain. We have a theorem which says that the length of the longest chain for a set of size n is n+1.

Infact theorem also says that the length of the longest anti-chain for a set of size n is n+1.

So the correct option is (b).

2.50 (a)

I: $f(x) = x + y - 3$

$x + a - 3 = x = a + x - 3$

so $a = 3$

Now 3 is unique, and $3 \in N$

So I has identity.

II: $f(x) = \max(x, y)$

$\max(x, a) = x = \max(a, x)$

In N, the only value of a which will satisfy above equation is $a = 1$.

Since 3 is unique, and $3 \in N$

So II has identity.

III: $f(x) = x^y$

$x^a = x = a^x$

Now $x^a = x \Rightarrow a = 1$, but $x = a^x$ has no solution for a in the set N.

So III has no identity.

So only I and II has identity.

2.51 (d)

Given $|X| = x$, $|Y| = y$ and $|Z| = z$

$$W = X \times Y$$

So $$|W| = xy$$

$$|E| = 2^{|W|} = 2^{xy}$$

So the number of functions for Z to E = $|E|^{|Z|}$

$$= (2^{xy})^z = 2^{xyz}$$

2.52 (c)

Let $A = \{1, 2, 3, 5, 7, 8, 9\}$

Construct the table for any $x, y \in A$ such that

$$x * y = (x \cdot y) \bmod 10$$

•	1	2	3	5	7	8	9
1	1	2	3	5	7	8	9
2	2	4	6	0	4	6	8
3	3	6	9	5	1	4	7
5	5	0	5	5	5	0	5
7	7	4	1	5	9	6	3
8	8	6	4	0	6	4	2
9	9	8	7	5	3	2	1

We know that $0 \notin A$. So it is not closed. Therefore, (a) is true.

The identity element = 1

$\therefore$ $(2 \cdot 2^{-1}) \bmod 10 = 1$

From the table we see that 2^{-1} does not exist.

Since, $(3 \cdot 7) \bmod 10 = 1$

$\therefore$ 7 is the inverse of 3 and $7 \in A$.

$\therefore$ (c) is false.

(d) is true since 8 does not have inverse.

2.53 (a)

$(x, y)\ R\ (u, v)$ iff $x < u$ and $y > v$

$(x, x) \not R (x, x)$ since $x \not< x$ and $x \not> x$

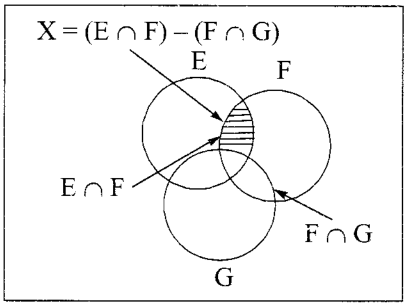
So R is not reflexive,

$\therefore$ R is neither a partial order, nor an equivalence relation.

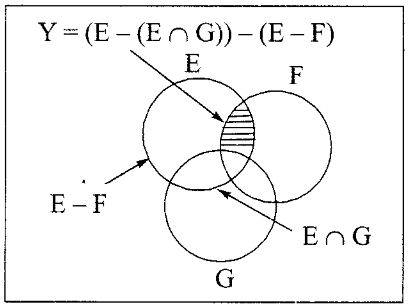
2.54 (c)

Consider the following Venn diagram for

$$X = (E \cap F) - (F \cap G)$$



$$Y = (E - (E \cap G)) - (E - F)$$



So $X = Y$

or alternatively the solution can be obtained from boolean algebra as follows:

$$\begin{aligned} X &= (E \cap F) - (F \cap G) \\ &= EF - FG \\ &= EF \cap (FG)' \\ &= EF \cdot (F' + G') \\ &= EFF' + EFG' \\ &= EFG' \end{aligned}$$

Similarly,

$$\begin{aligned} Y &= (E - (E \cap G)) - (E - F) \\ &= (E - EG) - (E \cdot F') \\ &= E \cdot (EG)' - EF' \\ &= E \cdot (E' + G') - EF' \\ &= EG' - EF' \\ &= EG' \cdot (EF')' \\ &= EG' \cdot (E' + F) \\ &= EE'G' + EFG' \\ &= EFG' \end{aligned}$$

Therefore, $X = Y$

2.55 (b)

The problem can be solved by considering the cases m = 4 and m = 5 etc.

Let $m = 4$

$$S = \{1, 2, 3, 4\}$$

n = number of 3 element subsets

$$= {}^4C_3 = {}^4C_1 = 4$$

$\therefore$ $n = 4$

The 4 subsets are {1, 2, 3}, {1, 2, 4}, {1, 3, 4} and {2, 3, 4} f(1) = number of subsets having 1 as an element = 3

f(2) = number of subsets having 2 as an element = 3

f(3) = 3 and f(4) = 3

$$\therefore \sum_{i=1}^{4} f(i) = 3 + 3 + 3 + 3 = 12$$

both choice (a) and choice (b) are matching the answer since

$$3m = 3n = 12$$

Now let us try m= 5

$$S = \{1, 2, 3, 4, 5\}$$

n = number of 3 element subsets = ${}^5C_3 = 10$

$$\therefore \quad n = 10$$

The 10 subsets are {1, 2, 3}, {1, 2, 4}, {1, 2, 5}, {1, 3, 4}, {1, 3, 5}, {1, 4, 5}, {2, 3, 4}, {2, 3, 5}, {2, 4, 5}, {3, 4, 5}

f(1) = f(2) = f(3) = f(4) = f(5) = 6

$$\sum_{i=1}^{5} f(i) = 6 + 6 + 6 + 6 + 6 = 30$$

Clearly 3 m = 3 × 5 = 15 {is not matching Σf(i)}

but 3n = 3 × 10 = 30 {is matching Σf(i) = 30}

∴ 3 n is the only correct answer.

Correct choice is (b).

The problem can also be solved in a more general way as follows:

$$\sum_{i=1}^{m} f(i) = f(1) + f(2) + ...+ f(m)$$

since f(1) = f(2) = ... = f(m) = $m - {}^1C_2$

$$\text{Therefore } \sum_{i=1}^{m} f(i) = m \times {}^{(m-1)}C_2$$

$$= \frac{m \times (m-1) \times (m-2)}{2}$$

$$= \frac{3 \times m \times (m-1) \times (m-2)}{1 \times 2 \times 3} = 3 \times {}^mC_3$$

Since n = Number of three elements subsets of a set of m elements = ${}^mC_3$

$$\text{Therefore } \sum_{i=1}^{m} f(i) = 3 \times {}^mC_3 = 3n$$

2.56 (b)

P Δ Q = pq′ + p′q

where Δ is symmetric difference between p and q.

I. LHS = P Δ Q ∩ R = pΔ(qr) = p(qr)′ + p′(qr) = p(q′ + r′) + p′qr

RHS = (P Δ Q) ∩ (P Δ R) = (pq′ + p′q) (pr′ + p′r) = pq′r′ + p′qr

LHS ≠ RHS

So statement I is false.

II. LHS = P ∩ Q Δ R = p(qr′ + q′r) = pqr′ + pq′r

RHS = (P ∩ Q) Δ (P ∩ R) = pq Δ pr = pq(pr)′ + (pq)′ pr = pq(p′ + r′) + (p′+q′)pr = pqr′ + pq′r

LHS = RHS

So statement II is true.

2.57 (b)

No's divisible by 2 in X = 61 [= floor (123/2)]

No's divisible by 3 in X = 41

No's divisible by 5 in X = 24

No's divisible by 2 and 3 .i,e by 6 = 20

No's divisible by 2 and 5 i.e by 10 = 12

No's divisible by 3 and 5 , i.e by 15 = 8

No's divisible by 2 and 3 and 5 ..ie by 30 = 4

No's divisible by either 2 or 3 or 5 = n (AUBUC)

= n(A) + n(B)+n(C) – n(A) ∩ B) – n(B ∩ C) – n(A ∩ C) + n(A ∩ B ∩ C)

= 61 + 41 + 24 – 20 – 12 – 8 + 4 = 90

X = {n ,1 ≤ n ≤ 123, n is **not divisible** by 2, 3 or 5}

Cardinality = 123 – 90 = **33**

2.58 (b)

Let S be a set of n elements say {1, 2, 3,..., n}. Now the smallest equivalence relation on S must contain all the reflexive elements {(1, 1), (2, 2), (3, 3),..., (n, n)} and its cardinality is therefore n. The largest equivalence relation on S is S × S, which has cardinality of n × n = n^2.

∴ The largest and smallest equivalence relations on S have cardinalities of n^2 and n respectively.

Correct choice is (b).

2.59 (c)

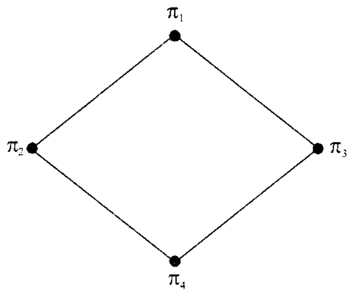
A partition P_1 is called a refinement of the partition P_2 if every set in P_1, is a subset of one of the sets in P_2.

π_4 is a refinement of π_2, π_3 and π_1

π_2 and π_3 are refinements of π_1

π_2 and π_3 are not comparable since neither is a refinement of the other.

So the poset diagram for (S′, <) would



Which is choice (c).

2.60 (d)

In this problem

a % b means the mod function (i.e. residue when a is divided by b).

a/b means integer division (i.e. quotient when a is divided by b)

(i) (101,22):

$101\%\ 10 \le 22\%\ 10 \Rightarrow 1 \le 2$ which is true.

(101/10, 22/10) = (10, 2) $\in$ P need to check

IS (10, 2) $\in$ P?

$10\%\ 10 \le 2\%\ 10 \Rightarrow 0 \le 2$ which is True.

Then (10/10, 2/10) = (1, 0) fails since $1 \not\le 0$.

$\therefore$ (101, 22) $\notin$ P.

(ii) (22, 101)

$22\%\ 10 \le 101\%\ 10 \Rightarrow 2 \le 1$ is False.

$\therefore$ (22, 101) $\notin$ P

(iii) (145, 265)

$145\%\ 10 \le 265\%\ 10 \Rightarrow 5 \le 5$ is true and (145/10, 265/10) = (14, 26) $\in$ P has to be checked.

Now consider (14, 26).

$14\%\ 10 \le 26\%\ 10 \Rightarrow 4 \le 6$ is true and (14/10, 26/10) = (1, 2) $\in$ P has to be checked.

Now consider (1, 2)

$1\%\ 10 \le 2\%\ 10 \Rightarrow 1 \le 2$ is true and (1/10, 2/10) = (0, 0) $\in$ P which is given to be true.

Therefore (145, 265) $\in$ P.

(iv) (0, 153):

$0\%\ 10 \le 153\%\ 10 \Rightarrow 0 \le 3$ is true.

Then (0/10, 153/10) = (0, 15) should be in P.

(0, 15):

$0\%\ 10 \le 15\%\ 10 \Rightarrow 0 \le 5$ is true.

Then (0/10, 15/10) = (0, 1) should be in P.

(0, 1):

$0\%\ 10 \le 1\%\ 10 \Rightarrow 0 \le 1$ is true.

Then (0/10, 1/10) = (0, 0) should be in P.

It is given that (0, 0) $\in$ P

Therefore (0, 153) $\in$ P.

So (iii) and (iv) are contained in P.

2.61 (d)

The given set theory expression can be converted into equivalent boolean algebra expression as follows:

$(p \cap q \cap r) \cup (p^C \cap q \cap r) \cup q^C \cup r^C$

$= p{\cdot}q{\cdot}r + p'{\cdot}q{\cdot}r + q' + r'$

$= qr\ (p + p') + q' + r'$

$= qr + q' + r'$

$= (q + q') \cdot (r + q') + r'$

$= r + q' + r'$

$= r + r' + q'$

$= 1 + q'$

$= 1 = U$

2.62 (a)

Hasse diagram is lattice when every pair of element have least upper bound and greatest lower bound in fig 2 and 3 some elements do not have both least upper bound and greatest lower bound.

So they are not lattices.

2.63 (a)

Group properties are closure, associativity existence of identity and existence of inverse for every element. Commutativity is not required for a mathematical structure to become a group.

2.64 (d)

Given, R = {(x, y), (x, z), (z, x), (z, y)} on set {x, y, z}, here (x, y) $\in$ R and (y, x) $\notin$ R.

$\therefore$ R is not symmetric

Also (x, z)$\in$R and (z, x)$\in$R.

$\therefore$ R is not antisymmetric.

R is neither symmetric nor anti-symmetric.

2.65 (c)

If an element is a generator, all elements must be obtained as powers of that element.

Try a, b, c, d one by one to see which are the generators.

$a = a$

$a^2 = a.a = a$

$a^3 = a.a^2 = a.a = a$ and so on.

$\therefore$ a is not the generator.

$b = b$

$b^2 = b.b = a$

$b^3 = b.b^2 = b.a = b$

$b^4 = b.b^3 = b.b = a$ and so on

$\therefore$ b is not the generator

$$c = c$$
$$c^2 = c.c = b$$
$$c^3 = c.c^2 = c.b = d$$
$$c^4 = c.c^3 = c.d = a$$

since all of a, b, c, d have been generated as powers of c

$\therefore$ c is a generator of this group.

Similarly

$$d = d$$
$$d^2 = d.d = b$$
$$d^3 = d.d^2 = d.b = c$$
$$d^4 = d.d^3 = d.c = a$$

$\therefore$ d is the other generator.

2.66 (c)

Number of reflexive relations on a set with n elements = 2^{n^2-n}.

Here n = 5. So, answer is $2^{5^2-5} = 2^{20}$.

2.67 (a)

The structure ({n, n^{th} roots of unity}, $\times$) always forms a group. When n = 3 we get the set ({1, w, w^2}, $\times$) which must also form a group.

or

The fact that ({1, w, w^2}, $\times$) forms a group can also be seen by the fact that it satisfies the four group properties.

1. Closure:

*	1	w	w^2
1	1	w	w^2
w	w	w^2	1
w^2	w^2	1	w

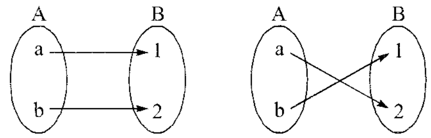
   From above operation table, we can see that the given operation is closed, on the given set.
2. Associative: "*" is an associative operation.
3. Identity: From operation table, we can see that the identity element, is "1".
4. Inverse: From operation table we can see that inverse of 1 is 1, inverse of w is w^2 and inverse of w^2 is w.

So, ({1, w, w^2}, *) is a group.

To be ring, integral domain or field, we need two binary operations to be specified, whereas here we have only one operation given. So, choice (a) is correct.

2.68 (c)

Let n = 2. There are only 2 onto functions as shown below:



For n = 2

Option (a) $2^n = 2^2 = 4$

Option (b) $2^n - 1 = 2^2 - 1 = 3$

Option (c) $2^n - 2 = 2^2 - 2 = 2$

Option (d) $2(2^n - 2) = 2(2^2 - 2) = 4$

So only option (c) gives correct answer.

Alternate method

The number of onto functions from a set A with m elements to set B with n elements where n < m is given by

$n^m - {}^nC_1 (n-1)^m + {}^nC_2 (n-2)^m \dots + {}^nC_{n-1} 1^m$

Here m = n = 2

So number of onto functions = $2^n - {}^2C_1 1^m$

$= 2^n - 2$

which is choice (c).

2.69 (a)

$$x \oplus y = x^2 + y^2$$
$$y \oplus x = y^2 + x^2$$

As '+' sign in commutative so $x^2 + y^2$ is equal to $y^2 + x^2$ so $x \oplus y$ is commutative.

Now check associativity

$$x \oplus (y \oplus z) = x \oplus (y^2 + z^2)$$
$$= x^2 + (y^2 + z^2)^2$$
$$= x^2 + y^4 + z^4 + 2y^2z^2$$
$$(x \oplus y) \oplus z = (x^2 + y^2) \oplus z$$
$$= (x^2 + y^2)^2 + z^2$$
$$= x^4 + y^4 + 2x^2y^2 + z^2$$
$$x \oplus (y \oplus z) \neq (x \oplus y) \oplus z$$

So not associative

Option (a) is correct.

2.70 Sol.

$f : \{0,1\}^4 \rightarrow \{0,1\} \Rightarrow$ S is the set of all functions from a 16 element set to a 2 element set.

$|S| = 2^{16}$

N = Number of functions from S to 2 element set {0, 1} = $2^{2^{16}}$]

$$N = 2^{2^{16}}$$

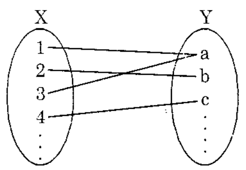
$\therefore$ $\log\log N = \log\log 2^{2^{16}} = \log 2^{16} = 16$

2.71 (a)

Given the following details:
$S = \{1,2,3,4,...2014\}$
$U < V$ if the minimum element in the symmetric difference of the two sets is in U.
S1: There is a subset of S that is larger than every other subset.
S2: There is a subset of S that is smaller than every other subset.
S1 is true since ϕ (which is subset of S) is larger then every other subset.
i.e. $\forall v \;\; v < \phi$ is true since the minimum element in $v \,\Delta\, \phi = v$, is in v.
(***Note:*** $v \,\Delta\, \phi = v\phi' + v'\phi = v$)
S2 is also true since S (which is subset of S) is smaller then every other subset.
i.e. $\forall v \;\; S < v$ is true since the minimum element in $S \,\Delta\, v = v'$, is in S.
(***Note:*** $S \,\Delta\, v = Sv' + S'v = 1v' + 0v = v'$)
$\therefore$ both S1 and S2 are correct.

2.72 (d)

Example:



Let $A = \{1, 2\}$, $B = \{3, 4\}$
(a) $|f(A \cup B)| = |f(A)| + |f(B)|$
$\Rightarrow$ 3 = 2 + 2 is false
(b) $f(A \cap B) = f(A) \cap f(B)$
$\Rightarrow$ $\phi = \{a\}$ is false
(c) $|f(A \cap B)| = \min\{|f(A)|, |f(B)|\}$
$\Rightarrow$ 0 = min (2, 2) = 2 is false
(d) Let $S = \{a, b\}$, $T = \{a, c\}$
$f^{-1}(S \cap T) = f^{-1}(S) \cap f^{-1}(T)$
$\Rightarrow \{1, 3\} = \{1, 2, 3\} \cap \{1, 3, 4\}$
$\Rightarrow \{1, 3\} = \{1, 3\}$ is true

2.73 Sol.

Order of subgroup divides order of group (Lagrange's theorem). So order of L has to be 1, 3, 5 or 15. As it is given that the subgroup has at least 4 elements and it is not equal to the given group, therefore the order of subgroup can't be 1, 3 or 15.
Hence it is 5.

2.74 (b)

Since it is given that $\forall i \;\; f(f(i)) = i$
It means $f \cdot f = I$ i.e. $f = f^{-1}$
That is f is a symmetric function.
Statement P means that every symmetric function is a identity function which is not true, since there are many symmetric functions other than identity function.
Example: {(0, 1), (1, 0), (2, 3), (3, 2),...(2013, 2014), (2014, 2013)} is a symmetric function but not the identity function.
Statement Q means that some symmetric function is identity function.
This is true, since identity function is one of the symmetric functions.
Statement R means that every symmetric function is onto which is true, since it is impossible to make an into function symmetric.

2.75 Sol.

(i) e is identity element
(ii) $x*x = e$, so $x = x^{-1}$.
(iii) $y*y = e$, so $y = y^{-1}$.
(iv) $(x*y)*(x*y) = e$, so $x*y = (x*y)^{-1}$ ...(i)
and $(y*x)*(y*x) = e$, so $y*x = (y*x)^{-1}$
Now $(x*y)*(y*x) = x*(y*y)*x = x*e*x$
$= x*x = e$
So $(x*y)^{-1} = (y*x)$ ...(ii)
From (i) and (ii) we get $x*y* = y*x$
There are only 4 distinct elements possible in this group
1. e
2. x
3. y
4. xy

All other combinations are equal to one of these four as can be seen below:
yx = xy (already proved)
xxx = xe = x
xyy = xe = x
xxy = ey = y
xyx = xxy = y
and so on...
So the group is $G = \{e, x, y, x*y\}$
$\Rightarrow \quad |G| = 4$

2.76 (a)

$$g(x) = 1 - x \;,\; h(x) = \frac{x}{x-1}$$

$$\frac{g(h(x))}{h(g(x))} = \frac{g\left(\frac{x}{x-1}\right)}{h(1-x)} = \frac{1-\frac{x}{x-1}}{\frac{1-x}{1-x-1}}$$

$$= \frac{\frac{x-1-x}{x-1}}{\frac{1-x}{-x}} = \frac{\frac{-1}{x-1}}{\frac{1-x}{-x}}$$

$$= \frac{x}{(x-1)(1-x)} = \frac{-x}{(1-x)^2}$$

$$\frac{h(x)}{g(x)} = \frac{\frac{x}{x-1}}{1-x} = \frac{x}{(1-x)(x-1)} = \frac{-x}{(1-x)^2}$$

$$\therefore \frac{g(h(x))}{h(g(x))} = \frac{h(x)}{g(x)}$$

2.77 (c)

$A = \{5, \{6\}, \{7\}\}$

1. $\phi \in 2^A$ is true. Any power set contains an empty set.
2. $\phi \subseteq 2^A$ is true. Empty set is subset of any set.
3. $\{5,\{6\}\} \in 2^A$ is true. power set of A contain $\{5, \{6\}\}$ as a 2 element subset of A.
4. $\{5, \{6\}\} \subseteq 2^A$ is false. Power set of A not contain 5 and $\{6\}$ as elements.

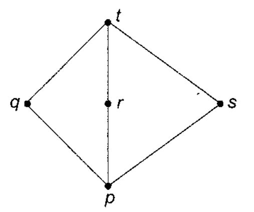
So $\{5, \{6\}\}$ can not be subset of 2^A.

2.78 (d)

$\mathcal{L}$ has 5 elements

$\mathcal{L}^3$ has all ordered triplets of elements of $\mathcal{L}$

$\Rightarrow \mathcal{L}^3$ contain $5 \times 5 \times 5 = 5^3 = 125$ elements.



If q,r,s are chosen, then only it will violate the distributive property.

Number of ways to choose q, r, s in any triplet order $= 3! = 3 \times 2 \times 1 = 6$.

$\therefore$ p(satisfying distributive property) = 1 − p(violate the distributive property)

$$= 1 - \frac{6}{5^3} = 0.952 \text{ which is between } 1/5 \text{ and } 1.$$

2.79 (d)

aRb iff a and b are distinct a and b have a common divisor other than 1.

(*i*) R is not reflexive since a and b are distinct i.e, $(a, a) \notin R$

(*ii*) R is symmetric

If a and b are distinct and have a common divisor other than 1, then b and a also are distinct and have a common divisor other than 1.

(*iii*) R is not transitive

If $(a, b) \in R$ and $(b, c) \in R$ then (a, c) need not be in R.

Example: $(2, 6) \in R$ and $(6, 2) \in R$, but $(2, 2) \notin R$

$\therefore$ R is symmetric but not reflexive and not transitive.

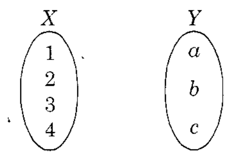
2.80 Sol.

Let $X = \{0, 1, 2, ...10\}$

$|X| = 11$

$|P(X)| = 2^{11} = 2048$

2.81 Sol.



The number of onto functions from $A \to B$ where $|A| = m$ and $|B| = n$ is given by the formula

$n^m - {}^nC_1 (n-1)^m + {}^nC_2(n-2)^m + \ldots + (-1)^{n-1}\,{}^nC_{n-1} 1^m$

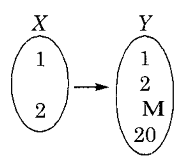
Here m = 4 and n = 3

Substituting in above formula we get,

$3^4 - {}^3C_1(3-1)^4 + {}^3C_2(3-2)^4$

$= 81 - 48 + 3 = 36$

2.82 Sol.



Total possible functions $= 20^2 = 400$

Number of one-to-one functions

$= 20P_2 = 20 \times 19$

$$\therefore \text{Required probability} = \frac{20 \times 19}{20 \times 20} = 0.95$$

2.83 (b)

$X \# Y = X' + Y'$

This is the NAND operation. NAND is known to be commutative but not associative.

So only S2 is true.

2.84 (d)

S = {1, 2, 3, 4, 5, 6}

U is the power set of S $\Rightarrow U = P(S)$

$U = \{\{\ \}\{1\}, \{2\}, \ldots\{1, 2\}, \{1, 3\}, \ldots\{1, 2, 3\}, \ldots\{1, 2, 3, 4\}, \ldots\{1, 2, 3, 4, 5\} \ldots\{1, 2, 3, 4, 5, 6\}\}$

$|U| = 2^6 = 64$ elements.

$T \in U \Rightarrow T' \in U$

$R \in U,\ T \setminus R = T - R$

(a) $\forall X \in U(|X| = |X'|)$ means that every subset of S has same size as its complement. Clearly this is False.
(For example, {1} ∈ U, and complement of {1} = {2, 3, 4, 5, 6})

(b) $\exists X \in U \exists Y \in U \quad (|X| = 5, |Y| = 5 \text{ and } X \cap Y = \phi)$ means that there are two 5 element subsets of S which have nothing in common. This is clearly False.
Since any two 5 element subsets will have atleast 4 elements in common.

(c) $\forall X \in U\ \forall Y \in U \quad (|X| = 2, |Y| = 3 \text{ and } X \setminus Y = \phi)$ means that every 2 element subset X and every 3 element subset Y will have $X - Y = \phi$ i.e., $X \subseteq Y$.
This is clearly False as can be seen from the example X = {1, 2}, Y = {3, 4, 5} where $X \subsetneq Y$.

(d) $\forall X \in U\ \forall Y \in U\ (X \setminus Y = Y' \setminus X')$ means that for any two subsets X and Y, $X \setminus Y = Y' \setminus X'$ i.e. $X - Y = Y' - X'$.

This is clearly True since by boolean algebra LHS = $X - Y = XY'$.

RHS = $Y' - X' = Y'X$ and therefore LHS = RHS.

2.85 (c)

$R = \{(p, q), (r, s) \mid p - s = q - r\}$

(i) Check reflexive property

$\forall (p, q) \in Z^+ \times Z^+ \quad ((p, q), (p, q)) \in R$

is true iff $p - q = q - p$ which is false.
So relation R is not reflexive.

(ii) Check symmetry property

If $((p, q), (r, s)) \in R$ then $((r, s), (p, q)) \in R$

$((p, q), (r, s)) \in R \Rightarrow p - s = q - r$

$((r, s), (p, q)) \in R \Rightarrow r - q = s - p$

If $p - s = q - r$ is true, then $r - q = s - p$ is also true by rearranging the equation.

$\therefore R$ is symmetric.

2.86 Sol.

$f(n) = f\left(\frac{n}{2}\right)$ if n is even

$f(n) = f(n + 5)$ if n is odd

$f : N^+ \to N^+$

Now $f(2) = f\left(\frac{2}{2}\right) = f(1)$

$f(3) = f(3 + 5) = f(8) = f\left(\frac{8}{2}\right) = f(4)$

$= f\left(\frac{4}{2}\right) = f(2) = (1)$

So $f(1) = f(2) = f(3) = f(4) = f(8)$

Now let us find $f(5) = f(5 + 5) = f(10) = f\left(\frac{10}{2}\right)$

$= f(5)$ so $f(5) = f(10)$

Now let us find $f(9)$

$f(9) = f(9 + 5) = f(14) = f\left(\frac{14}{2}\right) = f(7)$

$= f(7 + 5) = f(12) = f\left(\frac{12}{2}\right) = f(6)$

So $f(9) = f(7) = f(6)$

For $n > 10$, it will be equal to one of $f(1)$, $f(2) \ldots f(10)$

So the maximum no. of distinct values f takes is only 3.
First is $f(1) = f(2) = f(3) = f(4) = f(8)$
Second is $f(5) = f(10)$
Third is $f(6) = f(7) = f(9)$
All other n values will give only one of these three values.

2.87 (b)

(a, b) R (c, d) if $a \leq c$ or $b \leq d$
P : R is reflexive
Q : R is transitive
Since, (a, b) R $(a, b) \Rightarrow a \leq a$ or $b \leq b$
$\Rightarrow$ t or t
Which is always True, R is reflexive.
Now let us check transitive property
Let(a,b) R (c, d) $\Rightarrow$ a $\leq$ c or b $\leq$ d
and(c, d) R $(e, f) \Rightarrow c \leq e$ or $d \leq f$
Now let us take a situation
$a \leq c$ (True) or $b \leq d$ (false)
and $c \leq e$ (False) or $d \leq f$ (True)
Now we can get neither $a \leq e$ nor $b \leq f$
So, (a, b) R (c, d) and (c, d) R (e, f) $\not\Rightarrow$ (a, b) R (e, f). So clearly R is not transitive.
So P is true and Q is false. Choice (b) is correct.

2.88 (b)

I. If there is a subset of 9 component out of 23 component each of which reacts with exactly 3 components, it does not mean that every component of U reacts with odd number of components. So I is false.

II. Since there are 9 components in U which react with exactly 3 (odd) components , So II is true.

III. is false, since some of components in U reacts with odd number of components.

So only II is true.

3 Combinatiorics

3.1 (a) Solve the recurrence equations
$T(n) = T(n-1) + n$
$T(1) = 1$
(b) What is the generating function G(z) for the sequence of Fibonacci numbers?
[1987 : 2 Marks]

3.2 Solve the recurrence equations:

$$T(n) = T\left(\frac{n}{2}\right) + 1$$
$$T(1) = 1$$

[1988 : 2 Marks]

3.3 How many substrings can be formed from a character string of length n?
[1989 : 2 Marks]

3.4 The number of binary strings of n zeros and k ones that no two ones are adjacent is
(a) ${}^{n+1}C_k$
(b) ${}^{n}C_k$
(c) ${}^{n}C_{k+1}$
(d) None of these
[1990 : 1 Mark]

3.5 The number of substrings (of all lengths inclusive) that can be formed from a character string of length n is
(a) n
(b) n^2
(c) $\frac{n(n-1)}{2}$
(d) $\frac{n(n+1)}{2}$
[1994 : 2 Marks]

3.6 In a room containing 28 people, there are 18 people who speak English, 15 people who speak Hindi and 22 people who speak Kannada. 9 persons speak both English and Hindi, 11 person speak both Hindi and Kannada whereas 13 persons speak both Kannada and English. How many people speak all three languages?
(a) 9
(b) 8
(c) 7
(d) 6
[1998 : 2 Marks]

3.7 Solve the following recurrence relation
$x_n = 2x_{n-1} - 1, n > 1$
$x_1 = 2$
[1998 : 2 Marks]

3.8 Two girls have picked 10 roses, 15 sunflowers and 14 daffodils. What is the number of ways they can divide the flowers among themselves?
(a) 1638
(b) 2100
(c) 2640
(d) None of these
[1999 : 2 Marks]

3.9 The minimum number of cards to be dealt from an arbitrarily shuffled deck of 52 cards to guarantee that three cards are from some same suit is .
(a) 3
(b) 8
(c) 9
(d) 12
[2000 : 1 Mark]

3.10 How many 4 digit even numbers have all 4 digits distinct?
(a) 2240
(b) 2296
(c) 2620
(d) 4536
[2001 : 2 Marks]

3.11 The solution to the recurrence equation $T(2^k) = 3T(2^{k-1}) + 1$, $T(1) = 1$ is
(a) 2^k
(b) $\frac{(3^{k+1}-1)}{2}$
(c) $3^{\log_2 k}$
(d) $2^{\log_3 k}$
[2002 : 1 Mark]

3.12 The minimum number of colours required to colour the vertices of a cycle with n nodes in such a way that no two adjacent nodes have the same colour is
(a) 2
(b) 3
(c) 4
(d) $n - 2\left\lfloor \frac{n}{2} \right\rfloor + 2$
[2002 : 1 Mark]

3.13 Let A be a sequence of 8 distinct integers sorted in ascending order. How many distinct pairs of sequences, B and C are there such that (i) each is sorted in ascending order, (ii) B has 5 and C has 3 elements, and (iii) the result of merging B and C gives A?
(a) 2
(b) 30
(c) 56
(d) 256
[2003 : 1 Mark]

3.14 n couples are invited to a party with the condition that every husband should be accompanied by his wife. However, a wife need not be accompanied by her husband. The number of different gatherings possible at the party is

(a) $\binom{2n}{n} * 2^n$ (b) 3^n

(c) $\frac{(2n)!}{2^n}$ (d) $\binom{2n}{n}$

[2003 : 1 Mark]

3.15 Mala has a colouring book in which each English letter is drawn two times. She wants to paint each of these 52 prints with one of k colours, such that the colour-pairs used to colour any two letters are different. Both prints of a letter can also be coloured with the same colour. What is the minimum value of k that satisfies this requirement?

(a) 9 (b) 8
(c) 7 (d) 6

[2004 : 2 Marks]

3.16 In how many ways can we distribute 5 distinct balls, $B_1, B_2, \ldots, B_5$ in 5 distinct cells, $C_1, C_2, \ldots, C_5$ such that Ball B_i is not in cell $C_i, \forall_i = 1, 2, \ldots, 5$ and each cell contains exactly one ball?

(a) 44 (b) 96
(c) 120 (d) 3125

[IT-2004 : 2 Marks]

3.17 Let $n = p^2q$, where p and q are distinct prime numbers. How many numbers m satisfy $1 \le m \le n$ and gcd (m, n) = 1? Note that gcd (m, n) is the greatest common divisor of m and n.

(a) $p(q - 1)$ (b) pq
(c) $(p^2 - 1)(q - 1)$ (d) $p(p - 1)(q - 1)$

[IT-2005 : 2 Marks]

3.18 What is the minimum number of ordered pairs of non-negative numbers that should be chosen to ensure that there are two pairs (a, b) and (c, d) in the chosen set such that $a \equiv c \bmod 3$ and $b \equiv d \bmod 5$

(a) 4 (b) 6
(c) 16 (d) 24

[2005 : 2 Marks]

3.19 Let $G(x) = 1/(1 - x)^2 = \sum_{i=0}^{\infty} g(i)x^i$ where $|x| < 1$. What is g(i)?

(a) i (b) i + 1
(c) 2i (d) 2^i

[2005 : 2 Marks]

3.20 For each element in a set of size 2n, an unbiased coin is tossed. The 2n coin tossed are independent. An element is chosen if the corresponding coin toss were head. The probability that exactly n elements are chosen is

(a) $\binom{2n}{n} \Big/ 4^n$ (b) $\binom{2n}{n} \Big/ 2^n$

(c) $1 \Big/ \binom{2n}{n}$ (d) $\frac{1}{2}$

[2006 : 2 Marks]

Common Data for Q.3.21 & Q.3.22

Suppose that a robot is placed on the Cartesian plane. At each step it is allowed to move either one unit up or one unit right, i.e., if it is at (i, j) then it can move to either (i + 1, j) or (i, j + 1).

3.21 How many distinct paths are there for the robot to reach the point (10, 10) starting from the initial position (0, 0)?

(a) $\binom{20}{10}$ (b) 2^{20}

(c) 2^{10} (d) None of these

[2007 : 2 Marks]

3.22 Suppose that the robot is not allowed to traverse the line segment from (4, 4) to (5, 4). With this constraint, how many distinct paths are there for the robot to reach (10, 10) starting from (0, 0)?

(a) 2^9 (b) 2^{19}

(c) $\binom{8}{4} \times \binom{11}{5}$ (d) $\binom{20}{10} - \binom{8}{4} \times \binom{11}{5}$

[2007 : 2 Marks]

3.23 Let

$$p = \sum_{\substack{1 \le i \le 2k \\ i \text{ odd}}} i \text{ and } Q = \sum_{\substack{1 \le i \le 2k \\ i \text{ even}}} i,$$

where k is positive integer. Then

(a) P = Q – k (b) P = Q + k
(c) P = Q (d) P = Q + 2k

[2008 : 2 Marks]

Common Data for Q.3.24 & Q.3.25

Let x_n denote the number of binary strings of length n that conatin no consecutives 0s.

3.24 Which of the following recurrences does x_n satisfy?

(a) $x_n = 2x_{n-1}$ (b) $x_n = x_{[n/2]} + 1$

(c) $x_n = x_{[n/2]} + n$ (d) $x_n = x_{n-1} + x_{n-2}$

[2008 : 2 Marks]

3.25 The value of x_5 is

(a) 5 (b) 7

(c) 8 (d) 13

[2008 : 2 Marks]

3.26 The exponent of 11 in the prime factorization of 300! is

(a) 27 (b) 28

(c) 29 (d) 30

[IT-2008 : 2 Marks]

3.27 In how many ways can b blue balls and r red balls be distributed in n distinct boxes?

(a) $\dfrac{(n+b-1)!(n+r-1)!}{(n-1)!\,b!(n-1)!\,r!}$

(b) $\dfrac{(n+(b+r)-1)!}{(n-1)!(n-1)!\,(b+r)!}$

(c) $\dfrac{n!}{b!\,r!}$

(d) (n + (b + r) –1)! /n!(b + r –1)

[2008 : 2 Marks]

3.28 A pennant is a sequence of numbers, each number being 1 or 2. An n-pennant is a sequence of numbers with sum equal to n. For example, (1,1,2) is a 4-pennants. The set of all possible 1-pennant is {(1)}, the set of all possible 2-pennants is {(2), (1,1)}and the set of all 3-pennants is {(2,1), (1,1,1), (1,2)}. Note that the pennant (1,2) is not the same as the pennant (2,1). The number of 10-pennants is ______.

[2014 (Set-1) : 2 Marks]

3.29 Each of the nine words in the sentence

"The quick brown fox jumps over the lazy dog"

is written on a separate piece of paper. These nine pieces of paper are kept in a box. One of the pieces is drawn at random from the box. The expected length of the word drawn is ______. (The answer should be rounded to one decimal place).

[2014 (Set-2) : 1 Mark]

3.30 The number of distinct positive integral factors of 2014 is ______.

[2014 (Set-2) : 2 Marks]

3.31 $\sum_{x=1}^{99} \dfrac{1}{x(x+1)} =$ ______.

[2015 (Set-1) : 2 Marks]

3.32 Let a_n represent the number of bit strings of length n containing two consecutive 1s. What is the recurrence relation for a_n?

(a) $a_{n-2} + a_{n-1} + 2^{n-2}$

(b) $a_{n-2} + 2a_{n-1} + 2^{n-2}$

(c) $2a_{n-2} + a_{n-1} + 2^{n-2}$

(d) $2a_{n-2} + 2a_{n-1} + 2^{n-2}$

[2015 (Set-1) : 2 Marks]

3.33 The number of divisors of 2100 is ______.

[2015 (Set-2) : 1 Mark]

3.34 The number of 4 digit numbers having their digits in non-decreasing order (from left to right) constructed by using the digits belonging to the set {1, 2, 3} is ____.

[2015 (Set-3) : 1 Mark]

3.35 Let a_n be the number of n-bit strings that do NOT contain two consecutive 1's. Which one of the following is the recurrence relation for a_n?

(a) $a_n = a_{n-1} + 2a_{n-2}$ (b) $a_n = a_{n-1} + a_{n-2}$

(c) $a_n = 2a_{n-1} + a_{n-2}$ (d) $a_n = 2a_{n-1} + 2a_{n-2}$

[2016 (Set-1) : 1 Mark]

3.36 The coefficient of x^{12} in $(x^3 + x^4 + x^5 + x^6 + ...)^3$ is ______.

[2016 (Set-1) : 2 Marks]

3.37 Consider the recurrence relation $a_1 = 8$, $a_n = 6n^2 + 2n + a_{n-1}$. Let $a_{99} = K \times 10^4$. The value of K is ______.

[2016 (Set-1) : 2 Marks]

■■■■

Answers Combinatiorics

3.4 (a)	3.5 (d)	3.6 (d)	3.8 (c)	3.9 (c)	3.10 (b)	3.11 (b)	3.12 (d)	3.13 (c)
3.14 (b)	3.15 (d)	3.16 (a)	3.17 (d)	3.18 (c)	3.19 (b)	3.20 (a)	3.21 (a)	3.22 (d)
3.23 (a)	3.24 (d)	3.25 (d)	3.26 (b)	3.27 (a)	3.32 (a)	3.35 (b)		

Explanations Combinatiorics

3.1 Sol.

(a) $T(n) = T(n-1) + n$

$T_n - T_{n-1} = n$

For **Homogeneous solution**

$T_n - T_{n-1} = 0$

$t - 1 = 0$

$t = 1$

Therefore, homogenous solution is

$T_n = C(1)^n = C$

For **Particular solution**

Let particular solution be $(d_0 + d_1 n)n$

$\Rightarrow (d_0 + d_1 n)n - (d_0 + d_1(n-1))(n-1) = n$

$\Rightarrow d_0 n + d_1 n^2 - d_0 n + d_0 - d_1(n-1)^2 = n$

$\Rightarrow d_0 n + d_1 n^2 - d_0 n + d_0 - d_1(n^2 - 2n + 1) = n$

$\Rightarrow d_0 + 2d_1 n - d_1 = n$

$d_0 - d_1 = 0$ and $2d_1 = 1$

$\Rightarrow d_0 = d_1$ and $d_1 = \frac{1}{2}$

$\therefore d_0 = d_1 = \frac{1}{2}$

So particular solution is

$$\left(\frac{1}{2} + \frac{1}{2}n\right) \times n = \frac{n(n+1)}{2} = \frac{n^2+n}{2}$$

So complete solution is

$$T(n) = C + \frac{1}{2}n(n+1)$$

Given $T(1) = 1$

$$1 = C + \frac{1}{2} \times 1 \times (1+1)$$

$1 = C + 1$

$C = 0$

Therefore complete solution of the recurrence relation is $T(n) = \frac{n(n+1)}{2}$.

(b) The Fibonacci numbers are defined as

$a_0 = 1,\ a_1 = 1$

$a_r = a_{r-1} + a_{r-2} \quad r \geq 2$

$$\Rightarrow \sum_{r=2}^{\infty} a_r x^r = \sum_{r=2}^{\infty} a_{r-1} x^r + \sum_{r=2}^{\infty} a_{r-2} x^r$$

$$\Rightarrow \sum_{r=2}^{\infty} a_r x^r = x\sum_{r=2}^{\infty} a_{r-1} x^{r-1} + x^2 \sum_{r=2}^{\infty} a_{r-2} x^{r-2}$$

$\Rightarrow A(x) - a_0 - a_1 x = x(A(x) - a_0) + x^2 A(x)$

Since $a_0 = 1$ and $a_1 = 1$

$\Rightarrow A(x) - 1 - x = x(A(x) - 1) + x^2 A(x)$

$$\Rightarrow A(x) = \frac{1}{1 - x - x^2}$$

3.2 Sol.

$$T(n) = T\left(\frac{n}{2}\right) + 1$$

$$\Rightarrow T(n) - T\left(\frac{n}{2}\right) = 1$$

Let $n = 2^k$.

$$\Rightarrow T(2^k) - T\left(\frac{2^k}{2}\right) = 1$$

$\Rightarrow T(2^k) - T(2^{k-1}) = 1$

Let $T(2^k) = x_k$

$x_k - x_{k-1} = 1$

For Homogeneous solution

$x_k - x_{k-1} = 0$

$t - 1 = 0$

$t = 1$

So homogeneous solution is: $x_k = C(1)^k = C$

For Particular solution

Let particular solution be $d_1 k$

$d_1 k - d_1(k-1) = 1$

$\Rightarrow d_1 = 1$

Particular solution is k

$\therefore$ Complete solution is

$x_k = C + k$

$T(2^k) = C + k$

$T(n) = C + \log_2 n$

Given $T(1) = 1$

$\Rightarrow C = 1$

$\therefore$ Complete solution is: $T(n) = \log_2 n + 1$

3.3 Sol.

Let the string be of length 4 : abcd

Number of substrings of length 0 = 1 (only ε)

Number of substrings of length 1 = 4

a, b, c, d

Number of substrings of length 2 = 3

ab, bc, cd

Number of substrings of length 3 = 2

abc, bcd

Number of substrings of length 4 = 1

abcd

∴ Total number of substrings

= 1 + (4 + 3 + 2 + 1)

= 1+ (Sum of 4 natural number)

$$= 1 + \frac{4\times(4+1)}{2} = 11$$

Therefore, total number of substrings (maximum) that can be formed from a character string of length $1 + \frac{n(n+1)}{2}$.

3.4 (a)

First arranging all n zeros in a row. There is only 1 way for arranging n zeros in a row. By arranging n zeros in a row, we get (n + 1) positions to place ones.

So number of ways arranging k ones in (n+1) positions = ${}^{n+1}C_k$.

∴ Required number of binary strings of n zeroes and k ones that no two ones are adjacent = $1 \times {}^{n+1}C_k = {}^{n+1}C_k$.

3.5 (d)

For a string of length n:

The number of substrings of length 1 = n

The number of substrings of length 2 = n − 1

The number of substrings of length 3 = n − 2

and so on...

The number of substrings of length n is 1

So total number of substrings

= n + (n − 1) + .. +1

= Sum of n natural numbers

$$= \frac{n(n+1)}{2}.$$

3.6 (d)

E : Persons speaks English

H : Persons speaks Hindi

K : Persons speaks Kannada

Assume that everyone in the room speaks at least one of the languages.

Given, $n(E \cup H \cup K) = 28$

$$n(E) = 18$$
$$n(H) = 15$$
$$n(K) = 22$$
$$n(E \cap H) = 9$$
$$n(H \cap K) = 11$$
$$n(K \cap E) = 13$$
$$n(K \cap E \cap H) = ?$$

By principle of inclusion and exclusion

$n(E \cup H \cup K) = n(E) + n(H) + n(K) - n(E \cap H) - n(H \cap E) - n(K \cap E) + n(K \cap E \cap H)$

$28 = 18 + 15 + 22 - 9 - 11 - 13 + n(K \cap E \cap H)$

$n(k \cap E \cap H) = 28 - 55 + 33 = 61 - 55 = 6$

Therefore, the number of people who speak all three languages are 6.

3.7 Sol.

$$x_n = 2x_{n-1} - 1$$

For Homogeneous solution

$$x_n - 2x_{n-1} = 0$$
$$t - 2 = 0$$
$$t = 2$$

So homogeneous solution is

$$x_n = C(2)^n$$

For Particular solution

Let particular solution be d_0.

$$d_0 = 2d_0 - 1$$
$$d_0 = 1$$

So particular solution is 1

∴ Complete solution = Homogeneous solution + Particular solution

Complete solution = $C(2)^n + 1$

⇒ $x_n = C(2)^n + 1$

Given initial condition is $x_1 = 2$

$$2 = C(2)^1 + 1$$
$$1 = 2C$$
$$C = \frac{1}{2}$$
$$x_n = \frac{1}{2}(2)^n + 1$$

∴ $x_n = 2^{n-1} + 1$

3.8 **(c)**

Number of ways for distributing r similar things among n different things = ${}^{(n-1+r)}C_r$

The number of ways for distributing 10 roses among the two girls = ${}^{(2-1+10)}C_{10} = 11$.

Similarly number of ways for distributing 15 sunflowers among two girls = ${}^{(2-1+15)}C_{15}$

$$= {}^{16}C_{15} = {}^{16}C_1 = 16$$

Number of ways for distributing 14 daffodils among the two girls

$$= {}^{(2-1+14)}C_{14} = {}^{15}C_{14} = {}^{15}C_1 = 15$$

$\therefore$ Total number of ways = $11 \times 16 \times 15 = 2640$

3.9 **(c)**

Let the number of cards to be dealt from an arbitratily shuffled deck of 52 cards be n.

Number of suits = 4

Required number of cards from the same suit = 3.

So by pigeonhole principle

$$\left\lfloor \frac{n-1}{4} \right\rfloor + 1 = 3$$

$$\left\lfloor \frac{n-1}{4} \right\rfloor = 2$$

$$\lfloor n-1 \rfloor = 8$$

$$n \geq 9$$

So the minimum number of cards to be dealt from an arbitrarily shuffled deck of 52 cards to guarantee that three cards are from some same suit is 9.

3.10 **(b)**

The digits are given to be distinct i.e. no repetition.

4 digit even numbers cannot start with 0 and must end with 0,2,4,6 or 8.

Since there is a condition for 0 in starting as well as ending we will count the even numbers ending with 0 seperately.

So the total number of 4 digit even number = 4 digit even numbers ending with zero + 4 digit even numbers ending with 2,4,6,8.

4 digit even numbers ending with 0 =

9 × 8 × 7 × 1 = 504 ways

$\cancel{0}$			0

4 digit even numbers ending with 2,4,6,8 =

8 × 8 × 7 × 4 = 1792 ways

$\cancel{0}$			2,4,6,8

So the total number of 4 digit even numbers

$$= 504 + 1792 = 2296.$$

3.11 **(b)**

$$T(2^k) = 3T(2^{k-1}) + 1$$

Let $T(2^k) = x_n$

$\Rightarrow$ $x_n = 3x_{n-1} + 1$

$\Rightarrow$ $x_n - 3x_{n-1} = 1$

So for **Homogenous solution**

$$x_n - 3x_{n-1} = 0$$

$$n - 3 = 0$$

$$n = 3$$

Homogenous solution is

$$x_n = C_1(3)^n$$

$$T(2^k) = C_1(3)^k$$

For **Particular solution**

Let d be the particular solution

$$d - 3d = 1$$

$$2d = -1$$

$$d = -\frac{1}{2}$$

Therefore, the complete solution is

$$T(2^k) = C_1(3)^k - \frac{1}{2}$$

Given, $T(1) = 1$

$$1 = C_1(3)^0 - \frac{1}{2}$$

$$1 = C_1 - \frac{1}{2}$$

$$C_1 = \frac{3}{2}$$

So the complete solution is

$$T(2^k) = \frac{3}{2}(3)^k - \frac{1}{2}$$

$$T(2^k) = \frac{3^{k+1} - 1}{2}$$

3.12 **(d)**

The minimum number of colours required to colour the vertices of a cycle with n nodes

= 2, when n is even

= 3, when n is odd

Therefore $n - 2\left\lfloor \frac{n}{2} \right\rfloor + 2$ gives 2 when n is even, and 3 when n is odd.

3.13 **(c)**

This corresponds to an ordered partition of 8 elements into two groups, the first with 5 elements and second with 3 elements. The number of ways of doing this is

$$P(8; 5, 3) = \frac{8!}{5!\,3!} = 56$$

3.14 (b)

For each of the n couples invited to the party one of three thing is possible

1. both husband and wife attend the party.
2. wife only attends the party.
3. neither husband nor wife attends the party. Since there are n such couples, total number of possibilities = 3^n.

3.15 (d)

The problem reduces to finding how many distinct ordered colour pairs (C_1, C_2) are possible with k colors.

Since the first color C_1 can be any one of the k colours and the second color C_2 also can be any one of the k colors (both prints of a letter can be colored with same color), the total no. of such order color pairs is equal to $k \times k = k^2$.

Since each pair of letters must be colored with different color pairs, at least 26 color pairs are required to do this.

Therefore the requirement is $k^2 \geq 26$.

The minimum value of k that satisfies this equation is k = 6.

3.16 (a)

We want every one of the 5 balls to be in the wrong box. This is nothing but the number of derangements of a set of 5 elements = D_5.

i.e. we need to compute D_5

$$D_n = \sum_{r=2}^{n}(-1)^r \frac{n!}{r!}$$

$$D_5 = \sum_{r=2}^{5}(-1)^r \frac{5!}{r!} = \frac{5!}{2!} - \frac{5!}{3!} + \frac{5!}{4!} - \frac{5!}{5!}$$

= 60 − 20 + 5 − 1 = 44 ways.

3.17 (d)

The number of numbers from 1 to n, which are relatively prime to n i.e., gcd (m, n) = 1, is given by the Euler Totient function $\phi(n)$. If n is broken down into its prime factors as $n = p_1^{n_1} \cdot p_2^{n_2} \ldots$ where p_1, p_2 etc. are distinct prime numbers, then

$\phi(n) = \phi(p_1^{n_1})\, \phi(p_2^{n_2}) \ldots$

here, $n = p^2 q$,

so, $\phi(n) = \phi(p^2) \times \phi(q)$ . . . (*i*)

now, using the property $\phi(p^k) = p^k - p^{k-1}$,

$\phi(p^2) = p^2 - p^1 = p^2 - p$ and

$\phi(q) = q^1 - q^0 = q - 1$

Substituting these in eq. (*i*), we get

$\phi(n) = (p^2 - p)\,(q - 1) = p(p - 1)(q - 1)$

3.18 (c)

The number of combinations of pairs (a mod 3, b mod 5) is 3 × 5 = 15 (since a mod 3 can be 0, 1, or 2) and b mod 5 can be 0, 1, 2, 3 or 4)

∴ If 16 different ordered pairs are chosen at least 2 of them must have (a mod 3, b mod 5) as same (basic pigeon hole principle).

Let such two pairs be (a, b) and (c, d) then

a mod 3 ≡ c mod 3 ⇒ a ≡ c mod 3,

and b mod 5 ≡ d mod 5 ⇒ b ≡ d mod 5

3.19 (b)

$$\frac{1}{(1-x)^n} = \sum_{r=0}^{\infty} {}^{(n-1+r)}C_r\, x^r$$

Put n = 2,

$$\frac{1}{(1-x)^2} = \sum_{r=0}^{\infty} {}^{(2-1+r)}C_r\, x^r$$

$$= \sum_{r=0}^{\infty} {}^{(r+1)}C_r\, x^r$$

$$= \sum_{r=0}^{\infty} {}^{(r+1)}C_1\, x^r$$

$$= \sum_{r=0}^{\infty} (r+1)\, x^r$$

$$= \sum_{i=0}^{\infty} (i+1)\, x^i$$

(Since r is a dummy variable, r can be replaced by i)

$$= \sum_{r=0}^{\infty} g(i)\, x^i$$

$\therefore \quad g(i) = i + 1$

3.20 (a)

The probability that exactly n elements are chosen = The probability of getting n heads out of 2n tosses

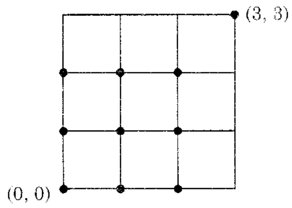
$= {}^{2n}C_n\,(1/2)^n\,(1/2)^{2n-n}$ (*Binomial formula*)

$= {}^{2n}C_n\,(1/2)^n\,(1/2)^n$

$$= \frac{{}^{2n}C_n}{2^{2n}} = \frac{{}^{2n}C_n}{(2^2)^n} = \frac{{}^{2n}C_n}{4^n}$$

3.21 (a)

Consider the following diagram.



The robot can move only right or up as defined in problem.

Let us denote right move by 'R' and up move by 'U'. Now to reach (3, 3) from (0, 0), the robot has to make exactly 3 'R' moves and 3'U' moves in any order.

Similarly to reach (10, 10) from (0, 0), the robot has to make 10 'R' moves and 10 'U' moves in any order. The number of ways this can be done is same as number of per mutations of a word consisting of 10 'R's and 10 'U's.

Applying formula of permutation with limited repetitions we get the answer as $\frac{20!}{10!\,10!} = {}^{20}C_{10}$.

3.22 (d)

The robot can reach (4, 4) from (0, 0) in ${}^{8}C_4$ ways as argued in previous problem. Now after reaching (4, 4), robot is not allowed to go to (5, 4). Let us count how many paths are there from (0, 0) to (10, 10) if robot goes from (4, 4) to (5, 4) and then we can subtract this from total number of ways to get the answer.

Now there are $8C_4$ ways for robot to reach (4, 4) from (0, 0) and then robot takes the 'U' move from (4, 4) to (5, 4). Now from (5, 4) to (10, 10) the robot has to make 5 'U' moves and 6 'R' moves in any order which can be done in $\frac{11!}{5!\,6!}$ ways $= {}^{11}C_5$ ways.

∴ The number of ways robot can move from (0, 0) (10, 10) via (4, 4) – (5, 4) move is

$${}^{8}C_4 \times {}^{11}C_5 = \binom{8}{4} \times \binom{11}{5}$$

∴ Number of ways robot can move from (0, 0) to (10, 10) without using (4, 4) to (5, 4) move is

$$\binom{20}{10} - \binom{8}{4} \times \binom{11}{5} \text{ ways.}$$

which is option (d).

3.23 (a)

$P = \Sigma\, i = 1 + 3 + 5 + 7 + \ldots + (2k - 1)$

$1 \le i \le 2k$, i is odd

$Q = \Sigma\, i = 2 + 4 + 6 + \ldots 2k,\ 1 \le i \le 2k$, i is even.

P is in A.P with a = 1, d = 2 and n = k

Q is in A.P with a = 2, d = 2 and n = k

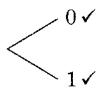
$$P = \frac{n}{2}(2a + (n-1)d)$$
$$= \frac{k}{2}(2 \times 1 + (k-1) \times 2)$$
$$= \frac{k}{2}(2 + 2k - 2) = k^2$$
$$Q = \frac{n}{2}(2a + (n-1)d)$$
$$= \frac{k}{2}(2 \times 2 + (k-1) \times 2)$$
$$= \frac{k}{2}(4 + 2k - 2)$$
$$= \frac{k}{2}(2k + 2)$$
$$= k^2 + k$$

Clearly, $P = Q - k$.

3.24 (d)

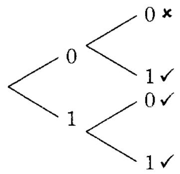
The ✓ represents those strings with no consecutive 0's.

Let $n = 1$



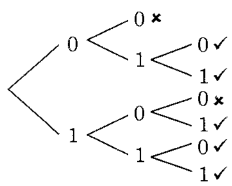
$x_1 = 2$

Let $n = 2$



$x_2 = 3$

Let $n = 3$



$x_3 = 5$

Now, substituting n = 3 in all of the answers only choice (d) $x_n = x_{n-1} + x_{n-2}$ satisfies the numbers obtained from the tree counting.

3.25 (d)

$x_1 = 2, x_2 = 3$

$\Rightarrow \quad x_3 = x_1 + x_2 = 2 + 3 = 5$

$\Rightarrow \quad x_4 = x_2 + x_3 = 3 + 5 = 8$

$\Rightarrow \quad x_5 = x_3 + x_4 = 5 + 8 = 13$

So option (d) is correct.

3.27 (b)

300! is $1 \times 2 \times 3 \times ... \times 300$

$$\left\lfloor \frac{300}{11} \right\rfloor = 27$$

So there are 27 multiples of 11 from 1 to 300, so they will include 11 as a prime factor atleast once.

Only 121 will contain an extra 11, all other will contain 11 as a factor only once.

So total number of 11's = 27 + 1 = 28.

So exponent of 11 is 28.

3.27 (a)

r red balls can be distributed into n distinct boxes in C(n–1+r, r) = (n–1+r)!/(n–1)! r!

b blue balls can be distributed in C(n–1 + b, b) = (n – 1 + b)! / (n – 1)! b!

Since they are independent, by product rule, total ways = (n–1 + b)! (n–1 + r)! /(n – 1)! b! (n – 1)! r!.

3.28 Sol.

In a 10-pennant let there we x_1 ones and x_2 twos. So we need to find all the solutions of

$$x_1 + 2x_2 = 10$$

Put $\quad x_1 = 0 \Rightarrow x_2 = 10/2 = 5$

So, (0, 5) is a solution i.e. a 10-pennant could have 0 ones and 5 twos. The number of ordered permutations of 0 ones and 5 twos

$$= 5! / 5! = 1$$

Now x1 cannot be 1 since in that case $x_2 = 9/2 = 4.5$ (is not an integer).

Put $\quad x_1 = 2 \Rightarrow x_2 = 8/2 = 4$

So, (2, 4) is a solution i.e. a 10-pennant could have 2 ones and 4 twos. The number of ordered permutations of 2 ones and 4 twos

$$= \frac{6!}{2!\,4!} = 15$$

Similarly (4, 3), (6, 2), (8, 1) and (10,0) are the other four solutions and the number of pennants for each is respectively

$$\frac{7!}{4!\,3!} = 35,\ \frac{8!}{6!\,2!} = 28,\ \frac{9!}{8!\,1!} = 9,\ \frac{10!}{10!\,0!} = 1$$

So the total number of 10-pennants

= 1 + 15 + 35 + 28 + 9 + 1 = 89

3.29 Sol.

"The quick brown fox jumps over the lazy dog"
(3) (5) (5) (3) (5) (4) (3) (4) (3)

Now let x be the number of letters in the word that is randomly picked.

Now, we make a probability distribution table for x

x	3	4	5
$p(x)$	4/9	2/9	3/9

From this table we can easily find the expected value of x.

$$E(x) = \sum x\,p(x) = 3 \times \frac{4}{9} + 4 \times \frac{2}{9} + 5 \times \frac{3}{9}$$

$= \frac{35}{9} = 3.88 = 3.9$ (after rounding to one decimal accuracy).

3.30 Sol.

Factorizing 2014 in primes by successively dividing by primes, we get

$$2014 = 2^1 \times 19^1 \times 53^1$$

Now we use the **formula**:

If $n = p_1^{n_1} \cdot p_2^{n_2} \cdots p_r^{n_r}$ is the prime factorization of n, then the number of distint factors of n is given by $(n_1+1) \times (n_2+1) \ldots (n_r+1)$.

Now since $2014 = 2^1 \times 19^1 \times 53^1$, the number of distint factors of 2014 = (1+1) (1+1) (1+1)

$$= 2 \times 2 \times 2 = 8$$

3.31 Sol.

$$\sum_{x=1}^{99} \frac{1}{x(x+1)} = \sum_{x=1}^{99} \left[\frac{1}{x} - \frac{1}{x+1}\right]$$

$$= \left[\frac{1}{1} - \frac{1}{2}\right] + \left[\frac{1}{2} - \frac{1}{3}\right] + \ldots$$

$$+ \left[\frac{1}{97} - \frac{1}{98}\right] + \left[\frac{1}{98} - \frac{1}{99}\right] + \left[\frac{1}{99} - \frac{1}{100}\right]$$

$= 1 - \frac{1}{100}$ (all the terms in above series except the first and last terms cancels out)

$$= \frac{99}{100} = 0.99$$

3.32 (a)

$a_1 = 0$ [no strings of length 1 contain two consecutive 1's]

$a_2 = 1$ [$\therefore$ strings = 11]

$a_3 = 3$ [$\therefore$ strings are : 011, 110, 111]

$a_4 = 8$ [$\therefore$ strings are : 0011, 0110, 0111, 1011, 1100, 1101, 1110, 1111]

Option (a):

$a_n = a_{n-2} + a_{n-1} + 2^{n-2}$

$\Rightarrow a_4 = a_{4-2} + a_{4-1} + 2^{4-2}$

$= a_2 + a_3 + 2^2 = 1 + 3 + 4 = 8$ which is True.

Option (b):

$a_n = a_{n-2} + 2a_{n-1} + 2^{n-2}$

$\Rightarrow a_4 = a_2 + 2a_3 + 2^2$

$= 1 + 2 \times 3 + 4 = 11$ which is False.

Option (c):

$a_n = 2a_{n-2} + a_{n-1} + 2^{n-2}$

$\Rightarrow a_4 = 2a_2 + a_3 + 2^2$

$= 2 \times 1 + 3 + 4 = 9$ which is False.

Option (d):

$a_n = 2a_{n-2} + 2a_{n-1} + 2^{n-2}$

$\Rightarrow a_4 = 2a_2 + 2a_3 + 2^2$

$= 2 \times 1 + 2 \times 3 + 4 = 12$ which is False.

$\therefore$ Option (a): $a_n = a_{n-2} + a_{n-1} + 2^{n-2}$ is correct.

3.33 Sol.

Dividing 2100 successively by prime numbers, we get that, the prime factors of 2100 are $2 \times 2 \times 3 \times 5 \times 5 \times 7$

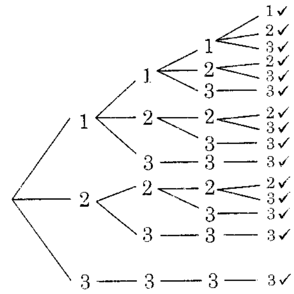
$\Rightarrow 2^2 \times 3^1 \times 5^2 \times 7^1$

If $N = p^{k_1} \times q^{k_2} \times r^{k_3} \ldots \times y^{k_n}$ then the number of factors of N are $(k_1+1)(k_2+1) \ldots (k_n+1)$

$\therefore$ For 2100 we have $(2+1)(1+1)(2+1)(1+1) = 36$ factors.

3.34 Sol.

To satisfy the non-decreasing order condition allow 1, 2, 3 after 1, allow only 2, 3 after 2 and allow only 3 after 3. The following tree diagram gives all the allowed numbers satisfying the given condition.



All the tick marked numbers satisfy the non-decreasing order condition.

$\Rightarrow$ Number of 4-digit numbers = Number of tick marks = 15.

3.35 (b)

Let a_n be the number of n-bit strings that do not contain two consecutive 1's. we wish to develop a recurrence relation for a_n.

Consider 1 bit strings 0, 1

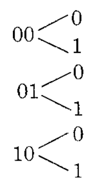
So $a_1 = 2$

Consider 2 bit strings 00, 01, 10, 11

Out of minimum only 00, 01, 10 do not contain two consecutive 1's.

So $a_2 = 3$

Consider 3 bit strings



Out of minimum six strings only 000, 001, 010, 100 and 101 five strings satisfy do not contain two consecutive 1's.

So $a_3 = 5$. Three numbers a_1, a_2, a_3 satisfy clearly only (b) $a_n = a_{n-1} + a_{n-2}$ is correct.

3.36 Sol.

We wish to find coefficient of x^{12} in $(x^3 + x^4 + x^5 + \ldots)^3$

$$= (x^3(1 + x^1 + x^2 + \ldots))^3$$

$$= x^9(1 + x + x^2 \ldots\ldots)^3$$

$$= \frac{x^9}{(1-x)^3} = x^9 \sum_{r=0}^{\infty} {}^{3-1+r}C_r X^r$$

$$= x^9 \sum_{r=0}^{\infty} {}^{r+2}C_r X^r$$

Now to make x^{12} we need to put $r = 3$

So coefficient of x^{12} is ${}^{3+2}C_3 = {}^5C_3 = {}^5C_2 = 10$

3.37 Sol.

Given $a_n = 6n^2 + 2n + a_{n-1}$ and $a_1 = 8$

We wish to find a_{99}

Now

$$a_2 = 6 \times 2^2 + 2 \times 2 + a_1$$

$$a_3 = 6 \times 3^2 + 2 \times 3 + a_2$$

$$= 6 \times 3^2 + 2 \times 3 + 6 \times 2^2 + 2 \times 2 + a_1 \ldots$$

$$a_{99} = 6 \times 99^2 + 2 \times 99 + 6 \times 98^2 + 2 \times 98 \ldots$$
$$\ldots + 6 \times 2^2 + 2 \times 2 + a_1$$

Since $a_1 = 8$

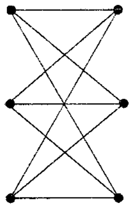
$$a_{99} = 6 \times 99^2 + 2 \times 99 + 6 \times 98^2 + 2 \times 98 \ldots$$
$$\ldots + 6 \times 2^2 + 2 \times 2 + 8$$
$$= 6 \times 99^2 + 2 \times 99 + 6 \times 98^2 + 2 \times 98 \ldots$$
$$\ldots 6 \times 2^2 + 2 \times 2 + 6 \times 1^2 + 2 \times 1$$
$$= 6(1^2 + 2^2 + 3^2 \ldots 99^2) + 2.(1 + 2 + 3 \ldots 99)$$
$$= 6 \cdot \frac{(99(99+1)(2 \times 99 + 1))}{6} + 2\left(\frac{99(99+1)}{2}\right)$$
$$= 99 \times 100 \times 199 + 99 \times 100$$
$$= 100 \times 99\ (199 + 1)$$
$$= 100 \times 99 \times 200$$
$$= 2 \times 99 \times 10^4$$
$$= 198 \times 10^4$$

So if $a_{99} = K \times 10^4$ then $K = 198$.

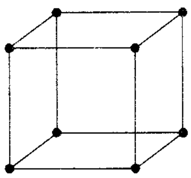
■■■■

4 Graph Theory

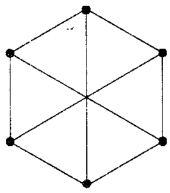
4.1 Which of the following graphs is/are planar? (see Figure)



G1



G2



G3

[1989 : 2 Marks]

(a) G1 only
(b) G1 and G2
(c) G2 only
(d) G2 and G3

4.2 A graph is planar if and only if,
(a) if does not contain subgraphs homeomorphic to K_5 and $K_{3,3}$.
(b) it does not contain subgraphs isomorphic to K_5 or $K_{3,3}$.
(c) it does not contain subgraphs isomorphic to K_5 and $K_{3,3}$.
(d) it does not contain subgraph homeomorphic to K_5 or $K_{3,3}$.

[1990 : 2 Marks]

4.3 The maximum number of possible edges in an undirected graph with 'a' vertices and 'k' components is _______.

[1991 : 2 Marks]

4.4 A non-planar graph with minimum number of vertices has
(a) 9 edges, 6 vertices
(b) 6 edges, 4 vertices
(c) 10 edges, 5 vertices
(d) 9 edges, 5 vertices

[1992 : 1 Mark]

4.5 Maximum number of edges in a planar graph with n vertices is ______

[1992 : 2 Marks]

4.6 The number of distinct simple graphs with upto three nodes is
(a) 15
(b) 10
(c) 7
(d) 9

[1994 : 1 Mark]

4.7 Prove that in a finite graph, the number of vertices of odd degree is always even.

[1995 : 2 Marks]

4.8 Let G be an undirected connected graph with distinct edge weights. Let e_{max} be the edge with maximum weight and e_{min} be the edge with minimum weight. Which of the following statements is false?
(a) Every minimum spanning tree of G must contain e_{min}
(b) If e_{max} is in a minimum spanning tree, then its removal must disconnect G
(c) No minimum spanning tree contains e_{max}
(d) G has a unique minimum spanning tree

[2000 : 2 Marks]

4.9 How many undirected graphs (not necessary connected) can be constructed out of a given set $V = \{v_1, v_2, ... v_n\}$ of n vertices?
(a) $\frac{n(n-1)}{2}$
(b) 2^n
(c) $n!$
(d) $2^{\frac{n(n-1)}{2}}$

[2001 : 2 Marks]

4.10 Maximum number of edges in a n-node undirected graph without self loops is
(a) n^2
(b) $n(n-1)/2$
(c) $n-1$
(d) $(n+1)(n)/2$

[2002 : 1 Mark]

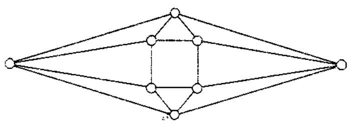
4.11 Let G be an arbitrary graph with n nodes and k components. If a vertex is removed from G, the number of components in the resultant graph must necessarily lie between.
(a) k and n
(b) $k-1$ and $k+1$
(c) $k-1$ and $n-1$
(d) $k+1$ and $n-k$

[2003 : 1 Mark]

4.12 How many perfect matchings are there in a complete graph of 6 vertices?
(a) 15 (b) 24
(c) 30 (d) 60
[2003 : 2 Marks]

4.13 A Graph G = (V, E) satisfies $|E| \leq 3\ |V| - 6$. The min-degree of G is defined as $\min_{v \in V}$ {degree (v)}. Therefore, min-degree of G cannot be
(a) 3 (b) 4
(c) 5 (d) 6
[2003 : 2 Marks]

4.14 The minimum number of colours required to colour the following graph, such that no two adjacent vertices are assigned the same colour, is



(a) 2 (b) 3
(c) 4 (d) 5
[2004 : 2 Marks]

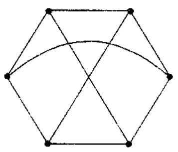
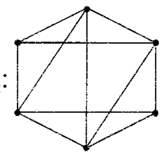
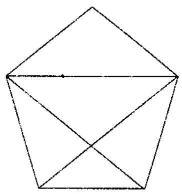
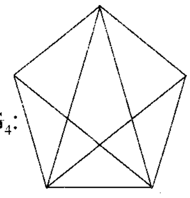
4.15 How many graphs on n labeled vertices exist which have at least $(n^2 - 3n)/2$ edges?
(a) ${}^{(n^2-n)/2}C_{(n^2-3n)/2}$ (b) $\sum_{k=0}^{(n^2-3n)/2} {}^{(n^2-n)}C_k$
(c) ${}^{(n^2-n)/2}C_n$ (d) $\sum_{k=0}^{n} {}^{\frac{(n^2-n)}{2}}C_k$
[2004 : 2 Marks]

4.16 What is the maximum number of edges in an acyclic undirected graph with n vertices?
(a) n − 1 (b) n
(c) n + 1 (d) 2n − 1
[IT-2004 : 1 Mark]

4.17 What is the number of vertices in an undirected connected graph with 27 edges, 6 vertices of degree 2, 3 vertices of degree 4 and remaining of degree 3?
(a) 10 (b) 11
(c) 18 (d) 19
[IT-2004 : 2 Marks]

4.18 Let G be a simple connected planar graph with 13 vertices and 19 edges. Then, the number of faces in the planar embedding of the graph is
(a) 0 (b) 8
(c) 9 (d) 13
[2005 : 1 Mark]

4.19 Which one of the following graphs is NOT plannar?

(a) G_1: 
(b) G_2: 
(c) G_3: 
(d) G_4: 

[2005 : 2 Marks]

4.20 If all the edge weights of an undirected graph are positive, then any subset of edges that connects all the vertices and has minimum total weight is a
(a) Hamiltonian cycle (b) grid
(c) hypercube (d) tree
[IT-2006 : 1 Mark]

4.21 Consider the undirected graph G defined as follows. The vertices of G are bit strings of length *n*. We have an edge between vertex *u* and vertex *v* if and only if *u* and *v* differ in exactly one bit position (in other words, *v* can be obtained from u by flipping a single bit). The ratio of the chromatic number of G to the diameter of G is
(a) $1/(2^{n-1})$ (b) 1/n
(c) 2/n (d) 3/n
[IT-2006 : 2 Marks]

Common Data for Q.4.22 & Q.4.24

The 2^n vertices of graph G correspond to all subsets of a set of size n, for $n \geq 6$. Two vertices of G are adjacent if and only if the corresponding sets intersect in exactly two elements.

4.22 The number of vertices of degree zero in G is
(a) 1 (b) n
(c) n + 1 (d) 2^n
[2006 : 2 Marks]

4.23 The maximum degree of a vertex in G is

(a) $\binom{n/2}{2} 2^{n/2}$ (b) 2^{n-2}

(c) $2^{n-3} \times 3$ (d) 2^{n-1}

[2006 : 2 Marks]

4.24 The number of connected components in G is

(a) n (b) n + 2

(c) $2^{n/2}$ (d) $2^n / n$

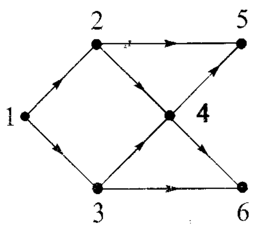
[2006 : 2 Marks]

4.25 Let G be the non-planar graph with the minimum possible number of edges. Then G has

(a) 9 edges and 5 vertices
(b) 9 edges and 6 vertices
(c) 10 edges and 5 vertices
(d) 10 edges and 6 vertices

[2007 : 1 Mark]

4.26 Consider the DAG with V = {1, 2, 3, 4, 5, 6}, shown below



Which of the following is NOT a topological ordering?

(a) 1 2 3 4 5 6 (b) 1 3 2 4 5 6

(c) 1 3 2 4 5 6 (d) 3 2 4 1 6 5

[2007 : 1 Mark]

4.27 Which of the following graphs has an Eulerian circuit?

(a) Any k-regular graph where k is an even number
(b) A complete graph on 90 vertices
(c) The complement of a cycle on 25 vertices
(d) None of the above

[2007 : 2 Marks]

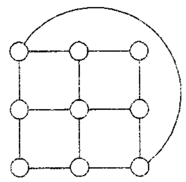
4.28 What is the size of the smallest MIS (Maximal Independent Set) of a chain of nine nodes?

(a) 5 (b) 4

(c) 3 (d) 2

[IT-2008 : 1 Mark]

4.29 What is the chromatic number of the following graph?



(a) 2 (b) 3

(c) 4 (d) 5

[IT-2008 : 1 Mark]

4.30 G is a simple, connected, undirected graph. Some vertices of G are of odd degree. Add a node v to G and make it adjacent to each odd degree vertex of G. The resultant graph is sure to be

(a) Regular (b) Complete

(c) Hamiltonian (d) Euler

[IT-2008 : 2 Marks]

4.31 What is the chromatic number of an n-vertex simple connected graph which does not contain any odd length cycle? Assume $n \geq 2$.

(a) 2 (b) 3

(c) n – 1 (d) n

[2009 : 1 Mark]

4.32 Which one of the following is TRUE for any simple connected undirected graph with more than 2 vertices?

(a) No two vertices have the same degree
(b) At least two vertices have the same degree
(c) At least three vertices have the same degree
(d) All vertices have the same degree

[2009 : 1 Mark]

4.33 Let G = (V, E) be a graph. Define $\xi(G) = \sum_d i_d \times d$, where i_d is the number of vertices of degree d in G. If S and T are two different trees with $\xi(S) = \xi(T)$, then

(a) $|S| = 2|T|$ (b) $|S| = |T| - 1$

(c) $|S| = |T|$ (d) $|S| = |T| + 1$

[2010 : 1 Mark]

4.34 In a binary tree with n nodes, every node has an odd number of descendants. Every node is considered to be its own descendant. What is the number of nodes in the tree that have exactly one child?

(a) 0 (b) 1

(c) (n – 1)/2 (d) n – 1

[2010 : 1 Mark]

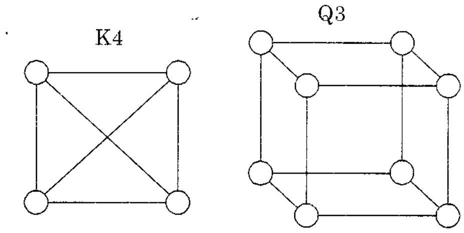
4.35 The degree sequence of a simple graph is the sequence of the degrees of the nodes in the graph in decreasing order. Which of the following sequences can not be the degree sequence of any graph?

I. 7, 6, 5, 4, 4, 3, 2, 1
II. 6, 6, 6, 6, 3, 3, 2, 2
III. 7, 6, 6, 4, 4, 3, 2, 2
IV. 8, 7, 7, 6, 4, 2, 1, 1

(a) I and II (b) III and IV
(c) IV only (d) II and IV

[2010 : 2 Marks]

4.36 K4 and Q3 are graphs with the following structures:



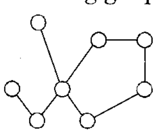
Which one of the following statements is TRUE in relation to these graphs?
(a) K4 is planar while Q3 is not
(b) Both K4 and Q3 are planar
(c) Q3 is planar while K3 is not
(d) Neither K4 nor Q3 is planar

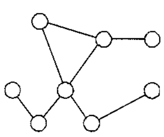
[2010 : 2 Marks]

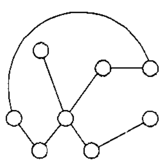
4.37 Let G be a simple undirected planar graph on 10 vertices with 15 edges. If G is a connected graph, then the number of bounded faces in any embedding of G on the plane is equal to
(a) 3 (b) 4
(c) 5 (d) 6

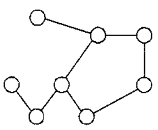
[2012 : 1 Mark]

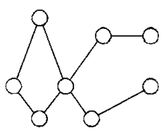
4.38 Which of the following graphs is isomorphic to



(a)


(b)


(c)


(d)


[2012 : 2 Marks]

4.39 Let G be a complete undirected graph on 6 vertices. If vertices of G are labeled, then the number of distinct cycles of length 4 in G is equal to
(a) 15 (b) 30
(c) 90 (d) 360

[2012 : 2 Marks]

4.40 Consider an undirected random graph of eight vertices. The probability that there is an edge between a pair of vertices is 1/2. What is the expected numbers of unordered cycles of length three?
(a) 1/8 (b) 1
(c) 7 (d) 8

[2013 : 1 Mark]

4.41 Which of the following statements is/are TRUE for undirected graph?
P: Number of odd degree vertices is even.
Q: Sum of degrees of all vertices is even.
(a) P only (b) Q only
(c) Both P and Q (d) Neither P nor Q

[2013 : 1 Mark]

4.42 Let $G = (V, E)$ be a directed graph where V is the set of vertices and E the set of edges. Then which one of the following graphs has the same strongly connected components as G?
(a) $G_1 = (V, E_1)$ where $E_1 = \{(u,v) \mid (u,v) \notin E\}$
(b) $G_2 = (V, E_2)$ where $E_2 = \{(u,v) \mid (v,u) \in E\}$
(c) $G_3 = (V, E_3)$ where $E_3 = \{(u,v) \mid$ there is a path of length ≤ 2 from u to v in E$\}$
(d) $G_4 = (V_4, E)$ where V_4 is the set of vertices in G which are not isolated

[2014 (Set-1) : 1 Mark]

4.43 Consider an undirected graph G where self-loops are not allowed. The vertex set of G is $\{(i, j) : 1 \leq i \leq 12, 1 \leq j \leq 12)$. There is an edge between (a, b) and (c, d) if $|a - c| \leq 1$ and $|b - d| \leq 1$.

The number of edges in this graph is____.

[2014 (Set-1) : 2 Marks]

4.44 An ordered n-tuple $(d_1, d_2, \ldots, d_n)$ with $d_1 \geq d_2 \geq \ldots \geq d_n$ is called *graphic* if there exists a simple undirected graph with n vertices having degrees $d_1, d_2, \ldots, d_n$ respectively. Which of the following 6-tuples is NOT graphic?

(a) (1, 1, 1, 1, 1, 1) (b) (2, 2, 2, 2, 2, 2)
(c) (3, 3, 3, 1, 0, 0) (c) (3, 2, 1, 1, 1, 0)

[2014 (Set-1) : 2 Marks]

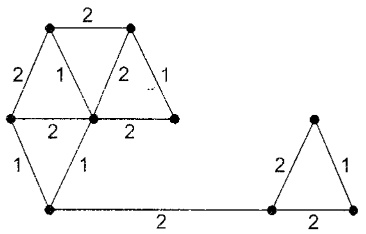
4.45 The maximum number of edges in a bipartite graph on 12 vertices is ______.

[2014 (Set-2) : 1 Mark]

4.46 A cycle on n vertices is isomorphic to its complement. The value of n is ______.

[2014 (Set-2) : 2 Marks]

4.47 The number of distinct minimum spanning trees for the weighted graph below is ______



[2014 (Set-2) : 2 Marks]

4.48 If G is a forest with n-vertices and k connected components, how many edges does G have?

(a) $\lfloor n/k \rfloor$ (b) $\lceil n/k \rceil$
(c) $n - k$ (d) $n - k + 1$

[2014 (Set-3) : 2 Marks]

4.49 Let δ denote the minimum degree of a vertex in a graph. For all planar graphs on n vertices with $\delta \geq 3$ which one of the following is **TRUE**?

(a) In any planar embedding, the number of faces is at least $\frac{n}{2}+2$

(b) In any planar embedding, the number of faces is less than $\frac{n}{2}+2$

(c) There is a planar embedding in which the number of faces is less than $\frac{n}{2}+2$

(d) There is a planar embedding in which the number of faces is at most $\frac{n}{\delta+1}$

[2014 (Set-3) : 2 Marks]

4.50 Let G be a connected planar graph with 10 vertices. If the number of edges on each face is three, then the number of edges in G is ______.

[2015 (Set-1) : 2 Marks]

4.51 A graph is self-complementary if it is isomorphic to its complement. For all self-complementary graphs on n vertices, n is

(a) A multiple of 4
(b) Even
(c) Odd
(d) Congruent to 0 mod 4, or 1 mod 4

[2015 (Set-2) : 2 Marks]

4.52 In a connected graph, a bridge is an edge whose removal disconnects a graph. Which one of the following statements is True?

(a) A tree has no bridges
(b) A bridge cannot be part of a simple cycle
(c) Every edge of a clique with size ≥ 3 is a bridge (A clique is any complete subgraph of a graph)
(d) A graph with bridges cannot have a cycle

[2015 (Set-2) : 2 Marks]

4.53 Consider a binary tree T that has 200 leaf nodes. Then, the number of nodes in T that have exactly two children are ______.

[2015 (Set-3) : 1 Mark]

4.54 Let G be connected undirected graph of 100 vertices and 300 edges. The weight of a minimum spanning tree of G is 500. When the weight of each edge of G is increased by five, the weight of a minimum spanning tree becomes ______.

[2015 (Set-3) : 2 Marks]

4.55 The minimum number of colours that is sufficient to vertex-colour any planar graph is ______.

[2016 (Set-2) : 1 Mark]

■■■■

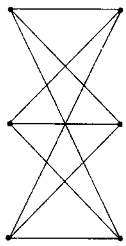
Answers Graph Theory

4.1	(c)	4.2	(d)	4.4	(c)	4.6	(c)	4.8	(c)	4.9	(d)	4.10	(b)	4.11	(c)	4.12	(a)
4.13	(d)	4.14	(c)	4.15	(d)	4.16	(a)	4.17	(d)	4.18	(b)	4.19	(a)	4.20	(d)	4.21	(c)
4.22	(c)	4.23	(c)	4.24	(b)	4.25	(b)	4.26	(d)	4.27	(c)	4.28	(b)	4.29	(b)	4.30	(d)
4.31	(a)	4.32	(b)	4.33	(c)	4.34	(a)	4.35	(d)	4.36	(b)	4.37	(d)	4.38	(b)	4.39	(d)
4.40	(c)	4.41	(c)	4.42	(b)	4.44	(c)	4.48	(c)	4.49	(a)	4.51	(d)	4.52	(b)		

Explanations Graph Theory

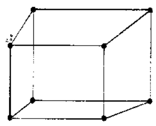
4.1 (c)

G_1:

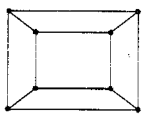


G_1 is $k_{3,3}$ which is a well-known non-planar graph.

G_2:

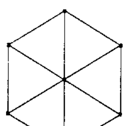


Graph G_2 is isomorphic to the following graph:



The above graph is planar. So G_2 is planar.

G_3:



G_3 is isomorphic to $K_{3,3}$ which is well known non-planar graph. Therefore, G_3 is a non-planar graph.

4.2 (d)

Kuratowski's theorem: A graph is planar if and only if, it does not contain subgraph homeomorphic to K_5 or $K_{3,3}$.

4.3 Sol.

The maximum number of possible edges in an undirected graph with "a" vertices and "k" components is $\frac{(a-k+1)(a-k)}{2}$.

4.4 (c)

K_5 is smallest non-planar graph in terms of number of vertices.

The number of vertices in K_5 is 5 and number of edges in K_5 is $\frac{5 \times 4}{2} = 10$.

4.5 Sol.

Maximum number of edges in a connected, planar, simple graph with n vertices is $3n - 6$.

4.6 (c)

Assume that the vertices are unlabelled. Number of distinct simple graphs with 1 node = 1

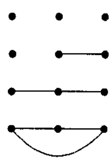
(i) •

Number of distinct simple graphs with 2 nodes = 2

(i) • • (No edges)

(ii) •—• (1 edge)

Number of distinct simple graphs with 3 nodes = 4



(i) (No edges)
(ii) (1 edge)
(iii) (2 edges)
(iv) (3 edges)

Therefore, total number of distinct simple graphs upto three nodes = 1 + 2 + 4 = 7.

4.7 Sol.

According to Handshaking theorem, for a graph G = (V, E) with n vertices and e edges then

$$\sum_{v \in V} \deg(v) = 2e$$

Let V_e and V_0 respectively the set of vertices of even degree and the set of vertices of odd degree in an undirected graph G = (V, E) then

$$\sum_{v \in V} \deg(v) = \sum_{v \in V_e} \deg(v) + \sum_{v \in V_o} \deg(v) = 2e$$

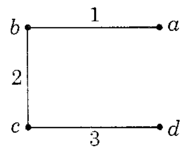
Since deg (v) is even for $v \in V_e$, the first sum in the right hand side of the equality is even. The total sum must be 2e, which is even, so the second sum must be even too. But all of its terms are odd, so there must be an even number of them.

4.8 (c)

(a) All the edge weights are distinct, so every minimum spanning tree of G must contain $e_{min.}$ (kruskal's algorithm will pick e_{min} first)

(b) If e_{max} is in a minimum spanning tree, then surely its removal must disconnect G (i.e. e_{max} must be a cut edge).

(c) A minimum spanning tree can contain e_{max} if its removal disconnect G (i.e. it is a cut edge). So this option is false.
For *example*, in the graph given below, the edge *cd* is a maximum weight edge but still is present in the minimum spanning tree.



(d) Since all edge weights are distinct G has a unique minimum spanning tree.

4.9 (d)

In a graph G with n vertices, maximum number of edges possible $= \dfrac{n(n-1)}{2}$.

There are two ways for a edge, (the edge may appear in graph or may absent in graph). So there are two options for each edge.

Total number of graphs with n vertices $= 2^{\frac{n(n-1)}{2}}$.

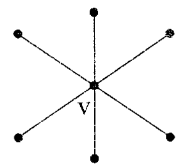
4.10 (b)

The graph containing maximum number of edges in a n-node undirected graph without self loops is complete graph.

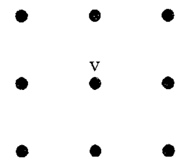
The number of edges in complete graph with n-node, k_n is $\dfrac{n(n-1)}{2}$.

4.11 (c)

Maximum components will result after removal of a node if graph G is a star graph as shown below.



or a null graph of n vertices as shown below:



In either case, if node v is removed, the number of components will be n – 1, where n is the total number of nodes in the star graph.

∴ n – 1 is the maximum number of components possible. Minimum components will result if the node being removed is a lone vertex in which case, the number of components will be k – 1.

∴ The number of components must necessarily lie between k – 1 and n – 1.

4.12 (a)

The number of perfect matchings in a complete graph of n vertices, where n is even, reduces to the problem of finding unordered partitions of the vertex set of the type p(2n; 2, 2, 2, ... n times)

$$= \frac{(2n)!}{(2!)^n \, n!}$$

For n = 3, 2n = 6, i.e. complete graph K_6, we have

$$\text{Number of perfect matchings} = \frac{6!}{(2!)^3 \, 3!}$$

$$= \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 2 \times 2 \times 6} = 15$$

4.13 (d)

Given $\quad |E| \le 3|V| - 6 \quad$ ...(i)

Let δ be the minimum degree of the graph.

Now δ cannot exceed the average degree of the graph.

so, $\quad \delta \le \dfrac{2|E|}{|V|} \quad$ ...(ii)

Substitute eq. (i) in eq. (ii) and get

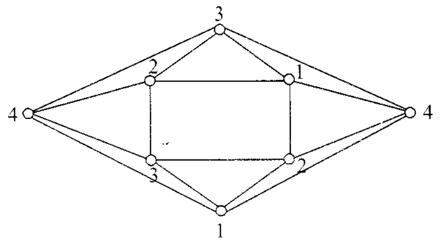
$$\delta \le \frac{2}{|V|}(3|V|-6)$$

$$\Rightarrow \qquad \delta \le 6 - \frac{12}{|V|}$$

Clearly the minimum degree has to be less than 6 always and hence cannot be equal to 6.

4.14 (c)

An assignment of the colors 1, 2, 3 and 4 to the vertices of the graph is shown below such that the graph is properly colored.



So 4 colours are required.

(***Note:*** The graph is a planar graph. The four colour theorem says that the chromatic number of a planar graph is at most = 4).

4.15 (d)

Take number of edges available in n labelled vertices is

$$nC_2 = \frac{n(n-1)}{2} = \frac{n^2-n}{2} \text{ edges}$$

Now from this we need to choose $\frac{n^2-3n}{2}$ edges or more upto a maximum of $\frac{n^2-n}{2}$ edges. Each such choice of edges represents a distinct graph on n labelled vertices.

Total number of such graphs

$$= {}^{\frac{n^2-n}{2}}C_{\frac{n^2-3n}{2}} + {}^{\frac{n^2-n}{2}}C_{\frac{n^2-3n}{2}+1} \ldots + {}^{\frac{n^2-n}{2}}C_{\frac{n^2-n}{2}}$$

Now since ${}^nC_r = {}^nC_{n-r}$

$${}^{\frac{n^2-n}{2}}C_{\frac{n^2-3n}{2}} = {}^{\frac{n^2-n}{2}}C_{\left[\frac{n^2-n}{2}\right]-\left[\frac{n^2-3n}{2}\right]}$$

$$= {}^{\frac{n^2-n}{2}}C_n$$

Similarly,

$${}^{\frac{n^2-n}{2}}C_{\left[\frac{n^2-3n}{2}+1\right]} = {}^{\frac{n^2-n}{2}}C_{n-1}$$

and so on until,

$${}^{\frac{n^2-n}{2}}C_{\frac{n^2-n}{2}} = {}^{\frac{n^2-n}{2}}C_0$$

So the required summation reduced to

$${}^{\frac{n^2-n}{2}}C_n + {}^{\frac{n^2-n}{2}}C_{n-1} + \ldots + {}^{\frac{n^2-n}{2}}C_0$$

$$= {}^{\frac{n^2-n}{2}}C_0 + {}^{\frac{n^2-n}{2}}C_1 + \ldots + {}^{\frac{n^2-n}{2}}C_n$$

$$= \sum_{k=0}^{n} {}^{\frac{n^2-n}{2}}C_k$$

which is choice (d).

4.16 (a)

Max number of edges in a connected graph is n – 1, which doesn't form any cycle.

4.17 (d)

Sum of degree of all the vertices = 2 × Number of edges

$2 \times 6 + 4 \times 3 + 3 \times x = 27 \times 2$

$x = 10.$

Number of vertices $= 6 + 3 + x = 19$

4.18 (b)

Given V = 13, E = 19

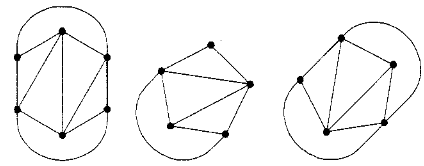
Let R be the number of regions.

$$R = E - V + 2$$

$$\Rightarrow \quad R = 19 - 13 + 2 = 8$$

4.19 (a)

G_1 is same as $K_{3,3}$ which is known to be non planar. G_2, G_3 and G_4 can be redrawn as follows so that they are planar.



G_2 G_3 G_4

4.20 (d)

If subset of edges that connects all the vertices and it has minimum total weight then it never forms a cycle. So it is called as tree.

4.21 (c)

For the given condition we can simply design a K-MAP and mark an edge between every two adjacent cells in K-Map (adjacency has to seen just as we do for minimization). That will give us a Bipartite graph. chromatic number for this = 2.

Also from the same we can conclude that we need ,for a 'n' bit string, to traverse NO MORE than (n – 1) edges or 'n' vertices to get a path between two arbitrary points.

So ratio is $2/n$.

Alternate Method

The hamming distance relation on bit strings has a digraph which will be always an n-cube where n is the number of bits.

Chromatic number of n-cube = 2 (since n-cube is always bipartite). Also the diameter of n-cube = n.

So the ratio of chromatic number to diameter of the n-cube = 2/n.

4.22 (c)

Let S contains n elements then S have 2^n subsets. Graph G contains 2^n vertices.

Let S = $\{v_1, v_2, ..., v_n\}$. Two vertices of G are adjacent if and only if the corresponding sets intersect in exactly two elements.

So $|\{V_i\} \cap \{V_j\}| = 2$

For this to happen, the subset must have at least 2 elements.

There are n sets which contains a single elements for V_1 to V_n who doesn't intersect another set such that it contains two elements. Therefore the degree of all these n vertices is zero. G also contains a vertex ϕ whose degree is zero. So the number of vertices whose degree is zero is n + 1.

4.23 (c)

Let the set be S = {1, 2, 3, 4, ..., n}

Consider a subset containing 2 elements of the form {1, 2}. Now {1, 2} will be adjacent to any subset with which it has exactly 2 elements in common. These sets can be formed by adding zero or more elements from remaining n – 2 elements, to the set {1, 2}. Since each of these elements may be either added or not added, number of ways of making such sets containing 1 and 2 is 2^{n-2}.

∴ Vertices with 2 elements will have 2^{n-2} degrees.

Now consider subsets of 3 elements say {1, 2, 3}. Since we want exactly 2 elements in common, we choose these in ${}^3C_2$ ways and then we can add or not add remaining n–3 elements. This can be done in 2^{n-3} ways.

∴ Total number of subsets with at least 2 elements common with {1, 2, 3} is given by ${}^3C_2 \times 2^{n-3}$.

Similarly, we can argue that the number of degrees of 4 element subsets is ${}^4C_2 \times 2^{n-4}$ and for 5 element subsets is ${}^5C_2 \times 2^{n-5}$ and so on.

Out of these $2^{n-2} = 2 \cdot 2^{n-3}$ is less than ${}^3C_2 \times 2^{n-3} = 3 \times 2^{n-3}$.

Then ${}^3C_2 \times 2^{n-3} = 3 \times 2^{n-3}$ is same as

${}^4C_2 \times 2^{n-4} = 6 \times 2^{n-4} = 3 \times 2^{n-3}$

and ${}^4C_2 \times 2^{n-4} = 3 \times 2^{n-3}$ is greater than

${}^5C_2 \times 2^{n-5} = 10 \times 2^{n-5} = 2.5 \times 2^{n-3}$

∴ maximum degree in this graph is occurring for 3 element and 4 element subsets both of which have $3 \times 2^{n-3}$ degree.

4.24 (b)

The number of connected component of G is determined by the degree and edges of vertices there are n + 1 vertices whose degree is zero, so they can form n + 1 connected component. The remaining vertices of the graph G are all connected as a single component. So total number of connected component is n + 2.

4.25 (b)

K_5 and $K_{3,3}$ are the smallest non planar graphs. K_5 has 5 vertices and ${}^5C_2 = 10$ edges and $K_{3,3}$ has 6 vertices and 3 × 3 = 9 edges. So, the non planar graph with minimum number of edges is $K_{3,3}$ with 9 edges and 6 vertices.

Note: K_5 is the non planar graph with minimum number of vertices.

4.26 (d)

In topological sorting the partial ordering of the DAG, must be preserved i.e. if a ≤ b in the DAG, then in the topological order, b must come after a, not before. Consider the ordering 3 2 4 1 6 5 . 1 ≤ 4 in the given DAG but 4 is coming before 1 in 3 2 4 1 6 5 order which means that 3 2 4 1 6 5 is not a topological order of the given DAG.

4.27 (c)

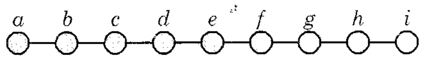
Whenever in a graph all vertices have even degrees, it will surely have an Euler circuit.

(a) Since in a k-regular graph, every vertex has exactly k degrees and if k is even, every vertex in the graph has even degrees. k-regular graph need not be connected, hence k-regular may not contain Euler circuit.

(b) Complete graph on 90 vertices not contains an Euler circuit, because every vertex degree is odd (89)

(c) C_{25} has 24 edges and each vertex has exactly 2 degrees. So every vertex in the complement of C_{25} will have $24 - 2 = 22$ degrees which is an even number.
Since every vertex in the complement of C_{25} has even degrees. Also, the complements of all C_n with $n \geq 5$ is connected. So, it is an Euler graph.

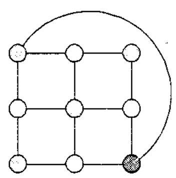
4.28 (b)



Maximum independent sets:
{a, c, e, g, i} and {b, d, f, h}.
Smallest maximum independent set
{b, d, f, h}.
Size = 4

4.29 (b)

Since there is a 5-cycle (odd cycle) in this graph, this graph is not bipartite and hence its chromatic number is atleast 3.
A proper colouring with 3 colours, is shown below:



Hence minimum number of colors needed to color given graph is equal to 3.

4.30 (d)

In any simple undirected graph, the number of vertices having odd degrees must be even.
Now Single vertex v is connected to all these even number of vertices (which have odd degrees).
So degree of v is also even. Moreover, now degree of all vertices which are connected to v is increased by 1, hence vertices which had odd degree earlier now have even degree.
So now, all vertices in graph have even degrees, and since the graph is connected, this graph meets the necessary and sufficient condition for Euler graph and hence is an Euler graph.

4.31 (a)

If a n-vertex simple connected graph contains no cycles of odd length, then its chromatic number is two, since the vertices can be alternately colored with the first color, then the second color, then the first color and then the second color and so on.
Alternatively, since a simple connected graph with no cycles of odd length must be bipartite, and since the chromatic number of a non-null bipartite graph is always 2 (in a bipartite graph each partition requires one color (there are no edges within a partition of a bipartite graph) and there are only two partitions).
Note: Since it is given that the graph is connected, it cannot be a null graph (ϕ_n)

4.32 (b)

In a simple connected undirected graph (with more than two vertices), at least 2 vertices must have same degree, since if this is not true, then all vertices would have different degrees. A graph with all vertices having different degrees is not possible to construct (can be proved as a corollary to the Havell-Hakimi theorem). Notice that it is possible to construct graphs satisfying choices a, c and d.

4.33 (c)

Given, $\xi(G) = \sum_{d} i_d \times d$ = Sum of degrees
By handshaking theorem, $\xi(G) = 2|E_G|$
where, $|E_G|$ is the number of edges in G.
If S and T are two trees with $\xi(S) = \xi(T)$.
$\Rightarrow \quad 2|E_S| = 2|E_T|$
$\Rightarrow \quad |E_S| = |E_T|$
In a tree, $|E_S| = |S| - 1$ and $|E_T| = |T| - 1$
Where $|S|$ is number of vertices of tree S and $|T|$ is number of vertices of tree T.
$\therefore \; |S| - 1 = |T| - 1$
$\Rightarrow |S| = |T|$

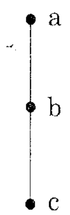
4.34 (a)

A tree with 1 node is not possible, since it is given that every node has exactly 1 child.

Now consider a tree with 2 nodes (a is the root)

Now a has exactly one child. Number of descendents of a = 2. But this contradicts the given fact that every node has an odd number of descendents.

Now consider a tree with 3 nodes. Since, every-node has exactly one child, it must be of the form,



Here a has 3 descendents, b has 2 descendents and c has one. Again we have contradiction in that b does not have odd number of descendents. Similarly can show that for tree with 4, 5, 6 ... nodes, it is not possible to have all nodes with odd number of descendents. So correct answer is the tree has 0 nodes, i.e., choice (a).

4.35 (d)

Havell-Hakimi algorithm can be used to check whether a given degree sequence is a graph or not.

The algorithm is

1. remove top node of the sequence.
2. subtract "1" from as many nodes in remaining sequence as the degree of top node that was removed.
3. rearrange this sequence in non increasing order.
4. check if resulting sequence is a graph.
5. proceed again to step 1.

   If the given sequence is not a graph we will see a violation in step 4, such as presence of negative degrees in the sequence. Otherwise the algorithm will bottom out with a degree sequence consisting of only even number of 1's and any number of 0's.

   Now applying the algorithm to the degree sequences I, II, III and IV, one by one:

I. 7, 6, 5, 4, 4, 3, 2, 1

6, 5, 4, 4, 3, 2, 1	(Step 1)
5, 4, 3, 3, 2, 1, 0	(Step 2)
5, 4, 3, 3, 2, 1, 0	(Step 3)
Sequence is a graph	(Step 4)
4, 3, 3, 2, 1, 0	(Step 1)
3, 2, 2, 1, 0, 0	(Step 2)
3, 2, 2, 1, 0, 0	(Step 3)
Sequence is a graph	(Step 4)
2, 2, 1, 0, 0	(Step 1)
1, 1, 0, 0, 0	(Step 2)
1, 1, 0, 0, 0	(Step 3)
Sequence is a graph	(Step 4)

Now, the algorithm ends, since the sequence has only 0's and even number of 1's.

The final sequence corresponds to following valid graph

Similarly for sequence II.

II. 6, 6, 6, 6, 3, 3, 2, 2

6, 6, 6, 3, 3, 2, 2	(Step 1)
5, 5, 5, 2, 2, 1, 2	(Step 2)
5, 5, 5, 2, 2, 2, 1	(Step 3)
Sequence is a graph	(Step 4)
5, 5, 2, 2, 2, 1	(Step 1)
4, 4, 1, 1, 1, 1	(Step 2)
4, 4, 1, 1, 1, 1	(Step 3)
Sequence is a graph	(Step 4)
4, 1, 1, 1, 1	(Step 1)
3, 0, 0, 0, 1	(Step 2)
3, 1, 0, 0, 0	(Step 3)
Sequence is a graph	(Step 4)
1, 0, 0, 0	(Step 1)
0, −1, −1, 0	(Step 2)
0, 0, −1, −1	(Step 3)

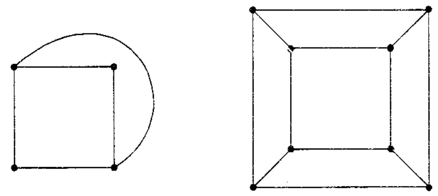
The sequence is not a graph (Step 4), since negative degrees not possible in a valid graph. So, algorithm ends.

II is cannot be the degree sequence of any graph. Similarly we can show that III is degree sequence of some graph and IV is not a degree sequence of any graph.

So, option (d) is correct.

4.36 (b)

The planar embedding of K_4 and Q_3 is shown below:



So both graphs are planar.

4.37 (d)

n = 10

e = 15

In a simple connected planar graph, Euler's formula gives the total number of regions as e – n + 2 = 15 – 10 + 2 = 7.

Out of this, one region is unbounded and the other 6 are bounded.

So correct answer is 6, which is option (d).

4.38 (b)

Check invariants are one by one.

Step 1: All 4 choice have same number of vertices and edges as given graph.

Step 2: So we find degree sequence of given graph which is (1, 1, 2, 2, 2, 2, 2, 4).

Degree sequence of graph in option (a) is (1, 1, 1, 2, 2, 2, 3, 4).

Degree sequence of graph in option (b) is (1, 1, 2, 2, 2, 2, 2, 4)

Degree sequence of graph in option (c) is (1, 1, 2, 2, 2, 2, 3, 3)

Degree sequence of graph in option (d) is (1, 1, 2, 2, 2, 2, 2, 4).

So only options (a) and (c) are not isomorphic to given graph, since degree sequence of these graphs is not same as given graph.

Step 3: Now to decide between options (b) and (d), which one is isomorphic to given graph, we check the number of cycles.

In given Graph there is one cycle of length 5 but in Graph (d), there is no cycle of length 5.

Graph (b) has one cycle of length 5.

So only Graph (b) can be isomorphic to given Graph.

4.39 (d)

The graph given is K_6.

In K_6, every cycle of length 4 corresponds to selecting 4 vertices out of 6 vertices, which can be done in ${}^6C_4$ ways and then ordering the 4 vertices in circular permutation in 3! ways (since vertices are labeled). So final answer is ${}^6C_4 \times 3! = 90$.

4.40 (c)

We need to find unordered cycle of length 3 so we choose any 3 vertices from 8 vertices. This can be done in ${}^8C_3$ ways

To make cycle we need to choose edge between the selected vertices probability of choosing any edge is 1/2.

So for three edges = $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

Expected no of cycle = np

Here n = ${}^8C_3$ and p = $\frac{1}{8}$

Expected no of cycles = ${}^8C_3 \times \frac{1}{8} = 7$.

4.41 (c)

Q is *Hand-shaking theorem* and hence true.

P is a corollory to the *Hand-shaking theorem* and hence also true.

4.42 (b)

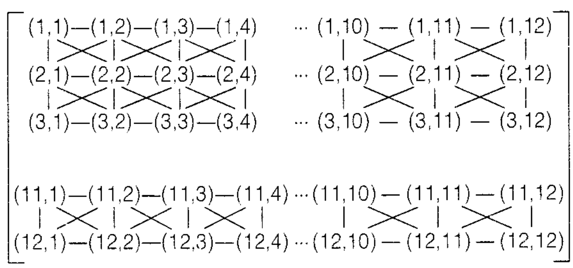
$G = (V, E)$ is directed graph

$G_2 = (V, E_2)$ where $E_2 = \{(u, v) \mid (v,u) \in E\}$

G and G_2 has same strongly connected components only the difference in G_2 is that all edges of G have been reversed the direction.

4.43 Sol.

The given condition translates into the graph shown below where every vertex is connected only with its neighbours.



From above diagram

(*i*) The four corner vertices have each 3 degrees which gives $4 \times 3 = 12$ degrees.

(*ii*) The 40 side vertices have 5 degrees each contributing a total of $40 \times 5 = 200$ degrees.

(*iii*) The 100 interior vertices each have 8 degrees contributing a total of $100 \times 8 = 800$ degrees.

So total degree of the graph

$12 + 200 + 800 = 1012$ degrees.

Now the number of edges in any undirected graph = Total degrees / 2

Therefore the number of edges in this graph = $1012 / 2 = 506$

4.44 (c)

Use the ***Havell-hakimi*** algorithm.

The sequence of steps in the algorithm for each graph is shown below:

(a) (1,1,1,1,1,1) is simple graph

(b) (2,2,2,2,2,2) ⇒ (1,1,2,2,2)
⇒ (2,2,2,1,1)
⇒ (1,1,1,1) is simple graph

(c) (3,3,3,1,0,0) ⇒ (2,2,0,0,0)
⇒ (1,−1,0,0)
cannot be a graph since negative degree not possible.

(d) (3,2,1,1,1,0) ⇒ (1,0,0,1,0)
⇒ (1,1,0,0,0) is simple graph

4.45 Sol.

Bipartite graph {*u*, *v* } where u and v are partition on vertices:

(a) {1, 11} ⇒ $1 \times 11 = 11$ edges maximum

(b) {2, 10} ⇒ $2 \times 10 = 20$ edges maximum

(c) {3, 9} ⇒ $3 \times 9 = 27$ edges maximum

(d) {4, 8} ⇒ $4 \times 8 = 32$ edges maximum

(e) {5, 7} ⇒ $5 \times 7 = 35$ edges maximum

(f) {6, 6} ⇒ $6 \times 6 = 36$ edges maximum

∴ Maximum number of edges in a bipartite graph on 12 vertices = 36 edges.

4.46 (5)

If a graph is isomorphic to its own complement, then it is a self complementary graph. In a self complementary graph

$$e = \frac{n(n-1)}{4}$$

but in the cycle C_n, $e = n$.

Som if a cycle is self complementary, then

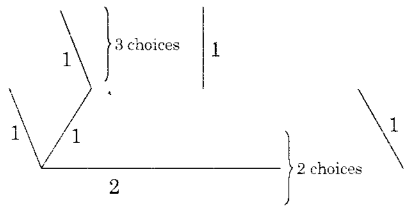
$$n = \frac{n(n-1)}{4}$$

The solution of the above equation is $n = 5$.

So, C_5 is the only cycle graph which is self complementary.

4.47 Sol.

The edges with weight 1, do not form a cycle. Take all of them. The weight 2 edges can be choosen in $3 \times 2 = 6$ ways, as shown below.



Total there are six possible minimum spanning trees. [$3 \times 2 = 6$]

4.48 (c)

The number of edges in spanning forest of a graph G with n vertices and k components = Rank (G) = n − k.

4.49 (a)

$$\sum \deg = 2e$$

Given $\delta \geq 3$ also $\delta \leq 2e/n \Rightarrow e \geq n\delta/2$

$\Rightarrow e \geq 3n/2$

Euler's formula:

$r = e - n + 2 \geq 3n/2 - n + 2 \geq n/2 + 2$

So the number of faces is atleast $n/2 + 2$

4.50 Sol.

Number of vertices (n) = 10

$d(r_i) = 3$

Number of edges (e) = ?

$r = e - v + 2$ (euler's formula for connected planar graphs)

$r = e - 10 + 2 = e - 8$ ...(1)

Since every region is bounded by exactly 3 edges and since every edge is exactly double counted, we have the equation

$e = 3r/2 \Rightarrow r = 2e/3$

Substituting this in equation (1) we get,

$2e/3 = e - 8$

$2e = 3e - 24 \Rightarrow e = 24$

4.51 (d)

An n-vertex self-complementary graph has exactly half number of edges of the complete graph i.e. $n(n-1)/4$ edges.

Since $n(n-1)$ must be divisible by 4, n must be congruent to 0 mod 4 or 1 mod 4.

4.52 (b)

An edge is called bridge iff it's removal will disconnect the graph into components.

In a cycle there are two paths between every pair of vertices and so removal of an edge from a cycle does not disconnect the cycle. So a bridge cannot be part of a simple cycle.

4.53 Sol.

n_0 : Number of leaf nodes (degree 0)

n_1 : Number of nodes with degree 1

n_2 : Number of nodes with degree 2

We have following two equations for binary tree

$n = n_0 + n_1 + n_2$ (where n is the total number of nodes in the tree) ...(1)

Accounting for total nodes as sum of those nodes which are children and the root which is not a child of any node,

we get $n = n_1 \times 1 + n_2 \times 2 + 1$

$= n_1 + 2n_2 + 1$ ....(2)

Combining equations (1) and (2) we get

$n_0 + n_1 + n_2 = n_1 + 2n_2 + 1$

$\Rightarrow n_2 = n_0 - 1$

Therefore the number of nodes of degree two in any binary tree = number of leaf nodes −1.

$n_2 = 200 - 1 = 199$

4.54 Sol.

G has $|V| = 100$ vertices and

$|E| = 300$ edges

the weight of MST of G = 500

In MST:

Number of vertices = 100

Number of edges = 99

If each edge of G is increased by 5 then MST weight also increased. For each edge of old MST increased with 5.

Total 99 edges. So $99 \times 5 = 445$ is increased

Total weight of new MST

$= 500 + 445 = 995$

4.55 Sol.

Four color theorem says that every planar graph can color with 4 colors i.e. four colors are sufficient to properly color any planar graph.

5 Probability

5.1 Let A and B be any two arbitrary events, then, which one of the following is true?

(a) $P(A \cap B) = P(A)\,P(B)$

(b) $P(A \cup B) = P(A) + P(B)$

(c) $P(A \mid B) = P(A \cap B)/P(B)$

(d) $P(A \cup B) < P(A) + P(B)$

[1994 : 1 Mark]

5.2 A bag contains 10 white balls and 15 black balls. Two balls are drawn in succession. The probability that one of them is black and the other is white is

(a) 2/3 (b) 4/5

(c) 1/2 (d) 1/3

[1995 : 2 Marks]

5.3 Two dice are thrown simultaneously. The probability that at least one of them will have 6 facing up is

(a) $\frac{1}{36}$ (b) $\frac{1}{3}$

(c) $\frac{25}{36}$ (d) $\frac{11}{36}$

[1996 : 1 Mark]

5.4 The probability that top and bottom cards of a randomly shuffled deck are both aces is

(a) $\frac{4}{52} \times \frac{4}{52}$ (b) $\frac{4}{52} \times \frac{3}{52}$

(c) $\frac{4}{52} \times \frac{3}{51}$ (d) $\frac{4}{52} \times \frac{4}{51}$

[1996 : 2 Marks]

5.5 The probability that it will rain today is 0.5. The probability that it will rain tomorrow is 0.6. The probability that it will rain either today or tomorrow is 0.7. What is the probability that it will rain today and tomorrow?

(a) 0.3 (b) 0.25

(c) 0.35 (d) 0.4

[1997 : 1 Mark]

5.6 A die is rolled three times. The probability that exactly one odd number turns up among the three outcomes is

(a) 1/6 (b) 3/8

(c) 1/8 (d) 1/2

[1998 : 1 Mark]

5.7 Suppose that the expectation of a random variable X is 5. Which of the following statements is true?

(a) There is a sample point at which X has the value 5.

(b) There is a sample point at which X has value greater than 5.

(c) There is a sample point at which X has a value greater than or equal to 5.

(d) None of the above

[1999 : 1 Mark]

5.8 Consider two events E_1 and E_2 such that probability of E_1, $\Pr[E_1] = \frac{1}{2}$, probability of E_2, $\Pr[E_2] = \frac{1}{3}$, and probability of E_1 and E_2, $\Pr[E_1 \text{ and } E_2] = \frac{1}{5}$. Which of the following statements is/are true?

(a) $\Pr[E_1 \text{ or } E_2]$ is $\frac{2}{3}$

(b) Events E_1 and E_2 are independent

(c) Events E_1 and E_2 are not independent

(d) $\Pr\left[\frac{E_1}{E_2}\right] = \frac{4}{5}$

[1999 : 2 Marks]

5.9 E1 and E2 are events in a probability space satisfying the following constraints:

- Pr(E1) = Pr(E2)
- $\Pr(E1 \cup E2) = 1$
- E1 and E2 are independent

The value of Pr(E1), the probability of the event E1, is

(a) 0 (b) $\frac{1}{4}$

(c) $\frac{1}{2}$ (d) 1

[2000 : 2 Marks]

5.10 Seven (distinct) car accidents occurred in a week. What is the probability that they all occurred on the same day?

(a) $\frac{1}{7^7}$ (b) $\frac{1}{7^6}$

(c) $\frac{1}{2^7}$ (d) $\frac{7}{2^7}$

[2001 : 2 Marks]

5.11 Four fair coins are tossed simultaneously. The probability that at least one head and one tail turn up is

(a) $\frac{1}{16}$ (b) $\frac{1}{8}$

(c) $\frac{7}{8}$ (d) $\frac{15}{16}$

[2002 : 2 Marks]

5.12 Let P (E) denote the probability of the event E. Given P (A) = 1, P(B) = 1/2, and A and B are independent, the values of P (A| B) and P(B|A) respectively are

(a) 1/4, 1/2 (b) 1/2, 1/4

(c) 1/2, 1 (d) 1, 1/2

[2003 : 1 Mark]

5.13 A program consists of two modules executed sequentially. Let f_1 (t) and f_2 (t) respectively denote the probability density functions of time taken to execute the two modules. The probability density function of the overall time taken to execute the program is given by

(a) $f_1(t) + f_2(t)$ (b) $\int_0^t f_1(x)f_2(x)dx$

(c) $\int_0^t f_1(x)f_2(t-x)dx$ (d) $\max\{f_1(t), f_2(t)\}$

[2003 : 2 Marks]

5.14 If a fair coin is tossed four times. What is the probability that two heads and two tails will result?

(a) 3/8 (b) 1/2

(c) 5/8 (d) 3/4

[2004 : 1 Mark]

5.15 In a population of N families, 50% of the families have three children, 30% of the families have two children and the remaining families have one child. What is the probability that a randomly picked child belongs to a family with two children?

(a) 3/23 (b) 6/23

(c) 3/10 (d) 3/5

[2004 : 1 Mark]

5.16 Let X and Y be two exponentially distributed and independent random variables with mean α and β, respectively. If Z = min (X,Y), then the mean of Z is given by

(a) $(1/(\alpha + \beta))$ (b) $\min(\alpha, \beta)$

(c) $(\alpha\beta/(\alpha + \beta))$ (d) $\alpha + \beta$

[2004 : 2 Marks]

5.17 An examination paper has 150 multiple-choice questions of one mark each, with each question having four choices. Each incorrect answer fetches – 0.25 mark. Suppose 1000 students choose all their answers randomly with uniform probablity. The sum total of the expected marks obtained by all these students is

(a) 0 (b) 2550

(c) 7525 (d) 9375

[2004 : 2 Marks]

5.18 Two n bit binary strings, S_1 and S_2 are chosen randomly with uniform probablity. The probability that the Hamming distance between these strings (the number of bit positions where the two strings differ) is equal to d is

(a) ${}^nC_d/2^n$ (b) ${}^nC_d/2^d$

(c) $d/2^n$ (d) $1/2^d$

[2004 : 2 Marks]

5.19 A point is randomly selected with uniform probability in the X-Y. Plane within the rectangle with corners at (0, 0), (1, 0), (1, 2) and (0, 2). If p is the length of the position vector of the point, the expected value of p^2 is

(a) 2/3 (b) 1

(c) 4/3 (d) 5/3

[2004 : 2 Marks]

5.20 Let f(x) be the continuous probability density function of a random variable X. The probability that $a < X \leq b$, is

(a) f (b – a) (b) f(b) – f(a)

(c) $\int_a^b f(x)dx$ (d) $\int_a^b xf(x)dx$

[2005 : 1 Mark]

5.21 A bag contains 10 blue marbles, 20 green marbles and 30 red marbles. A marble is drawn from the bag, its colour recorded and it is put back in the bag. This process is repeated 3 times. The probability that no two of the marbles drawn have the same colour is

(a) 1/36 (b) 1/6
(c) 1/4 (d) 1/3

[IT-2005 : 1 Mark]

5.22 An unbiased coin is tossed repeatedly until the outcome of two successive tosses is the same. Assuming that the trials are independent, the expected number of tosses is

(a) 3 (b) 4
(c) 5 (d) 6

[IT-2005 : 2 Marks]

5.23 In a certain town, the probability that it will rain in the afternoon is known to be 0.6. Moreover, meteorological data indicates that if the temperature at noon is less than or equal to 25°C, the probability that it will rain in the afternoon is 0.4. The temperature at noon is equally likely to be above 25°C, or at/below 25°C.
What is the probability that it will rain in the afternoon on a day when the temperature at noon is above 25°C?

(a) 0.4 (b) 0.6
(c) 0.8 (d) 0.9

[IT-2006 : 1 Mark]

5.24 When a coin is tossed, the probability of getting a Head is p, $0 < p < 1$. Let N be the random variable denoting the number of tosses till the first Head appears, including the toss where the Head appears. Assuming that successive tosses are independent, the expected value of N is

(a) 1/P (b) 1/(1 − P)
(c) $1/P^2$ (d) $1/(1 - P^2)$

[IT-2006 : 2 Marks]

5.25 Suppose there are two coins. The first coin gives heads with probability 5/8 when tossed, while the second coin gives heads with probability 1/4. One of the two coins is picked up at random with equal probability and tossed. What is the probability of obtaining heads?

(a) 7/8 (b) 1/2
(c) 7/16 (d) 5/32

[IT-2007 : 1 Mark]

5.26 Suppose we uniformly and randomly select a permutation from the 20! permutations of 1, 2, 3,..., 20. What is the probability that 2 appears at an earlier position that any other even number in the selected permutation?

(a) $\frac{1}{2}$ (b) $\frac{1}{10}$
(c) $\frac{9!}{20!}$ (d) None of these

[2007 : 2 Marks]

5.27 In a multi-user operating system on an average, 20 requests are made to use a particular resource per hour. The arrival of requests follows a Poisson distribution. The probability that either one, three or five requests are made in 45 minutes is given by :

(a) $6.9 \times 10^6 \times e^{-20}$ (b) $1.02 \times 10^6 \times e^{-20}$
(c) $6.9 \times 10^3 \times e^{-20}$ (d) $1.02 \times 10^3 \times e^{-20}$

[IT-2007 : 2 Marks]

5.28 A sample space has two events A and B such that probabilities

$P(A \cap B) = 1/2$, $P(\bar{A}) = 1/3$, $P(\bar{B}) = 1/3$.

What is $P(A \cup B)$?

(a) 11/12 (b) 10/12
(c) 9/12 (d) 8/12

[IT-2008 : 1 Mark]

5.29 What is the probability that in a randomly chosen group of r people at least three people have the same birthday?

(a) $1 - \frac{365.364...(365 - r + 1)}{365^r}$

(b) $1 - \frac{365.364...(365 - r + 1)}{365^r} + {}^rC_2 \cdot 365 \cdot \frac{364.363...(364 - (r - 2) + 1)}{364^{r-2}}$

(c) $1 - \frac{365.364...(365 - r + 1)}{365^r} - {}^rC_2 \cdot 365 \cdot \frac{364.363...(364 - (r - 2) + 1)}{364^{r-2}}$

(d) $\frac{365.364...(365 - r + 1)}{365^r}$

[IT-2008 : 2 Marks]

5.30 Aishwarya studies either computer science or mathematics everyday. If she studies computer science on a day, then the probability that she studies mathematics the next day is 0.6. If she studies mathematics on a day, then the probability that she studies computer science the next day is 0.4. Given that Aishwarya studies computer science on Monday, what is the probablity that she studies computer science on Wednesday?

(a) 0.24 (b) 0.36
(c) 0.4 (d) 0.6

[2008 : 2 Marks]

5.31 Let X be a random variable following normal distribution with mean +1 and variance 4. Let Y be another normal variable with mean –1 and variance unknown,. If $P(X \le -1) = P(Y \ge 2)$ the standard deviation of Y is

(a) 3 (b) 2
(c) $\sqrt{2}$ (d) 1

[2008 : 2 Marks]

5.32 An unbalanced dice (with 6 faces, numbered from 1 to 6) is thrown. The probability that the face value is odd is 90% of the probability that the face value is even. The probability of getting any even numbered face is the same.
If the probability that the face is even given that it is greater than 3 is 0.75, which one of the following options is closest to the probability that the face value exceeds 3?

(a) 0.453 (b) 0.468
(c) 0.485 (d) 0.492

[2009 : 2 Marks]

5.33 Consider a company that assembles computers. The probability of a faulty assembly of any computer is p. The company therefore subjects each computer to a testing process. This testing process gives the correct result for any computer with a probability of q. What is the probability of a computer being declared faulty?

(a) $pq + (1-p)(1-q)$ (b) $(1-q)p$
(c) $(1-p)q$ (d) pq

[2010 : 2 Marks]

5.34 What is the probability that a divisor of 10^{99} is a multiple of 10^{96}?

(a) 1/625 (b) 4/625
(c) 12/625 (d) 16/625

[2010 : 2 Marks]

5.35 If two fair coins are flipped and at least one of the outcomes is known to be a head, what is the probability that both outcomes are heads?

(a) 1/3 (b) 1/4
(c) 1/2 (d) 2/3

[2011 : 2 Marks]

5.36 If the difference between the expectation of the square of a random variable $(E[x^2]$ and the square of the expectation of the random variable $(E[x])^2$ is denoted by R, then

(a) $R = 0$ (b) $R < 0$
(c) $R \ge 0$ (d) $R > 0$

[2010 : 2 Marks]

5.37 Consider the finite sequence of random values $X = [x_1, x_2, ..., x_n]$. Let μ_x be the mean and σ_x be the standard deviation of X. Let another finite sequence Y of equal length be derived from this as $y_i = a * x_i + b$, where a and b are positive constant. Let μ_y be the mean and σ_y be the standard deviation of this sequence. Which one of the following statements INCORRECT?

(a) Index position of mode of X in X is the same as the index position of mode of Y in Y.
(b) Index position of median of X in X is the same as the index position of median of Y in Y
(c) $\mu_y = a\mu_x + b$
(d) $\sigma_y = a\sigma_x + b$

[2011 : 2 Marks]

5.38 A deck of 5 cards (each carrying a distinct number from 1 to 5) is shuffled thoroughly. Two cards are then removed one at a time from the deck. What is the probability that the two cards are selected with the number on the first card being one higher than the number on the second card?

(a) 1/5 (b) 4/25
(c) 1.4 (d) 2/5

[2011 : 2 Marks]

5.39 Consider a random variable X that takes values +1 and –1 with probability 0.5 each. The values of the cumulative distribution function F(x) at x = – 1 and +1 are

(a) 0 and 0.5 (b) 0 and 1
(c) 0.5 and 1 (d) 0.25 and 0.75

[2012 : 1 Mark]

5.40 Suppose a fair six-sided die is rolled once. If the value on the die is 1, 2 or 3 then die is rolled a second time. What is the probability that the sum total of values that turn up is at least 6?

(a) 10/21 (b) 5/12
(c) 2/3 (d) 1/6

[2012 : 2 Marks]

5.41 Suppose p is the number of cars per minute passing through a certain road junction between 5 PM, and p has Poisson distribution with mean 3. What is the probability of observing fewer than 3 cars during any given minute in this interval?

(a) $8/(2e^3)$ (b) $9/(2e^3)$
(c) $17/(2e^3)$ (d) $26/(2e^3)$

[2013 : 1 Mark]

5.42 Suppose you break a stick of unit length at a point chosen uniformly at random. Then the expected length of the shorter stick is_______.

[2014 (Set-1) : 1 Mark]

5.43 Four fair six-sided dice are rolled. The probability that the sum of the results being 22 is $X/1296$. The value of X is ______.

[2014 (Set-1) : 2 Marks]

5.44 The security system at an IT office is composed of 10 computers of which exactly four are working. To check whether the system is functional, the officials inspect four of the computers picked at random (without replacement). The system is deemed functional if at least three of the four computers inspected are working. Let the probability that the system is deemed functional be denoted by p.
Then $100p$ = _________.

[2014 (Set-2) : 1 Mark]

5.45 The probability that a given positive integer lying between 1 and 100 (both inclusive) is NOT divisible by 2, 3 or 5 is _______.

[2014 (Set-2) : 2 Marks]

5.46 Let S be a sample space and two mutually exclusive events A and B be such that $A \cup B = S$. If $P(\cdot)$ denotes the probability of the event, the maximum value of $P(A)\ P(B)$ is _______.

[2014 (Set-3) : 2 Marks]

5.47 Suppose X_i for i = 1, 2, 3 are independent and identically distributed random variables whose probability mass functions are $Pr[X_i = 0] = Pr[X_i = 1] = 1/2$ for i = 1, 2, 3. Define another random variable $Y = X_1X_2 \oplus X_3$, where $\oplus$ denotes XOR. Then $Pr[Y = 0 \mid X_3 = 0]$ = ______.

[2015 (Set-3) : 2 Marks]

5.48 A probability density function on the interval $[a, 1]$ is given by $1/x^2$ and outside this interval the value of the function is zero. The value of a is_____.

[2016 (Set-1) : 1 Mark]

5.49 Consider the following experiment.
Step 1. Flip a fair coin twice.
Step 2. If the outcomes are (TAILS, HEADS) then output Y and stop.
Step 3. If the outcomes are either (HEADS, HEADS) or (HEADS, TAILS), then output N and stop.
Step 4. If the outcomes are (TAILS, TAILS), then go to Step 1.

The probability that the output of the experiment is Y is _____ (up to two decimal places).

[2016 (Set-1) : 2 Marks]

5.50 Suppose that a shop has an equal number of LED bulbs of two different types. The probability of an LED bulb lasting more than 100 hours given that it is of Type 1 is 0.7, and given that it is of Type 2 is 0.4. The probability that an LED bulb chosen uniformly at random lasts more than 100 hours is______.

[2016 (Set-2) : 1 Mark]

Answers Probability

5.1 (c)	5.2 (c)	5.3 (d)	5.4 (c)	5.5 (d)	5.6 (b)	5.7 (c)	5.8 (c)	5.9 (d)
5.10 (b)	5.11 (c)	5.12 (d)	5.13 (c)	5.14 (a)	5.15 (b)	5.16 (c)	5.17 (d)	5.18 (a)
5.19 (d)	5.20 (c)	5.21 (b)	5.22 (a)	5.23 (c)	5.24 (a)	5.25 (c)	5.26 (d)	5.27 (b)
5.28 (b)	5.28 (c)	5.30 (c)	5.30 (a)	5.32 (b)	5.33 (a)	5.34 (a)	5.35 (a)	5.36 (c)
5.37 (d)	5.38 (a)	5.39 (c)	5.40 (b)	5.41 (c)				

Explanations Probability

5.1 (c)

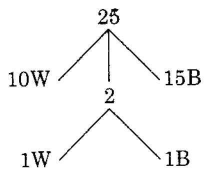
(a) $P(A \cap B) = P(A)\,P(B)$ is false since this is true if and only if A and B are independent events.

(b) $P(A \cup B) = P(A) + P(B)$ is false since $P(A \cap B)$ is zero if and only if A and B are mutually exclusive.

(c) $P(A \mid B) = P(A \cap B)/P(B)$ is true.

(d) $P(A \cup B) < P(A) + P(B)$ is false.
Since $P(A \cup B) \le P(A) + P(B)$

5.2 (c)



Bag contains 10 white balls and 15 black balls.
Required probability

$$= \frac{{}^{10}C_1 \times {}^{15}C_1}{{}^{25}C_2} = \frac{10\times 15}{\frac{25\times 24}{2}} = \frac{1}{2}$$

5.3 (d)

P(atleast one of dice will have 6 facing
= 1 – P(none of dice have 6 facing up)

$$= 1-\left[\frac{5}{6}\times\frac{5}{6}\right] = 1-\frac{25}{36} = \frac{11}{36}$$

5.4 (c)

The probability that the bottom card of a randomly shuffled deck is ace $= \frac{4}{52}$ because there are 4 aces out of total 52 cards.
From the remaining 3 aces out of 51 cards the probability that the top card is also an ace $= \frac{3}{51}$.

So required probability $= \frac{4}{52}\times\frac{3}{51}$.

5.5 (d)

P(Rain today) = 0.5
P(Rain tomorrow) = 0.6
P(Rain today $\cup$ Rain tomorrow) = 0.7
P(Rain today $\cap$ Rain tomorrow) = ?
P(rain today $\cap$ Rain tomorrow)
= P(Rain today) + P(rain tomorrow) – P(Rain today $\cap$ Rain tomorrow)
So; 0.7 = 0.5 + 0.6 – P(Rain today $\cap$ Rain tomorrow)
P(Rain today $\cap$ Rain tomorrow)
= 0.5 + 0.6 – 0.7 = 0.4
So, the probability that it will rain today and tomorrow is 0.4.

5.6 (b)

Probability of getting an odd number in rolling of a die $= \frac{3}{6} = \frac{1}{2}$.

Now using binomial distribution
P(Exactly one odd number among three outcomes)

$$= {}^3C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^2$$

$$= 3\times\left(\frac{1}{2}\right)^3 = \frac{3}{8}$$

5.7 (c)

If all the points have $X < 5$, then expectation of a random variable X is surely less than 5. So according to this there should be atleast a sample point at which $X \ge 5$.

5.8 (c)

$$P(E_1) = \frac{1}{2},\; P(E_2) = \frac{1}{3}$$

and $P(E_1 \cap E_2) = \frac{1}{5}$

(a) $P(E_1 \textbf{ or } E_2)$

$= P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$= \frac{1}{2} + \frac{1}{3} - \frac{1}{5} = \frac{19}{30}$

However given in option (a) is 2/3.

So option (a) is not true.

(b) For independent events

$P(E_1 \cap E_2) = P(E_1)\,P(E_2)$

Here, $P(E_1 \cap E_2) = \frac{2}{5}$

$P(E_1)\,P(E_2) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

So; $(P(E_1 \cap E_2) \neq P(E_1)\,P(E_2)$

So event E_1 and E_2 are not indeperdent.

Option (b) is not true.

(c) Since E_1 and E_2 are not independent

So option (c) is true.

(d) $P(E_1/E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)} = \frac{3}{5}$

So option (d) $P(E_1/E_2) = 4/5$ is false.

5.9 (d)

Constraints are

(i) $P(E_1) = P(E_2) = x$

(ii) $P(E_1 \cup E_2) = 1$

(iii) E_1 and E_2 are independent so

$P(E_1 \cap E_2) = P(E_1)\,P(E_2)$

$= x \times x = x^2$

Now,

$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$1 = x + x - x^2$

$1 = 2x - x^2$

$x^2 - 2x + 1 = 0$

$(x-1)^2 = 0$

$x = 1$

So; $P(E_1) = P(E_2) = x = 1$

5.10 (b)

Sample space = 7^7

All accidents on the same day = 7 ways (all on Monday, all on Tuesday. . .)

So, required probability = $\frac{7}{7^7} = \frac{1}{7^6}$.

5.11 (c)

p(atleast one head and one tail)

= 1 – p(no head or no tail)

= 1 – [p(no head) + p(no tail) – p(no head and no tail)]

= 1– [p (all tails) + p(all heads) – 0]

$= 1 - \left[\frac{1}{2^4} + \frac{1}{2^4} - 0\right] = 1 - \frac{2}{16} = 1 - \frac{1}{8} = \frac{7}{8}$.

Or

Alternate Method:

We need atleast one head (≥ 1 H) and atleast one tail (≥ 1 T). First we satisfy ≥ 1H as follows.

1 H, 3 T

2 H, 2 T

3 H, 1 T

and 4 H, 0 T

now to satisfy the second condition of ≥ 1 T, we have to remove 4 H, 0 T.

So, the favourable cases are only 1 H, 2 H and 3 H.

The probability of this by binomial distribution formula is

$$= \frac{^4C_1}{2^4} + \frac{^4C_2}{2^4} + \frac{^4C_3}{2^4} = \frac{7}{8}$$

5.12 (d)

Given P(A) = 1

P(B) = 1/2

Since both events are independent

$P(A \mid B) = P(A) = 1$

$P(B \mid A) = P(B) = 1/2$

5.13 (c)

Let the time taken for first and second modules be represented by x and y and total time = t.

∴ $t = x + y$ is a random variable.

Now the joint density function

$$g(t) = \int_0^t f(x, y)dx$$

$$= \int_0^t f(x, t-x)dx$$

$$= \int_0^t f_1(x)\, f_2(t-x)dx$$

which is also called as convolution of f_1 and f_2, abbreviated as $f_1 * f_2$.

Correct answer is therefore, choice (c).

5.14 (a)

Given $P(H) = 1/2$

$P(T) = 1/2$

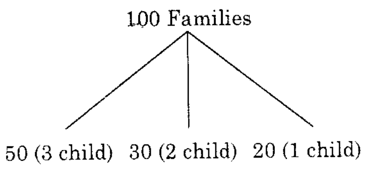
Apply Bernoulli's formula for binomial distribution,

$$P(X=2) = {}^4C_2(1/2)^2\left(1-\frac{1}{2}\right)^{4-2}$$

$$= {}^4C_2\left(\frac{1}{2}\right)^2(1/2)^2$$

$$= \frac{{}^4C_2}{2^4} = \frac{6}{16} = \frac{3}{8}$$

5.15 (b)



Total number of children

$= 50 \times 3 + 30 \times 2 + 20 \times 1 = 230$

Favorable cases = The number of children who come from 2 children families

$= 30 \times 2 = 60$

So the probability that a randomly picked child belongs to a 2 children families

$= 60/230 = 6/23$

5.16 (c)

$$f(x) = \lambda.e^{-\lambda x},\ x \ge 0$$

X and Y are two independent exponentially distributed random variables. Let λ_1 and λ_2 parameters of X and Y respectively.

$P(X \ge x) = e^{-\lambda_1 x},\quad x > 0$

$P(Y \ge x) = e^{-\lambda_2 x},\quad x > 0$

Given $Z = \min(X, Y)$

$P(Z \ge x) = P(X \ge x,\ Y \ge x)$

$= P(X \ge x)\ P(Y \ge x)$

$= e^{-\lambda_1 x} \cdot e^{-\lambda_2 x}$

$= e^{-(\lambda_1+\lambda_2)x}$

Since mean of exponential distribution = 1/Parameter

So $\alpha = 1/\lambda_1 \Rightarrow \lambda_1 = 1/\alpha$

$\beta = 1/\lambda_2 \Rightarrow \lambda_2 = 1/\beta$

$\therefore$ Z is random variable with parameter

$$\text{Mean of } Z = \frac{1}{\lambda_1+\lambda_2} = \frac{1}{\frac{1}{\alpha}+\frac{1}{\beta}} = \frac{\alpha\beta}{\alpha+\beta}.$$

5.17 (d)

Let the marks obtained per question be a random variable X.

Its probability distribution table is given below:

X	1	-0.25
p(X)	1/4	3/4

Expected marks per question

$= E(x) = \Sigma X\, P(X)$

$= 1 \times 1/4 + (-0.25) \times 3/4$

$= 1/4 - 3/16$

$= 1/16$ marks

Total marks expected for 150 questions

$= 1/16 \times 150 = \frac{75}{8}$ marks per student

Total expected marks of 1000 students

$= \frac{75}{8} \times 1000 = 9375$ marks

So correct answer is (d).

5.18 (a)

If hamming distance between two n bit strings is d, we are asking that d out of n trials to be success (success here means that the bits are different). So this is a binomial distribution with n trials and d successes and probability of success

$p = 2/4 = 1/2$

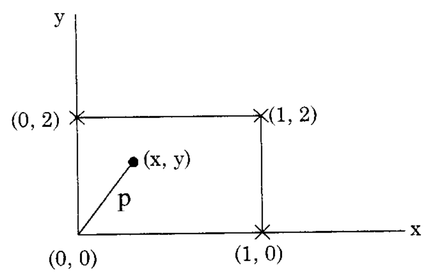
(Since out of the 4 possibilities {(0, 0), (0, 1), (1, 0), (1, 1)} only two of them (0, 1) and (1, 0) are success)

So $p(X=d) = {}^nC_d\,(1/2)^d\,(1/2)^{n-d}$

$$= \frac{{}^nC_d}{2^n}$$

Correct choice is therefore (a).

5.19 (d)



Length of position vector of point

$= p = \sqrt{x^2 + y^2}$

$$p^2 = x^2 + y^2$$
$$E(p^2) = E(x^2 + y^2)$$
$$= E(x^2) + E(y^2)$$

Now x and y are uniformly distributes $0 \le x \le 1$ and $0 \le y \le 2$

Probability density function of

$$x = \frac{1}{1-0} = 1$$

Probability density function of

$$y = \frac{1}{2-0} = 1/2$$

$$E(x^2) = \int_0^1 x^2\, p(x)\, dx$$
$$= \int_0^1 x^2 \cdot 1 \cdot dx$$
$$= \left[\frac{x^3}{3}\right]_0^1 = \frac{1}{3}$$
$$E(y^2) = \int_0^2 y^2\, p(y)\, dy$$
$$= \int_0^2 y^2 \cdot 1/2 \cdot dy$$
$$= \left[\frac{y^3}{6}\right]_0^2 = \frac{8}{6} = \frac{4}{3}$$
$$\therefore\ E(p^2) = E(x^2) + E(y^2)$$
$$= \frac{1}{3} + \frac{4}{3} = \frac{5}{3}$$

5.20 (c)

If f(x) is the continuous probability density function of a random variable X then,

$$p(a < x \le b) = p(a \le x \le b)$$
$$= \int_a^b f(x)dx$$

5.21 (b)

The given condition corresponds to sampling with replacement and without order.

No two marbles have the same color means, the final outcome of the three draws must be a permutation of Blue, Green, Red.

There are 3! = 6 such permutations possible.

Now, probability of getting a Blue first, Green second and Red third

$$= \frac{10}{60} \times \frac{20}{60} \times \frac{30}{60}$$

Required probability

$$= 6 \times \frac{10}{60} \times \frac{20}{60} \times \frac{30}{60} = \frac{1}{6}$$

5.22 (a)

$$E(X) = S(X_i \times P_i)$$

Where X_i = number of tosses till you get successive HEAD or TAIL.

P_i = Probability that you get in X_i tosses.

You need atleast 2 tosses to get 2 heads/tails.

Out of HT,HH,TH,TT, favourable cases are HH, TT.

$$p(X = 2) = 2/4$$

Similary for X = 3 only THH and HTT are favourable out of total 8 outcomes.

So, p(X = 3) = 2/8

To generalize, you can see that in every case you will have only two favourable cases and 2^n sample space. So for n th throw probability is $2/2^n$.

So the probability distribution table for X (the number of Tosses) is given below:

X	2	3	4	5	...
$P(X)$	2/4	2/8	2/16	2/32	

E(X) = 2*2/4 + 3*2/8 + 4*2/16+. . .

It is combination of AP and GP form. So multiplying E(X) by 1/2 and subtracting from E(X)

E(X) = 2*1/2 + 3*1/4 +4*1/8 +...

1/2*E(X) = 2*1/4 +3*1/8+...subtracting,

we get 1/2 *E(X) = 1+1/4+1/8+1/16+...

$$0.5 * E(X) = 1 + \frac{1/4}{(1-1/2)}$$

$$\Rightarrow\ E(x) = 3$$

5.23 (c)

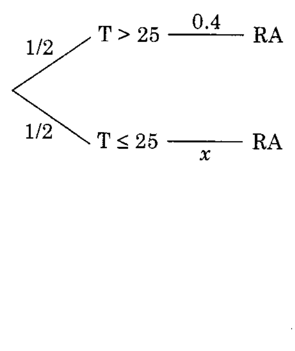
Let RA : Rain in the afternoon

T > 25 : Temperature more than 25°C

Let the desired probability =

$P(RA \mid T \le 25) = x$

The tree diagram for this problem is given below:



Given P(RA) = 0.6
by rule of total probability P(RA) =

$$\frac{1}{2}\times 0.4+\frac{1}{2}\times x = 0.6$$

$\Rightarrow \quad x = 0.8$

5.24 (a)

The number of attempts to first success follows geometric distribution. It is well known that the expected value in geometric distribution E(X) = 1/p
where p is the probability of success in any one attempt.

Alternate Method

Let X be the number of attempts to first success. Let p be the probability of success in any one attempt.
Now the probability distribution table of X is given below:

X	1	2	3	4	...
$P(X)$	p	$(1-p)p$	$(1-p)^2p$	$(1-p)^3p$	

$$E(X) = \sum xp(x)$$

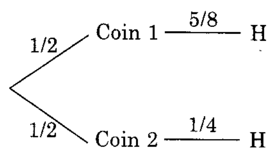
$= 1\times p+2\times(p-1)p+3\times(p-1)^2p+...$

This is arithmetico-geometric series which can be solved as

E(X) = 1/p

5.25 (c)

The tree diagram for the problem is given below:



$$P(H) = \frac{1}{2}\times\frac{5}{8}+\frac{1}{2}\times\frac{1}{4} = \frac{7}{16}$$

5.26 (d)

Number of permutations with '2' in the first position = 19!
Number of permutations with '2' in the second position = 10 × 18!
(Fill the first space with any of the 10 odd numbers and the 18 spaces after the 2 with 18 of the remaining numbers in 18! ways)
Number of permutations with '2' in 3rd position = 10 × 9 × 17!
(Fill the first 2 places with 2 of the 10 odd numbers and then the remaining 17 places with remaining 17 numbers) and so on until '2' is in 11th place. After that it is not possible to satisfy the given condition, since there are only 10 odd numbers available to fill before the '2'.
So the desired number of permutations which satisfies the given condition is
19! + 10 × 18! + 10 × 9 × 17! + 10 × 9 × 8 × 16! + ... + 10! × 9!
Now the probability of this happening is given by

$$\frac{19!+10\times18!+10\times9\times17!...+10!\times9!}{20!}$$

Which is clearly not choices (a), (b) or (c)
∴ Answer is (d) none of these.

5.27 (b)

The requests follow Poisson distribution with α = 20 requests/hr
The observation period = ΔT = 45 minutes
= 45/60 = 3/4 hr
So the parameter for the poisson distribution = λ = αΔT
= 20 × 3/4 = 15
The required probability =
P(X = 1) + P(X = 3) + P(X = 5)

$$= \frac{e^{-15}15^1}{1!}+\frac{e^{-15}15^3}{3!}+\frac{e^{-15}15^5}{5!}$$

$= 1.02 \times 10^6 \times e^{-20}$

5.28 (b)

P(A U B) = P(A) + P(B) – P(A ∩ B)
= (1 – P(A′)) + (1 – P(B′)) – P(A ∩ B)
= (1 – 1/3) + (1 – 1/3) – 1/2
= 4/3 – 1/2
= 5/6 = 10/12

5.28 (c)

P(at least three people have the same birthday) = 1 – P (all have different b'days) – P(exactly two people have same b'day)
Now, P(all have different b'days)

$$= \frac{365.364...(365-r+1)}{365^r}$$

P(exactly two people have same b'day)

$$= {}^rC_2 \cdot 365 \cdot \frac{364.363...(364-(r-2)+1)}{364^{r-2}}$$

[${}^rC_2$ ways to choose who those two people with same b'day are, 365 ways to choose what the b'day is]

Now,

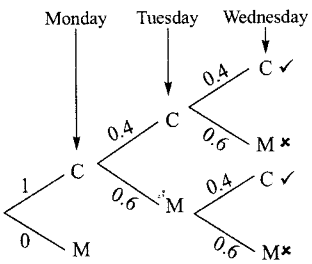
P (at least three people have the same birthday)

$$= 1 - \frac{365.364...(365-r+1)}{365^r} - {}^rC_2 \cdot 365 \cdot \frac{364.363...(364-(r-2)+1)}{364^{r-2}}$$

Which is option (c).

5.30 (c)

Let C denote computes science study and M denotes maths study. The tree diagram for the problem can be represented as shown below:



Now by rule of total probability we total up the desired branches (✓) and get the answer as shown below:

p(C on monday and C on wednesday)

= p(C on monday, C on tuesday and C on wednesday) + p(C on monday, M on tuesday and C on wednesday)

$= 1 \times 0.4 \times 0.4 + 1 \times 0.6 \times 0.4$

$= 0.16 + 0.24 = 0.40$

5.30 (a)

Given $\mu_X = 1$, $\sigma_x^2 = 4 \Rightarrow \sigma_x = 2$ and $\mu_Y = -1$. σ_Y is unknown

given, $p(X \le -1) = p(Y \ge 2)$

Converting into standard normal variates,

$$p\left(z \le \frac{-1-\mu_x}{\sigma_x}\right) = p\left(z \ge \frac{2-\mu_y}{\sigma_y}\right)$$

$$p\left(z \le \frac{-1-1}{2}\right) = p\left(z \ge \frac{2-(-1)}{\sigma_y}\right)$$

$$p(z \le -1) = p\left(z \ge \frac{3}{\sigma_y}\right) \quad \text{... (i)}$$

Now since us know that in standard normal distribution,

$$p(z \le -1) = p(z \ge 1) \quad \text{... (ii)}$$

Comparing (i) and (ii) we can say that

$$\frac{3}{\sigma_y} = 1 \Rightarrow \sigma_y = 3$$

5.32 (b)

It is given that

$p(\text{odd}) = 0.9\, p(\text{even})$

Now since $\Sigma p(x) = 1$

$\therefore$ $p(\text{odd}) + p(\text{even}) = 1$

$\Rightarrow$ $0.9\, p(\text{even}) + p(\text{even}) = 1$

$\Rightarrow$ $p(\text{even}) = \frac{1}{1.9} = 0.5263$

Now, it is given that p (any even face) is same

i.e. $p(2) = p(4) = p(6)$

Now since,

$$p(\text{even}) = p(2) \text{ or } p(4) \text{ or } p(6) = p(2) + p(4) + p(6)$$

$$\therefore \quad p(2) = p(4) = p(6) = \frac{1}{3} p(\text{even}) = \frac{1}{3}(0.5263) = 0.1754$$

It is given that

$p(\text{even} \mid \text{face} > 3) = 0.75$

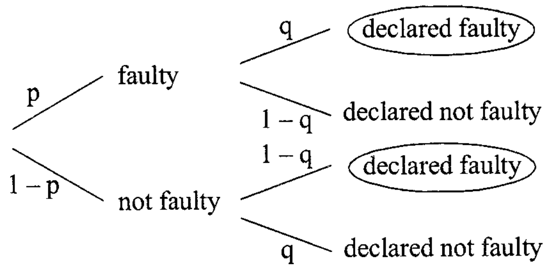
$$\Rightarrow \frac{p(\text{even} \cap \text{face} > 3)}{p(\text{face} > 3)} = 0.75$$

$$\Rightarrow \frac{p(\text{face} = 4, 6)}{p(\text{face} > 3)} = 0.75$$

$$\Rightarrow \quad p(\text{face} > 3) = \frac{p(\text{face} = 4, 6)}{0.75}$$

$$= \frac{p(4) + p(6)}{0.75} = \frac{0.1754 + 0.1754}{0.75} = 0.4677 \simeq 0.468$$

5.33 (a)



The tree diagram of probabilities is shown above.

From above tree, by rule of total probability,

$p(\text{declared faulty}) = pq + (1-p)(1-q)$

5.34 (a)

p(multiple of 10^{96} | divisor of 10^{99})

$$= \frac{p(\text{multiple of } 10^{96} \text{ \& divisor of } 10^{99})}{p(\text{divisor of } 10^{99})}$$

$$= \frac{n(\text{multiple of } 10^{96} \text{ \& divisor of } 10^{99})}{n(\text{divisor of } 10^{99})}$$

Since $10 = 2 . 5$

$10^{99} = 2^{99} . 5^{99}$

Any divisor of 10^{99} is of the form $2^a . 5^b$ where $0 \le a \le 99$ and $0 \le b \le 99$.

The number of such divisors is given by $(99+1) \times (99+1) = 100 \times 100$.

So, no. of divisors of $10^{99} = 100 \times 100$.

Any number which is a multiple of 10^{96} as well as divisor of 10^{99} is of the form $2^a . 5^b$ where $96 \le a \le 99$ and $96 \le b \le 99$. The number of such combinations of 4 values of a and 4 values of b is 4×4 combinations, each of which will be a multiple of 10^{96} as well as a divisor of 10^{99}.

$\therefore$ p(multiple of 10^{96} | divisor of 10^{99})

$$= \frac{4 \times 4}{100 \times 100} = \frac{1}{625}$$

5.35 (a)

Let A be the event of head in one coin.
B be the event of head in second coin.
The required probability is

$$p(A \cap B \mid A \cup B) = \frac{p\{(A \cap B) \cap (A \cup B)\}}{p(A \cup B)}$$

$$= \frac{p(A \cap B)}{p(A \cup B)}$$

$p(A \cap B)$ = p(both coin heads)

$$= p(H, H) = \frac{1}{4}$$

$p(A \cup B)$ = p(at least one head)

$$p(HH, HT, TH) = \frac{3}{4}$$

So required probability $= \frac{1/4}{3/4} = \frac{1}{3}$

5.36 (c)

$V(x) = E(x^2) - [E(x)]^2 = R$

where V(x) is the variance of x,

Since variance is σ_x^2 and hence never negative, $R \ge 0$.

5.37 (d)

Standard deviation is affected by scale but not by shift of origin.

So $\quad y_i = ax_i + b$

$\Rightarrow \quad \sigma_y = a\sigma_x$

if a could be negative then $\sigma_y = |a|\sigma_x$ is more correct since standard deviation cannot be negative)

Clearly, $\sigma_y = a\sigma_x + b$ is false

So (d) is incorrect.

5.38 (a)

The five cards are {1, 2, 3, 4, 5}

Sample space = 5×4 ordered pairs.

[Since there is a Ist card and IInd card we have to take ordered pairs]

p(Ist card = IInd card + 1)

= P{(2, 1), (3, 2), (4, 3), (5, 4)}

$$= \frac{4}{5 \times 4} = \frac{1}{5}$$

5.39 (c)

The p.d.t of the random variable is

x	−1	+1
P(x)	0.5	0.5

The cumulative distribution function F(x) is the probability upto x as given below:

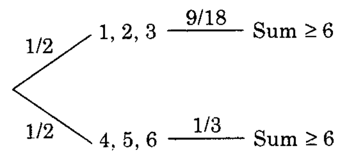
x	−1	+1
F(x)	0.5	1.0

So correct option is (c).

5.40 (b)

If first throw is 1, 2 or 3 then sample space is only 18 possible ordered pairs. Out of this only (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 5) and (3, 6) i.e. 9 out of 18 ordered pairs gives a Sum ≥ 6.

If first throw is 4, 5 or 6 then second throw is not made and therefore the only way Sum ≥ 6 is if the throw was 6. Which is one out of 3 possible. So the tree diagram becomes as follows:



. From above diagram

$$p(\text{sum} \geq 6) = \frac{1}{2}\times\frac{9}{18}+\frac{1}{2}\times\frac{1}{3} = \frac{15}{36} = \frac{5}{12}$$

5.41 (c)

Poisson formula for (P = x) given as

$$\frac{e^{-\lambda}\lambda^x}{x!}$$

$\alpha = 3$ cars/minute
$\Delta T = 1$ minute
So $\lambda = \alpha\Delta T = 3 \times 1 = 3$
Probability of observing fewer than 3 cars
$= P(X < 3) = P(X \leq 2)$
$= P(X = 0) + P(X = 1) + P(X = 2)$

$$= \frac{e^{-3}3^0}{0!}+\frac{e^{-3}3^1}{1!}+\frac{e^{-3}3^2}{2!} = \frac{17}{2e^3}$$

Option (c) is correct.

5.42 Sol.

Suppose you break a stick of unit length at a point chosen uniformly at random, then let the length of the shorter stick = x
x has uniform distribution in the interval [0, 1/2]
i.e. $\alpha = 0$ and $\beta = 1/2$
In uniform distribution, $E(x) = (\beta + \alpha)/2$
So the expected length of the shorter stick = $E(x)$
$= (1/2 + 0)/2 = 1/4 = 0.25$

5.43 Sol.

Sample space = $6^4 = 1296$

6, 6, 6, 4 ⇒ 4 ways $\left(\frac{4!}{3!} = 4\right)$

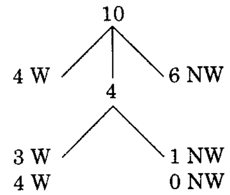
6, 6, 5, 5 ⇒ 6 ways $\left(\frac{4!}{2!2!} = 6\right)$

$$\text{Probability of sum to be } 22 = \frac{6+4}{1296} = \frac{x}{1296}$$

$\Rightarrow \quad x = 10$

5.41 Sol.

The tree diagram for the problem is shown below:



Required probability

$$= \frac{{}^4C_3 \cdot {}^6C_1}{{}^{10}C_4} + \frac{{}^4C_4 \cdot {}^6C_0}{{}^{10}C_4}$$

$$= \frac{24}{210}+\frac{1}{210} = \frac{25}{210}$$

$$p = 0.1190$$

$$\Rightarrow \quad 100p = 11.90$$

5.45 Sol.

$1 \leq x \leq 100$
P(x is not divisible by 2, 3 or 5) = 1 − P(x is divisible by 2, 3 or 5)

$$= 1 - \left[\left\lfloor\frac{100}{2}\right\rfloor + \left\lfloor\frac{100}{3}\right\rfloor + \left\lfloor\frac{100}{5}\right\rfloor - \left\lfloor\frac{100}{6}\right\rfloor - \left\lfloor\frac{100}{15}\right\rfloor - \left\lfloor\frac{100}{10}\right\rfloor + \left\lfloor\frac{100}{30}\right\rfloor\right]$$

$$1 - \frac{74}{100} = 0.26$$

5.46 Sol.

It is given that A and B are mutually exclusive also it is given that $A \cup B = S$
which means that A and B are collectively exhaustive.
Now if two events A and B are both mutually exclusive and collectively exhaustive, then P(A) + P(B) = 1 ⇒ P(B) = 1 − P(A)
Now we wish to maximize P(A) P(B)
= P(A) (1 − P(A))
Let P(A) = x
Now P(A) (1 − P(A)) = x(1 − x) = $x - x^2$
Say $y = x - x^2$

$$\frac{dy}{dx} = 1 - 2x = 0 \Rightarrow x = \frac{1}{2}$$

$$= \frac{d^2y}{dx^2} = -2 < 0; \left(\frac{d^2y}{dx^2}\right)_{x=\frac{1}{2}} = -2 < 0$$

y has maximum at $x = 1/2$

$$y_{max} = \frac{1}{2} - \left(\frac{1}{2}\right)^2 = 0.25$$

5.47 Sol.

The p.d.t. for X_1, X_2 and X_3 as given in the problem is shown below.

X_1	0	1
$p(X_1)$	$\frac{1}{2}$	$\frac{1}{2}$

X_2	0	1
$p(X_2)$	$\frac{1}{2}$	$\frac{1}{2}$

X_3	0	1
$p(X_3)$	$\frac{1}{2}$	$\frac{1}{2}$

Given $Y = X_1X_2 \oplus X_3$.

The required probability = $p(Y = 0 | X_3 = 0)$

$$= \frac{p(Y = 0 \cap X_3 = 0)}{p(X_3 = 0)} \quad ...(1)$$

Now,

$p(X_3 = 0) = 1/2$ (from p.d.t. of X_3) ...(2)

$p(Y = 0 \cap X_3 = 0)$

$= p(X_1X_2 \oplus X_3 = 0 \cap X_3 = 0)$

$= p((X_1X_2 \oplus 0) = 0 \cap X_3 = 0)$

$= p(X_1X_2 = 0 \cap X_3 = 0)$

$= p(X_1 = 0, X_2 = 0, X_3 = 0) + p(X_1 = 0, X_2 = 1, X_3 = 0) + p(X_1 = 1, X_2 = 0, X_3 = 0)$

$$= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \frac{3}{8}$$

So $p(Y = 0 \cap X_3 = 0) = \frac{3}{8}$ ...(3)

Now substituting (2) and (3) in (1), we get

The required probability =

$p(Y = 0 | X_3 = 0)$

$$= \frac{3/8}{1/2} = \frac{3}{4} = 0.75$$

5.48 Sol.

Given, $f(x) = \frac{1}{x^2} \quad a \le x \le 1$

$= 0$ elsewhere

So $\int_a^1 f(x) = 1$

$$\Rightarrow \int_a^1 \frac{1}{x^2} = 1$$

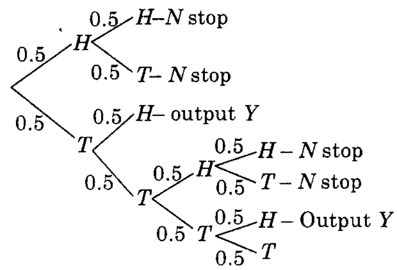
$$\Rightarrow \left[\frac{-1}{x}\right]_a^1 = 1$$

$$-\left[\frac{1}{1} - \frac{1}{a}\right] = 1$$

$$\Rightarrow \frac{1}{a} = 2$$

$$\Rightarrow a = \frac{1}{2} = 0.5$$

5.49 Sol.



The tree diagram for the problem is given above.

The desired output is Y.

Now by rule of total probability

$p(\text{output} = Y) = 0.5 \times 0.5 + 0.5 \times 0.5 \times 0.5 \times 0.5 + ...$

Infinite geometric series with

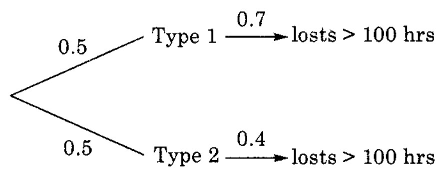
$a = 0.5 \times 0.5$

and $r = 0.5 \times 0.5$

so $p(\text{output} = Y) = \frac{0.5 \times 0.5}{1 - 0.5 \times 0.5} = \frac{0.25}{0.75}$

$\frac{1}{3} = 0.33$ (upto 2 decimal places)

5.50 Sol.



$P(\text{losts} > 100 \text{ hr}) = 0.5 \times 0.7 + 0.5 \times 0.4$

$= 0.35 + 0.2 = 0.55$

6 Linear Algebra

6.1 If a, b and c are constants, which of the following is a linear inequality?

(a) $ax + bcy = 0$ (b) $ax^2 + cy = 21$
(c) $abx + a^2y \geq 15$ (d) $xy + ax \geq 20$

[1987 : 2 Marks]

6.2 A square matrix is singular whenever:

(a) The rows are linearly independent
(b) The columns are linearly independent
(c) The rows are linearly dependent
(d) None of the above

[1987 : 2 Marks]

6.3 If A and B are real symmetric matrices of size $n \times n$. Then, which one of the following is true?

(a) $AA^t = I$ (b) $A = A^{-1}$
(c) $AB = BA$ (d) $(AB)^t = BA$

[1994 : 2 Marks]

6.4 The rank of the following $(n + 1) \times (n + 1)$ matrix, where a is a real number is

$$\begin{bmatrix} 1 & a & a^2 & \cdot & \cdot & \cdot & a^n \\ 1 & a & a^2 & \cdot & \cdot & \cdot & a^n \\ \cdot & \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & \cdot & & & & \cdot \\ 1 & a & a^2 & \cdot & \cdot & \cdot & a^n \end{bmatrix}$$

(a) 1
(b) 2
(c) n
(d) Depends on the value of a

[1995 : 1 Mark]

6.5 Let $AX = B$ be a system of linear equations where A is an $m \times n$ matrix and b is a $m \times 1$ column vector and X is a $n \times 1$ column vector of unknown.

Which of the following is false?

(a) The system has a solution if and only if, both A and the augmented matrix [A B] have the same rank.
(b) If $m < n$ and B is the zero vector, then the system has infinitely many solutions.
(c) If $m = n$ and B is non-zero vector, then the system has a unique solution.
(d) The system will have only a trivial solution when $m = n$, B is the zero vector and rank $(A) = n$.

[1996 : 1 Mark]

6.6 The matrices $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ and $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ commute under multiplication

(a) if $a = b$ or $\theta = n\pi$, n is an integer
(b) always
(c) never
(d) if $a \cos\theta \neq b \sin\theta$

[1996 : 2 Marks]

6.7 The determinant of the matrix

$$\begin{bmatrix} 6 & -8 & 1 & 1 \\ 0 & 2 & 4 & 6 \\ 0 & 0 & 4 & 8 \\ 0 & 0 & 0 & -1 \end{bmatrix} \text{ is}$$

(a) 11 (b) -48
(c) 0 (d) -24

[1997 : 1 Mark]

6.8 Let $a = (a_{ij})$ be an n-rowed square matrix and I_{12} be the matrix obtained by interchanging the first and second rows of the n-rowed Identity matrix. Then AI_{12} is such that its first

(a) row the same as its second row
(b) row is the same as the second row of A
(c) column is the same as the second column A
(d) row is all zero

[1997 : 2 Marks]

6.9 Consider the following set of equations

$x + 2y = 5$
$4x + 8y = 12$
$3x + 6y + 3z = 15$

This set

(a) has a unique solution
(b) has no solution
(c) has finite number of solutions
(d) has infinite number of solutions

[1998 : 1 Mark]

6.10 The rank of the matrix given below is

$$\begin{bmatrix} 1 & 4 & 8 & 7 \\ 0 & 0 & 3 & 0 \\ 4 & 2 & 3 & 1 \\ 3 & 12 & 24 & 21 \end{bmatrix}$$

(a) 3 (b) 1
(c) 2 (d) 4

[1998 : 2 Marks]

6.11 Consider the following determinant

$$\Delta = \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$$

Which of the following is a factor of Δ?
(a) $a + b$ (b) $a - b$
(c) $a + b + c$ (d) abc

[1998 : 2 Marks]

6.12 An $n \times n$ array v is defined as follows:
$v[i, j] = i - j$ for all $i, j,\ 1 \le i \le n,\ 1 \le j \le n$
The sum of the elements of the array v is
(a) 0 (b) $n - 1$
(c) $n^2 - 3n + 2$ (d) $n^2 \dfrac{(n+1)}{2}$

[2000 : 1 Mark]

6.13 The determinant of the matrix $\begin{bmatrix} 2 & 0 & 0 & 0 \\ 8 & 1 & 7 & 2 \\ 2 & 0 & 2 & 0 \\ 9 & 0 & 6 & 1 \end{bmatrix}$ is

(a) 4 (b) 0
(c) 15 (d) 20

[2001 : 1 Mark]

6.14 Consider the following statements:
S_1: The sum of two singular $n \times n$ matrices may be non-singular
S_2: The sum of two $n \times n$ non-singular matrices may be singular
Which of the following statements is correct?
(a) S_1 and S_2 are both true
(b) S_1 is true, S_2 is false
(c) S_1 is flase, S_2 is true
(d) S_1 and S_2 are both false

[2002 : 1 Mark]

6.15 The rank of the matrix $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ is

(a) 4 (b) 2
(c) 1 (d) 0

[2002 : 1 Mark]

6.16 Obtain the eigen values of the matrix

$$A = \begin{bmatrix} 1 & 2 & 34 & 49 \\ 0 & 2 & 43 & 94 \\ 0 & 0 & -2 & 104 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

[2002 : 2 Marks]

6.17 Consider the following system of linear equations

$$\begin{bmatrix} 2 & 1 & -4 \\ 4 & 3 & -12 \\ 1 & 2 & -8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \alpha \\ 5 \\ 7 \end{bmatrix}$$

Notice that the second and the third columns of the coefficient matrix are linearly dependent. For how many values of α, does this system of equations have infinitely many solutions?
(a) 0 (b) 1
(c) 2 (d) infinitely many

[2003 : 2 Marks]

6.18 Let A, B, C, D be $n \times n$ matrices, each with non-zero determinant, If $ABCD = I$, then B^{-1} is
(a) $D^{-1} C^{-1} A^{-1}$
(b) CDA
(c) ADC
(d) does not necessarily exist

[2004 : 1 Mark]

6.19 What values of x, y and z satisfy the following system of linear equations?

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 2 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \\ 12 \end{bmatrix}$$

(a) $x = 6,\ y = 3,\ z = 2$
(b) $x = 12,\ y = 3,\ z = -4$
(c) $x = 6,\ y = 6,\ z = -4$
(d) $x = 12,\ y = -3,\ z = 0$

[IT-2004 : 1 Mark]

6.20 Let A be an $n \times n$ matrix of the following form.

$$A = \begin{bmatrix} 3 & 1 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 1 & 3 & 1 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 1 & 3 & 1 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 & \dots & 0 & 0 & 0 \\ \dots \\ \dots \\ 0 & 0 & 0 & 0 & 0 & \dots & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 & 1 & 3 \end{bmatrix}_{n \times n}$$

What is the value of the determinant of A?

(a) $\left(\frac{5+\sqrt{3}}{2}\right)^{n-1}\left(\frac{5\sqrt{3}+7}{2\sqrt{3}}\right)+\left(\frac{5-\sqrt{3}}{2}\right)^{n-1}\left(\frac{5\sqrt{3}-7}{2\sqrt{3}}\right)$

(b) $\left(\frac{7+\sqrt{5}}{2}\right)^{n-1}\left(\frac{7\sqrt{5}+3}{2\sqrt{5}}\right)+\left(\frac{7-\sqrt{5}}{2}\right)^{n-1}\left(\frac{7\sqrt{5}-3}{2\sqrt{5}}\right)$

(c) $\left(\frac{3+\sqrt{7}}{2}\right)^{n-1}\left(\frac{3\sqrt{7}+5}{2\sqrt{7}}\right)+\left(\frac{3-\sqrt{7}}{2}\right)^{n-1}\left(\frac{3\sqrt{7}-5}{2\sqrt{7}}\right)$

(d) $\left(\frac{3+\sqrt{5}}{2}\right)^{n-1}\left(\frac{3\sqrt{5}+7}{2\sqrt{5}}\right)+\left(\frac{3-\sqrt{5}}{2}\right)^{n-1}\left(\frac{3\sqrt{5}-7}{2\sqrt{5}}\right)$

[IT-2004 : 2 Marks]

6.21 If matrix $X = \begin{bmatrix} a & 1 \\ -a^2+a-1 & 1-a \end{bmatrix}$ and $X^2 - X + I = O$ (I is the identity matrix and O is the zero matrix), then the inverse of X is

(a) $\begin{bmatrix} 1-a & -1 \\ a^2 & a \end{bmatrix}$

(b) $\begin{bmatrix} 1-a & -1 \\ a^2-a+1 & a \end{bmatrix}$

(c) $\begin{bmatrix} -a & 1 \\ -a^2+a-1 & 1-a \end{bmatrix}$

(d) $\begin{bmatrix} a^2-a+1 & a \\ 1 & 1-a \end{bmatrix}$

[IT-2004 : 2 Marks]

6.22 How many solutions does the following system of linear equations have?

$$-x + 5y = -1$$
$$x - y = 2$$
$$x + 3y = 3$$

(a) infinitely many
(b) two distinct solutions
(c) unique
(d) none of these

[2004 : 2 Marks]

6.23 The determinant of the matrix given below is

$$\begin{bmatrix} 0 & 1 & 0 & 2 \\ -1 & 1 & 1 & 3 \\ 0 & 0 & 0 & 1 \\ 1 & -2 & 0 & 1 \end{bmatrix}$$

(a) −1
(b) 0
(c) 1
(d) 2

[IT-2005 : 1 Mark]

6.24 Consider the following system of equations in three real variables x_1, x_2 and x_3

$$2x_1 - x_2 + 3x_3 = 1$$
$$3x_1 - 2x_2 + 5x_3 = 2$$
$$-x_1 - 4x_2 + x_3 = 3$$

This system of equations has
(a) no solution
(b) a unique solution
(c) more than one but a finite number of solutions
(d) an infinite number of solutions

[2005 : 2 Marks]

6.25 What are the eigenvalues of the following 2×2 matrix?

$$\begin{bmatrix} 2 & -1 \\ -4 & 5 \end{bmatrix}$$

(a) −1 and 1
(b) 1 and 6
(c) 2 and 5
(d) 4 and −1

[2005 : 2 Marks]

6.26 F is an $n \times n$ real matrix. b is an $n \times 1$ real vector. Suppose there are two $n \times 1$ vectors, u and v such that $u \neq v$, and $Fu = b$, $Fv = b$.

Which one of the following statements is false?
(a) Determinant of F is zero
(b) There are an infinite number of solutions to $Fx = b$
(c) There is an $x \neq 0$ such that $Fx = 0$
(d) F must have two identical rows

[2006 : 2 Marks]

6.27 What are the eigenvalues of the matrix P given below

$$P = \begin{pmatrix} a & 1 & 0 \\ 1 & a & 1 \\ 0 & 1 & a \end{pmatrix}$$

(a) $a, a-\sqrt{2}, a+\sqrt{2}$
(b) a, a, a
(c) 0, a, 2a
(d) −a, 2a, 2a

[IT-2006 : 2 Marks]

6.28 Let A be the matrix $\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$. What is the maximum value of $x^T Ax$ where the maximum is taken over all x that are the unit eigenvectors of A?

(a) 3 (b) $(5+\sqrt{5})/2$

(c) 3 (d) $(5-\sqrt{5})/2$

[IT-2007 : 1 Mark]

6.29 Consider the set of (column) vectors defined by $X=\{x\in R^3 \mid x_1 + x_2 + x_3 = 0$, where $X^T = [x_1, x_2, x_3]^T\}$. Which of the following is TRUE?

(a) $\{[1, -1, 0]^T, [1, 0, -1]^T\}$ is a basis for the subspace X.

(b) $\{[1, -1, 0]^T, [1, 0, -1]^T\}$ is a linearly independent set, but it does not span X and therefore is not a basis of X

(c) X is not a subspace for R^3

(d) None of the above

[2007 : 2 Marks]

6.30 The following system of equations

$$x_1 + x_2 + 2x_3 = 1$$
$$x_1 + 2x_3 + 3x_3 = 2$$
$$x_1 + 4x_2 + ax_3 = 4$$

has a unique solution. The only possible value(s) for a is/are

(a) 0

(b) either 0 or 1

(c) one of 0,1 or –1

(d) any real number other than 5

[2008 : 1 Mark]

6.31 How many of the following matrices have an eigenvalue 1?

$$\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}, \begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}, \begin{bmatrix}1 & -1\\1 & 1\end{bmatrix} \text{ and } \begin{bmatrix}-1 & 0\\1 & -1\end{bmatrix}$$

(a) one (b) two

(c) three (d) four

[2008 : 2 Marks]

6.32 If M is a square matrix with a zero determinant, which of the following assertion (s) is (are) correct?

S1: Each row of M can be represented as a linear combination of the other rows

S2: Each column of M can be represented as a linear combination of the other columns

S3: MX = O has a nontrivial solution

S4: M has an inverse

(a) S3 and S2 (b) S1 and S4

(c) S1 and S3 (d) S1, S2 and S3

[IT-2008 : 2 Marks]

6.33 Consider the following matrix.

$$A = \begin{bmatrix}2 & 3\\x & y\end{bmatrix}$$

If the eigenvalues of A are 4 and 8, then

(a) x = 4, y = 10 (b) x = 5, y = 8

(c) x = –3, y = 9 (d) x = –4, y = 10

[2010 : 2 Marks]

6.34 Consider the matrix as given below:

$$\begin{bmatrix}1 & 2 & 3\\0 & 4 & 7\\0 & 0 & 3\end{bmatrix}$$

Which one of the following options provides the CORRECT values of the eigenvalues of the matrix?

(a) 1, 4, 3 (b) 3, 7, 3

(c) 7, 3, 2 (d) 1, 2, 3

[2011 : 2 Marks]

6.35 Let A be the 2 × 2 matrix with elements $a_{11} = a_{12} = a_{21} = +1$ and $a_{22} = -1$.

Then the eigenvalues of the matrix A^{19} are

(a) 1024 and – 1024

(b) $1024\sqrt{2}$ and $-1024\sqrt{2}$

(c) $4\sqrt{2}$ and $-4\sqrt{2}$

(d) $512\sqrt{2}$ and $-512\sqrt{2}$

[2012 : 1 Mark]

6.36 Which one of the following does NOT equal

$$\begin{vmatrix}1 & x & x^2\\1 & y & y^2\\1 & z & z^2\end{vmatrix}?$$

(a) $\begin{vmatrix}1 & x(x+1) & x+1\\1 & y(y+1) & y+1\\1 & z(z+1) & z+1\end{vmatrix}$ (b) $\begin{vmatrix}1 & x+1 & x^2+1\\1 & y+1 & y^2+1\\1 & z+1 & z^2+1\end{vmatrix}$

(c) $\begin{vmatrix}0 & x-y & x^2-y^2\\0 & y-z & y^2-z^2\\1 & z & z^2\end{vmatrix}$ (d) $\begin{vmatrix}2 & x+y & x^2+y^2\\2 & y+z & y^2+z^2\\1 & z & z^2\end{vmatrix}$

[2013 : 1 Mark]

6.37 Consider the following system of equations:

$$3x + 2y = 1$$
$$4x + 7z = 1$$
$$x + y + z = 3$$
$$x - 2y + 7z = 0$$

The number of solutions for this system is ____.

[2014 (Set-1) : 1 Mark]

6.38 The value of the dot product of the eigenvectors corresponding to any pair of different eigen values of a 4-*by*-4 symmetric positive definite matrix is________.

[2014 (Set-1) : 1 Mark]

6.39 If the matrix A is such that

$$A = \begin{bmatrix} 2 \\ -4 \\ 7 \end{bmatrix} [1\ 9\ 5]$$

then the determinant of A is equal to ____.

[2014 (Set-2) : 1 Mark]

6.40 The product of the non-zero eigenvalues of the matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

is ________.

[2014 (Set-2) : 2 Marks]

6.41 Which one of the following statements is TRUE about every $n \times n$ matrix with only real eigenvalues?

(a) If the trace of the matrix is positive and the determinant of the matrix is negative, at least one of its eigenvalues is negative.

(b) If the trace of the matrix is positive, all its eigenvalues are positive.

(c) If the determinant of the matrix is positive, all its eigenvalues are positive.

(d) If the product of the trace and determinant of the matrix is positive, all its eigenvalues are positive.

[2014 (Set-3) : 1 Mark]

6.42 If V_1 and V_2 are 4-dimensional subspaces of a 6-dimensional vector space V, then the smallest possible dimension of $V_1 \cap V_2$ is ________.

[2014 (Set-3) : 1 Mark]

6.43 The minimum number of arithmetic operations required to evaluate the polynomial $P(X) = X^5 + 4X^3 + 6X + 5$ for a given value of X, using only one temporary variable is ________.

[2014 (Set-3) : 1 Mark]

6.44 Consider the following 2×2 matrix A where two elements are unknown and are marked by 'a' and 'b'. The eigenvalues of this matrix are –1 and 7. What are the values of 'a' and 'b' ?

$$A = \begin{pmatrix} 1 & 4 \\ b & a \end{pmatrix}$$

(a) a = 6, b = 4 (b) a = 4, b = 6
(c) a = 3, b = 5 (d) a = 5, b = 3

[2015 (Set-1) : 2 Marks]

6.45 The larger of the two eigenvalues of the matrix $\begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$ is _______.

[2015 (Set-2) : 1 Mark]

6.46 Perform the following operations on the matrix

$$\begin{bmatrix} 3 & 4 & 45 \\ 7 & 9 & 105 \\ 13 & 2 & 195 \end{bmatrix}.$$

1. Add the third row to the second row.
2. Subtract the third column from the first column.

The determinant of the resultant matrix is_____.

[2015 (Set-2) : 2 Marks]

6.47 In the given matrix $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}$, one of the eigenvalues is 1. The eigenvectors corresponding to the eigenvalue 1 are

(a) $\{\alpha(4,2,1) \mid \alpha \neq 0, \alpha \in \mathbb{R}\}$

(b) $\{\alpha(-4,2,1) \mid \alpha \neq 0, \alpha \in \mathbb{R}\}$

(c) $\{\alpha(\sqrt{2}, 0,1) \mid \alpha \neq 0, \alpha \in \mathbb{R}\}$

(d) $\{\alpha(-\sqrt{2}, 0,1) \mid \alpha \neq 0, \alpha \in \mathbb{R}\}$

[2015 (Set-3) : 1 Mark]

6.48 In the LU decomposition of the matrix $\begin{bmatrix} 2 & 2 \\ 4 & 9 \end{bmatrix}$, if the diagonal elements of U are both 1, then the lower diagonal entry l_{22} of L is ______.

[2015 (Set-1) : 1 Mark]

6.49 If the following system has non-trivial solution,

$$px + qy + rz = 0$$
$$qx + ry + pz = 0$$
$$rx + py + qz = 0$$

then which one of the following options is TRUE?

(a) $p - q + r = 0$ or $p = q = -r$
(b) $p + q - r = 0$ or $p = -q = r$
(c) $p + q + r = 0$ or $p = q = r$
(d) $p - q + r = 0$ or $p = -q = -r$

[2015 (Set-3) : 2 Marks]

6.50 Two eigenvalues of a 3×3 real matrix P are $(2 + \sqrt{-1})$ and 3. The determinant of P is ______.

[2016 (Set-1) : 1 Mark]

6.51 Consider the systems, each consisting of m linear equations in n variables.

I. If $m < n$, then all such systems have a solution.
II. If $m > n$, then none of these sytems has a solution.
III. If $m = n$, then there exists a system which has a solution

Which one of the following is **CORRECT**?

(a) I, II and III are true
(b) Only II and III are true
(c) Only III is true
(d) None of them is true

[2016 (Set-2) : 1 Mark]

6.52 Suppose that the eigenvalues of matrix A are 1, 2, 4. The determinant of $(A^{-1})^T$ is______.

[2016 (Set-2) : 1 Mark]

■■■■

Answers Linear Algebra

6.1 (c)	6.2 (c)	6.3 (d)	6.4 (a)	6.5 (c)	6.6 (a)	6.7 (b)	6.8 (c)	6.9 (b)	
6.10 (a)	6.11 (b)	6.12 (a)	6.13 (a)	6.14 (a)	6.15 (c)	6.17 (b)	6.18 (b)	6.19 (c)	
6.20 (d)	6.21 (b)	6.22 (c)	6.23 (c)	6.24 (b)	6.25 (b)	6.26 (d)	6.27 (a)	6.28 (b)	
6.29 (b)	6.30 (d)	6.31 (a)	6.32 (d)	6.33 (d)	6.34 (a)	6.35 (d)	6.36 (a)	6.41 (a)	
6.44 (d)	6.47 (b)	6.49 (c)	6.51 (c)						

Explanations Linear Algebra

6.1 (c)

(a) Equation ax + bcy = 0 is linear equality not linear inequality.
(b) $ax^2 + cy = 21$ is not linear inequality because here degree of x is 2 and also it is equality not inequality.
(c) $abx + a^2y \geq 15$ is linear inequality because both x and y is in its first degree.
(d) $xy + ax \geq 20$ is not linear inequality because xy term is of degree 2 not of degree 1.

6.2 (c)

If the rows (or columns) of a square matrix are linearly dependent, then the determinant of matrix becomes zero.

Therefore, whenever the rows are linearly dependent, the matrix is singular.

6.3 (d)

The matrix M is said to be symmetric iff $M^t = M$

$$(AB)^t = B^tA^t = BA$$

[Since A and B are symmetric, $A^t = A$ & $B^t = B$].

6.4 (a)

All the rows of the given matrix is same. So the matrix has only one independent row.

Rank of the matrix = No. of independent rows of the matrix.

$\therefore$ Rank of given matrix = 1

6.5 (c)

Following are the possibilities for a system of linear equations:

(i) If matrix A and augmented matrix [AB] have same rank, then the system has solution otherwise there is no solution.

(ii) If matrix A and augmented matrix [AB] have same rank which is equal to the no. of variables, then the system has unique solution and if B is zero vector then the system have only a trivial solution.

(iii) If matrix A and matrix [AB] have same rank which is less than the number of variables, then the system has infinite solution.

Therefore, option (c) is false because if m = n and B is non-zero vector, then it is not necessary that system has a unique solution, because m is the number of equations (quantity) and not the number of linearly independent equations (quality).

6.6 (a)

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

and $$B = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

$$AB = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

$$= \begin{bmatrix} a\cos\theta & -b\sin\theta \\ a\sin\theta & b\cos\theta \end{bmatrix}$$

$$BA = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} a\cos\theta & -a\sin\theta \\ b\sin\theta & b\cos\theta \end{bmatrix}$$

AB = BA iff $-b\sin\theta = -a\sin\theta$

and $a\sin\theta = b\sin\theta$

Both are same equation which is

$(a - b)\sin\theta = 0$

whose solution is $a = b$ or $\theta = \pm n\pi$

6.7 (b)

The given matrix is

$$\begin{bmatrix} 6 & -8 & 1 & 1 \\ 0 & 2 & 4 & 6 \\ 0 & 0 & 4 & 8 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Above matrix is upper triangular matrix and for upper triangular matrix the determinant is product of principle diagonal elements.

Determinant of matrix

$$= 6 \times 2 \times 4 \times -1 = -48$$

6.8 (c)

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

I_{12} is the matrix obtained by inter-changing the first and second row of the Identity Matrix I.

So $$I_{12} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$AI_{12} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} b & a \\ d & c \end{bmatrix}$$

AI_{12} is the matrix having first column same as the second column of A.

6.9 (b)

Set of equations is

$$x + 2y = 5$$
$$4x + 8y = 12$$
$$3x + 6y + 3z = 15$$

Above set of equations can be written as

$$\begin{bmatrix} 1 & 2 & 0 \\ 4 & 8 & 0 \\ 3 & 6 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ x \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \\ 15 \end{bmatrix}$$

Augmented matrix [AB] is given as

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 5 \\ 4 & 8 & 0 & 12 \\ 3 & 6 & 3 & 15 \end{array}\right]$$

Performing gauss-Elimination on the above matrix

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 5 \\ 4 & 8 & 0 & 12 \\ 3 & 6 & 3 & 15 \end{array}\right] \xrightarrow[R_3 \to R_3 - 3R_1]{R_2 \to R_2 - 4R_1}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 5 \\ 0 & 0 & 0 & -8 \\ 0 & 0 & 3 & 0 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_3}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 5 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & -8 \end{array}\right]$$

The rank of matrix A is 2 and rank of matrix [AB] is 3.

Therefore, the system is inconsistent and has no solutions.

6.10 (a)

The given matrix is

$$\begin{bmatrix} 1 & 4 & 8 & 7 \\ 0 & 0 & 3 & 0 \\ 4 & 2 & 3 & 1 \\ 3 & 12 & 24 & 21 \end{bmatrix}$$

Since $R_4 = 3R_1$

Rank $\neq 4$

Now try for a rank of 3

$$\begin{vmatrix} 1 & 4 & 8 \\ 0 & 0 & 3 \\ 4 & 2 & 3 \end{vmatrix} = -3 \times \begin{vmatrix} 1 & 4 \\ 4 & 2 \end{vmatrix}$$

$$= -3 \times -14 = 52 \neq 0$$

$\therefore$ Rank of given matrix = 3

6.11 (b)

The determinant of a matrix can't be affected by elementary row operations

So; $$\Delta = \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$\Delta = \begin{vmatrix} 1 & a & bc \\ 0 & (b-a) & (ca-bc) \\ 0 & (c-a) & (ab-bc) \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a & bc \\ 0 & (b-a) & -c(b-a) \\ 0 & (c-a) & -b(c-a) \end{vmatrix}$$

$$= \begin{vmatrix} (b-a) & -c(b-a) \\ (c-a) & -b(c-a) \end{vmatrix}$$

$$= 1(b-a)(c-a)\begin{vmatrix} 1 & -c \\ 1 & -b \end{vmatrix}$$

$$= (b-a)(c-a)(-b+c)$$

$$= (b-a)(c-a)(c-b)$$

$$= (a-b)(b-c)(c-a)$$

So $(a - b)$ is a factor of Δ.

6.12 (a)

The matrix V can be defined as

$$\begin{bmatrix} 0 & -1 & -2 & \cdots\cdots & (n-1) \\ 1 & 0 & -1 & \cdots\cdots & (n-2) \\ 2 & 1 & 0 & & \vdots \\ \vdots & \vdots & \vdots & & \vdots \\ (n-1) & (n-2) & \vdots & \cdots\cdots & 0 \end{bmatrix}$$

So above is antisymmetric matrix and the sum of the elements of any antisymmetric matrix is 0.

OR

Alternate Method:

$$\sum_{j=1}^{n}\sum_{i=1}^{n} i - j = \sum_{j=1}^{n} \frac{n(n+1)}{2} - jn$$

$$= \frac{n(n+1)}{2} \times n - \frac{n^2(n+1)}{2} = 0$$

6.13 (a)

The matrix is $\begin{bmatrix} 2 & 0 & 0 & 0 \\ 8 & 1 & 7 & 2 \\ 2 & 0 & 2 & 0 \\ 9 & 0 & 6 & 1 \end{bmatrix}$

Finding the determinant by expanding the first row of the matrix

$$\text{Determinant} = 2\begin{vmatrix} 1 & 7 & 2 \\ 0 & 2 & 0 \\ 0 & 6 & 1 \end{vmatrix} = 2 \times 1\begin{vmatrix} 2 & 0 \\ 6 & 1 \end{vmatrix}$$

$$= 2 \times 1 \times (2) = 4$$

Therefore, the determinant of given matrix is 4.

6.14 (a)

S_1 is true

Consider two singular matrices

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Sum of A and B is given as

However (A + B) is a non-singular matrix

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. So, S1 is true.

Now, consider two non-singular matrices

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } D = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$C + D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

However (C+ D) is a singular matrix. So S_2 is also true.
Therefore, both S_1 and S_2 are true.

6.15 (c)

The given matrix is $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

Above matrix has only 1 independent row, so the given matrix has rank 1.

6.16 Sol.

Matrix is $\begin{bmatrix} 1 & 2 & 34 & 49 \\ 0 & 2 & 43 & 94 \\ 0 & 0 & -2 & 104 \\ 0 & 0 & 0 & -1 \end{bmatrix}$

The given matrix is upper triangular matrix and the eigen values of the matrix are diagonal elements for upper triangular matrix.
So the eigen values of the matrix A are 1, 2, –2, –1.

6.17 (b)

The augmented matrix for the given system is

$$\left[\begin{array}{ccc|c} 2 & 1 & -4 & \alpha \\ 4 & 3 & -12 & 5 \\ 1 & 2 & -8 & 7 \end{array}\right]$$

Performing Gauss-Elimination on the above matrix

$$\left[\begin{array}{ccc|c} 2 & 1 & -4 & \alpha \\ 4 & 3 & -12 & 5 \\ 1 & 2 & -8 & 7 \end{array}\right] \xrightarrow[R_3 - 1/2 R_1]{R_2 - 2R_1}$$

$$\left[\begin{array}{ccc|c} 2 & 1 & -4 & \alpha \\ 0 & 1 & -4 & 5 - 2\alpha \\ 0 & 3/2 & -6 & 7 - \alpha/2 \end{array}\right]$$

$$\xrightarrow{R_3 - 3/2 R_2} \left[\begin{array}{ccc|c} 2 & 1 & -4 & \alpha \\ 0 & 1 & -4 & 5 - 2\alpha \\ 0 & 0 & 0 & \dfrac{5\alpha - 1}{2} \end{array}\right]$$

Now for infinite solution it is necessary that at least one row must be completely zero.

$$\therefore \quad \frac{5\alpha - 1}{2} = 0$$

$\alpha = 1/5$ is the solution

$\therefore$ There is only one value of α for which infinite solution exists.

6.18 (b)

A, B, C, D is n × n matrix.
Given ABCD = I

$$\Rightarrow ABCDD^{-1}C^{-1} = D^{-1}C^{-1}$$
$$\Rightarrow \quad AB = D^{-1}C^{-1}$$
$$\Rightarrow \quad A^{-1}AB = A^{-1}D^{-1}C^{-1}$$
$$\Rightarrow \quad B = A^{-1}D^{-1}C^{-1}$$
$$B^{-1} = (A^{-1}D^{-1}C^{-1})^{-1}$$
$$= (C^{-1})^{-1} \cdot (D^{-1})^{-1} \cdot (A^{-1})^{-1}$$
$$= CDA$$

6.19 (c)

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 2 & 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \\ 12 \end{bmatrix}$$

$1 \times x + 2^* y + 3 * z = 6$
$1 \times x + 3 * y + 4 * z = 8$
$2 \times x + 2^* y + 3 * z = 12$

Put $x = 6$, $y = 6$, $z = -4$, the above three equations are satisfied.

6.20 (d)

$$A = \begin{bmatrix} 3 & 1 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 1 & 3 & 1 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 1 & 3 & 1 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 & \dots & 0 & 0 & 0 \\ \dots \\ \dots \\ 0 & 0 & 0 & 0 & 0 & \dots & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 & 1 & 3 \end{bmatrix}_{n \times n}$$

A can be expanded using first row as $|A_n| = 3^*|A_{n-1}| + 1^*|A_{n-2}|$.
We find that the remaining sub-matrix is same as the given matrix with a LOWER order.
say $|A_n|$ is denoted as T_n.
Then recurrence relation is

$$T_n = 3T_{n-1} + T_{n-2}$$
$$\Rightarrow T_n - 3T_{n-1} - T_{n-2} = 0$$

This is a *Homogenous* system whose solution is

$$T_n = C_1\left(\frac{3+\sqrt{5}}{2}\right)^n + C_2\left(\frac{3-\sqrt{5}}{2}\right)^n \quad ...(i)$$

Now putting the initial conditions

$$T_1 = |3| = 3 \text{ and } T_2 = \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} = 8$$

we get,

$$\left(\frac{3+\sqrt{5}}{2}\right)^{n-1}\left(\frac{3\sqrt{5}+7}{2\sqrt{5}}\right)+\left(\frac{3-\sqrt{5}}{2}\right)^{n-1}\left(\frac{3\sqrt{5}-7}{2\sqrt{5}}\right)$$

Alternately, since we know that for $n = 1$ and $n = 2$

$$A_1 = |3| = 3 \text{ and } A_2 = \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} = 8$$

can put n = 1 and n = 2 in those answers which are in the required format i.e. equation (*i*) and see that only choice (d) goes to 3 and 8 respectively.

6.21 (b)

Given $X = \begin{bmatrix} a & 1 \\ -a^2+a-1 & 1-a \end{bmatrix}$

and $X^2 - X + I = O \quad ...(i)$

we need to find X^{-1}.

Solving (*i*) for I we get,

$$I = X - X^2$$

Multiplying by X^{-1} on both sides

$$X^{-1} = I - X$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} a & 1 \\ -a^2+a-1 & 1-a \end{bmatrix}$$

$$= \begin{bmatrix} 1-a & -1 \\ a^2-a+1 & a \end{bmatrix}$$

Which is option (b).

6.22 (c)

$$-x + 5y = -1$$
$$x - y = 2$$
$$x + 3y = 3$$

The augmented matrix is

$$\left[\begin{array}{cc|c} -1 & 5 & -1 \\ 1 & -1 & 2 \\ 1 & 3 & 3 \end{array}\right]$$

Using gauss-elimination on above matrix we get,

$$\left[\begin{array}{cc|c} -1 & 5 & -1 \\ 1 & -1 & 2 \\ 1 & 3 & 3 \end{array}\right] \xrightarrow[R_3+R_1]{R_2+R_1} \left[\begin{array}{cc|c} -1 & 5 & -1 \\ 0 & 4 & 1 \\ 0 & 8 & 2 \end{array}\right]$$

$$\xrightarrow{R_3-2R_2} \left[\begin{array}{cc|c} -1 & 5 & -1 \\ 0 & 4 & 1 \\ 0 & 0 & 0 \end{array}\right]$$

Rank [A | B] = 2 (number of non zero rows in [A | B])

Rank [A] = 2 (number of non zero rows in [A])

Rank [A | B] = Rank [A] = 2 = number of variables

∴ Unique solution exists. Correct choice is (c).

6.23 (c)

$$A = \begin{vmatrix} 0 & 1 & 0 & 2 \\ -1 & 1 & 1 & 3 \\ 0 & 0 & 0 & 1 \\ 1 & -2 & 0 & 1 \end{vmatrix}$$

Expanding A by third row (which contains lot of zeros)

$$A = -1 \times \begin{vmatrix} 0 & 1 & 0 \\ -1 & 1 & 1 \\ 1 & -2 & 0 \end{vmatrix}$$

$$= -1 \times 1 \times \begin{vmatrix} -1 & 1 \\ 1 & 0 \end{vmatrix}$$

$$= -1 \times 1 \times -1 = 1$$

6.24 (b)

The augmented matrix for the given system is

$$\left[\begin{array}{ccc|c} 2 & -1 & 3 & 1 \\ 3 & -2 & 5 & 2 \\ -1 & -4 & 1 & 3 \end{array}\right].$$

Using gauss-elimination method on above matrix we get,

$$\left[\begin{array}{ccc|c} 2 & -1 & 3 & 1 \\ 3 & -2 & 5 & 2 \\ -1 & -4 & 1 & 3 \end{array}\right] \xrightarrow[R_3+\frac{1}{2}R_1]{R_2-\frac{3}{2}R_1} \left[\begin{array}{ccc|c} 2 & -1 & 3 & 1 \\ 0 & -1/2 & 1/2 & 1/2 \\ 0 & -9/2 & 5/2 & 7/2 \end{array}\right]$$

$$\xrightarrow{R_3-9R_2} \left[\begin{array}{ccc|c} 2 & -1 & 3 & 1 \\ 0 & -1/2 & 1/2 & 1/2 \\ 0 & 0 & -2 & -1 \end{array}\right]$$

Rank ([A | B]) = 3

Rank ([A]) = 3

Since Rank ([A | B]) = Rank ([A]) = number of variables, the system has unique solution.

6.25 (b)

$$A = \begin{bmatrix} 2 & -1 \\ -4 & 5 \end{bmatrix}$$

The characteristic equation of this matrix is given by

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & -1 \\ -4 & 5-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)(5-\lambda) - 4 = 0$$

$$\lambda^2 - 7\lambda + 6 = 0$$

$$\lambda = 1, 6$$

$\therefore$ The eigen values of A are 1 and 6.

6.26 (d)

Given that Fu = b and Fv = b

If F is non singular, then it has a unique inverse.

Now, $u = F^{-1} b$ and $v = F^{-1} b$

Since F^{-1} is unique u = v but it is given that u ≠ v. This is a contradiction. So F must be singular.

This means that

(a) Determinate of F is zero is true. Also

(b) There are infinite number of solution to Fx = b is true since |F| = 0.

(c) There is an X ≠ 0 such that F X = 0 is also true, since X has infinite number of solutions, including the X = 0 solution.

(d) F must have 2 identical rows is false, since a determinant may become zero, even if two identical columns are present. It is not necessary that 2 identical rows must be present for |F| to become zero.

6.27 (a)

$$P = \begin{bmatrix} a & 1 & 0 \\ 1 & a & 1 \\ 0 & 1 & a \end{bmatrix}$$

Since P is a square matrix

Sum (eigen values) = Trace (P)

= a + a + a = 3a

Product of eigen values

$= |P| = a(a^2 - 1) - a = a^3 - 2a$

Only choice (a) has sum of eigen values = 3a and product of eigen values = $a^3 - 2a$.

So (a) is the correct option.

6.28 (b)

Theorem: The maximum value of $x^T Ax$ where the maximum is taken over all x that are the unit eigen-vectors of A is the maximum eigen value of A.

Find the eigen value of A.

$$\begin{vmatrix} 3-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix} = 0$$

$$\lambda^2 - 5\lambda + 5 = 0$$

The eigen values of A = $\left\{\frac{5+\sqrt{5}}{2}, \frac{5-\sqrt{5}}{2}\right\}$

The required answer is the maximum eigen value of A = $\frac{5+\sqrt{5}}{2}$

6.29 (b)

$$X = \{x \in R^3 \mid x_1 + x_2 + x_3 = 0\}$$

$X^T = [x_1, x_2, x_3]^T$ then

$\{[1, -1, 0]^T, [1, 0, -1]^T\}$ is a linearly independent set because one cannot be obtained from another by scalar multiplication. However (1, –1, 0) and (1, 0, –1) do not span X, since all such combinations (x_1, x_2, x_3) such that $x_1 + x_2 + x_3 = 0$ cannot be expressed as linear combinations of (1, –1, 0) and (1, 0, –1) [we need (0, 1, –1) also to span X].

6.30 (d)

The augmented matrix for above system is

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 1 & 2 & 3 & 2 \\ 1 & 4 & a & 4 \end{array}\right] \xrightarrow[R_3 - R_1]{R_2 - R_1} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 3 & a-2 & 3 \end{array}\right]$$

$$\xrightarrow{R_3 - 3R_2} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & a-5 & 0 \end{array}\right]$$

Now as long as a – 5 ≠ 0, rank (A) = rank (A | B) = 3

$\therefore$ a can take any real value except 5.

Closest correct answer is (d).

6.31 (a)

Eigen values of $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

$$\begin{vmatrix} 1-\lambda & 0 \\ 0 & 0-\lambda \end{vmatrix} = 0$$

$$(1-\lambda) * -\lambda = 0$$

$$\lambda = 0 \text{ or } \lambda = 1$$

Eigen values of $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$$\begin{bmatrix} -\lambda & 1 \\ 0 & -\lambda \end{bmatrix} = 0$$

$$\lambda^2 = 0$$

$$\lambda = 0, 0$$

Eigen values of $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

$$\begin{bmatrix} 1-\lambda & -1 \\ 1 & 1-\lambda \end{bmatrix} = 0$$

$$(1-\lambda)^2 + 1 = 0$$

$$(1-\lambda)^2 = -1$$

$$1-\lambda = i \text{ or } -i$$

$$\lambda = 1-i \text{ or } 1+i$$

Eigen values of $\begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix}$

$$\begin{bmatrix} -1-\lambda & 0 \\ 1 & -1-\lambda \end{bmatrix}$$

$$(-1-\lambda)(-1-\lambda) = 0$$

$$(1+\lambda)^2 = 0$$

$$\lambda = -1, -1$$

Only one matrix has an eigen value of 1 which is

$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$.

Correct choice is (a).

6.32 (d)

S1 and S2:

Since M has zero determinant, its rank is not full i.e. if M is of size 3 × 3, then its rank is not 3. So there is a linear combination of rows which evaluates to 0 i.e. $k_1R_1 + k_2R_2 + ... + k_nR_n = 0$ and there is a linear combination of columns which evaluates to 0 i.e.

$$k_1C_1 + k_2C_2 + ... + k_nC_n = 0$$

Now any row R_i can be written as linear combination of other rows as:

$$R_i = \frac{k_1}{k_i}R_1 - \frac{k_2}{k_i}R_2 - ...$$

$$-\frac{k_{i-1}}{k_i}R_{i-1} - \frac{k_{i+1}}{k_i}R_{i+1} - ... - \frac{k_n}{k_i}R_n$$

Similar is the case for columns.

So S1 and S2 are true.

S3: MX = O is a homogenous equation and such an equation when |M| = 0 has non-trivial solution.

S4: Since M is singular, M has no inverse. So S4 is false.

So S1, S2 and S3 only are true.

6.33 (d)

Sum of eigen values = Trace (A) = 2 + y

Product of eigen values = |A| = 2y – 3x

$$\therefore \quad 4 + 8 = 2 + y \quad \text{... (i)}$$

$$4 \times 8 = 2y - 3x \quad \text{... (ii)}$$

$$\therefore \quad 2 + y = 12 \quad \text{... (i)}$$

$$2y - 3x = 32 \quad \text{... (ii)}$$

∴ Solving (i) and (ii) we get x = –4 and y = 10.

6.34 (a)

Since the given matrix is upper triangular, its eigen values are the diagonal elements themselves, which are 1, 4 and 3.

6.35 (d)

$$A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Eigen (A) are the roots of the characteristic polynomial given below:

$$\begin{vmatrix} 1-\lambda & 1 \\ 1 & -1-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(-1-\lambda) - 1 = 0$$

$$-(1-\lambda)(1+\lambda) - 1 = 0$$

$$\lambda^2 - 2 = 0$$

$$\lambda = \pm\sqrt{2}$$

Eigen values of A are $\sqrt{2}$ and $-\sqrt{2}$ respectively.

So eigen values of $A^{19} = \left(\sqrt{2}\right)^{19}$ and $\left(-\sqrt{2}\right)^{19}$

$$= 2^{19/2} \text{ and } -2^{19/2}$$

$$= 2^9.2^{1/2} \text{ and } -2^9.2^{1/2}$$

$$= 512\sqrt{2} \text{ and } -512\sqrt{2}.$$

6.36 (a)

The given matrix can be transformed into the matrix given in options (b)(c) and (d) by elementry operations of the type of $R_i \pm kR_j$ or $C_i \pm kC_j$ only as shown below:

Option (b):

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} \xrightarrow[C_3+C_1]{C_2+C_1} \begin{vmatrix} 1 & x+1 & x^2+1 \\ 1 & y+1 & y^2+1 \\ 1 & z+1 & z^2+1 \end{vmatrix}$$

Option (c):

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} \xrightarrow[R_2-R_3]{R_1-R_2} \begin{vmatrix} 0 & x-y & x^2-y^2 \\ 0 & y-z & y^2-z^2 \\ 1 & z & z^2 \end{vmatrix}$$

Option (d):

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} \xrightarrow[R_2+R_3]{R_1+R_2} \begin{vmatrix} 2 & x+y & x^2+y^2 \\ 2 & y+z & y^2+z^2 \\ 1 & z & z^2 \end{vmatrix}$$

Option (a): We can show the given matrix can not be converted into option (a) without doing a column exchange which will change the sign of the determinant as can be seen below:

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} \xrightarrow[C_3+C_2]{C_2+C_1} \begin{vmatrix} 1 & x+1 & x(x+1) \\ 1 & y+1 & y(y+1) \\ 1 & z+1 & z(z+1) \end{vmatrix}$$

$$= -\begin{vmatrix} 1 & x(x+1) & x+1 \\ 1 & y(y+1) & y+1 \\ 1 & z(z+1) & z+1 \end{vmatrix}$$

6.37 Sol.

Performing gauss elimination on the augmented matrix shown below:

$$\left[\begin{array}{ccc|c} 3 & 2 & 0 & 1 \\ 4 & 0 & 7 & 1 \\ 1 & 1 & 1 & 3 \\ 1 & -2 & 7 & 0 \end{array}\right]$$

We can reduce it to

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -1 & -3 & -8 \\ 0 & 0 & 15 & 21 \\ 0 & 0 & 15 & 21 \end{array}\right] \xrightarrow{R_4-R_3} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -1 & -3 & -8 \\ 0 & 0 & 15 & 21 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Rank (A) = 3

Rank (A/B) = 3

Rank (A) = Rank (A/B) = Number of variables = 3.

So unique solution.

So, number of solutions = 1.

6.38 Sol.

The value of the dot product of the eigenvectors corresponding to any pair of different eigen values of any symmetric positive definite matrix is 0.

6.39 Sol.

$$A = \begin{bmatrix} 2 \\ -4 \\ 7 \end{bmatrix} [1 \ \ 9 \ \ 5]$$

$A = [X]_{3\times1} [Y]_{1\times3}$

So size of A is $[3 \times 3]$

$$A = \begin{bmatrix} 2 & 18 & 10 \\ -4 & -36 & -20 \\ 7 & 63 & 35 \end{bmatrix}$$

Since $R_2 = -2R_1$, rows are linearly dependent, $|A| = 0$.

6.40 Sol.

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}, \text{ let } X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$$

$\boldsymbol{Ak = Xk}$

$\Rightarrow x_1 + x_5 = kx_1 = kx_5$

$\Rightarrow x_2 + x_3 + x_4 = kx_2 = kx_3 = kx_4$

(i) $k \neq 0$

say, $x_1 = x_5 = a$

$x_2 = x_3 = x_4 = b$

$\Rightarrow x_1 + x_5 = kx_1 \Rightarrow 2a = ka$

$\Rightarrow k = 2$

$\Rightarrow x_2 + x_3 + x_4 = kx_2 \Rightarrow 3b = kb$

$\Rightarrow k = 3$

(ii) $k = 0$

$\Rightarrow$ Eigen value $k = 0$

$\therefore$ There are 3 distinct eigen values: 0,2,3

Product of non-zero eigen values: $2 \times 3 = 6$

6.41 (a)

Sum (eigen values) = Trace (A)
Product of eigen values = |A|

(a) If the trace of the matrix is positive and the determinant of the matrix is negative, at least one of its eigenvalues is negative.
This is true, since if this was is not true then all eigen values would be positive and determinant also therefore positive.

(b) If the trace of the matrix is positive, all its eigenvalues are positive.
This is false, since the sum of positive and negative eigen values could also be positive.

(c) If the determinant of the matrix is positive, all its eigenvalues are positive.
This is false, since product of 2 negative numbers can be positive.

(d) If the product of the trace and determinant of the matrix is positive, all its eigenvalues are positive.

Trace and determinant both could be negative to give a positive product and determinant is negative then all eigen values need not be positive.

6.42 Sol.

$V = \{a, b, c, d, e, f\}$
$\Rightarrow V_1 = \{a, b, c, d\}$ and $V_2 = \{e, f, \square, \square\}$
Smallest possible dimension of $V_1 \cap V_2 = 2$

6.43 Sol.

$P(X) = X^5 + 4X^3 + 6X + 5$
$= X(X^4 + 4X^2 + 6) + 5 = X(X(X^3 + 4X) + 6) + 5$
$= X(X(X(X^2 + 4)) + 6) + 5$
Only one temporary variable can be used.
temp = X*X
temp = temp + 4;
temp = X*temp;
temp = X*temp;
temp = temp + 6;
temp = X*temp;
temp = temp + 5
∴ 7 arithmetic operations needed.

6.44 (d)

Trace = Sum of eigen values
1 + a = 6
⇒ **a = 5**
Determinant = Product of eigen values
(a − 4b) = −7
5 − 4b = −7
−4b = −12
⇒ **b = 3**
∴ a = 5, b = 3

6.45 Sol.

$|A - \lambda I| = 0$

$$\begin{vmatrix} 4-\lambda & 5 \\ 2 & 1-\lambda \end{vmatrix} = (4-\lambda)(1-\lambda) - 10 = 0$$

$\lambda^2 - 5\lambda - 6 = 0$
$(\lambda - 6)(\lambda + 1) = 0$
$\Rightarrow \lambda = 6, -1$
∴ The Larger eigen value is '6'.

6.46 Sol.

Since operations 1 and 2 are elementary operations of the type of $R_i \pm kR_j$ and $C_i \pm kC_j$ respectively, the determinant will be unchanged from the original determinant.

So the required determinant

$$= \begin{vmatrix} 3 & 4 & 45 \\ 7 & 9 & 105 \\ 13 & 2 & 195 \end{vmatrix}$$

$$\begin{vmatrix} 3 & 4 & 45 \\ 7 & 9 & 105 \\ 13 & 2 & 195 \end{vmatrix} \xrightarrow{C_3 - 15C_1} \begin{vmatrix} 3 & 4 & 0 \\ 7 & 9 & 0 \\ 13 & 2 & 0 \end{vmatrix} = 0$$

So the required determinant = 0.

6.47 (b)

$$\text{Let } A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

Given eigen value $\lambda = 1$.
Lex X be the vector. Then $(A - \lambda I)X = 0$

$$\begin{bmatrix} 1-\lambda & -1 & 2 \\ 0 & 1-\lambda & 0 \\ 1 & 2 & 1-\lambda \end{bmatrix} X = 0$$

put $\lambda = 1$

$$\begin{bmatrix} 0 & -1 & 2 \\ 0 & 0 & 0 \\ 1 & 2 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \Rightarrow \begin{bmatrix} -x_2 + 2x_3 \\ 0 \\ x_1 + 2x_2 \end{bmatrix} = 0$$

putting $x_1 = k$
we get $x_2 = -k/2$ and $x_3 = -k/4$.

So the eigen vector = $k\begin{bmatrix} 1 \\ -1/2 \\ -1/4 \end{bmatrix}$

The ratios are $x_1/x_2 = \dfrac{-1}{-1/2} = -2$

and $x_2/x_3 = \dfrac{-1/2}{-1/4} = 2$

Only option (b) (–4, 2, 1) has the same ratios and therefore is a correct eigen vector.

or

Alternate Method:

$AX = \lambda X$

Check one-by-one each eigen vector until the equation is satisfied. For *example*,

Choice (a): $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix} = 1 \times \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix}$ **(false)**

Choice (b): $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} -4 \\ 2 \\ 1 \end{bmatrix} = 1 \times \begin{bmatrix} -4 \\ 2 \\ 1 \end{bmatrix}$ **(true)**

and so on...

only choice (b) satisfies and so it is the correct answer.

6.48 Sol.

$$\begin{bmatrix} L_{11} & 0 \\ L_{21} & L_{22} \end{bmatrix}\begin{bmatrix} 1 & U_{12} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 4 & 9 \end{bmatrix}$$

This is crout's LU decomposition, since diagonal elements of U are 1. So we will setup the equations for the elements of the matrix taken column-wise, as follows

$L_{11} = 2,\ L_{21} = 4$

$L_{11} \times U_{12} = 2$

$\Rightarrow \quad U_{12} = 1,$

$L_{21} \times U_{12} + L_{22} = 9$

$\Rightarrow \quad 4 + L_{22} = 9$

$\Rightarrow \quad L_{22} = 5$

6.49 (c)

$px + qy + rz = 0$

$qx + ry + pz = 0$

$rx + py + qz = 0$

Let $A = \begin{bmatrix} p & q & r \\ q & r & p \\ r & p & q \end{bmatrix}$. The system is $A\hat{x} = O$

This is a homogenous system. Such a system has non-trivial solution iff $|A|=0$.

So, $\begin{vmatrix} p & q & r \\ q & r & p \\ r & p & q \end{vmatrix} = 0$

$p(qr - p^2) - q(q^2 - pr) + r(pq - r^2) = 0$

$p^3 + q^3 + r^3 - 3pqr = 0$

p = q = r satisfies the above equation.

Also if $p + q + r = 0$ then a can be transformed into one of the row as completely 0's as shown below.

$$\begin{vmatrix} p & q & r \\ q & r & p \\ r & p & q \end{vmatrix} \xrightarrow{R_1+R_2+R_3} \begin{vmatrix} p+q+r & p+q+r & p+q+r \\ q & r & p \\ r & p & q \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 0 & 0 \\ q & r & p \\ r & p & q \end{vmatrix} = 0$$

Therefore the correct option is (c) which is $p + q + r = 0$ or $p = q = r$.

6.50 Sol.

Two eigen values are $2 + i$ and 3 of a 3×3 matrix. The third eigen value must be $2 - i$

Now $\Pi\lambda_i = |A|$

$\Rightarrow \quad |A| = (2 + i)(2 - i) \times 3 = (4 - i^2) \times 3 = 5 \times 3 = 15$

6.51 (c)

I. $m < n$ (system may still be inconsistent so incorrect)

II. $m > n$ (rank may still be equal to n of hence solution may exist so incorrect).

III. $m = n$ (some system rank may be equal to n and hence may have solution so correct).

So only III is correct.

6.52 Sol.

Eigen(A) = 1, 2, 4 $\Rightarrow |A| = 1 \times 2 \times 4 = 8$

Now, $\left|\left(A^{-1}\right)^t\right| = \left|A^{-1}\right| = \dfrac{1}{|A|} = \dfrac{1}{8} = 0.125$

■■■■

7 Calculus

7.1 If at every point of a certain curve, the slope of the tangent equals $\frac{-2x}{y}$ the curve is

(a) A straight line (b) A parabola
(c) A circle (d) An ellipse

[1995 : 1 Mark]

7.2 The formula used to compute an approximation for the second derivative of a function f at a point x_0 is

(a) $\frac{f(x_0+h)+f(x_0-h)}{2}$

(b) $\frac{f(x_0+h)-f(x_0-h)}{2h}$

(c) $\frac{f(x_0+h)+2f(x_0)+f(x_0-h)}{h^2}$

(d) $\frac{f(x_0+h)-2f(x_0)+f(x_0-h)}{h^2}$

[1996 : 1 Mark]

7.3 What is the maximum value of the function $f(x) = 2x^2 - 2x + 6$ in the interval [0, 2]?

(a) 6 (b) 10
(c) 12 (d) 5.5

[1997 : 2 Marks]

7.4 Consider the function $y = |x|$ in the interval [–1, 1]. In this interval, the function is

(a) continuous and differentiable
(b) continuous but not differentiable
(c) differentiable but not continuous
(d) neither continuous nor differentiable

[1998 : 1 Mark]

7.5 What is the value of $\int_0^{2\pi} (x-\pi)^3(\cos x)dx$

(a) –1 (b) 0
(c) 1 (d) π

[IT-2005 : 2 Marks]

7.6 The following definite integral evaluates to

$$\int_{-\infty}^{0} e^{-\left(\frac{x^2}{2}\right)}dx$$

(a) 1/2 (b) $\pi\sqrt{10}$
(c) $\sqrt{\pi/2}$ (d) π

[IT-2006 : 2 Marks]

7.7 If $f(x)$ is defined as follows, what is the minimum value of $f(x)$ for $x \in (0, 2]$?

$$f(x) = \begin{cases} \frac{25}{8x} \text{ when } x \le \frac{3}{2} \\ x + \frac{1}{x} \text{ otherwise} \end{cases}$$

(a) 2 (b) 2(1/12)
(c) 2(1/6) (d) 2(1/2)

[IT-2008 : 2 Marks]

7.8 Consider the following two statements about the function $f(x) = |x|$

P: f(x) is continuous for all real values of x
Q: f(x) is differentiable for all real values of x

Which of the following is TRUE?

(a) P is true and Q is false
(b) P is false and Q is true
(c) Both P and Q are true
(d) Both P and Q are false

[2007 : 1 Mark]

7.9 $\lim_{x\to\infty} \frac{x - \sin x}{x + \cos x}$ equals

(a) 1 (b) –1
(c) ∞ (d) $-\infty$

[2008 : 1 Mark]

7.10 A point on a curve is said to be an extremum if it is a local minimum or a local maximum. The number of distinct extrema for the curve $3x^4 - 16x^3 + 4x^2 + 37$ is

(a) 0 (b) 1
(c) 2 (d) 3

[2008 : 2 Marks]

7.11 $\int_0^{\pi/4} (1-\tan x)/(1+\tan x)\,dx$ evaluates to

(a) 0 (b) 1
(c) ln 2 (d) 1/2 ln 2

[2009 : 2 Marks]

7.12 What is the value of $\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^{2n}$?

(a) 0 (b) e^{-2}
(c) $e^{-1/2}$ (d) 1

[2010 : 1 Mark]

7.13 Given $i=\sqrt{-1}$, what will be the evaluation of the definite integral

$$\int_0^{\pi/2}\frac{\cos x+i\sin x}{\cos x-i\sin x}dx\ ?$$

(a) 0 (b) 2
(c) –i (d) i

[2011 : 2 Marks]

7.14 Consider the function f(x) = sin (x) in the interval $x\in[\pi/4, 7\pi/4]$. The number and location(s) of the local minima of this function are

(a) One, at $\pi/2$
(b) One, at $3\pi/2$
(c) Two, at $\pi/2$ and $3\pi/2$
(d) Two, at $\pi/4$ and $3\pi/2$

[2012 : 1 Mark]

7.15 Which one of the following functions is continuous at x = 3?

(a) $f(x)=\begin{cases}2, & \text{if } x=3\\ x-1, & \text{if } x>3\\ \frac{x+3}{3}, & \text{if } x<3\end{cases}$

(b) $f(x)=\begin{cases}4, & \text{if } x=3\\ 8-x & \text{if } x\neq 3\end{cases}$

(c) $f(x)=\begin{cases}x+3, & \text{if } x\leq 3\\ x-4 & \text{if } x>3\end{cases}$

(d) $f(x)=\frac{1}{x^3-27}$, if $x\neq 3$

[2013 : 1 Mark]

7.16 Let the function

$$f(q)=\begin{vmatrix}\sin\theta & \cos\theta & \tan\theta\\ \sin(\pi/6) & \cos(\pi/6) & \tan(\pi/6)\\ \sin(\pi/3) & \cos(\pi/3) & \tan(\pi/3)\end{vmatrix}$$

where $\theta\in\left[\frac{\pi}{6},\frac{\pi}{3}\right]$ and $f'(\theta)$ denote the derivative of f with respect to θ. Which of the following statements is/are TRUE?

I: There exists $\theta\in\left(\frac{\pi}{6},\frac{\pi}{3}\right)$ such that $f'(\theta)=0$.

II: There exists $\theta\in\left(\frac{\pi}{6},\frac{\pi}{3}\right)$ such that $f'(\theta)\neq 0$.

(a) I only (b) II only
(c) Both I and II (d) Neither I nor II

[2014 (Set-1) : 1 Mark]

7.17 The function $f(x)=x\sin x$ satisfies the following equation: $f''(x)+f(x)+t\cos x=0$. The value of t is ________.

[2014 (Set-1) : 2 Marks]

7.18 A function $f(x)$ is continuous in the interval [0, 2]. It is known that $f(0)=f(2)=-1$ and $f(1)=1$. Which one of the following statements must be true?

(a) There exists a y in the interval (0, 1) such that $f(y)=f(y+1)$
(b) For every y in the interval (0, 1), $f(y)=f(2-y)$
(c) The maximum value of the function in the interval (0, 2) is 1
(d) There exists a y in the interval (0, 1) such that $f(y)=-f(2-y)$

[2014 (Set-1) : 2 Marks]

7.19 A non-zero polynomial $f(x)$ of degree 3 has roots at $x=1$, $x=2$ and $x=3$. Which one of the following must be TRUE?

(a) $f(0)f(4)<0$ (b) $f(0)f(4)>0$
(c) $f(0)+f(4)>0$ (d) $f(0)+f(4)<0$

[2014 (Set-2) : 1 Mark]

7.20 If $\int_0^{2\pi}|x\sin x|dx=k\pi$, then the value of k is equal to ________.

[2014 (Set-3) : 1 Mark]

7.21 The value of the integral given below is

$$\int_0^{\pi}x^2\cos x\,dx$$

(a) -2π (b) π
(c) $-\pi$ (d) 2π

[2014 (Set-3) : 2 Marks]

7.22 $\lim_{x\to\infty}x^{1/x}$ is

(a) ∞ (b) 0
(c) 1 (d) Not defined

[2015 (Set-1) : 1 Mark]

7.23 $\int_{1/\pi}^{2/\pi} \frac{\cos(1/x)}{x^2} dx$ = ________.

[2015 (Set-1) : 2 Marks]

7.24 Let $f(x) = x^{-(1/3)}$ and A denote the area of the region bounded by $f(x)$ and the X-axis, when x varies from –1 to 1. Which of the following statements is/are True?

1. f is continuous in [–1, 1]
2. f is not bounded in [–1, 1]
3. A is nonzero and finite

(a) 2 only (b) 3 only
(c) 2 and 3 only (d) 1, 2 and 3

[2015 (Set-2) : 2 Marks]

7.25 The value of $\lim_{x\to\infty}(1+x^2)^{e^{-x}}$ is

(a) 0 (b) 1/2
(c) 1 (d) ∞

[2015 (Set-3) : 1 Mark]

7.26 If for non-zero x, $af(x) + bf\left(\frac{1}{x}\right) = \frac{1}{x} - 25$ where $a \neq b$ then $\int_1^2 f(x)dx$ is

(a) $\frac{1}{a^2-b^2}\left[a(\ln 2 - 25) + \frac{47b}{2}\right]$

(b) $\frac{1}{a^2-b^2}\left[a(2\ln 2 - 25) - \frac{47b}{2}\right]$

(c) $\frac{1}{a^2-b^2}\left[a(2\ln 2 - 25) + \frac{47b}{2}\right]$

(d) $\frac{1}{a^2-b^2}\left[a(\ln 2 - 25) - \frac{47b}{2}\right]$

[2015 (Set-3) : 2 Marks]

7.27 $\lim_{x\to 4}\frac{\sin(x-4)}{x-4}$ = ______.

[2016 (Set-1) : 1 Mark]

7.28 Let $f(x)$ be a polynomial and $g(x) = f'(x)$ be its derivative. If the degree of $(f(x) + f(-x))$ is 10, then the degree of $(g(x) - g(-x))$ is______.

[2016 (Set-2) : 1 Mark]

■■■■

Answers Calculus

7.1	(d)	7.2	(d)	7.3	(b)	7.4	(b)	7.5	(b)	7.6	(c)	7.7	(b)	7.8	(a)	7.9	(a)
7.10	(b)	7.11	(d)	7.12	(b)	7.13	(d)	7.14	(b)	7.15	(a)	7.16	(c)	7.18	(a)	7.19	(a)
7.21	(a)	7.22	(c)	7.24	(c)	7.25	(c)	7.26	(a)								

Explanations Calculus

7.1 (d)

Given that at every point of a certain curve, the slope of tangent is $-\frac{2x}{y}$

i.e., $\frac{dy}{dx} = -\frac{2x}{y}$

$ydy = -2xdx$

Integrating both side

$$\frac{y^2}{2} = -2\times\frac{x^2}{2} + c$$

$$\frac{y^2}{2} = -x^2 + c$$

$$\frac{y^2}{2} + x^2 = c$$

It is in the form of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = c$

So the curve is an ellipse with a = 1 and b = $\sqrt{2}$.

7.2 (d)

The formula used to compute an approximation of first derivative of a function f at a point x_0 is

$$f'(x_0) = \frac{f(x_0+h) - f(x_0)}{h}$$

So an approximation for the second derivative of a function f at a point x_0 is

$$f''(x_0) = \frac{f'(x_0) - f'(x_0 - h)}{h}$$

$$= \frac{\left[\left(\frac{f(x_0+h) - f(x_0)}{h}\right) - \left(\frac{f(x_0) - f(x_0 - h)}{h}\right)\right]}{h}$$

$$= \frac{f(x_0+h) - 2f(x_0) + f(x_0 - h)}{h^2}$$

7.3 (b)

We need absolute maximum of

$$f(x) = 2x^2 - 2x + 6$$

in the interval [0, 2]

First find local maximum if any by putting $f'(x) = 0$

$$f'(x) = 4x - 2 = 0$$

$$4x = 2$$

$$x = \frac{1}{2}$$

$$f''(x) = 4 > 0$$

So, $x = \frac{1}{2}$

is a point of local minimum. So there is no point of local maximum.

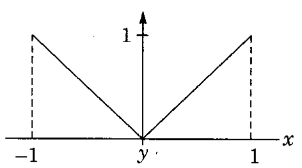
Now tabulate the values of f at end point of interval and at local maximum if any (in this case no point of local maximum).

x	$f(x)$
0	6
2	10

Clearly the absolute maxima is at $x = 2$ and absolute maximum value is 10.

7.4 (b)

The function y = |x| in the interval [–1, 1] is



| x | is continuous and differentiable every where except at x = 0, where it is continuous but not differentiable.

Since [–1, 1] contains 0, in this interval it is continuous but not differentiable.

7.5 (b)

In the integral $\int_0^{2\pi} (x-\pi)^3 \cos x \, dx$

$f(x) = (x - \pi)^3 \cos x$

$f(2\pi - x) = (2\pi - x - \pi)^3 \cos(2\pi - x)$

$= (\pi - x)^3 \cos x = -(x - \pi)^3 \cos x = -f(x)$

So $\int_0^{2\pi} (x-\pi)^3 \cos x \, dx = 0$

Since

$\int_0^{2\pi} f(x) dx = 0$ whenever $f(2\pi - x) = -f(x)$

7.6 (c)

$$I = \int_{-\infty}^{0} e^{(-x^2/2)} dx$$

Comparing with area under the standared normal curve from $-\infty$ to 0.

We get

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0} e^{-\frac{x^2}{2}} dx = 0.5$$

So, the required integral

$$\int_{-\infty}^{0} e^{-\frac{x^2}{2}} dx = 0.5 \times \sqrt{2\pi}$$

$$= \sqrt{\pi/2}$$

which is option (c).

7.7 (b)

For the function 25/8x the minimum value will come when x is maximum since it is a decreasing function.

The maximum value of x is 3/2.

At x = 3/2 the function has the value

$f(x) = 25/(8*3/2)$

$= 2.0833 = 2 + (1/12)$

So, $\infty \geq f(x) \geq 2(1/12)$ for $x \leq 3/2$

$x + 1/x \geq 2$

But since for this function $x \geq 3/2$, putting 3/2 in this function we get the minimum of this function which is

3/2+1/(3/2) = 13/6 = 2.166

Now comparing the minimum value 2.0833 of the first function with minimum value 2.166 of the second function, we get the overall minimum of this function to be 2.0833 = 2(1/12) which is option (b).

7.8 (a)

$|x|$ is continuous and differentiable every where except at $x = 0$, where it is continuous but not differentiable.
Since 0 is a real number, for all real values it is continuous but not differentiable.

7.9 (a)

$$\lim_{x\to\infty}\frac{x-\sin x}{x+\cos x} = \lim_{x\to\infty}\frac{1-\sin x/x}{1+\cos x/x}$$

$$= \frac{\lim_{x\to\infty}(1-\sin x/x)}{\lim_{x\to\infty}(1+\cos x/x)}$$

$$= \frac{1-\lim_{x\to\infty}\frac{\sin x}{x}}{1+\lim_{x\to\infty}\frac{\cos x}{x}}$$

$$= \frac{1-0}{1+0} = 1$$

7.10 (b)

$$y = 3x^4 - 16x^3 + 24x^2 + 37$$

$$\frac{dy}{dx} = 12x^3 - 48x^2 + 48x = 0$$

$$x(12x^2 - 48x + 48) = 0$$

$$x = 0$$

or $12x^2 - 48x + 48 = 0$

$$x^2 - 4x + 4 = 0$$

$$(x-2)^2 = 0$$

$$\frac{d^2y}{dx^2} = 36x^2 - 96x + 48$$

Now at $x = 0$

$$\frac{d^2y}{dx^2} = 48 > 0$$

$\therefore$ f(x) has a minimum at $x = 0$

$$\left[\frac{d^2y}{dx^2}\right]_{x=2} = 0 \text{ and } \left[\frac{d^3y}{dx^2}\right]_{x=2} = 48 \neq 0$$

So $x = 2$ is a saddle point (point of inflection)
$\therefore$ f(x) has no extremum at $x = 2$. So f(x) has only one point of extremum (at $x = 0$).

7.11 (d)

Since

$$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

$$\therefore \quad I = \int_0^{\pi/4}\frac{1-\tan x}{1+\tan x}dx$$

$$= \int_0^{\pi/4}\frac{1-\tan\left(\frac{\pi}{4}-x\right)dx}{1+\tan\left(\frac{\pi}{4}-x\right)}$$

Since $\tan(A-B) = \dfrac{\tan A - \tan B}{1+\tan A\tan B}$

$$\therefore \quad I = \int_0^{\pi/4}\frac{1-\left[\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right]}{1+\left[\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right]}dx$$

$$= \int_0^{\pi/4}\frac{1-\left[\frac{1-\tan x}{1+\tan x}\right]}{1+\left[\frac{1-\tan x}{1+\tan x}\right]}dx$$

$$= \int_0^{\pi/4}\frac{(1+\tan x)-(1-\tan x)}{(1+\tan x)+(1-\tan x)}dx$$

$$= \int_0^{\pi/4}\frac{2\tan x}{2}dx$$

$$= \int_0^{\pi/4}\tan x\,dx$$

$$= \left[\log(\sec x)\right]_0^{\pi/4}$$

$$= ln\left(\sec\frac{\pi}{4}\right) - ln(\sec 0)$$

$$= ln(\sqrt{2}) - ln(1)$$

$$= ln(2^{1/2}) - 0 = \frac{1}{2}ln 2$$

7.12 (b)

$$\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^{2n} = \lim_{n\to\infty}\left[\left(1-\frac{1}{n}\right)^n\right]^2$$

$$= \left[\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^n\right]^2$$

Now use the standard limit:

$$\lim_{x\to\infty}\left(1+\frac{a}{x}\right)^x = e^a$$

Required limit $= (e^{-1})^2 = e^{-2}$

7.13 (d)

According to **Demoivre's** theorem
$\cos x + i \sin x = e^{ix}$

$$\text{So, } \int_0^{\pi/2} \frac{\cos x + i\sin x}{\cos x - i\sin x}dx = \int_0^{\pi/2} \frac{e^{ix}}{e^{-ix}}dx$$

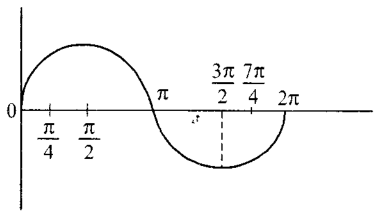
$$= \int_0^{\pi/2} e^{2ix}\,dx$$

$$= \left[\frac{e^{2ix}}{2i}\right]_0^{\pi/2} = \frac{1}{2i}\left[e^{i\pi} - e^{o}\right]$$

$$= \frac{1}{2i}[-1-1](\text{since } e^{i\pi} = \cos\pi + i\sin\pi = -\pi)$$

$$= \frac{-2}{2i} = \frac{-1}{i} = i$$

7.14 (b)



From the plot of sin x given above, we can easily see that in the range [π/4, 7π/4], there is only one local minima, at 3π/2.

7.15 (a)

$$\begin{cases} 2, & \text{if } x = 3 \\ x-1, & \text{if } x > 3 \\ \dfrac{x+3}{3}, & \text{if } x < 3 \end{cases}$$

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \frac{x+3}{3}$$

$$= \frac{3+3}{3} = 2$$

$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} x-1$$

$$= 3 - 1 = 2$$

Also, $f(3) = 2$

So, $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x) = f(3)$

So it is continuous at x = 3

Option (a) is correct.

7.16 (c)

$$f(\theta) = \begin{vmatrix} \sin\theta & \cos\theta & \tan\theta \\ \sin(\pi/6) & \cos(\pi/6) & \tan(\pi/6) \\ \sin(\pi/3) & \cos(\pi/3) & \tan(\pi/3) \end{vmatrix}$$

$f(\pi/6) = 0$

Since if we put θ = π/6 in above determinant it will evaluate to zero, since I and II row will become same.

$f(\pi/3) = 0$

Since if we put θ = π/3 in above determinant it will evaluate to zero, since I and III row will become same.

So f(π/6) = f(π/3). Also in the interval [π/6, π/3] the function f(θ).is continuous and differentiable (***note*** that the given interval doesn't contain any odd multiple of π/2 where tan θ is neither continuous nor differentiable).

Since all the three conditions of **Roll's theorem** are satisfied the conclusion of Rolls theorem is true i.e.

I: $\exists\theta \in \left(\frac{\pi}{6}, \frac{\pi}{3}\right)$ such that $f'(\theta) = 0$ is true

Now the statement

II: $\exists\theta \in \left(\frac{\pi}{6}, \frac{\pi}{3}\right)$ such that $f'(\theta) \neq 0$

is also true, since the only way it can be false is if $f'(\theta) = 0$ for all values of θ, which is possible only if f(θ) is a constant which is untrue.

Therefore, both (I) and (II) are correct.

7.17 Sol.

$f(x) = x \sin x$
$f'(x) = x \cos x + \sin x$
$f''(x) = (-x \sin x + \cos x) + \cos x$
$f''(x) + f(x) + t\cos x = 0$
$\Rightarrow -x \sin x + \cos x + \cos x + x \sin x + t \cos x = 0$
$\Rightarrow (2 + t) \cos x = 0$
$\Rightarrow t + 2 = 0$
$\Rightarrow t = -2$

7.18 (a)

Consider option (a)

(a) There exists a y in the interval (0, 1) such that $f(y) = f(y + 1)$.

This can be rewritten as

There exists a y in the interval (0, 1) such that

$$g(y) = f(y) - f(y+1) = 0$$

Now intermediate value theorem says that the above statement is true, if $g(0)\,g(1) < 0$.

Now we check $g(0)\,g(1)$

$$g(0) = f(0) - f(1) = -1 - 1 = -2$$
$$g(1) = f(1) - f(2) = 1 - (-1) = 2$$
$$g(0)\,g(1) = -2 \times 2 = -4 < 0$$

So by intermediate value theorem, option (a) is true.

7.19 (a)

$x = 1$, $x = 2$ and $x = 3$ are roots.

$f(x) = \alpha(x-1)(x-2)(x-3)$

$\Rightarrow \quad f(0) = \alpha(0-1)(0-2)(0-3) = -6\alpha$

$\Rightarrow \quad f(4) = \alpha(4-1)(4-2)(4-3) = 6\alpha$

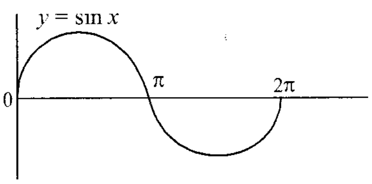
$\quad f(0) \times f(4) = -36\,\alpha^2 < 0$

$\therefore \quad f(0) \times f(4) < 0$

Alternate method

Since polynomials are continuous and since there is a root (actually 3 roots) in the interval (0, 4), by intermediate value theorem, surely $f(0) \times f(4) < 0$.

7.20 Sol.



$$\Rightarrow \int_0^{2\pi} |x \sin x|\, dx = k\pi$$

$$\Rightarrow \int_0^{\pi} |x \sin x|\, dx + \int_{\pi}^{2\pi} |x \sin x|\, dx = k\pi$$

$$\Rightarrow \int_0^{\pi} x \sin x\, dx + \int_{\pi}^{2\pi} -(x \sin x)\, dx = k\pi$$

$$\Rightarrow (-x\cos x + \sin x)\Big|_0^{\pi} - (-x\cos x + \sin x)\Big|_{\pi}^{2\pi} = k\pi$$

$$\Rightarrow 4\pi = k\pi$$

$$\Rightarrow k = 4$$

7.21 (a)

Integrating by parts once we get,

$$\int_0^{\pi} x^2 \cos x\, dx = [x^2(\sin x)]_0^{\pi} - 2\int_0^{\pi} x \sin x\, dx$$

$$= 0 - 2\int_0^{\pi} x \sin x\, dx = -2\int_0^{\pi} x \sin x\, dx = -2\pi$$

[Since, integrating by parts again we get

$$\int_0^{\pi} x \sin x\, dx = (-x\cos x + \sin x)\Big|_0^{\pi} = \pi]$$

7.22 (c)

$$y = \lim_{x\to\infty} x^{1/x}$$

$$\log y = \lim_{x\to\infty} \log x^{1/x}$$

$$\log y = \lim_{x\to\infty} \frac{\log x}{x}$$

∞/∞ form, use L' Hospital's rule

$$\log y = \lim_{x\to\infty} \frac{1/x}{1}$$

$$\log y = 0 \Rightarrow y = e^0 = 1$$

7.23 Sol.

$$\int_{1/\pi}^{2/\pi} \frac{\cos(1/x)}{x^2}\, dx = (-1)\int_{1/\pi}^{2/\pi} \left(\frac{-1}{x^2}\right)\cdot \cos\left(\frac{1}{x}\right)$$

Let $\dfrac{1}{x} = t \Rightarrow dt = -1/x^2$

Substituting in the integral and change limits we get,

$$(-1)\int_{\pi}^{\pi/2} \cos t\, dt = \int_{\pi/2}^{\pi} \cos t\, dt$$

$$= [\sin t]_{\pi/2}^{\pi} = (0 - 1) = -1$$

7.24 (c)

$$f(x) = \frac{1}{\sqrt[3]{x}}$$

Statement 1: f is continuous in [–1, 1]. Let us check this statement.

We need to check continuity at x = 0

$$\text{Let limit} = \lim_{x\to 0^-} \frac{1}{\sqrt[3]{x}} = \lim_{h\to 0} \frac{1}{\sqrt[3]{0-h}}$$

$$= \lim_{h\to 0} \frac{-1}{\sqrt[3]{h}} = -\infty$$

$$\text{Right limit} = \lim_{x\to 0^+} \frac{1}{\sqrt[3]{x}} = \lim_{h\to 0} \frac{1}{\sqrt[3]{0+h}}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt[3]{h}} = +\infty$$

[illegible] ht limit

[illegible] s false.

Statement 2: f is not bounded is [–1, 1]. Since at x = 0 it goes to $-\infty$ and $+\infty$ the function is not bounded.

$\therefore$ Statement 2 is true.

Statement 3: A is non zero and finite.

$$A = \left|\int_{-1}^{0} x^{-1/3}dx\right| + \left|\int_{0}^{1} x^{-1/3}dx\right|$$

$$= \left|\frac{3}{2}\left[x^{2/3}\right]_{-1}^{0}\right| + \left|\frac{3}{2}\left[x^{2/3}\right]_{0}^{1}\right|$$

$$= \left|\frac{3}{2}\right| + \left|\frac{3}{2}\right| = 3$$

So A is non zero and finite.

$\therefore$ Statement 3 is true.

7.25 (c)

Let $\quad y = \lim_{x\to\infty}(1+x^2)^{e^{-x}}$

$$\log y = \lim_{x\to\infty} \log(1+x^2)^{e^{-x}}$$

$$= \lim_{x\to\infty}\left[\frac{\log(1+x^2)}{e^x}\right]$$

∞/∞ form apply L' Hospital's rule

$$\Rightarrow \quad \log y = \lim_{x\to\infty}\frac{\frac{1}{1+x^2}(2x)}{e^x}$$

$$\Rightarrow \quad \log y = \lim_{x\to\infty}\frac{2x}{(1+x^2)e^x}$$

Again we are getting ∞/∞ form apply L' Hospital's rule

$$\log y = \lim_{x\to\infty}\frac{2}{(1+x^2)e^x + e^x\cdot 2x}$$

$$\log y = \frac{2}{\infty} = 0 \quad \Rightarrow \quad y = 1$$

7.26 (a)

$$a\,f(x) + b\,f\left(\frac{1}{x}\right) = \frac{1}{x} - 25 \qquad \text{...(1)}$$

Put $x = \dfrac{1}{x}$ in equation (1)

$$a\,f\left(\frac{1}{x}\right) + b\,f(x) = x - 25 \qquad \text{...(2)}$$

Solving equations (1) and (2) simultaneously we get $f(x)$ as follows:

Equation (1) $\times\, a$ – equation (2) $\times$ b

$$(1)\times a : \Rightarrow \; a^2f(x) + ba\,\cancel{f\left(\frac{1}{x}\right)} = \frac{a}{x} - 25a$$

$$(2)\times b : \Rightarrow \; ab\,\cancel{f\left(\frac{1}{x}\right)} + b^2 f(x) = bx - 25b$$

$$a^2f(x) - b^2f(x) = \frac{a}{x} - 25a - bx + 25b$$

$$\Rightarrow \quad (a^2 - b^2)\,.\,f(x) = \frac{a}{x} - bx + 25(b-a)$$

$$\Rightarrow \quad f(x) = \frac{1}{a^2-b^2}\left[\frac{a}{x} - bx + 25(b-a)\right]$$

$$\Rightarrow \int_1^2 f(x).dx$$

$$= \frac{1}{a^2-b^2}\left[a.\int_1^2\frac{1}{x}.dx - b\int_1^2 x.dx + 25(b-a)\int_1^2 1.dx\right]$$

$$= \frac{1}{a^2-b^2}\left[a\ln 2 - \frac{3}{2}b + 25(b-a)\right]$$

$$= \frac{1}{a^2-b^2}\left[a\ln 2 - 25a + \frac{47b}{2}\right]$$

$$= \frac{1}{a^2-b^2}\left[a(\ln 2 - 25) + \frac{47b}{2}\right]$$

7.27 Sol.

$$\lim_{x\to 4}\frac{\sin(x-4)}{x-4}$$

Let $x - 4 = t$ not as $x \to 4$

So the requires limit is $\lim_{t\to 0}\dfrac{\sin(t)}{t} = 1$

7.28 Sol.

If $f(x) + f(-x)$ is degree 10

$f(x) = a_{10}x^{10} + a_9x^9 \ldots\ldots + a_1x + a_0$

$f(-x) = a_{10}x^{10} - a_9x^9 \ldots - a_1x + a_0$

$f(x) + f(-x) = a_{10}x^{10} + a_8x^8 + \ldots + a_0$

Now $g(x) = f'(x) = 10a_{10}x^9 + 9a_9x^8 + \ldots + a_1$

$g(-x) = f'(-x) = -10a_{10}x^9 + 9a_9x^8 + \ldots + a_1$

$g(x) - g(-x) = 20a_{10}x^9 + \ldots$

Clearly degree of $(g(x) - g(-x))$ is 9.

Unit ■ II

Theory of Computation

■ **Contents**

UNIT II

Theory of Computation

Syllabus : Regular expressions and finite automata. Context-free grammars and push-down automata. Regular and contex-free languages, pumping lemma. Turing machines and undecidability.

Analysis of Previous GATE Papers

Exam Year	1 Mark Ques.	2 Marks Ques.	3 Marks Ques.	5 Marks Ques.	Total Marks
1990	–	3		–	**6**
1991	–	2		–	**4**
1992	–	5		–	**10**
1993	–	–		–	**–**
1994	1	4		1	**14**
1995	1	2		–	**5**
1996	3	2		–	**7**
1997	1	3		–	**7**
1998	5	3		–	**11**
1999	3	1		–	**5**
2000	2	1		–	**4**
2001	3	2		–	**7**
2002	1	6		1	**18**
2003	3	6		–	**15**
2004	1	4		–	**9**
2005	–	7		–	**14**
2006	2	5		–	**12**

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2007	2	5	**12**
2008	3	5	**13**
2009	4	3	**11**
2010	1	3	**7**
2011	3	3	**9**
2012	4	1	**6**
2013	2	3	**8**
2014 Set-1	2	2	**6**
2014 Set-2	2	2	**6**
2014 Set-3	2	2	**6**
2015 Set-1	1	2	**5**
2015 Set-2	1	3	**7**
2015 Set-3	1	1	**3**
2016 Set-1	3	3	**9**
2016 Set-2	3	4	**11**

1 Finite Automata : Regular Languages

1.1 How many substrings (of all lengths inclusive) can be formed from a character string of length n? Assume all characters to be distinct. Prove your answer.

[1989 : 2 Marks]

1.2 Let R_1 and R_2 be regular sets defined over the alphabet then
(a) $R_1 \cap R_2$ is not regular
(b) $R_1 \cup R_2$ is not regular
(c) $\Sigma^* - R_1$ is regular
(d) R_1^* is not regular

[1990 : 2 Marks]

1.3 Let $r = 1\ (1 + 0)^*$, $s = 11^*0$ and $t = 1^*\ 0$ be three regular expressions. Which one of the following is true?
(a) $L(s) \subseteq L(r)$ and $L(s) \subseteq L(t)$
(b) $L(r) \subseteq L(s)$ and $L(s) \subseteq L(t)$
(c) $L(s) \subseteq L(t)$ and $L(s) \subseteq L(r)$
(d) $L(t) \subseteq L(s)$ and $L(s) \subseteq L(r)$

[1991 : 2 Marks]

1.4 Which of the following regular expression identifies are true?
(a) $r\ (^*) = r^*$
(b) $(r^* s^*)^* = (r + s)^*$
(c) $(r + s)^* = r^* + s^*$
(d) $r^* s^* = r^* + s^*$

[1992 : 2 Marks]

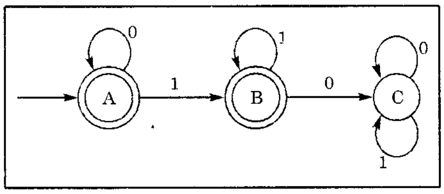
1.5 State True or False with one line explanation.
A FSM (Finite State Machine) can be designed to add two integers of any arbitrary length (arbitrary number of digits).

[1994 : 1 Mark]

1.6 The number of substrings (of all lengths inclusive) that can be formed from a character string of length n is
(a) n
(b) n^2
(c) $\frac{n(n-1)}{2}$
(d) $\frac{n(n+1)}{2} + 1$

[1994 : 2 Marks]

1.7 The regular expression for the language recognized by the finite state automation



[1994 : 2 Marks]

1.8 Which of the following definitions below generates the same language as L
Where $L = \{x^n y^n \mid n \geq 1\}$
(i) $E \rightarrow xEy \mid xy$
(ii) $xy \mid (x^+xyy^+)$
(iii) x^+y^+
(a) (i) only
(b) (i) and (ii)
(c) (ii) and (iii)
(d) (ii) only

[1995 : 2 Marks]

1.9 A finite state machine with the follows state table has a single input X and a single output Z.

present state	next state, z	next state, z
	x=1	x=0
A	D,0	B,0
B	B,1	C,1
C	B,0	D,1
D	B,1	C,0

It the initial state is unknown, then the shortest input sequence to reach the final state C is
(a) 01
(b) 10
(c) 101
(d) 110

[1995 : 2 Marks]

1.10 Which two of the following four regular expressions are equivalent?
(i) $(00)^*(\varepsilon + 0)$
(ii) $(00)^*$
(iii) 0^*
(iv) $0(00)^*$
(a) (i) and (ii)
(b) (ii) and (iii)
(c) (i) and (iii)
(d) (iii) and (iv)

[1996 : 1 Mark]

1.11 Let $L \subseteq \Sigma^*$ where $\Sigma = \{a, b\}$, which of the following is true?

(a) $L = \{x \mid x$ has an equal number of a's and b's$\}$ is regular
(b) $L = \{a^n b^n \mid n \geq 1\}$ is regular
(c) $L = \{x \mid x$ has more a's than b's$\}$ is regular
(d) $L = \{a^m b^n \mid m \geq, n \geq 1\}$ is regular

[1996 : 1 Mark]

1.12 Given $\Sigma = \{a,b\}$, which one of the following sets is not countable?
(a) Set of all strings over Σ
(b) Set of all languages over Σ
(c) Set of all regular languages over Σ
(d) Set of all languages over Σ accepted by Turing Machines

[1997 : 1 Mark]

1.13 Which one of the following regular expressions over {0,1} denotes the set of all strings not containing 100 as a substring?
(a) 0*(1 + 0)*   (b) 0*1010*
(c) 0*1*01*   (d) 0*(10 + 1)*

[1997 : 2 Marks]

1.14 If the regular set A is represented by A = (01 + 1)* and the regular set 'B' is represented by B = ((01) *1*)*, which of the following is true?
(a) $A \subset B$
(b) $B \subset A$
(c) A and B are incomparable
(d) A = B

[1998 : 1 Mark]

1.15 Which of the following set can be recognized by a Deterministic Finite state Automaton?
(a) The numbers 1, 2, 4, 8, ... 2^n, ... written in binary
(b) The numbers 1, 2, 4, ... 2^n, ... written in unary
(c) The set of binary string in which the number of zeros is the same as the number of ones.
(d) The set {1, 101, 11011, 1110111, ...}

[1998 : 1 Mark]

1.16 The string 1101 does not belong to the set represented by
(a) 110*(0 + 1)   (b) 1(0 + 1)*101
(c) (10)*(01)*(00 + 11)*   (d) (00 + (11)* 0)*

[1998 : 1 Mark]

1.17 How many substrings of different lengths (non-zero) can be formed from a character string of length n?
(a) n   (b) n^2
(c) 2^n (d) $\frac{n(n+1)}{2}$

[1998 : 1 Mark]

1.18 Let L be the set of all binary strings whose last two symbols are the same. The number of states in the minimum state deterministic finite state automaton accepting L is
(a) 2   (b) 5
(c) 8   (d) 3

[1998 : 2 Marks]

1.19 Consider the regular expression (0 + 1) (0 + 1) ... n times. The minimum state finite automation that recognizes the language represented by this regular expression contains
(a) n states   (b) n + 1 states
(c) n + 2 states   (d) None of these

[1999 : 1 Mark]

1.20 Let S and T be language over $\Sigma = \{a, b\}$ represented by the regular expressions (a + b*)* and (a + b)*, respectively. Which of the following is true?
(a) $S \subset T$ (b) $T \subset S$
(c) S = T   (d) $S \cap T = \phi$

[2000 : 1 Mark]

1.21 Let L denotes the language generated by the grammar $S \to 0S0 \mid 00$. Which of the following is true?
(a) $L = 0^+$
(b) L is regular but not 0^+
(c) L is context free but not regular
(d) L is not context free

[2000 : 1 Mark]

1.22 What can be said about a regular language L over {a} whose minimal finite state automation has two states?
(a) Can be $\{a^n \mid n$ is odd$\}$
(b) Can be $\{a^n \mid n$ is even$\}$
(c) Can be $\{a^n \mid \geq 0\}$
(d) Either L can be $\{a^n \mid n$ is odd$\}$ or L can be $\{a^n \mid n$ is even$\}$

[2000 : 2 Marks]

1.23 Consider the following statements:
S_1: $\{0^{2n} \mid n \geq 1\}$ is a regular language
S_2: $\{0^m 1^n 0^{m+n} \mid m \geq 1$ and $n \geq 1\}$ is a regular language

Which of the following is true about S_1 and S_2?
(a) Only S_1 is correct
(b) Only S_2 is correct
(c) Both S_1 and S_2 are correct
(d) None of S_1 and S_2 is correct

[2001 : 1 Mark]

1.24 Given an arbitrary non-deterministic finite automaton (NFA) with N states, the maximum number of states in an equivalent minimized DFA is at least
(a) N^2 (b) 2^N
(c) 2N (d) N!

[2001 : 1 Mark]

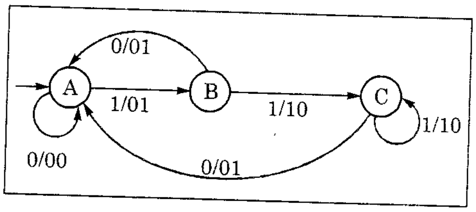
1.25 Consider a DFA over $\Sigma = \{a,b\}$ accepting all strings which have number of a's divisible by 6 and number of b's divisible by 8. What is the minimum number of states that the DFA will have?
(a) 8 (b) 14
(c) 15 (d) 48

[2001 : 2 Marks]

1.26 Consider the following languages:
$L_1 = \{ww \mid w \in \{a,b\}^*\}$
$L_2 = \{ww^R \mid w \in \{a, b\}^*, w^R \text{ is the reverse of } w\}$
$L_3 = \{0^{2i} \mid i \text{ is an integer}\}$
$L_4 = \{0^{i^2} \mid i \text{ is an integer}\}$
Which of the languages are regular?
(a) Only L_1 and L_2 (b) Only L_2, L_3 and L_4
(c) Only L_3 and L_4 (d) Only L_3

[2001 : 2 Marks]

1.27 The finite state machine described by the following state diagram with A as starting state, where an arc label is x/y and x stands for 1-bit input and y stands for 2-bit output



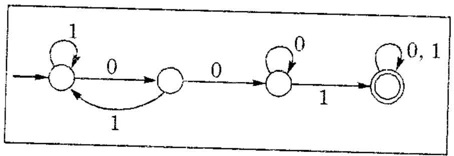
(a) Outputs the sum of the present and the previous bits of the input
(b) Outputs 01 whenever the input sequence contains 11
(c) Output 00 whenever the input sequence contains 10
(d) None of the above

[2002 : 2 Marks]

1.28 The smallest finite automation which accepts the language $L = \{x \mid \text{length of } x \text{ is divisible by } 3\}$ has
(a) 2 states (b) 3 states
(c) 4 states (d) 5 states

[2002 : 2 Marks]

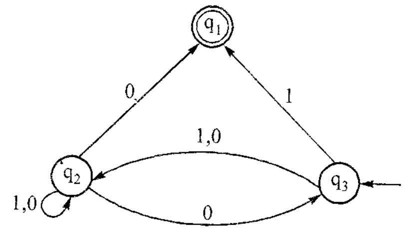
1.29 Consider the following deterministic finite state automaton M.



Let S denote the set of seven bit binary strings in which the first, the fourth, and the last bits are 1. The number of strings in S that are accepted by M is
(a) 1 (b) 5
(c) 7 (d) 8

[2003 : 2 Marks]

1.30 Consider the NFA M shown below.



Let the language accepted by M be L. Let L_1 be the language accepted by the NFA M_1, obtained by changing the accepting state of M to a non-accepting state and by changing the non-accepting state of M to accepting states. Which of the following statements is true?
(a) $L_1 = \{0, 1\}^* - L$ (b) $L_1 = \{0, 1\}^*$
(c) $L_1 \subseteq L$ (d) $L_1 = L$

[2003 : 2 Marks]

1.31 Which one of the following regular expressions is NOT equivalent to the regular expression $(a + b + c)^*$?
(a) $(a^* + b^* + c^*)^*$ (b) $(a^*b^*c^*)^*$
(c) $((ab)^* + c^*)^*$ (d) $(a^*b^* + c^*)^*$

[IT-2004 : 1 Mark]

1.32 Let $M = (K, \Sigma, \delta, s, F)$ be a finite state automaton, where
$K = \{A, B\}$, $\Sigma = \{a, b\}$, $s = A$, $F = \{B\}$,
$\delta(A, a) = A$, $\delta(A, b) = B$, $\delta(B, a) = B$ and $\delta(B, b) = A$

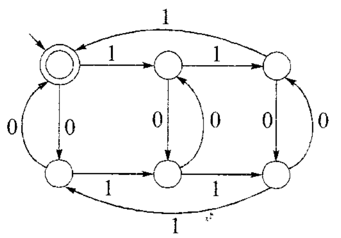
A grammar to generate the language accepted by M can be specified as G = (V, Σ, R, S), Where V = K ∪ Σ, and S = A

Which one of the following set of rules will make L(G) = L(M)?

(a) {A→aB, A→bA, B→bA, B→aA, B→ε}
(b) {A→aA, A→bB, B→aB, B→bA, B→ε}
(c) {A→bB, A→aB, B→aA, B→bA, B→ε}
(d) {A→aA, A→bA, B→aB, B→bA, A→ε}

[IT-2004 : 2 Marks]

1.33 The following finite state machine accepts all those binary strings in which the number of 1's and 0's are respectively



(a) divisible by 3 & 2
(b) odd and even
(c) even and odd
(d) divisible by 2 & 3

[2004 : 2 Marks]

1.34 Which of the following statements is TRUE about the regular expression 01*0?

(a) It represents a finite set of finite strings.
(b) It represents an infinite set of finite strings.
(c) It represents a finite set of infinite strings.
(d) It represents an infinite set of infinite strings.

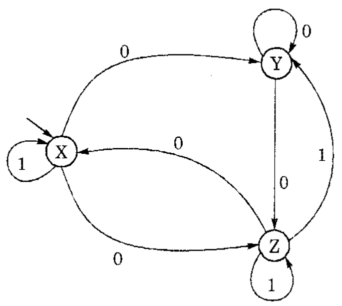
[IT-2005 : 1 Mark]

1.35 The language $\{0^n 1^n 2^n \mid 1 \le n \le 10^6\}$ is

(a) Regular
(b) Context-free but not regular.
(c) Context-free but its complement is not context-free.
(d) Not context-free.

[IT-2005 : 1 Mark]

1.36 Consider the non-deterministic finite automaton (NFA) shown in the figure.



State X is the starting state of the automaton. Let the language accepted by the NFA with Y as the only accepting state be L1. Similarly, let the language accepted by the NFA with Z as the only accepting state be L2. Which of the following statements about L1 and L2 is TRUE?

(a) L1 = L2
(b) L1 ⊂ L2
(c) L2 ⊂ L1
(d) None of these

[IT-2005 : 2 Marks]

1.37 Consider the regular grammar:

S → Xa | Ya
X → Za
Z → Sa | ε
Y → Wa
W → Sa

where S is the starting symbol, the set of terminals is {a} and the set of non-terminals is {S, W, X, Y, Z}. We wish to construct a deterministic finite automaton (DFA) to recognize the same language. What is the minimum number of states required for the DFA?

(a) 2
(b) 3
(c) 4
(d) 5

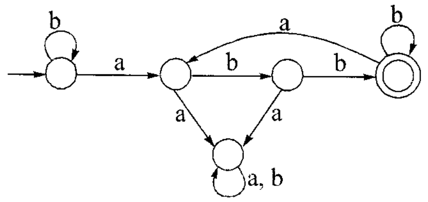
[IT-2005 : 2 Marks]

1.38 A language L satisfies the Pumping Lemma for regular languages, and also the Pumping Lemma for context-free languages. Which of the following statements about L is TRUE?

(a) L is necessarily a regular language.
(b) L is necessarily a context-free language, but not necessarily a regular language.
(c) L is necessarily a non-regular language.
(d) None of the above

[IT-2005 : 2 Marks]

1.39 Consider the machine M



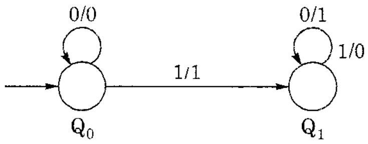
The language recognized by M is

(a) {w ∈ {a, b}* | every a in w is followed by exactly two b's}
(b) {w ∈ {a, b}* every a in w is followed by at least two b's}
(c) {w ∈ {a, b}* w contains the substring 'abb'

(d) $\{w \in \{a, b\}^*$ w does not contain 'aa' as a substring$\}$

[2005 : 2 Marks]

1.40 The following diagram represents a finite state machine which takes as input a binary number from the least significant bit



Which one of the following is TRUE?

(a) It computes 1's complement of the input number

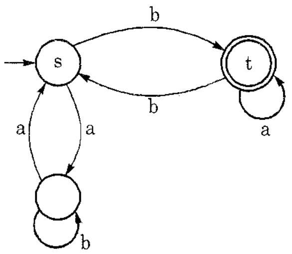
(b) It computes 2's complement of the input number

(c) It increments the input number

(d) It decrements the input number

[2005 : 2 Marks]

1.41 In the automaton below, s is the start state and t is the only final state.



Consider the strings u = abbaba, v = bab, and w = aabb. Which of the following statements is true?

(a) The automaton accepts u and v but not w

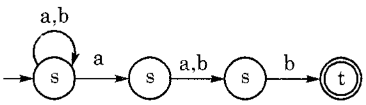
(b) The automaton accepts each of u, v, and w

(c) The automaton rejects each of u, v, and w

(d) The automaton accepts u but rejects v and w

[IT-2006 : 1 Mark]

1.42 Which regular expression best describes the language accepted by the non-deterministic automaton below?



(a) (a+b) * a(a+b)b

(b) (abb)*

(c) (a+b)* a(a+b) * b(a+b)*

(d) (a+b)* **[IT-2006 : 1 Mark]**

1.43 Consider the regular grammar below

$S \rightarrow bS \mid aA \mid \varepsilon$

$A \rightarrow aS \mid bA$

The Myhill-Nerode equivalence classes for the language generated by the grammar are

(a) $\{w \in (a+b)^* \mid \#_a(w)$ is even$\}$ and $\{w \in (a+b)^* \mid \#_a(w)$ is odd$\}$

(b) $\{w \in (a+b)^* \mid \#_b(w)$ is even$\}$ and $\{w \in (a+b)^* \mid \#_b(w)$ is odd$\}$

(c) $\{w \in (a+b)^* \mid \#_a(w) = \#_b(w)\}$ and $\{w \in (a+b)^* \mid \#_a(w) \neq \#_b(w)\}$

(d) $\{\varepsilon\}, \{wa \mid w \in (a+b)^*\}$ and $\{wb \mid w \in (a+b)^*\}$

[IT-2006 : 2 Marks]

1.44 Which of the following statements about regular languages is NOT true?

(a) Every language has a regular superset

(b) Every language has a regular subset

(c) Every subset of a regular language is regular

(d) Every subset of a finite language is regular

[IT-2006 : 2 Marks]

Directions for Q.1.45 to Q.1.46:

Let L be a regular language. Consider the constructions on L below:

I. repeat (L) = $\{ww \mid w \in L)$

II. prefix (L) = $\{u \mid \exists v : uv \in L\}$

III. suffix (L) = $(v \mid \exists u : uv \in L\}$

IV. half (L) = $\{u \mid v : \mid \exists v \mid = \mid u \mid$ and $uv \in L\}$

1.45 Which of the constructions could lead to a non-regular language?

(a) Both I and IV (b) Only I

(c) Only IV (d) Both II and III

[IT-2006 : 2 Marks]

1.46 Which choice of L is best suited to support your answer above?

(a) (a+b)* (b) $\{\varepsilon$, a, ab, bab$\}$

(c) (ab)* (d) $\{a^n b^n \mid n \geq 0\}$

[IT-2006 : 2 Marks]

1.47 If s is a string over $(0+1)^*$, then let $n_0(s)$ denote the number of 0's in s and $n_1(s)$ the number of 1's in s. Which one of the following languages is not regular?

(a) $L = \{s \in (0+1)^* \mid n_0(s)$ is a 3-digit prime$\}$

(b) $L = \{s \in (0+1)^* \mid$ for every prefix s' of s, $\mid n_0(s') - n_1(s') \mid \leq 2\}$

(c) $L = \{s \in (0+1)^* \mid \mid n_0(s) - n_1(s) \leq 4\}$

(d) L = {s ∈ (0 + 1)* | $n_0(s)$ mod 7 = $n_1(s)$ mod 5 = 0}

[IT-2006 : 2 Marks]

1.48 Consider the regular language L = (111 + 11111)*. The minimum number of states in any DFA accepting this languages is
(a) 3 (b) 5
(c) 8 (d) 9

[IT-2006 : 2 Marks]

1.49 Which of the following is TRUE?
(a) Every subset of a regular set is regular
(b) Every finite subset of a non-regular set is regular
(c) The union of two non-regular sets is not regular
(d) Infinite union of finite sets is regular

[IT-2007 : 1 Mark]

1.50 A minimum state deterministic finite automation accepting the language L = {w | w ∈ (0, 1)*, number of 0s & 1s in w are divisible by 3 and 5, respectively} has
(a) 15 states (b) 11 states
(c) 10 states (d) 9 states

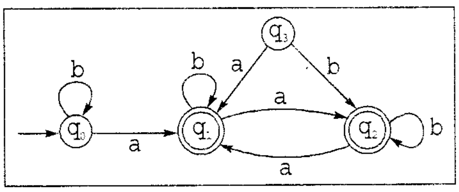
[2007 : 2 Marks]

1.51 Which of the following languages is regular?
(a) $\{ww^R \mid w \in \{0, 1\}^+\}$
(b) $\{ww^Rx \mid x, w \in (0, 1)^+\}$
(c) $\{wxw^R \mid x, w \in (0, 1)^+\}$
(d) $\{xww^R \mid x, w \in (0, 1)^+\}$

[2007 : 2 Marks]

Common Data for Q.1.52 & Q.1.53

Consider the following Finite State Automation



1.52 The language accepted by this automaton is given by the regular expression
(a) b * ab * ab * ab* (b) (a + b)*
(c) b * a (a + b) * (d) b * ab * ab *

[2007 : 2 Marks]

1.53 The minimum state automaton equivalent to the above FSA has the following number of states
(a) 1 (b) 2
(c) 3 (d) 4

[2007 : 2 Marks]

1.54 The two grammars given below generate a language over the alphabet {x, y, z}

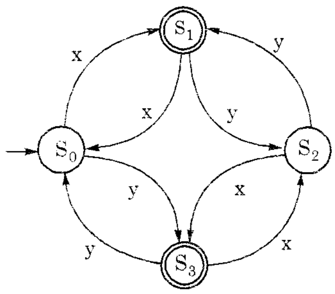
G1: S → x | z | xS | zS | yB
B → y | z | yB | zB

G2: S → y | z | yS | zS | xB
B→y | yS

Which one of the following choices describes the properties satisfied by the strings in these languages?
(a) G1 : No y appears before any x
G2 : Every x is followed by at least one y
(b) G1 : No y appears before any x
G2 : No x appears before any y
(c) G1 : No y appears after any x
G2 : Every x is followed by at least one y
(d) G1 : No y appears after any x
G2 : Every y is followed by at least one x

[IT-2007 : 2 Marks]

1.55 Consider the following DFA in which s_0 is the start state and s_1, s_3 are the final states.



What language does this DFA recognize?
(a) All strings of x and y
(b) All strings of x and y which have either even number of x and even number of y or odd number or x and odd number of y
(c) All strings of x and y which have equal number of x and y
(d) All strings of x and y with either even number of x and odd number of y or odd number of x and even number of y

[IT-2007 : 2 Marks]

1.56 Consider the grammar given below
S → xB | yA
A → x | xS | yAA
B → y | yS | yBB

Consider the following strings.

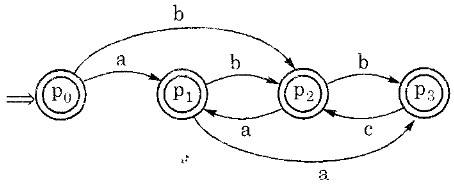
(i) xxyyx (ii) xxyyxy
(iii) xyxy (iv) yxxy
(v) yxx (vi) xyx

Which of the above strings are generated by the grammar?

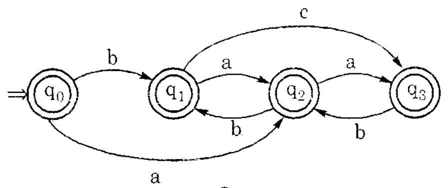
(a) (i), (ii), and (iii) (b) (ii), (v), and (vi)
(c) (iii) and (iv) (d) (i), (iii), and (iv)

[IT-2007 : 2 Marks]

1.57 Consider the following finite automata P and Q over the alphabet {a, b, c}. The start states are indicated by a double arrow and final states are indicated by a double circle. Let the languages recognized by them be denoted by L(P) and L(Q) respectively.

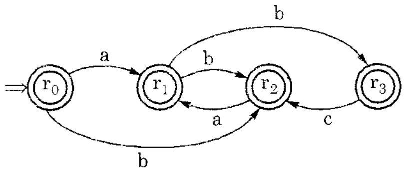


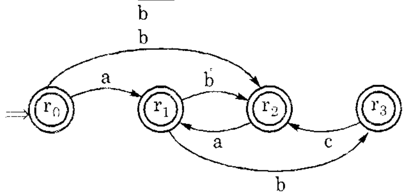
P

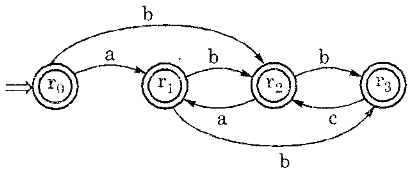


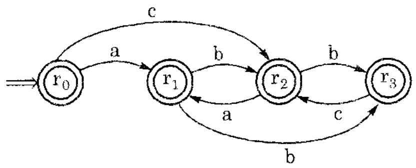
Q

The automation which recognizes the language L(P) ∩ L(Q) is :

(a) 

(b) 

(c) 

(d) 

(e) None of the above

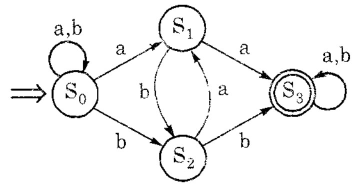
[IT-2007 : 2 Marks]

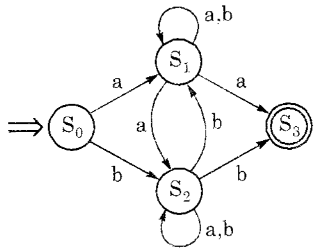
Common Data for Q.1.58 to Q.1.60:

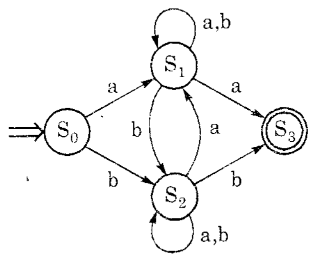
Consider the regular expression :

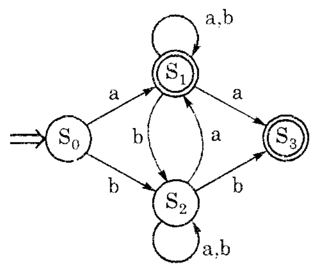
$$R = (a+b)^* \ (aa+bb) \ (a+b)^*$$

1.58 Which of the following non-deterministic finite automata recognizes the language defined by the regular expression R? Edges labeled λ denote transitions on the empty string.

(a) 

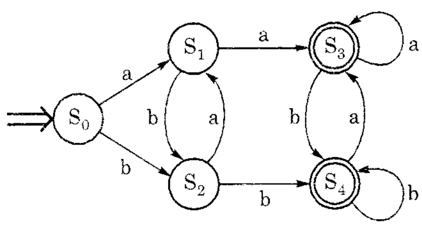
(b) 

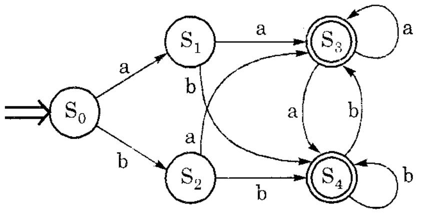
(c) 

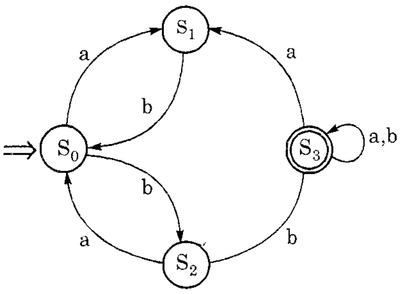
(d) 

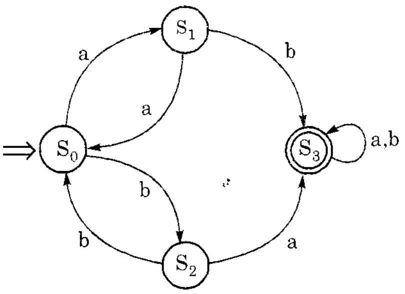
[IT-2007 : 2 Marks]

1.59 Which deterministic finite automaton accepts the language represented by the regular expression R?

(a) 

(b) 

(c) 

(d) 

[IT-2007 : 2 Marks]

1.60 Which one of the regular expressions given below defines the same language as defined by the regular expression R?

(a) $(a(ba)^* + b(ab)^*)(a + b)^+$

(b) $(a(ba)^* + b(ab)^*)^*(a + b)^*$

(c) $(a(ba)^* (a + bb) + b(ab)^*(b + aa))(a + b)^*$

(d) $(a(ba)^*(a + bb) + b(ab)^*(b + aa))(a + b)^+$

[IT-2007 : 2 Marks]

1.61 Which of the following regular expressions describes the language over {0, 1} consisting of strings that contain exactly two 1's?

(a) $(0 + 1)^* 11(0 + 1)^*$

(b) $0^* 110^*$

(c) $0^* 10^* 10^*$

(d) $(0 + 1)^* 1(0 + 1)^* 1 (0 + 1)^*$

[IT-2008 : 1 Mark]

1.62 Let N be an NFA with n states and let M be the minimized DFA with m states recogniz-ing the same language. Which of the following in NECESSARILY true?

(a) $m \leq 2^n$

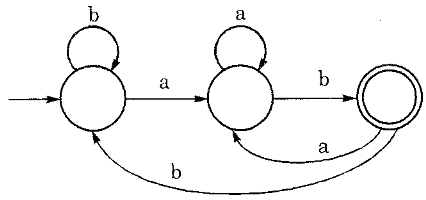
(b) $n \leq m$

(c) M has one accept state

(d) $m = 2^n$

[IT-2008 : 1 Mark]

1.63 If the final states and non-final states in the DFA below are interchanged, then which of the following languages over the alphabet {a, b} will be accepted by the new DFA?



(a) Set of all strings that do not end with ab

(b) Set of all strings that begin with either an a or ab

(c) Set of all strings that do not contain the substring ab,

(d) The set described by the regular expression $b^*aa^*(ba)^*b^*$

[IT-2008 : 2 Marks]

1.64 Which of the following languages is (are) non-regular?

$L_1 = \{0^m1^n \mid 0 \leq m \leq n \leq 10000\}$

L_2 = {w | w reads the same forward and backward}

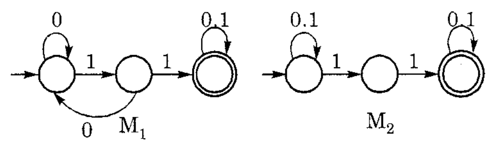
$L_3 = \{w \in \{0, 1\}^* \mid$ w contains an even number of 0's and an even number of 1's$\}$

(a) L_2 and L_3 only (b) L_1 and L_2 only

(c) L_3 only (d) L_2 only

[IT-2008 : 2 Marks]

1.65 Consider the following two finite automata. M_1 accepts L_1 and M_2 accepts L_2. Which one of the following is TRUE?



(a) $L_1 = L_2$ (b) $L_1 \cap L_2 = \phi$

(c) $L_1 \cap \overline{L_2} = \phi$ (d) $L_1 \cup L_2 \neq L_1$

[IT-2008 : 2 Marks]

1.66 Given below are two finite state automata (→ indicates the start and F indicates a final state)

Y :

	a	b
→1	1	2
2(F)	2	1

Z :

	a	b
→1	2	2
2(F)	1	1

Which of the following represents the product automaton Z × Y?

(a)

	a	b
→P	S	R
Q	R	S
R(F)	Q	P
S	P	Q

(b)

	a	b
→P	S	Q
Q	R	S
R(F)	Q	P
S	P	Q

(c)

	a	b
→P	Q	S
Q	R	S
R(F)	Q	P
S	Q	P

(d)

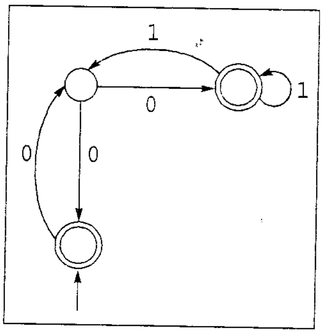
	a	b
→P	S	Q
Q	S	R
R(F)	Q	P
S	Q	P

[2008 : 2 Marks]

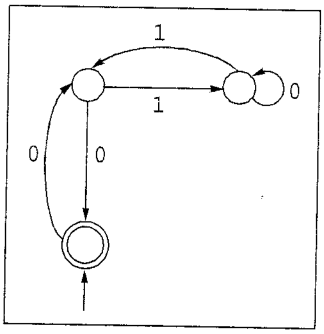
1.67 Match **List-I** with **List-II** and select the correct answer using the codes given below the lists:

List-I

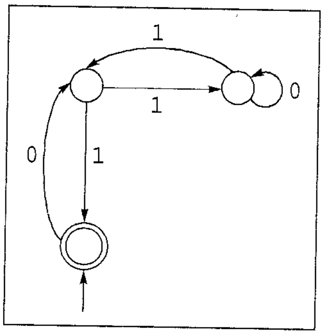
A.



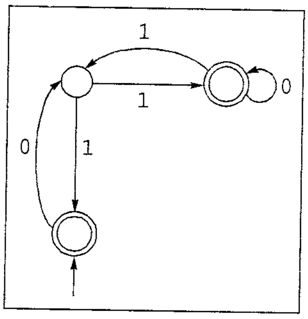
B.



C.



D.



List-II

1. $\varepsilon + 0(01^*1 + 00)^*01^*$
2. $\varepsilon + 0(10^*1 + 00)^*0$
3. $\varepsilon + 0(10^*1 + 10)^*1$
4. $\varepsilon + 0(10^*1 + 10)^*10^*$

Codes:

	A	B	C	D
(a)	2	1	3	4
(b)	1	3	2	4
(c)	1	2	3	4
(d)	3	2	1	4

[2008 : 2 Marks]

1.68 Which of the following are regular sets?

1. $\{a^n b^{2m} \mid n \geq 0, m \geq 0\}$
2. $\{a^n b^m \mid n = 2m\}$
3. $\{a^n b^m \mid n \neq m\}$
4. $\{xcy \mid x, y \in \{a, b\}^*\}$

(a) 1 and 4 only
(b) 1 and 3 only
(c) 1 only
(d) 4 only

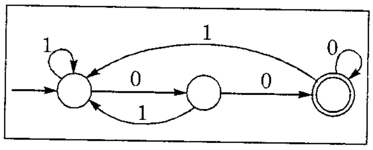
[2008 : 2 Marks]

1.69 Which one of the following languages over the alphabet {0, 1} is described by the regular expression: $(0 + 1)^*0(0 + 1)^*0(0 + 1)^*$?

(a) The set of all strings containing the substring 00
(b) The set of all strings containing at most two 0's
(c) The set of all strings containing at least two 0's
(d) The set of all strings that begin and end with either 0 or 1

[2009 : 1 Mark]

1.70 The following DFA accepts the set of all strings over {0, 1} that



(a) begin either with 0 or 1
(b) end with 0
(c) end with 00
(d) contain the substring 00

[2009 : 2 Marks]

1.71 Let $L = \{\omega \in (0 + 1)^* \mid \omega$ has even number of 1s$\}$, i.e., L is the set of all bit strings with even number of 1s. Which one of the regular expressions below represents L?

(a) $(0^*10^*1)^*$
(b) $0^*(10^*10^*)^*$
(c) $0^*(10^*1)^*0^*$
(d) $0^*1(10^*1)^*10^*$

[2010 : 2 Marks]

1.72 Let ω be any string of length n in $\{0, 1\}^*$. Let L be the set of all substrings of ω. What is the minimum number of states in a non-deterministic finite automaton that accepts L?

(a) $n - 1$
(b) n
(c) $n + 1$
(d) 2^{n+1}

[2010 : 2 Marks]

1.73 The lexical analysis for a modern computer language such as Java needs the power of which one of the following machine models in a necessary and sufficient sense?

(a) Finite state automata
(b) Deterministic pushdown automata
(c) Non-deterministic pushdown automata
(d) Turing machine

[2011 : 1 Mark]

1.74 Let P be a regular language and Q be a context-free language such that $Q \subseteq P$. (For example, let P be the language represented by the regular expression p^*q^* and Q be $\{p^n q^n \mid n \in N\}$. Then which of the following is ALWAYS regular?

(a) $P \cap Q$
(b) $P - Q$
(c) $\Sigma^* - P$
(d) $\Sigma^* - Q$

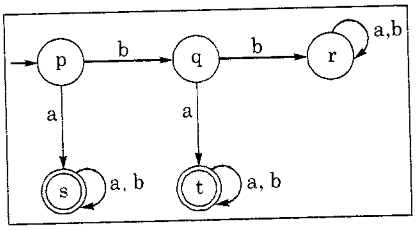
[2011 : 1 Mark]

1.75 Definition of a language L with alphabet $\{a\}$ is given as follows:
$L = \{a^{nk} \mid k > 0$, and n is a positive integer constant$\}$.
What is the minimum number of states needed in a dfa to recognize L?

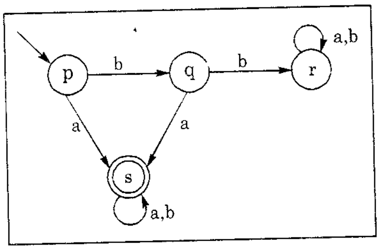
(a) $k + 1$
(b) $n + 1$
(c) 2^{n+1}
(d) 2^{k+1}

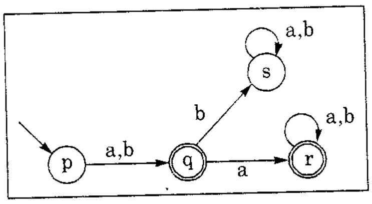
[2011 : 2 Marks]

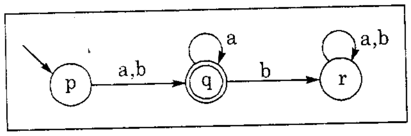
1.76 A deterministic finite automaton (DFA) D with alphabet $\Sigma = \{a, b\}$ is given below:

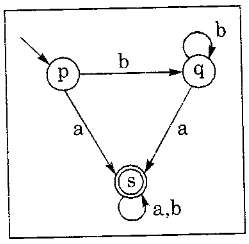


Which of the following finite state machines is a valid minimal DFA which accepts the same language as D?

(a)


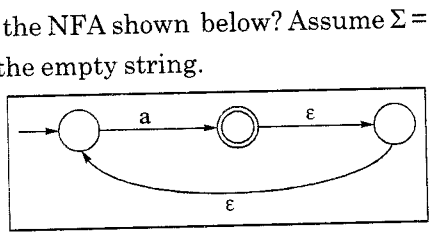
(b)


(c)


(d)


[2011 : 2 Marks]

1.77 What is the complement of the language accepted by the NFA shown below? Assume $\Sigma = \{a\}$ and ε is the empty string.



(a) $\varnothing$
(b) $\{\varepsilon\}$
(c) a^*
(d) $\{a, \varepsilon\}$

[2012 : 1 Mark]

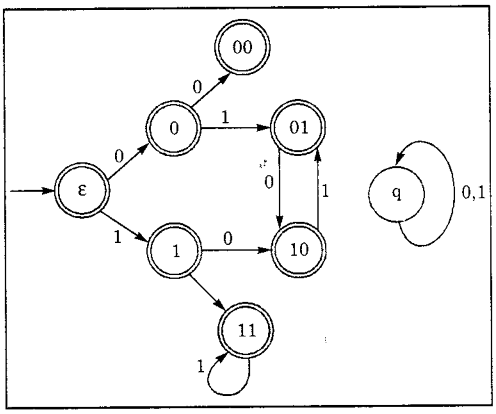
1.78 Given the language $L = \{ab, aa, baa\}$, which of the following strings are in L^*?

1. abaabaaabaa 2. aaaabaaaa
3. baaaaabaaaab 4. baaaaabaa

(a) 1. 2 and 3 (b) 2, 3 and 4
(c) 1, 2 and 4 (d) 1, 3 and 4

[2012 : 1 Mark]

1.79 Consider the set of strings on {0, 1} in which, every substring of 3 symbols has at most two zeros. For example, 001110 and 011001 are in the language, but 100010 is not. All strings of length less than 3 are also in the language. A partially complete DFA that accepts this language is shown below.
The missing arcs in the DFA are



(a)

	00	01	10	11	q
00	1	0			
01				1	
10	0				
11			0		

(b)

	00	01	10	11	q
00		0			1
01		1			
10				0	
11		0			

(c)

	00	01	10	11	q
00		1			0
01		1			
10			0		
11		0			

(d)

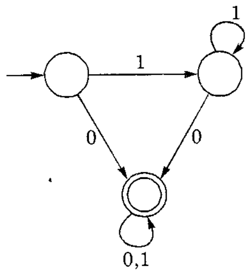
	00	01	10	11	q
00		1			0
01				1	
10	0				
11			0		

[2012 : 2 Marks]

1.80 Consider the languages $L_1 = \phi$ and $L_2 = \{a\}$. Which one of the following represents $L_1L_2^* \cup L_1^*$?
(a) $\{\varepsilon\}$ (b) ϕ
(c) a^* (d) $\{\varepsilon, a\}$

[2013 : 1 Mark]

1.81 Consider the DFA A is given below.



Which of the following are False?
1. Complement of L(A) is context-free.
2. L(A) = L((11 *0 + 0) (0 + 1)*0*1*)
3. For the language accepted by A, A is the minimal DFA.
4. A accepts all strings over {0, 1} of length at least 2.

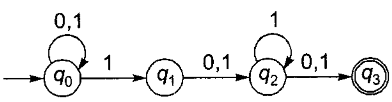
(a) 1 and 3 only (b) 2 and 4 only
(c) 2 and 3 only (d) 3 and 4 only

[2013 : 2 Marks]

1.82 Which one of the following is TRUE?
(a) The language $L = \{a^n b^n \mid n \geq 0\}$ is regular.
(b) The language $L = \{a^n \mid n \text{ is prime}\}$ is regular.
(c) The language $L = \{w \mid w$ has $3k + 1$ b's for some $k \in \mathbb{N}$ with $\Sigma = \{a, b\}\}$ is regular.
(d) The language $L = \{ww \mid w \in \Sigma^*$ with $\Sigma = \{0, 1\}\}$ is regular.

[2014 (Set-1) : 1 Mark]

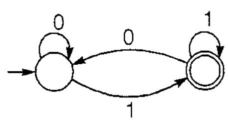
1.83 Consider the finite automaton in the following figure.



What is the set of reachable states for the input string 0011?
(a) $\{q_0, q_1, q_2\}$ (b) $\{q_0, q_1\}$
(c) $\{q_0, q_1, q_2, q_3\}$ (d) $\{q_3\}$

[2014 (Set-1) : 1 Mark]

1.84 Which of the regular expressions given below represent the following DFA?



I. 0*1 (1 + 00* 1)*
II. 0*1*1 +11* 0* 1
III. (0 + 1)* 1

(a) I and II only (b) I and III only
(c) II and III only (d) I, II, and III

[2014 (Set-1) : 2 Marks]

1.85 If $L_1 = \{a^n \mid n \geq 0\}$ and $L_2 = \{b^n \mid n \geq 0\}$, consider

I. $L_1 \cdot L_2$ is a regular language
II. $L_1 \cdot L_2 = \{a^n b^n \mid n \geq 0\}$

Which one of the following is CORRECT?

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

[2014 (Set-2) : 1 Mark]

1.86 Let $L_1 = \{w \in \{0,1\}^* \mid w$ has at least as many occurrences of (110)'s as (011)'s}. Let $L_2 = \{w \in \{0,1\}^* \mid w$ has at least as many occurrences of (000)'s as (111)'s}. Which one of the following is TRUE?

(a) L_1 is regular but not L_2
(b) L_2 is regular but not L_1
(c) Both L_1 and L_2 are regular
(d) Neither L_1 nor L_2 are regular

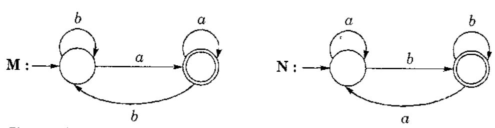
[2014 (Set-2) : 2 Marks]

1.87 The length of the shortest string NOT in the language (over $\Sigma = \{a, b\}$) of the following regular expression is ________.

$$a^*b^*(ba)^*a^*$$

[2014 (Set-3) : 1 Mark]

1.88



Consider the DFAs M and N given above. The number of states in a minimal DFA that accepts the language $L(M) \cap L(N)$ is ________.

[2015 (Set-1) : 2 Marks]

1.89 Consider alphabet $\Sigma = \{0, 1\}$, the null/empty string λ and the sets of strings X_0, X_1 and X_2 generated by the corresponding non-terminals of a regular grammar. X_0, X_1 and X_2 are related as follows:

$X_0 = 1X_1$
$X_1 = 0X_1 + 1X_2$
$X_2 = 0X_1 + \{\lambda\}$

Which one of the following choices precisely represents the strings in X_2?

(a) 10 (0* + (10)*)1
(b) 10 (0* + (10)*)*1
(c) 1 (0 + 10)*1
(d) 10 (0 + 10)*1 + 110 (0 + 10)*1

[2015 (Set-2) : 2 Marks]

1.90 Which of the following languages is/are regular?

L_1: $\{wxw^R \mid w, x \in \{a, b\}^*$ and $|w|, |x| > 0\}$ w^R is the reverse of string w

L_2: $\{a^n b^m \mid m \neq n$ and $m, n \geq 0\}$

L_3: $\{a^p b^q c^r \mid p, q, r \geq 0\}$

(a) L_1 and L_3 only (b) L_2 only
(c) L_2 and L_3 only (d) L_3 only

[2015 (Set-2) : 2 Marks]

1.91 The number of states in the minimal deterministic finite automaton corresponding to the regular expression (0+ 1)* (10) is ________.

[2015 (Set-2) : 2 Marks]

1.92 Let L be the language represented by the regular expression $\Sigma^*0011\Sigma^*$ where $\Sigma = \{0, 1\}$. What is the minimum number of states in a DFA that recognizes $\bar{L}$ (complement of L)?

(a) 4 (b) 5
(c) 6 (d) 8

[2015 (Set-3) : 1 Mark]

1.93 Which of the following languages is generated by the given grammar?

$$S \rightarrow aS \mid bS \mid \varepsilon$$

(a) $\{a^n b^m \mid n, m \geq 0\}$
(b) $\{w \in \{a, b\}^* \mid w$ has equal number of a's and b's$\}$
(c) $\{a^n \mid n \geq 0\} \cup \{b^n \mid n \geq 0\} \cup \{a^n b^n \mid n \geq 0\}$
(d) $\{a, b\}^*$

[2016 (Set-1) : 1 Mark]

1.94 Which of the following decision problems are undecidable?

I. Given NFAs N_1 & N_2, is $L(N_1) \cap L(N_2) = \Phi$?
II. Given a CFG $G = (N, \Sigma, P, S)$ and a string $x \in \Sigma^*$, does $x \in L(G)$?
III. Given CFGs G_1 and G_2, is $L(G_1) = L(G_2)$?
IV. Given a TM M, is $L(M) = \Phi$?

(a) I and IV only (b) II and III only
(c) III and IV only (d) II and IV only

[2016 (Set-1) : 1 Mark]

1.95 Which one of the following regular expressions represents the language: *the set of all binary strings having two consecutive 0s and two consecutive 1s?*

(a) $(0+1)^*0011(0+1)^* + (0+1)^*1100(0+1)^*$
(b) $(0+1)^*(00(0+1)^*11 + 11(0+1)^*00)(0+1)^*$
(c) $(0+1)^*00(0+1)^* + (0+1)^*11(0+1)^*$
(d) $00(0+1)^*11+11(0+1)^*00$

[2016 (Set-1) : 1 Mark]

1.96 The number of states in the minimum sized DFA that accepts the language defined by the regular expression $(0+1)^*(0+1)(0+1)^*$ is ______.

[2016 (Set-2) : 1 Mark]

1.97 Consider the following two statements:

I. If all states of an NFA are accepting states then the langauge accepted by the NFA is Σ^*.
II. There exists a regular language A such that for all languages B, $A \cap B$ is regular.

Which one of the following is **CORRECT**?

(a) Only I is true
(b) Only II is ture
(c) Both I and II are true
(d) Both I and II are false

[2016 (Set-2) : 2 Marks]

Answers Finite Automata : Regular Languages

1.2	(c)	1.3	(a, c)	1.4	(b)	1.6	(d)	1.8	(a)	1.9	(b)	1.10	(c)	1.11	(d)	1.12	(b)
1.13	(d)	1.14	(d)	1.15	(a)	1.16	(c,d)	1.17	(d)	1.18	(b)	1.19	(b)	1.20	(c)	1.21	(b)
1.22	(d)	1.23	(a)	1.24	(b)	1.25	(d)	1.26	(d)	1.27	(a)	1.28	(b)	1.29	(c)	1.30	(b)
1.31	(c)	1.32	(b)	1.33	(a)	1.34	(b)	1.35	(a)	1.36	(a)	1.37	(b)	1.38	(d)	1.39	(b)
1.40	(b)	1.41	(d)	1.42	(a)	1.43	(a)	1.44	(c)	1.45	(b)	1.46	(a)	1.47	(c)	1.48	(d)
1.49	(b)	1.50	(a)	1.51	(c)	1.52	(c)	1.53	(b)	1.54	(a)	1.55	(d)	1.56	(c)	1.57	(b)
1.58	(b)	1.59	(a)	1.60	(c)	1.61	(c)	1.62	(a)	1.63	(a)	1.64	(d)	1.65	(a)	1.66	(a)
1.67	(c)	1.68	(a)	1.69	(c)	1.70	(c)	1.71	(b)	1.72	(c)	1.73	(a)	1.74	(c)	1.75	(b)
1.76	(a)	1.77	(b)	1.78	(c)	1.79	(d)	1.80	(a)	1.81	(d)	1.82	(c)	1.83	(a)	1.84	(b)
1.85	(a)	1.86	(a)	1.89	(c)	1.90	(a)	1.92	(b)	1.93	(d)	1.94	(c)	1.95	(b)	1.97	(b)

Explanations Finite Automata : Regular Languages

1.1 Sol.

Number of substrings (of all lengths inclusive) that can be formed from a character strings of length n is $\frac{n(n+1)}{2}+1$.

Example: Let the length of string be 4, $|w| = 4$ and w = ABCD

No. of substrings of length 0 is 1(ε)
No. of substrings of length 1 is 4(A, B, C, D)
No. of substrings of length 2 is 3(AB, BC, CD)
No. of substrings of length 3 is 2(ABC, BCD)
No. of substrings of length 4 is 1 (ABCD)

So, no.of substrings of all length by a string of length 4 is

$$1 + (4 + 3 + 2 + 1) = 1 + \frac{4\times(4+1)}{2} = 11$$

1.2 (c)

Since regular languages are closed under union, intersection, complementation, concatenation as well as Kleene closure.

So (c) is correct and (a), (b) and (d) are false.

1.3 (a, c)

$$r = 1(1+0)^*$$

So the language corresponds to **r** is all strings starting with 1.

$$s = 11^*0$$

The language corresponds to s is all strings starting with 1 followed by any number of 1 and end with 0. So **s** has more restrictions as compared to r.

So, $L(s) \subseteq L(r)$

$$t = 1^*0$$

The language corresponds to t is all strings ending with 0. Since **s** has to start with atleast one 1 but t can start with zero 1s. So s has more restriction compared to **t**.

So $L(\mathbf{s}) \subseteq L(\mathbf{t})$

Note: Choices (a) and (c) are actually same.

1.4 (b)

According to identities of regular expression

$(r^* s^*)^* = (r + s)^*$

1.5 Sol.

FALSE, A FSM (Finite State machine) can't be designed to add two integers of any arbitrary length because FSM have finite memory and can't store integers of any arbitrary length.

1.6 (d)

Number of substrings (of all lengths inclusive) that can be formed from a character strings of length n is $\frac{n(n+1)}{2}+1$.

Example: Let the length of string be 4, $|w| = 4$ and w = ABCD

No. of substrings of length 0 is 1(ε)

No. of substrings of length 1 is 4(A, B, C, D)

No. of substrings of length 2 is 3(AB, BC, CD)

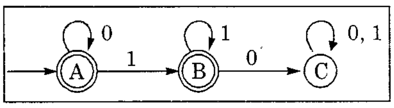
No. of substrings of length 3 is 2(ABC, BCD)

No. of substrings of length 4 is 1 (ABCD)

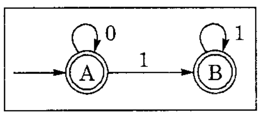
So, no.of substrings of all length by a string of length 4 is

$$1 + (4 + 3 + 2 + 1) = 1 + \frac{4 \times (4+1)}{2} = 11$$

1.7 Sol.



Since C is dead state so just remove this



Regular expression = RE of A + RE of B

$= 0^* + 0^*\ 11^*$

$= 0^* (\varepsilon + 11^*)$

$= 0^*\ 1^*$

1.8 (a)

$L = \{x^n y^n \mid n \geq 1\}$

The language L produces all strings having equal number of x and y and y follows the x.

(i) $E \rightarrow x\ Ey \mid xy$

The language produced by above grammar contain all strings having equal no. of x & y and y follows x.

(ii) $xy \mid x^+xy\ y^+$

The above can generate string xxxxyy which doesn't contain equal no. of x and y

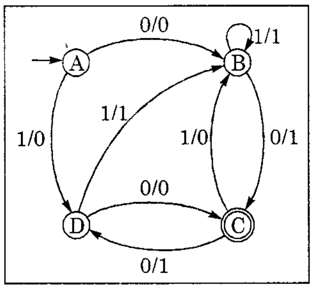
(iii) x^+y^+

The above regular expression generates all strings in which y follows x and contain x and y greater than one. Above RE can generate string xxxy which doesn't contain equal no of x & y.

So only (i) generates the same language as L.

1.9 (b)

The state diagram which represents the given state table is



From state A to reach C, the shortest sequence is

$$A \xrightarrow{0} B \xrightarrow{0} C$$

$$A \xrightarrow{1} D \xrightarrow{0} C$$

From B to reach C

$$B \xrightarrow{0} C$$

$$B \xrightarrow{1} B \xrightarrow{0} C$$

From C to reach C

$$C \xrightarrow{0} D \xrightarrow{0} C$$

$$C \xrightarrow{1} B \xrightarrow{0} C$$

From D to reach C

$$D \xrightarrow{0} C$$

$$D \xrightarrow{1} B \xrightarrow{0} C$$

So 10 is the shortest input sequence to reach the final state C, whatever be the initial state.

1.10 (c)

(i) $(00)^*(\varepsilon + 0)$
$\equiv (00)^* \varepsilon + (00)^* 0 = 0^*$
Even no. of 0's and Odd no. of 0's i.e., any number of 0's

(ii) $(00)^* \equiv$ Even no. of 0's

(iii) $0^* \equiv$ any no. of 0's

(iv) $0(00)^* \equiv$ Odd no. of 0's

So (i) and (iii) are same.

1.11 (d)

(a) L = {x|x has an equal number of a's and b's} is context free language (since there is comparison between number of a's and number of b's)

(b) $L = \{a^n b^n | n \geq 1\}$ is context free language (since there is comparison between number of a's and number of b's)

(c) L = {x|x has more a's than b's} is context free language (since there is comparison between number of a's and number of b's)

(d) $L = \{a^m b^n | m \geq 1, n \geq 1\}$ is regular (since m and n are independent and hence there is no comparison). The regular expression is aa^*bb^*.

1.12 (b)

(a) The set of all strings over Σ is Σ^* which is countably infinite.

(b) Set of all languages over Σ is 2^{Σ^*}. According to Cantor's theorem if S be an countably infinite set, then its power set 2^S is uncountable.

So 2^{Σ^*} is uncountable because Σ^* is countably infinite.

(c) Set of all regular languages over Σ is countably infinite.

(d) Set of all languages over Σ accepted by turing machine is the set of all RE languages which is countably infinite.

1.13 (d)

(a) r.e. (regular expression) = $0^*(1 + 0)^*$ can generate string 100, which contains substring 100

(b) r.e. (regular expression) = $0^*\ 1010^*$ can generate string 10100, which contain 100 as a substring. Also, this regular expression cannot generate ε which is in the given language.

(c) r.e. (regular expression) = $0^*1^*01^*$ generates strings which doesn't contain 100 as substring. However, ε is the smallest string which doesn't contain 100 as substring but above RE can't generate ε.

(d) r.e. (regular expression) = $0^*(10 + 1)^*$ generates all strings which doesn't contain 100 as substring.

1.14 (d)

$$(a^*b^*)^* \equiv (a + b)^*$$

So $\quad B = ((01)^*\ 1^*)^*$

$\equiv (01 + 1)^* \equiv A$

So, $\quad A = B$

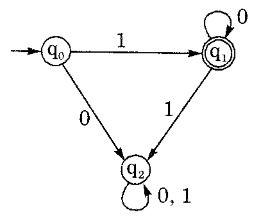
1.15 (a)

The numbers 1, 2, 4, 8,...2^n ... are represented by 1, 10, 100, 1000,....

The pattern in the regular expression is 1 followed by 0's.

The regular expression for above is 10^*

The DFA for above language is



So the numbers 1, 2, 4, 8,.... 2^n ... written in binary can be recognized by a deterministic finite state automaton.

1.16 (c,d)

(c) The string 1101 doesn't belong to set represented by $(10)^*\ (01)^*\ (00 + 11)^*$ because once 11 appears in string then 1 and 0 only appears in pairs.

(d) $(00 + (11)^*\ 0)^*$ can generate only strings with even number of 1's and hence cannot generate 1101.

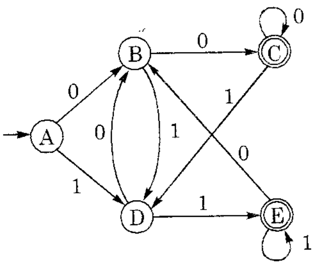
1.17 (d)

Number of substrings (of all lengths inclusive) that can be formed from a character strings of length n is $\frac{n(n+1)}{2} + 1$.

Since we do not want to count the substring of zero length (i.e. null string), the number of substrings becomes $\frac{n(n+1)}{2}$.

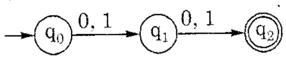
1.18 (b)

The number of states in the minimum state deterministic finite state automation accepting all binary strings whose last two symbols are the same is 5.



1.19 (b)

The minimum state finite automation that recognizes the language represented by regular expression (0 + 1) (0 + 1) ... n times is n + 1.
This language contains strings with exactly length n.
(n + 1) states are required to count length upto n. No trap state is required since we are making minimal FA, not minimal DFA.
For ***example***, for n = 2 the design is shown below.



1.20 (c)

According to rules of regular expressions

$$(r_1 + r_2)^* \equiv (r_1 + r_2^*)^*$$

Therefore $(a + b^*)^* \equiv (a + b)^*$

So $S = T$

1.21 (b)

The language generated by the grammar

$S \rightarrow 0S0 \mid 00$ is

$L = \{0^2, 0^4, 0^6, 0^8, ...\}$

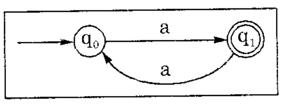
$= \{0^{2n+2} \mid n \geq 0\}$

$\Rightarrow = \{0^{2n} \mid n \geq 1\} = 00(00)^*$

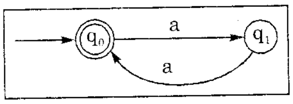
So above language is regular but not 0^+.

1.22 (d)

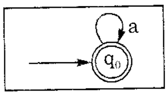
$L = \{a^n \mid n \text{ is odd}\}$



$L = \{a^n \mid n \text{ is even}\}$



$L = \{a^n \mid n \geq 0\}$



So for two states minimal finite state automation, L can be $\{a^n \mid n \text{ is odd}\}$ or L can be $\{a^n \mid n \text{ is even}\}$

1.23 (a)

$L_1 = \{0^{2n} \mid n \geq 1\}$

L_1 produces language having even number of 0's which is regular language.
Regular expression for S_1 is $00(00)^*$
$L_2 = \{0^m 1^n 0^{m+n} \mid m \geq 1 \text{ and } n \geq 1\}$ is context free language not regular. (m occurs in two places, so there is comparison of count).
So, S_1 is correct but S_2 is not correct.

1.24 (b)

For an arbitrary NFA with N states, the maximum number of states in an equivalent minimized DFA is 2^N.

1.25 (d)

A DFA over $\Sigma = \{a, b\}$ accepting all strings which have no. of a's divisible by 6 and number of b's divisible by 8 is a grid machine (product automata) having $6 \times 8 = 48$ states.

1.26 (d)

$L_1 = \{ww \mid w \in \{a, b\}^*\}$

is context sensitive language (CSL) (since there is infinite string matching in straight order).

$L_2 = \{ww^R \mid w \in (a, b)^*, w^R$ is the reverse of w$\}$

is context free language (since there is infinite string matching in reverse order).

$L_3 = \{0^{2i} \mid i \text{ is an integer}\} = (00)^*$

is regular language which contains all strings having even number of 0's.

$L_4 = \{0^{i^2} \mid i \text{ is an integer}\}$

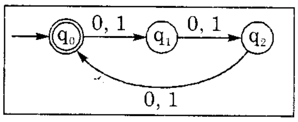
is context sensitive language (CSL) (since the power is infinite and non linear).

1.27 **(a)**

The state diagram represents the FSM which outputs the sum of the present and previous bits of the input.
State A represents previous bit is a 0 and B and C represents previous bit is a 1.

1.28 **(b)**

The minimal finite automation with 3 states which accepts the language $L = \{x \mid \text{length of } x \text{ is divisible by } 3\}$ is as follows:



1.29 **(c)**

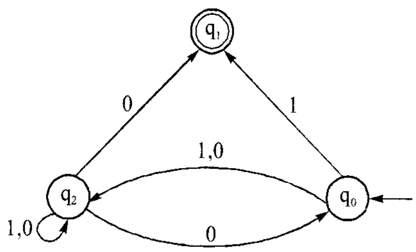
The given bit pattern can be represented as:
1 — — 1 — — 1
The four blanks can be filled in $2^4 = 16$ ways. Therefore there are 16 such strings in this pattern. Not all of these are accepted by the machine. The strings and its acceptance is given below:

							accepted
1	0	0	1	0	0	1	✓
1	0	0	1	0	1	1	✓
1	0	0	1	1	0	1	✓
1	0	0	1	1	1	1	✓
1	0	1	1	0	0	1	✓
1	1	0	1	0	0	1	✓
1	1	1	1	0	0	1	✓

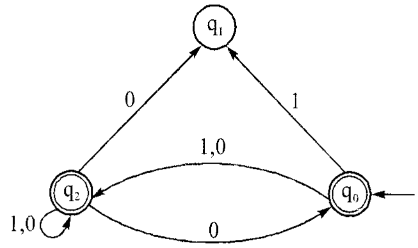
Only these seven strings given above are accepted. The other strings (9 of them) in this pattern are rejected, since they dont reached the final state.
∴ Correct answer is (c).

1.30 **(b)**

The given machine M is



Now the complementary machine $\overline{M}$ is



In the case of dfa, $L(\overline{M}) = \overline{L(M)}$ but in the case of nfa this is not true. In fact $L(\overline{M})$ and $L(M)$ have no connection.

∴ To find $L_1 = L(\overline{M})$ we have to look at $\overline{M}$ and directly find its language.

$$L_1 = L(\overline{M}) = \varepsilon + (0 + 1)(0 + 1)^* + \ldots$$
$$= (0 + 1)^* + \ldots = (0 + 1)^*$$

1.31 **(c)**

In choices (a), (b) and (d), inside the parenthesis we can generate "a", "b" and "c" seperately and hence all three are same as $(a + b + c)^*$.
In choice (c) the strings "a" and "b" cannot be generated seperately since "ab" is always together.
So, choice (c) is not same as $(a + b + c)^*$.

1.32 **(b)**

$$\delta(A, a) = A \equiv A \to aA$$
$$\delta(A, b) = B \equiv A \to bB$$
$$\delta(B, a) = B \equiv B \to aB$$
$$\delta(B, b) = A \equiv B \to bA$$

Since B is final state, so we need to put $B \to \varepsilon$.
So the correct grammar is choice (b) which is $\{A \to aA,\ A \to bB,\ B \to aB,\ B \to bA,\ B \to \varepsilon\}$.

1.33 **(a)**

The given finite state machine accepts any string $w \in (0, 1)^*$ in which the number of 1s is multiple of 3 and the number of 0s is multiple of 2.

1.34 **(b)**

Given regular expression is infinite set (because of *) of finite strings. A regular expression cannot generate any infinite string (since string is always finite in length by definition).

1.35 **(a)**

Given language is finite. Hence it is regular language.

1.36 (a)

Writing Y and Z interms of incoming arrows (Arden's method), we get

$$Y = X0 + Y0 + Z1$$
$$Z = X0 + Z1 + Y0$$

Clearly, Y = Z

1.37 (b)

The given grammar after substitution of X and Y becomes

$$S \rightarrow Zaa \mid Waa$$
$$Z \rightarrow Sa \mid \in$$
$$W \rightarrow Sa$$

Which after substituting Z and W is equivalent to $S \rightarrow Saaa \mid aa \mid Saaa$.
Which is equivalent to $S \rightarrow Saaa \mid aa$.
So, L(G) = (aaa)*aa
So the language generated by the grammar is the set of strings with a's such that number of a mod 3 is 2.So the number of states required should be 3 to maintain the count of number of a's mod 3.

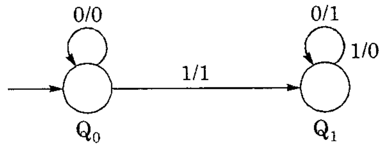
1.38 (d)

If L is regular ⇒ L satisfies the pumping lemma for regular languages.
If L is CFL ⇒ L satisfies the pumping lemma for CFLs.
By satisfying pumping lemma, we can never say that a language is regular or CFL. It can only be used to prove that a certain language is not regular or not CFL in case the language violates the corresponding pummping lemma.
So, both regular and non-regular languages can satisfy pumping lemma for regular language.
Similarly, both CFLs and non-CFLs can satisfy pumping lemma for CFLs.
So satisfying pumping lemma doesn't prove anything about the type of langauge.

1.39 (b)

(a) is false since M is accepting "abbb".
(b) is true.
(c) is false since "abba" contains "abb" as a substring, but is being rejected by the machine.
(d) is false, since λ does not contain "aa" as a substring, but λ is being rejected by M.

1.40 (b)

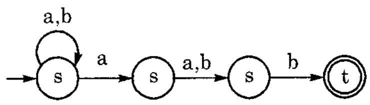


The given machine, executes the algorithm for 2's complement when input is given from LSB.

1.41 (d)

u = abbaba: Accepted by automata.
v = bab: Not accepted by automata.
w = aabb: Not accepted by automata.

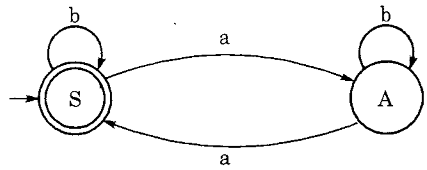
1.42 (a)



(a + b)*a (a + b)b.

1.43 (a)

The given right-linear grammar can be converted to the following DFA.



The machine accepts all strings over the alphabet {a, b} which have an even number of a's. It is a minimal DFA.
So Myhill-Nerode equivalnce classes for the language is nothing but the set of strings reaching S and A respectively.

$S = \{w \in (a + b)^* \mid \#_a(w) \text{ is even}\}$
$A = \{w \in (a + b)^* \mid \#_a(w) \text{ is odd}\}$

1.44 (c)

(a) Every language has a regular superset: True. Σ* is such a superset.
(b) Every language has a regular subset: True. φ is such a subset.
(c) Every subset of a regular language is regular: False $a^nb^n \subseteq \Sigma^*$, but a^nb^n is not Regular.
(d) Every subset of a finite language is regular: True. Every subset of a finite set must be finite by definition. Every finite set is regular. Hence, every subset of a finite language is regular.

1.45 (b)

Prefix (L), suffix (L) and Half (L) are regular languages.

Repeat (L) is not a regular language but a CSL.

1.46 (a)

Option (a): If L = (a + b)*, then repeat (L) = {ww | w∈ (a+b)*} is clearly not regular. So option (a) is best suited to show that repeat (L) need not be regular.

Option (b): If L = {ε, a, ab, bab}, then repeat (L) = {ww | w ∈ L} becomes finite and hence regular. So option (b) is not suited to show that repeat (L) need not be regular.

Option (c): If L = (ab)*, then repeat (L) = {ww | w∈ (ab)*} = (ab)* which is regular.

So option (c) is not suited to show that repeat (L) need not be regular.

Option (d): L = $\{a^n b^n \mid n \geq 0\}$, is not suited since it is not regular.

1.47 (c)

Choice (a) is regular since it is finite.

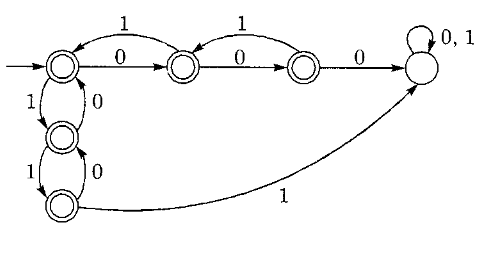
Choice (b) is regular since although comparison is made between 0's and 1's, it is for all prefixes and this can be done by DFA.

Note: $|n_0(s') - n_1(s')| \leq 2$ is same as $n_0(s') - n_1(s') \geq 2$ or $n_1(s') - n_0(s') \geq 2$.

Choice (c) involves comparison of number of 0's and 1's, but for the string as a whole, and this cannot be done by a dfa, since it has finite memory and has no stack for counting upto infinity. Therefore, choice (c) is not regular.

Choice (d) is regular since $n_0(s) \bmod 7 = n_1(s) \bmod 5 = 0$ means number of 0's is divisible by 7 and number of 1's is divisible by 5 and this can be accepted by a dfa with 7 × 5 = 35 states.

A minimal DFA that will accept the language of choice (b) is shown below:



1.48 (d)

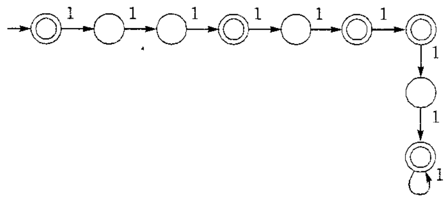
L = (111 + 11111)*

Here Σ = {1}

Note that λ ∈ L, 1 ∉ L, 11 ∉ L, 111 ∈ L, 1111 ∉ L, 11111 ∈ L, 111111 ∈ L, 1111111 ∉ L, 11111111 ∈ L.

Notice also that L includes all w such that $n_1(w) \geq 8$, since all words with more than eight 1's can be generated by some combination of 111 and 11111.

Therefore, the required dfa is shown below:



This dfa has 9 states.

1.49 (b)

(a) "Every subset of a regular set is regular" is false, since $L_1 = \Sigma^*$ and $L_2 = \{a^n b^n, n \geq 0\}$. Here, $L_2 \subseteq L$, but L_2 is not regular.

(b) "Every finite subset of a non-regular set is regular", is true, since all finite sets are regular.

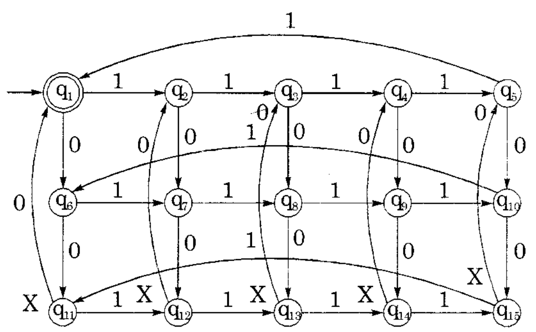
(c) "The union of two non-regular sets is not regular" is false, since if you take $L_1 = \{a^n b^n; n \geq 0\}$ and L_1^c, neither of these is regular but $L_1 \cup L_1^c = \Sigma^*$, is regular.

(d) "Infinite union of finite sets is regular" is false, since regular sets are not closed under infinite union.

1.50 (a)

L = {w | s∈ {0, 1}* number if 0's and 1's in w are divisible by 3 and 5 respectively}

The minimum state deterministic finite automation accepting the language L has 3 × 5 = 15 states.



1.51 (c)

$L = \{w\,x\,w^R \mid x, w, \in (0, 1)^+\}$is regular. Put w as 0 and 1 which are its minimal string values and get a minimal expression

$$r = [0(0 + 1)^+ 0] + [1(0 + 1)^+ 1]$$

Now putting w as any other string will not create any new string not generated by r.

Example if we put w as say 01, then $wxw^R = 01(0 + 1)^+ 10$ which is already generated by the first part of r.

Similarly if we put w as say 10, then $wxw^R = 10(0 + 1)^+ 01$ which is already generated by the second part of r.

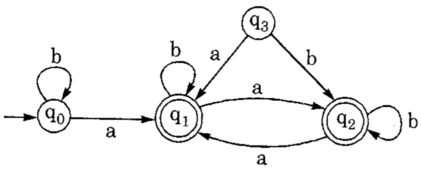
So, $L = L(r)$

So the regular expression for L is

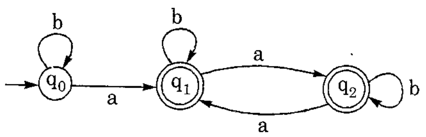
$$r = [0(0 + 1)^+ 0] + [1(0 + 1)^+ 1]$$

That is, L reduces to the language of words which starts and ends with the same symbol. Since we are able to write a regular expression for L, therefore L is regular.

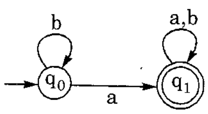
1.52 (c)



q_3 is unreachable from starting state and hence can be deleted to give the following diagram.



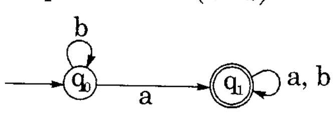
q_0 is the initial state. From q_0 to q_1 the regular expression is b* a. After that, every combination of a and b is accepted by q_1 or q_2. So q_1 and q_2 together can be collapsed into a permanent accept state and now the diagram becomes.



$\therefore \quad r = b^*a(a + b)^*.$

1.53 (b)

The minimum state FSA is given below for the regular expression $b^*a(a + b)^*$



So FSA contains minimum two states q_0 and q_1.

1.54 (a)

G1: $S \rightarrow x \mid z \mid xS \mid zS \mid yB$

$B \rightarrow y \mid z \mid yB \mid zB$

$B \rightarrow (y + z)^+$

Substitute in S to get

$S \rightarrow x \mid z \mid xS \mid zS \mid y(y + z)^+$

Now solution of S is

$S \rightarrow (x + z)^*S$

$S \rightarrow (x + z)^*(x + z + y\ (y + z)^+)$

So $\quad L(G1) = S = (x+z)^+ + (x + z)^*\ y(y + z)^+$

G1 generates every string in which "no y appears before any x".

G2: $S \rightarrow y \mid z \mid yS \mid zS \mid xB$

$B \rightarrow y \mid yS$

Substitute B in S to get

$S \rightarrow y \mid z \mid yS \mid zS \mid xy \mid xyS$

Now solution of S is

$S \rightarrow (y + z + xy)^*S$

$S \rightarrow (y + z + xy)^*\ (y + z + xy)$

So $\quad L(G2) = S = (y + z + xy)^+$

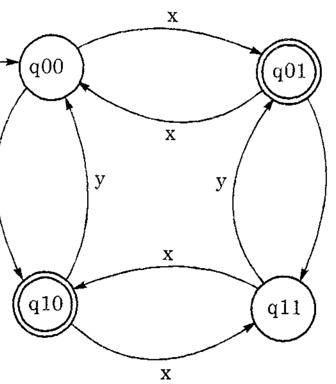
G2 generates every string in which "every x followed by atleast one y".

1.55 (d)

Given DFA can be redesigned as S0 as q00, S1 as q10, S2 as q11, S3 as q01.

Each state is q_{ab} [$a = n_a$ mod2, $b = n_b$ mod2].

q00 as $n_a$mod2 = 0, $n_b$mod2 = 0 [number of x is even, number of y is even].



q01 is final state mean where number of x is even and number of y is odd.

q10 is final state mean where number of x is odd and number of y is even.

1.56 (c)

This grammar cannot generate any string starting with *xx* or ending with *xx* so (i), (ii) and

(v) cannot be generated by the grammar. xyx also cannot be generated by the grammar. The derivation for (iii) $xyxy$ and (iv) $yxxy$ is shown below.

$S \to xB \to xyS \to xyxB \to xyxy$

$S \to yA \to yxS \to yxxS \to yxxy$

So only (iii) and (iv) can we derived from this grammar.

1.57 (b)

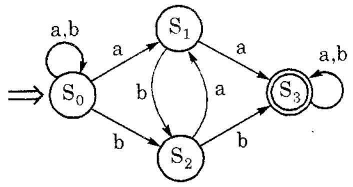
L(P) ∩ L(Q) must contain all strings common to P and Q. Note that 'aa' is common to both P and Q and hence must be accepted by any FA accepting L(P) ∩ L(Q). However, all of the given machine reject 'aa'. Therefore option (e) i.e. none of these is the right answer.

1.58 (a)

The language of the given regular expression R is 'containing the substring aa or bb'. Option (a) is the correct machine for this language.

1.59 (a)

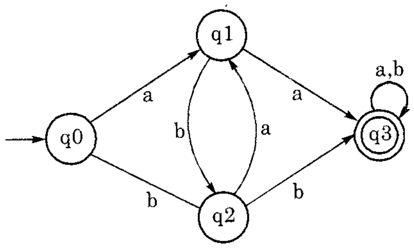
In option (a) S3 and S4 together act as a permanent accept and can therefore be collapsed into a single permanent accept state as shown below.



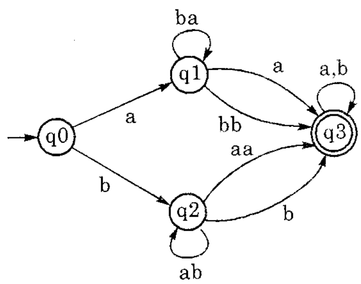
This machine clearly accepts all strings containing the substring 'aa' or 'bb', which is same as regular expression R.

1.60 (c)

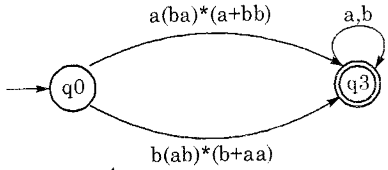
R = (a + b)*(aa + bb)(a + b)* has following equivalent DFA



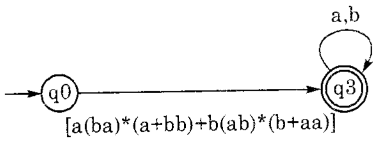
Which is equivalent Transition graph [by removing transition from q1 to q2 and q2 to q1 but does not effect on language]



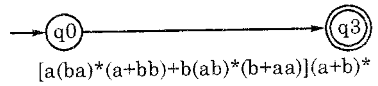
That is equivalent to :



which is equivalent to :



which is equivalent to :



Equivalent regular expression is [a(ba)* (a+bb) + b(ab)*(b+aa)](a+b)*.

1.61 (c)

(a) with at least 2 consecutive 1's, any no of 0's and any no of 1's
(b) exactly two consecutive 1's
(c) exactly two 1's but need not be consecutive
(d) Any no of 1's and 0's with at least two 1's

1.62 (a)

A state in a DFA will be a subset of the set of states of the equivalent NFA.

So, the maximum number of states in the equivalent DFA of an NFA, will be 2^n, where n is the number of states in NFA, as a set with n elements has maximum 2^n subsets.

So, number of states in equivalent minimal DFA ≤ number of states in equivalent DFA ≤ 2^n.

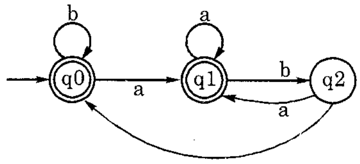
1.63 (a)

Interchanging final and non-final states of DFA is used for complementation.

Given DFA generates all strings end with ab.

Complement of DFA accepts all strings do not end with ab.

Complement of given DFA is:



It accepts all strings that do not end with ab.

1.64 (d)

L1 is regular. Since 10000 is finite.
L3 is also regular (mod machine, since finite number of residue combinations).
L2 = $\{w \mid w = w^R\}$ which is the Palindrome language which is known to be non regular.

1.65 (a)

M1 is a DFA accepting all string containing 2 consecutive 1's (containing the substring '11').
M2 is a NFA accepting the same language. Containing 2 consecutive 1's.
So, L1 = L2.

1.66 (a)

The transition tables for Y and Z is as follows:

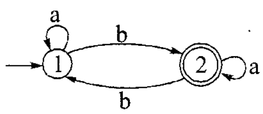
Y.

	a	b
→1	1	2
2(F)	2	1

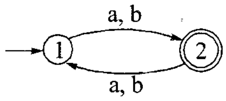
Z.

	a	b
→1	2	2
2(F)	1	1

Z contains 2 states and Y contains 2 states so the product automation Z × Y contains 2 × 2 = 4 states.



Y



Z

The table for Z × Y is shown below

	a	b
→(1, 1)	(2, 1)	(2, 2)
(1, 2)	(2, 2)	(2, 1)
(2, 1)	(1, 1)	(1, 2)
(F)(2, 2)	(1, 2)	(1, 1)

State (1, 1) has to be P since (1, 1) is starting state and P is also starting state in answer given.
State (2, 2) has to be R since (2, 2) is the final state and R is also final state in answer given.

Now the remaining 2 states (1, 2) & (2, 1) can be assigned the letters Q & S or S & Q resulting in 2 possible tables.

Table 1.

	a	b
P	S	R
Q(1, 2)	R	S
S(2, 1)	P	Q
R	Q	P

Table 2.

	a	b
P	S	R
S(1, 2)	R	Q
Q(2, 1)	P	S
R	Q	P

Rewriting these two table in same order (P, Q, R, S) as given in answers we get

Table 1.

	a	b
→P	S	R
Q	R	S
R(F)	Q	P
S	P	Q

Table 2.

	a	b
→P	S	R
Q	P	S
R(F)	Q	P
S	R	Q

Now choice (a) is exactly same as the table (1) shown above and hence choice (a) is acceptable as a correct answer.

1.67 (c)

A : $\in$ + 0(01*1+00)* (01*+0) which is same as expression 1.
B : Same as expression 2.
C : Same as expression 3.
D : $\in$ + 0(10*1+10)* (10*+1) which is same as expression 4.

A-1, B-2, C-3, D-4

Note: In each NFA the initial state of NFA is also the final state of NFA so it contains ε in each regular expression.

1.68 (a)

1. $\{a^n b^{2m} \mid n \geq 0, m \geq 0\}$ is regular, since we can write L as a regular expression a* (bb)*.
2. $\{a^n b^m \mid n = 2m\}$ is a DCFL, but not regular since here, we need to count the a's and compare with b's.

3. $\{a^n b^m \mid n \neq m\}$ is same as $\{a^n b^m \mid n < m\} \cup \{a^n b^m \mid n > m\}$ each of which is a CFL and the union is also a CFL (Since CFLs are closed under union). However this language is not regular since we have to count a's and b's and compare them which cannot be done by a finite state machine.
4. $\{xcy \mid x, y \in (a, b)^*\}$ is regular since we can write a regular expression $(a + b)^*c\,(a + b)^*$ for it.

1.69 (c)

$(0 + 1)^*\ 0\ (0 + 1)^*\ 0\ (0 + 1)^*$ is the regular expression for atleast two zero's.

1.70 (c)

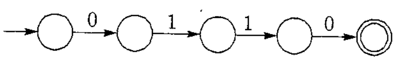
The given DFA is the standard construction for language given by choice (c) "ending with 00".

1.71 (b)

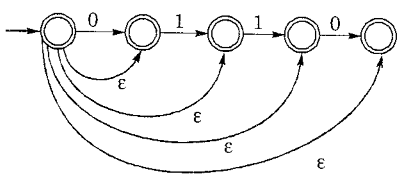
Choice (a) $(0^*\ 1\ 0^*\ 1)^*$
will always generate strings ending with 1. But we want an expression for bit strings with even no of 1's, which includes strings like "110" which ends with 0.
So, choice (a) is not correct.
Choice (c) $0^*\ (1\ 0^*\ 1)^*\ 0^*$
"1010101" $\notin 0^*(1\ 0^*\ 1)^*\ 0^*$
but "1010101" has even number of 1's. So choice (c) is incorrect.
Choice (d) $\lambda \notin 0^*\ 1(1\ 0^*\ 1)^*\ 10^*$
but "λ" is a bit string with even no of 1's (zero 1's). So choice (d) is incorrect.
Choice (b) can generate all bit strings with even no of 1's. So choice (b) is correct.

1.72 (c)

Let w = "0110". To design a minimal NFA to accept all the substrings of this string, first accept the string itself as follows.



Now, to accept all the other substrings of "0110", make every state as starting state by using null moves and also make every state as final state. The final NFA is shown below.



Since a string of length 4 requires 5 states, a string of length n will require n + 1 states.

1.73 (a)

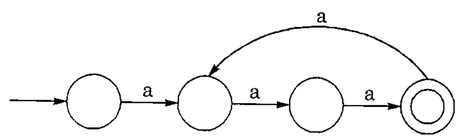
Lexical analysis only requires the power of FA.

1.74 (c)

$\Sigma^* - P = P^c$
If P is regular, then P^c is also regular, since regular language is closed under complementation.

1.75 (b)

Let $n = 3$ (say)
then $L = \{a^{3k} \mid k > 0\} = \{a^{3k} \mid k \geq 1\}$
The dfa for this will be

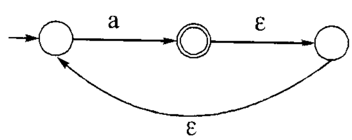


which has 4 states.
Similarly for $L = \{a^{nk} \mid k > 0\}$, we will need n + 1 states.

1.76 (a)

The language accepted by given dfa is
$L = a(a + b)^* + ba(a + b)^*$
Choice (a) is accepting same language and is minimal.
"aba" is rejected by choice (b) and it belongs to L. So choice (b) is incorrect.
Choice (c) is an NFA and hence not the right answer.
"bba" is accepted by choice (d) and is rejected by L. So choice (d) is incorrect.

1.77 (b)



Language for NFA $L = a^+$
Now $\bar{L} = \Sigma^* - L$
$\Rightarrow \bar{L} = a^* - a^+$
$\Rightarrow \bar{L} = \{\varepsilon\}$

1.78 (c)

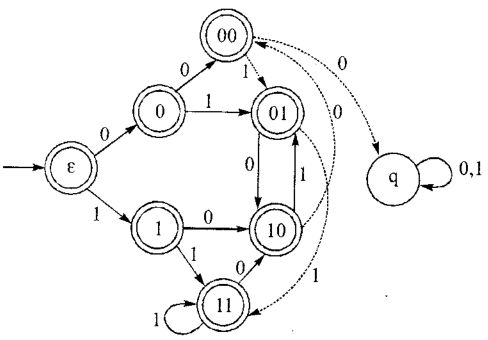
L = {ab, aa, baa}
The breakdown of the strings 1, 2, 4 in terms of ab, aa and baa is shown below:
1. ab aa baa ab aa

2. aa aab aa aa
3. baa aa ab aa

String no (3) has no breakdown in terms of strings in L and hence string (3) does not belong to L*.
Only 1, 2 and 4 belongs to L*.

1.79 (d)

Notice that the state names are given based on ending bits of the string, which has been processed.
The arc from 00 labeled "0" should go to trap state q (since at most 2 zeros are allowed in any substring).
Based on this fact, option (a) and (b) are incorrect.
Between option (c) and (d), if you look at arc labeled "1" from state 01, this arc should go to state 11 since the string at this point is ending with 11. So option (c) is wrong and option (d) is correct.
The dfa corresponding to correct option (d) is shown below with missing arrows shown in dotted lines.



1.80 (a)

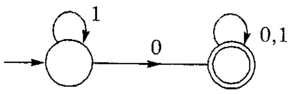
$L_1 = \phi$
$L_2 = \{a\}$

So $L_1 L_2^* \cup L_1^* = \phi \cdot a^* \cup \phi^* = \phi \cup \varepsilon = \varepsilon$

1.81 (d)

1. A is a NFA. So L(A) is regular. Complement of L(A) is also regular, since regular languages are closed under complement. So, L(A) is context-free is true.
2. L(A) = L((11 *0 + 0) (0 + 1)*0*1*) : true because the r.e. for the language accepted by above automata is (11*0 + 0) (0 + 1)*.
3. For the language accepted by A, A is the minimal DFA : false

Writing the r.e. for the language accepted by the given DFA and simplifying, we get,
L(A) = 11*0 (0 + 1)* + 0(0 + 1)*
= (11* + ε) 0(0 + 1)* = 1*0(0 + 1)*
The minimal DFA for the above language is



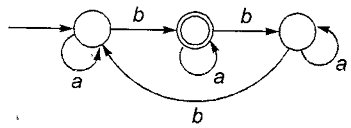
4. Machine A accepts all the string over {0, 1} of length at least 2 : False

Above DFA accepts string "0" whose length is less than 2.

1.82 (c)

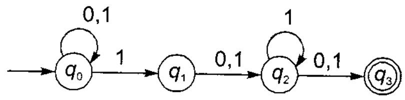
$L = \{a^n b^n \mid n \geq 0\}$ is not regular
$L = \{a^n \mid n \text{ is prime}\}$ is not regular
$L = \{ww \mid w \in \Sigma^*, \Sigma = \{0, 1\}$ is not regular
$L = \{w \mid w$ has $3k+1$ b's for some $k \in N$, $\Sigma = \{a, b\}\}$
$= \{w \mid$ Number of b's in w are 1 modulo 3$\}$ is regular since it can be accepted by a mod 3 machine as shown below.

DFA :

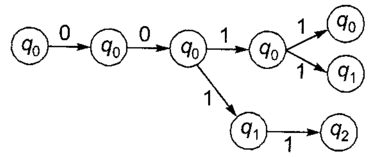


1.83 (a)

Given FA is

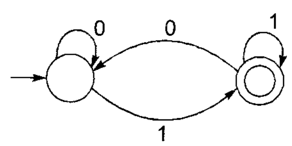


For input string 0011



∴ $\{q_0, q_1, q_2\}$ are reachable states.

1.84 (b)



I. 0*1 (1 + 00*1)* is a correct r.e. for the given machine.
II. 0*1*1+11*0*1 is not correct.
0101 is accepted by DFA, but can not be generated by this regular expression

III. $(0 + 1)^*\ 1$ is correct since the language accepted by the given DFA is all strings ending with 1.

$\therefore$ I and III are equivalent to given DFA.

1.85 (a)

$$L_1 = \{a^n \mid n \geq 0\}$$
$$L_2 = \{b^n \mid n \geq 0\}$$

I. $L_1 \cdot L_2 = \{a^m b^n \mid m, n \geq 0\} = a^*b^*$ is regular.

II. $L_1 \cdot L_2 \neq \{a^n b^n \mid n \geq 0\}$

$\Rightarrow$ II is not correct.

$\therefore$ Only I is correct.

1.86 (a)

A machine which accepts L_1 needs only finite memory since there is no need to keep the number of 110's in memory. The reason is because whenever two consecutive 110's come in a row then a 011 always come in between as shown below.

1 1 0 1 1 0

So the difference between number of 110 and number of 011 can only be 1, 0, −1.

So only finite memory needed and therefore L_1 is regular.

L_2 requires infinite memory since any number of 000's can come in a row without any 111 in between. So the difference between the number of 000's and 111's can go upto ∞ or $-\infty$. So this language requires inifinite memory. But FA has only finite memory.

So L_2 is not regular.

1.87 Sol.

Check the string one-by-one starting from ε, a, b, aa, ab, ba, bb,... until we reach the first string that is not generated by the given regular expression a*b*(ba)*a*.

In this case smallest string NOT generated by the given regular expression is 'bab', whose length is 3.

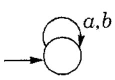
1.88 Sol.

L(M) = (a + b)*a (strings ending with a)

L(N) = (a + b)*b (strings ending with b)

L(M) $\cap$ L(N) = ϕ (nothing in common)

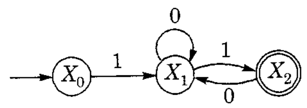
$\therefore$ Number of states = 1



minimal DFA which accepts empty language.

1.89 (c)

Converting the given transitions to a state diagram we get,



From this diagram we can write,

$$X_0 = 1(0+10)^*1$$

1.90 (a)

$L_1 = \{wxw^R \mid w, x \in \{a, b\}^* \text{ and } |w|, |x| > 0\}$

w cannot be put as "∈" since $|w| > 0$. So we put w as its smallest string which is 'a' and 'b' and get the regular expression:

$$r = a(a + b)^+a + b(a + b)^+ b.$$

Now putting w as any other string like say "ab" will not add any new string to the expression r, since any such string so generated will be either already generated by either $a(a+b)^+a$ or $b(a+b)^+b$.

So the given language = $a(a+b)^+a + b(a+b)^+b$ which is clearly regular.

$\therefore$ L_1 is regular.

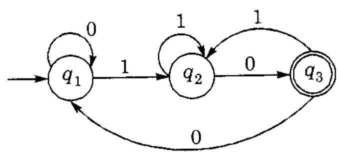
$L_2 = \{a^nb^m \mid m \neq n\}$. This language has infinite comparisons between number of a's and b's. L_2 is not regular language.

L_3: $\{a^pb^qc^r \mid p, q, r \geq 0\} = a^*b^*c^*$ is a regular language.

1.91 Sol.

The given regular expression (0+1)*(10) corresponds to binary string ending with "10". To accept the minimal string "10", we need 3 states. No trap state is required since this is a machine which accepts "ending with" type strings.

So, we need only 3 states. The design of the minimal DFA is shown below.



$\therefore$ Minimum number of states required for DFA = 3.

1.92 **(b)**

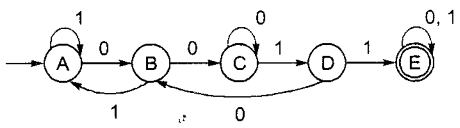
- $\Sigma^* 0011\ \Sigma^* = (0 + 1)^*\ 0011(0 + 1)^*$

Every string of the language L contains the substring 0011.

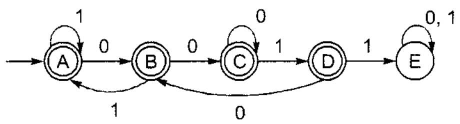
To accept the minimal string "0011" we need 5 states. Since substring machines don't required trap state, we need only 5 states.

Since the complementary DFA will accept complementary language, and since complementary DFA has same number of states as the original DFA, therefore $\bar{L}$ also has only 5 states as shown below.

Minimized DFA for

$L =$ 

Minimized DFA for

$\bar{L} =$ 

1.93 **(d)**

G: $S \to aS \mid bS \mid \varepsilon$

$L(G) = \{a, b\}^*$.

1.94 **(c)**

I. Disjointedness problem of regular = Decidable

II. Membership of CFL's = Decidable

III. Equivalence of CFL's = Undecidable

IV. Emptiness of RE language's = Undecidable

So, III and IV only is correct answer.

1.95 **(b)**

We wish to find regular expression "for all binary strings containing two consecutive 0's and two consecutive 1's".

Now, choice (a) cannot generate "00011"

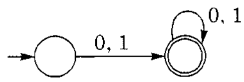
Choice (b) is correct

Choice (c) "00" which does not belong to given language.

Choice (d) always ends with 11 or 00 and hence cannot generate "001101".

1.96 **Sol.**

$L = (0 + 1)^*(0 + 1)(0 + 1)^* = (0 + 1)^+$

Min DFA : 

2 states.

1.97 **(b)**

I. Incorrect, since even if all states of NFA are accepting, dead configuration may be there and L need not be Σ^*.

II. Correct, since ϕ is regular and $\phi \cap B = \phi$ which is regular for all B.

■■■■

2 Push Down Automata: CFL & DCFL

2.1 A context-free grammar is ambiguous if
(a) the grammar contains useless non-terminals
(b) it produces more than one parse tree for some sentence
(c) some production has two non-terminals side by side on the right-hand side
(d) None of the above

[1987 : 2 Marks]

2.2 Fortran is
(a) regular language
(b) context free language
(c) context sensitive language
(d) None-of the above

[1987 : 2 Marks]

2.3 Context free languages and regular languages are both closed under the operation(s) of
(a) union (b) intersection
(c) concatenation (d) complementation

[1989 : 2 Marks]

2.4 Context-free languages are
(a) closed under union
(b) closed under complementation
(c) closed under intersection
(d) closed under Kleene closure

[1992 : 2 Marks]

2.5 If G is a context-free grammar and w is a string of length n in L(G), how long is a derivation of w in G, if G is Chomsky normal form?
(a) 2n (b) 2n + 1
(c) 2n – 1 (d) n

[1992 : 2 Marks]

2.6 Which of the following features cannot be captured by context-free grammar?
(a) Syntax of if-then-else statements
(b) Syntax of recursive procedures
(c) Whether a variable has been declared before its use
(d) Variable names of arbitrary length.

[1994 : 2 Marks]

2.7 Consider the grammar with the following productions.

$S \rightarrow a\,\alpha\, b \mid b\,\alpha\, c \mid ab$
$S \rightarrow \alpha\, S \mid b$
$S \rightarrow \alpha\, bb \mid ab$
$S\alpha \rightarrow bdb \mid bd$

the above grammar is
(a) context free grammar
(b) regular grammar
(c) context sensitive grammar
(d) LR(k)

[1995 : 1 Mark]

2.8 If L_1 and L_2 are context free languages and R a regular set, one of the languages below is not necessarily a context free language. Which one?
(a) L_1L_2 (b) $L_1 \cap L_2$
(c) $L_1 \cap R$ (d) $L_1 \cup L_2$

[1996 : 2 Marks]

2.9 Let G be a context free grammar with productions given below.

$S \rightarrow ABAC$
$A \rightarrow aA \mid \varepsilon$
$B \rightarrow bB \mid \varepsilon$
$C \rightarrow d$

Transform the grammar G to an equivalent form G′ that has no ε productions and no unit productions.

[1996 : 2 Marks]

2.10 Which of the following language over {a,b,c} is accepted by a deterministic push down automata?
(a) $\{wcw^R \mid w \in \{a, b\}^*\}$
(b) $\{ww^R \mid w \in \{a, b, c\}^*\}$
(c) $\{a^n b^n c^n \mid n \geq 0\}$
(d) {w | w is a palindrome over {a,b,c}}

[1997 : 2 Marks]

2.11 Which of the following statement is false?
(a) Every finite subset of a non-regular set is regular
(b) Every subset of a regular set is regular
(c) Every finite subset of a regular set is regular
(d) The intersection of two regular sets is regular

[1998 : 2 Marks]

2.12 Context free languages are closed under
(a) union, intersection
(b) union, kleene closure
(c) intersection, complement
(d) complement, kleene closure
[1999 : 1 Mark]

2.13 Let L_D be the set of all language accepted by a PDA by final state and L_E the set of all languages accepted by empty stack. Which of the following is true?
(a) $L_D = L_E$ (b) $L_D \supset L_E$
(c) $L_D \subset L_E$ (d) None of the above
[1999 : 1 Mark]

2.14 If L_1 is a context free language and L_2 is a regular language which of the following is are false?
(a) $L_1 - L_2$ is not context free
(b) $L_1 \cap L_2$ is context free
(c) $\sim L_1$ is context free
(d) ~ L2 is regular **[1999 : 2 Marks]**

2.15 Which of the following statement is true?
(a) if a language is context free it can always be accepted by a deterministic push-down automation
(b) the union of two context free language is context free
(c) the intersection of two context free languages is context free
(d) the complement of a context free languages is context free
[2001 : 1 Mark]

2.16 The language accepted by a Pushdown Automation in which the stack is limited to 10 items is best described as
(a) context free
(b) regular
(c) deterministic context free
(d) recursive
[2002 : 1 Mark]

2.17 Let G = ({S}, {a, b}, R, S) be a context free grammar where the rule set R is
$S \to a\,S\,b \mid S\,S \mid \varepsilon$
Which of the following statements is true?
(a) G is not ambiguous
(b) There exist x, y, ∈ L(G) such that $xy \notin L(G)$
(c) There is a deterministic pushdown automaton that accepts L(G)
(d) We can find a deterministic finite state automaton that accepts L(G)
[2003 : 2 Marks]

2.18 Which one of the following statements is FALSE?
(a) There exist context free languages such that all the context free grammars generating them are ambiguous
(b) An unambiguous context-free grammar always has a unique parse tree for each string of the language generated by it
(c) Both deterministic and non-deterministic pushdown automata always accept the same set of languages
(d) A finite set of strings from some alphabet is always a regular language
[IT-2004 : 1 Mark]

2.19 Let M = (K, Σ, Γ, Δ, s, F) be a pushdown automaton, where
K = (s, f), F = {f}, Σ = {a, b}, Γ = {a} and
$\Delta = \{((s, a, \varepsilon), (s, a)), ((s, b, \varepsilon), (s, a)), ((s, a, \varepsilon), (f, \varepsilon)), ((f, a, a), (f, \varepsilon)), ((f, b, a), (f, \varepsilon))\}$
Which one of the following strings is not a member of L(M)?
(a) aaa (b) aabab
(c) baaba (d) bab
[IT-2004 : 2 Marks]

2.20 The language $\{a^m b^n c^{m+n} \mid m, n \geq 1\}$ is
(a) regular
(b) context-free but not regular
(c) context sensitive but not context free
(d) type-0 but not context sensitive
[2004 : 2 Marks]

2.21 Consider the following grammar C
$S \to bS \mid aA \mid b$
$A \to bA \mid aB$
$B \to bB \mid aS \mid a$
Let $N_a(w)$ and $N_b(w)$ denote the number of a's and b's in a string w respectively. The language $L(G) \subseteq \{a, b\}^+$ generated by G is
(a) $\{w \mid N_a(w) > 3N_b(w)\}$
(b) $\{w \mid N_b(w) > 3N_a(w)\}$
(c) $\{w \mid N_a(w) = 3k, k \in \{0, 1, 2, ...\}\}$
(d) $\{w \mid N_b(w) = 3k, k \in \{0, 1, 2, ...\}\}$
[2004 : 2 Marks]

2.22 Let L be a regular language and M be a context-free language, both over the alphabet Σ. Let L^c and M^c denote the complements of L and M respectively.

Which of the following statements about the language $L^c \cup M^c$ is TRUE?

(a) It is necessarily regular but not necessarily context-free.
(b) It is necessarily context-free.
(c) It is necessarily non-regular.
(d) None of the above

[IT-2005 : 1 Mark]

2.23 Let P be a non-deterministic push-down automaton (NPDA) with exactly one state, q, and exactly one symbol, Z, in its stack alphabet. State q is both the starting as well as the accepting state of the PDA. The stack is initialized with one Z before the start of the operation of the PDA. Let the input alphabet of the PDA be Σ.

Let L(P) be the language accepted by the PDA by reading a string and reaching its accepting state. Let N(P) be the language accepted by the PDA by reading a string and emptying its stack. Which of the following statements is TRUE?

(a) L(P) is necessarily Σ^* but N(P) is not necessarily Σ^*.
(b) N(P) is necessarily Σ^* but L(P) is not necessarily Σ^*.
(c) Both L(P) and N(P) is necessarily Σ^*.
(d) Neither L(P) nor N(P) are necessarily Σ^*.

[IT-2005 : 2 Marks]

2.24 Let N_f and N_p denote the classes of languages accepted by non-deterministic finite automata and non-deterministic push-down automata, respectively. Let D_f and D_p denote the classes of languages accepted by deterministic finite automata and deterministic push-down automata, respectively. Which one of the following is TRUE?

(a) $D_f \subset N_f$ and $D_p \subset N_p$
(b) $D_f \subset N_f$ and $D_p = N_p$
(c) $D_f = N_f$ and $D_p = N_p$
(d) $D_f = N_f$ and $D_p \subset N_p$

[2005 : 2 Marks]

2.25 Consider the languages

$L_1 = \{a^n b^n c^m \mid n, m > 0\}$ and $L_2 = \{a^n b^m c^m \mid n, m > 0\}$

Which one of the following statements is FALSE?

(a) $L_1 \cap L_2$ is a context-free language
(b) $L_1 \cup L_2$ is a context-free language
(c) L_1 and L_2 are context-free language
(d) $L_1 \cap L_2$ is a context sensitive language

[2005 : 2 Marks]

2.26 Consider the languages

$L_1 = \{ww^R \mid w \in \{0, 1\}^*\}$

$L_2 = \{w \# w^R \mid w \in \{0, 1\}^*\}$, where # is a special symbol

$L_3 = \{ww \mid w \in \{0, 1\}^*\}$

Which one of the following is TRUE?

(a) L_1 is a deterministic CFL
(b) L_2 is a deterministic CFL
(c) L_3 is a CFL, but not a deterministic CFL
(d) L_3 is a deterministic CFL

[2005 : 2 Marks]

2.27 Let $L_1 = \{0^{n+m} 1^n 0^m \mid n, m \geq 0\}$,

$L_2 = \{0^{n+m} 1^{n+m} 0^m \mid n, m \geq 0\}$, and

$L_3 = \{0^{n+m} 1^{n+m} 0^{n+m} \mid n, m \geq 0\}$.

Which of these languages are NOT context free?

(a) L_1 only
(b) L_3 only
(c) L_1 and L_2
(d) L_2 and L_3

[2006 : 1 Mark]

2.28 In the context-free grammar below, S is the start symbol, a and b are terminals, and ε denotes the empty string

$$S \rightarrow aSa \mid bSb \mid a \mid b \mid \varepsilon$$

Which of the following strings is NOT generated by the grammar?

(a) aaaa
(b) baba
(c) abba
(d) babaaabab

[IT-2006 : 1 Mark]

2.29 Which of the following languages is accepted by a non-deterministic pushdown automaton (PDA) but NOT by a deterministic PDA?

(a) $\{a^n b^n c^n \mid n \geq 0\}$
(b) $\{a^l b^m c^n \mid l \neq m \text{ or } m \neq n\}$
(c) $\{a^n b^n \mid n \geq 0\}$
(d) $\{a^m b^n \mid m, n \geq 0\}$

[IT-2006 : 2 Marks]

2.30 Let L be a context-free language and M a regular language. Then the language $L \cap M$ is

(a) always regular
(b) never regular

(c) always a deterministic context-free language
(d) always a context-free language

[IT-2006 : 2 Marks]

2.31 Consider the pushdown automaton (PDA) below which runs over the input alphabet (a, b, c). It has the stack alphabet $\{Z_0, X\}$ where Z_0 is the bottom-of-stack marker. The set of states of the PDA is {s, t, u, f} where s is the start state and f is the final state. The PDA accepts by final state. The transitions of the PDA given below are depicted in a standard manner. For example, the transition $(s, b, X) \rightarrow (t, XZ_0)$ means that if the PDA is in state s and the symbol on the top of the stack is X, then it can read b from the input and move to state t after popping the top of stack and pushing the symbols Z_0 and X (in that order) on the stack.

$(s, a, Z_0) \rightarrow (s, XXZ_0)$
$(s, \varepsilon, Z_0) \rightarrow (f, \varepsilon)$
$(s, a, X) \rightarrow (s, XXX)$
$(s, b, X) \rightarrow (t, \varepsilon)$
$(t, b, X) \rightarrow (t, \varepsilon)$
$(t, c, X) \rightarrow (u, \varepsilon)$
$(u, c, X) \rightarrow (u, \varepsilon)$
$(u, \varepsilon, Z_0) \rightarrow (f, \varepsilon)$

The language accepted by the PDA is
(a) $\{a^l b^m c^n \mid l = m = n\}$
(b) $\{a^l b^m c^n \mid l = m\}$
(c) $\{a^l b^m c^n \mid 2l = m + n\}$
(d) $\{a^l b^m c^n \mid m = n\}$

[IT-2006 : 2 Marks]

2.32 In the context-free grammar below, S is the start symbol, a and b are terminals, and ε denotes the empty string.

$S \rightarrow aSAb \mid \varepsilon$
$A \rightarrow bA \mid \varepsilon$

The grammar generates the language
(a) $((a + b)^* b)^*$
(b) $\{a^m b^n \mid m \leq n\}$
(c) $\{a^m b^n \mid m = n\}$
(d) $a^* b^*$

[IT-2006 : 2 Marks]

2.33 Consider the following statements about the context-free grammar,
$G = \{S \rightarrow SS, S \rightarrow ab, S \rightarrow ba, S \rightarrow \varepsilon\}$
1. G is ambiguous.
2. G produces all strings with equal number of a's and b's.
3. G can be accepted by a deterministic PDA.

Which combination below expresses all the true statements about G?
(a) 1 only
(b) 1 and 3 only
(c) 2 and 3 only
(d) 1, 2 and 3

[2006 : 2 Marks]

2.34 Consider an ambiguous grammar G and its disambiguated version D. Let the language recognized by the two grammars be denoted by L(G) and L(D) respectively. Which one of the following is true?
(a) $L(D) \subset L(G)$
(b) $L(D) \supset L(G)$
(c) $L(D) = L(G)$
(d) L(D) is empty

[2007 : 1 Mark]

2.35 Consider the following grammars. Names representing terminals have been specified in capital letters.

G1: stmnt → WHILE (expr) stmnt
stmnt → OTHER
expr → ID

G2: stmnt → WHILE (expr) stmnt
stmnt → OTHER
expr → expr + expr
expr → expr * expr
expr → ID

Which one of the following statements is true?
(a) G_1 is context-free but not regular and G_2 is regular
(b) G_2 is context-free but not regular and G_1 is regular
(c) Both G_1 and G_2 are regular
(d) Both G_1 and G_2 are context-free but neither of them is regular

[IT-2007 : 2 Marks]

2.36 The language $L = \{0^i 2 1^i \mid i \geq 0\}$ over the alphabet {0, 1, 2} is
(a) not recursive
(b) is recursive and is a deterministic CFL
(c) is a regular language
(d) is not a deterministic CFL but a CFL

[IT-2007 : 2 Marks]

2.37 Which of the following statements is false?
(a) Every NFA can be converted to an equivalent DFA
(b) Every non-deterministic Turing machine can be converted to an equivalent deterministic Turing machine

(c) Every regular language is also a context-free language
(d) Every subset of a recursively enumerable set is recursive

[2008 : 2 Marks]

2.38 Which of the following statements are true?
1. Every left-recursive grammar can be converted to a right-recursive grammar and vice-versa
2. All ε-productions can be removed from any context-free grammar by suitable transformations
3. The language generated by a context-free grammar all of whose productions are of the form $X \rightarrow w$ or $X \rightarrow wY$ (where, w is a string of terminals and Y is a non-terminal), is always regular
4. The derivation trees of strings generated by a context-free grammar in Chomsky Normal Form are always binary trees

(a) 1, 2, 3 and 4 (b) 2, 3 and 4 only
(c) 1, 3 and 4 only (d) 1, 2 and 4 only

[2008 : 2 Marks]

2.39 Match **List-I** with **List-II** and select the correct answer using the codes given below the lists:

List-I
A. Checking that identifiers are declared before their use
B. Number of formal parameters in the declaration of a function agrees with the number of actual parameters in a use of that function
C. Arithmetic expressions with matched pairs of parentheses
D. Palindromes

List-II
1. $L = \{a^n b^m c^n d^m \mid n \geq 1, m \geq 1\}$
2. $X \rightarrow X b X \mid X c X \mid d X f \mid g$
3. $L = \{wcw \mid w \in (a \mid b)^*\}$
4. $X \rightarrow b X b \mid c X c \mid \varepsilon$

Codes:

	A	B	C	D
(a)	1	3	2	4
(b)	3	1	4	2
(c)	3	1	2	4
(d)	1	3	4	2

[2008 : 2 Marks]

2.40 Consider the following languages.
$L_1 = \{a^i b^j c^k \mid i = j, k \geq 1\}$
$L_2 = \{a^i b^j \mid j = 2i, i \geq 0\}$

Which of the following is true?
(a) L_1 is not a CFL but L_2 is
(b) $L_1 \cap L_2 = \phi$ and L_1 is non-regular
(c) $L_1 \cup L_2$ is not a CFL but L_2 is
(d) There is a 4 state PDA that accepts L_1, but there is no DPDA that accepts L_2

[IT-2008 : 2 Marks]

2.41 Consider a CFG with the following productions.
$S \rightarrow AA \mid B$
$A \rightarrow 0A \mid A0 \mid 1$
$B \rightarrow 0B00 \mid 1$

S is the start symbol, A and B are non-terminals and 0 and 1 are the terminals. The language generated by this grammar is
(a) $\{0^n 10^{2n} \mid n \geq 1\}$
(b) $\{0^i 10^j 10^k \mid i, j, k \geq 0\} \cup \{0^n 10^{2n} \mid n \geq 1\}$
(c) $\{0^i 10^j \mid i, j \geq 0\} \cup \{0^n 10^{2n} \mid n \geq 1\}$
(d) The set of all strings over {0, 1} containing at least two 0's

[IT-2008 : 2 Marks]

Directions for Question 2.42 to 2.43:
A CFG G is given with the following productions where S is the start symbol, A is a non-terminal and a and b are terminals.
$S \rightarrow aS \mid A$
$A \rightarrow aAb \mid bAa \mid \varepsilon$

2.42 Which of the following strings is generated by the grammar above?
(a) aabbaba (b) aabaaba
(c) abababb (d) aabbaab

[IT-2008 : 2 Marks]

2.43 For the correct answer in above Question, how many steps are required to derive the string and how many parse trees are there?
(a) 6 and 1 (b) 6 and 2
(c) 7 and 2 (d) 4 and 2

[IT-2008 : 2 Marks]

2.44 $S \rightarrow aSa \mid bSb \mid a \mid b$
The language generated by the above grammar over the alphabet {a, b} is the set of

(a) all palindromes
(b) all odd length palindromes
(c) strings that begin and end with the same symbol
(d) all even length palindromes

[2009 : 1 Mark]

2.45 Which one of the following is FALSE?
(a) There is a unique minimal DFA for every regular language
(b) Every NFA can be converted to an equivalent PDA
(c) Complement of every context-free language is recursive
(d) Every nondeterministic PDA can be converted to an equivalent deterministic PDA

[2009 : 1 Mark]

2.46 Match all items in Group I with correct options from those given in Group 2

Group 1
P. Regular expression
Q. Pushdown automata
R. Dataflow analysis
S. Register allocation

Group 2
1. Syntax analysis
2. Code generation
3. Lexical analysis
4. Code Optimization

(a) P-4, Q-1, R-2, S-3
(b) P-3, Q-1, R-4, S-2
(c) P-3, Q-4, R-1, S-2
(d) P-2, Q-1, R-4, S-3

[2009 : 1 Mark]

2.47 Given the following state table of an FSM with two states A and B, one input and one output:

Present State A	Present State B	Input	Next State A	Next State B	Output
0	0	0	0	0	1
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	1	0	0
0	0	1	0	1	0
0	1	1	0	0	1
1	0	1	0	1	1
1	1	1	0	0	1

If the initial state is A = 0, B = 0, what is the minimum length of an input string which will take the machine to the state A = 0, B = 1 with Output =1?
(a) 3
(b) 4
(c) 5
(d) 6

[2009 : 2 Marks]

2.48 Let $L = L_1 \cap L_2$, where L_1 and L_2 are languages as defined below:

$L_1 = \{a^m b^m c a^n b^n \mid m, n \geq 0\}$
$L_2 = \{a^i b^j c^k \mid i, j, k \geq 0\}$

Then L is
(a) not recursive
(b) regular
(c) context-free but not regular
(d) recursively enumerable but not context-free

[2009 : 2 Marks]

2.49 Consider the language $L1 = \{0^i1^j \mid i \neq j\}$, $L2 = \{0^i1^j \mid i = j\}$, $L3 = (0^i1^j \mid i = 2j + 1\}$, $L4 = \{0^i1^j \mid i \neq 2j\}$. Which one of the following statements is true?
(a) Only L2 is context free
(b) Only L2 and L3 are context free
(c) Only L1 and L2 are context free
(d) All are context free

[2010 : 2 Marks]

2.50 Consider the languages L1, L2 and L3 are given below:
$L1 = \{0^p1^q \mid p, q \in N\}$,
$L2 = \{0^p1^q \mid p, q \in N \text{ and } p = q\}$ and
$L3 = \{0^p1^q0^r \mid p, q, r \in N \text{ and } p = q = r\}$
Which of the following statements is NOT TRUE?
(a) Push Down Automata (PDA) can be used to recognize L1 and L2
(b) L1 is a regular language
(c) All the three languages are context free
(d) Turing machines can be used to recognize all the languages

[2011 : 2 Marks]

2.51 Consider the following languages
$L_1 = \{0^p 1^q 0^r \mid p, q, r \geq 0\}$
$L_2 = \{0^p 1^q 0^r \mid p, q, r \geq 0, p \neq r\}$
Which one of the following statements is FALSE?

(a) L_2 is context-free
(b) $L_1 \cap L_2$ is context-free
(c) Complement of L_2 is recursive
(d) Complement of L_1 is context-free but not regular

[2013 : 2 Marks]

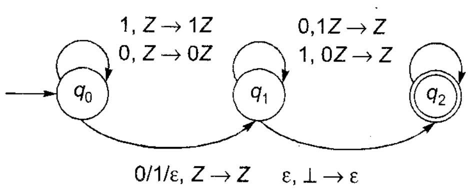
2.52 Consider the following languages over the alphabet $\Sigma = \{0, 1, c\}$:

$$L_1 = \{0^n 1^n \mid n \geq 0\}$$
$$L_2 = \{wcw^r \mid w \in \{0, 1\}^*\}$$
$$L_3 = \{ww^r \mid w \in \{0, 1\}^*\}$$

Here, w^r is the reverse of the string w. Which of these languages are deterministic Context-free languages?
(a) None of the languages
(b) Only L_1
(c) Only L_1 and L_2
(d) All the three languages

[2014 (Set-3) : 2 Marks]

2.53 Consider the NPDA $\langle Q = \{q_0, q_1, q_2\}, \Sigma = \{0, 1\}, \Gamma = \{0, 1, \perp\}, \delta, q_0, \perp, F = \{q_2\}\rangle$, where (as per usual convention) Q is the set of states, Σ is the input alphabet, Γ is stack alphabet, δ is the state transition function, q_0 is the initial state, $\perp$ is the initial stack symbol, and F is the set of accepting states, The state transition is as follows:



Which one of the following sequences must follow the string 101100 so that the overall string is accepted by the automaton?
(a) 10110 (b) 10010
(c) 01010 (d) 01001

[2015 (Set-1) : 2 Marks]

2.54 Which of the following languages are context-free?

$$L_1 = \{a^m b^n a^n b^m \mid m, n \geq 1\}$$
$$L_2 = \{a^m b^n a^m b^n \mid m, n \geq 1\}$$
$$L_3 = \{a^m b^n \mid m = 2n + 1\}$$

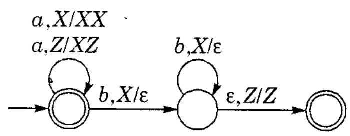
(a) L_1 and L_2 only (b) L_1 and L_3 only
(c) L_2 and L_3 only (d) L_3 only

[2015 (Set-3) : 1 Mark]

2.55 Consider the following context-free grammars:
G_1: $S \to aS \mid B, B \to b \mid bB$
G_2: $S \to aA \mid bB, A \to aA \mid B \mid \varepsilon, B \to bB \mid \varepsilon$
Which one of the following pairs of languages is generated by G_1 and G_2, respectively?
(a) $\{a^m b^n \mid m > 0 \text{ or } n > 0\}$ and $\{a^m b^n \mid m > 0 \text{ and } n > 0\}$
(b) $\{a^m b^n \mid m > 0 \text{ and } n > 0\}$ and $\{a^m b^n \mid m > 0 \text{ or } n \geq 0\}$
(c) $\{a^m b^n \mid m \geq 0 \text{ or } n > 0\}$ and $\{a^m b^n \mid m > 0 \text{ and } n > 0\}$
(d) $\{a^m b^n \mid m \geq 0 \text{ and } n > 0\}$ and $\{a^m b^n \mid m > 0 \text{ or } n > 0\}$

[2016 (Set-1) : 2 Marks]

2.56 Consider the transition diagram of a PDA given below with input alphabet $\Sigma = \{a, b\}$ and stack alphabet $\Gamma = \{X, Z\}$. Z is the initial stack symbol. Let L denote the language accepted by the PDA.



Which one of the following is TRUE?
(a) $L = \{a^n b^n \mid n \geq 0\}$ and is not accepted by any finite automata
(b) $L = \{a^n \mid n \geq 0\} \cup \{a^n b^n \mid n \geq 0\}$ and is not accepted by any deterministic PDA
(c) L is not accepted by any Turing machine that halts on every input
(d) $L = \{a^n \mid n \geq 0\} \cup \{a^n b^n \mid n \geq 0\}$ and is deterministic context-free

[2016 (Set-1) : 2 Marks]

2.57 Consider the following langauges:

$$L_1 = \{a^n b^m c^{n+m} : m, n \geq 1\}$$

$$L_2 = \{a^n b^n c^{2n} : n \geq 1\}$$

Which one of the following is **TRUE**?
(a) Both L_1 and L_2 are context-free.
(b) L_1 is context-free while L_2 is not context-free
(c) L_2 is context-free while L_1 is not context-free
(d) Neither L_1 nor L_2 is context-free

[2016 (Set-2) : 2 Marks]

2.58 Language L_1 is defined by the grammar:

$$S_1 \to aS_1b \mid \varepsilon$$

Language L_2 is defined by the grammar:

$$S_2 \to abS_2 \mid \varepsilon$$

Consider the following statements:

P: L_1 is regular

Q: L_2 is regular

Which one of the following is TRUE?

(a) Both P and Q are true
(b) P is true and Q is false
(c) P is false and Q is true
(d) Both P and Q are false

[2016 (Set-2) : 1 Mark]

2.59 Which one of the following grammars is free from left recursion?

(a) $S \to AB$, $A \to Aa \mid b$, $B \to c$

(b) $S \to Ab \mid Bb \mid c$, $A \to Bd \mid \varepsilon$, $B \to e$

(c) $S \to Aa \mid B$, $A \to Bb \mid Sc \mid \varepsilon$, $B \to d$

(d) $S \to Aa \mid Bb \mid c$, $A \to Bd \mid \varepsilon$, $B \to Ae \mid \varepsilon$

[2016 (Set-2) : 2 Marks]

Answers Push Down Automata: CFL & DCFL

2.2	(c)	2.3	(a, c)	2.4	(a,d)	2.5	(c)	2.6	(c)	2.7	(c)	2.8	(b)	2.10	(a)	2.11	(b)		
2.12	(b)	2.13	(a)	2.14	(a,c)	2.15	(b)	2.16	(b)	2.17	(c)	2.18	(c)	2.19	(b)	2.20	(b)		
2.21	(c)	2.22	(d)	2.23	(d)	2.24	(d)	2.25	(a)	2.26	(b)	2.27	(d)	2.28	(b)	2.29	(b)		
2.30	(d)	2.31	(c)	2.32	(b)	2.33	(b)	2.34	(c)	2.35	(d)	2.36	(b)	2.37	(d)	2.38	(c)		
2.39	(c)	2.40	(b)	2.41	(b)	2.42	(d)	2.43	(a)	2.44	(b)	2.45	(d)	2.46	(b)	2.47	(a)		
2.48	(c)	2.49	(d)	2.50	(c)	2.51	(d)	2.52	(c)	2.53	(b)	2.54	(b)	2.55	(d)	2.56	(a)		
2.57	(b)	2.58	(c)	2.59	(b)														

Explanations Push Down Automata: CFL & DCFL

2.1 Sol.

A context-free grammar is ambiguous if

1. It produces more than one parse tree for some sentence.

or

2. It produces more than one leftmost derivation for some sentence.

or

3. It produces more than one rightmost derivation for some sentence.

2.2 (c)

The languages like C, Fortran and Pascal are context sensitive languages. Due to presence of the following two features which cannot be handled by a PDA:

1. Variable declared before use
2. Matching formal and actual parameters of functions.

2.3 (a, c)

Regular language is closed under the all the operations of union, inter-section, concatenation, complementation and Kleene closure. However context free languages are closed under the operations of union, concatenation and Kleene closure but not closed under inter-section and complementation.

2.4 (a,d)

Context free languages are closed under union, concatenation, and Kleene closure. However, context free languages are not closed under complementation and intersection.

2.5 (c)

A context-free grammar G is in Chomsky normal form if all productions are in one of two simple form, either:

1. $A \to BC$ where A, B and C are variables, or
2. $A \to a$ where A is a variable and a is a terminal

So for any string of length n first production of type $A \to BC$ is used $n - 1$ times to produce sentential form of length n containing only variables and then each variable is replaced by a terminal using productions of type $A \to a$, n times. So the length of derivation of string w of length n in CNF is

$$(n - 1) + n = 2n - 1$$

Example:

$S \to XY$
$Y \to XZ$
$X \to a$
$Z \to b$

To derive the string "aab" by using above CNF

$S \to XY$
$\Rightarrow aY$
$\Rightarrow aXZ$
$\Rightarrow aaZ$
$\Rightarrow aab$

It requires 5 steps

$= 2 \times 3 - 1 = 2 * |aab| - 1$
$= 2n - 1$

2.6 (c)

(a) The language corresponding to syntax of if-then-else statements is context-free language. So syntax of if-then-else statements can be captured by context-free grammar.

(b) The language corresponding to syntax of recursive procedures is context-free language. So syntax of recursive procedures can be captured by context-free-grammar.

(c) The language corresponding to variable has been declared before its use is
$L = \{ww \mid w \text{ is variable}\}$
L is context sensitive language. So the feature that a variable has been declared before its use can't be captured by context free grammar.

(d) The feature that variable names of arbitrary length can be captured by context free grammar.

2.7 (c)

$S \to a\alpha b \mid b \to c \mid ab$
$S \to \alpha S \mid b$
$S \to abb \mid ab$
$S\alpha \to bdb \mid bd$

Since the LHS of the production $S\alpha \to bdb \mid bd$ which is $S\alpha$ does not belong to variable set V, the grammar is not a CFG (type 2). However since in all the productions length of LHS $\leq$ length of RHS, the grammar is type 1 (CSG).

2.8 (b)

(a) If L_1 and L_2 are context free language then L_1L_2 is also context free language because context free languages are closed under concatenation.

(b) Since context free languages are not closed under intersection, $L_1 \cap L_2$ is not necessarily a context free language.

(c) If L_1 is context free language and R is regular set then $L_1 \cap R$ is surely context free language because context fee languages are closed under regular intersection.

(d) If L_1 and L_2 are context free languages then $L_1 \cup L_2$ is surely context free language because context free languages are closed under union.

2.9 Sol.

Nullable variables are
$V_N = \{A, B\}$

Putting A and B to null in every possible combination, we get

G': $S \to ABAC \mid ABC \mid BAC \mid AAC \mid AC \mid C \mid BC$
$A \to aA \mid a$
$B \to bB \mid b$
$C \to d$

The above equivalent grammar G' contain unit production

$S \to C$

Mix $S \to C$ with $C \to d$ and get $S \to d$.
So replacing above unit production as

$S \to d$

So resultant grammar G' without ε-production and unit production is

G': $S \to ABAC \mid ABC \mid BAC \mid AAC \mid AC \mid BC \mid d$
$A \to aA \mid a$
$B \to bB \mid b$
$C \to d$

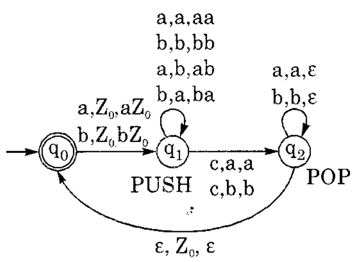
2.10 (a)

The language

$$L = \{wcw^R \mid w \in (a, b)^*\}$$

is accepted by DPDA (since there is a clear clue "c" available in the string that tells the machine exactly where the pop starts) and so it is DCFL. For above language until "c" appears everything is pushed into the stack and when c appears it is skipped and from then every symbol in w^R is matched with top of stack and popped. At the end when input string is empty and stack contain initial stack symbol, the string is accepted.

The DPDA for above language is



2.11 (b)

(a) and (c) Since for every finite set we can create DFA by brute force. So every finite subset of a non-regular or regular is regular.

(b) Let $\Sigma = (a, b)$. Σ^* is regular set

$$L_1 = \{a^n b^n \mid n \geq 0\}$$
$$L_2 = \{a^n b^m \mid nm \geq 0\}$$

Both L_1 & L_2 are subset of Σ^*

$$L_1 \subset \Sigma^* \ \& \ L_2 \subset \Sigma^*$$

However L_1 is CFL and L_2 is regular. So by above illustration every subset of a regular set need not to be regular.

(d) Since regular languages are closed under intersection. So the intersection of two regular sets is regular.

2.12 (b)

Since context free language are closed under union and kleene closure and not closed under intersection and complement.

2.13 (a)

There are two approaches by which PDA accepts its input

(a) Acceptance by empty stack.

(b) Accepting by entering final state.

Since there exist algorithm to convert PDA accepting by final state to PDA accepting by empty stack and vice-versa, the acceptance or recognition power of PDA of both approaches are same. Therefore $L_D = L_E$

2.14 (a,c)

(a) Since CFLs are closed under regular difference, so $L_1 - L_2$ = CFL – regular = CFL. So option (a) is false.

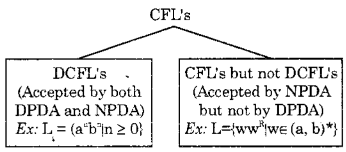
(b) Since context free language is closed under regular intersection so $L_1 \cap L_2$ = CFL $\cap$ REG = CFL.
So option (b) is true.

(c) Context free language is not closed under complementation. So $\sim L_1$ is not context free language. So option (c) is false.

(d) Regular languages are closed under complementation. So $\sim L_2$ is regular.
So option (d) is true.

2.15 (b)

(a) False, CFL's can be divided into two categories



So, if a language is context free it is not always true that it is accepted by a deterministic pushdown automation.

(b) True, since CFL's are closed under union, the union of two CFL is context free

(c) & (d) False, since CFL's are not closed under intersection and complement.

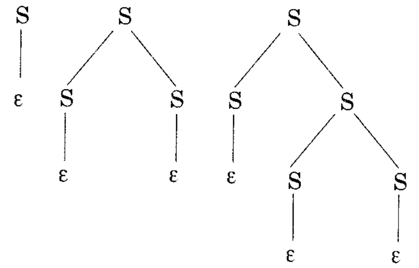
2.16 (b)

A PDA with a stack limited to containing 10 items, has finite memory and hence is equivalent to a DFA. So the language accepted by a PDA in which the stack is limited to 10 items is best described as regular.

2.17 (c)

The grammar is $S \rightarrow aSb \mid SS \mid \varepsilon$

(a) G is not ambiguous is false, since ε which belongs to L(G), has infinite number of derivation trees, which makes G ambiguous. Some derivation trees are given below



(b) There exists $x, y \in L(G)$ such that $xy \notin L(G)$ is false, since $S \rightarrow SS$ can be used to derive xy, whenever $x \in L(G)$ and $y \in L(G)$.

(c) is true since this language is $L(G) = \{w \mid n_a(w) = n_b(w)$ and $n_a(v) \geq n_b(v)$ where v is any prefix of w$\}$ which is equal to $(a^nb^n)^*$.
This language happens to be a deterministic context free language.
$\therefore$ There exists a DPDA that accepts it.

(d) is false, as the given language is not regular (comparison between number of 'a's and number of 'b's is there).
$\therefore$ No DFA exists to accept it.

2.18 (c)

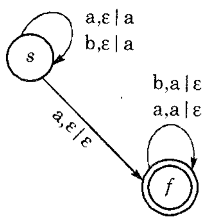
(a) This is true, since some CFLs are inherently ambiguous.

(b) Always correct, that's why it is called unambiguous.

(c) The set of all languages accepted by all NPDAs is the set of CFLs, which is a proper superset of the set of all languages accepted by all DPDAs, which is the set of DCFLs. Hence option (c) is false.

(d) Finite language is always regular. So, true.

2.19 (b)



This machine accepts the language
$L = \{w_1aw_2 \mid |w_1| = |w_2|\}$
The language allows only strings whose centre bit is "a" and the strings on either side have equal length i.e., strings of odd length with centre bit "a".

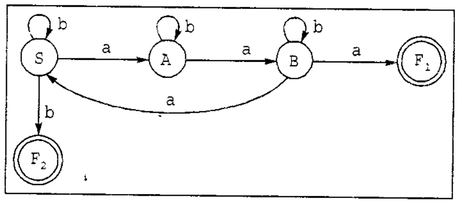
The string "aabab" is odd length but centre bit is not "a".
So, this string is not a member of L(M).

2.20 (b)

$L = \{a^m b^n c^{m+n} \mid m, n \geq 1\}$
A DPDA can accept this language. When a's and b's are in input, these are pushed into the stack and when c's appear in input the a's and b's are popped out. After all c's are finished, if stack is empty, this means $n_c(w) = n_a(w) + n_b(w)$ and the string is accepted. Else it is rejected. Now since a DPDA exists, the language is DCFL and hence, context-free. Clearly the language is not regular, since we must count and compare c's with a's and b's which cannot be done by any FA.

2.21 (c)

Draw the fa of the given grammar as shown below:



Clearly, the machine is accepting $N_a(w) = 3k$, where $k \in \{0, 1, 2, 3, ...\}$

2.22 (d)

$L^C \cup M^C = (\text{Reg})^C \cup (\text{CFL})^C = \text{Reg} \cup (\text{CSL})^C$
$= \text{Reg} \cup \text{CSL} = \text{CSL}$
A CSL may or may not be regular. So, options (a) and (c) are false.
A CSL need not be a CFL. So, option (b) is false.
So, answer is (d) none of these.

2.23 (d)

In NPDA we may have a dead configuration. This mean we may not give any transition to any alphabet from this state.
We say that a string is accepted if PDA is in final state after reading the final symbol in the string or after it has read '\$' symbol denoting end of the string and it is in final state.
Question never says that we have transitions defined for all the alphabet symbols in the PDA.
Neither L(P) nor N(P) are necessarily Σ^*.

2.24 (d)

Nf : Non-deterministic finite automata.
Np: Non-deterministic push-down automata.
Df : Deterministic finite automata.
Dp : Deterministic push down automata.
According to "Subset Construction" theorem every language accepted by Non-deterministic-finite automata (Nf) is also accepted by some Deterministic-Finite automata (Df) so Df = Nf.
Deterministic push-down automata (Dp) recognizes the DCFLs which is a proper subset of the language of context-free languages and the non-deterministic push down automata recognizes the context-free languages.
So, Dp $\subset$ Np.

2.25 (a)

L_1 and L_2 are context free languages and therefore $L_1 \cap L_2$ may or may not be context free, since CFL's are not closed under intersection. Now let us look at $L_1 \cap L_2$.
$L_1 \cap L_2 = \{a^n b^n c^n \mid n > 0\}$
Which is clearly not context free but is context sensitive.
$\therefore$ (a) is false.

2.26 (b)

L_1 is CFL but not a DCFL, since accepting ww^R necessarily involves finding the middle of the string, which involves non-determinism.
$\therefore$ (a) is false.
L_2 is a DCFL is true since "#" is a special symbol and middle of string can now be surely found by using "#", thereby eliminating the need for non-deterministic guessing. So (b) is true.
L_3 is a CSL and not a CFL. So (c) is false.
L_3 is not a DCFL either. So (d) is false.

2.27 (d)

$L_1 = \{0^{n+m} 1^n 0^m \mid n, m \geq 0\}$ is context free (first n + m 0's are pushed into the stack, then for each of the 1s and 0s following, we will pop the 0's in the stack one by one until at the end of the word if the stack is empty then the word is accepted).
$L_2 = \{0^{n+m} 1^{n+m} 0^m \mid n, m \geq 0\}$ is not context free, since two comparisons have to be made here to determine if $w \in L_2$.

1. 0's and 1's are equal.
2. Since $m < (n + m)$, we have to ensure that, 0's which come after $0^{n+m} 1^{n+m}$, are less in number, compared to n + m.

$L_3 = \{0^{n+m} 1^{n+m} 0^{n+m} \mid n_1\ m \geq 0\}$ is clearly context sensitive since have also 2 comparisons have to be made. Infact this language is same as $L_3 = \{0^x 1^x 0^x \mid x \geq 0\}$, which is context sensitive.

2.28 (b)

L(G) = Set of all palindrome strings.
"baba" is not a palindrome string.

2.29 (b)

To be accepted by NPDA but not DPDA, the language must be a CFL but not DCFL.
(a) is wrong, it is not context free.
(b) $L = \{a^l b^m c^n \mid l \neq m \text{ or } m \neq n\}$
This language has double comparison on m with 'or'. So the language is CFL but not DCFL.
(c) In this language only one comparison is there and push and pop position are clear. This is a DCFL.
(d) a*b* is regular. Every regular language is a DCFL. So this language is also a DCFL.

2.30 (d)

L is CFL and M is Regular. Intersection of CFL and Regular is always a CFL, since all languages are closed under regular intersection.

2.31 (c)

For every *a*, we put two X in stack [at state *s*]
For every *b* we pop out one X [reach to state *t* (getting *b* after *a*)]
For every *c* we pop out one *X* [reach to state *u* (getting *c* after *b*)]
If all *X* are popped out then reached to final state *f*, means that.
Sum of number of *b's* and number of *c's* = twice of number of *a's*
i.e. $L = \{a^l b^m c^n \mid 2l = m + n\}$

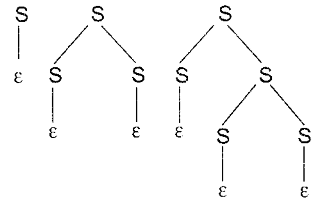
2.32 (b)

$A \rightarrow bA \mid \epsilon$
$L(A) = b^*$
$S \rightarrow aSAb \mid \epsilon$
Now, substituting A solution into this gives
$S \rightarrow aSb^*b \mid \epsilon$
$S \rightarrow aSb \mid aSbb \mid aSbbb... \mid \epsilon$
$L(S) = \{a^m b^n \mid m \leq n\}$

2.33 (b)

$G = \{S \rightarrow SS, S \rightarrow ab, S \rightarrow ba, S \rightarrow \varepsilon\}$

1. "G is ambiguous", is true since "ε" has infinite number of derivations some of which are shown below:



2. "G produces all strings with equal number of a's and b's", is false since $L(G) = (ab + ba)^*$ and this language cannot produce all strings with equal number of a's and b's. For example aabb has equal number of a's and b's but aabb $\notin L(G)$.
3. "G can be accepted by a DPDA" is true, since $L(G) = (ab + ba)^*$ and this is a regular language, since we have written a regular expression for it. Since every regular language is also a DCFL, a DPDA exists, which accepts this language.

2.34 (c)

Grammar may change but language remain the same in the grammar and its disambiguated version.

$$L(D) = L(G)$$

2.35 (d)

Using capital letters for variables and small letters for terminals, the given grammar G1 becomes

$S \rightarrow w(E)S$

$S \rightarrow o$

$E \rightarrow i$

Since LHS of every production is a single variable, G1 is context free grammar.

G2 becomes

$S \rightarrow w(E)S$

$S \rightarrow o$

$E \rightarrow E+E$

$E \rightarrow E^*E$

$E \rightarrow i$

Since LHS of every production is a single variable, G2 is also a context free grammar. RHS of some of the productions in G1 and G2 has two non-terminals, hence it is not regular grammar.

2.36 (b)

$\{0^i\, 21^i \mid i \geq 0\}$ is a DCFL since, a DPDA can accept this language. 0's are pushed into the stack and then when 2 appears in input, state is changed and immediately after that for every 1, a zero is popped out of the stack. In the end, if stack has start stack symbol only, then the string is accepted. Else it is rejected. Since every DCFL is recursive, we can say that the language is recursive, and is a DCFL.

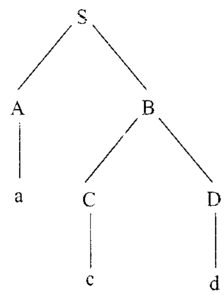
2.37 (d)

(a) True
(b) True
(c) True
(d) Every subset of a recursively enumerable set is recursive, is false, since a set is a subset of itself and there are RE languages, which are not REC.

2.38 (c)

1. True
2. All ε-productions can be removed from only context free grammars that produce λ-free CFL's.
   If $\lambda \in L(G)$, then all ε-productions cannot be successfully removed. So 2 is false.
3. True
4. True, since in Chomsky normal form, every production is of the form of $A \rightarrow BC$ or $A \rightarrow a$.

An example of a binary tree generated by CNF derivation is shown below:



2.39 (c)

In $\{w\,c\,w \mid w \in (a, b)^*\}$, leftmost w is the identifier checking and rightmost w is the use of w.

In $a^n b^m c^n d^m$ the actual parameters are a^n and b^m and the formal parameters are c^n and d^n such that the number of arguments of a and b are equal to c and d respectively.

The grammar $\{X \to XbX \mid XcX \mid dXf \mid g\}$ generates the arithmetic expressions with matched pair of parentheses (d is left parenthesis and f is right parenthesis).

The grammar $\{X \to bXb \mid cXc \mid \varepsilon\}$ generates the palindromic strings.

2.40 (b)

L1 is CFL [push a's onto stack , and pop a with each b]

L2 is CFL [push b's onto stack and pop b with each c]. So, option (a) is false.

In L1, every string has atleast one "c".

In L2, every string has no "c".

So, $L1 \cap L2 = \phi$. So option (b) is true.

Union of two CFLs is a CFL and so option (c) is false.

L2 is DCFL and hence acceptable by a DPDA. So, option (d) is false.

2.41 (b)

$S \to B$

$B \to 0B00 \mid 1$

So, S generates $\{0^n10^{2n} \mid n \geq 1\}$

$S \to AA$

$A \to 0A \mid A0 \mid 1$

$A \to \{0^m10^n \mid m, n \geq 0\}$

So, S generates $\{0^m10^n0^p10^q \mid m,n,p,q \geq 0\}$.

$= \{0^m10^x10^q \mid m, x, q \geq 0\}$

$= \{0^i10^j10^k \mid i, j, k \geq 0\}$

$L(S) = \{0^i10^j10^k \mid i, j, k \geq 0\} \cup \{0^n10^{2n} \mid n \geq 1\}$

Which is same as option (b).

2.42 (d)

$S \to aS$

$S \to aA$

$S \to aaAb$

$S \to aabAab$

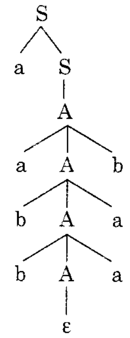
$S \to aabbAaab$

$S \to aabbaab$

2.43 (a)

$S \to aS$	1
$S \to aA$	2
$S \to aaAb$	3
$S \to aabAab$	4
$S \to aabbAaab$	5
$S \to aabbaab$	6

Thus 6 steps are needed and only one parse tree shown below.



2.44 (b)

$S \to aSa \mid bSb \mid a \mid b$

is the grammar corresponding to all odd length palindromes.

Notice that choice (c) is not correct since "abaa" is a string that begins and ends with the same symbol but cannot be generated by above grammar.

Since all strings generates by this grammar have odd length, choices (a) and (d) are also incorrect.

2.45 (d)

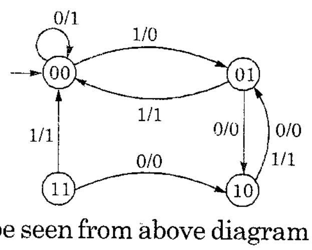
Since it is known that the set of DCFL's are a proper sub class of the set of CFL's, NPDA's which corresponds in general to class of CFL's cannot in general be converted to equivalent DPDA's, which correspond to the class of DCFL's.

2.46 (b)

P-3, Q-1, R-4, S-2

2.47 (a)

The transition diagram for the given FSM is given below.



As can be seen from above diagram the minimum length of the input string that will take machine from 00 to 01 with output 1, is three.

In fact the minimum length input string is "101". Which will give an output of 1, while reaching state 01.

2.48 (c)

$L_1 = \{a^m b^m c a^n b^n | m, n \geq 0\}$
$L_2 = \{a^i b^j c^k | i, j, k \geq 0\}$
$L = L_1 \cap L_2$

L_1 is CFL. L_2 is regular. First use closure property to get a estimate.

$L = L_1 \cap L_2 = \text{CFL} \cap \text{Reg} = \text{CFL}$

However, since one of the option (b) is regular is stronger than CFL answer obtained by closure property, we need to find the actual intersection. Any string belonging to both must have equal number of a's & b's followed by a single c followed by no a's or b's; which is the only string allowed by both L_1 and L_2.

i.e. $L = L_1 \cap L_2 = \{a^m b^m c\}$

Now this is clearly context free, but not regular.

2.49 (d)

$L_1 = \{0^i 1^j ; i \neq j\}$
$L_2 = \{0^i 1^j ; i = j\}$
$L_3 = \{0^i 1^j ; i = 2j + 1\}$
$L_4 = \{0^i 1^j ; i \neq 2j\}$

All of L_1, L_2, L_3 and L_4 are context free, since, each of these languages has a single linear comparison between i and j and any linear comparison between i and j can be performed, in a PDA.

2.50 (c)

Since L_3 involves two comparison p = q and q = r, it cannot be context free.
So statement (c) is not true.

2.51 (d)

$L_1 = \{0^p 1^q 0^r \mid p, q, r \geq 0\}$: regular language
$L_2 = \{0^p 1^q 0^r \mid p, q, r \geq 0, p \neq r\}$: CFL

(a) L_2 is CFL : True
(b) $L_1 \cap L_2$ is context free : True, because CFL is closed on regular intersection.
(c) Complement of L_2 is recursive:

$$\overline{L_2} = \overline{\text{CFL}} = \overline{\text{CSL}} = \text{CSL}$$

Now, every CSL is REC. So, this statement is true.

(d) Complement of L_1 is context free but not regular: false, because regular language are closed under complement and so the complement of L_1 is regular.

2.52 (c)

$L_1 = \{0^n 1^n | n \geq 0\}$ is DCFL
$L_2 = \{wcw^r | w \in \{0, 1\}^*\}$ is DCFL
$L_3 = \{ww^r | w \in \{0, 1\}^*\}$ is CFL but not DCFL

2.53 (b)

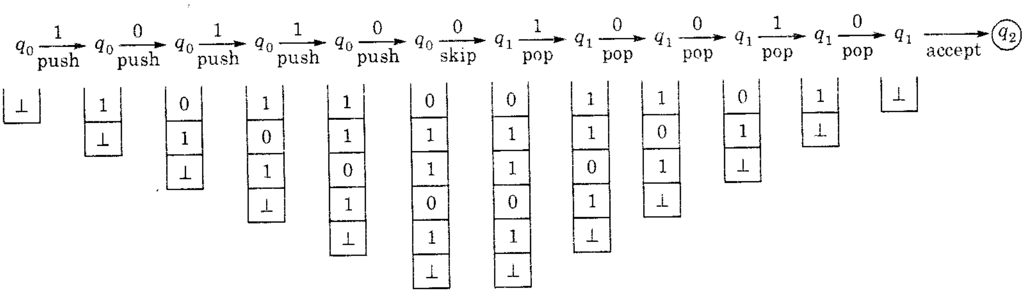
The language accepted is

L(M) = $\{wx\bar{w}^R | x \in \{0, 1, \varepsilon\}$, where $\bar{w}$ is the 1's complement of $w\}$.

Only the string "101100 10010" is in the language and hence will be accepted.

The following diagram shows how this string will be accepted by the PDA.

$$\underset{\text{Given string}}{\underline{101100}} \quad \underset{\text{Option (b) string}}{\underline{10010}}$$



2.54 (b)

$L_1 = \{a^m\ b^n\ a^n\ b^m \mid m, n \geq 1\}$ is CFL (push "a" and "b" and then first "a" pops "b" and then "b" pops "a").

$L_2 = \{a^m\ b^n\ a^m\ b^n \mid m, n \geq 1\}$ is non-CFL (Alternate comparison not possible in a PDA)

$L_3 = \{a^m\ b^n \mid m = 2n + 1\}$ is CFL (Single comparison is possible in a PDA)

2.55 (d)

G_1: $S \to aS \mid B$
$B \to b \mid bB$

G_2: $S \to aA \mid bB$
$A \to aA \mid B \mid \varepsilon$
$B \to bB \mid \varepsilon$

G: $B \to b \mid bB \Rightarrow B \to b^+$

Now substitute in $S \to aS \mid B$

We get $S \to aS \mid b^+ \Rightarrow S \to a^*\ b^+$

So, $L(G_1) = \{a^m b^n \mid m \geq 0 \text{ and } n > 0\}$

$G_2 = B \to bB \mid \varepsilon \Rightarrow B \to b^*$

Substitute in $A \to aA \mid B \mid \varepsilon \Rightarrow A \to aA \mid b^* \mid \varepsilon$

$\Rightarrow A \to aA \mid b^*$

$\Rightarrow A \to a^* b^*$

Now substitue A and B in $S \to aA \mid bB$

$\Rightarrow S \to aa^*\ b^* \mid bb^*$

$S \to aa^*\ b^* + bb^*$

So $\quad L(G_2) = \{a^m b^n \mid m > 0 \text{ or } n > 0\}$

So correct answer is choice (d).

2.56 (a)

The first state accepts only null string. Only a's will not be accepted since on first state ε, X will be dead reject.

The third state accept is $\{a^n b^n \mid n \geq 1\}$

So $\quad L = \{\varepsilon\} \cup \{a^n b^n \geq 1\}$

$= \{a^n b^n \mid n \geq 0\}$

Clearly this is a non-regular CFL and hence not accepted by any FA.

2.57 (b)

L_1 has only one comparisons and hence is CFL, but L_2 has 2 comparisons and hence not a CFL, but a CSL.

So answer (b) is correct.

2.58 (c)

$G_1 : S_1 \to aS_1b \mid \varepsilon$

$G_2 : S_2 \to abS_2 \mid \varepsilon$

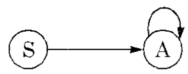
$$L_1 = L(G_1) = \{a^n b^n \mid n \geq 0\}$$

$$L_2 = L(G_2) = \{(ab)^n \mid n \geq 0\} = (ab)^*$$

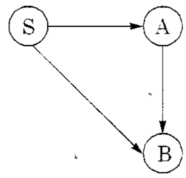
Clearly L_1 is regular is false of L_2 is regular is True. So correct answer is (c).

2.59 (b)

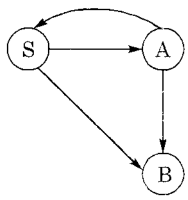
Draw left recussion variable dependency graph

(a) 

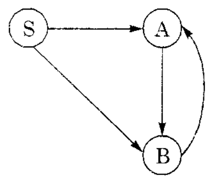
(Left recussion on A)

(b) 

(No left recursions on any variable i.e. no cycles)

(c) 

(Left recursions on S i.e., cycle on S)

(d) 

(Left recursions on A i.e., cycle on A)

So only (b) is free from left recursion on any variable.

■■■■

3 Turing Machine: RE, REC and Undecidability

3.1 Which of the following problems are undecidable?
(a) Membership problem in context-free languages
(b) Whether a given context-free language is regular
(c) Whether a finite state automation halts on all inputs
(d) Membership problem for type 0 languages
[1989 : 2 Marks]

3.2 It is undecidable whether
(a) an arbitrary Turing machine halts within 10 steps
(b) a turing machine prints a specific letter
(c) a turing machine computes the product of two numbers
(d) None of the above
[1990 : 2 Marks]

3.3 Recursive languages are:
(a) a proper superset of context free languages
(b) always recognizable by pushdown automata
(c) also called type-0 languages
(d) recognizable by turing machines
[1990 : 2 Marks]

3.4 Which one of the following is the strongest correct statement about a finite language over some finite alphabet Σ?
(a) It could be undecidable
(b) It is turing-machine recognizable
(c) It is a regular language
(d) It is a context-sensitive language
[1991 : 2 Marks]

3.5 In which of the cases stated below is the following statement true?
"For every non-deterministic machine M_1 there exists an equivalent deterministic machine M_2 recognizing the same language".
(a) M_1 is non-deterministic finite automaton
(b) M_1 is a non-deterministic PDA
(c) M_1 is a non-deterministic Turing machine
(d) For no machine M_1 use the above statement true
[1992 : 2 Marks]

3.6 Which of the following conversions is not possible (algorithmically)?
(a) Regular grammar to context free grammar
(b) Non-deterministic FSA to deterministic FSA
(c) Non-deterministic PDA to deterministic PDA
(d) Non-deterministic Turing machine to deterministic Turing machine
[1994 : 2 Marks]

3.7 Which of the following statements is false?
(a) The halting problem for Turing machine is undecidable
(b) Determining whether a context free grammar is ambiguous is undecidable
(c) Given two arbitrary context free grammars G_1 and G_2, it is undecidable whether $L(G_1) = L(G_2)$
(d) Given two regular grammars G_1 and G_2, it is undecidable whether $L(G_1) = L(G_2)$
[1996 : 1 Mark]

3.8 Which one of the following is not decidable?
(a) Given a Turing machine M, a strings s and an integer k, M accepts s within k steps
(b) Equivalence of two Turing machines
(c) Language accepted by a given finite state machine is non empty
(d) Language accepted by a context free grammar is non empty
[1997 : 2 Marks]

3.9 Regarding the power of recognition of languages, which of the following statements is false?
(a) The non-deterministic finite-state automata are equivalent to deterministic finite-state automata
(b) Non-deterministic push-down automata are equivalent to deterministic push-down automata
(c) Non-deterministic Turning machines are equivalent to deterministic Turning machines
(d) Multi-tape Turning machines are equivalent to Single-tape Turning machines.
[1998 : 1 Mark]

3.10 Which of the following is true?
(a) The complement of a recursive language is recursive
(b) The complement of a recursively enumerable language is recursively enumerable
(c) The complement of a recursively enumerable language is either recursive or recursively enumerable
(d) The complement of a context-free language is context-free

[2002 : 2 Marks]

3.11 The C language is
(a) A context free language
(b) A context sensitive language
(c) A regular language
(d) Parsable fully only by a Turing machine

[2002 : 2 Marks]

3.12 Consider the following decision problems
P_1: Does a given finite state machine accept a given string
P_2: Does a given context free grammar generate an infinite number of strings.
Which of the following statement is true?
(a) P_1 and P_2 are decidable
(b) Neither P_1 nor P_2 are decidable
(c) only P_1 is decidable
(d) only P_2 is decidable

[2002 : 2 Marks]

3.13 Consider the following Problem X.
"Given a Turing machine M over the input alphabet Σ, any state q of M and a word Σ^*, does the computation of M on w visit the state q"
Which of the following statements about X is correct?
(a) X is decidable
(b) X is undecidable but partially decidable
(c) X is undecidable and not even partially decidable
(d) X is not a decision problem

[2002 : 2 Marks]

3.14 Nobody knows yet if P = NP. Consider the language L defined as follows

$$L = \begin{cases} (0+1)^* \text{ if } P = NP \\ \phi \text{ otherwise} \end{cases}$$

Which of the following statements is true?
(a) L is recursive
(b) L is recursively enumerable but not recursive
(c) L is not recursively enumerable
(d) Whether L is recursive or not will be known after we find out if P = NP

[2003 : 1 Mark]

3.15 If the strings of a language L can be effectively enumerated in lexicographic (i.e., alphabetic) order, which of the following statements is true?
(a) L is necessarily finite
(b) L is regular but not necessarily finite
(c) L is context free but not necessarily regular
(d) L is recursive but not necessarily context free

[2003 : 1 Mark]

3.16 A single tape Turing Machine M has two states q_0 and q_1, of which q_0 is the starting state. The tape alphabet of M is {0, 1, B} and its input alphabet is {0, 1}. The symbol B is the blank symbol used to indicate end of an input string. The transition function of M is described in the following table

	0	1	B
q_0	q_1, 1, R	q_1, 1, R	Halt
q_1	q_1, 1, R	q_0, 1, L	q_0, B, L

The table is interpreted as illustrated below. The entry (q_1, 1, R) in row q_0 and column 1 signifies that if M is in state q_0 and reads 1 on the current tape square, then it writes 1 on the same tape square, moves its tape head one position to the right and transitions to state q_1. Which of the following statements is true about M?
(a) M does not halt on any string in $(0+1)^+$
(b) M does not halt on any string in $(00+1)^*$
(c) M halts on all string ending in a 0
(d) M halts on all string ending in a 1

[2003 : 2 Marks]

3.17 Consider three decision problems P_1, P_2 and P_3. It is known that P_1 is decidable and P_2 is undecidable. Which one of the following is TRUE?
(a) P_3 is decidable if P_1 is reducible to P_3
(b) P_3 is undecidable if P_3 is reducible to P_2
(c) P_3 is undecidable if P_2 is reducible to P_3
(d) P_3 is decidable if P_3 is reducible to P_2's complement

[2005 : 2 Marks]

3.18 Let L_1 be a recursive language, and let L_2 be a recursively enumerable but not a recursive language. Which one of the following is TRUE?

(a) $\overline{L_1}$ is recursive and $\overline{L_2}$ is recursively enumerable

(b) $\overline{L_1}$ is recursive and $\overline{L_2}$ is not recursively enumerable

(c) $\overline{L_1}$ and $\overline{L_2}$ are recursively enumerable

(d) $\overline{L_1}$ is recursively enumerable and $\overline{L_2}$ is recursive

[2005 : 2 Marks]

3.19 For $s \in (0 + 1)^*$ let d(s) denote the decimal value of s (e.g. d (101) = 5).

Let $L = \{s \in (0 + 1)^* \mid d(s) \bmod 5 = 2 \text{ and } d(s) \bmod 7 \neq 4\}$

Which one of the following statements is true?

(a) L is recursively enumerable, but not recursive
(b) L is recursive, but not context-free
(c) L is context-free, but not regular
(d) L is regular

[2006 : 2 Marks]

3.20 Let L_1 be regular language, L_2 be a deterministic context-free language and L_3 a recursively enumerable, but not recursive, language. Which one of the following statements is false?

(a) $L_1 \cap L_2$ is a deterministic CFL
(b) $L_3 \cap L_1$ is recursive
(c) $L_1 \cup L_2$ is context free
(d) $L_1 \cap L_2 \cap L_3$ is recursively enumerable

[2006 : 2 Marks]

3.21 Which of the following problems is undecidable?

(a) Membership problem for CFGs
(b) Ambiguity problem for CFGs
(c) Finiteness problem for FSAs
(d) Equivalence problem for FSAs

[2007 : 1 Mark]

3.22 Which of the following is true for the language $\{a^p \mid p \text{ is a prime}\}$?

(a) It is not accepted by a Turning Machine
(b) It is regular but not context-free
(c) It is context-free but not regular
(d) It is neither regular nor context-free, but accepted by a Turing machine

[2008 : 1 Mark]

3.23 If L and $\overline{L}$ are recursively enumerable then L is

(a) regular (b) context-free
(c) context-sensitive (d) recursive

[2008 : 1 Mark]

3.24 Which of the following are decidable?

1. Whether the intersection of two regular languages is infinite
2. Whether a given context-free language is regular
3. Whether two push-down automata accept the same language
4. Whether a given grammar is context-free

(a) 1 and 2 (b) 1 and 4
(c) 2 and 3 . (d) 2 and 4

[2008 : 1 Mark]

3.25 Let L1 be a recursive language. Let L2 and L3 be language that are recursively enumerable but not recursive. Which of the following statements is not necessarily true?

(a) L2-L1 is recursively enumerable
(b) L1-L3 is recursively enumerable
(c) $L2 \cap L3$ is recursively enumerable
(d) $L2 \cup L3$ is recursively enumerable

[2010 : 1 Mark]

3.26 Which of the following pairs have **DIFFERENT** expressive power?

(a) Deterministic finite automata (DFA) and Non-deterministic finite automata (NFA)
(b) Deterministic push down automata (DPDA) and Non-deterministic push down automata (NPDA)
(c) Deterministic single-tape Turing machine and Non-deterministic single-tape Turing machine
(d) Single-tape Turing machine and multi-tape Turing machine

[2011 : 1 Mark]

3.27 Which of the following problems are decidable?

1. Does a given program ever produce an output?
2. If L is a context-free language, then, is $\overline{L}$ also context-free?
3. If L is a regular language, then, is $\overline{L}$ also regular
4. If L is a recursive language, then, is $\overline{L}$ also recursive?

(a) 1, 2, 3, 4 (b) 1, 2
(c) 2, 3, 4 (d) 3, 4

[2012 : 1 Mark]

3.28 Which of the following statements is/are FALSE?

1. For every non-deterministic Turing machine, there exists an equivalent deterministic Turing machine.
2. Turing recognizable languages are closed under union and complementation.
3. Turing decidable languages are closed under intersection and complementation.
4. Turing recognizable languages are closed under union and intersection.

(a) 1 and 4 only (b) 1 and 3 only
(c) 2 only (d) 3 only

[2013 : 1 Mark]

3.29 Which of the following is/are undecidable?

1. G is CFG. Is L(G) = ϕ?
2. G is a CFG. Is L(G) = Σ^*?
3. M is a Turing machine. Is L(M) regular?
4. A is a DFA and N is an NFA. Is L(A) = L(N)?

(a) 3 only (b) 3 and 4 only
(c) 1. 2 and 3 only (d) 2 and 3 only

[2013 : 2 Marks]

3.30 Let L be a language and $\bar{L}$ be its complement. Which one of the following is NOT a viable possibility?

(a) Neither L nor $\bar{L}$ is recursively enumerable (r.e.).
(b) One of L and $\bar{L}$ is r.e. but not recursive; the other is not r.e.
(c) Both L and $\bar{L}$ are r.e. but not recursive.
(d) Both L and $\bar{L}$ are recursive.

[2014 (Set-1) : 2 Marks]

3.31 Let $<M>$ be the encoding of a Turing machine as a string over $\Sigma = \{0,1\}$. Let $L = \{<M> \mid M$ is a *Turing machine that accepts a string of length* 2014$\}$. Then, L is

(a) decidable and recursively enumerable
(b) undecidable but recursively enumerable
(c) undecidable and not recursively enumerable
(d) decidable but not recursively enumerable

[2014 (Set-2) : 2 Marks]

3.32 Let $A \leq_m B$ denotes that language A is mapping reducible (also known as many-to-one reducible) to language B. Which one of the following is FALSE?

(a) If $A \leq_m B$ and B is recursive then A is recursive.
(b) If $A \leq_m B$ and A is undecidable then B is undecidable.
(c) If $A \leq_m B$ and B is recursively enumerable then A is recursively enumerable.
(d) If $A \leq_m B$ and B is not recursively enumerable then A is not recursively enumerable.

[2014 (Set-2) : 1 Mark]

3.33 Let Σ be a finite non-empty alphabet and let 2^{Σ^*} be the power set of Σ^*. Which one of the following is TRUE?

(a) Both 2^{Σ^*} and Σ^* are countable
(b) 2^{Σ^*} is countable and Σ^* is uncountable
(c) 2^{Σ^*} is uncountable and Σ^* is countable
(d) Both 2^{Σ^*} and Σ^* are uncountable

[2014 (Set-3) : 1 Mark]

3.34 Which one of the following problems is undecidable?

(a) Deciding if a given context-free grammar is ambiguous.
(b) Deciding if a given string is generated by a given context-free grammar.
(c) Deciding if the language generated by a given context-free grammar is empty.
(d) Deciding if the language generated by a given context-free grammar is finite.

[2014 (Set-3) : 2 Marks]

3.35 For any two languages L_1 and L_2 such that L_1 is context free and L_2 is recursively enumerable but not recursive, which of the following is/are necessarily true?

1. $\bar{L}_1$ (complement of L_1) is recursive
2. $\bar{L}_2$ (complement of L_2) is recursive
3. $\bar{L}_1$ is context-free
4. $\bar{L}_1 \cup L_2$ is recursively enumerable

(a) 1 only (b) 3 only
(c) 3 and 4 only (d) 1 and 4 only

[2015 (Set-1) : 1 Mark]

3.36 Consider the following statements:

1. The complement of every Turning decidable language is Turning decidable
2. There exists some language which is in NP but is not Turing decidable
3. If L is a language in NP, L is Turing decidable

Which of the above statements is/are True?

(a) Only 2 (b) Only 3
(c) Only 1 and 2 (d) Only 1 and 3

[2015 (Set-2) : 1 Mark]

3.37 Let X be a recursive language and Y be a recursively enumerable but not recursive language. Let W and Z be two languages such that $\bar{Y}$ reduces to W, and Z reduces to $\bar{X}$ (reduction means the standard many-one reduction). Which one of the following statements is TRUE?

(a) W can be recursively enumerable and Z is recursive.
(b) W can be recursive and Z is recursively enumerable.
(c) W is not recursively enumerable and Z is recursive.
(d) W is not recursively enumerable and Z is not recursive.

[2016 (Set-1) : 2 Marks]

3.38 Consider the following types of languages L_1: Regular, L_2: Context-free, L_3: Recursive, L_4: Recursively enumerable.

Which of the following is/are **TRUE**?

I. $\overline{L_3} \cup L_4$ is recursively enumerable
II. $\overline{L_2} \cup L_3$ is recursive
III. $L_1^* \cap L_2$ is context-free
IV. $L_1 \cup \overline{L_2}$ is context-free

(a) I only (b) I and III only
(c) I and IV only (d) I, II and III only

[2016 (Set-2) : 1 Mark]

3.39 Consider the following languages.

- $L_1 = \{\langle M\rangle \mid M$ takes at least 2016 steps on some input$\}$
- $L_2 = \{\langle M\rangle \mid M$ takes at least 2016 stps on all inputs$\}$ and
- $L_3 = \{\langle M\rangle \mid M$ accepts $\varepsilon\}$

where for each turning machine M, $\langle M\rangle$ denotes a specific encoding of M. Which one of the following is **TRUE**?

(a) L_1 is recursive and L_2, L_3 are not recursive
(b) L_2 is recursive and L_1, L_3 are not recursive
(c) L_1, L_2 are recursive and L_3 is not recursive
(d) L_1, L_2, L_3 are recursive

[2016 (Set-2) : 2 Marks]

■■■■

Answers Turing Machine: RE, REC and Undecidability

3.1	(b, d)	3.2	(b, c)	3.3	(a, d)	3.4	(c)	3.5	(a, c)	3.6	(c)	3.7	(d)						
3.8	(b)	3.8	(b)	3.10	(a)	3.11	(b)	3.12	(a)	3.13	(b)	3.14	(a)	3.15	(d)	3.16	(a)		
3.17	(c)	3.18	(b)	3.19	(d)	3.20	(b)	3.21	(b)	3.22	(d)	3.23	(d)	3.24	(b)	3.25	(b)		
3.26	(b)	3.27	(d)	3.28	(c)	3.29	(d)	3.30	(c)	3.31	(b)	3.32	(d)	3.33	(c)	3.34	(a)		
3.35	(d)	3.36	(d)	3.37	(c)	3.38	(d)	3.39	(c)										

Explanations Turing Machine: RE, REC and Undecidability

3.1 (b,d)

(a) **Decidable**: Membership means "given any context free grammar (CFG) G and a word $w \in \Sigma^*$, does $w \in L(G)$ i.e., can G generate given string w ?" Membership problem in CFL's is decidable.

(b) **Undecidable**: Regularity of context free language is undecidable because there exist no algorithm to check whether a CFL is regular.

(c) **Decidable:** Finite state automaton is a decider. Decider is a machine that always halts. So finite automaton halts on all inputs is a trivially decidable problem, since the answer is always yes.

(d) **Undecidable:** Type-0 language is recursive enumerable language (RE) and membership problem for RE language is undecidable. So membership problem for type0 languages is undecidable problem.

3.2 (b,c)

(a) It is decidable whether an arbitrary turing machine halts within 10 steps. This can be decided by running the TM program on UTM for finite number (10) of steps.

(b) It is undecidable whether a turing machine prints a specific letter because for printing specific letter turing machine has to enter a specific state on an input. State entry problem can be reduced to this problem. State entry problem is undecidable so this problem also become undecidable.

(c) It is undecidable whether a given turing machine computes the product of two numbers. To decide this we have to first of all decide whether the given TM program halts. But the halting problem is undecidable. So, this problem is also undecidable.

3.3 (a,d)

(a) **True.** Every CFL is recursive language whereas there exist some recursive languages which are not CFL's. So recursive languages are a proper superset of context free languages.
$CFL \subset REC$

(b) **False.** Since there exist some recursive languages which are not context free. So recursive languages are not always recognizable by pushdown automata.

(c) **False.** Recursive enumerable languages are called as type 0 languages.

(d) **True.** Recursive languages are recognizable as well as decidable by turing machine.

3.4 (c)

Every finite language over some finite alphabet Σ is regular because for every finite language DFA or NFA can be created by brute force.

3.5 (a,c)

(a) There exist an algorithm to convert every NFA to DFA. So recognition power of NFA and DFA, is same.

(b) Since there exist no algorithm to convert NPDA to DPDA and recognition power of both are not same.

(c) The recognition power of NTM (non-deterministic turing machine) and DTM (deterministic turing machine) is same.

(d) Since for turing machines and finite automation the above statement is true so (d) is not true.

3.6 (c)

(a) Every regular grammar is also context free grammar. So this is trivially decidable.

(b) There exist algorithm to convert non-deterministic FSA to deterministic FSA since their recognition power is same.

(c) Since the recognition power of non-deterministic PDA and deterministic PDA is not same i.e., there exists some languages like $L = \{ww^R \mid w \in (a, b)^*\}$ which are accepted by non-deterministic PDA but not by deterministic PDA. So, there exist no algorithm for converting non-deterministic PDA to deterministic PDA.

(d) Since the recognition power of non-deterministic turing machine and deterministic turing machine are same. So there exist an algorithm to convert every non-deterministic turing machine to deterministic turing machine.

3.7 (d)

(a) True, the halting problem which states that: "Given a turing machine M and an input w, does M eventually halt when run with w or will run forever" is undecidable problem.

(b) True, an ambiguous grammar is a grammar for which there exists a string that can have more than one parse tree. Since there is no algorithm for finding whether context free grammar is ambiguous or not that why, this problem is undecidable.

(c) True, since equivalence problem for CFLs is undecidable.

(d) False, because there exist an algorithm for determining whether $L(G_1) = L(G_2)$ for any two regular grammars G_1 and G_2. So, equivalence of two regular grammars is a decidable problem.

3.8 (b)

(a) Given a turing machine M, a strings S and an integer k, M accepts S within k steps is decidable problem, since the turing machine M can be simulated upto a finite number of steps in finite time on a universal turing machine (UTM).

(b) Equivalence of two turing machines is undecidable problem.

(c) Emptiness of regular language is decidable problem. So the non emptiness problem of regular language is also decidable, since the complement of a decidable (REC) problem is always decidable (REC).

(d) Emptiness of CFL's is decidable problem.

3.8 (b)

(a) Since there exist an algorithm to convert any NFA into equivalent DFA. So the recognition power of NFA and DFA is same.

(b) Since there exist no algorithm for converting every NPDA to DPDA and there exist some context free languages which are accepted by NPDA but not by DPA. So the recognition power of NPDA and DPDA is not same.
Example:
$L = \{ww^R \mid w \in (a, b)^*\}$ is accepted by NPDA but not by DPDA.

(c) Nondeterministic turing machines are equivalent with deterministic turing machines. So the recognition power of NTM is equal to the DTM.

(d) Mutlitape turing machine is a turing machine with several tapes. The multitape turing machine is equivalent to the standard turing machine which is a single tape turing machine. So the recognition power of multitape and single tape turing machine is same.

3.10 (a)

The complement of recursive language is recursive because recursive languages are closed under complementation. RE and CFL are not closed under complementation.
So, options (b), (c) and (d) are false.

3.11 (b)

The C language is context sensitive language. All the feature of C language comes under CFL except following two which makes the C language as CSL:

(a) Declaration of variable before use

(b) The matching of actual and formal parameters.

3.12 (a)

P_1 is membership of FSM i.e., regular language.
P_2 is finite/infiniteness of context free grammar.
Membership of FSM and finiteness of CFG, both are decidable problems.
So P_1 and P_2 both are decidable.

3.13 (b)

Problem X is state entry problem. Halting problem of turing machine can be reduced to state entry problem. Since halting problem of turing machine is undecidable, the state-entry problem must also be undecidable. The language corresponding to state-entry problem is RE but not REC. So X is undecidable but partially decidable.

3.14 (a)

If P = NP then L = (0+1)* which is regular.
If P ≠ NP then L = ϕ which is also regular.
Either way L is regular and hence recursive.

3.15 (d)

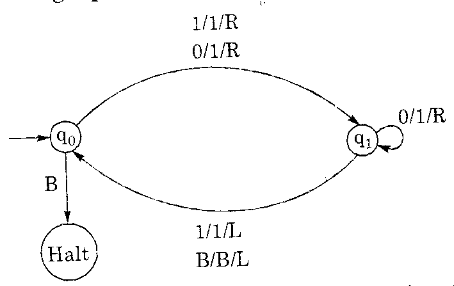
Theorem: A language is recursive if and only if it can be effectively enumerated in lexicographic order.
Based on this theorem clearly the answer is (d).

3.16 (a)

Transition function of M

	0	1	B
q_0	q_1, 1, R	q_1, 1, R	Halt
q_1	q_1, 1, R	q_0, 1, L	q_0, B, L

The graphic representation of M is as follows:



This machine hangs on every string in $(0 + 1)^+$. Example is shown below for the strings "00" and "01".

$$\underset{q_0}{B00B} \rightarrow \underset{q_1}{B10B} \rightarrow \underset{q_1}{B11B} \rightarrow \underset{q_0}{B11B} \rightarrow \underset{q_1}{B11B} \rightarrow \text{Hangs}$$

$$\underset{q_0}{B01B} \rightarrow \underset{q_1}{B11B} \rightarrow \underset{q_0}{B11B} \rightarrow \underset{q_1}{B11B} \rightarrow \text{Hangs}$$

3.17 (c)

Note that in general if $L_1 \le L_2$,
L_2 decidable ⇒ L_1 decidable and
L_1 undecidable ⇒ L_2 is undecidable.
Given that P_1 is decidable and P_2 is undecidable.
Consider (a) P_3 is decidable if P_1 is reducible to P_3
i.e. $P_1 \le_p P_3$
and if P_1 is undecidable, then P_3 would be undecidable.
But it is given that P_1 is decidable, therefore we cannot use this theorem.
(a) is false.
Consider (b) P_3 is undecidable if P_3 is reducible to P_2
i.e. $P_3 \le_p P_2$
Now if P_3 is undecidable then P_2 is undecidable but nothing is given regarding P_3.
Also if P_2 is decidable, then P_3 would be decidable, but it is given that P_2 is undecidable, so we can't use this.
(b) is false.
Consider (c) P_3 is undecidable if P_2 is reducible to P_3
i.e. $P_2 \le_p P_3$
Now if P_2 is undecidable, this would mean P_3 is undecidable.. Since it is given that P_2 is undecidable, therefore, surely P_3 is undecidable is correct.
Choice (c) is correct.

3.18 (b)

Given that L_1 is REC and L_2 is RE but not REC.
Consider choice (a) $\overline{L_1}$ is REC and $\overline{L_2}$ is RE.
Since L_1 is REC, clearly $\overline{L_1}$ is also REC.
Since L_2 is RE but not REC, its complement has to be not RE.
∴ But (a) claims that $\overline{L_2}$ is RE.
Therefore (a) is false.
Consider (b) $\overline{L_1}$ is REC and $\overline{L_2}$ is not RE.
If L_1 is REC, then $\overline{L_1}$ is REC.
Also, if L_2 is RE but not REC, then $\overline{L_2}$ is not RE.
∴ (b) is true.
Clearly (c) and (d) false since $\overline{L_2}$ is not RE.

3.19 (d)

There is a DFA corresponding to D(S) mod 5 = 2 as well as D(S) mod 7 = 4. Therefore, both of them are regular languages. Let they be L_1 and L_2. The given language is $L_1 \cap L_2^c$.
Since regular languages are closed under intersection and complementation, the given language is also regular.

3.20 (b)

Given L_1 be regular, L_2 is DCFL and L_3 is RE but not REC.

1. "$L_1 \cap L_2$ is a DCFL" is true since DCFL's are closed under regular intersection.
2. "$L_3 \cap L_1$ is recursive" is false since L_3 is RE but not REC and therefore $L_3 \cap L_1$ is surely RE according to closure of RE under regular intersection, but we cannot be sure that $L_3 \cap L_1$ is REC.

3. "$L_1 \cup L_2$ is CFL" is true, since L_2 is DCFL and hence a CFL and L_1 is regular. Therefore, by closure of CFL's on regular union, we can say that $L_1 \cup L_2$ is surely CFL.
4. "$L_1 \cap L_2 \cap L_3$ is RE" is true since all three are surely RE and RE languages are closed under intersection.

3.21 (b)

(a) Membership problem for CFG's is decidable (CYK algorithm exists).
(b) Ambiguity problem of CFG's is undecidable.
(c) Finiteness problem of FSA's is decidable, since there exists an algorithm to check if a given regular language L is finite or infinite.
(d) Equivalence problem for FSA's is decidable, since there exists an algorithm to check where $L(M_1) = L(M_2)$ or not.

3.22 (d)

Let $L = \{a^p \mid p \text{ is a prime}\}$. There is no DFA which recognizes L, since to check whether a number is prime or not, requires division to be performed, which neither FA nor PDA can do. It is a CSL and can be accepted by a turing machine.

3.23 (d)

There is a theorem which states that whenever L and $\overline{L}$ are both RE, then both will be REC.

3.24 (b)

1. "Intersection of two regular languages is infinite" is decidable, since we can construct a product dfa of the 2 given dfas and then, using the algorithm to check finiteness or infiniteness on this dfa, we can solve the problem.
2. "Whether a given CFL is regular" is undecidable.
3. "Whether two PDA's accept the same language" is undecidable, since equivalence of CFL's is undecidable.
4. "Whether a given grammar is context-free" is decidable since we can easily check using a TM, whether the LHS of every production has a single variable and, then it is a CFG, else it is not a CFG.

$\therefore$ 1 and 4 are decidable.

3.25 (b)

Given, L_1 is REC and L_2 and L_3 are RE but not REC.

Choice (a)

$$\begin{aligned} L_2 - L_1 &= L_2 \cap L_1^C \\ &= (\text{RE but not REC}) \cap (\text{REC})^C \\ &= (\text{RE but not REC}) \cap (\text{REC}) \\ &= \text{RE} \cap \text{REC} \\ &= \text{RE} \cap \text{RE} \\ &= \text{RE} \end{aligned}$$

Choice (a) is true.

Choice (b): $L_1 - L_3 = L_1 \cap L_3^C$
$= \text{REC} \cap (\text{RE but not REC})^C$

Now the complement of a RE but not REC language has to be "not RE".

$\therefore$ $L_1 - L_3 = \text{REC} \cap \text{not RE} = \text{not RE}$

$\therefore$ Choice (b) is false.

Choice (c):

$$\begin{aligned} L_2 \cap L_3 &= (\text{RE but not REC}) \cap (\text{RE but not REC}) \\ &= \text{RE} \cap \text{RE} = \text{RE} \end{aligned}$$

So, choice (c) is true.

Choice (d):

$$\begin{aligned} L_2 \cup L_3 &= (\text{RE but not REC}) \cup (\text{RE but not REC}) \\ &= \text{RE} \cup \text{RE} = \text{RE} \end{aligned}$$

$\therefore$ Choice (d) is true.

3.26 (b)

DFA $\equiv$ NFA
DPDA $\neq$ NPDA
DTM $\equiv$ NTM
Single tape TM $\equiv$ Multi-tape TM

3.27 (d)

Statement (1) is **undecidable** (since it is the halting problem of TM).

Complement of a context free language, may or may not be context free. So statement number (2) is not only a non trivial problem but is also **undecidable**.

If L is regular, $\overline{L}$ is surely regular.

So statement number (3) is (trivially) **decidable**.

Also if L is recursive, $\overline{L}$ is also surely recursive.

So statement number (4) is also (trivially) **decidable**.

So, correct option is (d) i.e. only 3 and 4 are **decidable**.

3.28 (c)

1. Non-deterministic turning machine can be simulated into an equivalent deterministic turing machine. So for every non deterministic turing machine there exits an equivalent deterministic turing machine. So true.
2. Turing recognizable languages (RE languages) are not closed under complementation so false.
3. Turing decidable languages (REC languages) are closed under both inter-section and complementation. So, true.
4. Turing recognizable languages (RE languages) are closed under union and intersection. So true.

So option (c) is correct.

3.29 (d)

1. G is CFG, Is $L(G) = \phi$?.
   This is the emptiness problem of CFLs which is **decidable**.
2. G is CFG, Is $L(G) = \Sigma^*$.
   This is the Kleene closedness problem of CFLs which is **undecidable**.
3. M is turning machine, Is L(M) regular.
   This is the regularity problem of REs which is **undecidable**.
4. A is a DFA and N is NFA, Is L(A) = L(N)?.
   This is the equivalence problem of regular languages which is **decidable**.

So, only 2 and 3 are **undecidable**.

3.30 (c)

Option (c): Both L and $\bar{L}$ are r.e. but not recursive, is not a viable possibility, because if both L and $\bar{L}$ were r.e. then both would become recursive due to the $L, \bar{L}$ theorem.

Option (a), (b) and (d) are allowed by the $L, \bar{L}$ theorem.

3.31 (b)

Whether a TM accepts a finite length string or not is a non trivial question and hence by Rice's theorem the problem is undecidable.

But the problem however falls in recursively enumerable since the condition given is a positive condition.

3.32 (d)

If $A \leq_m B$ and B is RE $\rightarrow$ A is RE is a one way theorem, which is true.

Option (d): B is not RE $\rightarrow$ A is not RE is the inverse of this theorem and hence false.

3.33 (c)

Canter's Theorem: If a set S is countably infinite then 2^S is uncountably infinite.

Now, Σ^* is countably infinite. So by Cantor's theorem, 2^{Σ^*} is uncountably infinite.

3.34 (a)

Ambiguity problem of context free grammar is undecidable.

3.35 (d)

L_1 is CFL $\Rightarrow \bar{L}_1 = \overline{CFL} = \overline{CSL} = CSL$

L_2 is RE but not REC $\Rightarrow \bar{L}_2$ is not RE

Statement 1: $\bar{L}_1$ is recursive is **true** because, compliment of CFL is always a CSL and hence recursive.

Statement 2: $\bar{L}_2$ is recursive is **false** because, $\bar{L}_2$ is not RE.

Statement 3: $\bar{L}_1$ is CFL is **false** ($\bar{L}_1$ is need not be CFL).

Statement 4: $\bar{L}_1 \cup L_2$ is RE is **true** because,

$\overline{CFL} \cup RE = \overline{CSL} \cup RE = CSL \cup RE = RE \cup RE = RE$

So only statements 1 and 4 are **true**.

3.36 (d)

Statement 1: True, because decidable languages (REC) are closed under complement operation.

Statement 2: False, because $P \subseteq NP \subseteq REC$. So every P and every NP language is decidable (REC).

Statement 3: True, every language in NP is decidable.

3.37 (c)

$X \rightarrow$ REC

$Y \rightarrow$ RE but not REC

$\bar{Y} \leq W$ and $Z \leq \bar{X}$

Now we know that RE, REC both go in reverse direction on reduction.

Now, if Y is RE but not REC, then $\bar{Y}$ is not RE.

Now $\bar{Y} \leq W$

If W is RE $\Rightarrow \bar{Y}$ is RE

Contrapositive is $\bar{Y}$ is not RE $\Rightarrow W$ is not RE

Now $Z \leq \bar{X}$

If X is REC, $\bar{X}$ is also REC

So $\bar{X}$ REC $\Rightarrow Z$ is REC

So W is not RE and Z is REC.

Choice (c) is correct.

3.38 (d)

$L_1 \rightarrow$ Regular
$L_2 \rightarrow$ CFL
$L_3 \rightarrow$ REC
$L_4 \rightarrow$ RE

I: $\bar{L}_3 \cup L_4$ is RE

$$\bar{L}_3 \cup L_4 = \overline{\text{REC}} \cup \text{RE} = \text{REC} \cup \text{RE}$$
$$= \text{RE} \cup \text{RE} = \text{RE}$$

So I is true

II: $\bar{L}_2 \cup L_3$ is recusive

$$\bar{L}_2 \cup L_3 = \overline{\text{CFL}} \cup \text{REC} = \overline{\text{CSL}} \cup \text{REC}$$
$$= \text{CSL} \cup \text{REC}$$
$$= \text{REC} \cup \text{REC} = \text{REC}$$

So II is True.

III: $L_1^* \cap L_2$ is CFL

$$L_1^* \cap L_2 = (\text{REG})^* \cap \text{CFL}$$
$$= \text{REG} \cap \text{CFL} = \text{CFL}$$

So III is True.

IV: $L_1 \cup \bar{L}_2$ is CFL

$$L_1 \cup \bar{L}_2 = \text{REG} \cup \overline{\text{CFL}} = \text{REG} \cup \overline{\text{CSL}}$$
$$= \text{REG} \cup \text{CSL} = \text{CSL}$$

Since, A CSL need not be a CFL, so IV is False.

So only I, II and III are true.

3.39 (c)

$$L_3 = \{M \mid \varepsilon \in L(M)\}$$

Since the question is a non-trivial question on RE language, it is undecidable by Rice's theorem. So L_3 is not recursive.

L_1, L_2 are decidable (recursive) since 2016 is finite and that many steps can be simulated on a UTM in finite time.

■■■■

Unit ■ III

Digital Logic

■ Contents

UNIT III

Digital Logic

Syllabus : Logic functions, Minimization, Design and synthesis of combinational and sequential circuits; Number representation and computer arithmetic (fixed and floating point).

Analysis of Previous GATE Papers

Exam Year	1 Mark Ques.	2 Marks Ques.	3 Marks Ques.	5 Marks Ques.	Total Marks
1990	–	8		3	**31**
1991	–	1		2	**12**
1992	–	–		–	–
1993	–	–		–	–
1994	–	–		–	–
1995	–	–		–	–
1996	3	4		2	**21**
1997	1	3		–	**7**
1998	–	–		–	–
1999	2	2		–	**6**
2000	1	4		–	**9**
2001	1	3		–	**7**
2002	4	4		1	**17**
2003	1	4		–	**9**
2004	4	5		–	**14**
2005	4	4		–	**12**
2006	1	5		–	**11**

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2007	3	5	**13**
2008	4	1	**9**
2009	2	–	**2**
2010	3	2	**7**
2011	2	3	**8**
2012	2	–	**2**
2013	3	–	**3**
2014 Set-1	2	1	**4**
2014 Set-2	3	1	**5**
2014 Set-3	2	2	**6**
2015 Set-1	1	2	**5**
2015 Set-2	1	2	**5**
2015 Set-3	–	3	**6**
2016 Set-1	3	2	**7**
2016 Set-2	3	–	**3**

1 Logic Functions and Minimization

1.1 Let * be defined as $x*y = \bar{x} + y$. Let $z = x*y$ value of $z*x$ is

(a) $\bar{x} + y$ (b) x

(c) 0 (d) 1

[1997 : 1 Mark]

1.2 Let $f(x, y, z) = \bar{x} + \bar{y}z + xz$ be a switching function. Which one of the following is valid?

(a) $\bar{y}x$ is a prime implicant of f

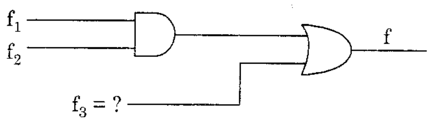
(b) xz is a minterm of f

(c) xz is an implicant of f

(d) y is a prime implicant of f

[1997 : 2 Marks]

1.3 Consider the logic circuit shown in figure below. The function f_1, f_2 and f(in canonical sum of products form in decimal notation) are



$f_1(w, x, y, z) = \Sigma(8, 9, 10)$

$f_2(w, x, y, z) = \Sigma(7, 8, 12, 13, 14, 15)$

$f(w, x, y, z) = \Sigma(8, 9)$

The function f_3 is

(a) $\Sigma 9, 10$ (b) $\Sigma 9$

(c) $\Sigma 1, 8, 9$ (d) $\Sigma 8, 10, 18$

[1997 : 2 Marks]

1.4 The simultaneous equations on the Boolean variables x, y, z and w,

$$x + y + z = 1$$
$$xy = 0$$
$$xz + w = 1$$
$$xy + \bar{z}\bar{w} = 0$$

have the following solution for x, y, z and w, respectively:

(a) 0100 (b) 1101

(c) 1011 (d) 1000

[2000 : 2 Marks]

1.5 Given the following Karnaugh map, which one of the following represents the minimal Sum-of-Products of the map?

yz \ wx	00	01	11	10
00	0	X	0	X
01	X	1	X	1
11	0	X	1	0
10	0	1	X	0

(a) xy + y'z (b) wx'y' + xy + xz

(c) w'x + y'z + xy (d) xz + y

[2001 : 1 Mark]

1.6 Minimum SOP for f(w, x, y, z) shown in Karnaugh-map below is

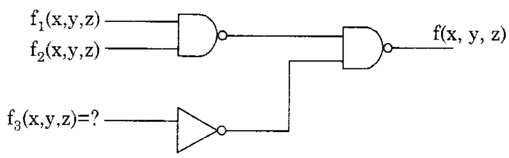
yz \ wx	00	01	11	10
00	0	1	1	0
01	x	0	0	1
11	x	0	0	1
10	0	1	1	x

(a) xz + y'z (b) xz' + zx'

(c) x'y + zx' (d) None of these

[2002 : 1 Mark]

1.7 Consider the following logic circuit whose inputs are functions f_1, f_2, f_3 and output is f. Given that



$f_1(x, y, z) = \Sigma(0, 1, 3, 5)$,

$f_2(x, y, z) = \Sigma(6, 5)$, and

$f(x, y, z) = \Sigma(1, 4, 5)$,

f_3 is

(a) $\Sigma(1,4, 5)$ (b) $\Sigma(6, 7)$

(c) $\Sigma(0, 1, 3, 5)$ (d) None of these

[2002 : 2 Marks]

1.8 f(A, B) = A′ + B. Simplified expression for function f(f(x + y, y), z) is

(a) x′ + z (b) xyz

(c) xy′ + z (d) None of these

[2002 : 2 Marks]

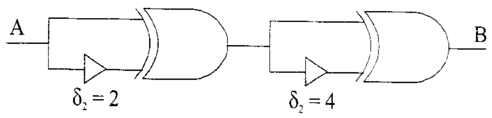
1.9 The literal count of a boolean expression is the sum of the number of times each literal appears in the expression. For example, the literal count of (xy + xz) is 4. What are the minimum possible literal counts of the product-of-sum and sum-of-product representations respectively of the function given by the following Karnaugh map? Here, X denotes "don't care"

xy \ zw	00	01	11	10
00	X	1	0	1
01	0	1	X	0
11	1	X	X	0
10	X	0	0	X

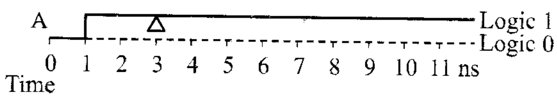
(a) (11, 9) (b) (9, 13)
(c) (9, 10) (d) (11, 11)

[2003 : 2 Marks]

1.10 Consider the following circuit composed of XOR gates and non-inverting buffers.



The non-inverting buffers have delays $\delta_1 = 2$ ns and $\delta_2 = 4$ ns as shown in the figure. Both XOR gates and all wires have zero delay. Assume that all gate inputs, outputs and wires are stable at logic level 0 at time 0. If the following waveform is applied at input A, how many transition (s) (change of logic levels) occur (s) at B during the interval from 0 to 10 ns?



(a) 1 (b) 2
(c) 3 (d) 4

[2003 : 2 Marks]

1.11 The Boolean function x′ y′ + xy + x′ y is equivalent to

(a) x′ + y′ (b) x + y
(c) x + y′ (d) x′ + y

[2004 : 1 Mark]

1.12 What is the minimum number of NAND gates required to implement a 2-input EXCLUSIVE-OR function without using any other logic gate?

(a) 3 (b) 4
(c) 5 (d) 6

[2004 : 1 Mark]

1.13 The function AB′C + A′BC + ABC′+A′B′C + AB′C′ is equivalent to

(a) AC′ + AB + A′C (b) AB′ + AC′ + A′C
(c) A′B + AC′ + AB′ (d) A′B + AC + AB′

[2004 : 2 Marks]

1.14 Which are the essential prime implicants of the following Boolean function?

f(a, b, c) = a′c + ac′ + b′c

(a) a′c and ac′ (b) a′c and b′c
(c) a′c only (d) ac′ and bc′

[2004 : 2 Marks]

1.15 Which of the following expressions is equivalent to $(A \oplus B) \oplus C$

(a) $(A + B + C)(\bar{A} + \bar{B} + \bar{C})$

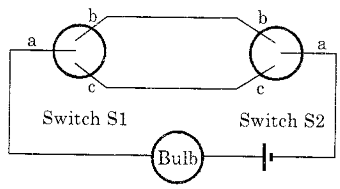
(b) $(A + B + C)(\bar{A} + \bar{B} + C)$

(c) $ABC + \bar{A}(B \oplus C) + \bar{B}(A \oplus C)$

(d) None of these

[2005 : 1 Mark]

1.16 A two-way switch has three terminals a, b and c. In ON position (logic value 1), *a* is connected to b, and in OFF position, *a* is connected to c. Two of these two-way switches S1 and S2 are connected to a bulb as shown below.

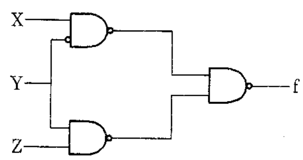


Which of the following expressions, if true, will always result in the lighting of the bulb?

(a) $S1 \cdot \overline{S2}$ (b) S1 + S2
(c) $\overline{S1 \oplus S2}$ (d) $S1 \oplus S2$

[2005 : 1 Mark]

1.17 Consider the following circuit.



Which one of the following is TRUE?

(a) f is independent of X
(b) f is independent of Y
(c) f is independent of Z
(d) None of X, Y, Z is redundant

[2005 : 1 Mark]

1.18 The switching expression corresponding to $f(A, B, C, D) = \Sigma(1, 4, 5, 9, 11, 12)$ is
(a) $BC'D' + A'C'D + AB'D$
(b) $ABC' + ACD + B'C'D$
(c) $ACD' + A'BC' + AC'D'$
(d) $A'BD + ACD' + BCD'$

[2005 : 1 Mark]

1.19 Given two 3-bit numbers $a_2a_1a_0$ and $b_2b_1b_0$ and c, the carry in, the function that represents the carry generate function when these two numbers are added is
(a) $a_2b_2 + a_2a_1b_1 + a_2a_1a_0b_0 + a_2a_0b_1b_0 + a_1b_2b_1 + a_1a_0b_2b_0 + a_0b_2b_1b_0$
(b) $a_2b_2 + a_2b_1b_0 + a_2a_1b_1b_0 + a_1a_0b_2b_1 + a_1a_0b_2 + a_1a_0b_2b_0 + a_2a_0b_1b_0$
(c) $a_2 + b_2 + (a_2 \oplus b_2)[a_1 + b_1 + (a_1 \oplus b_1)(a_0 + b_0)]$
(d) $a_2b_2 + \overline{a_2}\,a_1b_1 + \overline{a_2a_1}\,a_0b_0 + \overline{a_2}\,a_0\,\overline{b_1}\,b_0 + a_1\,\overline{b_2}\,b_1 + \overline{a_1}\,a_0\,\overline{b_2}\,b_0 + a_0\,\overline{b_2b_1}\,b_0$

[2006 : 2 Marks]

1.20 Consider a boolean function f(w, x, y, z). Suppose that exactly one of its inputs is allowed to change at a time. If the function happens to be true for two input vectors $i_1 = \langle w_1, x_1, y_1, x_1 \rangle$ and $i_2 = \langle w_2, x_2, y_2, z_2 \rangle$, we would like the function to remain true as the input changes from i_1 to i_2 (i_1 and i_2 differ in exactly one bit position), without becoming false momentarily. Let $f(w, x, y, z) = \Sigma(5, 7, 11, 12, 13, 15)$. Which of the following cube covers of f will ensure that the required property is satisfied?
(a) $\bar{w}xz$, $wx\bar{y}$, $x\bar{y}z$, xyz, wyz
(b) wxy, $\bar{w}xz$, wyz
(c) $wx\overline{yz}$, xz, $w\bar{x}yz$
(d) wyz, wxz, $\bar{w}xz$, $x\bar{y}z$, xyz

[2006 : 2 Marks]

1.21 We consider the addition of two 2's complement numbers $b_{n-1}b_{n-2} \dots b_0$ and $a_{n-1}a_{n-2} \dots a_0$. A binary adder for adding unsigned binary numbers is used to add the two numbers. The sum is denoted by $c_{n-1}c_{n-2} \dots c_0$ and the carryout by c_{out}.
Which one of the following options correctly identifies the overflow condition?
(a) $c_{out}\overline{(a_{n-1} \oplus b_{n-1})}$
(b) $a_{n-1}\,b_{n-1}\,\overline{c_{n-1}} + \overline{a_{n-1}\,b_{n-1}}\,c_{n-1}$
(c) $c_{out} \oplus c_{n-1}$
(d) $a_{n-1} \oplus b_{n-1} \oplus c_{n-1}$

[2006 : 2 Marks]

1.22 Consider numbers represented in 4-bit gray code. Let $h_3h_2h_1h_0$ be the gray code representation of a number n and let $g_3g_2g_1g_0$ be the gray code of (n + 1) (modulo 16) value of the number. Which one of the following functions is correct?
(a) $g_0(h_3\,h_2\,h_1\,h_0) = \Sigma(1, 2, 3, 6, 10, 13, 14, 15)$
(b) $g_1(h_3\,h_2\,h_1\,h_0) = \Sigma(4, 9, 10, 11, 12, 13, 14, 15)$
(c) $g_2(h_3\,h_2\,h_1\,h_0) = \Sigma(2, 4, 5, 6, 7, 12, 13, 15)$
(d) $g_3(h_3\,h_2\,h_1\,h_0) = \Sigma(0, 1, 6, 7, 10, 11, 12, 13)$

[2006 : 2 Marks]

1.23 What is the maximum number of different Boolean functions involving in 'n' Boolean variables?
(a) n^2
(b) 2^n
(c) 2^{2^n}
(d) 2^{n^2}

[2007 : 1 Mark]

1.24 Consider the following Boolean function of four variables
$f(w, x, y, z) = \Sigma(1, 3, 4, 6, 9, 11, 12, 14)$
The function is
(a) independent of one variable
(b) independent of two variables
(c) independent of three variables
(d) dependent on all the variables

[2007 : 1 Mark]

1.25 The following expression was to be realized using 2-input AND and OR gates. However, during the fabrication all 2-input AND gates were mistakenly substituted by 2-input NAND gates.

$$(a.b).c + (\bar{a}.c).d + (b.c).d + a.d$$

What is the function finally realized?
(a) 1
(b) $\bar{a} + \bar{b} + \bar{c} + \bar{d}$
(c) $\bar{a} + b + \bar{c} + \bar{d}$
(d) $\bar{a} + \bar{b} + c + \bar{d}$

[2007 : 2 Marks]

Linked Answer Q.1.26 & Q.1.27:

Consider the following expression:

$$a\bar{d} + \bar{a}\,\bar{c} + b\bar{c}d.$$

1.26 Which of the following Karnaugh Maps correctly represents the expression?

(a)

	$\bar{c}\bar{d}$	$\bar{c}d$	cd	$c\bar{d}$
$\bar{a}\bar{b}$	×	×		
$\bar{a}b$	×	×		
ab	×	×		×
$a\bar{b}$	×			×

(b)

	$\bar{c}\bar{d}$	$\bar{c}d$	cd	$c\bar{d}$
$\bar{a}\bar{b}$	×	×		
$\bar{a}b$	×			
ab	×	×		×
$a\bar{b}$	×	×		×

(c)

	$\bar{c}\bar{d}$	$\bar{c}d$	cd	$c\bar{d}$
$\bar{a}\bar{b}$	×	×		
$\bar{a}b$	×	×		×
ab	×	×		×
$a\bar{b}$	×			×

(d)

	$\bar{c}\bar{d}$	$\bar{c}d$	cd	$c\bar{d}$
$\bar{a}\bar{b}$	×	×		
$\bar{a}b$	×	×		
ab	×	×		×
$a\bar{b}$	×		×	×

[2007 : 2 Marks]

1.27 Which of the following expressions does not correspond to the Karnaugh Map obtained in above question?

(a) $\bar{c}\bar{d} + a\bar{d} + ab\bar{c} + \bar{a}\,\bar{c}\,d$

(b) $\bar{a}\,\bar{c} + \bar{c}\bar{d} + a\bar{d} + ab\bar{c}d$

(c) $\bar{a}\,\bar{c} + a\bar{d} + ab\bar{c} + \bar{c}d$

(d) $\bar{b}\bar{c}\bar{d} + ac\bar{d} + \bar{a}\,\bar{c} + ab\bar{c}$

[2007 : 2 Marks]

1.28 Let $f(w, x, y, z) = \Sigma(0, 4, 5, 7, 8, 9, 13, 15)$. Which of the following expressions are NOT equivalent to f?

(a) $x'y'z' + w'xy' + wy'z + xz$

(b) $w'y'z' + wx'y' + xz$

(c) $w'y'z' + wx'y' + xyz + xy'z$

(d) $x'y'z' + wx'y' + w'y$

[2007 : 2 Marks]

1.29 Define the connective * for the Boolean variable X and Y as : $X * Y = XY + X'Y'$

Let Z = X * Z. Consider the following expression P, Q and R.

P: $X = Y * Z$

Q: $Y = X * Z$

R: $X * Y * Z = 1$

Which of the following is TRUE?

(a) Only P and Q are valid

(b) Only Q and R are valid

(c) Only P and R are valid

(d) All P, Q, R are valid

[2007 : 2 Marks]

1.30 In a look-ahead carry generator, the carry generate function G_i and the carry propagate function P_i for inputs, A_i and B_i are given by

$$P_i = A_i \oplus B_i \text{ and } G_i = A_iB_i$$

The expressions for the sum bit S_i and carry bit C_{i+1} of the look-ahead carry adder are given by $S_i = P_i \oplus C_i$ and $C_{i+1} = G_i + P_iC_i$, where C_0 is the input carry.

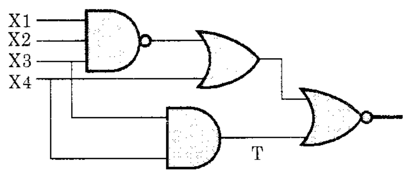
Consider a two-level logic implementation of the look-ahead carry generator. Assume that all P_i and G_i are available for the carry generator circuit and that the AND and OR gates can have any number of inputs. The number of AND gates and OR gates needed to implement the look-ahead carry generator for a 4-bit adder with S_3, S_2, S_1, S_0 and C_4 as its outputs are respectively

(a) 6, 3 (b) 10, 4

(c) 6, 4 (d) 10, 5

[2007 : 2 Marks]

1.31 The line T in the following figure is permanently connected to the ground.



Which of the following inputs (X1 X2 X3 X4) will detect the fault?

(a) 0000 (b) 0111

(c) 1111 (d) None of these

[2007 : 2 Marks]

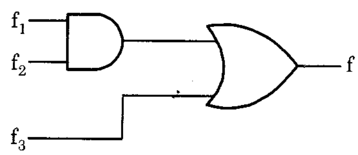
1.32 In the Karnaugh map shown below, X denoted a don't care term. What is the minimal form of the function represented by the Karnaugh map

cd \ ab	00	01	11	10
00	1	1		1
01	X			
11	X			
10	1	1		X

(a) $\bar{b}.\bar{d} + \bar{a}.\bar{d}$
(b) $\bar{a}.\bar{b} + \bar{b}.\bar{d} + \bar{a}.b.\bar{d}$
(c) $\bar{b}.\bar{d} + \bar{a}.b.\bar{d}$
(d) $\bar{a}.\bar{b} + \bar{b}.\bar{d} + \bar{a}.\bar{d}$

[2008 : 1 Mark]

1.33 Give F_1, F_3 and f in canonical sum of products form (in decimal) for the circuit



$f_1 = \Sigma m(4, 5, 6, 7, 8)$
$f_3 = \Sigma m(1, 6, 15)$
$f = \Sigma m(1, 6, 8, 15)$
Then f_2 is

(a) $\Sigma m(4, 6)$
(b) $\Sigma m(4, 8)$
(c) $\Sigma m(6, 8)$
(d) $\Sigma m(4, 6, 8)$

[2008 : 1 Mark]

1.34 A set of Boolean connectives is functionally complete if all Boolean functions can be synthesized using those. Which of the following sets of connectives is NOT functionally complete?

(a) EX-NOR
(b) implication, negation
(c) OR, negation
(d) NAND

[2008 : 1 Mark]

1.35 If P, Q, R are Boolean variables, then

$(P + \bar{Q})\,(P.\bar{Q} + P.R)\,(\bar{P}.\bar{R} + \bar{Q})$ simplifies to

(a) $P.\bar{Q}$
(b) $P.\bar{R}$
(c) $P.\bar{Q} + R$
(d) $P.\bar{R} + Q$

[2008 : 2 Marks]

1.36 What is the minimum number of gates required to implement the Boolean function (AB + C) if we have to use only 2-input NOR gates?

(a) 2
(b) 3
(c) 4
(d) 5

[2009 : 1 Mark]

1.37 The minterm expansion of

$f(P, Q, R) = PQ + Q\bar{R} + P\bar{R}$ is

(a) $m_2 + m_4 + m_6 + m_7$
(b) $m_0 + m_1 + m_3 + m_5$
(c) $m_0 + m_1 + m_6 + m_1$
(d) $m_2 + m_3 + m_4 + m_5$

[2010 : 1 Mark]

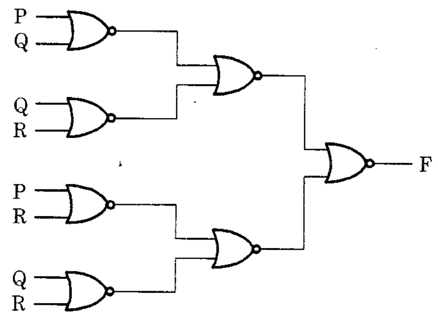
1.38 The simplified SOP (Sum of Product) form of the Boolean expression

$(P + \bar{Q} + \bar{R}) \cdot (P + \bar{Q} + R) \cdot (P + Q + \bar{R})$ is

(a) $(P + \bar{Q} + \bar{R})$
(b) $(P + \bar{Q} \cdot \bar{R})$
(c) $(\bar{P}.Q + R)$
(d) $(P \cdot Q + R)$

[2010 : 2 Marks]

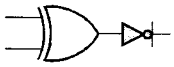
1.39 What is the boolean expression for the output f of the combinational logic circuit of NOR gates given below?

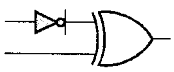


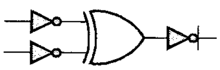
(a) $\overline{Q + R}$
(b) $\overline{P + Q}$
(c) $\overline{P + R}$
(d) $\overline{P + Q + R}$

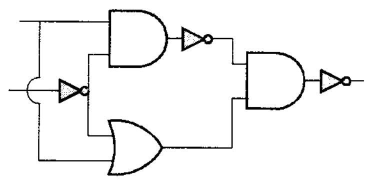
[2010 : 2 Marks]

1.40 Which one of the following circuits is NOT equivalent to a 2-input XNOR (exclusive NOR) gate?

(a) 

(b) 

(c) 

(d) 

[2011 : 1 Mark]

1.41 The decimal value 0.5 in IEEE single precision floating point representation has

(a) fraction bits of 000 ... 000 and exponent value of 0
(b) fraction bits of 000 ... 000 and exponent value of − 1
(c) fraction bits of 100 ... 000 and exponent value of 0
(d) no exact representation

[2012 : 1 Mark]

1.42 What is the minimal form of the Karnaugh map shown below? Assume that X denotes a don't care term.

cd \ ab	00	01	11	10
00	1	X	X	1
01	X			1
11				
10	1			X

(a) $\bar{b}\,\bar{d}$
(b) $\bar{b}\,\bar{d}+\bar{b}\,\bar{c}$
(c) $\bar{b}\,\bar{d}+a\,\bar{b}\,\bar{c}\,d$
(d) $\bar{b}\,\bar{d}+\bar{b}\,\bar{c}+\bar{c}\,\bar{d}$

[2012 : 2 Marks]

1.43 Which one of the following expression does NOT represent exclusive NOR of x and y?

(a) $xy + x'y'$
(b) $x \oplus y'$
(c) $x' \oplus y$
(d) $x' \oplus y'$

[2013 : 1 Mark]

1.44 Consider the following Boolean expression for F:

$$F(P, Q, R, S) = PQ+\bar{P}QR+\bar{P}Q\bar{R}S$$

The minimal sum-of-products form of F is

(a) $PQ + QR + QS$
(b) $P + Q + R + S$
(c) $\bar{P}+\bar{Q}+\bar{R}+\bar{S}$
(d) $\bar{P}R+\bar{P}\bar{R}S+P$

[2014 (Set-1) : 1 Mark]

1.45 The dual of a Boolean function $F(x_1, x_2, \ldots, x_n, +, \cdot, ')$, written as F^D, is the same expression as that of F with + and · swapped. F is said to be self-dual if $F = F^D$. The number of self-dual functions with n Boolean variables is

(a) 2^n
(b) 2^{n-1}
(c) 2^{2^n}
(d) $2^{2^{n-1}}$

[2014 (Set-2) : 1 Mark]

1.46 Consider the following minterm expression for F:

$$F(P,Q,R,S) = \sum 0, 2, 5, 7, 8, 10, 13, 15$$

The minterms 2, 7, 8 and 13 are 'do not care' terms. The minimal sum-of-products form for F is

(a) $Q\bar{S}+\bar{Q}S$
(b) $\bar{Q}\bar{S}+QS$
(c) $\bar{Q}\bar{R}\bar{S}+\bar{Q}R\bar{S}+Q\bar{R}S+QRS$
(d) $\bar{P}\bar{Q}\bar{S}+\bar{P}QS+PQS+P\bar{Q}\bar{S}$

[2014 (Set-3) : 1 Mark]

1.47 Let ⊕ denote the Exclusive OR (XOR) operation. Let '1' and '0' denote the binary constants.

Consider the following Boolean expression for F over two variables P and Q:

$$F(P, Q) = ((1 \oplus P) \oplus (P \oplus Q)) \oplus ((P \oplus Q) \oplus (Q \oplus 0))$$

The equivalent expression for F is

(a) $P + Q$
(b) $\overline{P+Q}$
(c) $P \oplus Q$
(d) $\overline{P \oplus Q}$

[2014 (Set-3) : 2 Marks]

1.48 Consider the operations

$$f(X,Y,Z) = X'YZ + XY' + Y'Z' \text{ and}$$
$$g(X,Y,Z) = X'YZ + X'YZ' + XY$$

Which one of the following is correct?

(a) Both {f} and {g} are functionally complete
(b) Only {f} is functionally complete
(c) Only {g} is functionally complete
(d) Neither {f} nor {g} is functionally complete

[2015 (Set-1) : 2 Marks]

1.49 The number of min-terms after minimizing the following Boolean expression is________.

$$[D' + AB' + A'C + AC'D + A'C'D]'$$

[2015 (Set-2) : 2 Marks]

1.50 The total number of prime implicants of the function $f(w,x,y,z) = \Sigma(0,2,4,5,6,10)$ is______.

[2015 (Set-3) : 2 Marks]

1.51 Given the function $F = P'+QR$, where F is a function in three Boolean variables P, Q and R and $P' = !P$, consider the following statements.

S1: $F = \Sigma\,(4, 5, 6)$
S2: $F = \Sigma\,(0, 1, 2, 3, 7)$
S3: $F = \Pi\,(4, 5, 6)$
S4: $F = \Pi\,(0, 1, 2, 3, 7)$

Which of the following is true?
(a) S1-False, S2-True, S3-True, S4-False
(b) S1-True, S2-False, S3-False, S4-True
(c) S1-False, S2-False, S3-True, S4-True
(d) S1-True, S2-True, S3-False, S4-False

[2015 (Set-3) : 2 Marks]

1.52 Consider the Boolean operator # with the following properties: $x\#0 = x, x\#1 = \bar{x}, x\#x = 0$ and $x\#\bar{x} = 1$. Then $x\#y$ is equivalent to

(a) $x\bar{y} + \bar{x}y$
(b) $x\bar{y} + \bar{x}\,\bar{y}$
(c) $\bar{x}y + xy$
(d) $xy + \bar{x}\,\bar{y}$

[2016 (Set-1) : 1 Mark]

1.53 Let $x_1 \oplus x_2 \oplus x_3 \oplus x_4 = 0$ where x_1, x_2, x_3, x_4 are Boolean variables, and $\oplus$ is the XOR operator. Which one of the following must always be TRUE?

(a) $x_1 x_2 x_3 x_4 = 0$
(b) $x_1 x_3 + x_2 = 0$
(c) $\bar{x}_1 \oplus \bar{x}_3 = \bar{x}_2 \oplus \bar{x}_4$
(d) $x_1 + x_2 + x_3 + x_4 = 0$

[2016 (Set-2) : 1 Mark]

Answers Logic Functions and Minimization

1.1	(b)	1.2	(c)	1.3	(b)	1.4	(c)	1.5	(a)	1.6	(b)	1.7	(a)	1.8	(c)	1.9	(c)
1.10	(d)	1.11	(d)	1.12	(b)	1.13	(b)	1.14	(a)	1.15	(c)	1.16	(c)	1.17	(d)	1.18	(a)
1.19	(a)	1.20	(a)	1.21	(c)	1.22	(c)	1.23	(c)	1.24	(b)	1.25	(c)	1.26	(a)	1.27	(c)
1.28	(d)	1.29	(d)	1.30	(b)	1.31	(d)	1.32	(a)	1.33	(c)	1.34	(a)	1.35	(a)	1.36	(b)
1.37	(a)	1.38	(b)	1.39	(a)	1.40	(d)	1.41	(b)	1.42	(b)	1.43	(d)	1.44	(a)	1.45	(d)
1.46	(b)	1.47	(d)	1.48	(b)	1.51	(a)	1.52	(a)	1.53	(c)						

Explanations Logic Functions and Minimization

1.1 (b)

$$z = x * y$$

Then value of $z * x$

$$z * x = (x * y) * x$$
$$= (\bar{x} + y) * x$$
$$= (\overline{\bar{x} + y}) + x$$
$$= x.\bar{y} + x$$

By absorption law $a + ab = a$

$$x.\bar{y} + x = x$$

Therefore

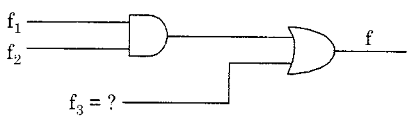
$$z * x = x$$

1.2 (c)

x \ yz	00	01	11	10
0	1	1	1	1
1		1	1	

Here there are two prime implicants, x' and z, both are also essential prime implicants.
xz is an implicant of f.

1.3 (b)



By the logic circuit, function f is given as

$$f = f_1 f_2 + f_3$$

$$f_1 f_2 = \Sigma(8, 9, 10)\Sigma(7, 8, 12, 13, 14, 15)$$

The common minterm in f_1 and f_2 is 8

Therefore,

$$f_1 f_2 = \Sigma(8)$$

For $f = \Sigma(8, 9)$, f_3 can be any one of the following

(i) $f_3 = \Sigma(8, 9)$

(ii) $f_3 = \Sigma 9$

Therefore option (b) is correct option.

1.4 (c)

Trying each option

(a) Given x = 0, y = 1, z = 0 and w = 0

- (i) x + y + z = 0 + 1 + 0 = 1
- (ii) xy = 0.1 = 0
- (iii) xz + w = 0.0 + 0 = 0

However given xz + w = 1

Therefore, option (a) is not the solution.

(b) Given x = 1, y = 1, z = 0 and w = 1

- (i) x + y + z = 1 + 1 + 0 = 1
- (ii) xy = 1.1 = 1

However given xy = 0

Therefore, option (b) is not the solution.

(c) Given x = 1, y = 0, z = 1 and w = 1

- (i) x + y + z = 1 + 0 + 1 = 1
- (ii) xy = 1.0 = 0
- (iii) xz + w = 1.1 + 1 = 1
- (iv) xy + z'w' = 1.0 + 0.0 = 0

(d) Given x = 1, y = 0, z = 0 and w = 0

- (i) x + y + z = 1 + 0 + 0 = 1
- (ii) xy = 1.0 = 0
- (iii) xz + w = 1.0 + 0 = 0

However given xz + w = 1

Therefore, option (d) is not the solution.

1.5 (a)

yz \ wx	00	01	11	10
00	0	x	0	x
01	x	1	x	1
11	0	x	1	0
10	0	1	x	0

Minimal SOP of the given k-map is

= xy + y'z

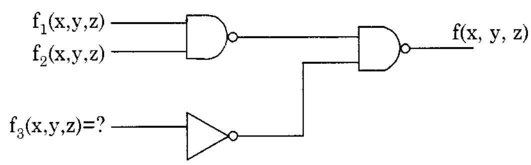
1.6 (b)

yz \ wx	00	01	11	10
00	0	1	1	0
01	x	0	0	1
11	x	0	0	1
10	0	1	1	x

Minimum SOP for f(w, x, y, z)

= xz' + zx'

1.7 (a)



$$f = \overline{\overline{f_1 \cdot f_2} \cdot \overline{f_3}} = f_1 \cdot f_2 + f_3 \qquad ...(1)$$

Given $f_1(x, y, z) = \Sigma m\ (0,1,3,5)$ and

$f_2(x, y, z) = \Sigma m(6, 5)$

$f_1 \cdot f_2 = \Sigma m\ (5)$ ...(2)

Common minterms of function f1 and f2 are minterms in $f_1 \cdot f_2$.

Hence f_3 can be $f_3(x, y, z) = \Sigma m\ (1, 4, 5)$

or $f_3 = (x, y, z) = \Sigma m\ (1, 4)$

From the given options answer is option (a).

1.8 (c)

= f(f(x + y, y), z)
= f((x + y)' + y, z)
= f(x'y' + y, z)
= f(x' + y, z)
= (x' + y)' + z = xy' + z

1.9 (c)

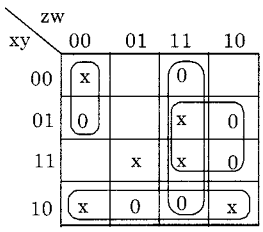
Find the sum-of-product form as follows

xy \ zw	00	01	11	10
00	x	1		1
01		1	x	
11	1	x	x	
10	x			x

= wy + w'y' + z'wx' + xyz'

Total literal count = 10 literals

Find product -of- sum form as follows



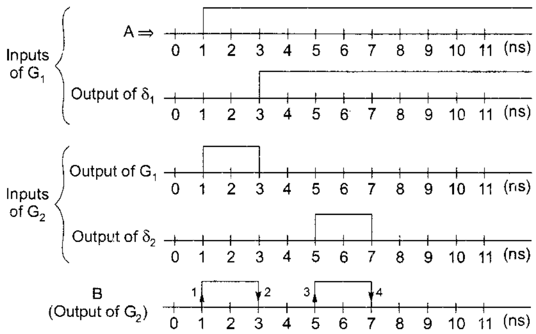
$f(z, w, x, y) = (y' + z')(w' + z')(z' + y)$
$(x + z + w)$

Total literal count = 9 literals

1.10 (d)

Consider left side EX-OR gate as G_1 and right side EX-OR gate as G_2.

1. To find number of transitions at B i.e. the output of gate G_2, it is required to identify the inputs of gate G_2.
2. To identify gate G_2 inputs it is required to find gate G_1 output waveform.
3. To find gate G_1 output waveform, it is required to identify δ_1 output waveform.



Total numbers of transitions at B during interval from 0 to 10ns are '4'.

Hence option (d).

1.11 (d)

$x'y' + xy + x'y = x'y' + x'y + xy$
$= x'(y' + y) + xy \quad [a + a' = 1]$
$= x' + xy \quad [a + a'b = a + b]$
$= x' + y$

1.12 (b)

Any 2-input Exclusive OR function can be implemented with the 4 NAND gates.

1st NAND gate:
Input: A, B
Output: (A B)′

2nd NAND gate:
Input: (AB)′, A
Output: A′ + AB

3rd NAND gate:
Input: (AB)′, B
Output: B′ + AB

4th NAND gate:
Input: A′ +AB, B′ + AB
Output: A′B + AB′ (Exclusive OR function)

1.13 (b)

K-map

	A′B′	A′B	AB	AB′
C′	0	0	1	1
C	1	1	0	1

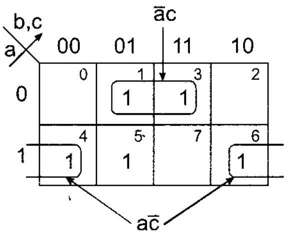
Equivalent expression of above K-map is A′C + AC′ + AB

1.14 (a)

$f(a, b, c) = \bar{a}c + a\bar{c} + \bar{b}c$

$= \Sigma m\ (1, 3, 4, 5, 6)$

The K-map of the function is



To cover the minterm m_3 the prime implicant $\bar{a}c$ is an essential prime implicant. To cover the minterm m_6 the prime implicant $a\bar{c}$ is an essential prime implicant. Hence answer is option (a).

1.15 (c)

$(A \oplus B) \oplus C$

$[A\bar{B} + B\bar{A}] \oplus C$

$[A\bar{B} + B\bar{A}]\bar{C} + [\overline{A\bar{B} + \bar{A}B}]C$

$A\bar{B}\bar{C} + \bar{A}B\bar{C} + [(\bar{A} + B)\cdot(A + \bar{B})]C$

$A\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}\bar{B}C + ABC$

$= ABC + \bar{A}[B \oplus C] + \bar{B}(A \oplus C)$

1.16 (c)

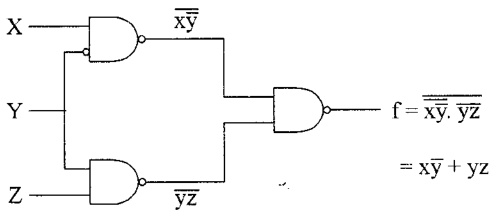
Bulb is On when both switch s1 and s2 are in same state, either off or on.

S1	S2	Bulb
0	0	On
0	1	Off
1	0	Off
1	1	On

Above truth table derives Ex-NOR operation.

1.17 (d)

The given circuit is



$$\begin{aligned} f &= x\overline{y} + yz \\ &= x\overline{y}\,(z + \overline{z}) + yz\,(x + \overline{x}) \quad [\because x + \overline{x} = 1] \\ &= x\overline{y}z + x\overline{y}\overline{z} + xyz + \overline{x}yz \end{aligned}$$

K-map:

x \ yz	00	01	11	10
0			1	
1	1	1	1	

$f = (xy' + yz)$

Therefore, none of x, y, z is redundant.

1.18 (a)

$f(A,B,C,D) = \Sigma(1,4,5,9,11,12)$

The K-map for f is

AB \ CD	00	01	11	10
00		1		
01	1	1		
11	1			
10		1	1	

$B\overline{C}\overline{D} + \overline{A}\overline{C}D + A\overline{B}D$

1.19 (a)

$$\begin{array}{llll} & c_2 & c_1 & c_0 \leftarrow \text{Input carry} \\ a = & a_2 & a_1 & a_0 \\ a = & b_2 & b_1 & b_0 \\ \hline c_3 & s_2 & s_1 & s_0 \leftarrow \text{Final output carry} \end{array}$$

$c_1 = a_0b_0 + b_0c_0 + a_0c_0 = a_0b_0$ ...(1)

(where input carry $c_0 = 0$)

$$\begin{aligned} c_2 &= a_1b_1 + b_1c_1 + a_1c_1 \\ &= a_1b_1 + a_0b_1b_0 + a_1a_0b_0 \quad \text{...(2)} \\ c_3 &= a_2b_2 + b_2c_2 + a_2c_2 \\ &= a_2b_2 + b_2[a_1b_1 + a_0b_1b_0 + a_1a_0b_0] \\ &\quad + a_2[a_1b_1 + a_0b_1b_0 + a_1a_0b_0] \\ &= a_2b_2 + a_1b_2b_1 + a_0b_2b_1b_0 + a_0b_2b_1b_0 + \\ &\quad a_1a_0b_2b_0 + a_2a_1b_1 + a_2a_0b_1b_0 + a_2a_1a_0b_0 \end{aligned}$$

1.20 (a)

$f(w, x, y, z) = \Sigma(5, 7, 11, 12, 13, 15)$

Draw the K-map

wx \ yz	00	01	11	10
00				
01		1	1	
11	1	1	1	
10			1	

$\overline{w}xz \rightarrow 0101 \quad (5, 13)$

$wx\overline{y} \rightarrow 1100 \quad (12)$

$x\overline{y}z \rightarrow 0101 \quad (5)$

$xyz \rightarrow 0111 \quad (7)$

$wyz \rightarrow 1011 \quad (11)$

1.21 (c)

$B = b_{n-1}\,b_{n-2}\ldots b_0$

$A = a_{n-1}a_{n-2}\ldots a_0$

$S = c_{n-1}c_{n-2}\ldots c_0$

The over flow condition V

$V = b'_{n-1}\,a'_{n-1}\,c_{n-2} + b_{n-1}a_{n-1}c'_{n-2}$

c_{n-1} is c_{out}, $V = c_{out} \oplus c_{n-1}$

So the over flow condition is

$c_{out} \oplus c_{n-1}$

1.22 (c)

For a binary number $b_3\,b_2\,b_1\,b_0$ the corresponding gray code $h_3\,h_2\,h_1\,h_0$ is given is Table-1.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
b_3	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
b_2	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
b_1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
b_0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
h_3	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
h_2	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0
h_1	0	0	1	1	1	1	0	0	0	0	1	1	1	1	0	0
h_0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0

Table-1

For a binary number 'n' the gray code is $h_3\,h_2\,h_1\,h_0$, the required output is $g_3\,g_2\,g_1\,g_0$ (it is corresponding gray code of binary number n+1), is given in Table-2.

	0	1	3	2	6	7	5	4	12	13	15	14	10	11	9	8
h_3	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
h_2	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0
h_1	0	0	1	1	1	1	0	0	0	0	1	1	1	1	0	0
h_0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0
g_3	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0
g_2	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0	0
g_1	0	1	1	1	1	0	0	0	0	1	1	1	1	0	0	0
g_0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0

Table-2

From the table-2, read the equations for g_3, g_2, g_1 and g_0.

The answer g_2 will match.

g_2 (h_3, h_2, h_1, h_0)

$= \Sigma m(2, 4, 5, 6, 7, 12, 13, 15)$

1.23 (c)

Let the binary constant a_i be the value of the function $f(x_1, x_2, ..., x_n)$ for the combination of variable whose decimal code is i. Then every switching function can be expressed in the form.

$f(x_1,x_2,...,x_n) = a_0x'_1...x'_n + a_1x'_1x'_2...x_n + a_rx_1x_2...x_n$.

A factor a_2 is either 0 or 1 if the corresponding minterm is contained in canonical from of the function. Then there are $r = 2^n$ coefficients each of which can have values either o or 1 hence there are 2^{2^n} possible assignments.

OR

Let us consider n = 2, A and B are two boolean variables. The possible combinations are four, they are

	0	1	2	3
A	0	0	1	1
B	0	1	0	1

1. Taking only one combination as a minterms the number of boolean functions are $4C_1$.
2. Taking any two combinations as minterms the number of boolean functions are $4C_2$.
3. Taking any three combinations are $4C_3$.
4. Taking all four combinations as minterms, the number of boolean functions are $4C_4$.
5. Taking no combinations as minterms, the number of boolean functions are $4C_0$.

Total number of functions on '2' boolean variables are

$4C_0 + 4C_1 + 4C_2 + 4C_3 + 4C_4$

$= 1 + 4 + 6 + 4 + 1 = 16 = 2^{2^2}$

Similar on 'n' boolean variables total number of boolean functions are $2^nC_0 + 2^nC_1 + 2^nC_2 + ... + 2^nC_{2^n}$ is equal to 2^{2^n}.

1.24 (b)

Given $f(w, x, y, z) = \Sigma(1,3,4,6,9,11,12,14)$

Corresponding K-map for f is

wx \ yz	00	01	11	10
00		1	1	
01	1			1
11	1			1
10		1	1	

$$\begin{aligned} f(w, x, y, z) &= w'xy'z' + wxy'z' + w'xyz' \\ &\quad + wxyz' + w'x'y'z \\ &\quad + w'x'yz + wx'y'z + wx'yz \\ &= xy'z'(w' + w) + xyz'(w' + w) \\ &\quad + w'x'z(y' + y) + wx'z(y' + y) \\ &= xy'z' + xyz' + w'x'z + wx'z \\ &= xz'(y' + y) + x'z(w' + w) \\ &= xz' + x'z \end{aligned}$$

So f(w, x, y, z) is independent of two variables w and y.

1.25 (c)

$(a \cdot b)c + (\bar{a} \cdot c)d + (bc)d + ad$

Substitude AND by NAND.

$= \overline{\overline{ab}\,c} + \overline{\overline{\bar{a}c}\,d} + \overline{\overline{bc}\,d} + \overline{ad}$

$= ab + \bar{c} + \bar{a}c + \bar{d} + bc + \bar{d} + \bar{a} + \bar{d}$

$= b + \bar{c} + \bar{d} + \bar{a} = \bar{a} + b + \bar{c} + \bar{d}$

Note: We used the following relues

$$\begin{aligned} \bar{a} + \bar{a}c &= \bar{a} \\ \bar{a} + ab &= \bar{a} + b \\ \bar{c} + bc &= \bar{c} + b \end{aligned}$$

1.26 (a)

$$f = a\bar{d} + \bar{a}\bar{c} + b\bar{c}d$$

$\bar{a}\bar{c}$

ab \ cd	00	01	11	10
00	1	1		
01	1	1		
11	1	1		1
10	1			1

$b\bar{c}d$

$a\bar{d}$

1.27 (c)

f = ad′ + a′+ c′ + bc′d

	c'd'	c'd	cd	cd'
a'b'	1	1	1	1
a'b	1	1	1	1
ab	1	1		1
ab'	1			1

In option (c):

$\bar{a}\bar{c} + a\bar{d} + ab\bar{c} + \bar{c}d$ expression, $\bar{c}d$ makes $a\bar{b}\bar{c}d$ as 1, which is not equivalent to given k-map.

1.28 (d)

f(w, x, y, z) = Σ(0, 4, 5, 7, 8, 9, 13, 15)

The K-map for f is

wx \ yz	00	01	11	10
00	1			
01	1	1	1	
11		1	1	
10	1	1		

w′xy′z + wxy′z + w′xyz + wxyz

= xy′z(w′ + w) + xyz (w′ + w)

= xy′z + xyz

= xz (y′ + y)

= xz

w′x′y′z′ + w′xy′z′

= w′y′z′ (x′ + x)

= w′y′z′

wx′y′z′ + wx′y′z

wx′y′(z′+ z)

wx′y′

∴ Minimal expression is

= w′y′z′ + wx′y′ +xz

The set of all minterms are = {xz, w′y′z′, wx′y′, x′y′z′}

consider a simple method let w = 1, x = 1, y = 1, z = 1 then the value of f is 1. Consider each statement

(a) x′y′z′ + w′xy′ + wy′z + xz = 0·0·0 +0·1·0 +1·0·1 + 1·1 = 1

(b) w′y′z′ + wx′y′ + xz = 0·0·0 +1·0·0 + 1·1 = 1

(c) w′y′z′ + wx′y′ + xyz + xy′z = 0·0·0 + 1·0·0 + 1·1·1 + 1·0·1 = 1

(d) x′y′z′ + wx′y′ + w′y = 0·0·0 + 1·0·0 + 0·1 = 0

So statement (d) is false because w =1, x =1, y =1, z = 1 the value of f is 0. (d) not contain the essential minterms.

1.29 (d)

X·Y = XY + X′Y′ ...(i)

Z = X·Z

Z = XZ + X′Z′ from eq. (i)

P : X = Y · Z

= YZ + Y′Z′ by eq. (i)

= Y(XZ + X′Z′) + Y′Z′

= XYZ + X′YZ′ + Y′Z

So P is valid

Q : Y = X · Z

Y = XZ + X′Z′ by eq. (i)

Y = XZ + X′Z′

So Q is a valid expression

R : X·Y·Z = 1 is also valid

if we will take the truth table for P, Q, R, then all are valid formulas.

1.30 (b)

The number of AND gates in carry generator circuit in 'n' bit adder = $\frac{n(n+1)}{2}$

If n = 4 ⇒ $\frac{4(5)}{2} = 10$.

The number of OR gates in carry generator circuit in 'n' bit adder = n.

If n = 4 ⇒ 4

1.31 (d)

None of the given options detect the fault.

1.32 (a)

The K-Map is

cd \ ab	00	01	11	10
00	1	1		1
01	x			
11	x			
10	1	1		x

a′d′ + b′d′

1.33 (c)

f_1 and f_2 are connected with AND gates so when f_2 = Σm(6,8) then f is true for Σm(1,6,8,15)

1.34 (a)

EX-NOR is not functionally complete {Implication, Negation} is functionally complete {OR, Negation} is functionally complete NAND is functionally complete.

1.35 (a)

$(P + Q')\ (PQ' + PR)\ (P'R' + Q')$

$= (PQ' + PR + PQ' + PRQ')\ (P'R' + Q')$

$= (PQ' + PR + PRQ')\ (P'R' + Q')$

$= (PQ' + PR(1+Q'))\ (P'R' + Q')$

$= (PQ' + PR)\ (P'R' + Q')$

$= PQ' + PRQ'$

$= PQ'(1 + R) = PQ'$

OR

$\underset{(1)}{(P+\overline{Q})}\quad \underset{(2)}{(P\overline{Q}+PR)}\quad \underset{(3)}{(\overline{P}\,\overline{R}+\overline{Q})}$

Perform AND operation between 1st and 3rd term

$(P+\overline{Q})(\overline{P}\,\overline{R}+\overline{Q}) = P\cancel{\overline{P}\overline{R}}+P\overline{Q}+\overline{P}\,\overline{R}\,\overline{Q}+\overline{Q}$

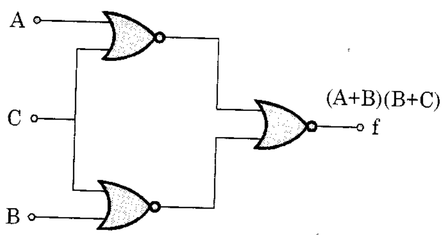
$= \overline{Q}\underbrace{[1+P+\overline{P}\,\overline{R}]}_{1} = \overline{Q}$

The given expression is

$(P\overline{Q}+PR)\overline{Q} = P\overline{Q}+P\overline{Q}R = P\overline{Q}$

1.36 (b)

$AB + C = (A+C)(B+C)$ using distributive property. The modified expression is in POS from. Hence 2 level NOR-NOR circuit is required using 3 NOR gates.



1.37 (a)

$f(P, Q, R) = PQ + Q\overline{R} + P\overline{R}$

Make this as a standard form

$= PQ(R+\overline{R}) + (P+\overline{P})Q\overline{R} + P(Q+\overline{Q})\overline{R}$

$= PQR + PQ\overline{R} + PQ\overline{R} + \overline{P}Q\overline{R} + PQ\overline{R} + P\overline{Q}\overline{R}$

$= PQR + PQ\overline{R} + \overline{P}Q\overline{R} + P\overline{Q}\overline{R}\qquad [\because A + A = A]$

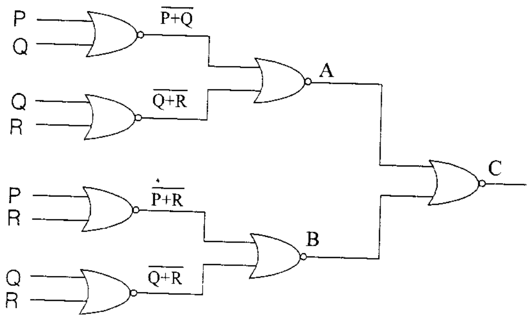
$= m_7 + m_6 + m_2 + m_4$

1.38 (b)

$(P+\overline{Q}+R)(P+\overline{Q}+R)(P+Q+\overline{R})$

Expanding the brackets and simplifying by boolean algebra we get, $P+\overline{Q}\,\overline{R}$.

1.39 (a)



So we take $A = \overline{\overline{P+Q}+\overline{Q+R}}$

$= \overline{\overline{\overline{P}}\cdot\overline{\overline{Q}}+\overline{\overline{Q}}\cdot\overline{\overline{R}}} = \overline{\overline{\overline{P}}\cdot\overline{\overline{Q}}}+\overline{\overline{\overline{Q}}\cdot\overline{\overline{R}}}$

$= \overline{\overline{(P+Q)(Q+R)}} = (P + Q)\ (Q + R)$

$B = \overline{\overline{P+R}+\overline{Q+R}}$

$= \overline{\overline{\overline{P}}\cdot\overline{\overline{R}}+\overline{\overline{Q}}\cdot\overline{\overline{R}}} = \overline{\overline{\overline{P}}\cdot\overline{\overline{R}}}\cdot\overline{\overline{\overline{Q}}\cdot\overline{\overline{R}}}$

$= \overline{\overline{(P+R)(Q+R)}} = (P + R)\ (Q + R)$

So $C = \overline{A + B}$

$= \overline{(P+Q)(Q+R) + (P+R)(Q+R)}$

$= \overline{PQ + QQ + PR + QR + PQ + RQ + PR + RR}$

$= \overline{PQ + Q + PR + QR + PQ + RQ + PR + R}$

$\therefore AA = A$
$1 + A = A$

$= \overline{Q + PR + QR + PQ + R}$

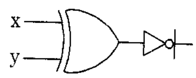
$= \overline{Q(1 + R) + R(1 + P) + PQ}$

$= \overline{Q + R + PQ}$

$= \overline{Q(1 + P) + R}$

$= \overline{Q + R}$

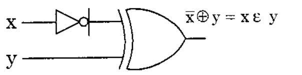
1.40 (d)

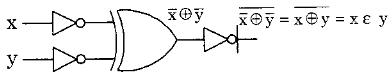
(a) 

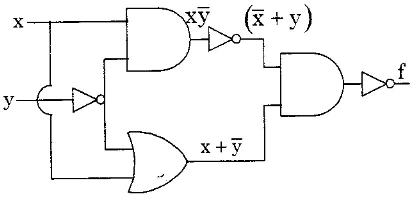
$\overline{x \oplus y} = \overline{\overline{x}y + \overline{y}x} = (x+\overline{y})(y+\overline{x})$

$= xy + x\overline{x} + \overline{y}y + \overline{y}\,\overline{x}$

$$= xy + \bar{x}\bar{y}$$
$$= x \odot y$$

(b) 

(c) 

(d) 

$$f = \overline{(\bar{x}+y)(x+\bar{y})}$$
$$= \overline{(\bar{x}+y)} + \overline{(x+\bar{y})}$$
$$= x.\bar{y} + \bar{x}y = x \oplus y$$
$$\neq x \odot y$$

$\therefore$ Option (d) is not XNOR.

1.41 (b)

$$0.5 = 0.1000 \times 2^0$$
$$= 1.0000 \times 2^{-1}$$

$\therefore$ exponent = – 1 and mantissa = 000000

$\therefore$ (b) is correct.

1.42 (b)

Two quads are possible here. So $\bar{b}\,\bar{d} + \bar{b}\,\bar{c}$ is the right answer.

cd \ ab	00	01	11	10
00	1	×	×	1
01	×			1
11				
10	1			×

1.43 (d)

Exclusive NOR of x and y

$$= \overline{(x \oplus y)}$$

$$\overline{(x \oplus y)} = (xy' + x'y)'$$
$$= ((x' + y)\,(x + y'))$$
$$= x'y' + xy \qquad \text{...(i)}$$

Option (a) : is same as (i)

Options (b) : $x \oplus y' = x'y' + xy$ same as (i)

Option (c) : $x' \oplus y = xy + x'y'$ same as (i)

Option (d) $x \oplus y = x'y + xy'$ not same as (i)

Option (d) is correct.

1.44 (a)

$$F = PQ + \bar{P}QR + \bar{P}Q\bar{R}S$$
$$F = PQ + QR + QS$$

PQ \ RS	00	01	11	10
00				
01		1	1	1
11	1	1	1	1
10				

QS, QR, PQ

F = PQ + QR + QS

1.45 (d)

n = 3 (3 boolean variables)

	A	B	C
0	0	0	0
1	0	0	1
2	0	1	0
3	0	1	1
4	1	0	0
5	1	0	1
6	1	1	0
7	1	1	1

$\left\{\begin{array}{l} 0 \rightarrow \bar{x}\,\bar{y}\,\bar{z} \\ 7 \rightarrow x\,y\,z \end{array}\right\}$ (0, 7) mutual exclusive terms

$\left\{\begin{array}{l} 1 \rightarrow \bar{x}\,\bar{y}\,z \\ 6 \rightarrow x\,y\,\bar{z} \end{array}\right\}$ (0, 6) mutual exclusive terms

$\left\{\begin{array}{l} 2 \rightarrow \bar{x}\,y\,\bar{z} \\ 5 \rightarrow x\,\bar{y}\,z \end{array}\right\}$ (2, 5) mutual exclusive terms

$\left\{\begin{array}{l} 3 \rightarrow \bar{x}\,y\,z \\ 4 \rightarrow x\,\bar{y}\,\bar{z} \end{array}\right\}$ (3, 4) mutual exclusive terms

In a self function

Number of minterms = Number of maxterms (i.e., the function must be neutral function)

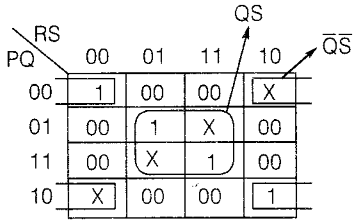
(0, 7), (1, 6), (2, 5), (3, 4) from these each mutual exclusional pair, only one combination must present to make the function is to be salf dual.

Hence (0, 7) (1, 6) (2, 5) (3, 4)

Posibilities: $2 \times 2 \times 2 \times 2 \times = 16 = 2^{2^{3-1}}$

similarly with n boolean variables.
$(i, j)\ (k, l)\ ()\ldots() = 2^{n-1}$ number of terms
Hence $2 \times 2 \times 2 \times \ldots \times 2 = 2^{2^{n-1}}$
Answer = $2^{2^{n-1}}$.

1.46 (b)



Minimal SOP form = $QS + \overline{Q}\overline{S}$

1.47 (d)

F(P, Q)

$= ((1 \oplus P) \oplus (P \oplus Q)) \oplus ((P \oplus Q) \oplus (Q \oplus 0))$

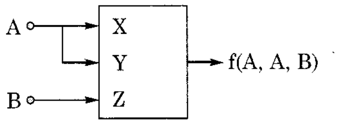
$= \left(\overline{P} \oplus (P\overline{Q} + \overline{P}Q)\right) \oplus (P \oplus Q) \oplus (Q \oplus 0))$

$= \left[\overline{P}\left(PQ + \overline{P}\overline{Q}\right) + \left(P\overline{Q} + \overline{P}Q\right)\right] \oplus$
$\left[\left(PQ + \overline{P}\overline{Q}\right) + \left(P\overline{Q} + \overline{P}Q\right)\overline{Q}\right]$

$= PQ + \overline{P}\overline{Q} = \overline{P \oplus Q}$

1.48 (b)

f(X,Y,Z) = X′ Y Z + X Y′ +Y′ Z′



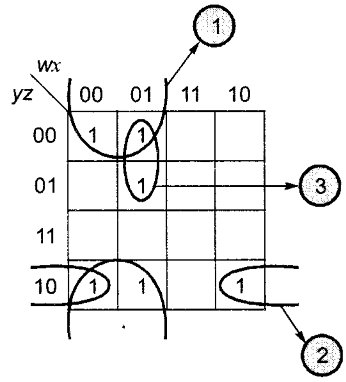
f(A, A, B) = A′AB + AA′ + A′B′ = A′B′ = (A + B)′ it is NOR operation.
Hence it is functionally complete.
Similarly if we try on g(x, y, z) it is possible to identify it is not functionally complete.

1.49 Sol.

[D′ + AB′ + A′C + AC′D + A′C′D]′
= [D′ + AC′D + AB′ + A′C + A′C′D]′
= [D′ + AC′ + AB′ + A′ [C + C′D]]′
= [D′ + AC′ + AB′ + A′ [C + D]]′
= [D′ + AC′ + AB′ + A′C + A′D]′ (∵ D′ + A′D = D′ + A′)
= [D′ + A′ + AC′ + AB′ + A′C]′
(∵ A′ + A′C = A′)
(∵ A′ + AC′ + AB′ = A′+A(C′+B′) = A′+C′ + B′)
= [D′ + A′ + C′ + B′]′
= ABCD

1.50 Sol.

$f(w, x, y, z) = \Sigma(0, 2, 4, 5, 6, 10)$



∴ 3 prime implicants.

1.51 (a)

$F = P' + QR = \Sigma(0, 1, 2, 3, 7) = \Pi(4, 5, 6)$
S1: $F = \Sigma(4, 5, 6)$ is false
S2: $F = \Sigma(0, 1, 2, 3, 7)$ is true
S3: $F = \Pi(4, 5, 6)$ is true
S4: $F = \Pi(0, 1, 2, 3, 7)$ is false

P \ QR	00	01	11	10
0	1^{0}	1^{1}	1^{3}	1^{2}
1	0^{4}	1^{5}	0^{7}	0^{6}

$\Rightarrow \quad F = \Sigma(0, 1, 2, 3, 7) = \pi(4, 5, 6)$

1.52 (a)

x	y	$x \# y$
0	0	0
0	1	1
1	0	1
1	1	0

$x \# 0 = x$ ...1

$x \# 1 = \overline{x}$ ...2

$x \# x = 0$ ...3

$x \# \overline{x} = 1$ ...4

$x \# y = x\overline{y} + \overline{x}y$

1.53 (c)

It is given $x_1 \oplus x_2 \oplus x_3 \oplus x_4 = 0$

	x_1	x_2	x_3	x_4	$x_1 \oplus x_2 \oplus x_3 \oplus x_4$
0	0	0	0	0	0 ←
1	0	0	0	1	1
2	0	0	1	0	1
3	0	0	1	1	0 ←
4	0	1	0	0	1
5	0	1	0	1	0 ←
6	0	1	1	0	0 ←
7	0	1	1	1	1
8	1	0	0	0	1
9	1	0	0	1	0 ←
10	1	0	1	0	0 ←
11	1	0	1	1	1
12	1	1	0	0	0 ←
13	1	1	0	1	1
14	1	1	1	0	1
15	1	1	1	1	0 ←

For the minterms

$m_0, m_3, m_5, m_6, m_9, m_{10}, m_{12}$ and m_{15}

$x_1 \oplus x_2 \oplus x_3 \oplus x_4 = 0$

[i.e. whenever even number of variables are 1, $x_1 \oplus x_2 \oplus x_3 \oplus x_4 = 0$]

Option (a) fails whenever

$$x_1 = x_2 = x_3 = x_4 = 1$$

Option (b) fails whenever

x_1, x_2, x_3 and x_4 are 0 1 0 0 [and for few more combinations]

Options (d) fails whenver

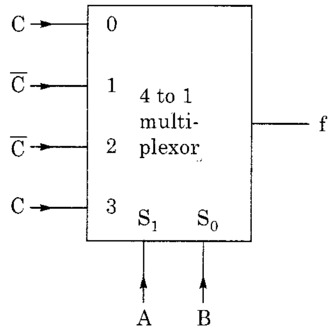
x_1, x_2, x_3 and x_4 are 0 0 1 1 [and for few more combinations]

Options (c) satisfies for every combination.

Hence option (c) is correct.

2 Combinational Circuits

2.1 Consider the circuit in figure shown. f implements



(a) $\bar{A}\bar{B}C + \bar{A}B\bar{C} + ABC$ (b) $A + B + C$

(c) $A \oplus B \oplus C$ (d) $AB + BC + CA$

[1996 : 2 Marks]

2.2 What is the equivalent Boolean expression in product-of-sums form for the Karnaugh map given in figure

CD \ AB	00	01	11	10
00		1	1	
01	1			1
11	1			1
10		1	1	

(a) $B\bar{D} + \bar{B}D$

(b) $(B + \bar{C} + D)(\bar{B} + C + \bar{D})$

(c) $(B + D)(\bar{B} + \bar{D})$

(d) $(B + \bar{D})(\bar{B} + D)$

[1996 : 2 Marks]

2.3 Which of the following functions implements the Karnaugh map shown below?

AB\CD	00	01	11	10
00	0	0	1	0
01	x	x	1	x
11	0	1	1	0
10	0	1	1	0

(a) $\bar{A}B + CD$

(b) $D(C + A)$

(c) $AD + \bar{A}B$

(d) $(C + D)(\bar{C} + D)(A + B)$

[1999 : 1 Mark]

2.4 Which of the following sets of component(s) is/are sufficient to implement any arbitrary Boolean function?

(a) XOR gates, NOT gates

(b) 2 to 1 multiplexors

(c) AND gates, XOR gates

(d) Three-input gates that output (A.B) + C for the inputs A. B and C.

[1999 : 2 Marks]

2.5 Which function does NOT implement the Karnaugh map given below?

wz →	00	01	11	10
xy ↓				
00	0	×	0	0
01	0	×	1	1
11	1	1	1	1
10	0	×	0	0

(a) $(w + x)y$

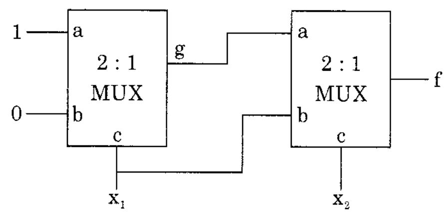
(b) $xy + yw$

(c) $(w + x)(w + y)(x + y)$

(d) None of the above

[2000 : 2 Marks]

2.6 Consider the circuit shown below. The output of a 2 : 1 Mux is given by the function (ac' + bc).



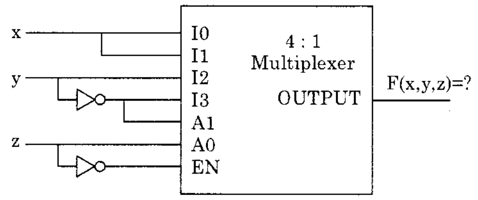
Which of the following is true?

(a) $f = x_1' + x_2$ (b) $f = x_1 x_2 + x_1 x_2'$

(c) $f = x_1 x_2 + x_1' x_2'$ (d) $f = x_1 + x_2$

[2001 : 2 Marks]

2.7 Consider the following multiplexer where I0, I1, I2, I3 are four data input lines selected by two address line combinations A1A0 = 00, 01, 10, 11 respectively and f is the output of the multiplexer, EN is the Enable input.

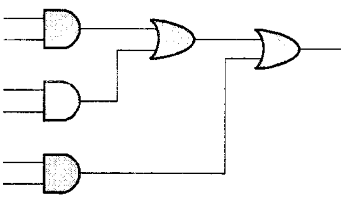


The function f(x, y, z) implemented by the above circuit is

(a) xyz′ (b) xy + z
(c) x + y (d) None of these

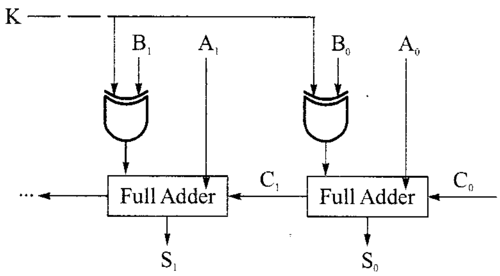
[2002 : 2 Marks]

2.8 Transform the following logic circuit (without expressing its switching function) into an equivalent logic circuit that employs only 6 NAND gates each with 2-inputs.



[2002 : 2 Marks]

2.9 Consider the ALU shown below



If the operands are in 2's complement representation, which of the following operations can be performed by suitably setting the control lines K and C_0 only (+ and – denote addition and subtraction respectively)?

(a) A + B, and A – B, but not A + 1
(b) A + B, and A + 1, but not A – B
(c) A + B, but not A – B, or A + 1
(d) A + B, and A – B, and A + 1

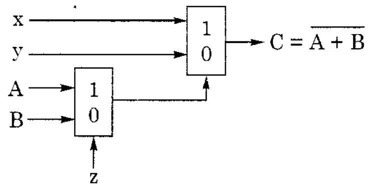
[2003 : 2 Marks]

2.10 A circuit outputs a digit in the form of 4 bits. 0 is represented by 0000, 1 by 0001, ..., 9 by 1001. A combinational circuit is to be designed which takes these 4 bits as input and outputs 1 if the digit ≥ 5, and 0 otherwise. If only AND, OR and NOT gates may be used, what is the minimum number of gates required?

(a) 2 (b) 3
(c) 4 (d) 5

[2004 : 2 Marks]

2.11 The circuit shown below implements a 2-input NOR gate using two 2 : 4 MUX (control signal 1 selects the upper input). What are the values of signals x, y and z?



(a) 1, 0, B (b) 1, 0, A
(c) 0, 1, B (d) 0, 1, A

[2005 : 2 Marks]

2.12 The boolean function for a combinational circuit with four inputs is represented by the following Karnaugh map.

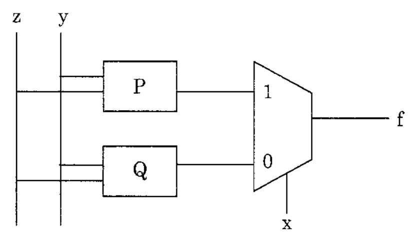
RS \ PQ	00	01	11	10
00	1	0	0	1
01	0	0	1	1
11	1	1	1	0
10	1	0	0	1

Which of the product terms given below is an essential prime implicant of the function?

(a) QRS (b) PQS
(c) PQ′S′ (d) Q′S′

[2006 : 2 Marks]

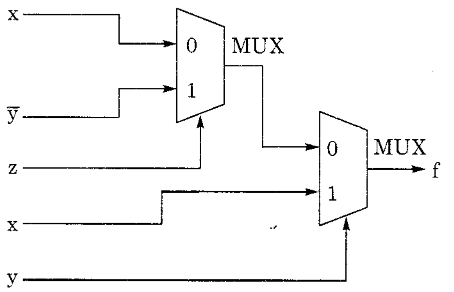
2.13 The majority function is a Boolean function f(x, y, z) that takes the value 1 whenever a majority of the variables x, y, z and 1. In the circuit diagram for the majority function shown below, the logic gates for the boxes labeled P and Q are, respectively,



(a) XOR, AND (b) XOR, XOR
(c) OR, OR (d) OR, AND

[2006 : 2 Marks]

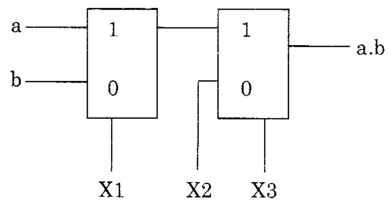
2.14 Consider the circuit below. Which one of the following options correctly represents f(x, y, z)?



(a) $x\bar{z} + xy + \bar{y}z$ (b) $x\bar{z} + xy + y\bar{z}$
(c) $xz + xy + \overline{yz}$ (d) $xz + x\bar{y} + \bar{y}z$

[2006 : 2 Marks]

2.15 The following circuit implements a two-input AND gate using two 2-1 multiplexers.



What are the values of X1, X2, X3?

(a) X1 = b, X2 = 0, X3 = a
(b) X1 = b, X2 = 1, X3 = b
(c) X1 = a, X2 = b, X3 = 1
(d) X1 = a, X2 = 0, X3 = b

[2007 : 1 Mark]

2.16 How many 3-to-8 line decoders with an enable input are needed to construct a 6-to-64 line decoder without using any other logic gates?

(a) 7 (b) 8
(c) 9 (d) 10

[2007 : 1 Mark]

2.17 Suppose only one multiplexer and one inverter are allowed to be used to implement any Boolean function of n variables. What is the minimum size of the multiplexer needed?

(a) 2^n line to 1 line (b) 2^{n+1} line to 1 line
(c) 2^{n-1} line to 1 line (d) 2^{n-2} line to 1 line

[2007 : 2 Marks]

2.18 Consider the following Boolean function of four variables:

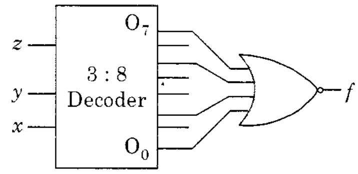
$f(A,B,C,D) = \Sigma(2, 3, 6, 7, 8, 9, 10, 11, 12, 13)$

The function is

(a) independent of one variable
(b) independent of two variables
(c) independent of three variable
(d) dependent on all the variables

[2008 : 1 Mark]

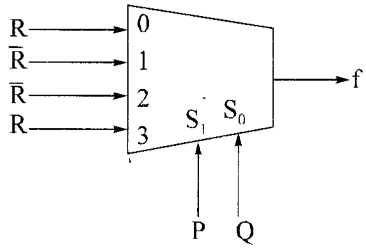
2.19 What Boolean function does the circuit below realize?



(a) $xz + \bar{x}\bar{z}$ (b) $x\bar{z} + \bar{x}z$
(c) $\bar{x}\bar{y} + yz$ (d) $xy + \bar{y}\bar{z}$

[2008 : 1 Mark]

2.20 The Boolean expression for the output f of the multiplexer shown below is



(a) $\overline{P \oplus Q \oplus R}$ (b) $P \oplus Q \oplus R$
(c) $P + Q + R$ (d) $\overline{P + Q + R}$

[2010 : 1 Mark]

2.21 In the following truth table, V = 1 if and only if the input is valid.

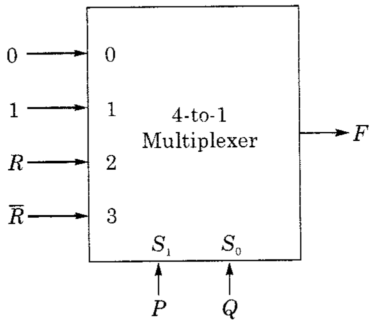
Inputs				Outputs		
D_0	D_1	D_2	D_3	X_0	X_1	V
0	0	0	0	x	x	0
1	0	0	0	0	0	1
x	1	0	0	0	1	1
x	x	1	0	1	0	1
x	x	x	1	1	1	1

What function does the truth table represent?

(a) Priority encoder (b) Decoder
(c) Multiplexer (d) Demultiplexer

[2013 : 1 Mark]

2.22 Consider the 4 to 1 multiplexer with two select lines S_1 and S_0 given below.



The minimal sum-of-products form of the Boolean expression for the output F of the multiplexer is

(a) $\bar{P}Q + Q\bar{R} + P\bar{Q}R$

(b) $\bar{P}Q + \bar{P}Q\bar{R} + PQ\bar{R} + P\bar{Q}R$

(c) $\bar{P}QR + \bar{P}Q\bar{R} + Q\bar{R} + P\bar{Q}R$

(d) $PQ\bar{R}$

[2014 (Set-1) : 2 Marks]

2.23 Consider the following combinational function block involving four Boolean variables x, y, a, b where x, a, b are inputs and y is the output.

$f(x, y, a, b)$
{
  if (x is 1) $y = a$;
  else $y = b$;
}

Which one of the following digital logic blocks is the most suitable for implementing this function?

(a) Full adder
(b) Priority encoder
(c) Multiplexor
(d) Flip-flop

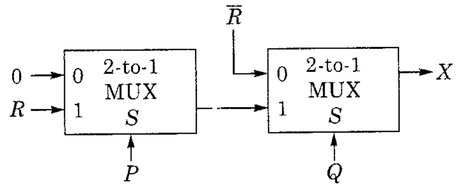
[2014 (Set-3) : 1 Mark]

2.24 A half adder is implemented with XOR and AND gates. A full adder is implemented with two half adders and one OR gate. The propagation delay of an XOR gate is twice that of an AND/OR gate. The propagation delay of an AND/OR gate is 1.2 microseconds.

A 4-bit ripple-carry binary adder is implemented by using four full adders. The total propagation time of this 4-bit binary adder in microseconds is ________.

[2015 (Set-2) : 2 Marks]

2.25 Consider the two cascaded 2-to-1 multiplexers as shown in the.figure.



The minimal sum of products form of the output X is

(a) $\bar{P}\bar{Q} + PQR$ (b) $\bar{P}Q + QR$

(c) $PQ + \bar{P}\bar{Q}R$ (d) $\bar{Q}\bar{R} + PQR$

[2016 (Set-1) : 2 Marks]

2.26 Consider a carry lookahead adder for adding two n-bit integers, built using gates of fan-in atmost two. The time to perform addition using this adder is

(a) $\Theta(1)$ (b) $\Theta(\log(n))$

(c) $\Theta(\sqrt{n})$ (d) $\Theta(n)$

[2016 (Set-1) : 2 Marks]

2.27 Consider an eight bit ripple-carry adder for computing the sum of A and B, where A and B are integers represented in 2's complement form. If the decimal value of A is one, the decimal value of B that leads to the longest latency for the sum to stabilize is ________.

[2016 (Set-2) : 1 Mark]

■■■■

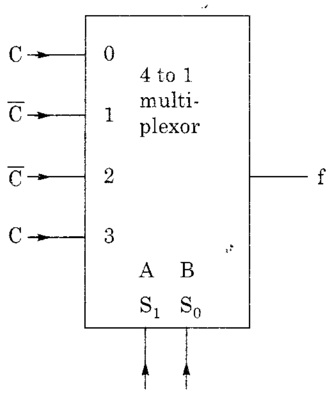
Answers Combinational Circuits

2.1 (a)	2.2 (c)	2.3 (b)	2.4 (b,c)	2.5 (c)	2.6 (c)	2.7 (a)	2.9 (d)	2.10 (b)	
2.11 (d)	2.12 (d)	2.13 (c)	2.14 (a)	2.15 (a)	2.16 (c)	2.17 (c)	2.18 (d)	2.19 (b)	
2.20 (b)	2.21 (a)	2.22 (a)	2.23 (c)	2.25 (d)	2.26 (b)				

Explanations Combinational Circuits

2.1 (a)

The circuit for function f(A, B, C) is given as follows



$f(A, B, C) = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC$

$= \Sigma m\ (1, 2, 4, 7)$

$= A \oplus B \oplus C$

2.2 (c)

The Karnaugh map for SOP form is given as

CD \ AB	00	01	11	10
00		1	1	
01	1			1
11	1			1
10		1	1	

The Karnaugh map for POS form can be given by

CD \ AB	00	01	11	10
00	0			0
01		0	0	
11		0	0	
10	0			0

POS form of f is given as

$f(A, B, C, D) = (B + D)(\bar{B} + \bar{D})$

2.3 (b)

Karnaugh map of the function is given as follows:

AB \ CD	00	01	11	10
00	0	0	1	0
01	x	x	1	x
11	0	1	1	0
10	0	1	1	0

Function from Karnaugh map is

$f(A, B, C, D) = CD + AD$

$= D(C + A)$

2.4 (b,c)

Functionally complete operation set is a set of logic functions from which any arbitrary boolean function can be realized. Examples of functionally complete operation set are:

FC_1 = {OR, AND, NOT}

FC_2 = {NAND}

FC_3 = {NOR}

FC_4 = {XOR, AND}

and 2 to 1 multiplexers.

2.5 (c)

For SOP form

xy \ wz	00	01	11	10
00	0	x	0	0
01	0	x	1	1
11	1	1	1	1
10	0	x	0	0

f in SOP can be given as

$f(x, y, w, z) = xy + yw$ or,

$f(x, y, w, z) = y(x + w)$

Therefore options (a) and (b) are true,

$f(x,y,w,z) = yx + yw = y(x + w)$

Thus, options (a) and (b) implements the given K-map.

For POS form

xy \ wz	00	01	11	10
00	0	x	0	0
01	0	x	1	1
11	1	1	1	1
10	0	x	0	0

POS form of f(x, y, w, z) is given as

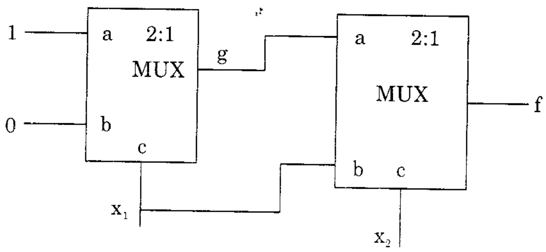
$f(x, y, w, z) = y(x + w)$

OR

$f(x, y, w, z) = (w + x)(x' + y)(x + y)$

Option (c) on simplification gives $y + wx$.

2.6 (c)



The output g is

$$g = 1\,x_1' + 0\,x_1 = x_1'$$

The output f is

$$f = gx_2' + x_1x_2$$

$$f = x_1'x_2' + x_1x_2$$

2.7 (a)

The output function f(x, y, z) is given as

f(x, y, z)

$= EN\left(\bar{A}_1\bar{A}_0I_0 + \bar{A}_1A_0I_1 + A_1\bar{A}_0I_2 + A_1A_0I_3\right)$

$= z'(yz'x + yzx + y'z'y + y'zy')$

$= z'(xyz' + xyz + 0 + y'z)$

$= xyz' + xyzz' + 0.z' + z'.zy')$

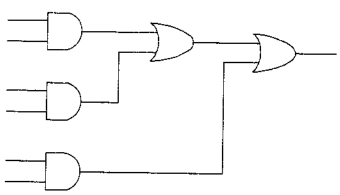
$= xyz' + 0 + 0 + 0$

$= xyz'$

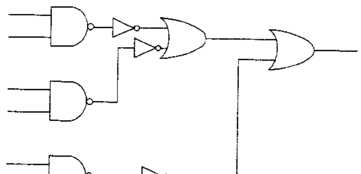
Therefore, option (a) is true.

2.8 Sol.

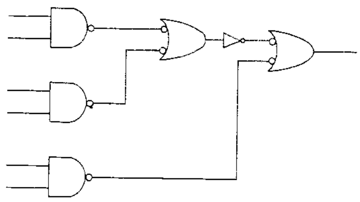
Given circuit is



Step-1: Converting AND into NAND

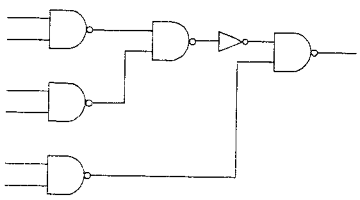


Step-2: Converting OR into bubbled OR

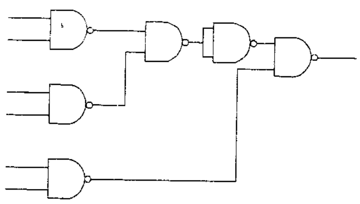


Since bubbled OR is equivalent to NAND

So given circuit can be given by



Step-3: Converting NOT into equivalent NAND gate



2.9 (d)

If $C_0 = 1$ and $K = 1$

then $S_0 = A_0B_0' + B_0A_0'$

and $C_1 = A_0 + A_0'B_0'$

and $S_1 = A_1 \oplus B_1' \oplus C_1$

if $C_0 = 1$ and $K = 0$

then $S_0 = A_0B_0 + A_0'B_0'$

and $C_1 = A_0 + A_0'B_0$

So it computes A + B, A – B and A + 1

2.10 (b)

Let the 4-input is w, x, y, z then

$f(w, x, y, z) = \Sigma(5, 6, 7, 8, 9)$

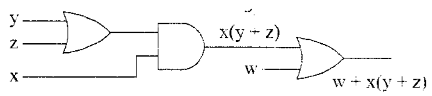
The decimal values 10, 11, 12, 13, 14, 15 are don't care condition so

$f(w, x, y, z) = \Sigma d(10, 11, 12, 13, 14, 15)$

K-map:

wx \ yz	00	01	11	10
00				
01		1	1	1
11	x	x	x	x
10	1	1	x	x

$$f(w, x, y, z) = w + xy + xz$$
$$f(w, x, y, z) = w + x(y + z)$$



So, 3-Gates are required.

2.11 (d)

It would be $f = Az + Bz'$ (as A will be selected when z is high).

So next function will become $g = xf + yf' = x(Az + Bz') + y(Az + Bz')'$

Putting $x = 0$, $y = 1$, $z = A$.

2.12 (d)

Here only $Q'S'$ is not grouped two times, so essential prime implicant is $Q'S'$.

RS \ PQ	00	01	11	10
00	1	0	0	1
01	0	0	1	1
11	1	1	1	0
10	1	0	0	1

2.13 (c)

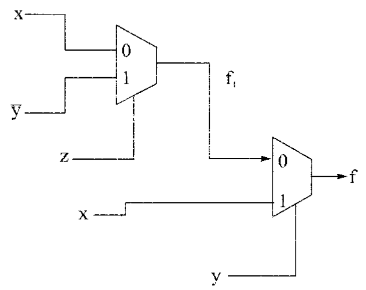
The value of 'f' should be 1 whenever a majority of the variables is 1.

If we select x as '1', either of z or y or both z and y needs to be 1 for the output 'f' to be '1'.

The output can be low only if less than 2 variables are high.

Atleast 2 variables are low. Hence z and y should be 0. Therefore AND gates can be used.

2.14 (a)



Let the output of first uppermost mux is f_1

$$f_1 = xz' + y'z$$

The output of second Mux is

$$\begin{aligned}
f &= (xz' + y'z)\,y' + xy \\
&= xy'z' + y'y'z + xy \\
&= xy'z' + y'z + xy & [a'.a'=a'] \\
&= y'z + xy'z' + xy(z + z') & [a + a' = 1] \\
&= y'z + xy'z' + xyz + xyz' \\
&= y'z + xy'z' + xyz' + xyz' + xyz & [a + a = a] \\
&= y'z + xz'(y' + y) + xy\,(z' + z) \\
&= y'z + xz' + xy & [a + a' = a] \\
&= xz' + xy + y'z
\end{aligned}$$

2.15 (a)

$$f = X3(\overline{X3}\,b + X1\,a) + \overline{X3}X2$$

Put $X1 = b$, $X2 = 0$, $X3 = a$

$$f = a(\overline{b}\cdot b + ba) + \overline{a}\cdot 0 = a\cdot b$$

2.16 (c)

For the construction of a 6×64 decoder by using 3×8 line decoder

$\frac{64}{8} = 8 + 1 \rightarrow$ extra decoder for combining the result $6 \times 64 \xrightarrow{8+1} 3 \times 8$

2.17 (c)

To implement n variable function we require a data selector with $n - 1$ select inputs and 2^{n-1} data inputs. So minimum size of multiplexer needed is 2^{n-1} line to 1 line.

2.18 (d)

	A'B'	A'B	AB	AB'
C'D'			1	1
C'D	1		1	1
CD		1		1
CD'	1	1		1

$f = AC' + AB' + B'C'D + A'CD' + A'BC$. Hence, dependent on all variables.

2.19 (b)

$$\left[\overline{\bar{Z}\bar{Y}\bar{X} + \bar{Z}Y\bar{X} + Z\bar{Y}X + ZYX}\right]$$

$$\left[\overline{\bar{X}\bar{Z}(Y+\bar{Y}) + ZX(Y+\bar{Y})}\right]$$

$$\left[\overline{\bar{X}\bar{Z} + XZ}\right]$$

$$\left[\bar{X}Z + X\bar{Z}\right]$$

2.20 (b)

In the given Multiplexer there are two select lines P and Q so

S_0(P)	S_1(Q)	Input
0	0	R
0	1	$\bar{R}$
1	0	$\bar{R}$
1	1	R

So the output f

$= \bar{P}\bar{Q}R + \bar{P}Q\bar{R} + P\bar{Q}\bar{R} + PQR$

From K map:

	00 $\bar{Q}\bar{R}$	01 $\bar{Q}R$	11 QR	10 $Q\bar{R}$
0 $\bar{P}$		1		1
1 P	1		1	

There is no simplification of this it is formula of

$= P \oplus Q \oplus R$

2.21 (a)

Truth table given represent the priority encoder i.e. 4 × 2 priority encoder V = 0 indicates output is invalid and V = 1 indicates output is valid.

2.22 (a)

Using given multiplexer:

$\Rightarrow \quad F = \bar{P}Q + P\bar{Q}R + PQ\bar{R}$

$Q[\bar{P} + Q\bar{R}] + P\bar{Q}R$

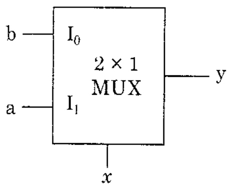
$Q[(\bar{P} + P)(\bar{P} + \bar{R})] + P\bar{Q}R$

$Q[\bar{P} + \bar{R}] + P\bar{Q}R$

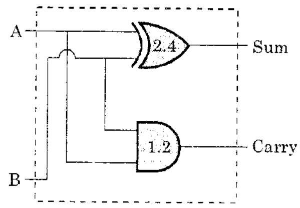
$\bar{P}Q + Q\bar{R} + P\bar{Q}R$

$\bar{P}Q + Q\bar{R} + P\bar{Q}R$

2.23 (c)

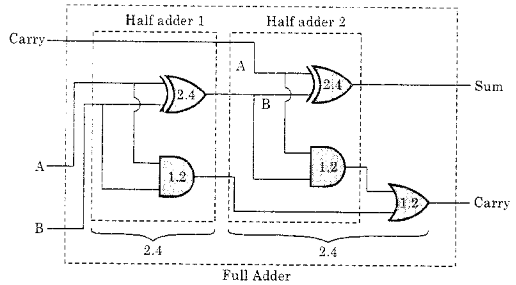


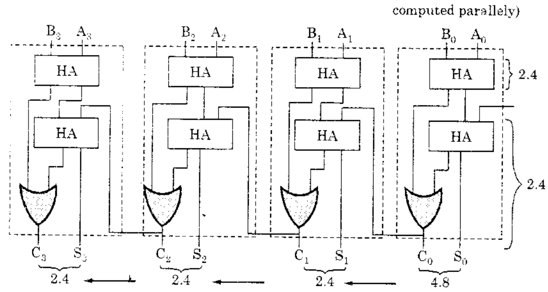
2.24 Sol.



Half Adder (2.4 µs)

F.A. implemented using 2 H.A. and 1 OR gate





Total delay = 4.8 + 2.4 + 2.4 + 2.4 = 12 µs

2.25 (d)

MUX-1 output $\Rightarrow \bar{P}(0) + P(R) = PR$

MUX-2 output $\Rightarrow X = \bar{Q}(\bar{R}) + Q(PR)$

$= \bar{Q}\bar{R} + PQR$

2.26 (b)

The gates to be used in CLA adder with the fan-in at most '2'. Therefore, time to perform addition using this adder is Θ(log (n)).

2.27 Sol.

A & B are 8 bit numbers in 2's complement form.

$A = +1 \Rightarrow$ **0** 0 0 0 0 0 0 1

B = ? so that longest latency in the 8 bit ripple carry adder if B = –1 then there will be longest latency whenever A = + 1.

Verification

A = +1	0 0000 001
A = –1	1 1111 111
Carry out → 1	0 0000 000

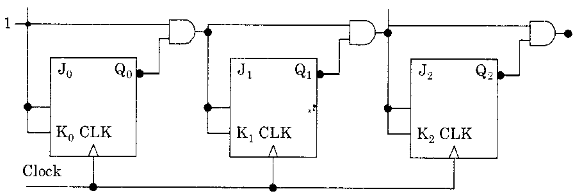
carry is propagating from LSB to MSB. Hence it is longest latency.

3 Sequential Circuits

3.1 RAM is a combinational Circuit and PLA is a sequential circuit.

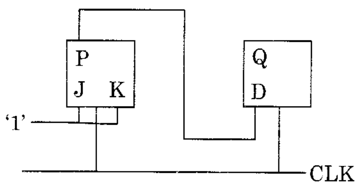
[1990 : 2 Marks]

3.2 Find the maximum clock frequency at which the counter in figure, can be operated. Assume that the propagation delay through each flip-flop and AND gate is 10 ns. Also assume that the setup time for the JK inputs of the flip-flops is negligible.



[1991 : 2 Marks]

3.3 The following arrangement of master-slave flip flops.

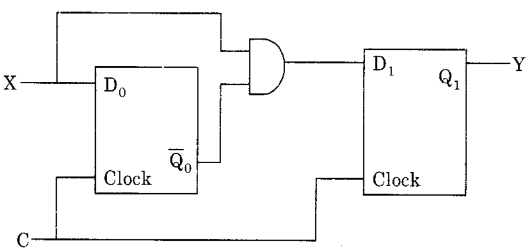


Has the initial state of P, Q as 0, 1 (respectively). After the clock cycles the output state P, Q is (respectively),

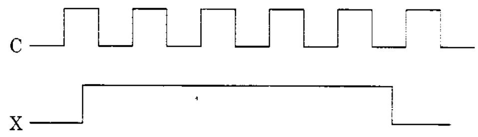
(a) 1, 0 (b) 1, 1
(c) 0, 0 (d) 0, 1

[2000 : 2 Marks]

3.4 Consider the following circuit with initial state $Q_0 = Q_1 = 0$. The D flip-flops are positive edged triggered and have set up time 20 ns and hold time is 0 ns.

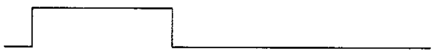


Consider the following timing diagrams of X and C; the clock period of C $\geq$ 40 nanosecond. Which one is the correct plot of Y?

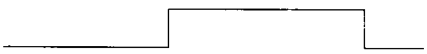


[2001 : 2 Marks]

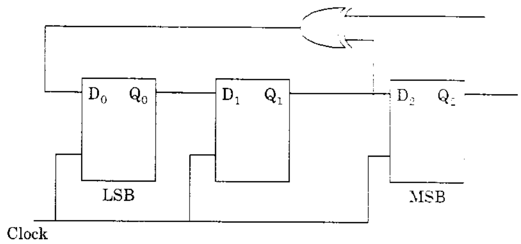
(a) 

(b) 

(c) 

(d) 

3.5 Consider the circuit given below with initial state $Q_0 = 1$, $Q_1 = Q_2 = 0$. The state of the circuit is given by the value $4Q_2 + 2Q_1 + Q_0$



Which one of the following is the correct state sequence of the circuit

(a) 1, 3, 4, 6, 7, 5, 2 (b) 1, 2, 5, 3, 7, 6, 4
(c) 1, 2, 7, 3, 5, 6, 4 (d) 1, 6, 5, 7, 2, 3, 4

[2001 : 2 Marks]

3.6 A 1-input, 2-output synchronous sequential circuit behaves as follows:

Let z_k, n_k denote the number of 0's and 1's respectively in initial k bits of the input ($z_k + n_k = k$). The circuit outputs 00 until one of the following conditions holds.

1. $z_k - n_k = 2$. In this case, the output at the k-th and all subsequency clock ticks is 10.

2. $n_k - z_k = 2$. In this case, the output at the k-th and all subsequent clock ticks is 01.

What in the minimum number of states required in the state transition graph of the above circuit?

(a) 5 (b) 6
(c) 7 (d) 8

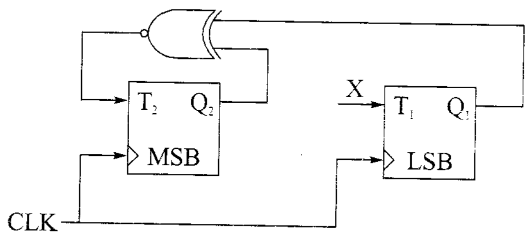
[2003 : 2 Marks]

3.7 In an SR latch made by cross-coupling two NAND gates, if both S and R inputs are set to 0, then it will result in

(a) $Q = 0, Q' = 1$ (b) $Q = 1, Q' = 0$
(c) $Q = 1, Q' = 1$ (d) Indeterminate states

[2004 : 1 Mark]

3.8 Consider the partial implementation of a 2-bit counter using T flip-flops following the sequence 0-2-3-1-0, as shown below:



To complete the circuit, the input X should be

(a) Q_2' (b) $Q_2 + Q_1$
(c) $(Q_1 \oplus Q_2)'$ (d) $Q_1 \oplus Q_2$

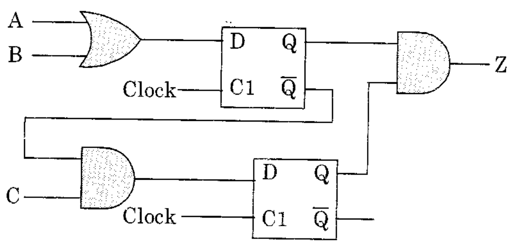
[2004 : 2 Marks]

3.9 How many pulses are needed to change the contents of a 8-bit upcounter from 10101100 to 00100111 (rightmost bit is the LSB)?

(a) 134 (b) 133
(c) 124 (d) 123

[2005 : 1 Mark]

3.10 Which of the following input sequences will always generate a 1 at the output z at the end of the third cycle?



(a) A B C
0 0 0
1 0 1
1 1 1

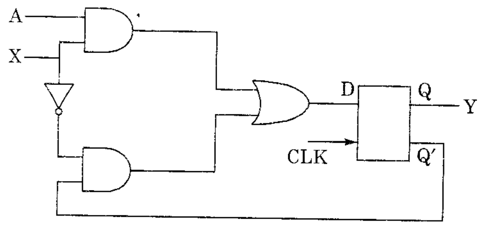
(b) A B C
1 0 1
1 1 0
1 1 1

(c) A B C
0 1 1
1 0 1
1 1 1

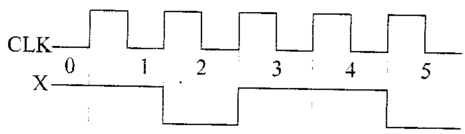
(d) A B C
0 0 1
1 1 0
1 1 1

[2005 : 2 Marks]

3.11 Consider the following circuit involving a positive edge triggered D FF.



Consider the following timing diagram. Let A_i represent the logic level on the line A in the i-th clock period.

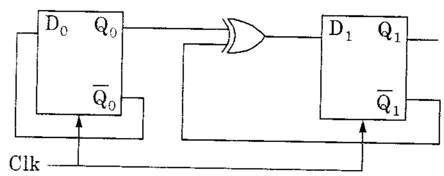


Let A′ represent the complement of A. The correct output sequence on Y over the clock periods 1 through 5 is

(a) $A_0\, A_1\, A_1'\, A_3\, A_4$ (b) $A_0\, A_1\, A_2'\, A_3\, A_4$
(c) $A_1\, A_2\, A_2'\, A_3\, A_4$ (d) $A_1\, A_2'\, A_3\, A_4\, A_5'$

[2005 : 2 Marks]

3.12 Consider the following circuit



The flip-flops are positive edge triggered D FFs. Each state is designated as a two bit string Q_0Q_1. Let the initial state be 00. The state transition sequence is

(a) 00 → 11 → 01

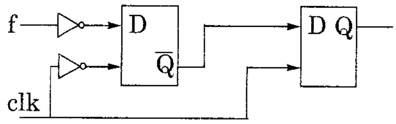
(b) 00 → 11

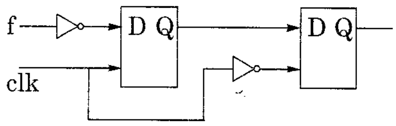
(c) 00 → 10 → 01 → 11

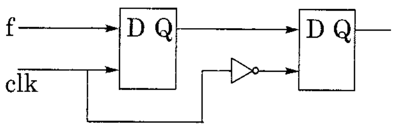
(d) 00 → 11 → 01 → 10

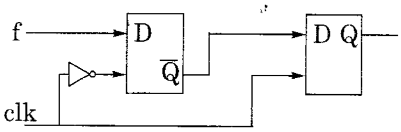
[2005 : 2 Marks]

3.13 You are given a free running clock with a duty cycle of 50% and a digital waveform f which changes only at the negative edge of the clock. Which one of the following circuits (using clocked D flip-flops) will delay the phase of f by 180°?

(a)


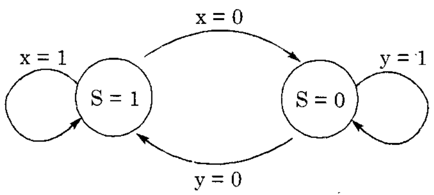
(b)


(c)


(d)


[2006 : 1 Mark]

3.14 For a state machine with the following state diagram the expression for the next state S^+ in terms of the current state S and the input variables x and y is



(a) $S^+ = S'\cdot y' + S\cdot x$
(b) $S^+ = S\cdot x\cdot y' + S'\cdot y\cdot x'$
(c) $S^+ = x\cdot y'$
(d) $S^+ = S'\cdot y + S\cdot x'$

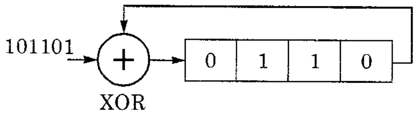
[2006 : 2 Marks]

3.15 Which of the following input sequences for a cross-coupled R-S flip-flop realized with two NAND gates may lead to an oscillation?

(a) 11, 00 (b) 01, 10
(c) 10, 01 (d) 00, 11

[2007 : 1 Mark]

3.16 What is the final value stored in the linear feedback shift register if the input is 101101?



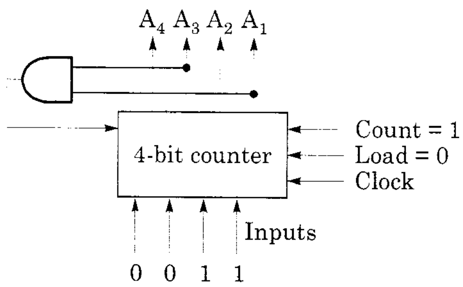
(a) 0110 (b) 1011
(c) 1101 (d) 1111

[2007 : 2 Marks]

3.17 The control signal functions of 4-bit binary counter are given below (where X is "don't care")

Clear	Clock	Load	Count	Function
1	X	X	X	Clear to 0
0	X	0	0	No change
0	↑	1	X	Load input
0	↑	0	1	Count next

The counter is connected as follows

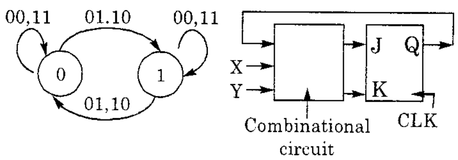


Assume that the counter and gate delays are negligible. If the counter starts at 0, then it cycles through the following sequence

(a) 0, 3, 4 (b) 0, 3, 4, 5
(c) 0, 1, 2, 3, 4 (d) 0, 1, 2, 3, 4, 5

[2007 : 2 Marks]

3.18 Consider the following state diagram and its realization by a JK flip flop.

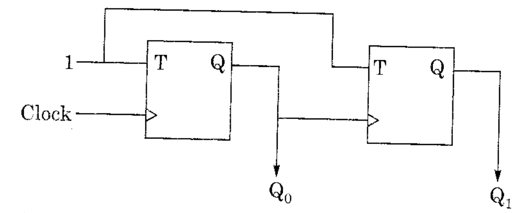


The combinational circuit generates J and K in terms of x, y and Q. The Boolean expressions for J and K are :

(a) $\overline{x \oplus y}$ and $\overline{x \oplus y}$ (b) $\overline{x \oplus y}$ and $x \oplus y$
(c) $x \oplus y$ and $\overline{x \oplus y}$ (d) $x \oplus y$ and $x \oplus y$

[2008 : 2 Marks]

3.19 In the sequential circuit shown below, if the initial value of the output Q_1Q_0 is 00, what are the next four values of Q_1Q_0?



(a) 11,10,01,00
(b) 10,11,01,00
(c) 10,00,01,11
(d) 11,10,00,01

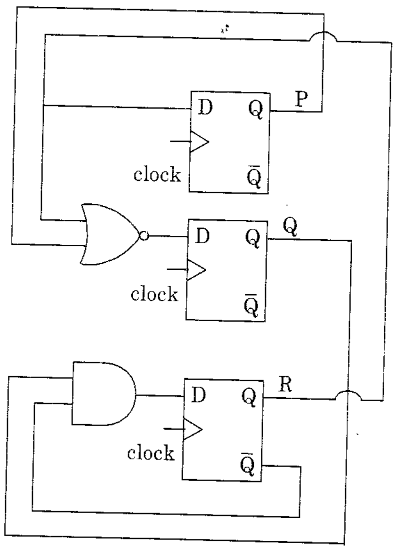
[2010 : 2 Marks]

3.20 The minimum number of D flip-flops needed to design a mod-258 counter is
(a) 9
(b) 8
(c) 512
(d) 258

[2011 : 1 Mark]

Common Data Q.3.21 & Q.3.22

Consider the following circuit involving three D-type flip-flops used in a certain type of counter configuration



3.21 If at some instance prior to the occurrence of the clock edge, P, Q and R have a value 0, 1 and 0 respectively, what shall be the value of PQR after the clock edge?
(a) 000
(b) 001
(c) 010
(d) 011

[2011 : 2 Marks]

3.22 If all the flip-flops were reset to 0 at power on, what is the total number of distinct outputs (states) represented by PQR generated by the counter?
(a) 3
(b) 4
(c) 5
(d) 6

[2011 : 2 Marks]

3.23 The truth table

X	Y	f(X,Y)
0	0	0
0	1	0
1	0	1
1	1	1

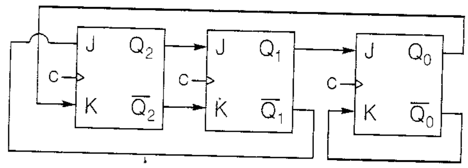
represents the Boolean function
(a) X
(b) X + Y
(c) $X \oplus Y$
(d) Y

[2012 : 1 Mark]

3.24 Let $k = 2^n$. A circuit is built by giving the output of an n-bit binary counter as input to an n-to-2^n bit decoder. This circuit is equivalent to a
(a) k-bit binary up counter.
(b) k-bit binary down counter.
(c) k-bit ring counter.
(d) k-bit Johnson counter.

[2014 (Set-2) : 1 Mark]

3.25



The above synchronous sequential circuit built using JK flip-flops is initialized with $Q_2Q_1Q_0 = 000$. The state sequence for this circuit for the next 3 clock cycles is
(a) 001, 010, 011
(b) 111, 110, 101
(c) 100, 110, 111
(d) 100, 011, 001

[2014 (Set-3) : 2 Marks]

3.26 Consider a 4 bit Johnson counter with an initial value of 0000. The counting sequence of this counter is
(a) 0, 1, 3, 7, 15, 14, 12, 8, 0
(b) 0, 1, 3, 5, 7, 9, 11, 13, 15, 0
(c) 0, 2, 4, 6, 8, 10, 12, 14, 0
(d) 0, 8, 12, 14, 15, 7, 3, 1, 0

[2015 (Set-1) : 1 Mark]

3.27 A positive edge-triggered D flip-flop is connected to a positive edge-triggered JK flip-flop as follows. The Q output of the D flip-flop is connected to both the J and K inputs of the JK flip-flop, while the Q output of the JK flip-flop is connected to the input of the D flip-flop. Initially, the output of the D flip-flop is set to logic one and the output of the JK flip-flop is cleared.

Which one of the following is the bit sequence (including the initial state) generated at the Q output of the JK flip-flop when the flip-flops are connected to a free-running common clock? Assume that J = K = 1 is the toggle mode and J = K = 0 is the state-holding mode of the JK flip-flop. Both the flip-flops have non-zero propagation delays.

(a) 0110110... (b) 0100100...
(c) 011101110... (d) 011001100...

[2015 (Set-1) : 2 Marks]

3.28 The minimum number of JK flip-flops required to construct a synchronous counter with the count sequence (0, 0, 1, 1, 2, 2, 3, 3, 0, 0,...) is____.

[2015 (Set-2) : 1 Mark]

3.29 We want to design a synchronous counter that counts the sequence 0-1-0-2-0-3 and then repeats. The minimum number of J-K flip-flops required to implement this counter is ______.

[2016 (Set-1) : 1 Mark]

■■■■

Answers Sequential Circuits

3.3	(a)	3.4	(a)	3.5	(b)	3.6	(a)	3.7	(c)	3.8	(d)	3.9	(d)	3.10	(b)	3.11	(a)
3.12	(d)	3.13	(c)	3.14	(a)	3.15	(a)	3.16	(a)	3.17	(c)	3.18	(d)	3.19	(a)	3.20	(a)
3.21	(d)	3.22	(b)	3.23	(a)	3.24	(c)	3.25	(c)	3.26	(d)	3.27	(a)				

Explanations Sequential Circuits

3.1 Sol.

False: A combinational circuit neither contains a periodic clock signal nor has any provisions for storage. There are no feedback involved and the output at all time is dependent on the inputs provided.

A sequential circuit involves feedback and has memory. It also has a periodic clock signal and hence the output is also a function of time in addition to being a function of inputs and previous outputs.

3.2 Sol.

All the flip flops are operated by same clock, together all takes one propagation delay.

All the AND gates consumes one propagation delay individually.

Total propagation delay

$= T_{CLK} \geq T_{\text{flip-flops}} + T_{\text{AND gates}}$

$= 10 \text{ ns} + (10 + 10 + 10) \text{ ns}$

$= 40 \text{ ns}$

Maximum clock frequency

$$= \frac{1}{T_{CLK}} = \frac{1}{40\text{ns}} = \frac{10^9}{40} = 25 \text{ MHz}$$

Maximum clock frequency at which the counter can operate is 25 MHz.

3.3 (a)

J (1)	K (1)	D (P)	P	Q
Initially			0	1
1	1	0	1	0
1	1	1	0	1

The circuit output PQ alternates between 01 and 10 states.

3.4 (a)

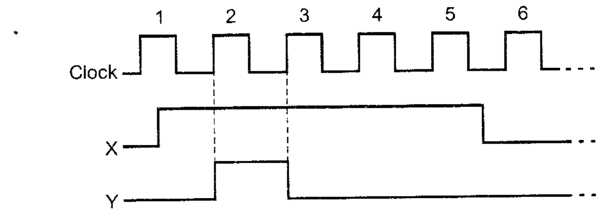
Flip-flops are having setup time 20 ns and hold time is 0 ns.

Flip-flops are positive edged triggered.

The input 'X' and clock 'C' waveform are given.

The output waveform Y i.e. the output of flip-flop 1, Q_1 is to be identified.

Clock	Q_0	Q_1	FF0 $D_0 = X$	FF1 $D_1 = X\bar{Q}_0$
Initial state	0	0	0	0
1	0	0	1	1
2	1	1	1	0
3	1	0	1	0
4	1	0	1	0
5	1	0	0	0
6	0	0		



3.5 (b)

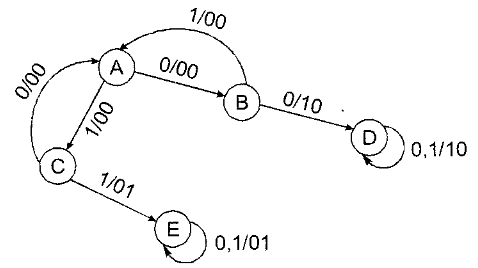
Clock	$D_2 = (Q_1)$ Initially	$D_1 = (Q_0)$	$D_0 = (Q_1 \oplus Q_2)$	Q_2	Q_1	Q_0	
				0	0	1	(1)
1	0	1	0	0	1	0	(2)
2	1	0	1	1	0	1	(5)
3	0	1	1	0	1	1	(3)
4	1	1	1	1	1	1	(7)
5	1	1	0	1	1	0	(6)
6	1	0	0	1	0	0	(4)
7	0	0	1	0	0	1	(1)

Therefore, the correct state sequence of the circuit is 1, 2, 5, 3, 7, 6, 4.

3.6 (a)

1. If $z_k - n_k = 2$. In the case, the output at the k^{th} and subsequency clock ticks is 10.
2. If $n_k - z_k = 2$. In this case, the output at the k^{th} and subsequency clock ticks is 01.

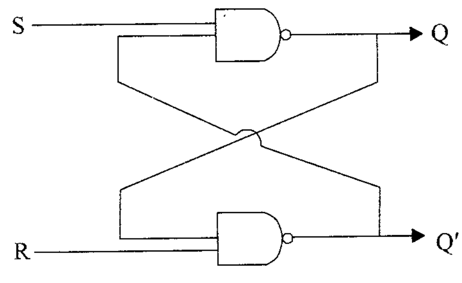
Based on sequential circuit description, the state diagram of the circuit is



$\therefore$ Total numbers of states are 5.

3.7 (c)

SR latch by cross-coupling two NAND gates



NAND LATCH

If the both S and R inputs are set 0 then the result will be:

Table for the SR Latch using the NAND gates:

S	R	Q
0	0	Invalid state (Q = Q′ = 1)
0	1	1
1	0	0
1	1	Previous state

$\therefore$ Q = 1 and Q′ = 1

3.8 (d)

	Clock	Q_2	Q_1	FF1 T_1
0		0	0	0
2	1	1	0	1
3	2	1	1	0
1	3	0	1	1
	4	0	0	

The required switching sequence of the counter is $0 \to 2 \to 3 \to 1 \to 0$.

$T1(Q_2, Q_1) = \Sigma m\ (1, 2)$

$$= \overline{Q}_2 Q_1 + Q_2 \overline{Q}_1 = Q_2 \oplus Q_1$$

3.9 (d)

In a 2^8 Counter the range would be from 0-255. Hence to go from 10101100 (172) to 00100111 (39), the counter has to go initially from 172 to 255 and then from 0 to 39.

Hence to go from 172 to 255, 255 − 172 = 83 Clock pulses would be required.

From 255 to 0 , again 1 clock pulse would be required.

Then from 0 to 39 , 39 clock pulses would be required.

Hence in total 83 + 1 + 39 = 123 Clock pulses would be required.

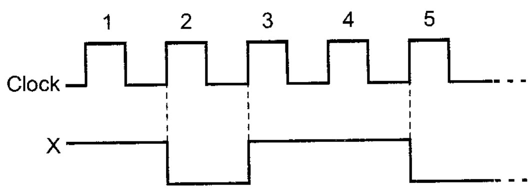
3.10 (b)

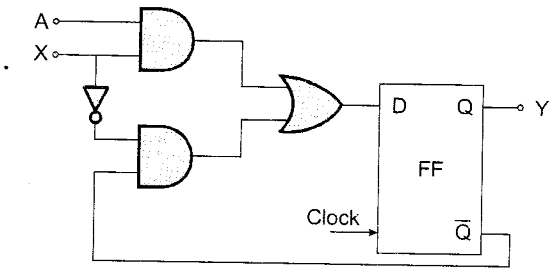
$$(A+B)\cdot\left[\bar{A}\bar{B}C\right] = 1$$

So by apply option 2 we always get output as 1.

3.11 (a)

Clock and input waveform X are given in problem description.





Clock	$Q_{n+1} = Y$	$D = AX + \overline{X}Q_n$
Initial state	0 or 1	A_0
1	A_0	A_1
2	A_1	A_1'
3	A_1'	A_3
4	A_3	A_4
5	A_4	

Before first clock the input is A_0 at 1st clock input is A_1 and so on.
Output Y is $A_0 A'_1 A'_1 A_3 A_4$ over the clock period 1 through 5.

3.12 (d)

Clock	Q_0	Q_1	FFO $D_0 = \overline{Q}_0$	FF1 $D_1 = \overline{Q}_1 \oplus Q_0$
Initial state	0	0	1	1
1	1	1	0	1
2	0	1	1	0
3	1	0	0	0
4	0	0		

The state transitions sequence is

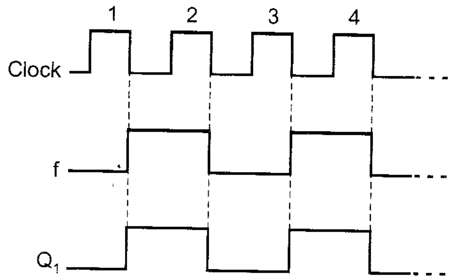
$$\rightarrow 00 \rightarrow 11 \rightarrow 01 \rightarrow 10 \rightarrow$$

3.13 (c)

1. In all the options consider left flip-flop as FF0 and right side flip-flop as FF1, FF0 output is Q_0 and FF1 output is Q_1.
2. Digital input 'f' changes at the negative edge of the clock. To get delay the phase of 'f' by 180°, the output i.e., Q_1 must change at the negative edge of the clock.
   Hence option (a) and (b) are not possible answer. [Q_1 is changing at the positive edge of the clock]
3. In option (b) and (c) FF0 will change at the clock rise and FF1 will change at clock falls.
4. FF1 input is changed output of FF0 after clock rise for both options (b) and (c).

Let us consider option (b):

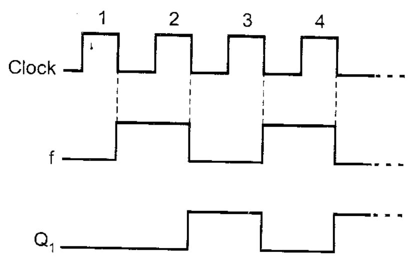
Clock	$D_0 = \overline{f}$	Q_0	Q_1	
		0	0	Initial state
1	1	1	1	
2	0	0	0	
3	1	1	1	
4	0	0	0	



Output Q_1 is in phase with the input f, hence it is not the answer.

Let us consider option (c):

Clock	$D_0 = f$	Q_0	Q_1	
		0	0	Initial state
1	1	0	0	
2	0	1	1	
3	1	0	0	
4	0	1	1	



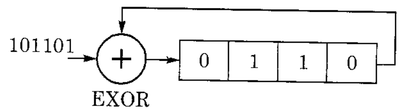
Output Q_1 is out of phase with input f by 180°, hence the output is 180° delayed by input f, hence the answer is option (c).

3.14 (a)

Present State	Inputs		Next state
S	x	Y	S^+
0	X	0	1
0	X	1	0
1	0	X	0
1	1	X	1

From the above table: $S^+ = S' \cdot y' + S \cdot x$

3.16 (a)



EXOR = $0 \oplus 1 = 1$ $0 \oplus 0 = 0$

$1 \oplus 0 = 1$ $1 \oplus 1 = 0$

• 10110 $\boxed{1}$

1^{st} time 1+0 = 1: Enter and shift all by one bit.

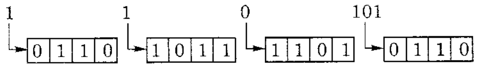
• 1011 $\boxed{1}$

2^{nd} time 0+1 = 1: Enter and shift all by one bit.

• 101 $\boxed{1}$

3^{rd} time 1+1 = 0: Enter and shift all by one bit.

101 same as 1^{st} 3 bits.



do same as 1^{st} 3 bit outputfrom LSB comes as 0110.

3.17 (c)

Count = 1, load = 0 externally connected to the counter. Based on the function table, whenever clear input is '0' the counter will increment its value by one and whenever clear input is 1, all the flip-flops is the counter will get value as '0'.

Clock	A_4	A_3	A_2	A_1	Clear = $A_3 \cdot A_1$
Initial state	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	0
3	0	0	1	1	0
4	0	1	0	0	0
5	0	1	0	1	1
	0	0	0	0	Transition

Hence the switching sequence is 0,1,2,3,4.

3.18 (d)

(i) $xy = 00 \Rightarrow J = x \oplus y = 0$

$xy = 11 \Rightarrow K = x \oplus y = 0$

If JK = 00 retains previous output.

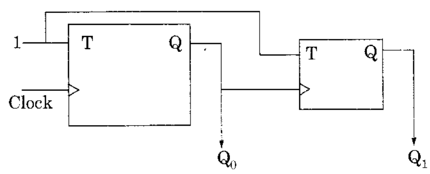
(ii) $xy = 01 \Rightarrow J = x \oplus y = 1$

$xy = 10 \Rightarrow K = x \oplus y = 1$

If JK = 11, toggles the previous output.

$\therefore J = x \oplus y,\ K = x \oplus y.$

3.19 (a)



The counter is a ripple counter with positive edged triggered T flip-flops, with T inputs are connected to 1.

Q_0 will change for every clock input.

Q_1 will change whenever Q_0 is changing from '0' to '1'.

Clock	Q_1	Q_0
Initial state	0	0
1	1	1
2	1	0
3	0	1
4	0	0

The sequence is 11, 10, 01, 00

3.20 (a)

$\lceil \log_2 258 \rceil = 9$ D flip-flops

3.21 (d)

	P Q R	$D_P = R$	$D_Q = \overline{P+R}$	$D_R = Q\overline{R}$
Initial state	0 1 0	0	1	1
1	0 1 1			

3.22 (b)

Clk	P Q R	$D_P = R$	$D_Q = \overline{P+R}$	$D_R = Q\overline{R}$
	0 0 0	0	1	0
1	0 1 0	0	1	1
2	0 1 1	1	0	0
3	1 0 0	0	0	0
4	0 0 0			

The counter switching sequence is

$\rightarrow 000 \rightarrow 010 \rightarrow 011 \rightarrow 100 \rightarrow$

The number of distinct states are 4.

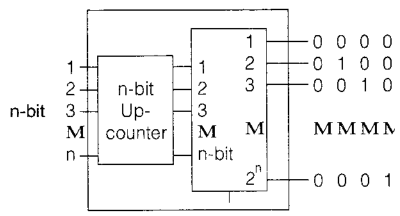
3.23 (a)

x	y	f(x,y)
0	0	0
0	1	0
1	0	1
1	1	1

Now equation for f(x, y) = $x\bar{y} + xy$

$\Rightarrow x(\bar{y} + y) \Rightarrow x$

3.24 (c)



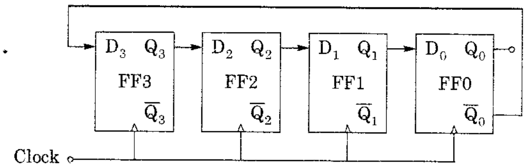
Ring counter of 2^n bits

3.25 (c)

Clock	Q_2 Q_1 Q_0	FF2 $J_2=\bar{Q}_1$	$K_2=Q_0$	FF1 $J_1=Q_2$	$K_1=\bar{Q}_2$	FF0 $J_0=Q_1$	$K_0=\bar{Q}_0$
Initial state	0 0 0	1	0	0	1	0	1
1	1 0 0	1	0	1	0	0	1
2	1 1 0	0	0	1	0	1	1
3	1 1 1						

$\therefore$ The state sequence is 100, 110, 111.

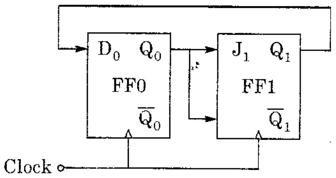
3.26 (d)



Clock	$D_3 = \overline{Q}_0$	Q_3	Q_2	Q_1	Q_0
		0	0	0	0 ← Initial state
1	1	1	0	0	0
2	1	1	1	0	0
3	1	1	1	1	0
4	1	1	1	1	1
5	0	0	1	1	1
6	0	0	0	1	1
7	0	0	0	0	1
8	0	0	0	0	0

Hence the switching sequence is : 0, 8, 12, 14, 15, 7, 3, 1, 0.

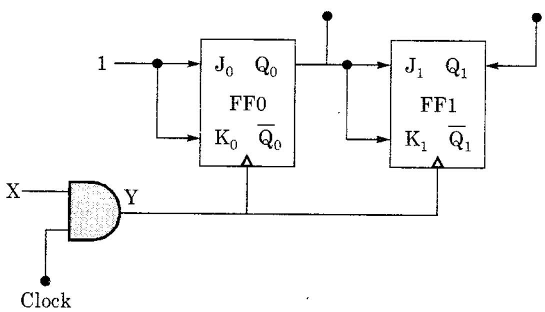
3.27 (a)

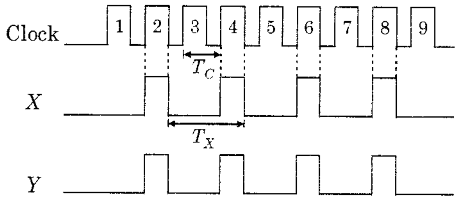


	Clock	Q_1	Q_0	$J_1 = Q_0$	$K_1 = Q_0$	$D_0 = Q_1$
				FF1		FF0
Initial state		0	1	1	1	0
	1	1	0	0	0	1
	2	1	1	1	1	1
	3	0	1	1	1	0
	4	1	0			

J-K flip-flop output is 0110110...

3.28 Sol.





Where $T_X = 2T_C$

X	Clock	Y	Q_1	Q_0
			0	0 ← Initial state
0	1	0	0	0
1	2	1	0	1
0	3	0	0	1
1	4	1	1	0
0	5	0	1	0
1	6	1	1	1
0	7	0	1	1
1	8	1	0	0
0	9	0	0	0

The switching sequence of the counter is 0,0,1,1,2,2,3,3,0,0, ...

$\therefore$ Only two flip-flops are required.

3.29 Sol.

Design synchronous counter using 4 flip-flops for the sequence

$\rightarrow 0 \rightarrow 1 \rightarrow 4 \rightarrow 2 \rightarrow 8 \rightarrow 3$

use J-K FF's.

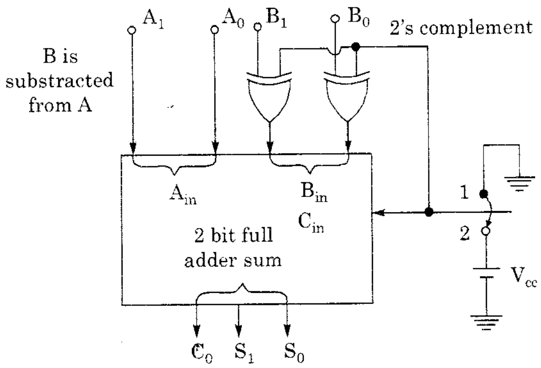
Read the output from FF1 and FFO only. So, that counter switching sequence is

$\rightarrow 0 \rightarrow 1 \rightarrow 0 \rightarrow 2 \rightarrow 0 \rightarrow 3$

Note: Don't read output from FF3 and FF2.

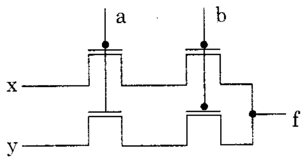
4 Number Systems

4.1 Fill in the blanks:
In the two bit full-adder/subtractor unit shown in figure, when the switch is in position 2. ...... using .... arithmetic



[1990 : 2 Marks]

4.2 For the digital in figure, the expression for the output f is ......



[1990 : 2 Marks]

4.3 Consider the number given by the decimal expression.

$$16^3 \times 9 + 16^2 \times 7 + 16 \times 5 + 3$$

The number of 1's in the unsigned binary representation of the number is ....

4.4 Using the 8087 arithmetic coprocessor with the 80186 CPU requires that the 8087 CPU is operated for ________

4.5 When two 4-bit binary number $A = a_3\, a_2\, a_1\, a_0$, $B = b_3\, b_2\, b_1\, b_0$ are multiplied, the digit c_1 of the product C is given by

4.6 A 32-bit floating-point number is represented by a 7-bit signed exponent, and a 24-bit fractional mantissa. The base of the scale factor is 16,
(a) the range of the exponent is ______
(b) the range of the exponent is ______ if the scale factor is represented in excess-64 format.

[1990 : 2 Marks]

4.7 A ROM is used to store the table for multiplication of two 8-bit unsigned integers. The size of ROM required is
(a) 256×16 (b) $64K \times 8$
(c) $4K \times 16$ (d) $64K \times 16$

[1996 : 1 Mark]

4.8 Booth's algorithm for integer multiplication gives worst performance when the multiplier pattern is
(a) 101010......1010 (b) 100000.....001
(c) 111111......1111 (d) 011111.....1110

[1996 : 1 Mark]

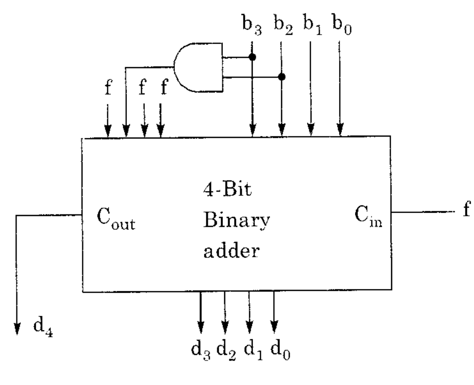
4.9 Consider the following floating point number representation.

31 24	23 0
Exponent	Mantissa

The exponent is in 2's complement representation and mantissa is in the sign magnitude representation. The range of the magnitude of the normalized numbers in this representation is
(a) 0 to 1 (b) 0.5 to 1
(c) 2^{-23} to 0.5 (d) 0.5 to $(1 - 2^{-23})$

[1996 : 1 Mark]

4.10 Consider the circuit in figure which has a four bit binary number $b_3\, b_2\, b_1\, b_0$ as input and a five bit binary number $d_4\, d_3\, d_2\, d_1\, d_0$ as output. The circuit implements:



(a) Binary to Hex conversion
(b) Binary to BCD conversion
(c) Binary to grey code conversion
(d) Binary to radix-12 conversion

[1996 : 2 Marks]

4.11 Given $\sqrt{(224)_r} = (13)_r$. The value of the radix' r is

(a) 10 (b) 8
(c) 5 (d) 6

[1997 : 2 Marks]

4.12 Booth's coding in 8 bits for the decimal number – 57 is

(a) $0 - 100 + 1000$ (b) $0 - 100 + 100 - 1$
(c) $0 - 1 + 100 - 10 + 1$ (d) $00 - 10 + 100 - 1$

[1999 : 1 Mark]

4.13 Zero has two representations in :

(a) Sign magnitude (b) 1's complement
(c) 2's complement (d) None of these

[1999 : 2 Marks]

4.14 The number 43 in 2's complement representation is

(a) 01010101 (b) 11010101
(c) 00101011 (d) 10101011

[2000 : 1 Mark]

4.15 Consider the values $A = 2.0 \times 10^{30}$, $B = -2.0 \times 10^{30}$, $C = 1.0$ and the sequence. What will be the values of X and Y if the following sequence of statements are carried out

X : = A + B
Y : = A + C
X : = X + C
Y : = Y + B

(a) X = 1.0, Y = 1.0 (b) X = 0.0, Y = 1.0
(c) X = 1.0, Y = 0.0 (d) X = 0.0, Y = 0.0

[2000 : 2 Marks]

4.16 The 2's complement representation of $(-539)_{10}$ in hexadecimal is

(a) ABE (b) DBC
(c) DE5 (d) 9E7

[2001 : 2 Marks]

4.17 The decimal value 0.25

(a) is equivalent to binary 0.1
(b) is equivalent to binary 0.01
(c) is equivalent to binary 0.00111
(d) can not be represented precisely in binary

[2002 : 1 Mark]

4.18 The 2's complement representation of the decimal value –15 is

(a) 1111 (b) 11111
(c) 111111 (d) 10001

[2002 : 1 Mark]

4.19 Sign extension is the step in

(a) Floating point multiplication
(b) Signed 16 bit integer addition
(d) Arithmetic left shift
(d) Converting a signed integer from one size to another

[2002 : 1 Mark]

4.20 Assuming all numbers are in 2's complement representation, which of the following numbers is divisible by 11111011?

(a) 11100111 (b) 11100100
(c) 11010111 (d) 11011011

[2003 : 1 Mark]

4.21 The following is a scheme for floating point number representation using 16 bits.

Bit position 15	14 9	8 0
s	e	m
Sign	Exponent	Mantissa

Let s. e, and m be the numbers represented in binary in the sign, exponent, and mantissa fields respectively. Then the floating point number represented is

$$\begin{cases}(-1)^s(1 + m \times 2^{-9})2^{e-31}, & \text{if the exponent} \neq 111111\\ 0 & \text{otherwise}\end{cases}$$

What is the maximum difference between two successive real numbers representable in this system?

(a) 2^{-40} (b) 2^{-9}
(c) 2^{22} (d) 2^{31}

[2003 : 2 Marks]

4.22 If 73_x (in base-x number system) is equal to 54_y, (in base-y number system), the possible values of x and y are

(a) 8, 16 (b) 10, 12
(c) 9, 13 (d) 8, 11

[2004 : 1 Mark]

4.23 What is the result of evaluating the following two expressions using three-digit floating point arithmetic with rounding?

(113. + –111.) + 7.51
113. + (–111. + 7.51)

(a) 9.51 and 10.0 respectively
(b) 10.0 and 9.51 respectively
(c) 9.51 and 9.51 respectively
(d) 10.0 and 10.0 respectively

[2004 : 1 Mark]

4.24 A 4-bit carry lookahead adder, which adds two 4-bit numbers, is designed using AND, OR, NOT, NAND, NOR gates only. Assuming that all the inputs are available in both complemented and uncomplemented forms and the delay of each gate is one time unit, what is the overall propagation delay of the adder? Assume that the carry network has been implemented using two-level AND-OR logic.

(a) 4 time units (b) 6 time units
(c) 10 time units (d) 12 time units

[2004 : 2 Marks]

4.25 Let A = 1111 1010 and B 0000 1010 be two 8-bit 2's complement numbers. Their product in 2's complement is

(a) 1100 0100 (b) 1001 1100
(c) 1010 0101 (d) 1101 0101

[2004 : 2 Marks]

4.26 The number $(123456)_8$ is equivalent to

(a) $(A72E)_{16}$ and $(22130232)_4$
(b) $(A72E)_{16}$ and $(22131122)_4$
(c) $(A73E)_{16}$ and $(22130232)_4$
(d) $(A62E)_{16}$ and $(22120232)_4$

[2004 : 2 Marks]

4.27 The range of integers that can be represented by an n bit 2's complement number system is

(a) -2^{n-1} to $(2^{n-1}-1)$
(b) $-(2^{n-1}-1)$ to $(2^{n-1}-1)$
(c) -2^{n-1} to 2^{n-1}
(d) $-(2^{n-1}+1)$ to $(2^{n-1}-1)$

[2005 : 1 Mark]

4.28 The hexadecimal representation of 657_8 is

(a) 1AF (b) D78
(c) D71 (d) 32F

[2005 : 1 Mark]

4.29 Using Booth's Algorithm for multiplication, the multiplier –57 will be recoded as

(a) 0 –1 0 0 1 0 0 –1 (b) 1 1 0 0 0 1 1 1
(c) 0 –1 0 0 1 0 0 0 (d) 0 1 0 0 –1 0 0 1

[2005 : 1 Mark]

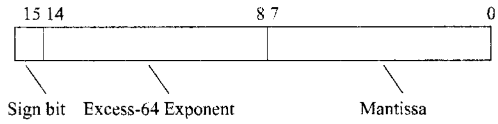
4.30 $(34.4)_8 \times (23.4)_8$ evaluates to

(a) $(1053.6)_8$ (b) $(1053.2)_8$
(c) $(1024.2)_8$ (d) None of these

[2005 : 2 Marks]

Common Data for Q.4.31 & Q.4.32

Consider the following floating point format



Mantissa is a pure fraction is sign-magnitude form.

4.31 The decimal number 0.239×2^{13} has the following hexadecimal representation without normalization and rounding off

(a) 0D 24 (b) 0D 4D
(c) 4D 0D (d) 4D 3D

[2005 : 2 Marks]

4.32 The normalized representation for the above format is specified as follows. The mantissa has an implicit 1 preceding the binary (radix) point. Assume that only 0's are padded in while shifting a field. The normalized representation of the above number (0.239×2^{13}) is

(a) 0A 20 (b) 11 34
(c) 4D D0 (d) 4A E8

[2005 : 2 Marks]

4.33 The addition of 4-bit, two's complement, binary numbers 1101 and 0100 results in

(a) 0001 and an overflow
(b) 1001 and no overflow
(c) 0001 and no overflow
(d) 1001 and an overflow

[2006 : 1 Mark]

4.34 When multiplicand Y is multiplied by multiplier $X = x_{n-1} x_{n-2}, \ldots, x_0$ using bit-pair recoding in Booth's algorithm, partial products are generated according to the following table.

Row	x_{i+1}	x_i	x_{i-1}	Partial Product
1	0	0	0	0
2	0	0	1	Y
3	0	1	0	Y
4	0	1	1	2Y
5	1	0	0	?
6	1	0	1	-Y
7	1	1	0	-Y
8	1	1	1	?

ıcts for rows 5 and 8 are

(a) 2Y and Y (b) –2Y and 2Y
(c) –2Y and 0 (d) 0 and Y

[2006 : 2 Marks]

4.35 $(C012.25)_H - (10111001110.101)_B =$
(a) $(135103.412)_O$ (b) $(564411.412)_O$
(c) $(564411.205)_O$ (d) $(135103.205)_O$

[2007 : 2 Marks]

4.36 The following bit pattern represents a floating point number in IEEE 754 single precision format:
1 10000011 10100000000000000000000
The value of the number in decimal form is
(a) –10 (b) –13
(c) –26 (d) None of these

[2008 : 1 Mark]

4.37 In the IEEE floating point representation the hexadecimal value 0x00000000 corresponds to
(a) the normalized value 2^{-127}
(b) the normalized value 2^{-126}
(c) the normalized value +0
(d) the special value +0

[2008 : 1 Mark]

4.38 Let a denote number system radix. The only value(s) of r that satisfy the equation $\sqrt{121} = 11$, is/are
(a) decimal 10 (b) decimal 11
(c) decimal 10 and 11 (d) any value > 2

[2008 : 1 Mark]

4.39 The two numbers given below are multiplied using the Booth's algorithm.
Multiplicand : 0101 1010 1110 1110
Multiplier : 0111 0111 1011 1101
How many additions/Subtractions are required for the multiplication of the above two numbers?
(a) 6 (b) 8
(c) 10 (d) 12

[2008 : 2 Marks]

4.40 $(1217)_8$ is equivalent to
(a) $(1217)_{16}$ (b) $(028F)_{16}$
(c) $(2297)_{10}$ (d) $(0B17)_{16}$

[2009 : 1 Mark]

4.41 P is a 16-bit signed integer. The 2's complement representation of P is $(F87B)_{16}$. The 2's complement representation of 8*P is
(a) $(C3D8)_{16}$ (b) $(187B)_{16}$
(c) $(F878)_{16}$ (d) $(987B)_{16}$

[2010 : 1 Mark]

4.42 The smallest integer that can be represent by and 8-bit number in 2's complement form is
(a) – 256 (b) – 128
(c) – 127 (d) 0

[2013 : 1 Mark]

4.43 The base (or radix) of the number system such that the following equation holds is_________.

$$\frac{312}{20} = 13.1$$

[2014 (Set-1) : 1 Mark]

4.44 Consider the equation $(123)_5 = (x8)_y$ with x and y as unknown. The number of possible solutions is_______.

[2014 (Set-2) : 1 Mark]

4.45 The value of a *float* type variable is represented using the single-precision 32-bit floating point format of IEEE-754 standard that uses 1 bit for sign, 8 bits for biased exponent and 23 bits for mantissa. A *float* type variable X is assigned the decimal value of –14.25. The representation of X in hexadecimal notation is
(a) C1640000H (b) 416C0000H
(c) 41640000H (d) C16C0000H

[2014 (Set-2) : 2 Marks]

4.46 Consider the equation $(43)_x = (y3)_8$ where x and y are unknown. The number of possible solutions is_______.

[2015 (Set-3) : 2 Marks]

4.47 The 16-bit 2's complement representation of an integer is 1111 1111 1111 0101; its decimal representation is ____.

[2016 (Set-1) : 1 Mark]

4.48 Let X be the number of distinct16-bit integers in 2's complement representation. Let Y be the number of distinct 16-bit integers in sign magnitude representation. Then $X - Y$ is ____.

[2016 (Set-2) : 1 Mark]

■■■■

Answers Number Systems

4.7 (d)	4.8 (a)	4.9 (d)	4.10 (d)	4.11 (c)	4.12 (b)	4.13 (a,b)	4.14 (c)	4.15 (c)
4.16 (c)	4.17 (b)	4.18 (d)	4.19 (d)	4.20 (a)	4.21 (c)	4.22 (d)	4.23 (a)	4.24 (a)
4.25 (a)	4.26 (a)	4.27 (a)	4.28 (a)	4.29 (a)	4.30 (d)	4.31 (d)	4.32 (d)	4.33 (a)
4.34 (c)	4.35 (a)	4.36 (c)	4.37 (d)	4.38 (d)	4.39 (b)	4.40 (b)	4.41 (a)	4.42 (b)
4.45 (a)								

Explanations Number Systems

4.1 Sol.

When the switch is in position 2, 2's complement of B is added to A.
This means B in subtracted from A.
Therefore, when the switch is in position 2 B is subtracted from A using 2's complement arithmetic.

4.2 Sol.

The expression for the output is $f = \bar{a}\bar{b}$.

4.3 Sol.

The number in decimal expression is given as

$= 16^3 \times 9 + 16^2 \times 7 + 16 \times 5 + 3$

$= 16^3 \times 9 + 16^2 \times 7 + 16^1 \times 5 + 6^0 \times 3$

The above number can be represented in hexadecimal as $= (9753)_H$

The binary equivalent of above hexadecimal is given as:

$$(9753)_H = (1001\ 0111\ 0101\ 0011)_B$$

The number of 1's in the unsigned binary representation of the number is 9.

4.4 Sol.

Floating point arithmetic.

4.5 Sol.

$$C = A \times B$$

$$C_7C_6C_5C_4C_3C_2C_1C_0 = (a_3a_2a_1a_0) \times (b_3b_2b_1b_0)$$

			$a_3a_2a_1a_0$			
			$\times\ b_3b_2b_1b_0$			
			a_3b_0	a_2b_0	a_1b_0	a_0b_0
		a_3b_1	a_2b_1	a_1b_1	a_0b_1	
	a_3b_2	a_2b_2	a_1b_2	a_0b_2		
a_3b_3	a_2b_3	a_1b_3	a_0b_3			
a_3b_3	$(a_3b_2+ a_2b_3)$	$(a_3b_1+ a_2b_2+ a_1b_3)$	$(a_3b_0+ a_2b_1+ a_1b_2+ a_0b_3)$	$(a_2b_0+ a_1b_1+ a_0b_2)$	$(a_1b_0+ a_0b_1)$	(a_0b_0)
C_6	C_5	C_4	C_3	C_2	C_1	C_0

The digit C_1 of the product C is given by

$$C_1 = a_1b_0 + a_0b_1$$

4.6 Sol.

The given floating point number format is

S (1)	EXP (7)	Fraction (Mantissa) (24)
31	30 – 24	23 – 0

Exponent = bias + 64

Above floating point architecture is IBM floating point architecture.

The number is represented as the following formula:

$$(-1)^S \times 0\,.\,\text{Mantissa} \times 10^{\text{EXP}-64}$$

(a) The range of the exponent is the range of the 7-bit signed numbers, which is –64 to +63.

(b) The range of the exponent if the scale factor is represented in excess-64 format is 0 to +127.

4.7 (d)

To store the table for multiplication of two 8-bit unsigned integers, size of ROM required is

$= (2^8 \times 2^8) \times (8 + 8)$

$= 2^{16} \times 16$

$= 2^6 \times 2^{10} \times 16$

$= 64\text{ K} \times 16$

4.8 (a)

Booth recording of a multiplier are

(a) 1 0 1 0 1 0 . . . . . 1 0 1 0

⇓

–1 +1 –1 +1 –1 +1 . . . . –1 +1 –1 0

(b) 1 0 0 0 0 0 . . . . 0 0 0 1

⇓

–1 0 0 0 0 0 . . . . 0 0 +1 –1

(c) 1 1 1 1 . . . . 1 1 1 1

⇓

0 0 –1

(d) 0 1 1 1 1 1 . . . . 1 1 1 0

$\Downarrow$

+1 0 0 0 0 0 . . . . 0 0 −1 0

Therefore, option (a) gives worst performance because it contains maximum number of 1's.

4.9 (d)

31 24	23 0
Exponent	Mantissa

The range of the magnitude of the normalized numbers is as follows; 0.5 to $(1-2^{-23})$.

4.10 (d)

Whenever both b_3 and b_2 is 1, then 01 00 is added to the number.

$b_3b_2b_1b_0 = (1100)_B = (12)_{10}$

So in all the numbers greater than equal to 12, 01 00 is added.

Therefore the circuit is binary to radix −12 converter.

4.11 (c)

(a) When the radix r is 10

Then $\sqrt{(224)_{10}} \approx 14.967 \neq (13)_{10}$

Option (a) is not correct

(b) When the radix r is 8

Then $\sqrt{(224)_8} = \sqrt{(148)_{10}} = (12.165)_{10}$

and $(13)_8 = (11)_{10}$

$\sqrt{(224)_8} \neq (13)_8$

Therefore, option (b) is not correct.

(c) When the radix r is 5

$\sqrt{(224)_5} = \sqrt{(64)_{10}} = (8)_{10}$

and $(13)_5 = (8)_{10}$

Therefore, $\sqrt{(224)_5} = (13)_5$

Option (c) is correct.

(d) When the radix r is 6

$\sqrt{(224)_6} = \sqrt{(88)_{10}} \approx (9.38)_{10}$

and $(13)_6 = (9)_{10}$

Therefore, $\sqrt{(224)_6} \approx (13)_6$

Option (d) is not correct.

Alternative method:

$\sqrt{(224)_r} = (13)_r$

$\Rightarrow 2r^2 + 2r + 4 = r^2 + 6r + 9$

$\Rightarrow r^2 - 4r - 5 = 0$

$\Rightarrow r^2 - 5r + r - 5 = 0$

$\Rightarrow r(r-5) + 1(r-5) = 0$

$\Rightarrow (r+1)(r-5) = 0$

$\Rightarrow r = -1, 5$

$\therefore$ r = 5 and choice (c) is correct.

4.12 (b)

−57 can be represented in 2's complement form in 8-bits as follows:

1 1 0 0 0 1 1 1

Booth's coding for decimal number −57 is

1 1 0 0 0 1 1 1 $\boxed{0}$

$\Downarrow$

0 −1 0 0 +1 0 0 −1

Alternative Method:

0 − 100 + 100 − 1 evaluates to 57 in Booth's coding as follows

$(0 \times 2^7) - (1 \times 2^6) + (0 \times 2^5) + (0 \times 2^4) + (1 \times 2^3) + (0 \times 2^2) + (0 \times 2^1) - (1 \times 2^0)$

$= -64 + 8 - 1$

$= -65 + 8 = -57$

4.13 (a,b)

Zero has two representations in

(a) Sign magnitude as

(i) 0 0 0 0

(ii) 1 0 0 0

(b) 1's complement as

(i) 0 0 0 0

(ii) 1 1 1 1

4.14 (c)

Positive 2's complement numbers are represented as the simple binary.

So, +43 in binary is represented as

0 0 1 0 1 0 1 1

4.15 (c)

$A = 2.0 \times 10^{30}$

$B = -2.0 \times 10^{30}$

$C = 1.0$

(i) $X = A + B$

$= 2.0 \times 10^{30} + (-2.0 \times 10^{30}) = 0.0$

(ii) $Y = A + C$
$= 2.0 \times 10^{30} + (1.0)$
$= 2.0 \times 10^{30} + 0.0000...1 \times 10^{30}$
$= 2.0000...1 \times 10^{30} \approx 2.0 \times 10^{30}$

(iii) $X = X + C = 0.0 + 1.0 = 1.0$

(iv) $Y = Y + B$
$= 2.000...1 \times 10^{30} - 2.0 \times 10^{30} = 0.0$

Therefore, after the sequence value of X and Y are 1.0 and 0.0 respectively.

4.16 (c)

2's complement representation of a negative number is determined as follows:

(a) Compute a binary representation of the magnitude of the number.

(b) Take 2's complement of binary representation.

$+539 = 0010\ 0001\ 1011$
$-539 = 1101\ 1110\ 0101 = (DE5)_H$

Therefore option (c) is correct.

4.17 (b)

$0.25 \times 2 = 0.50; \quad 0.50 \times 2 = 1.00$

Therefore, the decimal value 0.25 is equivalent to binary 0.01.

4.18 (d)

The 2's complement of negative numbers are represented as the binary number that when added to a positive number of the same magnitude equals zero

$+15 = 0\ 1\ 1\ 1\ 1$

The 2's complement representation of the decimal value −15 is

−15 in 2's complement = 10001

4.19 (d)

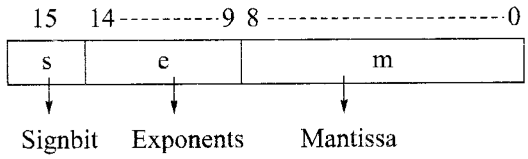
Sign extension is the operation, in computer arithmetic, of increasing the number of bits of a binary number while preserving the number's sign (positive/negative) and value.

4.20 (a)

	2's Complement	Decimal Value
(a)	11100111	25
(b)	11100100	28
(c)	11010111	41
(d)	11011011	37
(e)	11111011	5
	(Divider)	

So −25 is divisible by −5 so we can say 11100111 is divisible by 11111011.

4.21 (c)



Then the floating point number represented

$$\begin{cases} (-1)^s(1 + m \times 2^{-9})2^{e-31}, & \text{if the exponent } \neq 111111 = 63 \\ 0 \text{ otherwise} \end{cases}$$

Max. exponent = 111110 = 62 $\quad \because e = 62$

Maximum value is

$(-1)^s\,[1 + (m + 1) \times 2^{-9}] \times 2^{e-31}$

Next maximum value is

$(-1)^s\,[1 + m \times 2^{-9}] \times 2^{e-31}$

$\therefore$ Difference of last two successive numbers

$= (-1)^s[1 + (m+1) \times 2^{-9}] \times 2^{62-31} - (-1)^s$
$[1 + m \times 2^{-9}] \times 2^{62-31}$
$= [1 + (m + 1) \times 2^{-9}]\,2^{31} - [1 + m \times 2^{-9}]2^{31}$
$= 2^{31} \times 2^{-9} = 2^{22}$

4.22 (d)

$(73)_x$ if $x = 8$
$(73)_8 = 7 \times 8^1 + 3 \times 8^0$
$= 56 + 3 = 59$

$(54)_y$ if $y = 11$
$(54)_{11} = 5 \times 11^1 + 4 \times 11^0$
$= 55 + 4 = 59$

Alternative Method:

$(73)_x = (54)_y$
$\Rightarrow 7x + 3 = 5y + 4$
$\Rightarrow 7x - 5y = 1 \quad \text{...(i)}$

Put options (a), (b), (c), and (d) in (i) and check

$7 \times 8 - 5 \times 11 = 1$

$\therefore$ Option (d) is correct.

4.23 (a)

(113. + −111.) + 7.51
113. + (−111. + 7.51)

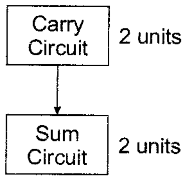
The exponent value is 10.0.

The Value evaluated by the expression is 9.51 and 10.0.

4.24 (a)

As per the problem description for generating carry the delay is 2 time units, and 2 time units delay is involved for generating sum output after knowing carry.

Hence the total delay is 4 time units.



4.25 (a)

$$\begin{aligned} A &= 11111010 \\ &= -2^7 \times 1 + 2^6 \times 1 + 2^5 \times 1 + 2^4 \times 1 \\ &\quad + 2^3 \times 1 + 2^2 \times 0 + 2^1 \times 0 + 2^0 \times 0 \\ &= -2^7 + 2^6 + 2^5 + 2^4 + 2^3 + 2^4 + 2^3 + 2^1 \\ &= -128 + 64 + 32 + 16 + 8 + 2 \\ &= -128 + 122 = -6 \\ B &= 00001010 \\ &= 2^3 \times 1 + 2^1 \times 1 \\ &= 8 + 2 = 10 \end{aligned}$$

$A \cdot B = (-6) \times 10 = -60$

So the value of AB in 2's complement representation

$AB = 11000100$

4.26 (a)

$$\begin{aligned} (123456)_8 &= (001\ 010\ 011\ 100\ 101\ 110)_2 \\ &= (00\ 1010\ 0111\ 0010\ 1110)_2 \\ &= (A72E)_{16} \\ &= (00\ 10\ 10\ 01\ 11\ 00\ 10\ 11\ 10)_2 \\ &= (22130232)_4 \end{aligned}$$

4.27 (a)

2's complement of a number represents the negative number. If a number is n-bit long the most significant digit is a sign bit so starting from -2^{n-1} because only $n-1$ place we represent the magnitude. So the range is -2^{n-1} to $(2^{n-1}-1)$ including 0.

4.28 (a)

$$\begin{aligned} (657)_8 &= (110\ 101\ 111)_2 \\ &= (1\ \ 1010\ \ 1111)_2 \\ &= (1\ \ A\ \ F)_{16} \end{aligned}$$

4.29 (a)

2's complement of −57 is 110001110.

Append 0 at the LSB and start from right end taking pairs of two symbols which are encoded as:

$00 \to 0$
$01 \to +1$
$10 \to -1$
$11 \to 0.$

4.30 (d)

$$\begin{aligned} 34.4_8 &= 011\ 100.1 \\ &= 16 + 8 + 4 + 0.5 = 28.5 \\ 23.4_8 &= 010\ 011.1 \\ &= 16 + 2 + 1 + 0.5 = 19.5 \end{aligned}$$

$28.5 \times 19.5 = 555.75 = 1053.6_8$

$(0.0.75 \times 8 = 0.6)$ and if we group together the remainders of dividing 555 by 8, we get 1053.

4.31 (d)

The decimal number is 0.239×2^{13}

We have to find hexadecimal representation without normalization.

Biased exponent = 13 + 64 = 77

Representing 77 in binary

$(77)_{10} = (1001101)_2$

Representing mantissa in binary

$(0.239)_{10} = 0.00111101000101$

Floating point representation is as follows:

Sign	Exponent	Mantissa
0	1001101	00111101

$$\underbrace{0100}_{4}\ \underbrace{1101}_{D}.\ \underbrace{0011}_{3}\ \underbrace{1101}_{D}$$

$= (4D3D)_H$

4.32 (d)

The decimal number is 0.239×2^{13}

We have to find hexadecimal representation with normalization.

Binary representation of 0.239 is 0.00111101000101

$\therefore$ N is $(0.00111101000101) \times 2^{13}$

After normalization we have

$1.11101000101 \times 2^{10}$

Then, biased exponent = 10 + 64 = 74

Representating biased exponent 74 in binary

$(74)_{10} = (1001010)_2$

Floating point representation of number is as follows:

Sign	Exponent	Mantissa
0	1001010	11101000

$$\underbrace{0100}_{4}\ \underbrace{1010}_{A}\ \underbrace{1110}_{E}\ \underbrace{1000}_{8}$$

$= (4AE8)_H$

4.33 (a)

$$\begin{array}{rr} & 1101 \\ & 0100 \\ \hline \text{Overflow} & 10001 \end{array}$$

Option (a) is correct.

4.34 (c)

Row	Multiplier bit triplet			Partial product
	X_{i+1}	X_i	X_{i-1}	
1	0	0	0	No operation
2	0	0	1	y add
3	0	1	0	y add
4	0	1	1	2y add
5	1	0	0	–2y (sub)
6	1	0	1	–y (sub)
7	1	1	0	–y (sub)
8	1	1	1	No operation

$$5 \quad \underbrace{1 \;\; 0} \;\; 0 \Rightarrow \begin{pmatrix} 2^1 \\ -1 \end{pmatrix}\begin{pmatrix} 2^0 \\ 0 \end{pmatrix} = -2y$$

$$8 \quad \underbrace{1 \;\; 1} \;\; 1 \Rightarrow 0 \;\; 0 = 0$$

4.35 (a)

$(C012.25)_H - (10111001110.101)_B$
$= 1100\ 0000\ 0001\ 0010.0010\ 0101$
$-\ 0000\ 0101\ 1100\ 1110.1010\ 0000$
$= 1011\ 1010\ 0100\ 0011.1000\ 0101$
$= 1\ 011\ 101\ 001\ 000\ 011.100\ 001\ 010$
$= (135103.412)_o$

Binary subtraction is like decimal subtraction:
$0 - 0 = 0$, $1 - 1 = 0$, $1 - 0 = 1$,
$0 - 1 = 1$ with 1 borrow.

4.36 (c)

Sign bit is 1 implies number is negative.
Exponent bits: 10000011
Exponent is added with 127 bias in IEEE single precision format.
So, Actual exponent = 10000011 – 127 = 131 – 127 = 4
Mantissa bits: 10100000000000000000000
In IEEE format, an implied 1 is before mantissa.
Hence the Number is $-1.101 * 2^4$
$= -(11010)_2 = -26$

4.37 (d)

An exponent of 0 together with a fraction of 0 with positive sign represents +0 which is a special value.

4.38 (d)

$$\sqrt{121_r} = (11)_r$$

So we will take base r any value that is greater than 2 since "the digit value must be less than radix".

4.39 (b)

Apply booth's algorithm:

0	1	1	1	0	1	1	1	1	0	1	1	1	1	0	1
(1–0)	(1–1)	(1–1)	(0–1)	(1–0)	(1–1)	(1–1)	(1–1)	(0–1)	(1–0)	(1–1)	(1–1)	(1–1)	(1–1)	(0–1)	(0–1)
+1	0	0	–1	+1	0	0	0	–1	+1	0	0	0	–1	+1	–1

4 additions and 4 substractions total 8 operations.

4.40 (b)

$$(1217)_8 = (028F)_{16}$$

$(1\ 2\ 1\ 7)_8$

001 010 001 111

2 8 F

4.41 (a)

2's complement representation of P is $(F8\,7B)_{16}$

$(F87B)_{16} = (1111100001111011)_2$

2's complement of P means $= \bar{P} + 1$

$\bar{P} + 1 = 1111\ 1000\ 0111\ 1011$

$\bar{P} = 1111\ 1000\ 0111\ 1010$

$P = 0000\ 0111\ 1000\ 0101$

$= 1 \times 2^0 + 1 \times 2^2 + 1 \times 2^7 + 1 \times 2^8 + 1 \times 2^9 + 1 \times 2^{10}$

$= 1 + 4 + 128 + 256 + 512 + 1024 = 1925$

$8 \times P = 8 \times 1925 = 15400$

Convert the into binary

2	15400	0
2	7700	0
2	3850	0
2	1925	1
2	962	0
2	481	1
2	240	0
2	120	0
2	60	0
2	30	0
2	15	1
2	7	1
2	3	1
	1	

$\underbrace{0011}\underbrace{1100}\underbrace{0010}\underbrace{1000}$

$8 \times P = 0011\ 1100\ 0010\ 1000$

1's complement of :

$8 \times P = 1100\ 0011\ 1101\ 0111$

2's complement of $8 \times P$:

$\underbrace{1100}_{C}\underbrace{0011}_{3}\underbrace{1101}_{D}\underbrace{1000}_{8} = (C3D8)_{16}$

Alternative Method:

2's complement of P is given as $(F87B)_{16}$

2's complement of $8 \times P = 2^3 \times (F87B)_{16}$

Left shift by 3 bits $= (C3D8)_{16}$

$(F87B)_{16} =$ 1111 1000 0111 1011

1100 0011 1101 1000

$(C \quad 3 \quad D \quad 8)_{16}$

4.42 (b)

Range of 2's complement no is

$(-2^{n-1}$ to $2^{n-1}-1)$

So minimum no represents by 8 bit is

$-2^{8-1} = -128$

maximum no represented by 8 bit is

$2^{8-1} - 1 = 127$

So option (b) is correct.

4.43 Sol.

$$\frac{(3x^2 + 1x^1 + 2x^0)}{(2x^1 + 0x^0)} = (1x^1 + 3x^0 + 1/x)$$

$\Rightarrow \quad 3x^2 + 1x + 2 = (x + 3 + 1/x)(2x)$

$\Rightarrow \quad 3x^2 + x + 2 = 2x^2 + 6x + 2$

$\Rightarrow \quad x^2 - 5x = 0$

$\Rightarrow \quad x = 5$

4.44 Sol.

$1*5^2 + 2*5^1 + 3*5^0 = x*y^1 + 8*y^0$

$25 + 10 + 3 = xy + 8$

$38 - 8 = xy$

$\Rightarrow \quad xy = 30$

Possible combinations:

[$\because$ The radix y should be greater than x]

$x = 1,\ y = 30$

$x = 2,\ y = 15$

$x = 3,\ y = 10$

$x = 5,\ y = 6$

Possible x-y values to satisfy given equation

1. $x = 1,\ y = 30$
2. $x = 2,\ y = 15$
3. $x = 3,\ y = 10$

$= 3$ possibilities

4.45 (a)

$14.25 = 1110.010 = 1.110010 \times 2^3$

$M = 110010$

$E = 127 + 3 = 130 = 10000010$

1	8	23	
S	E	M	= C1640000
1	10000010	11001000...	

4.46 Sol.

$(43)_x = (y3)_8$

$\Downarrow \quad \Downarrow$

$x > 4 \quad y < 8$

$\Rightarrow \quad 4x + 3 = 8y + 3$

$\Rightarrow \quad x = 2y,\ x > 4$ and $y < 8$

(i) $x = 6,\ y = 3$

(ii) $x = 8,\ y = 4$

(iii) $x = 10,\ y = 5$

(iv) $x = 12,\ y = 6$

(v) $x = 14,\ y = 7$

$\therefore$ Total 5 possible solutions.

4.47 Sol.

$\boxed{1}$111 1111 1111 0101 ← Given number in 2's complement form
No. is –ve

0000 0000 0000 1011 ← 2's of the number.

∴ Decimal equivalent is –11.

4.48 Sol.

Number of bits $n = 16$

In sign 2's complement representation the range is

$$= -2^{15} \text{ to } +(2^{15} - 1)$$
$$= -32768 \text{ to } +32767$$

$\therefore \quad X = 65536$

In sign magnitude representation the range is

$$= -(2^{15} - 1) \text{ to } +(2^{15} - 1)$$
$$= -32767 \text{ to } +32767$$

$\therefore \quad Y = 65535$

Hence

$X - Y = 1$

■■■

Unit ▪ IV

Computer Organization & Architecture

▪ Contents

UNIT IV

Computer Organization & Architecture

Syllabus : Machine instructions and addressing modes, ALU and data-path, CPU control design, Memory interface, I/O interface (Interrupt and DMA mode), Instruction pipelining, Cache and main memory, Secondary storage.

Analysis of Previous GATE Papers

Exam Year	1 Mark Ques.	2 Marks Ques.	3 Marks Ques.	5 Marks Ques.	Total Marks
1990	–	2		1	**9**
1991	–	–		–	**–**
1992	–	–		–	**–**
1993	–	–		–	**–**
1994	–	–		–	**–**
1995	–	–		–	**–**
1996	2	1		2	**14**
1997	6	1		–	**8**
1998	–	–		–	**–**
1999	1	5		2	**21**
2000	3	1		2	**15**
2001	5	2		2	**19**
2002	6	3		–	**9**
2003	2	3		–	**8**
2004	1	7		–	**15**
2005	4	8		–	**20**
2006	1	7		–	**15**

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2007	2	6	**14**
2008	–	12	**24**
2009	2	4	**10**
2010	1	4	**9**
2011	1	4	**9**
2012	2	2	**6**
2013	1	4	**9**
2014 Set-1	1	3	**7**
2014 Set-2	1	3	**7**
2014 Set-3	1	2	**5**
2015 Set-1	1	1	**3**
2015 Set-2	1	2	**5**
2015 Set-3	1	2	**5**
2016 Set-1	1	2	**5**
2016 Set-2	1	5	**11**

1 Cache and Main Memory

1.1 A block-set associative cache memory consists of 128 blocks divided into four block sets. The main memory consists of 16,384 blocks and each block contains 256 eight bit words.

(i) How many bits are required for addressing the main memory?

(ii) How many bits are needed to represent the TAG, SET and WORD fields?

[1990 : 2 Marks]

1.2 In a two-level virtual memory, the memory access time for main memory, $t_{A_1} = 10^{-8}$ sec, and the memory access time for the secondary memory, tag $= 10^{-3}$ sec. What must be the hit ratio, H such that the access efficiency is within 80 percent of its maximum value.

[1990 : 2 Marks]

1.3 The main memory of a computer has 2 cm blocks while the cache has 2 c blocks. If the cache uses the set associative mapping scheme with 2 blocks per set, then block k of the main memory maps to the set

(a) (k mod m) of the cache

(b) (k mod c) of the cache

(c) (k mod 2 c) of the cache

(d) (k mod 2 cm) of the cache

[1999 : 1 Mark]

1.4 Which of the following is/are advantage of virtual memory?

(a) Faster access to memory on an average.

(b) Processes can be given protected address spaces.

(c) Linker can assign addresses independent of where the program will be loaded in physical memory.

(d) Programs larger than the physical memory size can be run.

[1999 : 2 Marks]

1.5 A graphics card has on board memory of 1 MB. Which of the following modes can the card not support?

(a) 1600 × 400 resolution with 256 colours on a 17 inch monitor

(b) 1600 × 400 resolution with 16 million colours on a 14 inch monitor

(c) 800 × 400 resolution with 16 million colours on a 17 inch monitor

(d) 800 × 800 resolution with 256 colours on a 14 inch monitor

[2000 : 2 Marks]

1.6 Which of the following requires a device driver?

(a) Register (b) Cache

(c) Main memory (d) Disk

[2001 : 1 Mark]

1.7 More than one word are put in one cache block to

(a) exploit the temporal locality of reference in a program

(b) exploit the spatial locality of reference in a program

(c) reduce the miss penalty

(d) None of the above

[2001 : 1 Mark]

1.8 Which of the following statements is false?

(a) Virtual memory implements the translation of a program's address space into physical memory address space

(b) Virtual memory allows each program to exceed the size of the primary memory

(c) Virtual memory increases the degree of multiprogramming

(d) Virtual memory reduces the context switching overhead

[2001 : 1 Mark]

1.9 Consider a system with 2 level cache. Access times of Level 1 cache, Level 2 cache and main memory are 1 ns, 10 ns, and 500 ns, respectively. The hit rates of Level 1 and Level 2 caches are 0.8 and 0.9, respectively. What is the average access time of the system ignoring the search time within the cache?

(a) 13.0 ns (b) 12.8 ns

(c) 12.6 ns (d) 12.4 ns

[2004 : 1 Mark]

1.10 Consider a small two-way set-associative cache memory, consisting of four blocks. For choosing the block to be replaced, use the least recently used (LRU) scheme. The number of cache misses for the following sequence of block addresses is 8, 12, 0, 12, 8

(a) 2 (b) 3
(c) 4 (d) 5

[2004 : 2 Marks]

1.11 A dynamic RAM has a memory cycle time of 64 nsec. It has to be refreshed 100 times per msec and each refresh takes 100 nsec. What percentage of the memory cycle time is used for refreshing?

(a) 10 (b) 6.4
(c) 1 (d) 0.64

[2005 : 1 Mark]

1.12 Increasing the RAM of a computer typically improves performance because

(a) Virtual memory increases
(b) Larger RAMs are faster
(c) Fewer page faults occur
(d) Fewer segmentation faults occur

[2005 : 1 Mark]

1.13 Consider a direct mapped cache of size 32 KB with block size 32 bytes. The CPU generates 32 bit addresses. The number of bits needed for cache indexing and the number of tag bits are respectively

(a) 10, 17 (b) 10, 22
(c) 15, 17 (d) 5, 17

[2005 : 2 Marks]

1.14 A cache line is 64 bytes. The main memory has latency 32ns and bandwidth 1 GB/s. The time required to fetch the entire cache line from the main memory is

(a) 32 ns (b) 64 ns
(c) 96 ns (d) 128 ns

[2006 : 2 Marks]

1.15 A computer system has a level-1 instruction cache (1-cache), a level-1 data cache (D-cache) and a level-2 cache (L2-cache) with the following specifications:

	Capacity	Mapping method	Block size
1-cache	4K words	Direct mapping	4 Words
D-cache	4K words	2-way set-associative mapping	4 Words
L2-cache	64K words	4-way set-associative mapping	16 Words

Capacity Mapping methodBlock size 1-cache 4K words Direct mapping 4 Words D-cache 4K words 2-way set-associative mapping 4 Words L2-cache 64K words 4-way set-associative mapping 16 WordsThe length of the physical address of a word in the main memory is 30 bits. The capacity of the tag memory in the I-cache, D-cache and L2-cache is, respectively,

(a) 1K × 18-bit, 1 K × 19-bit, 4K × 16-bit
(b) 1K × 16-bit, 1 K × 19-bit, 4K × 18-bit
(c) 1K × 16-bit, 512 × 18-bit, 1K × 16-bit
(d) 1K × 18-bit, 512 × 18-bit, 1K × 18-bit

[2006 : 2 Marks]

1.16 A CPU has a cache with block size 64 bytes. The main memory has k banks, each bank being c bytes wide. Consecutive c-byte chunks are mapped on consecutive banks with warp-around. All the k banks can be accessed in parallel, but two accesses to the same bank must be serialized. A cache block access may involve multiple iterations of parallel bank accesses depending on the amount of data obtained by accessing all the k banks in parallel. Each iteration requires decoding the bank numbers to be accessed in parallel and this takes k/2 ns. The latency of one bank access is 80 ns. If c = 2 and k = 24, then latency of retrieving a cache block starting at address zero from main memory is

(a) 92 ns (b) 104 ns
(c) 172 ns (d) 184 ns

[2006 : 2 Marks]

Common Data for Q.1.17 & Q.1.18

Consider two cache organizations : The first one is 32 KB 2-way set associative with 32-byte block size. The second one is of the same size but direct mapped. The size of an address is 32 bits in both cases. A 2-to-1 multiplexer has latency of 0.6 ns while a k-bit comparator has a latency of k/10 ns. The hit latency of the set associative organization is h_1 while that of the direct mapped one is h_2.

1.17 The value of h_1 is

(a) 2.4 ns (b) 2.3 ns
(c) 1.8 ns (d) 1.7 ns

[2006 : 2 Marks]

1.18 The value of h_2 is

(a) 2.4 ns (b) 2.3 ns
(c) 1.8 ns (d) 1.7 ns

[2006 : 2 Marks]

Common Data for Q.1.19 & Q.1.20

A CPU has a 32 KB direct mapped cache with 128-byte block size. Suppose A is a two dimensional array of size 512 × 512 with elements that occupy 8-bytes each. Consider the following two C code segments, P_1 and P_2.

```
P1 : for (i = 0; i < 512; i ++)  {
         for (j = 0; j < 512; j ++) {
               x + = A [i] [j];
         }
     }
P2 : for (i = 0; i < 512; i++)  {
         for (j = 0 ; j < 512; j ++) {
               x + = A [j] [i];
         }
     }
```

P1 and P2 are executed independently with the same initial state, namely, the array A is not in the cache and i, j, x are in registers. Let the number of cache misses experienced by P1 be M_1 and that for P2 be M_2.

1.19 The value of M_1 is

(a) 0 (b) 2048
(c) 16384 (d) 262144

[2006 : 2 Marks]

1.20 The value of the ratio M_1/M_2 is

(a) 0 (b) 1/16
(c) 1/8 (d) 16

[2006 : 2 Marks]

1.21 Consider a 4-way set associative cache consisting of 128 lines with a line size of 64 words. The CPU generates a 20-bit address of a word in main memory. The number of bits in the TAG, LINE and WORD fields are respectively

(a) 9, 6, 5 (b) 7, 7, 6
(c) 7, 5, 8 (d) 9, 5, 6

[2007 : 1 Mark]

Common Data for Q. 1.22 & Q. 1.23

Consider a machine with a byte addressable main memory of 2^{16} bytes. Assume that a direct mapped data cache consisting of 32 lines of 64 bytes each is used in the system. A 50 × 50 two-dimensional array of bytes is stored in the main memory starting from memory location 1100 H. Assume that the data cache is initially empty. The complete array is accessed twice. Assume that the contents of the data cache do not change in between the two accesses.

1.22 How many data cache misses will occur in total?

(a) 48 (b) 50
(c) 56 (d) 59

[2007 : 2 Marks]

1.23 Which of the following lines of the data cache will be replaced by new blocks in accessing the array

(a) line 4 to line 11 (b) line 4 to line 12
(c) line 0 to line 7 (d) line 0 to line 8

[2007 : 2 Marks]

Common Data for Q. 1.24 & 1.25

Consider a computer with a 4-ways set-associative mapped cache of the following characteristics: a total of 1 MB of main memory, a word size of 1 byte, a block size of 128 words and a cache size of 8 KB.

1.24 The number of bits in the TAG, SET and WORD fields, respectively are:

(a) 7, 6, 7 (b) 8, 5, 7
(c) 8, 6, 6 (d) 9, 4, 7

[2008 : 2 Marks]

1.25 While accessing the memory location 00795H by the CPU, the contents of the TAG field of the corresponding cache line is

(a) 000011000 (b) 110001111
(c) 00011000 (d) 110010101

[2008 : 2 Marks]

1.26 For inclusion to hold between two cache level L1 and L2 in a multilevel cache hierarchy, which of the following are necessary?

1. L1 must be a write-through cache
2. L2 must be write-through cache
3. The associativity of L2 must be greater that of L1
4. The L2 cache must be at least as large as the L1 cache

(a) 4 only (b) 1 and 2 only
(c) 1, 2 and 4 only (d) 1, 2, 3 and 4

[2008 : 2 Marks]

1.27 In an instruction execution pipeline, the earliest that the data TLB (Translation Look a side Buffer) can be accessed is

(a) before effective address calculation has started
(b) during effective address calculation
(c) after effective address calculation has completed
(d) after data cache lookup has completed

[2008 : 2 Marks]

Common Data for Q.1.28 to 1.30

Consider a machine a 2-way set associative data cache of size 64 Kbytes and block size 16 bytes. The cache is managed using 32 bit virtual addressed and the page size is 4 Kbytes. A program to be run on this machine begins as follows:

```
double APR [1024] [1024]
int i, j ;
/ * Initalize array APR to 0.0 */
for ( i = 0; i < 1024; i ++)
    for ( j = 0; j < 1024; j++
        APR [i] [j] = 0.0;
```

The size of double 8 bytes. Array APR is in memory starting at the beginning of virtual page 0×FF000 and stored in row major order. The cache is initially empty and no pre-fetching is done. The only data memory references made by the program are those to array APR.

1.28 The total size of the tags in the cache directory is
(a) 32 kbits (b) 34 kbits
(c) 64 kbits (d) 68 kbits
[2008 : 2 Marks]

1.29 Which of the following array elements has the same cache index as APR[0][0]?
(a) APR[0][4] (b) APR[4][0]
(c) APR[0][5] (d) APR[5][0]
[2008 : 2 Marks]

1.30 The cache hit ratio for this initialization loop is
(a) 0% (b) 25%
(c) 50% (d) 75%
[2008 : 2 Marks]

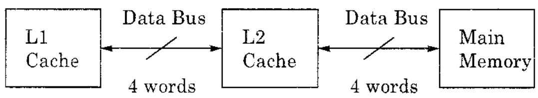
1.31 How many 32K × 1 RAM chips are needed to provide a memory capacity of 256 K-bytes?
(a) 8 (b) 32
(c) 64 (d) 128
[2009 : 1 Mark]

1.32 Consider a 4-way set associative cache (initially empty) with total 16 cache blocks. The main memory consists of 256 blocks and the request for memory blocks is in the following order:
0, 255, 1, 4, 3, 8, 133, 159, 216, 129, 63, 8, 48, 32, 73, 92, 155
Which one of the following memory block will NOT be in cache if LRU replacement policy is used?
(a) 3 (b) 8
(c) 129 (d) 216
[2009 : 2 Marks]

1.33 A main memory unit with a capacity of 4 megabytes is built using 1M × 1-bit DRAM chips. Each DRAM chip has 1K rows of cells with 1K cells in each row. The time taken for a single refresh operation is 100 nanoseconds. The time required to perform one refresh operation on all the cells in the memory unit is
(a) 100 nanoseconds
(b) $100*2^{10}$ nanoseconds
(c) $100*2^{20}$ nanoseconds
(d) $3200*2^{20}$ nanoseconds
[2010 : 1 Mark]

Common Data for Q. 1.34 & Q. 1.35

A computer system has an L1 and L2 cache, an L2 cache, and a main memory unit connected as shown below. The block size in L1 cache is 4 words. The block size in L2 cache is 16 words. The memory access times are 2 nanoseconds, 20 nanoseconds and 200 nanoseconds for L1 cache, L2 cache and main memory unit respectively.



1.34 When there is a miss in L1 cache and a hit in L2 cache, a block is transferred from L2 cache to L1 cache. What is the time taken for this transfer?
(a) 2 nanoseconds (b) 20 nanoseconds
(c) 22 nanoseconds (d) 88 nanoseconds
[2010 : 2 Marks]

1.35 When there is a miss in both L1 cache and L2 cache, first a block is transferred from main memory to L2 cache, and then a block is transferred from L2 cache to L1 cache. What is the total time taken for these transfers?
(a) 222 nanoseconds (b) 888 nanoseconds
(c) 902 nanoseconds (d) 968 nanoseconds
[2010 : 2 Marks]

1.36 An 8KB direct-mapped write back cache is organized as multiple blocks, each of size 32 bytes. The processor generates 32 bit addresses. The cache controller maintains the tag information for each cache block comprising of the following.

1 Valid bit
1 Modified bit
As many bits as the minimum needed to identify the memory block mapped in the cache.
What is the total size of memory needed at the cache controller to store meta-data (tags) for the cache?

(a) 4864 bits (b) 6144 bits
(c) 6656 bits (d) 5376 bits

[2011 : 2 Marks]

Linked Answer Q. 1.37 & Q. 1.38

A computer has a 256 Kbyte, 4-way set associative, write back data cache with block size of 32 Bytes. The processor sends 32 bit addresses to the cache controller. Each cache tag directory entry contains, in addition to address tag, 2 valid bits, 1 modified bit and 2 replacement bit.

1.37 The number of bits in the tag field of an address is

(a) 11 (b) 14
(c) 16 (d) 27

[2012 : 2 Marks]

1.38 The size of the cache tag directory is

(a) 160 Kbits (b) 136 Kbits
(c) 40 Kbits (d) 32 Kbits

[2012 : 2 Marks]

1.39 In a k-way set associative cache, the cache is divided into v sets, each of which consists of k lines. The lines of a set are placed in sequence on after another. The lines in set s are sequenced before the lines in set (s + 1). The main memory blocks are numbered 0 onwards. The main memory block numbered 'j' must be mapped to any one of the cache lines from

(a) (j mod v) * k to (j mod v) * k + (k – 1)
(b) (j mod v) * (j mod v) + (k – 1)
(c) (j mod k) to (j mod k) + (v – 1)
(d) (j mod k) * v (j mod k) * v + (v – 1)

[2013 : 1 Mark]

1.40 A RAM chip has capacity of 1024 words of 8 bits each (1K × 8). The number of 2 × 4 decoders with enable line needed to construct a 16 K × 16 RAM from 1K × 8 RAM is

(a) 4 (b) 5
(c) 6 (d) 7

[2013 : 2 Marks]

1.41 An access sequence of cache block address of length N and contains n unique block addresses. The number of unique block addresses between two consecutive accesses to the same block address is bounded above by k. What is the miss ratio if the access sequence is passed through a cache of associativity $A \geq k$ exercising least-recently used replacement policy?

(a) n/N (b) $1/N$
(c) $1/A$ (d) k/n

[2014 (Set-1) : 2 Marks]

1.42 A 4-way set-associative cache memory unit with a capacity of 16 KB is built using a block size of 8 words. The word length is 32 bits. The size of the physical address space is 4 GB. The number of bits for the TAG field is _______.

[2014 (Set-2) : 1 Mark]

1.43 In designing a computer's cache system, the cache block (or cache line) size is an important parameter. Which one of the following statements is correct in this context?

(a) A smaller block size implies better spatial locality
(b) A smaller block size implies a smaller cache tag and hence lower cache tag overhead
(c) A smaller block size implies a larger cache tag and hence lower cache hit time
(d) A smaller block size incurs a lower cache miss penalty

[2014 (Set-2) : 2 Marks]

1.44 If the associativity of a processor cache is doubled while keeping the capacity and block size unchanged, which one of the following is guaranteed to be NOT affected?

(a) Width of tag comparator
(b) Width of set index decoder
(c) Width of way selection multiplexor
(d) Width of processor to main memory data bus

[2014 (Set-2) : 2 Marks]

1.45 The memory access time is 1 nanosecond for a read operation with a hit in cache, 5 nanoseconds for a read operation with a miss in cache, 2 nanoseconds for a write operation with a hit in cache and 10 nanoseconds for a write operation with a miss in cache. Execution of a sequence of instructions involves 100 instruction fetch operations, 60 memory operand read operations and 40 memory operand write operations. The cache hit-ratio is 0.9. The average memory access time (in nanoseconds) in executing the sequence of instructions is ________.

[2014 (Set-3) : 2 Marks]

1.46 Assume that for a certain processor, a read request takes 50 nanoseconds on a cache miss and 5 nanoseconds on a cache hit. Suppose while running a program, it was observed that 80% of the processor's read requests result in a cache hit. The average read access time in nanoseconds is____________.

[2015 (Set-2) : 1 Mark]

1.47 Consider a machine with a byte addressable main memory of 2^{20} bytes, block size of 16 bytes and a direct mapped cache having 2^{12} cache lines. Let the addresses of two consecutive bytes in main memory be $(E201F)_{16}$ and $(E2020)_{16}$. What are the tag and cache line address (in hex) for main memory address $(E201F)_{16}$?

(a) E, 201 (b) F, 201
(c) E, E20 (d) 2, 01F

[2015 (Set-3) : 1 Mark]

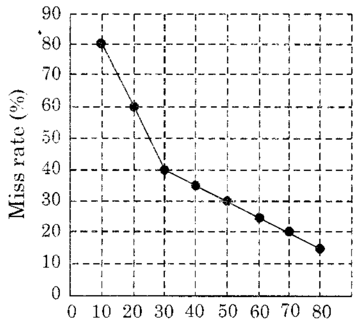
1.48 A processor can support a maximum memory of 4 GB, where the memory is word-addressable (a word consists of two bytes). The size of the address bus of the processor is at least ______ bits.

[2016 (Set-1) : 1 Mark]

1.49 The width of the physical address on a machine is 40 bits. The width of the tag field in a 512 KB 8-way set associative cache is _______bits.

[2016 (Set-2) : 2 Marks]

1.50 A file system uses an in-memory cache to cache disk blocks. The miss rate of the cache is shown in the figure. The latency to read a block from the cache is 1 ms and to read a block from the disk is 10 ms. Assume that the cost of checking whether a block exists in the cache is negligible. Available cache sizes are in multiples of 10 MB.



The smallest cache size required to ensure an average read latency of less than 6 ms is_________ MB.

[2016 (Set-2) : 2 Marks]

Answers Cache and Main Memory

1.1	(a)	1.2	(a)	1.3	(b)	1.4	(b)	1.6	(d)	1.7	(d)	1.8	(a)	1.9	(a)	1.10	(a)
1.3	(b)	1.4	(d)	1.5	(b)	1.6	(d)	1.7	(b)	1.8	(d)	1.9	(c)	1.10	(c)	1.11	(c)
1.12	(c)	1.13	(a)	1.14	(c)	1.15	(a)	1.16	(d)	1.17	(a)	1.18	(d)	1.19	(c)	1.20	(b)
1.21	(d)	1.22	(c)	1.23	(a)	1.24	(d)	1.25	(a)	1.26	(b)	1.27	(b)	1.28	(d)	1.29	(b)
1.30	(c)	1.31	(c)	1.32	(d)	1.33	(d)	1.34	(d)	1.35	(d)	1.36	(d)	1.37	(c)	1.38	(a)
1.39	(a)	1.40	(b)	1.41	(a)	1.43	(d)	1.44	(d)	1.47	(a)						

Explanations Cache and Main Memory

1.1 Sol.

(i) Main memory size

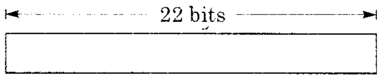
$= 16384$ blocks

$= 16384 \times 256$ words

$= 2^{14} \times 2^{8}$ words

$= 2^{22}$ words

Number of bits required for addressing the main memory is 22 bits



Physical Address

(ii) Set associative cache

Block size = 256 word = 2^8 words

$\therefore$ No. of bits in WORD OFFSET is 8 bits

No. of blocks in set-associative cache = 128

Number of blocks in one set = 4

$$\therefore \text{ No. of sets in cache} = \frac{128}{4} = 32 = 2^5$$

Number of bits in SET OFFSET is 5

22 bits

TAG	SET OFFSET	WORD OFFSET
9 bits	5 bits	8 bits

Number of bits in TAG = 22 – (8 + 5) = 9 bits

1.2 Sol.

Main memory access time,

$$t_{a_1} = 10^{-8} \text{ sec}$$

Secondary memory access time,

$$t_{a_2} = 10^{-3} \text{ sec}$$

Access efficiency,

$$\eta = 80\% = 0.8$$

Average access time,

$$t_{avg} = \eta \times t_{a_2} = 0.8 \times 10^{-3} \text{ sec}$$

Hit ratio, H

$$t_{avg} = H \times t_{a_1} + (1 - H) t_{a_2}$$

$$0.8 \times 10^{-3} = H \times 10^{-8} + (1 - H) \times 10^{-3}$$

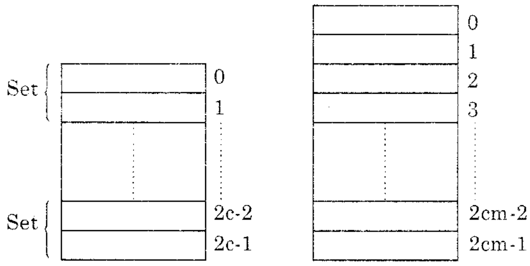
$$0.8 = H \times 10^{-5} + (1 - H)$$

$$H(1 - 10^{-5}) = 0.2$$

$$H = \frac{0.2}{(1 - 10^{-5})} = 0.200002 \approx 20\%$$

The hit ratio H must be 20%.

1.3 (b)



Set Associative Cache   Main Memory

# of set in set associative cache

$$= \frac{\text{\# of blocks in cache}}{\text{\# of blocks in one set}}$$

$$= \frac{2c}{2} = c$$

# of sets in cache = c

Therefore, the block k of the main memory maps to the set (k mod c) of the cache.

1.4 (d)

Virtual memory is a concept which allows programs larger than the physical memory size to run.

1.5 (b)

1 MB of memory supported by graphics card

(a) [1600*400*$\log_2 256$ bits] < 1 MB

(b) [1600*400*$\log_2 10^6$ bits] > 1 MB

(c) [800*400*$\log_2 10^6$ bits] < 1 MB

(d) [800*800*$\log_2 256$ bits] < 1 MB

$\therefore$ Option (b) can not supported by card.

1.6 (d)

Disk is the IO device attached externally to the processor. Therefore, disk requires a device driver.

1.7 (b)

There are two types of locality of reference:

1. **Temporal:** A recently executed instruction is likely to be executed again very soon.
2. **Spatial:** Instructions in close proximity to a recently executed instruction are also likely to be executed soon.

The spatial aspect suggests that instead of fetching just one item from the main memory to the cache, it is useful to fetch several items that reside at adjacent address as well.
Therefore more than one work are put in one cache block to exploit the spatial locality of reference in the program.

1.8 **(d)**

Techniques that automatically move program and data blocks into the physical main memory when they are required for execution are called virtual memory techniques. Programs, and hence the processor, reference an instruction and data space that is independent of the available physical main memory space. Virtual memory increases the degree of multi-programming.
However virtual memory increases the context switching overhead.

1.9 **(c)**

Access time
$= t_1 h_1 + (1 - h_1)h_2\, t_2 + (1 - h_1)\,(1 - h_2)t_m$
$= 1*0.8 + 0.2*0.9*10 + 0.2*0.1*500$
$= 0.8 + 1.8 + 10$
$= 12.6$ ns.

1.10 **(c)**

Sequence 8, 12, 0, 12, 8
Apply the LRU as follows

Page Frame

3		3		3		3		3	
2		2		2		2		2	
1		1	12	1	12	1	12	1	12
0	8	0	8	0	0	0	0	0	8
Miss		Miss		Miss		No Miss		Miss	

or

Set 0	~~8~~	~~8~~	8
	12		
Set 1			

Total number of miss = 4

1.11 **(c)**

Time for refresh = 100ns
Number of refreshes in a memory cycle
= 100*memory cycle time/1ms
= 100*64ns/1ms
$= 6.4*10^{-3}$
So, time for refresh in a memory cycle
$= 100\text{ns}*6.4*10^{-3}$
$= 64*10^{-2}$ ns
So, percentage of time spent for refresh
$= (64*10^{-2}/64)*100 = 1\%$.

1.12 **(c)**

Increasing the RAM means increase the primary memory which reduce the swapping so fewer page faults occur.

1.13 **(a)**

Size of cache = 32KB
$= 32 \times 2^{10}$ byte
$= 2^5 \times 2^{10}$ byte
$= 2^{15}$ byte = 15 bits
Size of tag = 32 – 15 = 17 bits
Cache indexing size = 10 bits

1.14 **(c)**

For 1 sec it is 10^9 bytes, for 64 bytes?
$= 64*1/10^9 = 64$ ns
MM latency is 32 ns.
Total time required to place cache line
= 64 + 32 = 96 ns.

1.15 **(a)**

1. **I-cache:** Number of blocks in cache
   $= 4\text{ K}/4 = 2^{10}$ blocks
   Bits to represent blocks = 10
   No. of words in a block $= 4 = 2^2$ words
   Bits to represent words = 2
   Tag bits = 30 – (10+2) = 18
   Each block will have it's own tag bits. So total tag bits = 1K × 18 bits.
2. **D-cache:** Number of blocks in cache
   $= 4\text{ K}/4 = 2^{10}$ blocks
   No. of sets in cache $= 2^{10}/2 = 2^9$ sets
   Bits to represent sets = 9
   No. of words in a block $= 4 = 2^2$ words
   Bits to represent words = 2
   Tag bits = 30 – (+2) = 19
   Each block will have it's own tag bits. So total tag bits = 1K × 19 bits.
3. **L2 cache:** Number of blocks in cache
   $= 64\text{K}/16 = 2^{12}$ blocks.
   No. of sets in cache $= 2^{12}/4 = 2^8$ sets
   Bits to represent sets = 8

No. of words in cache = 16 = 2^4 words

Bits to represent words = 4

Tag bits = 30 – (8+4) = 18

Each block will have it's own tag bits. So total tag bits = $2^{12} \times 18$ bits = 4K × 18 bits

1.16 (d)

Cache block size = 64 bytes

Main memory has K banks or k = 24

Each bank is 2 byte long because c = 2

Total time for one parallel access

$$T = K/2 + \text{Latency}$$
$$= 24/2 + 80$$
$$= 12 + 8 = 92 \text{ ns}$$

Total latency time = CT

$$= 2 \times 92 = 184 \text{ ns}$$

1.17 (a)

Tag	Set	Block
18	9	5

So h_1 = 18/10 + 0.6 ns
= 1.8 + 0.6 ns = 2.4 ns

1.18 (d)

Tag	Set	Block
17	10	5

So h_2 = 17/10 ns = 1.7 ns

1.19 (c)

$$\begin{bmatrix} a[0][0], a[0][1], \ldots\ldots, a[0][511] \\ a[1][0], a[1][1], \ldots\ldots, a[1][511] \\ \vdots \ldots\ldots\ldots\ldots\ldots \\ a[511][0], a[511][1], \ldots\ldots, a[511][511] \end{bmatrix}$$

One element size = 8 B

Block size = 128 B

$\therefore$ One block holds = $\frac{128}{8}$ = 16 element

1 row contains = 512 elements

$\therefore$ 32 block are required to carry row

Cache memory size = 32 kB

Block size = 128 B

$$\text{\# lines} = \frac{32\text{kB}}{128\text{B}} = \frac{2^{15}}{2^7} = 2^8 = 256$$

P1: In this array is accessed in row-major order like

a[0][0], a[0][1], a[0][2], ...

Accessing a[0][0] = Miss

Accessing a[0][1] = hit

Accessing a[0][2] = hit

⋮

Accessing a[0][15] = hit

Accessing a[0][16] = Miss

Accessing a[0][17] = hit

⋮

Accessing a[0][31] = hit

Accessing a[0][32] = Miss

⋮

$\therefore$ One row accessing = 32 miss operations

Total miss (M_1) = 32 × 512 = 16384

1.20 (b)

P_2: In this array is accessed in column-major order like.

a[0][0], a[1][0], a[2][0], ...

Accessing a[0][0] = Miss

Accessing a[1][0] = Miss

Accessing a[2][0] = Miss

⋮

Accessing a[511][0] = Miss

Accessing a[0][1] = Miss

⋮

Accessing a[511][1] = Miss

⋮

One column accessing = 512 miss operation

Total miss (M_2) = 512 * 512

$$\therefore \frac{M_1}{M_2} = \frac{16384}{512 \times 512} = \frac{1}{16}$$

1.21 (d)

4-way set associative

lines = 128

Block size = 64 words

← 20 bits →

TAG	SET OFFSET	WORD OFFSET
⇓	⇓	⇓
20 – (5 + 6) = 9 bits	$\log_2 32$ = 5 bits	$\log_2 (64)$ = 6 bits

$$\text{Number of set} = \frac{\text{\#lines}}{4} = \frac{128}{4} = 32$$

1.22 (c)

$$\begin{bmatrix} a[0][0],\ a[0][1],\ \ldots\ldots\ldots\ldots,\ a[0][49] \\ a[1][0],\ a[1][1],\ \ldots\ldots\ldots\ldots,\ a[1][49] \\ \vdots \\ a[49][0],\ a[49][1],\ldots\ldots\ldots,\ a[49][49] \end{bmatrix}$$

One element size = 1 B
Total array elements = 50 × 50 = 2500
So total array size = 2500 × 1 B = 2500 B
Block size = 64 B
∴ # block required to store the array

$$= \frac{2500\,\text{B}}{64\,\text{B}} = 39.04 \approx 40$$

lines in cache = 32
Block size = 64 B
Physical address = 1100 H

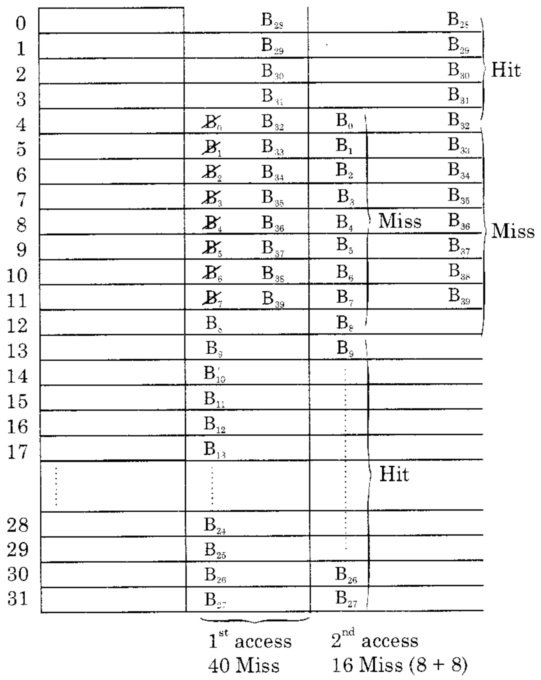
← 16 bits →

TAG	LINE OFFSET	WORD OFFSET
5 bit	5 bit	6 bit

$$(1100)_H = (0001000100000000)_B$$

TAG	LO	WO
00010	00100	000000

∴ LO = $(00100)_B = 4$
∴ Array is stored in the main memory starting from memory location of line 4.



∴ Total data cache misses will occur in total
= 40 + 16 = 56

1.23 (a)

The lines of the data cache that is replaced by new blocks in accessing the array = line 4 to line 11.

1.24 (d)

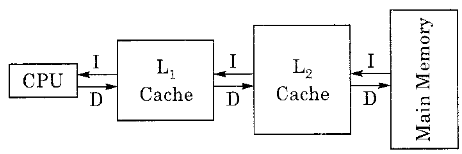
Number of cache blocks = 8KB/(128 * 1) = 64
Number of sets in cache (4-way set) = Number of cache blocks/4 = 64/4 = 16.
So, number of SET bits required = 4 (as 24 = 16, and with 4 bits we can get 16 possible outputs).
We can divide the physical memory into 16 regions so that, each set maps into only its assigned region. So, size of a region a set can address = 1MB/16 = 216 Bytes = 216 /128 = 29 cache blocks (as cache block size is 128 words = 128 bytes).
So, when an access comes to an cache entry, it must be able to determine which out of the 29 possible physical block it is. In short it needs 9 bits for TAG. Now, cache block size is 128 words and so to identify a word we need 7 bits for WORD.

1.25 (a)

We have 16 sets in cache and correspondingly 16 regions in physical memory to which each set is mapped.
WORD bit size is 7 as we need 7 bits to address 128 possible words in a cache block.
So, the lowest 7 bits of 0C795H will be used for this giving us the remaining bits as 0000 1100 0111 1 of these bits, the lower 4 are used for addressing the 16 possible sets so tag bits: 0000 1100 0.

1.26 (b)

Consider the following diagram



L_1 and L_2 both are write through caches. This two conditions are necessary.

1.27 (b)

In instruction execution pipeline, the earliest that the data TLB can be accessed during effective address calculation has started.

1.28 (d)

2 way set associative
Cache memory size = 64 kB
Block size = 16 B

$$\# \text{ lines} = \frac{64\,\text{kB}}{16\,\text{B}} = 4\text{ K} = 2^2 \times 2^{10} = 2^{12}$$

$$\#\text{sets} = \frac{2^{12}}{2} = 2^{11}$$

32-bit virtual address

← 32 bits →

TAG	SO	WO
⇓	⇓	⇓
32 − (4 + 11) = 17 bits	$\log_2 (2^{11})$ = 11 bits	$\log_2 (16)$ = 4 bits

Tag memory size
= Number of sets * No. of lines in a set * Number of tag bits
$= 2^{11} \times 2 \times 17$ bits
$= 68 \times 2^{10}$ bits = 68 K bits

1.29 (b)

Array is APR [1024][1024]

$$\begin{bmatrix} a[0][0],\ a[0][1],\ \ldots\ldots\ldots,\ a[0][1023] \\ a[1][0],\ a[1][1],\ \ldots\ldots\ldots,\ a[1][1023] \\ \vdots \\ a[1023][0],\ a[1023][1],\ \ldots a[1023][1023] \end{bmatrix}$$

1 element size = 8 B
Block size = 16 B
∴ One block holds 2 elements
One row size = 1024 element
∴ No. of blocks required to store one row
= 512 blocks

Sets:

0	APR [0], APR [0][1]	APR [4][0], APR [4][1]
1	APR [0][2], APR [0][3]	APR [4][2], APR [4][3]
2	APR [0][4], APR [0][5]	APR [4][4], APR [4][5]
511	⋮ 1st Row	5th Row
512	APR [0][1022], APR [0][1023]	APR [4][4], APR [4][5]
	APR [1][0], APR [1][1]	
	⋮ 2nd Row	6th Row
1023	a, APR[1][1023]	
1024	⋮ 3rd Row	7th Row
1535		
1536		
	⋮ 4th Row	8th Row
2047		

APR[0][0] has the same cache index as APR [4][0]

1.30 (c)

Row major access
a[0] [0] = Miss
a[0] [1] = hit
a[0] [2] = Miss
a[0] [3] = hit
⋮
Every miss is followed by a hit

$$\therefore \text{ Hit ratio} = \frac{1024}{1024+1024} = \frac{1024}{2048} = \frac{1}{2} = 50\%$$

1.31 (c)

$$\text{Basic RAM} = 32\text{ K} \times 1$$
$$\text{Design RAM} = 256\text{ K} \times 8$$
$$\text{No. of RAM} = \frac{256\text{ K} \times 8}{32\text{ K} \times 1} = \frac{2^{18} \times 2^3}{2^{15} \times 2^0} = 64$$

It require 8 parallel line and in each parallel line 8 serial RAM chip are required.

1.32 (d)

There are total 4 sets in the cache and each set contains 4 blocks.

Set 0	~~0~~ 48
	~~4~~ 32
	8
	~~216~~ 92
Set 1	1
	133
	129
	73
Set 2	
Set 3	~~255~~ 155
	3
	159
	63

0 mod 4 = 0 set 0
255 mod 4 = 3 set 3
1 mod 4 = 1 set 1
4 mod 4 = 0 set 0
3 mod 4 = 3 set 3
8 mod 4 = 0 set 0
133 mod 4 = 1 set 1

159 mod 4	=	3	set 3
216 mod 4	=	0	set 0
129 mod 4	=	1	set 1
63 mod 4	=	3	set 3
8 mod 4	=	0	set 0 (already in cache)
48 mod 4	=	0	set 0 48 will replace block 0 using LRU
32 mod 4	=	0	set 0 32 will replace block 4
73 mod 4	=	1	set 1
92 mod 4	=	0	set 0 92 will replace block 216
155 mod 4	=	3	set 3 155 will replace 255

1.33 (d)

Main memory unit has capacity = 4 MB

$$\text{No of DRAM chips} = \frac{4\text{ MB}}{1\text{M} \times 1\text{ bit}}$$

$$= \frac{4\text{ M} \times 8\text{ bit}}{1\text{ M} \times 1\text{ bit}} = 32$$

1 DRAM chip has 1K (2^{10}) Rows and 1 K(2^{10}) cells

In each row

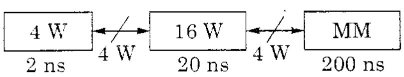
$$= 32 \times 2^{10} \times 2^{10} = 32 \times 2^{20}$$

Time taken in one refresh operation = 100 ns

Time required to perform one refresh operation on all the cells in MM unit

$$= 32 \times 2^{20} \times 100 = 3200 \times 2^{20}\text{ ns}$$

1.34 (d)



Memory access time to access L1 = 2 nanoseconds

Memory access time to access L2 = 20 nanoseconds

When there is a miss in L1 cache and hit in L2 cache so we have to access first L1 then after it L2 and the data bus size is 4 W. Now the access time is calculated as

2	Access time of L1 cache
20	Access time of L2 cache
2	Access time of L1 cache
20	Access time of L2 cache
2	Access time of L1 cache
20	Access time of L2 cache
2	Access time of L1 cache
20	Access time of L2 cache
88 ns	

1.35 (d)

When there is a miss in L1 and L2 cache then block transfer for mm to L2 cache, and then block is transferred from L2 cache to L1 cache- So total time for these transfer

Total Access time = Block transfer time from main memory to L2 cache + access time of L2 + Access time of L1

2	Access time of L1 cache
20	Access time of L2 cache
200	Access time of MM
20	Access time of L2 cache
200	Access time of MM
20	Access time of L2 cache
200	Access time of MM
20	Access time of L2 cache
200	Access time of MM
20	Access time of L2 cache
2	Access time of L1 cache
20	Access time of L2 cache
2	Access time of L1 cache
20	Access time of L2 cache
2	Access time of L1 cache
20	Access time of L2 cache
968 ns	

1.36 (d)

Cache memory size = 8KB

Block size = 32 bytes

Mapping technique is Direct.

Physical address size is 32 bits

cache lines= $2^{13} / 2^5 = 2^8 = 256$ lines

Memory address interpretation in direct mapping is:

Modified bit	Valid bit	tag	Line offset	Word offset

Word offset requires =5 bits

Line offset requires = 8 bits

Tag requires=32bits –(8+5)bits = 19 bits

The metadata present in cache memory is = data memory + tag size

Tag size = No of cache lines*No of bits in tag field for any mapping.

Hence the total size of memory needed at the cache controller to store metadata tags for the write back cache is:

= Number of cache lines*(tag bits + valid bit + modified bit)
= 256*(19+1+1) = 256*21 = 5376 bits.

1.37 **(c)**

$$\text{No. of cache lines} = \frac{2^8 \times 2^{10}}{2^5} = 2^{13}$$

$$\text{No. of sets} = \frac{2^{13}}{2^2} = 2^{11}$$

32

16	11	5
Tags	So	Word offset

No. of tag bits are 16 bits
∴ option 'C' is correct.

1.38 **(a)**

Tag directory size = Number of sets x no. of tag s in each sets x (no. of tag bits in each tag +valid bits + modified bit + replacement bit)
= $2^{11} \times 4 \times (16 + 2 + 1 + 1)$ = 160 KBits
∴ Option (a) is correct.

1.39 **(a)**

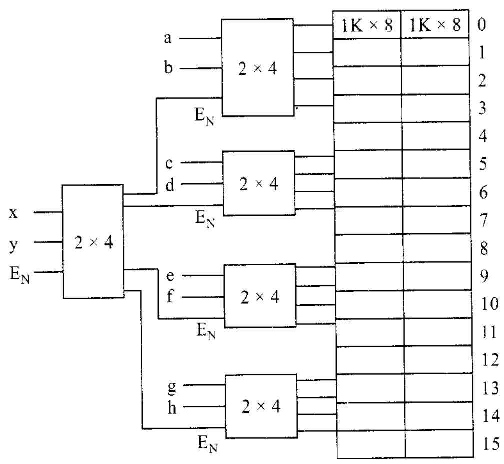
Main memory block 'j' mapped to any of cache line. Cache line range is:
(j mod v)*k → (j mod v)*k + (k – 1)

1.40 **(b)**

We need 16 K × 16 RAM from 1 K × 8 RAM
So, number of chips required

$$= \frac{16\text{K} \times 16}{1\text{K} \times 8} = 16 \times 2$$

So we need 16 output lines



∴ We need 5 decoders.

1.41 **(a)**

Miss ratio = n/N, if the access sequence is passed through a cache of associativity $A \geq k$ exercising least-recently used replacement policy.

1.42 **Sol.**

TAG	SET	WORD
20	7	5

← 32 bit →

WORD offset = 5 bits
[∵ word length = 32 bits]
SET offset = 7 bits

$$[\because \text{ \# blocks} = \frac{16\text{kB}}{8\,\text{Words}} = 512 \text{ blocks}$$

sets=512/4 = 128]
∴ # TAG bits = 32 – (7 + 5) = 20 bits.

1.43 **(d)**

A smaller cache block implies a larger cache tag and hence higher cache hit time.
So it incurs lower cache miss penalty.

1.44 **(d)**

By doubling the associativity of the cache, width of the processor to main memory databus is guaranteed to be not affected.

1.45 **Sol.**

Average read time
= 0.9 × 1ns + 0.1 × 5ns
= 0.9 + 0.5 = 1.4ns
In the execution sequence number of read operations = 160
So total time required for read operation
= 160 × 1.4ns = 224ns
Average write time = 0.9 × 2ns + 0.1 × 10ns
= 1.8 + 1 = 2.8ns
In the execution sequence number write operation = 40.
So, the total time required for write operation = 40 × 2.8ns = 112ns
Total time for instruction execution time for both read and write = 224 + 112 = 336ns.

200 times, access takes ____ 336 ns.
1 time, access takes ______ ?
Average memory access time

$$= \frac{336}{200} = 1.68 \text{ ns}$$

1.46 Sol.

$$T_{readmiss} = 50 \text{ nsec}$$
$$T_{readhit} = 5 \text{ nsec}$$
$$h = 0.8$$
$$T_{avgread} = h \times T_{readhit} + (1-h) \times T_{readmiss}$$
$$= 0.8 \times 5 + 0.2 \times 50$$
$$= 4 + 10 = 14 \text{ nsec}$$

1.47 (a)

Block size = 16 bytes = 2^4 bytes

offset = 4 bits

Index = 12 bits [$\because$ number of lines = 2^{12}]

Main memory size = 2^{20} bytes

$\Rightarrow$ Physical address = 20 bits

(4)	(12)	(4)
TAG	INDEX	OFFSET

20 bits

$$\underline{E}\quad \underline{2}\quad \underline{0}\quad \underline{1}\quad \underline{F}$$
$$1110\ \ 0010\ \ 0000\ \ 0001\ \ 1111$$

$\therefore$ E is tag and 201 is line address (index).

1.48 Sol.

Main memory size = 4 GB
Word size = 2 byte
1 word — 2 byte
? words — 4 GB
$\therefore$ Total 2 G words are present.
So, address size 2 G = 31 bits.

1.49 Sol.

40 bit

Tag	Set offset	word offset
21		

cache index ($\log_2 512$ K = 19 bits)
Block size not given so, cache index is considered

Or

Hypothetically, line size considered as cell size.

$\therefore$ Number of lines = 512 K

So, number of sets = $\dfrac{512\text{ K}}{8} \Rightarrow \dfrac{2^{19}}{2^3} = 2^{16}$

40 bit

Tag	Set offset	Word offset
24	16	Block size not given. So, no word offset.

So, TAG size is 24 bits.

1.50 Sol.

1. Miss rate = 0.8, 10 MB
   $T_c = 1$ ms
   $T_d = 10$ ms
   $T_{avg} = HT_c + (1 - H)(T_d + T_c) = 9$ ns
2. Miss rate = 0.6, 20 MB
   $T_c = 7$ ms
3. Miss rate = 0.4, 30 MB
   $T_{avg} = 5$ ms
4. Miss rate = 0.35, 40 MB
   $T_{avg} = 4.5$ ms
5. Miss rate = 0.3, 50 MB
   $T_{avg} = 4$ ms
6. Miss rate = 0.25, 60 MB
   $T_{avg} = 3.5$ ms
7. Miss rate = 0.2, 70 MB
   $T_{avg} = 3$ ms
8. Miss rate = 0.15, 80 MB
   $T_{avg} = 2.5$ ms

So, 30 MB will the answer.

2 Instructions : Pipelining and Addressing Modes

2.1 The total size of address space in a virtual memory system is limited by
(a) the length of MAR
(b) the available secondary storage
(c) the available main memory
(d) All of the above

[1991 : 2 Marks]

2.2 Comparing the time T1 taken for a single instruction on a pipelined CPU with time T2 taken on a non-pipelined but identical CPU, we can say that
(a) $T1 \leq T2$
(b) $T1 \geq T2$
(c) $T1 < T2$
(d) T1 plus T2 is the time taken for one instruction fetch cycle

[2000 : 1 Mark]

2.3 The most appropriate matching for the following pairs

X. Indirect addressing	1. Loops
Y. Immediate addressing	2. Pointers
Z. Auto decrement addressing	3. Constants

(a) X – 3 Y – 2 Z – 1 (b) X – 1 Y – 3 Z – 2
(c) X – 2 Y – 3 Z – 1 (d) X – 3 Y – 1 Z – 2

[2000 : 1 Mark]

2.4 Which of the following is not a form of memory?
(a) Instruction cache
(b) Instruction Register
(c) Instruction opcode
(d) Translation look aside buffer

[2002 : 1 Mark]

2.5 In serial data transmission, every byte of data is padded with a '0' in the beginning and one or two '1's at the end of byte because
(a) Receiver is to be synchronized for byte reception
(b) Receiver recovers lost '0's and '1's from these padded bits
(c) Padded bits are useful in parity computation
(d) None of the above

[2002 : 1 Mark]

2.6 In 2's complement addition, overflow
(a) is flagged whenever there is carry from sign bit addition
(b) cannot occur when a +ve value is added to a –ve value
(c) is flagged when the carries from sign bit and previous bit match
(d) None of the above

[2002 : 1 Mark]

2.7 The performance of a pipelined processor suffers if
(a) the pipeline stages have different delays
(b) consecutive instructions are dependent on each other
(c) the pipline stages share hardware resources
(d) All of the above

[2002 : 2 Marks]

2.8 For a pipelined CPU with a single ALU, consider the following situations
1. The $j+1^{st}$ instruction uses the result of the j^{th} instruction as an operand
2. The execution of a conditional jump instruction
3. The j^{th} and $j+1^{st}$ instructions require the ALU at the same time

Which of the above can cause a hazard?
(a) 1 and 2 only (b) 2 and 3 only
(c) 3 only (d) All the three

[2003 : 1 Mark]

Common Data for Q.2.9 & Q.2.10

Consider the following assembly language program for a hypothetical processor. A, B and C are 8 bit registers. The meanings of various instructions are shown as comments.

```
     MOV  B, #0  ;  B ← 0
     MOV  C, #8  ;  C ← 8
Z :  CMP  C, #0  ;  Compare C with 0
     JZ   X      ;  Jump to X if 0 flag is set
     SUB  C, #1  ;  C ← C – 1
     RRC  A, #1  ;  Rotate right A through carry by
                    one bit
```

```
    JC   Y     ; Jump to Y if any carry flag is
                 set
    JMP  Z     ; Jump to Z
Y : ADD  B,#1  ; B ← B + 1
    JMP  Z     ; Jump to Z
X :
```

2.9 If the initial value of register A is A_0, the value of register B after the program execution will be

(a) the number of 0 bits in A_0
(b) the number of 1 bits in A_0
(c) A_0
(d) 8

[2003 : 2 Marks]

2.10 Which of the following instructions when inserted at location X will ensure that the value of register A after program execution is the same as its initial value?

(a) RRC A, # 1
(b) NOP; no operation
(c) LRC A, # 1; left rotate A through carry flag by one bit
(d) ADD A, # 1

[2003 : 2 Marks]

2.11 Which of the following addressing modes are suitable for program relocation at run time?

1. Absolute addressing
2. Based addressing
3. Relative addressing
4. Indirect addressing

(a) 1 and 4 (b) 1 and 2
(c) 2 and 3 (d) 1, 2 and 4

[2004 : 1 Mark]

2.12 Using a 4-bit 2's complement arithmetic, which of the following additions will result in an overflow?

(i) 1100 + 1100 (ii) 0011 + 0111
(iii) 1111 + 0111

(a) (i) only (b) (ii) only
(c) (iii) only (d) (i) and (iii) only

[2004 : 2 Marks]

2.13 If we use internal data forwarding to speed up the performance of a CPU (R1, R2 and R3 are registers and M[100] is a memory reference), then the sequence of operations

R1 → M[100]
M[100] → R2
M[100] → R3

can be replaced by

(a) R1 → R3
R2 → M[100]
(b) M[100] → R2
R1 → R2
R1 → R3
(c) R1 → M[100]
R2 → R3
(d) R1 → R2
R1 → R3
R1 → M[100]

[2004 : 2 Marks]

2.14 Consider a pipeline processor with 4 stages S1 to S4. We want to execute the following loop : for (i = 1; i < = 1000; i++) {I1, I2, I3, I4} where the time taken (in ns) by instructions I1 to I4 for stages S1 to S4 are given below:

	S1	S2	S3	S4
I1:	1	2	1	2
I2:	2	1	2	1
I3:	1	1	2	1
I4:	2	1	2	1

The output of I1 for i = 2 will be available after

(a) 11 ns (b) 12 ns
(c) 13 ns (d) 28 ns

[2004 : 2 Marks]

2.15 In an enhancement of a design of a CPU, the speed of a floating point unit has been increased by 20% and the speed of a fixed point unit has been increased by 10%. What is the overall speedup achieved if the ratio of the number of floating point operations to the number of fixed point operations is 2 : 3 and the floating point operation used to take twice the time taken by the fixed point operation in the original design?

(a) 1.155 (b) 1.185
(c) 1.255 (d) 1.285

[2004 : 2 Marks]

Common Data for Q.2.16 to Q.2.18

Consider the following program segment for a hypothetical CPU having three user registers R1, R2 and R3.

Instruction	Operation	Instruction Size (in words)
MOV R1,5000 ;	R1 ← Memory [5000]	2
MOV R2,(R1) ;	R2 ← Memory [(R1)]	1
ADD R2, R3 ;	R2 ← R2 + R3	1
MOV 6000, R2;	Memory [6000] ← R2	2
HALT ;	Machine halts	1

2.16 Consider that the memory is byte addressable with size 32 bits, and the program has been loaded starting from memory location 1000 (decimal). If an interrupt occurs while the CPU has been halted after executing the HALT instruction, the return address (in decimal) saved in the stack will be

(a) 1007 (b) 1020
(c) 1024 (d) 1028

[2004 : 2 Marks]

2.17 Consider that the memory is word addressable with size 32 bits and the program has been loaded starting from memory location 1000 (decimal). If an interrupt occurs during the ADD instruction, what will be the return address pushed on to the stack.

(a) 1007 (b) 1004
(c) 1005 (d) 1016

[2004 : 2 Marks]

2.18 Let the clock cycles required for various operations be as follows:
Register to/from memory transfer:
3 clock cycles
ADD with both operands in register:
1 clock cycle
Instruction fetch and decode:
2 clock cycles per word
The total number of clock cycles required to execute the program is

(a) 29 (b) 24
(c) 23 (d) 20

[2004 : 2 Marks]

2.19 A 4-stage pipeline has the stage delays as 150, 120, 160 and 140 nanoseconds respectively. Registers that are used between the stages have a delay of 5 nanoseconds each. Assuming constant clocking rate, the total time taken to process 1000 data items on this pipeline will be

(a) 120.4 microseconds
(b) 160.5 microseconds
(c) 165.5 microseconds
(d) 590.0 microseconds

[2004 : 2 Marks]

2.20 Consider a three word machine instruction
ADD A[R_0], @ B
The first operand (destination) "A [R0]" uses indexed addressing mode with R0 as the index register. The second operand (source) "@B" uses indirect addressing mode. A and B are memory addresses residing at the second and the third words, respectively. The first word of the instruction specifies the opcode, the index register designation and the source and destination addressing modes. During execution of ADD instruction, the two operands are added and stored in the destination (first operand).
The number of memory cycles needed during the execution cycle of the instruction is

(a) 3 (b) 4
(c) 5 (d) 6

[2005 : 2 Marks]

2.21 Match **List-I** with **List-II** and select the correct answer using the codes given below the lists:

List-I	**List-II**
A. A[I] = B[J];	**1.** Indirect addressing
B. while[*A++];	**2.** Indexed addressing
C. int temp = *x;	**3.** Autoincrement

Codes:

	A	B	C
(a)	3	2	1
(b)	1	3	2
(c)	2	3	1
(d)	1	2	3

[2005 : 2 Marks]

2.22 A 5 stage pipelined CPU has the following sequence of stages
IF – Instruction fetch from instruction memory,
RD – Instruction decode and register read,
EX – Execute: ALU operation for data and address computation,
MA – Data memory access – for write access the register read at RD stage it used,
WB – Register write back.

Consider the following sequence of instructions:

I_1: L R0, Loc1;R0 $\Leftarrow$ M[Loc1]

I_2: A R0, R0 ;R0 $\Leftarrow$ R0 + R0

I_3: A R2, R0 ;R2 $\Leftarrow$ R2 – R0

Let each stage take one clock cycle.

What is the number of clock cycles taken to complete the above sequence of instructions starting from the fetch of I_1?

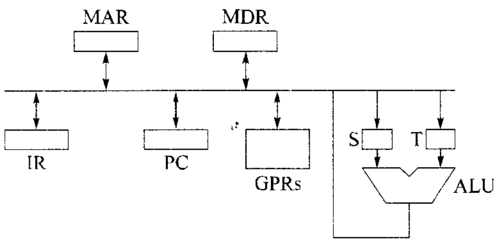
(a) 8 (b) 10

(c) 12 (d) 15

[2005 : 2 Marks]

Common Data for Q.2.23 & Q.2.24

Consider the following data path of a CPU



The, ALU, the bus and all the registers in the data path are of identical size. All operations including incrementation of the PC and the GPRs are to be carried out in the ALU. Two clock cycle are needed for memory read operation – the first one for loading address in the MAR and the next one for loading data from the memory but into the MDR.

2.23 The instruction "add R0, R1" has the register transfer interpretation R0 < = R0 + R1. The minimum number of clock cycles needed for execution cycle of this instruction is

(a) 2 (b) 3

(c) 4 (d) 5

[2005 : 2 Marks]

2.24 The instruction "call Rn, sub" is a two word instruction. Assuming that PC is incremented during the fetch cycle of the first word of the instruction, its register transfer interpretation is

Rn $\Leftarrow$ PC + 1;

PC $\Leftarrow$ M [PC];

The minimum number of CPU clock cycles needed during the execution cycle of this instruction is

(a) 2 (b) 3

(c) 4 (d) 5

[2005 : 2 Marks]

2.25 We have two designs D1 and D2 for a synchronous pipeline processor. D1 has 5 pipeline stages with execution times of 3 nsec, 2 nsec, 4 nsec, 2 nsec and 3 nsec while the design D2 has 8 pipeline stages each with 2 nsec execution time. How much time can be saved using design D2 over design D1 for executing 100 instructions?

(a) 214 nsec (b) 202 nsec

(c) 86 nsec (d) –200 nsec

[2005 : 2 Marks]

2.26 A CPU has 24-bit instructions. A program starts at address 300 (in decimal). Which one of the following is a legal program counter (all values in decimal)?

(a) 400 (b) 500

(c) 600 (d) 700

[2006 : 1 Mark]

2.27 Which of the following statements about relative addressing mode is FALSE?

(a) It enables reduced instruction size

(b) It allows indexing of array elements with same instruction

(c) It enables easy relocation of data

(d) It enables faster address calculations than absolute addressing

[2006 : 2 Marks]

2.28 The memory locations 1000, 1001 and 1020 have data values 18, 1 and 16 respectively before the following program is executed.

MOVI	Rs, 1	Move immediate
LOAD	Rd, 1000(Rs)	Load from memory
ADDI	Rd, 1000	Add immediate
STOREI	0(Rd), 20	Store immediate

Which of the statements below is TRUE after the program is executed?

(a) Memory location 1000 has value 20

(b) Memory location 1020 has value 20

(c) Memory location 1021 has value 20

(d) Memory location 1001 has value 20

[2006 : 2 Marks]

2.29 A CPU has five-stages pipeline and runs at 1 GHz frequency. Instruction fetch happens in the first stage of the pipeline. A conditional branch instruction computes the target address and evaluates the condition in the third stage of the pipeline. The processor stops fetching new instructions following a conditional branch until the branch outcome is known. A program executes 10^9 instructions out of which 20% are conditional branches. If each instruction takes one cycle to complete on average, then total execution time of the program is

(a) 1.0 second
(b) 1.2 seconds
(c) 1.4 seconds
(d) 1.6 seconds

[2006 : 2 Marks]

2.30 Consider a new instruction named branch-on-bit-set (mnemonic bbs). The instruction "bbs reg, pos, label" jumps to label if bit in position pos of register operand reg is one. A register is 32 bits wide and the bits are numbered 0 to 31, bit in position 0 being the least significant. Consider the following emulation of this instruction on a processor that does not have bbs implemented.

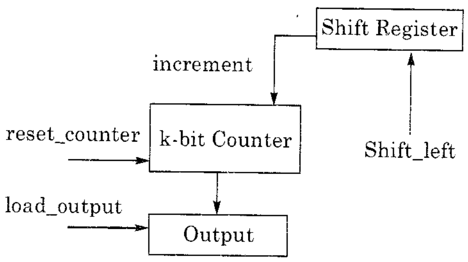
temp ← reg and mask
Branch to label if temp is non-zero

The variable temp is a temporary register. For correct emulation, the variable mask must be generated by

(a) mask ← 0x1 << pos
(b) mask ← 0x ffffffff >> pos
(c) mask ← pos
(d) mask ← 0xf

[2006 : 2 Marks]

2.31 The data path shown in the figure computes the number of 1s in the 32-bit input word corresponding to an unsigned even integer stored in the shift register. The unsigned counter, initially zero, is incremented if the most significant bit of the shift register is 1.



The microprogram for the control is shown in the table below with missing control words for microinstructions $I_1, I_2, \ldots, I_n$.

Micro-instruction	reset_counter	shift_left	load_output
BEGIN	1	0	0
I_1	?	?	?
:	:		:
I_n	?	?	?
END	0	0	1

The counter width (k), the number of missing microinstructions (n), and the control word for microinstructions $I_1, I_2, \ldots, I_n$ are, respectively,

(a) 32, 5, 010
(b) 5, 32, 010
(c) 5, 31, 011
(d) 5, 31, 010

[2006 : 2 Marks]

Common Data for Q.2.32 & 2.33

A pipelined processor uses a 4-stage instruction pipeline with the following stages: Instruction fetch (IF), Instruction decode (ID), Execute (EX) and Writeback (WB). The arithmetic operations as well as the load and store operations are carried out in the EX stage. The sequence of instructions corresponding to the statement X = (S – R * (P + Q)) / T is given below. The values of variables P, Q, R, S and T are available in the registers R0, R1, R2, R3 and R4 respectively, before the execution of the instruction sequence.

```
ADD    R5, R0, R1  ; R5 → R0 + R1
MUL    R6, R2, R5  ; R6 → R2 * R5
SUB    R5, R3, R6  ; R5 → R3 – R6
DIV    R6, R5, R4  ; R6 → R5/R4
STORE  R6, X       ; X ← R6
```

2.32 The number of Read-After-Write (RAW) dependencies, Write-After-Read (WAR) dependencies, and Write-After-Write (WAW) dependencies in the sequence of instructions are, respectively,

(a) 2, 2, 4
(b) 3, 2, 3
(c) 4, 2, 2
(d) 3, 3, 2

[2006 : 2 Marks]

2.33 The IF, ID and WB stages take 1 clock cycle each. The EX stage takes 1 clock cycle each for the ADD, SUB and STORE operations, and 3 clock cycles each for MUL and DIV operations. Operand forwarding from the EX stage to the ID stage is used. The number of clock cycles required to complete the sequence of instructions is

(a) 10
(b) 12
(c) 14
(d) 16

[2006 : 2 Marks]

2.34 A processor takes 12 cycles to complete an instruction 1. The corresponding pipelined processor uses 6 stages with the execution times of 3, 2, 5, 4, 6 and 2 cycles respectively. What is the asymptotic speedup assuming that a very large number of instructions are to be executed?

(a) 1.83 (b) 2
(c) 3 (d) 6

[2007 : 1 Mark]

2.35 Data forwarding techniques can be used to speed up the operation in presence of data dependencies. Consider the following replacements of LHS with RHS.

(i) R1 → Loc, Loc → R2 ≡ R1 → R2, R1 → Loc
(ii) R1 → Loc, Loc → R2 ≡ R1 → R2
(iii) R1 → Loc, R2 → Loc ≡ R1 → Loc
(iv) R1 → Loc, R2 → Loc ≡ R1 → Loc

In which of the following options, will the result of executing the RHS be the same as executing the LHS irrespective of the instructions that follow?

(a) (i) and (iii) (b) (i) and (iv)
(c) (ii) and (iii) (d) (ii) and (iv)

[2007 : 2 Marks]

2.36 Following table indicates the latencies of operations between the instruction producing the result and instruction using the result.

Instruction production the result	Instruction using the result	Latency
ALU Operation	ALU Operation	2
ALU Operation	Store	2
Load	ALU Operation	1
Load	Store	0

Consider the following code segment.

Load R1, Loc1;	Load R1 from memory location Loc1
Load R2, Loc2;	Load R2 from memory location Loc2
Add R1, R2, R1;	Add R1 and R2 and save result in R1
Dec R2;	Decrement R2
Dec R1;	Decrement R1
Mpy R1, R2, R3;	Multiply R1 and R2 and save result in R3
Store R3, Loc3;	Store R3 in memory location Loc3

What is the number of cycles needed to execute the above code segment assuming each instruction takes one cycle to execute?

(a) 7 (b) 10
(c) 13 (d) 14

[2007 : 2 Marks]

2.37 Consider a pipelined processor with the following four stages

IF : Instruction Fetch
ID : Instruction Decode and Operand Fetch
EX : Execute
WB : Write Back

The IF, ID and WB stages take one clock cycle each to complete the operation. The number of clock cycles for the EX stage depends on the instruction. The ADD and SUB instructions need 1 clock cycle and the MUL instruction need 3 clock cycles in the EX stage. Operand forwarding is used in the pipelined processor. What is the number of clock cycles taken to complete the following sequence of instructions?

ADD	R2,	R1,	R0	R2 ← R1 + R0
MUL	R4,	R3,	R2	R4 ← R3 * R2
SUB	R6,	R5,	R4	R6 ← R5 − R4

(a) 7 (b) 8
(c) 10 (d) 14

[2007 : 2 Marks]

Common Data for Q.2.38 to Q.2.40

Consider the following program segment. Here R1, R2 and R3 are the general purpose registers.

Instruction	Operation	Instruction size (no. of words)
MOV R1, (3000)	R1 ← M[3000]	2
LOOP:		
MOV R2, M[R3]	R2 ← M[R3]	1
ADD R2, R1	R2 ← R1 + R2	1
MOV (R3), R2	M[R3] ← R2	1
INC R3	R3 ← R3 + 1	1
DEC R1	R1 ← R1 − 1	1
BNZ LOOP	Branch on not zero	2
HALT		Stop

Assume that the content of memory location 3000 is 10 and the content of the register R3 is 2000. The content of each of the memory locations from 2000 to 2010 is 100. The program is loaded from the memory location 1000. All the numbers are in decimal.

2.38 Assume that the memory is word addressable. The number of memory references for accessing the data in executing the program completely is
(a) 10 (b) 11
(c) 20 (d) 21
[2007 : 2 Marks]

2.39 Assume that the memory is word addressable. After the execution of this program, the content of memory location 2010 is
(a) 100 (b) 101
(c) 102 (d) 110
[2007 : 2 Marks]

2.40 Assume that the memory is byte addressable and the word size is 32 bits. If an interrupt occurs during the execution of the instruction "INC R3", what return address will be pushed on to the stack?
(a) 1005 (b) 1020
(c) 1024 (d) 1040
[2007 : 2 Marks]

2.41 A processor that has carry, overflow and sign flag bits as part of its program status word (PSW) performs addition of the following two 2's complement numbers 01001101 and 11101001. After the execution of this addition operation, the status of the carry, overflow and sign flags, respectively will be:
(a) 1, 1, 0 (b) 1, 0, 0
(c) 0, 1, 0 (d) 1, 0, 1
[2008 : 1 Mark]

2.42 Assume that EA = (X)+ is the effective address equal to the contents of location X, with X incremented by one word length after the effective address is calculated; EA = –(X) is the effective address equal to the contents of location X, with X decremented by one word length before the effective address is calculated; EA = (X)– is the effective address equal to the contents of location X, with X decremented by one word length after the effective address is calculated. The format of the instruction is (opcode, source, destination), which means (destination ← source op destination). Using X as a stack pointer, which of the following instructions can pop the top two elements from the stack, perform the addition operation and push the result back to the stack.
(a) ADD (X)–, (X) (b) ADD (X), (X)–
(c) ADD –(X), (X)+ (d) ADD –(X), (X)
[2008 : 2 Marks]

2.43 A nonpipelined single cycle processor operating at 100 MHz is converted into a synchronous pipelined processor with five stages requiring 2.5 nsec, 1.5 nsec, 2 nsec, 1.5 nsec and 2.5 nsec, respectivley. The delay of the latches is 0.5 nsec. The speedup of the pipeline processor for a large number of instructions is
(a) 4.5 (b) 4.0
(c) 3.33 (d) 3.0
[2008 : 2 Marks]

2.44 Which of the following must be true for the RFE (Return from Exception) instruction on a general purpose processor.
1. It must be a trap instruction
2. It must be a privileged instruction
3. An exception can not be allowed to occur during execution of an RFE instruction.

(a) 1 only (b) 2 only
(c) 1 and 2 only (d) 1, 2 and 3 only
[2008 : 2 Marks]

2.45 Which of the following are NOT true in a pipelined processor?
1. Bypassing can handle all Raw hazards.
2. Register renaming can eliminate all register carried WAR hazards.
3. Control hazard penalties can be eliminated by dynamic branch prediction.

(a) 1 and 2 only (b) 1 and 3 only
(c) 2 and 3 only (d) 1, 2 and 3
[2008 : 2 Marks]

Common Data for Q.2.46 & Q.2.47
Delayed branching can help in the handling of control hazardous

2.46 For all delayed conditional branch instructions, irrespective of whether the condition evaluates to true or false, A
(a) the instruction following the conditional branch instruction in memory is executed
(b) the first instruction in the fall through path is executed
(c) the first instruction in the taken path is executed
(d) the branch takes longer to execute than any other instruction
[2008 : 2 Marks]

2.47 The following code is to run on a pipelined processor with one branch delay slot

I1: ADDR2 $\leftarrow$ R7 + R8
I2: SUB R4 $\leftarrow$ R5 – R6x
I3: ADD R1 $\leftarrow$ R2 + R3
I4: STORE Memory [R4]$\leftarrow$R1
BRANCH to Label if R1 = = 0

Which of the instruction I1, I2, I3 or I4 can legitimately occupy the delay slot without any other program modification?

(a) I1 (b) I2
(c) I3 (d) I4

[2008 : 2 Marks]

2.48 Consider a 4 stage pipeline processor. The number of cycles needed by the four instructions I1, I2, I3 , I4 in stages S1, S2, S3, S4 is shown below:

	S1	S2	S3	S4
I1	2	1	1	1
I2	1	3	2	2
I3	2	1	1	3
I4	1	2	2	2

What is the number of cycles needed to execute the following loop?

for (i = 1 to 2) {I1;I2; I3; I4;}

(a) 16 (b) 23
(c) 28 (d) 30

[2009 : 2 Marks]

2.49 A 5-stage pipelined processor has Instruction Fetch (IF), Instruction Decode (ID), Operand Fetch (OF), Perform Operation (PO) and Write Operand (WO) stages. The IF, ID, OF and WO stages take 1 clock cycle each for any instruction. The PO stage takes 1 clock cycle for ADD and SUB instructions 3 clock cycles for MUL instruction, and 6 clock cycles for DIV instruction respectively. Operand forwarding is used in the pipeline. What is the number of clock cycles needed to execute the following sequence of instructions?

Instruction	Meaning of Instruction
I_0: MUL R_2, R_0, R_1	$R_2 \leftarrow R_0 * R_1$
I_1: DIV R_5, R_3, R_4	$R_5 \leftarrow R_3/R_4$
I_2: ADD R_2, R_5, R_2	$R_2 \leftarrow R_5 + R_2$
I_3: SUB R_5, R_2, R_6	$R_5 \leftarrow R_2 - R_6$

(a) 13 (b) 15
(c) 17 (d) 19

[2010 : 2 Marks]

2.50 The program below uses six temporary variables a, b, c, d, e, f.

a = 1
b = 10
c = 20
d = a + b
e = c + d
f = c + e
b = c + e
e = b + f
d = 5 + e
return d + f

Assuming that all operations take their operands from registers, what is the minimum number of registers needed to execute this program without spilling?

(a) 2 (b) 3
(c) 4 (d) 6

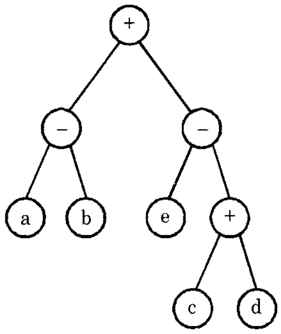
[2010 : 2 Marks]

2.51 Consider a hypothetical processor with a instruction of type LW R1, 20(R2), which during execution reads a 32-bit word from memory and stores it in a 32-bit register R1. The effective address of the memory location is obtained by the addition of a constant 20 and the contents of register R2. Which of the following best reflects the addressing mode implemented by this instruction for the operand in memory?

(a) Immediate Addressing
(b) Register Addressing
(c) Register Indirect Scaled Addressing
(d) Base Indexed Addressing

[2011 : 1 Mark]

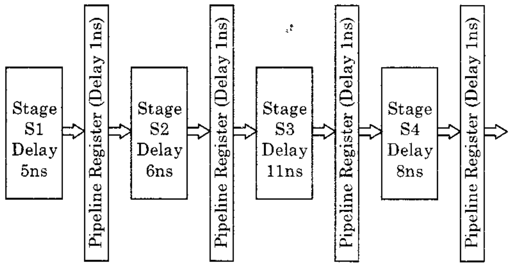
2.52 Consider evaluating the following expression tree on a machine with load-store architecture in which memory can be accessed only through load and store instructions. The variables a, b, c, d and e are initially stored in memory. The binary operators used in this expression tree can be evaluated by the machine only when the operands are in registers. The instructions produce result only in a register. If no intermediate results can be stored in memory, what is the minimum number of registers needed to evaluate this expression?



(a) 2
(b) 9
(c) 5
(d) 3

[2011 : 2 Marks]

2.53 Consider an instruction pipeline with four stages (S1, S2, S3 and S4) each with combinational circuit only. The pipeline registers are required between each stage and at the end of the last stage. Delays for the stages and for the pipeline registers are as given in the figure



What is the approximate speed up of the pipeline in steady state under ideal conditions when compared to the corresponding non-pipeline implementation?

(a) 4.0
(b) 2.5
(c) 1.1
(d) 3.0

[2011 : 2 Marks]

2.54 Register renaming is done in pipelined processors

(a) as an alternative to register allocation at compile time
(b) for efficient access to function parameters and local variables
(c) to handle certain kinds of hazards
(d) as part of address translation

[2012 : 1 Mark]

2.55 Consider the following sequence of micro-operations.

MBR ← PC
MAR ← X
PC ← Y
Memory ← MBR

Which one of the following is a possible operation performed by this sequence

(a) Instruction fetch
(b) Operand fetch
(c) Conditional branch
(d) Initiation of interrupt service

[2013 : 2 Marks]

2.56 Consider an instruction pipeline with five stages without any branch prediction: Fetch Instruction (FI), Decode Instruction (DI), Fetch Operand (FO), Execute Instruction (EI) and Write Operand (WO). The stage delays for FI, DI, FO, EI and WO are 5 ns, 7 ns, 10 ns, 8 ns and 6 ns, respectively. There are intermediate storage buffers after each stage and the delay of each buffer is 1 ns. A program consisting of 12 instruction I_1, I_2, I_3, ... I_{12} is execute in this pipelined processor. Instruction I_4 is the only branch instruction and its branch target is I_9. If the branch is taken during the execution of this program, the time (in ns) needed to complete the program is

(a) 132
(b) 165
(c) 176
(d) 328

[2013 : 2 Marks]

2.57 A machine has a 32-bit architecture, with 1-word long instructions. It has 64 registers, each of which is 32 bits long. It needs to support 45 instructions, which have an immediate operand in addition to two register operands. Assuming that the immediate operand is an unsigned integer, the maximum value of the immediate operand is ________.

[2014 (Set-1) : 1 Mark]

2.58 Consider a 6-stage instruction pipeline, where all stages are perfectly balanced. Assume that there is no cycle-time overhead of pipelining. When an application is executing on this 6-stage pipeline, the speedup achieved with respect to non-pipelined execution if 25% of the instructions incur 2 pipeline stall cycles is _______.

[2014 (Set-1) : 2 Marks]

2.59 Consider two processors P_1 and P_2 executing the same instruction set. Assume that under identical conditions, for the same input, a program running on P_2 takes 25% less time but incurs 20% more CPI (clock cycles per instruction) as compared to the program running on P_1. If the clock frequency of P_1 is 1GHz, then the clock frequency of P_2 (in GHz) is ________.

[2014 (Set-1) : 2 Marks]

2.60 Consider the following processors (ns stands for nanoseconds). Assume that the pipeline registers have zero latency.

P1: Four-stage pipeline with stage latencies 1 ns, 2 ns, 2 ns, 1 ns.

P2: Four-stage pipeline with stage latencies 1 ns, 1.5 ns, 1.5 ns, 1.5 ns.

P3: Five-stage pipeline with stage latencies 0.5 ns, 1 ns, 1 ns, 0.6 ns, 1 ns.

P4: Five-stage pipeline with stage latencies 0.5 ns, 0.5 ns, 1 ns, 1 ns, 1.1 ns.

Which processor has the highest peak clock frequency?

(a) P1 (b) P2
(c) P3 (d) P4

[2014 (Set-3) : 1 Mark]

2.61 An instruction pipeline has five stages, namely, instruction fetch (IF), instruction decode and register fetch (ID/RF), instruction execution (EX), memory access (MEM), and register writeback (WB) with stage latencies 1ns, 2.2ns, 2ns, 1ns, and 0.75ns, respectively (ns stands for nanoseconds). To gain in terms of frequency, the designers have decided to split the ID/RF stage into three stages (ID, RF1, RF2) each of latency 2.2/3 ns. Also, the EX stage is split into two stages (EX1, EX2) each of latency 1ns. The new design has a total of eight pipeline stages. A program has 20% branch instructions which execute in the EX stage and produce the next instruction pointer at the end of the EX stage in the old design and at the end of the EX2 stage in the new design. The IF stage stalls after fetching a branch instruction until the next instruction pointer is computed. All instructions other than the branch instruction have an average CPI of one in both the designs. The execution times of this program on the old and the new design are P and Q nanoseconds, respectively. The value of P/Q is ________.

[2014 (Set-3) : 2 Marks]

2.62 For computers based on three-address instruction formats, each address field can be used to specify which of the following:

S1: A memory operand

S2: A processor register

S3: An implied accumulator register

(a) Either S1 or S2 (b) Either S2 or S3
(c) Only S2 and S3 (d) All of S1, S2 and S3

[2015 (Set-1) : 1 Mark]

2.63 Consider a non-pipelined processor with a clock rate of 2.5 gigahertz and average cycles per instruction of four. The same processor is upgraded to a pipelined processor with five stages; but due to the internal pipeline delay, the clock speed is reduced to 2 gigahertz. Assume that there are no stalls in the pipeline. The speed up achieved in this pipelined processor is ____.

[2015 (Set-1) : 2 Marks]

2.64 Consider the sequence of machine instructions given below:

```
MUL    R5, R0, R1
DIV    R6, R2, R3
ADD    R7, R5, R6
SUB    R8, R7, R4
```

In the above sequence, R0 to R8 are general purpose registers. In the instructions shown, the first register stores the result of the operation performed on the second and the third registers. This sequence of instructions is to be executed in a pipelined instruction processor with the following 4 stages: (1) Instruction Fetch and Decode (IF), (2) Operand Fetch (OF), (3) Perform Operation (PO) and (4) Write back the Result (WB). The IF, OF and WB stages take 1 clock cycle each for any instruction. The PO stage takes 1 clock cycle for ADD or SUB instruction, 3 clock cycles for MUL instruction and 5 clock cycles for DIV instruction. The pipelined processor uses operand forwarding from the PO stage to the OF stage. The number of clock cycles taken for the execution of the above sequence of instructions is ________.

[2015 (Set-2) : 2 Marks]

2.65 Consider a processor with byte-addressable memory. Assume that all registers, including Program Counter (PC) and Program Status Word (PSW), are of size 2 bytes. A stack in the main memory is implemented from memory location $(0100)_{16}$ and it grows upward. The stack pointer (SP) points to the top element of the stack. The current value of SP is $(016E)_{16}$. The CALL instruction is of two words, the first word is the op-code and the second word is the starting address of the subroutine (one word = 2 bytes). The CALL instruction is implemented as follows:

- Store the current value of PC in the stack.
- Store the value of PSW register in the stack.
- Load the starting address of the subroutine in PC.

The content of PC just before the fetch of a CALL instruction is $(5FA0)_{16}$. After execution of the CALL instruction, the value of the stack pointer is

(a) $(016A)_{16}$ (b) $(016C)_{16}$
(c) $(0170)_{16}$ (d) $(0172)_{16}$

[2015 (Set-2) : 2 Marks]

2.66 Consider the following code sequence having five instructions I_1 to I_5. Each of these instructions has the following format.

OP Ri, Rj, Rk

where operation OP is performed on contents of registers Rj and Rk and the result is stored in register Ri.

I_1: ADD R1, R2, R3 I_2: MUL R7, R1, R3
I_3: SUB R4, R1, R5 I_4: ADD R3, R2, R4
I_5: MUL R7, R8, R9

Consider the following three statements:

S1: There is an anti-dependence between instructions I_2 and I_5.
S2: There is an anti-dependence between instructions I_2 and I_4.
S3: Within an instruction pipeline an anti-dependence always creates one or more stalls.

Which one of above statements is/are correct?

(a) Only S1 is true
(b) Only S2 is true
(c) Only S1 and S3 are true
(d) Only S2 and S3 are true

[2015 (Set-3) : 2 Marks]

2.67 Consider the following reservation table for a pipeline having three stages S_1, S_2 and S_3.

Time ⟶

	1	2	3	4	5
S_1	X				X
S_2		X		X	
S_3			X		

The minimum average latency (MAL) is________.

[2015 (Set-3) : 2 Marks]

2.68 The stage delays in a 4-stage pipeline are 800, 500, 400 and 300 picoseconds. The first stage (with delay 800 picoseconds) is replaced with a functionally equivalent design involving two stages with respective delays 600 and 350 picoseconds. The throughput increase of the pipeline is ______ percent.

[2016 (Set-1) : 2 Marks]

2.69 A processor has 40 distinct instructions and 24 general purpose registers. A 32-bit instruction word has an opcode, two register operands and an immediate operand. The number of bits available for the immediate operand field is______.

[2016 (Set-2) : 1 Mark]

2.70 Consider a processor with 64 registers and an instruction set of size twelve. Each instruction has five distinct fields, namely, opcode, two source register identifiers, one destination register identifier, and a twelve-bit immediate value. Each instruction must be stored in memory in a byte-aligned fashion. If a program has 100 instructions, the amount of memory (in bytes) consumed by the program text is ______.

[2016 (Set-2) : 2 Marks]

2.71 Consider a 3 GHz (gigahertz) processor with a three-stage pipeline and stage latencies τ_1, τ_2, and τ_3 such that $t_1 = 3\tau_2/4 = 2\tau_3$. If the longest pipeline stage is split into two pipeline stages of equal latency, the new frequency is ______ GHz, ignoring delays in the pipeline registers.

[2016 (Set-2) : 2 Marks]

2.72 Suppose the functions F and G can be computed in 5 and 3 nanoseconds by functional units U_F and U_G, respectively. Given two instances of U_F and two instances of U_G, it is required to implement the computation $F(G(X_i))$ for $1 \leq i \leq 10$. Ignoring all other delays, the minimum time required to complete this computation is ________ nanoseconds.

[2016 (Set-2) : 2 Marks]

Answers Instructions : Pipelining and Addressing Modes

2.1 (b)	2.2 (b)	2.3 (c)	2.4 (c)	2.5 (a)	2.6 (b)	2.7 (d)	2.8 (d)	2.9 (b)
2.10 (a)	2.11 (c)	2.12 (b)	2.13 (d)	2.14 (c)	2.15 (d)	2.16 (c)	2.17 (b)	2.18 (b)
2.19 (c)	2.20 (d)	2.21 (c)	2.22 (a)	2.23 (b)	2.24 (b)	2.25 (b)	2.26 (c)	2.27 (b)
2.28 (d)	2.29 (c)	2.30 (a)	2.31 (d)	2.32 (c)	2.33 (b)	2.34 (b)	2.35 (b)	2.36 (b)
2.37 (b)	2.38 (d)	2.39 (a)	2.40 (c)	2.41 (c)	2.42 (b)	2.43 (c)	2.44 (d)	2.45 (d)
2.46 (b)	2.47 (b)	2.48 (d)	2.49 (b)	2.50 (b)	2.51 (d)	2.52 (d)	2.53 (b)	2.54 (c)
2.55 (d)	2.56 (b)	2.60 (c)	2.62 (a)	2.65 (d)	2.66 (b)			

Explanations Instructions : Pipelining and Addressing Modes

2.1 (b)

Virtual memory concept is independent of size of main memory and depends only on the availability of the secondary storage.

2.2 (b)

In pipelined CPU, there will be buffer delays. So for 1 instruction nonpipelined CPU takes less time compared to pipelined CPU.

2.3 (c)

Indirect addressing mode is used for pointers access, immediate addressing mode is used for constant access and auto-decrement addressing mode is used for loops.

2.4 (c)

An opcode is the portion of a machine language instruction that specifies the operation to be performed.

2.5 (a)

Bits in the beginning and at the end of byte is known as start and stop bits respectively. Start and stop bits are used for synchronization purpose.

2.6 (b)

When two signed 2's complement numbers are added, overflow is detected if:

1. both operands are positive and the result is negative, or
2. both operands are negative and the result is positive.

Obviously; adding two numbers of opposite sign can't produce overflow because addition of different signed number is equal to subtraction.

2.7 (d)

Hazards are caused due to dependencies. Different dependencies in pipelined processor are:

1. **Structural dependency:** The pipeline stages have different delays.
2. **Control dependency:** Consecutive instructions are dependent as each other.
3. **Data dependency:** The pipeline stages shear hardware resources.

2.8 (d)

(i) The j+1st instruction uses the result of the jth instruction as an operand then read-after-write (RAW) hazard occurs. It is a part of data dependency.

(ii) The execution of a conditional jump instruction causes a flushing so conditional dependency occurs.

(iii) The jth and j+1st instructions require the ALU at the same time causes write-after-read (WAR) hazard.

2.9 (b)

$$A \leftarrow A_0$$

The Value of B after execution of the program is number of 1 bits in A_0.

2.10 (a)

Insert the instruction
RRC A, #

2.11 (c)

Based addressing mode & relative addressing mode are suitable for program relocation at run time due to following reason. In relative addressing scheme in which the operand field contains a relative address also called an offset or displacement D. If R_1, R_2...... R_k are CPU registers then A is computed as follows.

$$A := R + D$$

By changing contents of R, the processor can change the absolute address referred to by a block of instructions B. This address modification permits the processor to move (relocate) the entire block B from one region of main memory to another without invalidating the addresses in B. When used in this way R is referred to as a base register and it contents as a base addresses.

2.12 (b)

Overflow happens only when Sign bit of two input numbers is 0, and the result has sign bit 1.

Sign bit of two input numbers is 1, and the result has sign bit 0.

Overflow is important only for signed arithmetic while carry is important only for unsigned arithmetic. A carry happens when there is a carry to (or borrow from) the most significant bit. Here, (i) and (iii) cause a carry but only (ii) causes overflow.

2.13 (d)

Data forwarding means if CPU writes to a memory location and subsequently reads from the same memory location, the second instruction can fetch the value directly from the register used to do the write than waiting for the memory. So, this increases the performance.

Here, choices A, B and C doesn't really make any sense as the data was in R1 and it must be moved to R2, R3 and M[100].

2.14 (c)

Time	1	2	1	2	2	2	1	2
1	s1(1)	s2(2)	s3(1)	s4(2)				
I2		s1(2)	s2(1)	s3(2)	s4(1)			
I3			s1(1)	s2(1)	s3(2)	s4(1)		
I4				s1(2)	s2(1)	s3(2)	s4(1)	
I1					s1(1)	s2(2)	s3(1)	s4(2)

Total time = 13 ns

2.15 (d)

In Old system: 20 floating point operation and 30 fixed point operations.

Floating point takes 2 units and fixed point takes 1 unit.

Total time in old system = 20 * 2+ 30* 1 = 70 units

In Enhanced system: Floating point takes = 2 * 0.8 = 1.6 units

Fixed point takes = 1 * 0.9 = 0.9 units

Total time = 20 * 1.6 + 30 * 0.9 = 59 units

Therefore Speedup = 70/59 = 1.1864

2.16 (c)

Instruction	Instruction Size	Location (Decimal)
MOV R_1,5000	2	1000 to 1007
MOV R_0, (R_1)	1	1008 to 1011
ADD R_0, R_3	1	1012 to 1015
MOV 6000, R_2	2	1016 to 1023
Halt	1	1024 to 1027

If an interrupt occurs the CPU has been halted after executing the HALT instruction the return address 1024 saved in the stack.

2.17 (b)

Instruction	Instruction Size	Location (decimal)
MOV R_1, 5000	2	1000 to 1001
MOV R_2,(R_1)	1	1002
ADD R_2, R_3	1	1003
MOV 6000, R_2	2	1004 – 1005
Halt	1	1006

The return address pushed onto the stack is 1004.

2.18 (b)

Operation	Instruction Size	Required Clock cycles
R_0, Memory [5000]	2	$2 \times 3 + 2 = 8$
$R_2 \leftarrow$ Memory [(R_1)]	1	$1 \times 3 + 2 = 5$
$R_2 \leftarrow (R_2 + R_3)$]	1	$1 = 1$
Memory [6000] $\leftarrow R_2$	2	$3 \times 2 + 2 = 8$
Machine Halt	1	$1 + 1 = 2$
		Total = 24

2.19 (c)

K stage pipeline can process n tasks in T_K time

$T_K = [K + (n-1)]\tau$

When $\tau = \tau_m + d$ where τ_m is the maximum stage delay so max (150, 120, 160,140)

$= 160$

$\tau = 160 + 5 = 165$ ns

$\tau = 165 \times 10^{-3}$ms

$K = 4, n = 1000$

$T = [4 + (1000 - 1)]\tau$

$= 1003 \times 165 \times 10^{-3} = 165.5$ ms

2.20 (d)

ADD $A[R_0]$, @ B

$A[R_0]$ uses indexed addressing mode as follow such that R_0 as the index register

Regs[R3] ← Regs[R3] + Mem[Regs[B] + Regs [R0]] ...(i)

And @ B uses indirect as follows

ADD $A[R_0]$, @B

Regs $[R_0]$ ← Regs$[R_0]$ + Mem[Mem[B]] ...(ii)

So the first instruction needed 3 memory cycles and the second needed 2 and one memory cycle is needed for writing the result into the memory. So total 6 memory cycles are needed to execute ADD $A[R_0]$, @ B.

2.21 (c)

The correct matching is

1. A[I] = B[J] Indexed Addressing Mode
2. While[*A++] Autoincrement
3. int temp = * x ; Indirect Addressing Mode

2.22 (a)

This is RISC pipeline and in RISC pipeline dataforwarding is used as well as branch penalty is 1.

	CC_1	CC_2	CC_3	CC_4	CC_5	CC_6	CC_7	CC_8
I_1:	IF	ID	Ex	MA	WB			
I_2:		IF	ID	////	EX	MA	WB	
I_3:			IF	////	ID	EX	MA	WB

Number of cycle = 8 cycles.

2.23 (b)

RTL $R_0 \Leftarrow R_0 + R_1$

The sequence of microinstruction take place in following cycles.

I cycle: $R_{1\,out}$, S_{in}

II cycle: $R_{2\,out}$, T_{in}

III cycle: S_{out}, T_{out}, ALU_{add}, R_{in}

∴ 3 cycles will be required to execute.

2.24 (b)

RTL $R_n = PC + 1$

$P_c = M[Pc]$

The sequence of microinstrution take place in following cycles.

I cycle: PC_{out}, S_{in}, MAR_{in} (MAR can be loaded with PC_{out})

II cycle: S_{out}, $ALU_{increment}$, $R_{n\,in}$

III cycle: MDR_{out}, PC_{in} (MDR_{out} can be performed once MAR_{in} has been performed)

∴ 3 cycles will be required to execute.

2.25 (b)

Execution time for Pipeline = (K + n – 1)* execution_time

where k = Number of stages in pipeline,

n = Number of instructions execution time = Max(all stages execution time)

D1 = (5+100 – 1)*4 = 416

D2 = (8+100 – 1)*2 = 214

Time saved using D2 = 416 – 214 = 202.

2.26 (c)

Size of instruction 24 bits

Starting address of the program is 300. The size of instruction is 3 byte long So the address is always the multiple of 3 byte next address is 600 it is also the next instruction of the program.

2.27 (b)

Relative addressing cannot be faster than absolute addressing as absolute address must be calculated from relative address.

(a) is true as instead of absolute address we can use a much smaller relative address in instructions which results in smaller instruction size.

(b) By using the base address of array we can index array elements using relative addressing.

(c) is true as we only need to change the base address in case of relocation- instructions remain the same.

2.28 (d)

- Memory location 1001 has value 20.
  Rs ← 1
  Rd ← 1
  Rd ← 1001
  Store in address 1001 ← 20.

2.29 (c)

$$10^9 \text{ cycles} + \frac{20}{100} \times 2 \times 10^9$$
$$= 10^9 [1 + 0.4]$$
$$= 1.4 \times 10^9 \text{ cycles}$$
$$= 1.4 \times 1 \text{ sec} = 1.4 \text{ sec}$$

2.30 (a)

bbs reg, pos, label
temp ← reg and mask
Branch to label if temp is non zero. If we shift one position left by pos of ox1 then it evaluate the bbs instruction and mask ← $0x_1$ << pos

2.31 (d)

For 32 instructions counter require 5 bits and we require 31 shift left operation.
Instruction word would be 011 because for load_output we need not to load the OUTPUT for each shift because we need to count the number of 1's which will be available at the END or after the 31 left shift operations has been performed.

2.32 (c)

Their are 2 WAW dependancies present between I_1 and I_3, I_2 and I_6.
Their are 2 WAR dependancies present between I_2 and I_5, I_3 and I_6.
Their are 4 RAW dependancies present between I_1 and I_2, I_2 and I_3, I_2 and I_4, I_4 and I_5.
So, option (c) is correct.

2.33 (b)

	1	2	3	4	5	6	7	8	9	10	11	12
I_1	IF	ID	EX	WB								
I_2		IF	ID	EX	EX	EX	WB					
I_3			IF	–	–	ID	EX	WB				
I_4						IF	ID	EX	EX	EX	WB	
I_5							IF	–	–	ID	EX	WB

Their are 12 clock cycles are required to complete all instruction.

2.34 (b)

For non pipeline processor we have n instruction and each instruction take 12 cycle so total 12n instruction.
For pipeline processor we have each stage strict to 6ns so time to complete the n instruction is 6*6 + (n – 1)*6

$\lim_{x \to \infty} 12n / 36 + (n - 1)*6 = 12/6 = 2$

2.35 (b)

(iii) and (iv) are the same!! and both are wrong because R2 is writing last, not R1.
(i) is true. (ii) false, because R2 get the correct data, but location has not got updated.
No option.

2.36 (b)

The instruction of Type R1 ← R1 + R2 requires:
2 × (Load – ALU op) type
1 × (ALU op – store) type
Then the answer should be 14

Analysis:

Instr	Cost
1.	0
2.	0
3.	1 + 1 + 2 = 4
4.	1 + 2 = 3
5.	1 + 2 = 3
6.	1 + 1 + 2 = 4
7.	0

AND if, R1 ← R1+R2 is only an (ALU-store) type. Instructions 3 and 6 take 2 time units each resulting in the answer as 10.

2.37 (b)

Pipelined processor has 4 stages IF, ID, EX, WB

Clock Cycles	Instruction
1	ADD
1	SUB
3	MUL

Consider the following diagram

Clock Cyles	1	2	3	4	5	6	7	8
$R_2 \leftarrow R_1 + R_0$	IF	ID	EX	WB				
$R_4 \leftarrow R_3 * R_2$		IF	ID	EX	EX	EX	WB	
$R_6 \leftarrow R_5 - R_4$			IF	ID			EX	WB

So total required clocks cycle is 8.

2.38 (d)

Given $M[3000] = 10$

Instruction	Required Memory Reference
$R_1 \leftarrow M[3000]$	1
$R_2 \leftarrow M[R_3]$ or $M[R_3]$	10
$M[R_3] \leftarrow R_2$	10
	Total = 21

2.39 (a)

The register R_3 contain the memory location 2010 whose value is 100 at the end of loop BNZ Loop.

2.40 (c)

Location	Operation
1000-1007	$R_1 \leftarrow M[3000]$
1008-1011	
1012-1015	$R_2 \leftarrow M[R_3]$
1016-1019	$R_2 \leftarrow R_1 + R_2$
1020-1023	$M[R_3] \leftarrow R_2$
1024-1031	$R_3 \leftarrow R_3 + 1$

So if an interrupt occurs during the execution of the instruction "INC R_0, the return address 1024 will be pushed on to the stack.

2.41 (c)

Both numbers are given in 2's complement form.
01001101 is equivalent to decimal 77
11101001 is equivalent to decimal (–23)
After addition operation: 77 + (–23) = 54
11101001 + 01001101 = 00110110 (overflow flag set to 1)
Carry flag = 0, Overflow flag =1, and Sign flag = 0.

2.42 (b)

It should be A as 998 ← 1000 + 998 (only memory locations for sake of brevity).
Let's say sp is 1000 initially then after it calculates the EA of source(which is 1000 as it decrements after the EA) the destination becomes 998 and that is where we want to store the result as stack is decrementing. In case of C and D it becomes 998 ← 998 + 998.

2.43 (c)

For non pipeline system time required
= 2.5 + 1.5 + 2.0 + 1.5 + 2.5 = 10
For pipelined system = max(stage delay) + max (latch delay) = 2.5 + 0.5 = 3
Speedup = time in non-pipeline/time in pipeline
= 10/3 = 3.33.

2.44 (d)

A RFE (Return From Exception) instruction contains a trap instruction, privileged instruction and an exception can't be allowed to occur during the execution of an REF instruction.

2.45 (d)

All statement 1, 2 and 3 are false for a pipelined processor.

2.46 (b)

For all delayed conditional branch instruction the first instruction in the fall through path is executed and evaluates to true or false.

2.47 (b)

In pipelining result of 1 instruction is used for next in pipeline. Delay slot will be occupied by the next instruction in the fall through path. The branching instruction R1 = = 0 goto label X.
So in delay slot I_2 will be there.

2.48 (d)

No. of cycles needed to executed for loop when
$i = 1 \Rightarrow 2 + 1 + 3 + 2 + 2 + 3 + 2 = 15$
So total cycle needed to execute the loop
$= 2 \times 15 = 30$

2.49 (b)

	IF	ID	OF	PO	WO
MUL : I_D	1	1	1	3	1
DIV : I_1	1	1	1	6	1
ADD : I_2	1	1	1	1	1
SUB : I_3	1	1	1	1	1

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
I_o	IF	ID	OF	PO	PO	PO	WO								
I_1		IF	ID	OF	////	////	PO	PO	PO	PO	PO	PO	WO		
I_2			IF	ID	////	////	OF	////	////	////	////	////	PO	WO	
I_3				IF	///	///	ID	///	///	////	////	////	OF	PO	WO

∴ No. of cycles required = 15 cycles.

2.50 (b)

Register operations

$R1 \leftarrow a$	....	$(a = 1)$
$R2 \leftarrow b$	....	$(b = 10)$
$R3 \leftarrow c$	....	$(c = 11)$
$R1 \leftarrow R1 + R2$	....	$(d = a + b)$
$R1 \leftarrow R3 + R1$	....	$(e = c + d)$
$R2 \leftarrow R3 + R1$	....	$(f = c + e)$
$R3 \leftarrow R3 + R1$	....	$(b = c + e)$
$R1 \leftarrow R2 + R3$	....	$(e = b + f)$
$R3 \leftarrow 5 + R1$	....	$(d = 5 + e)$
return $(R2 + R3)$	....	return $(d + f)$

hence 3 registers needed only.

2.51 (d)

LW R1, 20(R2) Instruction shows the source address can be formed by adding the constant with the R2 content .It is the MIPS instruction. R2 points the base address of an array and 20 points the index address of an array. Hence the addressing mode used for the above is Based Indexed Addressing mode. This addressing mode is used to implement the Arrays.

2.52 (d)

Load r1,d
Load r2,c
Add r1,r2
Load r2,e
Sub r2,r1
Load r1,b
Load r3,a
Sub r3,r1
Add r3,r2

Tree is evaluated completely. It takes only 3 registers.

2.53 (b)

Speed up = Nonpipe line execution time/ Pipeline execution time.

Non pipeline Execution time
= S1 delay+S2 delay+S3 delay+S4 delay
= 5+6+11+8 =30ns

Pipeline Execution time
= Max(stage delay)+buffer delay
= max(5ns,6ns,11ns,8ns)+1ns
= 11ns+1ns = 12ns

Hence Speedup = 30/12 = 2.5

2.54 (c)

Register renaming is done in pipelining to handle certain hazards.

Operand forwarding (Bypassing), predictive buffer is also used to handle hazards.

∴ Option 'C' is true.

2.55 (d)

PC holds the value of next instruction to be executed we store the value of PC to MBR and then to memory. We are saving the value of PC in memory and new address value is loaded into PC. This can be done only in interrupt service option (d) is correct.

2.56 (b)

Number of stages = 5

Stage maximum delay = 10 + 1 = 11ns

WO stage of I_4 can overlap with FI stage of I_9.

Total time without overlap:

$$= ([\overset{\#\text{stages}}{5} + (\overset{I_1 \text{ to } I_4}{4} - 1)] + [\overset{\#\text{stages}}{5} + (\overset{I_9 \text{ to } I_{12}}{4} - 1)])*11 = 176$$

∴ Total time with overlap = 176 – 11(one stage delay) = 165

2.57 Sol.

(6)	(6)	(6)	
Opcode	R_1	R_2	Immediate

$32 - (6 + 6 + 6) = 14$ bits for immediate field

$\Rightarrow 2^{14} - 1 = 16383$ maximum possible value of immediate operand.

2.58 Sol.

$$\text{Speed up(S)} = \frac{\text{pipe depth}}{(1 + \#\text{stalls/instruction})}$$

Number of stalls/instruction

$$= 0.75 \times 0 + 0.25 \times 2 = 0.5$$

$$\therefore\ S = \frac{6}{1 + 0.5} = 4\,.$$

2.59 Sol.

$T_1 = \text{CPI} \times \text{Cycle time}_1 = \text{CPI} \times 1$ nsec

$T_2 = 1.2\ \text{CPI} \times \text{Cycle Time}_2$ ...(1)

$T_2 = 0.75 \times T_1 = 0.75 \times \text{CPI} \times 1$ nsec ...(2)

from (1) and (2)

$1.2\ \text{CPI} \times \text{Cycle time}_2 = 0.75\text{CPI} \times 1$ nsec

$$\Rightarrow \quad \text{Cycle time}_2 = \frac{0.75}{1.2} = 0.625 \text{ n sec}$$

$$\text{Frequency of } P_2 = \frac{1}{0.625 \text{ n sec}} = 1.6 \text{ GHz}$$

2.60 (c)

Clock cycle time= Maximum of stage delays.
For P1: clock cycle time = Maximum is 2
For P2: clock cycle time = Maximum is 1.5
For P3: clock cycle time = Maximum is 1
For P4: clock cycle time = Maximum is 1.1
Minimum clock cycle time gives the high clock rate.
Peak clock cycle rate

$$= \text{Maximum}\left(\frac{1}{2}, \frac{1}{1.5}, \frac{1}{1}, \frac{1}{1.1}\right) = 1$$

$\therefore$ P3 has peak clock cycle rate.

2.61 Sol.

$T_{avg} = (1 + \text{stall freq.} \times \text{stall cycle}) \times T_{clock}$
$TP_{avg} = (1 + 0.2 \times 2) \times 2.2\text{ns} = 3.08\text{ns}$
$TQ_{avg} = (1 + 0.2 \times 5) \times 1\text{ns} = 2\text{ns}$

$$\frac{P}{Q} = \frac{TP_{avg}}{TQ_{avg}} = \frac{3.08}{2} = 1.54$$

2.62 (a)

Address field of 3-address instruction can be either memory operand or processor register. Implied accumulator register can not be address field.

2.63 Sol.

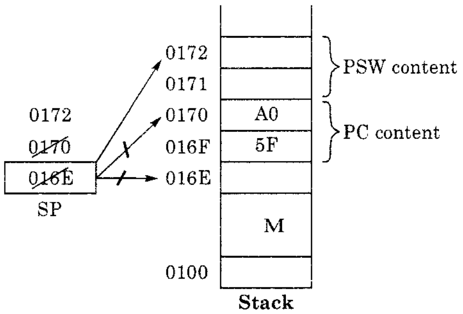
Non-pipelined processor:
For n instructions execution time
= (n × 4) / 2.5 = 1.6n nanoseconds.
Pipelined processor:
For n instructions execution time
= n / 2 = 0.5n nanoseconds.
Speedup = 1.6n / 0.5n = 3.2

2.64 Sol.

SUB				IF	–	–	–	–	–	–	OF	PO	WB
ADD			IF	–	–	OF	–	–	–	–	PO	WB	
DIV		IF	OF	–	–	PO	PO	PO	PO	PO	WB		
MUL	IF	OF	PO	PO	PO	WB							
	1	2	3	4	5	6	7	8	9	10	11	12	13

$\therefore$ It takes 13 clock cycles.

2.65 (d)



Just before CALL instruction execution, SP contains 016E
While CALL execution:
(*i*) PC contents are pushed i.e., SP incremented by 2 $\Rightarrow$ SP = 0170
(*ii*) PSW contents are pushed i.e., SP incremented by 2 $\Rightarrow$ SP = 0172
$\therefore$ The value of stack pointer is $(0172)_{16}$.

2.66 (b)

S1 is false
S2 is true because there is an anti-dependence between instructions I_2 and I_4.
S3 is false because anti-dependence stalls may be avoided when register renaming is used.

2.67 Sol.

- Collision vector (C) = (C_5 C_4 C_3 C_2 C_1)
- Forbiden latency creates collision. So it is non-permissible state i.e., 0.
- If permissible latency is 1 then, will initiate new task in 2^{nd} cycle.

Latency sequence with (1)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
S_1	1	2			1	2	3	4			3	4	5		
S_2		1	2	1	2			3	4	3	4			5	
S_3			1	2					3	4					5

Latency cycle:
(2 – 1) = 1
(7 – 2) = 5
(8 – 7) = 1
(12 – 7) = 5
i.e., (1, 5), (1, 5), (1, 5), . . .
so, average latency = 6/2 = 3

Latency sequence with (3)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
S_1	1			2	1		3	2		4	3		5	4		
S_2		1		1	2		2	3		3	4		4	5		5
S_3			1			2			3			4			5	

Latency cycle:
(4 – 1) = 3
(7 – 4) = 3
(10 – 7) = 3
so, average latency = 3.

Latency sequence with (5)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
S_1	1				1	2	3			2	3	4	5			4	5	
S_2		1		1			2	3	2	3			4	5	4	5		
S_3			1					2	3					4	5			

Latency cycle:
(6 – 1) = 5
(7 – 6) = 1
(12 – 7) = 5
(13 – 12) = 1
Latency cyle (5, 1), (5, 1), (5, 1), . . .
so, average latency = 3.
∴ MAL value = 3.

2.68 Sol.

P_1: 4-stage

$$t_p = \text{Max (stage delay)} = 800 \text{ ps}$$

1 instruction — 800 ps
? number of instruction — 1 sec
TP_R (throughput) = 1250 instruction/sec
P_2: 5-stage

$$t_p = 600 \text{ ps}$$

1 instruction — 600 ps
? number of instruction — 1 sec
TP_{P2} (throughput) = 1666 instruction/sec
1250 — 100% (old)
(1666 – 1250) — ? (New)

$$\Rightarrow \quad \frac{416}{1250} = 0.3328$$

i.e., 33.28%

2.69 Sol.

32 bit

opcode	Reg	Reg	Immediate
6 bit	5 bit	5 bit	16 bit

The number of bits for immediate operand field is 16 bits.

2.70 Sol.

34 bit

opcode	Reg	Reg	Reg	12 bit Immediate
4 bit	6	6	6	12

One instruction size = 34 bit = 5 bytes
100 instruction occupies 500 bytes.

2.71 Sol.

Given that, 3 stage pipeline with stage latencies

$$\tau_1 = \frac{3\tau_2}{4} = 2\tau_3$$

$$\tau_1 = \frac{3\tau_2}{4}, \frac{3\tau_2}{4} = 2\tau_3$$

$$\tau_1 : \tau_2 = 3 : 4,\ \tau_2 : \tau_3 = 8 : 3$$

Ratio of $\tau_1 : \tau_2 : \tau_3$ is $6x$, $8x$, $3x$ respectively. So, largest stage time $8x$.

So, calculate frequency = $\frac{1}{8x}$

$$\Rightarrow \quad \frac{1}{8x} = 3 \text{ GHz}$$

$$\Rightarrow \quad \frac{1}{x} = 24 \text{ GHz} \quad \text{...(1)}$$

Now, largest stage latency divide into 2 half parts i.e., $8x$ divide into $4x$ and $4x$.
So, 4 pipeline with stage latencies are $6x$, $4x$, $4x$, $3x$.
Now, largest stage time = $6x$.

So, calculate new frequency = $\frac{1}{6x}$.

$$\frac{1}{6x} = \frac{24}{6} = 4 \text{ GHz}$$

[∵ 1/x = 24 GHz from eq. (1)]

2.72 Sol.

Here pipelining concept is used. Given U_F take 5 ns while U_G take 3 ns to complete F and G respectively.
We have to do 10 computations i.e., $1 \le i \le 10$ with 2 instances of U_F and U_G. So,

U_F will take = $\frac{10 \times 5}{2} = \frac{50}{2} = 25$ (since 2 instances are used).

We have to wait 3 ns more to complete U_G to get the output.
So answer is 25 + 3 = 28 ns (rest of the instructions already complete at this time because pipelining is used).

■■■■

3 CPU Control Design and Interfaces

3.1 When an interrupt occurs, an operating system
(a) ignores the interrupt
(b) always changes state of interrupted process after processing the interrupt
(c) always resumes execution of interrupted process after processing the interrupt
(d) may change state of interrupted process to 'blocked' and schedule another process

[1997 : 1 Mark]

3.2 RAID configurations of disks are used to provide
(a) fault-tolerance (b) high speed
(c) high data density (d) None of these

[1999 : 2 Marks]

3.3 Arrange the following configuration for CPU in decreasing order of operating speeds: Hard wired control, vertical micro-programming, horizontal micro-programming.
(a) Hard wired control, vertical micropro-gramming, horizontal microprog-ramming.
(b) Hard wired control, horizontal micropro-gramming, vertical microprogramming.
(c) Horizontal micro-programming, vertical micro-programming, Hard wired control.
(d) Vertical micro-programming, horizontal micro-programming, hard wired control.

[1999 : 2 Marks]

3.4 A processor needs software interrupt to
(a) test the interrupt system of the processor
(b) implement co-routines
(c) obtain system services which need execution of privileged instructions
(d) return from subroutine

[2001 : 1 Mark]

3.5 Which is the most appropriate match for the items in the first column with the items in the second column

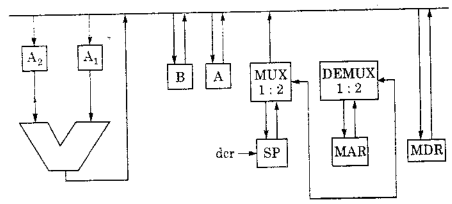
List-I
X. Indirect Addressing
Y. Indexed Addressing
Z. Base Register Addressing

List-II
I. Array implementation
II. Writing re-locatable code
III. Passing array as parameter

(a) (X, III) (Y, I) (Z, II)
(b) (X, II) (Y, III) (Z, I)
(c) (X, III) (Y, II) (Z, I)
(d) (X, I) (Y, III) (Z, II)

[2001 : 2 Marks]

3.6 Consider the following data path of a simple non-pipelined CPU. The registers A, B, A_1, A_2, MDR, the bus and the ALU are 8-bit wide. SP and MAR are 16-bit registers. The MUX is of size $8 \times (2:1)$ and the DEMUX is of size $8 \times (1:2)$. Each memory operation takes 2 CPU clock cycles and uses MAR (Memory Address Register) and MDR (Memory Data Register). SP can be decremented locally.



The CPU instruction "push r", where r = A or B, has the specification

$M[SP] \leftarrow r$
$SP \leftarrow SP - 1$

How many CPU clock cycles are needed to execute the "push r" instruction?
(a) 2 (b) 3
(c) 4 (d) 5

[2001 : 2 Marks]

3.7 A device employing INTR line for device interrupt puts the CALL instruction on the data bus while
(a) $\overline{\text{INTA}}$ is active (b) HOLD is active
(c) READY is active (d) None of these

[2002 : 1 Mark]

3.8 The absolute addressing mode
(a) the operand is inside the instruction
(b) the address of the operand is inside the instruction

(c) the register containing the address of the operand is specified inside the instruction
(d) the location of the operand is implicit

[2002 : 1 Mark]

3.9 Horizontal microprogramming
(a) does not require use of signal decoders
(b) results in larger sized microinstructions than vertical microprogramming
(c) uses one bit for each control signal
(d) All of the above

[2002 : 2 Marks]

3.10 Consider an array multiplier for multiplying two n bit numbers. If each gate in the circuit has a unit delay, the total delay of the multiplier is
(a) $\Theta(1)$ (b) $\Theta(\log n)$
(c) $\Theta(n)$ (d) $\Theta(n^2)$

[2003 : 1 Mark]

3.11 What is the minimum size of ROM required to store the complete truth table of an 8-bit × 8-bit multiplier?
(a) 32 K × 16 bits (b) 64 K × 16 bits
(c) 16 K × 32 bits (d) 64 K × 32 bits

[2004 : 1 Mark]

3.12 A CPU has only three instructions I1, I2 and I3, which use the following signals in time steps T1-T5:

I1: T1 : Ain, Bout, Cin
T2 : PCout, Bin
T3 : Zout, Ain
T4 : PCin, Bout
T5 : End

I2: T1 : Cin, Bout, Din
T2 : Aout, Bin
T3 : Zout, Ain
T4 : Bin, Cout
T5 : End

I3: T1 : Din, Aout
T2 : Din, Bout
T3 : Zout, Ain
T4 : Dout, Ain
T5 : End

Which of the following logic functions will generate the hardwired control for the signal Ain?
(a) T1.I1 + T2.I3 + T4.I3 + T3
(b) (T1 + T2 + T3).I3 + T1.I1
(c) (T1 + T2).I1 + (T2 + T4).I3 + T3
(d) (T1 + T2).I2 + (T1 + T3).I1 + T3

[2004 : 2 Marks]

3.13 Normally user programs are prevented from handling I/O directly by I/O instructions in them. For CPUs having explicit I/O instructions, such I/O protection is ensured by having the I/O instructions privileged. In a CPU with memory mapped I/O, there is no explicit I/O instruction. Which one of the following is true for a CPU with memory mapped I/O?
(a) I/O protection is ensured by operating system routine(s)
(b) I/O protection is ensured by a hardware trap
(c) I/O protection is ensured during system configuration
(d) I/O protection is not possible

[2005 : 1 Mark]

3.14 Which one of the following is true for a CPU having a single interrupt request line and a single interrupt grant line?
(a) Neither vectored interrupt nor multiple interrupting devices are possible
(b) Vectored interrupts are not possible but multiple interrupting devices are possible
(c) Vectored interrupts and multiple interrupting devices are both possible
(d) Vectored interrupt is possible but multiple interrupting devices are not possible

[2005 : 1 Mark]

3.15 A hardwired CPU uses 10 control signals S1 to S10 in various time steps T1 to T5 to implement 4 instructions I1 to I4 as shown below.

	T1	T2	T3	T4	T5
I1	S1, S3, S5	S2, S4, S6	S1, S7	S10	S3, S8
I2	S1, S3, S5	S8, S9, S10	S5, S6, S7	S6	S10
I3	S1, S3, S5	S7, S8, S10	S2, S6, S9	S10	S1, S3
I4	S1, S3, S5	S2, S6, S7	S5, S10	S6, S9	S10

Which of the following pairs of expressions represent the circuit for generating control signals S5 and S10 respectively [(IJ + Ik)Tn indicates that the control signal should be generated in time step Tn if the instruction being executed is IJ or lk]?
(a) S5 = T1 + I2.T3 and S10 = (I1 + I3).T4 + (I2 + I4).T5
(b) S5 = T1 + (I2 + I4).T3 and S10 = (I1 + I3).T4 + (I2 + I4).T5

(c) S5 = T1 + (I2 + I4).T3 and S10 = (I2 + I3+I4).T2 + (I1 + I3).T4 + (I2 + I4).T5

(d) S5 = T1 + (I2+I4).T3 and S10 = (I2 + I3).T2 + I4.T3 + (I1+I3).T4 + (I2+ I4).T5

[2005 : 2 Marks]

3.16 A line L in a circuit is said to have a stuck-at-0 fault if the line permanently has a logic value 0. Similarly a line L in a circuit is said to have a stuck-at-1 fault if the line permanently has a logic value 1. A circuit is said to have a multiple stuck-at fault if one or more lines have stuck at faults. The total number of distinct multiple stuck-at faults possible in a circuit with N lines is

(a) 3^N (b) $3^N - 1$

(c) $2^N - 1$ (d) 2^N

[2005 : 2 Marks]

3.17 An instruction set of a processor has 125 signals which can be divided into 5 groups of mutually exclusive signals as follows:

Group 1 : 20 signals, Group 2 : 70 signals, Group 3 : 2 signals, Group 4 : 10 signals, Group 5 : 23 signals.

How many bits of the control words can be saved by using vertical microprogramming over horizontal microprogramming?

(a) 0 (b) 103

(c) 22 (d) 55

[2005 : 2 Marks]

3.18 The floating point unit of a processor using a design D takes 2t cycles compared to t cycles taken by the fixed point unit. There are two more design suggestions D1 and D2. D1 uses 30% more cycles for fixed point unit but 30% less cycles for floating point unit as compared to design D. D2 uses 40% less cycles for fixed point unit but 10% more cycles for floating point unit as compared to design D. For a given program which has 80% fixed point operations and 20% floating point operations, which of the following ordering reflects the relative performances of three designs? (Di > Dj denotes that Di is faster than Dj)

(a) D1 > D > D2 (b) D2 > D > D1

(c) D > D2 > D1 (d) D > D1 > D2

[2007 : 2 Marks]

3.19 Consider a CPU where all the instructions require 7 clock cycles to complete execution. There are 140 instructions in the instruction set. It is found that 125 control signals are needed to be generated by the control unit. While designing the horizontal micro-programmed control unit, single address field format is used for branch control logic. What is the minimum size of the control word and control address register?

(a) 125, 7 (b) 125, 10

(c) 135, 9 (d) 135, 10

[2008 : 2 Marks]

3.20 Which of the following is/are true of the auto increment addressing mode?

1. It is useful in creating self relocating code
2. If it is included in an Instruction Set Architecture, then an additional ALU is required for effective address calculation
3. The amount of increment depends on the size of the data item accessed.

(a) 1 only (b) 2 only

(c) 3 only (d) 2 and 3 only

[2008 : 2 Marks]

3.21 The use of multiple register windows with overlap causes a reduction in the number of memory accesses for

1. function locals and parameters
2. register saves and restores
3. instruction fetches

(a) 1 only (b) 2 only

(c) 3 only (d) 1, 2 and 3

[2008 : 2 Marks]

3.22 A CPU generally handles an interrupt by executing an interrupt service routine

(a) as soon as an interrupt is raised

(b) by checking the interrupt register at the end of fetch cycle

(c) by checking the interrupt register after finishing the execution of the current instruction

(d) by checking the interrupt register at fixed time intervals

[2009 : 1 Mark]

3.23 On a non-pipelined sequential processor, a program segment, which is a part of the interrupt service routine, is given to transfer 500 bytes from an I/O device to memory.

```
Initialize the address register
Initialize the count to 500
LOOP:Load a byte from device
Store in memory at address given by
address register
Increment the address register
Decrement the court
If count! = 0 go to LOOP
```

Assume that each statement in this program is equivalent to a machine instruction which takes one clock cycle to execute if it is a non-load/store instruction. The load-store instructions take two clock cycles to execute.

The designer of the system also has an alternate approach of using the DMA controller to implement the same transfer. The DMA controller requires 20 clock cycles for initialization and other overhead. Each DMA transfer cycle takes two clock cycles to transfer one byte of data from interrupt driven program based input-output?

(a) 3.4
(b) 4.4
(c) 5.1
(d) 6.7

[2011 : 2 Marks]

3.24 The amount of ROM needed to implement at 4 bit multiplier is

(a) 64 bits
(b) 128 bits
(c) 1 Kbits
(d) 2 Kbits

[2012 : 1 Mark]

3.25 Consider a main memory system that consists of 8 memory modules attached to the system bus, which is one word wide. When a write request is made, the bus is occupied for 100 nanoseconds (ns) by the data, address, and control signals. During the same 100 ns, and for 500 ns thereafter, the addressed memory module executes one cycle accepting and storing the data. The (internal) operation of different memory modules may overlap in time, but only one request can be on the bus at any time.

The maximum number of stores (of one word each) that can be initiated in 1 millisecond is ________

[2014 (Set-2) : 2 Marks]

■■■■

Answers CPU Control Design and Interfaces

3.1	(d)	3.2	(a, b)	3.3	(b)	3.4	(c)	3.5	(a)	3.6	(a)	3.7	(a)	3.8	(b)	3.9	(d)
3.10	(c)	3.11	(b)	3.12	(a)	3.13	(a)	3.14	(b)	3.15	(d)	3.16	(b)	3.17	(b)	3.18	(a)
3.19	(b)	3.20	(c)	3.21	(c)	3.22	(c)	3.23	(a)	3.24	(d)						

Explanations CPU Control Design and Interfaces

3.1 (d)

An interrupt is a signal from a device attached to a computer or from a program within the computer that causes the main program that operates the computer to stop and figure out what to do next. After the interrupt signal is sensed, it may change state of interrupted process to 'blocked' and schedule another process.

3.2 (a,b)

RAID (Redundant array of independent disks) is a storage technology that combines multiple disk drive components into a logical unit. RAID configurations of disks are used to provide fault-tolerance and high speed.

3.3 (b)

Configurations for CPU in decreasing order of operating speeds:

Hardwired control > Horizontal micro-programming > Vertical micro-programming

3.4 (c)

A processor needs software interrupt to obtain system services which need execution of privileged instructions.

3.5 (a)

- Indirect addressing → Passing array as parameter

- Indexed addressing → Array implementation
- Base register addressing → Writing relocatable code

3.6 (a)

SP is decremented locally.
Therefore memory operation needs 2 clocks.

3.7 (a)

A device employing INTR line for device interrupt puts the CALL instruction on the data bus while $\overline{\text{INTA}}$ is active.

3.8 (b)

In absolute addressing mode, the address of the operand is inside the instruction.

3.9 (d)

Features of horizontal micro-programming are:
(a) doesn't require use of signal decoders
(b) results in larger sized micro-instructions than vertical micro-programming
(c) uses one bit for each control signal.

3.10 (c)

An array (n × n contains) multiplier contains (2n – 1) gate cells unit if each unit (cells) contain a $\theta(1)$ then delay then total delay is $(2n - 1)\,\theta(1)$ which is $\theta(n)$.

3.11 (b)

Multiplying two 8 bit numbers will give result in maximum 16 bits.
Total number of multiplications possible

$$= 2^8 \times 2^8 = 64\text{ K}.$$

Hence space required = 64 K × 16 bits.

3.12 (a)

We just have to see which all options give 1 whenever A_{in} is 1 and 0 otherwise.

A_{in} is 1 in T3 of I1, I2 and I3, also during T1 of I1, and T2 and T4 of I3.

T1.I1 + T2.I3 + T4.I3 + T3.I1 + T3.I2 + T3.I3

Since CPU is having only 3 instructions, T3.I1 + T3.I2 + T3.I3 can be replaced with T3 (we don't need to see which instruction and A_{in} will be activated in time step 3 of all the instructions).

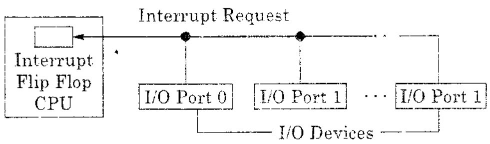
So, T1.I1 + T2.I3 + T4.I3 + T3

3.13 (a)

Memory-mapped I/O requires that memory locations and I/O ports share the same set of addresses, So address bit pattern that is assigned to memory can not also be assigned to an I/O port and vice versa. I/O protection in this approach is ensured by operating systems macros and routines.

3.14 (b)

Consider the following figure



In single line interrupt system contains a single interrupt request line and a interrupt grant line. In this system it may be possible that more than one output devices request interrupt at the same time for example I/O port 0 to I/O port n may request an interrupt at the same time but only one request will be granted according to priority. So, In single interrupt system vectored interrupts are not possible but multiple interrupting devices are possible.

3.15 (d)

If we look at the table, we need to find those time stamps and instructions which are using these control signals.

S5 = T1 has used control signal S5 for all the instructions or we can say irrespective of the instructions, S5 is used by instructions I2 and I4 for the time stamp T3, S5 = T1 + I2.T3 + I4.T3 = T1+(I2 + I3).T3.

Similarly calculate for S10 also.

It is an example of Hard wired CU programming used in RISC processors, gives accurate result and but not good for debugging, minor change will cause to restructure the control unit.

3.16 (b)

Total possible combinations (i.e a line may either be at fault (in 2 ways i.e stuck at fault 0 or 1) or it may not be, so there are only 3 possibilities for a line) is 3^N.

In only one combination the circuit will have all lines to be correct (i.e not at fault), Hence 3^{N-1} (as it has been said that circuit is said to have multiple stuck up fault if one or more line is at fault).

3.17 (b)

- In horizontal microprogramming we need 1 bit for every control word.

Total bits in horizontal microprogramming

$= 20 + 70 + 2 + 10 + 23 = 125$

In vertical microprogramming we use Decoder (n to 2^n) and output lines are equal to number of control words.

group 1 = # $\log_2 20$ # = 5
group 2 = # $\log_2 70$ # =7
group 3 = # $\log_2 2$ # = 1
group 4 = # $\log_2 10$ # = 4
group 5 = # $\log_2 23$ # = 5

Total bits required in vertical micro-programming = 5 + 7 + 1 + 4 + 5 = 22.

So number of bits of the control words can be saved by using vertical micro-programming over horizontal micro-programming

$= 125 - 22 = 103$

3.18 (a)

0.8 * (time taken in fixed point) + 0.2 (time taken in floating point)

D2 = 0.92
D = 1.2
D1 = 1.32

3.19 (b)

Since it uses horizontal micro-programmed that requires 1 bit/control signal.

For 125 control signal we need 125 bit.

Total number of microoperation instructions = 140 * 7 = 980.

It requires 10 bit.

3.20 (c)

In auto increment addressing mode the amount of increment depends on the size of the data item accessed. Consider the following example.

$\text{Regs}[R_1] \leftarrow \text{Regs}[R_1] + \text{Mem}[\text{Regs}[R_2]]$

$\text{Regs}[R_2] \leftarrow \text{Reg}[R_2] + d$

Where d is the size of the data item.

3.21 (c)

The use of multiple register windows with overlap causes a reduction in the number of memory access for instruction fetches only.

3.22 (c)

Interrupt handled by execution of interrupt service routine. It checks interrupt register after finishing the execution of current instruction.

3.23 (a)

Speedup = Execution time of the program in Interrupt driven IO/Execution time of the program in DMA.

Execution time of Interrupt Driven IO:

The program consists of 7 Instructions. The first 2 Instructions are in out of LOOP. So these Instructions takes 2 clock cycles.

Next 5 Instructions are in the LOOP. This LOOP will be executed upto 500 times. Each time 1 byte will be transferred to memory. We need to transfer 500 bytes from Disk to main memory. In the 5 Instructions 1 LOAD and 1 STORE Instruction present.

According to problem statement these instructions takes 2 clock cycles each. Hence These instructions takes 2000 clock cycles.

The remaining 3 Instruction in the LOOP will takes 1500 clock cycles.

The total time required for the above program in Interrupt driven IO is : 3502 clock cycles.

Execution time of DMA transfer:

According to the problem statement DMA initialization takes 20 clock cycles.

To transfer 1 Byte takes 2 clock cycles.

We need to transfer 500 bytes so the takes required for the complete transfer is : 2 * 500 = 1000 clock cycles.

Hence the total time required in DMA is: 1020 clock cycles

SPEED UP = INTERRUPT DRIVEN IO EXECUTION TIME / DMA EXECUTION TIME

i.e., speedup = 3502/1020 = 3.4

3.24 (d)

4-bit × 4-bit ⇒ 8-bit

ROM size = $2^4 \times 2^4 \times 8$ bits = 2 Kbits

3.25 Sol.

To send one word data to the bus takes 100 ns.

So, one words store → 100 ns

? Number of words store → 1 msec

∴ Number of words store

$$= \frac{1 \text{ msec}}{100 \text{ ns}} = 10000$$

■■■■

4 Secondary Memory & DMA

4.1 The data transfer between memory and I/O devices using programmed I/O is faster than interrupt driven I/O.

[1990 : 2 Marks]

4.2 The flags are affected when conditional CALL or JUMP instructions are executed.

[1990 : 2 Marks]

4.3 Transferring data in blocks from the main memory to the cache memory enables an interleaved main memory unit to operate unit at its maximum speed.

[1990 : 2 Marks]

4.4 A certain moving arm disk storage, with one head, has the following specifications.
Number of tracks/recording surface = 200
Disk rotation speed = 2400 rpm
Track storage capacity = 62,500 bits
The average latency of this device is P msec and the data transfer rate is Q bits/sec.
Write the value of P and Q.

[1991 : 2 Marks]

4.5 A certain moving arm disk storage, with one head, has the following specifications Number of tracks/recording surface = 200
Disk rotation speed = 2400 rpm
Track storage capacity = 62,500 bits
The average latency of this device is P msec and the data transfer rate is Q bits/sec. Write the value of P and Q.

[1993 : 2 Marks]

4.6 If the disk is rotating at 3600 rpm, determine the effective data transfer rate which is defined as the number of bytes transferred per second between disk and memory. (Given size of track = 512 bytes)

[1995 : 2 Marks]

4.7 For the daisy chain scheme of connecting I/O devices, which of the following statements is true?
(a) It gives non-uniform priority to various devices
(b) It gives uniform priority to all devices
(c) It is only useful for connecting slow devices to a processor device
(d) It requires a separate interrupt pin on the processor for each device

[1996 : 1 Mark]

4.8 A micro program control unit is required to generate a total of 25 control signals. Assume that during any microinstruction at most two control signals are active. Minimum number of bits required in the control word to generate the required control signal
(a) 2
(b) 25
(c) 10
(d) 12

[1996 : 2 Marks]

4.9 The correct matching for the following pairs is

A. DMA I/O	**1.** High speed RAM
B. Cache	**2.** Disk
C. Interrupt I/O	**3.** Printer
D. Condition code Register	**4.** ALU

Codes:
(a) A – 4, B – 3, C – 1, D – 2
(b) A – 2, B – 1, C – 3, D – 4
(c) A – 4, B – 3, C – 2, D – 1
(d) A – 2, B – 3, C – 4, D – 1

[1997 : 1 Mark]

4.10 I/O redirection
(a) implies changing the name of a file
(b) can be employed to use an existing file as input file for a program
(c) implies connection 2 programs through a pipe
(d) None of the above

[1997 : 1 Mark]

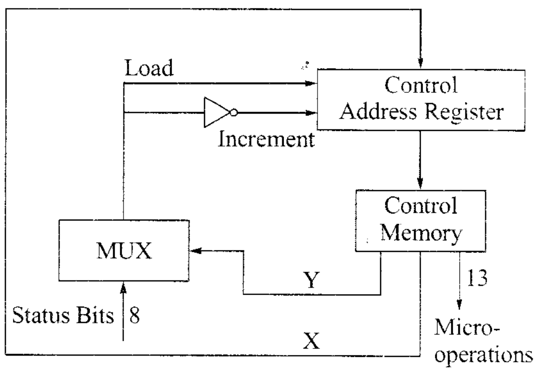
4.11 Formatting of floppy disk refers to
(a) arranging the data on the disk in contiguous fashion
(b) writing the directory
(c) erasing the system area
(d) writing identification information on all tracks and sectors

[1998 : 2 Marks]

4.12 The main differences(s) between a CISC and A RISC processor is/are that a RISC processor typically
(a) has fewer instructions
(b) has fewer addressing modes
(c) has more registers
(d) is easier to implement using hard-wired control logic

[1999 : 2 Marks]

4.13 The microinstructions stored in the control memory of a processor have a width of 26 bits. Each microinstruction is divided into three fields: a micro-operation field of 13 bits, a next address field (X), and a MUX select field (Y). There are 8 status bits in the inputs of the MUX.



How many bits are there in the X and Y fields, and what is the size of the control memory in number of words?
(a) 10, 3, 1024
(b) 8, 5, 256
(c) 5, 8, 2048
(d) 10, 3, 512

[2004 : 2 Marks]

4.14 A hard disk with a transfer rate of 10 M bytes/second is constantly transferring data to memory using DMA. The processor runs at 600 MHz, and takes 300 and 900 clock cycles to initiate and complete DMA transfer respectively. If the size of the transfer is 20 Kbytes, what is the percentage of processor time consumed for the transfer operation?
(a) 5.0%
(b) 1.0%
(c) 0.5%
(d) 0.1%

[2004 : 2 Marks]

4.15 The storage area of a disk has innermost diameter of 10 cm and outermost diameter of 20 cm. The maximum storage density of the disk is 1400bits/cm. The disk rotates at a speed of 4200 RPM. The main memory of a computer has 64-bit word length and 1μs cycle time. If cycle stealing is used for data transfer from the disk, the percentage of memory cycles stolen for transferring one word is
(a) 0.5%
(b) 1%
(c) 5%
(d) 10%

[2004 : 2 Marks]

Linked Answer Questions 4.16 and 4.17

A disk has 8 equidistant tracks. The diameters of the innermost and outermost tracks are 1 cm and 8 cm respectively. The innermost track has a storage capacity of 10 MB.

4.16 What is the total amount of data that can be stored on the disk if it is used with a drive that rotates it with (i) Constant Linear Velocity (ii) Constant Angular Velocity?
(a) (i) 80 MB (ii) 2040 MB
(b) (i) 2040 MB (ii) 80 MB
(c) (i) 80 MB (ii) 360 MB
(d) (i) 360 MB (ii) 80 MB

[2005 : 2 Marks]

4.17 If the disk has 20 sectors per track and is currently at the end of the 5^{th} sector of the inner-most track and the head can move at a speed of 10 meters/sec and it is rotating at constant angular velocity of 6000 RPM, how much time will it take to read 1 MB contiguous data starting from the sector 4 of the outer-most track?
(a) 13.5 ms
(b) 10 ms
(c) 9.5 ms
(d) 20 ms

[2005 : 2 Marks]

4.18 What is the swap space in the disk used for?
(a) Saving temporary html pages
(b) Saving process data
(c) Storing the superblock
(d) Storing device drivers

[2005 : 1 Mark]

4.19 A device with data transfer rate 10 KB/sec is connected to a CPU. Data is transferred byte-wise. Let the interrupt overhead be 4 μsec. The byte transfer time between the device interface register and CPU or memory is negligible. What is the minimum performance gain of operating the device under interrupt mode over operating it under program controlled mode?

(a) 15 (b) 25
(c) 35 (d) 45

[2005 : 2 Marks]

4.20 Consider a disk drive with the following specifications.
16 surfaces, 512 tracks/surface, 512 sectors/track, 1 KB/sector, rotation speed 3000 rpm. The disk is operated in cycle stealing mode whereby whenever one byte word is ready it is sent to memory; similarly, for writing, the disk interface reads a 4 byte word from the memory in each DMA cycle. Memory cycle time is 40 nsec. The maximum percentage of time that the CPU gets blocked during DMA operation is
(a) 10 (b) 25
(c) 40 (d) 50

[2005 : 2 Marks]

4.21 Which of the following DMA transfer modes and interrupt handling mechanisms will enable the highest I/O band-width?
(a) Transparent DMA and Polling interrupts
(b) Cycle-stealing and Vectored interrupts
(c) Block transfer and Vectored interrupts
(d) Block transfer and Polling interrupts

[2006 : 1 Mark]

4.22 Which of the following systems is a most likely candidate example of a pipe and filter architecture?
(a) Expert system
(b) DB repository
(c) Aircraft flight controller
(d) Signal processing

[2007 : 1 Mark]

4.23 Consider a disk pack with 16 surfaces, 128 tracks per surface and 256 sectors per track. 512 bytes of data are stored in a bit serial manner in a sector. The capacity of the disk pack and the number of bits required to specify a particular sector in the disk are respectively
(a) 256 Mbyte, 19 bits
(b) 256 Mbyte, 28 bits
(c) 512 Mbyte, 20 bits
(d) 64 Gbyte, 28 bits

[2007 : 1 Mark]

4.24 Consider a Direct Mapped Cache with 8 cache blocks (numbered 0-7). If the memory block requests are in the following order 3, 5, 2, 8, 0, 63, 9,16, 20, 17, 25, 18, 30, 24, 2, 63, 5, 82,17, 24. Which of the following memory blocks will not be in the cache at the end of the sequence?
(a) 3 (b) 18
(c) 20 (d) 30

[2007 : 2 Marks]

4.25 A hard disk system has the following parameters:
Number of tracks = 500
Number of sectors/track = 100
Number of bytes /sector = 500
Time taken by the head to move from one track to adjacent track = 1 ms Rotation speed = 600 rpm. What is the average time taken for transferring 250 bytes from the disk?
(a) 300.5 ms (b) 255.5 ms
(c) 255 ms (d) 300 ms

[2007 : 2 Marks]

4.26 For a magnetic disk with concentric circular tracks, the latency is not linearly proportional to the seek distance due to
(a) non-uniform distribution of requests
(b) arm starting and stopping inertia
(c) higher capacity of tracks on the periphery of the platter
(d) use of unfair arm scheduling policies.

[2008 : 2 Marks]

Common Data for Q. 4.27 and Q.4.28
A hard disk has 63 sectors per track, 10 platters each with 2 recording surfaces and 1000 cylinders. The address of a sector is given as a triple $\langle c, h, s \rangle$, where c is the cylinder number, h is the surface number and s is the sector number. Thus, the 0^{th} sector is addressed as $\langle 0, 0, 0 \rangle$, the 1st sector as $\langle 0, 0, 1 \rangle$, and so on.

4.27 The address $\langle 400, 16, 29 \rangle$ corresponds to sector number:
(a) 505035 (b) 505036
(c) 505037 (d) 505038

[2009 : 2 Marks]

4.28 The address of 1039th sector is

(a) $\langle 0, 15, 31 \rangle$ (b) $\langle 0, 16, 30 \rangle$

(c) $\langle 0, 16, 31 \rangle$ (d) $\langle 0, 17, 31 \rangle$

[2009 : 2 Marks]

4.29 An application loads 100 libraries at start-up. Loading each library requires exactly one disk access. The seek time of the disk to a random location is given as 10 ms. Rotational speed of disk is 6000 rpm. If all 100 libraries are loaded from random locations on the disk, how long does it take to load all libraries? (The time to transfer data from the disk block once the head has been positioned at the start of the block may be neglected.)

(a) 0.50 s (b) 1.50 s

(c) 1.25 s (d) 1.00 s

[2011 : 2 Marks]

4.30 Consider a hard disk with 16 recording surfaces (0 - 15) having 16384 cylinders (0-16383) and each cylinder contain 64 sectors (0-63). Data storage capacity in each sector is 512 bytes. Data are organized cylinder-wise and addressing format is <cylinder no., surface no., sector no>. A file of size 42797 KB is stored in the disk and the starting disk location of the file is < 1200, 9, 40>. What is the cylinder number of the last sector of the file, if it is stored in a contiguous manner?

(a) 1281 (b) 1282

(c) 1283 (d) 1284

[2013 : 2 Marks]

4.31 Consider a disk pack with a seek time of 4 milliseconds and rotational speed of 10000 rotations per minute (RPM). It has 600 sectors per track and each sector can store 512 bytes of data. Consider a file stored in the disk. The file contains 2000 sectors. Assume that every sector access necessitates a seek, and the average rotational latency for accessing each sector is half of the time for one complete rotation. The total time (in milliseconds) needed to read the entire file is ________.

[2015 (Set-1) : 2 Marks]

4.32 Consider a typical disk that rotates at 15000 rotations per minute (RPM) and has a transfer rate of 50×10^6 bytes/sec. If the average seek time of the disk is twice the average rotational delay and the controller's transfer time is 10 times the disk transfer time, the average time (in milliseconds) to read or write a 512 byte sector of the disk is ____________.

[2015 (Set-2) : 2 Marks]

4.33 The size of the data count register of a DMA controller is 16 bits. The processor needs to transfer a file of 29.154 kilobytes from disk to main memory. The memory is byte addressable. The minimum number of times the DMA controller needs to get the control of the system bus from the processor to transfer the file from the disk to main memory is _______.

[2016 (Set-1) : 2 Marks]

■■■■

Answers Secondary Memory & DMA

4.7 (a)	4.8 (c)	4.9 (b)	4.10 (c)	4.11 (d)	4.13 (a)	4.14 (d)	4.15 (c)	4.16 (d)	
4.17 (a)	4.18 (b)	4.19 (b)	4.20 (b)	4.21 (d)	4.22 (d)	4.23 (a)	4.24 (b)	4.25 (d)	
4.26 (c)	4.27 (c)	4.28 (c)	4.29 (b)	4.30 (d)					

Explanations Secondary Memory & DMA

4.1 Sol.

False: The data transfer between memory and I/O devices using interrupt driven I/O is faster than programmed I/O because programmed I/O technique require constant monitoring on peripheral by CPU, once data transfer is initiated, CPU have to wait for next transfer whereas in interrupt driven I/O once data transfer initiated, CPU executes next program without wasting time and the interface keep monitoring the device.

4.2 Sol.

False: Conditional CALL or JUMP instructions only test flags, they do not affect any of the flag values.

4.3 Sol.

True: If the main memory of a computer is structured as a collection of physically separate modules, each with its own address buffer register (ABR) and data buffer register (DBR), memory access operation may proceed in more than one module at the same time. Thus, the aggregate rate of transmission of words to and from the main memory system can be increased.

4.4 Sol.

Average latency,

$$P = \frac{1}{2} \times \text{Rotation time}$$

$$= \frac{1}{2} \times \frac{60}{2400} \text{sec}$$

$$= 12.5 \text{ msec}$$

$$\text{In } \frac{60}{2400} \text{sec} \rightarrow 200 \times 62500 \text{ bits}$$

$$1 \text{ sec} \rightarrow \frac{200 \times 62500 \times 2400}{60} \text{bits/sec}$$

$$\rightarrow 59.60 \text{ Mbps}$$

Data transfer rate, Q = 59.60 Mbps

4.5 Sol.

Disk rotation speed = 2400

2400 R in 60 sec

1 R = ?

$$1 \text{ R} = \frac{60}{2400} = \frac{1}{40}$$

So $\frac{1}{40}$ sec takes to transfer 62500 bits.

Transfer rate = In '1' sec how many data can be transferred i.e., = 40 × 62500 bits/sec

Avg. seek time latency is the time needed to traverse 100 tracks:

To traverse 200 tracks we have taken 5 sec as

$\frac{1}{40}$ sec we transfer = 62500 bits

? transfer 200 × 62500 bits

i.e., = 5 sec.

So for 100 tracks it is 2.5 sec

Hence, value of P and Q are

P: 2.5 sec

Q: 40 × 62500 bits/sec

4.6 Sol.

3600 Rotation — 60 sec

$$1 \text{ Rotation} — \frac{60}{3600} = \frac{1}{60}\text{sec}$$

Transfer rate = 60 × (size of track)/sec.

= 60 × 512 B/sec = 30 Kbps

4.7 (a)

The daisy-chinning method of establishing priority consists of a serial connection of all devices that request an interrupt. The device with the highest priority is placed in the first position, followed by lower-priority devices up to the device with the lowest priority, which is placed last in the chain. The farther the device is from the first position, the lower is its priority. Therefore daisy-chain gives non-uniform priority to various devices.

4.8 (c)

5 bits	5 bits

5 bits to identify first and 5 bits to identify second including the case when one of them not present.
So total bits required = 10

4.9 (b)

DMA I/O — Disk
Cache — High speed RAM
Interrupt I/O — Printer
Condition code register – ALU

4.10 (c)

I/O redirection implies connection two programs through a pipe.

4.11 (d)

Formatting of floppy disk refers to writing identification information on all tracks and sectors.

4.12 Sol.

(a), (b), (c), (d)
The major characteristics of a RISC processor are:

1. Relatively few instructions.
2. Relatively few addressing modes
3. More registers
4. Hardwired rather than micropro-grammed control

4.13 (a)

Total Size of micro instruction = 26 bits
Size of micro-operation = 13 bits
Mux input = 8 bit
So select line field size Y $\Rightarrow$ $8 = 2^3 \Rightarrow 3$ bits
Next address field size = 13 – 3
= 10 bits
Size of control memory
$= 2^{10} = 1024$

4.14 (d)

Data transfer rate = 10M bytes/sec
$= 10 \times 2^{10}$ K byte/sec
Size of transfer = 20 Kbytes

$$10 \times 2^{10} \times x\% = 20$$
$$10 \times 2^{10} \times x/100 = 20$$
$$x = \frac{20 \times 100}{10 \times 2^{10}}$$
$$x = \frac{200}{1024}$$
$$x = 0.19\%$$

or $x = 0.10\%$

4.15 (c)

As the max density is given so we take the inner most diameter as it is denser there ($2\pi r$). The capacity of each track = π * D* Density = 3.14*10*1400 bits = 14000π bits.
Rotational latency = 60/RPM
= 60 / 4200 = 1/70s
In 1/70 sec the disk can traverse an entire track and can read total 14000π bits.
Now the data transfer is done by the DMA controller which will be operated in memory cycle time i.e 1 micro sec and it has 64 bit word length. So in 1 cycle it is able to transfer total of 64 bits.
In 1 sec it can transfer 64×10^6 bits. (1sec memory cycle)
In 1 sec the disk can read total of 14000π*70 = 3.08×10^6 bits (considering $\pi = 22/7$)
So total memory cycle stolen is
$3.08 \times 10^6/(64 \times 10^6)\% = 5\%$ (approx).

4.16 (d)

With **Constant Linear Velocity, CLV,** the density of bits is uniform from cylinder to cylinder. Because there are more sectors in outer cylinders, the disk spins slower when reading those cylinders, causing the rate of bits passing under the read-write head to remain constant. This is the approach used by modern CDs and DVDs.
With **Constant Angular Velocity, CAV,** the disk rotates at a constant angular speed, with the bit density decreasing on outer cylinders. (These disks would have a constant number of sectors per track on all cylinders).
CLV = 10 + 20 + 30 + 40 +...80 = 360
CAV = 10*8 = 80 so answer should be d.

4.17 (a)

Radius of inner track is 0.5 cm (where the head is standing) and the radius of outermost track is 4 cm. So the header has to seek $(4 - 0.5) = 3.5$ cm.

Head can move 10 meters/sec, It will take 3.5 ms to seek 3.5 cm.

Since 6000 RPM, In 1 rotation, it takes 10ms to read 20 sectors.

Angular velocity is constant and header is now end of 5th sector. To keep the head at the beginning of 4th sector it must rotate 18 sectors.

If 20 sectors can read in 10 ms, 18 sectors can read in 9 ms.

In 1 Rotation, it takes 10ms to read 10MB data.

To read 1MB data, we require 1msec.

Total time required = 3.5 ms (to reach outermost track from innermost track) + 9 (to keep the head at the desired sector in the outermost track) +1 ms (to read 10 MB data)

Total time required = 13.5 msec

4.18 (b)

The swap space is basically used for saving process data. Consider CPU contains two process P_1 and P_2 if P_1 is running on the CPU then P_2 is swap out and all data (PC for process, content etc) are saved in the disk, when CPU executes process P_2 then all data items of P_1 saved on a disk.

4.19 (b)

$ET_{prog\text{-}IO}$: $\quad$ 10 kB $\Rightarrow$ 1 sec

$\qquad\qquad\qquad$ 1 B $\Rightarrow$? sec

$ET_{prog\text{-}IO} = 100\ \mu sec$

$ET_{INT\text{-}IO} = 4\ \mu sec$

$S = 100/4 = 25$

4.20 (b)

Revolution Per minute = 3000 RPM

Or $\qquad$ 3000/60 = 50RPS

In one track rotation it can read = 512 KB

In 50 RPS it can read $= 512 \times 50$

For 1 byte read $= 1/(512 \times 50) = 39.06$ ns

$\Rightarrow$ For 4 bytes it takes 156 ns

Percentage of time that the CPU gets blocked during DMA operation $= \dfrac{40}{156} \times 100 \simeq 25\%$.

4.21 (d)

In block transfer the entire block of data is transferred then only CPU again becomes the bus master.

In vectored Interrupts I/O device along with interrupts send vector address of Interrupt Service routine which guides CPU to execute for a specific I/O device.

Hence in both case BW will be required in a good amount.

4.22 (d)

Pipe is communication channel carring a signal messages, and filters are signal processing componants such as amplifiers noise filters etc.

4.23 (a)

$$\begin{aligned}
\text{No. of surfaces} &= 16 \\
\text{No. of tracks/sector} &= 128 \\
\text{No. of sector/track} &= 256 \\
\text{Total size of disk} &= 16 \times 128 \times 256 \times 512 \text{ bytes} \\
&= 2^4 \times 2^7 \times 2^8 \times 2^9 \text{ bytes} \\
&= 2^8 \times 2^{20} \text{ bytes} \\
&= 2^8 \text{ Mega bytes} \\
&= 256\text{ MB}
\end{aligned}$$

Total Number of Sector in the disk

$$\begin{aligned}
&= 16 \times 128 \times 256 \text{ bytes} \\
&= 2^4 \times 2^7 \times 2^8 \text{ bytes} \\
&= 2^{19} \text{ bytes}
\end{aligned}$$

So 19 bits are needed

4.24 (b)

Cache location (memory block) = (block req) MOD (number of cache blocks).

Since each block has only one location (associativity is 1) the last mod 8 request will be in cache assuming LRU replacement.

0 block - 8,0,16, at end contains 24

1- at ends contains 17

2- at end contains 82

3- 3

4- 20

5- 5

6- 30

7- 63.

4.25 (d)

- RPM = 600.

So, rotational delay = 60/600 = 0.1 sec.

In 1 rotation, we can transfer the whole data in a track.

Tack capacity = Track * bytes per track = 100 * 500 = 50,000 bytes.

In 0.1 sec, we can transfer 50,000 bytes.

Hence time to transfer 250 bytes = 0.1 * 250/ 50,000 = 0.5 ms

Avg. rotational delay = 0.5 * rotational delay = 0.5 * 0.1s = 50 ms

Average seek time = (0 + 1 + 2 +...+ 499)/500 (as time to move between successive tracks is 1 ms and we have 500 such tracks) = 499 * 250/ 500 = 249.5

Average time to transfer = Average seek time + Average rotational delay + Data transfer time

Average time for transferring 250 bytes = 249.5 + 50 + 0.5 = 300 ms.

4.26 (c)

In magnetic disk once head is in position, the desired cell may be in wrong part due to a high capacity of tracks. Some time is required for this cell to reach the read-write head so that data transfer can begin. The average time for this movement to take place is seek latency which is not linearly proportional to the seek distance because a higher capacity of tracks on the periphery of the platter.

4.27 (c)

The address <400, 16, 29> corresponds to sector no.

$400 \times 2 \times 10 \times 63 + 16 \times 63 + 29 = 505037$

4.28 (c)

The address <0, 16, 31> corresponds to sector no.

$16 \times 63 + 31 = 1039$

4.29 (b)

Rotational speed = 6000 rpm

$$\text{Average latency} = \left(\frac{60\,\text{sec}}{6000}\right)/2 = 5\text{msec}$$

Avg access time

= Avg latency + Avg seek time

= 5 ms + 10 ms = 15 msec

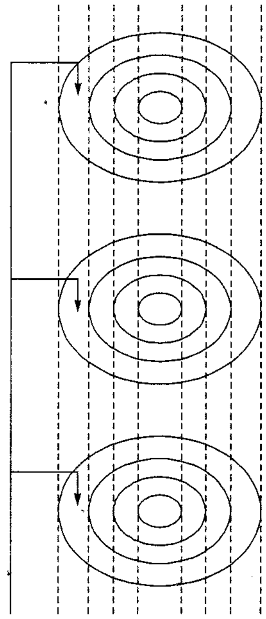
Time to load 1 library is 15 msec

Time to load 100 libraries

= 100 × 15 msec

= 1.5 sec

4.30 (d)



When head is moving then it is reading data from all 16 surface simultaneously.

Currently head is on 9th surface, sector no. 40.

How much data it read from 9th surface

$= 24 \times 512 = 12288$ bytes.

How much data it read from surface 10th to 15th

$= 5 \times 512 \times 64 = 163840$ bytes.

How much data it read from cylinder 1201 to 1283

$82 \times 16 \times 64 \times 512 = 42991616$ bytes.

Total data read from 1200th to 1283th cylinder

$12288 + 163840 + 42991616$

$= 13167744$ bytes $= 42156$ KB

But we need to real 42797 KB. So we need to go on 1284th cylinder.

4.31 Sol.

Seek time = 4 ms

$$\text{Rotational latency} = \frac{60}{10000}\text{sec} = 6 \text{ ms}$$

Average rotational latency = R/2 = 3 ms

For each sector, we require Seek time + Rotational Latency + Transfer time.

In one rotation, 512 B × 600 data is read in 6 ms

One sector of 512 B can read in $\frac{6\text{ ms}}{600}$

= 0.01 ms (transfer time for 1 sector)

Total time required for 1 sector

= 4 ms + 3 ms + 0.01 = 7.01

For 2000 sectors 7.01 × 2000 = 14020 ms

4.32 Sol.

1500 rotations → 1 minute

1 rotation → ?

$$\text{Rotational latency} = \frac{1}{15000} \times 1 \text{ minute}$$

$$= \frac{60\text{ sec}}{15000} = 4 \text{ msec}$$

(*i*) Average rotational latency $= \frac{1}{2} \times 4$ msec = 2 msec

(*ii*) Average seek time = 2 × Average rotational delay

= 2 × 2 = 4 msec

Transfer rate = 500 × 10^6 bytes/sec

Sector size = 512 byte

512 byte → ?

(*iii*) Transfer time $= \frac{512}{50 \times 10^6} \times 1$ sec

= 0.01024 msec

(*iv*) Controller's transfer time = 10 × Transfer time

= 10 × 0.01024 = 0.1024 msec

∴ Average time = Average rotational delay + Average seek time + Controller's time + Transfer time

= 2 + 4 + 0.01024 + 0.1024

= 6.11264

≅ 6.1 msec

4.33 Sol.

Data count register = 16 bits.

So, count value = 2^{16} = 64 K bytes

One time control, transfer — 64 K bytes

Number of controls to transfer — 29154 K bytes

So, number of time bus control required

$$= \left\lceil \frac{29154}{64} \right\rceil = 456$$

■■■■

Unit ■ V

Programming & Data Structures

■ **Contents**

UNIT V

Programming & Data Structures

Syllabus : Programming in C; Functions, Recursion, Parameter passing, Scope, Binding; Abstract data types, Arrays, Stacks, Queues, Linked Lists, Trees, Binary search trees, Binary heaps.

Analysis of Previous GATE Papers

Exam Year	1 Mark Ques.	2 Marks Ques.	3 Marks Ques.	5 Marks Ques.	Total Marks
1990	–	1		–	**2**
1991	1	2		3	**20**
1992	–	–		–	**–**
1993	–	5		3	**25**
1994	2	4		7	**45**
1995	2	3		2	**18**
1996	4	3		1	**15**
1997	2	3		2	**18**
1998	2	7		3	**31**
1999	–	4		2	**18**
2000	8	2		2	**22**
2001	–	3		1	**11**
2002	3	2		–	**7**
2003	5	7		–	**19**
2004	7	10		–	**27**
2005	5	6		–	**17**
2006	1	7		–	**15**

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2007	2	7	**16**
2008	3	7	**17**
2009	1	4	**9**
2010	1	5	**11**
2011	1	3	**7**
2012	1	4	**9**
2013	–	3	**6**
2014 Set-1	3	3	**9**
2014 Set-2	3	3	**9**
2014 Set-3	3	3	**9**
2015 Set-1	3	3	**6**
2015 Set-2	3	2	**4**
2015 Set-3	4	6	**16**
2016 Set-1	3	5	**13**
2016 Set-2	3	6	**15**

1 Programming

1.1 Study the following program written in a block-structured language:

```
var x, y : integer;
procedure P(n:integer);
begin
    x : = (n + 2) / (n – 3);
end;
procedure Q
var x, y : integer;
begin
    x : = 3;
    y : = 4;
    P(y);
    Write (x)                           ...(1)
end;
begin
    x : = 7;
    y : = 8;
    Q;
    Write (x)                           ...(2)
end
```

What will be printed by write statements marked (1) and (2) in the program if variables are statically scoped?

(a) 3, 6 (b) 6, 7
(c) 3, 7 (d) None of these

[1987 : 2 Marks]

1.2 In which of the following case(s) is it possible to obtain different results for call-by-reference and call-by-name parameter passing?

(a) Passing an expression as a parameter
(b) Passing an array as a parameter
(c) Passing a pointer as a parameter
(d) Passing an array element as a parameter

[1989 : 2 Marks]

1.3 Match the pairs in the following

List-I
A. Pointer data type
B. Activation record
C. Repeat-until
D. Coercion

List-II
P. Type conversion
Q. Dynamic data structure
R. Recursion
S. Nondeterministic loop

[1990 : 2 Marks]

1.4 Indicate all the true statements from the following:

(a) A programming language not supporting either recursion or pointer type does not need the support of dynamic memory allocation.
(b) Although C does not support call by name parameter passing, the effect can be correctly simulated in C.

[1991 : 1 Mark]

1.5 Match the pairs in the following:

List-I
A. Small talk
B. LISP
C. Prolog
D. VAL

List-II
P. Logic programming
Q. Data flow programming
R. Functional programming
S. Object-Oriented programming

[1993 : 2 Marks]

Common Data Questions 1.6 & 1.7:

Consider the Pascal program shown in below.

```
program PARAM (input, output):
var m, n : integer;
procedure P(var x, y : integer);
var m : integer;
begin
    m : = 1;
    x : = y + 1;
end;
procedure Q (x : integer; var y : integer);
begin
    x : = y + 1;
end;
```

```
begin
    m : = 0; P(m, m); write (m);
    n : = 0; Q (n*1, n) ; write (n)
end.
```

1.6 The value of m, output by the program PARAM is
(a) 1, because m is a local variable in P
(b) 0, because m is the actual parameter that corresponds to the formal parameter in p
(c) 0, because both x and y are just reference to m and y has the value 0
(d) 1, because both x and y are just references to m which gets modified in procedure P

[1993 : 2 Marks]

1.7 The value of n, output by the program PARAM is
(a) 0, because n is the actual parameter corresponding to x in procedure Q.
(b) 0, because n is the actual parameter to y in procedure Q.
(c) 1, because n is the actual parameter corresponding to x in procedure Q.
(d) 1, because n is the actual parameter corresponding to y in procedure Q.
(e) none of the above

[1993 : 2 Marks]

1.8 What is the scope of m declared in the main program?
(a) PARAM, P, Q (b) PARAM, P
(c) PARAM, Q (d) P, Q

[1993 : 2 Marks]

1.9 What does the following code do?

```
var a, b : integer;
begin
    a:=a+b;
    b:=a–b;
    a:=a –b
end;
```

(a) exchanges (a) and (b)
(b) doubles (a) and stores in (b)
(c) doubles (b) and stores in (a)
(d) leaves (a) and (b) unchanged

[1993 : 2 Marks]

1.10 FORTRAN implementation do not permit recursion because
(a) they use static allocation for variables
(b) they use dynamic allocation for variables
(c) stacks are not available on all machines
(d) it is not possible to implement recursion on all machines.

[1994 : 1 Mark]

1.11 An unrestricted use of the "goto" statement is harmful because
(a) it makes it more difficult to verify programs
(b) it increases the running time of the programs
(c) it increases the memory required for the programs
(d) it results in the complier generating longer machine code

[1994 : 1 Mark]

1.12 In which one of the following cases is it possible to obtain different results for call-by reference and call-by-name parameter passing methods?
(a) Passing a constant value as a parameter
(b) Passing the address of an array as a parameter
(c) Passing an array element as a parameter
(d) Passing an array

[1994 : 2 Marks]

1.13 What is the value of X printed by the following program?

```
program COMPUTE (input, output);
    var X:integer;
    procedure FIND (X:real);
        begin X: = sqrt(X); end;
        begin
            X: = 2
        Find(X)
        Writeln(X)
    end
```

(a) 1 (b) 2
(c) Run time error (d) None of these

[1995 : 2 Marks]

1.14 The correct matching for the following pairs is

List -I	List -II
A. Activation record	1. Linking loader
B. Location counter	2. Garbage collection
C. Reference counts	3. Subroutine call
D. Address relocation	4. Assembler

Codes:
(a) A - 3, B - 4, C - 1, D - 2
(b) A - 4, B - 3, C - 1, D - 2
(c) A - 4, B - 3, C - 2, D - 1
(d) A - 3, B - 4, C - 2, D - 1

[1996 : 2 Marks]

1.15 Heap allocation is required for languages
(a) that support recursion
(b) that support dynamic data structures
(c) that use dynamic scope rules
(d) None of the above

[1997 : 1 Mark]

1.16 Given the following Pascal like program segment

```
Procedure A;
    x, y:integer;
    Procedure B;
        x, z:real;
        S1
    end B;
    Procedure C;
        i:integer;
        S2
    end C;
end A;
```

The variables accessible in S1 and S2 are
(a) x of A, y, x of B and z in S1 and x of B, y and i in S2
(b) x of B, y and z in S1 and x of B, i and z in S2
(c) x of B, z and y in S1 and x of A, i and y in S2
(d) None of the above

[1997 : 2 Marks]

1.17 Faster access to non-local variables is achieved using an array of pointers to activation records called a
(a) stack
(b) heap
(c) display
(d) activation tree

[1998 : 2 Marks]

1.18 Given the programming constructs
(i) assignment
(ii) for loops where the loop parameter cannot be changed within the loop
(iii) if-then-else
(iv) forward go to
(v) arbitrary go to
(vi) non-recursive procedure call
(vii) recursive procedure/function call
(viii) repeat loop,

which constructs will you not include in a programming language such that it should be possible to program that terminates (i.e., halting) function in the same programming language.
(a) (ii), (iii), (iv)
(b) (v), (vii), (viii)
(c) (vi), (vii), (viii)
(d) (iii), (vii), (viii)

[1999 : 2 Marks]

1.19 Consider the following program in a language that has dynamic scoping:

```
var x: real;
procedure show:
    begin print(x);end;
procedure small;
    var x: real;
    begin x: = 0.125; show; end;
    begin
        x:=0.25;
        show; small
    end.
```

Then the output of the program is:
(a) 0.125 0.125
(b) 0.25 0.25
(c) 0.25 0.125
(d) 0.125 0.25

[1999 : 2 Marks]

1.20 A certain processor supports only the immediate and the direct addressing modes. Which of the following programming language features cannot be implemented on this processor?
(a) Pointers
(b) Arrays
(c) Records
(d) Recursive procedures with local variable

[1999 : 2 Marks]

1.21 Consider the following C function definition

```
int Trial (int a, int b, int c)
{
    if ((a > = b) && (c < b) return b;
    else if (a > = b) return Trial (a,c,b);
    else return Trial (b,a,c);
}
```

The function Trial:
(a) Finds the maximum of a, b and c
(b) Finds the minimum of a, b and c
(c) Finds the middle number of a, b and c
(d) None of the above

[1999 : 2 Marks]

1.22 The most appropriate matching for the following pairs

List-I	List-II
X. Indirect addressing	**1.** Loops
Y. Immediate addressing	**2.** Pointers
Z. Auto decrement addressing	**3.** Constants

(a) X – 3 Y – 2 Z – 1
(b) X – 1 Y – 3 Z – 2
(c) X – 2 Y – 3 Z – 1
(d) X – 3 Y – 1 Z – 2

[2000 : 1 Mark]

1.23 The following C declarations

```
struct node{
    int i:
  float j;
};
struct node *s[10];
```

define s to be

(a) An array, each element of which is a pointer to a structure of type node
(b) A structure of 2 fields, each field being a pointer to an array of 10 elements
(c) A structure of 3 fields: an integer, a float, and an array of 10 elements
(d) An array, each element of which is a structure of type node

[2000 : 1 Mark]

1.24 The most appropriate matching for the following pairs

List-I
X. m=malloc(5); m = NULL;
Y. free(n); n->value=5;
Z. char *p; *p='a';

List-II
1. using dangling pointers
2. using uninitialized pointers
3. lost memory

Codes:
(a) X – 1 Y – 3 Z – 2
(b) X – 2 Y – 1 Z – 3
(c) X – 3 Y – 2 Z – 1
(d) X – 3 Y – 1 Z – 2

[2000 : 1 Mark]

1.25 Aliasing in the context of programming languages refers to

(a) multiple variables having the same memory location
(b) multiple variables having the same value
(c) multiple variables having the same identifier
(d) multiple uses of the same variable

[2000 : 1 Mark]

1.26 Consider the following C declaration

```
struct {
    short s[5];
    union {
    float y;
    long z;
    } u;
} t;
```

Assume that objects of the type short, float and long occupy 2 bytes, 4 bytes and 8 bytes, respectively. The memory requirement for variable t, ignoring alignment considerations, is

(a) 22 bytes (b) 14 bytes
(c) 18 bytes (d) 10 bytes

[2000 : 1 Mark]

1.27 The value of j at the end of the execution of the following C program is ____

```
int incr (int i) {
    static int count = 0;
    count = count + i;
    return (count);
}
main () {
        int i,j;
        for (i = 0; i <=4; i++)
        j = incr(i);
}
```

(a) 10 (b) 4
(c) 6 (d) 7

[2000 : 2 Marks]

1.28 What is printed by the print statements in the program P1 assuming call by reference parameter passing?

```
Program P1()
{
    x = 10;
    y = 3;
    func1(y, x, x);
    print x;
```

```
        print y;
    }
    func1(x, y, z)
    {
        y = y + 4;
        z = x + y + z;
    }
```

(a) 10, 3 (b) 31, 3
(c) 27, 7 (d) None of these

[2001 : 2 Marks]

1.29 Consider the following three C functions:

```
[P1]    int *g (void)
        {
            int x = 10;
            return (&x);
        }
[P2]    int *g (void)
        {
            int *px;
            *px = 10;
            return px;
        }
[P3]    int *g(void)
        {
            int *px;
                px = (int *)malloc (size of (int));
            *px = 10;
            return px;
        }
```

Which of the above three functions are likely to cause problems with pointers?

(a) Only P3 (b) Only P1 and P3
(c) Only P1 and P2 (d) P1, P2 and P3

[2001 : 2 Marks]

1.30 Consider the following program

Program P2

```
    var n:int;
    procedure W(var x:int)
    begin
        x = x + 1;
        print x;
    end
    procedure D
    begin
        var n:int;
        n = 3;
        W(n);
    end
    begin // begin P2
        n = 10;
        D;
    end
```

If the language has dynamic scoping and parameters are passed by reference, what will be printed by the program?

(a) 10 (b) 11
(c) 3 (d) None of these

[2001 : 2 Marks]

1.31 The results returned by function under value-result and reference parameter passing conventions

(a) Do not differ
(b) Differ in the presence of loops
(c) Differ in all cases
(d) May differ in the presence of exception

[2002 : 1 Mark]

1.32 In the C language

(a) at most one activation record exists between the current activation record and the activation record for the main
(b) the number of activation records between the current activation record and the activation record for the main depends on the actual function calling sequence
(c) The visibility of global variables depends on the actual function calling sequence
(d) Recursion requires the activation record for the recursive function to be saved on a different stack before the recursive function can be called.

[2002 : 1 Mark]

1.33 Consider the following declaration of a two-dimensional array in C

char a[100][100];

Assuming that the main memory is byte-addressable and that the array is stored starting from memory address 0, the address of a [40] [50] is

(a) 4040 (b) 4050
(c) 5040 (d) 5050

[2002 : 2 Marks]

1.34 Assume the following C variable declaration
int*A[10], B[10][10];
Of the following expressions
1. A[2]
2. A[2] [3]
3. B[1]
4. B[2] [3]
Which will not give compile-time errors if used as left hand sides of assignment statements in a C program?
(a) 1, 2 and 4 only (b) 2, 3 and 4 only
(c) 2 and 4 only (d) 4 only
[2003 : 1 Mark]

1.35 In the following C program fragment j, k, n and TwoLog_n are integer variables, and A is an array of integers. The variable n is intialized to an integer ≥ 3, and TwoLog_n is initialized to the value of $2 * \lfloor \log_2(n) \rfloor$

```
for (k = 3; k < = n; k++)
    A [k] = 0;
for (k = 2; k < = TwoLog_n; k++)
    for (j = k + 1; j < = n; j++)
    A [j] = A [j] | | (j%k);
for (j = 3; j < = n; j++)
    if (!A[j]) print f ("%d", j);
```

The set of numbers printed by this program fragment is
(a) $\{m \mid m \leq n, (\exists i)\ [m = i!]\}$
(b) $\{m \mid m \leq n, (\exists i)\ [m = i^2]\}$
(c) $\{m \mid m \leq n, m \text{ is prime}\}$
(d) { }
[2003 : 2 Marks]

1.36 Consider the C program shown below.

```
# include <stdio.h>
# define print (x) printf ("%d", x)
int x;
void Q (int z) {
        z + = x; print (z);
}
void p (int *y) {
        int x = *y + 2;
        Q (x); *y = x – 1;
        print (x);
}
main (void) {
        x = 5;
        p (&x);
        print (x);
}
```

The output of this program is
(a) 12 7 6 (b) 22 12 11
(c) 14 6 6 (d) 7 6 6
[2003 : 2 Marks]

1.37 The goal of structured programming is to
(a) have well indented programs
(b) be able to infer the flow of control from the compiled code
(c) be able to infer the flow of control from the program text
(d) avoid the use of GOTO statements
[2004 : 1 Mark]

1.38 Consider the following C function

```
void swap (int a, int b)
    { int temp ;
    temp = a ;
    a = b ;
    b = temp ;
    }
```

In order to exchange the values of two variables x and y.
(a) call swap (x, y)
(b) call swap (&x, &y)
(c) swap (x, y) cannot be used as it does not return any value
(d) swap (x, y) cannot be used as the parameters are passed by value
[2004 : 1 Mark]

1.39 Let x be an integer which can take a value of 0 or 1. The statement

```
if (x = = 0) x = 1;
else x = 0;
```

is equivalent to which one of the following?
(a) x = 1 + x; (b) x = 1 – x;
(c) x = x – 1; (d) x = 1% x;
[2004 : 1 Mark]

1.40 Consider the following C program which is supposed to compute the transpose of a given 4 × 4 matrix M. Note that, there is an X in the program which indicates some missing statements. Choose the correct option to replace X in the program.

```
#include <stdio.h>
#define ROW 4
#define COL 4
int M[ROW][COL] = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10,
11, 12, 13, 14, 15, 16};
main( )
{
    int i, j, t;
    for (i = 0; i < 4; ++i)
    {
        X
    }
    for (i = 0; i < 4; ++i)
        for (j = 0; j < 4; ++j)
            printf ("%d", M[i][j]);
}
```

(a)
```
for(j = 0; j < 4; ++j)
{
    t = M[i][j];
    M[i][j] = M[j][i];
    M[j][i] = t;
}
```

(b)
```
for(j = 0; j < 4; ++j)
{
    M[i][j] = t;
    t = M[j][i];
    M[j][i] = M[i][j];
}
```

(c)
```
for(j = i; j < 4; ++j)
{
    t = M[j][i];
    M[i][j] = M[j][i];
    M[j][i] = t;
}
```

(d)
```
for(j = i; j < 4; ++j)
{
    M[i][j] = t;
    t = M[j][i];
    M[j][i] = M[i][j];
}
```

[2004 : 2 Marks]

1.41 What is the output of the following program?

```
#include <stdio.h>
int funcf (int x);
int funcg (int y);
main()
{
    int x = 5, y = 10, count;
    for(count = 1; count < = 2; ++count)
    {
        y + = funcf(x) + funcg(x);
        printf ("%d", y);
    }
}
funcf(int x)
{
    int y;
    y = funcg(g);
    return (y);
}
funcg(int x)
{
    static int y = 10;
    y + = 1;
    return (y + x);
}
```

(a) 43 80
(b) 42 74
(c) 33 37
(d) 32 32

[2004 : 2 Marks]

1.42 Consider the following C program:

```
#include <stclio.h>
typedef struct
{
    char *a;
    char *b;
} t;
void f1(t s);
void f2(t *p);
main( )
{
    static t s = {"A", "B"};
    printf("%s %s\n", s.a, s.b);
    f1(s);
    printf("%s %s\n", s.a, s.b);
    f2(&s);
}
void f1(t s)
{
    s.a = "U";
    s.b = "V"
    printf(%s %s\n", s.a, s.b);
    return;
}
```

```
void f2(t *p)
{
    p → a = "V";
    p → b = "W"
    printf("%s %s\n", p → a, p → b);
    return;
}
```

What is the output generated by the program?

(a) A B
U V
V W
V W

(b) A B
U V
A B
V W

(c) A B
U V
U V
V W

(d) A B
U V
V W
U V

[2004 : 2 Marks]

1.43 Consider the following C function

```
int f (int n)
{       static int i = 1 ;
        if (n > = 5) return n ;
        n = n + i ;
        i ++;
        return f (n) ;
}
```

The value returned by f(1) is

(a) 5 (b) 6
(c) 7 (d) 8

[2004 : 2 Marks]

1.44 Consider the following C program

```
main()
{
      int x, y, m, n;
      scanf ("%d %d", &x , &y);
      / * Assume x > 0 and y > 0* /
      m = x; n = y;
      while(m! = n)
      {
        if (m > n)
          m = m – n;
        else
          n = n – m;
      }
      printf ("% d", n);
}
```

The program computes

(a) $x \div y$, using repeated subtraction
(b) x mod y using repeated subtraction
(c) the greatest common divisor of x and y
(d) the least common multiple of x and y

[2004 : 2 Marks]

1.45 Choose the best matching between the programming styles in Group 1 and their characteristics in Group 2.

Group-I		Group-2
P. Functional	**1.**	Command-based, procedural
Q. Logic	**2.**	Imperative, abstract data types
R. Object-oriented	**3.**	Side-effect free, declarative, expression evaluation
S. Imperative	**4.**	Declarative, clausal represent, theorem proving

Codes:
(a) P-2, Q-3, R-4, S-1
(b) P-4, Q-3, R-2, S-1
(c) P-3, Q-4, R-1, S-2
(d) P-3, Q-4, R-2, S-1

[2004 : 2 Marks]

1.46 Consider the following program fragment for reversing the digits in a given integer to obtain a new integer. Let $n = d_1 d_2 \ldots d_m$.

```
int n, rev ;
rev = 0 ;
while (n > 0) {
        rev = rev * 10 + n % 10 ;
        n = n / 10 ;
}
```

The loop invariant condition at the end of the i^{th} iteration is

(a) $n = d_1\, d_2 \ldots d_{m-i}$ and $rev = d_m\, d_{m-1} \ldots d_{m-i+1}$
(b) $n = d_{m-i+1} \ldots d_{m-1}\, d_m$ or $rev = d_{m-i} \ldots d_2\, d_1$
(c) $n \neq rev$
(d) $n = d_1\, d_2 \ldots d_m$ or $rev = d_m \ldots d_2\, d_1$

[2004 : 2 Marks]

1.47 Consider the following C program segment:

```
char p [20] ;
char * s = "string" ;
int length = strlen (s) ;
for (i = 0 ; i < length; i++)
```

```
p[i] = s[length – i] ;
printf ("%s", p) ;
```

The output of the program is

(a) gnirts
(b) string
(c) gnirt
(d) no output is printed

[2004 : 2 Marks]

1.48 What does the following C-statement declare?

```
int (*f) (int *);
```

(a) A function that takes an integer pointer as argument and returns an integer
(b) A function that takes an integer pointer as argument and returns an integer pointer
(c) A pointer to a function that takes an integer pointer as argument and returns an integer
(d) A function that takes an integer pointer as argument returns a function pointer

[2005 : 1 Mark]

1.49 An Abstract Data Type (ADT) is

(a) same as an abstract class
(b) a data type that cannot be instantiated
(c) a data type for which only the operations defined on it can be used, but none else
(d) all of the above

[2005 : 1 Mark]

1.50 A common property of logic programming languages and functional languages is

(a) both are procedural language
(b) both are based on λ-calculus
(c) both are declarative
(d) all of the above

[2005 : 1 Mark]

1.51 Which of the following are essential features of an object-oriented programming languages?

1. Abstraction and encapsulation
2. Strictly-typedness
3. Type-safe property coupled with sub-type rule
4. Polymorphism in the presence of inheritance

(a) 1 and 2 only (b) 1 and 4 only
(c) 1, 2 and 4 only (d) 1, 3 and 4 only

[2005 : 1 Mark]

1.52 Consider the following C-program

```
double foo (double); /* Line 1*/
    int main () {
    double da, db;
    // input da
    db = foo (da);
    }
double foo (double a){
    return a;
    }
```

The above code complied without any error or warning. If Line 1 is deleted, the above code will show

(a) no compile warning or error
(b) some complier-warning not leading to unitended results
(c) Some complier-warning due to type-mismatch eventually leading to unitended results
(d) Complier errors

[2005 : 2 Marks]

1.53 Consider the following C-program

```
void foo (int n, int sum ) {
        int k = 0, j = 0;
        if (n = = 0) return;
        k = n % 10;
        j = n / 10;
        sum = sum + k;
        foo ( j, sum);
        printf ("% d", k);
        }
int main () {
        int a = 2048, sum = 0;
        foo (a, sum);
        printf ("%d\n", sum);
        }
```

What does the above program print?

(a) 8, 4, 0, 2, 14 (b) 8, 4, 0, 2, 0
(c) 2, 0, 4, 8, 14 (d) 2, 0, 4, 8, 0

[2005 : 2 Marks]

Linked Answer Questions 1.54 & 1.55:

A sink in a directed graph is a vertex i such that there is an edge from every vertex $j \neq i$ to i and there is no edge from i to any other vertex. A directed graph G with n vertices is represented by its adjacency matrix A, where A[i][j] = 1 if there is an edge directed from vertex i to j and 0 otherwise. The following algorithm determines whether there is a sink in the graph G.

```
i = 0;
do
{
    j = i + 1;
    while ((j < n) && E1) j++;
    if (j < n) E2;
} while (i < n);
flag = 1;
for (j = 0; j < n; j++)
if ((j! = i) && E3) flag = 0;
if (flag) printf("Sink exists");
else printf ("Sink does not exist");
```

1.54 Choose the correct expressions for E_1 and E_2

(a) E_1 : A[i][j] and E_2 : i = j;
(b) E_1 : !A[i][j] and E_2 : i = j + 1;
(c) E_1: !A[i][j] and E_2 : i = j;
(d) E_1 : A[i][j] and E_2 : i = j + 1;

[2005 : 2 Marks]

1.55 Choose the correct expression for E_3

(a) (A[i][j] && !A[j][i]) (b) (!A[i][j] && !A[j][i])
(c) (!A[i][j] | | A[j][i]) (d) (A[i][j] | | !A[j][i])

[2005 : 2 Marks]

1.56 The following C function takes two ASCII strings and determines whether one is an anagram of the other. An anagram of a string s is a string obtained by permuting the letters in s.

```
int anagram (char *a, char *b)
{
    int count [128], j;
    for (j = 0;  j < 128; j++)
        count[j] = 0;
    j = 0;
    while (a[j] && b[j])
    {
        A ;
        B ;
    }
    for (j = 0; j < 128; j++)
        if (count [j]) return 0;
    return 1;
}
```

Choose the correct alternative for statements A and B.

(a) A: count [a[j]]++ and
B: count[b[j]]– –
(b) A: count [a[j]]++ and
B: count[b[j]]++
(c) A: count [a[j++]]++ and
B: count[b[j]]– –
(d) A: count [a[j]]++ and
B: count[b[j++]]– –

[2005 : 2 Marks]

1.57 What is the output printed by the following program?

```
# include <stdio.h>
int f(int n, int k)
{
    if (n = = 0) return 0;
    else if (n% 2) return f(n/2, 2*k)+k;
    else return f(n/2, 2*k) – k;
}
int main( )
{
    printf("%d", f(20, 1));
    return 0;
}
```

(a) 5 (b) 8
(c) 9 (d) 20

[2005 : 2 Marks]

1.58 Which one of the choices given below would be printed when the following program is executed?

```
#include <stdio.h>
struct test
{
    int i;
    char *c;
} st[ ] = {5, "become", 4, "better", 6, "jungle", 8,
"ancestor", 7, "brother"};
main( )
{
    struct test *p = st;
    p+ = 1;
        ++p → c;
    printf("%s,", p++ → c);
    printf("%c,", *++p → c);
    printf("%d,", p[0].i);
    printf("%s \n", p → c);
```

(a) jungle, n, 8, ncestor
(b) etter, u, 6, ungle
(c) cetter, k, 6, jungle
(d) etter, u, 8, ncestor

[2006 : 2 Marks]

1.59 Which one of the choices given below would be printed when the following program is executed?

```
#include <stdio.h>
void swap (int *x, int *y)
{
    static int *temp;
    temp = x;
    x = y;
    y = temp;
}
void printab()
{
    static int i, a = -3, b = -6;
    i = 0;
    while (i < = 4)
{
    if ((i++)%2 = = 1) continue;
    a = a + i;
    b = b + i;
}
    swap (&a, &b);
    printf("a =  %d, b = %d\n", a, b);
}
main()
{
    printab();
    printab();
}
```

(a) a = 0, b = 3
 a = 0, b = 3

(b) a = 3, b = 0
 a = 12, b = 9

(c) a = 3, b = 6
 a = 3, b = 6

(d) a = 6, b = 3
 a = 15, b = 12

[2006 : 2 Marks]

1.60 Which one of the choices given below would be printed when the following program is executed?

```
#include <stdio.h>
int a1[] = {6, 7, 8, 18, 34, 67};
int a2[]= {23, 56, 28, 29};
int a3[] = {-12, 27, -31};
int *x[] = {a1, a2, a3};
void print(int *a[])
{
    printf("%d,", a[0][2]);
    printf("%d,", *a[2]);
    printf("%d,", *++a[0]);
    printf("%d,", *(++a)[0]);
    printf("%d\n", a[-1][+1]);
}
main()
{
    print(x);
}
```

(a) 8, –12, 7, 23, 8

(b) 8, 8, 7, 23, 7

(c) –12, –12, 27, –31, 23

(d) –12, –12, 27, –31, 56

[2006 : 2 Marks]

1.61 The following function computes the value of $\binom{m}{n}$ correctly for all legal values m and n ($m \geq 1$, $n \geq 0$ and $m > n$)

```
int func(int m, int n)
{
    if (E) return 1;
    else return(func(m -1, n) + func(m - 1, n -
    1));
}
```

In the above function, which of the following is the correct expression for E?

(a) (n = = 0) || (m = = 1)

(b) (n = = 0) && (m = = 1)

(c) (n = = 0) || (m = = n)

(d) (n = = 0) && (m = = n)

[2006 : 2 Marks]

1.62 Match the following concepts and their best possible descriptions.

Concept	Description
(i) overloading	(a) allows to define a class to have properties of another class
(ii) friend	(b) defining a set of similar functions
(iii) constructor	(c) used in dereferencing
(iv) protected	(d) used to give a non-member function access to the private parts of an object
(v) this	(e) a function which is automatically called when an object is created
(vi) inheritance	(f) allows a derived class to have access to the private parts of the base class
	(g) a pointer to the object associated with the current function
	(h) used to obtain object persistence

(1) i-b, ii-d, iii-e, iv-f, v-g, vi-a
(2) i-c, ii-a, iii-e, iv-d, v-h, vi-f
(3) i-c, ii-f, iii-h, iv-a, v-g, vi-d
(4) i-b, ii-e, iii-c, iv-f, v-g, vi-s

[2006 : 2 Marks]

1.63 Consider the following code written in a pass-by-reference language like FORTAN and these statements about the code.

```
      Subroutine swap (ix,iy)
            it = ix
L1 :        ix = iy
L2 :        iy = it
      end
      ia = 3
      ib = 8
      call swap (ia, ib + 5)
      print ia, ib
      end
```

S1: The compiler will generate code to allocate a temporary nameless cell, initialize it to 13, and pass the address of the cell to swap

S2: On execution the code will generate a runtime error on line L1

S3: On execution the code will generate a runtime error on line L2

S4: The program will print 13 and 8

S5: The program will print 13 and –2

Exactly the following set of statement (s) is correct:

(a) S1 and S2
(b) S1 and S4
(c) S3
(d) S1 and S5

[2006 : 2 Marks]

1.64 Consider the following C-function in which a [n] and b [m] are two sorted integer arrays and c [n + m] be another integer array.

```
void xyz (int a[], int b[], int c[]) {
    int i, j, k;
    i = j = k = 0;
    while ((i < n) && (j < m))
      if (a[i] < b[j]) c[k++] = a[i ++];
      else c[k ++] = b[j ++];
}
```

Which of the following condition (s) hold (s) after the termination of the while loop?

(i) $j < m$, $k = n + j - 1$, & $a[n-1] < b[j]$ if $i = n$
(ii) $i < n$, $k = m + i - 1$, & $b[m-1] \le a[i]$ if $j = m$

(a) only (i)
(b) only (ii)
(c) either (i) or (ii) but not both
(d) neither (i) nor (ii)

[2006 : 2 Marks]

1.65 Consider these two functions and two statements S1 and S2 about them.

```
int work1(int* a, int i, int j)
{
      int x=a[i+2];
      a[j]=x+1;
      return a[i+2]-3;
}
```

```
int work2(int* a, int i, int j)
{
      int t1=i+2;
      int t2=a[t1];
      a[j] = t2+1;
      return t2-3;
}
```

S1 : The transformation from work1 to work2 is valid, i.e., for any program state and input arguments, work2 will compute the same output and have the same effect on program state as work1

S2: All the transformations applied to work1 to get work2 will always improve the performance (i.e. reduce CPU time) of work2 compared to work1

(a) S1 is false and S2 is false
(b) S1 is false and S2 is true
(c) S1 is true and S2 is false
(d) S1 is true and S2 is true

[2006 : 2 Marks]

1.66 Consider this C code to swap two integers and these five statements : the code

```
void swap (int*px, int* py) {
        *px = *px – *py;
        *py = *px + *py;
        *px =*py –*px;
}
```

S1: will generate a compilation error

S2: may generate a segmentation fault at runtime depending on the arguments passed

S3: Correctly implements the swap procedure for all input pointers referring to integers stored in memory locations accessible to the process

S4: implements the swap procedure correctly for some but not all valid input pointers

S5: may add or subtract integers and pointers

(a) S1 (b) S2 and S3

(c) S2 and S4 (d) S2 and S5

[2006 : 2 Marks]

1.67 Consider the following C function:

```
int f(int n)
{       static int r = 0;
        if(n < = 0) return 1;
        if(n > 3)
        {
          r = n;
          return f(n – 2) + 2;
        }
        return f(n –1) + r;
}
```

What is the value of f(5)?

(a) 5 (b) 7

(c) 9 (d) 18

[2007 : 2 Marks]

1.68 The function f is defined as follows:

```
int f(int n)
{
    if (n < = 1)
    return 1;
    else if (n % 2  = =  0) return f(n/2);
    else return f(3n – 1);
```

Assuming that arbitrarily large integers can be passed as a parameter to the function, consider the following statements.

(i) The function f terminates for finitely many different values of $n \geq 1$.

(ii) The function f terminates for infinitely many different values of $n \geq 1$.

(iii) The function f does not terminate for finitely many different values of $n \geq 1$.

(iv) The function f does not terminate for infinitely many different values of $n \geq 1$.

Which one of the following options is true of the above?

(a) (i) and (iii) (b) (i) and (iv)

(c) (ii) and (iii) (d) (ii) and (iv)

[2007 : 2 Marks]

1.69 Consider the C program given below:

```
# include <stdio.h>
int main( )
{
    int sum = 0, maxsum = 0, i,  n = 6;
    int a( ) = {2, –2, –1, 3, 4, 2};
    for(i = 0; i < n; i++)
    {
        if(i = = 0| | a[i] < 0| | a[i] < a[i – 1])
        {
            if(sum > maxsum)
                maxsum = sum;
            sum = (a[i] > 0) ? a[i] : 0;
        }
        else sum + = a[i];
        if(sum > maxsum) maxsum = sum;
        printf ("%d\n", maxsum);
}
```

What is the value printed out when this program is executed?

(a) 9 (b) 8

(c) 7 (d) 6

[2007 : 2 Marks]

1.70 Consider the following C program:

```
# include <stdio.h>
# define EOF –1
void push (int); /* push the argument on the stack */
int pop  (void); /* pop the top of the stack */
void flagError ( );
int main( )
{
    int c, m, n, r;
    while ((c = getchar ( )) ! = EOF)
    {   if (isdigit (c)) push (c);
        else if ((c = = '+') | | (c = = '*'))
        {
            m = pop ( );
            n = pop ( );
            r = (c = = '+') ? n + m : n*m;
            push (r);
        }
        else if (c ! = ' ')
        flagError ( );
    }
    printf("% c", pop ( ));
}
```

What is the output of the program for the following input?

5 2 * 3 3 2 + * +

(a) 15 (b) 25
(c) 30 (d) 150

[2007 : 2 Marks]

1.71 Consider the program below in a hypothetical language which allows global variables and a choice of call by reference or call by value methods of parameter passing.

```
int i ;
program main( )
{   int j = 60;
    i = 50;
    call f(i, j);
    print i, j;
}
procedure f(x, y)
{
    i = 100;
    x = 10;
    y = y + i ;
}
```

Which one of the following options represents the correct output of the program for the two parameter passing mechanisms?

(a) Call by value : i = 70, j = 10; Call by reference : i = 60, j = 70
(b) Call by value : i = 50, j = 60; Call by reference : i = 50, j = 70
(c) Call by value : i = 10, j = 70; Call by reference : i = 100, j = 60
(d) Call by value : i = 100, j = 60; Call by reference : i = 10, j = 70

[2007 : 2 Marks]

1.72 Consider the program below in a hypothetical programming language which allows global variables and a choice of static or dynamic scoping.

```
int i ;
program main( )
{
    i = 10;
    call f( );
}
procedure f( )
{
    int i = 20;
    call g( );
}
procedure g( )
{
    print i;
}
```

Let x be the value printed under static scoping and y be the value printed under dynamic scoping. Then, x and y are

(a) x = 10, y = 10 (b) x = 20, y = 10
(c) x = 10, y = 20 (d) x = 20, y = 20

[2007 : 2 Marks]

1.73 Early binding refers to a binding performed at compile time and late binding refers to a binding performed at execution time. Consider the following statements:

(i) Static scope facilitates w1 bindings.
(ii) Dynamic scope requires w2 bindings.
(iii) Early bindings w3 execution efficiency.
(iv) Late bindings w4 execution efficiency.

The right choices of wl, w2, w3 and w4 (in that order) are

(a) Early, late, decrease, increase
(b) Late, early, increase, decrease
(c) Late, early, decrease, increase
(d) Early, late, increase, decrease

[2007 : 2 Marks]

1.74 Match the programming paradigms and languages given in the following table.

List-I (Paradigms)
I. Imperative
II. Object oriented
III. Functional
IV. Logic

List-II (Languages)
A. Prolog
B. Lisp
C. C, Fortran 77, pascal
D. C++, Smalltalk, Java

Codes:
(a) I-C, II-D, III-B, IV-A
(b) I-A, II-D, III-C, IV-B
(c) I-D, II-C, III-B, IV-A
(d) I-C, II-D, III-A, IV-B

[2008 : 1 Mark]

1.75 Which combination of the integer variables x, y and z makes the variable a get the value 4 in the following expression?

a = (x > y) ? ((x > z) ? x : z) : ((y>z) ? y : z)

(a) x = 3, y = 4, z = 2
(b) x = 6, y = 5, z = 3
(c) x = 6, y = 3, z = 5
(d) x = 5, y = 4, z = 5

[2008 : 1 Mark]

1.76 What is the output printed by the following C code?

```
# include <stdio.h>
int main( )
{
    char a[6] = "world";
    int i, j;
    for (i = 0, j = 5; i < j; a[i++] = a [j – –]);
    printf ("%s\n", a);
}
```

(a) dlrow (b) Null string
(c) dlrld (d) worow

[2008 : 2 Marks]

1.77 Consider the C program below. What does it print?

```
# include <stdio.h>
# define swapl (a,b) tmp = a; a = b; b =tmp;
void swap2 ( int a, int b)
{
    int tmp;
    tmp = a; a = b; b = tmp;
}
void swap3 (int*a, int*b)
{
    int tmp;
    tmp = *a; *a = *b; *b = tmp;
}
int main( )
{
    int num1 = 5, num2 = 4, tmp;
    if(num1 > num2)
    { swap1(num1, num2); }
    if (num1 < num2)
    { swap2(num1 + 1, num2); }
    if(num1 > = num2)
    { swap3 (&num1, &num2); }
    printf ("%d, %d", num1, num2);
}
```

(a) 5, 5 (b) 5, 4
(c) 4, 5 (d) 4, 4

[2008 : 2 Marks]

1.78 Consider the C program given below. What does it print?

```
# include <stdio.h>
int main( )
{
    int i, j;
    int a[8] = {1, 2, 3, 4, 5, 6, 7, 8};
    for(i = 0; i < 3; i++)
    {
        a[i] = a[i]+1;
        i++;
    }
    i – –;
    for (j = 7; j > 4, j– –)
    {
        int i = j/2;
        a[i] = a[i] –1;
    }
    printf ("%d, %d", i, a [i]);
}
```

(a) 2, 3 (b) 2, 4
(c) 3, 2 (d) 3, 3

[2008 : 2 Marks]

1.79 A C program is given below:

```
# include <stdio.h>
int main( )
{
    int i, j;
    char a[2][3] = {{'a', 'b', 'c'}, {'d', 'e', 'f'}};
    char b[3][2];
    char *p = *b;
    for (i = 0; i < 2; i++)
    {
        for (j = 0; j < 3; j++)
        {
            *(p + 2*j + i) = a [i] [j];
        }
    }
}
```

What should be the contents of the array b at the end of the program?

(a) a b
c d
e f

(b) a d
b e
c f

(c) a c
e b
d f

(d) a e
d c
b f

[2008 : 2 Marks]

Directions for Question 1.80 to 1.81:

Consider the code fragment written in C below :

```
void f(int n)
{
    if(n ≤ 1 )
    {
        printf("%d", n);
    }
    else
    {
        f(n/2);
        printf("%d", n%2);
    }
}
```

1.80 What does f(173) print?

(a) 010110101 (b) 010101101
(c) 10110101 (d) 10101101

[2008 : 2 Marks]

1.81 Which of the following implementations will produce the same output for f(173) as the one from above Question?

P1:

```
void f(int n)
{
  if(n/2)
  {
    f(n/2);
  }
  printf("%d", n%2);
}
```

P2:

```
void f(int n)
{
  if (n < = 1)
  {
    printf("%d", n); }
  else
  { printf("%d", n%2);
    f(n/2); }
}
```

(a) Both P1 and P2 (b) P2 only
(c) P1 only (d) Neither P1 nor P2

[2008 : 2 Marks]

1.82 What is printed by the following C program?

```
int f(int x, int *py, int ** ppz)
{ int y, z;
  **ppz + = 1; z = *ppz;
  *py + = 2; y = *py;
  x + = 3;
  return x + y + z;
}
void main( )
{ int c, *b, **a;
  c = 4; b = &c; a = & b;
  printf ("% d", f (c, b, a));
}
```

(a) 18 (b) 19
(c) 21 (d) 22

[2008 : 2 Marks]

1.83 Choose the correct option to fill ?1 and ?2 so that the program below prints an input string in reverse order. Assume that the input string is terminated by a newline character.

```
void reverse (void) {
    int c;
    if (?1) reverse ();
    ?2
}
main () {
    printf ("Enter Text") ; printf ("\n");
    reverse (); printf ("\ n");
}
```

(a) ? 1 is (getchar ( ) ! = '\n')
? 2 is getchar (c);
(b) ? 1 is (c = getchar ( )) ! = '\n')
? 2 is getchar (c)
(c) ? 1 is (c ! = '\n')
? 2 is putchar (c);
(d) ? 1 is ((c = getchar ( )) ! = '\n')
? 2 is putchar (c);

[2008 : 2 Marks]

Linked Answers Q. 1.84 and Q. 1.85

Consider the following C program that attempts to locate an element x in an array Y[ ] using binary search. The program is erroneous.

```
f(int Y[10] , int x) {
    int i, j, k;
    i = 0; j = 9;
    do {
        k = ( i + j) / 2;
        if (Y[K] < x)  i = k; else j = k;
    } while (( Y[k]  ! = x) && ( i < j));
    if (Y[k] = = x )
        printf("x is in the array") ;
    else
        printf("x is not in the array");
}
```

1.84 On which of the following contents of Y and x does the program fail?

(a) Y is [1 2 3 4 5 6 7 8 9 10] and x < 10
(b) Y is [1 3 5 7 9 11 13 15 17 19] and x < 1
(c) Y is [2 2 2 2 2 2 2 2 2 2] and x > 2
(d) Y is [2 4 6 8 10 12 14 16 18 20] and 2 < x < 20 and x is even

[2008 : 2 Marks]

1.85 The correction needed in the program to make it work properly is

(a) change line 6 to : if (Y[k] < x) i = k + 1; else j = k –1;

(b) change line 6 to : if (Y[k] < x) i = k –1; else j = k +1;

(c) change line 6 to : if (Y [k] < x) i = k; else j = k;

(d) change line 7 to : while ((Y [k] = = x) && (i < j));

[2008 : 2 Marks]

1.86 Consider the program below:

```
# include <stdio.h>
int fun (int n, int *f_p) {
   int t, f;
   if (n < = 1) {
     *f_p = 1 ;
     return 1 ;
   }
   t = fun(n–1, f_p) ;
   f = t + * f_p ;
   *f_p = t ;
   return f ;
}
int main( )
{
   int x = 15 ;
   printf ("% d\n", fun (5, &x)) ;
   return 0;
}
```

The value printed is

(a) 6 (b) 8

(c) 14 (d) 15

[2009 : 1 Mark]

1.87 What does the following program print?

```
#include < stdio.h>
void f(int *p, int *q)
{
   p = q;
   *p = 2;
}
int i = 0, j = 1;
int main( )
{
   f(&i, &j);
   printf ("%d %d\n", i, j);
   return 0;
}
```

(a) 2 2 (b) 2 1

(c) 0 1 (d) 0 2

[2010 : 1 Mark]

1.88 What is the value printed by the following C program?

```
#include < stdio.h>
int f(int *a, int n)
{
   if (n <= 0) return 0;
   else if (*a % 2 == 0)
       return *a + f(a + 1,  n – 1);
   else return *a – f(a + 1, n – 1);
}
int main ( )
{
   int a [ ] = {12, 7, 13, 4, 11, 6};
   printf("%d", f(a, 6));
   return 0;
}
```

(a) –9 (b) 5

(c) 15 (d) 19

[2010 : 2 Marks]

1.89 The following program is to be tested for statement coverage:

```
begin
   if (a = = b) {S1; exit;}
   else if (c = = d) {S2;}
   else {S3; exit;}
   S4;
end
```

The test cases T1, T2, T3 and T4 given below are expressed in terms of the properties satisfied by the values of variables a, b, c and d. The exact values are not given.

T1 : a, b, c and d are all equal

T2 : a, b, c and d are all distinct

T3 : a = b and c != d

T4 : a !=b and c = d

Which of the test suites given below ensures coverage of statements S1, S2, S3 and S4?

(a) T1, T2, T3 (b) T2, T4

(c) T3, T4 (d) T1, T2, T4

[2010 : 2 Marks]

1.90 What does the following fragment of C program print?

```
char c[] = "GATE2011";
char *p = c;
printf("%s", p + p[3] – p[1]);
```

(a) GATE 2011 (b) E2011
(c) 2011 (d) 011

[2011 : 1 Mark]

Common Data Question 1.91 and 1.92:

Consider the following recursive C function that takes two arguments.

```
unsigned int foo(unsigned int n, unsigned int r)
{
  if(n>0) return ((n%r)+foo(n/r,r));
  else return 0;
}
```

1.91 What is the return value of the function `foo` when it is called as `foo(345, 10)`?

(a) 345 (b) 12
(c) 5 (d) 3

[2011 : 2 Marks]

1.92 What is the return value of the function foo when it is called as foo(513, 2)?

(a) 9 (b) 8
(c) 5 (d) 2

[2011 : 2 Marks]

1.93 What will be the output of the following C program segment?

```
char inChar = 'A' ;
switch(inChar)
{
  case 'A' : printf ("Choice A\ n") ;
  case 'B' :
  case 'C' : printf ("Choice B") ;
  case 'D' :
  case 'E' :
  default   : printf (" NO Choice") ; }
```

(a) No Choice
(b) Choice A
(c) Choice A
Choice B No Choice
(d) Program gives no output as it is erroneous

[2012 : 1 Mark]

1.94 Consider the program given below, in a block-structured pseudo-language with lexical scoping and nesting of procedures permitted.

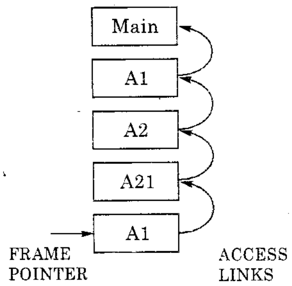
```
Program main;
Var ...
Procedure A1;
  Var ...
  Call A2;
End A1
Procedure A2;
  Var ...
  Procedure A21;
        Var ...
        Call A1,
  End A21
  Call A21;
End A2
Call A1;
End main;
```

Consider the calling chain:

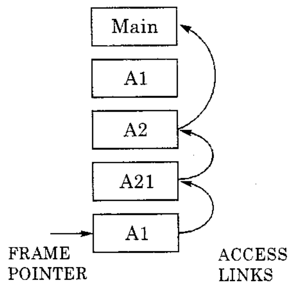
Main → A1 → A2 → A21 → A1

The correct set of activation records along with their access links is given by

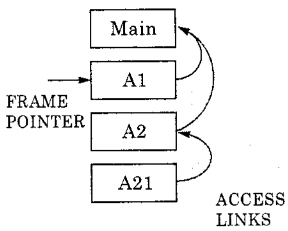
(a)



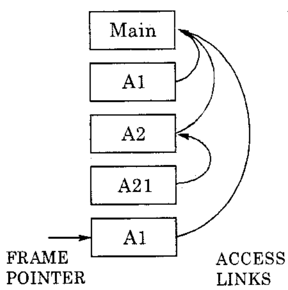
(b)



(c)



(d)



[2012 : 2 Marks]

Common Data for Questions 1.95 and 1.96:

Consider the following C code segment.

```
int a, b, c = 0;
void prtFun (void);
main()
{static int a = 1;/* Line 1*/
prtFun();
a+=1;
prtFun();
printf("\n %d %d ",a,b);}
void prtFun(void)
{static int a = 2;/*Line 2*/
int b = 1;
a += ++b;
printf("\n %d %d ",a,b);}
```

1.95 What output will be generated by the given code segment?

(a)	3	1	(b)	4	2
	4	1		6	1
	4	2		6	1
(c)	4	2	(d)	3	1
	6	2		5	2
	2	0		5	2

[2012 : 2 Marks]

1.96 What output will be generated by the given code segment if:

Line 1 is replaced by auto int a=1;

Line 2 is replaced by register int a = 2;

(a)	3	1	(b)	4	2
	4	1		6	1
	4	2		6	1
(c)	4	2	(d)	4	2
	6	2		4	2
	2	0		2	0

[2012 : 2 Marks]

1.97 What is the return value of f(p, p), if the value of p is initialized to 5 before the call? Note that the first parameter is passed by reference, whereas the second parameter is passed by value.

```
int f (int &x, int c)
{   c = c – 1;
    if (c = = 0) return 1;
    x = x + 1;
    return f(x, c) * x; }
```

(a) 3024 (b) 6561
(c) 55440 (d) 161051

[2013 : 2 Marks]

1.98 Consider the following program in C language:

```
#include <stdio.h>
main()
{
  int i;
  int *pi = &i;
  scanf("%d",pi);
  printf("%d\n", i+5);
}
```

Which one of the following statements is **TRUE**?

(a) Compilation fails.
(b) Execution results in a run-time error.
(c) On execution, the value printed is 5 more than the address of variable *i*.
(d) On execution, the value printed is 5 more than the integer value entered.

[2014 (Set-1) : 1 Mark]

1.99 Consider the following pseudo code. What is the total number of multiplications to be performed?

```
D = 2
for i = 1 to n do
   for j = i to n do
      for k = j + 1 to n do
         D = D * 3
```

(a) Half of the product of the 3 consecutive integers.
(b) One-third of the product of the 3 consecutive integers.
(c) One-sixth of the product of the 3 consecutive integers.
(d) None of the above.

[2014 (Set-1) : 2 Marks]

1.100 Consider the function func shown below:

```
int func(int num)
{  int count = 0;
   while (num)
   {  count++; num>>= 1;
   }
   return (count);
}
```

The value returned by function (435) is ______.

[2014 (Set-2) : 1 Mark]

1.101 Suppose n and p are unsigned int variables in a *C* program. We wish to set p to ${}^nC_3$. If n is large, which one of the following statements is most likely to set p correctly?

(a) $p = n*(n-1)*(n-2)/6$;
(b) $p = n*(n-1)/2*(n-2)/3$;
(c) $p = n*(n-1)/3*(n-2)/2$;
(d) $p = n*(n-1)*(n-2)/6.0$;

[2014 (Set-2) : 1 Mark]

1.102 Consider the following function

```
double f(double x) {
  if(abs(x*x – 3) < 0.01) return x;
  else return f(x/2 + 1.5/x);
}
```

Give a value q (to 2 decimals) such that $f(q)$ will return q : ________.

[2014 (Set-2) : 2 Marks]

1.103 Consider the C function given below.

```
int f(int j) {
    static int i = 50;
    int k;
    if (i == j) {
        printf("something");
        k = f(i);
        return 0;
    }
    else return 0;
}
```

Which one of the following is TRUE?

(a) The function returns **0** for all values of ***j***.
(b) The function prints the string something for all values of ***j***.
(c) The function returns **0** when ***j* = 50**.
(d) The function will exhaust the runtime stack or run into an infinite loop when ***j* = 50**.

[2014 (Set-2) : 2 Marks]

1.104 The output of the following C program is ______.

```
void f1 (int a, int b) {
    int c;
    c=a; a=b; b=c;
}
void f2 (int *a, int *b) {
    int c;
    c=*a; *a=*b; *b=c;
}
int main() {
    int a=4, b=5, c=6;
    f1 (a, b);
    f2 (&b, &c);
    printf ("%d", c – a – b);
}
```

[2015 (Set-1) : 1 Mark]

1.105 Consider the following pseudo code, where x and y are positive integers.

```
begin
    q := 0;
    r := x;
    while r ≥ y do
      begin r := r – y ; q := q + 1 ; end
end
```

The post condition that needs to be satisfied after the program terminates is

(a) $\{r = qx + y \wedge r < y\}$
(b) $\{x = qy + r \wedge r < y\}$
(c) $\{y = qx + r \wedge 0 < r < y\}$
(d) $\{q + 1 < r - y \wedge y > 0\}$

[2015 (Set-1) : 2 Marks]

1.106 Consider the following C function.

```
int fun (int n) {
    int x=1, k;
    if (n==1) return x;
    for (k=1; k<n; ++k)
        x=x + fun (k) * fun (n – k);
    return x;
}
```

The return value of fun (5) is ________.

[2015 (Set-2) : 1 Mark]

1.107 Consider the following function written in the C programming language.

```
void foo (char *a) {
    if (*a && *a != ' ') {
        foo (a+1);
        putchar (*a);
    }
}
```

The output of the above function on input "ABCD EFGH" is

(a) ABCD EFGH
(b) ABCD
(c) HGFE DCBA
(d) DCBA

[2015 (Set-2) : 1 Mark]

1.108 A Young tableau is a 2D array of integers increasing from left to right and from top to bottom. Any unfilled entries are marked with ∞, and hence there cannot be any entry to the right of, or below a ∞. The following Young tableau consists of unique entries.

1	2	5	14
3	4	6	23
10	12	18	25
31	∞	∞	∞

When an element is removed from a Young tableau, other elements should be moved into its place so that the resulting table is still a Young tableau (unfilled entries may be filled in with a ∞). The minimum number of entries (other than 1) to be shifted, to remove 1 from the given Young tableau is ______.

[2015 (Set-2) : 2 Marks]

1.109 Consider the following C program segment.

```
# include <stdio.h>
int main( ) {
    char s1[7] = "1234", *p;
    p = s1 + 2;
    *p = '0' ;
    printf ("%s", s1);
}
```

What will be printed by the program?

(a) 12 (b) 120400
(c) 1204 (d) 1034

[2015 (Set-3) : 1 Mark]

1.110 Consider the following C program.

```
# include < stdio.h>
int main( ) {
  static int a[ ] = {10, 20, 30, 40, 50};
  static int *p[ ] = {a, a+3, a+4, a+1, a+2};
    int **ptr = p;  ptr++;
    printf ("%d%d", prt-p, **ptr);
}
```

The output of the program is ______.

[2015 (Set-3) : 2 Marks]

1.111 Consider the following recursive C function.

```
void get (int n) {
    if (n < 1) return;
    get (n–1);
    get (n–3);
    printf ("%d", n);
}
```

If get(6) function is being called in main() then how many times will the get() function be invoked before returning to the main()?

(a) 15 (b) 25
(c) 35 (d) 45

[2015 (Set-3) : 2 Marks]

1.112 Consider the following C program:

```
# include <stdio.h>
int main( ) {
   int i, j, k = 0;
   j = 2 * 3 / 4 + 2.0 / 5 + 8 / 5;
   k – = – –j;
   for(i = 0; i < 5; i++) {
      switch(i + k)
      {
          case 1:
          case 2: printf("\n%d", i + k);
          case 3: printf("\n%d", i + k);
          default: printf("\n%d", i + k);
      }
   }
   return 0;
}
```

The number of times printf statement is executed is ______.

[2015 (Set-3) : 2 Marks]

1.113 Consider the following C program.

```
# include <stdio.h>
int f1(void);
int f2(void);
int f3(void);
int x = 10;
int main( )
{
     int x = 1;
     x +=f1( ) + f2( ) + f3( ) + f2( );
     pirntf("%d", x);
     return 0;
}
int f1( ) { int x = 25; x++; return x; }
int f2( ) { static int x = 50; x++; return x; }
int f3( ) { x *= 10; return x; }
```

The output of the program is ______.

[2015 (Set-3) : 2 Marks]

1.114 Consider the following C program.

```
void f (int, short);
void main()
{
    int i = 100;
    short s = 12;
    short *p = &s;
    __________;     // call to f ( )
}
```

Which one of the following expressions, when placed in the blank above, will NOT result in a type checking error?

(a) $f(s, *s)$ (b) $i = f(i, s)$
(c) $f(i, *s)$ (d) $f(i, *p)$

[2016 (Set-1) : 1 Mark]

1.115 Consider the following C program.

```
#include<stdio.h>
void mystery (int *ptra, int *ptrb)
{   int *temp;
    temp = ptrb;
    ptrb = ptra;
    ptra = temp;
}
int main()
{   int a = 2016, b = 0, c = 4, d = 42;
    mystery(&a, &b);
    if (a < c)
        mystery(&c, &a);
    mystery(&a, &d);
    printf("%d \ n", a);
}
```

The output of the program is _____.

[2016 (Set-1) : 1 Mark]

1.116 The following function computes the maximum value contained in an integer array p[ ] of size $n(n >= 1)$.

```
int max(int *p, int n)
{   int a = 0, b = n - 1;
    while ( ________ )
    {
        if (p[a] <= p[b])
        {
            a = a + 1;
        }
        else
        {
            b = b - 1;
        }
    }
    return p[a];
}
```

The missing loop condition is

(a) $a\ != n$ (b) $b\ != 0$
(c) $b > (a + 1)$ (d) $b\ != a$

[2016 (Set-1) : 2 Marks]

1.117 What will be the output of the following C program?

```
void count(int n)
{   static int d = 1;
    printf("%d", n);
    printf("%d", d);
    d++;
    if (n > 1) count (n-1);
    printf("%d", d);
}
void main()
{
    count (3);
}
```

(a) 3 1 2 2 1 3 4 4 4 (b) 3 1 2 1 1 1 2 2 2
(c) 3 1 2 2 1 3 4 (d) 3 1 2 1 1 1 2

[2016 (Set-1) : 2 Marks]

1.118 What will be the output of the following pseudo-code when parameters are passed by reference and dynamic scoping is assumed?

```
a = 3;
void n(x)
{
    x = x * a; print(x);
}
void m(y)
{   a = 1; a = y - a; n(a);
    print(a);
}
void main()
{
    m(a);
}
```

(a) 6, 2 (b) 6, 6
(c) 4, 2 (d) 4, 4

[2016 (Set-1) : 2 Marks]

1.119 The value printed by the following program is________.

```
void f(int* p, int m)
{
    m = m + 5;
    *p = *p + m;
    return;
}
void main ()
{
    int i = 5, j = 10;
```

```
    f(&i, j);
    printf ("%d", i + j) ;
}
```

[2016 (Set-2) : 1 Mark]

1.120 The following function computes X^Y for positive integers X and Y.

```
int exp (int X, int Y)
{
    int res = 1, a = X, b = Y;
    while (b ! = 0)
    {  if (b% 2 == 0)
       {  a = a*a; b = b/2;  }
          else {res = res*a; b = b – 1;
       }
    }
    return res;
}
```

Which one of the following conditions is TRUE befor every iteration of the loop?

(a) $X^Y = a^b$
(b) $(\text{res} * a)^Y = (\text{res} * X)^b$
(c) $X^Y = \text{res} * a^b$
(d) $X^Y = (\text{res} * a)^b$

[2016 (Set-2) : 2 Marks]

1.121 Consider the following program:

```
int f(int *p, int n)
{
    if (n < = 1) return 0;
    else return max (f(p+1, n–1), p[0] –p[1]);
}
int main ()
{
    int a[ ] = {3, 5, 2, 6, 4};
    print f("%d", f(a, 5));
}
```

Note: max $(x; y)$ returns the maximum of x and y.

The value printed by this program is ______.

[2016 (Set-2) : 2 Marks]

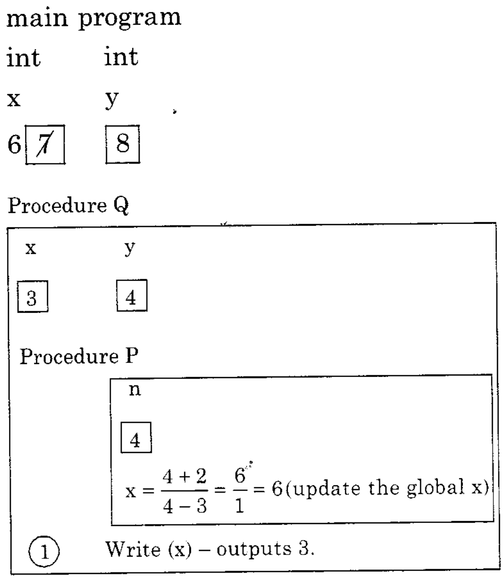
Answers Programming

1.1 (a)	1.2 (d)	1.6 (d) &	1.7 (b)	1.8 (c)	1.9 (a)	1.10 (a)	1.11 (a)	1.12 (c)
1.13 (b)	1.14 (d)	1.15 (b)	1.16 (c)	1.17 (d)	1.18 (b)	1.19 (c)	1.20 (a)	1.21 (d)
1.22 (c)	1.23 (a)	1.24 (d)	1.25 (a)	1.26 (c)	1.27 (a)	1.28 (b)	1.29 (c)	1.30 (d)
1.31 (b)	1.32 (d)	1.33 (b)	1.34 (a)	1.35 (d)	1.36 (a)	1.37 (c)	1.38 (d)	1.39 (b)
1.40 (c)	1.41 (a)	1.42 (b)	1.43 (c)	1.44 (c)	1.45 (d)	1.46 (a)	1.47 (d)	1.48 (c)
1.49 (c)	1.50 (c)	1.51 (b)	1.52 (d)	1.53 (d)	1.54 (c)	1.55 (c)	1.56 (d)	1.57 (c)
1.58 (b)	1.59 (d)	1.60 (a)	1.61 (c)	1.62 (a)	1.63 (b)	1.64 (d)	1.65 (b)	1.66 (b)
1.67 (d)	1.68 (d)	1.69 (c)	1.70 (b)	1.71 (d)	1.72 (c)	1.73 (d)	1.74 (c)	1.75 (a)
1.76 (b)	1.77 (c)	1.78 (b)	1.79 (b)	1.80 (d)	1.81 (c)	1.82 (b)	1.83 (d)	1.84 (c)
1.85 (a)	1.86 (b)	1.87 (d)	1.88 (c)	1.89 (d)	1.90 (c)	1.91 (b)	1.92 (d)	1.93 (c)
1.94 (d)	1.95 (c)	1.96 (d)	1.97 (b)	1.98 (d)	1.99 (c)	1.101 (b)	1.103 (d)	1.105 (b)
1.107 (d)	1.109 (c)	1.111 (b)	1.114 (d)	1.116 (d)	1.117 (a)	1.118 (d)	1.120 (c)	

Explanations Programming

1.1 **(a)**

Following is the execution sequence of program.



(2) write (x) – outputs 6

So the correct answer is option (a)

1.2 **(d)**

Passing an array element by call-by-name behaves like call-by-value. Therefore, there may be different results for call-by-reference and call-by-name parameter passing for passing an array elements.

1.3 **Sol.**

- In programming language, coercion is a way of implicitly or explicity, changing an entity of one datatype into another.
- Repeat-until is a nondeterministic loop.
- The portion of the stack used for an invocation of a function is called the function's stack frame or activation record. So activation record is used for recursion.
- Dynamic data structure is used for pointer data type.

  a-q, b-r, c-s, d-p

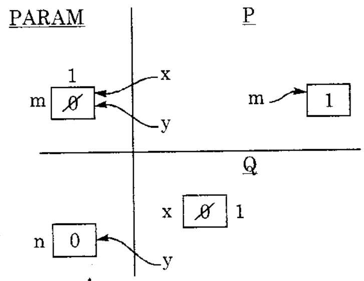
1.4 **Sol.**

Both are correct.

1.5 **Sol.**

A-S, B-R. C-P, D-Q

1.6 **(d) & 1.7 (b)**



In pascal, integer pointer variable is declared as "var p: integer" and integer variable is declared as "v: integer".

Procedure P(var x, y: integer): x and y are pointer variables, so P uses call by reference for x and y.

Procedure Q(x: integer, var y: integer): x is integer variable and y is pointer, so Q uses call by value for x and call by reference for y.

Prints m = 1 and n = 0.

1.8 **(c)**

m is declared in the main program and its scope is limited to PARAM and Q. m can not be accessible in P, because P has local variable with same name 'm'.

1.9 **(a)**

Given program is an example of swapping.

1.10 **(a)**

FORTRAN uses static allocation for variables and static variables don't support recursion.

1.11 **(a)**

A goto statement causes a program to unconditionally transfer control to the statement associated with the label specified on the goto statement. A goto statement is always undesirable because it makes program more difficult to verify.

1.12 **(c)**

Call by name and call by reference parameter passing techniques may differ when passes an array element as a parameter.

1.13 (b)

Because using 'static scoping' global variable value can not be changed.

1.14 (d)

Activation record → Subroutine call
Location counter → Assembler
Reference counts → Garbage collection
Address Relocation → Linking loader

1.15 (b)

Heap allocation is required to support dynamic data structure where memory can be allocated and deallocated at run time.

1.16 (c)

S_1 uses procedure B's 'x' variable.

1.17 (d)

Activation tree helps to access the non-local variables in faster using an array of pointers to activation records.

1.18 (b)

Arbitrary goto, recursive function calls, and repeat loops in programs, may causes the program to enter infinite loop. i.e., Program may not terminate.

1.19 (c)

Starting value of 'x' stores in global variable.
Var x = 0.25; real, after that put value according to question.

1.20 (a)

Pointers can not be implemented if the processor supports only immediate and direct addressing modes. Pointer requires indirect addressing modes.

1.21 (d)

Take some values and verify the options as 10, 6, 7.

1.22 (c)

Indirect addressing → Pointer;
MOV A, [100]
Immediate addressing → Constants;
MOV A, 10
Auto decrement → Loops;
for (i=0; i<10; i– –);

1.23 (a)

It defines an array each element of which is a pointer to a structure of type node.

1.24 (d)

m = malloc(S);
m = NULL; } → Lost memory: (memory is lost for m)

free(n);
n → values = 5; } → Dangling pointer: (No memory for n)

char * p;
*p = 'a'; } → Uninitialized pointer:
(p is not initilized yet. So trying for *p will fail)

1.25 (a)

Definition of aliasing.

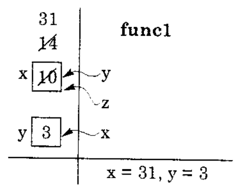
1.26 (c)

Here structure creates the minimum of array and union, but union only creates the minimum for only 'long z' which is max.
So total minimum required = 18.

1.27 (a)

Count is static variable so every time when value is changed it becomes the new value of count.
So, first count value is 0 then 1, then 3, then 6 at last 10.

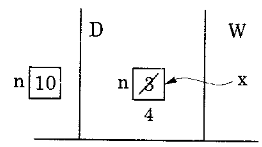
1.28 (b)



1.29 (c)

P_1 is wrong because parameter type is not defined and no type casting.
P_2 is wrong, variable pointer can't work like pointer function.

1.30 (d)



prints x value as 4

1.31 (b)

Call by value result and call by reference may differ in the presence of aliasing (sending the same parameter more than once).

1.32 (d)

Recursion requires activation records during execution.

1.33 (b)

Address of a [40][50] = Base address + 40 × 100 × element size + 50 * element size

= 0 + 4000 × 1 + 50 × 1 = 4050

Therefore, option (b) is correct.

1.34 (a)

Here int *A[10] is an array of 10 pointers.
Which can be A[2][3] as its left hand side.
It can also be A[2] as its left hand side and also B[2][3] can obviously be used as left hand side as it is given in declaration of B[10][10] only, but B[1] can not be used because it is declared as 2-dimensional array.

1.35 (d)

If (! A[j]) condition
If (A[j] = = 0)
Prints the value, but no zero in the array.

1.36 (a)

```
x = 5;
p (& x);
x = *y + 2 = 5 + 2 = 7;
Q(x);
z = 7;
z = z + x = 12   ⇒   print(z) = 12
*y = x – 1 = 6   ⇒   print(x) = 7
print(x) = 6
```

1.37 (c)

The goal of structured programming is able to infer the flow of control from the program text. It means user can execute the code according to his requirement. C and Pascal are good examples of structured programming. In structured programming control passes from one instruction to another instruction in the sequential manner.

1.38 (d)

Consider the given C function

```
void swap (int a, int b)
{   int temp;
    int temp;
    temp = a;
    a = b;
    b = temp;
}
```

The main program for given code is

```
# include <stdio.h>
void main()
{   int x, y;
    x = 1;
    y = 2;
    swap(x, y);
    printf("%d\n", a);
    printf("%d\n", y);
}
```

If we call swap (x, y) then there is no interchange in the value of x and y because the parameters are passed by value. There is interchange in formal parameters a and b but no interchange in actual parameter x and y because the scope of a and b lies with in the function but not in the main program so the value of x and y remain unchanged. If we will call it by reference then the value of x and y will be interchanged as below code.

```
void swap (int * a, int * b)
{   int temp ;
    temp = *a;
    *a = *b;
    *b = temp;
}
```

So option (d) is the current.

1.39 (b)

Put   x = 0
then   x = 1
so use option x = 1 – x
Put   x = 1
then   x = 0
so use option x = 1 – x
= 1 – 1 = 0
So option B is matching.

1.40 (c)

Look at the initial value of j, if j starts with 0, then double *for* loop will swap M[i][j] with M [j] [i] and also M [j] [i] and M[i][j] so the matrix M will remain unchanged, so to avoid this double swapping we need to initialize j = i and swap only upper triangular matrix with lower triangular matrix.

```
for (j = i; j < 4; ++j)
{   //code for swapping M[i][j] with M[j][i]
    t = M[i] [j];
    M[i][j] = M[j][i];
    M[j][i] = t;
}
```

1.41 (a)

funcf(x) + funcg(x)

funcf or funcg can be executed first. Let's assume funcf is executed first. It calls funcg.

So even if the order of call is revrsed, result will be same.

In first call of funcg, y becomes 11 and it returns 5+11 = 16.

In second call of funcg, y becomes 12 and it returns 5+12 = 17.

So, in main y is incremented by 16 + 17 = 33 to become 10+33 = 43 (option (a))

In the second iteration y will be incremented by 18 + 19 = 37 to give 43 + 37 = 80

1.42 (b)

First prints A B. f1 is call by value then changes applciable only for local.

From f1 U V is printed.

In main A B is printed.

In f2 V W is printed.

1.43 (c)

```
int f(int n)
{       static int i = 1;
        if (n >= 5) return n;
        n = n + i;
        i ++;
        return f(n)
}
```

	Iteration	n	i
	0	1	1
Initilize	1	1 + 1 = 2	2
	2	2 + 2 = 4	3
	3	4 + 3 = 7	4

So when we call f(1), the value i = 4 and n = 7, if condition is $7 \geq 5$ so returns the value 7.

1.44 (c)

The given C program is non-recursive. So we will get the value of n after some iteration. The main thing about the program is that.

1. If m = n then n is the GCD
2. If m > n then m = m − n
3. If m < n then n = n − m

For example let x = 40 and y = 32

m = x = 40 and n = y = 32

m > n so apply the 2nd condition

m = m − n

m = 40 − 32 = 8

m = 8 and n = 8

apply the first condition

GCD (40, 32) = 8

1.45 (d)

Functional → Expression evaluation

Logic → Theorem proving

Object oriented → Abstract data types

Imperative → Procedural

1.46 (a)

Loop invariant is the part of code motion. Loop invariant for while loop is a condition, if we assign this condition before the while loop then there is no effect in the code and produces same output.

Consider the given code.

```
int n, rev;
rev = 0;
while (n > 0){
    rev = rev * 10 + n%10;
    n = n/10;
}
```

Let input n = 12345

where $d_1 = 1$, $d_2 = 2$, $d_3 = 3$, $d_4 = 4$ and $d_5 = 5$

Iteration	n = n / 10	rev = rev * 10 + n%10
	$d_2d_2d_3d_4d_5$	
0	1 2 3 4 5	rev = 0
	$d_1d_2d_3d_4$	d_5
1	1 2 3 4	rev = 0 * 10 + 5 = 5
	$d_1d_2d_3$	d_5d_4
2	1 2 3	rev = 5 * 10 + 5 = 5 4
	d_1d_2	$d_5d_4d_3$
3	1 2	rev = 54 * 10 + 2 = 5 4 3
	d_1	$d_5d_4d_3d_2$
4	1	rev = 543 * 10 + 2 = 5 4 3 2
		$d_5d_4d_3d_2d_1$
5	0	rev = 5432 * 10 + 1 = 5 4 3 2 1

Lent n = d_1d_2 .... d_m in the i^{th} iteration

rev = $d_m.d_m - i + 1$

For example if m = 5 and i = 3 the above example produces n = d_1d_{5-3} = 12 and

rev = $d_5d_4d_{5-3+1}$ = 543

1.47 (d)

Because of first character is '\0' nothing will be printed.

1.48 (c)

int (*f) (int*);

return type int, (*f) is a pointer to a function the argument is (int*) an integer pointer So, int (*f) (int*) means a pointer to a function that takes an integer pointer as an argument and returns an integer.

1.49 (c)

The abstract data type (ADT) refers to a programmer defined data type together with a set of operations that can be performed on that data so the choice (c) is correct.

1.50 (c)

A common property of logic programming languages and functional languages is both are declarative because we declare any statement before we will use it.

1.51 (b)

Object oriented programming language is Object based PL + Abstraction + Inheritance.

The last two (abstraction and inheritance) are must for any L to be OOPL.

1.52 (d)

Without declaration we can't call any function. If you call it will assume by default return data type as integer. But here return data type is double. So it will give compile time error type mis-match.

1.53 (d)

```
void foo(int n, int sum){
        int k = 0, j = 0;
        if (n = = 0) return;
        k = n%10; j = n/10;
        sum = sum + k;
        foo(j, sum);
        printf("%d", k);
}
int main(){
        int a = 2048, sum = 0;
        foo(a, sum);
        printf("%d\n", sum);
}
```

The function foo is recursive function

When we call foo(a, sum) = foo (2048, 0)

k	j	sum
k = 2048%10 = foo(204,8)	j = 204	sum = 0 + 8 = 8
foo(204, 8)		
k = 204%10 = 4	j = 20	sum = 8 + 4 = 12
foo(20, 12)		
k = 20%10 = 0	j = 2	sum = 12 + 0 = 12
foo(2, 12)		
k = 2%10 = 2	j = 0	sum = 12 + 2 = 14

foo(0, 14) function will be terminated and value of k will print in stack way i.e. 2, 0, 4, 8 and sum = 0. Since sum is a local variable in the main function so the print sequence is 2, 0 , 4, 8, 0.

1.54 (c)

If there is a sink in the graph, the adjacency matrix will contain all 1's (except diagonal) in one column and all 0's (except diagonal) in the corresponding row of that vertex. The given algorithm is a smart way of doing this as it finds the sink in $O(n)$time complexity.

The first part of the code, is finding if there is any vertex which doesn't have any outgoing edge to any vertex coming after it in adjacency matrix. The smart part of the code is E2, which makes rows skip when there is no edge from i to it, making it impossible them to form a sink. This is done through

E1: !A[i][j] and E2: i = j;

E1 makes sure that there is no edge from i to j and i is a potential sink till A[i][j] becomes 1. If A[i][j] becomes 1, i can no longer be a sink, similarly all previous j can also not be a sink (as there was no edge from i to them and a sink requires an edge from all other vertices). Now, the next potential candidate for sink is j. So, in E2, we must make i = j.

1.55 (c)

The loop breaks when we found a potential sink that is a vertex which does not have any outgoing edge to any coming after it in adjacency matrix. So, if the column in which this vertex comes

in all 1 and the row is all 0's (except may be diagonal), this is the sink. Otherwise there is no sink in the graph.
So, E3 is checking this condition: !A[i][j] ensures row of this vertex is 0 and A[j][i] ensures column is 1.

1.56 (d)

A: Increments the Count by 1 at each index that is equal to the ASCII value of the alphabet it is pointing at

B: Decrements the count by 1 at each index that is equal to the ASCII value of the alphabet it is pointing at. Also it increments the loop counter for next iteration.
If one string is permutation of other ,there would have been equal increments and decrements at each index of array and So count should contain zero at each index.that is what the loop checks at last and if any non-zero elements is found, it returns 0 indicating that strings are not anagram to each other.

1.57 (c)

The sequence has to be followed.

6. f(20, 1) = 9
5. f(10, 2) – 1 = 9
4. f(5, 4) – 2 = 10
3. f(2, 8) + 4 = 12
2. f(1, 16) – 8 = 8
1. f(0, 32) + 16 = 16

1.58 (b)

etter, u, 6, ungle

1.59 (d)

First of all the swap function just swaps the pointers inside the function and has no effect on the variables being passed. Inside printab, a and b are added odd integers from 1 – 5, i.e., 1 + 3 + 5 = 9.
So, in first call to printab, a = –3 + 9 = 6 and b = –6 + 9 = 3.
Static variables have one memory throughout program run (initialized during program start) and they keep their values across function calls.
So, during second call to printab,
a = 6 + 9 = 15, b = 3 + 9 = 12.

1.60 (a)

a = {a1, a2, a3};
printf("%d,", a[0][2]);
a[0] is a1. So, this will print a1[2] = 8;
printf("%d,", *a[2]);
a[2] is a3. So, this will print *a3 = a3[0]= –12
([ ] has greater precedence than *)
printf("%d,", *++a[0]);
a[0] which is a1 is incremented. a1 is a pointer to int (base address of an integer array) and so increment means adding size of (int) and hence a1 now points to the second element in the array. So, *++a[0] prints second element of a1 which is 7 and now a1 starts from 7.
printf("%d,", *(++a)[0]);
++a will increment a, which being an array of pointers (to int) will add sizeof (pointer) to a. So, a now contains {a2, a3} and a[0] will be a2 and *a2 will be the first element in a2 which is 23
printf("%d\n", a[–1][+1]);
a[–1] will subtract a size of pointer from the base address of a. Normally this results in invalid memory access, but since we have incremented a previously, a[–1] is valid and will point to a1.
So, a[–1][+1] will be a1[1] which has the value 8 (a1 was incremented in 3rd printf and hence starts from 7 and not 6.

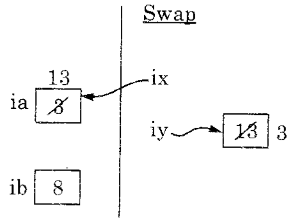
1.61 (c)

When $n = 0$, ${}^mC_n = {}^mC_0 = 1$
$m = n$, ${}^mC_n = {}^mC_m = 1$

1.62 (a)

1. Overloading is defining a set of similar functions.
2. Friend keyword is used to give a non member function access to the private part of an object.
3. Constructor is a function which is automatically called when an object is created.
4. Protect keyword allows a derived class to have access to the private parts of base class.
5. "This" is a pointer to the object associated with the current function.
6. Inheritance allows us to define a class to have properties of another class.

1.63 (b)



S1: The compiler will generate code to allocate a temporary nameless cell, initialize it to 13, and pass the address of the cell to swap

S4: The program will print ia = 13 and ib = 8

1.64 (d)

If array a and b contain identical elements then neither (i) nor (ii) holds.

1.65 (b)

Both will give different answer.
Example: Assume i = 3, j = 5, a[5] = 18
Work1 output is 16
Work2 output is 15
Comparing work1 work2 will take less time.

1.66 (b)

S2 and S3 are correct statements.

1.67 (d)

To compute f(5), initially r = 5.

$f(3) + 2 = 18$ $[(n > 3) : f(n-2) + 2]$
$\uparrow$
$f(2) + 5 = 16$ $[f(n-2) + r]$
$\uparrow$
$f(1) + 5 = 11$ $[f(n-2) + r]$
$\uparrow$
$f(0) + 5 = 6$ $[f(n-2) + r]$
$\uparrow$
1 $[n \leq 0]$

1.68 (d)

The function terminates for all powers of 2 (which is infinite), hence (i) is false and (ii) is TRUE.
Let n = 5.
Now, recursive calls will go like 5 – 14 – 7 – 20 – 10 – 5 and this goes into infinite recursion. And if we multiply 5 with any power of 2, also result will be infinite recursion. Since, there are infinite powers of 2 possible, there are infinite recursions possible (even considering this case only).
So, (iv) is TRUE and (iii) is false.
So, correct answer is (d)

1.69 (c)

synatax error in above code segment . right code given below

```
# include <stdio.h>
int main( )
{
    int sum = 0, maxsum = 0, i, n = 6;
    int a[ ]. = {2, –2, –1, 3, 4, 2};
    for (i = 0; i < n; i++)
    {
        if (i == 0| |a[i] < 0| |a[i] < a[i – 1])
        {
            if (sum > maxsum) maxsum = sum;
            sum = (a[i] > 0) ? a[i] : 0;
        }
        else
        sum += a[i];
    }
    if (sum > maxsum) maxsum = sum;
    printf ("%d\n", maxsum);
}
```

Simply execute and trace value of sum and maxsum for i = 0 to 5

i → initially	0	1	2	3	4	5	
sum	0	0	2	0	3	7	2
maxsum	0	0	2	2	2	2	7

so maxsum = 7

1.70 (b)

push 5
push 2
push 5*2 = 10. (pops 5 and 2)
push 3
push 3
push 2
push 3 + 2 = 5 (pops 2 and 3)
push 5 * 3 = 15 (pops (5 and 3)
push 15 + 10 = 25 (pops (15 and 10)

1.71 (d)

Call by value: 'i' is declared as global variable. Then in main( ) a local variable j as integer declared i.e j = 60 And global variable i initialized to 50 by i = 50. Now procedure f called and values of i and j are passed to it. i.e., in f(i, j) $\rightarrow$ f(x, y) content of memory location of i (here 50) is copied to memory location of x (which is different from i) and content of memory location of j (here, 60) is copied to memory location of y. Then in f(x, y) i = 100 changes the global i to 100, X= 10 changes the local X from 50 to 10 and Y = y + i means y = 60 + 100 = 160. Now when return back to main, i and j will be 100 and 60 respectively.

CALL BY REFERENCE: Now procedure f called and passed reference of i and j to it. i.e., in f(i, j) $\rightarrow$ f(x, y) x and y are new names (aliases) pointing to the same memory location of i and j respectively.

So, i = 100 changes the global i to 100 and x = 10 means x as well as global i = 10 (as the i being passed is the global variable and x and i share the same address).

y = y + i means y = 60 + 10 = 70 and this changes the value of j also to 70 as j and y have the same address. Now when return back to main, i and j will be 10 and 70 respectively.

1.72 (c)

In static scoping, the scope of an identifier is determined by its location in the code, and since that doesn't change, the scope doesn't either. In dynamic scoping, the scope is determined by the sequence of calls that has led to the use of an identifier, and since that can be different each time that use is reached, is dynamic.

Under static scoping: x = 10 (global i)

Under dynamic scoping: y = 20 (according to the sequence of calls, i.e 20).

1.73 (d)

(*i*) Static scoping facilitate early binding because it is done at compile time.

(*ii*) Dynamic scoping require late binding because it is done at runt time.

(*iii*) Early binding increases execution efficiency because both neccessory and uncessary things are load at compile time.

(*iv*) Late binding decrease execution efficiency because only nessessory things are loaded when they are needed.

1.74 (c)

Imperative: C++, Smalltalk, Java

Object oriented: C, Fortran 77, pascal

Functional: Lisp

Logic: Prolog

1.75 (a)

a = (x ≤ y) ? ((x ≤ z) ? x : z) : ((y > z) ? y : z)

In C "? : " is a ternary operator the syntax is (exp1 ? exp2 : exp3). It means if exp1 is true print exp2 else exp3.

Let x = 3, y = 4, z = 2.

$\Rightarrow$ a = (3 > 4) ? ((3 > 2) ? 3 : 2) : ((4 > 2) ? 4 : 2)

a = (3 > 4) ? 3 : 4

a = 4

1.76 (b)

char a[6] =

W	o	r	l	d	\0
a[0]	1	2	3	4	5

When Loop will execute first time,

a[0] = a[5]

a[0] = '\0'

printf("%s", a), the string starting at address a prints the string starting with '/0' and it indicates the end of string, so it will print null string.

1.77 (c)

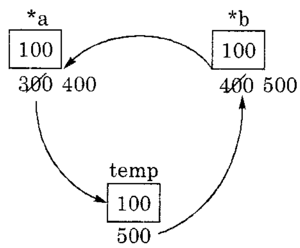
Num 1 = 5 num 1 = 4

5	4
num1	num2
(100)	(200)

1st condition fail

2nd condition fail

3rd condition execute because (num1> = num2)



So value of a = 4

value of b = 5

i.e., swapping done via call by reference.

1.78 (b)

Block Scope: A Block is a set of statements enclosed within left and right braces ({and} respectively). Blocks may be nested in C (a block may contain other blocks inside it). A variable declared in a block is accessible in the block and all inner blocks of that block, but not accessible outside the block.

If an inner block declares a variable with the same name as the variable declared by the outer block, then the visibility of the outer block variable ends at the point of declaration by inner block so here inner block int i has the scope in this block only and outer block int i visibility is not allowed in that block

j = 7 j > 4 (true)
i = 7/2 = 3
a[3] = a[3] – 1 = 4 – 1 = 3

j = 6 j > 4 (true)
i = 6/2 = 3
a[3] = a[3] – 1 = 3 – 1 = 2

j = 5 j > 4 (true)
i = 5/2 = 2
a[2] = a[2] – 1 = 4 – 1 = 3

j = 4 j > 4 (false)

Now when the for loop ends its variable named i scope is also end and the outer block variable now visible.

So the output would be: 3, 2.

1.79 (b)

First integer type two variables declared as i and j then an integer type 2-d array a[2][3] is declared and initialized and 2-d array b[3][2] is created but not initialized.

Address	value	address	value	
a[0][0]	2000	b[0][0]	3000	garbage value
a[0][1]	2001	b[0][1]	3001	garbage value
a[0][2]	2002	b[1][0]	3002	garbage value
a[1][0]	2003	b[1][1]	3003	garbage value
a[1][1]	2004	b[2][0]	3004	garbage value
a[1][2]	2005	b[2][1]	3005	garbage value

Now the char type pointer is declared and the base address of array b is put in it. so p = 3000 now the for loop is started where i is initialized to 0, so

When i = 0 : i < 2 (true)

j = 0; j < 3 (true)
*(3000 + 2*0 + 0) = a[0][0]
*(3000) = a
j++

j = 1; j < 3 (true)
*(3000 + 2*1 + 0) = a[0][1]
*(3002) = b
j++

j = 2; j < 3 (true)
*(3000 + 2*2 + 0) = a[0][2]
*(3004) = c
j++

j = 3; j < 3 (false)
i++

When i = 1 : i < 2 (true)

j = 0; j < 3 (true)
*(3000 + 2*0 + 1) = a[1][0]
*(3001) = d
j++

j = 1; j < 3 (true)
*(3000 + 2*1 + 1) = a[1][1]
*(3003) = e
j++

j = 2; j < 3 (true)
*(3000 + 2*2 + 1) = a[1][2]
*(3005) = f
j++

j = 3; j < 3 (false)
i++

Now the values in array b is

b[0][0]	3000	a
b[0][1]	3001	d
b[1][0]	3002	b
b[1][1]	3003	e
b[2][0]	3004	c
b[2][1]	3005	f

Hence the output will be (b) choice.

Note: *(p + 2*j + i)
p + size of inner dimension * j + i, hence is same as p[j] [i]. Hence with this statement we can identify that the code is transposing the matrix a and storing in b using pointer p.

1.80 (d)

10101101 i.e Option (d)
since recusive funtion calls will be
1st function call 173/2 = 86(int part only)

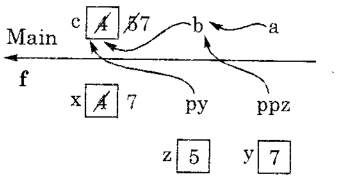
2nd function call 86/2 = 43
3rd function call 43/2 = 21(int part only)
4th function call 21/2 = 10(int part only),
5th function call 10/2 = 5,
6th function call 5/2 = 2 (integer part only),
7th function call 2/2 = 1,
In 7th function call condition if(n <=1) will become true so n will be printed (i.e 1 will be printed) now while returning every function will execute its remaining part of code i.e. printf ("%d", n%2);
So 6th function will print 2 mod 2 = 0,
5th function will print 5 mod 2 = 1,
4th function will print 10 mod 2 = 0,
3rd function will print 21 mod 2 = 1,
2nd function will print 43 mod 2 = 1,
1st function will print 86 mod 2 = 0,
Function call f(173) will print
173 mod 2 = 1

1.81 (c)

As P1 prints 10101101 same as given code in question but P2 prints the output in reverse order.

1.82 (b)

f(c, b, a) is called by the main() function the graphical execution of the program is given below.



$\Rightarrow x + y + z = 7 + 7 + 5 = 19$

int y, z;
**ppz = **ppz + 1;
z = *ppz = 5;
*py = *py + 2;
y = *py = 7
x = 4 + 3 = 7
return x + y + z = 19

1.83 (d)

Here we are using the '=' operator which has less priority than '! =' operator.
So (C = getchar( )) has to be in brackets and after reversing the string we use function putchar(c) for printing the character.

1.84 (c)

When Y is [2 2 2 2 2 2 2 2 2 2] and x > 2, given binary search can't search and the program fails.

1.85 (a)

If we will change line 6 to if (Y[k] < x) i = k + 1, else j = k – 1 then the program works properly for all the choices.

1.86 (b)

x = 15
fun (5, &x) let &x = 100
↓
fun (5, 100)
↓ t = 5, f = 5 + 3, *f_p = 5
return 8
fun (4, 100)
↓ t = 3, f = 3 + 2, *f_p = 3
return 5
fun (3, 100)
↓ t = 2, f = 2 + 1, *f_p = 2
return 3
fun (2, 100)
↓ t = 1; f = 1 + –1, *f_p = 1
return 2
fun (1, 100)
*f_p = 1
return 1
So fun(5, &x) will **return** 8 and it will be printed.

1.87 (d)

In the given program it begin from main function, i and j are globally initialized by 0 and 1. So when we call function f(& i, & j) the address of i and j are passed.
when p = q and *p = 2 means
*q = 2, so value of *q is passed.
and value of *q return to j.
Value of i and j are 0 and 2 respected.
So printf("% d %d", i, j) gives output (0, 2)

1.88 (c)

a[ ] =

12	7	13	4	11	6

So f(a, 6) → first this call
Thus is the number stored in the array and we can access these by using

*a – f(a, 6)
↓ 12
So12 + f(a + 1, n – 1) {even}
↓
7 – f(a + 1, n – 1) {odd}
↓
13 – f(a + 1, n – 1) {odd}
↓
4 + f(a + 1, n – 1) {even}
↓
11 – f(a + 1, n – 1) {odd}
↓
6 + f(a + 1, 0) {even}
↓
0

$= 12 + [7 - [13 - [4 + [11 - (6 + 0)]]]]$
$= 12 + [7 - [13 - [4 + [11 - 6]]]$
$= 12 + [7 - [13 - [4 + 5]]] = 12 + (7 - (13 - 9))$
$= 12 + (7 - 4)$
$= 12 + 3 = 15$

1.89 (d)

In given program we take the test cases and apply
From T_1 if all values are equal means a = b = c = d, a = b condition satisfied. S1 and S4 executed.
From T_2 when all a, b, c, d distinct:
S_1, S_2 not execute, S_3 executes.
From T_3 when a = b, S_1 execute but c = d.
S_2 will not execute but S_3 and S_4 executes.
Here no need of T_3 because we get all result from above two.
From T_4 if a! = b and c = d, S_1 not executes and S_2 and S_4 executes.
All of S_1, S_2, S_3, S_4 will execute and covered by T_1, T_2 and T_4.

1.90 (c)

C

2000	01	02	03	04	05	06	07	08
G	A	T	E	2	0	1	1	10
0	1	2	3	4	5	6	7	8

P
2000

P + P[3] – P[1] = 2000 + 69 – 65 = 2004
⇒ 2011 string will be printed.
(From 2004 Address)

1.91 (b)

Consider the following recursive C function foo (n, r)

```
{
    if (n > 0) return (n% r) + foo (n/r, r);
    else return 0;
}
```

Value return by the function foo (345, 10):

foo (345, 10)
⇓
5 + foo (34, 10)
⇓
4 + foo (3, 10)
⇓
3 + foo (0, 10)
⇓
0

The return value is 5 + 4 + 3 = 12

1.92 (d)

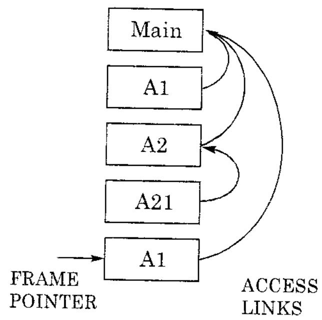
foo (513, 2)	
1 + foo (256, 2)	= 1 + 1 = 2
0 + foo (128, 2)	= 0 + 1 = 1
0 + foo (64, 2)	= 0 + 1 = 1
0 + foo (32, 2)	= 0 + 1 = 1
0 + foo (16, 2)	= 0 + 1 = 1
0 + foo (8, 2)	= 0 + 1 = 1
0 + foo (4, 2)	= 0 + 1 = 1
0 + foo (2, 2)	= 0 + 1 = 1
0 + foo (1, 2)	= 0 + 1 = 1
1 + foo (0, 2)	= 1 + 0 = 1
0	

foo(513, 2) = 2

1.93 (c)

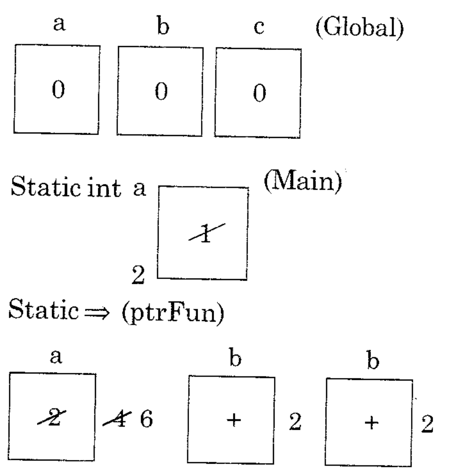
There is no break statement present in the code. Hence the switch case will jump to case : A and execute every statement from there.
∴ Choice A
Choice B No choice.

1.94 (d)



Given calling sequence from the program is:
Main → A1 → A2 → A21 → A1
A1 and A2 are defined in Main, so A1 and A2 access links are pointed to Main.
A21 definition is available in A2, hence A21 access link points to A2.

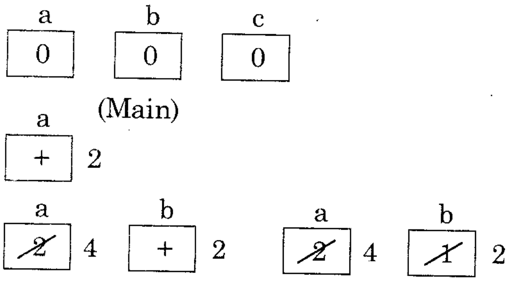
1.95 (c)



a = a + ++ b

O/P 4 2
6 2 ∴ Option 'C' is correct.
2 0

1.96 (d)



a = a+ + + b

O/P 4 2
4 2
2 0

1.97 (b)

Value of x is passed by reference so every time changes are reflected to original value

f(5, 5)
↓
x * f(6 * 4) $\Rightarrow 9^4$
↓
x * f(7 * 3) $\Rightarrow 9^3$
↓
x * f(8, 2) $\Rightarrow 9^2$
↓
x * f(9, 1) $\Rightarrow 9$
↓
1

when c = 1, x = 9
So $x * x * x * x = x^4 = 9^4 = 6561$
Option (b) is correct.

1.98 (d)

pi = &*i*; pi → $\boxed{}$ *i*

scanf("%d", pi); pi → $\boxed{10}$ *i*

printf("%d\n", *i* + 5); pi → $\boxed{15}$ *i*

The value entered is 10. It prints 15.

1.99 (c)

$$\sum_{i=0}^{n}\sum_{j=0}^{n}\sum_{k=j+1}^{n} 1 = \frac{1}{6} n \times (n+1)(n+2)$$

1.100 Sol.

$435 = 110110011$
num >>= 1; implies a num is shifted one bit right in every while loop execution. While loop is executed 9 times successfully and 10^{th} time num is zero.
So count is incremented 9 times.

1.101 (b)

$P = n*(n-1)/2*(n-2)/3$; P always results integer, because $n*(n-1)*(n-2)$ is always divisible by 6.
For larger value of n, option (b) is correct.

1.102 Sol.

$|x^2 - 3| < 0.01$

if $x = \sqrt{3}$, the given program returns x.

Therefore, $x = \sqrt{3} = 1.732$

For $q = 1.73 \Rightarrow f(q)$ returns q.

1.103 (d)

$f(50)$ goes into infinite loop by calling $f(i)$ recursively for $i = j = 50$.

1.104 Sol.

Here both f_1 and f_2 swaps the two arguments. f1 is call by value but f_2 is called by reference hence changes made by only f_2 are reflected in main function.

After f_2 is called, the values of a, b and c are 4, 6 and 5 respectively.

Hence the output is 5 – 6 – 4 = –5

1.105 (b)

```
q = 0
r = x
while (r ≥ y) do
    begin
        r = r - y
        q = q + 1
    end
```

Post condition: [after program termination]

$r < y$ and $r = r - qy$

$\Rightarrow r = x - qy$

$\Rightarrow x = r + qy$

$\therefore$ Post condition is: $(r < y) \wedge (x = qy + r)$

1.106 Sol.

Initially compute fun(1), fun(2), fun(3) and fun(4)

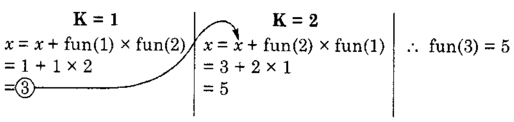
(*i*) fun(1) = 1

(*ii*) fun(2)

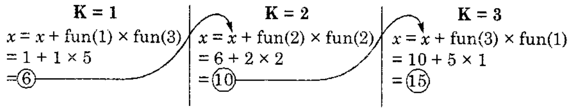
$k = 1$: $x = x + \text{fun}(1) \times \text{fun}(1) = 1 + 1 \times 1 = 2$

$\therefore$ fun(2) = 2

(*iii*) fun(3):

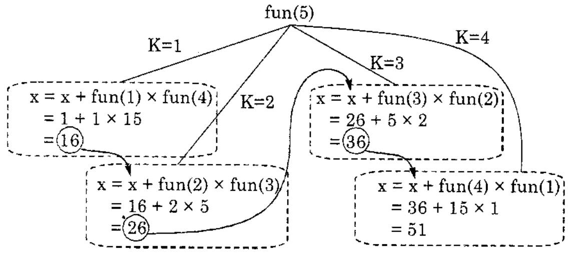


(*iv*) fun(4):



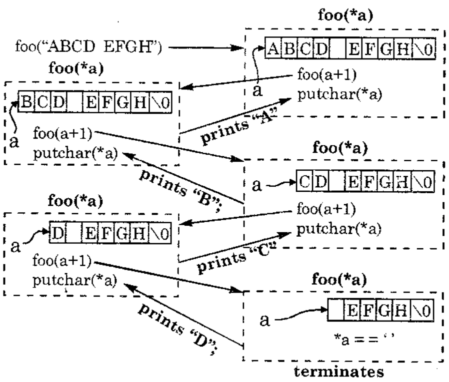
fun(4) = 15

Compute now fun(5)



fun(5) = 51

1.107 (d)



$\therefore$ Output is DCBA

1.108 Sol.

The minimum number of movements are 5. The following sequences will give minimum movements, after 1 moving.

Seq 1: Move 2 left, move 5 left, move 6 up, move 18 up, move 25 left.

Seq 2: Move 2 left, move 4 up, move 6 left, move 18 up, move 25 left.

2	4	5	14
3	6	18	23
10	12	25	∞
31	∞	∞	∞

1.109 (c)

1000	1001	1002	1003	1004	1005	1006
1	2	3	4	\0		

s_1 $\boxed{1000}$

$p = \boxed{1002}$

$*p$ = '0'; ⇒

1	2	0	4	\0		
1000	1001	1002	1003	1004	1005	1006

Therefore the output is 1204

1.110 Sol.

a	10	20	30	40	50
	a+0	a+1	a+2	a+3	a+4

p	a	a+3	a+4	a+1	a+2
	p+0	p+1	p+2	p+3	p+4

$ptr \boxed{P}$

ptr ++ ; ⇒ ptr is p + 1.

$ptr - p = (p+1) - p = \mathbf{1}$

$**ptr = *(*(p + 1)) = *(a + 3) = \mathbf{40}$

∴ Output is 1, 40 which is **140**.

1.111 (b)

Number of times invoked

get (–1) = **1** (itself)

get (0) = **1**

get (1) = get (0) + get (–2) + 1
= 1 + 1 + 1 = **3**

Ingeneral,

get (*n*) = get (*n*–1)+get (*n*–3) + 1

get (2) = get (1) + get (–1) + 1
= 3+1+1 = **5**

get (3) = get (2) + get (0) + 1
= 5+1+1 = **7**

get (4) = get (3) + get (2) + 1
= 7+3+1 = **11**

get (5) = get (4) + get (2) + 1
= 11+5+1 = **17**

get (6) = get (5) + get (3) + 1
= 17+7+1 = **25**

∴ Number of times get function invoked on get (6) = **25**

1.112 Sol.

$i = 0$, $k = -1$ only default executes $-i + k = -1$
⇒ executes 1 time.

$i = 1$, $k = -1$ only default executes $-i + k = 0$
⇒ executes 1 time.

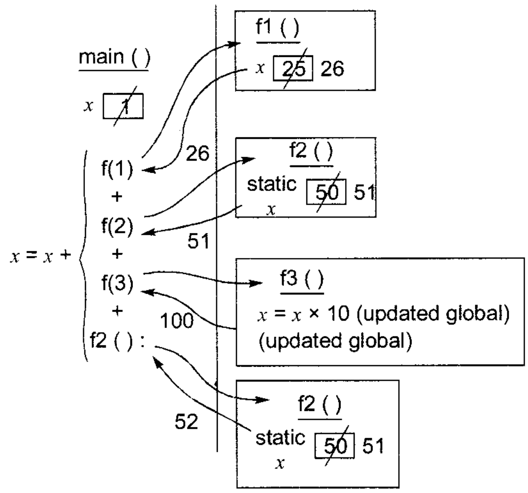
$i = 2$, $k = -1$ $- i + k = 1$ ⇒ executes 3 times (cases 1,2,3 and default)

$i = 3$, $k = -1$ $-i + k = 2$ ⇒ executes 3 times (cases 2,3 and default)

$i = 4$, $k = -1$ $-i + k = 3$ ⇒ executes 2 times (cases 3 and default)

∴ Total 10 times printf is executed.

1.113 Sol.



$x = 1 + 26 + 51 + 100 + 52 = 230$

1.114 (d)

$f(s, *s)$ since there is no pointer variable define with $*s$. So this function will give type checking error.

$i = f(i, s)$ since function prototype is void in void f (int, short) i.e., f is accepting argument int type and short type and its return type should be integer because we store the output of function in variable i which is integer type. So there must be type casting into integer type but type casting is not present. So this function will give type checking error.

$f(i, *s)$ since there is no pointer variable define with $*s$. So this function will give type checking error.

$f(i, *p)$ in this function to argument are pass one is int type and another is short type defined in the main function. So this function will not give type checking error.

1.115 Sol.

a	*b*	*c*	*d*
2016	0	4	42
100	200	300	400

1. mystery (&*a*, &*b*); address of '*a*' and '*b*' is passed.

*ptr *a* = &*a*; *ptr *b* = &*b*;

ptr *a*	ptr *b*	temp
100	~~200~~ 100	200
1000	2000	3000

1. temp = ptr *b*; ⇒ temp = 200
2. ptr *b* = ptr *a*; ⇒ ptr *b* = 100
3. ptr *a* = temp; ⇒ ptr *a* = 200

}No effect on variable '*a*' and '*b*'

2. if (2016 < 4) false

   So, mystery (&a, &b);

   *ptr a = &a; *ptr b = &d;

ptr a	ptr b	temp
100	~~200~~ 100	400
1000	2000	3000

1. temp = ptr b; ⇒ temp = 400
2. ptr b = ptr a; ⇒ ptr b = 100
3. ptr a = temp; ⇒ ptr a = 400

No effect on variable 'a' & 'd'

Print$f$("a"); ⇒ 2016

So, output of above program is 2016.

1.116 (d)

Option (a) is fail for descending array. Since n value is not decreament in program.

Option (b) is fail for ascending array. Since value of 'a' is increment every time and in while condition we check for value of 'b'. So it is incorrect.

Option (c) is fail for input 5 6 7 1 2 3. Since pointer a is pointing to 6 and 'b' will pointing to 7 i.e., 'a' and 'b' are adjacent to each other. Then while condition $b > (a + 1)$ is fail and it return 6 is maximum value which is incorrect.

Option (d) at the end of the program, pointer 'a' and 'b' pointed to same element then only while condition is false and return the maximum value of array.

1.117 (a)

D $\boxed{\cancel{1}\ \cancel{2}}$ $\cancel{3}$ 4

count (3)

n $\boxed{3}$

1. Print$f(n)$ = 3
2. Print$f(d)$ = 1

   d++;

   if (3 > 1) true

   count (2)

   n $\boxed{2}$

   3. Print$f(n)$ = 2
   4. Print$f(d)$ = 2

      d++;

      if (2 > 1) true

      count (1)

      $\boxed{n}$ 1

      5. Print$f(n)$ = 1
      6. Print$f(d)$ = 3

         d++;

         if (1 > 1) false

         count (1)

      7. Print$f(d)$ = 4

   8. Print$f(d)$ = 4

9. Print$f(d)$ = 4

So output will be printed in serial order given in solution i.e., 312213444.

1.118 (d)

a $\boxed{\cancel{3}1}$ globaly initialize.

1. $m(3)$

   y $\boxed{3}$

   1. $a = \cancel{1}\ \cancel{2}\ 4$
   2. $a = 3 - 1 = 2$

      $n(a)$;

      $n(2)$

      x $\boxed{2}$

      $x = x * a$;

      $= 2 * 2$ (here a is taken from global variable)

      $= 4$

   Print$f$(4); = 4

3. Print$f(a)$ = 4 since dyanmic scoping is used. So, take value of inner variable 'a'. So answer will be 4, 4.

1.119 Sol.

i $\boxed{\cancel{5}20}$ (1000) j $\boxed{10}$ (2000)

f(\$$i$, j);

*P $\boxed{1000}$ m $\boxed{\cancel{10}\ 15}$

1. $m = m + 5$; ∵ $m = 15$
2. $*P = *P + m$; ∵ $*P = 20$
3. Print$f(i + j)$; ∵ $20 + 10 = 30$

So value printed by above code is 30.

1.120 (c)

Take random value of X and Y i.e., X = 5 and Y = 3.

Initially X = 5, Y = 3, res = 1, a =X and b =Y.

Option (a): $X^Y = a^b$

$X^Y = a^b \equiv 5^3 = 5^3 \equiv 125 = 125$

After iteration 1

res = 5; a = 5; b = 2; X = 5; Y = 3

$X^Y = a^b \equiv 5^3 \neq 5^2 \equiv 125 \neq 25$

So, case fail. Option (a) cannot be answer.

Option (b): $(res * a)^Y = (res * X)^b$

$(1 \times 5)^3 = (1 \times 5)^3 \equiv 125 = 125$

After iteration 1

res = 5; a = 5; b = 2; X = 5; Y = 3

$(res * a)^Y = (res * X)^b \equiv (5 \times 5)^3 = (5 \times 5)^2$

$15625 \neq 625$ So, case fail. Option (b) cannot be answer.

Option (d): $X^Y = (res * a)^b$

$5^3 = (1 \times 5)^3 \equiv 125 = 125$

After iteration 1

res = 5; a = 5; b = 2; X = 5; Y = 3

$X^Y = (res * a)^b \equiv 5^3 = (5 \times 5)^2$

$125 \neq 625$ So, case fail.

Option (d) cannot be answer.

Option (c): $X^Y = res * a^b$

$5^3 = 1 \times 5^3 \equiv 125 = 125$

After iteration 1

res = 5; a = 5; b = 2; X = 5; Y = 3

$X^Y = res * a^b \equiv 5^3 = 5 \times 5^2 \equiv 125 = 125$

After iteration 2

res = 25; a = 5; b = 1; X = 5; Y = 3

$X^Y = res * a^b \equiv 5^3 = 25 \times 5^1 \equiv 125 = 125$

So, all cases are passes.

So option (c) will the answer.

1.121 Sol.

a					
	3	5	2	6	4

1. $f(a, 5)$ is a function contain 2 parameter one contain starting address of array and second parameter tell number of element in the array.
2. Every time 'n' value compare with 1 when it is less than equal to 1 return 0 and stop the program otherwise continue with recursive function call.

1. $f(a, 5)$
   $*P = a$; P pointed to same address pointed by 'a'.
   $n = 5$; n value greater than 1.
   So, max $(f(P + 1, 5 - 1), 3 - 5)$; or
   max $(f(P + 1, 4) - 2)$;
2. $f(P + 1, 4)$
   $*P = P + 1$; P is pointed to next element of array i.e., 5.
   $n = 4$; n value greater than 1.
   So, max (max$(f(P + 1, 4 - 1), 5 - 2), -2)$ or
   max (max$(f(P + 1)$ 3), 3), –2)
3. $f(P + 1, 3)$
   $*P = P + 1$; P is pointed to next element of array i.e., 2.
   $n = 3$; n value greater than 1.
   So, max(max(max$(f(P + 1, 3 - 1), 2 - 6), 3)$ $-2)$ or max(max(max$(f(P+1, 2), -4), 3) -2)$;
4. $f(P + 1, 2)$;
   $*P = P+1$; P is pointed to next element of array i.e., 6.
   $n = 2$; n value greater than 1.
   So, max(max(max(max$(f(P+1, 2-1), 2), -4)$, 3), –2) or
   max(max(max(max$(f(P+1, 1), 2), -4), 3), -2)$
5. $f(P + 1, 1)$;
   $*P = P+1$; P is pointed to next element of array i.e., 4.
   $n = 1$; n value equal to 1 so, return 0.
   So max(max(max(max(0, 2), –4), 3), –2)
   max(max(max(2, –4), 3), –2)
   max(max(2, 3), –2)
   max(3, –2) = 3.

So the value printed by given code is 3.

■■■■

2 Arrays

2.1 In a compact single dimensional array representation for lower triangular matrices (i.e. all the elements above the diagonal are zero) of size $n \times n$, non-zero elements (i.e. elements of the lower triangle) of each row are stored one after another, starting from the first row, the index of the $(i, j)^{th}$ element of the lower triangular matrix in this new representation is

(a) $i + j$ (b) $i + j - 1$

(c) $(j-1) + \frac{i(i-1)}{2}$ (d) $i + \frac{j(j-1)}{2}$

[1994 : 2 Marks]

2.2 Let A be a two-dimensional array declared as follows:

A : array [1 ... 10] [1 ... 15] of integer;

Assuming that each integer takes one memory location. The array is stored in row-major order and the first element of the array is stored at location 100, what is the address of the element A[i][j]?

(a) $15i + j + 84$ (b) $15j + i + 84$

(c) $10i + j + 89$ (d) $10j + i + 89$

[1998 : 2 Marks]

2.3 An $n \times n$ array v is defined as follows:

$v[i, j] = i - j$ for all i, j, $1 \le i \le n$, $1 \le j \le n$

The sum of the elements of the array v is

(a) 0 (b) $n - 1$

(c) $n^2 - 3n + 2$ (d) $n^2(n + 1)/2$

[2000 : 1 Mark]

2.4 Suppose you are given an array s[1...n] and a procedure reverse(s, i, j) which reverses the order of elements in between positions i and j (both inclusive). What does the following sequence do, where $1 \le k \le n$:

```
reverse (s, 1, k);
reverse (s, k + 1, n);
reverse (s, 1, n);
```

(a) Rotates s left by k positions

(b) Leaves s unchanged

(c) Reverses all elements of s

(d) None of the above

[2000 : 1 Mark]

2.5 An array of integers of size n can be converted into a heap by adjusting the heaps rooted at each internal node of the complete binary tree starting at the node $\lfloor (n-1)/2 \rfloor$, and doing this adjustment up to the root node (root node is at index 0) in the order $\lfloor (n-1)/2 \rfloor$, $\lfloor (n-3)/2 \rfloor$,..., 0. The time required to construct a heap in this manner is

(a) O(log n) (b) O(n)

(c) O (n log log n) (d) O(n log n)

[2004 : 2 Marks]

2.6 A program P reads in 500 integers in the range (0, 100) representing the scores of 500 students. If prints the frequency of each score above 50, what would be the best way for P to store the frequencies?

(a) An array of 50 numbers

(b) An array of 100 numbers

(c) An array of 500 numbers

(d) A dynamically allocated array of 550 numbers

[2005 : 1 Mark]

2.7 Let a be an array containing n integers in increasing order. The following algorithm determines whether there are two distinct numbers in the array whose difference is a specified number $S > 0$.

```
i = 0; j = 1;
while (j < n) {
    if (E) j++;
    else if (a[j] – a[i] = = S) break;
    else i++;
}
if (j < n) printf("yes");
else printf ("no");
```

Choose the correct expression for E.

(a) a[j] – a[i] > S (b) a[j] – a[i] < S

(c) a[i] – a[j] < S (d) a[i] – a[j] > S

[2005 : 2 Marks]

2.8 Let a and b be two sorted arrays containing n integers each, in non-decreasing order. Let c be a sorted array containing 2n integers obtained by merging the two arrays a and b. Assuming the arrays are indexed starting from 0, consider the following four statements

I. a[i] ≥ b[i] => c[2i] ≥ a[i]
II. a[i] ≥ b[i] => c[2i] ≥ b[i]
III. a[i] ≥ b[i] => c[2i] ≤ a[i]
IV. a[i] ≥ b[i] => c[2i] ≤ b[i]

Which of the following is TRUE

(a) only I and II (b) only I and IV
(c) only II and III (d) only III and IV

[2005 : 2 Marks]

2.9 Consider the following C function in which size is the number of elements in the array E:

```
int MyX(int *E, unsigned int size) {
    int Y = 0;
    int Z;
    int i, j, k;
    for(i = 0; i < size; i++)
        Y = Y + E[i];
    for(i = 0; i < size; i++)
        for(j = i; j < size; j++)
        {
            Z = 0;
            for(k = i; k <= j; k++)
                Z = Z + E[k];
            if (Z > Y)
                Y = Z;
        }
    return Y;
}
```

The value returned by the function **MyX** is the

(a) maximum possible sum of elements in any sub-array of array **E**.
(b) maximum element in any sub-array of array **E**.
(c) sum of the maximum elements in all possible sub-arrays of array **E**.
(d) the sum of all the elements in the array **E**.

[2014 (Set-1) : 2 Marks]

2.10 Let A be a square matrix of size $n \times n$. Consider the following pseudocode. What is the expected output?

```
C = 100;
for i=1 to n do
    for j=1 to n do
    {
        Temp = A[i][j] + C;
        A[i][j] = A[j][i];
        A[j][i] = Temp – C;
    }
for i=1 to n do
    for j =1 to n do
        output (A[i][j]);
```

(a) The matrix A itself
(b) Transpose of the matrix A
(c) Adding 100 to the upper diagonal elements and subtracting 100 from lower diagonal elements of A
(d) None of the above

[2014 (Set-3) : 1 Mark]

2.11 What is the output of the following C code? Assume that the address of x is 2000 (in decimal) and an integer requires four bytes of memory.

```
int main()
{   unsigned int x[4][3] =
  {{1, 2, 3}, {4, 5, 6}, {7, 8, 9}, {10, 11, 12}};
  printf("%u,%u, %u", x+3, *(x+3),*(x+2)+3);
}
```

(a) 2036, 2036, 2036 (b) 2012, 4, 2204
(c) 2036, 10, 10 (d) 2012, 4, 6

[2015 (Set-1) : 2 Marks]

2.12 Consider the following two C code segments. Y and X are one and two dimensional arrays of size n and $n \times n$ respectively, where $2 \le n \le 10$. Assume that in both code segments, elements of Y are initialized to 0 and each element $X[i][j]$ of array X is initialized to $i + j$. Further assume that when stored in main memory all elements of X are in same main memory page frame.

Code segment 1:

```
// initialize elements of Y to 0
// initialize elements X[i][j] of X to i+j
    for (i = 0; i < n; i++)
    Y[i] + = X[0][i];
```

Code segment 2:

```
// initialize elements of Y to 0
// initialize elements X[i][j] of X to i+j
    for (i = 0; i < n; i++)
    Y[i] + = X[i][0];
```

Which of the following statements is/are correct?

S1: Final contents of array Y will be same in both code segments.

S2: Elements of array X accessed inside the for loop shown in code segment 1 are contiguous in main memory.

S3: Elements of array X accessed inside the for loop shown in code segment 2 are contiguous in main memory.

(a) Only S2 is correct
(b) Only S3 is correct
(c) Only S1 and S2 are correct
(d) Only S1 and S3 are correct

[2015 (Set-3) : 2 Marks]

2.13 Suppose $c = \langle c[0], \ldots, c[k-1]\rangle$ is an array of length k, where all the entries are from the set $\{0, 1\}$. For any positive integers a and n, consider the following pseudocode.

```
DOSOMETHING (c, a, n)
z ← 1
for i ← 0 to k – 1
    do z ← z² mod n
    if c[i] = 1
        then z ← (z × a) mod n
return z
```

If $k = 4$, $c = \langle 1, 0, 1, 1\rangle$, $a = 2$ and $n = 8$, then the output of DOSOMETHING (c, a, n) is ______.

[2015 (Set-3) : 2 Marks]

Answers Arrays

2.1	(c)	**2.2**	(a)	**2.3**	(a)	**2.4**	(a)	**2.5**	(b)	**2.6**	(a)	**2.7**	(b)	**2.8**	(c)	**2.9**	(a)
2.10	(a)	**2.11**	(a)	**2.12**	(c)												

Explanations Arrays

2.1 (c)

LOC (i, j) formula of lower triangular matrix used

$$= (j-1) + \frac{i(i-1)}{2}.$$

2.2 (a)

Let r be number of elements in a row.

Address of the element A[i][j]

$= \text{Base address} + (i-1)^* r + (j-1)$
$= 100 + (i-1) \times 15 + (j-1)$
$= 100 + 15i - 15 + j - 1$
$= 100 + 15i + j - 16$
$= 84 + 5i + j$

2.3 (a)

Array is

$$\begin{bmatrix} 0 & -1 & -2 & \ldots & 1-n \\ 1 & 0 & -1 & \ldots & \\ 2 & 1 & 0 & & \\ 3 & 2 & 1 & & \\ \vdots & \vdots & \vdots & & \\ -(1-n) & -(2-n) & & & 0 \end{bmatrix}$$

Sum = 0 + 1 + 2 + ... + (–1) + (–2) + ... = 0.

2.4 (a)

Effect of the given 3 reversals for any k is equivalent to left rotation of the array of size n by k.

Let, S[1 .... 7]

1	2	3	4	5	6	7

∴ n = 7, k = 2

reverse (S, 1, 2) we get [2, 1, 3, 4, 5, 6, 7]
reverse (S, 3, 7) we get [2, 1, 7, 6, 5, 4, 3]
reverse (S, 1, 7) we get [3, 4, 5, 6, 7, 1, 2]

2.6 (a)

There are 500 students the score range is 0 to 100. Print the frequency of those student whose score above 50. So frequency range contains score from 50 to 100, so an array of 50 numbers is suitable for representing the frequency.

2.7 (b)

For some 'i' if we find that difference of (A[j] – A[i] < S) we increment 'j' to make this difference wider so that it becomes equal to S .If at times difference becomes greater than S we know that it wont reduce further for same 'i' and so we increment the 'i'.

We do it for each 'i' if not found in previous iteration until i = n.

2.8 (c)

a[i] ≥ b[i]. Since both a and b are sorted in the beginning, there are i elements smaller than a[i] (i starts from 0), and similarly i elements smaller than b[i].

So, a[i] ≥ b[i] means there are 2i elements smaller than a[i], and hence in the merged array a[i] can come only after these 2i elements (its index will be > 2i).

So, c[2i] ≤ a[i] (equality only comes when array contains repeated elements)

Similarly, a[i] ≥ b[i] says for b that, there are not more than 2i elements smaller than b[i] in the sorted array (i elements from b, and maximum another i elements from a).

So, b[i] ≤ c[2i]

So, II and III are correct → option (c).

2.9 (a)

```
for (i=0; i<size; i++)          → Sum of array stored in Y
Y=Y+E[i];
for (i=0; i<size; i++)          → Loop from 0 to (size-1)
{
  for (j=i; j<size; j++)        → Loop from i to (size-1)
  {
    Z=0;
    for (k=i; k<=j; k++)        → Computing sum of each
      Z=Z+E[K];                   subarray and stores in Z
    if (Z > Y)                  → If current sum of subarray is
      Y = Z;                      greater than previous then Y
  }                               holds new maximum sum of
}                                 subarray value.
```

∴ The value (Y) returned by function is the maximum possible sum of elements in any subarray of array E.

2.10 (a)

Temp = $A[i][j] + C$;

$A[i][j] = A[j][i]$;

$A[j][i] = \text{Temp} - C$;

This code swaps $A[i][j]$ and $A[j]\ [i]$ elements

For example

$i = 1, j = 2 \Rightarrow$ A[1][2] and $A[2][1]$ elements are exchanged

But when $i = 2, j = 1 \Rightarrow A[2][1]$ and $A[1][2]$ again swapped

∴ For the given code, the matrix A itself will be the output.

2.11 (a)

0^{th} row	2000 1	2004 2	2008 3
1^{st} row	2012 4	2016 5	2020 6
2^{nd} row	2024 7	2028 8	2032 9
3^{rd} row	2036 10	2040 11	2044 12

Arithmatic of two-dimensional array addresses.

'x' is the base address of the 0^{th} row.

'x+1' is the base address of the 1^{st} row.

'x+i' is the base address of the i^{th} row.

Similarly *(x+i) is the i^{th} row. So *(x+i) the address of the 0^{th} element in the 0^{th} row (by putting i = 0)

The outputs printed by printf statements are:

x+3 → 2036 (starting address of 3^{rd} row)

*(x+3) → 2036 (address of 0^{th} element in 3^{rd} row)

*(x+2)+3 → 2036 (gives address in 2^{nd} row but +3 makes the starting address of 2^{nd} row (2024) incremented by 3 × 4 = 12)

∴ Output is 2036, 2036, 2036.

2.12 (c)

Y is one dimensional array

X is two dimensional array

$$Y \; \begin{array}{|c|c|c|c|c|c|} \hline 0 & 0 & 0 & 0 & \ldots & 0 \\ \hline \end{array}$$
$$\begin{array}{cccc} 0 & 1 & \quad\quad & n-1 \end{array}$$

$$X \quad \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ \\ n-1 \end{array} \begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & 3 & \ldots & n-1 \end{array} \\ \begin{bmatrix} 0 & 1 & 2 & 3 & \ldots & n-1 \\ 1 & 2 & 3 & 4 & \ldots & n \\ 2 & 3 & 4 & 5 & \ldots & n+1 \\ 3 & 4 & 5 & 6 & \ldots & n+2 \\ .. & & & & & \\ n-1 & n & n+1 & n+2 & \ldots & n+n-2 \end{bmatrix}_{n \times n} \end{array}$$

Code segment 1:

```
for (i = 0; i < n; i++)
    Y[i] + = X[0] [i];
```

Code segment 2:

```
for (i = 0; i < n; i++)
      Y[i] + = X[i] [0];
```

S1: Final contents of array Y will be same in both code segments.

Y	0	1	2	3	4	5	6	7		$n-1$
	0	1	2	3	4	5	6	7	. . .	$n-1$

is the output of both segments.

S2: X contents are stored row wise in C-program hence code segment row '0' accessed contiguous in memory.

S3: In code segment 2, content of X are not accessed in contiguous.

2.13 Sol.

$K = 4$

0	1	2	3
1	0	1	1

$a = 2,\ n = 8$

Z = 1,

$i = 0$ Z = 1^2 mod 8 = 1

if (C[0] = = 1) Z = (1 × 2) mod 8 = 2

i = 1 Z = 2^2 mod 8 = 4

i = 2 Z = 4^2 mod 8 = 0

if (C[2] = = 1) Z = (0 × 2) mod 8 = 0

i = 3 Z = 0^2 mod 8 = 0

if (C[3] = = 1) Z = (0 × 2) mod 8 = 0

Returns 0

3 Stacks & Queues

3.1 The following sequence of operations is performed on a stack:
PUSH (10), PUSH (20), POP, PUSH (10), PUSH (20), POP, POP, POP, PUSH (20), POP
The sequence of values popped out is:
(a) 20, 10, 20, 10, 20 (b) 20, 20, 10, 10, 20
(c) 10, 20, 20, 10, 20 (d) 20, 20, 10, 20, 10
[1991 : 2 Marks]

3.2 Which of the following permutations can be obtained in the output (in the same order) using a stack assuming that the input is the sequence 1, 2, 3, 4, 5 in that order?
(a) 3, 4, 5, 1, 2 (b) 3, 4, 5, 2, 1
(c) 1, 5, 2, 3, 4 (d) 5, 4, 3, 1, 2
[1994 : 2 Marks]

3.3 The postfix expression for the infix expression A + B * (C + D) / F + D * E is
(a) AB + CD + *F/D+ E* (b) ABCD+*F/ + DE*+
(c) A *B + CD/F *DE++ (d) A+*BCD/F* DE++
[1995 : 2 Marks]

3.4 Consider the following statements:
(i) First-in-first out types of computations are efficiently supported by STACKS.
(ii) Implementing LISTS on linked lists is more efficient than implementing LISTS on an array for almost all the basic LIST operations.
(iii) Implementing QUEUES on a circular array is more efficient than implementing QUEUES on a linear array with two indices.
(iv) Last-in-first-out type of computations are efficiency supported by QUEUES.
(a) (ii) & (iii) are true (b) (i) & (ii) are true
(c) (iii) & (iv) are true (d) (ii) & (iv) are true
[1996 : 1 Mark]

3.5 Which of the following is essential for converting an infix expression to the post fix form efficiently?
(a) An operator stack
(b) An operand stack
(c) An operand stack and an operator stack
(d) A parse tree
[1997 : 1 Mark]

3.6 A priority queue Q is used to implement a stack that stores characters. PUSH (C) is implemented INSERT (Q, C, K) where K is an appropriate integer key chosen by the implementation. POP is implemented as DELETEMIN(Q). For a sequence of operations, the keys chosen are in
(a) non-increasing order
(b) non-decreasing order
(c) strictly increasing order
(d) strictly decreasing order
[1997 : 2 Marks]

3.7 Compute the postfix equivalent of the following expression:

$$3 * \log(x+1) - \frac{a}{2}$$

[1998 : 2 Marks]

3.8 What value would the following function return for the input x = 95?
Function fun (x:integer): integer;
Begin
If x > 100 then fun = x – 10
Else fun : = fun(fun (x + 11))
End;
(a) 89 (b) 90
(c) 91 (d) 92
[1998 : 2 Marks]

3.9 Let S be a stack of size $n \geq 1$. Starting with the empty stack, suppose we push the first n natural numbers in sequence, and then perform n pop operations. Assume that Push and Pop operation take X seconds each, and Y seconds elapse between the end of one such stack operation and the start of the next operation. For $m \geq 1$, define the stack-life of m is the time elapsed from the end of push(m) to the start of the pop operation that removes m from S. The average stack-life of an element of this stack is
(a) n (X + Y) (b) 3Y + 2X
(c) n (X + Y) – X (d) Y + 2X
[2003 : 2 Marks]

3.10 A single array A[1...MAXSIZE] is used to implement two stacks. The two stacks grow from opposite ends of the array. Variables top1 and top2 (top1 < top2) point to the location of the topmost element in each of the stacks. If the space is to be used efficiently, the condition for "stack full"is

(a) (top1 = MAXSIZE/2) and (top2 = MAXSIZE/2 + 1)
(b) top1 + top2 = MAXSIZE
(c) (top1 = MAXSIZE/2) or (top2 = MAXSIZE)
(d) top1 = top2 – 1

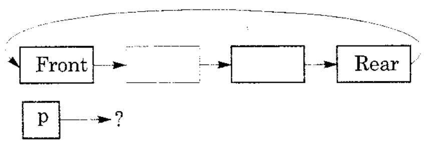
[2004 : 1 Mark]

3.11 The best data structure to check whether an arithmetic expression has balanced parentheses is a

(a) queue (b) stack
(c) tree (d) list

[2004 : 1 Mark]

3.12 A circularly linked list is used to represent a Queue. A single variable p is used to access the Queue. To which node should p point such that both the operations enQueue and deQueue can be performed in constant time?



(a) rear node
(b) front node
(c) not possible with a single pointer
(d) node next to front

[2004 : 2 Marks]

3.13 Assume that the operators +, –, × are left associative and ^ is right associative. The order of precedence (from highest to lowest) is ^, ×, +, –. The postfix expression corresponding to the infix expression a + b × c – d ^ e ^ f is

(a) abc × + def ^ ^ –
(b) abc × + de ^ f ^
(c) ab + c × d – e ^ f ^
(d) – + a × bc ^ ^ def

[2004 : 2 Marks]

3.14 A program attempts to generate as many permutations as possible of the string, 'abcd' by pushing the characters a, b, c, d in the same order onto a stack, but it may pop off the top character at any time. Which one of the following strings CANNOT be generated using this program?

(a) abcd (b) dcba
(c) cbad (d) cabd

[2004 : 2 Marks]

3.15 A function f defined on stacks of integers satisfies the following properties. $f(\phi) = 0$ and $f(push(S, i)) = max(f(S), 0) + i$ for all stacks S and integers i. If a stack S contains the integers 2, –3, 2, –1, 2 in order from bottom to top, what is f(S)?

(a) 6 (b) 4
(c) 3 (d) 2

[2005 : 1 Mark]

3.16 An implementation of a queue Q, using two stacks S1 and S2, is given below

```
void insert(Q, x)
{
    push(S1, x);
}
void delete(Q, x)
{
    if (stack-empty(S2)) then
        if (stack-empty(S1)) then
        {
            print("Q is empty");
            return;
        }
    else while (! stack-empty(S1))
    {
        x = pop(S1);
        push(S2, x);
    }
    x = pop(S2);
}
```

Let n insert and m (≤ n) delete operations be performed in an arbitrary order on an empty queue Q. Let x and y be the number of push and pop operations performed respectively in the processes. Which one of the following is true for all m and n?

(a) $n + m \le x \le 2n$ and $2m \le y \le n + m$
(b) $n + m \le x < 2n$ and $2m \le y \le 2n$
(c) $2m \le x < 2n$ and $2m \le y \le n + m$
(d) $2m \le x < 2n$ and $2m \le y \le 2n$

[2006 : 2 Marks]

3.17 The following postfix expression with single digit operands in evaluated using a stack

8 2 3 ^ / 2 3* + 5 1* –

Note that ^ is the exponentiation operator. The top two elements of the stack after the first* is evaluated are

(a) 6, 1 (b) 5, 7
(c) 3, 2 (d) 1, 5

[2007 : 2 Marks]

3.18 Suppose you are given an implementation of a queue of integers. The operations that can be performed on the queue are:

(i) isEmpty (Q) : returns true if the queue is empty, false otherwise.
(ii) delete (Q) : deletes the element at the front of the queue and returns its value.
(iii) insert (Q, i) : inserts the integer i at the rear of the queue.

Consider the following function:

```
void f (queue Q)
{
    int i;
    if (! is Empty (Q))
    {
        i = delete (Q);
        f(Q);
        insert (Q, i);
    }
}
```

What operation is performed by the above function f ?

(a) Leaves the queue Q unchanged
(b) Reverses the order of the elements in the queue Q
(c) Deletes the element at the front of the queue Q and inserts it at the rear keeping the other elements in the same order
(d) Empties the queue Q

[2007 : 2 Marks]

3.19 Suppose a circular queue of capacity (n - 1) elements is implemented with an array of n elements. Assume that the insertion and deletion operations are carried out using REAR and FRONT as array indeed variables, respectively. Initially, REAR = FRONT = 0. The conditions to detect queue full and queue empty are

(a) full : (REAR + 1) mod n == FRONT
empty : REAR == FRONT
(b) full : (REAR + 1) mod n == FRONT
empty : (FRONT + 1) mod n == REAR
(c) full : REAR == FRONT
empty : (REAR + 1) mod n == FRONT
(d) full : (FRONT + 1) mod n == REAR
empty : REAR == FRONT

[2012 : 2 Marks]

3.20 Consider the following operation along with Enqueue and Dequeue operations on queues, where k is a global parameter.

```
Multi-Dequeue (Q) {
    m = k;
    while (Q is not empty) and (m > 0) {
        Dequeue (Q);
        m = m – 1;
        }
}
```

What is the worst case time complexity of a sequence of n queue operations on an initially empty queue?

(a) $\Theta(n)$ (b) $\Theta(n + k)$
(c) $\Theta(nk)$ (d) $\Theta(n^2)$

[2013 : 2 Marks]

3.21 Suppose a stack implementation supports an instruction REVERSE, which reverses the order of elements on the stack, in addition to the PUSH and POP instructions. Which one of the following statements is TRUE *with respect to this modified stack*?

(a) A queue cannot be implemented using this stack.
(b) A queue can be implemented where ENQUEUE takes a single instruction and DEQUEUE takes a sequence of two instructions.
(c) A queue can be implemented where ENQUEUE takes a sequence of three instructions and DEQUEUE takes a single instruction.
(d) A queue can be implemented where both ENQUEUE and DEQUEUE take a single instruction each.

[2014 (Set-2) : 2 Marks]

3.22 Consider the C program below.

```
# include <stdio.h>
int *A, stkTop;
int stkFunc (int opcode, int val)
{
    static int size=0, stkTop=0;
    switch (opcode)
    {
        case -1: size = val; break;
        case 0: if (stkTop < size) A[stkTop++]
= val; break;
        default: if (stkTop) < return A[- -
stkTop];
    }
    return -1;
}
int main( )
{
    int B[20]; A=B; stkTop = -1
    stkFunc (-1, 10);
    stkFunc (0, 5);
    stkFunc (0, 10);
    printf ("%d\n", stkFunc (1, 0) + stkFunc
(1, 0));
}
```

The value printed by the above program is _____.

[2015 (Set-2) : 2 Marks]

3.23 The result evaluating the postfix expression 10 5 + 60 6 / * 8 – is

(a) 284 (b) 213
(c) 142 (d) 71

[2015 (Set-3) : 1 Mark]

3.24 A queue is implemented using an array such that ENQUEUE and DEQUEUE operations are performed efficiently. Which one of the following statements is CORRECT (n refers to the number of items in the queue)?

(a) Both operations can be performed in $O(1)$ time
(b) At most one operation can be performed in $O(1)$ time but the worst case time for the other operation will be $\Omega(n)$
(c) The worst case time complexity for both operations will be $\Omega(n)$
(d) Worst case time complexity for both operations will be $\Omega(\log n)$

[2016 (Set-1) : 1 Mark]

3.25 Let Q denote a queue containing sixteen numbers and S be an empty stack. Head(Q) returns the element at the head of the queue Q without removing it from Q. Similarly Top(S) returns the element at the top of S without removing it from S. Consider the algorithm given below.

while *Q is not Empty* do
if *S* is *Empty OR Top* (*S*) ≤ Head(*Q*) then
  x : = Dequeue (*Q*);
  Push (*S*, *x*);
  else
    x : = Pop (*S*);
    Enqueue (*Q*, *x*);
  end
end

The maximum possible number of iterations of the **while** loop in the algorithm is ________.

[2016 (Set-1) : 2 Marks]

3.26 Consider the following New-order stragety for traversing a binary tree:

- Visit the root;
- Visit the right subtree using New-order;
- Visit the left subtree using New-order;

The New-order traversal of the expression tree corresponding to the reverse polish expression 3 4 * 5 – 2 ^ 6 7 * 1 + – is given by

(a) + – 1 6 7 * 2 ^ 5 – 3 4 *
(b) – + 1 * 6 7 ^ 2 – 5 * 3 4
(c) – + 1 * 7 6 ^ 2 – 5 * 4 3
(d) 1 7 6 * + 2 5 4 3 * – ^ –

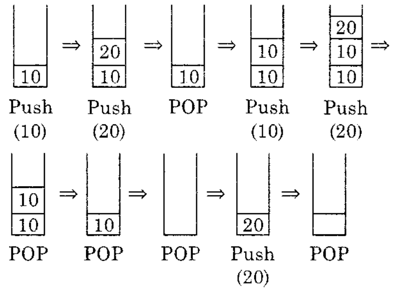
[2016 (Set-2) : 2 Marks]

Answers Stacks & Queues

3.1	(b)	3.2	(b)	3.3	(b)	3.4	(a)	3.5	(a)	3.6	(d)	3.8	(c)	3.9	(c)	3.10	(d)
3.11	(b)	3.12	(c)	3.13	(a)	3.10	(d)	3.15	(c)	3.16	(a)	3.10	(a)	3.10	(b)	3.19	(a)
3.20	(a)	3.21	(c)	3.23	(c)	3.24	(c)	3.26	(c)								

Explanations Stacks & Queues

3.1 (b)



∴ The sequence of values popped out is 20, 20, 10, 10, 20.
So option (b) is correct.

3.2 (b)

PUSH one by one element into Stack and POP accordingly desired output. Only second option can satisfy.

3.3 (b)

Infix: A+B*(C+D)/F+D*E
Postfix: ABCD + *F/ + DE*+
Therefore, option (b) is correct.

3.4 (a)

Stack is LIFO system so FIFO not supported by stack hence option statement 1 is incorrect and queue is using FIFO so statement 4 is also wrong.

3.5 (a)

Operands never change its position during expression conversion so only operator stack is needed.

3.6 (d)

In stack last element will be deleted first but according to question, it should be work like queue. So last element deleted at last, for that we give lower priority to last element. So all elements should be in strictly decreasing order.

3.7 Sol.

$$3 * \log(x+1) - \frac{a}{2}$$

Step by step conversion to postfix
= 3 * (x + 1) log – a/2
= 3 * x1+ log – a2/
= 3x1+ log * – a2/
= 3x1 + log * a2/–

3.8 (c)

fun(95)
fun (fun (106))
fun (96)
fun (fun (107))
fun (97)
fun (fun (108))
fun (98)
fun (fun (109))
fun (99)
fun (fun (200))
fun (100)
fun (fun (111))
fun (101)
fun = 101 – 10 = 91

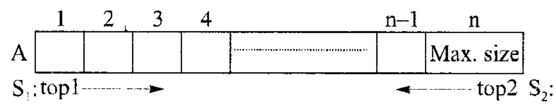
So the return value is 91.

3.9 (c)

Hint: Derive the answer taking few examples.

3.10 (d)

Consider the following diagram for the given problem



Let the two stacks are S_1 and S_2 and top1 and top2 are top of S_1 and S_2 respectively. The implementation of two stacks in a single array A is called colliding strategy. According to the question top1 <top2 means the location value of top2 is always greater than top1 if S_1 grows from initial location of A and S_2 grows from final location of A and both grows in opposite directions. Assume the array position initial value is 1 and Maxsize is n.
In this strategy there is decrement of 1 in each step of top2 and increment of 1 in each step of top1. So the condition for "stack full" must satisfy the following equation

$$\text{top2} - 1 - \text{top1} = 0$$

or $\boxed{\text{top1} = \text{top2} - 1}$

3.11 (b)

The stack is a data structure in which each push (insert) or pop (delete) operation takes constant θ(1) time. The best data structure to check whether an arithmetic expression has a balanced parentheses.

Step 1: Start with empty stack of currently open parentheses.

Step 2: Process each char of an expression string.

Step 3: If it is open, push it on the stack.

Step 4: If it is closed, check it against the top of stack element.

Step 4.1: If stack empty or not a matching parentheses then not balanced and return false.

Step 4.2: Otherwise it matches. pop the open parentheses then goto step 2.

Step 5: If stack is empty at the end, return true.

Step 6: If not empty then some parentheses is unmatched, return false.

3.12 (c)

If p points to rear, then enQueue and deQueue can be performed in constant time.

3.13 (a)

a + b × c − d ∧ e ∧ f
a + b × c − d ∧ e f ∧
a + b × c − def ∧∧
a + bc × − def ∧∧
abc × + − def ∧∧
abc × + def ∧∧−

3.10 (d)

(a) Push a and pop a, push b and pop b, push c and pop c, and finally push d and pop d. Sequence of popped elements will come to abcd.
(b) First push abcd, and after that pop one by one sequence of popped elements will come to dcba.
(c) Push abc, and after that pop one by one sequence of popped elements will come to cba, now push d and pop d, final sequence comes to cbad.
(d) This sequence is not possible because 'a' can not be popped before 'b' any how.

3.15 (c)

i: The element to be pushed
S: Stack
Initially f(S) = 0

f(S)	max(f(S),0)	i	Updated_f(S)=max(f(S),0)+i
0	0	2	2
2	2	−3	−1
−1	0	2	2
2	2	−1	1
1	1	2	**3**

So, option (c) is correct.

3.16 (a)

Number of push operations
= n(insert) + m(delete) = n + m
So, $n + m \le x$ but there are maximum 2n insert operations so $n + m \le x \le 2n$ ...(1)
No. of pop operations = n + m
But there are 2m delete operations which are less than no. of pop operations, hence
$2m \le n + m$ ...(2)
From (1) and (2): $n + m \le x \le 2n$ & $2m \le n + m$

3.17 (a)

Given postfix expression isWW
8 2 3 ∧ / 23 * + 5 1 * −

Expression	Opl	Op2	Value	top S(RL)
8				8
2				8, 2
3				8, 2, 3
∧	2	3	8	8, 8
/	8	8	1	1
2				1, 2
3				1, 2, 3
*	2	3	6	1, 6

So the top two elements of the stack are 6, 1 after the first * is evaluated.

3.18 (b)

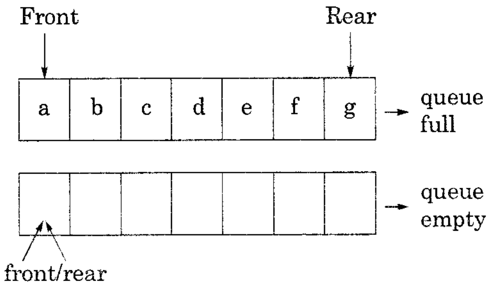
Case-1: When queue is empty then no change would happen on queue.

Case-2: 1st delete all elements one by one from queue then last deleted element enter at 1st position, 2nd last deleted element at 2nd position etc.

So finally we get reverse order of given (input queue).

3.19 (a)

So above condition can be specified as
Full : (Rear + 1) mod n = front
Empty : Rear = front



3.20 (a)

Since their are 3 queue operation i.e., enqueue, dequeue and Multi-dequeue 'n' enqueue operation will take Θ(n) time, 'n' dequeue operation will take Θ(n) time, 'n' Multi-dequeue operation will take Θ(n) time i.e., Min (n, k) but for worst case when n = k time complexity will be Θ(n).

3.21 (c)

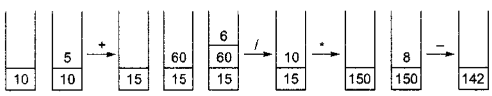
Three operations for enqueue (reverse, push, reverse). One operation for dequeue (pop).

3.22 Sol.

StkFunc (–1, 10); ⇒ size = 10; stkTop = 0 [case –1 executed]
StkFunc (0, 5); ⇒ A[0] sets to 5 and stkTop = 1 [case 0 executed]
StkFunc (0, 10); ⇒ A[1] sets to 10 and stkTop = 2 [case 0 executed]
StkFunc (1, 0); ⇒ returns **10**, stkTop = 1 [default case executed]
StkFunc (1, 0); ⇒ returns **5**, stkTop = 0 [default case executed]
∴ Program prints 10 + 5 = 15

3.23 (c)

10 5 + 60 6 / * 8 –



Result is 142.

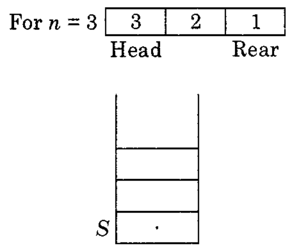
3.24 (c)

Implementing queue using array:
ENQUEUE Operation:
Check array full or not
if array is full
stop
else enter the element in the end of array;
which will take O(1) time.
DEQUEUE Operation:
Check array empty or not
if array is empty
stop
else delete the element from front of array and increment the head value (pointer to the starting element of array).
which will take O(1) time.
So for array implementation of queue, ENQUEUE and DEQUEUE operation takes O(1) time.

3.25 Sol.

The minimum number of iterations of the while loop in algorithm when use take queue contain element in ascending order i.e., 1, 2, 3, 4,..., 16 is 16.
The maximum number of iterations of while loop in algorithm when we take queue containing elements in descending order i.e., 16, 15, 14, ..., 1.
First 16 will push into stack and then enqueue it in the end of the queue. This process do till we get 1 as head element. When head point to 1 then simple push the 1 in stack. In this manner we have to push all element in stack in assending order, until queue is empty it will take 256 of iterations.

Example:



Sequence of operation with while loop execution.

1.	dequeue (3)	2.	pop (3)
3.	dequeue (2)		
	push (3)		enqueue (3)
	push (2)		
4.	pop (2)	5.	dequeue (1)
6.	dequeue (3)		
	enqueue (2)		push (1)
	push (3)		
7.	pop (3)	8.	dequeue (2)
9.	dequeue (3)		
	enqueue (3)		push (2)
	push (3)		

So for n = 3 it takes $3 \times 3 = 9$ iterations of while loop in algorithm.

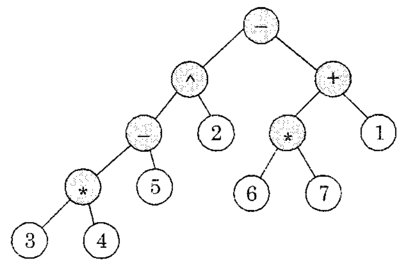
So, for $n = 16$ it will take $16 \times 16 = 256$ iterations of while loop.

3.26 (c)

The expression is given in reverse polish notation i.e., post order.

Expression is 3 4 * 5 – 2 ^ 6 7 * 1 + –

Expression tree for above post order expression is



From the above expression tree **NEW ORDER** traversal is – + 1 * 7 6 ^ 2 – 5 * 4 3.

4 Linked List

4.1 In a circular linked list organization, insertion of a record involves modification of

(a) One pointer (b) Two pointers
(c) Multiple pointers (d) No pointer

[1987 : 2 Marks]

4.2 Linked lists are not suitable data structures of which one of the following problems?

(a) Insertion sort
(b) Binary search
(c) Radix sort
(d) Polynomial manipulation

[1994 : 2 Marks]

4.3 In the worst case, the number of comparisons needed to search singly linked list of length n for a given element is

(a) $\log_2 n$ (b) n/2
(c) $\log_2 n - 1$ (d) n

[2002 : 1 Mark]

4.4 Consider the function f defined below.

```
struct item
{
  int data;
  struct item* next;
};
int f(struct item *p)
{
return((p = = NULL)||(p → next = =NULL)||
((p→data<=p→next→data)&& f(p→next)));
}
```

For a given linked list p, the function f returns 1 if and only if

(a) the list is empty or has exactly one element
(b) the elements in the list are sorted in non-decreasing order of data value
(c) the elements in the list are sorted in non-increasing order of data value
(d) not all elements in the list have the same data value

[2003 : 2 Marks]

4.5 Let P be a singly linked list. Let Q be the pointer to an intermediate node x in the list. What is the worst-case time complexity of the best-known algorithm to delete the node x from the list?

(a) O(n) (b) $O(\log^2 n)$
(c) O(log n) (d) O(1)

[2004 : 1 Mark]

4.6 The following C function takes a singly-linked list of integers as a parameter and rearranges the elements of the list. The list is represented as pointer to a structure. The function is called with the list containing the integers 1, 2, 3, 4, 5, 6, 7 in the given order. What will be the contents of the list after the function completes execution?

```
struct node {int value; struct node *next;);
void rearrange (struct node *list)
{
    struct node *p, *q;
    int temp;
    if (!list | | !list → next) return;
    p = list;
    q = list → next;
    while (q)
    {
        temp = p → value;
        p → value = q → value;
        q → value = temp;
        p = q → next;
        q = p ? p → next : 0;
    }
}
```

(a) 1, 2, 3, 4, 5, 6, 7 (b) 2, 1, 4, 3, 6, 5, 7
(c) 1, 3, 2, 5, 4, 7, 6 (d) 2, 3, 4, 5, 6, 7, 1

[2005 : 2 Marks]

4.7 The following C function takes a singly-linked list of integers as a parameter and rearranges the elements of the list. The function is called with the list containing the integers 1, 2, 3, 4, 5, 6, 7 in the given order. What will be the contents of the list after the function completes execution?

```
struct node {
    int value;
    struct node *next;
};
void rearrange (struct node *list) {
   struct node *p, *q ;
   int temp ;
   if (! list || ! list → next ) return;
   p = list ; q = list → next;
   while (q) {
      temp = p → value ;
      p → value = q → value;
      q → value = temp ; p = q → next;
      q = p? p → next : 0 ;
   }
}
```

(a) 1, 2, 3, 4, 5, 6, 7
(b) 2, 1, 4, 3, 6, 5, 7
(c) 1, 3, 2, 5, 4, 7, 6
(d) 2, 3, 4, 5, 6, 7, 1

[2008 : 2 Marks]

4.8 The following C function takes a singly-linked list as input argument. It modified the list by moving the last element to the front of the list and returns the modified list. Some part of the code is left blank.

```
typed of struct node
{
  int value;
  struct node *next;
} Node;
Node *mode_to_front (Node *head)
{
  Node *p, *q;
  if((head = = NULL)||(head→next = = NULL))
      return head;
  q = NULL;
  p = head;
  while (p→next ! = NULL)
  {
    q = p;
    p = p→next;
  }
  ____________________
  return head;
}
```

Choose the correct alternative to replace the blank line.

(a) q = NULL; p→next = head; head = p;
(b) q→next = NULL; head = p; p→next = head;
(c) head = p; p->next = q; q→next = NULL;
(d) q→next = NULL; p→next = head; head = p;

[2010 : 2 Marks]

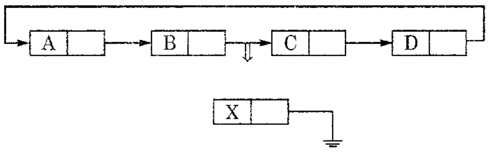
■■■■

Answers Linked List

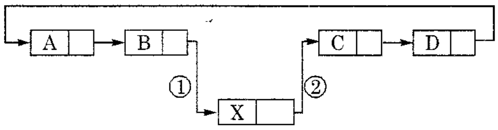
4.1	(b)	4.2	(b)	4.3	(d)	4.4	(b)	4.5	(a)	4.6	(b)	4.7	(b)	4.8	(d)

Explanations Linked List

4.1 (b)



In above linked list a new record X will be inserted in between B & C



Two pointers are modified to insert X record

4.2 (b)

Using linked list binary search will take O(n) time. So binary search is inefficient with linked list.

4.3 (d)

Linear search number of comparisons: O(n).

4.4 (b)

For a given linked list p, the function f returns 1 if and only if the elements in the list are stored in non-decreasing (increasing) order of the data value.
f returns 1 if p == NULL or p → Next == NULL or p → data <= p → next → data,
satisfies in linked list for every node p.

4.5 (a)

To delete node x, we should findout the previous node to the x. To find the previous nodes to the x will take O(n).
Therefore it takes O(n).

4.6 (b)

2 1 4 3 6 5 7: as p and q are swapping each other where q is p → next all the time.

4.7 (b)

The given C code is nonrecursive which interchange the two consecutive elements of the list. So the input sequence 1, 2, 3, 4, 5, 6, 7 produces the output 2, 1, 4, 3, 6 , 5, 7

4.8 (b)

This is the program by moving last element to front of list and returns modified list.
When while (P → next ! = NULL) execute then p pass to q and now p point to next. When "while" condition is false then we get q → next = NULL; p → next = head; head = p; means next to q made null, and points p → next is head and then make p as the head.

■■■■

5 Trees

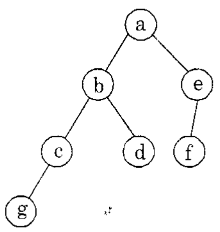
5.1 It is possible to construct a binary tree uniquely whose pre-order and post-order traversals are given. (True/False)

[1987 : 1 Mark]

5.2 If the number of leaves in a tree is not a power of 2, then the tree is not a binary tree.(True/False)

[1987 : 1 Mark]

5.3 If the binary tree in figure is traversed in inorder, then the order in which the nodes will be visited is ________.



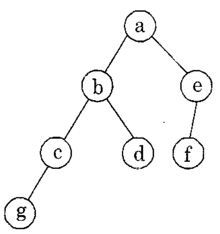
[1991 : 2 Marks]

5.4 A binary tree T has n leaf nodes. The number of nodes of degree 2 in T is

(a) $\log_2 n$ (b) $n - 1$
(c) n (d) 2^n

[1995 : 1 Mark]

5.5 In the balanced binary tree in the figure given below, how many nodes will become unbalanced when a node is inserted as a child of the node "g"?



(a) 1 (b) 3
(c) 7 (d) 8

[1996 : 1 Mark]

5.6 Which of the following sequences denotes the post order traversal sequence of the tree of above question?

(a) fegcbdba (b) gcbdafe
(c) gcdbfea (d) fedgcba

[1996 : 1 Mark]

5.7 A binary search tree is generated by inserting in order of the following integers: 50, 15, 62, 5, 20, 58, 91, 3, 8, 37, 60, 24. The number of nodes in the left subtree and right subtree of the root respectively is

(a) (4, 7) (b) (7, 4)
(c) (8, 3) (d) (3, 8)

[1996 : 2 Marks]

5.8 A binary search tree is used to locate the number 43. Which of the following probe sequences are possible and which are not? Explain.

(a) 61 52 14 17 40 43
(b) 2 3 50 40 60 43
(c) 10 65 31 48 37 43
(d) 81 61 52 14 41 43
(e) 17 77 27 66 18 43

[1996 : 2 Marks]

5.9 A binary search tree contains the values 1, 2, 3, 4, 5, 6, 7, 8. The tree is traversed in pre-order and the values are printed out. Which of the following sequences is a valid output?

(a) 53124786 (b) 53126487
(c) 53241678 (d) None of these

[1997 : 2 Marks]

5.10 Which of the following statements is false?

(a) A tree with n nodes has (n – 1) edges
(b) A labeled rooted binary tree can be uniquely constructed given its postorder and preorder traversal results.
(c) A complete binary tree with n internal nodes has (n + 1) leaves
(d) The maximum number of nodes in a binary tree of height h is $(2^{h+1} - 1)$

[1998 : 1 Mark]

5.11 A complete n-ary tree is one in which every node has 0 or n sons. If x is the number of internal nodes of a complete n-ary tree, the number of leaves in it is given by

(a) x (n – 1) +1 (b) xn – 1
(c) xn + 1 (d) x(n + 1)

[1998 : 2 Marks]

5.12 Consider the following nested representation of binary trees: (X Y Z) indicates Y and Z are the left and right subtrees, respectively, of node X. Note that Y and Z may be NULL, or further nested. Which of the following represents a valid binary tree?

(a) (1 2 (4 5 6 7)) (b) (1 ((234) 56)7)
(c) (1(234) (567)) (d) (1(23NULL) (45))

[2000 : 1 Mark]

5.13 Let LASTPOST, LASTIN and LASTPRE denote the last vertex visited in a postorder, inorder and preorder traversal. Respectively, of a complete binary tree. Which of the following is always true?

(a) LASTIN = LASTPOST
(b) LASTIN = LASTPRE
(c) LASTPRE = LASTPOST
(d) None of the above

[2000 : 2 Marks]

5.14 The number of leaf nodes in a rooted tree of n nodes, with each node having 0 or 3 children is

(a) n/2 (b) (n – 1)/2
(c) (n – 1)/2 (d) $\lceil (2n + 1)/3 \rceil$

[2002 : 2 Marks]

5.15 In a heap with n elements with the smallest element at the root, the 7th smallest element can be found in time

(a) $\Theta(n \log n)$ (b) $\Theta(n)$
(c) $\Theta(\log n)$ (d) $\Theta(1)$

[2003 : 1 Mark]

5.16 Let T(n) be the number of different binary search trees on n distinct elements.

Then $T(n) = \sum_{k=1}^{n} T(k-1)\ T(x)$, where x is

(a) n – k + 1 (b) n – k
(c) n – k – 1 (d) n – k – 2

[2003 : 1 Mark]

5.17 Suppose the numbers 7, 5, 1, 8, 3, 6, 0, 9, 4, 2 are inserted in that order into an initially empty binary search tree. The binary search tree uses the usual ordering on natural numbers. What is the inorder traversal sequence of the resultant tree?

(a) 7 5 1 0 3 2 4 6 8 9 (b) 0 2 4 3 1 6 5 9 8 7
(c) 0 1 2 3 4 5 6 7 8 9 (d) 9 8 6 4 2 3 0 1 5 7

[2003 : 1 Mark]

5.18 A data structure is required for storing a set of integers such that each of the following operations can be done in O(log n) time, where n is the number of elements in the set.

1. Deletion of the smallest element.
2. Insertion of an element if it is not already present in the set.

Which of the following data structures can be used for this purpose?

(a) A heap can be used but not a balanced binary search tree
(b) A balanced binary search tree can be used but not a heap
(c) Both balanced binary search tree and heap can be used
(d) Neither balanced binary search tree nor heap can be used

[2003 : 2 Marks]

5.19 The following numbers are inserted into an empty binary search tree in the given order: 10, 1, 3, 5, 15, 12, 16. What is the height of the binary search tree (the height is the maximum distance of a leaf node from the root)?

(a) 2 (b) 3
(c) 4 (d) 6

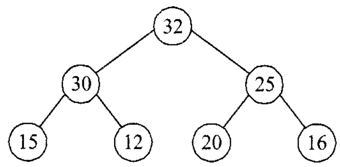
[2004 : 1 Mark]

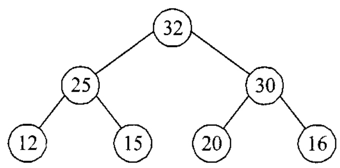
5.20 Level order traversal of a rooted tree can be done by starting from the root and performing

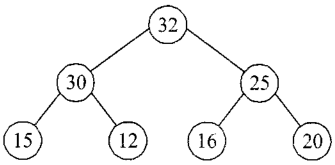
(a) preorder traversal (b) inorder traversal
(c) depth first search (d) breadth first search

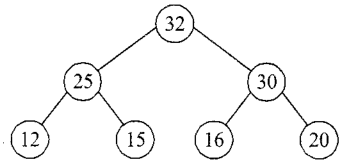
[2004 : 1 Mark]

5.21 The elements 32, 15, 20, 30, 12, 25, 16 are inserted one by one in the given order into a maxHeap. The resultant maxHeap is

(a)


(b)


(c) 

(d) 

[2004 : 2 Marks]

5.22 Consider the following C program segment

```
struct CellNode
{
  struct CellNode *leftChild ;
  int element;
  struct CellNode *rightChild ;
};
int DoSomething (struct CellNode *ptr)
{
  int value = 0;
  if (ptr ! = NULL)
  {
    if (ptr → leftChild ! = NULL)
      value = 1 + DoSomething (ptr → leftChild);
    if (ptr → rightChild ! = NULL)
      value = max (value, 1 + DoSomething (ptr
      → rightChild));
  }
  return (value);
}
```

The value returned by the function DoSomething when a pointer to the root of a non-empty tree is passed as argument is

(a) The number of leaf nodes in the tree
(b) The number of nodes in the tree
(c) The number of internal nodes in the tree
(d) The height of the tree

[2004 : 2 Marks]

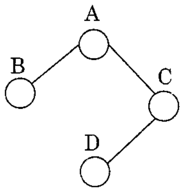
5.23 Consider the label sequences obtained by the following pairs of traversals on a labeled binary tree. Which of these pairs identify a tree uniquely?

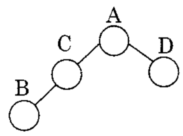
1. preorder and postorder
2. inorder and postorder
3. preorder and inorder
4. level order and postorder

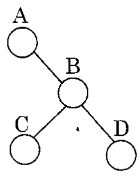
(a) 1 only (b) 2 and 3
(c) 3 only (d) 4 only

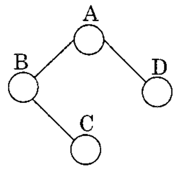
[2004 : 2 Marks]

5.24 Which one of the following binary trees has its inorder and preorder traversals as BCAD and ABCD, respectively?

(a) 

(b) 

(c) 

(d) 

[2004 : 2 Marks]

5.25 The numbers 1, 2,..., n are inserted in a binary search tree in some order. In the resulting tree, the right subtree of the root contains p nodes. The first number to be inserted in the tree must be

(a) p (b) p + 1
(c) n – p (d) n – p + 1

[2005 : 1 Mark]

5.26 In a binary tree, for every node the difference between the number of nodes in the left and right subtrees is at most 2. If the height of the tree is h > 0, then the minimum number of nodes in the tree is

(a) 2^{h-1} (2) $2^{h-1} + 1$
(c) $2^h - 1$ (4) 2^h

[2005 : 2 Marks]

5.27 A binary search tree contains the numbers 1, 2, 3, 4, 5, 6, 7, 8. When the tree is traversed in preorder and the values in each node printed out, the sequence of values obtained is 5, 3, 1, 2, 4, 6, 8, 7. If the tree is traversed in postorder, the sequence obtained would be

(a) 8, 7, 6, 5, 4, 3, 2, 1
(b) 1, 2, 3, 4, 8, 7, 6, 5
(c) 2, 1, 4, 3, 6, 7, 8, 5
(d) 2, 1, 4, 3, 7, 8, 6, 5

[2005 : 2 Marks]

5.28 Postorder traversal of a given binary search tree, T produces the following sequence of keys

10, 9, 23, 22, 27, 25, 15, 50, 95, 60, 40, 29

Which one of the following sequences of keys can be the result of an inorder traversal of the tree T?

(a) 9, 10, 15, 22, 23, 25, 27, 29, 40, 50, 60, 95
(b) 9, 10, 15, 22, 40, 50, 60, 95, 23, 25, 27, 29
(c) 29, 15, 9, 10, 25, 22, 23, 27, 40, 60, 50, 95
(d) 95, 50, 60, 40, 27, 23, 22, 25, 10, 0, 15, 29

[2005 : 2 Marks]

5.29 A priority-Queue is implemented as a Max-Heap. Initially, it has 5 elements. The level-order traversal of the heap is given below:

10, 8, 5, 3, 2

Two new elements '1' and '7' are inserted in the heap in that order. The level-order traversal of the heap after the insertion of the elements is

(a) 10, 8, 7, 5, 3, 2, 1 (b) 10, 8, 7, 2, 3, 1, 5
(c) 10, 8, 7, 1, 2, 3, 5 (d) 10, 8, 7, 3, 2, 1, 5

[2005 : 2 Marks]

5.30 How many distinct binary search trees can be created out of 4 distinct keys?

(a) 5 (b) 14
(c) 24 (d) 35

[2005 : 2 Marks]

5.31 In a complete k-ary, every internal node has exactly k children. The number of leaves in such a tree with n internal nodes is

(a) n k (b) (n – 1) k +1
(c) n(k – 1) + 1 (d) n (k –1)

[2005 : 2 Marks]

5.32 In a binary max heap counting n numbers, the smallest element can be found in time

(a) θ (n) (b) θ (log n)
(c) θ (log log n) (d) θ (1)

[2006 : 1 Mark]

5.33 In a binary tree, the number of internal nodes of degree 1 is 5, and the number of internal nodes of degree 2 is 10. The number of leaf nodes in the binary tree is

(a) 10 (b) 11
(c) 12 (d) 15

[2006 : 1 Mark]

5.34 Which of the following sequences of array elements forms a heap?

(a) {23, 17, 14, 6, 13, 10, 1, 12, 7, 5}
(b) {23, 17, 14, 6, 13, 10, 1, 5, 7, 12}
(c) {23, 17, 14, 7, 13, 10, 1, 5, 6, 12}
(d) {23, 17, 14, 7, 13, 10, 1, 12, 5, 7}

[2006 : 2 Marks]

5.35 Suppose that we have numbers between 1 and 100 in a binary search tree and want to search for the number 55. Which of the following sequences CANNOT be the sequence of nodes examined?

(a) {10, 75, 64, 43, 60, 57, 55}
(b) {90, 12, 68, 34, 62, 45, 55}
(c) {9, 85, 47, 68, 43, 57, 55}
(d) {79, 14, 72, 56, 16, 53, 55}

[2006 : 2 Marks]

Common Data for Q. 5.36 & Q. 5.37:

A 3-ary max heap is like a binary max heap, but instead of 2 children, nodes have 3 children. A 3-ary heap can be represented by an array as follows: The root is stored in the first location, a [0], nodes in the next level, from left to right, is stored from a [1] to a [3]. The nodes from the second level of the tree from left to right are stored from a [4] location onward. An item x can be inserted into a 3-ary heap containing n items by placing x in the location a [n] and pushing it up the tree to satisfy the heap property.

5.36 Which one of the following is a valid sequence of elements in an array representing 3-ary max heap?

(a) 1, 3, 5, 6, 8, 9 (b) 9, 6, 3, 1, 8, 5
(c) 9, 3, 6, 8, 5, 1 (d) 9, 5, 6, 8, 3, 1

[2006 : 2 Marks]

5.37 Suppose the elements 7, 2, 10, and 4 are inserted, in that order, into the valid 3-ary max heap found in the above question. Which one of the following is the sequence of items in the array representing the resultant heap?

(a) 10, 7, 9, 8, 3, 1, 5, 2, 6, 4
(b) 10, 9, 8, 7, 6, 5, 4, 3, 2, 1
(c) 10, 9, 4, 5, 7, 6, 8, 2, 1, 3
(d) 10, 8, 6, 9, 7, 2, 3, 4, 1, 5

[2006 : 2 Marks]

5.38 The height of a binary tree is the maximum number of edges in any root to leaf path. The maximum number of nodes is a binary tree of height h is

(a) 2^h (b) $2^{h-1}-1$
(c) $2^{h+1}-1$ (d) 2^{h+1}

[2007 : 1 Mark]

5.39 The maximum number of binary trees that can be formed with three unlabeled nodes is

(a) 1 (b) 5
(c) 4 (d) 3

[2007 : 1 Mark]

5.40 The inorder and preorder traversal of a binary tree are *dbeafcg* and *abdecfg* respectively. The postorder traversal of the binary tree is

(a) debfgca (b) edbgfca
(c) edbfgca (d) defgbca

[2007 : 2 Marks]

5.41 A complete n-ary tree is a tree in which each node has n children or no children. Let *I* be the number of internal nodes and L be the number of leaves in a complete n-ary tree. If L = 41, and *I* = 10, what is the value of n

(a) 3 (b) 4
(c) 5 (d) 6

[2007 : 2 Marks]

5.42 Consider the process of inserting an element into a Max Heap, where the Max Heap is represented by an array. Suppose we perform a binary search on the path from the new leaf to the root to find the position for the newly inserted element, the number of comparisons performed is

(a) $\theta(\log_2 n)$ (b) $\theta(\log_2 \log_2 n)$
(c) $\theta(n)$ (d) $\theta(n \log_2 n)$

[2007 : 2 Marks]

5.43 Consider the following C program segment where CellNode represents a node in a binary tree

```
struct CellNode {
struct CelloNode * leftchild;
int element;
struct CellNode *rightchild;
};
int GetValue (struct CellNode * ptr) {
   int value = 0;
if (ptr! = NULL) {
   if ((ptr → leftChild = = NULL) &&
      (ptr → rightChild = = NULL))
      Value = 1;
   else
    value = value +
            GetValue (ptr→leftChild)
          + GetValue(ptr→ rightChild);
}
   return (value);
}
```

The value returned by Get Value when a pointer to the root of a binary tree is passed as its argument is

(a) the number of nodes
(b) the number of internal nodes in the tree
(c) the number of leaf nodes in the tree
(d) the height of the tree

[2007 : 2 Marks]

5.44 When searching for the key value 60 in a binary search tree, nodes containing the key values 10, 20, 40, 50, 70 80, 90 are traversed, not necessarily in the order given. How many different orders are possible in which these key values can occur on the search path from the root to the node containing the value 60?

(a) 35 (b) 64
(c) 128 (d) 5040

[2007 : 2 Marks]

5.45 Which of the following is TRUE?

(a) The cost of searching an AVL tree is $\Theta(\log n)$ but that of a binary search tree is $O(n)$
(b) The cost of searching an AVL tree is $\Theta(\log n)$ but that of a complete binary tree is $\Theta(n \log n)$
(c) The cost of searching a binary search tree is $O(\log n)$ but that of an AVL tree is $\Theta(n)$
(d) The cost of searching an AVL tree is $\Theta(n \log n)$ but that of a binary search tree is $O(n)$

[2008 : 1 Mark]

Directions for Question 5.46 to 5.48:

A Binary Search Tree (BST) stores values in the range 37 to 573. Consider the following sequence of keys.

I. 81, 537, 102, 439, 285, 376, 305
II. 52, 97, 121, 195, 242, 381, 472
III. 142, 248, 520, 386, 345, 270, 307
IV. 550, 149, 507, 395, 463, 402, 270

5.46 Suppose the BST has been unsuccessfully searched for key 273. Which all of the above sequences list nodes in the order in which we could have encountered them in the search?

(a) II and III only (b) I and III only
(c) III and IV only (d) III only

[2008 : 2 Marks]

5.47 Which of the following statements is TRUE?

(a) I, II and IV are inorder sequences of three different BSTs
(b) I is a preorder sequence of some BST with 439 as the root
(c) II is an inorder sequence of some BST where 121 is the root and 52 is a leaf
(d) IV is a postorder sequence of some BST with 149 as the root

[2008 : 2 Marks]

5.48 How many distinct BSTs can be constructed with 3 distinct keys?

(a) 4 (b) 5
(c) 6 (d) 9

[2008 : 2 Marks]

Linked Questions 5.49 & 5.50:

A binary tree with $n > 1$ nodes has n_1, n_2 and n_3 nodes of degree one, two and three respectively. The degree of a node is defined as the number of its neighbours.

5.49 n_3 can be expressed as:

(a) $n_1 + n_2 - 1$ (b) $n_1 - 2$
(c) $[((n_1 + n_2)/2)]$ (d) $n_2 - 1$

[2008 : 2 Marks]

5.50 Starting with the above tree, while there remains a node v of degree two in the tree, add an edge between the two neighbours of v and then remove v from the tree.
How many edges will remain at the end of the process?

(a) $2 * n_1 - 3$ (b) $n_2 + 2 * n_1 - 2$
(c) $n_3 - n_2$ (d) $n_2 + n_1 - 2$

[2008 : 2 Marks]

5.51 You are given the postorder traversal, P of a binary search tree on the n elements 1, 2, ..., n. You have to determine the unique binary search tree that has P as its postorder traversal. What is the time complexity of the most efficient algorithm for doing this?

(a) $\Theta(\log n)$
(b) $\Theta(n)$
(c) $\Theta(n \log n)$
(d) none of the above, as the tree cannot be uniquely determined.

[2008 : 2 Marks]

5.52 We have a binary heap on n elements and wish to insert n more elements (not necessarily one after another) into this heap. The total time required for this is

(a) $\Theta(\log n)$ (b) $\Theta(n)$
(c) $\Theta(n \log n)$ (d) $\Theta(n^2)$

[2008 : 2 Marks]

5.53 The following three are known to be the preorder, inorder and postorder sequences of a binary tree. But it is not known which is which.

I. MBCAFHPYK **II.** KAMCBYPFH
III. MABCKYFPH

Pick the true statement from the following.

(a) I and II are preorder and inorder sequences, respectively
(b) I and III are preorder and postorder sequences, respectively
(c) II is the inorder sequence, but nothing more can be said about the other two sequences
(d) II and III are the preorder and inorder sequences, respectively

[2008 : 2 Marks]

5.54 What is the maximum height of any AVL-tree with 7 nodes? Assume that the height of a tree with a single node is 0.

(a) 2 (b) 3
(c) 4 (d) 5

[2009 : 2 Marks]

Linked Answer Question 5.55 and 5.56:

Consider a binary max-heap implemented using an array

5.55 Which one of the following array represents a binary max-heap?

(a) {25, 12, 16, 13, 10, 8, 14}
(b) {25, 14, 13, 16, 10, 8, 12}
(c) {25, 14, 16, 13, 10, 8, 12}
(d) {25, 14, 12, 13, 10, 8, 16}

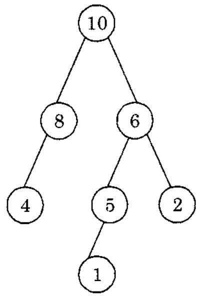
[2009 : 2 Marks]

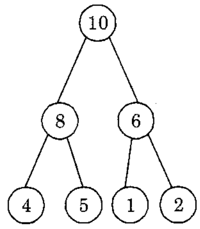
5.56 What is the content of the array after two delete operations on the correct answer to the previous question?

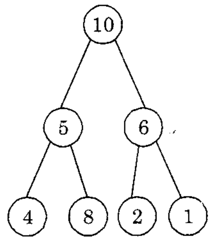
(a) {14, 13, 12, 10, 8} (b) {14, 12, 13, 8, 10}
(c) {14, 13, 8, 12, 10} (d) {14, 13, 12, 8, 10}

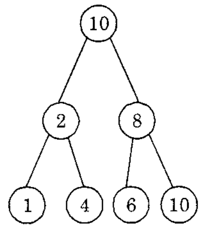
[2009 : 2 Marks]

5.57 A max-heap is a heap where the value of each parent is greater than or equal to the value of its children. Which of the following is a max-heap?

(a) 

(b) 

(c) 

(d) 

[2011 : 1 Mark]

5.58 We are given a set of n distinct elements and an unlabeled binary tree with n nodes. In how many ways can we populate the tree with the given set so that it becomes a binary search tree?

(a) 0 (b) 1

(c) n! (d) ${}^{2n}C_n/(n+1)$

[2011 : 2 Marks]

5.59 The height of a tree is defined as the number of edges on the longest path in the tree. The function shown in the pseudocode below is invoked as the height (root) to compute the height of a binary tree rooted at the tree pointer root.

```
int height(treeptr n)  {
   if(n==NULL)return -1;
   if(n → left==NULL)
      if(n→right==NULL)return0;
      else return [ B1 ];//Box1
else { h1=height(n → left);
      if(n → right==NULL) return(1+h1);
      else {h2=height(n → right);
         return[ B2 ];//Box2 }
   }
}
```

The appropriate expressions for the two boxes B1 and B2 are

(a) B1: (1 + height (n → right))
B2: (1 + max(h1, h2))

(b) B1: (height(n → right))
B2: (1 + max(h1, h2))

(c) B1: height(n → right)
B2: max(h1, h2)

(d) B1: (1 + height(n → right))
B2: max(h1, h2))

[2012 : 2 Marks]

5.60 The preorder traversal sequence of a binary search tree is 30, 20, 10, 15, 25, 23, 39, 35, 42. Which one of the following is the postorder traversal sequence of the same tree?

(a) 10, 20, 15, 23, 25, 35, 42, 39, 30
(b) 15, 10, 25, 23, 20, 42, 35, 39, 30
(c) 15, 20, 10, 23, 25, 42, 35, 39, 30
(d) 15, 10, 23, 25, 20, 35, 42, 39, 30

[2013 : 2 Marks]

5.61 Consider a rooted n node binary tree represented using pointers. The best upper bound on the time required to determine the number of subtrees having exactly 4 nodes is $O(n^a \log^b n)$. Then the value of $a + 10b$ is ________.

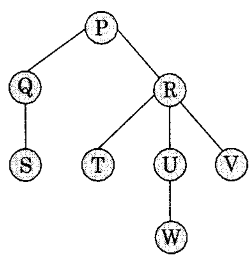
[2014 (Set-1) : 1 Mark]

5.62 A priority queue is implemented as a Max-Heap. Initially, it has 5 elements. The level-order traversal of the heap is: 10, 8, 5, 3, 2. Two new elements 1 and 7 are inserted into the heap in that order. The level-order traversal of the heap after the insertion of the elements is:

(a) 10, 8, 7, 3, 2, 1, 5 (b) 10, 8, 7, 2, 3, 1, 5
(c) 10, 8, 7, 1, 2, 3, 5 (d) 10, 8, 7, 5, 3, 2, 1

[2014 (Set-2) : 1 Mark]

5.63 Consider the following rooted tree with the vertex labeled P as the root:



The order in which the nodes are visited during an in-order traversal of the tree is

(a) SQPTRWUV (b) SQPTUWRV
(c) SQPTWUVR (d) SQPTRUWV

[2014 (Set-3) : 1 Mark]

5.64 Consider the pseudocode given below. The function **DoSomething()** takes as argument a pointer to the root of an arbitrary tree represented by the *leftMostChild-rightSibling* representation. Each node of the tree is of type **treeNode**.

```
typedef struct treeNode* treeptr;
struct treeNode
{
    treeptr leftMostChild, rightSibling;
};
int DoSomething (treeptr tree)
{
    int value=0;
    if (tree != NULL)
    {
        if (tree->leftMostChild == NULL)
            value = 1;
        else
            value = DoSomething(tree->
                                leftMostChild);
        value=value+DoSomething(tree->
                                rightSibling);
    }
    return(value);
}
```

When the pointer to the root of a tree is passed as the argument to **DoSomething**, the value returned by the function corresponds to the

(a) number of internal nodes in the tree.
(b) height of the tree.
(c) number of nodes without a right sibling in the tree.
(d) number of leaf nodes in the tree.

[2014 (Set-3) : 2 Marks]

5.65 Consider the *C* function given below. Assume that the array list A contains n (> 0) elements, sorted in ascending order.

```
int ProcessArray (int *listA, int x, int n)
{
    int i, j, k;
    i = 0;
    j = n – 1;
    do
    {
        k = (i +j )/2;
        if (x <= listA[k])
            j = k –1;
        if (listA[k] <= x)
            i = k+1;
    } while (i <= j);
    if (listA[k] == x)
        return(k);
    else
        return –1;
}
```

Which one of the following statements about the function **ProcessArray** is **CORRECT**?

(a) It will run into an infinite loop when x is not in **listA**.
(b) It is an implementation of binary search
(c) It will always find the maximum element in **listA**
(d) It will return –1 even when x is present in **listA**.

[2014 (Set-3) : 2 Marks]

5.66 Which of the following is/are correct inorder traversal sequence(s) of binary search tree(s)?

1. 3, 5, 7, 8, 15, 19, 25
2. 5, 8, 9, 12, 10, 15, 25
3. 2, 7, 10, 8, 14, 16, 20
4. 4, 6, 7, 9, 18, 20, 25

(a) 1 and 4 only
(b) 2 and 3 only
(c) 2 and 4 only
(d) 2 only

[2015 (Set-1) : 1 Mark]

5.67 The height of a tree is the length of the longest root-to-leaf path in it. The maximum and minimum number of nodes in a binary tree of height 5 are

(a) 63 and 6, respectively
(b) 64 and 5, respectively
(c) 32 and 6, respectively
(d) 31 and 5, respectively

[2015 (Set-1) : 1 Mark]

5.68 Consider a max heap, represented by the array: 40, 30, 20, 10, 15, 16, 17, 8, 4.

Array Index	1	2	3	4	5	6	7	8	9
Value	40	30	20	10	15	16	17	8	4

Now consider that a value 35 is inserted into this heap. After insertion, the new heap is

(a) 40, 30, 20, 10, 15, 16, 17, 8, 4, 35
(b) 40, 35, 20, 10, 30, 16, 17, 8, 4, 15
(c) 40, 30, 20, 10, 35, 16, 17, 8, 4, 15
(d) 40, 35, 20, 10, 15, 16, 17, 8, 4, 30

[2015 (Set-1) : 2 Marks]

5.69 A binary tree T has 20 leaves. The number of nodes in T having two children is ________.

[2015 (Set-2) : 1 Mark]

5.70 While inserting the elements 71, 65, 84, 69, 67, 83 in an empty binary search tree (BST) in the sequence shown, the element in the lowest level is

(a) 65 (b) 67

(c) 69 (d) 83

[2015 (Set-3) : 1 Mark]

5.71 Consider the following array of elements ⟨89, 19, 50, 17, 12, 15, 2, 5, 7, 11, 6, 9, 100⟩.

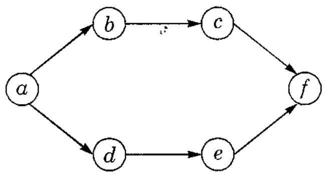
The minimum number of interchanges needed to convert it into a max-heap is

(a) 4 (b) 5

(c) 2 (d) 3

[2015 (Set-3) : 1 Mark]

5.72 Consider the following directed graph:



The number of different topological ordering of the vertices of the graph is______.

[2016 (Set-1) : 1 Mark]

5.73 An operator *delete* (*i*) for a binary heap data structure is to be designed to delete the item in the *i*-th node. Assume that the heap is implemented in an array and *i* refers to the *i*-th index of the array. If the heap tree has depth *d* (number of edges on the path from the root to the farthest leaf), then what is the time complexity to re-fix the heap efficiently after the removal of the element?

(a) $O(1)$

(b) $O(d)$ but not $O(1)$

(c) $O(2^d)$ but not $O(d)$

(d) $O(d2^d)$ but not $O(2d)$

[2016 (Set-1) : 2 Marks]

5.74 A complete binary min-heap is made by including each integer in [1, 1023] exactly once. The depth of a node in the heap is the length of the path from the root of the heap to that node. Thus, the root is at depth 0. The maximum depth at which integer 9 can appear is________.

[2016 (Set-2) : 2 Marks]

5.75 The number of ways in which the numbers 1, 2, 3, 4, 5, 6, 7 can be inserted in an empty binary search tree, such that the resulting tree has height 6, is_______. ***Note:*** The height of a tree with a single node is 0.

[2016 (Set-2) : 2 Marks]

Answers Trees

5.4	**(b)**	**5.5**	**(b)**	**5.6**	**(c)**	**5.7**	**(b)**	**5.9**	**(d)**	**5.10**	**(b)**	**5.11**	**(a)**	**5.4**	**(b)**	**5.5**	**(b)**
5.6	**(c)**	**5.7**	**(b)**	**5.9**	**(d)**	**5.10**	**(b)**	**5.11**	**(a)**	**5.12**	**(c)**	**5.13**	**(b)**	**5.14**	**(d)**	**5.15**	**(c)**
5.16	**(b)**	**5.17**	**(c)**	**5.18**	**(b)**	**5.19**	**(b)**	**5.20**	**(d)**	**5.21**	**(a)**	**5.22**	**(d)**	**5.23**	**(b)**	**5.24**	**(d)**
5.25	**(c)**	**5.26**	**(b)**	**5.27**	**(d)**	**5.28**	**(a)**	**5.29**	**(d)**	**5.30**	**(b)**	**5.31**	**(c)**	**5.32**	**(a)**	**5.33**	**(b)**
5.34	**(c)**	**5.35**	**(c)**	**5.36**	**(d)**	**5.37**	**(a)**	**5.38**	**(c)**	**5.39**	**(b)**	**5.40**	**(a)**	**5.41**	**(c)**	**5.42**	**(b)**
5.43	**(c)**	**5.44**	**(a)**	**5.45**	**(a)**	**5.46**	**(d)**	**5.47**	**(c)**	**5.48**	**(b)**	**5.49**	**(b)**	**5.50**	**(a)**	**5.51**	**(c)**
5.52	**(b)**	**5.53**	**(d)**	**5.54**	**(b)**	**5.55**	**(c)**	**5.56**	**(d)**	**5.57**	**(b)**	**5.58**	**(b)**	**5.59**	**(a)**	**5.60**	**(d)**
5.62	**(a)**	**5.63**	**(a)**	**5.64**	**(d)**	**5.65**	**(b)**	**5.66**	**(a)**	**5.67**	**(a)**	**5.68**	**(b)**	**5.70**	**(b)**	**5.71**	**(d)**
5.73	**(b)**																

Explanations Trees

5.1 Sol.

False

- For unique tree 'inorder' must be present along with pre-order or post-order.

5.2 Sol.

False

Tree may or may not be binary tree with given condition.

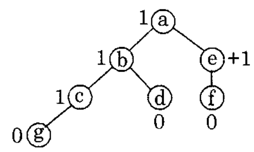
5.3 Sol.

g c b d a f e

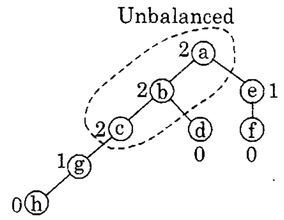
5.4 (b)

Because for 2 degree node, every time '2' leafs are added and no. of nodes increases is '1'.
So no. of nodes with degree 2 is always one less than no. of leafs present in tree.

5.5 (b)

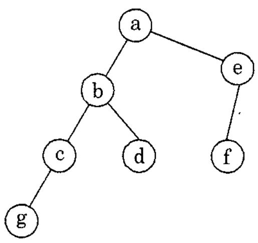


When a node is added as a child of g



Nodes a, b & c become unbalanced.
Therefore, option (b) is correct.

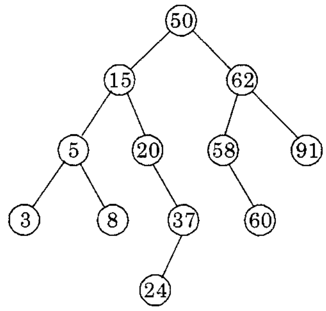
5.6 (c)



⇒ Postorder: gcdbfea

5.7 (b)

The binary search tree is



The number of nodes in the left subtree and right subtree of the root respectively is (7, 4)

5.8 Sol.

Possible sequence

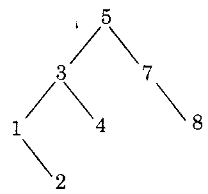
(a) 61 52 14 17 40 43
(c) 10 65 31 48 37 43
(d) 81 61 52 14 41 43

Not possible

(b) 2 3 50 40 60 43
(e) 17 77 27 66 18 43

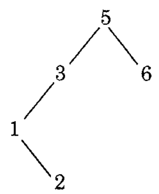
5.9 (d)

(a) 53124786



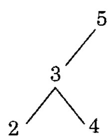
6 can not be child of 8.

(b) 53126487



4 can not be child of 6.

(c) 53241678



1 can not be child of 4 or 5.

5.10 (b)

A labelled rooted binary tree can not be constructed uniquely when inorder traversal is given along with post-order or pre-order traversal.

5.11 (a)

In n-ary tree the total number of vertices are

$$m = nx + 1$$

Where x: no. of internal nodes and let l be the number of leaf nodes.

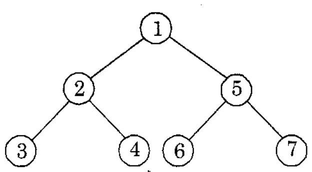
$$x + l = nx + 1$$

$$l = (n-1)x + 1$$

So number of leaf nodes in n-ary tree with x internal nodes is $(n-1)x + 1$.

5.12 (c)

(1 (2 3 4) (5 6 7))



5.13 (b)

Because of complete binary tree option (b) is always correct.

5.14 (d)

In m-ary tree

Total no. of nodes = m(internal nodes) + 1

Here, $n = 3i + 1$

Total nodes = internal nodes + leaf nodes

$$n = i + l$$

$$i = n - l$$

$$n = 3(n - l) + 1$$

$$[\because n = 3i + 1 \ \& \ i = n - l]$$

$$n = 3n - 3l + 1$$

$$3l = 2n + 1$$

$$l = \frac{2n+1}{3}$$

Where l is leaf nodes.

So option (d) is correct.

5.15 (c)

In a heap with n elements with the smallest element at the root this data structure is known as min heap or priority queue. We can implement priority queue with the help of 2–3 tree. Finding 7th smallest element means search the position of 7th smallest element. In 2–3 tree we can find the 7th smallest element in O (log n) time because member insert or delete instruction can be executed in at most O (log n) steps on a 2–3 tree with n leaves.

5.16 (b)

Left + Right + 1 = n

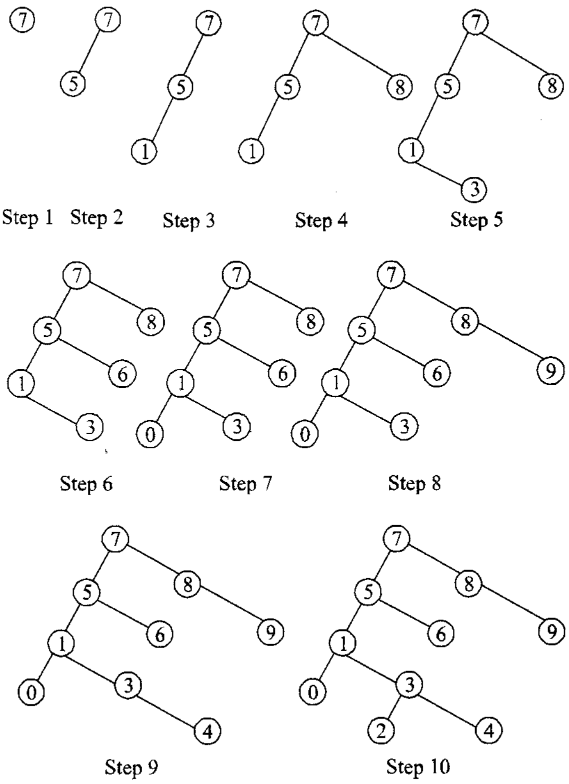
So k – 1 + x + 1 = n

x = n – k

5.17 (c)

Input sequence 7, 5, 1, 8, 3, 6, 0, 9, 4, 2

Construct the BST in which the value of left subtree is less then or equal to root and the value of right-subtree is greater than or equal to the root



In order traversal 0 1 2 3 4 5 6 7 8 9

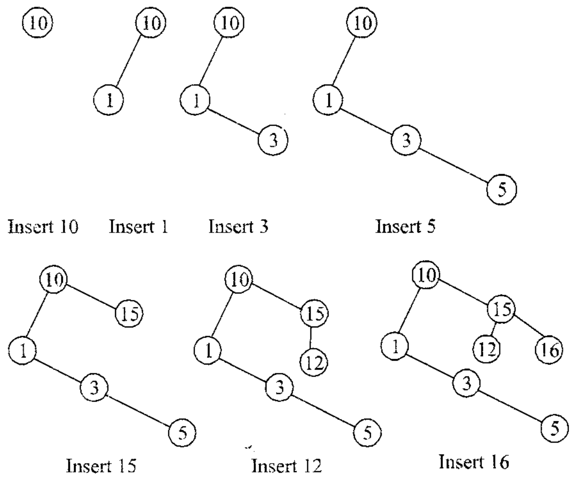
Note: The in-order traversal of a BST is in always sorted order.

5.18 (b)

Insertion and deletion in AVL and Min heap will take O(logn). But in this problem we have to check before inserting that element is already there are not, to do this Min heap will take O(n). AVL will take O(logn).

5.19 (b)

The binary search tree is initially empty.



So the maximum height of BST = 3 [Distance from element 5 to root 10 is 3]

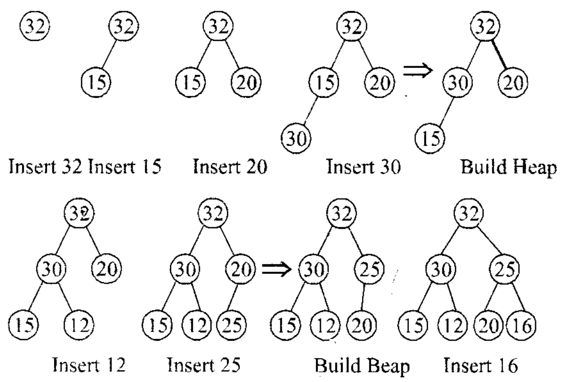
5.20 (d)

In breadth first search algorithm, the algorithm visited all vertices at distance k from root before visiting any vertices k + 1 due to this it is called level order traversal of a rooted tree.

5.21 (a)

Input sequence 32 ,15, 20, 30, 12, 25, 16

If i is the root then $A[i] \geq A[2i]$ for $1 \leq i \leq n/2$ and $A[i] \geq A[2i + 1]$ for $1 \leq i \leq n/2$ where 2i is the left of i and 2i + 1 is the right of i of a max heap.



5.22 (d)

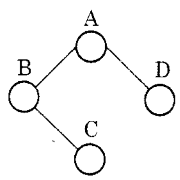
The function DoSomething is a recursive function. DoSomething(ptr →leftchild) computes height in left subtree recursively and the count stored in the variable value when we will add 1 then it includes the root nods. Same thing exist for right subtree. Finally max (value, 1 + DoSomething(ptr → rightchild)) returns the maximum height of left subtree or right subtree which is the height of a tree.

5.23 (b)

In a labeled binary tree preorder traversal produces prefix expression, postorder produces postfix expression and inorder produce infix expression. Preorder traversal root is first element and using inorder we can identity left subtree nodes and right subtree nodes for that root. So we can construct unique Binary tree using preorder and inorder similarly postorder and inorder. The postorder numbers assigned to the nodes have the useful property that the nodes in the subtree with root n are numbered consecutively from (postorder(n) – desc(n)) to postorder(n). To test if a vertex x is a descendent of vertex y then postorder(y) – desc(y) ≤ postorder(x) ≤ postorder(y)

A similar property holds for preorder and doesn't holds for inorder.

5.24 (d)



Inorder traversal is BCAD and preorder traversal is ABCD.

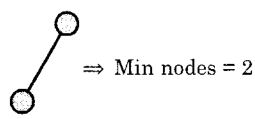
5.25 (c)

rom 1,... n elements p elements are on the right.

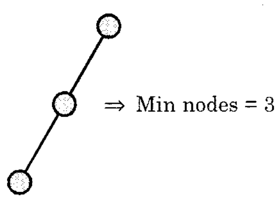
So root or first inserted will be at n–p.

5.26 (b)

For h = 1



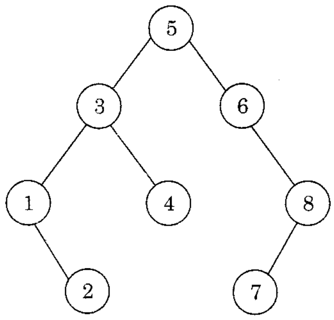
h = 2



So by putting in option we get option (b) is correct.

$2^{h-1} + 1$.

5.27 (d)

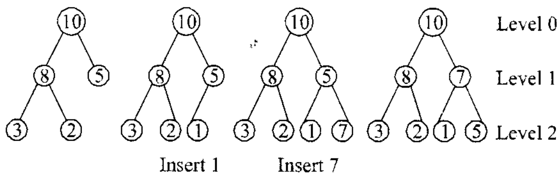


5.28 (a)

The inorder traversal of a binary search tree is always is in sorted order or increasing order of a given sequence so the inorder traversal of the tree T is 9, 10, 15, 22, 23, 25, 27, 29, 40, 50, 60, 55

5.29 (d)

Initially the sequence of Max Heap is



So, the sequence of level order traversal is 10, 8, 7, 3, 2, 15

5.30 (b)

Total number of distinct binary search tree with n nodes (key)

$$B_n = \frac{1}{n+1}\binom{2n}{n}$$

$$B_4 = \frac{1}{4+1}\binom{8}{4} = \frac{1}{5}\frac{\lfloor 8}{\lfloor 4 \lfloor 4}$$

$$B_4 = \frac{1}{5}\frac{8\times 7\times 6\times 5\times \lfloor 4}{4\times 3\times 2\times 1\times \lfloor 4} = 14$$

5.31 (c)

In a complete-k-ary tree. If every internal node has exacity k children then (k – 1) key contains no leaves. If there are n internal nodes, then number of leaves is n(k – 1) + 1.

5.32 (a)

MAX Heap used to identity max element in O(1) time for to identity min element require O(n), to apply the linear search.

5.33 (b)

Let number of vertices in bi.ary tree = n
Number of edges = n–1
So n – 1 = 1 × 5 + 2 × 10
n – 1 = 5 + 20
⇒ n = 26
Number of leaf nodes = 26 – 5 – 10 = 11

5.34 (c)

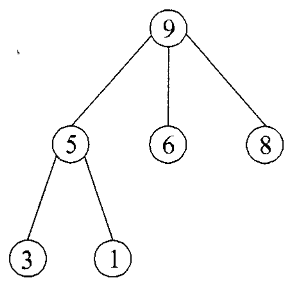
For a heap(max heap) parent should be greater than or equal to children. in a heap of [1..n] left child of ith node will be at 2*i th position and right child will be at 2*i + 1 position so for given options we can verify it.
Option (c) seems to be following the property.

5.35 (c)

In option (c) search sequence progress in ...47, 68, 43,..
At 47 we see that search key 55 is greater and it will be on right side of 47. So in further comparison a value less than 47 will not come, hence option (c) is wrong.

5.36 (d)

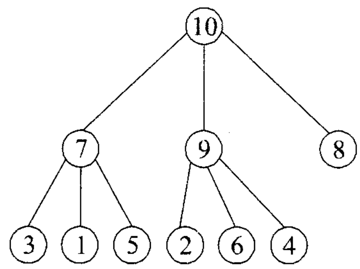
The 3 ary max heap



So the valid sequence is 9, 5, 6, 8, 3, 1

5.37 (a)

After inserting 7, 2, 10, 4 the resultant heap become
10, 7, 9, 8, 3, 1, 5, 2, 6, 4



5.38 (c)

The height of a binary tree is the maximum number of edges in any root to leaf path. Apply the induction

Height	No. of Nodes
0	$2^{0+1} - 1 = 2 - 1 = 1$ so it contains root itself
1	$2^{1+1} - 1 = 4 - 1 = 3$ root and one left & right child
h	$2^{h+1} - 1$

So maximum number of nodes in a binary tree of height h is $2^{h+1} - 1$.

5.39 (b)

The maximum number of Binary search contain n noes

$$= \frac{1}{n+1}\binom{2n}{n}$$

Given n = 3

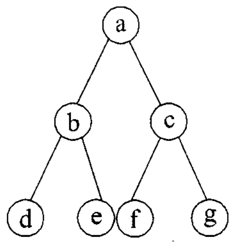
$$= \frac{1}{3+1}\,{}^6C_3 = \frac{1}{4}\cdot\frac{\lfloor 6}{\lfloor 3 \cdot \lfloor 3}$$

$$= \frac{1}{4} \times \frac{6 \times 5 \times 4 \times \lfloor 3}{3 \times 2 \times 1 \times \lfloor 3} = 5$$

5.40 (a)

The in order traversal sequence is d b e a f c g and the pre order traversal sequence is a b d e c f g.

So the tree is



In the post order traversal the sequence is d e b f g c a.

5.41 (c)

$I[n - 1] + 1 = L$

$$n = \frac{(L-1)}{I} + 1$$

$$n = \frac{41-1}{10} + 1 = \frac{40}{10} + 1$$

$$n = 4 + 1 = 5$$

5.42 (b)

After inserting an element at leaf in max heap we perform binary search on the path from the new leaf to the root to find the correct position for the newly inserted element. Comparisions will take θ(loglogn) but time will be θ(logn) because of swap operations.

5.43 (c)

In given C program segment int value = 0 it means the value is initilize to zero. Consider the statement if (ptr ! NULL) then inner part contains two statement with the help of if - else. We consider if part first if ((ptr → left child = = NULL) 88 (ptr → right child = = NULL)) value = 1 it means if a binary tree doesn't contains any left or right child, only contains the root node then number of leaf node in the tree is 1. Consider the else statement value = value + Get value (ptr → left child + Get value (ptr → right child) means if binary tree contains left & right child the recursively find the number of leaf nodes in left child as well as right child and return (value) return the number of leaf nodes in the tree.

5.44 (a)

10, 20, 40, 50, 70 80, 90

In BST search we if we go from say 10 to 40 while searching for 60, we will never encounter 20. So, 10, 20, 40 and 50 visited, means they are visited in order. Similarly, 90, 80 and 70 are visited in order.

x: Number of possible permutations of 7,

y: Number of possible permutations of numbers smaller than 60,

z: Number of possible permutations of numbers larger than 60.

Number of different orders = x! / (y! z!)

(Since only one permutation is valid for both the smaller set of number as well as larger set of numbers)7! / (4! 3!) = 35.

5.45 (a)

(a) is true as AVL tree is a balanced search tree that has time complexity of searching Θ(log n), but in binary search tree, we can have a completely left/right skewed tree, in which search is O(n).

5.46 (d)

Only III is the valid sequence.
Sequence I is wrong after 285.

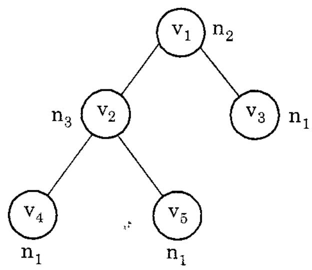
5.47 (c)

Inorder sequences are sorted. In preorder traversal root comes first and in postorder traversal root comes last.

5.48 (b)

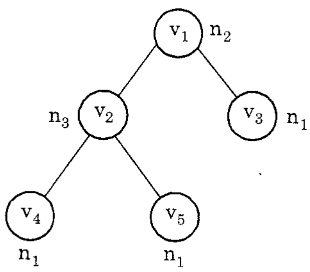
For distinct BST we apply this formula
C(2n, n) / n + 1
n = 3 here so C(6, 3) = 20 and 20/4 = 5

5.49 (b)

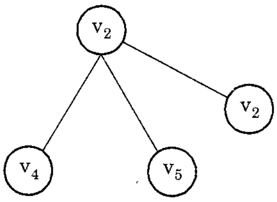


Assume the above tree so value of n_1 is 3, $n_2 = 1$, and $n_3 = 1$.
Hence $n_1 - 2$ gives n_3.

5.50 (a)



From above tree we will get the tree below.



Three edges remain in the tree.
$2 \times n_1 - 3 = 3$

5.51 (c)

We have post order traversal and the tree is binary search tree so in order traversal of binary search tree is assending order.

Using these two we can construct binary search tree with O(nlogn) [we can apply binary search].

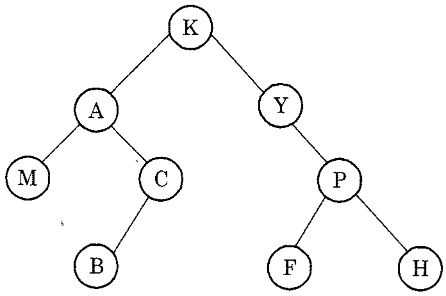
5.52 (b)

If we have a binary heap on elements and we wish to insert n more elements then we call a recursive call build heap which is responsible for constructing the heap.
The time spend by build heap is on the order of the sum over all vertices of the heights of the vertices. But at most $\lceil n/2^{i+1} \rceil$ vertices are of height i. So the total time spent by build heap is

$$\sum_{i=1}^{h} \frac{n}{2^i} i = \theta(n).$$

5.53 (d)

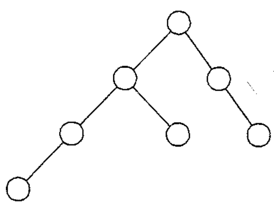
II. KAMCBYPFH (Preorder)
III. MABCKYFPH (Inorder)
If II is preorder and III is inorder then it is possible to construct binary tree.



⇒ I. MBCAFHPYK is postorder.

5.54 (b)

Maximum height of any AVL-tree with 7.

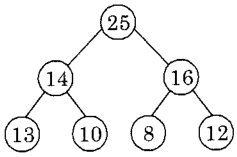


(There may be different way to draw AVL with 7 nodes).

5.55 (c)

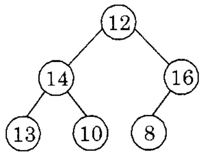
If i the node then $A[i] \geq A[2i]$ for $1 \leq i \leq n/2$ and $A[i] \geq A[2i + 1]$ for $1 \leq i \leq n/2$ where 2i is the left child and 2i + 1 right child of node i of a max heap.

Binary max-heap

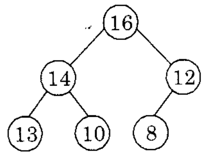


5.56 (d)

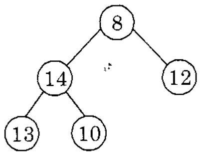
Deleter 25 : Replace it by 12



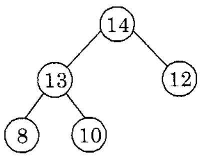
After heapfying :



Deleter 16 : Replace it by 8

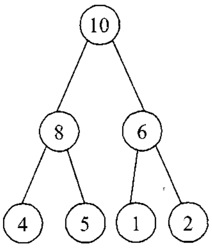


After heapfying :



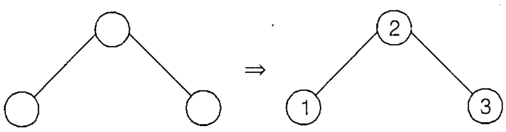
= {14, 13, 12, 8, 10}

5.57 (b)



5.58 (b)

n = {1, 2, 3}

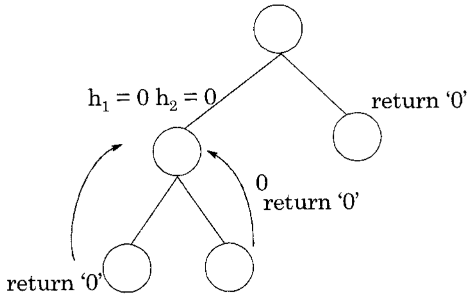


5.59 (a)

e.g.

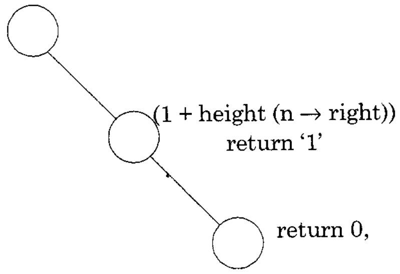
$h_1 = 1 + \max(0, 0)$, $h_2 = 0$,

return $1 + \max(h_1, h_2)$



another e.g.

return (1 + (height (n → right)

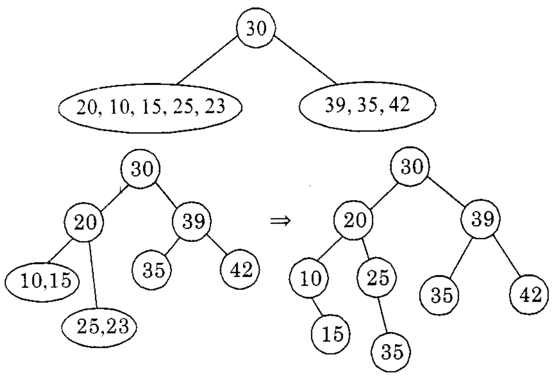


∴ option 'A' is correct by going through option.

5.60 (d)

Binary search tree and preorder is given

30, 20, 10, 15, 25, 23, 39, 35, 42



Postorder traversed of resultant tree is

15, 10, 23, 25, 20, 35, 42, 39, 30

5.61 Sol.

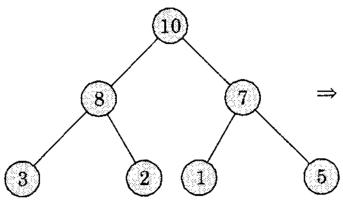
Upper bound on time required to determine the number of subtrees having exactly 4 nodes = $O(n)$

$= O(n^a \log^b n)$

$\Rightarrow \quad a = 1, b = 0$

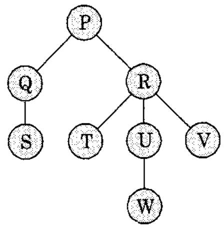
$\therefore \quad a + 10b = 1$

5.62 (a)



⇒ 10, 8, 7, 3, 2, 1, 5

5.63 **(a)**



Inorder : (Left, root, mid, right)

∴ **Inorder is :** S Q P T R W U V

5.64 **(d)**

Given program computes the number of leaf nodes in the tree.

5.65 **(b)**

Given program is implementation of binary search. x is used as key, K is in middle position of an array/subarray, i and j positions may change based on availability of key with comparison to listA[K] element.

5.66 **(a)**

The INORDER traversal of BST will always be sorted sequence (increasing order). Hence the sequences 1 and 4 are in increasing order.

5.67 **(a)**

Height is defined as path length i.e. (height +1) will give number of levels.

In a binary tree of height h = 5:

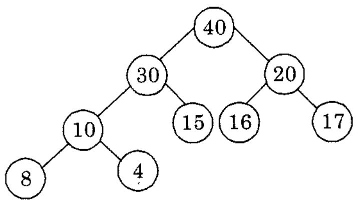
Maximum number of nodes = $2^{h+1}-1 = 2^{5+1}-1$

$= 64 - 1 = 63$

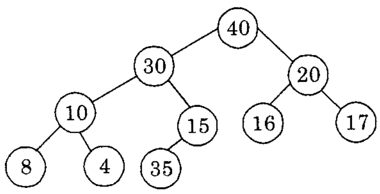
Minimum number of nodes = 6 i.e. one node at every level.

5.68 **(b)**

The existing max heap is

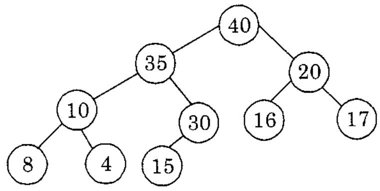


To insert 35 into the tree.



Now swap 15 and 35 to maintain max heap property.

Next swap 35 and 30 to maintain max heap property. Finally heap is



Array elements are : 40, 35, 20, 10, 30, 16, 17, 8, 4, 15.

5.69 **Sol.**

n_0 : Number of leaf nodes (degree 0)

n_1 : Number of nodes with degree 1

n_2 : Number of nodes with degree 2

We have following two equations for binary tree

$n = n_0 + n_1 + n_2$ (where n is the total number of nodes in the tree) ...(1)

Accounting for total nodes as sum of those nodes which are children and the root which is not a child of any node,

we get

$n = n_1 \times 1 + n_2 \times 2 + 1 = n_1 + 2n_2 + 1$ ....(2)

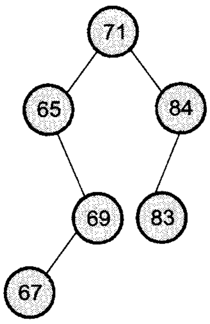
Combining equations (1) and (2) we get

$n_0 + n_1 + n_2 = n_1 + 2n_2 + 1 \Rightarrow n_2 = n_0 - 1$

Therefore the number of nodes of degree two in any binary tree = number of leaf nodes −1.

5.70 **(b)**

71, 65, 84, 69, 67, 83 are inserted into empty BST. Binary search tree:

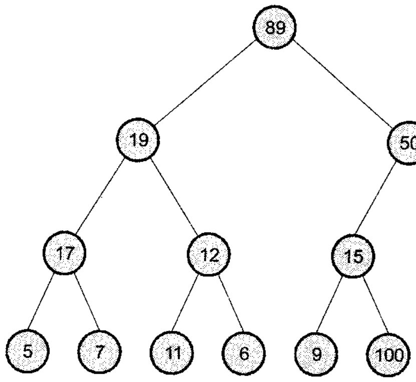


∴ 67 is present in the last level.

5.71 **(d)**

Array:

0	1	2	3	4	5	6	7	8	9	10	11	12
89	19	50	17	12	15	2	5	7	11	6	9	100



1st interchange : 100 & 15 (A[12] & A[5])

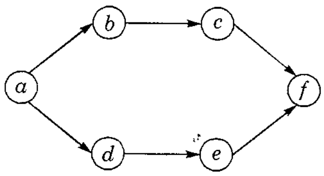
⇒ 2nd interchange : 100 & 50 (A[5] & A[27])

⇒ 3rd interchange : 100 & 89 (A[2] & A[0])

Maxheap Array:

100	19	89	17	12	50	2	5	7	11	6	9	15
0	1	2	3	4	5	6	7	8	9	10	11	12

5.72 Sol.



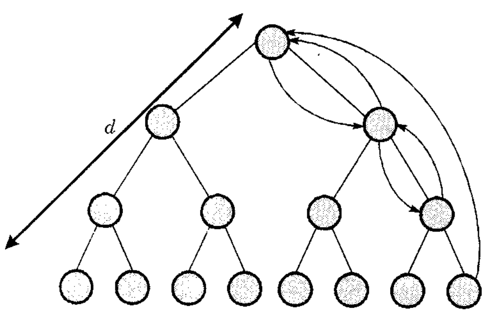
Number of topological orders: 6

a b c d e f
a b d e c f
a d b c e f
a d b c e f
a d e b c f
a d b e c f

5.73 (b)

Heap is implemented using array. If 'i' is parent element then '$2i$' is left child and '$2i$+1' is right child. So if an element is delete from last level of the heap then it will take $O(1)$ time. Since element can be deleted from any level of heap tree in worst case root element is deleted then at every level one element is exchange.

Example:



Minimum $O(d)$ time will take if a element is deleted in heap tree but not $O(1)$.

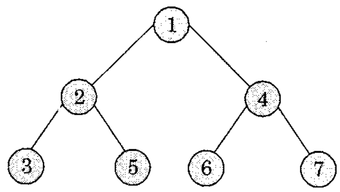
5.74 Sol.

Min heap has property "root element is greater than from its left child and from its right child (don't matter left child greater or smaller than right or vice-versa)".

With 1023 node complete binary tree have maximum 9 levels.

1st minimum element can be present at 1st level. 2nd minimum element can be present at 2nd level ... and so on. So 9th minimum in worst case can be present at 9th level. When level is 1 = height is 0. So when level is 9 = height 8.

Example: With 7 nodes, element 3 can be at level 3 in worst case but 2nd level at best case.



So integer 9 will be present at height 8.

5.75 Sol.

In question restriction on BST is height should be '6'. So we need 7 levels (given that root at height '0'). In creation of BST we have to use all element without repeatation.

At **1 level** = We have 2 choice i.e., either take 1 or 7.

At **2 level** = We have 2 choice for root 1 and 7 each. If 1 is root then 2 choice will be 6 and 2. If 7 is root then 2 choice will be 1 and 6.

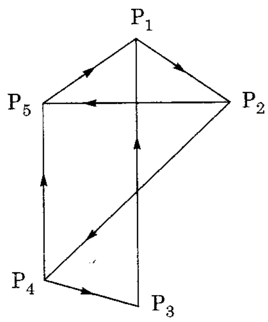
At **3 level** = If we take 1 at root, 6 at 2nd level than we have 2 choice i.e., 5 and 2 at 3rd level. If we take 1 at root, 2 at 2nd level than we have 2 choice i.e., 6 and 3 at 3rd level. Similarly if we take 7 as root element.

So till 6th level, we have two choice at every level and for last level we left with only 1 element. So number of BST with height 6

$$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 1 = 2^6 = 64.$$

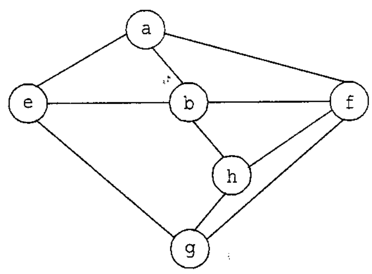
6 Graphs

6.1 Write the adjacency matrix representation of the graph given in figure.



[1998 : 2 Marks]

6.2 Consider the following graph



Among the following sequences

1. a b e g h f   **2.** a b f e h g
3. a b f h g e   **4.** a f g h b e

Which are depth first traversals of the above graph?

(a) 1, 2 and 4 only (b) 1 and 4 only
(c) 2, 3 and 4 only (d) 1, 3 and 4 only

[2003 : 1 Mark]

6.3 Let G = (V, E) be an undirected graph with a subgraph $G_1 = (V_1, E_1)$. Weights are assigned to edges of G as follows:

$$w(e) = \begin{cases} 0 \text{ if } e \in E_1 \\ 1 \text{ otherwise} \end{cases}$$

A single-source shortest path algorithm is executed on the weighted graph (V, E, w) with an arbitrary vertex v_1 of V_1 as the source. Which of the following can always be inferred from the path costs computed?

(a) The number of edges in the shortest paths from v_1 to all vertices of G
(b) G_1 is connected
(c) V_1 forms a clique in G
(d) G_1 is a tree

[2003 : 2 Marks]

6.4 Let G = (V, E) be a direction graph with n vertices. A path from v_i to v_j in G is sequence of vertices $(v_i, v_{i+1}, ..., v_j)$ such that $(v_k, v_{k+1}) \in E$ for all k in i through j – 1. A simple path is a path in which no vertex appears more than once. Let A be an n × n array initialized as follow:

$$A[j, k] = \begin{cases} 1 \text{ if } (j, k) \in E \\ 0 \text{ otherwise} \end{cases}$$

Consider the following algorithm

```
for i = 1 to n
for j = 1 to n
 for k = 1 to n
A [j, k] = max (A[j, k], (A[j, i] + A[i, k]));
```

Which of the following statements is necessarily true for all j and k after terminal of the above algorithm?

(a) $A[j, k] \le n$
(b) If $A[j, j] \ge n - 1$, then G has a Hamiltonian cycle
(c) If there exists a path from j to k, A[j, k] contains the longest path lengths from j to k
(d) If there exists a path from j to k, every simple path from j to k contain most A[j, k] edges

[2003 : 2 Marks]

6.5 In a depth-first traversal of a graph G with n vertices, k edges are marked as tree edges. The number of connected components in G is

(a) k (b) k + 1
(c) n – k – 1 (d) n – k

[2005 : 1 Mark]

6.6 Let G be a directed graph whose vertex set is the set of numbers from 1 to 100. There is an edge from a vertex i to a vertex j iff either j = i + 1 or j = 3i. The minimum number of edges in a path in G from vertex 1 to vertex 100 is

(a) 4 (b) 7
(c) 23 (d) 99

[2005 : 2 Marks]

6.7 Let T be a depth first search tree in a undirected graph G. Vertices u and v are leaves of this tree T. The degrees of both u and v in G are at least 2. Which one of the following statements is true?

(a) There must exist a vertex w adjacent to both u and v in G

(b) There must exist a vertex w whose removal disconnects u and v in G

(c) There must exist a cycle in G containing u and v

(d) There must exist a cycle in G containing u and all its neighbours in G

[2006 : 2 Marks]

6.8 What is the largest integer m such that every simple connected graph with n vertices and n edges contains at least m different spanning trees?

(a) 1 (b) 2

(c) 3 (d) n

[2007 : 2 Marks]

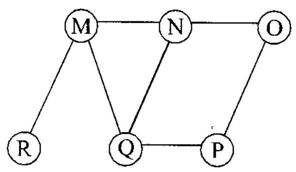
6.9 The most efficient algorithm for finding the number of connected components in an undirected graph on n vertices and m edges has time complexity.

(a) $\Theta(n)$ (b) $\Theta(m)$

(c) $\Theta(m + n)$ (d) $\Theta(mn)$

[2008 : 1 Mark]

6.10 The Breadth First Search algorithm has been implemented using the queue data structure. One possible order of visiting the nodes of the following graph is



(a) MNOPQR (b) NQMPOR

(c) QMNPRO (d) QMNPOR

[2008 : 1 Mark]

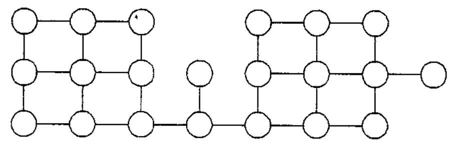
6.11 Let G of be a graph with n vertices and m edges. What is the tightest upper bound on the running time of Depth First Search on G, when G is represented as an adjacency matrix ?

(a) $\Theta(n)$ (b) $\Theta(n + m)$

(c) $\Theta(n)^2$ (d) $\Theta(m^2)$

[2014 (Set-1) : 1 Mark]

6.12 Suppose depth first search is executed on the graph below starting at some unknown vertex. Assume that a recursive call to visit a vertex is made only after first checking that the vertex has not been visited earlier. Then the maximum possible recursion depth (including the initial call) is.



[2014 (Set-3) : 1 Mark]

6.13 Breadth First Search (BFS) is started on a binary tree beginning from the root vertex. There is a vertex t at a distance four from the root. If t is the n-th vertex in this BFS traversal, then the maximum possible value of n is _______.

[2016 (Set-2) : 1 Mark]

6.14 In an adjacency list representation of an undirected simple graph $G = (V, E)$, each edge (u, v) has two adjacency list entries: $[v]$ in the adjacency list of u, and $[u]$ in the adjacency listof v. These are called twins of each other. A twin pointer is a pointer from an adjacency list entry to its twin. If $|E| = m$ and $|V| = n$, and the memory size is not a constraint, what is the time complexity of the most efficient algorithm to set the twin pointer in each entry in each adjacency list?

(a) $\Theta(n^2)$ (b) $\Theta(n + m)$

(c) $\Theta(m^2)$ (d) $\Theta(n^4)$

[2016 (Set-2) : 2 Marks]

Answers Graphs

6.2 (d) 6.3 (b) 6.4 (d) 6.5 (d) 6.6 (b) 6.7 (a) 6.8 (c) 6.9 (c) 6.10 (c)

6.11 (c) 6.14 (b)

Explanations Graphs

6.1 Sol.

Adjacency Matrix

$$\begin{array}{c|ccccc} & P_1 & P_2 & P_3 & P_4 & P_5 \\ \hline P_1 & 0 & 1 & 0 & 0 & 0 \\ P_2 & 0 & 0 & 0 & 1 & 0 \\ P_3 & 1 & 0 & 0 & 0 & 0 \\ P_4 & 0 & 0 & 1 & 0 & 1 \\ P_5 & 1 & 1 & 0 & 0 & 0 \end{array}$$

6.2 (d)

Consider each search

I. Search (a) = b   add (b) a,b
   Search (b) = e   add (e) a, b, e
   Search (e) = g   add (g) a, b, e, g
   Search (g) = h   add (h) a, b, e, h
   Search (h) = f   add (f) a, b, e, h, f

II. Search (a) = b   add (b) a, b
   Search (b) = f   add (f) a, b, f
   Search (f) = failure because there is no edge connected to e.

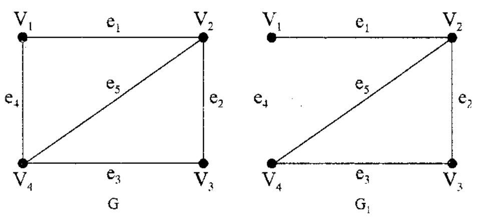
III. Search (a) = b   add (b) a, b
   Search (b) = f   add (f) a, b, f
   Search (f) = h   add (h) a, b, f, h
   Search (h) = g   add (g) a, b, f, h, g
   Search (g) = e   add (e) a, b, f, h, g, e

IV. Search (a) = f   add (f) a, f
   Search (f) = g   add (g) a, f, g
   Search (g) = h   add (b) a, f, g, h
   Search (h) = b   add (b) a, f, g, h, b
   Search (b) = e   add (e) a, f, g, h, b, e

6.3 (b)

Let G = (V, E) be an undirected graph $G_1 = (V, E_1)$ such that $E_1 \subseteq E$ and $V_1 \subseteq V$

Consider an example:



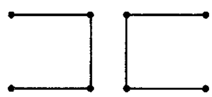
Consider the w (e) =1 if $e \notin E_1$ it means the cost of V_1 to V_4 is only 1 other edges having cost 0. It is notated that G_1 is connected. V_1 does not form of any clique in G a well as G_1 is not a tree. So choice (b) is correct.

6.4 (d)

for i = 1 to n
for j = 1 to n
for k = 1 to n
A[j, k] = max(A[j, k], (A[j, i] + A[i, k]))
A be n × n array and A[j, k] = 1 if (j, k) ∈ E.
So there exist a path from j to k and path must contain A[j, k] edges.

6.5 (d)

Number of vertices = n
Number of edges = k
Number of connected components = n − k
Ex. 8 vertex with 6 edges



So components = 8 − 6 = 2.

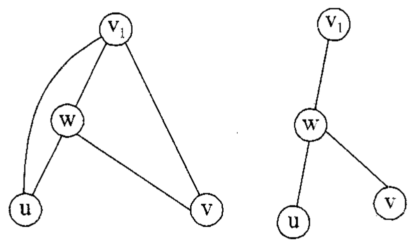
6.6 (b)

Edge set consists of edges from i to j using either (1) $j = i + 1$ OR (2) $j = 3i$.
The edge sequence with minimum number of edges is 1 − 3 − 9 − 10 − 11 − 33 − 99 − 100 which consists of 7 edges.

6.7 (a)

Consider the following graph G

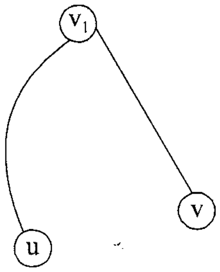
G: T: Depth First Search of G



According to the question u and v are leaves of the tree T and the degree of both u and v in G are 2. Consider each choice separately.

Choice (a); there must exist a vertex w adjacent to both u and v in G so it is true.

Choice (b); there must exist a vertex w whose removal disconnects u and v in G is false because when we remove w from G graph is connected.



Choice (c); there must exist a cycle in G containing u and v is false because in given graph there are two cycles v_1wvv_1 and v_1wuv_1 So both vertex u and v lie an different cycle.

Choice (d); there must exist a cycle in G containing u and v and all its neighbours in G is also false because both vertex u & v are not lie in a single cycle of G.

6.8 (c)

If a graph contain n vertices and n edges and it is simple connected graph then it forms a cycle. Atleast 3 vertices should participate hence the number of spanning trees will be atleast 3.

6.9 (c)

The most efficient algorithm for finding the number of connected components (articulation point) in an undirected graph on n vertices and m edges using depth-first search takes O(m + n) time. Assume n ≤ m.

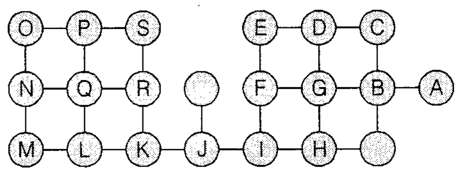
6.10 (c)

The BFS using Queue data structure is QMNPRO

6.11 (c)

Tightest upper bound on running time of DFS is $\Theta(n)^2$.

6.12 Sol.

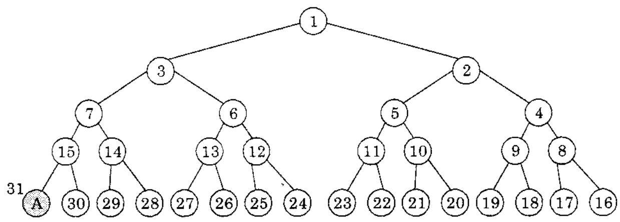


Maximum DFS recursion depth:

$A \to B \to C \to D \to E \to F \to G \to H \to I \to J \to K \to L \to M \to N \to O \to P \to Q \to R \to S = 19$

6.13 Sol.

In worst case binary tree with height 4 (i.e., distance from root node) look like.



Suppose we have to visit node A.

In worst case of BFS traversal n^{th} node (A) will be the last node at 4^{th} level i.e., we have to visit $2^{4+1} - 1 = 31$ node. So value of n will be 31.

6.14 (b)

By using BSF (Breadth First Search) traversal we can set the twin pointer in each entry in each adjacency list.

So it will take $\Theta(m + n)$ times (since adjacency list are using).

7 Hashing

7.1 A Hash table with ten buckets with one slot per bucket is shown in the following figure with the symbols S1 to S7 entered into it using some hashing function with linear probing. The worst case number of comparisons required when the symbol being searched is not in the table is

0	S_7
1	S_1
2	
3	S_4
4	S_2
5	
6	S_5
7	
8	S_6
9	S_3

[1989 : 2 Marks]

7.2 An advantage of chained hash table (external hashing) over the open addressing scheme is
(a) worst case complexity of search operations is less
(b) space used is less
(c) deletion is easier
(d) None of the above

[1996 :1 Mark]

7.3 Given the following input (4322, 1334, 1471, 9679, 1989, 6171, 6173, 4199) and the hash function x mod 10, which of the following statements are true?
1. 9679, 1989, 4199 hash to the same value
2. 1471, 6171 hash to the same value
3. All elements hash to the same value
4. Each element hashes to a different value
(a) 1 only (b) 2 only
(c) 1 and 2 only (d) 3 and 4 only

[2004 : 1 Mark]

7.4 A hash table contains 10 buckets and uses linear probing to resolve collisions. The key values are integers and the hash function used is key % 10. If the values 43, 165, 62, 123, 142 are inserted in the table, in what location would the key value 142 be inserted?
(a) 2 (b) 3
(c) 4 (d) 6

[2005 : 1 Mark]

7.5 Which of the following statement(s) is TRUE?
I. A hash function takes a message of arbitrary length and generates a fixed length code.
II. A hash function takes a message of fixed length and generates a code of variable length.
III. A hash function may give the same hash value for distinct messages.
(a) I only (b) II and III only
(c) I and III only (d) II only

[2006 : 1 Mark]

7.6 Consider a hash function that distributes keys uniformly. The hash table size is 20. After hashing of how many keys will the probability that any new key hashed collides with an existing one exceed 0.5.
(a) 5 (b) 6
(c) 7 (d) 10

[2007 : 2 Marks]

7.7 Consider the hash table of size seven, with starting index zero, and a hash function (3x + 4) mod 7. Assuming the has table is initially empty, which of the following is the contents of the table when the sequence 1, 3, 8, 10 is inserted into the table using closed hashing? Note that - denotes an empty location in the table
(a) 8, -, -, -, -,-, 10 (b) 1, 8, 10, -, -, -, 3
(c) 1, -, -, -, -, -, 3 (d) 1, 10, 8, -, -, -, 3

[2007 : 2 Marks]

7.8 Consider a hash table of size 11 that uses open addressing with linear probing. Let h(k) = k mod 11 be the hash function used. A sequence of records with keys 43 36 92 87 11 4 71 13 14 is inserted into an initially empty hash table, the bins of which are indexed from zero to ten. What is the index of the bin into which the last record is inserted?

(a) 3 (b) 4
(c) 6 (d) 7

[2008 : 2 Marks]

7.9 The keys 12, 18, 13, 2, 3, 23, 5 and 15 are inserted into an initially empty hash table of length 10 using open addressing with hash function h(k) = k mod 10 and linear probing. What is the resultant hash table?

(a)

0	
1	
2	2
3	23
4	
5	15
6	
7	
8	18
9	

(b)

0	
1	
2	12
3	13
4	
5	5
6	
7	
8	18
9	

(c)

0	
1	
2	12
3	13
4	2
5	3
6	23
7	5
8	18
9	15

(d)

0	
1	
2	12,2
3	13,3,23
4	
5	5,15
6	
7	
8	18
9	

[2009 : 2 Marks]

Linked Answer Questions 7.10 and 7.11

A has table of length 10 uses open addressing with hash function h(k) = k mod 10, and linear probing. After inserting 6 values into an empty hash table, the table is as shown below.

0	
1	
2	42
3	23
4	34
5	52
6	46
7	33
8	
9	

7.10 Which one of the following choices gives a possible order in which the key values could have been inserted in the table?

(a) 46, 42, 34, 52, 23, 33
(b) 34, 42, 23, 52, 33, 46
(c) 46, 34, 42, 23, 52, 33
(d) 42, 46, 33, 23, 34, 52

[2010 : 2 Marks]

7.11 How many different insertion sequences of the key values using the same hash function and linear probing will result in the hash table shown above?

(a) 10 (b) 20
(c) 30 (d) 40

[2010 : 2 Marks]

7.12 Consider a hash table with 9 slots. The hash function is $h(k) = k \bmod 9$. The collisions are resolved by chaining. The following 9 keys are inserted in the order: 5, 28, 19, 15, 20, 33, 12, 17, 10. The maximum, minimum, and average chain lengths in the hash table, respectively, are

(a) 3, 0, and 1 (b) 3, 3, and 3
(c) 4, 0, and 1 (d) 3, 0, and 2

[2014 (Set-1) : 2 Marks]

7.13 Consider a hash table with 100 slots. Collisions are resolved using chaining. Assuming simple uniform hashing, what is the probability that the first 3 slots are unfilled after the first 3 insertions?

(a) $(97 \times 97 \times 97)/100^3$
(b) $(99 \times 98 \times 97)/100^3$
(c) $(97 \times 96 \times 95)/100^3$
(d) $(97 \times 96 \times 95)/(3! \times 100^3)$

[2014 (Set-3) : 2 Marks]

7.14 Which one of the following hash functions on integers will distribute keys most uniformly over 10 buckets numbered 0 to 9 for i ranging from 0 to 2020?

(a) $h(i) = i^2 \bmod 10$
(b) $h(i) = i^3 \bmod 10$
(c) $h(i) = (11 * i^2) \bmod 10$
(d) $h(i) = (12 * i) \bmod 10$

[2015 (Set-2) : 2 Marks]

7.15 Given a hash table T with 25 slots that stores 2000 elements, the load factor α for T is ______.

[2015 (Set-3) : 1 Mark]

■■■■

Answers Hashing

7.2	(c)	7.3	(c)	7.4	(d)	7.5	(c)	7.6	(d)	7.7	(b)	7.8	(d)	7.9	(c)	7.10	(c)
7.11	(c)	7.12	(a)	7.13	(c)	7.14	(b)										

Explanations Hashing

7.1 Sol.

Maximum no. of comparison can be 5.
Starting with cell no. 8 to 2.

7.2 (c)

Deletion is easier, because chained hash table uses linked list.

7.3 (c)

Input (4322, 1334, 1471, 9679, 1989, 6171, 6173, 4199)
Given Hash function,

$h(a) = x \bmod 10$
$h(a) = h(1)$
$= \{1471, 6171\}$ hash to the same value
$h(a) = h(2) = \{4322\}$
$h(a) = h(3) = \{6173\}$
$h(a) = h(9)$
$= \{9679, 1989, 4199\}$ has to the same value.

h(0), h(5), h(6), h(7), h(8) are empty for given input.
So the statement (i) and (ii) are correct.

7.4 (d)

43, 165, 62 , 123 and 142 are inserted in the table.
43 mapped to location 3, 165 mapped to location 5, 62 mapped to location 2, 123 mapped to location 3 (occupied), so it goes to location 4, 142 mapped to location 2 (occupied), 3, 4, and 5 are also probes, so it goes to location 6.

7.5 (c)

A hash function takes a message of arbitrary length and generates a fixed length code. by taking mod (some value).
A hash function may give the same hash value for distinct messages.

7.6 (d)

Probability of k^{th} insertion > 0.5
$k/20 > 0.5$
$\Rightarrow k > 10$
After 10 keys, the probability of key exceed 0.5.

7.7 (b)

Size of hash table = 7
$h(x) = (3x + 4) \bmod 7$
$h(1) = (3.1 + 4) \bmod 7 = 7 \bmod 7$
$= 0$; insert at 0^{th} location.
$h(3) = (3.3 + 4) \bmod 7 = 13 \bmod 7$
$= 6$; insert at 6^{th} location
$h(8) = (3.8 + 4) \bmod 7 = 28 \bmod 7$
$= 0$; 0^{th} position is already filled by element 3 so insert 8 at next free location which is 1^{st} position
$h(10) = (3.10 + 4) \bmod 7 = 34 \bmod 7 = 6$ but 6^{th} position is already filled with the element 3 so insert 10 at next free location which is 2^{nd} position

1	8	10	–	–	–	3
0	1	2	3	4	5	6

7.8 (b)

43, 36, 92, 87, 11, 4, 71, 13, 14
$h(k) = k \bmod 11$

87	11	13	36	92	4	71	14			43
0	1	2	3	4	5	6	7	8	9	10

∴ At index 7, last record 14 is inserted.

7.9 (c)

12 mod 10 = 2
18 mod 10 = 8
13 mod 10 = 3
2 mod 10 = 2 collision
(2 + 1) mod 10 = 3 again collision (using linear probing)
(3 + 1) mod 10 = 4
3 mod 10 = 3 collision
(3 + 1) mod 10 = 4 collision (using linear probing)
(4 + 1) mod 10 = 5
23 mod 10 = 3 collision
(3 + 1) mod 10 = 4 collision
(4 + 1) mod 10 = 5 again collision
(5 + 1) mod 10 = 6
5 mod 10 = 5 collision

(5 + 1) mod 10 = 6 again collision
(6 + 1) mod 10 = 7
15 mod 10 = 5 collision
(5 + 1) mod 10 = 6 collision
(6 + 1) mod 10 = 7 collision
(7 + 1) mod 10 = 8 collision
(8 + 1) mod 10 = 9

So resulting hosh table.

0	
1	
2	12
3	13
4	2
5	3
6	23
7	5
8	18
9	15

7.10 (c)

In this method we simply checking the options by using linear probing

Option (a): 46, 42, 34, 52, 23, 33

0	
1	
2	42
3	52
4	34
5	23
6	46
7	33
8	
9	

According to linear probing this is hash table

Option (b): 34, 42, 23, 52, 33, 46

0	
1	
2	42
3	23
4	34
5	52
6	33
7	46
8	
9	

Hash table

Option (c): 46, 34, 42, 23, 52, 33

0	
1	
2	42
3	23
4	34
5	52
6	46
7	33
8	
9	

Hash table

Therefore option (c) is correct.

7.11 (c)

Different insertion sequences of key values using same hash function and linear probing.

46, 34, 42, 23, 52, 33

No problem to enter 46, 34, 42, 23

0	
1	
2	42
3	23
4	34
5	
6	46
7	
8	
9	

$\xrightarrow{52}$

0	
1	
2	42
3	23
4	34
5	52
6	46
7	
8	
9	

To enter 52 there are 6 possible options. After enter 52 we get hash table. Now to enter 23 there are 5 possible options so total

$= 6 \times 5 = 30$ possible options

7.12 (a)

0	1	2	3	4	5	6	7	8
	28 ↓ 19 ↓ 10	20	12		5	15 ↓ 33		17

Maximum length = 3
Minimum length = 0

$$\text{Avg length} = \frac{0+3+1+1+0+1+2+0+1}{9} = 1$$

7.13 (c)

P (First 3 slots are unfilled after the first 3 insertions) $= \frac{97}{100} \times \frac{96}{100} \times \frac{95}{100}$ becaues of uniform hashing.

7.14 (b)

$h(i) = i^3 \bmod 10$

M	M	M	M	M	M	M	M	M	M
30	31	38	37	34	35	36	33	32	39
20	21	28	27	24	25	26	23	22	29
10	11	18	17	14	15	16	13	12	19
0	1	8	7	4	5	6	3	2	9
0	1	2	3	4	5	6	7	8	9

$h(0) = 0^3 \bmod 10 = \mathbf{0} = h(10) = h(20) = h(30) = \ldots$

$h(1) = 1^3 \bmod 10 = \mathbf{1} = h(11) = h(21) = h(31) = \ldots$

$h(2) = 2^3 \bmod 10 = \mathbf{8} = h(12) = h(22) = h(32) = \ldots$

$h(3) = 3^3 \bmod 10 = \mathbf{7} = h(13) = h(23) = h(33) = \ldots$

$h(4) = 4^3 \bmod 10 = \mathbf{4} = h(14) = h(24) = h(34) = \ldots$

$h(5) = 5^3 \bmod 10 = \mathbf{5} = h(15) = h(25) = h(35) = \ldots$

$h(6) = 6^3 \bmod 10 = \mathbf{6} = h(16) = h(26) = h(36) = \ldots$

$h(7) = 7^3 \bmod 10 = \mathbf{3} = h(17) = h(27) = h(37) = \ldots$

$h(8) = 8^3 \bmod 10 = \mathbf{2} = h(18) = h(28) = h(38) = \ldots$

$h(9) = 9^3 \bmod 10 = \mathbf{9} = h(19) = h(29) = h(39) = \ldots$

7.15 Sol.

$$\text{Load factor } \alpha = \frac{\text{Number of elements}}{\text{Number of slots}} = \frac{2000}{25} = 80$$

Unit ▪ VI

Algorithms

▪ Contents

UNIT VI

Algorithms

Syllabus : Analysis, Asymptotic notation, Notions of space and time complexity, Worst and average case analysis; Design: Greedy approach, Dynamic programming, Divide-and-conquer; Tree and graph traversals, Connected components, Spanning trees, Shortest paths; Hashing, Sorting, Searching. Asymptotic analysis (best, worst, average cases) of time and space, upper and lower bounds, Basic concepts of complexity classes – P, NP, NP-hard, NP-complete.

Analysis of Previous GATE Papers

Exam Year	1 Mark Ques.	2 Marks Ques.	3 Marks Ques.	5 Marks Ques.	Total Marks
1990	–	1		–	**2**
1991	–	4		–	**8**
1992	–	3		–	**6**
1993	–	1		1	**7**
1994	–	3		2	**16**
1995	3	–		1	**8**
1996	1	5		2	**21**
1997	2	1		2	**14**
1998	2	–		–	**2**
1999	4	1		1	**9**
2000	1	1		–	**3**
2001	3	1		–	**5**
2002	2	5		–	**12**
2003	3	5		–	**13**
2004	1	7		–	**15**
2005	2	7		–	**16**
2006	7	5		–	**17**

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2007	2	8	**18**
2008	–	11	**22**
2009	3	5	**13**
2010	1	3	**7**
2011	1	3	**7**
2012	3	3	**9**
2013	4	3	**10**
2014 Set-1	2	2	**6**
2014 Set-2	2	2	**6**
2014 Set-3	1	3	**5**
2015 Set-1	3	3	**9**
2015 Set-2	3	3	**9**
2015 Set-3	2	3	**8**
2016 Set-1	2	3	**8**
2016 Set-2	2	2	**6**

1 Algorithm Analysis and Asymptotic Notations

1.1 Solve the recurrence equations
$T(n) = T(n-1) + n$
$T(1) = 1$

[1987 : 2 Marks]

1.2 What is the generating function G(z) for the sequence of Fibonacci numbers?

[1987 : 2 Marks]

1.3 $\sum_{1 \le k \le n} O(n)$, where O(n) stands for order n is

(a) O(n) (b) $O(n^2)$
(c) $O(m^3)$ (d) $O(3n^2)$

[1993 : 2 Marks]

1.4 Consider the following two functions:

$$g_1(n) = \begin{cases} n^3 \text{ for } 0 \le n \le 10{,}000 \\ n^2 \text{ for } n \ge 10{,}000 \end{cases}$$

$$g_2(n) = \begin{cases} n \text{ for } 0 \le n \le 100 \\ n^3 \text{ for } n > 100 \end{cases}$$

Which of the following is true?
(a) $g_1(n)$ is $O(g_2(n))$ (b) $g_1(n)$ is $O(n^3)$
(c) $g_2(n)$ is $O(g_1(n))$ (d) $g_2(n)$ is $O(n)$

[1994 : 2 Marks]

1.5 Which of the following is false?

(a) $100n \log n = O\left(\frac{n \log n}{100}\right)$

(b) $\sqrt{\log n} = O(\log \log n)$

(c) If $0 < x < y$ then $n^x = O(n^y)$

(d) $2n \neq O(nk)$

[1996 : 1 Mark]

1.6 The concatenation of two lists is to be performed in O(1) time. Which of the following implementations of a list should be used?
(a) Singly linked list
(b) Doubly linked list
(c) Circular doubly linked list
(d) Array implementation of list

[1997 : 1 Mark]

1.7 Let $f(n) = n^2 \log n$ and $g(n) = n(\log n)^{10}$ be two positive functions of n. Which of the following statements is correct?
(a) $f(n) = O(g(n))$ and $g(n) \neq O(f(n))$
(b) $g(n) = O(f(n))$ and $f(n) \neq O(g(n))$
(c) $f(n) \neq O(g(n))$ and $g(n) \neq O(f(n))$
(d) $f(n)=O(g(n))$ and $g(n) = O(f(n))$

[2001 : 1 Mark]

1.8 In the worst case, the number of comparisons needed to search singly linked list of length n for a given element is
(a) $\log_2 n$ (b) n/2
(c) $\log_2 n - 1$ (d) n

[2002 : 1 Mark]

1.9 Consider the following functions

$$f(n) = 3n^{\sqrt{n}}$$
$$g(n) = 2^{\sqrt{n}\log_2 n}$$
$$h(n) = n!$$

Which of the following is true?
(a) h(n) is O(f(n)) (b) h(n) is O(g(n))
(c) g(n) is not O(f(n)) (d) f(n) is O(g(n))

[2002 : 2 Marks]

1.10 Consider the following algorithm for searching for a given number x in an unsorted array A[l ..... n] having n distinct values:
1. Choose an i uniformly at random from l...n;
2. If A[i] = x then Stop else Goto 1;

Assuming that x is present on A, what is the expected number of comparisons made by the algorithm before it terminates?
(a) n (b) n – 1
(c) 2n (d) n/2

[2002 : 2 Marks]

1.11 The running time of the following algorithm
Procedure A(n)
If n < = 2 return (1) else return $\left(A\left(\lceil \sqrt{n} \rceil\right)\right)$;
is best described by
(a) O(n) (b) O(log n)
(c) O(log log n) (d) O(1)

[2002 : 2 Marks]

1.12 Consider the following three claims

1. $(n + k)^m = \Theta(n^m)$, where k and m are constants
2. $2^{n+1} = O(2^n)$
3. $2^{2n+1} = O(2^n)$

Which of these claims are correct?

(a) 1 and 2 (b) 1 and 3
(c) 2 and 3 (d) 1, 2 and 3

[2003 : 1 Mark]

1.13 Consider the following C function.

```
float f (float x, int y)
{
  float p, s; int i;
  for (s = 1, p = 1, i = 1; i < y; i ++)
  {
    p* = x/i;
    s + = p;
  }
  return s;
}
```

For large values of y, the return value of the function f best approximates

(a) x^y (b) e^x
(c) $ln(1 + x)$ (d) x^x

[2003 : 1 Mark]

1.14 The tightest lower bound on the number of comparisons, in the worst case, for comparison-based sorting is of the order of

(a) n (b) n^2
(c) nlogn (d) $n\log^2 n$

[2004 : 1 Mark]

1.15 What does the following algorithm approximate? (Assume $m > 1, \varepsilon > 0$).

```
x = m;
y = 1;
while (x – y > ε)
{
  x = (x+y)/2;
  y = m/x;
}
  print (x);
```

(a) log m (b) m^2
(c) $m^{1/2}$ (d) $m^{1/3}$

[2004 : 2 Marks]

1.16 Let A [1,..., n] be an array storing a bit (1 or 0) at each location, and f(m) is a function whose time complexity is $\Theta(m)$. Consider the following program fragment written in a C like language:

```
counter = 0;
for(i = 1; i < n; i++)
{
    if (A[i] = = 1) counter ++;
    else
    {
      f (counter);
      counter = 0;
    }
}
```

The complexity of this program fragment is

(a) $\Omega(n^2)$ (b) $\Omega(n \log n)$ and $O(n^2)$
(c) $\Theta(n)$ (d) $O(n \log n)$

[2004 : 2 Marks]

1.17 The time complexity of the following C function is (assume n > 0)

```
int recursive (int n)
{
  if (n = = 1)
    return (1);
  else
    return(recursive(n–1)+ recursive(n–1));
}
```

(a) O(n) (b) O (n log n)
(c) $O(n^2)$ (d) $O(2^n)$

[2004 : 2 Marks]

1.18 The recurrence equation

$T(1) = 1$

$T(n) = 2T(n-1) + n, n \geq 2$

evaluates to

(a) $2^{n+1} - n - 2$ (b) $2^n - n$
(c) $2^{n+1} - 2n - 2$ (d) $2^n + n$

[2004 : 2 Marks]

1.19 Let f(n), g(n) and h(n) be functions defined for positive integers such that f(n) = O(g(n), g(n)) ≠ O(f(n)), g(n) = O(h(n)), and h(n) = O(g(n)). Which one of the following statements is FALSE?

(a) $f(n) + g(n) = O(h(n)) + h(n))$
(b) $f(n) = O(h(n))$
(c) $h(n) \neq O(f(n))$
(d) $f(n)h(n) \neq O(g(n)h(n))$

[Year: 2004] 2 Marks

1.20 The time complexity of computing the transitive closure of a binary relation on a set of n elements is known to be

(a) O(n) (b) O(n log n)
(c) $O(n^{3/2})$ (d) $O(n^3)$

[2005 : 1 Mark]

1.21 Let T(n) be a function defined by the recurrence $T(n) = 2T(n/2) + \sqrt{n}$ for $n \geq 2$ and $T(1) = 1$. Which of the following statements is TRUE?
(a) $T(n) = \Theta(\log n)$ (b) $T(n) = \Theta(\sqrt{n})$
(c) $T(n) = \Theta(n)$ (d) $T(n) = \Theta(n \log n)$
[2005 : 2 Marks]

1.22 Suppose $T(n) = 2T(n/2) + n$, $T(0) = T(1) = 1$
Which one of the following is FALSE?
(a) $T(n) = O(n^2)$ (b) $T(n) = \Theta(n \log n)$
(c) $T(n) = \Omega(n^2)$ (d) $T(n) = O(n \log n)$
[2005 : 2 Marks]

Common Data for Q. 1.23 & Q. 1.24
Consider the following C function:

```
double foo(int n)
{
  int i;
  double sum;
  if (n = = 0) return 1.0;
  else
  {
    sum = 0.0;
    for (i = 0; i < n ; i ++)
    sum + = foo(i);
    return sum;
  }
}
```

1.23 The space complexity of the above function is
(a) O(1) (b) O(n)
(c) O(n!) (d) $O(n^n)$
[2005 : 2 Marks]

1.24 The space complexity of the above function is foo( ) and store the values of foo(i), 0 < = i < n, as and when they are computed. With this modification, the time complexity for function foo( ) is significantly reduced. The space complexity of the modified function would be
(a) O(1) (b) O(n)
(c) $O(n^2)$ (d) O(n!)
[2005 : 2 Marks]

1.25 Consider the following C-program fragment in which i, j, and n are integer variables.
for (i = n, j = 0; i > 0; i / = 2, j + = i);
Let Val (j) = denote the value stored in the variable j after termination of the for loop. Which one of the following is true?
(a) val (j) = Θ (log n) (b) val (j) = Θ ($\sqrt{n}$)
(c) val (j) = Θ (n) (d) val (j) = Θ (n log n)
[2006 : 1 Mark]

1.26 Consider the following is true?

$$T(n) = 2T\left(\left\lceil \sqrt{n} \right\rceil\right) + 1, T(1) = 1$$

Which one of the following is true?
(a) $T(n) = \Theta(\log \log n)$ (b) $T(n) = \Theta(\log n)$
(c) $T(n) = \Theta(\sqrt{n})$ (d) $T(n) = \Theta(n)$
[2006 : 2 Marks]

1.27 Consider the following segment of C code

```
int j, n;
j = 1;
while (j < = n)
j = j* 2;
```

The number of comparisons made in the execution of the loop for any n > 0 is
(a) $\lceil \log_2 n \rceil + 1$ (b) n
(c) $\lceil \log_2 n \rceil$ (d) $\lfloor \log_2 n \rfloor + 1$
[2007 : 1 Mark]

1.28 What is the time complexity of the following recursive function :

```
int DoSomething ( int n)
{
  if (n < = 2)
    return 1;
  else
    return DoSomething (floor (sqrt (n)))+n;
}
```

(a) $\Theta(n^2)$ (b) $\Theta(n \log_2 n)$
(c) $\Theta(\log_2 n)$ (d) $\Theta(\log_2 \log_2 n)$
[2007 : 2 Marks]

1.29 An array of n numbers is given, where n is an even number. The maximum as well as the minimum of these n numbers needs to be determined. Which of the following is true about the number of comparisons needed?
(a) At least 2n – c comparisons, for some constant c, are needed.
(b) At most 1.5n – 2 comparisons are needed.
(c) At least $n \log_2 n$ comparisons are needed.
(d) None of the above
[2007 : 2 Marks]

1.30 Consider the following C code segment :

```
int IsPrime(n)
{
  int i, n;
  for(i = 2; i < = sqrt(n); i ++)
  {
    if (n % i = = 0)
      printf ("Not Prime\n");
    return 0;
  }
  return 1;
}
```

Let T(n) denote the number of times the for loop is executed by the program on input n. Which of the following is TRUE?

(a) $T(n) = O(\sqrt{n})$ and $T(n) = \Omega(\sqrt{n})$

(b) $T(n) = O(\sqrt{n})$ and $T(n) = \Omega(1)$

(c) $T(n) = O(n)$ and $T(n) = \Omega(\sqrt{n})$

(d) None of these

[2007 : 2 Marks]

1.31 Arrange the following functions in increasing asymptotic order:

A. $n^{1/3}$ **B.** e^n
C. $n^{7/4}$ **D.** $n \log^9 n$
E. 1.0000001^n

(a) A, D, C, E, B (b) D, A, C, E, B
(c) A, C, D, E, B (d) A, C, D, B, E

[2008 : 1 Mark]

1.32 When $n = 2^{2k}$ for some $k \geq 0$, the recurrence relation $T(n) = \sqrt{2}\ T(n/2) + \sqrt{n}$, $T(1) = 1$ evaluates to :

(a) $\sqrt{n}$ (log n+1) (b) $\sqrt{n} \log n$
(c) $\sqrt{n} \log \sqrt{n}$ (d) $n \log \sqrt{n}$

[2008 : 2 Marks]

1.33 Consider the following functions:

$f(n) = 2^n$
$g(n) = n!$
$h(n) = n^{\log n}$

which of the following statements about the asymptotic behaviour of f (n), g(n), and h (n) is true?

(a) $f(n) = O(g(n))$; $g(n) = O(h(n))$
(b) $f(n) = \Omega(g(n))$; $g(n) = O(h(n))$
(c) $g(n) = O(f(n))$; $h(n) = O(f(n))$
(d) $h(n) = O(f(n))$; $g(n) = \Omega(f(n))$

[2008 : 2 Marks]

1.34 The minimum number of comparison required to determine if an integer appears more than n/2 times in a sorted array of n integers is

(a) $\Theta(n)$ (b) $\Theta(\log n)$
(c) $\Theta(\log^2 n)$ (d) $\Theta(1)$

[2008 : 2 Marks]

Common Data Questions Q.1.35 and Q.1.36

Consider the following C functions:

```
int f1(int n)
{
  if(n = = 0 || n = = 1) return n;
  else
    return (2* f1( n -1) + 3 * f1(n - 2));
}
int f2( int n)
{
  int i ;
  int X[N] , Y[N] , Z[N] ;
  X[1] = 1; Y[1] = 2; Z[1] = 3;
  for( i = 2; i < = n ; i ++ )
  {
    X[ i ] = Y[ i - 1] + Z[ i - 2] ;
    Y[ i ] = 2 * X[ i ] ;
    Z[ i ] = 3 * X[ i ];
  }
return X[n] ;
}
```

1.35 The running time of f1(n) and f2(n) are

(a) $\Theta(n)$ and $\Theta(n)$ (b) $\Theta(2^n)$ and O (n)
(c) $\Theta(n)$ and $\Theta(2^n)$ (d) $\Theta(2^n)$ and $\Theta(2^n)$

[2008 : 2 Marks]

1.36 f1(8) and f2(8) return the values

(a) 1661 and 1640 (b) 59 and 59
(c) 1640 and 1640 (d) 1640 and 1661

[2008 : 2 Marks]

1.37 What is the number of swaps required to sort n elements using selection sort, in the worst case?

(a) $\Theta(n)$ (b) $\Theta(n \log n)$
(c) $\Theta(n^2)$ (d) $\Theta(n^2 \log n)$

[2009 : 1 Mark]

1.38 The running time of an algorithm is represented by the following recurrence relation:

$$T(n) = \begin{cases} n & n \leq 3 \\ T\left(\frac{n}{3}\right) + cn & \text{otherwise} \end{cases}$$

Which one of the following represents the time complexity of the algorithm?

(a) $\Theta(n)$
(b) $\Theta(n \log n)$
(c) $\Theta(n^2)$
(d) $\Theta(n^2 \log n)$

[2009 : 2 Marks]

1.39 Let W(n) and A(n) denote respectively, the worst case and average case running time of an algorithm executed on an input of size n. Which of the following is ALWAYS TRUE?

(a) $A(n) = \Omega(W(n))$
(b) $A(n) = \Theta(W(n))$
(c) $A(n) = O(W(n))$
(d) $A(n) = o(W(n))$

[2012 : 1 Mark]

1.40 The recurrence relation capturing the optimal execution time of the Towers of Hanoi problem with n discs is

(a) $T(n) = 2T(n-2) + 2$
(b) $T(n) = 2T(n-1) + n$
(c) $T(n) = 2T(n/2) + 1$
(d) $T(n) = 2T(n-1) + 1$

[2012 : 1 Mark]

1.41 Consider the following function:

```
int unknown (int n)
{
  int i, j, k = 0;
  for (i = n/2; i <= n; i++)
    for (j = 2; j <= n; j = j*2)
      k = k + n/2;
  return (k);
}
```

The return value of the function is

(a) $\Theta(n^2)$
(b) $\Theta(n^2 \log n)$
(c) $\Theta(n^3)$
(d) $\Theta(n^3 \log n)$

[2013 : 2 Marks]

1.42 Which one of the following correctly determines the solution of the recurrence relation with $T(1) = 1$?

$$T(n) = 2T\left(\frac{n}{2}\right) + \log n$$

(a) $\Theta(n)$
(b) $\Theta(n \log n)$
(c) $\Theta(n^2)$
(d) $\Theta(\log n)$

[2014 (Set-2) : 1 Mark]

1.43 Suppose we have a balanced binary search tree T holding n-numbers. We are given two numbers L and H and wish to sum up all the numbers in T that lie between L and H. Suppose there are m such numbers in T.
If the tightest upper bound on the time to compute the sum is $O(n^a \log^b n + m^c \log^d n)$, the value of $a + 10b + 100c + 1000d$ is ____.

[2014 (Set-3) : 2 Marks]

1.44 Consider the following C function.

```
int fun1 (int n)
{   int i, j, k, p, q = 0;
    for (i = 1; i<n; ++i)
    {   p = 0;
        for (j=n; j>1; j=j/2)
        ++p;
        for (k=1; k<p; k=k*2)
        ++q;
    } return q;
}
```

Which one of the following most closely approximates the return value of the function fun1?

(a) n^3
(b) $n(\log n)^2$
(c) $n \log n$
(d) $n \log (\log n)$

[2015 (Set-1) : 2 Marks]

1.45 An algorithm performs $(\log N)^{1/2}$ find operations, N insert operations, $(\log N)^{1/2}$ delete operations, and $(\log N)^{1/2}$ decrease-key operations on a set of data items with keys drawn from a linearly ordered set. For a delete operation, a pointer is provided to the record that must be deleted. For the decrease-key operation, a pointer is provided to the record that has its key decreased. Which one of the following data structures is the most suited for the algorithm to use, if the goal is to achieve the best total asymptotic complexity considering all the operations?

(a) Unsorted array
(b) Min-heap
(c) Sorted array
(d) Sorted doubly linked list

[2015 (Set-1) : 2 Marks]

1.46 Consider a complete binary tree where the left and the right subtrees of the root are max-heaps. The lower bound for the number of operations to convert the tree to a heap is

(a) $\Omega(\log n)$
(b) $\Omega(n)$
(c) $\Omega(n \log n)$
(d) $\Omega(n^2)$

[2015 (Set-2) : 1 Mark]

1.47 An unordered list contains n distinct elements. The number of comparisons to find an element in this list that is neither maximum nor minimum is

(a) $\Theta(n \log n)$ (b) $\Theta(n)$
(c) $\Theta(\log n)$ (d) $\Theta(1)$

[2015 (Set-2) : 1 Mark]

1.48 Consider the equality $\sum_{i=0}^{n} i^3 = X$ and the following choices for X

I. $\Theta(n^4)$ II. $\Theta(n^5)$
III. $O(n^5)$ IV. $\Omega(n^3)$

The equality above remains correct if X is replace by

(a) Only I
(b) Only II
(c) I or III or IV but not II
(d) II or III or IV but not I

[2015 (Set-3) : 1 Mark]

1.49 Let $f(n) = n$ and $g(n) = n^{(1+\sin n)}$, where n is a positive integer. Which of the following statements is/are correct?

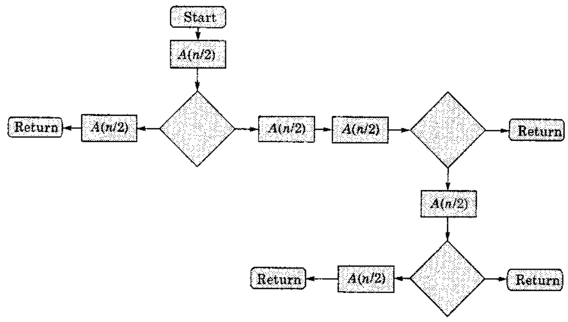
I. $f(n) = O(g(n))$ **II.** $f(n) = \Omega(g(n))$

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

[2015 (Set-3) : 2 Marks]

1.50 The given diagram shows the flow chart for a recursive function $A(n)$. Assume that all statements, except for the recursive calls, have $O(1)$ time complexity. If the words case time complexity of this function is $O(n^{\alpha})$, then the least possible value (accurate up to two decimal positions) of α is ________.

Flow chart for Recursive Function $A(n)$



[2016 (Set-2) : 2 Marks]

1.51 N items are stored in a sorted doubly linked list. For a *delete* operation, apointer is provided to the record to be deleted. For a *decrease-key* operation, a pointer is provided to the record on which the operation is to be performed. An algorithm performs the following operations on the list in this order: $\Theta(N)$ *delete*, $O(\log N)$ insert, $O(\log N)$ *find*, and $\Theta(N)$ *decrease-key*. What is the time complexity of all these operations put together?

(a) $O(\log^2 N)$ (b) $O(N)$
(c) $O(N^2)$ (d) $\Theta(N^2 \log N)$

[2016 (Set-2) : 1 Mark]

■■■■

Answers Algorithm Analysis and Asymptotic Notations

1.3	(b)	1.4	(a)	1.5	(b, d)	1.6	(c)	1.7	(b)	1.8	(d)	1.9	(d)	1.10	(a)	1.11	(c)
1.12	(a)	1.13	(b)	1.14	(c)	1.15	(c)	1.16	(c)	1.17	(d)	1.18	(a)	1.19	(d)	1.20	(d)
1.21	(c)	1.22	(c)	1.23	(b)	1.24	(b)	1.25	(c)	1.26	(b)	1.27	(d)	1.28	(d)	1.29	(b)
1.30	(b)	1.31	(a)	1.32	(a)	1.33	(d)	1.34	(b)	1.35	(b)	1.36	(c)	1.37	(a)	1.38	(a)
1.39	(c)	1.40	(d)	1.41	(b)	1.42	(a)	1.44	(d)	1.45	(a)	1.46	(a)	1.47	(d)	1.48	(c)
1.49	(d)	1.51	(c)														

Explanations Algorithm Analysis and Asymptotic Notations

1.1 Sol.

By using substitution method we get following series:

$n + (n-1) + (n-2) + (n-3) \ldots 3 + 2 + 1$

Which is sum of 'n' natural numbers.

$$= \frac{n(n+1)}{2} \Rightarrow O(n^2)$$

1.2 Sol.

The generating function for the Fibonacci numbers G(z) is

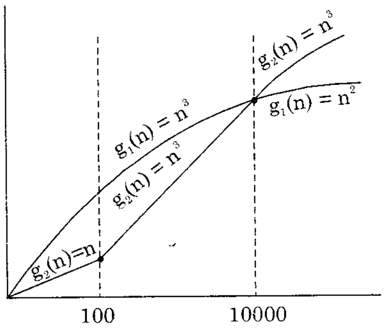
$$G(z) = \frac{z}{1 - z - z^2}$$

1.3 (b)

$$\sum_{1\le k\le n} O(n) = O(1) + O(2) + O(3) + \dots + O(n)$$

$$= O\left(\frac{n(n+1)}{2}\right) = O(n^2)$$

1.4 (a)



Therefore;

$n^2 \le n^3$ for $N \ge 10000$

$g_1(n) = O(g_2(n))$

Option (a) is correct.

1.5 (b,d)

(a) We know that

$f(n) = O(g(n))$ i.e.,

$f(n) \le k.g(n)$

for k, some positive integer and $n > n_0$

$$100\ n \log n \le 10000 \times \frac{n \log n}{100}$$

for k = 10000

$$\therefore\ 100\ n \log n = O\left(\frac{n \log n}{100}\right)$$

(b) $\sqrt{\log n} \le 1 * \log \log n$

$\therefore\ \sqrt{\log n} \ne O(\log \log n)$

(c) $n^x \le n^y$ as $0 < x < y$

$\therefore\ n^x = O(n^y)$

(d) $2n \le kn$ for $k \ge 2$

$\therefore\ 2n = O(nk)$

1.6 (c)

As list concatenation requires traversing at least one list to the end. So singly linked list and doubly linked list requires O(n) time complexity whereas circular doubly linked list required O(1) time.

1.7 (b)

$$f(n) = n^2 \log n$$
$$g(n) = n(\log n)^{10}$$
$$n(\log n)^{10} \le n^2 \log n$$
$$\therefore\quad g(n) = O(f(n))$$

Whereas

$$f(n) \ne O(g(n)) \text{ because}$$
$$n^2 \log n \not\le n(\log n)^{10}$$

1.8 (d)

Worst case of searching singly linked list is when given element doesn't present at all in the singly linked list. Using linear search then require "n" comparisons in worst case.

1.9 (d)

$$f(n) = 3n^{\sqrt{n}},\ g(n) = 2^{\sqrt{n}\log_2 n}$$

$$\Rightarrow\ g(n) = n^{\sqrt{n}\log_2 2}\left[a^{\log b} = b^{\log a}\right]$$

$$\Rightarrow\ n^{\sqrt{n}}$$

1.10 (a)

Let expected number of comparisons be E. Value of E is sum of following expression for all the possible cases:

Case-I If A[i] is found in the first attempt,
Number of comparisons = 1
Probability = 1/n.

Case-II If A[i] if found in the second attempt,
Number of comparisons = 2

$$\text{Probability} = \frac{(n-1)}{n} * \frac{1}{n}$$

Case-III If A[i] is found in the third attempt,
Number of comparisons = 3

$$\text{Probability} = \frac{(n-1)}{n} * \frac{(n-1)}{n} * \frac{1}{n}$$

There are actually infinite such cases. So, we have following infinite series for E.

$$E = \frac{1}{n} + \frac{n-1}{n} * \frac{1}{n} * 2 + \frac{(n-1)}{n} * \frac{(n-1)}{n} * \frac{1}{n} * 3 + \dots \text{ (i)}$$

After multiplying equation (i) with $\frac{(n-1)}{n}$, we get

$$E\frac{(n-1)}{n} = \frac{n-1}{n} * \frac{1}{n} + \frac{n-1}{n} * \frac{1}{n} * 2 + \dots \text{ (ii)}$$

Subtracting (ii) from (i), we get

$$\frac{E}{n} = \frac{1}{n} + \frac{n-1}{n} * \frac{1}{n} + \frac{n-1}{n} * \frac{n-1}{n} * \frac{1}{n} + \ldots$$

The expression on right side is a G.P with infinite elements.

So apply the sum formula $\left(\frac{a}{1-r}\right)$

$$\frac{E}{n} = \left(\frac{1}{n}\right) \bigg/ \left(\frac{n-(n-1)}{n}\right) = 1$$

$E = n$

Therefore, option (a) is correct.

1.11 (c)

Recursive relation for procedure A(n) is

$$T(n) = T(\sqrt{n}) + c1 \quad \text{if } n > 2$$

Let $n = 2^m$.

$\Rightarrow$ $T(n) = T(2^m)$

$\Rightarrow$ $T(2^m) = S(m)$

$\Rightarrow$ $T(n) = S(m)$

$\Rightarrow$ $T(\sqrt{n}) = T(2^{m/2}) = S(m/2)$

$$T(n) = T(\sqrt{n}) + c1 \quad \text{if } n > 2$$

$\therefore$ $S(m) = S\left(\frac{m}{2}\right) + c1 = O(\log m)$

$= O(\log \log n) \quad [\because n = 2^m \Rightarrow m = \log_2 n]$

$\Rightarrow$ $T(n) = S(m) = O(\log \log n)$

$\therefore$ Option (c) is correct.

1.12 (a)

Consider each statement separately

I. $f(n) = (n + k)^m$

so, $f(n) = (1 + n)^m$

(Assume k = 1 is constant)

$f(n) = 1 + {}^mC_1 n + {}^mC_2 n^2 + \ldots {}^mC_m n^m$

$f(n) = O(n^m)$

II. $f(n) = 2^{n+1}$

$f(n) = 2^n \cdot 2^1$

$f(n) = 2 \cdot 2^n$

$f(n) = O(2^n)$

III. $f(n) = 2^{2n+1}$

$f(n) = 2^{2n} \cdot 2^1$

$f(n) = 2 \cdot 2^{2n}$

$f(n) = O(2^{2n})$

Therefore I and II are correct.

1.13 (b)

The given function f is not recursive, so consider the following iteration method.

i	p	s
	$p = p * \frac{x}{i}$	$s = s + p$
Initilize 1	1	1
1	$p = x$	$s = 1 + x$
2	$p = x \cdot \frac{x}{2}$	$s = 1 + x + \frac{x^2}{2}$
3	$p = \frac{x^2}{2} \cdot \frac{x}{3}$	$s = 1 + x + \frac{x^2}{2} + \frac{x^3}{6}$
4	$p = \frac{x^3}{6} \cdot \frac{x}{4}$	$s = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24}$
5	$p = \frac{x^4}{24} \cdot \frac{x}{5}$	$s = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120}$

For large value of y assume $y \to \infty$ then i also tends to infinite it means increment of for loop may tends to infinite. In the given function we choose y as a large integer but not infinite. The return value of the function f is s.

$$s = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} \ldots + \infty$$

$$s = 1 + x + \frac{x^2}{!2} + \frac{x^3}{!3} + \frac{x^4}{!4} + \frac{x^5}{!5} + \ldots + \infty$$

$s = e^x$

1.14 (c)

Any decision tree that sorts n distinct elements has height at least $\log \lfloor\underline{n}$. So the tightest lower bound on the number of comparison based sorting is $\log \lfloor\underline{n}$ but from starling's approximation.

$\lfloor\underline{n} = (n/e)^n$

Taking log both sides

$\log \lfloor\underline{n} = \log (n/e)^n$

$\log \lfloor\underline{n} = n\log (n/e)$

$\log \lfloor\underline{n} = n(\log n - \log e)$

$\log \lfloor\underline{n} = n(\log n - 1.44)$

$\log \lfloor\underline{n} = n\log n - 1.44\, n$

So $\log \lfloor\underline{n} = O(n\log n)$

1.15 (c)

Let x = m = 9. The loop will be terminated when x − y = 0 or x − y < 0. Consider the following iteration for x = m = 9, y = 1

x − y > 0 ⇒	x = (x + y)/2,	y = m/x
9 − 1 = 8	x = 5.0,	y = 9/5.0 = 1.8
5.0 − 1.8 = 3.2	x = 3.4,	y = 9/3.4 = 2.6
3.4 − 2.6 = .80	x = 3.0,	y = 9/3.0 = 3.0

x − y = 3.0 − 3.0 = 0, loop terminated

So, m = 9 then x = 3

$x = (m)^{1/2} = (9)^{1/2} \Rightarrow x = 3$

So the algorithm computes $x = m^{1/2}$.

1.16 (c)

The given code is

```
1.      Counter = 0;
2.  for (i = 1; i < = n; i ++)
3.  {   if (A[i] = = 1) counter ++;
4.      else {f(counter); counter = 0;}
5.  }
```

The time complexity of the program fragment depends on the freqµency (Number of steps) of line 3 and 4. In line 4 the frequency depends on the variable counter and there is no increment in the counter variable which is initialize to 0, so f(0) then counter = 0 it means there is no cell in an array which having a bit 0, so all cells in the array contains 1. Consider the line 3 if (A[i] = = 1) counter ++; the value of i will be increases upto n so the value of counter will be n. Since n is the frequency of the line 3 and the frequency of line 4 is 0. So the time complexity of line 3 is O(n) on average n and f(0) = O(1) is the time complexity of line 4. So the time complexity of the prcgram fragment is maximum of line 3 and 4 which is O(n) on average.

1.17 (d)

The given C function is recursive. The best way to find the time complexity of recursive function is that convert the code (algorithm) into recursion equation and solution of the recursion equation is the time complexity of given algorithm.

```
1. int recursive (int n) {
2. if (n = = 1) return (1);
3. else
4. return (recursive(n–1)+recursive (n– 1));
5. }
```

Let recursive(n) = T(n)

According to line 2 if (n = = 1) return(1) then the recursive equation is

$$T(n) = 1, \quad n = 1$$

According to line 4 the recursion equation is

$$T(n) = T(n-1) + T(n-1), \; n > 1$$

So the complete recursion equation is

$$T(n) = 1, \quad n = 1$$
$$T(n) = T(n-1) + T(n-1), \; n > 1$$

or

$$T(n) = 2T(n-1), \; n > 1$$
$$T(1) = 1 = 2^{\circ}$$
$$T(2) = 2T(1) = 2.1 = 2^1$$
$$T(3) = 2T(2) = 2.2 = 2^2$$
$$T(4) = 2T(3) = 2.2^2 = 2^3$$

. . . .

. . . .

. . . .

$$T(n) = 2^{n-1}$$

Or, $T(n) = 2^n . \frac{1}{2}$

So, $T(n) = O(2^n)$

1.18 (a)

$$T(1) = 1$$
$$T(n) = 2T(n-1) + n \quad n \geq 2$$
$$T(2) = 2T(1) + 2 = 2.1 + 2 = 4$$
$$T(3) = 2T(2) + 3 = 2.4 + 3 = 11$$
$$T(4) = 2T(3) + 4 = 2.11 + 4 = 26$$
$$\vdots$$
$$T(n-1) = 2T(n-2) + n = 2^n - (n-1) - 2$$

So $T(n) = 2^{n+1} - n - 2$

1.19 (d)

We can verify as:

f ≤ g BUT g $\not\leq$ f. Therefore f < g

Also g = h as g = O(h) and h = O(g).

Therefore f < g and g = h

(a) f(n) + g(n) = O(h(n)) + h(n)) is true.

⇒ f + g = f + h < h + h

(b) f(n) = O(h(n)) is true.

⇒ f < h

(c) h(n) ≠ O(f(n)) is true.

⇒ h $\not<$ f is correct.

(d) f(n)h(n) ≠ O(g(n)h(n)) is false.

⇒ f · h < g · h implies fh = O(gh)

1.20 (d)

The time complexity of computing the transitive closure of a binary relation on a set of n elements is $O(n^3)$ apply the Warshall's algorithm to compute the transitive closure. The algorithm contains three nested for loops each having frequency n so time complexity is $O(n^3)$.

1.21 (c)

$$T(n) = 2T(n/2) + \sqrt{n}$$

By substitution method $T(n) = \Theta(n)$.

$$T(n) = 2 \cdot T(n/2) + \sqrt{n} \qquad \ldots(1)$$
$$= 2\left[2 \cdot T(n/4) + \sqrt{n/2}\right] + \sqrt{n}$$
$$= (2)^2 \cdot T(n/2^2) + \sqrt{n}(1+\sqrt{2}) \ldots(2)$$
$$= (2)^3 \cdot T(n/2^3) + \sqrt{n}(1+\sqrt{2}+2) \ldots(3)$$
$$\vdots$$
$$= (2)^k \cdot T(n/2^k) + \sqrt{n}\left(\sum_{i=0}^{k-1}\left(\sqrt{2}\right)^i\right)$$
$$= \Theta(n)$$

1.22 (c)

$$T(0) = T(1) = 1$$
$$T(n) = 2T(n/2) + n$$

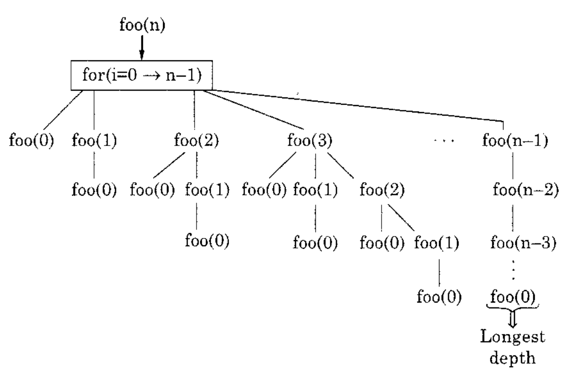
T(n) be computed as follows

$$T(n) = n\sum_{i=0}^{\log n}(2/2)^i$$
$$T(n) = n\sum_{i=0}^{\log n}(1)^i$$
$$T(n) = \Theta(n\log n)$$

[T(n) can also proved by Master Theorem]

If $T(n) = \Theta(n \log n)$ then it is also $O(n \log n)$ and $O(n^2)$ but it is not $\Omega(n^2)$.

1.23 (b)



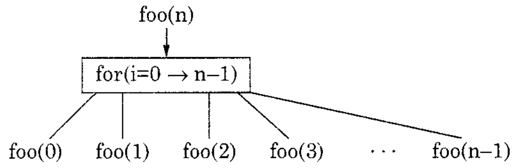
The longest calling sequence:

$$foo(n) \rightarrow foo(n-1) \rightarrow \ldots foo(0)$$

Therefore space complexity = O(n)

[***Note:*** Time complexity = $O(n^n)$]

1.24 (b)



Space required: foo(0), foo(1),...,foo(n−1) = O(n)

Even storing foo(i), 0 <= i < n the space complexity is O(n).

[***Note:*** Time complexity = O(n)]

1.25 (c)

The frequency of the for loop is i = i/2 = n/2 so after termination of for loop vol(j) = O(n) on average.

$$j = n + \frac{n}{2} + \frac{n}{2^2} + \ldots \frac{n}{2^{\log_2 n}} = \Theta(n)$$

1.26 (b)

Let $\quad n = 2^m$

$T(2^m) = T(2(m/2)) + 1$

Let $\quad T(2^m) = S(m)$

Then, $\quad S(m) = 2S\left(\frac{m}{2}\right) + 1$

Using Master's theorem

$S(m) = \Theta(m)$

$= \Theta(\log n)$ since $n = 2^m$

Now, $\quad T(n) = T(2^m) = S(m)$

$= \Theta(\log n)$

Therefore, option (b) is correct.

1.27 (d)

Let the increment of j is $2^0, 2^1, \ldots, 2^i$ for some value of i so according to the question for while loop: $2^i \le n$ **or** $i \le \log_2 n$.

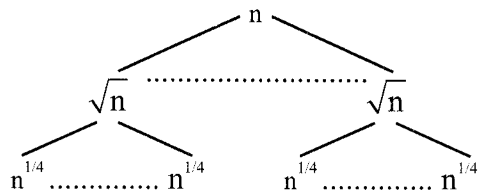
- One extra comparison required for the termination of while loop. So total number of comparisons = $i + 1 = \lfloor \log_2 n \rfloor + 1$

1.28 (d)

The given function is recursive so the equivalent recursion equation is

$$T(n) = 1; \quad n \le 2$$

$$T(n) = \lfloor \sqrt{n} \rfloor + n;\ n > 2$$



All the level sums are equal to n. The problem size at level k of the recursion tree is $n^{2^{-k}}$ and we stop recursing when this value is a constant. Setting $n^{2^{-k}} = 2$ and solving for k gives us

$$2^{-k}\log_2 n = 1$$
$$\Rightarrow \quad 2^k = \log_2 n$$
$$\Rightarrow \quad k = \log_2\log_2 n$$
So $\quad T(n) = \Theta(\log_2\log_2 n)$

1.29 (b)

When we apply the divide and conquer method such that dived n into two parts of n/2 then recurrence equation is

$T(n) = 1$ for $n = 2$

$T(n) = 2T(n/2) + 2$ for $n > 2$

The solution of T(n) is

$$T(n) = \frac{3}{2}n - 2$$
$$= 1.5\,n - 2 \text{ comparison}$$

1.30 (b)

T(n) denote the number of times the for loop is executed by the program on input n.

$$T(n) = \sqrt{n} + 1$$

When $n = 1$ then best case occurs so T(n) takes constant time i.e., $T(n) = \Omega(1)$.

In worst case,

$$T(n) = \sqrt{n} + 1 \text{ so } T(n) = O\left(\sqrt{n}\right).$$

1.31 (a)

A < C and A < D

E < B and C, D < E as E is exponential function.

Now, we just need to see if C or D is larger.

In C, term $n^{3/4}$ and correspondingly in D we have $\log^9 n$.

$n^{3/4}$ is larger than $\log^9 x$.

So, D < C.

1.32 (a)

$$T(n) = \sqrt{2}\cdot T(n/2) + \sqrt{n} \quad \text{...(1)}$$
$$= \sqrt{2}\left[\sqrt{2}\cdot T(n/4) + \sqrt{n/2}\right] + \sqrt{n}$$
$$= \left(\sqrt{2}\right)^2 \cdot T\left(n/2^2\right) + 2\sqrt{n} \quad \text{...(2)}$$
$$= \left(\sqrt{2}\right)^3 \cdot T\left(n/2^3\right) + 3\sqrt{n} \quad \text{...(3)}$$
$$\vdots$$
$$= \left(\sqrt{2}\right)^{2k} \cdot T\left(n/2^{2k}\right) + 2k\sqrt{n}$$
$$= \left(\sqrt{2}\right)^{\log n} \cdot 1 + \log n \cdot \sqrt{n} \quad [\because \text{Put } 2k = \log n]$$
$$= \sqrt{n} + \log n \cdot \sqrt{n}$$
$$= \sqrt{n}(\log n + 1)$$

1.33 (d)

$f(n) = 2^n \Rightarrow f(n) = O(2^n)$

$g(n) = n! \Rightarrow g(n) = O(n!)$

$h(n) = n^{\log n} \Rightarrow h(n) = O(n^{\log n})$

[$n < n^2$ means n grows more slowly than n^2]

The asymptotic order of function is as follows:

$1 < \log\log n < \log n < n^{\varepsilon} < n^c < n^{\log n} < c^n < n^n < c^{c^n} < n!$

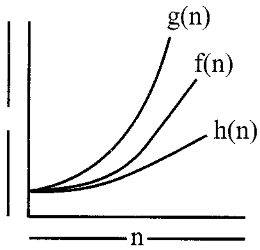
where $0 < \varepsilon < 1 < c < n$

$\Rightarrow \quad n^{\log n} < c^n < n!$

$\Rightarrow \quad n^{\log n} < 2^n < n!$

Assume c = 2

$h(n) < f(n) < g(n)$



$h(n) = O(f(n)) \Rightarrow h(n) \le k.f(n)$ is true

$g(n) = \Omega(f(n)) \Rightarrow g(n) \ge k.f(n)$ is also true

1.34 (b)

Consider an array of n elements which stored integers in sorted order, A[1], A[2], ..., A[n]. If we want to search an element x with the help of binary search then in expected case comparison is not greater than $\lceil \log(n+1) \rceil$. But if integer appears n/2 times in an array and array is already sorted then it take $\Theta(1)$ times for consecutive locations. So total no. of comparisons is not greater than $\Theta(\log n)$ time.

1.35 (b)

The $f_1(n)$ is a recursive function.
The recurrence equation for $f_1(n)$ is

$$T(0) = 0$$
$$T(1) = 1$$
$$T(n) = 2T(n-1) + 3T(n-2)$$

The solution of this linear equation contains a polynomial of 2^n so on average the running time of $f_1(n)$ is $\Theta(2^n)$.

1.36 (c)

$$\begin{aligned}
T(0) &= 0 \\
T(1) &= 1 \\
\mathbf{T(n)} &= \mathbf{2T(n-1) + 3T(n-2)} \\
T(2) &= 2T(1) + 3T(0) = 2.1 + 3.0 = 2 \\
T(3) &= 2T(2) + 3T(1) = 2.2 + 3.1 = 7 \\
T(4) &= 2T(3) + 3T(2) = 2.7 + 3.2 = 20 \\
T(5) &= 2T(4) + 3T(3) = 2.20 + 3.7 = 61 \\
T(6) &= 2T(5) + 3T(4) = 2.61 + 3.20 = 182 \\
T(7) &= 2T(6) + 3T(5) = 2.182 + 3.61 = 547 \\
T(8) &= 2T(7) + 3T(6) = 2.547 + 3.182 = 1640
\end{aligned}$$

So f(8) returns 1640, $f_2(n)$ is the nonrecursive version of $f_1(n)$ so the output of $f_2(8)$ is 1640.

1.37 (a)

In each pass there can be at most one swap. There are n such passes in selection sort i.e. maximum n swap will happen in worst case.

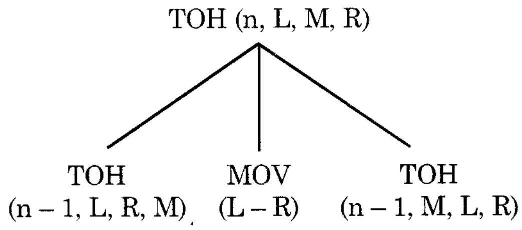
1.38 (a)

Complexity is decided for large values of n only,
So, $T(n) = T(n/3) + cn$ for $n > 3$
using master's theorem
here $a = 1, b = 3, \log_b a = \log_3 1 = 0$
$f(n) = cn = \Theta(n^1)$
since $n^{\log_b a} = n^0$ is below $f(n) = \Theta(n^1)$
this belongs to case III of master's theorem, where the solution is $T(n) = \Theta(f(n)) = \Theta(n)$

1.39 (c)

$A(n) \rightarrow$ Average case complexity
$W(n) \rightarrow$ Worst case complexity
We know
Best case $\leq$ Average case complexity $\leq$ Worst case complexity
and always $A(n) \leq k.W(n) \Rightarrow A(n) = O(W(n))$

1.40 (d)



$\Rightarrow T(n) = 2T(n-1) + 1$

1.41 (b)

Outer loop execute for $\frac{n}{2}+1$ iterations. Inner loop executes for $\log_2 n$ iterations. In every iteration of inner loop $\frac{n}{2}$ is added to k.

Return value $= \frac{n}{2} \times$ number of outer loops $\times$ number of inner loops

$$= \frac{n}{2} \times \left(\frac{n}{2}+1\right)(\log n)$$
$$= O(n^2 \log n)$$

1.42 (a)

$T(n) = 2.T(n/2) + \log n$
[by master's theorem case1]
$= \Theta(n)$

1.43 Sol.

$O(m + \log n) = O(n^0 \log^1 n + m^1 \log^0 n)$
$\Rightarrow a = 0, b = 1, c = 1, d = 0$
$\Rightarrow a + 10b + 100c + 1000d$
$= 10 + 100 = 110$

1.44 (d)

```
int fun1(int n)
{
    int i, j, k, p, q = 20
    for (i = 1; i < n; ++i)
    {
```

P = 0;

for (j=n; j<1; j=j/2) ⇒ log n times for each i ⇒ Total = n × logn

++p;

for (k=1; k<p; k=k×2) ⇒ log log n times for each i ⇒ Total = n log n

++q;

}

return q; ⇒ Return value = O(n log log n)

}

Time complexity of above program = O(n log n)

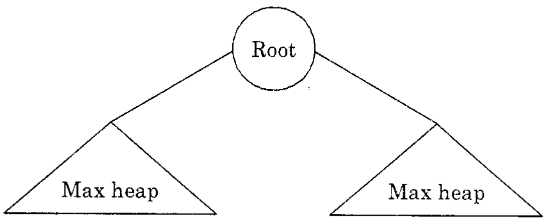
Return value q = O(n log log n)

1.45 (a)

	Find	Insert	Delete	Decrease key	Overall Asymptotic time complement
Unsorted Array	Each find operation in O(N) $O(N\sqrt{\log N})$	O(N)	$O(N\sqrt{\log N})$	$O(\sqrt{\log N})$	$O(N\sqrt{\log N})$
Min heap	$O(N\sqrt{\log N})$	O(N·logN)	$O(N\sqrt{\log N})$	$O(\log N)^{3/2}$	O(N logN)
Sorted array	$O(\log N)^{3/2}$	$O(N^2)$	$O(N\sqrt{\log N})$	$O(N\sqrt{\log N})$	$O(N^2)$
Sorted doubly linked list	$O(N\sqrt{\log N})$	$O(N^2)$	$O(\sqrt{\log N})$	$O(\sqrt{\log N})$	$O(N^2)$

∴ Unsorted array is best.

1.46 (a)



Heapify operation always takes O(logn) (for a complete binary tree).

Since left subtree and right subtree are max heaps it takes Ω(log n) to heapify.

1.47 (d)

Take first 3 elements. The middle of the 3 elements will be the element that is neither minimum nor maximum in the array.

Hence O(1) time to compare 3 elements.

Example: Assume 4, 6, 2 are first three elements. Then 4 is the middle element of 4, 6 and 2. Which is neither minimum nor maximum.

1.48 (c)

$$X = 1 + 2^3 + 3^3 + 4^3 + \dots n^3$$

$$X = \left[\frac{n(n+1)}{2}\right]^2 = \frac{n^2(n+1)^2}{4} = \Theta(n^4)$$

$X = \Theta(n^4)$

$X = O(n^5)$

$X = \Omega(n^3)$

1.49 (d)

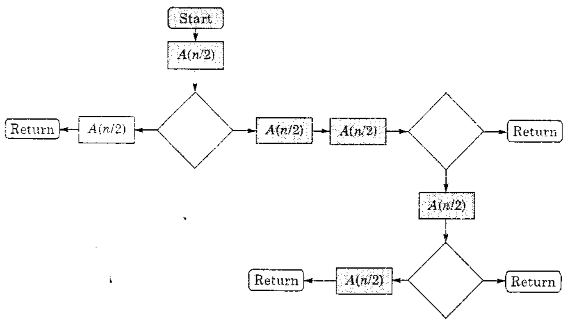
When ever the value of 'sin n' value is −1, then g(n) = 1. {sin n value ranges from −1 to +1}.

Hence f(n) = O(g(n)) is violated. Hence I is false.

and when ever the value of 'sin n' is +1, then g(n) = n^2

Hence f(n) = Ω(g(n)) is violated. Hence II is false.

1.50 Sol.



In worst case scenerio,

we have 5 A(n/2) function calls and other statements takes O(1) time.

So, the recurrrance relation will be,

A(n) = 5 A(n/2) + O(1)

By using Master theorem, we get $n^{\log_2 5}$ which is equivalent to $n^{2.32}$. So the value of α is 2.32.

1.51 (c)

Data structure used	Find	Insert	Delete	Decrease key	Overall asymptotic time
Sorted doubly linked list	O(N logN)	O(N logN)	O(N)	O(N)	O(N logN)

Sorted doubly linked list take maximum of of O(N logN) time, but since not present in option.

So, gо for nearest value $O(N^2)$.

2 Divide and Conquer

2.1 Let P be a quicksort program to sort numbers in ascending order. Let t1 and t2 be the time taken by the program for the inputs [1 2 3 4] and [5 4 3 2 1], respectively. Which of the following holds?

(a) $t_1 = t_2$ (b) $t_1 > t_2$
(c) $t_1 < t_2$ (d) $t_1 = t_2 + 5 \log 5$

[1987 : 2 Marks]

2.2 Find a solution to the following recurrence equation

$$T(n) = T\left(\frac{n}{2}\right) + n$$

$$T(1) = 1$$

[1987 : 2 Marks]

2.3 Assume that the last element of the set is used as partition element in Quicksort. If n distinct elements from the set [1.....n] are to be sorted, give an input for which Quicksort takes maximum time.

[1992 : 2 Marks]

2.4 Following algorithm (s) can be used to sort n integers in the range [1 ... n^3] in O(n) time?

(a) Heap sort (b) Quick sort
(c) Merge sort (d) Radix sort

[1992 : 2 Marks]

2.5 The recurrence relation that arises in relation with the complexity of binary search is

(a) $T(n) = T(n/2) + k$, k is a constant
(b) $T(n) = 2T(n/2) + k$, k is a constant
(c) $T(n) = T(n/2) + \log n$
(d) $T(n) = T(n/2) + n$

[1994 : 2 Marks]

2.6 Which one of the following statements is false?

(a) Optimal binary search tree construction can be performed efficiently using dynamic programming
(b) Breadth-first search cannot be used to find connected components of a graph
(c) Given the prefix and postfix walks over a binary tree, the binary tree cannot be uniquely constructed
(d) Depth-first search can be used to find connected components of a graph

[1994 : 2 Marks]

2.7 Merge sort uses

(a) Divide and conquer strategy
(b) Backtracking approach
(c) Heuristic search
(d) Greedy approach

[1995 : 1 Mark]

2.8 For merging two sorted lists of sizes m and n into a sorted list of size m+n, we required comparisons of

(a) O(m) (b) O(n)
(c) O(m + n) (d) O(logm + logn)

[1995 : 1 Mark]

2.9 Which of the following statements is true?

I. As the number of entries in a hash table increases, the number of collisions increases.
II. Recursive programs are efficient
III. The worst case complexity for Quicksort is $O(n^2)$
IV. Binary search using a linear linked list is efficient.

(a) I and II (b) II and III
(c) I and IV (d) I and III

[1995 : 1 Mark]

2.10 The recurrence relation

$T(1) = 2$

$$T(n) = 3T\left(\frac{n}{4}\right) + n$$

Has the solution T(n) equal to

(a) O(n) (b) O(log n)
(c) $O(n^{3/4})$ (d) None of these

[1996 : 2 Marks]

2.11 Quicksort is run on two inputs shown below to sort in ascending order

(i) 1, 2, 3 ..... n
(ii) n, n – 1, n – 2, ..., 2, 1

Let C_1 and C_2 be the number of comparisons made for the inputs (i) and (ii) respectively. Then,

(a) C1 < C2
(b) C1 > C2
(c) C1 = C2
(d) we cannot say anything for arbitrary n

[1996 : 2 Marks]

2.12 Let T(n) be the function defined by T(1) = 1, T(n) $= 2T\left(\left\lfloor \frac{n}{2} \right\rfloor\right) + \sqrt{n}$ for $n \geq 2$. Which of the following statements is true?

(a) $T(n) = O(\sqrt{n})$ (b) $T(n) = O(n)$
(c) $T(n) = O(\log n)$ (d) None of these

[1997 : 2 Marks]

2.13 A sorting technique is called stable if
(a) it takes O(nlog n) time
(b) it maintains the relative order of occurrence of non-distinct elements
(c) it uses divide and conquer paradigm
(d) it takes O(n) space

[1999 : 1 Mark]

2.14 If one uses straight two-way merge sort algorithm to sort the following elements in ascending order:

20, 47, 15, 8, 9, 4, 40, 30, 12, 17

then the order of these elements after second pass of the algorithm is
(a) 8, 9, 15, 20, 47, 4, 12, 17, 30, 40
(b) 8, 15, 20, 47, 4, 9, 30, 40, 12, 17
(c) 15, 20, 47, 4, 8, 9, 12, 30, 40, 17
(d) 4, 8, 9, 15, 20, 47, 12, 17, 30, 40

[1999 : 1 Mark]

2.15 Let s be a sorted array of n integers. Let t(n) denote the time taken for the most efficient algorithm to determine if there are two elements with sum less than 1000 in s. Which of the following statements is true?
(a) t (n) is O(1)
(b) $n \leq t(n) \leq n \log_2 n$
(c) m **[2000 : 1 Mark]**

2.16 Randomized quicksort is an extension of quicksort where the pivot is chosen randomly. What is the worst case complexity of sorting n numbers using randomized quicksort?

(a) O(n) (b) (n log n)
(c) $O(n^2)$ (d) O(n!)

[2001 : 1 Mark]

2.17 The usual $\Theta(n^2)$ implementation of Insertion Sort to sort an array uses linear search to identify the position where an element is to be inserted into the already sorted part of the array. If instead, we use binary search to identify the position, the worst case running time will

(a) remain $\Theta(n^2)$ (b) become $\Theta(n (\log n)^2)$
(c) become $\Theta(n \log n)$ (d) become $\Theta(n)$

[2003 : 1 Mark]

2.18 Consider a list of recursive algorithms and a list of recurrence relations as shown below. Each recurrence relation corresponds to exactly one algorithm and is used to derive the time complexity of the algorithm.

List-I (Recursive Algorithm)
P. Binary search
Q. Merge sort
R. Quick sort
S. Tower of Hanoi

List-II (Recurrence Relation)
I. $T(n) = T(n - k) + T(k) + cn$
II. $T(n) = 2T(n - 1) + 1$
III. $T(n) = 2T(n/2) + cn$
IV. $T(n) = T(n/2) + 1$

Which of the following is the correct match between the algorithms and their recurrence relations?

Codes:

	P	Q	R	S
(a)	II	III	IV	I
(b)	IV	III	I	II
(c)	III	II	IV	I
(d)	IV	II	I	III

[2004 : 2 Marks]

2.19 Suppose there are log n sorted lists of n/log n elements each. The time complexity of producing a sorted list of all these elements is:
(***Hint:*** Use a heap data structure)

(a) $O(n \log \log n)$ (b) $\Theta(n \log n)$
(c) $\Omega(n \log n)$ (d) $\Omega(n^{3/2})$

[2005 : 2 Marks]

2.20 The median of n elements can be found in O(n) time. Which one of the following is correct about the complexity of quick sort, in which remains is selected as pivot?

(a) $\Theta(n)$ (b) $\Theta(n \log n)$
(c) $\Theta(n^2)$ (d) $\Theta(n^3)$

[2006 : 2 Marks]

2.21 Which of the following sorting algorithms has the lowest worst-case complexity?

(a) Merge sort (b) Bubble sort
(c) Quick sort (d) Selection sort

[2007 : 1 Mark]

2.22 In the following C function, let $n \geq m$.

```
int gcd (n, m)
{
    if (n%m = = 0} return m;
    n = n%m;
    return gcd (m, n);
}
```

How many recursive calls are made by this function?

(a) $\Theta(\log_2 n)$ (b) $\Omega(n)$
(c) $\Theta(\log_2 \log_2 n)$ (d) $\Theta(\sqrt{n})$

[2007 : 2 Marks]

2.23 Consider the Quicksort algorithm. Suppose there is a procedure for finding a pivot element which splits the list into sub-lists each of which contains at least one-fifth of the elements. Let T(n) be the number of comparisons required to sort n elements. Then

(a) $T(n) \leq 2T(n/5) + n$
(b) $T(n) \leq T(n/5) + T(4n/5) + n$
(c) $T(n) \leq 2T(4n/5) + n$
(d) $T(n) \leq 2T(n/2) + n$

[2008 : 2 Marks]

2.24 In quick sort, for sorting n elements, the $(n/4)^{th}$ smallest element is selected as pivot using an O(n) time algorithm. What is the worst case time complexity of the quick sort?

(a) $\Theta(n)$ (b) $\Theta(n \log n)$
(c) $\Theta(n^2)$ (d) $\Theta(n^2 \log n)$

[2009 : 2 Marks]

2.25 Four matrices M_1, M_2, M_3 and M_4 of dimensions $p \times q$, $q \times r$, $r \times s$ and $s \times t$ respectively can be multiplied in several ways with different number of total scalar multiplications. For example when multiplied as $((M_1 \times M_2) \times (M_3 \times M_4))$, the total number of scalar multiplications is pqr + rst + prt. When multiplied as $(((M_1 \times M_2) \times M_3) \times M_4)$, the total number of scalar multiplications is pqr + prs + pst.

If p = 10, q = 100, r = 20, s = 5 and t = 80, then the minimum number of scalar multiplications needed is

(a) 248000 (b) 44000
(c) 19000 (d) 25000

[2011 : 2 Marks]

2.26 The worst case running time to search for an element in a balanced binary search tree with $n2^n$ elements is

(a) $\Theta(n \log n)$ (b) $\Theta(n2^n)$
(c) $\Theta(n)$ (d) $\Theta(\log n)$

[2012 : 1 Mark]

2.27 A list of n strings, each of length n, is sorted into lexicographic order using the merge sort algorithm. The worst case running time of this computation is

(a) $O(n \log n)$ (b) $O(n^2 \log n)$
(c) $O(n^2 + \log n)$ (d) $O(n^2)$

[2012 : 2 Marks]

2.28 Which one of the following is the tightest upper bound that represents the time complexity of inserting an object into a binary search tree of n nodes?

(a) $O(1)$ (b) $O(\log n)$
(c) $O(n)$ (d) $O(n \log n)$

[2013 : 1 Mark]

2.29 Let P be a quicksort program to sort numbers in ascending order using the first element as the pivot. Let t_1 and t_2 be the number of comparisons made by P for the inputs [1 2 3 4 5] and [4 1 5 3 2] respectively. Which one of the following holds?

(a) $t_1 = 5$ (b) $t_1 < t_2$
(c) $t_1 > t_2$ (d) $t_1 = t_2$

[2014 (Set-1) : 1 Mark]

2.30 The minimum number of comparisons required to find the minimum and the maximum of 100 numbers is ______.

[2014 (Set-1) : 2 Marks]

2.31 You have an array of n elements. Suppose you implement quicksort by always choosing the central element of the array as the pivot. Then the tightest upper bound for the worst case performance is

(a) $O(n^2)$ (b) $O(n\log n)$
(c) $\Theta(n\log n)$ (d) $O(n^3)$

[2014 (Set-3) : 1 Mark]

2.32 Which one of the following is the recurrence equation for the worst case time complexity of the Quicksort algorithm for sorting n($\geq$ 2) numbers? In the recurrence equations given in the options below, c is a constant.

(a) $T(n) = 2T(n/2) + cn$
(b) $T(n) = T(n-1) + T(1) + cn$
(c) $T(n) = 2T(n-1) + cn$
(d) $T(n) = T(n/2) + cn$

[2015 (Set-1) : 1 Mark]

2.33 What are the worst-case complexities of insertion and deletion of a key in a binary search tree?

(a) $\Theta(\log n)$ for both insertion and deletion
(b) $\Theta(n)$ for both insertion and deletion
(c) $\Theta(n)$ for insertion and $\Theta(\log n)$ for deletion
(d) $\Theta(\log n)$ for insertion and $\Theta(n)$ for deletion

[2015 (Set-1) : 1 Mark]

2.34 Suppose you are provided with the following function declaration in the C programming language.

```
int partition (int a[ ], int n);
```

The function treats the first element of a[ ] as a pivot, and rearranges the array so that all elements less than or equal to the pivot is in the left part of the array, and all elements greater than the pivot is in the right part. In addition, it moves the pivot so that the pivot is the last element of the left part. The return value is the number of elements in the left part.

The following partially given function in the C programming language is used to find the k^{th} smallest element in an array a[ ] of size n using the partition function. We assume $k \leq n$.

```
int kth_smallest (int a[ ], int n, int k)
{
    int left_end = partition (a, n);
    if (left_end+1==k)
    {
        return a [left_end];
    }
    if (left_end+1 > k)
    {
        return kth_smallest (______);
    }
    else
    {
        return kth_smallest (______);
    }
}
```

The missing argument lists are respectively

(a) (a, left_end, k) and (a+left_end+1, n–left_end–1, k–left_end–1)
(b) (a, left_end, k) and (a, n–left_end–1, k–left_end–1)
(c) (a+left_end+1, n–left_end–1, k–left_end–1) and(a, left_end, k)
(d) (a, n–left_end–1, k–left_end–1) and (a, left_end, k)

[2015 (Set-2) : 2 Marks]

2.35 Assume that a mergesort algorithm in the worst case takes 30 seconds for an input of size 64. Which of the following most closely approximates the maximum input size of a problem that can be solved in 6 minutes?

(a) 256 (b) 512
(c) 1024 (d) 2048

[2015 (Set-3) : 2 Marks]

2.36 The worst case running times of *Insertion sort*, *Merge sort* and *Quick sort*, respectively, are:

(a) $\Theta(n \log n)$, $\Theta(n \log n)$, and $\Theta(n^2)$
(b) $\Theta(n^2)$, $\Theta(n^2)$, and $\Theta(n \log n)$
(c) $\Theta(n^2)$, $\Theta(n \log n)$, and $\Theta(n \log n)$
(d) $\Theta(n^2)$, $\Theta(n \log n)$, and $\Theta(n^2)$

[2016 (Set-1) : 1 Mark]

2.37 Assume that the algorithms considered here sort the input sequences in ascending order. If the input is already in ascending order, which of the following are TRUE?

I. Quicksort runs in $\Theta(n^2)$ time
II. Bubblesort runs in $\Theta(n^2)$ time
III. Mergesort runs in $\Theta(n)$ time
IV. Insertion sort runs in $\Theta(n)$ time

(a) I and II only (b) I and III only
(c) II and IV only (d) I and IV only

[2016 (Set-2) : 1 Mark]

■■■■

Answers Divide and Conquer

2.1 (c)	2.4 (d)	2.5 (a)	2.6 (b)	2.7 (a)	2.8 (c)	2.9 (d)	2.10 (a)	2.11 (c)
2.12 (b)	2.13 (b)	2.14 (b)	2.15 (a)	2.16 (c)	2.17 (a)	2.18 (b)	2.19 (a)	2.20 (b)
2.21 (a)	2.22 (a)	2.23 (b)	2.24 (b)	2.25 (c)	2.26 (c)	2.27 (b)	2.28 (c)	2.29 (c)
2.31 (a)	2.32 (b)	2.33 (b)	2.34 (a)	2.35 (b)	2.36 (d)	2.37 (d)		

Explanations Divide and Conquer

2.1 (c)

t1 is less than t2
Sorting [1 2 3 4] takes less time compared to sorting [5 4 3 2 1] list. First list has 4 elements and second list has 5 elements. Both lists are worst cases of quick sort.

2.2 Sol.

By using substitution method we get following series:

$$\Rightarrow \quad T\left(\frac{n}{2^k}\right)+\frac{n}{2^{k-1}}+\frac{n}{2^{k-2}}\cdots\frac{n}{2^2}+\frac{n}{2^n}+n$$

$\Rightarrow$ So put $k = \log_2 n$ then we get

$$T\left(\frac{n}{2^{\log_2 n}}\right)+n\left(\frac{1}{2^0}+\frac{1}{2^1}+\frac{1}{2^2}\cdots\frac{1}{2^{\log_2 n-1}}\right)$$

$$= O(n)$$

2.3 Sol.

Quicksort gives worst case when all elements are already sorted. So give any sorted order input either in ascending or descending order.

2.4 (d)

Radix sort is a non-comparative integer sorting algorithm that sorts data with integer keys by grouping keys which share same position and value. So it take O(n) time.

2.5 (a)

Binary search only half of the array.

So, $T\left(\frac{n}{2}\right)+K$

2.6 (b)

Connected components of a graph can be computed in linear time by using either breadth-first search or depth-first search.
So option (b) is false.

2.7 (a)

A merge sort is comparison based sorting algorithm and divide-and-conquer algorithm.

2.8 (c)

The number of comparisons required in the worst case is O(m + n).

2.9 (d)

I. In hashing, as the number of entries in a hash table increases, the number of collisions increases.
II. It is not always true that recursive programs are efficient.
III. Quicksort is a comparison sort which is worst case makes $O(n^2)$ comparisons.
IV. Binary search can be used with linked list, however, it is inefficient.

So option (d) is correct.

2.10 (a)

Using Master Theorem

We get $n^{\log_4 3} < n$

2.11 (c)

Both of given cases are
Quicksort Worst cases problem, so comparisons are equal.

2.12 (b)

Use Master Theorem $n^{\log_2 2} \geq \sqrt{n}$

2.13 (b)

A sorting algorithm is called stable if it keeps elements with equal keys in the same relative order in the output as they were in the input.

For ***example*** in the following input the two 4's are indistinguishable. 1, 4_a, 3, 4_b, 2 and so the output of a stable sorting algorithm must be:

$$1, 2, 3, 4_a, 4_b$$

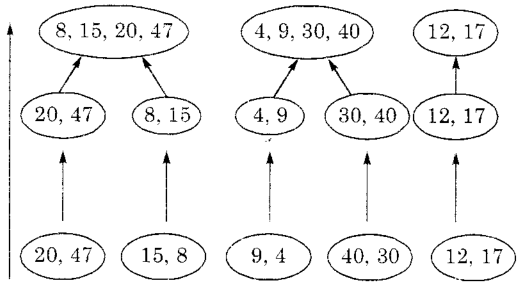
Bubble sort, merge sort, counting sort, insertion sort are stable sorting algorithms.

2.14 (b)

Given: 20, 47, 15, 8, 9, 4, 40, 30, 12, 17

2-way merge sort so group of 2 is taken at once.

2nd pass:



The order of elements after second pass of the algorithm is 8, 15, 20, 47, 4, 9, 30, 40, 12, 17.

2.15 (a)

Take starting 2 elements of array, if sum of these two are less than 1000 then return those elements, otherwise return false.

2.16 (c)

Randomized quick sort worst case time complexity = $O(n^2)$

2.17 (a)

The usual O (n^2) implementation of insertion sort to sort an array uses binary search to identify the position, the worst case running time will remain O (n^2). The time required to insert an element x into a binary search tree is bounded by a constant times the number of comparisons made between x and elements already in the tree. Thus we can measure time in terms of the number of comparisons made. In worst case implementation of insertion sort takes $O(n^2)$. So in the worst case adding n elements to a binary search tree could require O (n^2) time. In expected case means element are sorted in increasing order adding n elements to a binary search tree could require O (n log n) time.

2.18 (b)

Binary search : T(n) = T(n/2) + 1

Merge sort : T(n) = 2T(n/2) + cn

Quick sort : T(n) = T(n − k) + T(k) + cn

Tower of Hanoi : T(n) = 2T(n − 1) + 1

2.19 (a)

When we uses the max heap data structure then there are $\lceil \log n \rceil$ sorted lists of which each contains $\lfloor n/\log n \rfloor$ elements. Construction of heap takes O(log logn) time and if we have $\lfloor n/\log n \rfloor$ elements then worst case time complexity is O(n log log n).

2.20 (b)

If the median of n elements can be found in O(n) time and we select the median as pivot. Then quicksort sort n elements of an array A[1],..., A[n] as follows. We permute the elements in the array so that for some j all the records with key less than v appear in A[1],..., A[j] and all those with keys v or greater appear in A[j + 1],..., A[n]. We then apply quicksort recursively to A[1],..., A[j] and to A[j + 1],..., A[n].

The recursion equation becomes for some value of i

$$T(n) \le \frac{1}{n}\sum_{i=1}^{n}\left(T(i-1)+T(n-i)\right)$$

The solution of the $T(n) \le \Theta(n \log n)$

Given to find median O(n) time required.

2.21 (a)

Sorting Algorithm	Worst care complexity (with n inputs)
Merge sort	O(nlogn)
Bubble sort	$O(n^2)$
Quick sort	$O(n^2)$
Selection sort	$O(n^2)$

2.22 (a)

Let T(m, n) be the total number of steps.

So T(m, 0) = 0, T(m, n) = T(n, m mod n) on average

$$T_n = \frac{1}{n}\sum_{0 \le k \le n} T(k, n)$$

$$T_n \approx 1 + \frac{1}{n}(T_0 T_1 + \dots + T_{n-1})$$
$$T_n \approx S_n$$
$$S_n = 1 + \frac{1}{n}(S_0 S_1 + \dots + S_{n-1})$$
$$S_n = 1 + \frac{1}{n+1}(S_0 S_1 + \dots + S_n)$$
$$= 1 + \frac{1}{n+1}(n(S_{n-1}) + S_n)$$
$$= 1 + \frac{1}{n+1} = S_n + \frac{1}{n+1}$$

So $T_n \approx \Theta(\log_2 n) + 0(1)$

$T \approx \Theta(\log_2 n)$

2.23 (b)

If we wants to sort n elements with the help of Qucksort algorithm. If pivot elements which split the list into two sub-lists each in which one list contains one-fifth element or n/5 and other list contains 4n/5 and balancing takes n. So

$T(n) \le T(n/5) + T(4n/5) + n$

[**Note :** $n - \frac{n}{5} = \frac{5n-n}{5} = \frac{4n}{5}$]

2.24 (b)

The relation $T(n) = T(n/4) + T(3n/4) + n$

The pivot element is selected in such a way that it will divide the array into 1/4th and 3/4th always solving this relation give $\Theta(n \log n)$.

2.25 (c)

We get minimum number of multiplications using $((M1 \times (M2 \times M3)) \times M4)$.

Total number of multiplication

$= 100 * 20 * 5 + 10 * 100 * 5 + 10 * 5 * 80$

$= 19000$

2.26 (c)

We know that in balanced BST there are $\log_2 n$ levels in both worst as well as best case where 'n' is no. of elements

$\therefore \log_2 (n\, 2^n) = \log_2 n + \log_2 2^n$

$\log_2 n + n \log_2 2 = \log_2 n + n = \Theta(n)$

$\therefore$ Option 'C' is correct.

2.27 (b)

The recurrence tree for merge sort will have height n and $O(n^2)$ work will be done at each level of the recurrence tree (Each level involves n comparisons and a comparison takes $O(n)$ time in worst case). So time complexity of this merge sort will be $O(n^2 \log n)$.

2.28 (c)

In the worst case length of binary search tree can be $O(n)$.

So insertion of an object will take $O(n)$ time in worst case option (c) is correct.

2.29 (c)

Input1: [1 2 3 4 5]

Input2: [4 1 5 3 2]

(a) [1 2 3 4 5]: Already elements are in sorted order $\frac{5(5-1)}{2}$ comparisons are required.

$\Rightarrow t_1 = 10$

(b) [4 1 5 3 2]: Elements are in random order. Choosing first element as pivot will give less comparisons compared to t_1.

$\therefore t_2 < t_1$.

2.30 Sol.

$n = 100$

To find maximum and minimum, minimum number of comparisons using divide and conquer

$= \left\lceil \frac{3n}{2} \right\rceil - 2 = \left(\frac{3 * 100}{2} \right) - 2$

$= 148$ comparisons

2.31 (a)

Choosing the central element (middle) as the pivot always can not make the partition into equal parts.

$\therefore$ The worst case time complexity $= O(n^2)$

2.32 (b)

In worst case the pivot element position may be either first or last. In that case the elements are always divided in 1 : (n – 1) proportion. The recurrence relation for such a proportional division would be

$T(n) = T(1) + T(n-1) + O(n) = O(n^2)$

(a) $T(n) = 2T(n/2) + cn = O(n \log n)$

(b) $T(n) = T(n-1) + T(1) + cn = O(n^2)$

(c) $T(n) = 2T(n-1) + cn = O(2^n)$

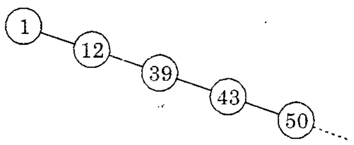
(d) $T(n) = T(n/2) + cn = O(n)$

2.33 (b)

In worst case the BST may be Skewed BST. This case occurs when we insert the elements in increasing order or decreasing order.

Example: Consider we insert elements, in the order 1, 12, 39, 43, 50, ...

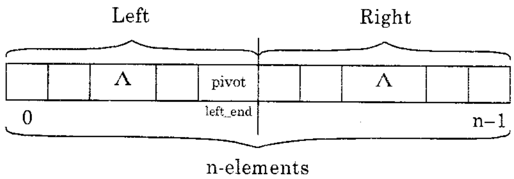
The BST would be



To find 'n' is the worst case we may have to traverse to bottom of tree which takes O(n) time.

Hence for both insertion and deletion worst case goes to $\Theta(n)$.

2.34 (a)



left_end (is pivot position) = partition (a, n);

Note: k^{th} smallest is same as an element at index (k–1) in the sorted array.

if (left_end+1 = = k) // k^{th} smallest found.

return a[left_end]; // return pivot element.

if (left_end+1 > k) // k^{th} smallest is before pivot.

return k^{th}_smallest (a, left_end, k); // search before pivot

else // k^{th} smallest is after pivot (right part)

return kth_smalest $\underbrace{(a+left\_end+1,}_{\text{address of first element after pivot (right part)}}\ \underbrace{n-(left\_end-1), k-(left\_end+1));}_{\text{Number of elements in right part}}$

2.35 (b)

For $n = 64$

$c_1.n\log n = 30$ sec

$c_1.64. \log 64 = 30$ sec

$$c_1 = \frac{5}{64} \qquad ...(1)$$

Now, $c_1.n\log n = 6 \times 60$ sec

$$n \log n = \frac{6 \times 60 \text{sec}}{c_1 \text{ sec}}$$

$$n \log n = \frac{6 \times 60}{5} \times 64$$

$n \log n = 4608$

$n \log n = 512 \times 9 \Rightarrow \quad n = 512$

2.36 (d)

The worst case time complexity of algo given are:

Insertion sort = $\Theta(n^2)$

Merge sort = $\Theta(n \log n)$

Quick sort = $\Theta(n^2)$

2.37 (d)

Quick sort takes $O(n^2)$ time to sort the input (ascending order) into ascending order. by using divide and conquer.

Insertion sort take O(n) time to sort input (ascending order) into ascending order since need one comparision every time and no swaping.

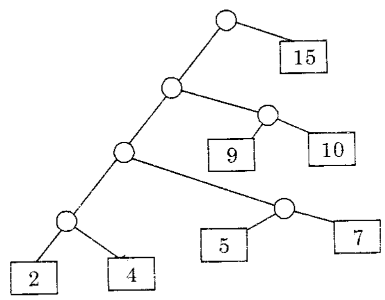
Merge sort take O(n log n) and bubble sort can take O(n) time.

So, option (d) is most appropriate.

■■■■

3 Greedy Method

3.1 The weighted external path length of the binary tree in figure is



[1991 : 2 Marks]

3.2 Kruskal's algorithm for finding a minimum spanning tree of a weighted graph G with n vertices and m edges has the time complexity of
(a) $O(n^2)$ (b) $O(mn)$
(c) $O(m + n)$ (d) $O(m\log n)$
(e) $O(m^2)$

[1991 : 2 Marks]

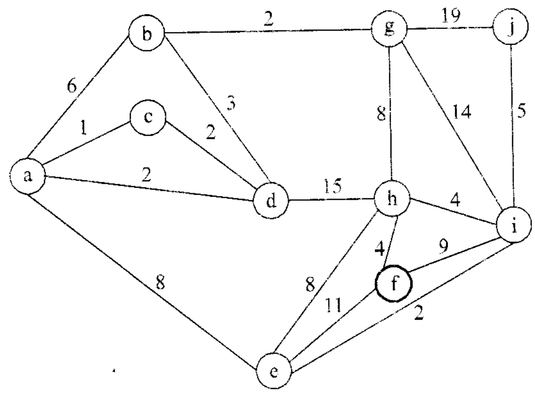
3.3 Complexity of Kruskal's algorithm for finding the minimum spanning tree of an undirected graph containing n vertices and m edges if the edges are sorted is ________.

[2002 : 2 Marks]

3.4 Let G be an undirected connected graph with distinct edge weights. Let e_{max} be the edge with maximum weight and e_{min} the edge with minimum weight. Which of the following statements is false?
(a) Every minimum spanning tree of G must contain e_{min}
(b) If e_{max} is in a minimum spanning tree, then its removal must disconnect G
(c) No minimum spanning tree contains e_{max}
(d) G has a unique minimum spanning tree

[2002 : 2 Marks]

3.5 What is the weight of a minimum spanning tree of the following graph?



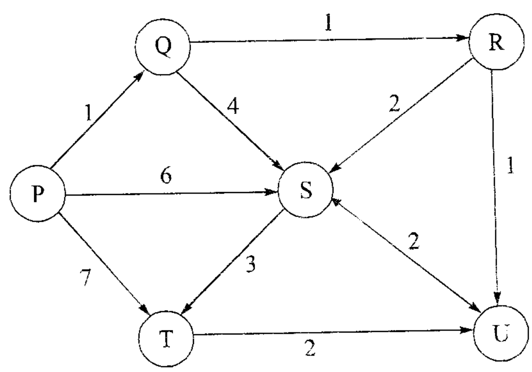
(a) 29 (b) 31
(c) 38 (d) 41

[2003 : 2 Marks]

3.6 The following are the starting and ending times of activities A, B, C, D, E, F, G, and H respectively in chronological order: "a_s b_s c_s a_e d_s c_e e_s f_s b_e d_e g_s e_e f_e h_s g_e h_e" Here, x_s denotes the starting time and x_e denotes the ending time of activity X. W need to schedule the activities in a set of rooms available to us. An activity can be scheduled in a room only if the room is reserved for the activity for its entire duration. What is the minimum number of rooms required?
(a) 3 (b) 4
(c) 5 (d) 6

[2003 : 2 Marks]

3.7 Suppose we run Dijkstra's single source shortest-path algorithm on the following edge-weighted directed graph with vertex P as the source.

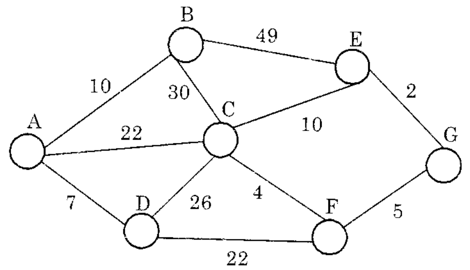


In what order do the nodes get included into the set of vertices for which the shortest path distances are finalized?

(a) P, Q, R, S, T, U (b) P, Q, R, U, S, T
(c) P, Q, R, U, T, S (d) P, Q, T, R, U, S

[2004 : 2 Marks]

3.8 Consider the undirected graph below:



Using Prim's algorithm to construct a minimum spanning tree starting with node A, which one of the following sequences of edges represents a possible order in which the edges would be added to construct the minimum spanning tree?

(a) (E,G), (C,F), (F,G), (A,D), (A,B), (A,C)
(b) (A,D), (A,B), (A,C), (C,F), (G,E), (F,G)
(c) (A,B), (A,D), (D,F), (F,G), (G,E), (F,C)
(d) (A,D), (A,B), (D,F), (F,C), (F,G), (G,E)

[2004 : 2 Marks]

3.9 In the following table, the left column contains the names of standard graph algorithms and the right column contains the time complexities of the algorithms. Match each algorithm with its time complexity.

List-I

1. Bellman-Ford algorithm
2. Kruskal's algorithm
3. Floyd-Warshall algorithm
4. Topological sorting

List-II

A. O(mlog n)
B. $O(n^3)$
C. O(nm)
D. O(n + m)

(a) 1-C, 2-A, 3-B, 4-D
(b) 1-B, 2-D, 3-C, 4-A
(c) 1-C, 2-D, 3-A, 4-B
(d) 1-B, 2-A, 3-C, 4-D

[2005 : 1 Mark]

3.10 A undirected graph G has n nodes. Its adjacency matrix is given by an n × n square matrix whose

1. diagonal elements are 0's, and
2. non-diagonal elements are 1's.

Which one of the following is TRUE?

(a) Graph G has no minimum spanning tree (MST)
(d) Graph G has a unique MST of cost n-1
(c) Graph G has multiple distinct MST's, each of cost n-1
(d) Graph G has multiple spanning trees of different costs

[2005 : 1 Mark]

3.11 Let G be a weighted undirected graph and e be an edge with maximum weight in G. Suppose there is a minimum weight spanning tree in G containing the edge e. Which of the following statements is always TRUE?

(a) There exists a cutset in G having all edges of maximum weight.
(b) There exists a cycle in G having all edges of maximum weight.
(c) Edge e cannot be contained in a cycle.
(d) All edges in G have the same weight.

[2005 : 2 Marks]

3.12 Let G (V, E) an undirected graph with positive edge weights. Dijkstra's single source-shortest path algorithm can be implemented using the binary heap data structure with time complexity?

(a) $O(|V|^2)$ (b) $O(|E|+|V|\log|V|)$
(c) $O(|V|\log|V|)$ (d) $O((|E|+|V|)\log|V|)$

[2005 : 2 Marks]

Common Data for Q. 3.13 & Q. 3.14

We are given 9 tasks $T_1, T_2 \ldots T_9$. The execution of each task requires one unit of time. We can execute one task at a time. T_i has a profit P_i and a deadline d_i profit P_i is earned if the task is completed before the end of the d_i^{th} unit of time.

Task	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9
Profit	15	20	30	18	18	10	23	16	25
Deadline	7	2	5	3	4	5	2	7	3

3.13 Are all tasks completed in the schedule that gives maximum profit?

(a) All tasks are completed
(b) T_1 and T_6 are left out
(c) T_1 and T_8 are left out
(d) T_4 and T_6 are left out

[2005 : 2 Marks]

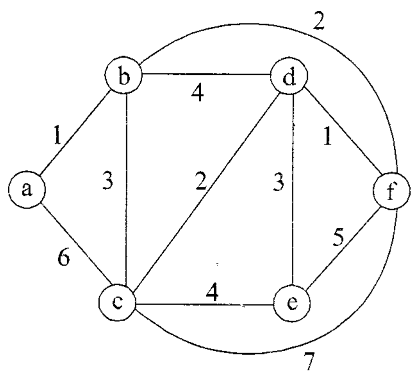
3.14 What is the maximum profit earned?
(a) 147 (b) 165
(c) 167 (d) 175
[2005 : 2 Marks]

3.15 Consider the polynominal $p(x) = a_0 + a_1x + a_2x^2 + a_3x^3$, where $a_i \neq 0, \forall i$. The minimum number of multiplcations needed to evaluate p on an input x is
(a) 3 (b) 4
(c) 6 (d) 9
[2006 : 1 Mark]

3.16 Consider a weighted complete graph G on the vertex set $\{v_1, v_2, ..., v_n\}$ such that the weight of the edge (v_i, v_j) is $2|i - j|$. The weight of a minimum spanning tree of G is
(a) $n - 1$ (b) $2n - 2$
(c) $\binom{n}{2}$ (d) n^2
[2006 : 1 Mark]

3.17 To implement Dijkstra's shortest path algorithm on unweighted graphs so that it runs in linear time, then data structure to be used is
(a) Queue (b) Stack
(c) Heap (d) B-Tree
[2006 : 1 Mark]

3.18 Consider the following graph:



Which one of the following cannot be the sequence of edges added, in that order, to a minimum spanning tree using Kruskal's algorithm?
(a) (a – b), (d – f), (b – f), (d – c), (d – e)
(b) (a – b), (d – f), (d – c), (b –f), (d – e)
(c) (d – f), (a – b), (d – c), (b – f), (d – e)
(d) (d – f), (a – b), (b – f), (d – e), (d – c)
[2006 : 2 Marks]

3.19 The characters a to h have the set of frequencies based on the first 8 Fibonacci numbers as follows:
a : 1, b : 1, c : 2, d : 3, e : 5, f : 8, g : 13, h : 21
A Huffman code is used to represent the characters. What is the sequence of characters corresponding to the following code?
110111100111010
(a) fdheg (b) ecgdf
(c) dchfg (c) fehdg
[2006 : 2 Marks]

3.20 In an unweighted, undirected connected graph, the shortest path from a node S to every other node is computed most efficiently, in terms of time complexity, by
(a) Dijkstra's algorithm starting from S.
(b) Warshall's algorithm
(c) performing a DFS starting from S
(d) performing a BFS starting from S
[2007 : 2 Marks]

3.21 Let w be the minimum weight among all edge weights in an undirected connected graph. Let e be a specific edge of weight w. Which of the following is FALSE?
(a) There is a minimum spanning tree containing e.
(b) If e is not in a minimum spanning tree T, then in the cycle formed by adding e to T, all edges have the same weight.
(c) Every minimum spanning tree has an edge of weight w
(d) e is present in every minimum spanning tree
[2007 : 2 Marks]

Common Data for Q. 3.22 & Q. 3.23

Suppose the letters a, b, c, d, e have probabilities $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$ respectively.

3.22 Which of the following is the Huffman code for the letters a, b, c, d, e?
(a) 0, 10, 110, 1110, 1111
(b) 11, 10, 011, 010, 001
(c) 11, 10, 01, 001, 0001
(d) 110, 100, 010, 000, 001
[2007 : 2 Marks]

3.23 What is the average length of the correct answer to above question?

(a) 3 (b) 2.1875
(c) 2.25 (d) 1.781

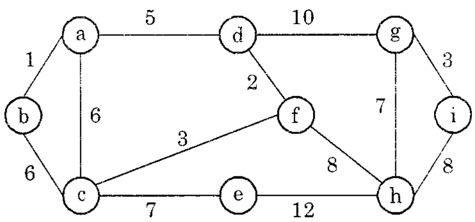
[2007 : 2 Marks]

3.24 Consider n jobs $J_1, J_2, \ldots, J_n$ such that job J_i has execution time t_i and a non-negative integer weight w_i. The weighted mean completion time of the jobs is defined to be $\sum_{i=1}^{n} w_i T_i \Big/ \sum_{i=1}^{n} w_i$ where T_i is the completion time of job J_i. Assuming that there is only one processor available, in what order must the jobs be executed in order to minimize the weighted mean completion time of the jobs?

(a) Non-decreasing order of t_i
(b) Non-increasing order of w_i
(c) Non-increasing order of $w_i t_i$
(d) None-increasing order of w_i/t_i

[2007 : 2 Marks]

3.25 For the undirected, weighted graph given below, which of the following sequences of edges represents a correct execution of Prim's algorithm to construct a Minimum Spanning Tree?



(a) (a, b), (d, f), (f, c), (g, i), (d, a), (g, h), (c, e), (f, h)
(b) (c, e), (c, f), (f, d), (d, a), (a, b), (g, h), (h, f), (g, i)
(c) (d, f), (f, c), (d, a), (a, b), (c, e), (f, h), (g, h), (g, i)
(d) (h, g), (g, i), (h, f), (f, c), (f, d), (d, a), (a, b), (c, e)

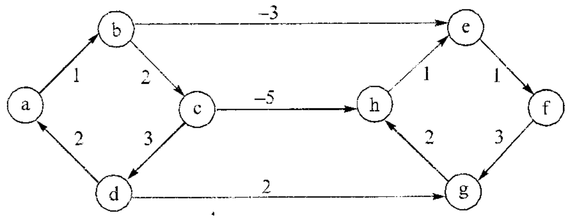
[2008 : 2 Marks]

3.26 G is a graph on n vertices and 2n – 2 edges. The edges of G can be partitioned into two edge-disjoint spanning trees. Which of the following is NOT true for G?

(a) For every subset of k vertices, the induced subgraph has a most 2k – 2 edges.
(b) The minimum cut in G has a least two edges
(c) There are two edges-disjoint paths between every pair of vertices
(d) There are two vertex-disjoint paths between every pair of vertices.

[2008 : 2 Marks]

3.27 Dijkstra's single source shortest path algorithm when run from vertex a in the below graph, computes the corrects shortest path distance to



(a) only vertex a
(b) only vertices a, e, f, g, h
(c) only vertices, a, b, c, d
(d) all the vertices

[2008 : 2 Marks]

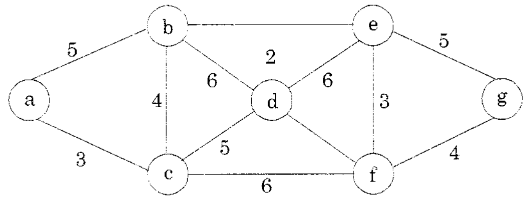
3.28 Which of the following statement(s) is/are correct regarding Bellman-Ford shortest path algorithm?

P. Always finds a negative weighted cycle, if one exists.
Q. Finds whether any negative weighted cycle is reachable from the source

(a) P only (b) Q only
(c) both P and Q (d) neither P nor Q

[2009 : 1 Mark]

3.29 Consider the following graph:



Which one of the following is NOT the sequence of edges added to the minimum spanning tree using Kruskal's algorithm?

(a) (b, e) (e, f) (a, c) (b, c) (f, g) (c, d)
(b) (b, e) (e, f) (a, c) (f, g) (b, c) (c, d)
(c) (b, e) (a, c) (e, f) (b, c) (f, g) (c, d)
(d) (b, e) (e, f) (b, c) (a, c) (f, g) (c, d)

[2009 : 2 Marks]

3.30 The weight of a sequence $a_0, a_1, \ldots, a_{n-1}$ of real numbers is defined as $a_0 + a_1/2 + \ldots a_{n-1}/2^{n-1}$. A subsequence of a sequence is obtained by deleting some elements from the sequence, keeping the order of the remaining elements the same. Let X denote the maximum possible weight of a subsequence of $a_0, a_1, \ldots, a_{n-1}$ and Y the maximum possible weight of a subsequence of $a_1, a_2, \ldots, a_{n-1}$. Then X is equal to

(a) max(Y, a_0 + Y) (b) max(Y, a_0 + Y/2)
(c) max(Y, a_0 + 2Y) (d) a_0 + Y/2

[2010 : 2 Marks]

Common Data for Questions 3.31 and 3.32

Consider a complete undirected graph with vertex set {0, 1, 2, 3, 4}. Entry W_{ij} in the matrix W below is the weight of the edge {i, j}.

$$W = \begin{pmatrix} 0 & 1 & 8 & 1 & 4 \\ 1 & 0 & 12 & 4 & 9 \\ 8 & 12 & 0 & 7 & 3 \\ 1 & 4 & 7 & 0 & 2 \\ 4 & 9 & 3 & 2 & 0 \end{pmatrix}$$

3.31 What is the minimum possible weight of a spanning tree T in this graph such that vertex 0 is a leaf node in the tree T?

(a) 7 (b) 8
(c) 9 (d) 10

[2010 : 2 Marks]

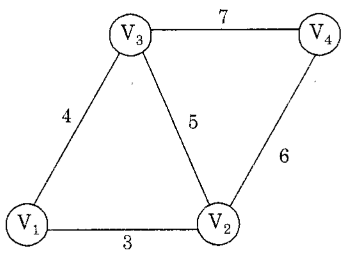
3.32 What is the minimum possible weight of a path P from vertex 1 to vertex 2 in this graph such that P contains at most 3 edges?

(a) 7 (b) 8
(c) 9 (d) 10

[2010 : 2 Marks]

Linked Data Question 3.33 and 3.34

An undirected graph G(V, E) contains n(n > 2) nodes named $v_1, v_2, \ldots, v_n$. Two nodes v_i, v_j are connected if and only if $0 < |i - j| \le 2$. Each edge (v_i, v_j) is assigned a weight i + j. A sample graph with n = 4 is shown below.



3.33 What will be the cost of the Minimum Spanning Tree (MST) of such a graph with n nodes?

(a) $\frac{1}{12}(11n^2 - 5n)$ (b) $n^2 - n + 1$
(c) 6n − 11 (d) 2n + 1

[2011 : 2 Marks]

3.34 The length of the path from v_5 to v_6 in the MST of previous question with n = 10 is

(a) 11 (b) 25
(c) 31 (d) 41

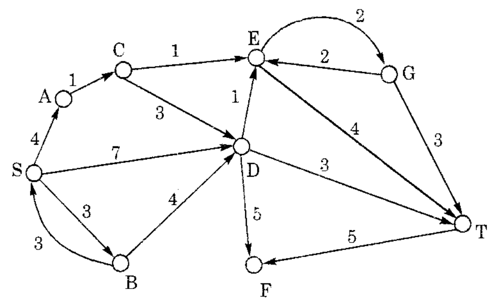
[2011 : 2 Marks]

3.35 Let G be a weighted graph with edge weights greater than one and G′ be the graph constructed by squaring the weights of edge in G. Let T and T′ be the minimum spanning trees of G and G′ respectively, with total weights t and t′. Which of the following statements is TRUE?

(a) T′ = T with total weight $t' = t^2$
(b) T′ = T with total weight $t' < t^2$
(c) T′ ≠ T but total weight $t' = t^2$
(d) None of the above

[2012 : 2 Marks]

3.36 Consider the directed graph shown in the figure below. There are multiple shortest paths between vertices S and T. Which one will be reported by Dijkstra's shortest path algorithm? Assume that, in any iteration, the shortest path to a vertex v is updated only when a strictly shorter path to v is discovered.



(a) SDT (b) SVDT
(c) SACDT (d) SACET

[2012 : 2 Marks]

3.37 Which one of the following is the tightest upper bound that represents the number of swaps required to sort *n* numbers using selection sort?

(a) O(log *n*) (b) O(*n*)
(c) O(*n* log *n*) (d) O(n^2)

[2013 : 1 Mark]

3.38 Consider the tree arcs of a BFS traversal from a source node *W* in an unweighted, connected, undirected graph. The tree T formed by the tree arcs is a data structure for computing

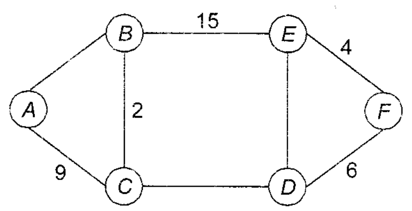
(a) the shortest path between every pair of vertices.
(b) the shortest path from W to every vertex in the graph.
(c) the shortest paths from W to only those nodes that are leaves of T.
(d) the longest path in the graph.

[2014 (Set-2) : 1 Mark]

3.39 Suppose P, Q, R, S, T are sorted sequences having lengths 20,24,30,35,50 respectively. They are to be merged into a single sequence by merging together two sequences at a time. The number of comparisons that will be needed in the worst case by the optimal algorithm for doing this is ________.

[2014 (Set-2) : 2 Marks]

3.40 The graph shown below has 8 edges with distinct integer edge weights. The minimum spanning tree (MST) is of weight 36 and contains the edges: {(A, C), (B, C), (B, E), (E, F), (D, F)}. The edge weights of only those edges which are in the MST are given in the figure shown below. The minimum possible sum of weights of all 8 edges of this graph is ________.



[2015 (Set-1) : 2 Marks]

3.41 Let G be a weighted connected undirected graph with distinct positive edge weights. If every edge weight is increased by the same value, then which of the following statements is/are TRUE?

P: Minimum spanning tree of G does not change
Q: Shortest path between any pair of vertices does not change

(a) P only
(b) Q only
(c) Neither P nor Q
(d) Both P and Q

[2016 (Set-1) : 1 Mark]

3.42 Let G be a complete undirected graph on 4 vertices, having 6 edges with weights being 1, 2, 3, 4, 5, and 6. The maximum possible weight that a minimum weight spanning tree of G can have is ________.

[2016 (Set-1) : 2 Marks]

3.43 $G = (V, E)$ is an undirected simple graph in which each edge has a distinct weight, and e is a particular edge of G. Which of the following statements about the minimum spanning trees (MSTS) of G is/are TRUE?

I. If e is the lightest edge of *some* cycle in G, then every MST of G *includes e*.
II. If e is the heaviest edge of *some* cycle in G, then every MST of G *excludes e*.

(a) I only
(b) II only
(c) both I and II
(d) neither I nor II

[2016 (Set-1) : 2 Marks]

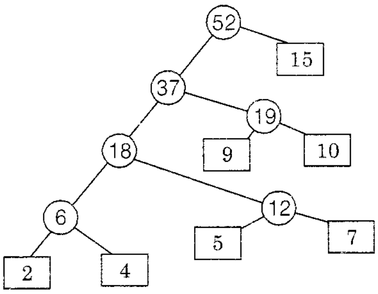
■■■■

Answers Greedy Method

3.2	(d)	3.4	(c)	3.5	(b)	3.6	(b)	3.7	(b)	3.8	(d)	3.9	(a)	3.10	(c)	3.11	(a)
3.12	(b)	3.13	(d)	3.14	(a)	3.15	(a)	3.16	(b)	3.17	(c)	3.18	(d)	3.19	(c)	3.20	(d)
3.21	(d)	3.22	(a)	3.23	(d)	3.24	(d)	3.25	(c)	3.26	(d)	3.27	(d)	3.28	(b)	3.29	(d)
3.30	(d)	3.31	(d)	3.32	(b)	3.33	(b)	3.34	(c)	3.35	(d)	3.36	(d)	3.37	(b)	3.38	(b)
3.41	(a)	3.43	(b)														

Explanations Greedy Method

3.1 Sol.



So the path length is 52.

3.2 (d)

Kruskal's algorithm time complexity
= O (e log v)
= O (m log n)

3.3 Sol.

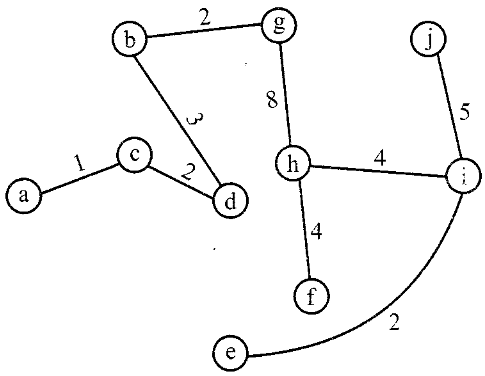
O(m + n), edges are sorted so no need to make min-heap. So deletion from min. heap time is saved.

3.4 (c)

All edges are distinct weights.
Removal of e_{max} may not disconnect G, because other edges may cover the vertices incident with e_{max}.

3.5 (b)

Number of vertices in the graph n = 10 so after adding n – 1 = 9 edges which contains all vertices and doesn't form a circuit in the minimum spanning tree of G.



$$\text{Total weight} = \sum_{i=1}^{9} w(i)$$

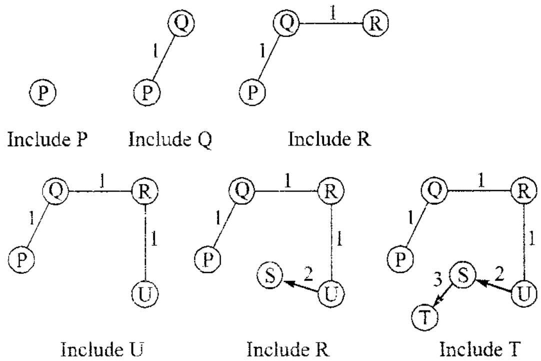
= 1 + 2 + 2 + 2 + 3 + 4 + 4 + 5 + 8 = 31

3.6 (b)

The activities are A, B, C, D, E, F, G and H. The starting and ending times are "$a_s\, b_s\, c_s\, a_e\, d_s\, c_e\, e_s\, f_s\, b_e\, d_e\, g_s\, e_s\, f_e\, h_s\, g_e\, h_e$".
The gap between starting and ending time of B is greater than all other activities. Consider the order $b_s c_s a_e d_s c_e e_s f_s b_e$ be the gap between b_s to b_e is 6 in which two activity a_e and c_e ended so minimum number of required rooms is 4.

3.7 (b)

In Dijkstra's single source shortest path the nodes are included in following sequence.



Total Distance (from source P)
= 1+ 1+ 1+ 2+ 3 = 8
Node sequence = P, Q, R, U, S, T

3.8 (d)

(a) and (b) produce disconnected components with the GIVEN order in options which is NEVER allowed by prim's algorithm.
(c) produces connected component every instant a new edge is added BUT when first vertex is chosen(first vertex is chosen randomly) first edge must be the minimum weight edge that is chosen. Therefore (a,d) MUST be chosen BEFORE (a,b). Therefore (c) is false.

3.9 (a)

Bellman-Ford algorithm : O(nm)
Kruskal's algorithm : O(mlog n)
Floyd-Warshall algorithm : $O(n^3)$
Topological sorting : O(n + m)

3.10 (c)

Undirected graph G contains n nodes.

$$\begin{array}{c|ccccc|} & 1 & 2 & 3 & \cdots & n \\ \hline 1 & 0 & 1 & 1 & \cdots & 1 \\ 2 & 1 & 0 & 1 & \cdots & 1 \\ 3 & 1 & 1 & 0 & \cdots & 1 \\ \vdots & & & & & \\ n & 1 & 1 & 1 & \cdots & 0 \\ \hline \end{array}$$

In undirected graph G, the diagonal elements are 0 means there is no self loop of any vertex. Where each vertex in G is connected through $n - 1$ vertices in the graph G so G is a complete graph. In a complete graph apply the Kelly's theorem for the total number of minimum spanning trees in a complete graph G with n vertices is n^{n-2} so G has multiple distinct MSTs. The cost of a spanning tree is sum of all edges. If G contains n vertices then we will must stop the algorithm after adding the $n-1$ edges so each spanning tree have a cost $n-1$.

3.11 (a)

Option (a) is always true.
Option (b) is not true when e is not part of a cycle.
Option (c) is not true when e is part of a cycle and all edge weights are same in that cycle
Option (d) is need not be true when e is not part of a cycle.
Option (a) is always true as only the min weight edge in a cut set will be part of a minimum spanning tree.

3.12 (b)

Dijkstra's implementation for single source shortest path

(i) Using binary heap takes $O((|E|+|V|)\log|V|)$

(ii) Using Fibonacci heap takes $O(|E|+|V|\log|V|)$

3.13 (d)

The given problem is job scheduling problem 9 tasks are $T_1, T_2, \ldots, T_9$.
J is initially empty then according to deadlines it includes $\{T_1, T_2, T_3, T_5, T_7, T_8, T_9\}$.
So T_4 and T_6 can't be included in J

3.14 (a)

Total profit earn by algorithm
$= 15 + 20 + 30 + 18 + 23 + 16 + 25 = 147$

3.15 (a)

$$p(x) = a_0 + a_1x + a_2x^2 + a_3x^3$$

Apply Horner's rule to evaluate the p(x)

$$p(x) = a_0 + ((a_3x + a_2)x + a_1)x$$
$$t = a_3 * x$$
$$t = t + a_2$$
$$t = t * x$$
$$t = t + a_1$$
$$t = t * x$$
$$t = t + a_0$$

So, 3 minimum number of multiplications needed to evaluate p on an input x.

3.16 (b)

Given vertex set of $G = \{v_1, v_2, \ldots, v_n\}$
Weight of an edge $= 2|i\text{-}j| : (v_i, v_j) \in G$

$v_1 \underset{2}{\text{——}} v_2 \underset{2}{\text{——}} v_3 \underset{2}{\text{——}} v_4 \underset{2}{\text{——}} v_5 \ldots v_n$ is MST.

In the case of minimum spanning tree of a graph G we will add upto $(n - 1)$ vertices, so the weight of a minimum spanning tree of G

$$= \sum_{i=1}^{n-1} 2|V_i - V_{i-1}|$$
$$= 2\sum_{i=1}^{n-1} |V_i - V_{i-1}| = 2\sum_{i=1}^{n-1} |1|$$
$$= 2\,|n-1|$$
$$= 2(n-1) = 2n - 2$$

3.17 (c)

Heap and priority queue are very neat data structures allowing:

- Add an element to heap with an associated priority.
- Remove the element from the heap or priority queue that has the highest priority, and return it.
- Peak at the element with highest priority without removing it.

A simple way to implement a heap or priority queue data type is to keep a list of elements, and search through the list for the highest priority which gives O(n) time to implement Dijkstra's shortest path algorithm on unweighted graph.

3.18 (d)

In Kruskal's algorithm we will add the edges of minimum weight whether they are adjacent or not. Consider each choice separately

(a) (a – b), (d – f), (b – f), (d – c), (d, e)
1 + 1 + 2 + 2 + 3 = 9

(b) (a – b), (d – f), (d – c), (b – f), (d, e)
1 + 1 + 2 + 2 + 3 = 9

(c) (d – f), (a – b), (d – c), (b – f), (d, e)
1 + 1 + 2 + 2 + 3 = 9

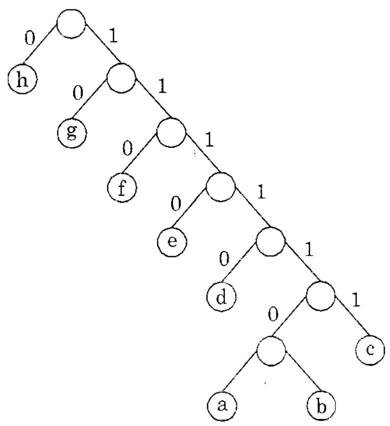
(d) (d – f), (a – b), (b – f), (d – e), (d – c)
1 + 1 + 2 + 3 + 2 = 9

In choice (d), we added (d – e) before (d – c) which violates algorithm, hence option (d) is not correct.

3.19 (c)

The two least frequent characters are taken as children of a newly made note and the frquency of the newly made node and the frequency of newly made node is made equal to the sum of those two childrens.

The same procedure is repeated till all nodes are finished.



3.20 (d)

In an unweighted graph performing a BFS starting from S takes O(n) time if graph contains n vertices. Dijkstra's algorithm only find the single source shortest path and takes $O(n^2)$ time but it is good for weighted graph. Warshall's algorithm for all pair shortest path takes $O(n^3)$ time but it is also applicable for weighted graph.

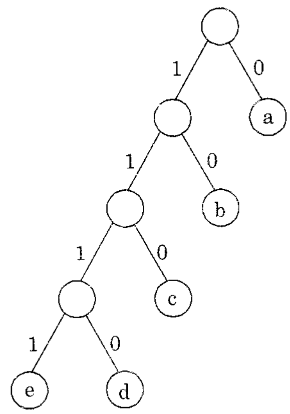
3.21 (d)

W be the minimum weight among all edge weights in an undirected connected graph. e is the specific edge of weight w. It may be possible that another edge in the graph having weight w which had been added to minimum spanning tree and when we add e to minimum spanning tree it form a simple circuit. So we can't include e in every minimum spanning tree.

3.22 (a)

The probabilities are $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}$

So, the Huffman code according to tree is unique. 0, 10, 110, 1110, 11110, 11111 represents a, b, c, d, e, f respectively.



3.23 (d)

The average length of the tree is

$$= 1 \times \frac{1}{2} + 2 \times \frac{1}{4} + 3 \times \frac{1}{8} + 4 \times \frac{1}{16} + 5 \times \frac{1}{32} = 1.781$$

3.24 (d)

The execution must follow SJF. Since we have to minimize $W_i * T_i$ where wi is a constant, so we can only minimize the product by atleast decreasing the T_i for a job by executing shorter jobs first because that will reduce completion time for the jobs that follow.

3.25 (c)

First thing that we note is that Prims Algorithm, during execution, always generates a subset of edges that is connected. So prim's algorithm always generates a connected component during its execution.

Second thing to note is that, only getting a connected graph does not suffice here. The edge chosen next must be the smallest possible edge as per GREEDY strategy. So let's check the options.

(a) (a, b) if fine .but second edge (d, f) is nowhere connected to previous edge. Therefore (a) is eliminated.

(b) I will straight away go to the point. In choosing edge (g, h) we scan previous edges and DON'T find 'g' or 'h' in any of the order pairs. So (g, h) is not adjacent to any of them. So (b) also eliminated. You can do this by marking the graph as well.

(c) This satisfies all the required properties.

(d) At every stage you will find the graph connected BUT let's check for other property: GREEDYNESS. When we choose (f, c) and (f, d) this order violate the greedy approach. Because (f, d) has cost 2 and should be included before (f, c). Therefore false.

3.26 (d)

Given graph G contains n vertices and $2n - 2 = 2(n - 1)$ edges. So spanning tree must contains $2n - 4$ edges. If G can be partitioned into two edge disjoint spanning tree. Then for any tree $2e = \sum_{i=1}^{n} d_i$ belongs to a tree if each d_i is positive and $e = n - 1$ but in given problem $e = 2n - 4 = 2(n - 2)$. So there is no two-vertex-disjoint paths between every pair of vertices.

3.27 (d)

Dijkstra's single source shortest path is not guaranteed to work for graphs with negative weight edges, but it works for the given graph.
Run the 1st pass: b1
b is minimum, so shortest distance to b is 1
After 1st pass, distances are: c3, e – 2
e is minimum, shortest distance to e is –2
After 2nd pass, distances are: c3, f0
f is minimum, so shortest distance to f is 0
After 3rd pass, distances are: c3, g3
Both are same, let us take g so shortest distance to g is 3
After 4th pass, distances are: c3, h5
c is minimum, so shortest distance to c is 3
After 5th pass, distance is: h – 2
h is minimum, so shortest distance to h is –2.

3.28 (b)

Bellman-Ford shortest path algorithm always finds any negative weighted cycle which is reachable from the source. If the negative weighted cycle is not reachable then, Bellman Ford algorithm not able to find that cycle.

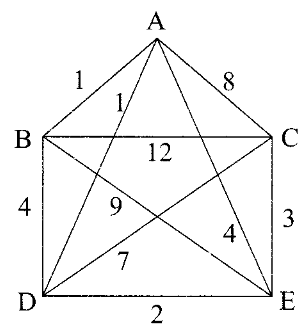
3.29 (d)

In Kruskal's algorithm, edges are added in non decreasing order, starting from lowest edge weight, as long as they don't form a cycle with already added edges.
In choice (d): (b, e) edge weight is 2 is added first. This step is ok, since 2 is smallest in edge weight. Edge (e, f) is then added, whose edge weight of 3, is next highest, and hence this is also ok.
Then edge (b, c) is added, whose edge weight is 4, and this is not ok is Kruskal's algorithm, since there is an edge (a, c) of edge weight 3, which is less than the edge weight of (b, c). So (a, c) has to be added before (b, c). Notice also that adding (a, c) does not form a cycle with already added edges.
∴ Choice (d) is not the sequence that would be allowed by Kruskal's algorithm.

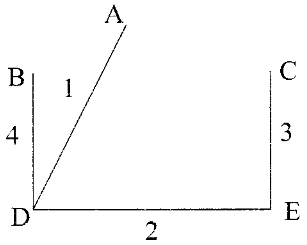
3.31 (d)

$$W = \begin{array}{c|ccccc} & A & B & C & D & E \\ A & 0 & 1 & 8 & 1 & 4 \\ B & 1 & 0 & 12 & 4 & 9 \\ C & 8 & 12 & 0 & 7 & 3 \\ D & 1 & 4 & 7 & 0 & 2 \\ E & 4 & 9 & 3 & 2 & 0 \end{array}$$

We draw a graph



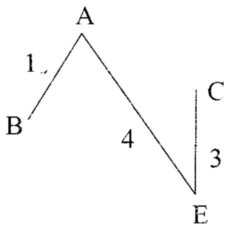
We have to make Minimum possible spanning tree having vertex A is a leaf node.



So Minimum possible weight
= 4 + 1 + 2 + 3 = 10

3.32 (b)

From vertex 1 to vertex 2 means from B to C. So we choose the path



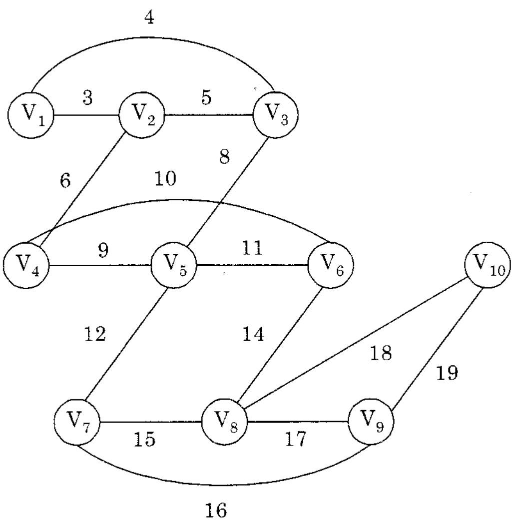
So weight = 1 + 4 + 3 = 8

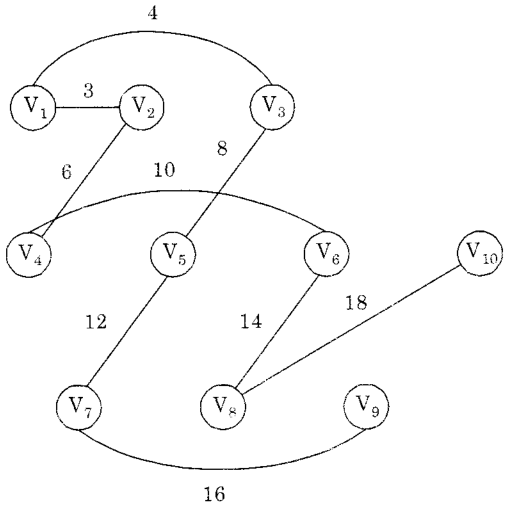
3.33 (b)

We observe a pattern in weight of MST being formed:
For n = 3 $\Rightarrow$ (1+2+3) + (1)
For n = 4 $\Rightarrow$ (1+2+3+4) + (1+2)
For n = 5 $\Rightarrow$ (1+2+3+4+5) + (1+2+3)
For n = 6 $\Rightarrow$ (1+2+3+4+5+6) + (1+2+3+4)
In general, total weight of MST for n:

$$= \sum_{i=1}^{n} i + \sum_{i=1}^{n-2} i = n^2 - n + 1$$

3.34 (c)





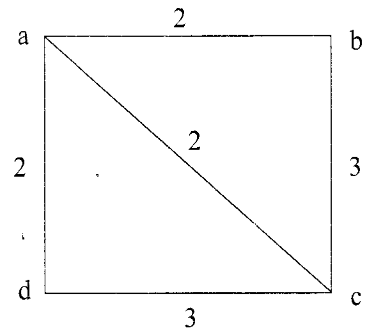
$$V_5 \rightarrow V_6 : (V_5 - V_3 - V_1 - V_2 - V_4 - V_6)$$
$$\Downarrow$$
$$31$$

$V_5 \rightarrow V_6 = 31$

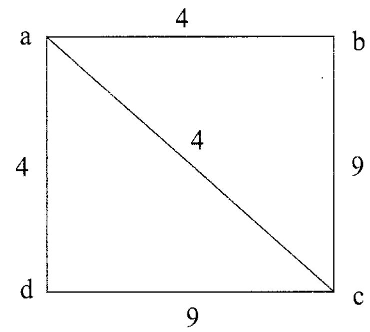
3.35 (d)

E.g.

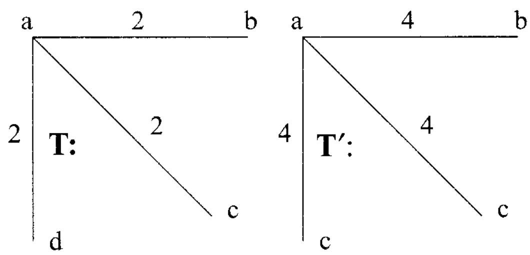
G:



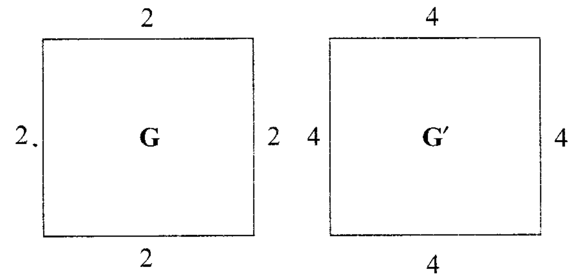
G′:



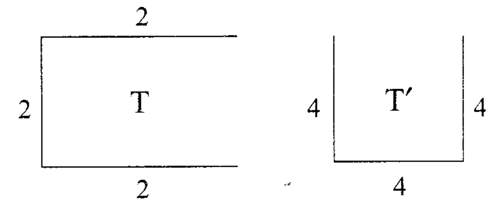
MST



E.g.



MST



∴ we can make different as well as same MST also.

∴ option (d) is correct.

3.36 (d)

	A	B	C	D	E	F	G	T	
S	4	3	∞	7	∞	∞	∞	∞	
B	4		∞	7	∞	∞	∞	∞	
A			5	7	∞	∞	∞	∞	
C				7	6	∞	∞	∞	
E				7		∞	8	10	⇒ SACET

∴ The shortest path from S → T is SACET.

3.37 (b)

In selection sort we starts with unsorted array, and mark the index from where we starts. Then we find the minimum element in the array and swap it with marked index. Now we mark the next index and again find the minimum in the array and swap with marked index.

In every iteration there is a swapping so in the worst case it will take O(n) swaps.

Option (b) is correct.

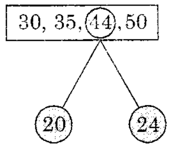
3.38 (b)

BFS traversal from *W* gives. Shortest path from *W* to every other vertex in the graph.

3.39 Sol.

Given files: 20, 24, 30, 35, 50

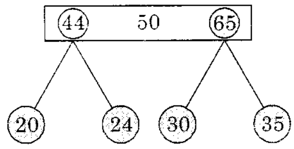
Step-I:



⇒ 43 Comparisons

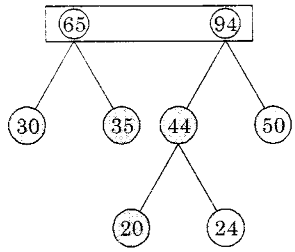
[20 + 24 – 1 comparisons maximum]

Step-II:



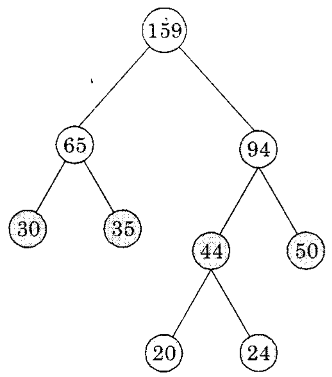
⇒ 64 Comparisons

Step-III:



⇒ 93 Comparisons

Step-IV:

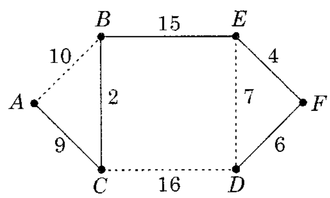


⇒ 158 Comparisons

∴ 158 + 93 + 64 + 43 = 358 comparisons

3.40 Sol.

|MST| = 36 and graph has distinct integer edge weights.

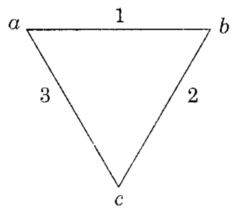


2 + 4 + 6 + 7 + 9 + 10 + 15 + 16 = 69

3.41 (a)

Statement P: Since every edge weight is positive and we increase the value of every edge weight by same constant values. So minimum spanning tree of G does not change.

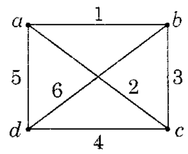
Statement Q: Taking an example:



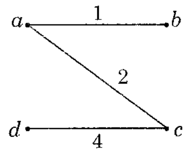
First path from 'a' to 'c' via 'b' have path value 3. But here path can be change 'a' to 'e' direct since paths value is same but path can be change.
So statement is wrong.

3.42 Sol.

Since graph will be complete graph contain 4 vertex and 6 edges with weight 1. 2, 3, 4, 5, 6.

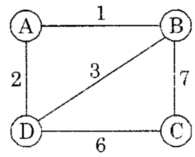


If we consider edge weights in such order, that form the cycle, then we have to choose two minimum we choose minimum value i.e., 4. So MST will be

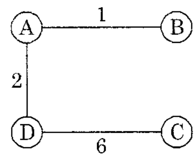


So max weight will be 1 + 2 + 4 = 7.

3.43 (b)



Here 'BD' is lightest edge in cycle 'BCD'. But here MST is not include edge 'BD' i.e.,



In second statement "if in any graph 'e' is the heaviest edge in then every MST must exclude it" is correct statement.

4 Dynamic Programming

4.1 A binary search tree is generated by inserting in order the following integers: 50, 15, 62, 5, 20, 58, 91, 3, 8, 37, 60, 24. The number of nodes in the left subtree and right subtree of the root respectively is

(a) (4, 7) (b) (7, 4)
(c) (8, 3) (d) (3. 8)

[1996 : 2 Marks]

4.2 Which one of the following algorithm design techniques is used in finding all pairs of shortest distances in a graph?

(a) Dynamic programming
(b) Backtracking
(c) Greedy
(d) Divide and Conquer

[1998 : 1 Mark]

4.3 A B-tree of order 4 is built from scratch by 10 successive insertions. What is the maximum number of node splitting operations that may take place?

(a) 3 (b) 4
(c) 5 (d) 6

[2008 : 2 Marks]

4.4 The subset-sum problem is defined as follows: Given a set S of n positive integers and a positive integer W; determine whether there is a subset of S whose elements sum to W.

An algorithm Q solves this problem in O(nW) time. Which of the following statements is false?

(a) Q solves the subset-sum problem in polynomial time when the input is encoded in unary
(b) Q solves the subset-sum problem is polynominal time when the input is encoded in binary
(c) The subset sum problem belongs to the class NP
(d) The subset sum problem in NP-hard

[2008 : 2 Marks]

Linked Answers Questions 4.5 and 4.6

The subset-sum problem is defined as follows. Given a set of n positive integers, $S = \{a_1, a_2, a_3, \ldots\ldots, a_n\}$ and a positive integer W is there a subset S whose elements sum of W? A dynamic program for solving this problem uses a 2-dimensional Boolean array, X with n rows and W-1 columns X [i, j], $1 \le i \le n$, $0 \le j \le W$, is TRUE if and only if there is a subset of $\{a_1, a_2\ldots\ldots, a_i\}$ whose elements sum to j.

4.5 Which of the following is valid for $2 \le i \le n$ and $a_i \le j \le W$?

(a) $X[i, j] = X[i-1, j] \vee X[i, j-a_i]$
(b) $X[i, j] = X[i-1, j] \vee X[i-1, j-a_i]$
(c) $X[i, j] = X[i-1, j] \wedge X[i, j-a_i]$
(d) $X[i, j] = X[i-1, j] \wedge X[i-1, j-a_i]$

[2008 : 2 Marks]

4.6 Which entry of the array X, if TRUE, implies that there is a subset whose elements sum to W?

(a) X [1, W] (b) X [n, 0]
(c) X [n, W] (d) X [n –1, n]

[2008 : 2 Marks]

Common Data for Question 4.7 and 4.8

A subsequence of a given sequence is just the given sequence with some elements (possibly none or all) left out. We are given two sequences X[m] and Y[n] of lengths m and n, respectively, with indexes of X and Y starting from 0.

4.7 We wish to find the length of the longest common subsequence (LCS) of X[m] and Y[n] as $l(m, n)$, where an incomplete recursive definition for the function $l(i, j)$ to compute the length of the LCS of X[m] and Y[n] is given below:

$l(i, j) = 0$, if either $i = 0$ or $j = 0$
= expr1, if $i, j > 0$ and $x[i-1] = Y[j-1]$
= expr2, if $i, j > 0$ and $x[i-1] \ne Y[j-1]$

Which one of the following options is correct?

(a) expr1 $\equiv l(i-1, j) + 1$
(b) expr1 $\equiv l(i, j-1)$
(c) expr2 $\equiv \max(l(i-1, j), l(i, j-1))$
(d) expr2 $\equiv \max(l(i-1, j-1), l(i, j))$

[2009 : 2 Marks]

4.8 The values of $l(i,j)$ could be obtained by dynamic programming based on the correct recursive definition of $l(i, j)$ of the form given above, using an array L [M, N], where M = m + 1 and N = n + 1, such that $L[i,j] = l(i, j)$.

Which one of the following statements would be TRUE regarding the dynamic programming solution for the recursive definition of $l(i, j)$?

(a) All elements of L should be initialized to 0 for the values of $l(i,j)$ to be properly computed
(b) The values of $l(i, j)$ may be computed in a row major order or column major order of L [M, N]
(c) The values of $l(i, j)$ cannot be computed in either row major order or column major order of L [M, N]
(d) L [p, q] needs to be computed before L [r, s] if either p < r or q < s

[2009 : 2 Marks]

4.9 An algorithm to find the length of the longest monotonically increasing sequence of numbers in an array A[0: n − 1] is given below.
Let L_i denotes the length of the longest monotonically increasing sequence starting at index i in the array
Initialize $L_{n-1} = 1$
For all i such that $0 \le i \le n - 2$

$$L_i = \begin{cases} 1 + L_{i+1}, & \text{If } A[i] < A[i+1] \\ 1 & \text{Otherwise} \end{cases}$$

Finally the length of the longest monotonically increasing sequence is $\text{Max}(L_0, L_1, \ldots\ldots, L_{n+1})$.
Which of the following statements is TRUE?

(a) The algorithm uses dynamic programming paradigm
(b) The algorithm has a linear complexity and uses branch and bound paradigm
(c) The algorithm has a non-linear polynomial complexity and uses branch and bound paradigm
(d) The algorithm uses divide and conquer paradigm

[2011 : 1 Mark]

4.10 What is the time complexity of Bellman -Ford single-source shortest path algorithm on a complete graph of n vertices?

(a) $\Theta(n^2)$ (b) $\Theta(n^2 \log n)$
(c) $\Theta(n^3)$ (d) $\Theta(n^3 \log n)$

[2013 : 1 Mark]

4.11 Consider two strings A = "$qpqrr$" and B = "$pqprqrp$". Let x be the length of the longest common subsequence (not necessarily contiguous) between A and B and let y be the number of such longest common subsequences between A and B. Then $x + 10y$ = ________.

[2014 (Set-2) : 2 Marks]

4.12 Consider the weighted undirected graph given by with 4 vertices, where the weight of edge $\{i, j\}$ is the entry W_{ij} in the matrix W.

$$W = \begin{bmatrix} 0 & 2 & 8 & 5 \\ 2 & 0 & 5 & 8 \\ 8 & 5 & 0 & x \\ 5 & 8 & x & 0 \end{bmatrix}$$

The largest possible integer value of x, for which at least one shortest path between some pair of vertices will contain the edge with weight x is________.

[2016 (Set-1) : 2 Marks]

4.13 The Floyd-Warshall algorithm for all-pair shortest paths computation is based on

(a) Greedy paradigm
(b) Divide-and-conquer paradigm
(c) Dynamic Programming paradigm
(d) Neither Greedy nor Divide-and-Conquer nor Dynamic Programming paradigm.

[2016 (Set-2) : 1 Mark]

4.14 Let A_1, A_2, A_3 and A_4 be four matrices of dimensions 10 × 5, 5 × 20, 20 × 10, and 10 × 5, respectively. The minimum number of scalar multiplications required to find the product $A_1 A_2 A_3 A_4$ using the basic matrix multiplication method is ______.

[2016 (Set-2) : 2 Marks]

■■■■

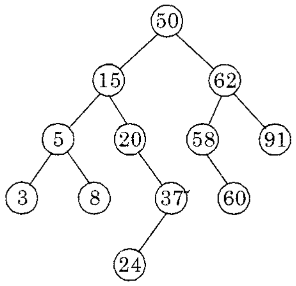
Answers Dynamic Programming

4.1 (b) 4.2 (a) 4.3 (a) 4.4 (d) 4.5 (b) 4.6 (c) 4.7 (c) 4.8 (b) 4.9 (a)
4.10 (c) 4.13 (c)

Explanations Dynamic Programming

4.1 (b)

The binary search tree is as follows:



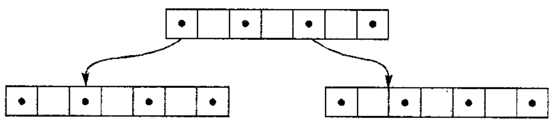
The number of nodes in the left subtree and right subtree of the root respectively is (7, 4). Therefore option (b) is correct.

4.2 (a)

Bellman-Ford algorithm is used to find all pairs shortest distances in a graph and it is dynamic programming technique.

4.3 (a)

A B-tree is similar to 2-3 tree. Consider a B-tree of order 4.



A B-tree of order m contains n records and if each contains b records on the average then the tree has about $\lceil n/b \rceil$ leaves. If we split k nodes along the path from leaves then

$$k \leq 1 + \log_{m/2} \lceil n/b \rceil$$

In given problem n = 10, b = 3, m = 4 So

$$k \leq 1 + \log_{4/2} \lceil 10/3 \rceil$$
$$k \leq 1 + \log_2 4$$
$$k \leq 1 + 2$$
$$k \leq 3$$

4.4 (d)

If an algorithm Q solves subset-sum problem in O(nW) time where w is an integer which is constant so Q solves the subset-sum problem in O(n) time because W is a constant and input must encoded in binary.

4.5 (b)

The subset-sum problem is NP-complete problem.

For example given a set $S = \{-7, -3, -2, 5, 8\}$ the algorithm determine whether the sum of the element of S′ is zero such that $S' \subseteq S$. For example if $S' = \{-3, -2, 5\}$ then return yes.

In given dynamic algorithm x is a two dimensional Boolean array contains n rows and W + 1 columns where W is a positive weight or sum.

$x[i, j]\ 1 \leq i \leq n,\ 0 \leq j \leq W$

The i^{th} row determines the element of subset of S, and j^{th} column determines the corresponding weight.

If $2 \leq i \leq n$ and $a \leq j \leq W$

Let subset $S' = \{a_1, a_2, .., a_i\}$ the weight as $a_1, ..., a_2$ is W.

So $X[i, j] = X[i-1, j] \vee X[i-1, j-a_i]$

The row starts from 2^{nd} position so $i = i - 1$. The column position starts from j but we will add upto a_ith position so $j = j - a_i$.

4.6 (c)

The entry X[n, W] is true, because we find the weight W of subset of S which contains n elements.

4.7 (c)

$l(i, j)$

$= 0$; if either $i = 0$ or $j = 0$

$= l(i-1, j-1)$;

if $i, j > 0$ and $x[i-1] = Y[j-1]$

$= \max(l(i-1, j), l(i, j-1))$;

if $i, j > 0$ and $x[i-1] \neq Y[j-1]$

$\therefore$ expr 2 $\equiv \max(l(i-1, j), l(i, j-1))$

4.8 (b)

$l(i, j)$ values can be computed in a row major order or column major order of $L[M, N]$.

4.9 (a)

The algorithm uses dynamic programming paradigm.

4.10 (c)

In a complete graph total no of edges is

$$= \frac{n(n-1)}{2}$$

Time complexity of Bellman-ford algo for a graph having n vertices and m edges = O(nm)
for a complete graph time complexity

$$O\left(n \times \frac{n(n-1)}{2}\right) = O(n^3)$$

So, option (c) is correct.

4.11 Sol.

$A = qpqrr$

$B = pqprqrp$

LCS $(A, B) = qpqr$

(or) $= qprr$

(or) $= pqrr$

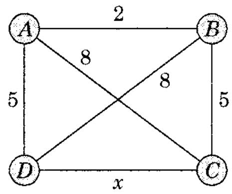
x = length of LCS $(A, B) = 4$

y = # LCS $(A, B) = 3$

$\therefore\ x + 10y = 4 + 10 \times 3 = 34$

4.12 Sol.

By drawing weighted undirected graph with 4 vertices from given matrix is



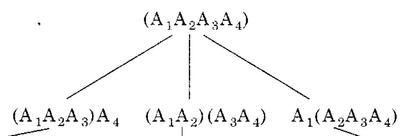
By analysis given graph we see their are two paths i.e., one is '$C\,B\,A\,D$' and '$C\,A\,D$' having weight 12 and 13 respectively to reach C to D.

Since we have to find atleast 1 (means more than 1 or 1) sortest path between C to D. So we have to consider 12 as value of x, by doing this we have atleast 1 path between C to D i.e., 1 path $CBAD$ and 2nd path CD having weight 12 each.

4.13 (c)

Floyd Warshall algorithm for all pair shortest paths computation is based on dynamic programming paradigm.

4.14 Sol.



1. $10 \times 5 \times 5 \times 20 = 1000$ (10×20)	1. $10 \times 5 \times 5 \times 20 = 1000$ (10×20)	1. $5 \times 20 \times 20 \times 10 = 1000$ (5×10)
2. $10 \times 20 \times 20 \times 10 = 2000$ (10×10)	2. $20 \times 10 \times 10 \times 5 = 1000$ (20×5)	2. $5 \times 10 \times 10 \times 5 = 250$ (5×10)
3. $10 \times 10 \times 10 \times 5 = 500$ (10×5)	3. $10 \times 20 \times 20 \times 5 = 1000$ (10×5)	3. $5 \times 10 \times 10 \times 5 = 250$ (5×10)
3500	3000	1500

The minimum number of multiplication required using basic matrix multiplication method will be 1500.

■■■■

5 P & NP Concepts

5.1 A problem in NP is NP-complete if
(a) it can be reduced to the 3-SAT problem in polynomial time
(b) the 3-SAT problem can be reduced to it in polynomial time
(c) it can be reduced to any other problem in NP in polynomial time
(d) some problem in NP can be reduced to it in polynomial time

[2006 : 1 Mark]

5.2 For problems X and Y, Y is NP-complete and X reduces to Y in polynomial time. Which of the following is TRUE?
(a) If X can be solved in polynomial time, then so can Y
(b) X is NP-complete
(c) X is NP-hard
(d) X is in NP, but not necessarily NP-complete

[2008 : 1 Mark]

5.3 Let π_A be a problem that belongs to the class NP. Then which one of the following is TRUE?
(a) There is no polynomial time algorithm for π_A
(b) If π_A can be solved deterministically in polynomial time, then $P = NP$
(c) If π_A is NP-hard, then it is NP-complete
(d) π_A may be undecidable

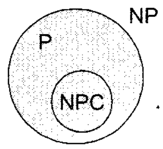
[2009 : 1 Mark]

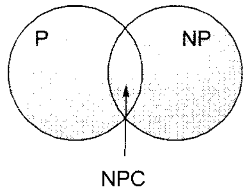
5.4 Which of the following statements are TRUE?
1. The problem of determining whether there exists a cycle in an undirected graph is in P.
2. The problem of determining whether there exists a cycle in an undirected graph is in NP.
3. If a problem A is NP-Complete, there exists a non-deterministic polynomial time algorithm to solve A.

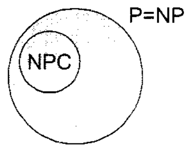
(a) 1, 2 and 3 (b) 1 and 3 only
(c) 2 only (d) 3 only

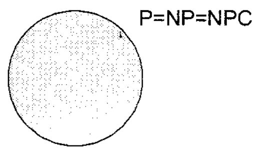
[2013 : 1 Mark]

5.5 Suppose a polynomial time algorithm is discovered that correctly computes the largest clique in a given graph. In this scenario, which one of the following represents the correct Venn diagram of the complexity classes P, NP and NP Complete (NPC)?

(a)


(b)


(c)


(d)


[2014 (Set-1) : 2 Marks]

5.6 Consider the decision problem *2CNFSAT* defined as follows:

$\{\Phi \mid \Phi$ is a satisfiable propositional formula in CNF with at most two literals per clause$\}$

For example, $\Phi = (x_1 \vee x_2) \wedge (x_1 \vee \overline{x_3}) \wedge (x_2 \vee x_4)$ is a Boolean formula and it is in *2CNFSAT*. The decision problem *2CNFSAT* is
(a) NP-Complete.
(b) solvable in polynomial time by reduction to directed graph reachability.
(c) solvable in constant time since any input instance is satisfiable.
(d) NP-hard, but not NP-complete.

[2014 (Set-3) : 2 Marks]

5.7 Consider two decision problems Q_1, Q_2 such that Q_1 reduces in polynomial time to 3-SAT and 3-SAT reduces in polynomial time to Q_2. Then which one of the following is consistent with the above statement?

(a) Q_1 is in NP, Q_2 is NP hard
(b) Q_2 is in NP, Q_1 is NP hard
(c) Both Q_1 and Q_2 are in NP
(d) Both Q_1 and Q_2 are NP hard

[2015 (Set-2) : 1 Mark]

5.8 Language L_1 is polynomial time reducible to language L_2. Language L_3 is polynomial time reducible to L_2, which in turn is polynomial time reducible to language L_4. Which of the following is/are True?

I. If $L_4 \in P$, $L_2 \in P$
II. If $L_1 \in P$ or $L_3 \in P$, then $L_2 \in P$
III. $L_1 \in P$, if and only if $L_3 \in P$
IV. If $L_4 \in P$, then $L_1 \in P$ and $L_3 \in P$

(a) II only
(b) III only
(c) I and IV only
(d) I only

[2015 (Set-3) : 2 Marks]

Answers P & NP Concepts

5.1	(a)	5.2	(b)	5.3	(c)	5.4	(a)	5.5	(d)	5.6	(b)	5.7	(a)	5.8	(c)

Explanations P & NP Concepts

5.1 (a)

3-SAT being an NPC problem, reducing NP problem to 3-SAT would mean that NP problem is NPC.

5.2 (b)

X is reducible to NPC. Hence X is also NPC.

5.3 (c)

It is given that $\pi_A \in NP$

$\therefore$ If π_A is NP- hard, and since it is given that $\pi_A \in NP$, this means that π_A is NP-complete

$\therefore$ choice (c) is correct.

Notice that choice (a) is incorrect since as $P \in NP$, some NP problems are actually P, and hence polynomial time algorithm can exist for these.

Choice (b) is incorrect since, If π_A can be solved deterministically in polynomial time, it does not generate that P=NP, unless of-course it is additionally known that π_A is NP-complete.

Choice (d) is incorrect since,

All problems belonging to P or NP have to be decidable.

5.4 (a)

1. By depth first search we can find whether there exits a cycle in an undirected graph in $O(n^2)$ times so it is P problem.
2. As $P \subseteq NP$ so this problem will also be considered as NP problem.
3. This is the definition of NP-complete so it is trivially true.

So option (a) is correct

5.5 (d)

Clique is NP-complete problem

If clique is P then P = NPC

$\therefore$ P = NP = NPC

5.6 (b)

2CNF is P-problem.

Therefore it is solvable in polynomial time by reduction to directed graph reachability.

5.7 (a)

Given $Q_1 \leq$ 3-SAT and 3-SAT $\leq Q_2$

3-SAT is NP-complete.

$\therefore$ It is NP as well as NP-hard.

$Q_1 \leq$ 3-SAT and 3-SAT is NP $\Rightarrow Q_1$ is NP-problem

3-SAT $\leq$ Q2 and 3-SAT is NP-hard. $\Rightarrow Q_2$ is NP-hard problem.

So the strongest statements is Q_1 is in NP and Q_2 is NP-hard.

5.8 (c)

$L_1 \leq_P L_2$, $L_3 \leq_P L_2$, and $L_2 \leq_P L_4$

I. If $L_4 \in P$, $L_2 \in P$ is **true**

$L_2 \leq_P L_4$

II. If $L_1 \in P$ or $L_3 \in P$, then $L_2 \in P$ is **false**

$L_1 \leq_P L_2$, and $L_3 \leq_P L_2$

III. $L_1 \in P$, if and only if $L_3 \in P$ is **false**

If $L_1 \in P$ we cannot say anything about L_2 from $L_1 \leq_P L_2$

Since we cannot say anything about L_2 therefore we cannot say anything about L_3 either from $L_3 \leq_P L_2$.

So Statement III is **false**.

IV. If $L_4 \in P$, then $L_1 \in P$ and $L_3 \in P$ is **true**.

If $L_4 \in P$ then from $L_2 \leq_P L_4$ we can get that $L_2 \in P$.

and from $L_3 \leq_P L_2$ we can now get that $L_3 \in P$

also from $L_1 \leq_P L_2$ we can get that $L_1 \in P$

So Statement IV is **true**.

■■■■

6 Miscellaneous Topics

6.1 Match the pairs in the following:

List-I

A. Straseen's matrix multiplication algorithm
B. Kruskal's minimum spanning tree algorithm
C. Biconnected components algorithm
D. Floyd's shortest path algorithm

List-II

P. Greedy method
Q. Dynamic programming
R. Divide and Conquer
S. Depth first search

[1990 : 2 Marks]

6.2 The minimum number of comparisons required to sort 5 elements is

[1991 : 2 Marks]

6.3 Following algorithm(s) can be used to sort n integers in the range [1 ... n^3] in O(n) time

(a) Heap sort (b) Quick sort
(c) Merge sort (d) Radix sort

[1992 : 2 Marks]

6.4 The minimum number of interchanges needed to convert the array
89, 19, 40, 17, 12, 10, 2, 5, 7, 11, 6, 9, 70 into a heap with maximum element at the root is

(a) 0 (b) 1
(c) 2 (d) 3

[1996 : 2 Marks]

6.5 The average number of key comparisons done on a successful sequential search in list of length n is

(a) log n (b) $\frac{n-1}{2}$
(c) n/2 (d) $\frac{n+1}{2}$

[1996 : 2 Marks]

6.6 The correct matching for the following pairs is

List-I

A. All pairs shortest paths
B. Quick Sort
C. Minimum weight spanning tree
D. Connected Components

List-II

1. Greedy
2. Depth-First search
3. Dynamic Programming
4. Divide and Conquer

Codes:

(a) A – 2 B – 4 C – 1 D – 3
(b) A – 3 B – 4 C – 1 D – 2
(c) A – 3 B – 4 C – 2 D – 1
(d) A – 4 B – 1 C – 2 D – 3

[1997 : 1 Mark]

6.7 Give the correct matching for the following pairs:

List-I	List-II
A. O(log n)	**P.** Selection
B. O(n)	**Q.** Insertion sort
C. O(n log n)	**R.** Binary search
D. O(n^2)	**S.** Merge sort

Codes:

(a) A – R B – P C – Q D – S
(b) A – R B – P C – S D – Q
(c) A – P B – R C – S D – Q
(d) A – P B – S C – R D – Q

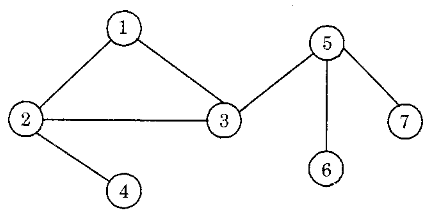
[1998 : 1 Mark]

6.8 Suppose we want to arrange the n numbers stored in an array such that all negative values occur before all positive ones. Minimum number of exchanges required in the worst case is

(a) n – 1 (b) n
(c) n + 1 (d) None of these

[1999 : 1 Mark]

6.9 The number of articulation points of the following graph is



(a) 0 (b) 1
(c) 2 (d) 3

[1999 : 1 Mark]

6.10 If $T_1 = O(1)$, give the correct matching for the following pairs:

(m) $T_n = T_{n-1} + n$ (u) $T_n = O(n)$

(n) $T_n = T_{\frac{n}{2}} + n$ (v) $T_n = O(n \log n)$

(o) $T_n = T_{\frac{n}{2}} + n \log n$ (w) $T_n = O(n^2)$

(p) $T_n = T_{n-1} + \log n$ (x) $T_n = O(\log^2 n)$

(a) m – w, n – v, o – u, p – x
(b) m – w, n – u, o – x, p – v
(c) m – v, n – w, o – x, p – u
(d) m – w, n – u, o – v, p – x

[1999 : 2 Marks]

6.11 Lt G be an undirected graph. Consider a depth-first traversal of G, and let T be the resulting depth-first search tree. Let u be a vertex in G and let v be the first new (unvisited) vertex visited after visiting u in the traversal. Which of the following statements is always true?

(a) {u,v} must be an edge in G, and u is a descendant of v in T
(b) {u,v} must be an edge in G, and v is a descendant of u in T
(c) If {u,v} is not an edge in G then u is a leaf in T
(d) If {u,v} is not an edge in G then u and v must have the same parent in T

[2000 : 2 Marks]

6.12 Consider any array representation of an n element binary heap where the elements are stored from index 1 to index n of the array. For the element stored at index i of the array ($i \le n$), the index of the parent is

(a) $i - 1$ (b) $\left\lfloor \frac{i}{2} \right\rfloor$

(c) $\left\lceil \frac{i}{2} \right\rceil$ (d) $\frac{(i+1)}{2}$

[2001 : 1 Mark]

6.13 Consider an undirected unweighted graph G. Let a breadth-first traversal of G be done starting from a node r. Let d(r, u) and d(r, v) be the lengths of the shortest paths from r to u and v respectively in G. If u is visited before v during the breadth-first traversal, which of the following statement is correct?

(a) $d(r, u) < d(r, v)$ (b) $d(r, u) > d(r, v)$
(c) $d(r, u) \le d(r, v)$ (d) None of these

[2001 : 2 Marks]

6.14 The minimum number of colours required to colour the vertices of a cycle with n nodes in such a way that no two adjacent nodes have the same colour is

(a) 2 (b) 3
(c) 4 (d) $n - 2\left\lfloor \frac{n}{2} \right\rfloor + 2$

[2002 : 1 Mark]

Common Data for Q. 6.15 & Q. 6.16

In a permutation $a_1 \ldots a_n$ of n distinct integers, an inversion is a pair (a_i, a_j) such that $i < j$ and $a_i > a_j$.

6.15 If all permutations are equally likely, what is the expected number of inversions in a randomly chosen permutation of 1...n?

(a) $n(n-1)/2$ (b) $n(n-1)/4$
(c) $n(n+1)/4$ (d) $2n[\log_2 n]$

[2003 : 2 Marks]

6.16 What would be the worst case time complexity of the insertion sort algorithm, if the inputs are restricted to permutations of 1...n with at most n inversions?

(a) $\Theta(n^2)$ (b) $\Theta(n \log n)$
(c) $\Theta(n^{1.5})$ (d) $\Theta(n)$

[2003 : 2 Marks]

6.17 The cube root of a natural number n is defined as the largest natural number m such that $m^3 \le n$. The complexity of computing the cube root of n (n is represented in binary notation) is

(a) $O(n)$ but not $O(n^{0.5})$
(b) $O(n^{0.5})$ but not $O((\log n)^k)$ for any constant $k > 0$.
(c) $O((\log n)^k)$ for some constant $k > 0$, but not $O((\log \log n)^m)$ for any constant $m > 0$
(d) $O((\log \log n)^k)$ for some constant $k > 0.5$, but not $O((\log \log n)^{0.5})$

[2003 : 2 Marks]

6.18 Two matrices M_1 and M_2 are to be stored in arrays A and B respectively. Each array can be stored either in row-major or column-major order in contiguous memory locations. The time complexity of an algorithm to compute $M_1 \times M_2$ will be

(a) best if A is in row-major, and B is in column-major order
(b) best if both are in row-major order
(c) best if both are in column-major order
(d) independent of the storage scheme

[2004 : 2 Marks]

6.19 Suppose each set is represented as a linked list with elements in arbitrary order. Which of the operations among union, intersection, membership, cardinality will be the slowest?
(a) union only
(b) intersection, membership
(c) membership, cardinality
(d) union, intersection

[2004 : 2 Marks]

6.20 A program takes as input a balanced binary search tree with n leaf nodes and computes the value of a function g(x) for each node x. If the cost of computing g(x) is min(number of leaf-nodes in left-subtree of x, number of leaf-nodes in right-subtree of x) then the worst case time complexity of the program is
(a) $\Theta(n)$
(b) $O(n \log n)$
(c) $O(n)^2$
(d) $O(2^n)$

[2004 : 2 Marks]

6.21 A scheme for storing binary trees in an array X is as follows. Indexing of X starts at 1 instead of 0. The root is stored at X[1]. For a node stored at X[i], the left child, if any, is stored in X[2i] and the right child, if any, in X[2i + 1]. To be able to store any binary tree on n vertices, the minimum size of X should be
(a) $\log_2 n$
(b) n
(c) 2n + 1
(d) 2n

[2006 : 1 Mark]

6.22 Which one the following inplace sorting algorithms needs the minimum number of swaps?
(a) Quicksort
(b) Insertion sort
(c) Selection sort
(d) Heap sort

[2006 : 1 Mark]

6.23 An element in an array X is called a leader if it is greater than all elements to the right of it in X. The best algorithm to find all leaders in an array.
(a) Solves it in linear time using a left to right pass of the array
(b) Solves in linear time using a right to left pass
(c) Solves it is using divide and conquer in time $\Theta(n \log n)$
(d) Solves it in time $\Theta(n^2)$

[2006 : 1 Mark]

6.24 A set X can be represented by an array x[n] as follows

$$x[i] = \begin{cases} 1; & \text{if } i \in X \\ 0; & \text{otherwise} \end{cases}$$

Consider the following algorithm in which x, y, and z are boolean arrays of size n:

```
algorithm zzz (x[ ], y[ ], z[ ])
{
  int i;
  for (i = 0; i < n; ++ i)
    z [i] = (x [i] ∧ ~ y[i]) ∨ (~x [i] ∧y[i]);
}
```

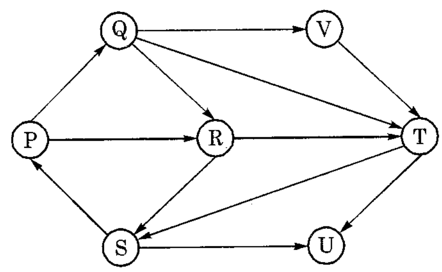
The set Z computed by the algorithm is
(a) $(X \cup Y)$
(b) $(X \cap Y)$
(c) $(X - Y) \cap (Y - X)$
(d) $(X - Y) \cup (Y - X)$

[2006 : 2 Marks]

6.25 Given two arrays of numbers $a_1,..., a_n$ and $b_1,..., b_n$ where each number is 0 or 1, the fastest algorithm to find the largest span (i, j) such that $a_i + a_{i+1} +...+ a_j = b_i + b_{i+1} +...+ b_j$, or report that there is no such span,
(a) Takes $O(3^n)$ and $\Omega(2^n)$ time if hashing is permitted
(b) Takes $O(n^3)$ and $\Omega(n^{2.5})$ time in the key comparison model
(c) Takes $\Theta(n)$ time and space
(d) Takes $O(\sqrt{n})$ time only if the sum of the 2n elements is an even number

[2006 : 2 Marks]

6.26 Which of the following is the correct decomposition of the directed graph given below into its strongly connected components?



(a) {P, Q, R, S}, {T}, {U}, {V}
(b) {P,Q, R, S, T, V}, {U}
(c) {P, Q, S, T, V}, {R}, {U}
(d) {P, Q, R, S, T, U, V}

[2006 : 2 Marks]

6.27 Consider the depth-first-search of an undirected graph with 3 vertices P, Q, and R. Let discovery time d(u) represent the time instant when the vertex u is first visited, and finish time f(u) represent the time instant when the vertex u is last visited. Given that

d(P) = 5 units, f(P) = 12 units
d(Q) = 6 units, f(Q) = 10 units
d(R) = 14 unit, f(R) = 18 units

which one of the following statements is TRUE about the graph

(a) There is only one connected component
(b) There are two connected components, and P and R are connected
(c) There are two connected components, and Q and R are connected
(d) There are two connected components, and P and Q are connected

[2006 : 2 Marks]

Directions for Question 6.28 to 6.30:

An array X of n distinct integers is interpreted as a complete binary tree. The index of the first element of the array is 0.

6.28 The index of the parent of element X [i], $i \neq 0$, is

(a) $\lfloor i / 2 \rfloor$ (b) $\lceil (i-1)/2 \rceil$
(c) $\lceil i/2 \rceil$ (d) $\lceil i/2 \rceil - 1$

[2006 : 2 Marks]

6.29 If only the root node does not satisfy the heap property, the algorithm to convert the complete binary tree into a heap has the best asymptotic time complexity of

(a) O(n) (b) O(log n)
(c) O(n log n) (d) O(n loglog n)

[2006 : 2 Marks]

6.30 If the root node is at level 0, the level of element X[i], $i \neq 0$, is

(a) $\lfloor \log_2 i \rfloor$ (b) $\lceil \log_2 (i+1) \rceil$
(c) $\lfloor \log_2 (i+1) \rfloor$ (d) $\lceil \log_2 i \rceil$

[2006 : 2 Marks]

6.31 A depth-first search is performed on a directed acyclic graph. Let d[u] denote the time at which vertex u is visited for the first time and f[u] the time at which the dfs call to the vertex u terminates. Which of the following statements is always true for all edges (u, v) in the graph?

(a) d[u] < d[v] (b) d[u] < f[v]
(c) f[u] < f[v] (d) f[u] > f[v]

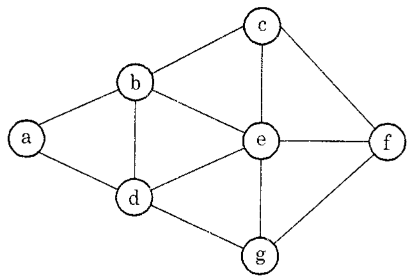
[2007 : 2 Marks]

6.32 If we use Radix Sort to sort n integers in the range $(n^{k/12}, n^k]$, for some $k > 0$ which is independent of n, the time taken would be

(a) $\Theta(n)$ (b) $\Theta(kn)$
(c) $\Theta(n \log n)$ (d) $\Theta(n^2)$

[2008 : 2 Marks]

6.33 Consider the following sequence of nodes for the undirected graph given below.



1. a b e f d g c 2. a b e f c g d
3. a d g e b c f 4. a d b c g e f

A Depth First Search (DFS) is started at node *a*. The nodes are listed in the order they are first visited. Which all of the above is (are) possible output(s)?

(a) 1 and 3 only (b) 2 and 3 only
(c) 2, 3 and 4 only (d) 1, 2 and 3 only

[2008 : 2 Marks]

6.34 Two alternative packages A and B are available for processing a database having 10^k records. Package A requires $0.0001\ n^2$ time units and package B requires $10n\log_{10}n$ time units to process n records. What is the smallest value of k for which package B will be preferred over A?

(a) 12 (b) 10
(c) 6 (d) 5

[2010 : 1 Mark]

6.35 The number of elements that can be stored in $\Theta(\log n)$ time using heap sort is

(a) $\Theta(1)$

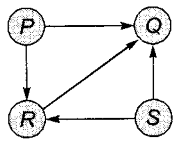
(b) $\Theta\left(\sqrt{\log n}\right)$

(c) $\Theta\left(\frac{\log n}{\log \log n}\right)$

(d) $\Theta(\log n)$

[2013 : 2 Marks]

6.36 Consider the directed graph given below.



Which one of the following is TRUE?

(a) The graph does not have any topological ordering.

(b) Both PQRS and SRQP are topological orderings.

(c) Both PSRQ and SPRQ are topological orderings.

(d) PSRQ is the only topological ordering.

[2014 (Set-1) : 1 Mark]

6.37 Suppose you want to move from 0 to 100 on the number line. In each step, you either move right by a unit distance or you take a *shortcut*. A shortcut is simply a pre-specified pair of integers i, j with $i < j$. Given a shortcut i, j if you are at position i on the number line, you may directly move to j. Suppose $T(k)$ denotes the smallest number of steps needed to move from k to 100. Suppose further that there is at most 1 shortcut involving any number, and in particular from 9 there is a shortcut to 15.

Let y and z be such that $T(9) = 1 + \min(T(y), T(z))$. Then the value of the product yz is ______.

[2014 (Set-3) : 2 Marks]

6.38 Match the following

List-I

A. Prim's algorithm for minimum spanning tree

B. Floyd-Warshall algorithm for all pairs shortest paths

C. Mergesort

D. Hamiltonian circuit

List-II

1. Backtracking

2. Greedy method

3. Dynamic programming

4. Divide and conquer

Codes:

	A	B	C	D
(a)	3	2	4	1
(b)	1	2	4	3
(c)	2	3	4	1
(d)	2	1	3	4

[2015 (Set-1) : 1 Mark]

6.39 Given below are some algorithms, and some algorithm design paradigms.

List-I

A. Dijkstra's Shortest Path

B. Floyd-Warshall algorithm to compute all pairs shortest path

C. Binary search on a sorted array

D. Backtracking search on a graph

List-II

1. Divide and Conquer

2. Dynamic Programming

3. Greedy design

4. Depth-first search

5. Breadth-first search

Match the above algorithms **(List-I)** to the corresponding design paradigm **(List-II)** they follow.

Codes:

	A	B	C	D
(a)	1	3	1	5
(b)	3	3	1	5
(c)	3	2	1	4
(d)	3	2	1	5

[2015 (Set-2) : 2 Marks]

Answers Miscellaneous Topics

6.3	(d)	6.4	(c)	6.5	(d)	6.6	(b)	6.7	(b)	6.8	(d)	6.9	(d)	6.10	(d)	6.11	(c)
6.12	(b)	6.13	(c)	6.14	(d)	6.15	(b)	6.16	(d)	6.17	(c)	6.18	(d)	6.19	(d)	6.20	(a)
6.21	(b)	6.22	(c)	6.23	(b)	6.24	(d)	6.25	(c)	6.26	(c)	6.27	(d)	6.28	(a)	6.29	(b)
6.30	(c)	6.31	(a)	6.32	(b)	6.33	(b)	6.34	(c)	6.35	(c)	6.36	(c)	6.38	(c)	6.39	(c)

Explanations Miscellaneous Topics

6.1 Sol.

A-R, B-P, C-S, D-Q

6.2 Sol.

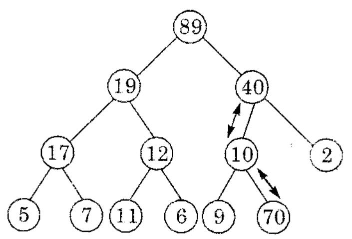
Use either selection or insertion sort, minimum comparisons is 4.

6.3 (d)

Radix sort take O(n) time and other takes O(n log n).

6.4 (c)

Tree representation of the given array is



So two exchanges are required.

6.5 (d)

$$\text{Average case} = \frac{O(1)+n}{2} = \frac{n+1}{2}$$

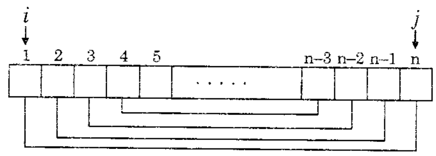
6.6 (b)

All pairs shortest paths → Dynamic programming
Quick sort → Divide and conquer
Minimum weight spanining tree → Greedy technique
Connected components → Depth-First-Search

6.7 (b)

Binary ← O(logn)
[Linear search] Selection ← O(n)
Merge sort ← O(n log n)
Insertion sort ← $O(n^2)$

6.8 (d)



In worst case, minimum exchanges required = $\left\lfloor \frac{n}{2} \right\rfloor$

(i) If (A[j] < A[j] && A[i] < 0) {i++;}
(ii) If (A[j] > A[i] && A[j] > 0) {j– –;}

Sorting is not needed, keep all negatives before positives. Worst case occurs when i and j reaches to middle position.

We require $\left\lfloor \frac{n}{2} \right\rfloor$ exchanges if array has first n/2 elements are positive and last n/2 elements are negative exactly.

6.9 (d)

Articulation point is that by removing that vertex, graph must be in disconnected state. 2, 3 and 5 are articulation points and also called as cut vertices.

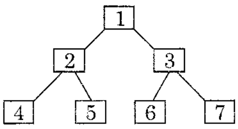
6.10 (d)

$T(n) = T(n-1)+n \to T(n) = O(n^2)$
$T(n) = T(n/2)+n \to T(n) = O(n)$
$T(n) = T(n/2)+n \log n \to T(n) = O(n \log n)$
$T(n) = T(n-1)+ \log n \to T(n) = O(n \log n)$

6.11 (c)

Draw some random graph and try to verify the options.

6.12 (b)



Partent (i) = $\left\lfloor \frac{i}{2} \right\rfloor$

Example:

Parent (7) = $\left\lfloor \frac{7}{2} \right\rfloor = 3$

Parent (6) = $\left\lfloor \frac{6}{2} \right\rfloor = 3$

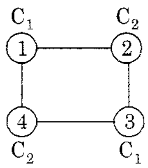
6.13 (c)

If u is visited before v, d(r, u) ≤ d(r, v) i.e., r to u is having shortest or equal length compared from r to v.

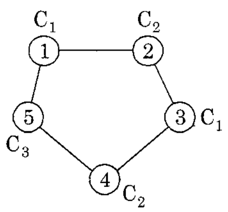
6.14 (d)

Cycle may contain even or odd number of vertices

(i) Let n = even. Then exactly 2 colors are required minimum



(ii) Let n = odd



Then 3 colors are required to color the cycle with odd.

Minimum number of colors = $n - 2\left\lfloor \frac{n}{2} \right\rfloor + 2$.

6.15 (b)

Let $a_1, a_2, a_3, \ldots, a_n$ be a permutation of a set {1, 2, ..., n}. If i <j and $a_i > a_j$ the pair (a_i, a_j) is called an inversion of the permutation. For example the permutation 3142 has three inversions : (3, 1), (3, 2) and (4, 2). If all permutations are equally likely then according to "O. Terquen" the average number of inversions in a random permutation is

$$= \frac{1}{2}\binom{n}{2} = \frac{1}{2}({}^nC_2)$$

$$= \frac{1}{2}\frac{n!}{2!\,(n-2)!}$$

$$= \frac{1}{2}\frac{n(n-1)\,(n-2)!}{2\cdot (n-2)!}$$

$$= \frac{n(n-1)}{4}$$

6.16 (d)

Insertion sort runs in O(n + f(n)) time where f(n) denotes number of inversions.

Therefore time complexity is Θ(n).

6.17 (c)

The cube root of a natural number n is defined as the largest natural number m such that $m^3 \leq n$. Complexity of computing the cube root of n is $O((\log n)^k)$ for some constant k > 0.

6.18 (d)

Access time equal for all elements if it is stored in array. Accessing any element takes constant time.

6.19 (d)

If each set is represented as a linked list with elements in arbitrary order. Membership cardinality takes O(1) time for an element in the set so the operation is very fast.

In the case of intersection, Let A and B are two sets. An operation such as A ∩ B requires time at least proportional to the sum of the sizes of the two sets. Since the list representing A and the list representant B must each be scanned at least once.

Similarly operation A ∩ B requires time at least proportional to the sum of the set sizes, since we must check for the some element appearing in both sets and delete one instance of each such element.

The well known algorithm using 2-3 tree (ordered or unordered) takes O(nlogn) if set contains n elements for union and intersection operation so it is slowest operation.

6.20 (a)

g(x) is min {number of leaf nodes in left subtree of x, number of leaf nodes in right subtree of x}. If balanced binary search tree is used and contains n nodes then in worst case time complexity of g(x) is O(n).

6.21 (b)

The root of the tree is stored at X[1],
Left of X[i] is stored at X[2i],
Right of X[i] is stored at X[2i + 1].
If tree is complete binary tree minimum it require n consecutive memory locations of x, x[1],...,x[n – 1], x[n].

6.22 (c)

In selection sort, minimum number of swaps taken compared to other sorting techniques.
Number of swaps = O(n)
Number of comparisons = $O(n^2)$
In finds the minimum of the subarray and exchanges only once in every pass.

6.23 (b)

Scan the array from right and select the leader. Now compare each element with the leader and find new leader. So it will take complete one scan of an array.
So it is in linear time only.

6.24 (d)

In given algorithm the for loop contains a logical expression

$z[i] = (x[i] \wedge \sim y[i]) \vee (\sim x[i] \wedge y[i])$

The equivalent set representation of given logical expression if we assume z[i] = z, x[i] = x and y[i] = y then

$z = (x \cap y') \cup (x' - y)$

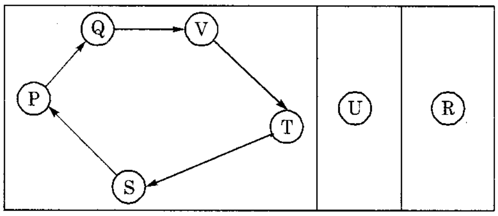
$z = (x - y) \cup (y - x) \quad [A \cap B' = A - B]$

6.25 (c)

If each a_i and b_i is either 0 or 1 and the numbers are stored in an array then we can find the largest span (i, j) on average in $\Theta(n)$ times because these is only need of search.

6.26 (c)

In strongly connected component, every two vetices must be reachable from one to other and it is maximal component.



{P, Q, S, T, V}, {R}, {U}

6.27 (d)

Since d(Q) = d(P) + 1 and f(Q) < f(P) which means P and Q are connected and R is separate.

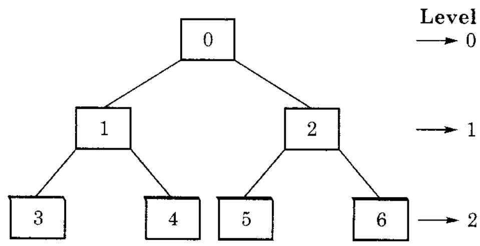
6.28 (a)

Left child of i^{th} element will be at 2*i+1 and right child at 2(i+1). The index of the parent of element X [i] is $\lfloor i / 2 \rfloor$.

6.29 (b)

We can just call heapify and it will also take *log n* time only as other nodes are already in place.

6.30 (c)



Index of Every node is shown in the box. Floor (log (i + 1)) draw the tree and realise that the last element at each level is the best choice to arrive at a conclusion.

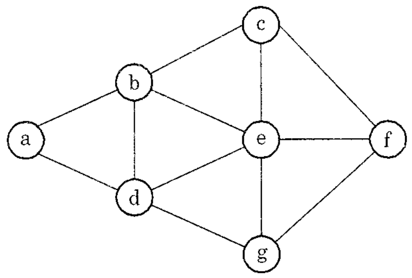
6.31 (a)

Since DFS is performed on directed acyclic graph so we don't have any back edges.
Since we are only having tree edges forward edges and cross edges.

6.32 (b)

Time complexity of radix sort = $\Theta(kn)$

6.33 (b)



DFS orders:

1. a–b–e–f–(d is not depth first order)
2. a–b–e–f–c–g–d is correct DFS order.
3. a–d–g–e–b–c–f is correct DFS order.
4. a–d–b–c– (g is not depth first order).

∴ 2 and 3 are correct DFS orders.

6.34 (c)

A requires $0.0001\ n^2$ time units. And B require $10\ n \log_{10} n$ time unit to process n records for package B will be preferred over A.

So $0.0001\ n^2 < 10\ n \log_{10} n$

$$\frac{n^2}{10^5} < 10\ n \log_{10} n$$

$$n^2 = 10^6\ n \log_{10} n$$

compare this constant with 10^K

So minimum value of K = 6 for which package B will be preferred over A.

6.35 (c)

If we have an array of x elements then time complexity to sort x elements using heap sort is $\Theta(x \log x)$.

We need to find value of x so that time complexity to sort x elements gets $\Theta(\log n)$ we can direct discard option (a) and (d).

Now check option (c)

$$= \Theta\left(\frac{\log n}{\log \log n} \log\left(\frac{\log n}{\log \log n}\right)\right)$$

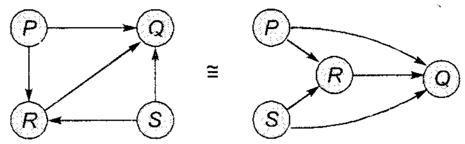
$$= \Theta\left(\frac{\log n}{\log \log n}(\log \log n - \log \log \log n)\right)$$

$$= \Theta\left(\log n\left(\frac{\log \log n - \log \log \log n}{\log \log n}\right)\right)$$

$= \Theta(\log n)$

Option (c) is correct

6.36 (c)



∴ Topological orderings: PSRQ and SPRQ

6.37 Sol.

Given that there is shortcut from 9 to 15

OR

It can move right by a unit distance (9 to 10)

$T(9) = 1 + \text{Min}(T(10), T(15))$

$\Rightarrow\ y = 10, z = 15$

$\therefore\ yz = 10 \times 15 = 150$

6.38 (c)

Prims algorithm for finding MST is a greedy algorithm. Floyd warshalls algorithm for finding all shortest paths is dynamic algorithm. Merge sort is a Divide and Conquer algorithm. Hamiltonian circuit is an NP-complete problem and done by using backtracking.

6.39 (c)

Dijktra's shortest path → Greedy design.

Floyd warshall algorithm to compute all pair shortest path → Dynamic programming.

Binary search on a sorted array → Divide and Conquer

Backtracking searching → Depth first search on a graph.

■■■■

Unit ■ VII

Compiler Design

■ Contents

UNIT VII

Compiler Design

Syllabus : Lexical analysis, Parsing, Syntax directed translation, Runtime environments, Intermediate and target code generation, Basics of code optimization.

Analysis of Previous GATE Papers

Exam Year	1 Mark Ques.	2 Marks Ques.	3 Marks Ques.	5 Marks Ques.	Total Marks
1990	2	–		–	**2**
1991	–	–		–	–
1992	–	1		–	**2**
1993	–	–		–	–
1994	1	–		–	**1**
1995	3	1		–	**5**
1996	1	–		–	**1**
1997	1	–		–	**1**
1998	2	–		1	**7**
1999	–	1		–	**2**
2000	3	–		–	**3**
2001	2	–		–	**2**
2002	–	–		–	–
2003	3	5		–	**13**
2004	3	1		–	**5**
2005	1	4		–	**9**
2006	1	5		–	**11**

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2007	1	5	**11**
2008	2	1	**4**
2009	1	–	**1**
2010	2	1	**4**
2011	1	1	**3**
2012	–	2	**4**
2013	1	3	**7**
2014 Set-1	1	1	**3**
2014 Set-2	2	2	**6**
2014 Set-3	2	1	**4**
2015 Set-1	1	2	**5**
2015 Set-2	2	1	**4**
2015 Set-3	1	1	**3**
2016 Set-1	1	2	**5**
2016 Set-2	1	1	**3**

1 Lexical Analysis

1.1 In a compiler the module that checks every character of the source text is called
(a) The code generator
(b) The code optimizer
(c) The lexical analyzer
(d) The syntax analyzer

[1988 : 1 Mark]

1.2 Match the followings

Group-I

A. Lexical analysis
B. Code optmization
C. Code generation
D. Abelian groups

Group-II

P. DAG's
Q. Syntax trees
R. Push down automation
S. Finite automaton

[1990 : 1 Mark]

1.3 Which of the following strings can definitely be said to be token without looking at the next input character while compiling a Pascal program?

I. begin **II.** Program **III.** <>

(a) I (b) II
(c) III (d) All of these

[1995 : 1 Mark]

1.4 In some programming languages an identifier is permitted to be a letter followed by any number of letters or digits. If L and D denotes the sets of letters and digits respectively. Which of the following expression defines an identifier?

(a) $(L + D)^+$ (b) $L(L + D)^*$
(c) $(L.D)^*$ (d) $L(L.D)^*$

[1995 : 1 Mark]

1.5 Type checking is normally done during
(a) Lexical analysis
(b) Syntax analysis
(c) Syntax directed translation
(d) Code optimization

[1998 : 1 Mark]

1.6 The number of tokens in the Fortran statement DO 10 I = 1.25 is

[1999 : 2 Marks]

1.7 The number of tokens in the following C statement printf("i = %d, &i = %x", i, &i); is

[2000 : 1 Mark]

1.8 The number of tokens in the following C statement
printf("i = %d, &i = %x", i, &i); is
(a) 3 (b) 26
(c) 10 (d) 21

[2000 : 1 Mark]

1.9 Which of the following is NOT an advantage of using shared, dynamically linked libraries as opposed to using statically linked libraries?
(a) Smaller sizes of executable
(b) Lesser overall page fault rate in the system
(c) Faster program startup
(d) Existing programs need not be re-linked to take advantage of newer versions of libraries

[2003 : 2 Marks]

1.10 Consider a program P that consists of two source modules M_1 and M_2 contained in two different files. If M_1 contains a reference to a function defined in M_2, the reference will be resolved at
(a) Edit-time (b) Compile-time
(c) Link-time (d) Load-time

[2004 : 1 Mark]

1.11 Consider line number 3 of the following C-program.

```
int main () {                 /* Line 1 */
  int i, n;                   /* Line 2 */
  fro (i = 0, i < n, i + +);  /* Line 3 */
}
```

Identify the compiler's response about this line while creating the object - module
(a) No compilation error
(b) Only a lexical error
(c) Only syntactic errors
(d) Both lexical and syntactic errors

[2005 : 2 Marks]

1.12 Which of the following statements are TRUE?
I. There exist parsing algorithms for some programming languages whose complexities are less than $\theta(n^3)$
II. A programming language which allows recursion can be implemented with static storage allocation
III. No L-attributed definition can be evaluated in the framework of bottom-up parsing
IV. Code improving transformations can be performed at both source language and intermediate code level
(a) I and II
(b) I and IV
(c) III and IV
(d) I, III and IV

[2009 : 1 Mark]

1.13 Which data structure in a complier is used for managing information about variables and their attributes?
(a) Abstract syntax tree
(b) Symbol table
(c) Semantic stack
(d) Parse table

[2010 : 1 Mark]

1.14 In a compiler, keywords of a language are recognized during
(a) parsing of the program
(b) the code generation
(c) the lexical analysis of the program
(d) dataflow analysis

[2011 : 1 Mark]

1.15 Match the following

List-I
P: Lexical analysis
Q: Top down parsing
R: Semantic analysis
S: Runtime environments

List-II
(i) Leftmost derivation
(ii) Type checking
(iii) Regular expressions
(iv) Activation records

Codes:
(a) P – i, Q – ii, R – iv, S – iii
(b) P – iii, Q – i, R – ii, S – iv
(c) P – ii, Q – iii, R – i, S – iv
(d) P – iv, Q – i, R – ii, S – iii

[2016 (Set-2) : 1 Mark]

■■■■

Answers Lexical Analysis

1.1 (c) **1.3** (c) **1.4** (b) **1.5** (c) **1.8** (c) **1.9** (c) **1.10** (c) **1.11** (c) **1.12** (b)
1.13 (b) **1.14** (c) **1.15** (b)

Explanations Lexical Analysis

1.1 (c)

Lexical analysis is a phase of compiler in which a sequence of characters get converted into a group of tokens. A program or function which performs lexical analysis is called a lexical analyzer. So lexical analyzer checks every character of the source text.

1.2 Sol.

(A)–(S), (B)–(P), (C)–(R), (D)–(Q)

1.3 (c)

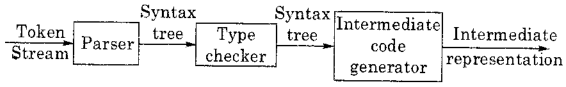
I. Begin can be followed by anything to form identifier, function name, etc.
II. Program can be followed by anything to form identifier, function name, etc.
III. < > can definitely be said to be token without looking at the next input character.

1.4 (b)

L : Set of letters
D: Set of digits
Identifier = letter (letter or digit)*
Identifier = L(L + D)*

1.5 (c)

Type checking verifies that a type of a construct matches that expected by its context.



Position of type checker

1.6 Sol.

In Fortran identifies can contain spaces.
'Do 10 I' is one token
'=' is another token and '1.25' is third token
∴ 3 tokens are identified in Fortran.
Whereas in C-language as following it counts:

DO	10	I	=	1.25
①	②	③	④	⑤

∴ The number of tokens in the C statement is 5.

1.7 Sol.

Printf	(	"i - %d, & i = %x"	,	i	,	&	i	)	;
①	②	③	④	⑤	⑥	⑦	⑧	⑨	⑩

∴ The number of tokens in the C statement is 10.

1.8 (c)

Inside the printf (" ") the part between (" – ") count as '1'.

1.9 (c)

Statically linked libraries program run faster from startup since the linking has been performed during the compile time only and there is no need for the since at run time of program and program can run faster.

1.10 (c)

Modules may be compiled separately and all linking between references is made at link time.

1.11 (c)

Error is underlined function fro( )
Which is a syntactical error rather than lexical so correct option is (c).

1.12 (b)

Recursion can not be implemented with static storage allocation.
L-attributed definition can be evaluated if all rules are at the end and all attributes are synthesized.
So I and IV are correct.

1.13 (b)

As we know symbol table is a Data structure containing record for each identifier, with fields for the attribute of the identifier.
The symbols entered into symbol table are nothing but identifier and operators. It contain information about variables and their attributes.

1.14 (c)

Lexical analysis of the program.

1.15 (b)

P. Lexical analysis → (iii) Regular expression
Q. Top down parsing → (i) LMD
R. Semantic Analysis → (ii) Type checking
S. Runtime Environments → (iv) Activation records

■■■■

2 Parsing Techniques

2.1 Consider the SLR(1) and LALR (1) parsing tables for a context free grammar. Which of the following statements is/are true?
(a) The goto part of both tables may be different.
(b) The shift entries are identical in both the tables.
(c) The reduce entries in the tables may be different.
(d) The error entries in the tables may be different.

[1992 : 2 Marks]

2.2 A shift reduce parser carries out the actions specified within braces immediately after reducing with the corresponding rule of grammar
$S \rightarrow xx\,W$ {print"1"}
$S \rightarrow y$ {print"2"}
$W \rightarrow Sz$ {print"3"}
What is the translation of xxxxyzz using the syntax directed translation scheme described by the above rules?
(a) 23131 (b) 11233
(c) 11231 (d) 33211

[1995 : 2 Marks]

2.3 The pass numbers for each of the following activities
(i) object code generation
(ii) literals added to literal table
(iii) listing printed
(iv) address resolution of local symbols that occur in a two pass assembler respectively are
(a) 1, 2, 1, 2 (b) 2, 1, 2, 1
(c) 2, 1, 1, 2 (d) 1, 2, 2, 2

[1996 : 1 Mark]

2.4 Which of the following statements is true?
(a) SLR parser is more powerful than LALR
(b) LALR parser is more powerful than Canonical LR parser
(c) Canonical LR parser is more powerful than LALR parser
(d) The parsers SLR, Canonical LR, and LALR have the same power

[1998 : 1 Mark]

2.5 The process of assigning load addresses to the various parts of the program and adjusting the code and data in the program to reflect the assigned addresses is called
(a) Assembly (b) Parsing
(c) Relocation (d) Symbol resolution

[2001 : 1 Mark]

2.6 Which of the following statements is false?
(a) An unambiguous grammar has same leftmost and rightmost derivation
(b) An LL(1) parser is a top-down parser
(c) LALR is more powerful than SLR
(d) An ambiguous grammar can never be LR(k) for any k

[2001 : 1 Mark]

2.7 Which of the following suffices to convert an arbitrary CFG to an LL(1) grammar?
(a) Removing left recursion alone
(b) Factoring the grammar alone
(c) Removing left recursion and factoring the grammar
(d) None of the above

[2003 : 1 Mark]

2.8 Assume that the SLR parser for a grammar G has n_1 states and the LALR parser for G has n_2 states. The relationship between n_1 and n_2 is
(a) n_1 is necessarily less than n_2
(b) n_1 is necessarily equal to n_2
(c) n_1 is necessarily greater than n_2
(d) None of the above

[2003 : 1 Mark]

2.9 Consider the grammar shown below
$S \rightarrow i\,E\,t\,S\,S' \mid \alpha$
$S' \rightarrow e\,S \mid \varepsilon$
$E \rightarrow b$
In the predictive parse table M, of this grammar, the entries M[S′, e] and M[S′, $] respectively are
(a) $\{S' \rightarrow e\,S\}$ and $\{S' \rightarrow \varepsilon\}$
(b) $\{S' \rightarrow e\,S\}$ and $\{\}$
(c) $\{S' \rightarrow \varepsilon\}$ and $\{S' \rightarrow \varepsilon\}$
(d) $\{S' \rightarrow e\,S, S' \rightarrow \varepsilon\}$ and $\{S' \rightarrow \varepsilon\}$

[2003 : 2 Marks]

2.10 Consider the grammar shown below.

$S \to C\ C$

$C \to c\ C \mid d$

The grammar is

(a) LL (1)

(b) SLR (1) but not LL (1)

(c) LALR (1) but not SLR (1)

(d) LR (1) but not LALR (1)

[2003 : 2 Marks]

2.11 Which of the following grammar rules violate the requirements of an operator grammar? P, Q, R are not terminals and r, s, t are terminals.

(i) $P \to Q\ R$ (ii) $P \to Q\ s\ R$

(iii) $P \to \varepsilon$ (iv) $P \to Q\ t\ R\ r$

(a) (i) only (b) (i) and (iii) only

(c) (ii) and (iii) only (d) (iii) and (iv) only

[2004 : 1 Mark]

2.12 The grammar $A \to AA \mid (A) \mid \varepsilon$ is not suitable for predictive-parsing because the grammar is

(a) ambiguous (b) Left-recursive

(c) right-recursive (d) an operator-grammar

[2005 : 1 Mark]

2.13 Consider the grammar

$S \to (S) \mid a$

Let the number of states in SLR (1), LR (1) and LALR(1) parsers for the grammar be n_1, n_2 and n_3 respectively. The following relationship holds good

(a) $n_1 < n_2 < n_3$ (b) $n_1 = n_3 < n_2$

(c) $n_1 = n_2 = n_3$ (d) $n_1 \geq n_3 \geq n_2$

[2005 : 2 Marks]

Linked Answer for Q. 2.14 & Q. 2.15:

Consider the context-free grammar

$E \to E + E$

$E \to (E * E)$

$E \to id$

where E is the starting symbol, the set of terminals is { id, (, +,), * } and the set of non-terminals is {E}.

2.14 Which of the following terminal strings has more than one parse tree when parsed according to the above grammar?

(a) id + id + id + id

(b) id + (id* (id * id))

(c) (id* (id * id)) + id

(d) ((id * id + id) * id)

[2005 : 2 Marks]

2.15 For the terminal string with more than one parse tree obtained as solution to in above Question, how many parse trees are possible?

(a) 5 (b) 4

(c) 3 (d) 2

[2005 : 2 Marks]

2.16 Consider the following grammar.

$S \to S \times E$

$S \to E$

$E \to F + E$

$E \to F$

$F \to id$

Consider the following LR(0) items corresponding to the grammar above.

(i) $S \to S \times .E$

(ii) $E \to F. + E$

(iii) $E \to F + .E$

Given the items above, which two of them will appear in the same set in the canonical sets-of-items for the grammar?

(a) (i) and (ii) (b) (ii) and (iii)

(c) (i) and (iii) (d) None of these

[2006 : 1 Mark]

2.17 Consider the following grammar

$S \to FR$

$R \to *S \mid \varepsilon$

$F \to id$

In the predictive parser table, M, of the grammar the entries M[S, id] and M[R, $] respectively

(a) $\{S \to FR\}$ and $\{R \to \varepsilon\}$

(b) $\{S \to FR\}$ and $\{\}$

(c) $\{S \to FR\}$ and $\{R \to *S\}$

(d) $\{F \to id\}$ and $\{R \to \varepsilon\}$

[2006 : 2 Marks]

Linked Data Questions 2.18 & 2.19

2.18 Which one of the following grammars generates the language $L = \{a^i b^j \mid i \neq j\}$?

(a) $S \to AC \mid CB$

$C \to aCb \mid a \mid b$

$A \to aA \mid \varepsilon$

$B \to Bb \mid \varepsilon$

(b) $S \to aS \mid Sb \mid a \mid b$

(c) $S \to AC \mid CB$

$C \to aCb \mid \varepsilon$

$A \to aA \mid \varepsilon$

$B \to Bb \mid \varepsilon$

(d) $S \rightarrow AC \mid CB$
$C \rightarrow aCb \mid \varepsilon$
$A \rightarrow aA \mid a$
$B \rightarrow bB \mid b$ **[2006 : 2 Marks]**

2.19 In the correct grammar above, what is the length of the derivation (number of steps starting from S) to generate the string $a^l b^m$ with $l \neq m$?
(a) max $(l, m) + 2$ (b) $l + m + 2$
(c) $l + m + 3$ (d) max $(l, m) + 3$
[2006 : 2 Marks]

2.20 Which one of the following is a top-down parser?
(a) Recursive descent parser
(b) Operator precedence parser
(c) An LR(k) parser
(d) An LALR(k) parser
[2007 : 1 Mark]

2.21 Consider the grammar with non-terminals N = {S, C, S_1}, terminals T = {a, b, i, t, e}, with S as the start symbol, and the following set of rules
$S \rightarrow iCtSS_1 \mid a$
$S_1 \rightarrow eS \mid \varepsilon$
$C \rightarrow b$
The grammar is NOT LL(1) because:
(a) It is left recursive
(b) It is right recursive
(c) It is ambiguous
(d) It is not context-free
[2007 : 2 Marks]

2.22 Consider the following two statements:
P: Every regular grammar is LL (1)
Q: Every regular set has LR (1) grammar
Which of the following is TRUE?
(a) Both P and Q are true
(b) P is true and Q is false
(c) P is false and Q is true
(d) Both P and Q are false
[2007 : 2 Marks]

Common Data for Q. 2.23 & Q. 2.24

Consider the CFG with {S, A, B} as the non-terminal alphabet, {a, b} as the terminal alphabet, S as the start symbol and the following set of production rules

$S \rightarrow bA$	$S \rightarrow aB$
$A \rightarrow a$	$B \rightarrow b$
$A \rightarrow aS$	$B \rightarrow bS$
$A \rightarrow bAA$	$B \rightarrow aBB$

2.23 Which of the following strings is generated by the grammar?
(a) aaaabb (b) aabbbb
(c) aabbab (d) abbbba
[2007 : 2 Marks]

2.24 For the correct answer string to above question how many derivation trees are there?
(a) 1 (b) 2
(c) 3 (d) 4
[2007 : 2 Marks]

2.25 Which of the following describes a handle (as applicable to LR-parsing) appropriately?
(a) It is the position in a sentential form where the next shift or reduce operation will occur
(b) It is a non-terminal whose production will be used for reduction in the next step
(c) It is a production that may be used for reduction in a future step along with a position in the sentential form where the next shift or reduce operation will occur.
(d) It is the production p that will be used for reduction in the next step along with a position in the sentential form where the right hand side of the production may be found
[2008 : 1 Mark]

2.26 An LALR(1) parser for a grammar G can have shift-reduce (S-R) conflicts if and only if
(a) the SLR(1) parser for G has S-R conflicts
(b) the LR(1) parser for G has S-R conflicts
(c) the LR(0) parser for G has S-R conflicts
(d) the LALR(1) parser for G has reduce-reduce conflicts
[2008 : 2 Marks]

2.27 The grammar $S \rightarrow aSa \mid bS \mid c$ is
(a) LL(1) but not LR(1)
(b) LR(1) but not LL(1)
(c) Both LL(1) and LR(1)
(d) Neither LL(1) nor LR(1)
[2010 : 2 Marks]

Linked Data Questions 2.28 and 2.29

For the grammar below, a partial LL(1) parsing table is also presented along with the grammar. Entries that need to be filled are indicated as E1, E2, and E3. ε is the empty string, \$ indicates end of input, and | separates alternate right hand sides of productions.

$S \rightarrow a\,A\,b\,B \mid b\,A\,a\,B \mid \varepsilon$
$A \rightarrow S$
$B \rightarrow S$

	a	b	$
S	**E1**	**E2**	$S \rightarrow \varepsilon$
A	$A \rightarrow S$	$A \rightarrow S$	error
B	$B \rightarrow S$	$B \rightarrow S$	**E3**

2.28 The FIRST and FOLLOW sets for the non-terminals A and B are

(a) FIRST (A) = {a, b, ε} = FIRST (B),
FOLLOW (A) = {a, b}
FOLLOW (B) = {a, b, $}

(b) FIRST (A) = {a, b, $}
FIRST (B) = {a, b, ε}
FOLLOW (A) = {a, b}
FOLLOW (B) = {$}

(c) FIRST (A) = {a, b, ε} = FIRST (B),
FOLLOW (A) = {a, b}
FOLLOW (B) = E

(d) FIRST (A) = {a, b} = FIRST (B)
FOLLOW (A) = {a, b}
FOLLOW (B) = {a, b}

[2012 : 2 Marks]

2.29 The appropriate entries for E1, E2 and E3 are

(a) E1 : $S \rightarrow aAbB$, $A \rightarrow S$
E2 : $S \rightarrow bAaB$, $B \rightarrow S$
E3 : $B \rightarrow S$

(b) E1 : $S \rightarrow aAbB$, $S \rightarrow \varepsilon$
E2 : $S \rightarrow bAaB$, $B \rightarrow \varepsilon$
E3 : $S \rightarrow \varepsilon$

(c) E1 : $S \rightarrow aAbB$, $S \rightarrow \varepsilon$
E2 : $S \rightarrow bAaB$, $S \rightarrow \varepsilon$
E3 : $B \rightarrow S$

(d) E1 : $A \rightarrow S$, $S \rightarrow \varepsilon$
E2 : $B \rightarrow S$, $S \rightarrow \varepsilon$
E3 : $B \rightarrow S$

[2012 : 2 Marks]

2.30 What is the maximum number of reduces moves that can be taken by a bottom up parser for a grammar without epsilon and unit productions (i.e. of type $A \rightarrow \varepsilon$ and $A \rightarrow a$) to parse a string with n tokens?

(a) n/2 (b) n-1
(c) 2n-1 (d) 2^n

[2013 : 1 Mark]

2.31 Consider the following two sets of LR(1) items of LR(1) grammar.

$X \rightarrow c.X,\ c/d$ $X \rightarrow c.X,\ \$$
$X \rightarrow .cX,\ c/d$ $X \rightarrow .cX,\ \$$
$X \rightarrow .d,\ c/d$ $X \rightarrow .d,\ \$$

Which of the following statement related to merging of the two sets in the corresponding parser is/are FALSE?

1. Cannot be merged since look aheads are different.
2. Can be merged but will result in S-R conflict.
3. Can be merged but will result in R-R conflict.
4. Cannot be merged since goto on c will lead to two different sets.

(a) 1 only (b) 2 only
(c) 1 and 4 only (d) 1, 2, 3 and 4

[2013 : 2 Marks]

2.32 A canonical set of items is given below

$S \rightarrow L \cdot > R$
$Q \rightarrow R \cdot$

On input symbol > the set has

(a) a shift-reduce conflict and a reduce-reduce conflict.
(b) a shift-reduce conflict but not a reduce-reduce conflict.
(c) a reduce-reduce conflict but not a shift-reduce conflict.
(d) neither a shift-reduce nor a reduce-reduce conflict.

[2014 (Set-1) : 2 Marks]

2.33 Consider the grammar defined by the following production rules, with two operators * and +

$S \rightarrow T * P$
$T \rightarrow U \mid T * U$
$P \rightarrow Q + P \mid Q$
$Q \rightarrow \text{Id}$
$U \rightarrow \text{Id}$

Which one of the following is TRUE?

(a) + is left associative, while * is right associative
(b) + is right associative, while * is left associative
(c) Both + and * are right associative
(d) Both + and * are left associative

[2014 (Set-2) : 1 Mark]

2.34 Which one of the following is TRUE at any valid state in shift-reduce parsing?

(a) Viable prefixes appear only at the bottom of the stack and not inside
(b) Viable prefixes appear only at the top of the stack and not inside
(c) The stack contains only a set of viable prefixes
(d) The stack never contains viable prefixes

[2015 (Set-1) : 1 Mark]

2.35 Match the following:

List-I

A. Lexical analysis
B. Parsing
C. Register allocation
D. Expression evaluation

List-II

1. Graph coloring
2. DFA minimization
3. Post-order traversal
4. Production tree

Codes:

	A	B	C	D
(a)	2	3	1	4
(b)	2	1	4	3
(c)	2	4	1	3
(d)	2	3	4	1

[2015 (Set-2) : 1 Mark]

2.36 Among simple LR (SLR), canonical LR, and look-ahead LR (LALR), which of the following pairs identify the method that is very easy to implement and the method that is the most powerful, in that order?
(a) SLR, LALR
(b) Canonical LR, LALR
(c) SLR, canonical LR
(d) LALR, canonical LR

[2015 (Set-3) : 1 Mark]

2.37 Consider the following grammar G.

$$S \rightarrow F \mid H$$
$$F \rightarrow p \mid c$$
$$H \rightarrow d \mid c$$

Where S, F and H are non-terminal symbols, p, d and c are terminal symbols. Which of the following statement(s) is/are correct?

S1: LL(1) can parse all strings that are generated using grammar G.
S2: LR(1) can parse all strings that are generated using grammar G.

(a) Only S1 (b) Only S2
(c) Both S1 and S2 (d) Neither S1 and S2

[2015 (Set-3) : 2 Marks]

Answers Parsing Techniques

2.1 (b), (c), (d) 2.2 (a) 2.3 (b) 2.4 (c) 2.5 (c) 2.6 (a) 2.7 (d) 2.8 (b)
2.9 (d) 2.10 (a) 2.11 (b) 2.12 (a) 2.13 (b) 2.14 (a) 2.15 (a) 2.16 (d) 2.17 (a)
2.18 (d) 2.19 (a) 2.20 (a) 2.21 (c) 2.22 (c) 2.23 (c) 2.24 (b) 2.25 (d) 2.26 (b)
2.27 (c) 2.28 (a) 2.29 (c) 2.30 (c) 2.31 (d) 2.32 (d) 2.33 (b) 2.34 (c) 2.35 (c)
2.36 (c) 2.37 (d)

Explanations Parsing Techniques

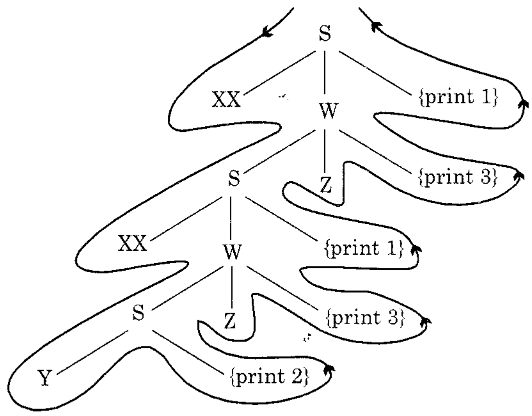
2.1 (b,c,d)

2.1 **(b), (c), (d)**

The difference in parsing tables for context free grammar of SLR & LALR parser is in reduce entires only. Due to difference in reduce entries there may be difference in error entries also.

2.1 (a)

Syntax directed tree of xxxxyzz is as follows:



So translation by rules is 23131

[The code has translation rules at the end, so the result of top-down and bottom-up parsing will be same.]

2.3 (b)

A multi-pass compiler is a type of compiler that processes the source code or abstract syntax tree of a program several times. Two-pass compiler, on the first pass checks the syntax of statements and constructs a table of symbols, while on the second pass it actually translates program statements into machine language.

(i) Object code generation – 2

(ii) Literals added to literal table – 1

(iii) Listing printed – 2

(iv) Address resolution of local symbols that occur in a two pass assembler respectively – 1

2.4 (c)

LR parsers in term of power is

CLR > LALR > SLR > LR(O)

2.5 (c)

Relocation is the process of assigning load address to various parts of a program and adjusting the code and data in the program to reflect the assigned addresses.

2.6 (a)

(a) An unambiguous grammar can have different leftmost and rightmost derivation. However, an unambiguous grammar has only one derivation tree. So option (a) is false.

(b) LL(1) is a top-down parser.

(c) LALR is more powerful than SLR.

(d) For any parser, grammar should be unambiguous.

2.7 (d)

An ambiguous grammar can't be LL(1). Removing left recursion and factoring the grammar not always guarantees the unambiguity of grammar.

Therefore, option (d) is correct.

2.8 (b)

SLR parser has n_1 states for a grammar G LALR parser has n_2 states for a grammar G. The states of SLR and LALR parser are the states of corresponding states in a deterministic finite automata which recognizes the viable prefixes and both deterministic finite automata contains the equal number of states so $n_1 = n_2$.

2.9 (d)

The grammar is

$$S \rightarrow iEtSS' \mid a$$
$$S' \rightarrow eS \mid \varepsilon$$
$$E \rightarrow b$$

The predictive parser table M is

Non-terminal	a	b	e	i	t	$
S	$S \rightarrow a$			$S \rightarrow iEtSS'$		
S′			$S' \rightarrow \varepsilon$ $S' \rightarrow eS$			$S' \rightarrow \varepsilon$
E		$E \rightarrow b$				

So $M[S', e] = \{S' \rightarrow \varepsilon, S' \rightarrow eS\}$

$M[S', \$] = S \rightarrow \varepsilon$

2.10 (a)

Consider the grammar

$S \rightarrow CC$

$C \rightarrow cC \mid d$

The above grammar is LL(1)

[it is unambiguous, not left recursive and left factored].

It is also LR(0) grammar, hence it is SLR(1), LALR(1) and CLR(1).

Therefore option (a) matches correctly.

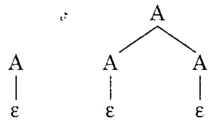
2.11 (b)

In operator grammar the grammar has the property that no production side is ε or it doesn't contain adjacent nonterminals.

So $P \rightarrow QR$ and $P \rightarrow \varepsilon$ violate the requirements of an operator grammar.

2.12 (a)

The string ε has more than one parse tree for the given grammar.



Given grammar is Ambiguous, that is the reason that it is not suitable for predictive parsing.

2.13 (b)

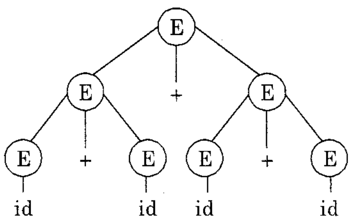
$S \rightarrow (S) \mid a$

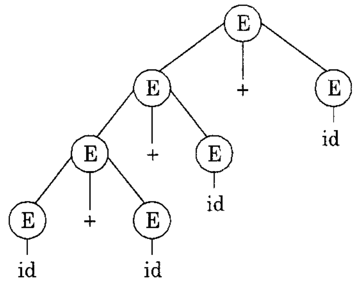
Parser	Number of states
SLR(1)	n_1
LR(1)	n_2
LALR(1)	n_3

The number of states of deterministic finite automate in SLR(1) and LALR(1) parsers are equal, so $n_1 = n_3$. The number of states of deterministic finite automata in LR(1) is greater than number of states of deterministic finite automata of SLR(1) and LALR(1)

So $n_1 = n_3 < n_2$.

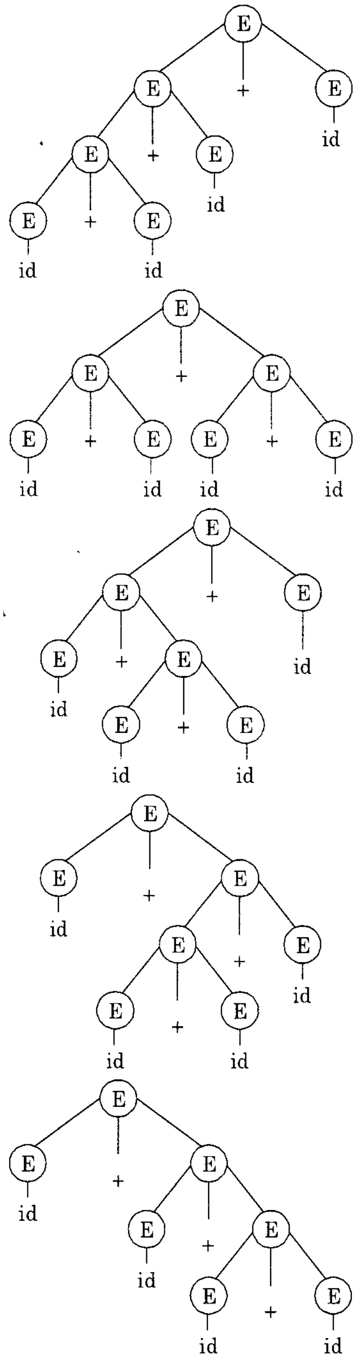
2.14 (a)





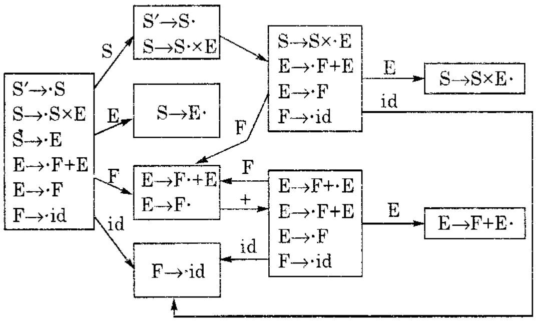
∴ id+id+id+id has more than one parse tree.

2.15 (a)



∴ 5 parse trees are exist for id+id+id+id

2.16 (d)



Given S→ S×•E, E→F•+E and E → F+•E. Any of these two items not present in the same state of above DFA.

2.17 (a)

Construct Predictive parser table as follows

Non-terminal	id	*	$
S	$S \to FR$		
R		$R \to *S$	$R \to \varepsilon$
F	$F \to id$		

So $M[S, id] = \{S \to FR\}$

and $M[R, \$] = \{R \to \varepsilon\}$

2.18 (d)

$$S \to AC \mid CB$$
$$C \to aCb \mid \varepsilon$$
$$A \to aA \mid a$$
$$B \to bB \mid b$$

From the above grammar,

$C \to aCb|\varepsilon \Rightarrow L(C) = \{a^n b^n\}$

$A \to aA|a \Rightarrow L(A) = a^+$

$B \to bB|b \Rightarrow L(B) = b^+$

By substituting C, A and B in S.

$S \to AC \Rightarrow L(S) = \{a^m b^n \mid m > n\}$

$S \to CB \Rightarrow L(S) = \{a^m b^n \mid m < n\}$

$L(S) = \{a^m b^n | m > n \text{ or } m < n\} = \{a^i b^j \mid i \neq j\}$

2.19 (a)

In the Q.No. 2.16 if grammar generates $a^n b^0$ or $a^0 b^n$ then the length is max(l, m) if it generates $a^l b^m$ initially we include two extra derivation so the total length is max(l, m) + 2.

2.20 (a)

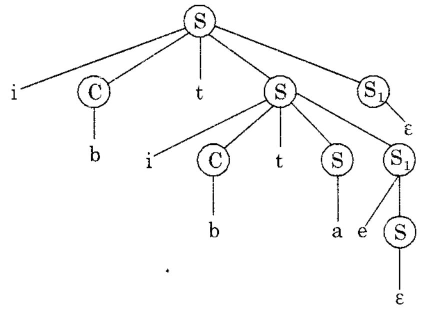
Predictive parser and recursive descent parser are example of top-down parser. Whereas LR(k) and LALR(k) parser are bottom up parser.

2.21 (c)

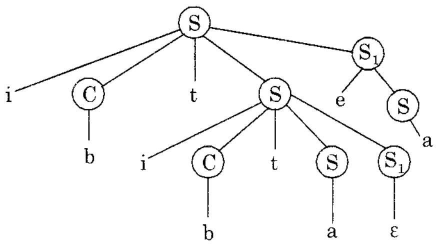
$$S \to iCtSS_1 \mid a$$
$$S_1 \to eS \mid \varepsilon$$
$$C \to b$$

Consider the string = ibtibtaea. There are two parse trees to derive this string.

Parse Tree-1



Parse Tree-2



For the same string more than one parse tree is present to derive the string. Hence the given grammar is ambiguous.

2.22 (c)

- A regular grammar can be ambiguous. An ambiguous grammar can't be LL(1). Therefore, it is false that every regular grammar is LL(1)
- Every regular language is unambiguous i.e., for every regular language there exist atleast one grammar which is unambiguous. Therefore, it is true that every regular set has LR(1) grammar.

∴ P is false and Q is true.

So option (c) is correct option.

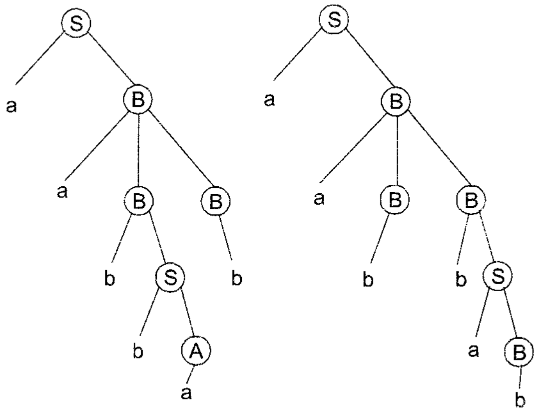
2.23 (c)

$S \Rightarrow aB \Rightarrow aaBB \Rightarrow aabB \Rightarrow aabbS$
$\Rightarrow aabbaB \Rightarrow aabbab.$

So aaabbab is generated by the grammar.

2.24 (b)

There are two parse trees for the string aabbab.



2.25 (d)

Definition of handle is given in (d) option.

2.26 (b)

LALR(1) parser uses the LR(1) items So LALR(1) parser for a grammar G can have S-R conflict if and only if the LR(1) Parser for G has S-R conflicts.

2.27 (c)

$S \rightarrow aSa \mid bS \mid c$

The above grammar is LL(1) because,

First [aSa] $\cap$ first [bS] = (a) $\cap$ (b) = ϕ

&&

First [bs] $\cap$ first [c] = (b) $\cap$ (c) = ϕ

&&

First [c] $\cap$ first [aSa] = (c) $\cap$ (a) = ϕ

As the above grammar is LL(1), also LR(1) because LL(1) grammar is always LR(1) grammar.

2.28 (a) & 2.29 (c)

$S \rightarrow aAbB \mid bAaB \mid \varepsilon$

$A \rightarrow S$

$B \rightarrow S$

First of S = {a, b, ε}

First of A = {a, b, ε}

First of B = {a, b, ε}

Follow of S = {$, a, b}

Follow A = {b, a}

Follow of B = {$, a, b}

LL(1) Parsing table:

	a	b	$
S	$S \rightarrow aAbB$ $S \rightarrow \varepsilon$	$S \rightarrow aAbB$ $S \rightarrow \varepsilon$	$S \rightarrow \varepsilon$
A	$A \rightarrow S$	$A \rightarrow S$	error
B	$B \rightarrow S$	$B \rightarrow S$	$B \rightarrow S$

2.30 (c)

Unit production : $A \rightarrow B$

[**Note:** There is typo error in question for unit production. Instead of $A \rightarrow B$, it was given as $A \rightarrow a$]

Null production: $A \rightarrow \varepsilon$

Ex. $S \rightarrow AB$

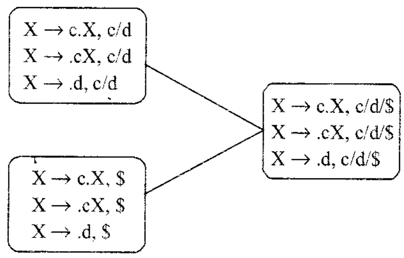
$A \rightarrow a$

$B \rightarrow b$

[2 length string] ab $\Rightarrow$ Ab $\Rightarrow$ AB $\Rightarrow$ S, it requires three reductions.

$\therefore$ For n length string (2n – 1) reductions required.

2.31 (d)

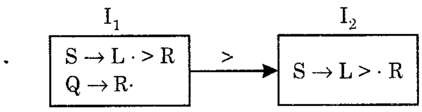


1. Cannot be merged since look-a-heads are different: FALSE, because merging does not depend on look ahead.
2. Can be merged but will results in S-R conflict: FALSE the two states are not containing reduce item, so after merging, merged state can not contain any S-R conflict.
3. Can be merged but will result in R-R conflict: FALSE, No RR conflict in merged state.
4. Cannot be merged since goto on c will lead to two different sets: FALSE, merging of states does not depends on goto.

2.32 (d)

$S \to L \cdot > R$

$Q \to R \cdot$



In above diagram, we see at I_2 state it has only one reduced item and there is no other item in the state. Therefore for input symbol '>', there is no conflict.

2.33 (b)

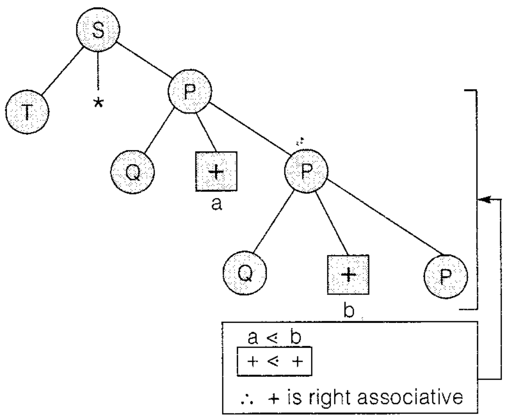
$S \to T * P$

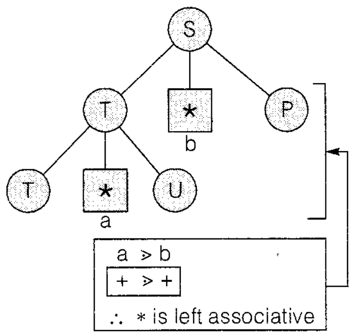
$T \to U | T * U$

$P \to Q + P | Q$

$Q \to \text{Id}$

$U \to \text{Id}$





2.34 (c)

Stack contains only a set of viable prefixes.

2.35 (c)

(A) Lexical Analysis → DFA minimization (2)

(B) Parsing → Production Tree (3)

(C) Register Allocation → Graph coloring (1)

(D) Expression evaluation → Post-order traversal (4)

2.36 (c)

Among SLR, CLR and LALR parsers

(i) **SLR** parser is simple and very easy to implement compared to other parsers.

(ii) **CLR** parser is most powerful than other parsers because it can accepts more languages than other.

2.37 (d)

$S \to F \mid H$

$F \to p \mid c$

$H \to d \mid c$

Given grammar is ambiguous, since string "c" can be generated in two ways.

An ambiguous grammar can be neither LL nor LR since every LL and every LR grammar is unambiguous.

∴ Neither S1 nor S2 is correct.

3 Syntax Directed Translation

3.1 Generation of intermediate code based on an abstract machine model is useful in compilers because

(a) it makes implementation of lexical analysis and syntax analysis easier

(b) syntax-directed translations can be written for intermediate code generation

(c) it enhances the portability of the front end of the complier

(d) it is not possible to generate code for real machines directly from high level language programs

[1994 : 1 Mark]

3.2 A linker is given object modules for a set of programs that were compiled separately. What information need to be included in an object module?

(a) Object code

(b) Relocation bits

(c) Names and locations of all external symbols defined in the object module

(d) Absolute addresses of internal symbols

[1995 : 1 Mark]

3.3 In the following grammar

$X ::= X \oplus Y/Y$

$Y ::= Z \odot Y/Z$

$Z ::= id$

Which of the following is true?

(a) '$\oplus$' is left associative while '$\odot$' is right associative

(b) Both '$\oplus$' and '$\odot$' is left associative

(c) '$\oplus$' is the right associative while '$\odot$' is left associative

(d) None of the above

[1997 : 1 Mark]

3.4 In a bottom-up evaluation of a syntax directed definition, inherited attributes can

(a) always be evaluated

(b) be evaluated only if the definition is L-attributed

(c) be evaluated only if the definition has synthesized attributes

(d) never be evaluated

[2003 : 1 Mark]

3.5 Consider the translation scheme shown below:

$S \rightarrow T\ R$

$R \rightarrow +\ T$ {print ('+');} $R \mid \varepsilon$

$T \rightarrow$ num {print (num.val);}

Here num is a token that represents an integer and num.val represents the corresponding integer value. For an input string '9 + 5 + 2', this translation scheme will print

(a) 9 + 5 + 2 (b) 9 5 + 2 +

(c) 9 5 2 + + (d) + + 9 5 2

[2003 : 2 Marks]

3.6 Consider the syntax directed definition shown below:

$S \rightarrow id := E$ {gen (id.place = E.place;);}

$E \rightarrow E_1 + E_2$ {t = newtemp ( );
gen (t = E_1. place + E_2. place;);
E.place = t}

$E \rightarrow id$ {E.place = id.place;}

Here, gen is a function that generates the output code, and newtemp is a function that returns the name of a new temporary variable on every call. Assume that t_i's are the temporary variable names generated by newtemp.

For the statement 'X: = Y + Z', the 3-address code sequence generated by this definition is

(a) $X = Y + Z$

(b) $t_1 = Y + Z; X = t_1$

(c) $t_1 = Y; t_2 = t_1 + Z; X = t_2$

(d) $t_1 = Y; t_2 = Z; t_3 = t_1 + t_2; X = t_3$

[2003 : 2 Marks]

3.7 Consider the grammar rule $E \rightarrow E_1 - E_2$ for arithmetic expressions. The code generated is targeted to a CPU having a single user register. The subtraction operation requires the first operand to be in the register. If E_1 and E_2 do not have any common subexpression, in order to get the shortest possible code

(a) E_1 should be evaluated first
(b) E_2 should be evaluated first
(c) Evaluation of E_1 and E_2 should necessarily be interleaved
(d) Order of evaluation of E_1 and E_2 is of no consequence

[2004 : 1 Mark]

3.8 Consider the grammar with the following translation rules and E as the start symbol.

$E \rightarrow E_1 \# T$ {E.value = E_1.value * T.value}
   | T   {E.value = T.value}
$T \rightarrow T_1$ & F   {T.value = T_1.value + F.value}
   | F   {T.value = F.value}
$F \rightarrow$ num   {F.value = num.value}

Compute E. value for the root of the parse tree for the expression: 2 # 3 & 5 # 6 & 4.

(a) 200 (b) 180
(c) 160 (d) 40

[2004 : 2 Marks]

3.9 Consider the grammar $E \rightarrow E + n \mid E \times n \mid n$
For a sentence $n + n \times n$, the handles in the right-sentential form of the reduction are
(a) n, E + n and E + n × n
(b) n, E + n and E + E × n
(c) n, n + n and n + n × n
(d) n, E + n and E × n

[2005 : 2 Marks]

Common Data for Q. 3.10 & Q. 3.11

Consider the following expression grammar. The semantic rules for expression calculation are stated next to each grammar production.

$E \rightarrow$ number   E.val = number.val
   | E '+' E $E^{(1)}$.val = $E^{(2)}$.val + $E^{(3)}$.val
   | E '×' E $E^{(1)}$.val = $E^{(2)}$.val × $E^{(3)}$.val;

3.10 The above grammar and the semantic rules are fed to a yacc tool (which is an LALR(1) parser generator) for parsing and evaluating arithmetic expressions. Which one of the following is true about the action of yacc for the given grammar?
(a) It detects recursion and eliminates recursion
(b) It detects reduce-reduce conflict, and resolves
(c) It detects shift-reduce conflict, and resolves the conflict in favor of a shift over a reduce action
(d) It detects shift-reduce conflict, and resolves the conflict in favor of a reduce over a shift action

[2005 : 2 Marks]

3.11 Assume the conflicts in Part (a) of this question are resolved and an LALR(1) parser is generated for parsing arithmetic expressions as per the given grammar. Consider an expression 3 × 2 + 1. What precedence and associativity properties does the generated parser realize?
(a) Equal precedence and left associativity; expression is evaluated to 7
(b) Equal precedence and right associativity; expression is evaluated to 9
(c) Precedence of '×' is higher than that of '+', and both operators are left associative; expression is evaluated to 7
(d) Precedence of '+' is higher than that of '×', and both operators are left associative; expression is evaluated to 9

[2005 : 2 Marks]

3.12 Consider the following translation scheme.

$S \rightarrow ER$
$R \rightarrow$ *E {print('*'): R | ε
$E \rightarrow$ F + E {print ('+'); | F
$F \rightarrow$ (S) | id {print (id. value);}

Here id is a token that represents an integer and id.value represents the corresponding integer value. For an input '2 * 3 + 4', this translation scheme prints
(a) 2 * 3 + 4 (b) 2 * + 3 4
(c) 2 3 * 4 + (d) 2 3 4 + *

[2006 : 2 Marks]

3.13 In a simplified computer the instructions are:
OP R_j, R_i – Performs R_j OP R_i and stores the result in register R_i
OP m, R_i – Performs val OP R_i and stores the result in R_i. val denotes the content of memory location m.
MOV m, R_i – Moves the content of memory location m to register R_i.
MOV Ri, m – Moves the content of register R_i to memory location m.
The computer has only two registers, and OP is either ADD or SUB.

Consider the following basic block:

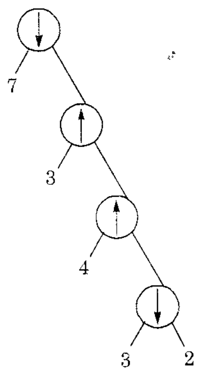
$t_1 = a + b$
$t_2 = c + d$
$t_3 = e - t_2$
$t_4 = t_1 - t_3$

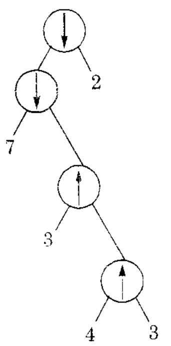
Assume that all operands are initially in memory. The final value of the computation should be in memory. What is the minimum number of MOV instructions in the code generated for this basic block?

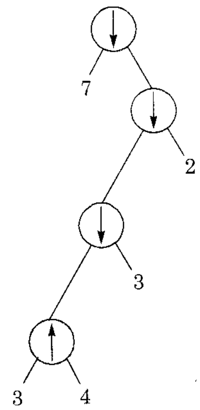
(a) 2 (b) 3
(c) 5 (d) 6

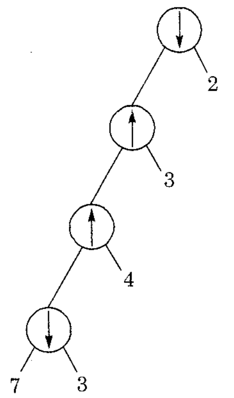
[2007 : 2 Marks]

3.14 Consider two binary operators '↑' and '↓' with the precedence of operator ↓ being lower than that of the operator ↑. Operator ↑ is right associative while operator ↓ is left associative. Which one of the following represents the parse tree for expression (7↓3↑4↑3↓2)

(a)


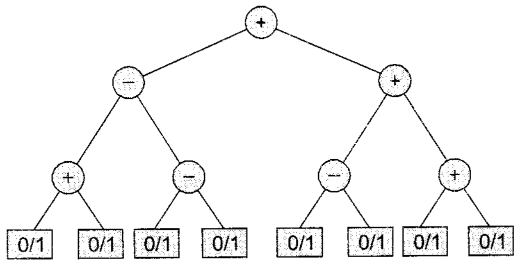
(b)


(c)


(d)


[2011 : 2 Marks]

3.15 Consider the expression tree shown. Each leaf represents a numerical value, which can either be 0 or 1. Over all possible choices of the values at the leaves, the maximum possible value of the expression represented by the tree is ______.



[2014 (Set-2) : 2 Marks]

3.16 One of the purposes of using intermediate code in compilers is to

(a) make parsing and semantic analysis simpler.
(b) improve error recovery and error reporting.
(c) increase the chances of reusing the machine-independent code optimizer in other compilers.
(d) improve the register allocation.

[2014 (Set-3) : 1 Mark]

3.17 The attributes of three arithmetic operators in some programming language are given below.

Operator	Precedence	Associativity	Arity
+	High	Left	Binary
–	Medium	Right	Binary
*	Low	Left	Binary

The value of the expression $2 - 5 + 1 - 7 * 3$ in this language is ______.

[2016 (Set-1) : 2 Marks]

3.18 Consider the following Syntax Directed Translation Scheme (SDTS), with non-terminals $\{S, A\}$ and terminals $\{a, b\}$.

$S \rightarrow aA$ {print 1}
$S \rightarrow a$ {print 2}
$A \rightarrow Sb$ {print 3}

Using the above SDTS, the output printed by a bottom-up parser, for the input *aab* is:

(a) 1 3 2 (b) 2 2 3
(c) 2 3 1 (d) syntax error

[2016 (Set-1) : 2 Marks]

■■■■

Answers Syntax Directed Translation

3.1	(a)	3.2	(d)	3.3	(a)	3.4	(c)	3.5	(b)	3.6	(b)	3.7	(b)	3.8	(c)	3.9	(d)
3.10	(c)	3.11	(b)	3.12	(d)	3.13	(b)	3.14	(b)	3.16	(c)	3.18	(c)				

Explanations Syntax Directed Translation

3.1 (a)

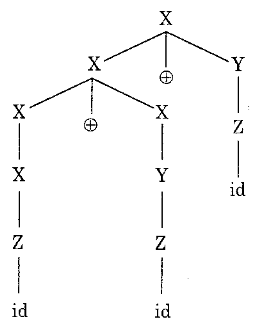
Generation of intermediate code based on an abstract machine model is useful in compilers because it makes implementation of lexical analysis and syntax analysis easier.

3.2 (d)

A linker is a computer program that takes one or more object files generated by a compiler and combines them into a single executable program. The linker also takes care of arranging the objects in a program's address space. Therefore absolute addresses of internal symbols need to be included in an object module.

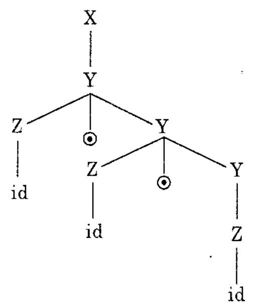
3.3 (a)

Creating syntax tree for id⊕id⊕d :



Therefore ⊕ is left associative.

Creating syntax tree for id⊙id ⊙id :



Therefore, ⊙ is right associative.

3.4 (c)

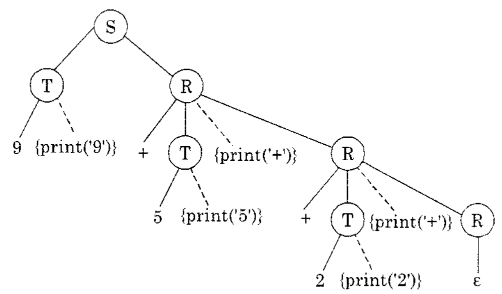
Every S(Synthesized) - attributed definition is L-attributed. For implementing inherited attributed during bottom-up parsing, extends to some, but not LR grammars. Consider the following example

Production	**Semantic Rule**
$S \rightarrow L$	L. count : = 0
$L \rightarrow L_1$ 1	L: count : = L. count : + 1
$L \rightarrow E$	print (L.count)

In the example above the nonterminal L in $L \rightarrow E$ inherits the count of the number of 1's generated by S. Since the production $L \rightarrow E$ is the first that a bottom- up parser would reduce by, the translator at the time can't know the number of 1's in the input. So in a bottom - up evaluation of a syntax directed definition, inherits attributes can't be evaluated if the definition is L-attributed in the given example. So we can say that L-attributed definition is based on simple LR(1) grammar, but it can't be implemented always but inherit attributes can be evaluated only if the definition has synthesized attributes.

3.5 (b)

For the input '9 + 5 + 2' the translation scheme is 95 + 2 + shown below:



3.6 (b)

gen( ) function will be used only two times for X = Y + Z and only one temp variable is created with newtemp( ).

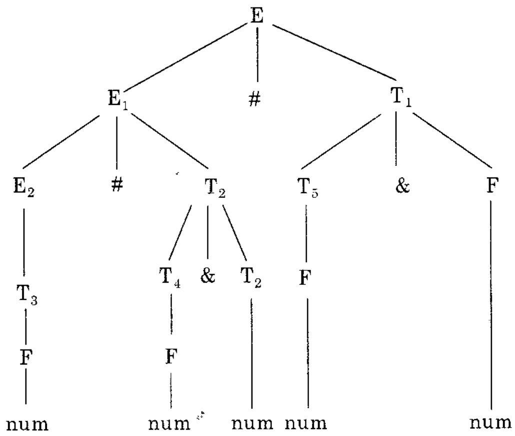
$\therefore t_1 = Y + Z; X = t_1$

3.7 (b)

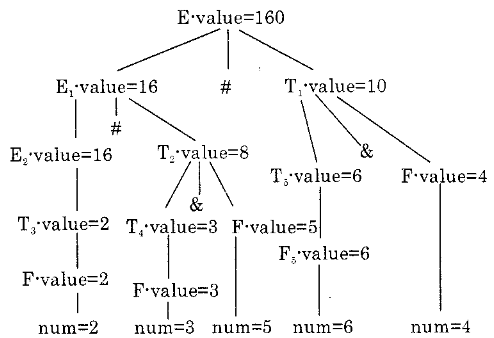
To optimize the solution evaluate the expression E_2. Then we can calculate E_1 and finally E_1 will be one of operands that will be in register and we can perform subtraction directly. But if we follow the opposite then we have to make move and store operations.

3.8 (c)

First we have to construct the parse tree.



Then we construct the annonated parse tree or parse tree with value at the leaf node.



3.9 (d)

$E \rightarrow E + n \mid E \times n \mid n$

Input String $n + n \times n$

$\Rightarrow n + n \times n$

$\Rightarrow E + n \times n$ reduction $E \rightarrow n$

$\Rightarrow E \times n$ reduction $E \rightarrow E + n$

$\Rightarrow E$ reduction $E \rightarrow E \times n$

So the reductions are n, E + n, E × n

3.10 (c)

$E \rightarrow \text{number} \mid E' + E' \mid E' \times E'$

Then YACC compiler detects shift-reduce conflict, and resolves the conflict in favor of a shift over a reduce action.

Consider the following configuration where shift-reduce conflict occurs

StackInput

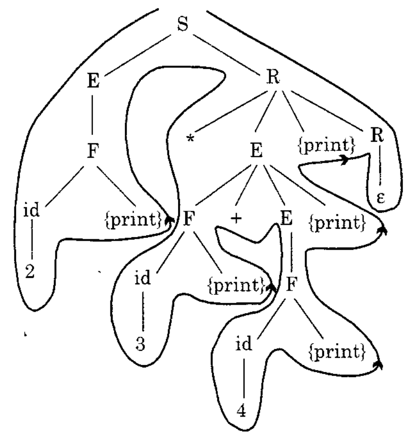
... E + E × .E...$

3.11 (b)

SR conflict is resolved in favour of shift. All operators are shifted and evaluated from right end, because × and + have equal precedence. Hence, × and + follows right associatively.

$3 \times 2 + 1 = 3 \times 3 = 9$

3.12 (d)



So an input 2 * 3 + 4, it prints from the above parse tree as 234 + *.

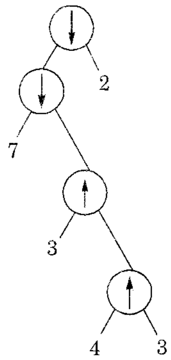
3.13 (b)

Checkit out using following code.

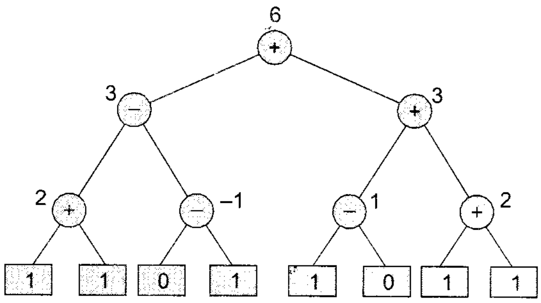
MOV	a, R_1	
opr	b, R_1	$t_1 = a + b$
MOV	d, R_2	
opr	c, R_2	$t_2 = c + d$
opr	e, R_2	$t_3 = e - t_2$
MOV	t_3, R_1	
opr	t_1, R_1	$t_4 = t_1 - t_3$

Minimum number of MOV instructions required = 3.

3.14 (b)



3.15 Sol.



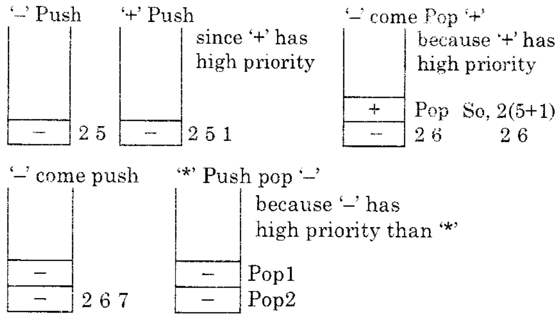
3.16 (c)

Intermediate code can be optimized using the machine-independent code optimizers. All compilers can use same machine independent code optimizers to optimize the intermediate code.

3.17 Sol.

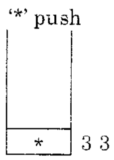
Since given expression in infix expression. So we use operator stack.

2 − 5 + 1 − 7 * 3



So, 2 − (6 − 7) ∴ '−' right associative.

2 − (−1) = 3



String finish so pop '*'

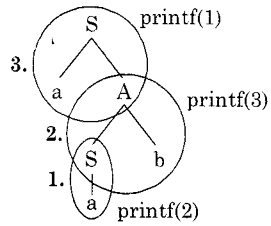
So, 3*3 = 9.

So, 2 − 5 + 1 − 7 * 3 evaluates to 9.

3.18 (c)

Input is '*aab*'

So tree for given input.



So output will be 2, 3 and 1 because printed order will be 1, 2, 3.

So output: 2 3 1

4 Code Generation and Optimization

4.1 Consider the following C code segment.

```
for (i = 0; i < n; i ++)
{
   for (j = 0; j < n; j ++)
   {
      if (i % 2)
      {  x + = (4 * j + 5 * i);
         y + = (7 + 4 *j);
      }
   }
}
```

which one of the following is false?

(a) The code contains loop invariant computation
(b) There is scope of common sub-expression elimination in this code
(c) There is scope strength reduction in this code
(d) There is scope of dead code elimination in this code

[2006 : 2 Marks]

4.2 Some code optimizations are carried out on the intermediate code because

(a) They enhance the portability of the complier to other target processors
(b) Program analysis is more accurate on intermediate code than on machine code
(c) The information from data flow analysis cannot otherwise be used for optimization
(d) The information from the front end cannot otherwise be used for optimization

[2008 : 1 Mark]

4.3 Which languages necessarily need heap allocation in the runtime environment?

(a) Those that support recursion
(b) Those that use dynamic scoping
(c) Those that allow dynamic data structure
(d) Those that use global variables

[2010 : 1 Mark]

Common Data for Questions 4.4 and 4.5

The following code segment is executed on a processor which allows only register operands in its instructions. Each instruction can have almost two source operands and one destinations operand. Assume that all variables are dead after this code segment.

```
c = a + b;
d = c * a;
e = c + a;
x = c * c;
if (x > a) {
    y = a * a;
}
else {
    d = d * d;
    e = e * e;
}
```

4.4 Suppose the instruction set architecture of the processor has only two registers. The only allowed complier optimization is code motion, which moves statements from one place to another while preserving correctness. What is the minimum number of spills to memory in the compiled code?

(a) 0 (b) 1
(c) 2 (d) 3

[2013 : 2 Marks]

4.5 What is the minimum number of registers needed in the instruction set architecture of the processor to compile this code segment without any spill to memory? Do not apply any optimization other than optimizing register allocation?

(a) 3 (b) 4
(c) 5 (d) 6

[2013 : 2 Marks]

4.6 Which one of the following is **FALSE**?

(a) A basic block is a sequence of instructions where control enters the sequence at the beginning and exits at the end.
(b) Available expression analysis can be used for common subexpression elimination.
(c) Live variable analysis can be used for dead code elimination.
(d) $x = 4 \times 5 \Rightarrow x$ is an example of common subexpression elimination.

[2014 (Set-1) : 1 Mark]

4.7 Which one of the following is NOT performed during compilation?
(a) Dynamic memory allocation
(b) Type checking
(c) Symbol table management
(d) Inline expansion

[2014 (Set-2) : 1 Mark]

4.8 For a C program accessing X[i][j][k], the following intermediate code is generated by a compiler. Assume that the size of an integer is 32 bits and the size of a character is 8 bits.

```
t0 = i * 1024
t1 = j * 32
t2 = k * 4
t3 = t1 + t0
t4 = t3 + t2
t5 = X[t4]
```

Which one of the following statements about the source code for the C program is CORRECT?
(a) X is declared as "int X[32][32][8]".
(b) X is declared as "int X[4][1024][32]".
(c) X is declared as "char X[4][32][8]".
(d) X is declared as "char X[32][16][2]".

[2014 (Set-2) : 2 Marks]

4.9 Which of the following statements are CORRECT?
(1) Static allocation of all data areas by a compiler makes it impossible to implement recursion.
(2) Automatic garbage collection is essential to implement recursion.
(3) Dynamic allocation of activation records is essential to implement recursion.
(4) Both heap and stack are essential to implement recursion.

(a) 1 and 2 only (b) 2 and 3 only
(c) 3 and 4 only (d) 1 and 3 only

[2014 (Set-3) : 1 Mark]

4.10 Consider the basic block given below.

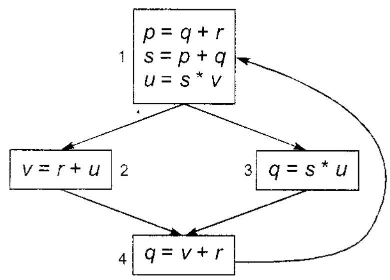
$$a = b + c$$
$$c = a + d$$
$$d = b + c$$
$$e = d - b$$
$$a = e + b$$

The minimum number of nodes and edges present in the DAG representation of the above basic block respectively are

(a) 6 and 6 (b) 8 and 10
(c) 9 and 12 (d) 4 and 4

[2014 (Set-3) : 2 Marks]

4.11 A variable x is said to be live at a statement S_i in a program if the following three conditions hold simultaneously:
1. There exists a statement S_j that uses x
2. There is a path from S_i to S_j in the flow graph corresponding to the program
3. The path has no intervening assignment to x including at S_i and S_j



The variables which are live both at the statement in basic block 2 and at the statement in basic block 3 of the above control flow graph are
(a) p, s, u (b) r, s, u
(c) r, u (d) q, v

[2015 (Set-1) : 2 Marks]

4.12 The least number of temporary variables required to create a three-address code in static single assignment form for the expression q+r/3 + s – t * 5+u * v/w is _____.

[2015 (Set-1) : 2 Marks]

4.13 In the context of abstract-syntax-tree (AST) and control-flow-graph (CFG), which one of the following is True?
(a) In both AST and CFG, let node N_2 be the successor of node N_1. In the input program, the code corresponding to N_2 is present after the code corresponding to N_1
(b) For any input program, neither AST nor CFG will contain a cycle
(c) The maximum number of successors of a node in an AST and a CFG depends on the input program
(d) Each node in AST and CFG corresponds to at most one statement in the input program

[2015 (Set-2) : 1 Mark]

4.14 Consider the intermediate code given below:

1. i = 1
2. j = 1
3. t1 = 5 * i
4. t2 = t1 + j
5. t3 = 4 * t2
6. t4 = t3
7. a[t4] = –1
8. j = j + 1
9. if j <= 5 goto (3)
10. i = i + 1
11. if i < 5 goto (2)

The number of nodes and edges in the control-flow-graph constructed for the above code, respectively, are

(a) 5 and 7 (b) 6 and 7
(c) 5 and 5 (d) 7 and 8

[2015 (Set-2) : 2 Marks]

4.15 Consider the following code segment.

$x = u - t;$
$y = x * v;$
$x = y + w;$
$y = t - z;$
$y = x * y;$

The minimum number of *total* variables required to convert the above code segment to *static single assignment* form is _____.

[2016 (Set-1) : 1 Mark]

4.16 A student wrote two context-free grammars $G1$ and $G2$ for generating a single C-like array declaration. The dimension of the array is at least one. For *example*,

int a [10] [3];

The grammars use D as the start symbol, and use six terminal symbols int; id [ ] num.

Grammar $G1$	**Grammar $G2$**
$D \rightarrow$ int L;	$D \rightarrow$ int L;
$L \rightarrow$ id [E	$L \rightarrow idE$
$E \rightarrow$ num]	$E \rightarrow E$ [num]
$E \rightarrow$ num] [E	[$E \rightarrow$ [num]

Which of the grammars correctly generate the declaration mentioned above?

(a) Both $G1$ and $G2$
(b) Only $G1$
(c) Only $G2$
(d) Neither $G1$ nor $G2$

[2016 (Set-2) : 2 Marks]

Answers Code Generation and Optimization

4.1	(d)	4.2	(b)	4.3	(c)	4.4	(b)	4.5	(b)	4.6	(d)	4.7	(a)	4.8	(a)	4.9	(d)
4.10	(a)	4.11	(*)	4.13	(c)	4.14	(b)	4.16	(a)								

Explanations Code Generation and Optimization

4.1 (d)

(a) i%2 is inner loop invariant, it can be moved before inner loop.
(b) 4*j is common sub-expression appeared in two statements.
(c) 4*j can be reduced to j<<2 by strength reduction.
(d) There is no dead code in given code segment. So there is no scope of dead code elimination in this code.

Hence only option (d) is FALSE.

4.2 (b)

Some code optimizations are carried out on the intermediate code because program analysis is more accurate on intermediate code than on machine code.

4.3 (c)

Runtime environment means we deal with dynamic memory allocation and Heap is a dynamic data structure.

So it is clear that those languages that allow dynamic data structure necessarily need heap allocation in the runtime environment.

4.4 (b)

c = a + b;
x = c * c; ...(i)
if (x > a) {
 y = a * a;
}
else {
 d = c * a; ...(ii)
 e = c + a;
 d = d * d;
 e = e * e;
}

(i) is to store c*c, it needs onememory spill.
(ii) is uses previous same(i) memory spill to store c*a.
Number of memory spills are used in above program is one. With the use of one memory cell and two registers the above optimized code can be executed.

4.5 (b)

$R_1 \leftarrow R_0 + R_1$	c = a + b
$R_2 \leftarrow R_0 * R_1$	d = c * a
$R_3 \leftarrow R_1 + R_0$	e = c + a
$R_1 \leftarrow R_1 * R_1$	x = c * c
if ($R_1 > R_0$)	if (x > a)
{	
$R_1 \leftarrow R_0 * R_0$	y = a * a
}	
else	
{	
$R_2 \leftarrow R_2 * R_1$	d = d * d
$R_3 \leftarrow R_3 * R_3$	e = e * a
}	

4 registers are required.
∴ Option (b) is correct.

4.6 (d)

$$x = 4 \times 5$$

$x = 20$ is an example for constant folding but not for common subexpression elimination.

4.7 (a)

Dynamic memory allocation performed during runtime whereas type checking, symbol table management and Inline expansion is performed during compilation.

4.8 (a)

Each integer element requires 4 bytes
int X [32] [32] [8];
To access X[i][j][k] :
X [(i*32*8+j*8 + k)*4] ...(1)
(integer size = 4)
From given intermediate code:
$X[t_4] = X[t_3 + t_2] = X[t_1+t_0+t_2]$
$= X[t_0 + t_1 + t_2]$
= X [(i*1024+j*32+k *4)] ...(2)
∴ (1) and (2) are equivalent.

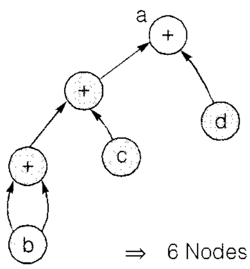
4.9 (d)

Recursion can not be implemented using static allocation.
Recursion can be implemented using dynamic allocation.

4.10 (a)

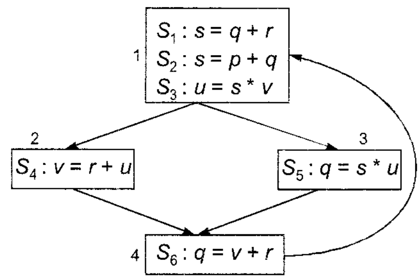
$a = b + c \Rightarrow a = e + b$
$c = a + d \Rightarrow a = d - b - b = d$
$d = b + c \Rightarrow a = b + c$
$e = d - b \Rightarrow a = b + a + d$
$a = e + b \Rightarrow a = b + b + c + d$

∴ $b + b + c + d$ is expression



⇒ 6 Nodes and 6 edges

4.11 (*)



(i) p is not live both at Block 2 and Block 3. There is a path from (S_4 & S_5) to S_2, but S_1 has an assignment to p.

(ii) q is not live both at Block 2 and Block 3. There is a path from (S_4 & S_5) to S_1, but S_6 has an assignment to q in between the path.

(iii) r is live both at Block 2 and Block 3. There is a path from (S_4 & S_5) to S_6, S_6 uses r and no assignment to r in this path.

(iv) u is not live both at Block 2 and Block 3. There is a path from S_4 to S_5 and S_5 to S_4 where both S_4 and S_5 uses u but S_3 has an assignment to u in this path.

(v) Similarly s and v are not live both at Block 2 and Block 3

∴ Only r is live both at basic Block 2 and basic Block 3.

4.12 Sol.

In static single assignment, every variable assigned only once and that variable can be used any number of times without assignment.

Expression: q+r/3+s – t*5+u * v/w

t_1=r / 3

t_2 = q + t_1

t_3= t_2 + s

t_4= t * 5

t_5= t_3 – t_4

t_6= u * v

t_7= t_6 / w

t_8= t_5 + t_7

∴ 8 temporary variables are required.

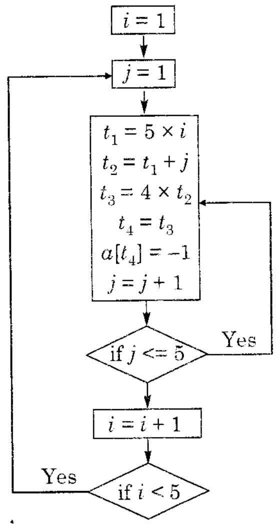
4.13 (c)

Maximum number of successors of a node in an AST and a CFG depends on input program.

CFG can contain a cycle.

A CFG node may corresponds to more than one statement.

4.14 (b)



Number of nodes = 6

Number of edges = 7

4.15 Sol.

$r_1 = r_2 - r_3;$

$r_4 = x * r_5;$

$x_6 = y + r_7;$

$r_8 = r - r_9;$

$r_{10} = x * y;$

In static single assignment form 1 temporary register will be assign to only 1 variable and every variable is defined before it is used (we have to assign the register at a time of define the variable) throughout the code segments. So here 10 variables are needed.

4.16 (a)

Both $G1$ and $G2$ can generate int a[10][3]; as follows:

G1 :D → int L; → int id[E ;
→ int id[num][E;
→ int id[num] [num];

G2 :D → int L ; → int id E;
→ int id E [num];
→ int id [num][num];

■■■■

Unit ■ VIII

Operating System

■ Contents

UNIT VIII

Operating System

Syllabus : Processes, Threads, Inter-process communication, Concurrency, Synchronization, Deadlock, CPU scheduling, Memory management and virtual memory, File systems, I/O systems, Protection and security.

Analysis of Previous GATE Papers

Exam Year	1 Mark Ques.	2 Marks Ques.	3 Marks Ques.	5 Marks Ques.	Total Marks
1990	–	7		2	**24**
1991	–	4		1	**13**
1992	1	2		1	**10**
1993	–	5		2	**20**
1994	–	2		2	**14**
1995	4	5		4	**34**
1996	4	4		2	**22**
1997	7	2		–	**11**
1998	8	4		2	**26**
1999	3	3		2	**19**
2000	1	1		–	**32**
2001	10	3		1	**1**
2002	1	5		4	**31**
2003	2	5		–	**12**
2004	3	4		–	**11**
2005	–	2		–	**4**
2006	1	8		–	**17**

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2007	2	6	**14**
2008	2	5	**12**
2009	2	5	**12**
2010	3	2	**7**
2011	4	2	**8**
2012	1	4	**9**
2013	2	4	**10**
2014 Set-1	2	3	**8**
2014 Set-2	1	3	**7**
2014 Set-3	1	3	**7**
2015 Set-1	2	4	**10**
2015 Set-2	2	3	**8**
2015 Set-3	2	2	**6**
2016 Set-1	1	4	**9**
2016 Set-2	1	3	**7**

1 Process Management-I

(Introduction, Process, Threads & CPU Scheduling)

1.1 State an undesirable characteristic of each of the following criteria for measuring performance of an operating system:
(a) Turn around time
(b) Waiting time

[1988 : 2 Marks]

1.2 The highest-response ratio next scheduling policy favours .... jobs, but it also limits the waiting time of .... jobs.

[1990 : 2 Marks]

1.3 Assume that the following jobs are to be executed on a single processor system

c

Job Id	CPU Brust time
p	4
q	1
r	8
s	1
t	2

The jobs are assumed to have arrived at time 0^+ and in the order p, q, r, s, t. Calculate the departure time (completion time) for job p if scheduling is round robin with time slice 1.

(a) 4 (b) 10
(c) 11 (d) 12

[1993 : 2 Marks]

1.4 Which scheduling policy is most suitable for a time-shared operating systems?

b

(a) Shortest Job First
(b) Round Robin
(c) First come first serve
(d) Elevator

[1995 : 1 Mark]

1.5 The sequence ..... is an optimal non-preemptive scheduling sequence for the following jobs which leaves the CPU idle for ..... unit(s) of time.

a

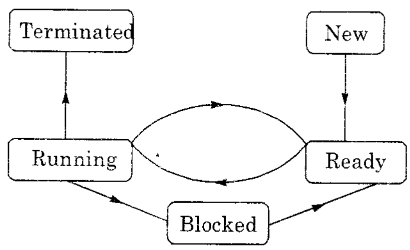
Job	Arrival time	Burst time
1	0.0	9
2	0.6	5
3	1.0	1

(a) {3, 2, 1}, 1 (b) {2, 1, 3}, 0
(c) {3, 2, 1}, 0 (d) {1, 2, 3}, 5

[1995 : 2 Marks]

1.6 The process state transition diagram in figure is representative of

b



(a) a batch operating system
(b) an operating system with a preemptive scheduler
(c) an operating system with a non-preemptive scheduler
(d) a uni-programmed operating system

[1996 : 1 Mark]

1.7 Which of the following is an example of spooled device?

a

(a) A line printer used to print the output of a number of jobs
(b) A terminal used to enter input data to a running program
(c) A secondary storage device in a virtual memory system
(d) A graphic display device

[1996 : 1 Mark]

1.8 Four jobs to be executed on a single processor system arrive at time 0^+ in the order A, B, C, D. Their burst CPU time requirements are 4, 1, 8, 1 time units respectively. The completion time of A under round robin scheduling with time slice of one time unit is

d

(a) 10 (b) 4
(c) 8 (d) 9

[1996 : 2 Marks]

1.9 Which of the following is an example of a spooled device?

b

(a) The terminal used to enter the input data for the C program being executed
(b) An output device used to print the output of a number of jobs.
(c) The secondary memory device in a virtual storage system
(d) The swapping area on a disk used by the swapper.

[1998 : 1 Mark]

1.10 Consider n processes sharing the CPU in a round-robin fashion. Assuming that each process switch takes s seconds, what must be the quantum size q such that the overhead resulting from process switching is minimized but at the same time each process is guaranteed to get its turn at the CPU at least every t seconds?

a

(a) $q \le \frac{t-ns}{n-1}$ (b) $q \ge \frac{t-ns}{n-1}$

(c) $q \le \frac{t-ns}{n+1}$ (d) $q \ge \frac{t-ns}{n+1}$

[1998 : 1 Mark]

1.11 Four jobs are waiting to be run. Their expected run times are 6, 3, 5 and x in what order should they be run to minimize the average response time?

[1998 : 2 Marks]

1.12 System calls are usually invoked by using

a

(a) a software interrupt
(b) polling
(c) an indirect jump
(d) a privileged instruction

[1999 : 1 Mark]

1.13 A multi-user, multi-processing operating system cannot be implemented on hardware that does not support

c

(a) Address translation
(b) DMA for disk transfer
(c) At least two modes of CPU execution (privileged and non-privileged)
(d) Demand paging

[1999 : 2 Marks]

1.14 Which of the following actions is/are typically not performed by the operating system when switching context from process A to process B?

c

(a) Saving current register values and restoring saved register values for process B.
(b) Changing address translation tables.
(c) Swapping out the memory image of process A to the disk.
(d) Invalidating the translation look-aside buffer.

[1999 : 2 Marks]

1.15 A processor needs software interrupt to

c

(a) Test the interrupt system of the processor
(b) Implement co-routines
(c) Obtain system services which need execution of privileged instructions
(d) Return from subroutine

[2001 : 1 Mark]

1.16 A CPU has two modes-privileged and non-privileged. In order to change the mode from privileged to non-privileged

d or b

(a) a hardware interrupt is needed
(b) a software interrupt is needed
(c) a privileged instruction (which does not generate an interrupt) is needed
(d) a non-privileged instruction (which does not generate an interrupt) is needed

[2001 : 1 Mark]

1.17 Consider a set of n tasks with known runtimes $r_1, r_2, \ldots, r_n$ to be run on a uniprocessor machine. Which of the following processor scheduling algorithms will result in the maximum throughput?

b

(a) Round-Robin
(b) Shortest-Job-First
(c) Highest-Response-Ratio-Next
(d) First-Come-First-Served

[2001 : 1 Mark]

1.18 Which of the following scheduling algorithms is non-preemptive?

b

(a) Round Robin
(b) First-in-First-out
(c) Multilevel Queue Scheduling
(d) Multilevel Queue Scheduling with Feedback

[2001 : 1 Mark]

1.19 Which of the following does not interrupt a running process?

a

(a) A device (b) Timer
(c) Scheduler process (d) Power failure

[2001 : 2 Marks]

1.20 Which combination of the following features will suffice to characterize an OS as a multi-programmed OS? (i) more than one program may

d

be loaded into main memory at the same time for execution. (ii) If a program waits for certain events such as I/O, another program is immediately scheduled for execution. (iii) If the execution of a program terminates, another program is immediately scheduled for execution

(a) (i)
(b) (i) and (ii)
(c) (i) and (iii)
(d) (i), (ii) and (iii)

[2002 : 2 Marks]

1.21 Draw the process state transition diagram of an OS in which (i) each process is in one of the five states: created, ready, running, blocked (i.e. sleep or wait), or terminated, and (ii) only non-preemptive scheduling is used by the OS. Label the transitions appropriately.

[2002 : 2 Marks]

1.22 A uni-processor computer system only has two processes, both of which alternate 10 ms CPU bursts with 90 ms I/O bursts. Both the processes were created at nearly the same time. The I/O of both processes can proceed in parallel. Which of the following scheduling strategies will result in the least CPU utilization (over a long period of time) for this system?

d

(a) First come first served scheduling
(b) Shortest remaining time first scheduling
(c) Static priority scheduling with different priorities for the two processes
(d) Round robin scheduling with a time quantum of 5 ms.

[2003 : 2 Marks]

1.23 Consider the following statements with respect to user-level threads and kernel-supported threads

a

(i) Context switch is faster with kernel-supported threads
(ii) For user-level threads, a system call can block the entire process
(iii) Kernel-supported threads can be scheduled independently
(iv) User-level threads are transparent to the kernel

Which of the above statements are true?

(a) (ii), (iii) and (iv) only
(b) (ii) and (iii) only
(c) (i), and (iii) only
(d) (i) and (ii) only

[2004 : 1 Mark]

1.24 Which one of the following is NOT shared by the threads of the same process?

a

(a) Stack
(b) Address Space
(c) File Descriptor Table
(d) Message Queue

[2004 : 1 Mark]

1.25 A process executes the following segment of code:

c

```
for(i = 1; i < = n; i++)
fork();
```

The number of new processes created is

(a) n
(b) ((n(n + 1))/2)
(c) $2^n - 1$
(d) $3^n - 1$

[2004 : 2 Marks]

1.26 Consider the following set of processes, with the arrival times and the CPU-burst times given in milliseconds

a

Process	Arrival time	Burst time
P1	0	5
P2	1	3
P3	2	3
P4	4	1

What is the average turnaround time for these processes with the preemptive Shortest Remaining Processing Time first (SRPT) algorithm?

(a) 5.50
(b) 5.75
(c) 6.00
(d) 6.25

[2004 : 2 Marks]

1.27 A user level process in Unix traps the signal sent on a Ctrl-C input, and has a signal handling routine that saves appropriate files before terminating the process. When a Ctrl-C input is given to this process, what is the mode in which the signal handling routine executes?

a

(a) kernel mode
(b) super user mode
(c) privileged mode
(d) user mode

[2005 : 1 Mark]

1.28 We wish to schedule three processes P1, P2 and P3 on a uniprocessor system. The priorities, CPU time requirements and arrival times of the processes are as shown below.

d

Process	Priority	CPU time required	Arrival time (hh:mm:ss)
P1	10 (highest)	20 sec	00:00:05
P2	9	10 sec	00:00:03
P3	8 (lowest)	15 sec	00:00:00

We have a choice of preemptive or non-preemptive scheduling. In preemptive scheduling, a late-arriving higher priority process can preempt a currently running process with lower priority. In non-preemptive scheduling, a late-arriving higher priority process must wait for the currently executing process to complete before it can be scheduled on the processor. What are the turnaround times (time from arrival till completion) of P2 using preemptive and non-preemptive scheduling respectively?

(a) 30 sec, 30 sec (b) 30 sec, 10 sec
(c) 42 sec, 42 sec (d) 30 sec, ~~40 sec~~ 42 sec

[2005 : 2 Marks]

1.29 (c) Two shared resources R_1 and R_2 are used by processes P_1 and P_2. Each process has a certain priority for accessing each resource. Let T_{ij} denote the priority of P_i for accessing R_j. A process P_i can snatch a resource R_k from process P_j if T_{ik} is greater than T_{jk}.

Given the following :

I. $T_{11} > T_{21}$ **II.** $T_{12} > T_{22}$
III. $T_{11} < T_{21}$ **IV.** $T_{12} < T_{22}$

Which of the following conditions ensures that P_1 and P_2 can never deadlock?

(a) (I) and (IV) (b) (II) and (III)
(c) (I) and (II) (d) None of these

[2005 : 2 Marks]

1.30 (d) Consider the following code fragment:

```
if (fork ( ) = = 0)
{a = a + 5; printf("%d, %d\n", a, &a);}
else{a = a – 5; printf("%d, %d\n", a, &a);}
```

Let u, v be the values printed by the parent process, and x, y be the values printed by the child process. Which one of the following is TRUE?

(a) u = x + 10 and v = y
(b) u = x + 10 and v ≠ y
(c) u + 10 = x and v = y
(d) u + 10 = x and v ≠ y

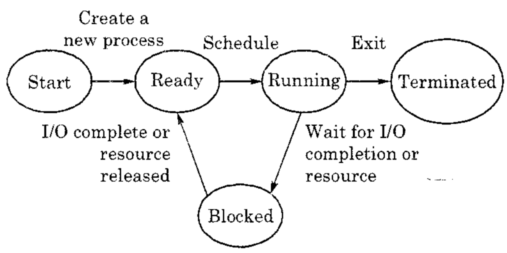
[2005 : 2 Marks]

1.31 (b) Consider three CPU-intensive processes, which require 10, 20 and 30 time units and arrive at times 0, 2, and 6, respectively. How many context switches are needed if the operating system implements a shortest remaining time first scheduling algorithm? Do not count the context switches at time zero and at the end

(a) 1 (b) 2
(c) 3 (d) 4

[2006 : 1 Mark]

1.32 (b) The process state transition diagram of an operating system is as given below.



Which of the following must be FALSE about the above operating system?

(a) It is a multiprogrammed operating system
(b) It uses preemptive scheduling
(c) It uses non-preemptive scheduling
(d) It is a multi-user operating system

[2006 : 1 Mark]

1.33 (b) The arrival time, priority, and durations of the CPU and I/O bursts for each of three processes P_1, P_2 and P_3 are given in the table below. Each process has a CPU burst followed by an I/O burst followed by another CPU burst. Assume that each process has its own I/O resource.

Process	Arrival time	Priority	Burst durations CPU, I/O CPU
P_1	0	2	1, 5, 3
P_2	2	3 (lowest)	3, 3, 1
P_3	3	1 (highest)	2, 3, 1

The multi-programmed operating system uses preemptive priority scheduling. What are the finish times of the processes P_1, P_2 and P_3?

(a) 11, 15, 9 (b) 10, 15, 9
(c) 11, 16, 10 (d) 12, 17, 11

[2006 : 2 Marks]

1.34 (b) Consider three processes, all arriving at time zero, with total execution time of 10, 20 and 30 units, respectively. Each process spends the first 20% of execution time doing I/O, the next 70% of time doing computation, and the last 10% of time doing I/O again. The operating system uses a shortest remaining compute time first scheduling algorithm and schedules a new process either when the running process get blocked on I/O or when the running process finishes its compute burst. Assume that all I/O operations can be overlapped as much as possible. For what percentage of time does the CPU remain idle?

(a) 0% (b) 10.6%
(c) 30.0% (d) 89.4%

[2006 : 2 Marks]

1.35 Consider three processes (process id 0, 1, 2, respectively) with compute time bursts 2, 4, and 8 time units. All processes arrive at time zero. Consider the longest remaining time first (LRTF) scheduling algorithm. In LRTF ties are broken by giving priority to the process with the lowest process id. The average turn around time is

(a) 13 units (b) 14 units
(c) 15 units (d) 16 units

[2006 : 2 Marks]

(a)

1.36 Let a memory have four free blocks of sizes 4k, 8k, 20k, 2k. These blocks are allocated following the best-fit strategy. The allocation requests are stored in a queue as shown below.

Request No.	Request sizes	Usage Time
J1	2k	4
J2	14k	10
J3	3k	2
J4	6k	8
J5	6k	4
J6	10k	1
J7	7k	8
J8	20k	6

The time at which the request for J7 will be completed will be

(a) 16 (b) 19
(c) 20 (d) 37

[2007 : 1 Mark]

1.37 **List-I** contains some CPU scheduling algorithms and **List-II** contains some applications. Match entries in **List-I** entries in **List-II**

List-I

A. Gang Scheduling
B. Rate Monotonic Scheduling
C. Fair Share Scheduling

List-II

1. Guaranteed Scheduling
2. Real-time Scheduling
3. Thread Scheduling

(a)

Code:

	A	B	C
(a)	3	2	1
(b)	1	2	3
(c)	2	3	1
(d)	1	3	2

[2007 : 1 Mark]

1.38 Consider the following statements about user level threads and kernel level threads. Which one of the following statements is FALSE?

(d)

(a) Context switch time is longer for kernel level threads than for user level threads
(b) User level threads do not need any hardware support
(c) Related kernel level threads can be scheduled on different processors in a multiprocessor system
(d) Blocking one kernel level thread blocks all related threads

[2007 : 1 Mark]

1.39 An operating system uses Shortest Remaining Time first (SRT) process scheduling algorithm. Consider the arrival times and execution times for the following processes

(b)

Process	Execution time	Arrival time
P1	20	0
P2	25	15
P3	10	30
P4	15	45

What is the total waiting time for process P2?

(a) 5 (b) 15
(c) 40 (d) 55

[2007 : 2 Marks]

1.40 If the time-slice used in the round-robin scheduling policy is more than the maximum time required to execute any process, then the policy will

(c)

(a) degenerate to shortest job first
(b) degenerate to priority scheduling
(c) degenerate to first come first serve
(d) None of the above

[2008 : 2 Marks]

1.41 Which of the following statements about synchronous and asynchronous I/O is NOT true?

(b)

(a) An ISR is invoked on completion of I/O in synchronous I/O but not in asynchronous I/O
(b) In both synchronous and asynchronous I/O an ISR (Interrupt Service Routine) is invoked after completion of the I/O

(c) A process making a synchronous I/O call waits until I/O is complete, but a process making an asynchronous I/O call does not wait for completion of the I/O

(d) In the case of synchronous I/O, the process waiting for the completion of I/O is woken up by the ISR that is invoked after the completion of I/O

[2008 : 2 Marks]

1.42 A process executes the following code for (i = 0; i < n; i ++) fork ( );
The total number of child processes created is

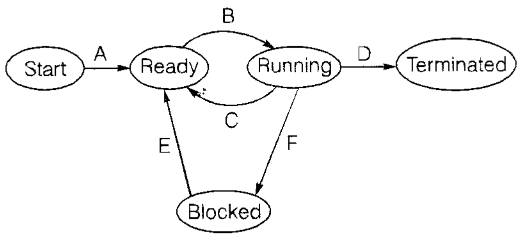
(b)

(a) n (b) $2^n - 1$
(c) 2^n (d) $2^{n+1}-1$

[2008 : 2 Marks]

1.43 In the following process state transition diagram for a uniprocessor system, assume that there are always some processes in the ready state:

(c)



Now consider the following statements:

I. If a process makes a transition D, it would result in another process making transition A immediately

II. A process P_2 in blocked state can make transition E while another process P_1 is in running state

III. The OS uses preemptive scheduling

IV. The OS uses non-preemptive scheduling

Which of the above statements are TRUE?

(a) I and II (b) I and III
(c) II and III (d) II and IV

[2009 : 2 Marks]

1.44 Which of the following statements are true?

(d)

I. Shortest remaining time first scheduling may cause starvation

II. Preemptive scheduling may cause starvation

III. Round robin is better than FCFS in terms of response time

(a) I only (b) I and III only
(c) II and III only (d) I, II and III

[2010 : 1 Mark]

1.45 Let the time taken to switch between user and kernel modes of execution be t_1 while the time taken to switch between two processes be t_2. Which of the following is TRUE?

(c)

(a) $t_1 > t_2$
(b) $t_1 = t_2$
(c) $t_1 < t_2$
(d) nothing can be said about the relation between t_1 and t_2

[2011 : 1 Mark]

1.46 A computer handles several interrupt sources of which the following are relevant for this question.

(d)

- Interrupt from CPU temperature sensor (raises interrupt if CPU temperature is too high)
- Interrupt from Mouse (raises interrupt if the mouse is moved or a button is pressed)
- Interrupt from Keyboard (raises interrupt when a key is pressed or released)
- Interrupt from Hard Disk (raises interrupt when a disk read is completed)

Which one of these will be handled at the **HIGHEST** priority?

(a) Interrupt from Hard Disk
(b) Interrupt from Mouse
(c) Interrupt from Keyboard
(d) Interrupt from CPU temperature sensor

[2011 : 1 Mark]

1.47 A thread is usually defined as a "light weight process" because an Operating System (OS) maintains smaller data structures for a thread than for a process. In relation to this, which of the following is TRUE?

(c)

(a) On per-thread basis, the OS maintains only CPU register state
(b) The OS does not maintain a separate stack for each thread
(c) On per-thread basis, the OS does not maintain virtual memory state
(d) On per-thread basis, the OS maintains only scheduling and accounting information

[2011 : 1 Mark]

1.48 Consider the following table of arrival time and burst time for three processes P0, P1 and P2.

(a)

Process	Arrival time	Burst time
P0	0 ms	9 ms
P1	1 ms	4 ms
P2	2 ms	9 ms

The pre-emptive shortest job first scheduling algorithm is used. Scheduling is carried out only at arrival or completion of processes. What is the average waiting time for the three processes?

(a) 5.0 ms (b) 4.33 ms
(c) 6.33 ms (d) 7.33 ms

[2011 : 2 Marks]

1.49 A process executes the code

fork();
fork();
fork();

(c)

The total number of child processes created is

(a) 3 (b) 4
(c) 7 (d) 8

[2012 : 1 Mark]

1.50 Consider the 3 processes, P1, P2 and P3 shown in the table.

(c)

Process Name	Arrival Time	Time Unit Required
P1	0	5
P2	1	7
P3	3	4

The completion order of the 3 processes under the policies FCFS and RR2 (round robin scheduling with CPU quantum of 2 time units) are:

(a) **FCFS:** P1, P2, P3 **RR2:** P1, P2, P3
(b) **FCFS:** P1, P3, P2 **RR2:** P1, P3, P2
(c) **FCFS:** P1, P2, P3 **RR2:** P1, P3, P2
(d) **FCFS:** P1, P3, P2 **RR2:** P1, P2, P3

[2012 : 2 Marks]

1.51 A scheduling algorithm assigns priority proportional to the waiting time of a process. Every process starts with priority zero (the lowest priority). The schedule reevaluates the process priorities every T time units and decides the next process to schedule. Which one of the following is TRUE if the processes have no I/O operations and all arrive at time zero?

(b)

(a) This algorithm is equivalent to the first come first serve algorithm
(b) This algorithm is equivalent to the round-round algorithm
(c) This algorithm is equivalent to the shortest-job-first algorithm
(d) This algorithm is equivalent to the shortest-remaining-time-first algorithm

[2013 : 1 Mark]

1.52 Which one of the following is **FALSE**?

(d)

(a) User level threads are not scheduled by the kernel.
(b) When a user level thread is blocked, all other threads of its process are blocked.
(c) Context switching between user level threads is faster than context switching between kernel level threads.
(d) Kernel level threads cannot share the code segment.

[2014 (Set-1) : 1 Mark]

1.53 Consider the following set of processes that need to be scheduled on a single CPU. All the times are given in milliseconds.

(7.2)

Process Name	Arrival Time	Execution Time
A	0	6
B	3	2
C	5	4
D	7	6
E	10	3

Using the *shortest remaining time first* scheduling algorithm, the average process turnaround time (in msec) is ________.

[2014 (Set-1) : 2 Marks]

1.54 Three processes *A*, *B* and *C* each execute a loop of 100 iterations. In each iteration of the loop, a process performs a single computation that requires t_c CPU milliseconds and then initiates a single I/O operation that lasts for t_{io} milliseconds. It is assumed that the computer where the processes execute has sufficient number of I/O devices and the OS of the computer assigns different I/O devices to each process. Also, the scheduling overhead of the OS is negligible. The processes have the following characteristics:

(1000)

Process id	t_c	t_{io}
A	100 ms	500 ms
B	350 ms	500 ms
C	200 ms	500 ms

The processes *A*, *B*, and *C* are started at times 0, 5 and 10 milliseconds respectively, in a pure time sharing system (round robin scheduling) that uses a time slice of 50 milliseconds. The time in milliseconds at which process C would complete its first I/O operation is ________.

[2014 (Set-2) : 2 Marks]

1.55 An operating system uses *shortest remaining time first* scheduling algorithm for pre-emptive scheduling of processes. Consider the following set of processes with their arrival times and CPU burst times (in milliseconds):

5.5

Process	Arrival Time	Burst Time
P1	0	12
P2	2	4
P3	3	6
P4	8	5

The average waiting time (in milliseconds) of the processes is ________.

[2014 (Set-3) : 2 Marks]

1.56 Consider a uniprocessor system executing three tasks T_1, T_2 and T_3, each of which is composed of an infinite sequence of jobs (or instances) which arrive periodically at intervals of 3, 7 and 20 milliseconds, respectively. The priority of each task is the inverse of its period, and the available tasks are scheduled in order of priority, with the highest priority task scheduled first. Each instance of T_1, T_2 and T_3 requires an execution time of 1, 2 and 4 milliseconds, respectively. Given that all tasks initially arrive at the beginning of the 1st millisecond and task preemptions are allowed, the first instance of T_3 completes its execution at the end of ________ milliseconds.

[2015 (Set-1) : 2 Marks]

1.57 The maximum number of processes that can be in *Ready* state for a computer system with n CPUs is

d

(a) n
(b) n^2
(c) 2^n
(d) Independent of n

[2015 (Set-3) : 1 Mark]

1.58 For the processes listed in the following table, which of the following scheduling schemes will give the lowest average turnaround time?

c

Process	Arrival Time	Processing Time
A	0	3
B	1	6
C	4	4
D	6	2

(a) First Come First Serve
(b) Non-preemptive Shortest Job First
(c) Shortest Remaining Time
(d) Round Robin with Quantum value two

[2015 (Set-3) : 2 Marks]

1.59 Consider an arbitrary set of CPU-bound processes with unequal CPU burst lengths submitted at the same time to a computer system. Which one of the following process scheduling algorithms would minimize the average waiting time in the ready queue?

a

(a) Shortest remaining time first
(b) Round-robin with time quantum less than the shortest CPU burst
(c) Uniform random
(d) Highest priority first with priority proportional to CPU burst length

[2016 (Set-1) : 1 Mark]

1.60 Consider the following processes, with the arrival time and the length of the CPU burst given in milliseconds. The scheduling algiritþm used is preemptive shortest remaining-time first.

8.25

Process	Arrival Time	Burst Time
P_1	0	10
P_2	3	6
P_3	7	1
P_4	8	3

The average turn around time of these processes is ________ milliseconds.

[2016 (Set-2) : 2 Marks]

Answers Process Management-I

1.3	(c)	1.4	(b)	1.5	(a)	1.6	(b)	1.7	(a)	1.8	(d)	1.9	(b)	1.10	(a)	1.12	(a)
1.13	(c)	1.14	(c)	1.15	(c)	1.16	(d)	1.17	(b)	1.18	(b)	1.19	(a)	1.20	(d)	1.22	(d)
1.23	(a)	1.24	(a)	1.25	(c)	1.26	(a)	1.27	(a)	1.28	(d)	1.29	(c)	1.30	(d)	1.31	(b)
1.32	(b)	1.33	(b)	1.34	(b)	1.35	(a)	1.36	(c)	1.37	(a)	1.38	(d)	1.39	(b)	1.40	(c)
1.41	(b)	1.42	(b)	1.43	(c)	1.44	(d)	1.45	(c)	1.46	(d)	1.47	(c)	1.48	(a)	1.49	(c)
1.50	(c)	1.51	(b)	1.52	(d)	1.57	(d)	1.58	(c)	1.59	(a)						

Explanations Process Management-I

1.1 Sol.

Undesirable characteristics of

(a) **Turn around time:** Burst time of the process should not be too long.

(b) **Waiting time:** Arrival time of the processes should not be same.

1.2 Sol.

Shorter job, longer jobs.

1.3 (c)

Create the gantt chart with Round Robin scheduling.

0	1	2	3	4	5	6	7	8	9	10	11
p	q	r	s	t	p	r	t	p	r	p	...

1.4 (b)

Round Robin scheduling working on time quantum, after certain time every process get back the CPU units for it's completion and the same phenomena used in time sharing system. So Round Robin is best for time sharing system.

1.5 (a)

In this sequence initially CPU ideal for '1' unit of time but after that, smaller jobs don't have to wait for so long time, so it is ideal.

1.6 (b)

Since there is an arrow from running to ready, so it is pre-emptive.

1.7 (a)

Spooled devices are those which fetches data from job waiting area one by one & spool means simultaneous peripheral operation on line, so printer is a spooling device.

1.8 (d)

Draw Gantt chart and place jobs according to round robin scheduling with time quantum '1' unit.

0	1	2	3	4	5	6	7	8	9
A	B	C	D	A	C	A	C	A	C

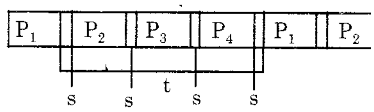
So correct ans is '9'.

1.9 (b)

Spooled devices is output device used to print the output of jobs.

1.10 (a)

Let $n = 4$



$$t \geq n \times s + q \times (n-1)$$

So, $$q \leq \frac{t - ns}{n-1}$$

1.11 Sol.

The average response time depends on the value of x,

If value of x < 3

then order of execution should be

x, 3, 5, 6

If value of x > 3 & x < 5

then order of execution should be

3, x, 5, 6

If value of x > 5 & x < 6

then order of execution should be

3, 5, x, 6

If value of x > 6

then order of execution should be

3, 5, 6, x

Note: Expected run times are in the increasing order.

1.12 (a)

System calls are invoked by using software interrupt.

1.13 (c)

The system has multi user and multi proccessing, the security is primary concern. Otherwise user program and operating system will be corrupted. That is the reason to ensure the security and protection hence two modes are required.

1.14 (c)

Swapping out the memory image of process A to the disk typically not performed by OS when switching context from process A to B.

1.15 (c)

To execute privileged instructions, system services can be obtained using software interrupt.

1.16 (d)

Because we want to change the mode from privileged to non-privileged, to the next instruction to be executed should be non-privileged instruction.

1.17 (b)

SJF executes shortest job very early so throughput increases in the case of SJF.

1.18 (b)

FCFS is non pre-emptive algorithm.

1.19 (a)

Device can not interrupt a running process but timer, scheduler and power failure can interrupt a running process.

1.20 (d)

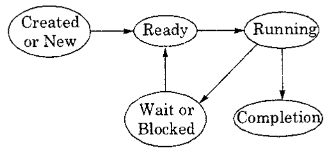
(i) Virtual memory helps to load more than one program.

(ii) Multi programming, wait for an I/O of one process can execute other process.

(iii) After completion of one process, other process executes, immediately.

1.21 Sol.

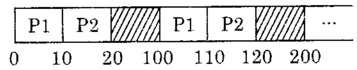
Just draw the diagram by using all criteria as below.



1.22 (d)

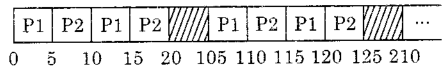
CPU = 10 ms
I/O = 90 ms

FCFS



So CPU idle time is 80

Round Robin



CPU idle time is 85.

Therefore, round robin scheduling with a time quantum of 5ms has least CPU utilization.

1.23 (a)

Consider the each statement

(i) Statement is false because there is no connection between kernel-supported threads and context switch.

(ii) Statement is true because it is the drawback of user-level threads blocking system call can block the entire process.

(iii) Statement is true because kernel-supported threads having own memory area and scheduled independently by the operating system.

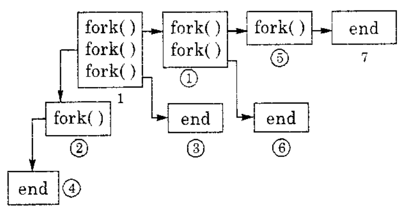
(vi) Statement is true because kernel is unware about user level threads and there is no kernel support to user-level threads. So user-level threads are transparent to the kernal.

1.24 (a)

Stack and registers are not shared by the threads of the same process while address space, message queue etc. are shared.

1.25 (c)

The number of process created will be $2^n - 1$.



$2^3 - 1 = 7$

1.25 (a)

Process	Arrival Time	Burst Time
P_1	0	5
P_2	1	3
P_3	2	3
P_4	4	1

The Gantt chart for SRPT CPU scheduling algorithm is

P_1	P_2	P_2	P_2	P_4	P_3	P_3	P_3	P_1	P_1	P_1	P_1

0 1 2 3 4 5 6 7 8 9 10 11 12

Process P_1 arrive at time 0, at time unit 1 the remaining time of P_1 is 4 and remaining time of P_2 is 3 so P_1 preempted. At the end of time unit 2 the remaining time of P_2 is 2 and P_3 is 3 so P_2 is not preempted At time unit 4 the remain time of P_2 is 0 and P_4 is scheduled. The same process repeated upto time unit 12.

The turnaround time is the interval between submission and completion of a given process.

Calculate turnaround time for each process as

$P_1 = 12 - 0 = 12$

$P_2 = 4 - 1 = 3$

$P_3 = 6, P_4 = 1$

So average turnaround time

$$= \frac{12+3+6+1}{4} = \frac{22}{4} = 5.50$$

1.27 (a)

When user level process trapping the Ctrl+C signal then the trap signal is going through system call and that's why mode changed to kernel mode from user mode and then the request is handling.

One more thing kernel mode and privilege mode are same, answer is kernel mode (privilege mode).

1.28 (d)

TAT = Completion Time – Arrival Time.

The Gantt Chart for Non Preemptive scheduling will be (0)P3, (15)P1, (35)P2(45).

From above this can be inferred easily that completion time for P2 is 45, for P1 is 35 and P3 is 15.

Gantt Chart for Preemptive: (0)P3, (1)P3, (2)P3, (3)P2, (4)P2, (5)P1, (25)P2, (33)P3(45).

Similarly take completion time from above for individual processes and subtract it from the Arrival time to get TAT.

1.29 (c)

By following I and II conditions the process P_1 will get both the resouces R_1 and R_2.

If R_1 and R_2 are allocated to the Process P_1, then it will complete it's job and release it. After that process P_2 will get both the resources and complete it's Job.

1.30 (d)

Fork( ) returns 0 is child process and process ID of child process is parent process.

In child(x), a = a + 5

In paraent(u), a = a – 5

Therefore, x = u + 10

The physical addresses of parent and child processes will be different, $v \neq y$.

1.31 (b)

Let three processes are P_1, P_2 and P_3

Process	Arrival Time	Burst Time
P_1	0	10
P_2	2	20
P_3	6	30

The Gantt chart for SRTF scheduling algorithm is

P_1	P_2	P_3

0 10 30 60

So there is only two context switches, at time unit 10 context switch from P_1 to P_2 and at time unit 30 context switch from P_2 to P_3.

1.32 (b)

If it were a preemptive scheduling then there would have been a transition from Running state to Ready state. So it is non-preemptive scheduling.

1.33 (b)

At t = 0 only P1 is present so execute it for 1 unit (**remaining CPU time of P1 = 0, now it will perform I/O**).

At t = 1 no process is available for CPU so CPU will remain idle and P1 will perform I/O **(remaining I/O time of P1 = 4)**

At t = 2 P2 arrives and is available for CPU so execute it for 1 unit and simultaneously P1 will do I/O (**remaining CPU time of P2 = 2 & remaining I/O time of P1 = 3**).

At t = 3 P3 also arrives and it is having highest priority among all processes available for CPU so we can execute P3 for its complete CPU burst (i.e 2 unit) as all processes has arrived and simultaneously P1 will do I/O(**remaining CPU time of P3 = 0, now P3 will perform I/O & remaining I/O time of P1 = 1**).

At t = 5 only P2 is available for CPU so it will execute for 1 unit and P1 and P3 will perform I/O(**remaining CPU time of P2 = 1, remaining I/O time of P3 = 2, remaining I/O time of P1 = 0, now P1 will perform CPU**).

At t = 6 P2 and P1 are available for CPU but P1 will be selected because it is having highest priority then P2 so execute P1 for 1 unit(**remaining CPU time of P1 = 2 & remaining I/O time of P3 is 1**)

At t = 7 again P1 will be executed for 1 unit(**remaining CPU time of P1 = 1 & remaining I/O time of P3 = 0, now P3 will perform CPU**).

At t = 8 now P1, P2 & P3 are available for CPU so P3 will be selected based on highest priority and will be executed for 1 unit (**remaining CPU time of P3 = 0, so P3 completed at t = 9**).

At t = 9 P1 will be executed on CPU for 1 unit(**remaining CPU time of P1 = 0, so P1 completed at t = 10**).

At t = 10 only P2 is available for CPU so it will execute its remaining CPU burst (i.e 1 unit) (**remaining CPU time of P2 = 0, now P2 will perform I/O.**)

At t = 11 P2 will perform I/O for 3 units and CPU will remain IDLE.

At t = 14 P2 will perform CPU again and **P2 is completed at t = 15**

P1	IDLE	P2	P3	P2	P1	P1	P3	P1	P2	IDLE	P2
0	1	2	3	5	6	7	8	9	10	11	14 15

1.34 (b)

Process	Total burst time	I/O time	CPU time	I/O time
P1	10	2	7	1
P2	20	4	14	2
P3	30	6	21	3

The Gantt chart is

Idle	P1	P2	P3	Idle

0 2 9 23 44 47

Total time spent = 47

Idle time = 2 + 3 = 5

$$\% \text{ of idle time} = \frac{5}{47} \times 100 = 10.6\%$$

1.35 (a)

Process	id	Arrival time	CPU Burst
P_0	0	0	2
P_1	1	0	4
P_2	2	0	8

	Remaining Time
P_0	2
P_1	4
P_2	8

The Gantt chart for LRTF CPU scheduling algorithm.

P_0	P_1	P_2	P_2	P_2	P_2	P_2	P_1	P_2	P_1	P_2	P_0	P_1	P_2

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14

Turn around time for P_0 = 12 – 0 = 12

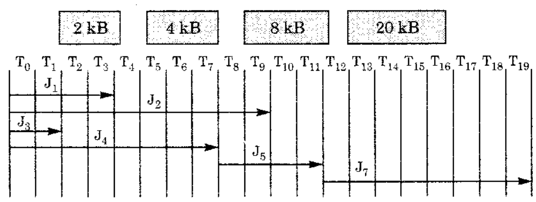
Turn around time for P_1 = 13 – 1 = 12

Turn around time for P_2 = 14 – 2 = 12

Average Turn around Time

= 36/3 = 12 ≈ 13

1.36 (c)



Timing Diagram

1.37 (a)

List-I

A. Gang Scheduling

B. Rate Monotonic Scheduling

C. Fair Show Scheduling

List-II
3. Thread Scheduling
2. Real-time Scheduling
1. Guaranteed Scheduling

1.38 (d)

In Kernel level threads, the blocking system call causes, the kernel can schedule another thread in the application for execution. So statement (d) is false about kernel level threads.

1.39 (b)

Process	Execution Time	Arrival Time
P_1	20	0
P_2	25	15
P_3	10	30
P_4	15	45

The Gantt chart for SRT Scheduling algorithm is

P_1	P_2	P_3	P_2	P_4

0 20 30 40 55 70

So the waiting time for

$$P_2 = (20 - 15) + (40 - 30)$$
$$= 5 + 10 = 15$$

1.40 (c)

When time quantum used in round robin scheduling is more than maximum time required to execute any process then its behave like First Come First serve.

1.41 (b)

Statement (b) is not true because an ISR (Interrupt Service Routine) is invoked after completion of I/O in synchronous but not in asynchronous I/O.

1.42 (b)

Fork( ) system call creates the child process initially number of processes is 0. After first fork( ), it creates a single process. After second fork( ), it creates one parent and two child processes. After n + 1 fork( ), the total number of processes is 2^n but we subtract the main process then total number of child processes is $2^n - 1$.

1.43 (c)

Transition C indicates that OS uses preemptive scheduling.
II is also true.

1.44 (d)

In the shortest remaining time first scheduling, process will not get a chance if that one has the high remaining time so due to this, starvation take place.
Shortest remaining time first scheduling will cause starvation, is true because if a large CPU brust required process is there and if consider that all the processes which are coming have less CPU-brust then the former have to wait untill all finish execution gives starvation.
In preemptive scheduling, always one process preempt due to low priority then due to this starvation take place.
Response time = first response - process submission time
In the round robin every process first response gets after certain time quantum so due to this it give more better than FCFS in term of response time.

1.45 (c)

Process switching includes mode switching.
Context switching can occur only in kernal mode.

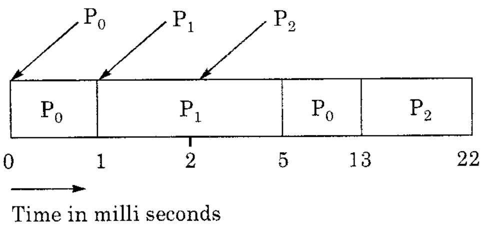
1.46 (d)

Interrupt from CPU temperature sensor is given top priority to protect system resources. When CPU temperature is too high, the BIOS initiate an interrupt and informs the Operating System. OS gives top priority to this interrupt and immediately shuts down the system.

1.47 (c)

(a) **False:** On per thread basics, OS maintains both CPU register state and stack. Hence option (a) is false, because only CPU register state mentioned.
(b) **False:** OS maintains a separate stack for each thread.
(c) **True:** OS does not maintain virtual memory state.
(d) **False:** OS does not maintain scheduling and accounting information.

1.48 (a)

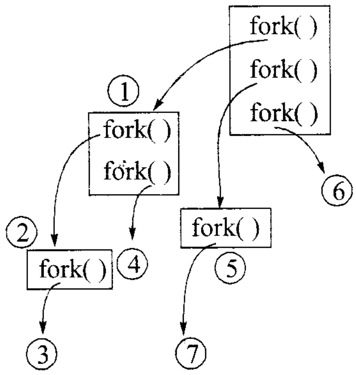


Average waiting time

$$= \frac{w_1 + w_2 + w_3}{3}$$

$$= \frac{(5-1)+0+(13-2)}{3} = 5.0 \text{ ms}$$

1.49 (c)



The number of child process created

$= 2^n - 1 = 2^3 - 1 = 7$

(where n is number of fork( ) statements)

1.50 (c)

RR Queue:

P1 P2 P1 P3 P2 P1 P3 P2

0 1 2 3 4 6 8 10

P1	P2	P1	P3	P2	P1	P3	P2

0 2 4 6 8 10 11 13 16

⇒ P1, P3, P2

FCFS: P1, P2, P3

Therefore option (c) is correct.

FCFS : P1, P2, P3

RR2 : P1, P3, P2

1.51 (b)

Lets take an example

Process	Execution time	Arrival time
1	10	0
2	8	1
3	7	2
4	6	3

Consider scheduler schedule processes priority after S times units so order of execution will be $P_1 P_2 P_3 P_4 P_1 P_2 P_3 P_4$ (S < execution time of any process) which is exactly same as round robin so answer is (b).

1.52 (d)

Kernel level threads shares the code segment.

1.53 Sol.

	A.T	E.T
A	0	6
B	3	2
C	5	4
D	7	6
E	10	3

Using SRTF:

A	B	A	C	E	D

0 3 5 8 12 15 21

T.A.T (A) = 8 – 0 = 8

T.A.T (B) = 5 – 3 = 2

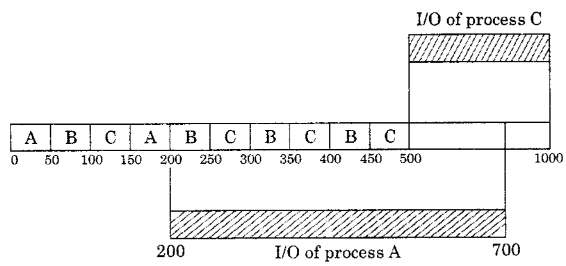
T.A.T (C) = 12 – 5 = 7

T.A.T (D) = 21 – 7 = 14

T.A.T (E) = 15 – 10 = 5

$$\text{Average T.A.T} = \frac{8+2+7+14+5}{5} = 7.2$$

1.54 Sol.



∴ At 1000 time units C completes its I/O.

1.55 Sol.

SRTF:

P1	P2	P3	P4	P1

0 2 6 12 17 27

Waiting time of P1 = 17 – 2 = 15
Waiting time of P2 = 2 – 2 = 0
Waiting time of P3 = 6 – 3 = 3
Waiting time of P4 = 12 – 8 = 4
Average waiting time

$$= \frac{15+0+3+4}{4} = 5.5$$

1.56 Sol.

Periodic arrival times of T1: 0, 3, 6, 9, 12, 15, 18, 21,...
Priority of T1=1/3, service time of $T_1 = 1$.

Periodic arrival times of T_2: 0, 7, 14, 21,...
Priority of T_2= 1/7, service time of $T_2 = 2$.

Periodic arrival times of T_3: 0, 20, 40,...
Priority of T_3=1/20, service time of $T_3 = 4$.

T_1 has highest priority and T3 has lowest priority.

Preemption of T_3

T_1	T_2	T_1	T_3	T_1	T_2	T_1	T_3	

0 1 2 3 4 5 6 7 8 9 10 11 12

T_1 arrives again T_1 arrives T_1 arrives

First instance of T_3 (4 units) completed at the end of 12 ms.

1.57 (d)

Maximum number of processes that can be in ready state is independent of number of processes (n).

1.58 (c)

Process	Arrival Time	B.T
A	0	3
B	1	6
C	4	4
D	6	2

(i) **FCFS:**

A	B	C	D

0 3 9 13 15

$$\text{Avg TAT} = \frac{3+8+9+9}{4} = \frac{29}{4} = 7.25$$

(ii) **Non-peemptive SJF:**

A	B	D	C

0 3 9 11 15

$$\text{Avg TAT} = \frac{3+8+5+11}{4} = 6.75$$

(iii) **SRTF:**

A	B	D	C	B

0 3 4 8 10 15

$$\text{Avg TAT} = \frac{3+14+4+4}{4} = \frac{25}{4}$$

$= 6.25$

(iv) **RR:**

A	B	A	C	B	D	C	B

0 2 4 5 7 9 11 13 15

$$\text{Avg TAT} = \frac{5+14+9+5}{4} = \frac{33}{4} = 8.25$$

∴ SRTF has lowest turn around time.

1.59 (a)

To minimize the average waiting time, we need to select the shortest remaining time process first, because all are arriving at the same time, and they have unequal CPU burst times.

All other options will not minimize the waiting time. So, the answer is SRTF algorithm.

1.60 Sol.

Gantt chart

P_1	P_2	P_3	P_2	P_4	P_1

0 3 7 8 10 13 20

Process Number	Arrival time	Burst time	Completion time	TAT
1	0	10	20	20
2	3	6	10	7
3	7	1	8	1
4	8	3	13	5
Average turn around time : 33/4 = 8.25				

Average turn around time is 8.25.

■■■■

2 Process Management-II

(IPC, Synchronization and Concurrency)

2.1 A critical region is

d

(a) One which is enclosed by a pair of P and V operations on semaphores.
(b) A program segment that has not been proved bug-free
(c) A program segment that often causes unexpected system crashes
(d) A program segment where shared resources are accessed.

[1987 : 2 Marks]

2.2 Semaphore operations are atomic because they are implemented within the OS......

[1990 : 2 Marks]

2.3 At a particular time of computation the value of a counting semaphore is 7. Then 20 P operations and 15 V operations were completed on this semaphore. The resulting value of the semaphore is:

b

(a) 42 (b) 2
(c) 7 (d) 12

[1992 : 2 Marks]

2.4 A critical section is a program segment.

c

(a) Which should run in certain specified amount of time
(b) Which avoids deadlocks
(c) Where shared resources are accessed
(d) Which must be enclosed by a pair of semaphore operations, P and V

[1996 : 1 Mark]

2.5 A solution to the Dining Philosophers Problem which avoids deadlock is

c

(a) ensure that all philosophers pick up the left fork before the right fork
(b) ensure that all philosophers pick up the right fork before the left fork
(c) ensure that one particular philosophers picks up the left fork before the right fork, and that all other philosophers pick up the right fork before the left fork
(d) None of the above

[1996 : 2 Marks]

2.6 Each process Pi, i = 1 ... 9 is coded as follows

d

```
repeat
    P(mutex)
    {critical section}
    V(mutex)
forever
```

The code for P_{10} is identical except that it uses V(mutex) in place of P(mutex). What is the largest number of processes that can be inside the critical section at any moment?

(a) 1 (b) 2
(c) 3 (d) None of these

[1997 : 2 Marks]

2.7 When the result of a computation depends on the speed of the processes involved there is said to be

b

(a) cycle stealing (b) race condition
(c) a time lock (d) a deadlock

[1998 : 1 Mark]

2.8 A counting semaphore was initialized to 10. Then 6 P (wait) operations and 4V (signal) operations were completed on this semaphore. The resulting value of the semaphore is

b

(a) 0 (b) 8
(c) 10 (d) 12

[1998 : 1 Mark]

2.9 Let m[0]...m[4] be mutexes (binary semaphores) and P[0]...P[4] be processes. Suppose each process P[i] executes the following:

b

wait (m[i]);wait (m[(i+1) mode 4]);
....
release (m[i]); release (m[(i+1)mod 4]);

This could cause

(a) Thrashing
(b) Deadlock
(c) Starvation, but not deadlock
(d) None of the above

[2000 : 1 Mark]

2.10 Suppose a processor does not have any stack pointer register. Which of the following statement is true?

a

(a) It cannot have subroutine call instruction
(b) It can have subroutine call instruction, but no nested subroutine calls
(c) Nested subroutine calls are possible, but interrupts are not
(d) All sequences of subroutine calls and also interrupts are possible

[2001 : 1 Mark]

2.11 Where does the swap space reside?

b

(a) RAM (b) Disk
(c) ROM (d) On-chip cache

[2001 : 1 Mark]

2.12 Consider Peterson's algorithm for mutual exclusion between two concurrent processes i and j. The program executed by process is shown below.

b

```
repeat
    flag[i]=true;
    turn=j;
    while (P) do no-op;
    Enter critical section, perform actions, then
    exit critical section
    flag[i]=false;
    Perform other non-critical section actions.
Until false;
```

For the program to guarantee mutual exclusion, the predicate P in the while loop should be

(a) flag[j] = true and turn = i
(b) flag [j] = true and turn = j
(c) flag [i] = true and turn = j
(d) flag [i] = true and turn = i

[2001 : 2 Marks]

Common Data for Q. 2.13 & 2.14

Suppose we want to synchronize two concurrent processes P and Q using binary semaphores S and T. The code for the processes P and Q is shown below.

Process P:	**Process Q:**
while (1) {	while (1) {
W:	Y:
print '0';	print '1' ;
print '0';	print '1' ;
X:	Z:
}	}

Synchronization statements can be inserted only at points W, X, Y and Z.

2.13 Which of the following will always lead to an output string with '001100110011'?

b

(a) P(S) at W, V(S) at X, P(T) at Y, V(T) at Z, S and T initially 1
(b) P(S) at W, V(T) at X, P(T) at Y, V(S) at Z, S initially 1, and T initially 0
(c) P(S) at W, V(T) at X, P(T) at Y, V(S) at Z, S and T initially 1
(d) P(S) at W, V(T) at X, P(T) at Y, V(T) at Z, S initially 1, and T initially 0

[2003 : 2 Marks]

2.14 Which of the following will ensure that the output string never contains a substring of the form 01^n0 or 10^n1 where n is odd?

c

(a) P(S) at W, V(S) at X, P(T) at Y, V(T) at Z, S and T initially 1
(b) P(S) at W, V(T) at X, P(T) at Y, V(S) at Z, S and T initially 1
(c) P(S) at W, V(S) at X, P(S) at Y, V(S) at Z, S initially 1
(d) V(S) at W, V(T) at X, P(S) at Y, P(T) at Z, S and T initially 1

[2003 : 2 Marks]

2.15 Consider two processes P_1 and P_2 accessing the shared variables X and Y protected by two binary semaphores S_X and S_Y respectively, both initialized to 1. P and V denote the usual semaphore operators, where P decrements the semaphore value, and V increments the semaphore value. The pseudocode of P_1 and P_2 is as follows:

d

P_1:	**P_2:**
while true do {	while true do {
L_1 :	L_3 : ....
L_2 :	L_4 : ....
X = X + 1;	Y = Y + 1;
Y = Y – 1;	X = X – 1;
$V(S_X)$;	$V(S_Y)$;
$V(S_Y)$; }	$V(S_X)$; }

In order to avoid deadlock, the correct operators at L_1, L_2, L_3 and L_4 are respectively

(a) $P(S_Y), P(S_X), P(S_X), P(S_Y)$
(b) $P(S_X), P(S_Y), P(S_Y), P(S_X)$
(c) $P(S_X), P(S_X), P(S_Y), P(S_Y)$
(d) $P(S_X), P(S_Y), P(S_X), P(S_Y)$

[2004 : 2 Marks]

2.16 The semaphore variables full, empty and mutex are initialized to 0, n and 1, respectively. Process P_1 repeatedly adds one item at a time to a buffer of size n, and process P_2 repeatedly removes one item at a time from the same buffer using the programs given below. In the programs, K, L, M and N are unspecified statements.

d

```
P1 : while (1)
{
    K;
    P(mutex);
    Add an item to the buffer;
    V(mutex);
    L;
}
P2 :  while (1)
{
    M;
    P(mutex);
    Remove an item from the buffer;
    V(mutex);
    N;
}
```

The statements K, L, M and N are respectively
(a) P(full), V(empty), P(full), V(empty)
(b) P(full), V(empty), P(empty), V(full)
(c) P(empty), V(full), P(empty), V(full)
(d) P(empty), V(full), P(full), V(empty)

[2004 : 2 Marks]

2.17 Given below is a program which when executed spawns two concurrent processes:

a

```
semaphore X : = 0 ;
/* Process now forks into concurrent processes
P1 & P2 */
P1: repeat forever     P2: repeat forever
V(X) ;                     P(X) ;
Compute ;                  Compute ;
P(X) ;                     V(X) ;
```

Consider the following statements about processes P1 and P2:
I: It is possible for process P1 to starve.
II: It is possible for process P2 to starve.
Which of the following holds?
(a) Both I and II are true.
(b) I is true but II is false.
(c) II is true but I is false
(d) Both I and II are false.

[2005 : 2 Marks]

2.18 Two concurrent processes P1 and P2 use four shared resources R1, R2, R3 and R4, as shown below.

b

P1:	**P2:**
Compute:	Compute;
Use R1;	Use R1;
Use R2;	Use R2;
Use R3;	Use R3;
Use R4;	Use R4;

Both processes are started at the same time, and each resource can be accessed by only one process at a time. The following scheduling constraints exist between the access of resources by the processes:

- P2 must complete use of R1 before P1 gets access to R1.
- P1 must complete use of R2 before P2 gets access to R2.
- P2 must complete use of R3 before P1 gets access to R3.
- P1 must complete use of R4 before P2 gets access to R4.

There are no other scheduling constraints between the processes. If only binary semaphores are used to enforce the above scheduling constraints; what is the minimum number of binary semaphores needed?
(a) 1 (b) 2
(c) 3 (d) 4

[2005 : 2 Marks]

2.19 The atomic *fetch-and-set x, y* instruction unconditionally sets the memory location x to 1 and fetches the old value of x in y without allowing any intervening access to the memory location x. Consider the following implementation of P and V functions on a binary semaphore S.

a

```
void P(binary_semaphore *S) {
    unsigned y;
unsigned *x = & (S → value);
    do {
      fetch-and-set x, y;
    } while (y);
}
void V (binary_semphore *S) {
    S → value = 0;
}
```

Which one of the following is true?

(a) The implementation may not work if context switching is disabled in P
(b) Instead of using fetch-and-set, a pair of normal load/store can be used
(c) The implementation of V is wrong
(d) The code does not implement a binary semaphore

[2006 : 2 Marks]

Common Data for Q. 2.20 & 2.21:

Barrier is a synchronization construct where a set of processes synchronizes globally i.e. each process in the set arrives at the barrier and waits for all others to arrive and then all processes leave the barrier. Let the number of processes in the set be three and S be a binary semaphore with the usual P and V functions. Consider the following C implementation of a barrier with line numbers shown on the left.

```
void barrier (void) {
 1 :  P (S) ;
 2 :  process_arrived + +;
 3 :  V (S) ;
 4 :  while (process_arrived ! = 3) ;
 5 :  P (S) ;
 6 :  process_left + + ;
 7 :  if (process_left = = 3){
 8 :      process_arrived = 0;
 9 :      process_left = 0;
10 :  }
11 :  V (S) ;
12 :  }
```

The variables process_arrived and process_left are shared among all processes and are initialized to zero. In a concurrent program all the three processes call the barrier function when they need to synchronize globally.

2.20 The above implementation of barrier is incorrect. Which one of the following is true?

b

(a) The barrier implementation is wrong due to the use of binary semaphore S
(b) The barrier implementation may lead to a deadlock if two barrier invocations are used in immediate succession
(c) Lines 6 to 10 need not be inside a critical section
(d) The barrier implementation is correct if there are only two processes instead of three

[2006 : 2 Marks]

2.21 Which one of the following rectifies the problem in the implementation?

b

(a) Lines 6 to 10 are simply replaced by process_arrived
(b) At the beginning of the barrier the first process to enter the barrier waits until process_arrived becomes zero before proceeding to execute P(S)
(c) Context switch is disabled at the beginning of the barrier and re-enabled at the end
(d) The variable process_left is made private instead of shared

[2006 : 2 Marks]

2.22 Processes P1 and P2 use critical_flag in the following routine to achieve mutual exclusion. Assume that critical_flag is initialized to FALSE in the main program.

c

```
get_exclusive_access ( )
{
   if (critical _flag = = FALSE)
   {
     critical_flag = TRUE ;
     critical_region ( ) ;
     critical_flag = FALSE;
   }
}
```

Consider the following statements.

(i) It is possible for both P1 and P2 to acess critical_region concurrently.
(ii) This may lead to a deadlock.

Which of the following holds?

(a) (i) is false and (ii) is true
(b) Both (i) and (ii) are false
(c) (i) is true and (ii) is false
(d) Both (i) and (ii) are true

[2007 : 1 Mark]

2.23 Two processes, P1 and P2, need to access a critical section of code. Consider the following synchronization construct used by the processes:

d

```
/* P1 */
  while (true) {
   wants1 = true;
   while (wants2 = = true);
   /*   Critical
        Section */
   wants 1 = false;
}
/* Remainder section */
```

```
/* P2 */
  while (true) {
    wants2 = true;
    while (wants1 = = true);
    /*  Critical
        Section */
    wants 2 = false;
}
/* Remainder section */
```

Here, wants1 and wants2 are shared variables, Which are initialized to false. Which one of the following statements is TRUE about the above construct?

(a) It does not ensure mutual exclusion.
(b) It does not ensure bounded waiting.
(c) It requires that processes enter the critical section in strict alternation.
(d) It does not prevent deadlocks, but ensures mutual exclusion.

[2007 : 2 Marks]

2.24 Synchronization in the classical readers and writers problem can be achieved through use of semaphores. In the following incomplete code for readers-writers problem, two binary semaphores mutex and wrt are used to obtain synchronization

c

```
wait (wrt)
writing is performed
signal (wrt)
wait (mutex)
readcount = readcount + 1
if readcount = 1 then S1
S2
reading is performed
S3
readcount = readcount - 1
if readcount = 0 then S4
Signal (mutex)
```

The values of S1, S2, S3, S4, (in that order) are

(a) signal (mutex), wait (wrt), signal (wrt), wait (mutex)
(b) signal (wrt), signal (mutex), wait (mutex), wait (wrt)
(c) wait (wrt), signal (mutex), wait (mutex), signal (wrt)
(d) signal (mutex), wait (mutex), signal (mutex), wait (mutex)

[2007 : 2 Marks]

2.25 The P and V operations on counting semaphores, where s is a counting semaphore, are defined as follows:

c

P(s): $s = s - 1$;
    if $s < 0$ then wait;
V(s): $s = s + 1$;
    if $s \leq 0$ then wakeup a process waiting on s;

Assume that P_b and V_b, the wait and signal operations on binary semaphores are provided. Two binary semaphores X_b and Y_b are used to implement the semaphore operations P(s) and V(s) as follows:

```
P(s):   Pb(Xb);
        s = s - 1;
        if (s < 0) {
          Vb (Xb);
          Pb(Yb);
        }
        else Vb(Xb);
V(s):   Pb(Xb);
        s = s + 1;
        if (s ≤ 0) Vb (Yb);
        Vb(Xb);
```

The initial values of X_b and Y_b are respectively

(a) 0 and 0
(b) 0 and 1
(c) 1 and 0
(d) 1 and 1

[2008 : 2 Marks]

2.26 The enter_CS ( ) and leave_CS ( ) functions to implement critical section of a process are realized using test-and-set instruction as follows:

a

```
Void enter_CS (X)
{
    while (test-and-set (X));
}
Void leave_CS (X)
{
    X = 0 ;
}
```

In the above solution, X is a memory location associated with the CS and is initialized to 0. Now consider the following statements

I. The above solution to CS problem is deadlock-free.
II. The solution is starvation free.
III. The processes enter CS in FIFO order.
IV. More than one process can enter CS at the same time.

Which of the above statements are TRUE?

(a) I only (b) I and II
(c) II and III (d) IV only

[2009 : 2 Marks]

2.27 Consider the methods used by processes P1 and P2 for accessing their critical sections whenever needed, as given below. The initial values of shared boolean variables S1 and S2 are randomly assigned.

a

Method used by P1	Method used by P2
while (S1==S2);	while (S1!=S2);
Critical Section	Critical Section
S1=S2;	S2=not (S1);

Which one of the following statements describes the properties achieved?

(a) Mutual exclusion but not progress
(b) Progress but not mutual exclusion
(c) Neither mutual exclusion nor progress
(d) Both mutual exclusion and progress

[2010 : 1 Mark]

2.28 The following program consists of 3 concurrent processes and 3 binary semaphores. The semaphores are initialized as S0 = 1, S1 = 0, S2 = 0.

a

Process P0	Process P1	Process P2

while (true) {
wait (S0);
print '0'
release (S1);
release (S2);
}
	wait (S1);
	release(S0);
		wait (S2)
		release(S0);

How many times will process P0 print '0'?

(a) At least twice (b) Exactly twice
(c) Exactly thrice (d) Exactly once

[2010 : 2 Marks]

2.29 Fetch_And_Add(X, i) is an atomic Read-Modify-Write instruction that reads the value of memory location X, increments it by the value i, and returns the old value of X, it is used in the pseudocode shown below to implement a busy-wait lock. L is unsigned integer shared variable initialized to 0. The value of 0 corresponds to lock being available, while any non-zero value corresponds to the lock being not available.

a
b

```
AcquireLock(L){
   while(Fetch_And_Add(L, 1))
   L=1;
}
ReleaseLock(L){
   L=0;
}
```

This implementation

(a) fails as L can overflow
(b) fails as L can take on a non-zero value when the lock is actually available
(c) works correctly but may starve some processes
(d) works correctly without starvation

[2012 : 2 Marks]

2.30 Three concurrent processes X, Y and Z execute three different code segments that access and update ceratin shared variables. Process X execute the P operation (i.e., wait) on semaphores a, b and c; process Y executes the P operation on semaphores b, c and d; process Z executes the P operation on semaphores c, d and a before entering the respective code segments. After completing the execution of its code segments, each process invokes the V operation (i.e., signal) on its three semaphores. All semaphores are binary semaphores initialized to one. Which one of the following represents a deadlock-free order of invoking the P operations by the processes?

b

(a) X : P(a)P(b)P(c)
Y : P(b)P(c)P(d)
Z : P(c)P(d)P(a)
(b) X : P(b)P(a)P(c)
Y : P(b)P(c)P(d)
Z : P(a)P(c)P(d)
(c) X : P(b)P(a)P(c)
Y : P(c)P(b)P(d)
Z : P(a)P(c)P(d)
(d) X : P(a)P(b)P(c)
Y : P(b)P(c)P(d)
Z : P(c)P(d)P(a)

[2013 : 1 Mark]

2.31 A shared variable x, initialized to zero, is operated on by four concurrent process W, X, Y, Z as follows. Each of the processes W and X reads x from memory, increments by one, stores it to memory, and then terminates. Each of the

d

processes Y and Z reads x from memory, decrements by two, stores it to memory ,and then terminates. Each process before reading x invokes the P operations (i.e., wait) on a counting semaphore S and invokes the V operation (i.e., signal) on the semaphore S after storing x to memory. Semaphore S is initialized to two. What is the maximum possible value of x after all processes complete execution?

(a) -2 (b) -1

(c) 1 (d) 2

[2013 : 2 Marks]

2.32 A certain computation generates two arrays a and b such that $a[i] = f$ for $0 \le i \le n$ and $b[i] = g(a[i])$ for $0 \le i < n$. Suppose this computation is decomposed into two concurrent processes X and Y such that X computes the array a and Y computes the array b. The processes employ two binary semaphores R and S, both initialized to zero. The array a is shared by the two processes. The structure of the processes are shown below.

c

Process X	**Process Y**
private i;	private i;
for (i = 0; i < n; i++)	for (i = 0; i < n; i++)
{	{
a[i] = f(i);	EntryY(R, S);
ExitsX (R, S);	b[i] = g(a[i]);
}	}

Which one of the following represents the **CORRECT** implementations of ExitsX and EntryY?

(a)
```
ExitsX (R, S) {
   P(R);
   V(S)
}
EntryY(R, S) {
   P(S);
   V(R);
}
```

(b)
```
ExitsX (R, S) {
   V(R);
   V(S);
}
EntryY(R, S) {
   P(R);
   P(S);
}
```

(c)
```
ExitsX (R, S) {
   P(S);
   V(R)
}
EntryY(R, S) {
   V(S);
   P(R);
}
```

(d)
```
ExitsX (R, S) {
   V(R);
   P(S);
}
EntryY(R, S) {
   V(S);
   P(R);
}
```

[2013 : 2 Marks]

2.33 Consider the procedure below for the *Producer-Consumer* problem which uses semaphores:

c

```
semaphore n = 0;
semaphore s = 1;

void producer( )              void consumer( )
{                             {
  while(true)                   while (true)
  {                             {
     produce( );                   semWait (s);
     semWait(s);                   semWait (n);
     addToBuffer( );               removeFromBuffer ( );
     semSignal(s);                 semSignal (s);
     semSignal(n);                 consume( );
  }                             }
}                             }
```

Which one of the following is TRUE?

(a) The producer will be able to add an item to the buffer, but the consumer can never consume it.

(b) The consumer will remove no more than one item from the buffer.

(c) Deadlock occurs if the consumer succeeds in acquiring semaphore s when the buffer is empty.

(d) The starting value for the semaphore n must be 1 and not 0 for deadlock-free operation.

[2014 (Set-2) : 2 Marks]

2.34 The following two functions P1 and P2 that share a variable B with an initial value of 2 execute concurrently.

P1( )	**P2( )**
{	{
C = B – 1;	D = 2 * B;
B = 2 * C;	B = D – 1;
}	}

The number of distinct values that B can possibly take after the execution is ________.

[2015 (Set-1) : 1 Mark]

2.35 Two processes *X* and *Y* need to access a critical section. Consider the following synchronization construct used by both the processes

a

Process *X*

```
/* other code for process X */
while (true)
{   varP = true;
    while (varQ == true)
    {  /* Critical Section */
       varP = false;
    }
}
/* other code for process X */
```

Process Y

```
/* other code for process Y */
while (true)
{   varQ = true;
    while (varP = = true)
    {  /* Critical Section */
       varQ = false;
    }
}
/* other code for process Y */
```

Here, varP and varQ are shared variables and both are initialized to false. Which one of the following statements is true?

(a) The proposed solution prevents deadlock but fails to guarantee mutual exclusion
(b) The proposed solution guarantees mutual exclusion but fails to prevent deadlock
(c) The proposed solution guarantees mutual exclusion and prevents deadlock
(d) The proposed solution fails to prevent deadlock and fails to guarantee mutual exclusion

[2015 (Set-3) : 1 Mark]

2.36 Consider the following proposed solution for the critical section problem. There are n processes: $P_0 \ldots P_{n-1}$. In the code, function *pmax* returns an integer not smaller than any of its arguments. For all i, $t[i]$ is initialized to zero.

(a)

Code for P_i:

```
do
{
    c[i]=1; t[i] = pmax (t[0],..., t[n-1]) +1; c[i] = 0;
    for every j ≠ i in {0,..., n – 1}
    {  while (c[j]);
       while (t[j] != 0 && t[j] <= t[i]);
    }
    Critical Section; t[i] = 0;
    Remainder Section;
}   while (true);
```

Which one of the following is TRUE about the above solution?

(a) At most one process can be in the critical section at any time
(b) The bounded wait condition is satisfied
(c) The progress condition is satisfied
(d) It cannot cause a deadlock

[2016 (Set-1) : 2 Marks]

2.37 Consider the following two-process synchronization solution

(c)

Process 0

Entry: loop while (turn == 1);
(critical section)
Exit: turn = 1;

Process 1

Entry : loop while (turn == 0);
(critical section)
Exit: turn = 0;

The shared variable turn in initialized to zero. Which one of the following is **TRUE**?

(a) This is a correct two-process synchronization solution
(b) This solution violates mutual exclusion requirement
(c) This solution violates progress requirement
(d) This solution violates bounded wait requirement

[2016 (Set-2) : 2 Marks]

2.38 Consider a non-negative counting semaphore S. The operation $P(S)$ decrements S, and $V(S)$ increments S. During an execution, 20 $P(S)$ operations and 12 $V(S)$ operations are issued in some order. The largest initial value of S for which at lest one $P(S)$ operation will remain blocked is _______.

[2016 (Set-2) : 2 Marks]

■■■■

Answers Process Management-II

2.1	(d)	2.3	(b)	2.4	(c)	2.5	(c)	2.6	(d)	2.7	(b)	2.8	(b)	2.9	(b)	2.10	(a)
2.11	(b)	2.12	(b)	2.13	(b)	2.14	(c)	2.15	(d)	2.16	(d)	2.17	[illegible]	2.18	(b)	2.19	(a)
2.20	(b)	2.21	(b)	2.22	[illegible]	2.23	(d)	2.24	(c)	2.25	(c)	2.26	(a)	2.27	(a)	2.28	(a)
2.29	(b)	2.30	(b)	2.31	(d)	2.32	(c)	2.33	(c)	2.35	(a)	2.36	(a)	2.37	(c)		

Explanations Process Management-II

2.1 (d)

A critical region is a program segment where shared resources are accessed.

2.2 Sol.

Kernel.

2.3 (b)

Initially semaphore value is 7 then apply down and up operation. So first 20 down so –13, then 15 up so at last 2.

2.4 (c)

Critical section is an area in a program where the resources can be accessed.

2.5 (c)

Ensure that one particular philosophers picks up the left fork before the right fork, and that all other philosophers pick up the right fork before the left fork.

2.6 (d)

Everytime when process P_{10} enters it performs the 'UP' operation so after that one more process can enter into it, so this phenomena continue till end. So 10 processes at a time can enter into critical section.

2.7 (b)

Race condition.

2.8 (b)

Initially semaphore is 10, then 6 down operations are performed means (10 – 6) and 4 up operation means (4 + 4) = 8.
So option (b) is correct.

2.9 (b)

All processes are in deadlock condition i.e., not a single one process can enter into critical section.

2.10 (a)

If stack pointer register is not available then activation records in the stack can not be created. So it cannot have subroutine call instruction.

2.11 (b)

Disk has swap space.

2.12 (b)

To ensure the mutual exclusion, we have to take care that 'turn' value which is 'j' so we should not allow that process in CS which is having flag [j] = true.
Hence option (b) is correct.

2.13 (b)

To get the expected output , The sequence of processes are P, Q, P, Q, P, Q and so on we must initialize S = 1 and T = 0.

2.14 (c)

If the output string never contains a substring of the form 01^n0 or 10^n1 where n is odd then the procedures becomes.

S = 1

Process P:	Process Q:
While(1){	While(1){
W : P(S)	Y : P(S)
print '0';	print '1';
print '0';	print '1';
X : V(S)	Z : V(S)
}	}

2.15 (d)

S_x and S_y are two binary semaphores if we assume P means wait and V means signal then for two process P_1 and P_2 we take alternate then there in no chance of dead lock.

2.16 (d)

Since **P(Empty)** at **M** then P2 removes an item from an empty buffer. Which fail the synchronisation problem i.e. P2 want to remove the item which is not present inside the buffer. So OPTION B,C are not be the answer.
If P(Full) then P1 cant enter an item into Buffer so Option A is not the answer.
Here option D ensures that P2 never removes an item from an empty buffer. While P1 enter the item easily.

2.17 (d)

None of the processes will ever starve. Starvation is said to be possible if in ALL the possible ways that a process want to enter CS, it is DENIED the access. But that can't happens here.
P1 is never starved because this is INFACT the first process that can enter. V(X) by P1 is always successful and hence it always enter the CS. So 1 is FASLE.
When P1 is in CS, X = 1. At this time P2 can make a successful DOWN operation P(X) and enter CS. So 2 is also FALSE.
But P2 must exit the CS before P1.

2.18 (b)

It needs two semaphores. x = 0, y = 0

P1:	**P2:**
P(X)	
R1	R1
	V(X)
	P(Y)
R2	R2
V(Y)	
P(X)	
R3	R3
	V(X)
	P(Y)
R4	R4
V(Y)	

2.19 (a)

The P and V function is as follows:

```
1.  void P (binary-semaphore * S){
2.      unsigned y;
3.      unsigned * x = & (S → value);
4.      do {
5.          fetch-and-set x, y;
6.      } while (y);
7.  }
8.  void V (binary-semaphore * S){
9.      S → value = 0;
10. }
```

fetch-and-set instruction always sets the memory location x = 1 and fetches the old value of x into y. The binary semphore * S takes only two values either 0 and 1. When we initiliaze S = 0 then in statement 3 this value will be start at location x and fetch-and-set instruction changes the value of x = 0 to x = 1 and y becomes 0. If there are more than two processes and context switching between processes is disabled in P then this implementation doesn't work properly and can't synchronize the processes.

2.20 (b)

The barrier implementation may lead to a deadlock if two barrier invocations are used in immediate succession which is due to line 3 and 7.

2.21 (b)

At the beginning of the barrier the first process to enter in line 8 the barrier waits until process_arrived becomes zero before proceeding to execute P(S)

2.22 (c)

Both process can run the critical section concurrently.
Let's say p1 starts and it enters inside if clause, and just after its entrance and before execution of critical_flag = TRUE, a context switch happens and p2 also gets entrance since the flag is still false.
Now both process are in critical section!!
So (i) is true, and (ii) is false.
There is no way that flat is true and no process' are inside the if clause, if someone enters the critical section, it will definitely make flag = false. So no deadlock occurs.

2.23 (d)

In process P_1 if wants1 is true then wants2 entered in critical section and in process P_2 if wants2 is true then wants1 entered in critical section but in both cases a deadlock can occur but it ensures the mutual exclusion.

2.24 (c)

S1: If readcount is 1 i.e some reader is reading, wait on wrt so that no writer can write.
S2: After readcount has been updated, signal on mutex is applied.
S3: Wait on mutex is applied to update readcount.
S4: If readcount is zero (no reader is reading), signal on wrt is applied to allow writer to write.

2.25 (c)

In given code, P(S) decrements the value of semaphore and V(S) increment the value of semaphore. P_b and V_b are wait and signal operations on binary semaphore. X_b and Y_b are binary semaphore. To avoid the mutual exclusion condition if the value of X_b is 1 and the value of Y_b = 0 then P(S) and V(S) works properly.

2.26 (a)

Only I is true.

2.27 (a)

P1	P2
While (S1 == S2); Critical section S1 = S2;	While (S1! = S2); Critical section S2 = Not(S1);

We initially take $S_1 = 1$, and $S_2 = 1$.
When P1 execute S1 = S2 then it do nothing and process P1 go into critical section after executing it make the value of S1 = S2 and when P1 execute then P2 not execute because that checks different condition in while so both show mutual exclusion. But progress is not shown by these two because in this one process can be delayed infinitely. Because again both P1 make its own condition true (S1 = S2 and S2 = not (S1);).

2.28 (a)

When $S_0 = 1$, $S_1 = 0$
So firstly the value of S_1 and $S_2 = 0$, P_1 and P_2 Not execute. The value of $S_0 = 1$, So it execute, process P_0 Now value of $S_0 = 0$
And it print zero one time when it release (S_1) call then value of S_1, S_2 increase by 1. So P_1 execute. It makes value of S_1 again zero and S_0 = 1, after it release (S_2) make value of $S_2 = 0$ by applying wait and $S_0 = 2$. So again P_0 process execute and wait (S_0) means it decrease by one so it print zero and this process continue. So P_0 print zero at least twice.

2.29 (b)

Assume P_1 executes until while condition and preempts before executing L = 1. Now P_2 executes all statements, hence L = 0. Then P_1 without checking L it makes L = 1 by executing the statement where it was preempted.
∴ It takes a non-zero value (L = 1) when the lock is actually available (L = 0)

2.30 (b)

Trick: To check deadlock in the given code proceed parallely i.e. first executed the first line of all the processes then executed the second line of all processes and so on.

(a)

X	Y	Z
1. P(a)	1. P(b)	1. P(c)
2. P(b)	2. P(c)	2. P(d)
3. P(c)	3. P(d)	3. P(a)

Value of a, b, c are 1 initially.
Execute first line of all processes i.e. $X_1 \rightarrow Y_1 \rightarrow Z_1$ after the executive of $X_1 \rightarrow Y_1 \rightarrow Z_1$ value of {a,b,c} = 0. Now no process is allowed to enter into the critical section. So deadlock.

(b)

X	Y	Z
1. P(b)	1. P(b)	1. P(a)
2. P(a)	2. P(c)	2. P(c)
3. P(c)	3. P(d)	3. P(d)

1. Execute X_1 so value of b gets 0.
2. As value of b = 0 when we execute Y_1 it get blocked (Y is blocked).

3. Now execute Z_1 so value of a get 0.
4. Now try to execute the second line of unblocked processes as value of a = 0 when we try to execute X_2 it get blocked (X is blocked)
5. Now execute Z_2, value of c gets 0.
6. As process X and Y are blocked so we can easily execute Z_3.

So no deadlock, like wise we can check option c and d, so option (b) is correct.

2.31 (d)

	X	W	Y	Z
1.	P(s)	P(s)	P(s)	P(s)
2.	R(x)	R(x)	R(x)	R(x)
3.	x = x +1	x = x + 1	x = x + 2	x = x + 2
4.	Store to MM	Store to MM	Store to MM	Store to MM
5.	V(s)	V(s)	V(s)	V(s)

1. Start with X. P(s) ⇒ s = 1 read x = 0, x = x+1
   Now before store into the memory process X preempt to process Y.
2. Y will perform P(s) ⇒ s = 0, read x = 0
   x = x – 2 then store x= – 2 and V(s)
   ⇒ s = 1
3. Z will perform P(s) ⇒ s = 0, read x= – 2
   x = x – 2
   then store x = – 4 and V(s) ⇒ s = 1
4. Now process x will store value of variable x into the MM so x = 1 and V(s) ⇒ s = 2
5. Now process W perform P(s) ⇒ s = 1 read x = 1
   x = x + 1, store x = 2, V(s) ⇒ s = 2,
   Option (d) is correct

2.32 (c)

The solution is using a binary semaphore. X and Y are two different processes whatever the values inserted by the processes in the array that will be used by process Y afterwards.

Option (a): It is wrong, because there exists a deadlock.

Option (b): It is wrong, because process X can insert multiple values in an array and terminate but process Y can consume only one value.

Option (c): It is correct, because each and every value inserted by the process X in the array will be immediately consumed by process Y.

Option (d): It is wrong, because process X can insert two values in the array but the process Y can consume only one value if the preemption occurs after V(S) operation in the entry seciton of process Y.

2.33 (c)

Consumer executes wait(S) then wait(n) and goes to sleep by decreasing n value.

After consumer sleep, producer goes to the sleep by executing wait(S).

2.34 Sol.

B = 2

C = B – 1 B = 2 × C D = 2 × B B = D – 1 ⇓ B = 3	D = 2 × B B = D – 1 C = B – 1 B = 2 × C ⇓ B = 4	C = B – 1 D = 2 × B B = 2 × C B = D – 1 ⇓ B = 3	C = B – 1 D = 2 × B B = D – 1 B = 2 × C ⇓ B = 2	D = 2 × B C = B – 1 B = D – 1 B = 2 × C ⇓ B = 2	D = 2 × B C = B – 1 B = 2 × C B = D – 1 ⇓ B = 3

⇒ B = 2, 3 or 4

∴ Three distinct values are possible.

2.35 (a)

(*i*) Mutual exclusion fail in the given program. First, process *X* executes all statements till var*P* = true. Second, process *Y* executes all statements till var*Q* = true; Now, both process *X* and process *Y* both can execute while condition as true and then enters critical section simultaneously. Therefore mutual exclusion fails.

(*ii*) Deadlock is prevented by the given code.

2.36 (a)

It satisfies the mutual exclusion, so only one process can be in the critical section at any time.

2.37 (c)

Given problem is strict alteration problem in which one process go into critical section or not is decided by non critical section of another process.

So, progress property is not satisfied.

2.38 Sol.

$$S - 20 + 12 = -1$$
$$S - 8 = -1$$
$$S = -1 + 8 = +7$$

So, the initial values of the semaphore should be '7'.

■■■■

3 Deadlock

3.1 A computer system has 6 tape drives, with n process completing for them. Each process may need 3 tape drives. The maximum value of n for which the system is guaranteed to be deadlock free is

(a) 2 (b) 3
(c) 4 (d) 1

[1992 : 2 Marks]

3.2 Consider a system having m resources of the same type. These resources are shared by 3 processes A, B and C, which have peak demands of 3, 4 and 6 respectively. For what value of m deadlock will not occur?

(a) 7 (b) 9
(c) 10 (d) 13

[1993 : 2 Marks]

3.3 An operating system contains 3 user processes each requiring 2 units of resource R. the minimum number of units of r such that no deadlocks will ever arise is

(a) 3 (b) 5
(c) 4 (d) 6

[1997 : 2 Marks]

3.4 A computer has six tape drives, with n processes competing for them. Each process may need two drives. What is the maximum value of n for the system to be deadlock free?

(a) 6 (b) 5
(c) 4 (d) 3

[1998 : 1 Mark]

3.5 Which of the following is NOT a valid deadlock prevention scheme?

(a) Release all resources before requesting a new resource.
(b) Number the resources uniquely and never request a lower numbered resource than the last one requested.
(c) Never request a resource after releasing any resource
(d) Request and all required resources be allocated before execution.

[2000 : 2 Marks]

3.6 In a certain operating system, deadlock prevention is attemped using the following scheme. Each process is assigned a unique timestamp, and is restarted with the same timestamp if killed. Let P_h be the process holding a resource R, P_r be a process requesting for the same resource R, and $T(P_h)$ and $T(P_r)$ be their timestamps respectively. The decision to wait or preempt one of the processes is based on the following algorithm.

if $T(P_r) < T(P_h)$ then
kill P_r
else
wait

Which one of the following is TRUE?

(a) The scheme is deadlock-free, but not starvation-free
(b) The scheme is not deadlock-free, but starvation-free
(c) The scheme is neither deadlock-free nor starvation-free
(d) The scheme is both deadlock-free and starvation-free

[2004 : 2 Marks]

3.7 Suppose n processes, $P_1, ..., P_n$ share m identical resource units, which can be reserved and released one at a time. The maximum resource requirement of process P_i is s_p where $s_i > 0$. Which one of the following is a sufficient condition for ensuring that deadlock does not occur?

(a) $\forall$ i, $s_i < m$ (b) $\forall$ i, $s_i < n$

(c) $\sum_{i=1}^{n} s_i < (m + n)$ (d) $\sum_{i=1}^{n} s_i < (m * n)$

[2005 : 2 Marks]

3.8 Consider the solution to the bounded buffer producer/consumer problem by using general semaphores S, F, and E. The semaphore S is the mutual exclusion semaphore initialized to 1. The semaphore F corresponds to the number of free slots in the buffer and is initialized to N.

The semaphore E corresponds to the number of elements in the buffer and is initialized to 0.

Producer Process	Consumer Process
Produce an item;	Wait (E);
Wait (F);	Wait (S);
Wait (S);	Remove an item from the buffer;
Append the item to the buffer;	Signal (S);
Signal (S);	Signal (F);
Signal (E);	Consumer the item;

Which of the following interchange operations may result in a deadlock?

(i) Interchanging Wait (F) and Wait (S) in the Producer process.

(ii) Interchanging Signal (S) and Signal (F) in the Consumer process.

(a) I only (b) II only

(c) Neither I nor II (d) Both I and II

[2006 : 2 Marks]

3.9 (b) Consider the following snapshot of a system running n processes. Process i is holding x_i instances of a resource R, for $1 \leq i \leq n$. Currently, all instances of R are occupied. Further, for all i, process i has placed a request for an additional y_i instances while holding the x_i instances it already has. There are exactly two processes p and q such that $y_p = y_q = 0$. Which one of the following can serve as a necessary condition to guarantee that the system is not approaching a deadlock?

(a) $\min(x_p, x_q) < \max_{k \neq p, q} y_k$

(b) $x_p + x_q \geq \max_{k \neq p, q} y_k$

(c) $\min(x_p, x_q) < 1$

(d) $\min(x_p, x_q) > 1$

[2006 : 2 Marks]

3.10 (c) A single processor system has three resource types X, Y, and Z, which are shared by three processes. There are 5 units of each resource type. Consider the following scenario, where the column alloc denotes the number of units of each resource type allocated to each process, and the column request denotes the number of units of each resource type requested by a process in order to complete execution. Which of these processes will finish LAST?

	alloc	request
	X Y Z	X Y Z
P0	1 2 1	1 0 3
P1	2 0 1	0 1 2
P2	2 2 1	1 2 0

(a) P0

(b) P1

(c) P2

(d) None of the above, since the system is in a deadlock

[2007 : 2 Marks]

3.11 (d) The following is a code with two threads, producer and consumer, that can run in parallel. Further, S and Q are binary semaphores equipped with the standard P and V operations.

```
semaphore S = 1, Q = 0;
integer x ;
producer :                consumer :
while (true) do           While (true) do
P(S) ;                    P(Q) ;
x = produce( ) ;          consume (x);
V(Q) ;                    V(S) ;
done                      done
```

Which of the following is TRUE about the program above?

(a) The process can deadlock

(b) One of the threads can starve

(c) Some of the items produced by the producer may be lost

(d) Values generated and stored in 'x' by the producer will always be consumed before the producer can generate a new value

[2008 : 2 Marks]

3.12 (b) An operating system implements a policy that requires a process to release all resources before making a request for another resource.

Select the TRUE statement from the following:

(a) Both starvation and deadlock can occur

(b) Starvation can occur but deadlock cannot occur

(c) Starvation cannot occur but deadlock can occur

(d) Neither starvation nor deadlock can occur

[2008 : 2 Marks]

3.13 (a) Which of the following is NOT true of deadlock prevention and deadlock avoidance schemes?

(a) In deadlock prevention, the request for resources is always granted if the resulting state is safe
(b) In deadlock avoidance, the request for resources is always granted if the resulting state is safe
(c) Deadlock avoidance is less restrictive than deadlock prevention
(d) Deadlock avoidance requires knowledge of resource requirements a priori

[2008 : 2 Marks]

3.14 (a) Consider a system with 4 types of resources R1 (3 units), R2 (2 units), R3 (3 units), R4 (2 units). A non-preemptive resource allocation policy is used. At any given instance, a request is not entertained if it cannot be completely satisfied. Three processes P1, P2, P3 request the resources as follows if executed independently.

Process P1 :	Process P2 :	Process P3 :
t = 0 : requests 2 units of R2	t = 0 : requests 2 units of R3	t = 0 : requests 1 unit of R4
t = 1 : requests 1 unit of R3	t = 2 : requests 1 unit of R4	t = 2 : requests 2 units of R1
t = 3 : requests 2 units of R1	t = 4 : requests 1 unit of R1	t = 5 : releases 2 units of R1
t = 5 : releases 1 unit of R2 and 1 unit of R1	t = 6 : releases 1 unit of R3	t = 7 : requests 1 unit of R2
t = 7 : releases 1 unit of R3	t = 8 : Finishes	t = 8 : requests 1 unit of R3
t = 8 : requests 2 units of R4		t = 9 : Finishes
t = 10 : Finishes		

Which one of the following statements is TRUE if all three processes run concurrently starting at time t = 0?

(a) All processes will finish without any deadlock
(b) Only P1 and P2 will be in deadlock
(c) Only P1 and P3 will be in deadlock
(d) All three processes will be in deadlock

[2009 : 2 Marks]

3.15 (b) A system has n resources $R_0, R_1, ..., R_{n-1}$, and k processes $P_0, P_1, ..., P_{k-1}$. The implementation of the resource request logic of each process P_i, is as follows:

```
if (i%2 == 0) {
    if (i < n) request R_i ;
    if (i+2 < n) request R_{i+2};
}
else {
    if (i < n) request R_{n-1};
    if (i+2 < n) request R_{n-i-2};
}
```

In which one of the following situations is a deadlock possible?

(a) n = 40, k = 26
(b) n = 21, k = 12
(c) n = 20, k = 10
(d) n = 41, k = 19

[2010 : 2 Marks]

3.16 (b) An operating system uses the *Banker's algorithm* for deadlock avoidance when managing the allocation of three resource types *X*, *Y*, and *Z* to three processes P0, P1, and P2. The table given below presents the current system state. Here, the *Allocation* matrix shows the current number of resources of each type allocated to each process and the *Max* matrix shows the maximum number of resources of each type required by each process during its execution.

	Allocation			Max		
	X	*Y*	*Z*	*X*	*Y*	*Z*
P0	0	0	1	8	4	3
P1	3	2	0	6	2	0
P2	2	1	1	3	3	3

There are 3 units of type *X*, 2 units of type *Y* and 2 units of type *Z* still available. The system is currently in a **safe** state. Consider the following independent requests for additional resources in the current state:

REQ1: P0 requests 0 units of *X*, 0 units of *Y* and 2 units of *Z*

REQ2: P1 requests 2 units of *X*, 0 units of *Y* and 0 units of *Z*

Which one of the following is **TRUE**?

(a) Only REQ1 can be permitted.
(b) Only REQ2 can be permitted.
(c) Both REQ1 and REQ2 can be permitted.
(d) Neither REQ1 nor REQ2 can be permitted.

[2014 (Set-1) : 2 Marks]

3.17 A system contains three programs and each requires three tape units for its operation. The minimum number of tape units which the system must have such that deadlocks never arise is ____.

[2014 (Set-3) : 2 Marks]

3.18 A system has 6 identical resources and N processes competing for them. Each process can request atmost 2 resources. Which one of the following values of N could lead to a deadlock?

(a) 1 (b) 2
(c) 3 (d) 4

[2015 (Set-2) : 1 Mark]

3.19 Consider the following policies for preventing deadlock in a system with mutually exclusive resources.

d

I. Processes should acquire all their resources at the beginning of execution. If any resource is not available, all resources acquired so far are released.

II. The resources are numbered uniquely, and processes are allowed to request for resources only in increasing resource numbers.

III. The resources are numbered uniquely, and processes are allowed to request for resources only in decreasing resource numbers.

IV. The resources are numbered uniquely. A process is allowed to request only for a resource with resource number larger than its currently held resources.

Which of the above policies can be used for preventing deadlock?

(a) Any one of I and III but not II or IV
(b) Any one of I, III and IV but not II
(c) Any one of II and III but not I or IV
(d) Any one of I, II, III and IV

[2015 (Set-3) : 2 Marks]

■■■■

Answers Deadlock

3.1	(a)	3.2	(d)	3.3	(c)	3.4	(b)	3.5	(c)	3.6	(a)	3.7	(c)	3.8	(a)	3.9	(b)
3.10	(c)	3.11	(d)	3.12	(b)	3.13	(a)	3.14	(a)	3.15	(b)	3.16	(b)	3.19	(d)		

Explanations Deadlock

3.1 (a)

If there are 2 processes then each process will hold 3 tape drives as there are 6 tape drives for which the system is guaranteed to be deadlock free.

3.2 (d)

If anyone peak demand satisfied then we can't get deadlock.

3.3 (c)

There are 3 processes, and demands '2' resources. If any one gets equal resources as it's demand, deadlock won't occur, and according to this question 4 resources are enough.

3.4 (b)

Every process needs 2 resources. With 5 resources atleast one can get whole 2 and executes. So after execution it frees 2 resources which can be used by other processes.

P_1	P_2	P_3	P_4	P_5
1	1	1	1	1

3.5 (c)

It is not a valid deadlock prevention scheme.

3.6 (a)

When the process wakes up again after it has been killed once or twice it will have same time-stamp as it had when it was killed first time and that time stamp can never be greater than a process that was killed after that or a new process that may have arrived.

So every time when the killed process wakes up again "it will always find a new process" that will say "your time stamp is less than me, which of course is as we know, and that process will again be killed", This may happen indefinitely if processes keep coming and killing that "innocent" every time it try to access.

So Starvation is possible. Deadlock is not possible.

3.7 (c)

There are n processes $P_1, P_2, \ldots, P_n$.
Number of identical resources = m
The maximum resource requirements for a process P_i is S_P, where $S_i > 0$.

If all processes are in safe state then system can allocate resources to each process P_i in some order and still avoid a dead lock. We can avoid the dead lock if each P_i can still request can be satisfied by the current available resources plus the resources held by process P_j such that $j < i$. The maximum resource requirement of n processes

is $\sum_{i=1}^{n} S_i$ and this amount must less than m + n.

$$\therefore \quad \sum_{i=1}^{n} S_i < (m+n)$$

3.8 (a)

Suppose the slots are full $\rightarrow$ F = 0.
Now, if Wait(F) and Wait(S) are interchanges and Wait(S) succeeds, The producer will wait for Wait(F) which is never going to succeed as Consumer would be waiting for Wait(S).
So, deadlock can happen.
If Signal(S) and Signal(F) are interchanged in Consumer, deadlock won't happen.
It will just give priority to a producer compared to the next consumer waiting.

3.9 (b)

Total number of processes = n
Process i is holding x_i instances of a resource R for $1 \le i \le n$.
Process i holding resource x_i and additional required y_i. There are two processes p and q such that $y_p = y_q = 0$.
For no dead lock can occur

$$x_p + x_q \geq (\max_{k \neq p,q} y_k)$$

3.10 (c)

Consider the matrix

	alloc	request
	XYZ	XYZ
P_0	121	103
P_1	201	012
P_2	221	120

So the available matrix is (0, 1, 2) and the requirement of process P_1 is (0, 1, 2) so P_1 can use the available resources after releasing the resources the matrix becomes:

	alloc	request
	XYZ	XYZ
P_0	121	103
P_1.	000	000
P_2	221	120

In this time available matrix is (2 1 3) and required matrix of P_0 is (1 0 3) so P_0 uses the resources of the releasing the resource. Tthe available matrix is (3, 3, 4) so P_2 will use the resources, P_2 will finish last.

3.11 (d)

Consumer can consume only once the producer has produced the item, and producer can produce (except the first time) only once the consumer has consumed the item.

3.12 (b)

Releasing all resources before making a request for another resource is a technique of deadlock prevention. So, deadlock cannot occur.
However, we cannot say anything about starvation, it may occur or may not.

3.13 (a)

In deadlock prevention, the request for a resource may not be granted even if the resulting state is safe.

3.14 (a)

Initially available resources

R1	R2	R3	R4
3	2	3	2

	Process	Allocate				Available			
t = 0		**R1**	**R2**	**R3**	**R4**	**R1**	**R2**	**R3**	**R4**
P_1 : request 2 unit of R2	P1	0	2	0	0	3	0	1	1
P_2 : request 2 unit of R3	P2	0	0	2	0				
P_3 : request 1 unit of R4	P3	0	0	0	1				
t = 1									
P_1 : request 1 unit of R3	P1	0	2	1	0	3	0	0	1
	P2	0	0	2	0				
	P3	0	0	0	1				
t = 2									
P_2 : request 1 unit of R4	P1	0	2	1	0	1	0	0	0
P_3 : request 2 unit of R1	P2	0	0	2	1				
	P3	2	0	0	1				
t = 3									
P_1 : request 2 unit of R1	P1	1	2	1	0	0	0	0	0
(wait for 1 unit of R1)	P2	1	0	2	1				
	P3	2	0	0	1				
t = 4									
P_2 : request 1 unit of R2 (wait)									
t = 5									
P_1 : release 1 unit of R2	P1	1	1	1	0	1	1	0	0
and 1 unit of R1	P2	1	0	2	1				
	P3	0	0	0	1				
P_3 : release 2 unit of R1									
now request of P_1, 1 unit of R1 will be granted and request of P_2, 1 unit of R1 will be granted									
t = 6									
P_2 : release 1 unit of R3	P1	1	1	1	0	1	1	1	0
	P2	1	0	1	1				
	P3	0	0	0	1				
t = 7									
P_1 : release 1 unit of R3	P1	1	1	0	0	1	0	2	0
P_3 : request 1 unit of R2	P2	1	0	1	1				
	P3	0	1	0	1				
t = 8									
P_2 : finish	P1	1	1	0	0	2	0	3	1
	P3	0	1	0	1				
P_1 : request 2 unit of R4 (wait)		1	1	0	0	2	0	2	1
P_3 : request 1 unit of R3		0	1	1	1				
t = 9									
P_3 : finish	P1	1	1	0	0	2	1	3	2
request of 2 unit of R4 by P1 will be granted.									
	P1	1	1	0	2	2	1	3	0
t = 10									
P_1 : finish						3	2	3	2

All processes will finish without any deedlock.

3.15 (b)

According to the given program, in all the cases for even sequences of processes only corresponding sequence number of recource will be alloted except case (b), where n = 21, k = 12, for $P_{10} \Rightarrow R_{10}$ can only be alloted and for $P_{11} \Rightarrow R_{21-11} \Rightarrow R_{10}$ can be alloted.

$P_i \Rightarrow R_i$ alloted and for odd sequence of process. i.e. $P_i \Rightarrow R_{n-i}$ that never conflict in any case except (b), when a same resource is requested by more than one process at the same time may lead to the deadlock.

3.16 (b)

	Allocation	Max	Need	Need with additional requests
	X Y Z	X Y Z	X Y Z	X Y Z
P0	0 0 1	8 4 3	8 4 2	8 4 4 (REQ1)
P1	3 2 0	6 2 0	3 0 0	5 0 0 (REQ2)
P2	2 1 1	3 3 3	1 2 2	1 2 2

Available: X Y Z
3 2 2 → P2 can satisfy it needs
2 1 1 → P2 releases its allocation
5 3 3 → P1 satisfies it needs
3 2 0 → P1 releases its allocation
8 5 3 → P0 can not satisfy with additional requests

∴ Only REQ2 can be permitted.

3.17 Sol.

(3*2 tape units) + 1 tape unit = 7

3.18 (*)

N is number of processes, 6 identical resources

$N \geq 6 \times (2 - 1)$

If $N \geq 6 \Rightarrow$ It could lead to a deadlock

No option matches with $N \geq 6$

3.19 (d)

All of the given policies can be used for preventing deadlock.

4 Memory Management and Virtual Memory

4.1 Match the pairs in the following questions.

Pair-I	Pair-II
A. Virtual Memory	**P.** Temporal locality
B. Shared memory	**Q.** Spatial locality
C. Look-ahead buffer	**R.** Address translation
D. Look-aside buffer	**S.** Mutual exclusion

[1989 : 2 Marks]

4.2 Under paged memory management scheme simple lock and key memory protection arrangement may still be required if the _________ processors do not have address mapping hardware.

[1990 : 2 Marks]

4.3 Match the pairs in the following Questions.

(a) Critical region	(p) Hoare's monitor
(b) Wait/Signal	(q) Mutual exclusion
(c) Working set	(r) Principle of locality
(d) Deadlock	(s) Circular wait

[1990 : 2 Marks]

4.4 State whether the following statements are TRUE or FALSE with reason. Transferring data in blocks from the main memory to the cache memory enables an interleaved main memory unit to operate unit at its maximum speed.

[1990 : 2 Marks]

4.5 State whether the following statements are TRUE or FALSE with reason. The Link-load-and-go loading scheme required less storage space than the Link-and-go loading scheme.

[1990 : 2 Marks]

4.6 Match the pairs in the following question by writing the corresponding letters only.

(a) Buddy system	(p) Run-time type specification
(b) Interpretation	(q) Segmentation
(c) Pointer type	(r) Memory allocation
(d) Virtual memory	(s) Garbage collection

[1991 : 2 Marks]

4.7 The total size of address space in a virtual memory system is limited by

(a) the length of MAR
(b) the available secondary storage
(c) the available main memory
(d) All of the above

[1991 : 2 Marks]

4.8 A "link editor" is a program that:

(a) Matches the parameters of the macro definition with locations of the parameters of the macro call.
(b) Matches external names of one program with their location in other programs.
(c) Matches the parameters of subroutine definition with the location of parameters of subroutine call.
(d) Acts as link between text editor and the user.
(e) Acts as a link between compiler and user program.

[1991 : 2 Marks]

4.9 Indicate all the false statements from the statements given below:

(a) The amount of virtual memory available is limited by the availability of secondary storage.
(b) Any implementation of a critical section requires the use of an indivisible machine-instruction, such as test-and-set.
(c) The LRU page replacement policy may cause hashing for some type of programs.
(d) The best fit techniques for memory allocation ensures the memory will never be fragmented.

[1991 : 2 Marks]

4.10 Which page replacement policy sometimes leads to more page faults when size of memory is increased?

(a) Optimal (b) LRU
(c) FIFO (d) None of these

[1992 : 1 Mark]

4.11 A simple two-pass assembler does the following in the first pass:

(a) it allocates space for the literals
(b) it computes the total length of the program
(c) it builds the symbol table for the symbols and their values
(d) it generates code for all the load and store register instructions.

[1993 : 2 Marks]

4.12 A memory page containing a heavily used variable that was initialized very early and is in constant use is removed when

(b)

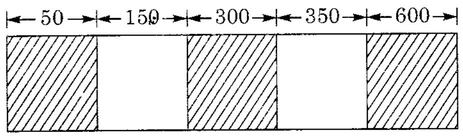
(a) LRU page replacement algorithm is used
(b) FIFO page replacement algorithm is used
(c) LFU page replacement algorithm is used
(d) None of these above

[1994 : 2 Marks]

4.13 Consider the following heap (figure) in which blank regions are not in use and hatched regions are in use.

(b)

The sequence of requests for blocks of size 300, 25, 125, 50 can be satisfied if we use



Increasing addresses

(a) either first fit or best fit policy (any one)
(b) first fit but not best fit policy
(c) best fit but not first fit policy
(d) none of the above

[1994 : 2 Marks]

4.14 The principle of locality justifies the use of

(a) Interrupts (b) DMA
(c) Polling (d) Cache Memory

[1995 : 1 Mark]

4.15 In a paged segmented scheme of memory management, the segment table itself must have a page table because

(a) the segment table is often too large to fit in one page
(b) each segment is spread over a number of pages
(c) segment tables point to page table and not to the physical locations of the segment
(d) the processor's description base register points to a page table

[1995 : 1 Mark]

4.16 A linker is given object modules for a set of programs that were compiled separately. What information need to be included in an object module?

(a)

(a) Object code
(b) Relocation bits
(c) Names and locations of all external symbols defined in the object module
(d) Absolute addresses of internal symbols

[1995 : 1 Mark]

4.17 The capacity of a memory unit is defined by the number of words multiplied by the number of bits/word. How many separate address and data lines are needed for a memory of 4 K × 16?

(a) 10 address, 16 data lines
(b) 11 address, 8 data lines
(c) 12 address, 16 data lines
(d) 12 address, 12 data lines

[1995 : 2 Marks]

4.18 The address sequence generated by tracing a particular program executing in a pure demand paging system with 100 records per page with 1 free main memory frame is recorded as follows. What is the number of page faults?

(c)

0100, 0200, 0430, 0499, 0510, 0530, 0560, 0120, 0220, 0240, 0260, 0320, 0370

(a) 13 (b) 8
(c) 7 (d) 10

[1995 : 2 Marks]

4.19 In a virtual memory system the address space specified by the address lines of the CPU must be ____ than the physical memory size and ____ than the secondary storage size.

(c)

(a) smaller, smaller (b) smaller, larger
(c) larger, smaller (d) larger, larger

[1995 : 2 Marks]

4.20 A ROM is used to store the table for multiplication of two 8-bit unsigned integers. The size of ROM required is

(a) 256 × 16 (b) 64 K × 8
(c) 4K × 16 (d) 64 K × 16

[1996 : 1 Mark]

4.21 A 1000 Kbyte memory is managed using variable partitions but no compaction. It currently has two partitions of sizes 200 Kbytes and 260 Kbytes respectively. The smallest allocation request in Kbytes that could be denied is for

(d)

(a) 151 (b) 181
(c) 231 (d) 541

[1996 : 2 Marks]

4.22 Locality of reference implies that the page reference being made by a process
(a) will always be to the page used in the previous page reference
(b) is likely to be to one of the pages used in the last few page references
(c) will always be to one of the pages existing in memory
(d) will always lead to a page fault

[1997 : 1 Mark]

4.23 Thrashing
(a) Reduces page I/O
(b) Decreases the degree of multipro-gramming
(c) Implies excessive page I/O
(d) Improve the system performance

[1997 : 1 Mark]

4.24 Dirty bit for a page in a page table
(a) Helps avoid unnecessary writes on paging device
(b) Helps maintain LRU information
(c) Allows only read on a page
(d) None of the above

a

[1997 : 1 Mark]

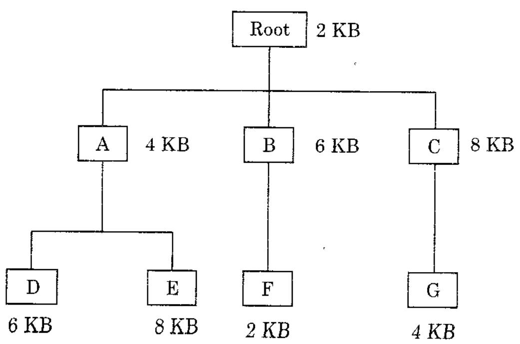
4.25 In a resident–OS computer, which of the following system software must reside in the main memory under all situations?
(a) Assembler (b) Linker
(c) Loader (d) Compiler

c

[1998 : 1 Mark]

4.26 The overlay tree for a program is as shown below:

b



What will be the size of the partition (in physical memory) required to load (and run) this program?
(a) 12 KB (b) 14 KB
(c) 10 KB (d) 8 KB

[1998 : 2 Marks]

4.27 If an instruction takes i microseconds and a page fault takes an additional j microseconds, the effective instruction time if on the average a page fault occurs every k instruction is

a

(a) $i + \frac{j}{k}$ (b) $i + j * k$
(c) $\frac{i + j}{k}$ (d) $(i + j) * k$

[1998 : 2 Marks]

4.28 Which of the following is/are advantage of virtual memory?

d

(a) Faster access to memory on an average.
(b) Processes can be given protected address spaces.
(c) Linker can assign addresses independent of where the program will be loaded in physical memory.
(d) Programs larger than the physical memory size can be run.

[1999 : 2 Marks]

4.29 Suppose the time to service a page fault is on the average 10 milliseconds, while a memory access takes 1 microsecond. Then a 99.99% hit ratio results in average memory access time of

d

(a) 1.9999 milliseconds
(b) 1 millisecond
(c) 9.999 microseconds
(d) 1.9999 microseconds

[2000 : 2 Marks]

4.30 Consider a machine with 64 MB physical memory and a 32-bit virtual address space. If the page size of 4 KB, what is the approximate size of the page table?

c

(a) 16 MB (b) 8 MB
(c) 2 MB (d) 24 MB

[2001 : 2 Marks]

4.31 Which of the following statements is/are false?
(a) Virtual memory implements the translation of a program's address space in to physical memory address space.
(b) Virtual memory allows each program to exceed the size of the primary memory.
(c) Virtual memory increases the degree of multiprogramming.
(d) Virtual memory reduces the context switching overhead.

[2001 : 1 Mark]

4.32 The process of assigning load addresses to the various parts of the program and adjusting the code and data in the program to reflect the assigned addresses is called
(a) assembly (b) parsing
(c) relocation (d) symbol resolution
[2001 : 1 Mark]

4.33 Consider a virtual memory system with FIFO page replacement policy. For an arbitrary page access pattern, increasing the number of page frames in main memory will
(a) always decrease the number of page faults
(b) always increase the number of page faults
(c) sometimes increase the number of page faults
(d) never affect the number of page faults
[2001 : 1 Mark]

4.34 Which of the following is not a form of memory?
(a) Instruction cache
(b) Instruction register
(c) Instruction opcode
(d) Translation look a side buffer
[2002 : 1 Mark]

4.35 Which of the following requires a device driver?
(a) Has not been used for the longest time in the past
(b) Will not be used for the longest time in the future
(c) Has been used least number of times
(d) Has been used most number of times
[2002 : 1 Mark]

4.36 Dynamic linking can cause security concerns because
(a) Security is dynamic
(b) The path for searching dynamic libraries is not known till runtime
(c) Linking is insecure
(d) Cryptographic procedures are not available for dynamic linking
[2002 : 2 Marks]

4.37 In a system with 32 bit virtual addresses and 1 KB page size, use of one-level page tables for virtual to physical address translation is not practical because of
(a) the large amount of internal fragmentation
(b) the large amount of external fragmentation
(c) the large memory overhead in maintaining page tables
(d) the large computation overhead in the translation process
[2003 : 1 Mark]

Common Data for Q. 4.38 & 4.39

A processor uses 2-level page tables for virtual to physical address translation. Page tables for both levels are stored in the main memory. Virtual and physical addresses are both 32 bits wide. The memory is byte addressable. For virtual to physical address translation, the 10 most significant bits of the virtual address are used as index into the first level page table while the next 10 bits are used as index into the second level page table. The 12 least significant bits of the virtual address are used as offset within the page. Assume that the page table entries in both levels of page tables are 4 bytes wide. Further, the processor has a translation look-aside buffer (TLB), with a hit rate of 96%. The TLB caches recently used virtual page numbers and the corresponding physical page numbers. The processor also has a physically addressed cache with a hit ratio of 90%. Main memory access time is 10 ns, cache access time is 1 ns, and TLB access time is also 1 ns.

4.38 Assuming that no page faults occur, the average time taken to access a virtual address is approximately (to the nearest 0.5 ns)
(a) 1.5 ns (b) 2 ns
(c) 3 ns (d) 4 ns
[2003 : 2 Marks]

4.39 Suppose a process has only the following pages in its virtual address space: two contiguous code pages starting at virtual address 0 × 00000000, two contiguous data pages starting at virtual address 0 × 00400000, and a stack page starting at virtual address 0 × FFFFF000. The amount of memory required for storing the page tables of this process is
(a) 8 KB (b) 12 KB
(c) 16 KB (d) 20 KB
[2003 : 2 Marks]

4.40 Consider an operating system capable of loading and executing a single sequential user process at a time. The disk head scheduling algorithm used is First Come First Served (FCFS). If FCFS is replaced by Shortest Seek Time First (SSTF), claimed by the vendor to given 50% better benchmark results, what is the expected improvement in the I/O performance of user programs?

(a) 50% (b) 40%
(c) 25% (d) 0%

[2004 : 1 Mark]

4.41 The minimum number of page frames that must be allocated to a running process in a virtual memory environment is determined by

(a) the instruction set architecture
(b) page size
(c) physical memory size
(d) number of processes in memory

[2004 : 1 Mark]

4.42 Consider a system with a two-level paging scheme in which a regular memory access takes 150 nanoseconds, and servicing a page fault takes 8 milliseconds. An average instruction takes 100 nanoseconds of CPU time, and two memory accesses. The TLB hit ratio is 90%, and the page fault rate is one in every 10,000 instructions. What is the effective average instruction execution time?

(a) 645 nanoseconds (b) 1050 nanoseconds
(c) 1215 nanoseconds (d) 1230 nanoseconds

[2004 : 2 Marks]

4.43 Consider a fully associative cache with 8 cache blocks (numbered 0-7) and the following sequence of memory block requests: 4, 3, 25, 8, 19, 6, 25, 8, 16, 35, 45, 22, 8, 3, 16, 25, 7.

If LRU replacement policy is used, which cache block will have memory block 7?

(a) 4 (b) 5
(c) 6 (d) 7

[2004 : 2 Marks]

4.44 In a virtual memory system, size of virtual address is 32-bit, size of physical address is 30-bit, page size is 4 Kbyte and size of each page table entry is 32-bit. The main memory is byte addressable. Which one of the following is the maximum number of bits that can be used for storing protection and other information in each page table entry?

(a) 2 (b) 10
(c) 12 (d) 14

[2004 : 2 Marks]

4.45 Consider a 2-way set associative cache memory with 4 sets and total 8 cache blocks (0-7) and a main memory with 128 blocks (0-127). What memory blocks will be present in the cache after the following sequence of memory block references if LRU policy is used for cache block replacement. Assuming that initially the cache did not have any memory block from the current job?

0 5 3 9 7 0 16 55

(a) 0 3 5 7 16 55 (b) 0 3 5 7 9 16 55
(c) 0 5 7 9 16 55 (d) 3 5 7 9 16 55

[2005 : 2 Marks]

4.46 For each of the four processes P_1, P_2, P_3 and P_4. The total size in kilobytes (KB) and the number of segments are given below.

Process	Total size (in KB)	Number of segments
P_1	195	4
P_2	254	5
P_3	45	3
P_4	364	8

The page size is 1 KB. The size of an entry in the page table is 4 bytes. The size of an entry in the segment table is 8 bytes. The maximum size of a segment is 256 KB. The paging method for memory management uses two-level paging, and its storage overhead is P. The storage overhead for the segmentation method is S. The storage overhead for the segmentation and paging method is T. What is the relation among the overheads for the different methods of memory management in the concurrent execution of the above four processes?

(a) $P < S < T$ (b) $S < P < T$
(c) $S < T < P$ (d) $T < S < P$

[2006 : 2 Marks]

4.47 A CPU generates 32-bit virtual addresses. The page size is 4 KB. The processor has a translation look-aside buffer (TLB) which can hold a total of 128 page table entries and is 4-way set associative. The minimum size of the TLB tag is

(a) 11 bits (b) 13 bits
(c) 15 bits (d) 20 bits

[2006 : 2 Marks]

4.48 A computer system supports 32-bit virtual addresses as well as 32-bit physical addresses.

Since the virtual address space is of the same size as the physical address space, the operating system designers decide to get rid of the virtual memory entirely. Which one of the following is true?

(a) Efficient implementation of multi-user support is no longer possible
(b) The processor cache organization can be made more efficient now
(c) Hardware support for memory management is no longer needed
(d) CPU scheduling can be made more efficient now

[2006 : 2 Marks]

4.49 The address sequence generated by tracing a particular program executing in a pure demand paging system with 100 bytes per page is 0100, 0200, 0430, 0499, 0510, 0530, 0560, 0120, 0220, 0240, 0260, 0320, 0410.
Suppose that the memory can store only one page and if x is the address which causes a page fault then the bytes from addresses x to x + 99 are loaded on to the memory. How many page faults will occur?

(a) 0 (b) 4
(c) 7 (d) 8

[2007 : 1 Mark]

4.50 A demand paging system takes 100 time units to service a page fault and 300 time units to replace a dirty page. Memory access time is 1 time unit. The probability of a page fault is p. In case of a page fault, the probability of page being dirty is also p. It is observed that the average access time is 3 time units. Then the value of p is

(a) 0.194 (b) 0.233
(c) 0.514 (d) 0.981

[2007 : 2 Marks]

Common Data for Q. 4.51 & Q. 4.52:

A process has been allocated 3 page frames. Assume that none of the pages of the process are available in the memory initially. The process makes the following sequence of page references (reference string) : 1, 2, 1, 3, 7, 4, 5, 6, 3, 1.

4.51 If optimal page replacement policy is used, how many page faults occur for the above reference string?

(a) 7 (b) 8
(c) 9 (d) 10

[2007 : 2 Marks]

4.52 Least Recently Used (LRU) page replacement policy is a practical approximation to optimal page replacement. For the above reference string, how many more page faults occur with LRU than with the optimal page replacement policy?

(a) 0 (b) 1
(c) 2 (d) 3

[2007 : 2 Marks]

4.53 A paging scheme uses a Translation Look-aside Buffer (TLB). A TLB-access takes 10 ns and a main memory access takes 50 ns. What is the effective access time(in ns) if the TLB hit ratio is 90% and there is no page-fault?

(a) 54 (b) 60
(c) 65 (d) 75

[2008 : 1 Mark]

4.54 A processor uses 36 bit physical addresses and 32 bit virtual addresses, with a page frame size of 4 Kbytes. Each page table entry is of size 4 bytes. A three level page table is used for virtual-to-physical address translation, where the virtual address is used as follows

- bits 30-31 are used to index into the first level page table,
- bits 21-29 are used to index into the second level page table
- bits 12-20 are used to index into the third level page table
- bits 0-11 are used as offset within the page

The number of bits required for addressing the next level page table (or page frame) in the page table entry of the first, second and third level page tables are respectively.

(a) 20, 20 and 20 (b) 24, 24, and 24
(c) 24, 24 and 20 (d) 25, 25 and 24

[2008 : 2 Marks]

4.55 Assume that a main memory with only 4 pages, each of 16 bytes, is initially empty. The CPU generates the following sequence of virtual addresses and uses the Least Recently Used (LRU) page replacement policy.

0, 4, 8, 20, 24, 36, 44, 12, 68, 72, 80, 84, 28, 32, 88, 92

How many page faults does this sequence cause? What are the page numbers of the pages present in the main memory at the end of the sequence?

(a) 6 and 1, 2, 3, 4 (b) 7 and 1, 2, 4, 5
(c) 8 and 1, 2, 4, 5 (d) 9 and 1, 2, 3, 5

[2008 : 2 Marks]

4.56 Match the following flag bits used in the context of virtual memory management on the List-I (Name of the bit) with the different purposes on the List-II (Purpose) of the table below.

List-I	List-II
I. Dirty	a. Page initialization
II. R/W	b. Write-back policy
III. Reference	c. Page protection
IV. Valid	d. Page replacement policy

Codes:

(a) I-d, II-a, III-b, IV-c
(b) I-b, II-c, III-a, IV-d
(c) I-c, II-d, III-a, IV-b
(d) I-b, II-c, III-d, IV-a

[2008 : 2 Marks]

4.57 In which one of the following page replacement policies, Bleady's anomaly may occur?

(a) FIFO (b) Optimal
(c) LRU (d) MRU

[2009 : 1 Mark]

4.58 The essential content(s) in each entry of a page table is/are

(a) virtual page number
(b) page frame number
(c) both virtual page number and page frame number
(d) access right information

[2009 : 1 Mark]

4.59 A system uses FIFO policy for page replacement. It has 4 page frames with no pages loaded to begin with. The system first accesses 100 distinct pages in some order and then accesses the same 100 pages but now in the reverse order. How many page faults will occur?

(a) 196 (b) 192
(c) 197 (d) 195

[2010 : 1 Mark]

4.60 Let the page fault service time be 10 ms in a computer with average memory access time being 20 ns. If one page fault is generated for every 10^6 memory accesses, what is the effective access time for the memory?

(a) 21 ns (b) 30 ns
(c) 23 ns (d) 35 ns

[2011 : 1 Mark]

4.61 Consider the virtual page reference string
1, 2, 3, 2, 4, 1, 3, 2, 4, 1
on a demand paged virtual memory system running on computer system that has main memory size of 3 page frames which are initially empty. Let LRU, FIFO and OPTIMAL denote the number of page faults under the corresponding page replacement policy. Then

(a) OPTIMAL < LRU < FIFO
(b) OPTIMAL < FIFO < LRU
(c) OPTIMAL = LRU
(d) OPTIMAL = FIFO

[2012 : 2 Marks]

Linked Answer Questions 4.62 and 4.63:

A computer uses 46-bit virtual address, 32-bit physical address, and a three-level paged page table organization. The page table base register stores the base address of the first-level (T_1), which occupies exactly one page. Each entry of T_1 stores the base address of a page of the second-level table (T_2). Each entry of T_2 stores the base address of a page of the third-level table (T_3). Each entry of T_3 stores a page table entry (PTE). The PTE is 32 it in size. The processor used in the computer has a 1 MB 16-way-set associative virtually indexed physical tagged cache the cache block size is 64 bytes.

4.62 What is the size of a page in KB in this computer?

(a) 2 (b) 4
(c) 8 (d) 16

[2013 : 2 Marks]

4.63 What is the minimum number of page colours needed to guarantee that no two synonyms map to different sets in the processor cache of this computer?

(a) 2 (b) 4
(c) 8 (d) 16

[2013 : 2 Marks]

4.64 Assume that there are 3 page frames which are initially empty. If the page reference string is 1, 2, 3, 4, 2, 1, 5, 3, 2, 4, 6, the number of page faults using the *optimal replacement policy* is ________.

[2014 (Set-1) : 2 Marks]

4.65 A computer has twenty physical page frames which contain pages numbered 101 through 120. Now a program accesses the pages numbered 1, 2,...,100 in that order, and repeats the access sequence THRICE.

d

Which one of the following page replacement policies experiences the same number of page faults as the optimal page replacement policy for this program?

(a) Least-recently-used
(b) First-in-first-out
(c) Last-in-first-out
(d) Most-recently-used

[2014 (Set-2) : 2 Marks]

4.66 A system uses 3 page frames for storing process pages in main memory. It uses the Least Recently Used (LRU) page replacement policy. Assume that all the page frames are initially empty. What is the total number of page faults that will occur while processing the page reference string given below?

4, 7, 6, 1, 7, 6, 1, 2, 7, 2

[2014 (Set-3) : 1 Mark]

4.67 Consider a paging hardware with a TLB. Assume that the entire page table and all the pages are in the physical memory. It takes 10 milliseconds to search the TLB and 80 milliseconds to access the physical memory. If the TLB hit ratio is 0.6, the effective memory access time (in milliseconds) is ________.

[2014 (Set-3) : 2 Marks]

4.68 Consider a system with byte-addressable memory, 32 bit logical addresses, 4 kilobyte page size and page table entries of 4 bytes each. The size of the page table in the system in megabytes is _______.

[2015 (Set-1) : 1 Mark]

4.69 Consider a main memory with five page frames and the following sequence of page references: 3, 8, 2, 3, 9, 1, 6, 3, 8, 9, 3, 6, 2, 1, 3. Which one of the following is true with respect to page replacement policies First-In-First Out (FIFO) and Least Recently Used (LRU)?

(a) Both incur the same number of page faults
(b) FIFO incurs 2 more page faults than LRU
(c) LRU incurs 2 more page faults than FIFO
(d) FIFO incurs 1 more page faults than LRU

[2015 (Set-1) : 2 Marks]

4.70 A computer system implements a 40 bit virtual address, page size of 8 kilobytes, and a 128-entry translation look-aside buffer (TLB) organized into 32 sets each having four ways. Assume that the TLB tag does not store any process id. The minimum length of the TLB tag in bits is _____.

[2015 (Set-2) : 1 Mark]

4.71 Consider six memory partitions of size 200 KB, 400 KB, 600 KB, 500 KB, 300 KB, and 250 KB, where KB refers to kilobyte. These partitions need to be allotted to four processes of sizes 357 KB, 210 KB, 468 KB and 491 KB in that order. If the best fit algorithm is used, which partitions are NOT allotted to any process?

(a) 200 KB and 300 KB
(b) 200 KB and 250 KB
(c) 250 KB and 300 KB
(d) 300 KB and 400 KB

[2015 (Set-2) : 2 Marks]

4.72 A Computer system implements 8 kilobyte pages and a 32-bit physical address space. Each page table entry contains a valid bit, a dirty bit, three permission bits, and the translation. If the maximum size of the page table of a process is 24 megabytes, the length of the virtual address supported by the system is ______________ bits.

[2015 (Set-2) : 2 Marks]

4.73 Consider a computer system with 40-bit virtual addressing and page size of sixteen kilobytes. If the computer system has a one-level page table per process and each page table entry requires 48 bits, then the size of the per-process page table is _______ megabytes.

[2016 (Set-1) : 2 Marks]

■■■■

Answers Memory Management and Virtual Memory

4.7	(b)	4.8	(e)	4.9	(b, d)	4.10	(c)	4.11	(a)	4.12	(b)	4.13	(b)	4.14	(d)	4.15	(a)
4.16	~~(c)~~ d	4.17	(c)	4.18	(c)	4.19	(c)	4.20	(d)	4.21	(d)	4.22	(b)	4.23	(c)	4.24	(a)
4.25	(c)	4.26	(b)	4.27	(a)	4.28	(d)	4.29	(d)	4.30	(c)	4.31	(d)	4.32	(c)	4.33	(c)
4.34	(c)	4.35	(d)	4.36	(b)	4.37	(c)	4.38	(d)	4.39	(c)	4.40	(d)	4.41	(a)	4.42	(d)
4.43	(b)	4.44	(d)	4.45	(c)	4.46	(b)	4.47	(c)	4.48	(c)	4.49	(c)	4.50	(a)	4.51	(a)
4.52	(c)	4.53	(c)	4.54	(b)	4.55	(b)	4.56	(d)	4.57	(a)	4.58	(b)	4.59	(a)	4.60	(b)
4.61	(b)	4.62	(c)	4.63	(c)	4.65	(d)	4.69	(a)	4.71	(a)						

Explanations Memory Management and Virtual Memory

4.1 Sol.

A-Q, B-S, C-P, D-R

4.2 Sol.

Input/output

4.3 Sol.

a-q, b-p, c-r, d-s

4.4 Sol.

Statement is true, main memory transfers the data in the form of block to cache and cache transfers the data in the form of words, so it enables the interleaved main memory unit to operate unit at maximum speed.

4.5 Sol.

False because in link load and go scheme some portion of memory is occupied by assembler which is simply wastage of memory. As this scheme is combination of assembler and loader activities so it uses large amount of memory.

4.6 Sol.

a-r, b-p, c-s, d-q

4.7 (b)

Since virtual memory is implemented on secondary storage.

4.8 (e)

Link editor is another name of 'linker'. Linker is a program which links the object code with it's library.

4.9 (b,d)

(b) Without indivisible machine instruction, critical section can be implemented like using monitors.

(d) Best fit also suffers from fragmentation.

4.10 (c)

It's statement of Bleady's Anamoly. More frames in FIFO = More page faults.

4.11 (a)

2-pass assembler

In 1st pass: allocate space for literals.

In 2nd pass: object program is generated.

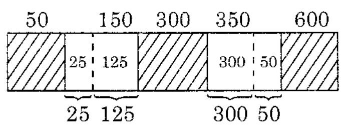
4.12 (b)

First in First out.

4.13 (b)

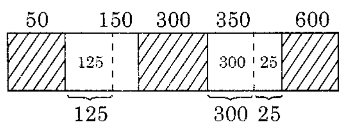
Requests: 300, 25, 125, 50

First fit:



$\Rightarrow$ First fit can satisfy all the requests.

Best fit:



$\Rightarrow$ 50 can not be satisfied by best fit

4.14 (d)

The principle of locality justifies the use of cache memory.

4.15 (a)

Segment table is often too large to fit in one page. So segment table itself must have a page table in a paged segmented scheme.

4.16 (d)

Linker definition: Linker is a program which links the compiled code with library files.

4.17 (c)

Size of memory $= 4K \times 16$ bits

$= 2^{12} \times 16$ bits

$\Rightarrow$ 12 address lines and 16 data lines are needed.

4.18 (c)

Given that only one frame is available which can hold 100 records at one time (0-99).

100	200	430	499	510	530
×	×	×	✓	✓	×
560	120	220	240	260	320
✓	×	×	✓	✓	×
370					
✓					

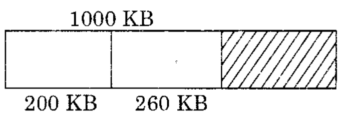
4.19 (c)

In virtual memory system, Address Space specified by the address lines of CPU must be larger than the physical memory size and smaller than the secondary storage size.

4.20 (d)

$2^8 \times 2^8 = 2^{16} \times 16$

4.21 (d)



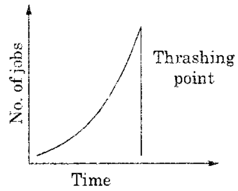
(a) 151 can fit in any of 200 KB and 260KB
(b) 181 can fit in any of 200 KB and 260KB
(c) 231 can fit in 260 KB
(d) 541 can not fit in any of partitions.
$\therefore$ The request 541 is denied.

4.22 (b)

Locality of reference is a phenomenon describing the same value, related storage location being frequently accessed.

4.23 (c)

Thrashing: After reaching the maximum number of jobs allowed in main memory. CPU efficiency drastically getting down this situation is called Trashing.



Thrashing implies excessive page I/O.

4.24 (a)

Which page has Dirty bit only those pages being written back in main memory, so it avoids unnecessary writing.

4.25 (c)

Loader is used to load the program into main memory.

4.26 (b)

Maximum space that can be needed at a time is 14 kB (using E). The maximum space to load the program is decided by the maximum size taken by rootine call sequence from the root. i.e., Root $\rightarrow$ A $\rightarrow$ E.

4.27 (a)

$EMAT = p \times s + (1 - p) \times m$

p: page fault rate

s: page fault service time

m: main memory access time

$$\Rightarrow \frac{(i+j)}{R} + \left(1 - \frac{1}{R}\right) \times i = \frac{j + iR}{R} = j + \frac{i}{R}$$

4.28 (d)

Virtual memory allows the user can run the programs larger than the physical memory size.

4.29 (d)

Average memory access time = [(% of page miss) * (time of service a page fault) + (% of page hit) * (memory access time)]/100

So, average memory access time in microseconds is

(99.99 * 1 + 0.01 * 10 * 1000)/100

= (99.99 + 100)/1000

= 199.99/1000 = 1.9999 μsec

4.30 (c)

Number of entries in page table $= \frac{2^{32}}{2^{12}} = 2^{20}$

Frame size $= \frac{2^{26}}{2^{12}} = 2^{14}$

$\therefore$ PT have to be stored in one frame so entry size must be 2 bytes, hence size of PT

$= 2^{20} \times 2 = 2^{21} = 2$ MB

4.31 (d)

In a system with virtual memory context switch includes extra overhead in switching of address spaces.

4.32 (c)

Relocation can change the assigned address of data and code in the program.

4.33 (c)

Bleady's Anamoly more number of frames = More page faults.

4.34 (c)

Instruction cache, instruction register and TLB are memories, but instruction opcode is the part of instruction that specify the operation to perform.

4.35 (d)

Device driver is required if any device has been used most number of times like register.

4.36 (b)

Linker comes into the picture at run time.

4.37 (c)

Given, Page Size = 1 KB = 2^{10} bytes

Virtual address is 32 bit long

So required number of pages

$= 2^{32}/2^{10} = 2^{22}$

2^{22} is a very large so the large memory overhead in maintaining page tables.

4.38 (d)

Average time taken to access a virtual address

$= [(96/100)(1 + 0.9 \times 1 + 0.1 \times (1 + 10))]$

$+ [(0.04)(21 + 0.9 \times 1 + 0.1 \times (1 + 10))]$

$= [(.96)(1 + 0.9 + 0.1 \times 11)] + [(0.04)$

$(21 + 0.9 + 0.1 \times 11)]$

$= 3.87$ ns $= 4$ ns

4.39 (c)

Logical address = 32 bits

P_1	P_2	d
10 bits	10 bits	12 bits

P_1 = 1st level page table

P_2 = 2nd level page table

d = Page offset

First virtual address given

0 × 0 0 0 0 0 0 0 0

0000 0000 00 00 0000 0000 0000 0000 0000

P_1 P_2 d

Address will be found in first level page table first block.

Second virtual address given

0 × 0 0 4 0 0 0 0 0

0000 0000 01 00 0000 0000 0000 0000 0000

P_1 P_2 d

Address will be found in first level page table second block.

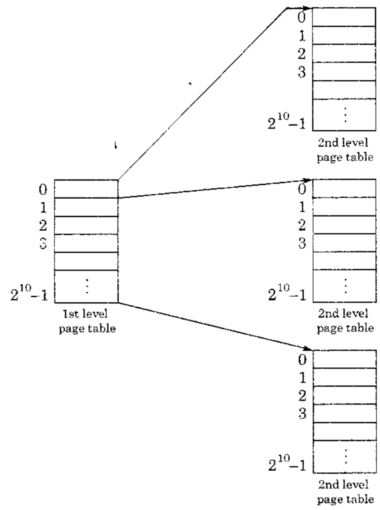
Third virtual address given

0 × F F F F F 0 0 0

1111 1111 11 11 1111 1111 0000 0000 0000

P_1 P_2 d

Address will be found in first level page table last block.



TO execute the process we need to bring all the above 4 page tables in main memory.

$4 \times 2^{10} \times 4$ B = 16 kB

4.40 (d)

The I/O performance of a user program determined by many input and output devices not only by the disk. When we uses FCFS and replace it by SSTF it improves only disk driver performance not the entire I/O performance. So I/O improvement performance of user program is 0%.

4.41 (a)

Minimum number of page frames allocated to a running process is determined by the instruction set architecture.

4.42 (d)

Page fault rate $P = 1/10^4$

Effective memory access time (EMA)

$= 0.9 \times 150$ ns $+ 0.1 \times [150+150] = 165$ ns

Average instruction execution time = CPU time $+ P \times S + (1 - P)$ EMA

$= 100 + 10^{-4} \times 8$ ms $+ (1 - 10^{-4}) \times 2 \times 165$ ns

$= 1229.997 \simeq 1230$ ns

4.43 (b)

When 45 comes, the cache contents are:

4 3 25 8 19 6 16 35

LRU array (first element being least recently used)

[4 3 19 6 25 8 16 35]

So, 45 replaces 4

45 3 25 8 19 6 16 35

[3 19 6 25 8 16 35 45]

Similarly 22 replaces 3 to give

45 22 25 8 19 6 16 35

[19 6 25 8 16 35 45 22]

8 hits in cache

45 22 25 8 19 6 16 35

[19 6 25 16 35 45 22 8]

3 replaces 19

45 22 25 8 3 6 16 35

[6 25 16 35 45 22 8 3]

16 and 25 hits in cache

45 22 25 8 3 6 16 35

[6 35 45 22 8 16 25 3]

Finally 7 replaces 6, which is in block 5.

4.44 (d)

Page table entry must contain bits for representing frames and other bits for storing information like dirty bit, reference bit etc.

Number of frames (number of possible pages)

= Physical memory size/ Page size $= 2^{30}/2^{12}$

$= 2^{18}$

$18 + x = 32$ (PT entry size = 32 bit)

$x = 14$ bits.

4.45 (c)

128 main memory blocks are mapped to 4 sets in cache. So, each set maps 32 blocks each. And in each set there is place for two blocks (2-way set). Now, we have 4 sets meaning 2 index bits. Also, 32 blocks going to one set means 5 tag bits. Now, these 7 bits identify a memory block and tag bits are placed before index bits (otherwise adjacent memory references spatial locality- will hamper cache performance).

So, based on the two index bits (lower 2 bits) blocks will be going to sets as follows:

Set Number	Block Numbers
0	0, 16
1	5, 9
2	-
3	3, 7, 55

Since, each set has only 2 places, 3 will be throw out as its the least recently used block. So, final content of cache will be 0 5 7 9 16 55.

4.46 (b)

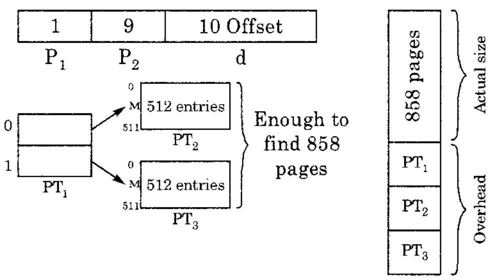
Overhead is considered as memory wasted by maintaining the page tables.

So, we need to find the overhead for every case in the given question.

Case-1: Overhead using two level paging.

Each page size is 1 KB.

Total pages = 195 + 254 + 45 + 364 = 858 KB pages.



Each page table entry = 4 bytes.

Size of page $table_1 = 2 \times 4 = 8$ bytes

Size of page $table_2 = 2^9 \times 4 = 2048$ bytes

Size of page $table_3 = 2^9 \times 4 = 2048$ bytes

$\therefore$ Overhead of paging (P) $= 8 + 2048 + 2048$

$= 4104$ bytes.

Case-2: Overhead using segmentation.

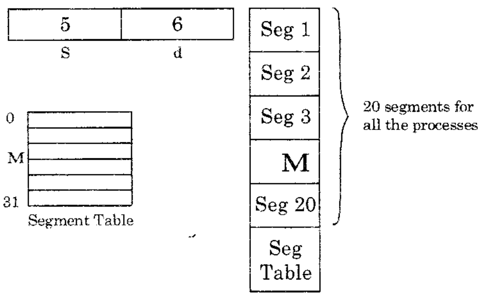
Number of segments = 4 + 5 + 3 + 8 = 20 (5 bits for segment number)

Maximum segment size

$$= \max\left(\frac{195}{4}, \frac{254}{5}, \frac{45}{3}, \frac{364}{8}\right)$$

= max(49,51,15,46) = 51 bytes (helps to find segment offset)

Number of bits required to find the limit (segment offset) = 6



Size of segment table (aprox)

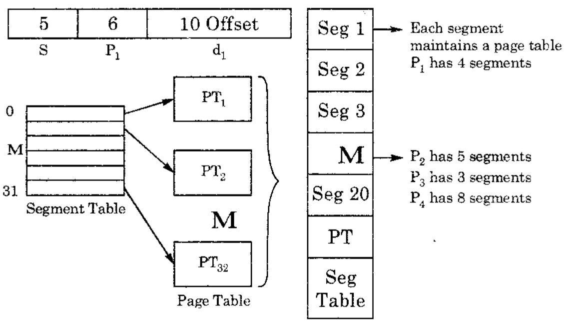
$= 32 \times 8 = 256$ bytes

$\therefore$ Overhead using segmentation (S) = 256 bytes

Case-3: Overhead using segmentation and paging.

Number of segments = 20 $\Rightarrow$ 5 bits required for segment number.

Maximum segment size = 51 bytes $\Rightarrow$ 6 bits required for page offset.



20 page tables are required.

Each segment needed 1 page table.

Size of segment table = 32×8 = 256 bytes.

Size of each page table = 64×4 = 256 bytes.

$\therefore$ Segmentation and paging overhead (T)

= 256 B + 20 × 256 B = 5376 B

S: Simple segmentation takes less overhead.

P: Two level paging takes little more overhead because of two page tables.

T: Segmented paging takes more overhead because we have paging on both segment and segment table.

So, S < P < T.

4.47 (c)

Size of virtual address = 32-bit

Page size = 4 kB

So offset bits = 12

Number of bits used for indexing

= 32 – 12 = 20.

Number of sets = 128/4 = 32 requires 5 bits

Total tag bits = 20 – 5 = 15 bits

4.48 (c)

If the computer system supports 32-bit virtual addresses as well as 23-bit physical addresses. Then address translation from virtual to physical is done by hardware called Memory Management Unit (MMU) is not needed or hardware support for memory management is no longer needed.

4.49 (c)

0100 page fault, addresses till 199 in memory
0200 page fault, addresses till 299 in memory
0430 page fault, addresses till 529 in memory
0499 no page fault
0510 no page fault
0530 page fault, addresses till 629 in memory
0560 no page fault
0120 page fault, addresses till 219 in memory
0220 page fault, addresses till 319 in memory
0240 no page fault
0260 no page fault
0320 page fault, addresses till 419 in memory
0410 no page fault.

4.50 (a)

p(p*300+(1 – p)*100) + (1 – p)*1 = 3

p(300p+100 – 100p) + 1 – p = 3

$200p^2 + 99p - 2 = 0$

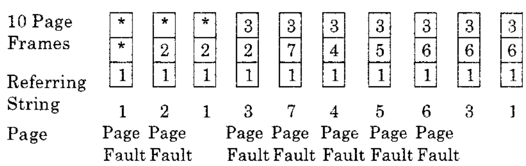
p ≈ 0.0194

4.51 (a)

Number of page frames = 3

Consider the optimal page replacement algorithm as follows:

1, 2, 1, 3, 7, 4, 5, 6, 3, 1



So total number of page faults = 7

4.52 (c)

Consider the LRU page replacement policy

10 Page Frames				3	3	3	5	5	5	1
		2	2	2	7	7	7	6	6	6
	1	1	1	1	1	4	4	4	3	3
Referring String	1	2	1	3	7	4	5	6	3	1
Page	Page Fault	Page Fault		Page Fault	Page Fault	Page Fault	Page Fault	Page Fault	Page Fault	Page Fault

So total number of page fault = 9

Number of page fault in LRU – Number of page fault in optional = 9 – 7 = 2 more page fault.

4.53 (c)

Effective access time = hit ratio * (TLB access time + Memory access time) + miss ratio * (TLB access time + Page table access time + Memory access time)

= 0.9 * (10+50) + 0.1 *(10 + 50 + 50)

= 54 + 11 = 65

4.54 (b)

No. of bits required for first level = 24

No. of bits required for second level = 24

No. of bits required for third level = 24

4.55 (b)

0: Page fault -1,
Pages in memory - 0

4: Page faults - 1,
Pages in memory - 0

8: Page faults - 1,
Pages in memory - 0

20: Page faults - 2,
Pages in memory - 0, 1

24: Page faults - 2,
Pages in memory - 0, 1

36: Page faults - 3,
Pages in memory - 0, 1, 2

44: Page faults - 3,
Pages in memory - 0, 1, 2

12: Page faults - 3,
Pages in memory - 1, 2, 0

68: Page faults - 4,
Pages in memory - 1, 2, 0, 4

72: Page faults - 4,
Pages in memory - 1, 2, 0, 4

80: Page faults - 5,
Pages in memory - 2, 0, 4, 5

84: Page faults - 5,
Pages in memory - 2, 0, 4, 5

28: Page faults - 6,
Pages in memory - 0, 4, 5, 1

32: Page faults - 7,
Pages in memory - 4, 5, 1, 2

88: Page faults - 7,
Pages in memory - 4, 1, 2, 5

92: Page faults - 7,
Pages in memory - 4, 1, 2, 5

4.56 (d)

Dirty and R/W is well known. Reference bit is used in a version of FIFO called second chance (SC) policy, in order to avoid replacement of heavily used page. It is set to one when a page is used heavily and periodically set to 0. Since it is used in a version FIFO which is a page replacement policy, this bit is come under category of page replacement.

Valid bit is not used for page replacement. It is not used in any page replacement policy. It tells the page in the memory is valid or not. If it is valid it is directly used and if it is not then a fresh page is loaded.

So basically it is page initialization. because we are not replacing, it is initializing, we not knocking out somebody, we are filling empty space. so initialization.

4.57 (a)

In Bleady's Anomaly if number of frames are increased then number of page faults increases. This behaviour found only with FIFO.

4.58 (b)

Page frame numbers are most important, virtual page number maynot be stored entirely.

4.59 (a)

FIFO policy for page replacement used.

Access 100 distinct pages by taking some example: 2 3 4 5 6 7 8 9

So by loading it get

$$\begin{array}{|c|}\hline 5 \\ \hline 4 \\ \hline 3 \\ \hline 2 \\ \hline \end{array} = 4 \text{ page fault}, \quad \begin{array}{|c|}\hline 9 \\ \hline 8 \\ \hline 7 \\ \hline 6 \\ \hline \end{array} = 4 \text{ page fault}$$

and now access these page in reverse so

$\underbrace{9\ 8\ 7\ 6}_{\text{So no page fault for these}}$ $\quad$ $\underbrace{5\ 4\ 3\ 2}_{\text{page fault for these}}$

9	5	
8	4	4 pagefault
7	3	
6	2	

So total = 4 + 4 + 4 = 12 pagefault
for 8 pages = 2 × 8 – 4 = 12
so for n pages = 2n – 4
so for 100 pages = 2 × (100) – 4 = 196

4.60 (b)

Effective access time = [(1 – p)* access time when no page fault +p* access time during page fault]

$$= \left[1-\left(\frac{1}{10^6}\right)\right]*20\text{ns}+\left(\frac{1}{10^6}\right)*10\text{ms}$$

$$= \left(\frac{10^6-1}{10^6}\right)*20\text{ns}+\left(\frac{1}{10^6}\right)*10*10^6\text{ns}$$

$\approx$ 30 ns

4.61 (b)

OPTIMAL

		3	3	3	3	3	2	2	2
	2	2	2	4	4	4	4	4	4
1	1	1	1	1	1	1	1	1	1
MISS	MISS	MISS	MISS	MISS	HIT	H	M	H	H

5 miss

FIFO

		3	3	3	3	3	2	2	2
	2	2	2	2	1	1	1	1	1
1	1	1	1	4	4	4	4	4	4
M	M	M	H	M	M	H	M	H	H

6 miss

LRU

		3	3	3	1	1	1	4	4
	2	2	2	2	2	3	3	3	1
1	1	1	1	4	4	4	2	2	2
M	M	M	H	M	M	M	M	M	M

$\Rightarrow$ 9 MISS
So optimal < FIFO < LRU

4.62 (c)

Let page size be x for 3 levels.

$$\frac{2^{46}}{x^3}\times 4^3 = x$$

$\Rightarrow \quad x = 2^{13} = 8$ KB

4.63 (c)

Since each cache line can contain 16,648 blocks, assuming best case 1024 bytes can accomodate in one line hence $2^{13}/2^{10} = 2^3$

$\Rightarrow$ Number of colours needed = 8.

4.64 Sol.

Optimal page replacement policy:

✓	✓	✓	✓	×	×	✓	✓	×	×	✓
		3	4	4	4	4	4	4	4	6
	2	2	2	2	2	2	2	2	2	2
1	1	1	1	1	1	5	3	3	3	3
1	2	3	4	2	1	5	3	2	4	6

$\therefore$ 7 page faults

4.65 (d)

Pages:
1,2,...20,...100, 1,...20,...100, 1,...20,...100

Optimal page replacement:

1	2	3	...	19	20

⇓ 20 page faults after 1 to 20 pages

1	2	3	...	19	100

⇓ 80 page faults after 21 to 100 pages

Therefore for first 100 pages there are 100 page faults

1	2	3	...	19	100

⇓ No page faults from 1 to 19 pages

~~20~~	~~21~~	~~22~~	...	~~19~~	100
~~40~~	~~41~~	~~42~~		~~59~~	
~~60~~	~~61~~	~~62~~		~~79~~	
80	81	82	...	99	100

80 page faults after 20 to 99 pages and then no page fault for 100

Therefore for second 100 pages there are 80 page faults
Similarly, for third 100 pages there will be 80 page faults.
$\therefore$ Total page faults for 300 pages = 100 + 80 + 80 = 260

Most recently used:

1	2	3	...	19	20

⇓ 20 page faults after 1 to 20 pages

1	2	3	...	19	100

⇓ 80 page faults after 21 to 100 pages

Therefore for first 100 pages there are 100 page faults

1	2	3	...	19	100

⇓ No page faults from 1 to 19 pages — Most recently used

20	21	22	...	~~19~~	100
				~~20~~	
				⋮	
20	21	22	...	99	100

80 page faults after 20 to 99 pages and then no page fault for 100

Therefore for second 100 pages there are 80 page faults
Similarly, for third 100 pages there will be 80 page faults.
$\therefore$ Total page faults for 300 pages = 100 + 80 + 80 = 260

4.66 Sol.

✓	✓	✓	✓	×	×	×	✓	✓	×
		6	6	6	6	6	6	7	7
	7	7	7	7	7	7	2	2	2
4	4	4	1	1	1	1	1	1	1
4	7	6	1	7	6	1	2	7	2

∴ 6 page faults will occur using LRU

4.67 Sol.

Effective memory access time

$= 0.6 * (10 + 80) + 0.4 * (10 + 80 + 80) = 122$

4.68 (4)

Page table size = Number of page table entries × Entry size

= Number of pages × 4 bytes

$$= \frac{2^{32}}{2^{12}} \times 4\,\text{bytes} = 4\text{ MB}$$

4.69 (a)

3, 8, 2, 3, 9, 1, 6, 3, 8, 9, 3, 6, 2, 1, 3

	1	1	1	1	1
	9	9	9	9	2
FIFO	2	2	2	8	8
	8	8	3	3	3
	3	6	6	6	6

FIFO 9 page faults.

LRU

				8	2	2	9	1	1	1	8	9	3
			8	2	3	9	1	6	6	8	9	3	9
		3	2	3	9	1	6	3	8	9	3	6	6
	3	8	3	9	1	6	3	8	9	3	6	2	2
3	8	2	9	1	6	3	8	9	3	6	2	1	1

Here LRU also incurs 9 page faults.

Both FCFS and LRU incurs the same number of page faults.

4.70 Sol.

Page size = 8 kB = 2^{13} bytes

⇒ 13 bits required for offset.

Number of sets in cache = 32

⇒ 5 bits required for index (set offset).

40 bit

22 bits for tag	Set offset	Page offset
	5	13

∴ 22 tag bits are required.

4.71 (a)

The best fit algorithm finds the smallest sufficient partition.

200 KB	400 KB	600 KB	500 KB	300 KB	250 KB
	357	491	468		210

∴ The memory partitions 200 KB and 300 KB are not allotted.

4.72 Sol.

Page size = 8 KB ⇒ 13 bit offset

Number of frame bits = 32 – 13 = 19 bits

Page table entry = Valid + Dirty + Permission bits + Translation (frame bits)

= 1 + 1 + 3 + Frame bits

= 5 + 19 = 24 bits

Page table size = 24 Mbytes

Number of pages = Number of page table entries

$$= \frac{24\text{ Mbytes}}{24\text{ Bits}} = 8\text{ M} = 2^{23}\text{ pages}$$

∴ 23 bits needed for page and 13 bits offset

Length of virtual address = 23 + 13 = 36 bits.

4.73 Sol.

Page table size = Number of entries in page table × Page table entry size

$$= \left(\frac{2^{40}}{2^{14}}\right) \times 48\text{ bits} = 2^{26} \times 6\text{ bytes}$$

$= 64\text{ M} \times 6\text{ B} = 384\text{ MB}$

■■■■

5 File System and Device Management

5.1 On receiving an interrupt from an I/O device the CPU:
(a) halts for predetermined time
(b) hands over control of address bus and data bus to the interrupting device.
(c) branches off to the interrupt service routine immediately.
(d) branches off to the interrupt service routine after completion of the current instruction.

[1987 : 2 Marks]

5.2 Disk requests come to disk driver for cylinders 10, 22, 20, 2, 40, 56 and 38 in that order at a time when the disk drive is reading from cylinder 20. The seek time is 6 msec per cylinder. Compute the total seek time if the disk arm scheduling algorithm is
(a) First come first served
(b) Closest cylinder next

[1989 : 2 Marks]

5.3 State whether the following statements are TRUE or FALSE with reason. The data transfer between memory and I/O devices using programmed I/O is faster than interrupt-driven I/O.

[1990 : 2 Marks]

5.4 The root directory of a disk should be placed
(a) at a fixed address in main memory
(b) at a fixed location on the disk
(c) any where on the disk
(d) at a fixed location on the system disk

[1993 : 2 Marks]

5.5 The correct matching for the following pairs is

List-I	List-II
A. DMA I/O	**1.** High speed RAM
B. Cache	**2.** Disk
C. Interrupt I/O	**3.** Printer
D. Condition Code Register	**4.** ALU

Codes:
(a) A-4 B-3 C-1 D-2
(b) A-2 B-1 C-3 D-4
(c) A-4 B-3 C-2 D-1
(d) A-2 B-3 C-4 D-1 **[1997 : 1 Mark]**

5.6 The correct matching for the following pairs is

List-I	List-I
A. Disk scheduling	**1.** Round robin
B. Batch processing	**2.** SCAN
C. Time sharing	**3.** LIFO
D. Interrupt processing	**4.** FIFO

Codes:
(a) A-3, B-4, C-2, D-1
(b) A-4, B-3, C-2, D-1
(c) A-2, B-4, C-1, D-3
(d) A-3, B-4, C-3, D-2

[1997 : 1 Mark]

5.7 I/O redirection
(a) implies changing the name of a file
(b) can be employed to use an existing file as input file for a program
(c) implies connecting 2 programs through a pipe
(d) None of the above **[1997 : 1 Mark]**

5.8 When an interrupt occurs, an operating system
(a) ignores the interrupt
(b) always changes state of interrupted process after processing the interrupt
(c) always resumes execution of interrupted process after processing the interrupt
(d) may change state of interrupted process to 'blocked' and schedule another process

[1997 : 1 Mark]

5.9 Which of the following devices should get higher priority in assigning interrupts?
(a) Hard disk (b) Printer
(c) Keyboard (d) Floppy disk

[1998 : 1 Mark]

5.10 Which of the following is true?
(a) Unless enabled, a CPU will not be able to process interrupts.
(b) Loop instructions cannot be interrupted till they complete.
(c) A processor checks for interrupts before executing a new instruction.
(d) Only level triggered interrupts are possible on microprocessors

[1998 : 1 Mark]

5.11 Listed below are some operating system abstractions (in the left column) and the hardware components (in the right column)?

A. Thread	**1.** Interrupt
B. Virtual address space	**2.** Memory
C. File system	**3.** CPU
D. Signal	**4.** Disk

[1999 : 1 Mark]

5.12 Which of the following disk scheduling strategies is likely to give the best throughput?
(a) Farthest cylinder next
(b) Nearest cylinder next
(c) First come first served
(d) Elevator algorithm

[1999 : 1 Mark]

5.13 Which of the following requires a device driver?
(a) Register (b) Cache
(c) Main memory (d) Disk

[2001 : 1 Mark]

5.14 In the index allocation scheme of blocks to a file, the maximum possible size of the file depends on
(a) the size of the blocks, and the size of the address of the blocks.
(b) the number of blocks used for the index, and the size of the blocks
(c) the size of the blocks, the number of blocks used for the index, and the size of the address of the blocks
(d) None of the above **[2002 : 2 Marks]**

5.15 Using a larger block size in a fixed block size file system leads to
(a) better disk throughput but poorer disk space utilization
(b) better disk throughput and better disk space utilization
(c) poorer disk throughput but better disk space utilization
(d) poorer disk throughput and poorer disk space utilization

[2003 : 1 Mark]

5.16 A Unix-style I-node has 10 direct pointers and one single, one double and one triple indirect pointers. Disk block size is 1 Kbyte, disk block address is 32 bits, and 48-bit integers are used. What is the maximum possible file size?
(a) 2^{24} bytes (b) 2^{32} bytes
(c) 2^{34} bytes (d) 2^{48} bytes

[2004 : 2 Marks]

5.17 A disk has 200 tracks (numbered 0 through 199). At a given time, it was servicing the request of reading data from track 120, and at the previous request, service was for track 90. The pending requests (in order of their arrival) are for track numbers.

30 70 115 130 110 80 20 25.

How many times will the head change its direction for the disk scheduling policies SSTF (Shortest Seek Time First) and FCFS (First Come Fist Serve)?
(a) 2 and 3 (b) 3 and 3
(c) 3 and 4 (d) 4 and 4

[2004 : 2 Marks]

5.18 (b) In a particular Unix OS, each data block is of size 1024 bytes, each node has 10 direct data block addresses and three additional addresses: one for single indirect block, one for double indirect block and one for triple indirect block. Also, each block can contain addresses for 128 blocks. Which one of the following is approximately the maximum size of a file in the file system?
(a) 512 MB (b) 2 GB
(c) 8 GB (d) 16 GB

[2004 : 2 Marks]

5.19 A student wishes to create symbolic links in a computer system running Unix. Three text files named "file1", "file2" and "file3" exist in her current working directory, and the student has read and write permissions for all three files. Assume that file1 contains information about her hobbies, file2 contains information about her friends and file3 contains information about her courses. The student executes the following sequence of commands from her current working directory

```
ln -s file1 file2
ln -s file2 file3
```

Which of the following types of information would be lost from her file system?
I. Hobbies II. Friends
III. Courses
(a) (I) and (II) only (b) (II) and (III) only
(c) (II) only (d) (I) and (III) only

[2005 : 1 Mark]

5.20 The shell command

find -name password -print

is executed in /etc directory of a computer system running Unix. Which of the following shell commands will give the same information as the above command when executed in the same directory?

(a) ls passwd
(b) cat passwd
(c) grep name passwd
(d) grep print passwd **[2005 : 1 Mark]**

5.21 In a computer system, four files of size 11050 bytes, 4990 bytes, 5170 bytes and 12640 bytes need to be stored. For storing these files on disk, we can use either 100 byte disk blocks or 200 byte disk blocks (but can't mix block sizes). For each block used to store a file, 4 bytes of bookkeeping information also needs to be stored on the disk. Thus, the total space used to store a file is the sum of the space taken to store the file and the space taken to store the book keeping information for the blocks allocated for storing the file. A disk block can store either bookkeeping information for a file or data from a file, but not both.

What is the total space required for storing the files using 100 byte disk blocks and 200 byte disk blocks respectively?

(a) 35400 and 35800 bytes
(b) 35800 and 35400 bytes
(c) 35600 and 35400 bytes
(d) 35400 and 35600 bytes

[2005 : 2 Marks]

5.22 In the working-set strategy, which of the following is done by the operating system to prevent thrashing?

I. It initiates another process if there are enough extra frames.
II. It selects a process to suspend if the sum of the sizes of the working-sets.
III. Execeeds the total number of available frames.

(a) I only (b) II only
(c) Neither I nor II (d) Both I and II

[2006 : 1 Mark]

Directions for Question 5.23 to 5.24:

The head of a hard disk serves requests following the shortest seek time first (SSTF) policy. The head is initially positioned at track number 180.

5.23 Which of the request sets will cause the head to change its direction after servicing every request assuming that the head does not change direction if there is a tie in SSTF and all the requests arrive before the servicing starts?

(a) 11, 139, 170, 178, 181, 184, 201, 265
(b) 10, 138, 170, 178, 181, 185, 201, 265
(c) 10, 139, 169, 178, 181, 184, 201, 265
(d) 10, 138, 170, 178, 181, 185, 200, 265

[2007 : 2 Marks]

5.24 What is the maximum cardinality of the request set, so that the head changes its direction after servicing every request if the total number of tracks are 2048 and the head can start from any track?

(a) 9 (b) 10
(c) 11 (d) 12

[2007 : 2 Marks]

5.25 A virtual memory system uses First In First Out (FIFO) page replacement policy and allocates a fixed number of frames to a process. Consider the following statements:

P : Increasing the number of page frames allocated to a process sometimes increases the page fault rate.

Q : Some programs do not exhibit locality of reference.

Which one of the following is TRUE?

(a) Both P and Q are true, and Q is the reason for P
(b) Both P and Q are true, but Q is not the reason for P
(c) P is false, but Q is true
(d) Both P and Q are false

[2007 : 2 Marks]

5.26 The data block of a very large file in the Unix file system are allocated using

(a) Contiguous allocation
(b) Linked allocation
(c) indexed allocation
(d) an extension of indexed allocation

[2008 : 1 Mark]

5.27 Consider a disk system with 100 cylinders. The requests to access the cylinders occur in following sequence:

4, 34, 10, 7, 19, 73, 2, 15, 6, 20

Assuming that the head is currently at cylinder 50, what is the time taken to satisfy all requests if it takes 1 ms to move from one cylinder to adjacent one and shortest seek time first policy is used?

b

(b)

(a) 95 ms (b) 119 ms
(c) 233 ms (d) 276 ms

[2009 : 2 Marks]

5.28 A multilevel page table is preferred in comparison to a single level page table for translating virtual address to physical address because

(a) it reduces the memory access time to read or write a memory location
(b) it helps to reduce the size of page table needed to implement the virtual address space of a process
(c) it is required by the translation look a side buffer
(d) it helps to reduce the number of page faults in page replacement algorithms.

[2009 : 2 Marks]

5.29 A file system with 300 GByte disk uses a file descriptor with 8 direct block addresses, I indirect block address and 1 doubly indirect block address. The size of each disk block is 128 Bytes and the size of each disk block address is 8 Bytes. The maximum possible file size in this file system is

(a) 3 KBytes
(b) 35 KBytes
(c) 280 KBytes
(d) dependent on the size of the disk

[2012 : 2 Marks]

5.30 Suppose a disk has 201 cylinders, numbered from 0 to 200. At some time the disk arm is at cylinder 100, and there is a queue of disk access requests for cylinders 30, 85, 90, 100, 105, 110, 135 and 145. If Shortest-Seek Time First (SSTF) is being used for scheduling the disk access, the request for cylinder 90 is serviced after servicing _______ number of requests.

[2014 (Set-1) : 1 Mark]

5.31 A FAT (file allocation table) based file system is being used and the total overhead of each entry in the FAT is 4 bytes in size. Given a 100×10^6 bytes disk on which the file system is stored and data block size is 10^3 bytes, the maximum size of a file that can be stored on this disk in units of 10^6 bytes is _____.

[2014 (Set-2) : 1 Mark]

5.32 Suppose the following disk request sequence (track numbers) for a disk with 100 tracks is given: 45, 20, 90, 10, 50, 60, 80, 25, 70. Assume that the initial position of the R/W head is on track 50. The additional distance that will be traversed by the R/W head when the Shortest Seek Time First (SSTF) algorithm is used compared to the SCAN (Elevator) algorithm (assuming that SCAN algorithm moves towards 100 when it starts execution) is ________ tracks.

[2015 (Set-1) : 2 Marks]

5.33 Consider a disk queue with requests for I/O to blocks on cylinders 47, 38, 121, 191, 87, 11, 92, 10. The C-LOOK scheduling algorithm is used. The head is initially at cylinder number 63, moving towards larger cylinder numbers on its servicing pass. The cylinders are numbered from 0 to 199. The total head movement (in number of cylinders) incurred while servicing these requests is _______.

[2016 (Set-1) : 2 Marks]

5.34 Consider a computer system with ten physical page frames. The system is provided with an access sequence $(a_1, a_2, ..., a_{20}, a_1, a_2, \ldots, a_{20})$, where each a_i is a distinct virtual page number. The difference in the number of page faults between the last-in-first-out page replacement policy and the optimal page replacement policy is________.

[2016 (Set-1) : 2 Marks]

5.35 In which one of the following page replacement algorithms it is possible for the page fault rate to increase even when the number of allocated frames increases?

(a) LRU (Least Recently Used)
(b) OPT (Optimal Page Replacement)
(c) MRU (Most Recently Used)
(d) FIFO (First In First Out)

[2016 (Set-2) : 1 Mark]

■■■■

Answers File System and Device Management

5.1	(d)	5.4	(d)	5.5	(b)	5.6	(c)	5.7	(b)	5.8	(d)	5.9	(a)	5.10	(a, c)	5.12	(b)
5.13	(d)	5.14	(c)	5.15	(a)	5.16	(c)	5.17	(c)	5.18	(b)	5.19	(b)	5.20	(a)	5.21	(c)
5.22	(b)	5.23	(b)	5.24	(b)	5.25	(b)	5.26	(d)	5.27	(b)	5.28	(b)	5.29	(b)	5.35	(d)

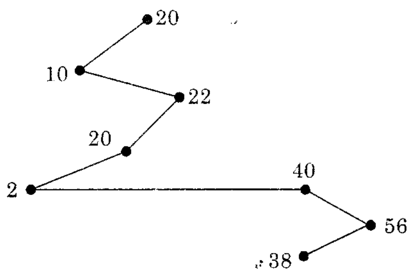
Explanations File System and Device Management

5.1 (d)

After getting the interrupt CPU first executes current instruction then services the interrupt.

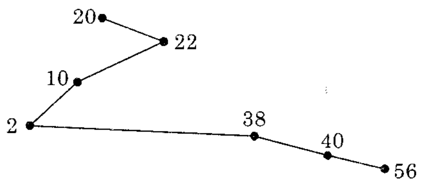
5.2 Sol.

(a) FCFS



$6 \times (10 + 12 + 2 + 18 + 38 + 16 + 18) = 684$

(b) Closest cylinder next



$6 \times (2 + 12 + 8 + 36 + 2 + 16) = 456$

5.3 Sol.

Statement is false, because in programmed I/O the processor has to wait a long time for the I/O module of concern to be reads for either reception or transmission of data; while in interrupt driven I/O CPU is in waiting state when interrupt comes. So takes less time to transfer the data and in interrupt driven I/O, the interface between memory and I/O works like a CPU.

5.4 (d)

The root directory of a disk is placed at a fixed location on the system disk.

5.5 (b)

DMA I/O → Disk
Cache → High speed RAM
Interrupt I/O → Printer
Condition code register → ALU

5.6 (c)

Disk scheduling → SCAN
Batch processing → FIFO (Queue)
Time sharing → Round Robin (uses time slice)
Interrupt processing → LIFO (stack)

5.7 (b)

I/O redirection helps to use an existing file as input file for a program.

5.8 (d)

When an interrupt occurs, may change state of interrupted process to 'blocked' and schedule another process.

5.9 (a)

Hard disk should be higher priority compared to printer, keyboard, and floppy disk. Because all user programs and OS stored at hard disk.

5.10 (a,c)

(a) When interrupt is enabled, a CPU will be able to process the interrupt.

(b) CPU checks for interrupts before executing a new instruction.

5.11 Sol.

Thread → CPU
Virtual address space → Memory
File system → Disk
Signal → Interrupt

5.12 (b)

Shortest seek time next or nearest cylinder next gives the best throughput.

5.13 (d)

Disk requires a device driver in computer system.

5.14 (c)

$\frac{\text{DB size}}{\text{DBA}}$ = Number of DBA's possible in one disk block.

5.15 (a)

In a fixed block size file system, larger block size gives better disk throughput but poorer disk space utilization.

5.16 (c)

The number of disk block pointers that will be fit in one block

$$= \frac{2^{10}\text{ byte}}{32\text{ bit}} = \frac{2^{10}\text{ byte}}{4\text{ byte}} = 256$$

Maximum file size due to single indirection pointer

$= 256 \times 1 = 256$ KB

Maximum file size due to direct pointer

$= 10 \times 1 = 10$ KB

Maximum file size due to double indirection pointer

$= 256 \times 256 \times 1$ KB

Maximum file size due to triple indirection pointer

$= 256 \times 256 \times 256 \times 1$ KB

So the maximum file size

$= 256 \times 256 \times 256 \times 1$ KB

$= 2^8 \times 2^8 \times 2^8 \times 2^{10}$ bytes

$= 2^{24} \times 2^{10}$ bytes

$= 2^{34}$ bytes

5.17 (c)

SSTF: Direction changes at 120,110,130

FCFS: Direction changes at 120,30,130,20.

5.18 (b)

Maximum file size = [direct DBA + Number of (Data block size/DBA) + Number of (Data block size/DBA)^2 + Number of (Data block size/DBA)^3]* Data block size.

Data Block size = 1024 byte

(Disk Block size/DBA) = Number of Disk Block address Store inside One Block.

Maximum file size = [10+1*(128) + 1*(128*128*) + 1*(128*128*128)]*1024 Byte

= Approx 2 GB.

5.19 (b)

As ln -s is symbolic link. In this case File3 ⇒ File2 ⇒ File1 ⇒ Hobbies(actual data). So File2 and File3 content are lost.

5.21 (c)

For 100 bytes block:

11050 = 111 blocks requiring 111 * 4 = 444 bytes of bookkeeping info which requires another 5 disk blocks. So, totally 111 + 5 = 116 disk blocks.

Similarly,

4990 = 50 + (50*4)/100 = 52

5170 = 52 + (52*4)/100 = 55

12640 = 127 + (127*4/100) = 133

356 × 100 = 35600 bytes

For 200 bytes block:

56 + (56*4/200) = 58

25 + (25*4/200) = 26

26 + (26*4/200) = 27

64 + (64*4/200) = 66

177 × 200 = 35400.

5.22 (b)

If there are enough extra frames, another process can be initiated. If the sum of the working-set sizes increases, exceeding the total number of available frames, the operating system selects a process to suspend. The process's pages are written out (swapped), and its frames are reallocated to other processes. The suspended process can be restarted later."

5.23 (b)

When the head starts from 180... it seeks the nearest track which is 181. Then from 181 it seeks the nearest one which is 178 and 184. But the difference in both from 181 is same and as given in the question... if there is a tie then the head won't change its direction, and therefore to change the direction we need to consider 178 and thus we can eliminate option A and C. Coming next to option B and D.. following the above procedure you'll see that option D is eliminated on similar ground and thus you can say option B is correct.

5.24 (b)

Since it uses horizontal micro-programmed that requires 1 bit / control signal.
For 125 control signals we need 125 bits.
Total number of micro-operation instructions are 140*7 = 980 that requires 10 bit.

5.25 (b)

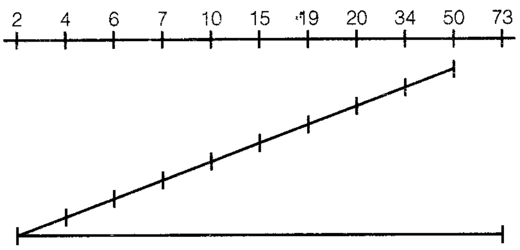
P and Q both are true but Q is not the reason for P because increasing the number of page frames allocated to a process sometimes increases the page fault rate or it is not concern with replacement policy.

5.26 (d)

The data blocks of a very large file in the unix file system are allocated using an extension of indexed allocation or EXT2 file system.

5.27 (b)

Head is currently at cylinder 50,



$$\begin{aligned} 50-34 &= 16 \\ 34-20 &= 14 \\ 20-19 &= 1 \\ 19-15 &= 4 \\ 15-10 &= 5 \\ 10-7 &= 3 \\ 7-6 &= 1 \\ 6-4 &= 2 \\ 4-2 &= 2 \\ 2-73 &= 71 \\ \text{Total moves} &= 16+14+1+4+5+3 \\ &\quad +1+2+2+71 \\ &= 119 \end{aligned}$$

5.28 (b)

Multilevel page table in preferred to reduce the size of page table needed to implement the virtual address space of a process.

5.29 (b)

Maximum possible size

$$= \left[\begin{pmatrix}\text{address pointed by} \\ \text{doubly indrect block}\end{pmatrix}^2\right.$$

$$+\begin{pmatrix}\text{addres pointed by} \\ \text{single address}\end{pmatrix}$$

+ Address points by single direct address]
× [block size]

$$= \left[\left(\frac{128}{8}\right)^2 + \left(\frac{128}{8}\right) + 8\right] \times 128 \text{ B}$$

$$= [2^8 + 2^4 + 2^3] \times 128 \text{ B}$$
$$= 32 \text{ kB} + 2 \text{ kB} + 1 \text{ kB}$$
$$= 35 \text{ kB}$$

5.30 Sol.

$$100 \longrightarrow 105 \longrightarrow 110 \longrightarrow 90$$

90 is serviced after servicing the 3 requests.

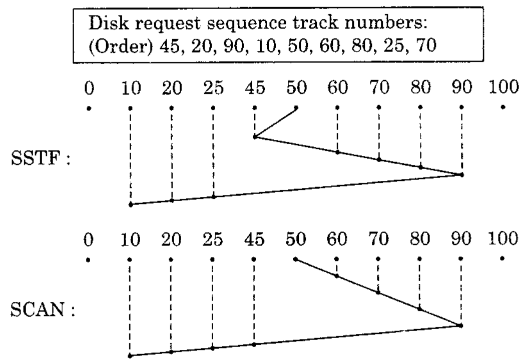
5.31 Sol.

Overhead of each entry in FAT = 4 bytes
Block size = 10^3 bytes
Total size for each entry = 1004 bytes
Number of entries in FAT

$$= \frac{100 \times 10^6}{1004} = 0.099601$$

Maximum size of a file
$$= 0.099601 \times 10^3 \text{ bytes}$$
$$= 99.601 \times 10^6 \text{ bytes}$$

5.32 Sol.



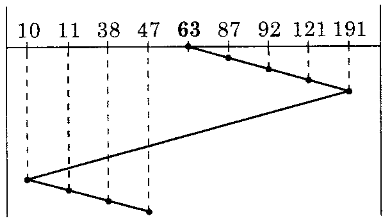
SSTF distance = (50 – 45)+(90 – 45)+(90 – 10)
= 5 + 45 + 80 = 130

SCAN distance = (90 – 50) + (90 – 10)
= 40 + 80 = 120

∴ SSTF distance – SCAN distance
= 130 – 120 = 10

5.33 Sol.

C-Look



(84 – 63) + (92 – 87) + (121 – 92) + (191 – 121)
+ (191 – 10) + (11 – 10) + (38 – 11) + (47 – 38)
= 24 + 5 + 29 + 70 + 181 + 1 + 27 + 9 = 346

5.34 Sol.

This question is little tricks, but not difficult. You can solve the problem by taking smaller example with smaller values.

Example: 1, 2, 3, 4, 1, 2, 3, 4 with '2' frames.

LIFO: 1, 2, 3, 4, 1, 2, 3, 4

	$\cancel{2}$	$\cancel{3}$	$\cancel{4}$	$\cancel{4}$	$\cancel{2}$	3	4
1	1	1	1	1	1	1	1
F	F	F	F		F	F	F

Total page faults = 7

Optimal: 1, 2, 3, 4, 1, 2, 3, 4

	$\cancel{2}$	$\cancel{3}$	4	4	4	4	4
1	1	1	1	$\cancel{1}$	$\cancel{2}$	3	3
F	F	F	F		F	F	

Total page faults = 6

7 – 6 = 1.

5.35 (d)

Because of belady's anamoly, it will happen in FIFO phase replacement algorithm.

■■■■

Unit ▪ IX

Databases

▪ Contents

UNIT IX

Databases

Syllabus : ER-model, relational model (relational algebra, tuple calculus), Database design (integrity constraints, normal forms), Query languages (SQL). File structures (sequential files, indexing, B and B+ trees), Transactions and concurrency control.

Analysis of Previous GATE Papers

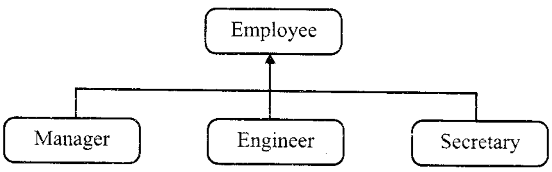
Exam Year	1 Mark Ques.	2 Marks Ques.	3 Marks Ques.	5 Marks Ques.	Total Marks
1990	–	–			–
1991	–	–			–
1992	–	–			–
1993	–	–			–
1994	3	–			**3**
1995	–	1			**2**
1996	–	1			**2**
1997	–	2			**4**
1998	2	3			**8**
1999	4	3			**10**
2000	2	3			**8**
2001	2	2			**6**
2002	2	3			**8**
2003	2	3			**8**
2004	2	5			**12**
2005	3	4			**11**
2006	1	4			**9**

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2007	–	6	**12**
2008	1	5	**11**
2009	–	5	**10**
2010	2	2	**6**
2011	–	2	**4**
2012	2	5	**12**
2013	–	3	**6**
2014 Set-1	2	3	**8**
2014 Set-2	2	3	**8**
2014 Set-3	2	3	**8**
2015 Set-1	2	2	**6**
2015 Set-2	2	2	**6**
2015 Set-3	2	2	**6**
2016 Set-1	4	1	**6**
2016 Set-2	2	2	**6**

1 ER-Model

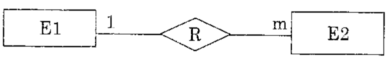
1.1 It is desired to design an object-oriented employee record system for a company. Each employee has a name, unique id and salary. Employees belong to different categories and their salary is determined by their category. The functions getName, getId and computeSalary are required. Given the class hierarchy below, possible locations for these functions are:

(i) getId is implemented in the superclass
(ii) getId is implemented in the subclass
(iii) getName is an abstract function in the superclass
(iv) getName is implemented in the superclass
(v) getName is implemented in the subclass
(vi) getSalary is an abstract function in the superclass
(vii) getSalary is implemented in the superclass
(viii) getSalary is implemented in the subclass



Choose the best design
(a) (i), (iv), (vi), (viii)
(b) (i), (iv), (vii)
(c) (i), (iii), (v), (vi), (viii)
(d) (ii), (v), (viii) **[2004 : 2 Marks]**

1.2 Consider the following entity relationship diagram (ERD), where two entities E1 and E2 have a relation R of cardinality 1 : m

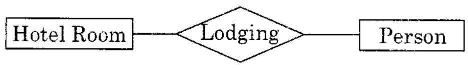


The attributes of E1 are A11, A12 and A13 where A11 is the key attribute. The attributes of E2 are A21, A22 and A23 where A21 is the key attribute and A23 is a multi-valued attribute. Relation R does not have any attribute. A relational database containing minimum number of tables with each table satisfying the requirements of the third normal form (3NF) is designed from the above ERD. The number of tables in the database is

(a) 2 (b) 3
(c) 5 (d) 4

[2004 : 2 Marks]

1.3 Consider the entities 'hotel room', and 'person' with a many to many relationship 'lodging' as shown below:



If we wish to store information about the rent payment to be made by person (s) occupying different hotel rooms, then this information should appear as an attribute of

(a) Person (b) Hotel Room
(c) Lodging (d) None of these

[2005 : 1 Mark]

1.4 Let E_1 and E_2 be two entities in an E/R diagram with simple single-valued attributes. R_1 and R_2 are two relationships between E_1 and E_2, where R_1 is one-to-many and R_2 is many-to-many. R_1 and R_2 do not have any attributes of their own. What is the minimum number of tables required to represent this situation in the relational model?

(a) 2 (b) 3
(c) 4 (d) 5

[2005 : 2 Marks]

1.5 The following table has two attributes A and C where A is the primary key and C is the foreign key referencing A with on-delete cascade.

A	C
2	4
3	4
4	3
5	2
7	2
9	5
6	4

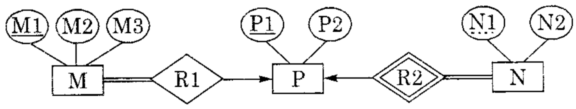
The set of all tuples that must be additionally deleted to preserve referential integrity when the tuple (2, 4) is deleted is:

(a) (3, 4) and (6, 4)
(b) (5, 2) and (7, 2)
(c) (5, 2) (7, 2) and (9, 5)
(d) 1

[2005 : 2 Marks]

Common Data for Q.1.6 & Q.1.7

Consider the following ER diagram



1.6 The minimum number of tables needed to represent M, N, P, R1, R2 is

(a) 2 (b) 3
(c) 4 (d) 5

[2008 : 2 Marks]

1.7 Which of the following is a correct attribute set for one of the tables for the correct answer to the above question?

(a) {M1, M2, M3, P1}
(b) {M1, P1, N1, N2}
(c) {M1, P1, N1}
(d) {M1, P1}

[2008 : 2 Marks]

1.8 Given the basic ER and relational models, which of the following is INCORRECT?

(a) An attribute of an entity can have more than one value
(b) An attribute of an entity can be composite
(c) In a row of a relational table, an attribute can have more than one value
(d) In a row of a relational table, an attribute can have exactly one value or a NULL value

[2012 : 1 Mark]

1.9 Consider an Entity-Relationship (ER) model in which entity sets E_1 and E_2 are connected by an m : n relationship R_{12}. E_1 and E_3 are connected by a 1 : n (1 on the side of E_1 and n on the side of E_3) relationship R_{13}.

E_1 has two single-valued attributes a_{11} and a_{12} of which a_{11} is the key attribute. E_2 has two single-valued attributes a_{21} and a_{22} of which a_{21} is the key attribute. E_3 has two single-valued attributes a_{31} and a_{32} of which a_{31} is the key attribute. The relationships do not have any attributes.

If a relational model is derived from the above ER model, then the minimum number of relations that would be generated if all the relations are in 3NF is ________.

[2015 (Set-1) : 2 Marks]

■■■■

Answers ER-Model

1.1	(a)	1.2	(b)	1.3	(c)	1.4	(b)	1.5	(c)	1.6	(b)	1.7	(a)	1.8	(c)

Explanations ER-Model

1.1 (a)

Superclass hide the information to the outside world so each id of an employee must be implemented in superclass.

Name is also an important attribute of employee so must be implemented in the superclass. Salary of an employee is differ by category and department so it is a function must be implemented in subclass.

1.2 (b)

One table for E1, two tables for E2(A21, A22 and A21, A23). We need to make a separate table for multi-valued attribute to satisfy minimum 1NF condition.

Now, Relation table can be merged with (A21,A22). Tables are:

(a) E1 (A11, A12, A13)
(b) E21 (A11, A21, A22) and
(c) E22 (A21, A23).

Number of tables = 3.

1.3 (c)

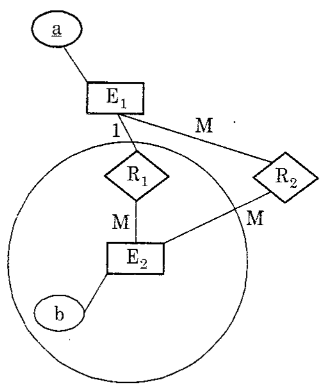
It is many to many. Rent cannot be an attribute of room or person entities alone.

If depending on number of persons sharing a room the rent for each person for the room will be different.

Otherwise rent can be attribute of room.

Hence attribute is Lodging.

1.4 (b)



R_1 key is b, R_2 key is ab, E_1 key is a and E_2 key is b.

R_1 and E_2 can merge because keys are same.

$\therefore$ E_1, R_1E_2 and R_2 tables are required.

1.5 (c)

If (2, 4) is deleted then 2 is the primary key but in (5, 2) and (7, 2), 2 is the foreign key so these must be deleted. The primary key for (5, 2) and (7, 2) is 5 and 7 respectively but in (9, 5), 5 is the foreign key so it is also deleted.

1.6 (b)

The minimum number of tables needed for given ER diagram to represent M, N, P, R1, R2 is three.

1.7 (a)

The correct attribute set is {M1, M2, M3, P1}

1.8 (c)

Option 'A' is correct as multivalued attribute e.g. phone no (attribute)

Option 'B' is also correct.

Option 'D' 'Null' values are allowed in a row of relational database (Null value arc constraints only for primary keys.

As in 1NF also we remove multivalued attribute.

$\therefore$ Option 'C' is incorrect.

1.9 Sol.

For E_1 relation: $\langle a_{11}, a_{12}\rangle$
For E_2 relation: $\langle a_{21}, a_{22}\rangle$
For E_3 and R_{13} relationship: $\langle a_{31}, a_{32}, a_{11}\rangle$
For R_{12} : $\langle a_{11}, a_{21}\rangle$

■■■■

2 Database Design: Functional Dependencies and Normalization

2.1 State True or False with reason

There is always a decomposition into Boyce-Codd normal form (BCNF) that is lossless and dependency preserving.

[1994 : 1 Mark]

2.2 (a) Consider the relation scheme R(A, B, C) with the following functional dependencies:

$AB \rightarrow C$

$C \rightarrow A$

Show that the scheme R is the Third Normal Form (3NF) but not in Boyce- Code Normal Form (BCNF).

(b) Determine the minimal keys of relation R.

[1995 : 2 Marks]

2.3 For a database relation R(a, b, c, d), where the domains of a, b, c, d include only atomic values, only the following functional dependencies and those that can be inferred from them hold:

$a \rightarrow c$

$b \rightarrow d$

This relation is

(a) in first normal form but not in second normal form

(b) in second normal form but not in third normal form

(c) in third normal form

(d) None of the above

[1997 : 2 Marks]

2.4 Let R (a, b, c) and S(d, e, f) be two relations in which d is the foreign key of S that refers to the primary key of R. Consider the following four operations on R and S

(i) Insert into R (ii) Insert into S

(iii) Delete from R (iv) Delete from S

Which of the following is true about the referential integrity constraint above?

(a) None of (i), (ii), (iii) or (iv) can cause its violation

(b) All of (i), (ii), (iii) and (iv) can cause its violation

(c) Both (i) and (iv) can cause its violation

(d) Both (ii) and (iii) can cause its violation

[1997 : 2 Marks]

2.5 Which normal form is considered adequate for normal relational database design?

(a) 2 NF (b) 5 NF

(c) 4 NF (d) 3 NF

[1998 : 1 Mark]

2.6 There are five records in a database.

Name	Age	Occupation	Category
Rama	27	CON	A
Abdul	22	ENG	A
Jeniffer	28	DOC	B
Maya	32	SER	D
Dev	24	MUS	C

There is an index file associated with this and it contains the values 1, 3, 2, 5 and 4. Which one of the fields is the index built from?

(a) Age (b) Name

(c) Occupation (d) Category

[1998 : 1 Mark]

2.7 Consider the following database relations containing the attributes

Book_id

Subject_Category_of_book

Name_of_Author

Nationality_of_Author

With book_id as the primary key.

(a) What is the highest normal form satisfied by this relation?

(b) Suppose the attributes Book_title and Author_address are added to the relation, and the primary key is changed to {Name_of_Author, Book_title}, what will be the highest normal form satisfied by the relation?

[1998 : 2 Marks]

2.8 Let R = (A, B, C, D, E, F) be a relation scheme with the following dependencies $C \rightarrow F$, $E \rightarrow A$, $EC \rightarrow D$, $A \rightarrow B$. Which of the following is a key for R?

(a) CD (b) EC
(c) AE (d) AC

[1999 : 1 Mark]

2.9 Consider the schema R = (S T U V) and the dependencies $S \rightarrow T$, $T \rightarrow U$, $U \rightarrow V$ and $V \rightarrow S$. Let R = (R1 and R2) be a decomposition such that $R1 \cap R2 = \phi$. The decomposition is

(a) not in 2NF
(b) in 2NF but not 3NF
(c) in 3NF but not in 2NF
(d) in both 2NF and 3NF

[1999 : 2 Marks]

2.10 Given the following relation instance

X	Y	Z
1	4	2
1	5	3
1	6	3
3	2	2

Which of the following functional dependencies are satisfied by the instance?

(a) $XY \rightarrow Z$ and $Z \rightarrow Y$
(b) $YZ \rightarrow X$ and $Y \rightarrow Z$
(c) $YZ \rightarrow X$ and $X \rightarrow Z$
(d) $XZ \rightarrow Y$ and $Y \rightarrow X$

[2000 : 2 Marks]

2.11 Consider a schema R(A,B,C,D) and functional dependencies $A \rightarrow B$ and $C \rightarrow D$. Then the decomposition of R into $R_1(AB)$ and $R_2(CD)$ is

(a) dependency preserving and lossless join
(b) lossless join but not dependency preserving
(c) dependency preserving but not lossless join
(d) not dependency preserving and not lossless join

[2001 : 1 Mark]

2.12 R(A, B, C, D) is a relation. Which of the following does not have a lossless join, dependency preserving BCNF decomposition?

(a) $A \rightarrow B$, $B \rightarrow CD$
(b) $A \rightarrow B$, $B \rightarrow C$, $C \rightarrow D$
(c) $AB \rightarrow C$, $C \rightarrow AD$
(d) $A \rightarrow BCD$

[2001 : 2 Marks]

2.13 Relation R with an associated set of functional dependencies, F, is decomposed into BCNF. The redundancy (arising out of functional dependencies) in the resulting set of relation is

(a) Zero
(b) More than zero but less than that of an equivalent 3NF decomposition
(c) Proportional to the size of F^+
(d) Indeterminate

[2002 : 1 Mark]

2.14 Relation R is decomposed using a set of functional dependencies, F, and relation S is decomposed using another set of functional dependencies, G. One decomposition is definitely BCNF, the other is definitely 3 NF, but it is not known which is which. To make a guaranteed identification, which one of the following tests should be used on the decompositions? (Assume that the closures of F and G are available).

(a) Dependency-preservation
(b) Lossless-join
(c) BCNF definition
(d) 3 NF definition

[2002 : 2 Marks]

2.15 From the following instance of a relation scheme R(A, B, C), we can conclude that:

A	B	C
1	1	1
1	1	0
2	3	2
2	3	2

(a) A functionally determines B and B functionally determines C
(b) A functionally determines B and B does not functionally determines C
(c) B does not functionally determines C
(d) A does not functionally determines B and B does not functionally determines C

[2002 : 2 Marks]

2.16 Consider the following functional dependencies in a database:

Data_of_Birth $\rightarrow$ Age
Age $\rightarrow$ Eligibility
Name $\rightarrow$ Roll_number
Roll_number $\rightarrow$ Name
Course_number $\rightarrow$ Course_name

Course_number → Instructor

(Roll_number; Course_number) → Grade

The relation (Roll_number; Name, Date_of_birth, Age) is

(a) in second normal form but not in third normal form

(b) in third normal form but not in BCNF

(c) in BCNF

(d) in none of the above

[2003 : 2 Marks]

2.17 Consider the following relation schema pertaining to a students database:

Student (rollno, name, address)

Enroll (rollno, courseno, coursename)

where the primary keys are shown underlined. The number of tuples in the student and Enroll tables are 120 and 8 respectively. What are the maximum and minimum number of tuples that can be present in (Student * Enroll), where '*' denotes natural join?

(a) 8, 8 (b) 120, 8

(c) 960, 8 (d) 960, 120

[2004 : 1 Mark]

2.18 The relation scheme student Performance (name, courseNo, rollNo, grade) has the following functional dependencies:

name, courseNo → grade

RollNo, courseNo → grade

name → rollNo

rollNo → name

The highest normal form of this relation scheme is

(a) 2 NF (b) 3 NF

(c) BCNF (d) 4 NF

[2004 : 2 Marks]

2.19 A relation Empdtl is defined with attributes empcode (unique), name, street, city, state and pincode. For any pincode, there is only one city and state. Also, for any given street, city and state, there is just one pincode. In normalization terms, Empdtl is a relation in

(a) 1NF only

(b) 2NF and hence also in 1NF

(c) 3NF and hence also in 2NF and 1NF

(d) BCNF and hence also in 3NF, 2NF an 1NF

[2004 : 2 Marks]

2.20 A table has fields Fl, F2, F3, F4, F5 with the following functional dependencies

F1 → F3, F2 → F4, (F1, F2) → F5

In terms of Normalization, this table is in

(a) 1 NF (b) 2 NF

(c) 3 NF (d) None of these

[2005 : 1 Mark]

2.21 Which-one of the following statements about normal forms is FALSE?

(a) BCNF is stricter than 3 NF

(b) Loss less, dependency-preserving decomposition into 3 NF is always possible

(c) Loss less, dependency-preserving decomposition into BCNF is always possible

(d) Any relation with two attributes is BCNF

[2005 : 1 Mark]

2.22 Consider a relation scheme R = (A, B, C, D, E, H) on which the following functional dependencies hold:

{A → B, BC → D, E → C, D → A}

What are the candidate keys of R?

(a) AE, BE (b) AE, BE, DE

(c) AEH, BEH, BCH (d) AEH, BEH, DEH

[2005 : 2 Marks]

2.23 In a schema with attributes A, B, C, D and E following set of functional dependencies are given

A → B

A → C

CD → E

B → D

E → A

Which of the following functional dependencies is NOT implied by the above set

(a) CD → AC (b) BD → CD

(c) BC → CD (d) AC → BC

[2005 : 2 Marks]

2.24 Consider the relations r_1(P,Q,R) and r_2 (R,S,T) with primary keys P and R respectively. The relation r_1 contains 2000 tuples and r_2 contains 2500 tuples. The maximum Size of the join $r_1 \bowtie r_2$ is :

(a) 2000 (b) 2500

(c) 4500 (d) 5000

[2006 : 1 Mark]

2.25 Consider a relation R with five attributes V, W, X, Y, and Z. The following functional dependencies hold : VY → W, WX → Z, and ZY → V. Which of the following is a candidate key for R?

(a) VXZ (b) VXY
(c) VWXY (d) VWXYZ

[2006 : 2 Marks]

2.26 The following functional dependencies are given: $AB \to CD$, $AF \to D$, $DE \to F$, $C \to G$, $F \to E$, $G \to A$.

Which one of the following options is false?

(a) $\{CF\}^+ = \{ACDEFG\}$ (b) $\{BG\}^+ = \{ABCDG\}$
(c) $\{AF\}^+ = \{ACDEFG\}$ (d) $\{AB\}^+ = \{ACDFG\}$

[2006 : 2 Marks]

2.27 Which one of the following statements is FALSE?

(a) Any relation with two attributes is in BCNF
(b) A relation in which every key has only one attribute is in 2NF
(c) A prime attribute can be transitively dependent on a key in 3NF relation
(d) A prime attribute can be transitively dependent on a key in a BCNF relation.

[2007 : 2 Marks]

2.28 Consider the following implications relating to functional and multivalued dependencies given below, which may or may not be correct.

(i) If $A \to\to B$ and $A \to\to C$ then $A \to BC$
(ii) If $A \to B$ and $A \to C$ then $A \to\to BC$
(iii) If $A \to\to BC$ and $A \to B$ then $A \to C$
(iv) If $A \to BC$ and $A \to B$ then $A \to\to C$

Exactly how many of the above implications are valid?

(a) 0 (b) 1
(c) 2 (d) 3

[2007 : 2 Marks]

2.29 Let R (A, B, C, D) be a relational schema with the following functional dependencies:

$A \to B$, $B \to C$, $C \to D$ and $D \to B$.

The decomposition of R into (A, B), (B, C), (B, D)

(a) Gives a lossless join, and is dependency preserving
(b) Gives a lossless join, but is not dependency preserving
(c) Does not give a lossless join, but is dependency preserving
(d) Does not give a lossless join and is not dependency preserving

[2008 : 2 Marks]

2.30 Let R (A, B, C, D, E, P, G) be a relational schema in which the following functional dependencies are known to hold:

$AB \to CD$, $DE \to P$, $C \to E$, $P \to C$ and $B \to G$.

The relational schema R is

(a) in BCNF
(b) in 3NF, but not in BCNF
(c) in 2NF, but not in 3NF
(d) not in 2NF

[2008 : 2 Marks]

2.31 Consider the following relational schemes for a library database:

Book (Title, Author, Catalog_no, Publisher, Year, price)

Collection (Title, Author, Catalog_no)

which the following functional dependencies:

I. Title Author $\to$ Catalog_no
II. Catalog_no $\to$ Title Author Publisher Year
III. Publisher Title Year $\to$ Price

Assume {Author, Title} is the key for both schemes: which of the following statements is true?

(a) Both Book and Collection are in BCNF
(b) Both Book and Collection are in 3NF only
(c) Book is in 2NF and Collection is in 3NF
(d) Both Book and Collection are in 2NF only

[2008 : 2 Marks]

2.32 The following functional dependencies hold for relations R(A, B, C) and S(B, D, E):

$B \to A$
$A \to C$

The relation R contains 200 tuples and the relation S contains 100 tuples. What is the maximum number of tuples possible in the natural join $R \bowtie S$?

(a) 100 (b) 200
(c) 300 (d) 2000

[2010 : 2 Marks]

2.33 Consider a relational table with a single record from each registered student with the following attributes.

1. Registration_Num: Unique registration number of each registered student
2. UID: Unique identity number, unique at the national level for each citizen
3. BankAccount_Num: Unique account number at the bank. A student can have multiple accounts or joint accounts. This attribute stores the primary account number
4. Name: Name of the student
5. Hostel_Room: Room number of the hostel

Which of the following options is INCORRECT?
(a) BankAccount_Num is a candidate key
(b) Registration_Num can be a primary key
(c) UID is a candidate key if all students are from the same country
(d) If S is a superkey such that S ∩ UID is NULL then S ∪ UID is also a superkey

[2011 : 1 Mark]

2.34 Which of the following is TRUE?
(a) Every relation in 3NF is also in BCNF
(b) A relation R is in 3NF if every non-prime attribute of R is fully functionally dependent on every key of R
(c) Every relation in BCNF is also in 3NF
(d) No relation can be in both BCNF and 3NF

[2012 : 1 Mark]

Linked Answer Questions 2.35 and 2.36

Relation R has eight attributes ABCDEFGH. Fields of R contain only atomic values.
$F = \{CH \rightarrow G, A \rightarrow BC, B \rightarrow CFH, E \rightarrow A, F \rightarrow EG\}$ is a set of functional dependencies (FDs) so that F is exactly the set of FDs that hold for R.

2.35 How many candidate keys does the relation R have?
(a) 3 (b) 4
(c) 5 (d) 6

[2013 : 2 Marks]

2.36 The relation R is
(a) in 1NF, but not in 2NF.
(b) in 2NF, but not in 3NF.
(c) in 3NF, but not in BCNF.
(d) in BCNF.

[2013 : 2 Marks]

2.37 Consider the relation scheme $R = (E, F, G, H, I, J, K, L, M, N)$ and the set of functional dependencies $\{\{E,F\} \rightarrow \{G\}, \{F\} \rightarrow \{I,J\}, \{E,H\} \rightarrow \{K, L\}, \{K\} \rightarrow \{M\}, \{L\} \rightarrow \{N\}\}$ on R. What is the key for R?
(a) $\{E, F\}$ (b) $\{E, F, H\}$
(c) $\{E, F, H, K, L\}$ (d) $\{E\}$

[2014 (Set-1) : 1 Mark]

2.38 Given the following statements:
S1: A foreign key declaration can always be replaced by an equivalent check assertion in SQL.
S2: Given the table R(a,b,c) where a and b together form the primary key, the following is a valid table definition.

```
CREATE TABLE S (
a INTEGER,
d INTEGER,
e INTEGER,
PRIMARY KEY (d),
FOREIGN KEY (a) references R)
```

Which one of the following statements is **CORRECT**?
(a) S1 is TRUE and S2 is FALSE.
(b) Both S1 and S2 are TRUE.
(c) S1 is FALSE and S2 is TRUE.
(d) Both S1 and S2 are FALSE.

[2014 (Set-1) : 1 Mark]

2.39 Given the following two statements:
S1: Every table with two single-valued attributes is in 1NF, 2NF, 3NF and BCNF.
S2: $AB \rightarrow C, D \rightarrow E, E \rightarrow C$ is a minimal cover for the set of functional dependencies $AB \rightarrow C, D \rightarrow E, AB \rightarrow E, E \rightarrow C$.

Which one of the following is **CORRECT**?
(a) S1 is TRUE and S2 is FALSE.
(b) Both S1 and S2 are TRUE.
(c) S1 is FALSE and S2 is TRUE.
(d) Both S1 and S2 are FALSE.

[2014 (Set-1) : 2 Marks]

2.40 The maximum number of superkeys for the relation schema R(E,F,G,H) with E as the key is_____.

[2014 (Set-2) : 1 Mark]

2.41 Given an instance of the STUDENTS relation as shown below:

StudentID	StudentName	StudentEmail	StudentAge	CPI
2345	Shankar	shankar@math	X	9.4
1287	Swati	swati@ee	19	9.5
7853	Shankar	shankar@cse	19	9.4
9876	Swati	swati@mech	18	9.3
8765	Ganesh	ganesh@civil	19	8.7

For **(StudentName, StudentAge)** to be a key for this instance, the value X should NOT be equal to _________.

[2014 (Set-2) : 1 Mark]

2.42 A prime attribute of a relation scheme R is an attribute that appears
(a) in all candidate keys of R.
(b) in some candidate key of R.
(c) in a foreign key of R.
(d) only in the primary key of R.

[2014 (Set-3) : 1 Mark]

2.43 Consider the relation $X(P, Q, R, S, T, U)$ with the following set of functional dependencies

F = {
$\{P, R\} \rightarrow \{S, T\}$
$\{P, S, U\} \rightarrow \{Q, R\}$
}

Which of the following is the trivial functional dependency in F^+ is closure of F?

(a) $\{P, R\} \rightarrow \{S, T\}$
(b) $\{P, R\} \rightarrow \{R, T\}$
(c) $\{P, S\} \rightarrow \{S\}$
(d) $\{P, S, U\} \rightarrow \{Q\}$

[2015 (Set-3) : 1 Mark]

2.44 Which of the following is NOT a superkey in a relational schema with attributes V, W, X, Y, Z and primary key $V Y$?

(a) $VXYZ$
(b) $VWXZ$
(c) $VWXY$
(d) $VWXYZ$

[2016 (Set-1) : 1 Mark]

2.45 A database of research articles in a journal uses the following schema.

(VOLUME, NUMBER, STARTPAGE, ENDPAGE, TITLE, YEAR, PRICE)

The primary key is (VOLUME, NUMBER, STARTPAGE, ENDPAGE) and the following functional dependencies exist in the schema.

(VOLUME, NUMBER, STARTPAGE, ENDPAGE) $\rightarrow$ TITLE

(VOLUME, NUMBER) $\rightarrow$ YEAR

(VOLUME, NUMBER, STARTPAGE, ENDPAGE) $\rightarrow$ PRICE

The database is redesigned to use the following schemas.

(VOLUME, NUMBER, STARTPAGE, ENDPAGE, TITLE, PRICE)

(VOLUME, NUMBER, YEAR)

Which is the weakest normal form that the new database satisfies, but the old one does not?

(a) 1NF
(b) 2NF
(c) 3NF
(d) BCNF

[2016 (Set-1) : 1 Mark]

■■■■

Answers Database Design: Functional Dependencies and Normalization

2.3	(a)	2.4	(d)	2.5	(d)	2.6	(c)	2.8	(b)	2.9	(a)	2.10	(b)	2.11	(c)	2.12	(c)
2.13	(a)	2.14	(c)	2.15	(b)	2.16	(d)	2.17	(a)	2.18	(b)	2.19	(b)	2.20	(a)	2.21	(c)
2.22	(d)	2.23	(b)	2.24	(a)	2.25	(b)	2.26	(c, d)	2.27	(d)	2.28	(c)	2.29	(c)	2.30	(d)
2.31	(c)	2.32	(a)	2.33	(a)	2.34	(c)	2.35	(b)	2.36	(a)	2.37	(b)	2.38	(d)	2.39	(a)
2.42	(b)	2.43	(c)	2.44	(b)	2.45	(b)										

Explanations Database Design: Functional Dependencies and Normalization

2.1 Sol.

False

Sometimes, there is no decomposition into BCNF that is dependency-preserving.

For example: Consider the relation schema ABD, if we have the FD's

AB $\rightarrow$ D

D $\rightarrow$ B

Then ABD is not in BCNF because D is not a key. If we try to decompose it, however, we can't preserve the dependency AB $\rightarrow$ D.

2.2 Sol.

(a) Relation scheme R(A, B, C) has following functional dependencies:

AB $\rightarrow$ C

C $\rightarrow$ A

The candidate keys of relation scheme R(A, B, C) is {AB, BC}

Functional dependency

AB $\rightarrow$ C

is allowed in both BCNF & 3NF

Functional dependency

C $\rightarrow$ A

Here both C & A are proper subset of candidate key. Therefore functional dependency $C \rightarrow A$ is allowed in 3NF however, not allowed in BCNF.

Therefore R is in 3NF but not in BCNF.

(b) The minimal keys of relation R are {AB, BC}

2.3 (a)

Relation R(a, b, c, d) has following functional dependencies:

$a \rightarrow c$

$b \rightarrow d$

The candidate key of relation R is {ab}

Attribute a & b are prime attribute whereas c & d are non-prime attribute.

Both the FD's of R is of kind

"Proper subset of candidate key $\rightarrow$ Non-prime attribute"

Above type of FD's are not allowed in 2NF therefore, relation R is in 1NF but not in 2NF.

2.4 (d)

Insertion into S can cause inconsistency, since it has foreign key which refers to the primary key of R.

Deletion from R can cause inconsistency because it's primary key is the foreign key for S.

2.5 (d)

There is not always a decomposition into BCNF that is lossless and dependency preserving. So 3NF is considered adequate for normal relational database design.

2.6 (c)

The index is built from field occupation where,

CON = 1 index value

ENG = 2 index value

DOC = 3 index value

SER = 4 index value

MUS = 5 index value

2.7 Sol.

(a) BCNF, (b) 1NF

2.8 (b)

We can choose the correct option by finding the closure of each option

(a) CD

$(CD)^+ = \{C, D, F\}$

$\therefore$ CD is not a key for R.

(b) $(EC)^+ = \{A, B, C, D, E, F\}$

$\therefore$ EC is a key for R.

(c) $(AE)^+ = \{A, B, E\}$

$\therefore$ AE is not a key for R

(d) $(AC)^+ = \{A, B, C, F\}$

$\therefore$ AC is not a key for R

Therefore, EC is a key for R.

2.9 (a)

R = (STUV)

$S \rightarrow T, T \rightarrow U, U \rightarrow V, V \rightarrow S$

Decomposed into R_1 and R_2 such that $R_1 \cap R_2 = \phi$ is lossy join decomposition which is not allowed in 2NF, 3NF, BCNF.

2.10 (b)

(a) $XY \rightarrow Z$ is satisfied whereas $Z \rightarrow Y$ is not satisfied.

(b) $YZ \rightarrow X$ & $Y \rightarrow Z$ both are satisfied

(c) $YZ \rightarrow X$ is satisfied whereas $X \rightarrow Z$ is not satisfied

(d) $XZ \rightarrow Y$ is not satisfied whereas $Y \rightarrow X$ is satisfied.

2.11 (c)

R(A, B, C, D)

R_1(AB) and R_2(CD)

$F_1 = (A \rightarrow B)$ and $F_2 = \{C \rightarrow D\}$

All the original functional dependency can be derived from F1 and F2, hence the decomposition is dependency preserving. The attribute B doesnot functionally determine either 'A' or 'C' or 'D'. The attribute D doesnot functionally determine either 'A' or 'B' or 'C'.

$\therefore$ It is not lossless join (or it is lossy).

2.12 (c)

(a) {AB, BCD} is BCNF decomposition is loseless join and dependency preserving.

(b) {AB, BC, CD} is BCNF decomposition is loseless join and dependency preserving.

(c) {CAD, BC} is BCNF decomposition is loseless join but not dependency preserving.

(d) Given functional dependecies in BCNF.

2.13 (a)

Degree of redundancy over functional dependencies (single valued FD's) is zero percent.

2.14 (c)

The relation which is in BCNF is also in 3NF whereas the relation which is in 3NF may be or may not be in BCNF. Therefore, to make a guaranteed identification, BCNF definition should be used on the decomposition.

2.15 (b)

For 1 value of A, B is 1 and when A is 2, B is 3 in all entries.
However when B is 1 C is 1 and 0.
Therefore, A functionally determines B and B does not functionally determines C.

2.16 (d)

Relation (Roll_number, Name, Date-of-birth, Age)
Fd's which member of the relation
Date-of-birth $\rightarrow$ Age
Name $\rightarrow$ Roll_number
Roll_number $\rightarrow$ Name

Candidate keys {Name, date-of-birth}, {Rollnumber, date-of-birth}

Date-of-birth $\rightarrow$ Age is partial dependency which not allowed in 2NF. Highest NF of given relation 1 NF.

2.17 (a)

Student (rno, name, address) {rno} unique.
Enroll (rno, cno, Cname) {rnocno} unique.
Each record of Enroll can be mapped one record of student based on natural join condition. So maximum 8 tuples in natural join and rno of Enroll foreign key references to student. So minimum also 8 tuples in natural join.

2.18 (b)

Student (name, courseNo, rollno, grade)
Rollno, CourseNo $\rightarrow$ grade
Rollno, CourseNo $\rightarrow$ grade
Name $\rightarrow$ rollno
rollno $\rightarrow$ name
candidate keys {name, courseNo}, {rollno, CourseNo}
Highest NF of given relation 3NF but not BCNF.

2.19 (b)

It is in 2NF.
For 2NF, all non prime attributes should be fully functionally dependent on key.

key is empcode. Key contains only one attribute hence no partial dependency, but there is transitive dependency.
So it is not in 3 NF.

2.20 (a)

key is F1F2
F1 $\rightarrow$ F3,F2 $\rightarrow$ F4 are partial dependencies.

2.21 (c)

Lossless join decomposition into BCNF always possible. But dependency preserving BCNF decomposition may not possible for all relations.

2.22 (d)

R = (A, B, C, D, E, H)
FD = {A $\rightarrow$ B, BC $\rightarrow$ D, E $\rightarrow$ C, D $\rightarrow$ A,}
The candidate keys of R is AEH, BEH, DEH because both of these generates the {A, B, C, D, E, H}

2.23 (b)

Apply membership test for all the given Functional Dependencies.

1. CD $\rightarrow$ AC
   $CD^+ = CDEAB$
2. BD $\rightarrow$ CD
   $BD^+ = BD$

i.e. BD cannot derive CD and hence is not implied.
Similarly for rest two can be done.

2.24 (a)

Relation r_1 contains 2000 tuples, and r_2 contains 2500 tuples.
Maximum common R values are possible for join of r_1 and r_2 is 2000.

2.25 (b)

$(VY)^+ = VYW$
$(VYX)^+ = VYWXZ$
We have a dependencies ZY $\rightarrow$ V and WX $\rightarrow$ Z.
$\therefore$ VYX, ZYX and WXY are candidate keys.
$\Rightarrow$ VXY is candidate key for R.

2.26 (c,d)

(c) $(AF)^+$ = {AFED} but given
$(AF)^+$ = {ACDEFG} which is wrong.
(d) $(AB)^+$ = {ABCDG} but given
$(Ab)^+$ = {ACDFG} which is wrong.

2.27 (d)

Statement (d) is false because in BCNF relation a prime attribute can't be transitively dependent on a key.

2.28 (c)

(i) If $A \rightarrow\rightarrow B$ and $A \rightarrow\rightarrow C$ then $A \rightarrow BC$ not valid
(ii) If $A \rightarrow B$ and $A \rightarrow C$ then $A \rightarrow BC$, therefore $A \rightarrow\rightarrow BC$ is also valid.
(iii) If $A \rightarrow\rightarrow BC$ and $A \rightarrow B$, then $A \rightarrow C$ not valid
(iv) If $A \rightarrow BC$ and $A \rightarrow B$ then $A \rightarrow C$, therefore $A \rightarrow\rightarrow C$ also valid.

Note: Every FD is a special case of multi-valued dependency.

2.29 (c)

(A,B) and (B,C) have common attribute as B. Due to $B \rightarrow C$, B is a key for (B,C).
Hence ABC can be loss lessly decomposed into (A,B) and (B,C).
(A, B, C) (B, D), common attribute is B.
$B \rightarrow D$ is a FD (via $B \rightarrow C$, $C \rightarrow D$), and hence B is a key for (B, D).
So, decomposition of (A, B, C, D) into (A, B, C) (B, D) is lossless.
Thus the given decomposition is lossless.
The given decomposition is also dependency preserving:
$A \rightarrow B$ is is present in (A, B), $B \rightarrow C$ is present in (B, C), $D \rightarrow B$ is present in (B, D), and $C \rightarrow D$ is indirectly present via $C \rightarrow B$ in (B, C) and $B \rightarrow D$ in (B, D).

2.30 (d)

Not in 2NF because here candidate key is AB and in FD's proper subset of C.K. determine the non prime attribute i.e. $B \rightarrow G$.

2.31 (c)

Book (Title, Author, Catalog-no., Publisher, Year, Price)
Collection (Title, Author, Catalog-no.)
The FD are
I. Title Author $\rightarrow$ Catalog-no.
II. Catalog-no. $\rightarrow$ Title Author Publisher Year
III. Publisher Title Year $\rightarrow$ Price
In the relation book there is a transitivity dependency by I, II and III so Book is in 2NF.
In the relation Collection there is n fully transitive dependency so Collection is in 3NF.

2.32 (a)

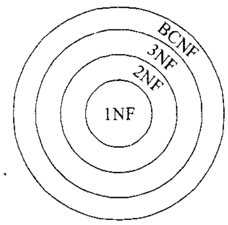
$B \rightarrow A$ where R(A, B, C) and S(B, D, E)
$A \rightarrow C$
R contains 200 tuples
S contains 100 tuples
Natural join $\bowtie = R \bowtie S = \pi[\sigma(R \times S)]$
R = 200 tuples and S = 100 tuples.
$R \bowtie S = 100$
[common in both or we can say Distinct equality between all common Attribute]

2.33 (a)

BankAccount_Num can not be a key, because if students has only joint accounts then it can not identify uniquely.

2.34 (c)

BCNF is more restrictive than 3NF. A relation is in BCNF iff $X \rightarrow Y$ where 'X' is super key and in 3NF iff other 'X' is super key or 'Y' is prime attribute.
$\therefore$ Every relation in BCNF is also in 3NF.



So option (c) is correct.

2.35 (b)

Relation contain 8 attributes ABCDEFGH
F = {$CH \rightarrow G$, $A \rightarrow BC$, $B \rightarrow CFH$, $E \rightarrow A$, $F \rightarrow EG$}
Candidate key : AD, ED, BD, FD
option (b) is correct

2.36 (a)

Option (a) is correct because partial dependencies exist in functional dependency.

2.37 (b)

$\{E, F, H\} \rightarrow \{E, F, H, G, I, J, K, L, M, N\}$
$\therefore$ *EFH* is a key for the relation *R*.

2.38 (d)

S1 is FALSE. A foreign key declaration can not always be replaced by an equivalent check assertion in SQL.

S2: CREATE TABLE S (
a INTEGER,
d INTEGER,
e INTEGER,
PRIMARY KEY (d),
FOREIGN KEY (a) references R)

S2 referenced attribute set (set of key attribute used for foreign reference) and referencing attribute set (set of foreign key attribute) must be same.

2.39 (a)

S1: Every two attribute relation is BCNF

$\therefore$ S1 is true

S2: $FD = \{AB \rightarrow C, D \rightarrow E, AB \rightarrow E, E \rightarrow C\}$

$\{AB \rightarrow C, D \rightarrow E, E \rightarrow C\}$ is not a minimal cover.

AB can not determine E.

$\therefore$ S1 is TRUE and S2 is FALSE

2.40 Sol.

Super keys:

= {E, EF, EG, EH, EFG, EFH, EGH, EFGH}

2.41 Sol.

(StudentName, StudentAge) is a key

$\left.\begin{matrix}\text{Shankar} & X \\ \text{Shankar} & 19\end{matrix}\right\}$ here X should not be 19

2.42 (b)

A prime attribute is an attribute that appears in some candidate key of given relation R.

2.43 (c)

$PS \rightarrow S$ is trivial FD, because $\{P, S\} \supseteq \{S\}$.

2.44 (b)

In $V\,W\,X\,Z$, no complete candidate key exists. So, it is not super key.

2.45 (b)

Journal (Volume, Number, Startpage, Endpage, Title, Year, Price)

Primary key: Volume, Number, Startpage, Endpage

FD's: Volume Number Startpage Endpage $\rightarrow$ Title

Volume number $\rightarrow$ Year

Volume Number, Startpage Endpage $\rightarrow$ Price

Given relation 1NF but not 2NF. This DB is redesigned following schemas

R_1(Volume, Number Startpage Endpage Title Price) which has FD's

Volume Number, Startpage Endpage $\rightarrow$ Title

Volume Number Startpage Endpage $\rightarrow$ Price

Which is in BCNF.

R_2(Volume, Number, Year)

Volume Number $\rightarrow$ Year

Which is also in BCNF.

Journal in 1NF

R_1 R_2 in BCNF

Weakest NF which satisfy R_1 and R_2 and fails for journal is 2NF.

■■■■

3 Structured Query Language (SQL)

3.1 Suppose we have a database consisting of the following three relations.
FREQUENTS (student, parlor) giving the parlors each student visits.
SERVES (parlor, ice-cream) indicating what kind of ice-creams each parlor serves.
LIKES (student, ice-cream) indicating what ice-creams each student likes.
(Assume that each student likes at least one ice-cream and frequents at least one parlor)
Express the following in SQL:
Print the students that frequent at least one parlor that serves some icecream that they like.
[1998 : 2 Marks]

3.2 Which of the following is/are correct?
(a) An SQL query automatically eliminates duplicates
(b) An SQL query will not work if there are no indexes on the relations
(c) SQL permits attribute names to be repeated in the same relation
(d) None of the above
[1999 : 2 Marks]

3.3 Consider the set of relations
EMP (Employee-no, Dept-no, Employee-name, Salary)
DEPT (Dept-no, Dept-name, Location)
Write an SQL query to:
(a) Find all employee names who work in departments located at 'Calcutta' and whose salary is greater than Rs.50,000.
(b) Calculate, for each department number, the number of employees with a salary greater than Rs.1,00,000.
[1999 : 2 Marks]

3.4 In SQL, relations can contain null values, and comparisons with null values are treated as unknown. Suppose all comparisons with a null value are treated as false. Which of the following pairs is not equivalent?
(a) $x = 5$, not (not $(x = 5)$)
(b) $x = 5$, $x > 4$ and $x < 6$, where x is an integer
(c) $x \neq 5$, not $(x = 5)$
(d) None of the above
[2000 : 2 Marks]

3.5 Given relations r(w, x) and s(y, z), the result of
select distinct w, x
from r, s
is guaranteed to be same as r, provided
(a) r has no duplicates and s is non-empty
(b) r and s have no duplicates
(c) s has no duplicates and r is non-empty
(d) r and s have the same number of tuples
[2000 : 2 Marks]

3.6 Consider the following SQL query
select distinct $a_1, a_2, \ldots, a_n$
from $r_1, r_2 \ldots, r_m$
where ρ
For an arbitrary predicate ρ, this query is equivalent to which of the following relational algebra expressions?
(a) $\Pi_{a_1,a_2,\ldots,a_n} \sigma_\rho(r_1 \times r_2 \times \ldots \times r_m)$
(b) $\Pi_{a_1,a_2,\ldots,a_n} \sigma_\rho(r_1 \bowtie r_2 \bowtie \ldots \bowtie r_m)$
(c) $\Pi_{a_1,a_2,\ldots,a_n} \sigma_\rho(r_1 \cup r_2 \cup \ldots \cup r_m)$
(d) $\Pi_{a_1,a_2,\ldots,a_n} \sigma_\rho(r_1 \cap r_2 \cap \ldots \cap r_m)$
[2003 : 1 Mark]

3.7 Consider the set of relations shown below and the SQL query that follow:
Students:
(Roll_number, Name, Date_of_birth)
Courses:
(Course_number, Course_name, Instructor)
Grades:
(Roll_number, Course_number, Grade)
select distinct Name
from Students, Courses, Grades
where
Students.Roll_number= Grades.Roll_number

and
Courses.Instructor=Korth and
Courses.Course_number=
Grades.Course_number and
Grades. grade=A

Which of the following sets is computed by the above query?

(a) Names of students who have got an A grade in all courses taught by Korth
(b) Names of students who have got an A grade in all courses
(c) Name of students who have got an A grade in at least one of the courses taught by Korth
(d) None of the above

[2003 : 2 Marks]

3.8 The employee information in a company is stored in the relation

Employee (name, sex, salary, deptName)

Consider the following SQL query

```
select deptName
from Employee
where sex = 'M'
group by deptName
having avg (salary)>
  (select avg (salary) from Employee)
```

It returns the names of the department in which

(a) the average salary is more than the average salary in the company
(b) the average salary of male employees is more than the average salary of all male employees in the company
(c) the average salary of male employees is more than the average salary of employees in the same department
(d) the average salary of male employees is more than the average salary in the company

[2004 : 2 Marks]

3.9 A relational database contains two tables student and department in which student table has columns roll_no, name and dept_id and department table has columns dept_id and dept_name. The following insert statements were executed successfully to populate the empty tables:

Insert into department values (1, 'Mathematics')
Insert into department values (2, 'Physics')
Insert into student values (l, 'Navin', 1)
Insert into student values (2, 'Mukesh', 2)
Insert into student values (3, 'Gita', 1)

How many rows and columns will be retrieved by the following SQL statement? Select * from student, department

(a) 0 row and 4 columns
(b) 3 rows and 4 columns
(c) 3 rows and 5 columns
(d) 6 rows and 5 columns

[2004 : 2 Marks]

3.10 A table T1 in a relational database has the following rows and columns:

roll no.	marks
1	10
2	20
3	30
4	Null

The following sequence of SQL statements was successfully executed on table T1.

Update T1 set marks = marks +5
Select avg(marks) from T1

What is the output of the select statement?

(a) 18.75
(b) 20
(c) 25
(d) Null

[2004 : 2 Marks]

3.11 Consider two tables in a relational database with columns and rows as follows:

Table : Student

Roll_no	Name	Dept_id
1	ABC	1
2	DEF	1
3	GHI	2
4	JKL	3

Table : Department

Dept_id	Dept_name
1	A
2	B
3	C

Roll_no is the primary key of the Student table, Dept_id is the primary key of the Department table and Student.Dept_id is a foreign key from Department.Dept_id

What will happen if we try to execute the following two SQL statements?

(i) update Student set Dept_id = Null where Roll_no = 1
(ii) update Department set Dept_id = Null where Dept_id = 10

(a) Both (i) and (ii) will fail
(b) (i) will fail but (ii) will succeed
(c) (i) will succeed but (ii) will fail
(d) Both (i) and (ii) will succeed

[2004 : 2 Marks]

3.12 A company maintains records of sales made by its salespersons and pays them commission based on each individual's total sales made in a year. This data is maintained in a table with following schema:

salesinfo = (salespersonid, totalsales, commission)

In a certain year, due to better business results, the company decides to further reward its salespersons by enhancing the commission paid to them as per the following formula:

If commission < = 50000, enhance it by 2%
If 50000 < commission < = 100000, enhance it by 4%
If commission > 100000, enhance it by 6%

The IT staff has written three different SQL scripts to calculate enhancement for each slab, each of these scripts is to run as a separate transaction as follows:

```
T1 Update salesinfo
   Set commission = commission * 1.02
Where commission < = 50000;
T2 Update salesinfo
Set commission = commission * 1.04
   Where commission > 50000 and commission
   is < = 100000;
T3 Update salesinfo
   Set commission = commission * 1.06
   Where commission > 100000;
```

Which of the following options of running these transactions will update the commission of all salespersons correctly?

(a) Execute T1 followed by T2 followed by T3
(b) Execute T2, followed by T3; T1 running concurrently throughout
(c) Execute T3 followed by T2; T1 running concurrently throughout
(d) Execute T3 followed by T2 followed by T1

[2005 : 2 Marks]

3.13 In an inventory management system implemented at a trading corporation, there are several tables designed to hold all the information. Amongst these, the following two tables hold information on which items are supplied by which suppliers, and which warehouse keeps which items along with the stock-level of these items.

Supply = (supplierid, itemcode)
Inventory = (itemcode, warehouse, stocklevel)

For a specific information required by the management, following SQL query has been written

```
Select distinct STMP.supplierid
From Supply as STMP
Where not unique (Select ITMP.supplierid
From Inventory, Supply as ITMP
Where STMP.supplierid = ITMP.supplierid
And ITMP.itemcode = Inventory.itemcode
And Inventory.warehouse = 'Nagpur');
```

For the warehouse at Nagpur, this query will find all suppliers who

(a) do not supply any item
(b) supply exactly one item
(c) supply one or more items
(d) supply two or more items

[2005 : 2 Marks]

3.14 The relation book (title, price) contains the titles and prices of different books. Assuming that no two books have the same price, what does the following SQL

```
select title
from book as B
where (select count (*)
   from book as T
  where T. price > B. Price) < 5
```

(a) Titles of the four most expensive books
(b) Title of the fifth most inexpensive book
(c) Title of the fifth most expensive book
(d) Titles of the five most expensive books

[2005 : 2 Marks]

Directions for Question 3.15 to 3.16:
Consider a database with three relation instances shown below. The primary keys for the Drivers and Cars relation are *did* and *cid* respectively and the records are stored in ascending order of these primary keys as given in the tables. No indexing is available in the database.

D : Drivers relation

did	dname	rating	age
22	Karthikeyan	7	25
29	Salman	1	33
31	Boris	8	55
32	Amoldt	8	25
58	Schumacher	10	35
64	Sachin	7	35
71	Senna	10	16
74	Sachin	9	35
85	Rahul	3	25
95	Ralph	3	53

R : Reserves relation

did	cid	day
22	101	10/10/06
22	102	10/10/06
22	103	08/10/06
22	104	07/10/06
31	102	10/11/06
31	103	06/11/06
31	104	12/11/06
64	101	05/09/06
64	102	08/09/06
74	103	08/09/06

C : Cars relation

cid	cname	colour
101	Renault	blue
102	Renault	red
103	Ferrari	green
104	Jaguar	red

3.15 What is the output of the following SQL query?

```
select D.dname
from Drivers D
where D.did in (
    select R.did
    from Cars C, Reserves R
    where R.cid=C.cid and C.colour = 'red'
    intersect
    select R.did
    from Cars C, Reserves R
    where R.cid=C.cid and C.colour= 'green'
    )
```

(a) Karthikeyan, Boris
(b) Sachin, Salman
(c) Karthikeyan, Boris, Sachin
(d) Schumacher, Senna

[2006 : 2 Marks]

3.16 Let n be the number of comparisons performed when the above SQL query is optimally executed. If linear search is used to locate a tuple in a relation using primary key, then n lies in the range

(a) 36 – 40 (b) 44 – 48
(c) 60 – 64 (d) 100 – 104

[2006 : 2 Marks]

3.17 Consider the relation account (customer, balance) where customer is a primary key and there are no null values. We would like to rank customers according to decreasing balance. The customer with the largest balance gets rank 1. Ties are not broken but ranks are skipped: if exactly two customers have the largest balance they each get rank 1 and rank 2 is not assigned.

Query 1: Select A.customer, count (B.customer) from account A, account B where A.balance < = B.balance Group by A.customer

Query 2: Select A.customer, 1 + count (B.customer) from account A, account B where A.balance < B.balance group by A.customer

Consider these statements about Query 1 and Query 2.

1. Query 1 will produce the same row set as Query 2 for some but not all databases
2. Both Query 1 Query 2 are correct implementations of the specification
3. Query 1 is a correct implementation of the specification but Query 2 is not
4. Neither Query 1 nor Query 2 is a correct implementation of the specification
5. Assigning rank with a pure relational Query takes less time than scanning in decreasing balance order and assigning ranks using ODBC

Which two of the above statements are correct?

(a) 2 and 5 (b) 1 and 3
(c) 1 and 4 (d) 3 and 5

[2006 : 2 Marks]

3.18 Consider the relation enrolled (student, course) in which (student, course) is the primary key, and the relation paid (student, amount) where student is the primary key. Assume no null values and no foreign keys or integrity constraints. Given the following four queries:

Query 1: Select student from enrolled where student in (select student form paid)

Query 2: Select student from paid where student in (select student from enrolled)

Query 3: Select E.student from enrolled E, paid P where E.student = P.student

Query 4: Select student from paid where exists (select * from enrolled where enrolled.student = paid.student)

which one of the following statements is correct?

(a) All queries return identical row sets for any database

(b) Query 2 and Query 4 return identical row sets for all databases but there exist databases for which Query 1 and Query 2 return different row sets

(c) There exist databases for which Query 3 returns strictly fewer rows than Query 2

(d) There exist databases for which Query 4 will encounter an intergrity violation at runtime

[2006 : 2 Marks]

3.19 Consider the table employee (empId, name, department, salary) and the two queries Q_1, Q_2 below. Assuming that department 5 has more than one employee, and we want to find the employees who get higher salary than anyone in the department 5, which one of the statements is TRUE for any arbitrary employee table?

$\mathbf{Q_1}$: Select e.empId
From employee e
Where not exists
(Select * From employee s Where s.department = "5" and s.salary >= e.salary)

$\mathbf{Q_2}$: Select e. empId
From employee e
Where e.salary > Any
(Select distinct salary From employee s Where s.department = "5")

(a) Q_1 is the correct query

(b) Q_2 is the correct query

(c) Both Q_1 and Q_2 produce the same answer

(d) Neither Q_1 nor Q_2 is the correct query

[2007 : 2 Marks]

Directions for Question 3.20 to 3.21:

Consider the following relational schema:
Student (school-id, sch-roll-no, sname, saddress)
School (school-id, sch-name, sch-address, sch-phone)
Enrolment(school-id sch-roll-no, erollno, examname)
ExamResult (erollno, examname, marks)

3.20 What does the following SQL query output?

```
SELECT   sch-name, COUNT (*)
FROM     School C, Enrolment E, ExamResult
         R
WHERE    E.school-id C.school-id
         AND
         E.examname = R.examname AND
         E.Erollno = R.erollno
         AND
         R.marks = 100 AND S.school-id IN
         (SELECT school-id
         FROM student
         GROUP BY school-id
         HAVING COUNT (*) > 200)
GROUP By school-id
```

(a) for each school with more than 200 students appearing in exams, the name of the school and the number of 100s scored by its students

(b) for each school with more than 200 students in it, the name of the school and the number of 100s scored by its students

(c) for each school with more than 200 students in it, the name of the school and the number of its students scoring 100 in at least one exam

(d) nothing; the query has a syntax error

[2008 : 2 Marks]

3.21 Consider the following tuple relational calculus query.

$\{t \mid \exists E \in \text{Enrolment } t = E.\text{school-id} \wedge \mid \{x \mid x\ \text{Enrolment} \wedge x.\text{school-id} = t \wedge (\exists B\ \text{ExamResult } B.\text{erollno} = x.\text{erollno} \wedge B.\text{examname} = x.\text{examname} \wedge B.\text{marks} > 35)\} \mid \div \mid \{x \mid x\ \text{Enrolment} \wedge x.\text{school-id} = t\} \mid * 100 > 35\}$

If a student needs to score more than 35 marks to pass an exam, what does the query return?

(a) The empty set

(b) schools with more than 35% of its students enrolled in some exam or the other

(c) schools with a pass percentage above 35% over all exams taken together

(d) schools with a pass percentage above 35% over each exam

[2008 : 2 Marks]

3.22 Consider a database table T containing two columns X and Y each of type integer. After the creation of the table, one record (X = 1, Y = 1) is inserted in the table.

Let MX and MY denote the respective maximum values of X and Y among all records in the table at any point in time. Using MX and MY, new records are inserted in the table 128 times with X and Y values being MX + 1, 2*MY + 1 respectively. It may be noted that each time after the insertion, values of MX and MY change.

What will be the output of the following SQL query after the steps mentioned above are carried out?

SELECT Y FROM T WHERE X = 7;

(a) 127 (b) 255

(c) 129 (d) 257

[2011 : 2 Marks]

3.23 Database table by name Loan_Records is given below:

Borrower	Bank_Manager	Loan_Amount
Ramesh	Sunderajan	10000.00
Suresh	Ramgopal	5000.00
Mahesh	Sunderajan	7000.00

What is the output of the following SQL query?

SELECT count(*)
FROM (
(SELECT Borrower, Bank_Manager FROM Loan_Records) AS S NATURAL JOIN
(SELECT Bank-Manager, Loan_Amount FROM Loan_Records) AS T
);

(a) 3 (b) 9

(c) 5 (d) 6

[2011 : 2 Marks]

3.24 Which of the following statements are TRUE about an SQL query?

P : An SQL query can contain a HAVING clause even if it does not have a GROUP BY clause

Q : An SQL query can contain a HAVING clause only if it has a GROUP BY clause

R : All attributes used in the GROUP BY clause must appear in the SELECT clause

S : Not all attributes used in the GROUP BY clause need to appear in the SELECT clause

(a) P and R (b) P and S

(c) Q and R (d) Q and S

[2012 : 1 Mark]

Common Data for Questions 3.25 and 3.26

Consider the following relations A, B and C:

A.

Id	Name	Age
12	Arun	60
15	Shreya	24
99	Rohit	11

B.

Id	Name	Age
15	Shreya	24
25	Hari	40
98	Rohit	20
99	Rohit	11

C.

Id	Phone	Age
10	2200	02
99	2100	01

3.25 How many tuples does the result of the following relational algebra expression contain? Assume that the schema of $A \cup B$ is the same as that of A.

$$(A \cup B) \bowtie_{A.Id > 40 \vee C.Id < 15} C$$

(a) 7 (b) 4

(c) 5 (d) 9

[2012 : 2 Marks]

3.26 How many tuples does the result of the following SQL query contain?

SELECT A.Id
FROM A
WHERE A.Age >
All (SELECT B.Age
FROM B
WHERE B.Name='Arun')

(a) 4 (b) 3

(c) 0 (d) 1

[2012 : 2 Marks]

3.27 Given the following schema:

employees(emp-id, first-name, last-name, hire-date, dept-id, salary)

departments(dept-id, dept-name, manager-id, location-id)

You want to display the last names and hire dates of all latest hires in their respective departments in the location ID 1700. You issue the following query:

```
SQL > SELECT last-name, hire-date
      FROM employees
      WHERE (dept-id, hire-date) IN
      (SELECT dept-id, MAX(hire-date) FROM
      employees JOIN departments
      USING(dept-id)
      WHERE location-id = 1700
      GROUP BY dept-id);
```

What is the outcome?

(a) It executes but does not give the correct result.
(b) It executes and gives the correct result.
(c) It generates an error because of pairwise comparison.
(d) It generates an error because the GROUP BY clause cannot be used with table joins in a subquery.

[2014 (Set-1) : 2 Marks]

3.28 SQL allows duplicate tuples in relations, and correspondingly defines the multiplicity of tuples in result of joins. Which one of the following queries always gives the same answer as the nested query shown below:

select * from R where a in (select S.a from S)

(a) select R.* from R, S where R.a = S.a
(b) select distinct R.* from R,S where R.a = S.a
(c) select R.* from R,(select distinct a from S) as S1 where R.a = S1.a
(d) select R.* from R,S where R.a = S.a and is unique R

[2014 (Set-2) : 2 Marks]

3.29 What is the optimized version of the relation algebra expression $\pi_{A1}(\pi_{A2}(\sigma_{F1}(\sigma_{F2}(r))))$, where $A1, A2$ are sets of attributes in r with $A1 \subset A2$ and $F1, F2$ are Boolean expressions based on the attributes in r?

(a) $\pi_{A1}(\sigma_{(F1 \wedge F2)}(r))$
(b) $\pi_{A1}(\sigma_{(F1 \vee F2)}(r))$
(c) $\pi_{A2}(\sigma_{(F1 \wedge F2)}(r))$
(d) $\pi_{A2}(\sigma_{(F1 \vee F2)}(r))$

[2014 (Set-3) : 1 Mark]

3.30 Consider the following relational schema:

employee(**empId**,empName,empDept)
customer(**custId**,custName,salesRepId,rating)

salesRepId is a foreign key referring to **empId** of the employee relation. Assume that each employee makes a sale to at least one customer. What does the following query return?

```
SELECT empName
FROM employee E
WHERE NOT EXISTS (SELECT custId
        FROM customer C
        WHERE C.salesRepId = E.empId
        AND C.rating <> 'GOOD');
```

(a) Names of all the employees with at least one of their customers having a 'GOOD' rating.
(b) Names of all the employees with at most one of their customers having a 'GOOD' rating.
(c) Names of all the employees with none of their customers having a 'GOOD' rating.
(d) Names of all the employees with all their customers having a 'GOOD' rating.

[2014 (Set-3) : 2 Marks]

3.31 Consider the following relations:

Student

Roll_No	Student_Name
1	Raj
2	Rohit
3	Raj

Performance

Roll_No	Course	Marks
1	Math	80
1	English	70
2	Math	75
3	English	80
2	Physics	65
3	Math	80

Consider the following SQL query.

```
SELECT S. Student_Name, sum (P.Marks)
FROM Student S, Performance P
WHERE S. Roll_No =P.Roll_No
GROUP BY S.Student_Name
```

The number of rows that will be returned by the SQL query is ________.

[2015 (Set-1) : 2 Marks]

3.32 SELECT operation in SQL is equivalent to

(a) the selection operation in relational algebra
(b) the selection operation in relational algebra, except that SELECT in SQL retains duplicates
(c) the projection operation in relational algebra
(d) the projection operation in relational algebra, except that SELECT in SQL retains duplicates

[2015 (Set-1) : 1 Mark]

3.33 Consider the following relation

Cinema (*theater, address, capacity*)

Which of the following options will be needed at the end of the SQL query

SELECT P1.address
FROM Cinema P1

such that it always finds the addresses of theaters with maximum capacity?

(a) WHERE P1.*capacity* > = All (select P2. *capacity* from Cinema P2)

(b) WHERE P1.*capacity* > = Any (select P2. *capacity* from Cinema P2)

(c) WHERE P1.*capacity* > All (select max (P2. *capacity*) from Cinema P2)

(d) WHERE P1.*capacity* > Any (select max (P2. *capacity*) from Cinema P2)

[2015 (Set-3) : 1 Mark]

3.34 Consider the following database table named water_schemes:

water_schemes		
scheme_no	district_name	capacity
1	Ajmer	20
1	Bikaner	10
2	Bikaner	10
3	Bikaner	20
1	Churu	10
2	Churu	20
1	Dungargarh	10

the number of tuples returned by the following SQL query is:

with total (name, capacity) **as**
select district_name, **sum** (capacity)
from water_schemes
group by district_name
with total_avg (capacity) **as**
select avg (capacity)
from total
select name
from total, total_avg
where total.capacity ≥ total_avg. capacity

[2016 (Set-2) : 2 Marks]

■■■■

Answers Structured Query Language (SQL)

3.2	(d)	3.4	(c)	3.5	(a)	3.6	(a)	3.7	(c)	3.8	(d)	3.9	(d)	3.10	(c)	3.11	(c)		
3.12	(d)	3.13	(d)	3.14	(d)	3.15	(a)	3.16	(c)	3.17	(c)	3.18	(b)	3.19	(b)	3.20	(b)		
3.21	(c)	3.22	(a)	3.23	(c)	3.24	(c)	3.25	(a)	3.26	(b)	3.27	(b)	3.28	(c)	3.29	(a)		
3.30	(d)	3.32	(d)	3.33	(a)														

Explanations Structured Query Language (SQL)

3.1 Sol.

Select Student
From FREQUENTS
Where parlor in (select parlor from SERVES where ice-cream in (select ice-cream from LIKES where LIKES.student = FREQEUNTS.student))

3.2 (d)

To eliminate duplicate rows, we need to use "DISTNICT" keyword with SELECT in SQL query.

SQL not permits attribute names to be repeated in the same relation. Therefore option (d) is correct.

3.3 Sol.

(a) Select Employee-name
From EMP
Where Salary > 50,000
and Dept-no in (Select Dept-no from Dept where Location = 'Calcutta')

or

Select EMP. Employee-name
From EMP, DEPT
Where EMP. Dept-no=DEPT.Dept-no
and DEPT.Location= 'Calcutta' and EMP.Salary > 50,000

(b) Select Dept-no, count (Employee-no)
From EMP
Where Salary > 1,00,000
group by Dept-no.

3.4 (c)

(c) {x ≠ 5 and (x = 5) are not equivalent.
If all are null values x ≠ 5 gives false but not (x = 5) gives true.

(a) x = 5 and not (not (x = 5)) are equivalent

(b) x = 5 and (x > 4 and x < 6) are equivalent.

3.5 (a)

If any of the relation is empty in cross-join then the overall result is empty. Therefore S need to be non-empty. To have same rows of 'r' then 'r' should not have duplicates.

3.6 (a)

The given SQL is
Select distinct $a_1, a_2, ..., a_n$
from $r_1, r_2, ..., r_m$
where P
All possible combination of tuples from r_1, r_2, r_n is denoted by $r_1 \times r_2 \times ... \times r_m$
If P is a predicate then to select the all, condition is denoted by $\sigma_P(r_1 \times r_2 \times ... \times r_m)$
If we wants to select only some tuples in the relation then composite expression for above SQL is

$$\Pi_{a_1, a_2, ..., a_n} \sigma_\rho(r_1 \times r_2 \times ... \times r_m)$$

3.7 (c)

There are three relations
Select distinct name, select the name of students and then there are three predicates
Courses. instructor = korth specify the courses taught by korth. The other two predicates specify that student can earn a grade at least A from courses so the SQL query compute.
Name of students who have got an A grade in at least one of the courses taught by korth.

3.8 (d)

Select deptName
from employee
where sex = 'M'
grouply deptName

$$\underbrace{\text{having avg (salary)}}_{\text{[Avg salary of male employees in each department]}} > \underbrace{\text{(select avg (salary) from employees)}}_{\text{[Avg salary in the company]}}$$

It returns the names of departments in which the average salary of male employees is more than the average salary in the company.

3.9 (d)

There is no specific joining condition specified, so it will retrieve Cartesian product of the tables.
Number of rows = Product of number of rows in each relation = 3 * 2 = 6
Number of columns = Sum of number of columns = 3 + 2 = 5.

3.10 (c)

Update on null gives null.
Average function ignores null values.
So, Average will produce (15 + 25 + 35)/3 = 25.

3.11 (c)

(i) If we update in STUDENT table dept id = NULL, it will not cause any problem to referenced table.

(ii) If we set in DEPARTMENT table Dept_id = NULL, it will produce inconsistency because in STUDENT table we still have the tuples containing the Dept_id = 1.

3.12 (d)

T3 followed by T2 followed by T1 will be correct execution sequence other cases some people will get two times increment.
Example: Suppose T1 followed by T2.
If initial commission is 49500 then he is belonging to < 50000
So 49500*1.02 = 50490.
He is eligible in second category then 50490*1.04 = 52509.6. So he will get increment two times. but he is eligible for only one slab of commission.

3.13 (d)

Nested query ensures that for only those suppliers it returns true which supplies more than 1 item in which case supplier id in inner query will be repeated for that supplier.

3.14 (d)

The given SQL query compute "Titles of the five most expensive books".

3.15 (a)

For color = "Red", did = {22, 22, 31, 64}
For color = "Green", did = {22, 31, 74}
Intersection of Red and Green will give = {22, 31}
which is Karthikeyan and Boris.

3.16 (c)

Sub query:
Select R.did
FROM Cars C, Reserves R
WHERE R.Cid = C.Cid and R.color='Red':
If query Executes optimally

(i) Select R.did
FROM (Select Cid
From Cars
Where color = Red)C, Reserve R
Where R.Cid = C.Cid
Number of comparisions for above query:
- 4 comparision for Red car selection.
- 20 comparision for R.Cid = C.Cid.

(ii) Select R.did
FROM (Select Cid
From Cars
Where color = Green)C, Reserve R
Where R.Cid = C.Cid
Number of comparisions for above query:
- 4 comparision for Green car selection.
- 20 comparision for R.Cid = C.Cid.

(iii) Intersection comparision:
{22, 22, 31, 31, 64} result of (i) sub query.
{22, 31, 74} result of (ii) sub query.
6 comparision for intersection by using linear search to search element.

(iv) In operation of outer query:
20 comparisions are required.
Total comparision = 24+14+6+20 = 64

3.17 (c)

Consider the query 1.
Query 1:
Select A·customer
count (B·customer)
from account A, account B
Where A·Balance < = B·balance
group by A·customer
The query1 computes the group of customer where the balance of first group is less than or equal to another group.

Query 2:
Select A·customer, 1 + count (B · customer)
from account A,account B
Where A·balance < B·balance group by A·customer
So some rows of Query1 is same as Query 2 but not all results are equal.
Both Query1 and Query2 doesn't compute the required problem rank the customers according to decreasing balance and the customer with the largest balance gets rank 1.

3.18 (b)

Enroll (student, course) paid (student, amount) student column not unique for enroll and unique for paid relation.
Query 1: Retrieving student from enroll for which student in paid. [May result duplicate student values].
Query 2: Retrieving student from paid for which students in enroll. [result distance student values]
Query 3: Retrieving student by joining enroll and paid which results same as query 1.
Query 4: Retrieving students from paid if exists in enroll, which results same as query 2.

3.19 (b)

Query 1: Retrieves employees who gets more salary than every employee of dept 5.
Query 2: Retrieves employees who gets more salary than some employee of dept 5.

3.20 (b)

If Select clause consist aggregate and non aggregate columns.
All non aggregate columns in the Select clause must appear in Group By clause.
But in this query Group by clause consists school-id instead of school-name.

3.22 (a)

Initially the table is

X	Y
1	1

Then MX = 1, MY = 1
Using MX and MY new records inserted is $(1 + 1,\ 2 \times 1 + 1) = (2, 3)$

X	Y
1	1
2	3

Now MX = 2, MY = 3

New record is $(2 + 1, 3 \times 2 + 1) = (3, 7)$

X	Y
1	1
2	3
3	7

Records are inserted in the table 128 times and looks like as follows.

X	Y
1	1
2	3
3	7
4	15
5	31
6	63
7	127
8	255
⋮	

Therefore, when X= 7, Y is 127.

So option (a) is correct.

3.23 (c)

Firstly, inner queries get executed

(SELECT Borrower, Bank-Manager Loan_Records) AS S

S

Borrower	Bank-Manager
Ramesh	Sunderajan
Suresh	Ramgopal
Mahesh	Sunderajan

(SELECT Bank-Manager, Loan_Amount FROM Loan_Records) AS T

T

Bank-Manager	Loan_Amount
Sunderajan	10000.00
Ramgopal	5000.00
Sunderajan	7000.00

S Natural Join T

Borrower	Bank-Manager	Loan_Amount
Ramesh	Sunderajan	10000.00
Ramesh	Sunderajan	7000.00
Suresh	Ramgopal	5000.00
Mahesh	Sunderajan	7000.00
Mahesh	Sunderajan	10000.00

Therefore, the output of the SQL query contains 5 records.

3.24 (c)

As per SQL standard HAVING clause allowed only if GROUP BY clause exists.

Like basic clauses:

SELECT [DISTINCT] Atributes
FROM relations
[WHERE condition]
[GROUP BY attributes
[HAVING condition]]

All attributes used in the GROUP BY clause must appear in the SELECT clause.

3.25 (a)

$A \cup B$ =

ID	Name	Age
12	Arun	60
15	Shreya	24
99	Rohit	11
25	Hari	40
98	Rohit	20

$(A \cup B) \bowtie_{A.Id > 40 \vee C.Id < 15} C$

ID	Name	Age	ID	Phone	Area
12	Arun	60	10	2200	02
15	Shreya	24	10	2200	02
99	Rohit	11	10	2200	02
25	Hari	40	10	2200	02
98	Rohit	20	10	2200	02
99	Rohit	11	99	2100	01
98	Rohit	20	99	2100	01

3.26 (b)

In this (select B. Age
From B
where B. Name = Arun)

$\Downarrow$

ϕ

So all A. Age will be selected, so Ans 3.

3.27 (b)

Given SQL query will correctly display the last names and hire dates of all latest hires in their respective departments in the location ID 1700.

3.28 (c)

Select * from R where a in (select s.a from S)

$\cong$

Select R.* from R, (select distinct a from S) as $S1$ where $R.a = S1.a$.

3.29 (a)

(i) $\pi_{A1}(\pi_{A2}(X)) = \pi_{A1}(X)$, since $A1 \subset A2$

(ii) $\sigma_{F1}(\sigma_{F2}(X)) = \sigma_{F1 \wedge F2}(X)$

$\therefore \pi_{A1}(\pi_{A2}(\sigma_{F1}(\sigma_{F2}(r)))) = \pi_{A1}(\sigma_{F1 \wedge F2}(r))$

3.30 (d)

SELECT empName
FROM employee E
WHERE NOT EXISTS

```
(SELECT CustId
FROM Customer C
WHERE C.SalesRepId = E.empId
AND C.rating<>
```

where, the SQL query in the box represents all customers having other than 'good' while the complete query represents name of all employees with all their customers having a 'good rating'.

3.31 Sol.

Student_Name	Marks
Raj	310
Rohit	140

3.32 (d)

Π in relation algebra is similar to SELECT in SQL but the only difference is that 'Π' gives distinct rows by eliminating duplicates but SELECT does not remove duplicates.

3.33 (a)

The following query finds the addresses of theaters with maximum capacity:

SELECT P1.address

FROM Cinema P1

WHERE P1.*capacity* > = All (select P2. *capacity* from Cinema P2)

3.34 Sol.

Total

Name	Capacity
Ajmer	20
Bikaner	40
Churu	30
Dungargarth	10

is result of first query.

Total average

Capacity
25

is result of second subquery.

Select name
from Total, Total_Avg
Where total capacity ≥ Total_Avg capcity
Query results two records.

■■■■

4 Relational Model: Relational Algebra and Tuple Calculus

4.1 An instance of a relational scheme R(A, B, C) has distinct values for attributes A. Can you conclude that A is a candidate key for R?

[1994 : 1 Mark]

4.2 Give a relational algebra expression using only the minimum number of operators from $\{\cup, -\}$ is equivalent to $R \cap S$.

[1994 : 1 Mark]

4.3 A library relational database system uses the following schema
USERS (User #, UserName, HomeTown)
BOOK (Book #, BookTitle, AuthorName)
ISSUED (Book #, User #, Date)
Explain in one English sentence, what each of the following relational algebra queries is designed to determine.

(a) $\sigma_{User\#=6}(\pi_{User\#,\,BookTitle}((USERS \bowtie ISSUED) \bowtie BOOK))$

(b) $\sigma_{AuthorName}(BOOK \bowtie (\sigma_{HomeTown=Delhi}(USERS \bowtie ISSUED)))$

[1996 : 2 Marks]

4.4 Which of the following query transformations (i.e. replacing the LHS expression by the RHS expression) is incorrect? R_1 and R_2 are relations, C_1, C_2 are selection conditions and A_1, A_2 are attributes of R_1?

(a) $\sigma_{c1}(\sigma_{c2}(R_1)) \rightarrow \sigma_{c2}(\sigma_{c1}(R_1))$

(b) $\sigma_{c1}(\pi_{A1}(R_1)) \rightarrow \pi_{A1}(\sigma_{c1}(R_1))$

(c) $\sigma_{c1}(R_1 \cup R_2) \rightarrow \sigma_{c1}(R_1) \cup \sigma_{c1}(R_2)$

(d) $\pi_{A1}(\sigma_{c1}(R_1)) \rightarrow \sigma_{c1}(\pi_{A1}(R_1))$

[1998 : 2 Marks]

4.5 Consider the join of a relation R with a relation S. If R has m tuples and S has n tuples then the maximum and minimum sizes of the join respectively are

(a) m + n and 0

(b) mn and 0

(c) m + n and $|m - n|$

(d) mn and m + n

[1999 : 1 Mark]

4.6 The relational algebra expression equivalent to the following tuple calculus expression:
$\{t \mid t \in r \wedge (t[A] = 10 \wedge t[B] = 20)\}$ is

(a) $\sigma_{(A=10 \vee B=20)}(r)$

(b) $\sigma_{(A=10)}(r) \cup \sigma_{(B=20)}(r)$

(c) $\sigma_{(A=10)}(r) \cap \sigma_{(B=20)}(r)$

(d) $\sigma_{(A=10)}(r) - \sigma_{(B=20)}(r)$

[1999 : 1 Mark]

4.7 Given the relations
employee (name, salary, deptno), and
department (deptno, deptname, address)
Which of the following queries cannot be expressed using the basic relational algebra operations $(\sigma, \pi, \times, \bowtie, \cup, \cap, -)$?

(a) Department address of every employee

(b) Employees whose name is the same as their department name

(c) The sum of all employees salaries

(d) All employees of a given department

[2000 : 1 Mark]

4.8 Which of the following relational calculus expressions is not safe?

(a) $\{t \mid \exists u \in R_1\,(t[A] = u[A]) \wedge \neg \exists s \in R_2\,(t[A] = s[A])\}$

(b) $\{t \mid \forall u \in R_1\,(u[A] = \text{“x”} \Rightarrow \exists s \in R_2\,(t[A] = s[A] \wedge s[A] = u[A]))\}$

(c) $\{t \mid \neg\,(t \in R_1)\}$

(d) $\{t \mid \exists u \in R_1\,(t[A] = u[A]) \wedge \exists s \in R_2\,(t[A] = s[A])\}$

[2001 : 2 Marks]

4.9 With regard to the expressive power of the formal relational query languages, which of the following statements is true?

(a) Relational algebra is more powerful than relational calculus

(b) Relational algebra has the same power as relational calculus

(c) Relational algebra has the same power as safe relational calculus

(d) None of the above

[2002 : 1 Mark]

4.10 Let $R_1(A, B, C)$ and $R_2(D, E)$ be two relation schema, where the primary keys are shown underlined, and let C be a foreign key in R_1 referring to R_2. Suppose there is no violation of the above referential integrity constraint in the corresponding relation instances r_1 and r_2. Which one of the following relational algebra expressions would necessarily produce an empty relation?

(a) $\Pi_D(r_2) - \Pi_C(r_1)$

(b) $\Pi_C(r_1) - \Pi_D(r_2)$

(c) $\Pi_D(r_1 \bowtie_{C \neq D} R_2) - \Pi_C(r_1)$

(d) $\Pi_C(r_1 \bowtie_{C = D} R_2)$

[2004 : 1 Mark]

4.11 Consider the relation Student (name, sex, marks), where the primary key is shown underlined, pertaining to students in a class that has at least one boy and one girl. What does the following relational algebra expression produce? (Note: ρ is rename operator)

$$\Pi_{name}(\sigma_{sex=female}(\text{Student})) - \Pi_{name}(\text{Student} \bowtie_{(sex\ =\ female\ \wedge\ x\ =\ male\ \wedge\ marks\ \leq\ m)} \rho_{n,x,m}(\text{Student}))$$

(a) names of girl students with the highest marks
(b) names of girl students with more marks than some boy student
(c) names of girl students with marks not less than some boy student
(d) names of girl students with more marks than all the boy students

[2004 : 2 Marks]

4.12 Let r be a relation instance with schema R = (A, B, C, D). We define $r_1 = \Pi_{A,B,C}(r)$ and $r_2 = \Pi_{A,D}(r)$. let $S = r_1 * r_2$ where * denotes natural join. Given that the decomposition of r into r_1 and r_2 is lossy, which one of the following is TRUE?

(a) $s \subset r$
(b) $r \cup s = r$
(c) $r \subset s$
(d) $r * s = s$

[2005 : 1 Mark]

4.13 A table 'student' with schema (roll, name, hostel, marks), and another table 'hobby' with schema (roll, hobbyname) contains records as shown below:

Table Student

Roll	Name	Hostel	Marks
1798	Manoj Rathod	7	95
2154	Soumic Banerjee	5	68
2369	Gumma Reddy	7	86
2581	Pradeep Pendse	6	92
2643	Suhas Kulkarni	5	78
2711	Nitin Kadam	8	72
2872	Kiran Vora	5	92
2926	Manoj Kunkalikar	5	94
2959	Hemant Karkhanis	7	88
3125	Rajesh Doshi	5	82

Table Hobby

Roll	Hobbyname
1798	chess
1798	music
2154	music
2369	swimming
2581	cricket
2643	chess
2643	hockey
2711	volleyball
2872	football
2926	cricket
2959	photography
3125	music
3125	chess

The following SQL query is executed on the above tables:

select hostel
from student natural join hobby
where marks > = 75 and roll between 2000 and 3000;

Relations S and H with the same schema as those of these two tables respectively contain the same information as tuples. A new relation S′ is obtained by the following relational algebra operation:

$S' = \Pi_{hostel}((\sigma_{s.roll = H.roll}(\sigma_{marks > 75 \text{ and } roll > 2000 \text{ and } roll < 3000}(S)) \times (H))$

The difference between the number of rows output by the SQL statement and the number of tuples in S′ is

(a) 6
(b) 4
(c) 2
(d) 0

[2005 : 2 Marks]

4.14 Which of the following relational query languages have the same expressive power?

I. Relational algebra.

II. Tuple relational calculus restricted to safe expressions.

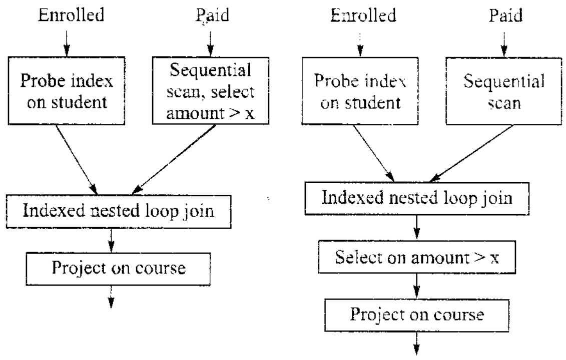
III. Domain relational calculus restricted to safe expressions.

(a) II and III only (b) I and II only

(c) I and III only (d) I, II and III

[2006 : 1 Mark]

4.15 Consider the relation enrolled (student, course) in which (student, course) is the primary key, and the relation paid (student, amount) where student is the primary key. Assume no null values and no foreign keys or integrity constraints. Assume that amounts 6000, 7000, 8000, 9000 and 10000 were each paid by 20% of the students. Consider these query plans (Plan 1 on left, Plan 2 on right) to "list all courses taken by students who have paid more than x."



A disk seek takes 4 ms.disk data transfer bandwidth is 300 MB/s and checking a tuple to see if amount is greater than x takes 10 μs. Which of the following statements is correct?

(a) Plan 1 and Plan 2 will not output identical row sets for all databases

(b) A course may be listed more than once in the output of Plan 1 for some databases

(c) For x = 5000, Plan 1 executes faster than Plan 2 for all databases

(d) For x = 9000, Plan 1 executes slower than Plan 2 for all databases

[2006 : 2 Marks]

4.16 Consider a selection of the form $\sigma_{A \le 100}(r)$, where r is a relation with 1000 tuples. Assume that the attribute values for A among the tuples are uniformly distributed in the interval [0, 500]. Which one of the following options is the best estimate of the number of tuples returned by the given selection query?

(a) 50 (b) 100

(c) 150 (d) 200

[2007 : 2 Marks]

4.17 Consider the following relation schemas:

b-Schema = (b-name, b-city, assets)

a-Schema = (a-num, b-name, bal)

d-Schema = (c-name, a-number)

Let branch, account and depositor be respectively instances of the above schemas. Assume that account and depositor relations are much bigger than the branch relation.

Consider the following query:

$\Pi_{\text{c-name}}$ ($\sigma_{\text{b-city="Agra"} \wedge \text{bal}<0}$ (branch $\bowtie$ (account $\bowtie$ depositor))

Which one of the following queries is the most efficient version of the above query?

(a) $\Pi_{\text{c-name}}$ ($\sigma_{\text{bal} < 0}$ ($\sigma_{\text{b-city = "Agra"}}$ branch $\bowtie$ account) $\bowtie$ depositor)

(b) $\Pi_{\text{c-name}}$ ($\sigma_{\text{b-city = "Agra"}}$ branch $\bowtie$ ($\sigma_{\text{bal} < 0}$ account $\bowtie$ depositor))

(c) $\Pi_{\text{c-name}}$ ($\sigma_{\text{b-city = "Agra"}}$ branch $\bowtie$ $\sigma_{\text{b-city= "Agra"} \wedge \text{bal} < 0}$ account $\bowtie$ depositor)

(d) $\Pi_{\text{c-name}}$ ($\sigma_{\text{b-city = "Agra"}}$ branch $\bowtie$ ($\sigma_{\text{b-city= "Agra"} \wedge \text{bal} < 0}$ account $\bowtie$ depositor))

[2007 : 2 Marks]

4.18 Information about a collection of students is given by the relation studInfo (studId, name, sex). The relation enroll (studId, CourseId) gives which student has enrolled for (or taken) what course(s). Assume that every course is taken by at least one male and at least one female student. What does the following relational algebra expression represent?

Π_{courseId} ((Π_{studId} ($\sigma_{\text{sex = " female"}}$(studInfo) $\times$ Π_{courseId}(enroll)) – enroll)

(a) Courses in which all the female students are enrolled

(b) Courses in which a proper subset of female students are enrolled

(c) Courses in which only male students are enrolled

(d) None of the above

[2007 : 2 Marks]

4.19 Consider the relation employee (name, sex, supervisorName) with name as the key. supervisorName gives the name of the supervisor of the employee under consideration. What does the following Tuple Relational Calculus query produce?

{e.name | employee (e) ∧
(∀ x) [¬ employee (x) ∨ x.supervisorName
≠ e.name ∨ x.sex = "male"]}

(a) Names of employees with a male supervisor
(b) Names of employees with no immediate male subordinates
(c) Names of employees with no immediate female subordinates
(d) Names of employees with a female supervisor

[2007 : 2 Marks]

4.20 Let R and S be two relations with the following schema
R(P, Q, R1, R2, R3)
S(P, Q, S1, S2)
where {P, Q} is the key for both schemas. Which of the following queries are equivalent?

I. $\Pi_P(R \bowtie S)$ **II.** $\Pi_P(R) \bowtie \Pi_P(S)$
III. $\Pi_P(\Pi_{P,Q}(R) \cap \Pi_{P,Q}(S))$
IV. $\Pi_P(\Pi_{P,Q}(R) - (\Pi_{P,Q}(R) - \Pi_{P,Q}(S)))$

(a) Only I and II (b) Only I and III
(c) Only I, II and III (d) Only I, III and IV

[2008 : 2 Marks]

4.21 Let R and S be relational schemes such that R = {a, b, c} and S = {c}. Now consider the following queries on the database :

I. $\pi_{R-S}(r) - \pi_{R-S}(\pi_{R-S}(r) \times S - \pi_{R-S,S}(r))$
II. $\{t | t \in \pi_{R-S}(r) \wedge \forall u \in s(\exists v \in r(u = v[s] \wedge t = v[R - S]))\}$
III. $\{t | t \in \pi_{R-S}(r) \wedge \forall v \in r(\exists u \in s(u = v[s] \wedge t = v[R - S]))\}$
IV. Select R.a, R.b
from R, S
where R.c = S.c

Which of the above queries are equivalent?

(a) I and II (b) I and III
(c) II and IV (d) III and IV

[2009 : 2 Marks]

Common Data for Questions 4.22 and 4.23

Consider the following relational schema:
Suppliers(sid: integer, sname: string, city: string, street: string)
Parts(pid:integer, pname:string, color:string)
Catalog(sid:integer, pid:integer, cost:real)

4.22 Consider the following relational query on the above database:

```
SELECT S.sname
FROM  Suppliers S
WHERE S. sid NOT IN (SELECT C.sid
    FROM  Catalog C
    WHERE C.pid NOT IN (SELECT P.pid
    FROM Parts P
    WHERE P.color <> 'blue'))
```

Assume that relations corresponding to the above schema are not empty. Which one of the following is the correct interpretation of the above query?

(a) Find the names of all suppliers who have supplied a non-blue part
(b) Find the names of all suppliers who have not supplied a non-blue part
(c) Find the names of all suppliers who have supplied only blue parts
(d) Find the names of all suppliers who have not supplied only blue parts

[2009 : 2 Marks]

4.23 Assume that, in the suppliers relation above, each supplier and each street within a city has a unique name, and (sname, city) forms a candidate key. No other functional dependencies are implied other than those implied by primary and candidate keys. Which one of the following is TRUE about the above schema?

(a) The schema is in BCNF
(b) The schema is in 3NF but not in BCNF
(c) The schema is in 2NF but not in 3 NF
(d) The schema is not in 2NF

[2009 : 2 Marks]

4.24 Suppose R_1 (A, B) and R_2(C, D) are two relation schemes. Let r1 and r2 be the corresponding relation instances. B is a foreign key that refers to C in R2. If data in r1 and r2 satisfy referential integrity constraints, which of the following is ALWAYS TRUE?

(a) $\Pi_B(r_1) - \Pi_C(r_2) = E$ (b) $\Pi_C(r_2) - \Pi_B(r_1) = E$
(c) $\Pi_B(r_1) = \Pi_C(r_2)$ (d) $\Pi_B(r_1) - \Pi_C(r_2) \neq E$

[2012 : 2 Marks]

4.25 Consider the following relational schema.
Student (rollno: integer: sname string)
Courses (courseno: integer, cname: string)
Registration (rollno: integer, courseno: integer, percent: real)
Which of the following queries are equivalent to this query in English?
"Find the distinct names of all students who score more than 90% in the course numbered 107"

(I) SELECT DISTINCT S.sname
FROM Students as S, Registration as R
WHERE R.rollno=S.roll.no AND R.courseno=107 and R.percent>90

(II) $\Pi_{sname}(\sigma_{courseno=107 \wedge percent>90}$ (Regis-tration $\bowtie$ Student))

(III) {T | ∃S∈ Students, ∃R∈ Registration (S.rollno=R.rollno ∧ R.courseno=107 ∧ R.percent>90∧T.sname=S.sname)}

(IV) $\{<S_N> \mid \exists S_R \exists R_P\ (<S_R, S_N> \in \text{Student} \wedge <S_R, 107, R_P> \in \text{Registration} \wedge R_P > 90)\}$

(a) I, II, III and IV (b) I, II and III only
(c) I, II and IV only (d) I, III and IV only

[2013 : 2 Marks]

4.26 Consider a join (relation algebra) between relations *r*(R) and *s*(S) using the nested loop method. There are 3 buffers each of size equal to disk block size, out of which one buffer is reserved for intermediate results.
Assuming size(*r*(R)) < size(*s*(S)), the join will have fewer number of disk block accesses if
(a) relation *r*(R) is in the outer loop.
(b) relation *s*(S) is in the outer loop.
(c) join selection factor between *r*(R) and *s*(S) is more than 0.5.
(d) join selection factor between *r*(R) and *s*(S) is less than 0.5.

[2014 (Set-2) : 2 Marks]

4.27 Consider the relational schema given below, where **eId** of the **dependent** is a foreign key referring to **empId** of the relation **employee**. Assume that every employee has at least one associated dependent in the **dependent** relation.

employee (empId, empName, empAge)
dependent(depId, eId, depName, depAge)

Consider the following relational algebra query:

$$\Pi_{empId}(\text{employee}) - \Pi_{empId}(\text{employee} \bowtie_{(empId = eId) \wedge (empAge \leq depAge)} \text{dependent})$$

The above query evaluates to the set of empIds of employees whose age is greater than that of
(a) some dependent.
(b) all dependents.
(c) some of his/her dependents.
(d) all of his/her dependents.

[2014 (Set-3) : 2 Marks]

4.28 Consider two relations R_1 (A, B) with the tuples (1, 5), (3, 7) and R_2(A, C) = (1, 7), (4, 9).
Assume that R(A,B,C) is the full natural outer join of R_1 and R_2. Consider the following tuples of the form (A,B,C): a = (1, 5, null), b = (1, null, 7), c = (3, null, 9), d = (4, 7, null), e = (1, 5, 7), f = (3, 7, null), g = (4, null, 9). Which one of the following statements is correct?
(a) R contains a, b, e, f, g but not c, d
(b) R contains all of a, b, c, d, e, f, g
(c) R contains e, f, g but not a, b
(d) R contains e but not f, g

[2015 (Set-2) : 1 Mark]

■■■■

Answers Relational Model: Relational Algebra and Tuple Calculus

4.4	(d)	4.5	(b)	4.6	(c)	4.7	(c)	4.8	(c)	4.9	(c)	4.10	(b)	4.11	(d)	4.12	(c)
4.13	(b)	4.14	(d)	4.15	(c)	4.16	(d)	4.17	(b)	4.18	(b)	4.19	(c)	4.20	(d)	4.21	(c)
4.22	(a)	4.23	(b)	4.24	(a)	4.25	(a)	4.26	(a)	4.27	(d)	4.28	(c)				

Explanations Relational Model: Relational Algebra and Tuple Calculus

4.1 Sol.

- No

For given instance "A" values unique doesn't mean candidate key.
Because candidate key identification based on constraints of the DB design not based on particular instance.

4.2 Sol.

$R \cap S = R - (R - S)$

4.3 Sol.

(a) Determines titles of the books issued to user with user # 6

(b) determines name of authors of books that are issued to users who have home town 'Delhi'.

4.4 (d)

$\pi_{A1}(\sigma_{c1}(R_1)) \rightarrow \sigma_{c1}(\pi_{A1}(R_1))$

If the selection condition (c_1) is on attribute over A_2, then we can not replace LHS expression by RHS expression. In RHS first projection is on A_1 then selection condition c_1 will fail because no A_2.

4.5 (b)

Whenever there is no foreign key constraint between two tables then the max. & min. no. of tuples in their join is mn and 0 respectively.

4.6 (c)

The given tuple calculus expression give tuples where A = 10 & B = 20 of relation r.
Therefore, relational algebra expression $\sigma_{A=10}(r) \cap \sigma_{B=20}(r)$ is equivalent to given tuple calculus expression.

4.7 (c)

Aggregate operations like sum, average, count can't be expressed in terms of basic relational algebra operations. Aggregate function requires extended relational algebra. (Min. Max can be expressed using basic RA operations)

4.8 (c)

The query $\{t \mid \neg(t \in R_1)\}$ is syntactically correct. However, it ask for all tuples t such that t is not in R_1.
That set of such t tuples is obviously infinite, in the context of such as the infinite domain set of all integers. Therefore, this is unsafe query.

4.9 (c)

Every query that can be expressed using a safe relational calculus query can also be expressed as a relational algebra query.
Therefore, relational algebra has the same power as safe relational calculus.

4.10 (b)

$R_1(A, B, C)$
$R_2(D, E)$
C is the foreign key in R_1 and referring to primary key of R_2.
D is the primary key of R_2.
In relation R_2 column D values are superset or equal of R_1 column C. Because of integrity constraints. So $\pi_C(R_1) - \pi_D(R_2)$ is empty.

4.11 (d)

The given query computes the names of girl students with more marks than all the boy students.

4.12 (c)

Given

$$R = (A, B, C, D)$$
$$r_1 = \pi_{A, B, C}(r)$$
$$r_2 = \pi_{A, D}(r)$$
$$S = r_1 * r_2$$

All tuples in r is also tuples of S so $r \subset S$.

4.13 (b)

SQL query will return the following output.

Roll	Hostel
2369	7
2581	6
2643	5

2643	5 Duplicate Row is present in Hobby table
2872	5
2926	5
2959	7

Total 7 rows are selected

In Relation Algebra, only distinct values of hostels are selected i.e. 5,6,7.

Therefore SQL row count RA row count = 7 − 3 = 4.

4.14 (d)

Given three relational query languages have same expressive power.

4.15 (c)

There are two plans mainly plan1 and plan2. In plan 1 first all the records pair is selected then they are joined but in plan2 all paid records are joined first and then the records are checked. The seek time of disk is 4 ms and data transfer rate is 300 MB/sec. So if x = 5000 then plan 1 executes faster than plan 2 for all databases.

4.16 (d)

Number of tuples in relation r = 1000

Each value of A is appeared twice, because uniformly distributed in [0, 500]

$\sigma_{A \le 100}$ (r) returns 200 tuples.

4.17 (b)

As b is very small compared to a and d which is also taken in account.

$\sigma_{\text{b-city}=\text{“Agra”}}$ branch which already filter city as agra making it small, and before that $\sigma_{bal<0}$ account $\bowtie$ depositor filter and give selected tables so $\bowtie$ between them will give same result and better one.

4.18 (b)

Let the tables are like

Stud Info

StudId	Name	Sex
1	Sita	F
2	Ram	M
3	Geeta	F
4	Shyam	M
5	Radha	F

Enroll

StudId	CourseId
1	A
3	A
1	B
2	B
3	B
5	B
1	C
4	C
2	D
4	D

Now the query $\pi_{\text{studId}}(\sigma_{\text{sex} = \text{“females”}}(\text{Stud Info}))$ results in

S

StudId
1
3
5

π_{courseId}(Enroll)

C

CourseId
A
B
C
D

$((S \times C)\_\text{enroll})$

StudentId	CourseId
1	A
1	B
1	C
1	D
3	A
3	B
3	C
3	D
5	A
5	B
5	C
5	D

T

StudentId	CourseId
1	D
3	C
3	D
5	A
5	C
5	D

Selecting course-id from above

CourseId
D
C
A

A, C, D are courses which is enrolled by proper subset of female students.

4.19 (c)

The query

{e·name | employee (e) ∧ (∀x) [¬employee (x) ∨ x·supervisorName ≠ e·name ∨ x.sex = "male"]} computes the names of employees with no immediate female subordinates.

4.20 (d)

In I, Ps from natural join of R & S are selected. In III, all Ps from intersection of (P, Q) pairs present in R and S. IV is also equivalent to III because (R-(R-S)) = R ⋈ S.

II is not equivalent as it may also include Ps where Qs are not same in R and S.

4.21 (c)

II & IV are equivalent.

4.22 (a)

Supplier sells other than blue part it can be red or green or both.

4.23 (b)

Suppliers (Sid, Sname, City, Street)

1. Each supplier and each street within a city has a unique name

   Sid Street City → Sname
2. (Sname, city) forms candidate key so that

   (Sname city) → Sid Street
3. Sid primary key so that

   Sid → Sname City Street

   Each FD of above satisfy BCNF.

4.24 (a)

R_1 (A, B)

R_2 (C, D)

B is a foreign key and referring to C and C is candidate key

So, $\Pi_B(r_1) - \Pi_C(r_2) = E$

E.g.

R_1

A	B
1	5
2	5
3	7
4	7

R_2

C	D
5	1
6	2
7	3
8	4

Now, (B) $\Pi_C(r_2) - \Pi_B(r_1) = \{6, 8\}$

(C) $\Pi_B(r_1) \neq \Pi_C(r_2)$

{5, 7} {5, 6, 7, 8}

(D) $\Pi_B(r_1) - \Pi_C(r_2) = E$

4.25 (a)

There is no problem in any of given query all the queries are working perfectly for given English statement.

4.26 (a)

Join will have fewer number of disk block accesses if outer loop has smaller relation ($r(R)$).

4.27 (d)

$\pi_{empId}(\text{employee}) - \pi_{empId}(\text{employee} \bowtie_{(empId=eId) \wedge (empAge \leq depAge)} \text{dependent})$

Where,

(*i*) $\pi_{empId}(\text{employee} \bowtie_{(empId=eId) \wedge (empAge \leq depAge)} \text{dependent})$

Means

All employees whose age is less than or equal to that of all of his dependents.

(*ii*) $\pi_{empId}(\text{employee}) - \pi_{empId}(\text{employee} \bowtie_{(empId=eId) \wedge (empAge \leq depAge)} \text{dependent})$.

Means

All employees whose age is greater than that of all of his dependents.

4.28 (c)

R_1

A	B
1	5
3	7

R_2

A	C
1	7
4	9

$R = R_1 ⟗ R_2$

R

A	B	C	
1	5	7	→ e
3	7	Null	→ f
4	Null	9	→ g

R contains e, f, g but not a, b.

5 Transactions & Concurrency Control

5.1 For the schedule given below, which of the following is correct?

1	Read A	
2		Read B
3	Write A	
4		Read A
5		Write A
6		Write B
7	Read B	
8	Write B	

(a) This schedule is serialzable and can occur in a scheme using 2PL protocol.
(b) This schedule is serialzable but cannot occur in a scheme using 2PL protocol.
(c) This schedule is not serializable but can occur in a scheme using 2PL protocol.
(d) This schedule is not serializable and cannot occur in a scheme using 2PL protocol.

[1999 : 2 Marks]

5.2 Which of the following scenarios may lead to an irrecoverable error in a database system?
(a) A transaction writes a data item after it is read by an uncommitted transaction
(b) A transaction reads a data item after it is read by an uncommitted transaction
(c) A transaction reads a data item after it is written by a committed transaction
(d) A transaction reads a data item after it is written by an uncommitted transaction

[2003 : 1 Mark]

5.3 Consider three data items D1, D2, and D3, and the following execution schedule of transactions T1, T2 and T3. In the diagram, R(D) and W(D) denote the actions reading and writing the data item D respectively.

T1	T2	T3
	R(D3);	
	R(D2);	
	W(D2);	
		R(D2);
		R(D3);
R(D1);		
W(D1);		
		W(D2);
		W(D3);
	R(D1);	
R(D2);		
W(D2);		
	W(D1);	

(a) The schedule is serializable as T2; T3; T1;
(b) The schedule is serializable as T2; T1; T3;
(c) The schedule is serializable as T3; T2; T1;
(d) The schedule is not serializable

[2003 : 2 Marks]

5.4 Which level of locking provides the highest degree of concurrency in a relational database?
(a) Page
(b) Table
(c) Row
(d) Page, table and row level locking allow the same degree of concurrency

[2004 : 1 Mark]

5.5 Consider the following schedule S of transactions T1 and T2:

T1	T2
Read(A)	
A = A–10	
	Read(A)
	Temp = 0.2 * A
	Write(A)
	Read(B)
Write(A)	
Read(B)	
B = B + 10	
Write(B)	
	B = B + Temp
	Write(B)

Which of the following is TRUE about the schedule S ?
(a) S is serializable only as T1, T2
(b) S is serializable only as T2, T1
(c) S is serializable both as T1, T2 and T2, T1
(d) S is not serializable either as T1 or as T2

[2004 : 2 Marks]

5.6 Amongst the ACID properties of a transaction, the 'Durability' property requires that the changes made to the database by a successful transaction persist
(a) Except in case of an Operating System crash
(b) Except in case of a Disk crash
(c) Except in case of a power failure
(d) Always, even if there is a failure of any kind

[2005 : 1 Mark]

5.7 Consider the following log sequence of two transactions on a bank account, with initial balance 12000, that transfer 2000 to a mortgage payment and, then apply a 5% interest.

1. T1: start
2. T1: B old = 12000, new = 10000
3. T1: M old = 0, new = 2000
4. T1: commit
5. T2: start
6. T2: B old = 10000, new = 10500
7. T2: commit

Suppose the database system crashed just before log record 7 is written. When the system is restarted, which one statement is true of the recovery procedure?

(a) We must redo log record 6 to set B to 10500
(b) We must undo log record 6 to set B to 10000 and then redo log records 2 and 3
(c) We need not redo log records 2 and 3 because transaction T1 has committed
(d) We can apply redo and undo operations in arbitrary order because they are idempotent

[2006 : 1 Mark]

5.8 Consider the following schedules involving two transactions. Which one of the following statements is TRUE?

S_1 : $r_1(X); r_1(Y); r_2(X); r_2(Y); w_2(Y); w_1(X)$
S_2 : $r_1(X); r_2(X); r_2(Y); W_2(Y); r_1(Y); w_1(X)$

(a) Both S_1 and S_2 are conflict serializable
(b) S_1 is conflict serializable and S_2 is not conflict serializable
(c) S_1 is not conflict serializable and S_2 is conflict serializable
(d) Both S_1 and S_2 are not conflict serializable

[2007 : 2 Marks]

5.9 Consider the following two transactions : T_1 and T2.

T1:	T2:
read(A);	read(B);
read(B);	read(A);
If A = 0 then B ← B + 1;	If B ≠ 0 then A ← A – 1;
write(B);	write(A);

Which of the following schemes, using shared and exclusive locks, satisfy the requirements for strict two phase locking for the above transactions?

(a)

S1 :	**S2 :**
lock S(A);	lock S(B);
read (A);	read (B);
lock S(B);	lock S(A);
read (B);	read (A);
if A = 0	if B ≠ 0
then B ← B+1;	then A ← A–1;
write (B);	write (A);
commit;	commit;
unlock (A);	unlock (B);
unlock (B);	unlock (A);

(b)

S1 :	**S2 :**
lock X(A);	lock X(B);
read (A);	read (B);
lock X(B);	lock X(A);
read (B);	read (A);
if A = 0	if B ≠ 0
then B ← B+1;	then A ← A–1;
write (B);	write (A);
unlock (A);	unlock (A);
commit;	commit;
unlock (B);	unlock (A);

(c)

S1 :	**S2 :**
lock S(A);	lock S(B);
read (A);	read (B);
lock X(B);	lock X(A);
read (B);	read (A);
if A = 0	if B ≠ 0
then B ← B+1;	then A ← A– 1;
write (B);	write (A);
unlock (A);	unlock (A);
commit;	commit;
unlock (B);	unlock (A);

(d)

S1 :	**S2 :**
lock S(A);	lock S(B);
read (A);	read (B);
lock X(B);	lock X(A);
read (B);	read (A);
if A = 0	if B ≠ 0
then B ← B+1;	then A ← A–1;
write (B);	write (A);
unlock (A);	unlock (A);
unlock (B);	unlock (B);
commit;	commit;

[2007 : 2 Marks]

5.10 Consider the following three schedules of transactions T1, T2 and T3. [Notation: In the following NYO represents the action Y (R for read, W for write) performed by transaction N on object O].

S1: 2RA 2WA 3RC 2WB 3WA 3WC
1RA 1RB 1WA 1WB

S2: 3RC 2RA 2WA 2WB 3WA 1RA
1RB 1WA 1WB 3WC

S3: 2RA 3RC 3WA 2WA 2WB 3WC
1RA 1RB 1WA 1WB

Which of the following statements is TRUE?
(a) S1, S2 and S3 are all conflict equivalent to each other
(b) No two of S1, S2 and S3 are conflict equivalent to each other
(c) S2 is conflict equivalent to S3, but not to S1
(d) S1 is conflict equivalent to S2, but not to S3

[2008 : 2 Marks]

5.11 Consider two transactions T_1 and T_2, and four schedules S_1, S_2, S_3, S_4 of T_1 and T_2 as given below:

$T_1 : R_1[x]\ W_1[x]\ W_1[y]$
$T_2 : R_2[x]\ R_2[y]\ W_2[y]$
$S_1 : R_1[x]\ R_2[x]\ R_2[y]\ W_1[x]\ W_1[y]\ W_2[y]$
$S_2 : R_1[x]\ R_2[x]\ R_2[y]\ W_1[x]\ W_2[y]\ W_1[y]$
$S_3 : R_1[x]\ W_1[x]\ R_2[x]\ W_1[y]\ R_2[y]\ W_2[y]$
$S_4 : R_2[x]\ R_2[y]\ R_1[x]\ W_1[x]\ W_1[y]\ W_2[y]$

Which of the above schedules are conflict-serializable?
(a) S_1 and S_2
(b) S_2 and S_3
(c) S_3 only
(d) S_4 only

[2009 : 2 Marks]

5.12 Which of the following concurrency control protocols ensure both conflict serializability and freedom from deadlock?
I. 2-phase locking
II. Time-stamp ordering
(a) I only
(b) II only
(c) Both I and II
(d) Neither I nor II

[2010 : 1 Mark]

5.13 Consider the following schedule for transactions T1, T2 and T3:

T1	T2	T3
Read (X)		
	Read (Y)	
		Read (Y)
	Write (Y)	
Write (X)		
		Write (X)
	Read (X)	
	Write (X)	

Which one of the schedules below is the correct serialization of the above?
(a) T1 → T3 → T2
(b) T2 → T1 → T3
(c) T2 → T3 → T1
(d) T3 → T1 → T2

[2010 : 2 Marks]

5.14 Consider the following transactions with data items P and Q initialized to zero:

T_1 : read (P);
read (Q);
if P = 0 then Q: = Q + 1;
write (Q);

T_2 : read (Q);
read (P);
if Q = 0 then P: = P + 1;
write (P);

Any non-serial interleaving of T_1 and T_2 for concurrent execution leads to
(a) a serializable schedule
(b) a schedule that is not conflict serializable
(c) a conflict serializable schedule
(d) a schedule for which a precedence graph cannot be drawn

[2012 : 2 Marks]

5.15 An index is clustered, if
(a) it is on a set of fields that form a candidate key.
(b) it is on a set of fields that include the primary key.
(c) the data record of the file are organized in the same order as the date entries of the index
(d) the data records of the file are organized not in the same order as the data entries of the index.

[2013 : 1 Mark]

5.16 Consider the following four schedules due to three transactions (indicated by the subscript) using *read* and *write* on a data item x, denoted by $r(x)$ and $w(x)$ respectively. Which one of them is conflict serializable?

(a) $r_1(x); r_2(x); w_1(x); r_3(x); w_2(x)$
(b) $r_2(x); r_1(x); w_2(x); r_3(x); w_1(x)$
(c) $r_3(x); r_2(x); r_1(x); w_2(x); w_1(x)$
(d) $r_2(x); w_2(x); r_3(x); r_1(x); w_1(x)$

[2014 (Set-1) : 2 Marks]

5.17 Consider the following schedule S of transactions T1, T2, T3 and T4:

T1	T2	T3	T4
	Reads(X)		
		Writes(X)	
		Commit	
Writes(X)			
Commit			
	Writes(Y)		
	Reads(Z)		
	Commit		
			Reads(X)
			Reads(Y)
			Commit

Which one of the following statements is CORRECT?
(a) S is conflict-serializable but not recoverable
(b) S is not conflict-serializable but is recoverable
(c) S is both conflict-serializable and recoverable
(d) S is neither conflict-serializable nor is it recoverable

[2014 (Set-2) : 2 Marks]

5.18 Consider the transactions $T1$, $T2$, and $T3$ and the schedules $S1$ and $S2$ given below.

$T1$: $r1(X); r1(Z); w1(X); w1(Z)$
$T2$: $r2(Y); r2(Z); w2(Z)$
$T3$: $r3(Y); r3(X); w3(Y)$
S1 : $r1(X); r3(Y); r3(X); r2(Y); r2(Z); w3(Y); w2(Z); r1(Z); w1(X); w1(Z)$
S2 : $r1(X); r3(Y); r2(Y); r3(X); r1(Z); r2(Z); w3(Y); w1(X); w2(Z); w1(Z)$

Which one of the following statements about the schedules is **TRUE**?
(a) Only S1 is conflict-serializable.
(b) Only S2 is conflict-serializable.
(c) Both S1 and S2 are conflict-serializable.
(d) Neither S1 nor S2 is conflict-serializable.

[2014 (Set-3) : 2 Marks]

5.19 Consider the following transaction involving two bank accounts x and y.

read (x) ; $x := x - 50$; write (x); read (y); $y := y + 50$; write (y)

The constraint that the sum of the accounts x and y should remain constant is that of
(a) Atomicity
(b) Consistency
(c) Isolation
(d) Durability

[2015 (Set-2) : 1 Mark]

5.20 Consider a simple checkpointing protocol and the following set of operations in the log.

(start, T4); (write, T4, y, 2, 3); (start, T1); (commit, T4); (write, T1, z, 5, 7);
(checkpoint);
(start, T2); (write, T2, x, 1, 9); (commit, T2); (start, T3); (write, T3, z, 7, 2);

If a crash happens now and the system tries to recover using both undo and redo operations, what are the contents of the undo list and the redo list
(a) Undo: T3, T1; Redo: T2
(b) Undo: T3, T1; Redo: T2, T4
(c) Undo: none; Redo: T2, T4, T3, T1
(d) Undo: T3, T1, T4; Redo: T2

[2015 (Set-2) : 1 Mark]

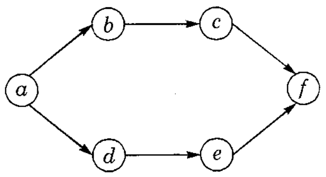
5.21 Consider the following partial Schedule S involving two transactions $T1$ and $T2$. Only the *read* and the *write* operations have been shown. The *read* operation on data item P is denoted by *read(P)* and the *write* operation on data item P is denoted by *write(P)*.

Time instance	Transaction-id	
	T1	T2
1	*read(A)*	
2	*write(A)*	
3		*read(C)*
4		*write(C)*
5		*read(B)*
6		*write(B)*
7		*read(A)*
8		*commit*
9	*read(B)*	

Suppose that the transaction $T1$ fails immediately after time instance 9. Which one of the following statements is correct?
(a) $T2$ must be aborted and then both $T1$ and $T2$ must be restarted to ensure transaction atomicity
(b) Schedule S is nonrecoverable and cannot ensure transaction atomicity
(c) Only $T2$ must be aborted and then restarted to ensure transaction atomicity
(d) Schedule S is recoverable and can ensure atomicity and nothing else needs to be done

[2015 (Set-3) : 2 Marks]

5.22 Consider the following directed graph:



The number of different topological ordering of the vertices of the graph is ______.

[2016 (Set-1) : 1 Mark]

5.23 Which one of the following is NOT a part of the ACID properties of database transactions?

(a) Atomicity (b) Consistency
(c) Isolation (d) Deadlock-freedom

[2016 (Set-1) : 1 Mark]

5.24 Consider the following two phase locking protocol. Suppose a transaction T accesses (for read or write operations), a certain set of objects $\{O_1,...,O_k\}$. This is done in the following manner:

Step 1. T acquires exclusive locks to $O_1,...,O_k$ in increasing order of their addresses.

Step 2. The required operations are performed.

Step 3. All locks are released.

This protocol will

(a) guarantee serializability and deadlock-freedom
(b) guarantee neither serializability nor deadlock-freedom
(c) guarantee serializability but not deadlock-freedom
(d) guarantee deadlock-freedom but not serializability

[2016 (Set-1) : 2 Marks]

5.25 Suppose a database schedule S involves transactions $T_1,..., T_n$. Construct the precedence graph of S with vertices representing the transactions and edges representing the conflicts. If S is serializable, which one of the following orderings of the vertices of the precedence graph is guaranteed to yield a serial schedule?

(a) Topological order
(b) Depth-first order
(c) Breadth-first order
(d) Ascending order of transaction indices

[2016 (Set-2) : 1 Mark]

5.26 Consider the following database schedule with two transactions, T_1 and T_2.

$S = r_2(X); r_1(X); r_2(Y); w_1(X); r_1(Y); w_2(X); a_1; a_2$

where $r_i(Z)$ denotes a *read* operation by transaction T_i on a variable Z, $w_i(Z)$ denotes a *write* operation by T_i on a variable Z and a_i denotes an *abort* by transation T_i.

Which one of the following statements about the above schedule is **TRUE**?

(a) S is non-recoverable
(b) S is recogverable, but has a cascading abort
(c) S does not have a cascading abort
(d) S is strict

[2016 (Set-2) : 2 Marks]

■■■■

Answers Transactions & Concurrency Control

5.1 (d)	5.2 (d)	5.3 (d)	5.4 (c)	5.5 (d)	5.6 (d)	5.7 (c)	5.8 (c)	5.9 (c)
5.10 (d)	5.11 (b)	5.12 (b)	5.13 (a)	5.14 (b)	5.15 (c)	5.16 (d)	5.17 (c)	5.18 (a)
5.19 (b)	5.20 (a)	5.21 (b)	5.23 (d)	5.24 (a)	5.25 (a)	5.26 (c)		

Explanations Transactions & Concurrency Control

5.1 (d)

Initial read of A is done by T_1 whereas final write of B is also done by T_1. Therefore $W_1(A)$, $R_2(A)$ and $W_2(B)$, $R_1(B)$ are conflicting pairs. The schedule is neither $T_1 \rightarrow T_2$ nor $T_2 \rightarrow T_1$ serializable. Since schedule is not serializable, it can't occur in a scheme using 2PL protocol.

5.2 (d)

A transaction reads a data item after it is written then we can't recover the errors in this situation by an uncommitted transaction.

5.3 (d)

We have sequence W2(D2) → W1(D1)
Which means T2 → T1 whereas we also have W1(D1) → R2(D1) which means T1 → T2. Both are not possible. Therefore, schedule is not serializable.

5.4 (c)

Row level locking provides more concurrency. Because different transactions can access different rows in a table/page at same time.

5.5 (d)

S is not serializable because T1 and T2 forms a cycle.

5.6 (d)

Irrespective of any failure the successful result of transaction should persist.

5.7 (c)

In data base transaction system if transaction is commit then it becomes permanent there is no effect of any failure so we need not redo log records 2 and 3 because transaction T_1 has committed.

5.8 (c)

Consider the table for transaction

time	S_1	S_2
↓	$r_1(x)$;	$r_1(x)$;
	$r_1(y)$;	$r_2(x)$;
	$r_2(x)$;	$r_2(y)$;
	$r_2(y)$;	$w_2(y)$;
	$w_2(y)$;	$r_1(y)$;
	$w_1(x)$;	$w_1(x)$;

There is no serializable conflict in transaction S_1 but for statement w_1 (x); there is a conflict in S_2 but it is also serializable.

5.9 (c)

Requirement to follow Strict 2PL :

1. Exclusive locks should be released after the commit .
2. No Locking can be done after the first Unlock and vice versa.

In 2PL, deadlock may occur but it may be that it doesn't occur at all.

Consider that in option (c) if both execute in serial order without concurrency then that is perfectly valid and YES it follows Strict 2PL.

5.10 (c)

(i) Schedule S1: 2RA, 2WA, 3RC, 2WB, 3WA, 3WC, 1RA, 1RB, 1WA, 1WB.

T1	T2	T3
	R(A)	
	W(A)	
		R(C)
	W(B)	
		W(A)
		W(C)
R(A)		
R(B)		
W(A)		
W(B)		

S1 is conflict serializable.
T2 → T3 → T1

(ii) Schedule S2: 3RC, 2RA, 2WA, 2WB, 3WA, 1RA, 1RB, 1WA, 1WB, 3WC.

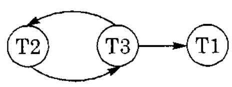
T1	T2	T3
		R(C)
	R(A)	
	W(A)	
	W(B)	
		W(A)
R(A)		
R(B)		
W(A)		
W(B)		
		W(C)

S2 is conflict serializable.
T2 → T3 → T1
S1 is conflict equivalent to S2.

(iii) Schedule S3: 2RA, 3RC, 3WA, 2WA, 3WB, 3WC,1RA, 1RB,1WA, 1WB.

T1	T2	T3
	R(A)	
		R(C)
		W(A)
	W(A)	
	W(B)	
		W(C)
R(A)		
R(B)		
W(A)		
W(B)		

S3 is not conflict serializable.



Therefore S1 and S2 are conflict cycle equivalent schedules but S3 is not equivalent to S1 and S3.

5.11 (b)

Schedule S2

T1	T2
R[X]	
	R[X]
	R[Y]
W[X]	
	W[Y]
W[Y]	

dependency graph 

S2 have no cycles.

Schedule S3

T1	T2
R[X]	
W[X]	
	R[X]
W[Y]	
	R[Y]
	W[Y]

dependency graph 

S3 also have no cycles.

So S2 & S3 are conflict-serializable.

5.12 (b)

In 2-phase locking concurrency control protocol it ensures the conflict serializable schedule but it may not free from deadlock. Ex.

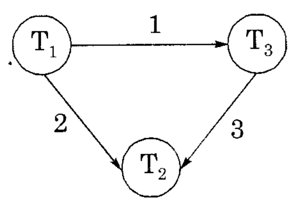
T_1	T_2
l(A)	l(A)
W(A)	W(B)
u(A)	
Deny → l(B)	l(A) ← Denied
W(B)	W(A)

T_1 in waiting T_2 for unlock

In time stamp ordering protocol it ensure conflict serializablity and free from dead lock.

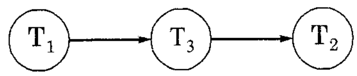
5.13 (a)

By using the precedence graph we solve thus problem



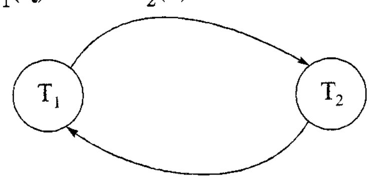
1. Read (x) in T_1 is followed by write (x) in T_2
2. Read (x) in T_1 is followed by write (x) in T_3
3. Read (y) in T_3 is followed by write (y) in T_2

So it in clear from precedence graph



5.14 (b)

T_1	T_2
r_1(P)	r_2(Q)
r_4(Q)	r_2(P)
w_1(Q)	w_2(P)



Cycle present so not conflict serializable.

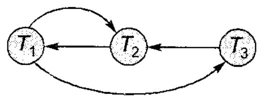
5.15 (c)

Clustered index sort the data in the table based on their key values of the column on which clustered index is created.

So option (c) is correct.

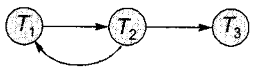
5.16 (d)

(a) $r_1(x); r_2(x); w_1(x); r_3(x); w_2(x)$



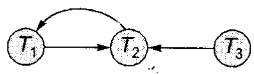
Contains cycle; Not conflict serializable

(b) $r_2(x); r_1(x); w_2(x); r_3(x); w_1(x)$



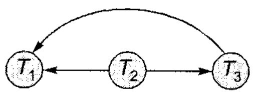
Contains cycle; Not conflict serializable

(c) $r_3(x); r_2(x); r_1(x); w_2(x); w_1(x)$



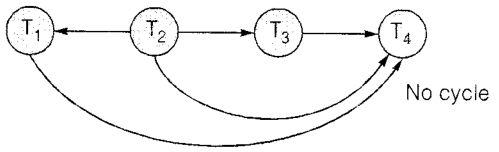
Contains cycle; Not conflict serializable

(d) $r_2(x); w_2(x); r_3(x); r_1(x); w_1(x)$



Not contains cycle; conflict serializable.

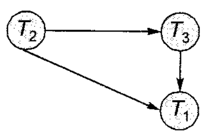
5.17 (c)



∴ Conflict serializable and it is also recoverable.

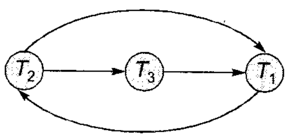
5.18 (a)

S1: $r1(X); r3(Y); r3(X); r2(Y); r2(Z); w3(Y); w2(Z); r1(Z); w1(X); w1(Z)$



No cycle ⇒ S1 is conflict serializable.

S2: $r1(X); r3(Y); r2(Y); r3(X); r1(Z); r2(Z); w3(Y); w1(X); w2(Z); w1(Z)$



Cycle ⇒ S2 is not conflict serializable.

5.19 (b)

Consistency ensures the given constraint.

5.20 (a)

The transactions that committed before checkpoint can be ignored as their updations are retained in data base.

Those transactions that are committed after checkpoint can be repeated (REDO). The transactions that are not committed after checkpoint have to be UNDONE UNDO : T3, T1 // T1 and T3 are not committed after checkpoint

REDO : T2 // T2 committed after checkpoint

5.21 (b)

T_1 writes on A.

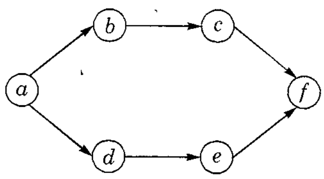
T_2 is performing read operation on A, and T_2 commits before T_1.

Hence it's DIRTY Read or Uncommitted Read.

Hence it's non-recoverable as T_2 is committed before T_1 commit.

It can not ensure transaction atomicity.

5.22 Sol.



Number of topological orders: 6

a	b	c	d	e	f
a	b	d	e	c	f
a	d	b	c	e	f
a	d	b	c	e	f
a	d	e	b	c	f
a	d	b	e	c	f

5.23 (d)

ACID stands for

A: Atomicity

C: Consistency

I: Isolation

D: Durability

So, deadlock-freedom is not the ACID property.

5.24 (a)

2PL over objects $O_1 \dots O_k$

Step-1: T acquires exclusive lock to $O_1 \dots O_k$ in increasing order of their address.

Step-2: The required operations are performed.

Step-3: All locks are released.

Because of 2PL it guarantee serializability and objects locks in increasing order of address and all objects locks before read/write which avoids deadlock.

5.25 (a)

For acyclic precedence graph of conflict serializable schedule equal serial schedule is topological order of acyclic precedence graph.

5.26 (c)

No uncommitted reads so that its casecadeless rollback recoverable because $T_1 w_1(x)$ before T_1 commit / Rollback $T_2 w_2(x)$.

So not strict recoverable.

T_1	T_2
	$r_2(x)$
$r_1(x)$	
	$r_2(y)$
$w_1(x)$	
$r_1(y)$	
	$w_2(x)$
a_1	
	a_2

6 File Structures

6.1 Which of the following is correct?

(a) B-trees are for storing data on disk and B^+ trees are for main memory.

(b) Range queries are faster on B^+ trees.

(c) B-trees are for primary indexes and B^+ trees are for secondary indexes.

(d) The height of a B^+ tree is independent of the number of records.

[1999 : 1 Mark]

6.2 B^+ –trees are preferred to binary trees in databases because

(a) Disk capacities are greater than memory capacities

(b) Disk access is much slower than memory access

(c) Disk data transfer rates are much less than memory data transfer rates

(d) Disks are more reliable than memory

[2000 : 1 Mark]

6.3 A B^+ tree index is to be built on the Name attribute of the relation STUDENT. Assume that all student names are of length 8 bytes, disk blocks are of size 512 bytes, and index pointers are of size 4 bytes. Given this scenario, what would be the best choice of the degree (i.e. the number of pointers per node) of the B^+ - tree?

(a) 16 (b) 42

(c) 43 (d) 44

[2002 : 2 Marks]

6.4 The order of an internal node in a B^+ tree index is the maximum number of children it can have. Suppose that a child pointer takes 6 bytes, the search field value takes 14 bytes, and the block size is 512 bytes. What is the order of the internal node?

(a) 24 (b) 25

(c) 26 (d) 27

[2004 : 2 Marks]

6.5 Consider a table T in a relational database with a key field K. A B-tree of order p is used as an access structure on K, where p denotes the maximum number of tree pointers in a B-tree index node. Assume that K is 10 bytes long; disk block size is 512 bytes; each data pointer P_D is 8 bytes long and each block pointer P_B is 5 bytes long. In order for each B-tree node to fit in a single disk block, the maximum value of p is

(a) 20 (b) 22

(c) 23 (d) 32

[2004 : 2 Marks]

6.6 Which one of the following is a key factor for preferring B^+-trees to binary search trees for indexing database relations?

(a) Database relations have a large number of record

(b) Database relations are sorted on the primary key

(c) B^+-trees require less memory than binary search trees

(d) Data transfer from disks is in blocks

[2005 : 1 Mark]

6.7 A B-Tree used as an index for a large database table has four levels including the root node. If a new key is inserted in this index, then the maximum number of nodes that could be newly created in the process are

(a) 5 (b) 4

(c) 3 (d) 2

[2005 : 1 Mark]

Linked Answer Questions 6.8 and 6.9:

A database table T1 has 2000 records and occupies 80 disk blocks. Another table T2 has 400 records and occupies 20 disk blocks. These two tables have to be joined as per a specified join condition that needs to be evaluated for every pair of records from these two tables. The memory buffer space available can hold exactly one block of records for T1 and one block of records for T2 simultaneously at any point in time. No index is available on either table.

6.8 If Nested-loop join algorithm is employed to perform the join, with the most appropriate choice of table to be used in outer loop, the number of block accesses required for reading the data are

(a) 800000 (b) 40080
(c) 32020 (d) 100

[2005 : 2 Marks]

6.9 If, instead of Nested-loop join, Block nested-loop join is used, again with the most appropriate choice of table in the outer loop, the reduction in number of block accesses required for reading the data will be

(a) 0 (b) 30400
(c) 38400 (d) 798400

[2005 : 2 Marks]

6.10 In a database file structure, the search key field is 9 bytes long, the block size is 512 bytes, a record pointer is 7 bytes and a block pointer is 6 bytes. The largest possible order of a non-leaf node in a B+ tree implementing this file structure is

(a) 23 (b) 24
(c) 34 (d) 44

[2006 : 2 Marks]

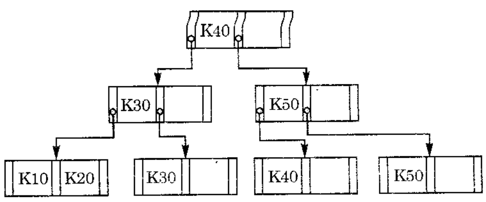
6.11 The order of a leaf node in a B^+–tree is the maximum number of (value, data record pointer) pairs it can hold. Given that the block size is 1K bytes, data record pointer is 7 bytes long, the value field is 9 bytes long and a block pointer is 6 bytes long, what is the order of the leaf node?

(a) 63 (b) 64
(c) 67 (d) 68

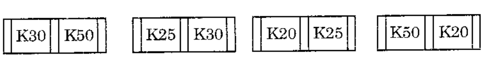
[2007 : 2 Marks]

Directions for question 6.12 to 6.13:

Consider the B^+ tree in the adjoining figure, where each node has at most two keys and three links.



6.12 Keys K15 and then K25 are inserted into this tree in that order. Exactly how many of the following nodes (disregarding the links) will be present in the tree after the two insertions?



(a) 1 (b) 2
(c) 3 (d) 4

[2007 : 2 Marks]

6.13 Now the key K50 is deleted from the B^+ tree resulting after the two insertions made earlier. Consider the following statements about the B^+ tree resulting after this deletion.

(i) The height of the tree remains the same.
(ii) The node $\boxed{k20}$ (disregarding the links) is present in the tree.
(iii) The root node remains unchanged (disregarding the links).

Which one of the following options is true ?

(a) Statements (i) and (ii) are true
(b) Statements (ii) and (iii) are true
(c) Statements (iii) and (i) are true
(d) All the statements are false

[2007 : 2 Marks]

6.14 A clustering index is defined on the fields which are of type

(a) Non-key and ordering
(b) Non-key and non-ordering
(c) Key and ordering
(d) Key and non-ordering

[2008 : 1 Mark]

6.15 Consider a file of 16384 records. Each record is 32 bytes long and its key field is of size 6 bytes. The file is ordered on a non-key field, and the file organization is unspanned. The file is stored in a file system with block size 1024 bytes, and the size of a block pointer is 10 bytes. If the secondary index is built on the key field of the file, and a multilevel index scheme is used to store the secondary index, the number of first-level and second-level blocks in the multilevel index are respectively

(a) 8 and 0 (b) 128 and 6
(c) 256 and 4 (d) 512 and 5

[2008 : 2 Marks]

6.16 The following key values are inserted into a B^+ - tree in which order of the internal nodes is 3, and that of the leaf nodes is 2, in the sequence given below. The order of internal nodes is the maximum number of tree pointers in each node, and the order of leaf nodes is the maximum number of data items that can be stored in it. The B^+ - tree is initially empty.

10, 3, 6, 8, 4, 2, 1

The maximum number of times leaf nodes would get split up as a result of these insertions is

(a) 2 (b) 3
(c) 4 (d) 5

[2009 : 2 Marks]

6.17 Consider a B^+-tree in which the maximum number of keys in a node is 5. What is the minimum number of keys in any non-root node?

(a) 1 (b) 2
(c) 3 (d) 4

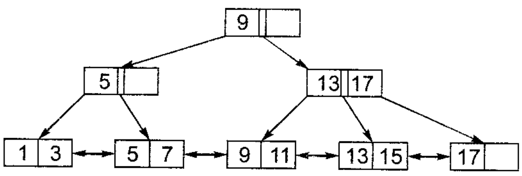
[2010 : 1 Mark]

6.18 A file is organized so that the ordering of data records is the same as or close to the ordering of data entries in some index. Then that index is called

(a) Dense (b) Sparse
(c) Clustered (d) Unclustered

[2015 (Set-1) : 1 Mark]

6.19 With reference to the B^+ tree index of order 1 shown below, the minimum number of nodes (including the Root node) that must be fetched in order to satisfy the following query: "Get all records with a search key greater than or equal to 7 and less than 15" is _______.



[2015 (Set-2) : 1 Mark]

6.20 Consider a B^+ tree in which the search key is 12 bytes long, block size is 1024 bytes, record pointer is 10 bytes long and block pointer is 8 bytes long. The maximum number of keys that can be accommodated in each non-leaf node of the tree is _______.

[2015 (Set-3) : 2 Marks]

6.21 B^+ Trees are considered BALANCED because

(a) the lengths of the paths from the root to all leaf nodes are all equal.
(b) the lengths of the paths from the root to all leaf nodes differ from each other by at most 1.
(c) the number of children of any two non-leaf sibling nodes differ by at most 1.
(d) the number of records in any two leaf nodes differ by at most 1.

[2016 (Set-2) : 1 Mark]

■■■■

Answers Transactions & Concurrency Control

6.1	**(b)**	**6.2**	**(b)**	**6.3**	**(c)**	**6.4**	**(c)**	**6.5**	**(c)**	**6.6**	**(d)**	**6.7**	**(a)**	**6.8**	**(c)**	**6.9**	**(b)**
6.10	**(c)**	**6.11**	**(a)**	**6.12**	**(a)**	**6.13**	**(a)**	**6.14**	**(a)**	**6.15**	**(c)**	**6.16**	**(a)**	**6.17**	**(b)**	**6.18**	**(c)**
6.21	**(a)**																

Explanations File Structures

6.1 (b)

Most database systems use indexes built on some form of a B^+-tree due to its many advantages, in particular its support for range queries. Leaf nodes are linked together in B^+ trees hence range queries are faster.

6.2 (b)

Disk access is slow and B^+ tree provide search in less number of disk hits. This is primarily because unlike binary search trees, B^+ trees have very high fanout (typically on the order of 100 or more), which reduces the number of I/O operations required to find an element in the tree.

6.3 (c)

Let n be the degree

Given, k, key size (length of the name = 8 byte attribute of student)

Disk block size, B = 512 bytes

Index pointer size, b = 4 bytes

Degree of B^+ tree can be calculated if we know the maximum number of key a internal node can have the formula for that is

$(n - 1)\ k + n * b = \text{block size}$

$(n - 1)\ 8 + n * 4 = 512$

$12n = 520$

$$n = \frac{520}{12} = 43$$

So, the answer is option (c).

6.4 (c)

Size of child pointer = 6 byte

Size of search field value takes 14 bytes

Block size = 512

The order of internal node = P

$\Rightarrow (P - 1)14 + P * 6 \leq 512$

$P = 26$

6.5 (c)

(p – 1) (key_ptr_size + record_ptr_size) + p(block_ptr_size) ≤ 512

Therefore p = 23.

6.6 (d)

The transfer of data from disk to primary memory is in the form of data blocks, if a data block is larger than indexing is easy due to this B^+ tree is better than binary search tree data structure if large amount of data can be access.

6.7 (a)

Suppose all nodes are completely full that means every node has n – 1 keys.

Tree has 4 levels. If a new key is inserted then at every level new node will be created.

In worst case root node will also be broken into two parts and we have 4 levels.

So answer should be 5 because tree will be increased with one more level.

6.8 (c)

Nested Loop algorithm will involve $n_r * b_s + b_r$ block transfers.

Either T1 can be R or T2.

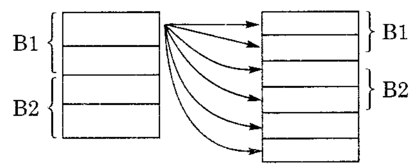
If R is T1 then total number of block access is 2000*20 + 80 = 40080

If R is T2 then total number of block access is 400*80 + 20 = 32020

Better one is the second case, total number of block accesses = (32020).

6.9 (b)

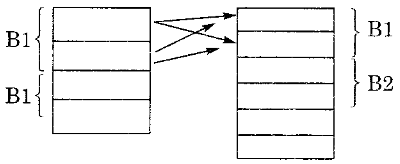
In Nested loop join for each tuple in first table we scan through all the tuples in second table.



Here we will take table T2 as the outer table in nested loop join algorithm. The number of block accesses then will be

$$20 + (400 \times 80) = 32020.$$

In block nested loop join we keep 1 block of T1 in memory and 1 block of T2 in memory and do join on tuples.



For every block in T1 we need to load all blocks of T2. So number of block accesses is

$$80*20 + 20 = 1620$$

The difference is $32020 - 1620 = 30400$

6.10 (c)

From the structure of B+ tree we can get the following equation:

$n*p + (n - 1)*(k + q) \leq B$

where n = order,

p = tree/block/index pointer,

q = record/data pointer,

B = size of block

In B+ tree, non leaf node has no record pointer: (by putting $q = 0$)

$n*p + (n - 1)k \leq B$

$n*6 + (n - 1)*9 \leq 512$

$n \leq 34.77$

Largest possible order of a non-leaf node = 34.

6.11 (a)

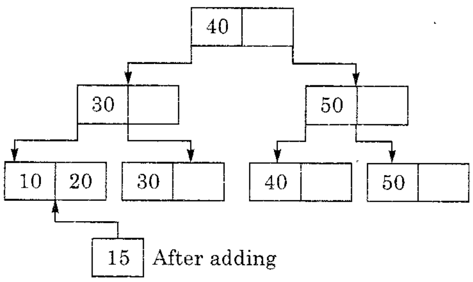
Let the order of leaf node is n.

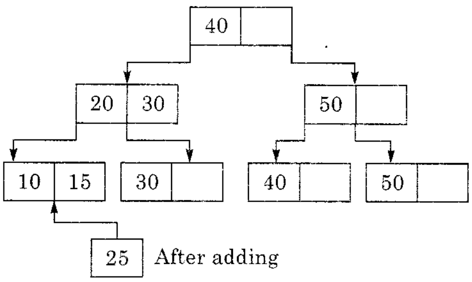
Block size = 1K = 1024 bytes

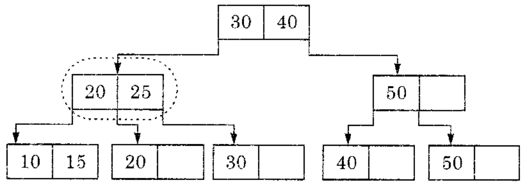
So, $6 + 7n + (n)\,9 \leq 1024$

$n = 63$

6.12 (a)







Matching node is $\boxed{20 \mid 25}$ which is one only.

6.13 (a)

Only (i) and (ii) are correct.

After deleting 50 from the tree we are left with node (20,40) with 40 having no right subtree except 40 itself. Nodes can't be combined because that would overflow the node as they are already half-full or full.

So key 40 can be out in node containing 30. Height remains same with 20 at root.

6.14 (a)

If records of a file are physically ordered on a nonkey field which doesn't have a distinct value for each record that field is called the clustering field.

6.15 (c)

Content of index < key, BP > = 6 + 10 = 16

In first level entry for each record,

Total size = 16384×16.

Number of blocks in first level

= Size of first level index / Block size

$$= \frac{16384}{1024} \times 256$$

In second level

Total size = No. of entries × Size of entry

$= 256 \times 16$

Number of blocks in second level

= Size of second level index / Block size

$= 256 \times 16 / 1024 = 4$

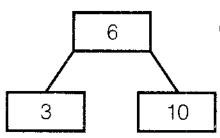
6.16 (a)

B^+-tree is initially empty

insert 10 : $\boxed{10}$

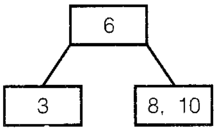
insert 3 : $\boxed{3,\ 10}$

insert 6 :

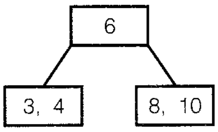


one split

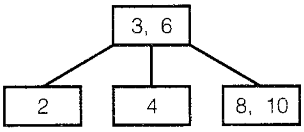
insert 8 :



insert 4 :

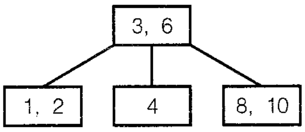


insert 2 :



one more split

insert 1 :



So total splits = 2.

6.17 (b)

In a B^+ tree the root node have minimum two block pointer and maximum P block pointer where P = order, key = order – 1 and in the non-root node the minimum number of keys $= \left\lceil \frac{P}{2} \right\rceil - 1$

So in the question key = 5, order = 6

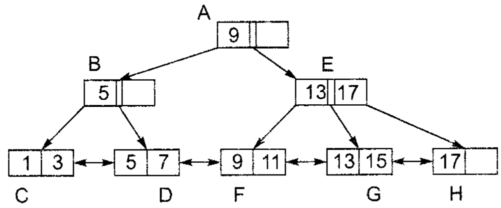
So minimum number of keys in non-root node =

$\left\lceil \frac{6}{2} \right\rceil - 1 = 2$

6.18 (c)

File is organized so that the ordering of data records is the same as or close to the ordering of data entries in some index. Then that index is called clustered index.

6.19 (5)



We need three node accesses to search for key 7.

In B^+ tree all neighbouring leaf nodes are connected (Via block pointers) and it is possible to access remaining records after 7 using linear search for records. Hence two more nodes needed to fetch records less than 15.

Nodes A, B, D, F and G are needed to get all records greater than or equal to 7 and less than 15.

6.20 Sol.

For Non-leaf node:

$p \times \text{key size} + (p + 1)\, B_p \leq \text{block size}$

$p \times 12 + (p + 1)8 \leq 1024$

$20p \leq 1016$

$p \leq \lceil 50.8 \rceil$

$p = 51$

Number of keys $= p - 1 = 51 - 1 = 50$

6.21 (a)

B^+ tree balanced because the length of the paths from the root to all leaf nodes are all equal and every internal node must filled by.

■■■■

Unit ■ X

Computer Networks

■ Contents

UNIT X

Computer Networks

Syllabus : ISO/OSI stack, LAN technologies (Ethernet, Token ring), Flow and error control techniques, Routing algorithms, Congestion control, TCP/UDP and sockets, IP (v4), Application layer protocols (icmps, dns, smtp, pop, ftp, http); Basic concepts of hubs, switches, gateways and routers.

Network Security: Basic concepts of public key and private key cryptography, digital signature, firewalls.

Analysis of Previous GATE Papers

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2003	2	3	**8**
2004	3	4	**11**
2005	5	2	**15**
2006	1	5	**11**
2007	2	6	**14**
2008	1	4	**9**
2009	–	5	**10**
2010	2	3	**8**
2011	2	2	**6**
2012	3	3	**9**
2013	3	2	**7**
2014 Set-1	2	3	**8**
2014 Set-2	3	2	**7**
2014 Set-3	3	3	**9**
2015 Set-1	4	2	**8**
2015 Set-2	2	3	**8**
2015 Set-3	2	3	**8**
2016 Set-1	2	4	**10**
2016 Set-2	3	4	**11**

1 ISO/OSI Stack and SWP

1.1 Host A is sending data to host B over a full duplex link. A and B are using the sliding window protocol for flow control. The send and receive window sizes are 5 packets each. Data packets (sent only from A to B) are all 1000 bytes long and the transmission time for such a packet is 50 µs. Acknowledgment packets (sent only from B to A) are very small and require negligible transmission time. The propagation delay over the link is 200 µs. What is the maximum achievable throughput in this communication?

(a) 7.69×10^6 bps (b) 11.11×10^6 bps
(c) 12.33×10^6 bps (d) 15.00×10^6 bps

[2003 : 2 Marks]

1.2 Choose the best matching between **List-I** and **List-II**.

List-I

P. Data link layer
Q. Network layer
R. Transport layer

List-II

1. Ensures reliable transport of data over a physical point-to-point link
2. Encodes / decodes data for physical transmission
3. Allow end-to-end communication between two processes
4. Routes data from one network node to the next

	A	B	C
(a)	1	4	3
(b)	2	4	1
(c)	2	3	1
(d)	1	3	2

[2004 : 1 Mark]

1.3 How many 8-bit characters can be transmitted per second over a 9600 baud serial communication link using asynchronous mode of transmission with one start bit, eight data bits, and one parity bit and two stop bits?

(a) 600 (b) 800
(c) 876 (d) 1200

[2004 : 1 Mark]

1.4 A serial transmission T1 uses 8 information bits, 2 start bits, 1 stop bit and 1 parity bit for each character. A synchronous transmission T2 uses 3 eight-bit sync characters followed by 30 eight-bit information characters. If the bit rate is 1200 bits/second in both cases, what are the transfer rates of T1 and T2?

(a) 100 characters/sec, 153 characters/sec
(b) 80 characters/sec, 136 characters/sec
(c) 100 characters/sec, 136 characters/sec
(d) 80 characters/sec, 153 characters/sec

[2004 : 2 Marks]

1.5 In a data link protocol, the frame delimiter flag is given by 0111. Assuming that bit stuffing is employed, the transmitter sends the data sequence 01110110 as

(a) 01101011 (b) 011010110
(c) 011101100 (d) 0110101100

[2004 : 2 Marks]

1.6 In a sliding window ARQ scheme, the transmitter's window size is N and the receiver's window size is M. The minimum number of distinct sequence numbers required to ensure correct operation of the ARQ scheme is

(a) min (M, N) (b) max (M, N)
(c) M + N (d) MN

[2004 : 2 Marks]

1.7 A 20 Kbps satellite link has a propagation delay of 400 ms. The transmitter employs the "go back n ARQ" scheme with n set to 10. Assuming that each frame is 100 bytes long, what is the maximum data rate possible?

(a) 5 Kbps (b) 10 Kbps
(c) 15 Kbps (d) 20 Kbps

[2004 : 2 Marks]

1.8 Consider a parity check code with three data bits and four parity check bits. Three of the code words are 0101011, 1001101 and 1110001.

Which of the following are also code words?

I. 0010111 II. 0110110
III. 1011010 IV. 0111010

(a) I and III (b) I, II and III
(c) II and IV (d) I, II, III and IV

[2004 : 2 Marks]

1.9 The maximum window size for data transmission using the selective reject protocol with n-bit frame sequence numbers is

(a) 2^n (b) 2^{n-1}
(c) $2^n - 1$ (d) 2^{n-2}

[2005 : 1 Mark]

1.10 Consider the following message M = 1010001101. The cyclic redundancy check (CRC) for this message using the divisor polynomial $x^5 + x^4 + x^2 + 1$ is :

(a) 01110 (b) 01011
(c) 10101 (d) 10110

[2005 : 2 Marks]

1.11 A channel has a bit rate of 4 kbps and one-way propagation delay of 20 ms. The channel uses stop and wait protocol. The transmission time of the acknowledgement frame is negligible. To get a channel efficiency of at least 50%, the minimum frame size should be

(a) 80 bytes (b) 80 bits
(c) 160 bytes (d) 160 bits

[2005 : 2 Marks]

1.12 In the 4B/5B encoding scheme, every 4 bits of data are encoded in a 5-bit codeword. It is required that the codewords have at most 1 leading and at most 1 trailing zero. How many such codewords are possible?

(a) 14 (b) 16
(c) 18 (d) 20

[2006 : 2 Marks]

1.13 On a wireless link, the probability of packet error is 0.2. A stop-and-wait protocol is used to transfer data across the link. The channel condition is assumed to be independent from transmission to transmission. What is the average number of transmission attempts required to transfer 100 packets?

(a) 100 (b) 125
(c) 150 (d) 200

[2006 : 2 Marks]

1.14 Station A uses 32 byte packets to transmit messages to Station B using a sliding window protocol. The round trip delay between A and B is 80 milliseconds and the bottleneck bandwidth on the path between A and B is 128 kbps. What is the optimal window size that A should use?

(a) 20 (b) 40
(c) 160 (d) 320

[2006 : 2 Marks]

1.15 Station A needs to send a message consisting of 9 packets to Station B using a siding window (window size 3) and go-back-n error control strategy. All packets are ready and immediately available for transmission. If every 5^{th} packet that A transmits gets lost (but no acks from B ever get lost), then what is the number of packets that A will transmit for sending the message to B?

(a) 12 (b) 14
(c) 16 (d) 18

[2006 : 2 Marks]

1.16 The message 11001001 is to be transmitted using the CRC polynomial $x^3 + 1$ to protect it from errors. The message that should be transmitted is:

(a) 11001001000 (b) 11001001011
(c) 11001010 (d) 110010010011

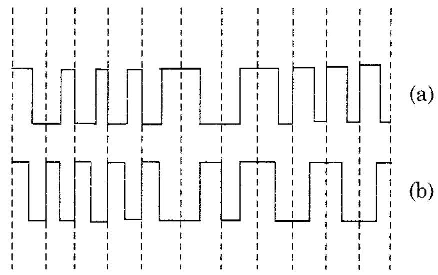
[2007 : 2 Marks]

1.17 An error correcting code has the following code words: 00000000, 00001111, 01010101, 10101010, 11110000. What is the maximum number of bit errors that can be corrected ?

(a) 0 (b) 1
(c) 2 (d) 3

[2007 : 2 Marks]

1.18 In the waveform (a) given below, a bit stream is encoded by Manchester encoding scheme. The same bit stream is encoded in a different coding scheme in wave form (b). The bit stream and the coding scheme are



(a) 1000010111 and Differential Manchester respectively
(b) 0111101000 and Differential Manchester respectively
(c) 1000010111 and Integral Manchester respectively
(d) 0111101000 and Integral Manchester respectively

[2007 : 2 Marks]

1.19 Let us consider a statistical time division multiplexing of packets. The number of sources is 10. In a time unit, a source transmits a packet of 1000 bits. The number of sources sending data for the first 20 time units is 6, 9, 3, 7, 2, 2, 2, 3, 4, 6, 1, 10, 7, 5, 8, 3, 6, 2, 9, 5 respectively. The output capacity of multiplexer is 5000 bits per time unit. Then the average number of backlogged of packets per time unit during the given period is

(a) 5 (b) 4.45
(c) 3.45 (d) 0

[2007 : 2 Marks]

1.20 Your are given the following four bytes :

10100011 00110111 11101001 10101011

Which of the following are substrings of the base 64 encoding of the above four bytes?

(a) zdp (b) fpq
(c) qwA (d) oze

[2007 : 2 Marks]

1.21 How many bytes of data can be sent in 15 seconds over a serial link with baud rate of 9600 in asynchronous mode with odd parity and two stop bits in the frame?

(a) 10,000 bytes (b) 12,000 bytes
(c) 15,000 bytes (d) 27,000 bytes

[2008 : 1 Mark]

1.22 A 1 Mbps satellite link connects two ground stations. The altitude of the satellite is 36,504 km and speed of the signal is 3×10^8 m/s. What should be the packet size for a channel utilization of 25% for a satellite link using go-back-127 sliding window protocol?

Assume that the acknowledgment packets are negligible in size and that there are no errors during communication.

(a) 120 bytes (b) 60 bytes
(c) 240 bytes (d) 90 bytes

[2008 : 2 Marks]

1.23 Data transmitted on a link uses the following 2D parity scheme for error detection:

Each sequence of 28 bits is arranged in a 4×7 matrix (rows r_0 through r_3, and columns d_7 through d_1) and is padded with a column d_0 and row r_4 of parity bits computed using the Even parity scheme. Each bit of column d_0 (respectively, row r_4) gives the parity of the corresponding row (respectively, column). These 40 bits are transmitted over the data link.

	d_7	d_6	d_5	d_4	d_3	d_2	d_1	d_0
r_0	0	1	0	1	0	0	1	1
r_1	1	1	0	0	1	1	1	0
r_2	0	0	0	1	0	1	0	0
r_3	0	1	1	0	1	0	1	0
r_4	1	1	0	0	0	1	1	0

The table shows data received by a receiver and has n corrupted bits. What is the minimum possible value of n?

(a) 1 (b) 2
(c) 3 (d) 4

[2008 : 2 Marks]

1.24 Let G(x) be the generator polynomial used for CRC checking. What is the condition that should be satisfied by G(x) to detect odd number of bits in error?

(a) G(x) contains more than two terms
(b) G(x) does not divide $1 + x^k$, for any k not exceeding the frame length
(c) 1 + x is a factor of G(x)
(d) G(x) has an odd number of terms

[2009 : 2 Marks]

Linked Answer for Q.1.25 and Q. 1.26

Frames of 1000 bits are sent over a 10^6 bps duplex link between two hosts. The propagation time is 25 ms. Frames are to be transmitted into this link to maximally pack them in transit (within the link).

1.25 What is the minimum number of bits (l) that will be required to represent the sequence numbers distinctly? Assume that no time gap needs to be given between transmission of two frames.

(a) $l = 2$ (b) $l = 3$
(c) $l = 4$ (d) $l = 5$

[2009 : 2 Marks]

1.26 Suppose that the sliding window protocol is used with the sender window size of 2^l, where l is the number of bits identified in the earlier part and acknowledgments are always piggy backed. After sending 2^l frames, what is the minimum time the sender will have to wait before starting transmission of the next frame? (Identify the closest choice ignoring the frame processing time).

(a) 16 ms (b) 18 ms
(c) 20 ms (d) 22 ms

[2009 : 2 Marks]

1.27 In the following pairs of OSI protocol layer/sub-layer and its functionality, the **INCORRECT** pair is

(a) Network layer and Routing
(b) Data Link Layer and Bit synchronization
(c) Transport layer and End-to-end process communication
(d) Medium Access Control sub-layer and Channel sharing

[2014 (Set-3) : 1 Mark]

1.28 A bit-stuffing based framing protocol uses an 8-bit delimiter pattern of 01111110. If the output bit-string after stuffing is 01111100101, then the input bit-string is

(a) 0111110100 (b) 0111110101
(c) 0111111101 (d) 0111111111

[2014 (Set-3) : 1 Mark]

1.29 Suppose that the stop-and-wait protocol is used on a link with a bit rate of 64 kilobits per second and 20 milliseconds propagation delay. Assume that the transmission time for the acknowledgment and the processing time at nodes are negligible. Then the minimum frame size in bytes to achieve a link utilization of at least 50% is ________.

[2015 (Set-1) : 2 Marks]

1.30 A link has a transmission speed of 10^6 bits/sec. It uses data packets of size 1000 bytes each. Assume that the acknowledgment has negligible transmission delay, and that its propagation delay is the same as the data propagation delay. Also assume that the processing delays at nodes are negligible. The efficiency of the stop-and-wait protocol in this setup is exactly 25%. The value of the one-way propagation delay (in milliseconds) is ________.

[2015 (Set-2) : 1 Mark]

1.31 Consider a network connecting two systems located 8000 kilometers apart. The bandwidth of the network is 500×10^6 bits per second. The propagation speed of the media is 4×10^6 meters per second. It is needed to design a Go-Back-N sliding window protocol for this network. The average packet size is 10^7 bits. The network is to be used to its full capacity.

Assume that processing delays at nodes are negligible. Then, the minimum size in bits of the sequence number field has to be _____.

[2015 (Set-3) : 2 Marks]

1.32 Two hosts are connected via a packet switch with 10^7 bits per second links. Each link has a propagation delay of 20 microseconds. The switch begins forwarding a packet 35 microseconds after it receives the same. If 10000 bits of data are to be transmitted between the two hosts using a packet size of 5000 bits, the time elapsed between the transmission of the first bit of data and the reception of the last bit of the data in microseconds is ________.

[2015 (Set-3) : 2 Marks]

■■■■

Answers ISO/OSI Stack and SWP

1.1	(b)	1.2	(a)	1.3	(b)	1.4	(c)	1.5	(d)	1.6	(c)	1.7	(b)	1.8	(b)	1.9	(b)
1.10	(a)	1.11	(d)	1.12	(c)	1.13	(b)	1.14	(b)	1.15	(c)	1.16	(b)	1.17	(d)	1.18	(c)
1.19	(c)	1.21	(c)	1.22	(c)	1.23	(c)	1.24	(c)	1.25	(d)	1.26	(b)	1.27	(b)	1.28	(b)

Explanations ISO/OSI Stack and SWP

1.1 (b)

Given

Window size

$n = 5$ packets

Packet size = 1000 byte

Total packet size = $5 \times 1000 = 5000$ bytes

Total time = Transmission Time + Propagation Time

$= 5 \times 50 + 200\ \mu s$

$= 250 + 200\ \mu s$

$= 450\ \mu s$

$= 450 \times 10^{-6}$ s

Maximum Achievable throughput

$$= \frac{\text{Total Size}}{\text{Total Time}}$$

$$= \frac{5000}{450 \times 10^{-6}}$$

$$= \frac{5000 \times 10^{6}}{450}$$

$= 11.11 \times 10^{6}$ bps

1.2 (a)

P. Datal link layer:

1. Ensures reliable transport of data over a physical point-to point link- Reliable mens error correction & detection done by Data link layer.

Q. Network layer:

4. Routes data from one network node to the next according to routing algorithm.

R. Transport layer:

3. Allows end-to- end communication between two processes with the help of TCP and UDP protocol.

1.2 (b)

Total number of bits = 12

modulation Rate = 9600 baud

Number of characters (8 bit character) are transmitted $= \frac{9600}{12\text{bits}} = 800$

1.4 (c)

T_1: 1 character = (8 + 2 + 1 + 1) = 12 bit

Transfer Rate = 1200/12 = 100 char/sec.

T_2: Transfer character in bits = 24 + 240 = 264 bits

If 264 bits for 30 characters,

Then 1200 bits = ?

264/30 = 1200/X

Therefore X = 136.3 character/sec.

1.5 (d)

Three consecutive ones are used for delimiter so whenever in data two consecutive one comes stuff a zero after them. Data is 01110110, After stuffing 0110101100.

1.6 (c)

M+N: Because $W_s + W_r \leq$ Sequence numbers (because the maximum number of unacknowledged packets at sender will be W_s and at the receiver it will be W_r, similar to the sequence numbering in Selective Repeat) where W_s is size of sender window and W_r is receiver window's size.

1.7 (b)

Tx = 100*8 bits /20 Kbps = 40 ms

Tp = 400ms,

a = Tp / Tx = 400/40 = 10

Efficiency of GBN = W/(1 + 2a),

where w = window size

= 10 = 10/(1+20) = 10/21

BW utilization or throughput or max data rate = efficiency * BW = (10/21)*20

It is nearly 10 Kbps

1.9 (b)

In case of the selective Reject Protocol, the maximum window size $= \frac{2^n}{2} = 2^{n-1}$

In the case of Selective Reject the window size will be half.

1.10 (a)

Generator polynomial is of degree 5 so append 5 0's to the end of data and then divide new data by generator polynomial $x^5 + x^4 + x^2 + 1$ = 110101.

```
110101 ) 101000110100000 (
         110101
         0111011
          110101
          0111011
           110101
           00111110
             110101
             00101100
               110101
               0110010
                110101
                0001110
```

Remainder is 01110

1.11 (d)

Efficiency of stop and wait = 1/(1 + 2a).
If 1/(1 + 2a) = 0.5
$\Rightarrow 2*T_p = T_x$
$\Rightarrow L = 2*B*T_p = 160$ bits.

1.12 (c)

It says we have 5 bit codeword such that "it can't have two consecutive zeros in first and second bit" and also" can't have two consecutive zeros in last two bits.
Code word with first two bits zero = 0 | 0 | x | x | x |
= 8
Code word with last two bits zero = | x | x | x | 0 | 0 |
= 8
Code word with first and last two bits zero = 0 | 0 | x | 0 | 0 | = 2
Code word with first OR last two bits zero = 8 + 8 – 2 = 14
Therefore possible codewords = 32 –14 = 18

1.13 (b)

Error rate is 0.2
In stop and wait protocol: sender will transmits $100*(1+(0.2)^1 + (0.2)^2 + (0.2)^3 + (0.2)^4 +...)$ packets.
= 100 *(1/(1 – 0.02)) = 100/0.8
= 125 (sum of infinite G.P. is a/(a–r))

1.14 (b)

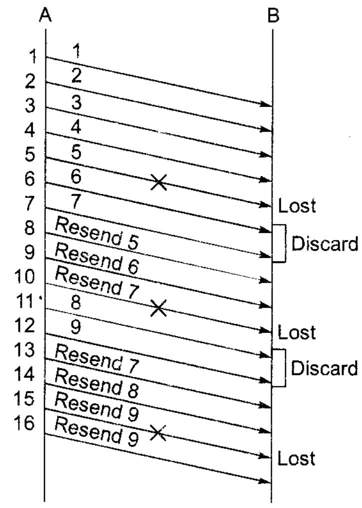
Given round trip delay t = 80 ms

$$= 80 \times 10^{-3} \text{ sec}$$
$$R = 128 \text{ kbps}$$
$$= 128 \times 10^3 \text{ bps}$$
$$L = Rt$$
$$= 128 \times 10^3 \times 80 \times 10^{-3}$$
$$= 128 \times 80$$

So, optional window size = n

$$= \frac{128 \times 80}{32 \times 8} = \frac{10240}{256}$$
$$n = 40$$

1.15 (c)



1.16 (b)

P(x) = 11001001
divisor D(x) = 1001 and CRC remainder is 011.
So the transmitted message is 11001001011.

1.17 (d)

The maximum hamming distance among the gives code words is 8. i.e.

```
  01010101
+ 10101010
  11111111
```

Th maximum number of bit errors that can be corrected are 'd'.
The hamming distance to correct the 'd' errors should be 2d+1.
$\therefore 2d + 1 = 8$
$d \simeq 3$
$\therefore$ Maximum 3 bit errors can be corrected.

1.18 (c)

'0' – Low to high.
'1' – High to low.
There is a transition at the middle of each bit period.
$\therefore$ The bit pattern is 100 00 10111.

1.19 (c)

In statistical TDM, the bandwidth is divided into slots each for a source if the source requires. There is no dedicated slot for each source in the bandwidth. STDM do not reserve a time slot for each terminal, rather it assign a slot when the terminal is requires the slot to send its data.
Multiplexer bandwidth = 5000 bits per sec.
If the number sources sending are greater than 5 no body can send and packets are back logged.
∴ Avg number of block logged packets are:
= 6 + 9 + 7 + 6 + 10 + 7 + 8 + 9 / 20
= 3.4

1.22 (c)

Efficiency for a sliding window protocol is

$$\eta = \frac{N}{1+2a}$$

where N is the window size.
⇒ N = 127 − 1 = 126
Utilization is given as 0.25

$$\therefore \quad 0.25 = \frac{126}{1+2a} \qquad ...(1)$$

$$\text{where } a = \frac{t_p}{t_d}$$

$$t_d = \frac{x \text{ bytes}}{1 \text{ Mbps}}$$

$$t_p = \frac{2 \times 36504 \times 10^3}{3 \times 10^3}$$

Substituting in 1 and solving for x we get $x \simeq$ 240.

1.24 (c)

The polynomial generator used for CRC checking must satisfy at least two condition to detect odd numbers of errors:

1. It should be not divisible by x.
2. It should be divisible by 1 + x.

Therefore (1 + x) is a factor of G(x).

1.25 (d)

The link is a Duplex hence we need not wait for twice the propagation time for sending the frame belonging to next window. If the sender window is of size N.
Transmitting 10^6 bits require = 1 sec

$$N \times 1000 \text{ bits require} = \frac{1}{10^6} N \times 10^3$$

$$= N \times 10^{-3} \text{ sec} = N \text{ m sec}$$

Nm sec = 25 m sec, N = 25 < 2^5
∴ Minimum number of bits required is 5 to represent sequence numbers disinctly.

1.26 (b)

Time taken to send10^6 bits = 1 sec
∴ Time taken to send 2^5 frames = 32 m sec (1 frame = 1000 bits)
Time taken for the first frame to be acknowledged = 25 × 2 = 50 ms
Then waiting time = 50 – 32 = 18 ms

1.27 (b)

Bit synchronization is in physical layer.

1.28 (b)

Input string: 0111110101
Output string: 01111100101
After five consecutive 1's in the input, bit 0 is inserted.

1.29 Sol.

For atleast 50% efficiency: t_d = 2 p_d
∴ Minimum frame size = 64 kbps × 40 ms
= 2560 bits = 320 bytes.

1.30 Sol.

$$\text{Efficiency of Stop \& Wait} = \frac{1}{1+2a}$$

$$\Rightarrow \quad 0.25 = \frac{1}{1+2a}$$

$$a = \frac{p_d}{t_d}, \; t_d = 8 \text{ ms}$$

$$\therefore \; 0.25 = \frac{1}{1+2\times\frac{p_d}{8\,\text{ms}}} \quad \Rightarrow p_d = 12 \text{ ms}$$

1.31 Sol.

$$d = 8000 \text{ km.}$$

$$\text{Band width} = 500 \times 10^6 \text{ bps}$$

Propagation delay

$$P_d = \frac{8000 \times 10^3}{4 \times 10^6} = 2 \text{ sec}$$

$$\text{Packet size} = 10^7 \text{ bits}$$

Transmission delay

$$t_d = \frac{10^7}{500 \times 10^6}$$

$$= \frac{1}{50} \text{ sec}$$

$$n = 100\%$$

$$\frac{100}{100} = \frac{N}{1+2a}$$

$$a = \frac{P_d}{t_d} = \frac{2}{1/50} \text{ sec} = 100$$

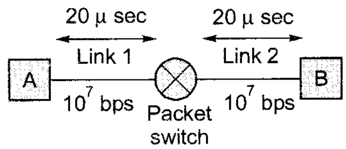
$$\frac{100}{100} = \frac{N}{201}$$

$$N = 201$$

$\Rightarrow$ Number of packets = 201

$\therefore$ 8 bits are required for sequence number.

1.32 Sol.



Extra delay at switch = 35 μsec for each packet.

Data = 10000 bits

$$\text{Number of packets} = \frac{10000}{5000} = 2 \text{ packet}$$

Transmission delay for one packet = 500 μsec.

At t = **500** μsec, last bit of packet 1 is placed on link 1 by A and Transmission of packet begins.

At t = **520**, last bit of packet 1 reaches switch.

At t = **555**, first bit of packet 1 is placed on link 2 by switch.

At t = **1000**, last bit of packet 2 is placed on link 1 by A.

At t = **1020**, last bit of packet 2 reaches switch.

Note: pkt 2 need to wait upto 1055 μsec before switch transfers it.

Last bit of packet1 will be placed on link 2 by switch at 1055 μsec.

Hence No additional delay for packet 2.

At t = **1055** packet two first bit is placed on link 2.

At t = **1575** last bit of packet 2 reaches B.

$\therefore$ 1575 μsec is required.

■■■■

2 LAN

2.1 A 2 km long broadcast LAN has 10^7 bps bandwidth and uses CSMA/CD. The signal travels along the wire at 2×10^8 m/s. What is the minimum packet size that can be used on this network?

(a) 50 bytes (b) 100 bytes
(c) 200 bytes (d) None of these

[2003 : 2 Marks]

2.2 A host is connected to a Department network which is part of a University network. The University network, in turn, is part of the Internet. The largest network in which the Ethernet address of the host is unique is

(a) the subnet to which the host belongs
(b) the Department network
(c) the University network
(d) the Internet

[2004 : 1 Mark]

2.3 Consider a simplified time slotted MAC protocol, where each host always has data to send and transmits with probability p = 0.2 in every slot. There is no backoff and one frame can be transmitted in one slot. If more than one host transmits in the same slot, then the transmissions are unsuccessful due to collision. What is the maximum number of hosts which this protocol can support, if each host has to be provided a minimum through put of 0.16 frames per time slot?

(a) 1 (b) 2
(c) 3 (d) 4

[2004 : 2 Marks]

2.4 A and B are the only two stations on an Ethernet. Each has a steady queue of frames to send. Both A and B attempt to transmit a frame, collide, and A wins the first backoff race. At the end of this successful transmission by A, both A and B attempt to transmit and collide. The probability that A wins the second backoff race is

(a) 0.5 (b) 0.625
(c) 0.75 (d) 1.0

[2004 : 2 Marks]

2.5 In a network of LANs connected by bridges, packets are sent from one LAN to another through intermediate bridges. Since more than one path may exist between two LANs, packets may have to be routed through multiple bridges. Why is the spanning tree algorithm used for bridge-routing?

(a) For shortest path routing between LANs
(b) For avoiding loops in the routing paths
(c) For fault tolerance
(d) For minimizing collisions

[2005 : 1 Mark]

2.6 Which of the following statements is TRUE about CSMA/CD

(a) IEEE 802.11 wireless LAN runs CSMA/CD protocol
(b) Ethernet is not based on CSMA/CD protocol
(c) CSMA/CD is not suitable for a high propagation delay network like satellite network
(d) There is no contention in a CSMA/CD network

[2005 : 1 Mark]

2.7 Which of the following statements is FALSE regarding a bridge

(a) Bridge is a layer 2 device
(b) Bridge reduces collision domain
(c) Bridge is used to connect two or more LAN segments
(d) Bridge reduces broadcast domain

[2005 : 1 Mark]

2.8 A network with CSMA/CD protocol in the MAC layer is running at 1 Gbps over a 1 km cable with no repeaters. The signal speed in the cable is 2×10^8 m/sec. The minimum frame size for this network should be

(a) 10000 bits (a) 10000 bytes
(b) 5000 bits (d) 5000 bytes

[2005 : 2 Marks]

2.9 In a TDM medium access control bus LAN, each station is assigned one time slot per cycle for transmission. Assume that the length of each time slot is the time to transmit 100 bits plus the end-to-end propagation delay. Assume a propagation speed of 2×10^8 m/sec. The length of the LAN is 1 km with a bandwidth of 10 Mbps. The maximum number of stations that can be allowed in the LAN so that the throughput of each station can be 2/3 Mbps is

(a) 3 (b) 5
(c) 10 (d) 20

[2005 : 2 Marks]

2.10 A router has two full-duplex Ethernet interfaces each operating at 100 Mb/s. Ethernet frames are at least 84 bytes long (including the Preamble and the Inter-Packet-Gap).
The maximum packet processing time at the router for wirespeed forwarding to be possible is (in micro-seconds)

(a) 0.01 (b) 3.36
(c) 6.72 (d) 8

[2006 : 2 Marks]

2.11 In Ehernet when Manchester encoding is used, the bit rate is

(a) Half the baud rate
(b) Twice the baud rate
(c) Same as the baud rate
(d) None of these

[2007 : 1 Mark]

2.12 A broadcast channel has 10 nodes and total capacity of 10 Mbps. It uses polling for medium access. Once a node finishes transmission, there is a polling delay of 80 ìs to poll the next node. Whenever a node is polled, it is allowed to transmit a maximum of 1000 bytes. The maximum throughput of the broadcast channel is

(a) 1 Mbps (b) 100/11 Mbps
(c) 10 Mbps (d) 100 Mbps

[2007 : 2 Marks]

2.13 There are n stations in a slotted LAN. Each station attempts to transmit with a probability p in each time slot. What is the probability that ONLY one station transmits in a given time slot?

(a) $np(1-p)^{n-1}$ (b) $(1-p)^{n-1}$
(c) $p(1-p)^{n-1}$ (d) $1-(1-p)^{n-1}$

[2007 : 2 Marks]

2.14 The distance between two stations M and N is L kilometres. All frames are K bits long. The propagation delay per kilometre is t seconds. Let R bits/second be the channel capacity. Assuming that processing delay is negligible, the minimum number of bits for the sequence number field in a frame for maximum utilization, when the sliding window protocol is used, is:

(a) $\left\lceil \log_2 \frac{2LtR + 2K}{K} \right\rceil$ (b) $\left\lceil \log_2 \frac{2LtR}{K} \right\rceil$

(c) $\left\lceil \log_2 \frac{2LtR + K}{K} \right\rceil$ (d) $\left\lceil \log_2 \frac{2LtR + K}{2K} \right\rceil$

[2007 : 2 Marks]

2.15 The minimum frame size required for a CSMA/CD based computer network running at 1 Gbps on a 200m cable with a link speed of 2×10^8m/s is

(a) 125 bytes (b) 250 bytes
(c) 500 bytes (d) None of these

[2008 : 2 Marks]

2.16 Consider a source computer (S) transmitting a file of size 10^6 bits to a destination computer (D) over a network of two routers (R_1 and R_2) and three links (L_1, L_2 and L_3). L_1 connects S to R_1; L_2 connects R_1 to R_2; and L_3 connects R_2 to D. Let each link be of length 100 km. Assume signals travel over each link at a speed of 10^8 meters per second. Assume that the link bandwidth on each link is 1 Mbps. Let the file be broken down into 1000 packets each of size 1000 bits. Find the total sum of transmission and propagation delays in transmitting the file from S to D?

(a) 1005 ms (b) 1010 ms
(c) 3000 ms (d) 3003 ms

[2012 : 2 Marks]

2.17 Determine the maximum length of the cable (in km) for transmitting data at a rate of 500 Mbps in an Ethernet LAN with frames of size 10,000 bits. Assume the signal speed in the cable to be 2,00,000 km/s.

(a) 1 (b) 2
(c) 2.5 (d) 5

[2013 : 2 Marks]

2.18 Consider a LAN with four nodes S_1, S_2, S_3 and S_4. Time is divided into fixed-size slots, and a node can begin its transmission only at the beginning of a slot. A collision is said to have occurred if more than one node transmit in the same slot. The probabilities of generation of a frame in a time slot by S_1, S_2, S_3 and S_4 are 0.1, 0.2, 0.3 and 0.4, respectively. The probability of sending a frame in the first slot without any collision by any of these four stations is ______.

[2015 (Set-1) : 2 Marks]

2.19 Consider a CSMA/CD network that transmits data at a rate of 100 Mbps (10^8 bits per second) over a 1 km (kilometer) cable with no repeaters. If the minimum frame size required for this network is 1250 bytes, what is the signal speed (km/sec) in the cable?

(a) 8000 (b) 10000
(c) 16000 (d) 20000

[2015 (Set-3) : 1 Mark]

2.20 Which one of the following protocols is NOT used to resolve one form of address to another one?

(a) DNS (b) ARP
(c) DHCP (d) RARP

[2016 (Set-1) : 2 Marks]

2.21 Which of the following is/are example(s) of statefull application layer protocols?

(*i*) HTTP (*ii*) FTP
(*iii*) TCP (*iv*) POP3

(a) (*i*) and (*ii*) only (b) (*ii*) and (*iii*) only
(c) (*ii*) and (*iv*) only (d) (*iv*) only

[2016 (Set-1) : 2 Marks]

2.22 A sender uses the Stop-and-Wait ARQ protocol for reliable transmission of frames. Frames are of size 1000 bytes and the transmission rate at the sender is 80 Kbps (1 Kbps = 1000 bits/second). Size of an acknowledgment is 100 bytes and the transmission rate at the receiver is 8 Kbps. The one-way propagation delay is 100 milliseconds.

Assuming no frame is lost, the sender throughput is ______ bytes/second.

[2016 (Set-1) : 2 Marks]

2.23 In an Ethernet local area network, which one of the following statements is **TRUE**?

(a) A station stops to sense the channel once it starts transmitting a frame.
(b) The purpose of the jamming signal is to pad the frames that are smaller than the minimum frame size.
(c) A station continues to transmit the packet even after the collision is detected.
(d) The exponential backoff mechanism reduces the probability of collision on retransmissions.

[2016 (Set-2) : 2 Marks]

2.24 A network has a data transmission bandwidth of 20×10^6 bits per second. It uses CSMA/CD in the MAC layer. The maximum signal propagation time from one node to another node is 40 microseconds. The minimum size of a frame in the network is ______ bytes.

[2016 (Set-2) : 2 Marks]

2.25 Consider a 128×10^3 bits/second satellite communication link with one way propagation delay of 150 milliseconds. Selective retransmission (repeat) protocol is used on this link to send data with a frame size of 1 kilobyte. Neglect the transmission time of acknowledgement. The minimum number of bits required for the sequence number field to achieve 100% utilization is ______.

[2016 (Set-2) : 2 Marks]

■■■■

Answers LAN

2.1	(d)	2.2	(d)	2.3	(b)	2.4	(b)	2.5	(b)	2.6	(c)	2.7	(d)	2.8	(a)	2.9	(c)
2.10	(c)	2.11	(b)	2.12	(a)	2.13	(a)	2.14	(c)	2.15	(a)	2.16	(a)	2.17	(b)	2.19	(d)
2.20	(c)	2.21	(c)	2.23	(d)												

Explanations LAN

2.1 (d)

In CSMA/CD the minimum frame size

= 2×z×data rate (Bandwidth)

$$\because \quad z = \frac{d}{v} = \frac{\text{distance}}{\text{velocity}}$$

$d = 2\text{ km} = 2 \times 10^3$ m

$v = 2 \times 10^8$ m/sec.

Data rate = 10^7 bps

$$z = \frac{d}{v} = \frac{2 \times 10^3 \text{m}}{2 \times 10^8 \text{m/sec}} = 10^{-5} \text{ sec}$$

$\therefore$ Minimum packet size = $2 * 10^{-5} * 10^7$

$= 2 * 10^2$

$= 200 \text{ bits} = \frac{200}{8} = 25$ bytes

2.2 (d)

Ethernet address is nothing but MAC Address which is present on NIC and it is unique for every system or host in the internet.

2.3 (b)

Let there be N such hosts.
Then when one host is transmitting then others must be silent.
So the throughput per host:

$0.16 = 0.2*(0.8)^{(N-1)}$

$0.8 = 0.8^{(N-1)}$

It implies $N - 1 = 1$, so $N = 2$.

2.4 (b)

There are two conditions when A will win the first back-off (0, 1). In second back-off there are four conditions (0, 1, 2, 3). If A choose 1, 0 in first then required probability

$$= \frac{1}{2}.\frac{3}{4} + \frac{1}{2}.\frac{1}{2} = \frac{3}{8} + \frac{1}{4} = \frac{5}{8} = 0.625$$

2.5 (b)

The spanning tree approach is a mechanism in which bridges automatically develop a routing table and update that table in response to changing topology. The algorithm consists of three mechanisms

1. Frame Forwarding
2. Address Learning
3. Loop resolution.

2.6 (c)

CSMA/CD was used in early days,802.3 not in 802.11.
There will be contention in this protocol.
Ethernet is based on CSMA/CD early in 1980s.

2.7 (d)

Bridge does not reduce broadcast domain, it remains same.

2.8 (a)

Let S is minimum packet size.

$T_p = (1 \text{ km})/(2 \times 10^8 \text{ m/s})$

$= 5 \times 10^{-6}$ seconds.

Minimum frame size can be found by formula

$T_x = 2 \, T_p \, S/1$ Gbps

$S = 10^9 \times 10^{-5} = 10^4$ bits.

2.9 (c)

Each slot is equal to transmission time of 100 bits + propagation delay.
Propagation delay = 1 km/2*10^8ms = 5μs.
T_x = 100/10 Mbps = 10μs.
Let there are maximum N number of station then Length of cycle is = N*(10+5) = 15N μs.
In a whole cycle each user transmit for only 10μs.
Therefore efficiency is (10/15N).
Throughput of each station is (10/15N)* 10 Mbps
... s 2/3 Mbps
... 5*2) = 10

2.10 (c)

For maintaining the speed of forwarding of wire, i.e. 100 Mbps.
Processing time should be at most same as minimum transmission time.
84*8 bits / 100 Mbps = 6.72 micro seconds.

2.11 (b)

In Manchester encoding, two signal changes to represent a bit. Therefore baud rate (number of signals/sec) = 2*bit rate (number of bits/sec). Hence, bit rate is half the baud rate.

2.12 (a)

T_x = 1000 bytes/10Mbps = 800 μs.
Delay because of polling is = 80 μs.
Efficiency of channel e = transmission delay/(total delay) = 800/(800 + 80) = 10/11
Maximum throughput is
= (10/11)*10 Mbps = 100/11 Mbps.

2.13 (a)

If there are n stations in a slotted LAN. Each station attempts to transmit with a probability p in each time slot and therefore acquires the medium. This is just the binomial probability distribution that any one station attempts to transmit and the others do not. Let the event is denoted by P(E)

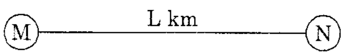
$$P(E) = {}^{n}C_1\, p^1 (1-p)^{n-1}$$

$$= \frac{\lfloor n}{\lfloor 1 \lfloor n-1}\, p(1-p)^{n-1}$$

$$= \frac{n \lfloor n-1}{\lfloor n-1}\, p(1-p)^{n-1}$$

$$= np(1-p)^{n-1}$$

2.14 (c)



Frame size K bit long
Propagation delay t sec/km
Channel capacity = R bits/sec

$$U = \frac{w \cdot \frac{K}{R}\,\text{sec}}{\frac{K}{R}\,\text{sec} + 2\,Lt}$$

$$1 = \frac{\frac{wK}{R}\,\text{sec}}{\frac{K + 2\,LtR}{R}}$$

$$w = \frac{K + 2\,LtR}{K}$$

$$2^n = \frac{K + 2\,LtR}{K}$$

$$n = \left\lceil \log_2 \frac{K + 2\,LtR}{K} \right\rceil$$

2.15 (a)

The minimum frame size for a CSMA/CD is

$= 2 \times Pd \times BW$

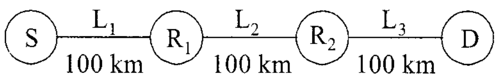
$= 2 \times \frac{200}{2 \times 10^8} \times 1\text{ Gbps}$

$= 2 \times 10^{-6} \times 10^9$

= 2000 bits

= 250 bytes

2.16 (a)



Propagation delay to travel from S to R_1

$$= \frac{\text{Distance}}{\text{Link Speed}} = \frac{10^5}{10^8} = 1\text{ ms}$$

Total propagation delay to travel from S to D = 3 * 1 ms = 3 ms

Total transmission delay for 1 packet

$$= 3 * \frac{(\text{Number of bits})}{\text{Bandwidth}}$$

$$= 3 * \frac{(1000)}{10^6} = 3\text{ ms}$$

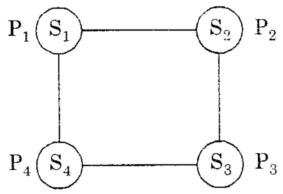
So the first packet will take 6 ms to reach D. While first packet was reaching D, other packets must have been processing in parallel. So D will receive remaining packets 1 packet per 1 ms from R_2. So remaining 999 packets will take 999 ms and total time will be 999 + 6 = 1005 ms.

2.17 (b)

$$\frac{10000\text{ bit}}{500 \times 10^6\text{ bits/sec}} = \frac{2 \times L}{2 \times 10^5\text{ km/sec}}$$

$$L = 2\text{ km}$$

2.18 Sol.



P_i is the probability of frame being generated by station S_i

Required probability = Only one station generation frames

$= (P_1 \cdot \bar{P}_2 \cdot \bar{P}_3 \cdot \bar{P}_4) + (\bar{P}_1 \cdot P_2 \cdot \bar{P}_3 \cdot \bar{P}_4) +$

$(\bar{P}_1 \cdot \bar{P}_2 \cdot P_3 \cdot \bar{P}_4) + (\bar{P}_1 \cdot \bar{P}_2 \cdot \bar{P}_3 \cdot P_4)$

$= (0.1 \times 0.8 \times 0.7 \times 0.6) + (0.9 \times 0.2 \times 0.7 \times 0.6) + (0.9 \times 0.8 \times 0.3 \times 0.6) + (0.9 \times 0.8 \times 0.7 \times 0.4)$

$= 0.0336 + 0.0756 + 0.1296 + 0.2016 = 0.4404$

2.19 (d)

Band width $= 100 \times 10^6$ bps

$d = 1000$ m

For CSMA/CD: $t_d \geq 2P_d$

$$\frac{1250 \times 8}{100 \times 10^6} = 2 \times \frac{1000}{x \ m/s}$$

$$x = \frac{2 \times 1000 \times 100 \times 10^6}{1250 \times 8}$$

$= 2 \times 10^7 = 20000$ (km/sec)

2.20 (a)

- DNS is used for mapping host name to IP address.
- ARP is address resulution protocol used to map IP address with MAC address.
- RARP is reverse address resolution protocol used to map MAC address with IP address.
- DHCP is also used to map MAC address with IP address.

Since all options are used to find one address to another address but option (c) is most appropriate answer.

2.21 (c)

FTP has control and data connection it requires authorization. HTTP is stateless protocol. TCP is not application layer but it is statefull. POP3 is application protocol and it gets state with help of TCP.

2.22 Sol.

$$\text{Sender throughput} = \frac{\text{Data}}{\text{Total time}}$$

$$= \frac{1000 \text{ bytes}}{0.1 + 0.1 + 0.1 + 0.1} = 2500 \text{ bytes/sec}$$

2.23 (d)

Exponential back of algorithm reduce the possibility of collisions in next iteration.

2.24 Sol.

Bandwidth $= 20 \times 16^6$ bps

Propagation time $= 40$ μsec

For ethernet,

Transmission time $= 2 \times$ Propagation time

$$\frac{\text{Frame size}}{20 \times 10^6 \text{bits/sec}} = 2 \times 40 \ \mu\text{sec}$$

Frame size $= 20 \times 10^6 \times 2 \times 40 \times 10^{-6}$

$= 1600$ bits

$= \frac{1600}{8} = 200$ bytes

2.25 Sol.

$$\text{Transmission time} = \frac{\text{Frame size}}{\text{Bandwidth}}$$

$$\text{T.T.} = \frac{1024 \times 8}{128 \times 10^3} = \frac{1028}{16} \text{msec}$$

$= 64$ msec [$\therefore$ 1 K $= 2^{10}$]

$$\text{Efficiency} = \text{w.s.} \times \frac{\text{T.T.}}{\text{T.T.} + 2\text{P.T.}}$$

$$100\% = \text{w.s.} \times \frac{1}{1 + 2\frac{\text{P.T.}}{\text{T.T.}}}$$

$$1 + 2\frac{\text{P.T.}}{\text{T.T.}} = \text{w.s.}$$

$$1 + 2\left[\frac{150}{64}\right] = \text{w.s.}$$

$1 + 2[2.34] = $ w.s.

w.s. $= \lceil 5.68 \rceil = 6$

In selective repeate ARQ.

Total window size $\geq \log_2$ (S.w.s. + R.w.s.) and

Sender w.s. = Reciever w.s.

S[illegible] w.s. $\geq \log_2 [6 + 6] = \log_2[12] = 4$

■■■■

3 TCP, UDP and IP

3.1 Which of the following assertions is FALSE about the Internet Protocol (IP)?
(a) It is possible for a computer to have multiple IP addresses
(b) IP packets from the same source to the same destination can take different routes in the network
(c) IP ensures that a packet is farwarded if it is unable to reach its destination within a given number of hopes
(d) The packet source cannot set the route of an outgoing packets; the route is determined only by the routing tables in the routers on the way

[2003 : 1 Mark]

3.2 Which of the following functionalities must be implemented by a transport protocol over and above the network protocol?
(a) Recovery from packet losses
(b) Detection of duplicate packets
(c) Packet delivery in the correct order
(d) End to end connectivity

[2003 : 1 Mark]

3.3 The subnet mask for a particular network is 255.255.31.0 Which of the following pairs of IP addresses could belong to this network?
(a) 172.57.88.62 and 172.56.87.23.2
(b) 10.35.28.2 and 10.35.29.4
(c) 191.203.31.87 and 191.234.31.88
(d) 128.8.129.43 and 128.8.161.55

[2003 : 2 Marks]

3.4 Which of the following is NOT true with respect to a transparent bridge and a router?
(a) Both bridge and router selectively farward data packets
(b) A bridge uses IP addresses while a router uses MAC addresses
(c) A bridge builds up its routing table by inspecting incoming packets
(d) A router can connect between a LAN and a WAN

[2004 : 1 Mark]

3.5 Which one of the following statements is FALSE?
(a) TCP guarantees a minimum communication rate
(b) TCP ensures in-order delivery
(c) TCP reacts to congestion by reducing sender window size
(d) TCP employs retransmission to compensate for packet loss

[2004 : 1 Mark]

3.6 A subnet has been assigned a subnet mask of 255.255.255.192. What is the maximum number of hosts that can belong to this subnet?
(a) 14 (b) 30
(c) 62 (d) 126

[2004 : 1 Mark]

3.7 In TCP, a unique sequence number assigned to each
(a) byte (b) word
(c) segment (d) message

[2004 : 1 Mark]

3.8 In the TCP/IP protocol suite, which one of the following is NOT part of the IP header?
(a) Fragment Offset
(b) Source IP address
(c) Destination IP address
(d) Destination port number

[2004 : 2 Marks]

3.9 A TCP message consisting of 2100 bytes is passed to IP for delivery across two networks. The first network can carry a maximum payload of 1200 bytes per frame and the second network can carry a maximum payload of 400 bytes per frame, excluding network overhead. Assume that IP overhead per packet is 20 bytes. What is the total IP overhead in the second network for this transmission?
(a) 40 bytes (b) 80 bytes
(c) 120 bytes (d) 160 bytes

[2004 : 2 Marks]

3.10 Suppose that the maximum transmit window size for a TCP connection is 12000 bytes. Each packet consists of 2000 bytes. At some point of time, the connection is in slow-start phase with a current transmit window of 4000 bytes. Subsequently, the transmitter receives two acknowledgement. Assume that no packets are lost and there are no time-outs. What is the maximum possible value of the current transmit window?

(a) 4000 bytes (b) 8000 bytes
(c) 10000 bytes (d) 12000 bytes

[2004 : 2 Marks]

3.11 The routing table of a router is shown below:

Destination	*Subnet Mask*	*Interface*
128.75.43.0	255.255.255.0	Eth0
128.75.43.0	255.255.255.128	Eth1
192.12.17.5	255.255.255.255	Eth3
default		Eth2

On which interfaces will the router farward packets addressed to destinations 128.75.43.16 and 192.12.17.10 respectively?

(a) Eth1 and Eth2 (b) Eth0 and Eth2
(c) Eth0 and Eth3 (d) Eth1 and Eth3

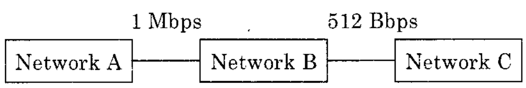
[2004 : 2 Marks]

Common Data Questions Q. 3.12 & Q. 3.13

Consider three IP networks A, B and C. Host H_A in network A sends messages each containing 180 bytes of application data to a host H_C in network C. The TCP layer prefixes a 20 byte header to the message. This passes through an intermediate network B. The maximum packet size, including 20 byte IP header, in each network is

A : 1000 bytes
B : 100 bytes
C : 1000 bytes

The network A and B are connected through a 1 Mbps link, while B and C are connected by a 512 Kbps link (bps = bits per second).



3.12 Assuming that the packets are correctly delivered, how many bytes, including headers, are delivered to the IP layer at the destination for one application message, in the best case? Consider only data packets.

(a) 200 (b) 220
(a) 240 (b) 260

[2004 : 2 Marks]

3.13 What is the rate at which application data is transferred to host H_c? Ignore errors, acknowledgments, and other overheads.

(a) 325.5 Kbps (b) 354.5 Kbps
(c) 409.6 Kbps (d) 512.0 Kbps

[2004 : 2 Marks]

3.14 Packets of the same session may be routed through different paths in

(a) TCP, but not UDP
(b) TCP and UDP
(c) UDP, but not TCP
(d) Neither TCP, nor UDP

[2005 : 1 Mark]

3.15 The address resolution protocol (ARP) is used for

(a) Finding the IP address from the DNS
(b) Finding the IP address of the default gateway
(c) Finding the IP address that corresponds to a MAC address
(d) Finding the MAC address that corresponds to an IP address

[2005 : 1 Mark]

3.16 An organization has a class B network and wishes to form subnets for 64 departments. The subnet mask would be

(a) 255.255.0.0 (b) 255.255.64.0
(c) 255.255.128.0 (d) 255.255.252.0

[2005 : 1 Mark]

3.17 In a packet switching network, packets are routed from source to destination along a single path having two intermediate nodes. If the message size is 24 bytes and each packet contains a header of 3 bytes, then the optimum packet size is

(a) 4 (b) 6
(c) 7 (d) 9

[2005 : 2 Marks]

3.18 Suppose the round trip propagation delay for a 10 Mbps Ethernet having 48-bit jamming signal is 46.4 μs. The minimum frame size is:

(a) 94 (b) 416
(c) 464 (d) 512

[2005 : 2 Marks]

3.19 On a TCP connection, current congestion window size is Congestion Window = 4 KB. The window size advertised by the receiver is Advertise Window = 6 KB. The last byte sent by the sender is LastByteSent = 10240 and the last byte acknowledged by the receiver is LastByteAcked = 8192. The current window size at the sender is
(a) 2048 bytes (b) 4096 bytes
(c) 6144 bytes (d) 8192 bytes

[2005 : 2 Marks]

3.20 In a communication network, a packet of length L bits takes link L1 with a probability of p_1 or link L2 with a probability of p_2. Link L1 and L2 have bit error probability of b_1 and b_2 respectively. The probability that the packet will be received without error via either L1 or L2 is
(a) $(1 - b_1)^L p_1 + (1 - b_2)^L p_2$
(b) $[1 - (b_1 + b_2)^L]p_1p_2$
(c) $(1 - b_1)^L (1 - b_2)^L p_1p_2$
(d) $1 - (b_1^L p_1 + b_2^L p_2)$

[2005 : 2 Marks]

3.21 A company has a class C network address of 204.204.204.0. It wishes to have three subnets, one with 100 hosts and two with 50 hosts each. Which one of the following options represents a feasible set of subnet address/subnet mask pairs?
(a) 204.204.204.128/255.255.255.192
204.204.204.0/255.255.255.128
204.204.204.64/255.255.255.128
(b) 204.204.204.0/255.255.255.192
204.204.204.192/255.255.255.128
204.204.204.64/255.255.255.128
(c) 204.204.204.128/255.255.255.128
204.204.204.192/255.255.255.192
204.204.204.224/255.255.255.192
(d) 204.204.204.128/255.255.255.128
204.204.204.64/255.255.255.192
204.204.204.0/255.255.255.192

[2005 : 2 Marks]

3.22 Which of the following statements is TRUE?
(a) Both Ethernet frame and IP packet include checksum fields
(b) Ethernet frame includes a checksum field and IP packet includes a CRC field
(c) Ethernet frame includes a CRC field and IP packet includes a checksum field
(d) Both Ethernet frame and IP packet include CRC fields

[2006 : 1 Mark]

3.23 For which one of the following reasons does Internet Protocol (IP) use the time-to-live (TTL) field in the IP datagram header?
(a) Ensure packets reach destination within that time
(b) Discard packets that reach later than that time
(c) Prevent packets from looping indefinitely
(d) Limit the time for which a packet gets queued in intermediate routers

[2006 : 1 Mark]

3.24 A router uses the following routing table:

Destination	Mask	Interface
144.16.0.0	255.255.0.0	eth0
144.16.64.0	255.255.224.0	eth1
144.16.68.0	255.255.255.0	eth2
144.16.68.64	255.255.255.224	eth3

A packet bearing a destination address 144.16.68.117 arrives at the router. On which interface will it be forwarded?
(a) eth0 (b) eth1
(c) eth2 (c) eth3

[2006 : 2 Marks]

3.25 Suppose that it takes 1 unit of time to transmit a packet (of fixed size) on a communication link. The link layer uses a window flow control protocol with a window size of N packets. Each packet causes an ack or a nak to be generated by the receiver, and ack/nak transmission times are negligible. Further, the round trip time on the link is equal to N units. Consider time i > N. If only acks have been received till time i(no naks), then the goodput evaluated at the transmitter at time i(in packets per unit time) is
(a) 1 – N/i (b) i/(N + i)
(c) 1 (d) $1 - e^{(i/N)}$

[2006 : 2 Marks]

3.26 A link of capacity 100 Mbps is carrying traffic from a number of sources. Each source generates an on-off traffic stream; when the source is on, the rate of traffic is 10 Mbps, and when the source is off, the rate of traffic is zero. The duty cycle, which is the ratio of on-time to off-time, is 1 : 2. When there is no buffer at the link, the minimum number of sources that can be multiplexed on the link so that link capacity is not wasted and no data loss occurs is S_1. Assuming that all sources are synchronized and that the link is provided with a large buffer, the maximum number of sources that can be multiplexed so that no data loss occurs is S_2. The values of S_1 and S_2 are, respectively,

(a) 10 and 30 (b) 12 and 25
(c) 5 and 33 (d) 15 and 22

[2006 : 2 Marks]

3.27 A program on machine X attempts to open a UDP connection to port 5376 on a machine Y, and a TCP connection to port 8632 on machine Z. However, there are no applications listening at the corresponding ports on Y and Z. An ICMP Port Unreachable error will be generated by

(a) Y but not Z (b) Z but not Y
(c) Neither Y nor Z (d) Both Y and Z

[2006 : 2 Marks]

3.28 A subnetted Class B network has the following broadcast address : 144.16.95.255. Its subnet mask

(a) is necessarily 255.255.224.0
(b) is necessarily 255.255.240.0
(c) is necessarily 255.255.248.0.
(d) could be any one of 255.255.224.0, 255.255.240.0, 255.255.248.0

[2006 : 2 Marks]

3.29 Two computers C1 and C2 are configured as follows. C1 has IP address 203. 197.2.53 and netmask 255.255.128.0. C2 has IP address 203.197.75.201 and netmask 255.255.192.0. Which one of the following statements is true?

(a) C1 and C2 both assume they are on the same network
(b) C2 assumes C1 is on same network, but C1 assumes C2 is on a different network
(c) C1 assumes C2 is on same network, but C2 assumes C1 is on a different network
(d) C1 and C2 both assume they are on different networks

[2006 : 2 Marks]

3.30 Consider the following statements about the timeout value used in TCP.

(i) The timeout value is set to the RTT (Round Trip Time) measured during TCP connection establishment for the entire duration of the connection.
(ii) Appropriate RTT estimation algorithm is used to set the timeout value of a TCP connection.
(iii) Timeout value is set to twice the propagation delay from the sender to the receiver.

Which of the following choices hold?

(a) (i) is false, but (ii) and (iii) are true
(b) (i) and (iii) are false, but (ii) is title
(c) (i) and (ii) are false, but (iii) is true
(d) (i), (ii) and (iii) are false

[2007 : 1 Mark]

3.31 Consider a TCP connection in a state where there are no outstanding ACKs. The sender sends two segments back to back. The sequence numbers of the first and second segments are 230 and 290 respectively. The first segment was lost, but the second segment was received correctly by the receiver.

Let X be the amount of data carried in the first segment (in bytes), and Y be the ACK number sent by the receiver. The values of X and Y (in that order) are

(a) 60 and 290 (b) 230 and 291
(c) 60 and 231 (d) 60 and 230

[2007 : 1 Mark]

3.32 The address of a class B host is to be split into subnets with a 6-bit subnet number. What is the maximum number of subnets and the maximum number of hosts in each subnet?

(a) 62 subnets and 262142 hosts
(b) 64 subnets and 262142 hosts
(c) 62 subnets and 1022 hosts
(d) 64 subnets and 1024 hosts

[2007 : 2 Marks]

3.33 What is the maximum size of data that the application layer can pass on to the TCP layer below?

(a) Any size
(b) 2^{16} bytes – size of TCP header
(c) 2^{16} bytes
(d) 1500 bytes

[2008 : 1 Mark]

3.34 Which of the following system calls results in the sending of SYN packets?

(a) socket (b) bind
(c) listen (d) connect

[2008 : 1 Mark]

3.35 Which of the following statements are TRUE?

S1: TCP handles both congestion and flow control

S2: UDP handles congestion but not flow control

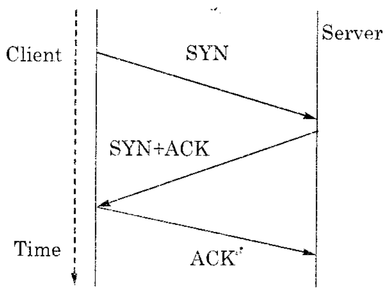
S3: Fast retransmit deals with congestion but not flow control

S4: Slow start mechanism deals with both congestion and flow control

(a) S1, S2 and S3 only
(b) S1 and S3 only
(c) S3 and S4 only
(d) S1, S3 and S4 only

[2008 : 2 Marks]

3.36 The three way handshake for TCP connection establishment is shown below.



Which of the following statements are TRUE?

S1: Loss of SYN + ACK from the server will not establish a connection

S2: Loss of ACK from the client cannot establish the connection

S3: The server moves LISTEN → SYN_RCVD → SYN_SENT → ESTABLISHED in the state machine on no packet loss

S4: The server moves LISTEN → SYN_RCVD → ESTABLISHED in the state machine on no packet loss.

(a) S2 and S3 only (b) S1 and S4 only
(c) S1 and S3 only (d) S2 and S4 only

[2008 : 2 Marks]

Directions for Question 3.37 to 3.38:

Host X has IP address 192.168.1.97 and is connected through two routers R1 and R2 to another host Y with IP address 192.168.1.80. Router R1 has IP addresses 192.168.1.135 and 192.168.1.110. R2 has IP addresses 192.168.1.67 and 192.168.1.155. The netmask used in the network is 255.255.255.224.

3.37 Given the information above, how many distinct subnets are guaranteed to already exist in the network?

(a) 1 (b) 2
(c) 3 (d) 6

[2008 : 2 Marks]

3.38 Which IP address should X configure its gateway as?

(a) 192.168.1.67 (b) 192.168.1.110
(c) 192.168.1.135 (d) 192.168.1.155

[2008 : 2 Marks]

3.39 In the slow start phase of the TCP congesting control algorithm, the size of the congestion window

(a) Does not increase
(b) Increases linearly
(c) Increases quadratically
(d) Increases exponentially

[2008 : 2 Marks]

3.40 If a class B network on the Internet has a subnet mask of 255.255.248.0, what is the maximum number of hosts per subnet?

(a) 1022 (b) 1023
(c) 2046 (d) 2047

[2008 : 2 Marks]

3.41 A client process P needs to make a TCP connection to a server process S. Consider the following situation: the server process S executes a socket ( ), a bind ( ) and a listen ( ) system call in that order, following which it is preempted. Subsequently, the client process P executes a socket ( ) system call followed by connect ( ) system call to connect to the server process S. The server process has not executed any accept ( ) system call. Which one of the following events could take place?

(a) connect( ) system call returns successfully
(b) connect( ) system call blocks
(c) connect( ) system call returns an error
(d) connect( ) system call results in a core dump

[2008 : 2 Marks]

3.42 While opening a TCP connection, the initial sequence number is to be derived using a time-of-day (ToD) clock that keeps running even when the host is down. The low order 32 bits of the counter of the ToD clock is to be used for the initial sequence numbers. The clock counter increments once per millisecond. The maximum packet lifetime is given to be 64s.

Which one of the choices given below is closest to the minimum permissible rate at which sequence numbers used for packets of a connection can increase?

(a) 0.015/s (b) 0.064/s
(c) 0.135/s (d) 0.327/s

[2009 : 2 Marks]

3.43 One of the header fields in an IP datagram is the Time-to-Live (TTL) field. Which of the following statements best explains the need for this field?

(a) It can be used to prioritize packets
(b) It can be used to reduce delays
(c) It can be used to optimize throughput
(d) It can be used to prevent packet looping

[2010 : 1 Mark]

3.44 Suppose computers A and B have IP addresses 10.105.1.113 and 10.105.1.91 respectively and they both use the same netmask N. Which of the values of N given below should not be used if A and B should belong to the same network?

(a) 255.255.255.0
(b) 255.255.255.128
(c) 255.255.255.192
(d) 255.255.255.224

[2010 : 2 Marks]

3.45 A layer-4 firewall (a device that can look at all protocol headers up to the transport layer) CANNOT

(a) block entire HTTP traffic during 9:00 pm and 5:00 am
(b) block all ICMP traffic
(c) stop incoming traffic from a specific IP address but allow outgoing traffic to the same IP address
(d) block TCP traffic from a specific user on a multi-user system during 9:00 pm and 5:00 am

[2011 : 1 Mark]

3.46 Which of the following transport layer protocols is used to support electronic mail?

(a) SMTP (b) IP
(c) TCP (d) UDP

[2012 : 1 Mark]

3.47 In the IPv4 addressing format, the number of networks allowed under Class C addresses is

(a) 2^{14} (b) 2^{7}
(c) 2^{21} (d) 2^{24}

[2012 : 1 Mark]

3.48 An Internet Service Provider (ISP) has the following chunk of CIDR-based IP addresses available with it : 245.248.128.0/20. The ISP wants to give half of this chunk of addresses to Organization A, and a quarter of Organization B, while retaining the remaining with itself. Which of the following is a valid allocation of addresses to A and B?

(a) 245.248.136.0/21 and 245.248.128.0/22
(b) 245.248.128.0/21 and 245.248.128.0/22
(c) 245.248.132.0/22 and 245.248.132.0/21
(d) 245.248.136.0/24 and 245.248.132.0/21

[2012 : 2 Marks]

3.49 Consider an instance of TCP's Additive Increase Multiplicative Decrease (AIMD) algorithm where the windows size at the start of the slow start phase is 2 MSS and the threshold at the start of the first transmission is 8 MSS. Assume that a time-out occurs during the fifth transmission. Find the congestion window size at the end of the tenth transmission.

(a) 8 MSS (b) 14 MSS
(c) 7 MSS (d) 12 MSS

[2012 : 2 Marks]

3.50 In an IPv4 datagram, the M bit is 0, the value of HLEN is 10, the value of total length is 400 and the fragment offset value if 300. The position of the datagram, the sequence numbers of the first and the last bytes of the payload, respectively are

(a) Last fragment, 2400 and 2789
(b) First fragment, 2400 and 2759
(c) Last fragment, 2400 and 2759
(d) Middle fragment, 300 and 689

[2013 : 2 Marks]

3.51 Let the size of congestion window of a TCP connection be 32 KB when a timeout occurs. The round trip time of the connection is 100 msec and the maximum segment size used is 2 KB. The time taken (in msec) by the TCP connection to get back to 32 KB congestion window is ____

[2014 (Set-1) : 2 Marks]

3.52 Consider a selective repeat sliding window protocol that uses a frame size of 1 KB to send data on a 1.5 Mbps link with a one-way latency of 50 msec. To achieve a link utilization of 60%, the minimum number of bits required to represent the sequence number field is ________.

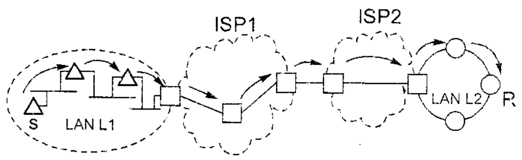
[2014 (Set-1) : 2 Marks]

3.53 Which one of the following socket API functions converts an unconnected active TCP socket into a passive socket?

(a) connect (b) bind
(c) listen (d) accept

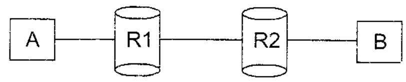
[2014 (Set-2) : 1 Mark]

3.54 In the diagram shown below, L1 is an Ethernet LAN and L2 is a Token-Ring LAN. An IP packet originates from sender S and traverses to R, as shown. The links within each ISP and across the two ISPs, are all point-to-point optical links. The initial value of the TTL field is 32. The maximum possible value of the TTL field when R receives the datagram is _______.



[2014 (Set-2) : 1 Mark]

3.55 Consider the store and forward packet switched network given below. Assume that the bandwidth of each link is 10^6 bytes/sec. A user on host A sends a file of size 10^3 bytes to host B through routers $R1$ and $R2$ in three different ways. In the first case a single packet containing the complete file is transmitted from A to B. In the second case, the file is split into 10 equal parts, and these packets are transmitted from A to B. In the third case, the file is split into 20 equal parts and these packets are sent from A to B. Each packet contains 100 bytes of header information along with the user data. Consider only transmission time and ignore processing, queuing and propagation delays. Also assume that there are no errors during transmission. Let $T1$, $T2$ and $T3$ be the times taken to transmit the file in the first, second and third case respectively. Which one of the following is CORRECT?



(a) $T1 < T2 < T3$ (b) $T1 > T2 > T3$
(c) $T2 = T3$, $T3 < T1$ (d) $T1 = T3$, $T3 > T2$

[2014 (Set-2) : 2 Marks]

3.56 Host A (on TCP/IP v4 network A) sends an IP datagram D to host B (also on TCP/IP v4 network B). Assume that no error occurred during the transmission of D. When D reaches B, which of the following IP header field(s) may be different from that of the original datagram D?

(*i*) TTL (*ii*) Checksum
(*iii*) Fragment Offset

(a) (*i*) only (b) (*i*) and (*ii*) only
(c) (*ii*) and (*iii*) only (d) (*i*), (ii) and (*iii*)

[2014 (Set-3) : 1 Mark]

3.57 An IP router implementing Classless Inter-domain Routing (CIDR) receives a packet with address 131.23.151.76. The router's routing table has the following entries:

Prefix	Output Interface Identifier
131.16.0.0/12	3
131.28.0.0/14	5
131.19.0.0/16	2
131.22.0.0/15	1

The identifier of the output interface on which this packet will be forwarded is _______.

[2014 (Set-3) : 2 Marks]

3.58 Every host in an IPv4 network has a 1-second resolution real-time clock with battery backup. Each host needs to generate up to 1000 unique identifiers per second. Assume that each host has a globally unique IPv4 address. Design a 50-bit globally unique ID for this purpose. After what period (in seconds) will the identifiers generated by a host wrap around?

[2014 (Set-3) : 2 Marks]

3.59 An IP router with a Maximum Transmission Unit (MTU) of 1500 bytes has received an IP packet of size 4404 bytes with an IP header of length 20 bytes. The values of the relevant fields in the header of the third IP fragment generated by the router for this packet are

(a) MF bit: 0, Datagram Length: 1444; Offset: 370
(b) MF bit: 1, Datagram Length: 1424; Offset: 185
(c) MF bit: 1, Datagram Length: 1500; Offset: 370
(d) MF bit: 0, Datagram Length: 1424; Offset: 2960

[2014 (Set-3) : 2 Marks]

3.60 Suppose two hosts use a TCP connection to transfer a large file. Which of the following statements is/are FALSE with respect to the TCP connection?

1. If the sequence number of a segment is m, then the sequence number of the subsequent segment is always m+1.
2. If the estimated round trip time at any given point of time is t sec, the value of the retransmission timeout is always set to greater than or equal to t sec.
3. The size of the advertised window never changes during the course of the TCP connection.
4. The number of unacknowledged bytes at the sender is always less than or equal to the advertised window.

(a) 3 only (b) 1 and 3 only
(c) 1 and 4 only (d) 2 and 4 only

[2015 (Set-1) : 1 Mark]

3.61 Which one of the following fields of an IP header is NOT modified by a typical IP router?
(a) Checksum (b) Source address
(c) Time to Live (TTL) (d) Length

[2015 (Set-1) : 1 Mark]

3.62 Identify the correct order in which a server process must invoke the function calls **accept**, **bind**, **listen**, and **recv** according to UNIX socket API.
(a) listen, accept, bind recv
(b) bind, listen, accept, recv
(c) bind, accept, listen, recv
(d) accept, listen, bind, recv

[2015 (Set-2) : 1 Mark]

3.63 Assume that the bandwidth for a TCP connection is 1048560 bits/sec. Let α be the value of RTT in milliseconds (rounded off to the nearest integer) after which the TCP window scale option is needed. Let β be the maximum possible window size with window scale option. Then the values of α and β are
(a) 63 milliseconds 65535×2^{14}
(b) 63 milliseconds 65535×2^{16}
(c) 500 milliseconds 65535×2^{14}
(d) 500 milliseconds 65535×2^{16}

[2015 (Set-2) : 2 Marks]

3.64 Consider the following routing table at an IP router:

Network No.	Net Mask	Next Hop
128.96.170.0	255.255.254.0	Interface 0
128.96.168.0	255.255.254.0	Interface 1
128.96.166.0	255.255.254.0	R2
128.96.164.0	255.255.252.0	R3
0.0.0.0	Default	R4

For each IP address in Group-I identify the correct choice of the next hop from Group-II using the entries from the routing table above.

List-I		List-II
A. 128.96.171.92		**1.** Interface 0
B. 128.96.167.151		**2.** Interface 1
C. 128.96.163.121		**3.** R2
D. 128.96.165.121		**4.** R3
		5. R4

Codes:

	A	B	C	D
(a)	1	3	5	4
(b)	1	4	2	5
(c)	2	3	4	5
(d)	2	3	5	4

[2015 (Set-2) : 2 Marks]

3.65 Host A sends a UDP datagram containing 8880 bytes of user data to host B over an Ethernet LAN. Ethernet frames may carry data up to 1500 bytes (i.e. MTU = 1500 bytes). Size of UDP header is 8 bytes and size of IP header is 20 bytes. There is no option field in IP header. How may total number of IP fragments will be transmitted and what will be the contents of offset field in the last fragment?
(a) 6 and 925 (b) 6 and 7400
(c) 7 and 1110 (d) 7 and 8880

[2015 (Set-2) : 2 Marks]

3.66 Consider the following statements.

I. TCP connections are full duplex.

II. TCP has no option for selective acknowledgment

III. TCP connections are message streams.

(a) Only I is correct
(b) Only I and II are correct
(c) Only II and III are correct
(d) All of I, II and III are correct

[2015 (Set-3) : 1 Mark]

3.67 In the network 200.10.11.144/27, the fourth octet (in decimal) of the last IP address of the network which can be assigned to a host is ________.

[2015 (Set-3) : 2 Marks]

3.68 An IP datagram of size 1000 bytes arrives at a router. The router has to forward this packet on a link whose MTU (maximum transmission unit) is 100 bytes. Assume that the size of the IP header is 20 bytes. The number of fragments that the IP datagram will be divided into for transmission is _______.

[2016 (Set-1) : 2 Marks]

3.69 For a host machine that uses the token bucket algorithm for congestion control, the token bucket has a capacity of 1 megabyte and the maximum output rate is 20 megabytes per second. Tokens arrive at a rate to sustain output at a rate of 10 megabytes per second. The token bucket is currently full and the machine needs to send 12 megabytes of data.

The minimum time required to transmit the data is _______ seconds.

[2016 (Set-1) : 2 Marks]

Answers TCP, UDP and IP

3.1	(d)	3.2	(d)	3.3	(c)	3.4	(b)	3.5	(a)	3.6	(c)	3.7	(d)	3.8	(d)	3.9	(c)
3.10	(b)	3.11	(a)	3.12	(d)	3.13	(b)	3.14	(b)	3.15	(d)	3.16	(d)	3.17	(d)	3.18	(c)
3.19	(b)	3.20	(a)	3.21	(d)	3.22	(c)	3.23	(c)	3.24	(c)	3.25	(a)	3.26	(a)	3.27	(a)
3.28	(d)	3.29	(c)	3.30	(c)	3.31	(d)	3.32	(c)	3.33	(a)	3.34	(d)	3.35	(d)	3.36	(c)
3.37	(b)	3.38	(a)	3.39	(d)	3.40	(c)	3.41	(c)	3.42	(b)	3.43	(d)	3.44	(d)	3.45	(d)
3.46	(c)	3.47	(c)	3.48	(a)	3.49	(c)	3.50	(c)	3.53	(c)	3.55	(d)	3.56	(d)	3.59	(a)
3.60	(b)	3.61	(b)	3.62	(b)	3.63	(c)	3.64	(a)	3.65	(c)	3.66	(a)				

Explanations TCP, UDP and IP

3.1 (d)

Consider each choice separately

Choice (a): It is possible for a computer to have multiple IP addresses-IP addresses specify the network connection not to a host computer so if a host computer moves from one network to another, its IP address must change. In the network the IP address for a computer is unique but when we move the host computes from one network to another network, its IP address must be changed.

Choice (b): IP packets from the same source to the same destination can take different routes in the network. In packet switching network the routes are determined by routing algorithms. It may be possible that different networks follow different routing algorithms so the statement is true.

Choice (c): IP ensures that a packet is discarded if it is unable to reach its destination within a given number of hops, so statement is true.

Choice (d): The packet source cannot set the route of an outgoing packet, the route is determined only by the routing tables in the routers on the way. The usual IP routing algorithm employs an internet routing table on each machine (computer) that stores informations about possible destination and how to reach them. Because both hosts (computer) and routers route datagrams both have IP routing tables so the statement is false.

3.2 (d)

The transport protocol provides an end-to end connectivity that shields network layer protocol from the details of the intervening network or networks. A transport protocol can be either connection oriented such as TCP, or connectionless such as UDP.

3.3 (c)

The subnet mask is

255 · 255 · 31 · 0

The given subnet mask belongs to class B network because first two octets are all one's which specify the physical network. The third octet also specify the local physical network connection and the fourth octet specify the host computer. The lowest address of class B network is 128 · 0 · 0 · 0 and the highest address is 191 · 255 · 0 · 0. So 191 · 203 · 31 · 87 and 191 · 234 · 31 · 88 belongs to class B network

3.4 (b)

Choice (b) is not true.

A bridge operates at layer 2 (Data Link Layer) so it use MAC address while routes operates at layer 3 (Network Layer) so it uses IP addresses.

3.5 (a)

Some of the services that TCP *does not* provide: it does not guarantee a minimum transmission rate, a sending process is not permitted to transmit at any rate it wishes, the sending rate is regulated by TCP congestion control, or it may force the sender to send at a low average rate.

Hence 1st statement is false. All other statements are true.

3.6 (c)

Since you have 6 subnet bits so we can make (64 – 2) = 62 hosts.

3.7 (d)

TCP sequences each byte in the packet. Assigning a sequence number to indicate the first byte in a multi-byte packet does this. The second packet will have a sequence number equal to the first sequence number plus the number of bytes in the first packet.

3.8 (d)

Destination Port number is NOT present in IP header.

Because the IP header has nothing to do with the port number.

Port numbers are used by the transport layer to ensure process to process delivery.

3.9 (c)

In the question they have directly given the payload so 2100 will be divided into 1200 and 904 (900 is not a multiple of 8 so we have to pad 4 bits in order to make it a multiple of 8).

Now in second network payload is 400B.

So 1200 bytes packet will be divided in 400, 400, 400 with each having 20B header, and 900 will be divided into 400, 400 and 104 (4 bits padded) each having 20 B header.

So total overhead is 20*6 = 120 B

3.10 (b)

In slow-start phase, for each ACK, the sender increases the current transmit window by Maximum Segment Size (MSS).

In the question it is given a packet consists of 2000 bytes and that can be taken as MSS.

So, after two ACKs, current transmit window = 4000 + 2000 + 2000 = 8000

3.11 (a)

Solution for IP address: 128.75.43.16

$$\begin{array}{r} 10000000.01010011.00101011.00010000 \\ 11111111.11111111.11111111.00000000 \\ \hline 10000000.01010011.00101011.00000000 \end{array}$$

i.e. 128.75.43.0 i.e. Eth0

128.75.43.0

255.255.255.128

128.75.43.0 Eth1

But number of 1's in Eth1 is more than Eth0

So answer will (a).

3.12 (d)

Network A sends a message of size 180 byte. Network B having the maximum packet size limit is 100 byte including 20 byte header. So the possible combination of sending packet from network A to C is

Header		Header		Header	
20	80	20	80	20	40

So the packet size
= 100 + 100 + 60 = 260 byte

3.13 (b)

Apply Nyquist Theorem

$C = 2W\log_2 M$

for $H_C = 354.5$ kbps

3.14 (b)

TCP and UDP are not network layer protocols. TCP and UDP both are transport layer protocols. Routing packets from source to destination done by network layer and depends on routing algorithm so there is no interface of TCP and UDP at network layer.

3.15 (d)

The Address Resolution Protocol (ARP), allows a host to find the MAC (Physical) address of a target host on the same physical network, given only the targets IP address.

3.16 (d)

In a class B network initial two octets are all 1's but the third octet specifies the physical network for subnet of 64 department or 2^6 so initial 6 bits of third octets are 1's.

11111111 · 11111111 · 11111100 · 00000000
255 · 255 · 252 · 0

3.17 (d)

Let S denotes the source station and D denotes the destination station. x and y are two intermediate nodes between S and D.

S x y D

Message size = 24 bytes, Header (control information) = 3 bytes consider the first choice (a).

Packet size = 4 then message size = 4 – 3 = 1 byte so it required 24 messages each containtaing 3 byte header so the transmission time for header overhead increases. Consider the second, third and fourth choices.

(b) Packet size = 6 then message size = 6 – 3 = 3 bytes (required 8 packets)

(c) Packet size = 7 then message size = 7 – 3 = 4 bytes (Required 6 packets)

(d) Packet size = 9 then message size = 9 – 3 = 6 bytes (Required 4 packets)

So 4 packet is the optional message size & 9 is the optional packet size.

3.18 (c)

Round trip propagation delay is $2*T_p$ minimum frame size of ethernet can be found by using formula

Frame size = $(2T_p)$ * (Bandwidth)
= 46.4 ms * 10 Mbps
= 464 k bits

3.19 (b)

Current Window Size = Min (congestion window, advertised window).

3.20 (a)

Required probability = Probability of selecting a link L1 × (Probability of number of bit errors in L1)^L + Probability of selecting a link L2 × (Probability of number of bit errors in L2)^L

$= p_1(1 - b_1)^L + p_2(1 - b_2)^L$

3.21 (d)

In Class C network first three octet are reserved for net id, so we have total 8 bits for host and subnets.

If we want to distribute addresses in subnets so first we should consider the subnet with maximum host, here which is subnet with 100 hosts.

For 100 hosts we require 7 bits for host id, one bit remains for subnet, which we fix to 1 for this subnet.

Subnet mask for this subnet is 255.255.255.10000000 = 255.255.255.128 and subnet address is 204.204.204.128.

For first subnet we have fixed 17^{th} bit to 1 now for second and third subnet 17^{th} bit will be 0. 50 host require 6 bit for host id , two bit remain for subnet id in which one bit is already fixed to zero.
We can configure 18^{th} bit only for these subnets; for second subnet we fix 18^{th} bit to 1 and for third subnet to 0.
Subnet mask for 2^{nd} **subnet** and 3^{rd} **subnet** is 255.255.255.11000000 = 255.255.255.192.
Subnet address for 2^{nd} **subnet** is 204.204.204.64 and 3^{rd} **subnet** is 204.204.204.0.

3.22 (c)

Ethernet uses a **Cyclic Redundancy Check** (CRC) algorithm to detect transmission errors. The Internet Protocol (IP) and most higher-layer protocols of the Internet Protocol Suite (ICMP, IGMP, UDP, UDP-Lite, TCP) use a common checksum algorithm to validate the integrity of the packets that they exchange.

3.23 (c)

TTL is a mechanism that limits the lifespan or lifetime of data in a network. Once the prescribed event count or timespan has elapsed, data is discarded. TTL prevents a data packet from circulating indefinitely.

3.24 (c)

When a packet comes to a router, it matches destination network with those available in routing tables to route the packet.
The route which has longest network address match will be used to route the packet.
The longest matching mask is 255.255.255.0 and hence forwarded to eth2.

3.25 (a)

In computer networks, Goodput is application level throughput, is the number of useful information bits delivered by the network to a certain destination per unit of time. Successful delivery of packet can be assured if ACK has been received for it.
So till time 'i' we would have transmitted 'i' packets but only (i–N) can be acknowledged.
Because minimum time for a packet to get Acknowledged is N.
So successfully delivered packets = (i–N)
Time for transmission = i
Goodput = Successfully delivered data/ Time = (i – N)/i = 1 – N/i.

3.26 (a)

Since there is no buffer, and constraint given, there should not be any data loss and no wastage of capacity as well.
Calculate for the extreme case when all sources are on time (that in transmitting).
10 Mbps * n_station = 100 Mbps
n_station = 10.
The link is provided with large buffer, so calculate expected value of bandwidth usage:
E = 1/3*10+1/3*10+ ... n_station times
E ≤ 100 Mbps
1/3*10 * n_station ≤ 100 Mbps
Therefore n_station = 30.

3.27 (a)

TCP does not rely on ICMP for error control, but UDP does.
UDP does not have its own Error Control policies.

3.28 (d)

In the broadcast address for a subnet, all the host bits are set to 1. So as long as all the bits to the right are 1, bits left to it can be taken as possible subnet.
Broadcast address for subnet is 95.255.0101 1111.1111 1111 (as in Class B, 16 bits each are used for network and host).
So we can take minimum 3 bits (from left) as subnet and make rest as host bits(as they are 1).
.224.0 1110 0000. 0000 0000 (leftmost 3 bits for subnet)
.240.0 1111 0000. 0000 0000 (leftmost 4 bits for subnet)
.248.0 1111 1000. 0000 0000 (...5 bits for subnet)

3.29 (c)

The two computer C_1 and C_2 are configured as follows:

	C_1		C_2
IP Address AND Netmask	203.197.2.53	AND	203.197.75.201
	255.255.128.0		255.255.192.0
	203.197.0.0		203.197.64.0
	Network Address		Network Address

203.197.2.53 → 203.197.00000010.00111001
255.255.192.0 → 255.255.11000000.00000000
203.197.0.0   203.197.0.0

203.197.75.201
255.255.128.0
203.192.0.0

$\therefore$ C_1 assumes C_2 is on same network but C_2 assumes C_1 is on different network.

3.30 (c)

(i) is false because RTT is measured every time an ack is received, because propagation delay changes from time to time depending on the traffic.

(ii) Time out value is set to twice the propagation delay from sender to the receiver.

(iii) True for same reason.

3.31 (d)

In TCP sequence number is assign to each data bytes.
Number of bytes in first segment is
290 − 230 = 60 bytes
In TCP ACK number send by receiver is next expected sequence number which is 230 here, 230 is first sequence number of first byte in first segment.
First segment is lost and second received correctly, but receiver send ACK with sequence number of first byte of first segment until first segment received correctly.

3.32 (c)

The class B is defined as follows

0		16	31
1 0	netid		hostid

Maximum number of subnets
$= 2^6 - 2 = 64 - 2 = 62$
Maximum number of hosts in each subnet
$= 2^{16-6} - 2 = 2^{10} - 2 = 1022$

3.33 (a)

Since there is no maximum limit of data that application can send, so option (a) is correct.

3.34 (d)

Connect system call is responsible for synchronize the packets.

3.35 (d)

S1 is true because TCP has its congestion control and flow control mechanisms.
UDP itself does note have any flow control or congestion control mechanism. Therefore S2 is false.
Fast retransmit and fast recovery are part of TCP congestion control algorithms. Therefore S3 is True.
In slow start algorithm we use congestion and advertised windows and these act as flow control windows.
The congestion windows flow control imposed by the sender while the advertised window is flow control imposed by receiver.
$\therefore$ S4 is True.

3.36 (c)

Initially server is in LISTEN mode when a SYN is received, the server sends SYN+Ack and goes to SYN_RCVD state. When ack from client is received it moves to ESTABLISHED state.
$\therefore$ S3 is false and S4 is true.
The loss of SYN+Ack from server will not allow the client to move to ESTABLISHED state.
Hence connection can not be established.

3.37 (b)

By performing bitwise AND between given IP addresses and subnetmask such as

192.168. 1 .97	192.168. 1 .80
255.255.255.224	255.255.255.224
192.168. 1 .01100001	192.168. 1 .01010000
192.168. 1 .135	192.168. 1 .110
255.255.255.224	255.255.255.224
192.168. 1 .10000111	192.168. 1 .01101110
192.168. 1 .67	192.168. 1 .155
255.255.255.224	255.255.255.224
192.168. 1 .01000011	192.168. 1 .10011011

Their are 3 distinct subnet present i.e., 011, 010, 100.
So answer should be option (b).

3.38 (a)

X with IP 192.168.1.97 and subnet mask 255.255.255.224 generate subnet ID to reach the gateway is 192.168.1.96.

192.168. 1 .97
255.255.255.224
192.168. 1 .01100001

Now, the gateway must also have the same subnet number. We have to check subnet ID from both IP addresses of R1 router and the one which has same subnet ID to X will be its gateway ID.

192.168. 1 .135	192.168. 1 .110
255.255.255.224	255.255.255.224
192.168. 1 .10000111	192.168. 1 .01101110

So IP 192.168.1.110 of R1 router has same subnet ID. So it will be the gateway ID (we can simply check the three MSB of last octet).
So option (a) is correct.

3.39 (d)

In the slow-start (additive) phase of the TCP congestion control algorithm, the size of congestion window increases exponentially TCP acts the window size as follows allowed-window = min (receiver_advertisment, congestion-window)
In slow-start (additive) phase whenever starting traffic on a new connection or increasing traffic after a period of congestion, start the congestion window at the size of a single segment and increases the congestion window by one segment each time an acknowledgment arrives.

3.40 (c)

11111111 11111111 11111000 00000000
So the number of hosts per subnet
$= 2^{11} - 2 = 2048 - 2 = 2046$

3.41 (c)

Connect ( ) system call returns an error.

3.42 (b)

Maximum packet lifetime = 64 seconds
Sequence of one packet change after 64 sec (otherwise packet duplication possible) and clock counter increment per millisec. therefore min. possible rate = 0.064/sec.

3.43 (d)

In the IP datagram there is time to live (TTL) field. It is mainly used to prevent packet looping. Means every packet is associated with certain time stamp. If one packet is received after certain time stamp (TTL) then this packet discarded so it can be used to prevent packet looping.

3.44 (d)

IP address of A = 10.105.1.113
= 10.105.1.01110001
IP address of B = 10.105.1.91
= 10.105.1.01011011

For finding network address we take AND of IP address with the netmask N. So we check one by one

(a) (i) 255.255.255.0 = N
10.105.1.01110001 Taking AND → IP address of A
10.105.1.00000000

(ii) By taking and with B
255.255.255.00000000
10.105.1.01011011
10.105.1.00000000
So both belong to same network

(b) (i) 255.255.255.128
255.255.255.10000000
10.105.1.01110001 IP of A
10.105.1.00000000

(ii) 255.255.255.10000000
10.105.1.01011011 IP of B
10.105.1.00000000
So both belong to same N/W

(d) (i) 255.255.255.11100000
10.105.1.01110001 IP of A
10.105.1.01100000

(ii) 255.255.255.11100000
10.105.1.01011011 IP of B
10.105.1.01100000

Both are not same so it not belong to same network.

3.45 (d)

To block TCP traffic from specific user, requires information of specific user which is done by application layer. To block HTTP data, layer-4, i.e., transport layer can be used becuase it can block port used by HTTP.

3.46 (c)

UDP and TCP are transport layer protocol. TCP supports electronic mail.

$\therefore$ option 'C' is correct.

3.47 (c)

No. of Networks in class 'C' are 2^{21} as in class 'C' there

$\underbrace{\quad\quad}_{\substack{\text{24 bits} \\ \text{Net id}}} \quad \underbrace{\quad\quad}_{\substack{\text{8 bits} \\ \text{host id}}}$

out of 24 bits 3-bits are used for representation class 'C' i.e. 110

$\therefore$ 21 bits with 21 bits we can make 2^{21} networks.

3.48 (a)

It has 20 bit mask, so total number of hosts are 2^{12}.

It wants half of the chunk to A, therefore mask of A should be 21 bits, so it will contain 2^{11} hosts.

$\therefore$ Two choices for A

$\Rightarrow$ 245.248.1000 0000.0/21

245.248.1000 1000.0/21

$\therefore$ A can be assigned

$\Rightarrow$ 245.248.136.0/21

245.248.128.0/21

Now, one fourth needs to be assigned to B, hence its mask should be of 22 bits. But if we look at choice (b), the host in A and B are conflicting, therefore option (a) is correct.

3.49 (c)

Threshold = 8 MSS

$\left.\begin{array}{l}\text{Window size for} \\ \text{1st transmission = 2 MSS} \\ \text{Window size for} \\ \text{2nd transmission = 4 MSS} \\ \text{Windows size for} \\ \text{3rd transmission = 8 MSS}\end{array}\right\}\text{Slow start}$

Threshold reached, increase linearly (according to AIMD)

Window size for 4th transmission = 9 MSS

Window size for 5th transmission = 10 MSS

Time out occur so new threshold = $\frac{10}{2}$ = 5 MSS

Window size for 6th transmission = 2 MSS

Windows size for 7th transmission = 4 MSS

Threshold reached

Window size for 8th transmission = 5 MSS

Window size for 9th transmission = 6 MSS

Window size for 10th transmission = 7 MSS

3.50 (c)

M = 0 : Means no more fragmentation so it represent the last fragment.

HLEN = 10 :

Header length = $10 \times 4 = 40$ bytes

Payload : $400 - 40 = 360$ bytes [0 to 359]

Fragment offset: = 300 means 300×8

= 2400 bytes

Sequence number of first fragment = 2400

Sequence number of last fragment

= 2400 + 359 = 2759

Option (c) is correct.

3.51 Sol.

Congestion window = 32KB.

Threshold = 16KB

$2 \rightarrow 4 \rightarrow 8 \rightarrow 16 \rightarrow 18 \rightarrow 20 \rightarrow 22 \rightarrow 24 \rightarrow 26 \rightarrow 28 \rightarrow 30 \rightarrow 32$

Time taken to reach 32KB = 12 segments $\times$ 100 msec = 1200 msec.

3.52 Sol.

1RTT = $2 \times t_{prop} = 2 \times 50$ msec = 100 msec

Bandwidth = 105×10^6 bps

1.5×10^6 bits $\rightarrow$ 1sec

(1 packet) 1000 bytes $\rightarrow$?

$\Rightarrow$ 1 packet transmitted in: $\frac{1000 \times 8}{1.5 \times 10^6}$ sec

$\frac{1000 \times 8}{1.5 \times 10^6}$ sec $\rightarrow$ 1 packet transmitted

100 msec $\rightarrow$? Number of packets

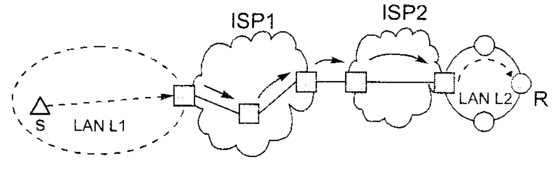
$\Rightarrow$ # packets = $\frac{100 \text{ msec}}{1000 \times 8} \times 1.5 \times 10^6 = 18.75$

$\therefore$ 5-bits are needed for sequence number [i.e., $\log_2 18.75 = 5$]

3.53 (c)

Listen() converts an unconnected active TCP socket into a passive socket.

3.54 Sol.



32 – 6 = 26 is maximum possible value of TTL.

3.55 **(d)**

Bandwidth : 10^6 bps
Packet size: 1000 bytes
1^{st} transmission: 1000 +100 = 1100 bytes transmitted at a time from A
10^6 bits $\rightarrow$ 1 sec
1100 bytes ?
Transmission time at A=1100 × 8 /10^6 = 8.8 msec.
From A, R1 and R2 it takes T1 = 8.8 + 8.8 + 8.8 = **26.4 msec** to reach B
2^{nd} transmission: 100 +100 = 200 bytes transmitted 10 times from A
10^6 bits $\rightarrow$ 1 sec
200 bytes ?
Transmission time at A for 200 bytes
= 200 × 8 /10^6 = 1.6 msec.
From A transmission time for entire packet
= 1.6 × 10 = 16 msec
Note: It uses store and forward so when 10^{th} packet is transmitted from A, B receives 8^{th} packet.
From A, R1 and R2 it takes T2 = 16 + 1.6+ 1.6 = **19.2 msec** to reach B
3^{rd} transmission: 50 +100 = 150 bytes transmitted 20 times from A
10^6 bits $\rightarrow$ 1 sec
150 bytes ?
Transmission time at A for 150 bytes
= 150 × 8 /10^6 = 1.2 msec.
From A transmission time for entire packet
= 1.2 × 20 = 24 msec
Note: It uses store and forward so when 10^{th} packet is transmitted from A, B receives 8^{th} packet.
From A, R1 and R2 it takes T3 = 24 + 1.2+ 1.2 = **26.4 msec** to reach B.
T1 = T3 = 26.4 msec
T2 = 19.2 msec, T2 < T3

3.56 **(d)**

TTL changes from one hop to the next
Check sum changes on each hop due to TTL change.
Fragment offset may be computed again.
∴ TTL, check sum and fragment offset may be changed in the transmission.

3.57 **Sol.**

131.23.151.76 = 10000011.00010111.10010111.01001100

131.16.0.0/12 = 10000011.0001 0000.0000 0000.0000 0000

131.22.0.0/15 = 10000011.00010110. 0000 0000.0000 0000

Longest prefix match is: 131.22.0.0/15
∴ Interface 1 will be selected to forward the packet.

3.58 **Sol.**

Size of IP address of IPv4 is 32 bits.
So number of host present on network = 2^{32}.
Each host generate 1000 packet in 1 second (simultaneously).
To number of packet will be number of host on × Number of packet network generated.
$= 2^{32} \times 1000$
We have two build global unique ID of 50 bits. So number of hosts = 2^{50}.
So time to generate host wrap arround

$$= \frac{2^{50}}{2^{32} \times 1000} = 262.14 \equiv 262.$$

3.59 **(a)**

MTU = 1500 bytes
Header length = 20 bytes
Packet size = 4404 bytes
Fragment 1: 1480 + 20
Fragment 2: 1480 + 20
Fragment 3: 1444 + 20
Datagram length for third IP fragment = 1444
MF bit = 0
Offset = 1480/4 = 370

3.60 **(b)**

Statement 1 is false because, the sequence number of the subsequent segment depends on the number of byte characters in the current segment.
Statement 2 True, because RTO timer is always ≥ Estimated RTT.
Statement 3 False. Receiver window is used to give the sender an idea of how much free buffer space is available at the receiver.
Statement 4 is true, TCP is not permitted to overflow the allocated receiver buffer.

Hence when the sender cannot send any more data receiver window would be 0 and hence. The buffer would have unacknowledged data.

3.61 (b)

Only source address field of IP header is NOT modified by a router.

3.62 (b)

Order of function calls invoked: Bind( ), Listen( ), Accept( ) and Recv( ).

3.63 (c)

Maximum windows size is the amount of data that can be transmitted in an RTT.

$$\therefore \text{ RTT} = \frac{65535 \times 8}{1048560 \text{ bps}} = 500 \text{ ms}$$

For scaling factor 14 bits are used

$\therefore$ Option (c) is correct.

3.64 (a)

IP Address	Net Mask	Network No.	Next Hop
128.96.171.92	255.255.254.0	128.96.170.0	Interface 0
128.96.167.151	255.255.254.0	128.96.166.0	R2
128.96.163.151	default	0.0.0.0	R4
128.96.165.121	255.255.252.0	128.96.164.0	R3

3.65 (c)

User data in UDP datagram = 8880 bytes

UDP header = 8 bytes

IP header = 20 bytes

We need to break (8880+8) bytes into small fragments.

Ethernet frame can hold (1500 – IP header)

= 1500 – 20 = 1480 payload of data

1st fragment 0 to 184 (1480 bytes)

2nd fragment 185 to 369 (1480 bytes)

3rd fragment 370 to 554 (1480 bytes)

4th fragment 555 to 739 (1480 bytes)

5th fragment 740 to 924 (1480 bytes)

6th fragment 925 to 1109 (1480 bytes)

7th fragment 1110 to 1117 (8 bytes)

$\therefore$ 7 fragments with 1110 as the offset of last fragment.

3.66 (a)

TCP is a byte stream protocol. Hence III is false. TCP can use both selective ACK and Cumulative Acknowledgment. Hence II is false. TCP connection are full duplex.

$\therefore$ Statement I is correct.

3.67 Sol.

200.10.11.144/27

255.255.255.224

$$224 \rightarrow \underbrace{111}\,00000$$

$$144 \rightarrow \underbrace{100}_{\text{N/w ID.}}\,10000$$

$$\text{First IP} \rightarrow \underbrace{100}\,00001$$

$$\vdots$$

$$\text{Last IP} \rightarrow \underbrace{100}\,11110$$

i.e. last IP is 200.10.11.158

$\therefore$ 158 is the fourth octet of the last IP address.

3.68 Sol.

MTU is 100 bytes, IP header is 20 bytes, IP datagram is 1000 bytes.

So, number of fragments are 13.

3.69 Sol.

Time taken to transmit 1 MB when output rate is 20 MBps, capacity is 1 MB and token arrival rate is 10 MBps is

$$C + \rho S = MS$$

$$1 \text{ MB} + (10 \text{ MBps}) \times S \Leftarrow (20 \text{ MBps}) \times S$$

$$S = \frac{1 \text{ MB}}{(20-10)\text{MBps}}$$

$$S = \frac{1 \text{ MB}}{10 \text{ MBps}} = 0.1 \text{sec}$$

In 0.1 sec data transmit = 0.1 × Output rate

= 0.1 × 20 MBps = 2 MB

Remaining data = (12 MB – 2 MB) = 10 MB

So to transmit 1 MB takes 0.1 sec

then for 10 MB = 10 × 0.1 sec = 1 sec

Total time = (0.1 + 1) sec = 1.1 sec

4 Routing, Application Layer and Network Security

4.1 Which one of the following statements is FALSE?
(a) Packet switching leads to better utilization of bandwidth resources than circuit switching
(b) Packet switching results in less variation in delay than circuit switching
(c) Packet switching requires more per-packet processing than circuit switching
(d) Packet switching can lead to reordering unlike in circuit switching

[2004 : 1 Mark]

4.2 A sender is employing public key cryptography to send a secret message to a receiver. Which one of the following statements is TRUE?
(a) Sender encrypts using receiver's public key
(b) Sender encrypts using his own public key
(c) Receiver decrypts using sender's public key
(d) Receiver decrypts using his own public key

[2004 : 1 Mark]

4.3 Consider the three commands : PROMPT, HEAD and RCPT. Which of the following options indicate a correct association of these commands with protocols where these are used?
(a) HTTP, SMTP, FTP
(b) FTP, HTTP, SMTP
(c) HTTP, FTP, SMTP
(d) SMTP, HTTP, FTP

[2005 : 1 Mark]

4.4 Traceroute reports a possible route that is taken by packets moving from some host A to some other host B. Which of the following options represents the technique used by traceroute to identify these hosts
(a) By progressively querying routers about the next router on the path to B using ICMP packets, starting with the first router
(b) By requiring each router to append the address to the ICMP packet as it is forwarded to B. The list of all routers en-route to B is returned by B in an ICMP reply packet
(c) By ensuring that an ICMP reply packet is returned to A by each router en-route to B, in the ascending order of their hop distance from A
(d) By locally computing the shortest path from A to B

[2005 : 1 Mark]

4.5 Count to infinity is a problem associated with
(a) link state routing protocol.
(b) distance vector routing protocol.
(c) DNS while resolving host name.
(d) TCP for congestion control.

[2005 : 1 Mark]

4.6 Assume that "host1.mydomain.dom" has an IP address of 145.128.16.8. Which of the following options would be most appropriate as a subsequence of steps in performing the reverse lookup of 145.128.16.8? In the following options "NS" is an abbreviation of "nameserver".
(a) Query a NS for the root domain and then NS for the "dom" domains
(b) Directly query a NS for "dom" and then a NS for "mydomain.dom" domains
(c) Query a NS for in-addr.arpa and then a NS for 128.145.in-addr.arpa domains
(d) Directly query a NS for 145.in-addr.arpa and then a NS for 128.145.in-addr.arpa domains

[2005 : 2 Marks]

4.7 Suppose that two parties A and B wish to setup a common secret key (D-H key) between themselves using the Diffie-Hellman key exchange technique. They agree on 7 as the modulus and 3 as the primitive root. Party A chooses 2 and party B chooses 5 as their respective secrets. Their D-H key is
(a) 3 (b) 4
(c) 5 (d) 6

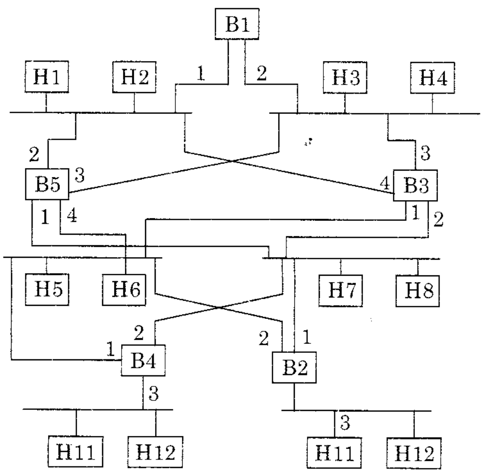
[2005 : 2 Marks]

4.8 HELO and PORT, respectively, are commands from the protocols
(a) FTP and HTTP
(b) TELNET and POP3
(c) HTTP and TELNET
(d) SMTP and FTP

[2006 : 2 Marks]

Linked Data for Q.4.9 and Q.4.10

Consider the diagram shown below where a number of LANs are connected by (transparent) bridges. In order to avoid packets looping through circuits in the graph, the bridges organize themselves in a spanning tree. First, the root bridge is identified as the bridge with the least serial number. Next, the root sends out (one or more) data units to enable the setting up of the spanning tree of shortest paths from the root bridge to each bridge.



Each bridge identifies a port (the root port) through which it will forward frames to the root bridge. Port conflicts are always resolved in favour of the port with the lower index value. When there is a possibility of multiple bridges forwarding to the same LAN (but not through the root port), ties are broken as follows : bridges closest to the root get preference and between such bridges, the one with the lowest serial number is preferred.

4.9 For the given connection of LANs by bridges, which one of the following choices represents the depth first traversal of the spanning tree of bridges?
(a) B1, B5, B3, B4, B2
(b) B1, B3, B5, B2, B4
(c) B1, B5, B2, B3, B4
(d) B1, B3, B4, B5, B2

[2006 : 2 Marks]

4.10 Consider the correct spanning tree for the previous question. Let host H1 send out a broadcast ping packet. Which of the following options represents the correct forwarding table on B3?

(a)

Hosts	Port
H1, H2, H3, H4	3
H5, H6, H9, H10	1
H7, H8, H11, H12	2

(b)

Hosts	Port
H1, H2	4
H3, H4	3
H5, H6	1
H7, H8, H9, H10 H11, H12	2

(c)

Hosts	Port
H3, H4	3
H5, H6, H9, H10	1
H1, H2	4
H7, H8, H11, H12	2

(d)

Hosts	Port
H1, H2, H3, H4	3
H5, H7, H9, H10	1
H7, H8, H11, H12	4

[2006 : 2 Marks]

4.11 Which one of the following uses UDP as the transport protocol?1
(a) HTTP (b) Telnet
(c) DNS (d) SMTP

[2007 : 1 Mark]

4.12. Consider the following two statements:
(i) A hash function (these are often used for computing digital signatures) is an injective function.
(ii) An encryption technique such as DES performs a permutation on the elements of its input alphabet.

Which one of the following options is valid for the above two statements?
(a) Both are false
(b) Statement (i) is true and the other is false
(c) Statement (ii) is true and the other is false
(d) Both are true

[2007 : 1 Mark]

4.13 The minimum positive integer p such that 3^p modulo 17 = 1 is

(a) 5 (b) 8
(c) 12 (d) 16

[2007 : 1 Mark]

4.14 Exponentiation is a heavily used operation in public key cryptography. Which of the following options is the tightest upper bound on the number of multiplications required to compute b^n modulo m, $0 \le b, n \le m$?

(a) $O(\log n)$ (b) $O(\sqrt{n})$
(c) $O(n/\log n)$ (d) $O(n)$

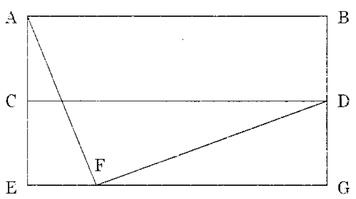
[2007 : 1 Mark]

4.15 A firewall is to be configured to allow hosts in a private network to freely open TCP connections and send packets on open connections. However, it will only allow external hosts to send packets on existing open TCP connections or connections that are being opened (by internal hosts) but not allow them to open TCP connections to hosts in the private network. To achieve this the minimum capability of the firewall should be that of

(a) A combinational circuit
(b) A finite automaton
(c) A pushdown automaton with one stack
(d) A pushdown automaton with two stacks

[2007 : 1 Mark]

4.16 For the network given in the figure below, the routing tables of the four nodes A, E, D and G are shown. Suppose that F has estimated its delay to its neighbors, A, E, D and G as 8, 10, 12 and 6 msecs respectively and updates its routing table using distance vector routing technique.



Routing Table of A	
A	0
B	40
C	14
D	17
E	21
F	9
G	24

Routing Table of E	
A	24
B	27
C	7
D	20
E	0
F	11
G	22

Routing Table of D	
A	20
B	8
C	30
D	0
E	14
F	7
G	22

Routing Table of G	
A	21
B	24
C	22
D	19
E	22
F	10
G	0

(a)

A	8
B	20
C	17
D	12
E	10
F	0
G	6

(b)

A	21
B	8
C	7
D	19
E	14
F	0
G	22

(c)

A	8
B	20
C	17
D	12
E	10
F	16
G	6

(d)

A	8
B	8
C	7
D	12
E	10
F	0
G	6

[2007 : 2 Marks]

4.17 A group of 15 routers are interconnected in a centralized complete binary tree with a router at each tree node. Router i communicates with router j by sending a message to the root of the tree. The root then sends the message back down to router j. The mean number of hops per message, assuming all possible router pairs are equally likely is

(a) 3 (b) 4.26
(c) 4.53 (d) 5.26

[2007 : 2 Marks]

4.18 Consider the following clauses:

(i) Not inherently suitable for client authentication.
(ii) Not a state sensitive protocol.
(iii) Must be operated with more than one server.
(iv) Suitable for structured message organization.
(v) May need two ports on the serve side for proper operation.

The option that has the maximum number of correct matches is

(a) IMAP-(i), FTP-(ii), HTTP-(iii), DNS-(iv), POP3-(v)
(b) FTP-(i), POP3-(ii), SMTP-(iii), HTTP-(iv), IMAP-(v)
(c) POP3-(i), SMTP-(ii), DNS-(iii), IMAP-(iv), HTTP-(v)
(d) SMTP-(i), HTTP-(ii), IMAP-(iii), DNS-(iv), FTP-(v)

[2007 : 2 Marks]

4.19 Match the following:

P. SMTP	**1.** Application layer
Q. BGP	**2.** Transport layer
R. TCP	**3.** Data link layer
S. PPP	**4.** Network layer
	5. Physical layer

Codes:

(a) P-2, Q-1, R-3, S-5 (b) P-1, Q-4, R-2, S-3
(c) P-1, Q-4, R-2, S-5 (d) P-2, Q-4, R-1, S-3

[2007 : 2 Marks]

4.20 Two popular routing algorithms are Distance Vector(DV) and Link State (LS) routing. Which of the following are true?

S1: Count to infinity is a problem only with DV and not LS routing
S2: In LS, the shortest path algorithm is run only at one node
S3: In DV, the shortest path algorithm is run only at one node
S4: DV requires lesser number of network messages than LS

(a) S1, S2 and S4 only
(b) S2 and S3 only
(c) S1, S2 and S3 only
(d) S1 and S4 only

[2008 : 2 Marks]

4.21 The total number of keys required for a set of n individuals to be able to communicate with each other using secret key and public key cryptosystems, respectively are:

(a) n(n–1) and 2n
(b) 2n and ((n(n–1))/2)
(c) ((n(n–1))/2) and 2n
(d) ((n(n–1))/2) and n

[2008 : 2 Marks]

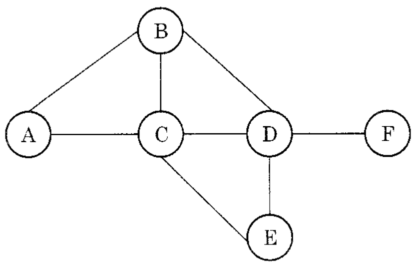
Linked Answer Questions 4.22 and 4.23

Consider a simple graph with unit edge costs. Each node in the graph represents a router. Each node maintains a routing table indicating the next hop router to be used to relay a packet to its destination and the cost of the path to the destination through that router. Initially, the routing table is empty. The routing table is synchronously updated as follows. In each updation interval, three tasks are performed.

(i) A node determines whether its neighbours in the graph are accessible. If so, it sets the tentative cost to each accessible neighbour as 1. Otherwise, the cost is set to ∞.

(ii) From each accessible neighbour, it gets the costs to relay to other nodes via that neighbour (as the next hop).

(iii) Each node updates its routing table based on the information received in the previous two steps by choosing the minimum cost.



4.22 For the graph given above, possible routing tables for various nodes after they have stabilized, are shown in the following options. Identify the correct table.

(a)

Table for node A		
A	–	–
B	B	1
C	C	1
D	B	3
E	C	3
F	C	4

(b)

Table for node C		
A	A	1
B	B	1
C	–	–
D	D	1
E	E	1
F	E	3

(c)

Table for node B		
A	A	1
B	–	–
C	C	1
D	D	1
E	C	2
F	D	2

(d)

Table for node D		
A	B	3
B	B	1
C	C	1
D	–	–
E	E	1
F	F	1

[2005 : 2 Marks]

4.23 Continuing from the earlier problem, suppose at some time t, when the costs have stabilized, node A goes down. The cost from node F to node A at time (t + 100) is :

(a) > 100 but finite (b) ∞
(c) 3 (d) > 3 and $\leq$ 100

[2005 : 2 Marks]

4.24 A computer on a 10 Mbps network is regulated by a token bucket. The token bucket is filled at a rate of 2 Mbps. It is initially filled to capacity with 16 Megabits. What is the maximum duration for which the computer can transmit at the full 10 Mbps?

(a) 1.6 seconds (b) 2 seconds
(c) 5 seconds (d) 8 seconds

[2008 : 2 Marks]

4.25 In the RSA public key cryptosystem, the private and public keys are (e, n) and (d, n) respectively, where n = p*q and p and q are large primes. Besides, n is public and p and q are private. Let M be an integer such that 0 < M < n and $\phi(n) = (p-1)(q-1)$. Now consider the following equations.

I. $M' = M^e \bmod n$
$M = (M')^d \bmod n$
II. $ed \equiv 1 \bmod n$
III. $ed \equiv 1 \bmod \phi(n)$
IV. $M' = M^e \bmod \phi(n)$
$M = (M')^d \bmod \phi(n)$

Which of the above equations correctly represent RSA cryptosystem?

(a) I and II (b) I and III
(c) II and IV (d) III and IV

[2009 : 2 Marks]

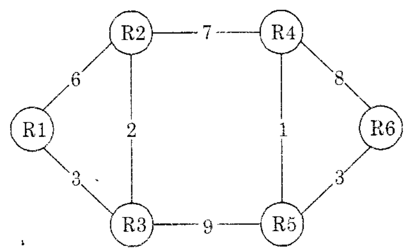
4.26 Which one of the following is not a client-server application?

(a) Internet chat (b) Web browsing
(c) E-mail (d) Ping

[2010 : 1 Mark]

Linked Data Questions Q. 4.27 and Q. 4.28

Consider a network with 6 routers R1 to R6 connected with links having weights as shown in the following diagram.



4.27 All the routers use the distance vector based routing algorithm to update their routing tables. Each router starts with its routing table initialized to contain an entry for each neighbour with the weight of the respective connecting link. After all the routing tables stabilize, how many links in the network will never be used for carrying any data?

(a) 4 (b) 3
(c) 2 (d) 1

[2010 : 2 Marks]

4.28 Suppose the weights of all unused links in the previous question are changed to 2 and the distance vector algorithm is used again until all routing tables stabilize. How many links will now remain unused?

(a) 0 (b) 1
(c) 2 (d) 3

[2010 : 2 Marks]

4.29 Consider different activities related to email

m1: Send an email from a mail client to a mail server

m2: Download an email from mailbox server to a mail client

m3: Checking email in a web browser

(a) m1:HTTP m2:SMTP m3:POP

(b) m1:SMTP m2:FTP m3:HTTP

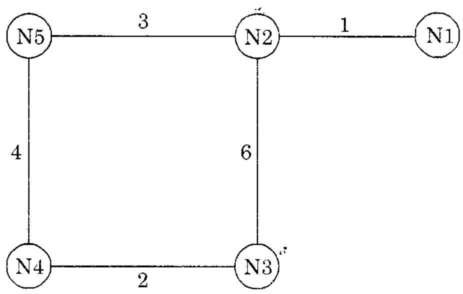
(c) m1:SMTP m2:POP m3:HTTP

(d) m1:POP m2:SMTP m3:IMAP

[2011 : 1 Mark]

Linked Data Question Q.4.30 and Q.4.31

Consider a network with five nodes, N1 to N5, as shown below.



The network uses a Distance Vector Routing protocol. Once the routes have stabilized, the distance vectors at different nodes are as following.

N1: (0, 1, 7, 8, 4)

N2: (1, 0, 6, 7, 3)

N3: (7, 6, 0, 2, 6)

N4: (8, 7, 2, 0, 4)

B5: (4, 3, 6, 4, 0)

Each distance vector is the distance of the best known path at that instance to nodes, N1 to N5, where the distance to itself is 0. Also, all links are symmetric and the cost is identical in both directions. In each round, all nodes exchange their distance vectors with their respective neighbors. Then all nodes update their distance vectors. In between two rounds, any change in cost of a link will cause the two incident nodes to change only that entry in their distance vectors.

4.30 The cost of link N2-N3 reduces to 2(in both directions). After the next round of updates, what will be the new distance vector at node, N3?

(a) (3, 2, 0, 2, 5) (b) (3, 2, 0, 2, 6)

(c) (7, 2, 0, 2, 5) (d) (7, 2, 0, 2, 6)

[2011 : 2 Marks]

4.31 After the update in the previous question, the link N1-N2 goes down. N2 will reflect this change immediately in its distance vector as cost ∞. After the NEXT ROUND of update, what will be the cost to N1 in the distance vector of N3?

(a) 3 (b) 9

(c) 10 (d) ∞

[2011 : 2 Marks]

4.32 The Protocol Data Unit (PDU) for the application layer in the Internet stack is

(a) Segment (b) Datagram

(c) Message (d) Frame

[2012 : 1 Mark]

4.33 The transport layer protocols used for real time multimedia, file transfer, DNS and email respectively are

(a) TCP, UDP, UDP and TCP

(b) UDP, TCP, TCP and UDP

(c) UDP, TCP, UDP and TCP

(d) TCP, UDP, TCP and UDP

[2013 : 1 Mark]

4.34 Using public key cryptography, X adds a digital signature σ to message M, encrypts <M, σ >, and sends it to Y, where it is decrypted. Which one of the following sequence of keys is used for the operations?

(a) Encryption : X's private key followed by Y's private key; Decryption : X's public key followed by Y's public key

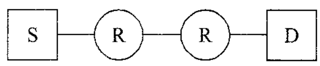
(b) Encryption : X's private key followed by Y's private key; Decryption : X's public key followed by Y's private key

(c) Encryption : X's public key followed by Y's private key; Decryption : Y's public key followed by X's private key

(d) Encryption : X's private key followed by Y's public key; Decryption : Y's private key followed by X's public key

[2013 : 1 Mark]

4.35 Assume that source S and destination D are connected through two intermediate routers labels R. Determine how many times each packet has to visit the network layer and the data link layer during a transmission form S to D.



(a) Network layer - 4 times and Data link layer - 4 times
(b) Network layer - 4 times and Data link layer - 3 times
(c) Network layer - 4 times and Data link layer - 6 times
(d) Network layer - 2 times and Data link layer - 6 times

[2013 : 1 Mark]

4.36 Consider the following three statements about link state and distance vector routing protocols, for a large network with 500 network nodes and 4000 links.

S1: The computational overhead in link state protocols is higher than in distance vector protocols.

S2: A distance vector protocol (with split horizon) avoids persistent routing loops, but not a link state protocol.

S3: After a topology change, a link state protocol will converge faster than a distance vector protocol.

Which one of the following is correct about S1, S2, and S3?

(a) S1, S2, and S3 are all true.
(b) S1, S2, and S3 are all false.
(c) S1 and S2 are true, but S3 is false.
(d) S1 and S3 are true, but S2 is false.

[2014 (Set-1) : 1 Mark]

4.37 Which of the following are used to generate a message digest by the network security protocols?

(P) RSA (Q) SHA-1
(R) DES (S) MD5

(a) P and R only (b) Q and R only
(c) Q and S only (d) R and S only

[2014 (Set-1) : 1 Mark]

4.38 Which one of the following is TRUE about the interior gateway routing protocols - Routing Information Protocol (RIP) and Open Shortest Path First (OSPF)

(a) RIP uses distance vector routing and OSPF uses link state routing
(b) OSPF uses distance vector routing and RIP uses link state routing
(c) Both RIP and OSPF use link state routing
(d) Both RIP and OSPF use distance vector routing

[2014 (Set-2) : 1 Mark]

4.39 An IP machine Q has a path to another IP machine H via three IP routers $R1$, $R2$, and $R3$.

$$Q — R1 — R2 — R3 — H$$

H acts as an HTTP server, and Q connects to H via HTTP and downloads a file. Session layer encryption is used, with DES as the shared key encryption protocol. Consider the following four pieces of information:

[I1] The URL of the file downloaded by Q
[I2] The TCP port numbers at Q and H
[I3] The IP addresses of Q and H
[I4] The link layer addresses of Q and H

Which of I1, I2, I3, and I4 can an intruder learn through sniffing at $R2$ alone?

(a) Only I1 and I2 (b) Only I1
(c) Only I2 and I3 (d) Only I3 and I4

[2014 (Set-2) : 2 Marks]

4.40 In one of the pairs of protocols given below, both the protocols can use multiple TCP connections between the same client and the server. Which one is that?

(a) HTTP, FTP (b) HTTP, TELNET
(c) FTP, SMTP (d) HTTP, SMTP

[2015 (Set-1) : 1 Mark]

4.41 Suppose that everyone in a group of N people wants to communicate secretly with the N–1 others using symmetric key cryptographic system. The communication between any two persons should not be decodable by the others in the group. The number of keys required in the system as a whole to satisfy the confidentiality requirement is

(a) 2N (b) N(N – 1)
(c) N(N – 1)/2 (d) $(N-1)^2$

[2015 (Set-1) : 1 Mark]

4.42 Consider that B wants to send a message m that is digitally signed to A. Let the pair of private and public keys for A and B be denoted by K_x^- and K_x^+ for $x = A, B$, respectively. Let $K_x(m)$ represent the operation of encrypting m with a key K_x and $H(m)$ represent the message digest. Which one of the following indicates the CORRECT way of sending the message m along with the digital signature to A?

(a) $\{m, K_B^+(H(m))\}$ (b) $\{m, K_B^-(H(m))\}$
(c) $\{m, K_A^-(H(m))\}$ (d) $\{m, K_A^+(m)\}$

[2016 (Set-1) : 2 Marks]

4.43 Anarkali digitally signs a message and sends it to Salim. Verification of the signature by Salim requires
(a) Anarkali's public key
(b) Salim's public key
(c) Salim's private key
(d) Anarkali's private key

[2016 (Set-2) : 1 Mark]

4.44 Identify the correct sequence in which the following packets are transmitted on the network by a host when a browser requests a webpage from a remote server, assuming that the host has just been restarted.
(a) HTTP GET request, DNS query, TCP SYN
(b) DNS query, HTTP GET request, TCP SYN
(c) DNS query, TCP SYN, HTTP GET request
(d) TCP SYN, DNS query, HTTP GET request

[2016 (Set-2) : 1 Mark]

4.45 The value of the expression 13^{99}(mod 17), in the range 0 to 16, is________.

[2016 (Set-2) : 2 Marks]

4.46 For the IEEE 802.11 MAC protocol for wireless communication, which of th following statements is/are **TRUE**?
I. At least three non-overlapping channels are available for transmission.
II. The RTS-CTS mechanism is used for collision detection.
III. Unicast frames are ACKed
(a) All I, II, and III
(b) I and III only
(c) II and III only
(d) II only

[2016 (Set-2) : 2 Marks]

Answers Routing, Application Layer and Network Security

4.1	(c)	4.2	(a)	4.3	(a)	4.4	(a)	4.5	(b)	4.6	(c)	4.7	(d)	4.8	(d)	4.9	(a)
4.10	(a)	4.11	(c)	4.12	(c)	4.13	(d)	4.14	(a)	4.15	(d)	4.16	(a)	4.17	(d)	4.18	(d)
4.19	(b)	4.20	(d)	4.21	(c)	4.22	(c)	4.23	(b)	4.24	(b)	4.25	(b)	4.26	(d)	4.27	(c)
4.28	(b)	4.29	(c)	4.30	(a)	4.31	(c)	4.32	(c)	4.33	(c)	4.34	(d)	4.35	(c)	4.36	(d)
4.37	(c)	4.38	(a)	4.39	(c)	4.40	(a)	4.41	(c)	4.42.	(b)	4.43.	(a)	4.44.	(c)	4.46.	(b)

Explanations Routing, Application Layer and Network Security

4.2 (a)

In public key cryptography if sender uses receiver's public key for encryption then the decryption of message is possible only by using private key of receiver.
Hence 1st is true and remaining are false.
If sender encrypts using his own public key then no one except the sender can decrypt it and only sender knows his private key no one else.

4.3 (a)

RCPT: Recipient to, As the name suggest it is used in SMTP(Simple Mail Transfer protocol).
HEAD: This is used in HTTP to get the meta-information, to decide the category of packet.
Prompt: Turns off prompting for individual files when using the mget or mput commands.

4.4 (a)

Traceroute works by sending packets with gradually increasing TTL value, starting with TTL value of 1.

The first router receives the packet, decrements the TTL value and drops the packet because it then has TTL value zero.
The router sends an ICMP Time Exceeded message back to the source.
The next set of packets are given a TTL value of 2, so the first router forwards the packets, but the second router drops them and replies with ICMP Time Exceeded.
Proceeding in this way, traceroute uses the returned ICMP Time Exceeded messages to build a list of routers that packets traverse, until the destination is reached and returns an ICMP Echo Reply message.

4.5 (b)

In distance vector routing the count-to-infinity problem happens when a router is unable to reach an adjoining network. A second router, 1 hop away from the first router, thinks that the unreachable network is 2 hops away.
Meanwhile, the first router then updates its records to say it is 3 hops away from the unreachable network based on the fact it is 1 hop from the second router, which says it is 2 hops from the unreachable network.
The routers continue incrementing their hop count until the maximum (15), "infinity", is reached.

4.6 (c)

The appropriate sequence of steps performing the reverse lookup of 145.128.16.8 are:
***Step*-1:** Query a Name server for in-addr.arpa. Because information regarding the IP address 145.128.16.8 is located at domain name. 145.128.16.8 in-addr.arpa.
***Step*-2:** Second setup is request Name server for 128.45.in-addr.arpa domain.
This is because the query is for PTR records. The resolver reverses the address and appends the in-addr.arpa to the end of the reversed address. This forms the fully qualified domain name for which to be searched is reverse lookup zone.

4.7 (d)

D-H key: $g^{AB} \bmod n = 3^{2\times5} \bmod 7 = 3^{10} \bmod 7 = 4$.

4.8 (d)

HELO: Initiates a conversation with the mail server. When using this command you can specify your domain name so that the mail server knows who you are.
The PORT command is sent by an FTP client to establish a secondary connection (address and port) for data to travel over.

4.9 (a)

B1-B5-B2-B3-B4

4.10 (a)

Use spanning tree generated in previous question.

4.11 (c)

Domain Name System (DNS) maps a name onto an IP address, an application program calls a library procedure called the resolver, passing name as a parameter. The resolver sends a UDP packet to a load DNS server, which then looks up the name and returns the IP address to the resolver, which then returns it to caller. So DNS uses transport layer protocol UDP.

4.12 (c)

During hash generation more than one message may generate the same HASH value.
So hash function is Many-to-One.

4.13 (d)

By Fermat's theorem, 3^{17-1} modulo 17 = 1.

So p = 16.

4.14 (a)

One can compute b, b^2, b^4, b^8,... and combine them to the extent possible.
This means a continuous division of n by 2 and so give a complexity of O(log n).

4.15 (d)

A pushdown automata with two stacks can simulate a general purpose computer.
The number of internal hosts and external hosts is not bounded, and we need comparison so (a) and (b) cannot help as they deal with bounded numbers.

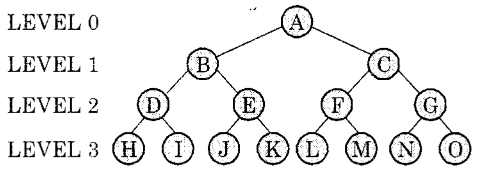
4.16 (a)

	A	B	C	D	E	F	G
F via A	8	48	22	25	29	0	32
F via B							
F via C							
F via D	32	20	42	12	26	0	34
F via E	34	37	17	30	10	0	32
F via G	27	30	28	25	28	0	6
Min values	≤ 8	≤ 20	≤ 17	≤ 12	≤ 10	0	≤ 6

Based on the given options only (a) mathces correct.

4.17 (d)

Consider Complete tree:



Forward packet to the router at Level 3: Total 6 hops are required if A wants to communicate with any level 3 node. Similarly, 5 hops are required if H wants to communicate with any level 2 node, 4 hops are required if H wants to communicate with any level 1 node and 3 hops are required if H wants to communicate with any level 0 node.

Hops required if H wants to communicates with all other nodes

$= (8 - 1)*6 + 4*5 + 2*4 + 1*3 = 73.$

If all 8 nodes of level 3 communicates with all other nodes then hops required = 73 * 8 = 584.

Similarly, Hops required if D wants to communicates with all other nodes = 8*5 +(4 – 1)*4 + 2*3 + 1*2 = 60.

If all 4 nodes of level 2 communicates with all other nodes then hops required = 60*4 = 240.

Hops required if B wants to communicates with all other nodes

$= 8*4 + 4*3 + (2 - 1)*2 + 1*1 = 47.$

If all 2 nodes of level 1 communicates with all other nodes then hops required = 47*2 = 94.

Hops required if A wants to communicates with all other nodes = 8*3+4*2+2*1 = 34.

Total hops required when all nodes communicate with all other nodes

$$= 584 + 240 + 94 + 34 = 952.$$

Total number of messages

$$= 2*{}^{15}C_2 = 2*(15*14/2) = 2*105 = 210.$$

(Here 2 is multiplied with ${}^{15}C_2$, because in communication between A and B, A sends message to B and B sends message to A)

Mean number of hops per message

= 952/210 = 4.53

4.18 (d)

IMAP : It distributes mail boxes across multiple servers.

FTP: Requires two ports: 20 and 21 for FTP-data and FTP-control respectively.

HTTP: HTTP is a stateless protocol. Hence not a state sensitive protocol.

DNS: This protocol maintains it's database is a structured and hierarchial manner.

SMTP: Intersect standard for e-mail transmission and not suitable for client server communication.

4.19 (b)

P. SMTP = Application Layer protocol

Q. BGP = Network Layer protocol

R. TCP = Transport Layer protocol

S. PPP = Data Link Layer protocol

4.20 (d)

Count to infinity problem accurs only in distance vector algorithm.

Hence S1 is true.

In link stae we use flooding to share updates hence S4 is true.

S2 and S3 are both false because routing algorithm need to be executed at each and every node.

4.21 (c)

For private key cryptography for communication between each pair of individuals on secret key will be required.

If an individual wants to communicate with other n – 1 individuals he should have n – 1 secret keys, so the total number of secret keys for private encryption is (n*(n – 1))/2.

For public key encryption each individual needs to have a public and private key, the total keys required in 2*n.

4.24 (b)

Data transfer rate of token bucket = 10 Mbps
Initially filled to capacity 16 Megabits
Maximum duration = 16/18 = 2 seconds

4.25 (b)

I and III equations correctly represent RSA cryptosystem.

4.26 (d)

Internet chat, web browsing and E-mail all are client-server application. Ping is a utility. It is mainly used to check the connection between two computers, there is a chance both are client or one is client and another server. In chat system first user authentication required and it requires server.

4.27 (c)

Routing table

for R1

		Next hop
R2	5	R3
R3	3	R3
R4	12	R3
R5	12	R3
R6	16	R3

for R2

		Next hop
R1	5	R3
R3	2	R3
R4	7	R4
R5	8	R4
R6	12	R4

for R3

		Next hop
R1	3	R1
R2	2	R2
R4	9	R4
R5	9	R5
R6	13	R5

for R4

		Next hop
R1	12	R2
R2	7	R2
R3	9	R2
R5	1	R5
R6	5	R5

for R5

		Next hop
R1	12	R3
R2	8	R4
R3	9	R3
R4	1	R4
R6	4	R6

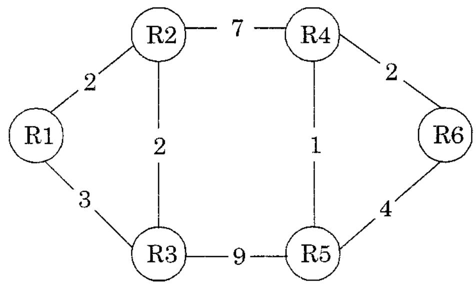
for R6

		Next hop
R1	16	R5
R2	12	R5
R3	13	R5
R4	5	R5
R5	4	R5

So it is clear visualize from the all routing table construction that we never use the direct path between R1 —6— R2 and R4 —8— R6.

So two links never be used for carrying data.

4.28 (b)



As in the previous question

R1 —— R2 and

R4 —— R6 not used.

So we give them weight = 2 by Apply.
Same method only one link R5-R6 not used.

4.29 (c)

SMTP is typically used by user clients for sending mails. POP is used by clients for receiving mails. Checking mails in web browser is a simple HTTP process.

4.30 (a)

In the next round, every node will send and receive distance vectors to and from neighbours, and update its distance vector. N3 will receive (1, 0, 2, 7, 3) from N2 and it will update distances to N1 and N5 as 3 and 5 respectively.

4.31 (c)

In the next round, N3 will receive distance from N2 to N1 as infinite. It will receive distance from N4 to N1 as 8.
So it will update distance to N1 as 8 + 2 = 10.

4.32 (c)

Message → Application layer
Segment → Transport layer
Datagram → Network layer
Frame → Data link layer

4.33 (c)

UDP: Transport layer protocol which is unreliable but fast.
TCP: Transport layer connection oriented protocol which is secure and reliable but comparatively slow.
So for real time multimedia we need fast processing so UDP is suitable for it.

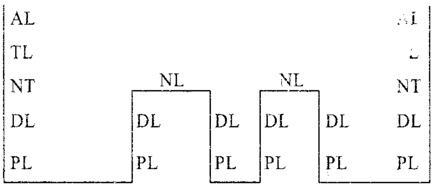
For file transfer we need security so TCP is suitable for file transfer.
DNS always use UDP.
For email we need security so uses TCP.
Option (c) is correct.

4.34 (d)

The message over the network should be encrypted by y's public key.
So order of encryption is x's private key and y's public key.
On receiving the encrypted message, y will decrypt it using its private key and x's public key for signature.
So order of decrypting is y's private key followed by x's public key.
So (d) correct answer.

4.35 (c)

In the layered architecture sender and receiver uses all the 5 layers and router do processing upto the network layer.



So network layer is visited 4 times and data link layer is visited 6 times.

4.36 (d)

S1 is TRUE: Processing time in link state protocol is high because of link state database.
S2 is FALSE: Link state protocol is a loop free protocol. Link state protocol avoids persistent routing loops.
S3 is TRUE: Link state protocol is fast convergence because of link state database i.e., it contains complete information of the network. This information is given to the router by the link state packets.

4.37 (c)

SHA-1 and MD5 are used to generate a message digest.

4.38 (a)

RIP uses distance vector routing
OSPF uses link state routing

4.39 (c)

Given R_1, R_2 and R_3 are routers
At R_2 intruder can learn the TCP port numbers and IP address of Q and H.

4.40 (a)

HTTP and FTP protocols can use multiple TCP connections between the same client and the server. FTP uses data and control connections used with two separate TCP connections.

4.41 (c)

Every pair of nodes need a separate key. There are ${}^{N}C_2$ pairs required for N people.

$$\therefore \quad {}^{N}C_2 = \frac{N(N-1)}{2} \text{ keys are required.}$$

4.42 (b)

In digital signature Message is digested represented by $H(m)$ and encrypted with sender's private key i.e., $K_B^-(H(m))$ to create sign and send it along with the original message m.
So, the correct answer is $\{m, K_B^-(H(m))\}$.

4.43 (a)

Verification or authorization can be done by encrypting sender's private key and decrypting at receiver with sender's public key.

4.44 (c)

DNS query, TCP SYN, HTTP GET request.

4.45 Sol.

$$13^{99} \bmod 17 \Rightarrow (13^3 \bmod 17)^{33}$$
$$\Rightarrow (4 \bmod 17)^{33}$$
$$\Rightarrow (4^3 \bmod 17)^{11}$$
$$\Rightarrow (4^{11} \bmod 17)^3$$
$$\Rightarrow 4$$

4.46 (b)

RTS-CTS is used for collision avoidance but not for collision detection.

■■■■

Unit ■ XI

General Aptitude

UNIT XI

General Aptitude

Syllabus :

Verbal Ability: English grammar, sentence completion, verbal analogies, word groups, instructions, critical reasoning and verbal deduction.

Numerical Ability: Numerical computation, numerical estimation, numerical reasoning and data interpretation.

Analysis of Previous GATE Papers

Exam Year	2010	2011	2012	2013	2014 Set-1	2014 Set-2	2014 Set-3	2015 Set-1	2015 Set-2	2015 Set-3	2016 Set-1	2016 Set-2
1 Mark Ques.	5	5	5	5	5	5	5	5	5	5	5	5
2 Mark Ques.	5	5	5	5	5	5	5	5	5	5	5	5
Total Marks	**15**	**15**	**15**	**15**	**15**	**15**	**15**	**15**	**15**	**15**	**15**	**15**

1 General Aptitude

1.1 Which of the following options is the closest in meaning to the word below?

Circuitous

(a) Cyclic (b) Indirect
(c) Confusing (d) Crooked

[2010 : 1 Mark]

1.2 The question below consists of a pair of related words followed by four pairs of words. Select the pair that best expresses the relation in the original pair.

Unemployed: Worker

(a) Fallow : Land
(b) Unaware : Sleeper
(c) Wit : Jester
(d) Renovated : House

[2010 : 1 Mark]

1.3 Choose the most appropriate word from the options given below to complete the following sentence:

If we manage to .... our natural resources, we would leave a better planet for our children.

(a) uphold (b) restrain
(c) cherish (d) conserve

[2010 : 1 Mark]

1.4 Choose the most appropriate word from the options given below to complete the following sentence:

His rather casual remarks on politics .... his lack of seriousness about the subject.

(a) masked (b) belied
(c) betrayed (d) suppressed

[2010 : 1 Mark]

1.5 25 persons are in a room. 15 of them play hockey, 17 of them play football and 10 of them play both hockey and football. Then the number of persons playing neither hockey nor football is

(a) 2 (b) 17
(c) 13 (d) 3

[2010 : 1 Mark]

1.6 Modern warfare has changed from large scale clashes of armies to suppression of civilian populations. Chemical agents that do their work silently appear to be suited to such warfare; and regretfully, there exist people in militrary establishments who think that chemical agents are usefull fools for their cause.

Which of the following statements best sums up the meaning of the above passage?

(a) Modern warfare has resulted in civil strife.
(b) Chemical agents are useful in modern warfare.
(c) Use of chemical agents in warfare would be undesirable.
(d) People in militrary establishments like to use chemical agents in war.

[2010 : 2 Marks]

1.7 If 137 + 276 = 435 how much is 731 + 672?

(a) 534 (b) 1403
(c) 1623 (d) 1531

[2010 : 2 Marks]

1.8 5 skilled workers can build a wall in 20 days; 8 semi-skilled workers can build a wall in 25 days; 10 unskilled workers can build a wall in 30 days. If a team has 2 skilled, 6 semi-skilled and 5 unskilled workers, how long will it take to build the wall?

(a) 20 days (b) 18 days
(c) 16 days (d) 15 days

[2010 : 2 Marks]

1.9 Given digits 2, 2, 3, 3, 3, 4, 4, 4, 4 how many distinct 4 digit numbers greater than 3000 can be formed?

(a) 50 (b) 51
(c) 52 (d) 54

[2010 : 2 Marks]

1.10 Hari (H), Gita (G), Irfan (I) and Saira (S) are siblings (i.e. brothers and sisters). All were born on I^{st} January. The age difference between any two successive siblings (that is born one after another) is less than 3 years. Given the following facts:

1. Hari's age + Gita's age > Irfan's age + Saira's age.
2. The age difference between Gita and Saira is 1 year. However, Gita is not the oldest and Saira is not the youngest.
3. There are no twins.

In what order were they born (oldest first)?

(a) HSIG (b) SGHI
(c) IGSH (d) IHSG

[2010 : 2 Marks]

1.11 Which of the following options is the closest in the meaning to the word below:

Inexplicable

(a) Incomprehensible (b) Indelible
(c) Inextricable (d) Infallible

[2011 : 1 Mark]

1.12 If Log(P) = (1/2)Log(Q) = (1/3)Log(R), then which of the following options is TRUE?

(a) $P^2 = Q^3R^2$ (b) $Q^2 = PR$
(c) $Q^2 = R^3P$ (d) $R = P^2Q^2$

[2011 : 1 Mark]

1.13 Choose the most appropriate word(s) from the options given below to complete the following sentence.

In contemplated ________ Singapore for my vacation but decided against it.

(a) to visit (b) having to visit
(c) visiting (d) for a visit

[2011 : 1 Mark]

1.14 Choose the most appropriate word from the options given below to complete the following sentence.

If you are trying to make a strong impression on your audience, you cannot do so by being understated, tentative or

(a) hyperbolic (b) restrained
(c) argumentative (d) indifferent

[2011 : 1 Mark]

1.15 Choose the word from the options given below that is most nearly opposite in meaning to the given word:

Amalgamate

(a) merge (b) split
(c) collect (d) separate

[2011 : 1 Mark]

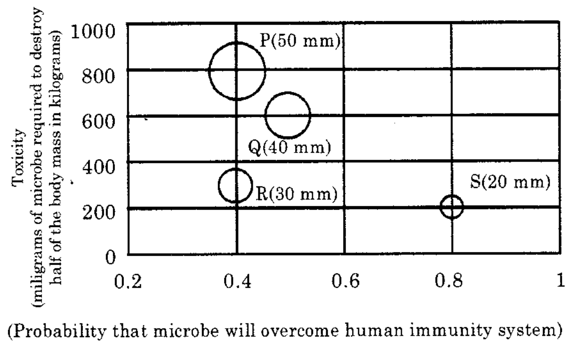
1.16 Few school curricula include a unit on how to deal with bereavement and grief, and yet all students at some point in their lives suffer from losses through death and parting.

Based on the above passage which topic would not be included in a unit on bereavement?

(a) how to write a letter of condolence
(b) what emotional stages are passed through in the healing process
(c) what the leading causes of death are
(d) how to give support to a grieving friend

[2011 : 2 Marks]

1.17 P, Q, R and S are four types of dangerous microbes recently found in a human habitat. The area of each circle with its diameter printed in brackets represents the growth of a single microbe surviving human immunity system within 24 hours of entering the body. The danger to human beings varies proportionately with the toxicity, potency and growth attributed to a microbe shown in the figure below:



A pharmaceutical company is contemplating the development of a vaccine against the most dangerous microbe. Which microbe should the company target in its first attempt?

(a) P (b) Q
(c) R (d) S

[2011 : 2 Marks]

1.18 The variable cost (V) of manufacturing a product varies according to the equation V = 4q, where q is the quantity produced. The fixed cost (F) of production of same product reduces with q according to the equation F = 100/q. How many units should be produced to minimize the total cost (V + F)?

(a) 5 (b) 4
(c) 7 (d) 6

[2011 : 2 Marks]

1.19 A transporter receives the same number of orders each day. Currently, he has some pending orders (backlog) to be shipped. If he uses 7 trucks, then at the end of the 4th day he can clear all the orders. Alternatively, if he uses only 3 trucks, then all the orders are cleared at the end of the 10th day. What is the minimum number of trucks required so that there will be no pending order at the end of the 5th day?

(a) 4 (b) 5
(c) 6 (d) 7

[2011 : 2 Marks]

1.20 A container originally contains 10 litres of pure spirit. From this container 1 litre of spirit is replaced with 1 litre of water. Subsequently, 1 litre of the mixture is again replaced with 1 litre of water and this processes is repeated one more time. How much spirit is now left in the container?

(a) 7.58 litres (b) 7.84 litres
(c) 7 litres (d) 7.29 litres

[2011 : 2 Marks]

1.21 The cost function for a product in a firm is given by $5q^2$, where q is the amount of production. The firm can sell the product at a market price of ₹ 50 per unit. The number of units to be produced by the firm such that the profit is maximized is

(a) 5 (b) 10
(c) 15 (d) 25

[2012 : 1 Mark]

1.22 Choose the most appropriate alternative from the options given below to complete the following sentence:

Despite several ____ the mission succeeded in its attempt to resolve the conflict.

(a) attempts (b) setbacks
(c) meetings (d) delegations

[2012 : 1 Mark]

1.23 Which one of the following options is the closest in meaning to the word given below?

Mitigate

(a) Diminish (b) Divulge
(c) Dedicate (d) Denote

[2012 : 1 Mark]

1.24 Choose the grammatically INCORRECT sentence:

(a) They gave us the money back less the service charges to Three Hundred rupees.
(b) This country's expenditure is not less than that of Bangladesh.
(c) The committee initially asked for a funding of Fifty Lakh rupees, but later settled for a lesser sum.
(d) This country's expenditure on educational reforms is very less.

[2012 : 1 Mark]

1.25 Choose the most appropriate alternative from the options given below to complete the following sentence:

Suresh's dog is the one ____ was hurt in the stampede.

(a) that (b) which
(c) who (d) whom

[2012 : 1 Mark]

1.26 Wanted Temporary, Part-time persons for the post of Field Interviewer to conduct personal interviews to collect and collate economic data. Requirements: High School-pass, must be available for Day, Evening and Saturday work. Transportation paid, expenses reimbursed.

Which one of the following is the best inference from the above advertisement?

(a) Gender-discriminatory
(b) Xenophobic
(c) Not designed to make the post attractive
(d) Not gender-discriminatory

[2012 : 2 Marks]

1.27 A political party orders an arch for the entrance to the ground in which the annual conventions is being held. The profile of the arch follows the equation $y = 2x - 0.1\,x^2$ where y is the height of the arch in meters. The maximum possible height of the arch is

(a) 8 meters (b) 10 meters
(c) 12 meters (d) 14 meters

[2012 : 2 Marks]

1.28 An automobile plant contracted to buy shock absorbers from two supplies X and Y. X supplies 60% and Y supplies 40% of the shock absorbers. All shock absorbers are subjected to a quality test. The ones that pass the quality test are considered reliable. Of X's shock absorbers, 96% are reliable. OF Y's shock absorbers, 72% are reliable. The probability that a randomly chosen shock absorber, which is found to be reliable, is made by Y is

(a) 0.288 (b) 0.334
(c) 0.667 (d) 0.720

[2012 : 2 Marks]

1.29 Which of the following assertions are CORRECT?

P: Adding 7 to each entry in a list adds 7 to the mean of the list
Q: Adding 7 to each entry in a list adds 7 to the standard deviation of the list
R: Doubling each entry n a list doubles the mean of the list
S: Doubling each entry in a list leaves the standard deviation of the list unchanged

(a) P, Q (b) Q, R
(c) P, R (d) R, S

[2012 : 2 Marks]

1.30 Given the sequence of terms, AD CG FK JP, the next time is

(a) OV (b) OW
(c) PV (d) PW

[2012 : 2 Marks]

1.31 Which one of the following options is the closest in meaning to the word given below?

Nadir

(a) Highest (b) Lowest
(c) Medium (d) Integration

[2013 : 1 Mark]

1.32 Complete the sentence :
Universalism is to particularism as diffuseness is to _____.

(a) specificity (b) neutrality
(c) generality (d) adaptation

[2013 : 1 Mark]

1.33 What will be the maximum sum of 44, 42, 40, ...?

(a) 502 (b) 504
(c) 506 (d) 500

[2013 : 1 Mark]

1.34 Were you a bird, you _____ in the sky.

(a) would fly (b) shall fly
(c) should fly (d) shall have flown

[2013 : 1 Mark]

1.35 Choose the grammatically INCORRECT sentence:

(a) He is of Asian origin.
(b) They belonged to Africa
(c) She is an European.
(d) They migrated from India to Australia.

[2013 : 1 Mark]

1.36 Find the sum of the expression

$$\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+.....+\frac{1}{\sqrt{80}+\sqrt{81}}$$

(a) 7 (b) 8
(c) 9 (d) 10

[2013 : 2 Marks]

1.37 Out of all the 2-digit integers between 1 and 100, a 2-digit number has to be selected at random. What is the probability that the selected number is not divisible by 7?

(a) 13/90 (b) 12/90
(c) 78/90 (d) 77/90

[2013 : 2 Marks]

1.38 After several defeats in wars, Robert Bruce went in exile and wanted to commit suicide. Just before committing suicide, the came across a spider attempting tirelessly to have its net. Time and again, the spider failed but the did not deter it to refrain from making attempts. Such attempts by the spider made Bruce curious. Thus, Bruce started observing the near-impossible goal of the spider to have the net. Ultimately, the spider succeeded in having its net despite several failures. Such act of the spider encouraged Bruce not to commit suicide. And then, Bruce went back again and won many a battle, and the rest is history.

Which one of the following assertions is best supported by the above information?

(a) Failure is the pillar of success
(b) Honesty is the best policy
(c) Life begins and ends with adventures
(d) No adversity justifies giving up hope

[2013 : 2 Marks]

1.39 A tourist covers half of this journey by train at 60 km/h, half of the remainder by bus at 30 km/h and the rest by cycle at 10 km/h. The average speed of the tourist in km/h during his entire journey is

(a) 36 (b) 30
(c) 24 (d) 18

[2013 : 2 Marks]

1.40 The current erection cost of a structure is Rs. 13,200. If the labour wages per day increase by 1/5 of the current wages and the working hours decrease by 1/24 of the current period, then the new cost of erection in Rs., is

(a) 16, 500 (b) 15, 180
(c) 11, 000 (d) 10, 120

[2013 : 2 Marks]

1.41 Which of the following options is the closest in meaning to the phrase underlined in the sentence below?

It is fascinating to see life forms **cope with** varied environmental conditions.

(a) adopt to (b) adapt to
(c) adept in (d) accept with

[2014 (Set-1) : 1 Mark]

1.42 Choose the most appropriate word from the options given below to complete the following sentence.

He could not understand the judges awarding her the first prize, because he thought that her performance was quite ______.

(a) superb (b) medium
(c) mediocre (d) exhilarating

[2014 (Set-1) : 1 Mark]

1.43 In a press meet on the recent scam, the minister said, "The buck stops here". What did the minister convey by the statement?

(a) He wants all the money
(b) He will return the money
(c) He will assume final responsibility
(d) He will resist all enquiries

[2014 (Set-1) : 1 Mark]

1.44 If $(z + 1/z)^2 = 98$, compute $(z^2 + 1/z^2)$.

[2014 (Set-1) : 1 Mark]

1.45 The roots of $ax^2 + bx + c = 0$ are real and positive. a, b and c are real. Then $ax^2 + b\ |x| + c = 0$ has

(a) no roots (b) 2 real roots
(c) 3 real roots (d) 4 real roots

[2014 (Set-1) : 1 Mark]

1.46 The Palghat Gap (or Palakkad Gap), a region about 30 km wide in the southern part of the Western Ghats in India, is lower than the hilly terrain to its north and south. The exact reasons for the formation of this gap are not clear. It results in the neighbouring regions of Tamil Nadu getting more rainfall from the South West monsoon and the neighbouring regions of Kerala having higher summer temperatures.

What can be inferred from this passage?

(a) The Palghat gap is caused by high rainfall and high temperatures in southern Tamil Nadu and Kerala
(b) The regions in Tamil Nadu and Kerala that are near the Palghat Gap are low-lying
(c) The low terrain of the Palghat Gap has a significant impact on weather patterns in neighbouring parts of Tamil Nadu and Kerala
(d) Higher summer temperatures result in higher rainfall near the Palghat Gap area

[2014 (Set-1) : 2 Marks]

1.47 Geneticists say that they are very close to confirming the genetic roots of psychiatric illnesses such as depression and schizophrenia, and consequently, that doctors will be able to eradicate these diseases through early identification and gene therapy.

On which of the following assumptions does the statement above rely?

(a) Strategies are now available for eliminating psychiatric illnesses
(b) Certain psychiatric illnesses have a genetic basis
(c) All human diseases can be traced back to genes and how they are expressed
(d) In the future, genetics will become the only relevant field for identifying psychiatric illnesses

[2014 (Set-1) : 2 Marks]

1.48 Round-trip tickets to a tourist destination are eligible for a discount of 10% on the total fare. In addition, groups of 4 or more get a discount of 5% on the total fare. If the one way single person fare is Rs 100, a group of 5 tourists purchasing round-trip tickets will be charged Rs ______.

[2014 (Set-1) : 2 Marks]

1.49 In a survey, 300 respondents were asked whether they own a vehicle or not. If yes, they were further asked to mention whether they own a car or scooter or both. Their responses are tabulated below. What percent of respondents do not own a scooter?

		Men	Women
Own vehicle	Car	40	34
	Scooter	30	20
	Both	60	46
Do not own vehicle		20	50

[2014 (Set-1) : 2 Marks]

1.50 When a point inside of a tetrahedron (a solid with four triangular surfaces) is connected by straight lines to its corners, how many (new) internal planes are created with these lines? ______.

[2014 (Set-1) : 2 Marks]

1.51 Choose the most appropriate phrase from the options given below to complete the following sentence. India is a post-colonial country because

(a) it was a former British colony
(b) Indian Information Technology professionals have colonized the world
(c) India does not follow any colonial practices
(d) India has helped other countries gain freedom

[2014 (Set-2) : 1 Mark]

1.52 Who ___ was coming to see us this evening?

(a) you said (b) did you say
(c) did you say that (d) had you said

[2014 (Set-2) : 1 Mark]

1.53 Match the columns.

Column 1	Column 2
(1) eradicate	(P) misrepresent
(2) distort	(Q) soak completely
(3) saturate	(R) use
(4) utilize	(S) destroy utterly

Codes:

(a) 1:S, 2:P, 3:Q, 4:R
(b) 1:P, 2:Q, 3:R, 4:S
(c) 1:Q, 2:R, 3:S, 4:P
(d) 1:S, 2:P, 3:R, 4:Q

[2014 (Set-2) : 1 Mark]

1.54 What is the average of all multiples of 10 from 2 to 198?

(a) 90 (b) 100
(c) 110 (d) 120

[2014 (Set-2) : 1 Mark]

1.55. The value of $\sqrt{12+\sqrt{12+\sqrt{12+\ldots}}}$ is

(a) 3.464 (b) 3.932
(c) 4.000 (d) 4.444

[2014 (Set-2) : 1 Mark]

1.56 The old city of Koenigsberg, which had a German majority population before World War 2, is now called Kaliningrad. After the events of the war, Kaliningrad is now a Russian territory and has a predominantly Russian population. It is bordered by the Baltic Sea on the north and the countries of Poland to the south and west and Lithuania to the east respectively. Which of the statements below can be inferred from this passage?

(a) Kaliningrad was historically Russian in its ethnic make up
(b) Kaliningrad is a part of Russia despite it not being contiguous with the rest of Russia
(c) Koenigsberg was renamed Kaliningrad, as that was its original Russian name
(d) Poland and Lithuania are on the route from Kaliningrad to the rest of Russia

[2014 (Set-2) : 2 Marks]

1.57 The number of people diagnosed with dengue fever (contracted from the bite of a mosquito) in north India is twice the number diagnosed last year. Municipal authorities have concluded that measures to control the mosquito population have failed in this region.

Which one of the following statements, if true, does not contradict this conclusion?

(a) A high proportion of the affected population has returned from neighbouring countries where dengue is prevalent
(b) More cases of dengue are now reported because of an increase in the Municipal Office's administrative efficiency
(c) Many more cases of dengue are being diagnosed this year since the introduction of a new and effective diagnostic test
(d) The number of people with malarial fever (also contracted from mosquito bites) has increased this year

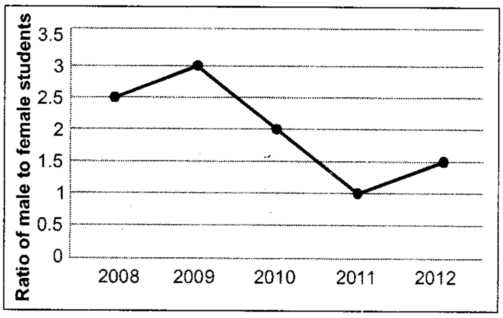
[2014 (Set-2) : 2 Marks]

1.58 If x is real and $|x^2 - 2x + 3| = 11$, then possible values of $|-x^3 + x^2 - x|$ include

(a) 2, 4 (b) 2, 14
(c) 4, 52 (d) 14, 52

[2014 (Set-2) : 2 Marks]

1.59 The ratio of male to female students in a college for five years is plotted in the following line graph. If the number of female students doubled in 2009, by what percent did the number of male students increase in 2009?



[2014 (Set-2) : 2 Marks]

1.60 At what time between 6 *a.m.* and 7 *a.m.* will the minute hand and hour hand of a clock make an angle closest to 60° ?

(a) 6 : 22 *a.m.* (b) 6 : 27 *a.m.*
(c) 6 : 38 *a.m.* (d) 6 : 45 *a.m.*

[2014 (Set-2) : 2 Marks]

1.61 $\underset{\text{(I)}}{\underline{\text{While trying to collect}}}$ an envelope $\underset{\text{(II)}}{\underline{\text{from under the table}}}$, $\underset{\text{(III)}}{\underline{\text{Mr. X fell down}}}$ and $\underset{\text{(IV)}}{\underline{\text{was losing consciousness}}}$.

Which one of the above underlined parts of the sentence is NOT appropriate?

(a) I (b) II
(c) III (d) IV

[2014 (Set-3) : 1 Mark]

1.62 If she ____ how to calibrate the instrument, she _____ done the experiment.

(a) knows, will have
(b) knew, had
(c) had known, could have
(d) should have known, would have

[2014 (Set-3) : 1 Mark]

1.63 Choose the word that is opposite in meaning to the word "coherent".

(a) sticky (b) well-connected
(c) rambling (d) friendly

[2014 (Set-3) : 1 Mark]

1.64 Which number does not belong in the series below?

2, 5, 10, 17, 26, 37, 50, 64

(a) 17 (b) 37
(c) 64 (d) 26

[2014 (Set-3) : 1 Mark]

1.65 The table below has question-wise data on the performance of students in an examination. The marks for each question are also listed. There is no negative or partial marking in the examination.

Q. No.	Marks	Answered Correctly	Answered Wrongly	Not Attempted
1	2	21	17	6
2	3	15	27	2
3	2	23	18	3

What is the average of the marks obtained by the class in the examination?

(a) 1.34 (b) 1.74
(c) 3.02 (d) 3.91

[2014 (Set-3) : 1 Mark]

1.66 A dance programme is scheduled for 10.00 a.m. Some students are participating in the programme and they need to come an hour earlier than the start of the event. These students should be accompanied by a parent. Other students and parents should come in time for the programme. The instruction you think that is appropriate for this is

(a) Students should come at 9.00 a.m. and parents should come at 10.00 a.m.
(b) Participating students should come at 9.00 a.m. accompanied by a parent, and other parents and students should come by 10.00 a.m.
(c) Students who are not participating should come by 10.00 a.m. and they should not bring their parents. Participating students should come at 9.00 a.m.
(d) Participating students should come before 9.00 a.m. Parents who accompany them should come at 9.00 a.m. All others should come at 10.00 a.m.

[2014 (Set-3) : 2 Marks]

1.67 By the beginning of the 20th century, several hypotheses were being proposed, suggesting a paradigm shift in our understanding of the universe. However, the clinching evidence was provided by experimental measurements of the position of a star which was directly behind our sun. Which of the following inference(s) may be drawn from the above passage?

(i) Our understanding of the universe changes based on the positions of stars
(ii) Paradigm shifts usually occur at the beginning of centuries
(iii) Stars are important objects in the universe
(iv) Experimental evidence was important in confirming this paradigm shift

(a) (i), (ii) and (iv) (b) (iii) only
(c) (i) and (iv) (d) (iv) only

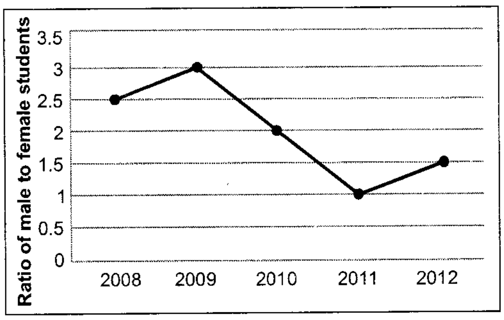
[2014 (Set-3) : 2 Marks]

1.68 The Gross Domestic Product (GDP) in Rupees grew at 7% during 2012-2013. For international comparison, the GDP is compared in US Dollars (USD) after conversion based on the market exchange rate. During the period 2012-2013 the exchange rate for the USD increased from Rs. 50/ USD to Rs. 60/ USD. India's GDP in USD during the period 2012-2013

(a) increased by 5 % (b) decreased by 13%
(c) decreased by 20% (d) decreased by 11%

[2014 (Set-3) : 2 Marks]

1.69 The ratio of male to female students in a college for five years is plotted in the following line graph. If the number of female students in 2011 and 2012 is equal, what is the ratio of male students in 2012 to male students in 2011?



(a) 1:1 (b) 2:1
(c) 1.5:1 (d) 2.5:1

[2014 (Set-3) : 2 Marks]

1.70 Consider the equation: $(7526)_8 - (Y)_8 = (4364)_8$, where $(X)_N$ stands for X to the base N. Find Y.

(a) 1634 (b) 1737
(c) 3142 (d) 3162

[2014 (Set-3) : 2 Marks]

1.71 Didn't you buy _____ when you went shopping?

(a) any paper (b) much paper
(c) no paper (d) a few paper

[2015 (Set-1) : 1 Mark]

1.72 Which of the following options is the closet in meaning to the sentence below?

She enjoyed herself immensely at the party.

(a) She had a terrible time at the party
(b) She had a horrible time at the party
(c) She had a terrific time at the party
(d) She had a terrifying time at the party

[2015 (Set-1) : 1 Mark]

1.73 Given Set A = {2, 3, 4, 5} and Set B = {11, 12, 13, 14, 15}, two numbers are randomly selected, one from each set. What is the probability that the sum of the two numbers equals 16?

(a) 0.20 (b) 0.25
(c) 0.30 (d) 0.33

[2015 (Set-1) : 1 Mark]

1.74 Based on the given statements, select the most appropriate option to solve the given question. If two floors in a certain building are 9 feet apart, how many steps are there in a set of stairs that extends from the first floor to the second floor of the building?

Statements:

1. Each step is 3/4 foot high.
2. Each step is 1 foot wide.

(a) Statement 1 alone is sufficient, but statement 2 alone is not sufficient
(b) Statement 2 alone is sufficient, but statement 1 alone is not sufficient
(c) Both statement together are sufficient, but neither statement alone is sufficient
(d) Statement 1 and 2 together are not sufficient

[2015 (Set-1) : 1 Mark]

1.75 Which one of the following combinations is incorrect?

(a) Acquiescence - Submission
(b) Wheedle - Roundabout
(c) Flippancy - Lightness
(d) Profligate - Extravagant

[2015 (Set-1) : 1 Mark]

1.76 The number of students in a class who have answered correctly, wrongly, or not attempted each question in an exam, are listed in the table below. The marks for each question are also listed. There is no negative or partial marking.

Q.No.	Marks	Answered Correctly	Answered Wrongly	Not Attempted
1	2	21	17	6
2	3	15	27	2
3	1	11	29	4
4	2	23	18	3
5	5	31	12	1

What is the average of the marks obtained by the class in the examination?

(a) 2.290
(b) 2.970
(c) 6.795
(d) 8.795

[2015 (Set-1) : 2 Marks]

1.77 Select the alternative meaning of the underlined part of the sentence.

The chain snatchers **took to their heels** when the police party arrived.

(a) took shelter in a thick jungle
(b) open indiscriminate fire
(c) took to flight
(d) unconditionally surrendered

[2015 (Set-1) : 2 Marks]

1.78 The given statement is following by some courses of action. Assuming the statement to be true, decide the correct option.

Statement: There has been a significant drop in the water level in the lakes supplying water to the city.

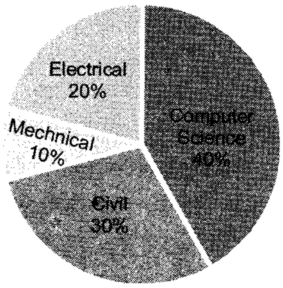
Course of action:

1. The water supply authority should impose a partial cut in supply to tackle the situation.
2. The government should appeal to all the residents through mass media for minimal use of water.
3. The government should ban the water supply in lower areas.

(a) Statement 1 and 2 follow
(b) Statement 1 and 3 follow
(c) Statement 2 and 3 follow
(d) All statements follow

[2015 (Set-1) : 2 Marks]

1.79 The pie chart below has the breakup of the number of students, from different departments in an engineering college for the year 2012. The proportion of male to female students in each department is 5 : 4. There are 40 males in Electrical Engineering. What is the different between the numbers of female students in the Civil department and the female students in the Mechanical department?



[2015 (Set-1) : 2 Marks]

1.80 The probabilities that a student passes in Mathematics, Physics and Chemistry are m, p, and c respectively. Of these subjects, the student has 75% chance of passing in at least one, a 50% chance of passing in at least two and a 40% chance of passing in exactly two. Following relations are drawn in m, p and c:

1. $p + m + c = 27 / 20$
2. $p+m+c = 13 / 20$
3. $(p) \times (m) \times (c) = 1 / 10$

(a) Only relation 1 is true
(b) Only relation 2 is true
(c) Relations 2 and 3 are true
(d) Relations 1 and 3 are true.

[2015 (Set-1) : 2 Marks]

1.81 We ________ our friend's birthday and we ________ how to make it up to him.

(a) completely forgot --- don't just know
(b) forgot completely --- don't just know
(c) completely forgot --- just don't know
(d) forgot completely --- just don't know

[2015 (Set-2) : 1 Mark]

1.82 Choose the statement where underlined word is used correctly.

(a) The industrialist had a personnel jet
(b) I write my experience in my personnel diary
(c) All personnel are being given the day off
(d) Being religious is a personnel aspect

[2015 (Set-2) : 1 Mark]

1.83 Consider a function $f(x) = 1 - |x|$ on $-1 \le x \le 1$. The value of x at which the function attains a maximum, and the maximum value of the function are:

(a) 0, –1 (b) –1, 0
(c) 0, 1 (d) –1, 2

[2015 (Set-2) : 1 Mark]

1.84 A generic term that includes various items of clothing such as a skirt, a pair of trousers and a shirt is

(a) fabric (b) textile
(c) fibre (d) apparel

[2015 (Set-2) : 1 Mark]

1.85 Based on the given statements, select the most appropriate option to solve the given question. What will be the total weight of 10 poles each of same weight?

Statements:

1. One fourth of the weight of a pole is 5 Kg.
2. The total weight of these poles is 160 Kg more than the total weight of two poles.

(a) Statement 1 alone is not sufficient
(b) Statement 2 alone is not sufficient
(c) Either 1 or 2 alone is sufficient
(d) Both statements 1 and 2 together are not sufficient

[2015 (Set-2) : 1 Mark]

1.86 If the list of letters, P, R, S, T, U is an arithmetic sequence, which of the following are also in arithmetic sequence?

1. 2P, 2R, 2S, 2T, 2U
2. P–3, R–3, S–3, T–3, U–3
3. P^2, R^2, S^2, T^2, U^2

(a) 1 only (b) 1 and 2
(c) 2 and 3 (d) 1 and 3

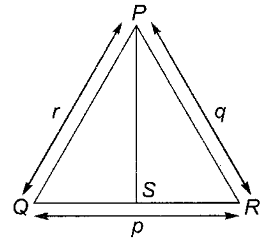
[2015 (Set-2) : 2 Marks]

1.87 Four branches of a company are located at M, N, O and P. M is north of N at a distance of 4 km; P is south of O at a distance of 2 km; N is southeast of O by 1 km. What is the distance between M and P in km?

(a) 5.34 (b) 6.74
(c) 28.5 (d) 45.49

[2015 (Set-2) : 2 Marks]

1.88 In a triangle PQR, PS is the angle bisector of ∠QPR and ∠QPS = 60°. What is the length of PS?



(a) $\dfrac{(q+r)}{qr}$ (b) $\dfrac{qr}{(q+r)}$

(c) $\sqrt{(q^2+r^2)}$ (d) $\dfrac{(q+r)^2}{qr}$

[2015 (Set-2) : 2 Marks]

1.89 If p, q, r, s are distinct integers such that:

f(p, q, r, s) = max (p, q, r, s)
g(p, q, r, s) = min (p, q, r, s)
h(p, q, r, s) = remainder of (p × q) /(r × s) if (p × q) > (r × s) or remainder of (r × s)/ (p × q) if (r × s) > (p × q)

Also a function fgh (p, q, r, s) = f(p, q, r, s) × g(p, q, r, s) × h(p, q, r, s).

Also the same operation are valid with two variable functions of the form f(p, q).

What is the value of fg(h(2, 5, 7, 3), 4, 6, 8)?

[2015 (Set-2) : 2 Marks]

1.90 Out of the following four sentences, select the most suitable sentence with respect to grammar and usage:

(a) Since the report lacked needed information, it was of no use to them
(b) The report was useless to them because there were no needed information in it
(c) Since the report did not contain the needed information, it was not real useful to them
(d) Since the report lacked needed information, it would not had been useful to them

[2015 (Set-2) : 2 Marks]

1.91 If ROAD is written as URDG, then SWAN should be written as:

(a) VXDQ (b) VZDQ
(c) VZDP (d) UXDQ

[2015 (Set-3) : 1 Mark]

1.92 The Tamil version of ________ John Abraham-starrer *Madras Cafe* _____ cleared by the Censor Board with no cuts last week, but the film's distributors ______ no takers among the exhibitors for a release in Tamil Nadu _______ this Friday.

(a) Mr., was, found, on
(b) a, was, found, at
(c) the, was, found, on
(d) a, being, find at

[2015 (Set-3) : 1 Mark]

1.93 Extreme focus on syllabus and studying for tests has become such a dominant concern of Indian students that they close their minds to anything ________ to the requirements of the exam.

(a) related (b) extraneous
(c) outside (d) useful

[2015 (Set-3) : 1 Mark]

1.94 Select the pair that best expresses a relationship similar to that expressed in the pair:

Children : Pediatrician

(a) Adult : Orthopaedist
(b) Females : Gynaecologist
(c) Kidney : Nephrologist
(d) Skin : Dermatologist

[2015 (Set-3) : 1 Mark]

1.95 A function f(x) is linear and has a value of 29 at $x = -2$ and 39 at $x = 3$. Find its value at $x = 5$.

(a) 59 (b) 45
(c) 43 (d) 35

[2015 (Set-3) : 1 Mark]

1.96 Alexander turned his attention towards India, since he had conquered Persia. Which one of the statements below is logically valid and can be inferred from the above sentence?

(a) Alexander would not have turned his attention towards India had he not conquered Persia
(b) Alexander was not ready to rest on his laurels, and wanted to march to India
(c) Alexander was completely in control of his army and could command it to move towards India.
(d) Since Alexander's kingdom extended to Indian borders after the conquest of Persia, he was keen to move further.

[2015 (Set-3) : 2 Marks]

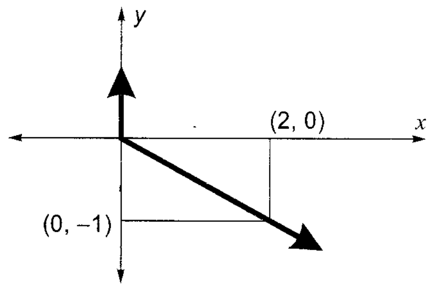
1.97 The head of a newly formed government desires to appoint five of the six selected members P, Q, R, S, T and U to portfolios of Home, Power, Defense, Telecom, and Finance. U does not want any portfolio if S gets one of the five. R wants either Home or Finance or no portfolio. Q says that if S gets either Power of Telecom, then she must get the other one. T insists on a portfolio if P gets one.

Which is the valid distribution of portfolios?

(a) P-Home, Q-Power, R-Defense, S-Telecom, T-Finance
(b) R-Home, S-Power, P-Defense, Q-Telecom, T-Finance
(c) P-Home, Q-Power, T-Defense, S-Telecom, U-Finance
(d) Q-Home, U-Power, T-Defense, R-Telecom, P-Finance

[2015 (Set-3) : 2 Marks]

1.98 Choose the most appropriate equation for the function drawn as a thick line, in the plot below.



(a) $x = y - |y|$ (b) $x = -(y - |y|)$
(c) $x = y + |y|$ (d) $x = -(y + |y|)$

[2015 (Set-3) : 2 Marks]

1.99 Most experts feel that in spite of possessing all the technical skills required to be a batsman of the highest order, he is unlikely to be so due to lack of requisite temperament.
He was guilty of throwing away his wicket several times after working hard to lay a strong foundation. His critics pointed out that until he addressed this problem, success at the highest level will continue to elude him.

Which of the statement(s) below is/are logically valid and can be inferred from the above passage?

(i) He was already a successful batsman at the highest level.
(ii) He has to improve his temperament in order to become a great batsman.

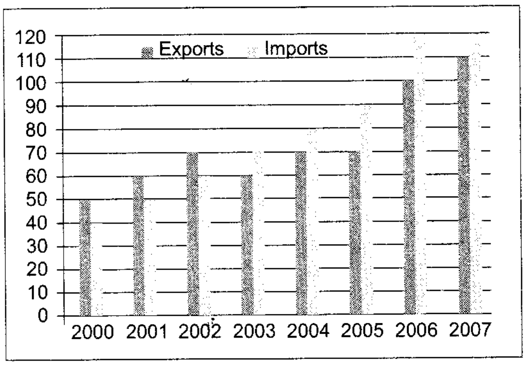
(iii) He failed to make many of his good starts count.

(iv) Improving his technical skills will guarantee success.

(a) (iii) and (iv) (b) (ii) and (iii)
(c) (i), (ii) and (iii) (d) (ii) only

[2015 (Set-3) : 2 Marks]

1.100 The exports and imports (in crores of Rs.) of a country from the year 2000 to 2007 are given in the following bar chart. In which year is the combined percentage increase in imports and exports the highest?



[2015 (Set-3) : 2 Marks]

1.101 Out of the following four sentences, select the most suitable sentence with respect to grammar and usage.

(a) I will not leave the place until the minister does not meet me.

(b) I will not leave the place until the minister doesn't meet me.

(c) I will not leave the place until the minister meet me.

(d) I will not leave the place until the minister meets me.

[2016 : 1 Mark, Set-I]

1.102 A rewording of something written or spoken is a ________.

(a) paraphrase (b) paradox
(c) paradigm (d) paraffin

[2016 : 1 Mark, Set-I]

1.103 Archimedes said, "Give me a lever long enough and a fulcrum on which to place it, and I will move the world." The sentence above is an example of a ________ statement.

(a) figurative (b) collateral
(c) literal (d) figurine

[2016 : 1 Mark, Set-I]

1.104 If 'relftaga' means carefree, 'otaga' means careful and 'fertaga' means careless, which of the following could mean 'aftercare'?

(a) zentaga (b) tagafer
(c) tagazen (d) relffer

[2016 : 1 Mark, Set-I]

1.105 A cube is built using 64 cubic blocks of side one unit. After it is built, one cubic block is removed from every corner of the cube. The resulting surface area of the body (in square units) after the removal is ________.

(a) 56 (b) 64
(c) 72 (d) 96

[2016 : 1 Mark, Set-I]

1.106 A shaving set company sells 4 different types of razors, Elegance, Smooth, Soft and Executive. Elegance sells at Rs. 48, Smooth at Rs. 63, Soft at Rs. 78 and Executive at Rs. 173 per piece. The table below shows the numbers of each razor sold in each quarter of a year.

Quarter/ Product	Elegance	Smooth	Soft	Executive
Q1	27300	20009	17602	9999
Q2	25222	19392	18445	8942
Q3	28976	22429	19544	10234
Q4	21012	18229	16595	10109

Which product contributes the greatest fraction to the revenue of the company in that year?

(a) Elegance (b) Executive
(c) Smooth (d) Soft

[2016 : 2 Marks, Set-I]

1.107 Indian currency notes show the denomination indicated in at least seventeen languages. If this is not an indication of the nation's diversity, nothing else is. Which of the following can be logically inferred from the above sentences?

(a) India is a country of exactly seventeen languages.

(b) Linguistic pluralism is the only indicator of a nation's diversity.

(c) Indian currency notes have sufficient space for all the Indian languages.

(d) Linguistic pluralism is strong evidence of India's diversity.

[2016 : 2 Marks, Set-I]

1.108 Consider the following statements relating to the level of poker play of four players P, Q, R and S. I. P always beats Q II. R always beats S III. S loses to P only sometimes IV. R always loses to Q. Which of the following can be logically inferred from the above statements? (i) P is likely to beat all the three other players (ii) S is the absolute worst player in the set

(a) (i) only (b) (ii) only
(c) (i) and (ii) (d) neither (i) nor (ii)

[2016 : 2 Marks, Set-I]

1.109 If $f(x) = 2x^7 + 3x - 5$, which of the following is a factor $f(x)$?

(a) $(x^3 + 8)$ (b) $(x - 1)$
(c) $(2x - 5)$ (d) $(x + 1)$

[2016 : 2 Marks, Set-I]

1.110 In a process, the number of cycles to failure decreases exponentially with an increase in load. At a load of 80 units, it takes 100 cycles for failure. When the load is halved, it takes 10000 cycles for failure. The load for which the failure will happen in 5000 cycles is ________.

(a) 40.00 (b) 46.02
(c) 60.01 (d) 92.02

[2016 : 2 Marks, Set-I]

1.111 The man who is now Municipal Commissioner worked as ________.

(a) the security guard at a university
(b) *a security guard at the university*
(c) a security guard at university
(d) the security guard at the university

[2016 : 1 Mark, Set-II]

1.112 Nobody knows how the Indian cricket team is going to cope with the difficult and seamer-friendly wickets in Australia. Choose the option which is closest in meaning to the underlined phrase in the above sentence.

(a) put up with (b) put in with
(c) put down to (d) put up against

[2016 : 1 Mark, Set-II]

1.113 Find the odd one in the following group of words.

mock, deride, praise, jeer

(a) mock (b) deride
(c) praise (d) jeer

[2016 : 1 Mark, Set-II]

1.114 Pick the odd one from the following options.

(a) CADBE (b) JHKIL
(c) XVYWZ (d) ONPMQ

[2016 : 1 Mark, Set-II]

1.115 In a quadratic function, the value of the product of the roots (α, β) is 4. Find the value of

$$\frac{\alpha^n + \beta^n}{\alpha^{-n} + \beta^{-n}}$$

(a) n^4 (b) 4^n
(c) 2^{2n-1} (d) 4^{n-1}

[2016 : 1 Mark, Set-II]

1.116 Among 150 faculty members in an institute, 55 are connected with each other through Facebook® and 85 are connected through WhatsApp®. 30 faculty members do not have Facebook® or WhatsApp® accounts. The number of faculty members connected only through Facebook® accounts is ________.

(a) 35 (b) 45
(c) 65 (d) 90

[2016 : 2 Marks, Set-II]

1.117 Computers were invented for performing only high-end useful computations. However, it is no understatement that they have taken over our world today. The internet, for example, is ubiquitous. Many believe that the internet itself is an unintended consequence of the original invention. With the advent of mobile computing on our phones, a whole new dimension is now enabled. One is left wondering if all these developments are good or, more importantly, required.

Which of the statement(s) below is/are logically valid and can be inferred from the above paragraph?

(i) The author believes that computers are not good for us.
(ii) Mobile computers and the internet are both intended inventions

(a) (i) only (b) (ii) only
(c) both (i) and (ii) (d) neither (i) nor (ii)

[2016 : 2 Marks, Set-II]

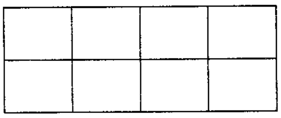
1.118 All hill-stations have a lake. Ooty has two lakes. Which of the statement(s) below is/are logically valid and can be inferred from the above sentences?

(i) Ooty is not a hill-station.
(ii) No hill-station can have more than one lake.

(a) (i) only
(b) (ii) only
(c) both (i) and (ii)
(d) neither (i) nor (ii)

[2016 : 2 Marks, Set-II]

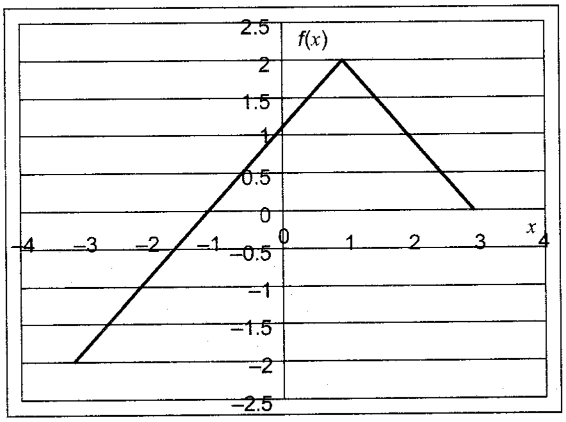
1.119 In a 2 × 4 rectangle grid shown below, each cell is a rectangle. How many rectangles can be observed in the grid?



(a) 21 (b) 27
(c) 30 (d) 36

[2016 : 2 Marks, Set-II]

1.120



Choose the correct expression for $f(x)$ given in the graph.

(a) $f(x) = 1 - |x - 1|$
(b) $f(x) = 1 + |x - 1|$
(c) $f(x) = 2 - |x - 1|$
(d) $f(x) = 2 + |x - 1|$

[2016 : 2 Marks, Set-II]

Answers General Aptitude

1.1 (b)	1.2 (a)	1.3 (d)	1.4 (c)	1.5 (d)	1.6 (d)	1.7 (c)	1.8 (d)	1.9 (b)	
1.10 (b)	1.11 (a)	1.12 (b)	1.13 (c)	1.14 (b)	1.15 (d)	1.16 (c)	1.17 (d)	1.18 (a)	
1.19 (c)	1.20 (d)	1.21 (a)	1.22 (b)	1.23 (a)	1.24 (b)	1.25 (a)	1.26 (d)	1.27 (b)	
1.28 (b)	1.29 (c)	1.30 (a)	1.31 (b)	1.32 (a)	1.33 (c)	1.34 (a)	1.35 (c)	1.36 (b)	
1.37 (d)	1.38 (d)	1.39 (c)	1.40 (b)	1.41 (b)	1.42 (c)	1.43 (c)	1.45 (d)	1.46 (c)	
1.47 (b)	1.51 (a)	1.52 (b)	1.53 (a)	1.54 (b)	1.55 (c)	1.56 (b)	1.57 (d)	1.58 (d)	
1.60. (a)	1.61 (d)	1.62 (c)	1.63 (c)	1.64 (c)	1.65 (c)	1.66 (b)	1.67 (d)	1.68 (d)	
1.69 (c)	1.70 (c)	1.71 (a)	1.72 (c)	1.73 (a)	1.74 (a)	1.75 (b)	1.76 (c)	1.77 (c)	
1.78 (a)	1.80 (d)	1.81 (c)	1.82 (c)	1.83 (c)	1.84 (d)	1.85 (c)	1.86 (b)	1.87 (a)	
1.88 (b)	1.90 (a)	1.92 (c)	1.93 (b)	1.94 (b)	1.95 (c)	1.96 (d)	1.97 (b)	1.98 (b)	
1.99 (b)	1.101 (d)	1.102 (a)	1.103 (a)	1.104 (c)	1.105 (d)	1.106 (b)	1.107 (d)	1.108 (d)	
1.109 (b)	1.110 (b)	1.111 (b)	1.112 (a)	1.113 (c)	1.114 (d)	1.115 (a)	1.116 (a)	1.117 (d)	
1.118 (d)	1.119 (c)	1.120 (c)							

Explanations General Aptitude

1.1 (b)

Circuitous: Deviating from a straight course ⇒ Indirect

(a) Cyclic: Recurring in cycle
(b) Indirect: Not leading by straight line
(c) Confusing: Lacking clarity
(d) Crooked: For shapes (irregular in shape)

1.2 (a)

Unemployed: Worker ⇒ Here one is opposite to other.

(a) Fallow: Land ⇒ Fallow means undeveloped land.
(b) Unaware: sleeper ⇒ Both are same unaware or asleep.
(c) Wit: Jester ⇒ Wit means ability to make jokes and jester is a joker.
(d) Renovated : House ⇒ Renovate means to make better and house can be renovated.

1.3 (d)

(a) Uphold: cause to remain ⇒ not appropriate
(b) Restrain: keep under control ⇒ not appropriate
(c) Cherish: be fond of ⇒ not related
(d) Conserve: Keep in safety and protect from harm, decay, loss, or destruction ⇒ most appropriate.

1.4 (c)

(a) Masked: Hide under a false appearance ⇒ opposite
(b) Belied: Be in contradiction with ⇒ not appropriate
(c) Betrayed: Reveal unintentionally ⇒ most appropriate
(d) Suppressed: To put down by force or authority ⇒ irrelevant

1.5 (d)

Using the set theory formula

n(A) : Number of people who play hockey = 15

n(B) : Number of people who play football = 17

n $(A \cap B)$: Persons who play both hockey and football = 10

n $(A \cup B)$: Persons who play either hockey or football or both

Using the formula

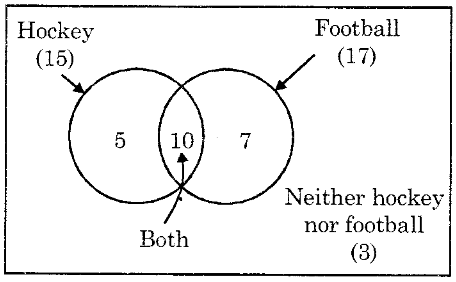
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A \cup B) = 15 + 17 - 10 = 22$

Thus people who play neither hockey nor football $= 25 - 22 = 3$

Alternative Method

Refer to Venn diagram given below:



1.6 (d)

(a) Modern warfare has resulted in civil strife: There is no direct consequence of warfare given, so it is not appropriate.
(b) Chemical agents are useful in modern warfare: Passage does not say whether chemical agents are useful or not, so not appropriate.
(c) Use of chemical agents in warfare would be undesirable: Given that people in military think these are useful, undesirable is wrong
(d) People in military establishments like to use chemical agents in war; Correct choice as last statement tells that military people think that chemical agents are useful tools for their cause (work silently in warfare).

1.7 (c)

$137 + 276 = 435$

This is a octahedral addition thus

$731 + 672(8) = 1623$

Alternative Method

7 and 6 added is becoming five means the given two numbers are added on base 8.

$$\begin{array}{r} (137)_8 \\ +(276)_8 \\ \hline (435)_8 \end{array}$$

Hence we have to add the another two given set of numbers also on base 8.

$$\begin{array}{r} (731)_8 \\ +(672)_8 \\ \hline (1623)_8 \end{array}$$

Hence the overall problem was based on identyfying base which was 8 and adding number on base 8.

1.8 (d)

Per day work or rate of 5 skilled workers = $\frac{1}{20}$

$\Rightarrow$ Per day work or rate of one skill worker =

$\frac{1}{5\times 20} = \frac{1}{100}$

Similarly Per day work or rate of 8 semiskilled

workers = $\frac{1}{25}$

$\Rightarrow$ Per day work or rate of one semi-skill worker

$= \frac{1}{8\times 25} = \frac{1}{200}$

And per day work or rate of 10 unskilled workers

$= \frac{1}{30}$

$\Rightarrow$ Per day work or rate of one semi-skill worker

$= \frac{1}{10\times 30} = \frac{1}{300}$

Thus total per day work of 2 skilled, 6 semiskilled and 5 unskilled workers

$= \frac{2}{100} + \frac{6}{200} + \frac{5}{300} = \frac{12+18+10}{600}$

$= \frac{40}{600} = \frac{1}{15}$

Thus time to complete the work is 15 days.

Alternative Method:

Let one day work of skilled semi-skilled and unskilled worker be a, b, c units respectively.

$5a \times 20 = 8b + 25 = 10c \times 30$ = Total unit of work

$100a = 200b = 300c$

$a = 2b = 3c$

$\Rightarrow \quad b = \frac{a}{2}$ and $c = \frac{a}{3}$

Given that 2 skilled, 6 semi-skilled and 5 unskilled workers are working. Let they finish the work in 'x' days.

$(2a + 6b + 5c)x = 5a + 20$

= Total units of work

$\left(2a + 3a + \frac{5}{3}a\right)x = 5a \times 20$

$\frac{20a}{3}x = 5a \times 20$

$x = 15$ days

1.9 (b)

We have to make 4 digit numbers, so the number should be start with 3 or 4, two cases possible;

Case (1) thousands digit is 3

Now other three digits may be any of 2, 2, 3, 3, 4, 4, 4, 4.

(a) Using 2, 2, 3
$\Rightarrow$223, 232, 322 -------
$\left(\frac{3!}{2!} = 3 \text{ numbers are possible}\right)$

(b) Using 2, 2, 4 $\Rightarrow$ 224, 242, 422
$\left(\frac{3!}{2!} = 3 \text{ numbers are possible}\right)$

(c) Using 2, 3, 3 $\Rightarrow$ 233, 323, 332
$\left(\frac{3!}{2!} = 3 \text{ numbers are possible}\right)$

(d) Using 2, 3, 4 $\Rightarrow$ 234, 243, 324, 342, 423, 432
(3! = 6 numbers are possible)

(e) Using 2, 4, 4 $\Rightarrow$ 244, 424, 442
$\left(\frac{3!}{2!} = 3 \text{ numbers are possible}\right)$

(f) Using 3, 3, 4 $\Rightarrow$ 334, 343, 433
$\left(\frac{3!}{2!} = 3 \text{ numbers are possible}\right)$

(g) Using 3, 4, 4 $\Rightarrow$ 344, 434, 443
$\left(\frac{3!}{2!} = 3 \text{ numbers are possible}\right)$

(h) Using 4, 4, 4 $\Rightarrow$ 444
$\left(\frac{3!}{3!} = 1 \text{ numbers are possible}\right)$

Total 4 digit numbers in case 1
$= 3 + 3 + 3 + 6 + 3 + 3 + 3 + 1 = 25$

Case (2): thousands digit is 4 ; Now other three digits may be any of 2, 2, 3, 3, 3, 4, 4, 4.

(a) Using 2, 2, 3 $\Rightarrow$ 223, 232, 322

$\left(\frac{3!}{2!} = 3 \text{ numbers are possible}\right)$

(b) Using 2, 2, 4 $\Rightarrow$ 224, 242, 422

$\left(\frac{3!}{2!} = 3 \text{ numbers are possible}\right)$

(c) Using 2, 3, 3 $\Rightarrow$ 233, 323, 332

$\left(\frac{3!}{2!} = 3 \text{ numbers are possible}\right)$

(d) Using 2, 3, 4 $\Rightarrow$ 234, 243, 324, 342, 423, 432

$\left(\frac{3!}{2!} = 3 \text{ numbers are possible}\right)$

(e) Using 2, 4, 4 $\Rightarrow$ 244, 424, 442

$\left(\frac{3!}{2!} = 3 \text{ numbers are possible}\right)$

(f) Using 3, 3, 3 $\Rightarrow$ 333

$\left(\frac{3!}{3!} = 1 \text{ numbers are possible}\right)$

(g) Using 3, 3, 4 $\Rightarrow$ 334, 343, 433

$\left(\frac{3!}{2!} = 3 \text{ numbers are possible}\right)$

(h) Using 3, 4, 4 $\Rightarrow$ 344, 434, 443

$\left(\frac{3!}{2!} = 3 \text{ numbers are possible}\right)$

(i) Using 4, 4, 4 $\Rightarrow$ 444

$\left(\frac{3!}{3!} = 1 \text{ numbers are possible}\right)$

Total 4 digit numbers in case (2)
= 3 + 3 + 3 + 6 + 3 + 3
= 1 + 3 + 1 = 26
Thus total 4 digits numbers using case (1) and case (2)
= 25 + 26 = 51

***Alternative Method / Short-cut method**

As the number is greater than 3000. So thousand's place can be tiehr 3 or 4.
Let's consider the following two cases

Case (I) When thousand's place is 3.
3 a b c
If there is no restricton on number of two's, three's and four's. Then each of a, b, c can be filled with 2 or 3 or 4 each in 3 ways.
So 3 × 3 × 3 = 27 numbers are there. Out of which 3222, 3333 are invalid as 2 can be used twice & three thrice only so number of such valid numbers begining with 3 are 27 – 2 = 25.

Case (II) When thousand's place is 4
4 a b c
Without restriction on number of 2's, 3's and 4's a, b, c (as explained in case I) can be filled in 27 ways.
Out of these 27 numbers, 4 2 2 2 is only invalid as two have to be used twice only.
So valid numbers are 27 – 1 = 26.
Total numbers from Case (I) & Case (II) 25 + 26 = 51.

1.10 (b)

Suppose: Hari's age : H, Gita's age : G, Saira's age : S, Irfan's age : I

- H +G > I + S
- Using Statement (2) both G – S = 1 or S – G=1; G can't be oldest and S can't be youngest.
- There are no twins thus using statement (2) either GS or SG possible.

(a) HSIG: not possible as there is I between S and G which is not possible using statement (3)

(b) SGHI: SG order is possible, S > G > H > I and G + H > S +I (possible)
Because if { S = G + 1; and G = H+1 and H = I +2 then G + (I+2) > (G+1) +I}

(c) IGSH: according to this I > G and S > H thus adding these both inequalities we get I + S > G + H which is opposite of statement (2) thus not possible.

(d) IHSG: according to this I > H and S > G thus adding both inequalities I + S > H + G which is opposite of statement (2) thus not possible.

1.12 (b)

$$\text{Log}(P) = (1/2)\text{Log}(Q) = (1/3)\text{Log}(R)$$
$$Q = P^2,$$
$$R = P^3,$$
$$Q = R^{2/3}$$
$$\Rightarrow \quad Q^2 = Q.Q = P^2.P^2 = P.P^3 = P.R$$

1.17 (d)

As we can understand that the danger of a microbe to human being will be directly proportional to potency and growth. At the same time, it will be inversely proportional to toxicity defined (more dangerous will a microbe be if lesser of its milligram is required). So level of dangerous

$\alpha \dfrac{P\uparrow \times G\uparrow}{T\downarrow}$ where P, G and T are the potency, growth and toxicity as defined in question.

So $D_i = \dfrac{KPG}{T}$ ...(i)

Where K is constant of proportionality.

So level of dangerous of S will be maximum given

by $D_S = \dfrac{0.8\times\pi(10\,\text{mm})^2}{200}$.

Similar calculation for D_P, D_Q, D_R can be done based on (i) to find out that D_S is maximum and so most dangerous among them.

1.18 (a)

(T.C.) Total cost = V + f

$$T.C = 4q + \frac{100}{q}$$

As we have to minimize total cost.

Using options

(a) $q = 5,\ T.C. = 4\times5+\dfrac{100}{5} = 40$

(b) $q = 4,\ T.C. = 4\times4+\dfrac{100}{4} = 41$

(c) $q = 7,\ T.C. = 4\times7+\dfrac{100}{7} = 42.285$

(d) $q = 6,\ T.C. = 4\times6+\dfrac{100}{6} = 40.\overline{66}$

Hence, T.C. is minimum at q = 5, **(a) ans.**

* always put options to get answer fast.

Alternate Solution

or $T.C. = 4q + \dfrac{100}{q}$

for minimum $\dfrac{d}{dq}\left(4q+\dfrac{100}{q}\right) = 0$

$\Rightarrow \quad q^2 = 25$ or $q = 5$

$$\frac{d}{dq}\left(4q+\frac{100}{q}\right)_{q=5} > 0$$

Hence, TC (Total cost) is minimum at q = 5 **(a) ans.**

1.19 (c)

Let 'y' we the backlog with transporter and 'x' be the number of orders each day. So, as per conditions given in question

$4x + y = 28$ ...(i)

$10x + y = 30$ ...(ii)

Solving (i) and (ii),

$$x = \frac{1}{3} \text{ and } y = \frac{80}{3}$$

Since, we need to find out number of trucks so that no pending order will be there at the end of 5th day.

$5x + y = n \times 5$

So we need to find 'n', where 'n' is the number of trucks required.

$$n = \frac{5x+y}{5} = \frac{5\times\frac{1}{3}+\frac{80}{3}}{5} = \frac{\frac{85}{3}}{5}$$

Hence, $5.\overline{66}$ truck will be required. As number of trucks have to be natural number. Hence, 6 trucks will be required.

1.20 (d)

Short-cut Method

Every time if we take 1 litre of mixture out and replace with water, content of pure spirit will keep on reducing by 10%.

So, final quantity of spirit after 3 such operations are

$10 \times 0.9 \times 0.9 \times 0.9 = 7.29$ litres

Alternate Solution

$$\frac{\text{Quantity of spirit left after } n^{th} \text{ operation}}{\text{Intial quantity of spirit}} = \left(\frac{a-b}{a}\right)^n = \left(1-\frac{b}{a}\right)^n$$

where 'a' is initial quantity of pure spirit and 'b' is quantity taken out and replaced every time.

Hence, quantity of spirit left after 3rd operation

$$= \text{initial quantity} \times \left(1 - \frac{1}{10}\right)^3$$

$$= 10 \times 0.9 \times 0.9 \times 0.9 = 7.29 \text{ litres}$$

1.21 (a).

Total Cost $C = 5q^2$

Total Sales Revenue,

$S = 50q$

Profit (P)= $(S - C) = 50q - 5q^2$

As the profit have to be maximized

$P = 50q - 5q^2$

Using options

(a) $q = 5$

$P = 50 \times 5 - 5 \times 5^2 = 125$

(b) $q = 10$

$P = 50 \times 10 - 5 \times 10^2 = 0$

(c) $q = 15$

$P = 50 \times 15 - 5 \times 15^2 = $ –ive

(d) $q = 25$

$P = 50 \times 25 - 5 \times 25^2 = $ –ive

Hence the maximum profit will happen at $q = 5$

Alternative solution:

$$P = 50q - 5q^2$$

For max/min $\frac{dp}{dq} = 0$,

$$\Rightarrow \quad 50 - 10q = 0$$

$$q = 10$$

$$\left.\frac{d^2p}{dq^2}\right]_{q=5} = -10 \text{ which is negative}$$

Hence maximum profit will happen at

$$q = 5$$

1.22 (b)

Setbacks

Despite several setbacks the mission succeeded in its attempt to resolve the conflict.

The word 'Despite' indicates that there has to be a contrast in the sentence, use of the word 'Setbacks' in the blank indicates that despite many problems the mission was successful.

1.23 (a)

Diminish

Mitigate means to reduce, to lessen etc. So only the word Diminish is close. Rest all choices have no link with the given word.

Divulge means to disclose or reveal which has no link with the given word.

1.24 (b)

The country's expenditure is not less than that of Bangladesh.

The correct statement should be: The country's expenditure is not less than that of Bangladesh's. Because the country's expenditure cannot be compared with Bangladesh. It should be compared with Bangladesh's expenditure.

1.25 (a)

Suresh's dog is the one that was hurt in the stampede.

That is used with restrictive clauses.

1.26 (d)

Not gender-discriminatory.

Choice (A) cannot be considered since there is no gender discrimination mentioned in the argument.

Choice (B) Xenophobic is one who has fear of foreigners, no link with the given argument.

Choice (C) It is wrong to say that the profile has not been designed to make the post attractive, since there are certain features which have been added to make the profile lucrative (which are given towards the end of the advertisement, like Transportation paid, expenses reimbursed).

1.27 (b)

$$y = 2x - 0.1x^2$$

For y (height) to be maximum $\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = 2x - 2x = 0$$

$$\Rightarrow \quad x = 10$$

$$\left.\frac{d^2y}{dx^2}\right]_{x=10} = -0.2 \text{ (–ive)}, \quad \frac{d^2y}{dx^2} < 0$$

Hence maximum height will be at $x = 10$

$$y = 2 \times 10 - 0.1 \times (10)^2 = 10$$

Maximum height = 10 meters.

1.28 (b)

Let total 100 shock absorbes are supplied

so X supplies = 60 (40% of 100)

Y supplies = 40 (60% of 100)

Reliable supply by X = 96% of 60 = 57.6
Reliable supply by Y = 72% of 40 = 28.8
So the required probability that random chosen reliable shock absorber is made by Y is
Required probability

$$= \frac{28.8}{57.6+28.8} = 0.333 \approx 0.334$$

1.29 (c)

Adding each and every observation by constant increases the arithmetic mean by same constant. Also multiplying each and every observation by constant mutiplies the arithmetic mean by same constant. While the standard deviation will not show similar effect.

Alternative Solution:

$$\frac{x_1 + x_2 + x_3 \cdots x_N}{N} = \bar{x}$$

$$\frac{(x_1+7)+(x_2+7)+\cdots+(x_n+7)}{N} = \bar{x}+7$$

and $$\frac{2(x_1 + x_2 \cdots x_N)}{N} = 2\bar{x}$$

Hence only P, R are correct.

1.30 (a)

AD, CG, FK, JP are given terms. The first letter in each term A, C, F, J are respectively 1, 3, 6, 10 letter of alphabetical sequence.

A = 1; 1st letter of alphabetical sequence
C = 1 + 2 = 3 ; 3rd letter of alphabetical sequence
F = 1 + 2 + 3 = 6 ; 6th letter of alphabetical sequence
G = 1 + 2 + 3 + 4 = 10; 10th letter of alphabetical sequence

The next term should begin with
1 + 2 + 3 + 4 + 5 = 15th letter of alphabetical sequence. So next term should begin with O.
Also in the series AD, CG, FK, JP, two letters in each term are separated by 2, 3, 4, 5 letters in between them respectively. Hence next term should be OV as there should be a gap of 6 alphabets between O and V.
Hence answer is OV.

1.31 (b)

Lowest
Nadir which is opposite to the word Zenith (the highest point) means the lowest point.

1.32 (a)

Specificity
Universalism and Particularism are opposite words. Similarly, we require a word that indicates an opposite to diffuseness.
Diffuseness refers to spreading or causing to spread in all directions. Specificity is an appropriate opposite.

1.33 (c)

For sum to be maximum, we will consider only positive terms

$44 + 42 + \ldots\ldots 2$
or $2 + 4 + 6 \ldots\ldots 42 + 44$
$2[1 + 2 + 3 \ldots\ldots 22]$

$$= \frac{2[22\times 23]}{2} = 506$$

Alternate:
Sum of even Natural Numbers
$2 + 4 + 6 \ldots 44 = n(n + 1)$
$22 \times 23 = 506$

1.34 (a)

Would fly
This is a sentence based on Conditional Sentences. In the first clause imagination (Were) has been used, so the main clause should comprise of 'would + I form of verb' usage.

1.35 (c)

She is an European.
The usage of articles a and an is not decided by the first alphabet of the word but the sound / pronunciation of the word. European though starts with 'E' which is a vowel but it does not yield a vowel sound. When read aloud, it leaves the sound of 'Y' which is a consonant sound.
So the correct sentence should be, "She is a European".

1.36 (b)

The question can be written as..

$$\frac{1}{\sqrt{2}+\sqrt{1}} + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{\sqrt{4}+\sqrt{3}} \cdots \frac{1}{\sqrt{8}+\sqrt{80}}$$

Rationalize each term by multiplying it with its congugate in both numerator and denominator.

$$\left[\frac{1}{\sqrt{2}+\sqrt{1}} \times \frac{\sqrt{2}-\sqrt{1}}{\sqrt{2}-\sqrt{1}}\right] \times \left[\frac{1}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}\right] \times \ldots\ldots$$

$$\ldots\ldots\left[\frac{1}{\sqrt{8}\times\sqrt{80}}\times\frac{\sqrt{81}-\sqrt{80}}{\sqrt{81}-\sqrt{80}}\right]$$

Which can be resolved as

$$\left[\sqrt{2}-\sqrt{1}\right]+\left[\sqrt{3}-\sqrt{2}\right]\ldots\ldots\ldots\left[\sqrt{81}-\sqrt{80}\right]$$

(as denominator in each term will become 1)
Now cancelling like terms we will be left with

$$\sqrt{81}-\sqrt{1}=(9-1)=8$$

1.37 (d)

There are Ninety (90) two digited numbers between 1 and 100 (typically 10 to 99). Among these numbers which are divisible by 7 are (14, 21, ...98) only 13 numbers.

$$\left[\because\left(\frac{100}{7}\right)-1=13\right]$$

So, the required probability that the number is not divisible by

$$1-\frac{13}{90}=\frac{77}{90}$$

1.38 (d)

No adversity justifies giving up hope.
Options (b) and (c) can be clearly discarded since honesty and adventure making have no relationship with the given argument.
Option (a) may appear close. But remember, every failure may not result into success as it happens in the case of spider.

1.39 (c)

$$\text{Average speed}=\frac{\text{Total distance}}{\text{Total time}}$$

$$=\frac{D}{\frac{D/2}{60}+\frac{D/4}{30}+\frac{D/4}{10}}=24\text{ kmph}$$

1.40 (b)

Now cost

$$=13200\left[1+\frac{1}{5}\right]\left[1-\frac{1}{24}\right]$$

$$=13200\left[\frac{6}{5}\right]\times\left[\frac{23}{24}\right]=15,180$$

1.44 Sol.

$$(z+1/z)^2 = 98$$
$$\Rightarrow\quad z^2+1/z^2+2 = 98$$
$$\Rightarrow\quad z^2+1/z^2 = 96$$

1.45 (d)

Given $ax^2+bx+c=0$
roots are real and positive
For e.g.
$x^2-5x+6=0$ roots are 2, 3
Then roots of $ax^2+b|x|+c=0$
then roots of $x^2-5|x|+6=0$
$\therefore\ |x|^2-5|x|+6=0$

$$\left.\begin{array}{l}\text{if } x>0\\ |x|=x\end{array}\right\}\quad x^2-5x+6=0$$

$(x-2)(x-3)=0$ $\qquad x=2, 3$ are roots

$$\left.\begin{array}{l}\text{If } x<0\\ |x|=-x\end{array}\right\}\quad x^2+5x+6=0$$

$(x+2)(x+3)=0$ $\qquad x=-2, -3$ are roots
So if $x^2-5x+6=0$ has two real roots
then $|x|^2-5|x|+6=0$ has four real roots
$ax^2+bx+c=0$ has real roots
then $ax^2+b|x|+c=0$ has 4 real roots.

1.48 Sol.

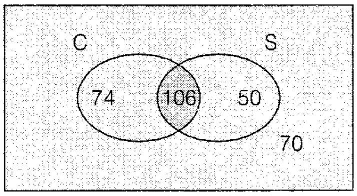
One way fare of single person = 100
Undiscounted round trip fare of singe person = 200
Undiscounted round trip fare of five persons = 1000
Round trip will result in a discount of 10%. Also additional discount of 5% on total fare for 4 or more person.
So a term of 5 person will be eligible for overall discount of (10% + 5%) = 15% discount
Discounted round trip for 5 persons
$1000\times 0.85=850$ Rs.

1.49 Sol.

Following table can be redrawn:

		Men	Women	Total
Own vehicle	Car	40	34	74
	Scooter	30	20	50
	Both	60	46	106
Do not own vehicle		20	50	70

Now following venn-diagram can be drawn



So % of people not owing a scooter

$$\left[\left(\frac{74+70}{300}\right)\times 100\right] = \frac{144}{300}\times 100 = 48\%$$

1.50 Sol.

By visualization we can see 6 new internal planes will be created through these lines.

1.54 (b)

There are 19 multiples of 10 from 2 to 198 typically 10, 20, 30 ⋯ 190.

Average of these number will be

$$= \frac{10+20+30+\cdots+190}{19}$$

$$= 10\frac{[1+2+3+\cdots+19]}{19}$$

$$= 10\frac{[19\times 20]}{19\times 2} = 100$$

1.55 (c)

Let, $y = \sqrt{12+\sqrt{12+\sqrt{12+\cdots}}}$

So, $y = \sqrt{12+y}$

Squaring $y^2 = 12 + y$

$y^2 - y - 12 = 0$

$\Rightarrow$ $(y-4)(y+3) = 0$

$y = 4$

Option (c) $\rightarrow$ 4.000

1.58 (d)

$x^2 - 2x + 3 = 11$

$\Rightarrow x^2 - 2x - 8 = 0$

$\Rightarrow x^2 - 4x + 2x - 8 = 0$

$\Rightarrow x(x-4) + 2(x-4) = 0$

$\Rightarrow x = 4, -2$

(i) put $x = 4$ in $|-x^3 + x^2 - x|$

$\Rightarrow |-(4)^3 + (4)^2 - (4)| = 52$

(ii) put $x = -2$ in $|-x^3 + x^2 - x|$

$\Rightarrow |-(-2)^3 + (-2)^2 - (-2)| = 14$

Possible values = 14, 52

1.59 Sol.

Given ratio of male to female is 2.5 in year 2008.
So let us assume male as 500 and female as 200.

So ratio $\frac{M}{F}$ in 2008 = 2.5

Now female doubles up in 2009

Number of female in 2009 = 2 × 200 = 400

$\frac{M}{F}$ ratio in 2009 is 3

$$\frac{M}{400} = 3 \Rightarrow M = 1200$$

So number of males in 2009 is 1200.

% increase in number of males from 2008 to 2009

$$\%\text{ increase} = \frac{1200-500}{500}\times 100 = 140\%$$

1.60 (a)

Any angle and corresponding time can be driven by

$$\left[5x - \left(\frac{D^0}{6}\right)\right]\times\frac{12}{11}$$

Between 6 and 7 pm angle have to be 60°

Put x = 6 and D = 60°

$$\left[5\times 6 - \frac{60^\circ}{6}\right]\times\frac{12}{11} = 20\times\frac{12}{11} = 21\frac{9}{11}$$

So at $4:21\frac{9}{11}$ angle between minutes and hour hand will be 60°.

In given question angle have to close to 60° nearest option is 6 : 22.

1.64 (c)

Given series: 2, 5, 10, 17, 26, 37, 50, 64

$1^2+1, 2^2+1, 3^2+1, 4^2+1, 5^2+1, 6^2+1, 7^2+1, 8^2+1$

$\Rightarrow$ 2, 5, 10, 17, 26, 37, 50, 65

$\therefore$ 64 is not in the series.

1.65 (c)

Q.No.	Marks	Answered Correctly	Answered Wrongly	Not Attemped	Total Students
1	2	21	17	6	44
2	3	15	27	2	44
3	2	23	18	3	44

$$\text{Average} = \frac{\text{Total marks scored}}{\text{Total number of students}}$$

$$= \frac{21 \times 2 + 15 \times 3 + 23 \times 2}{44} = 3.022727 \simeq 3.02$$

1.68 (d)

Let GDP before year 2012-2013 be 100 Rs.
During 2012-2013 GDP increased by 7%.
So, GDP in year 2013-2013 = 107 Rs.
Now exchange rate for USD change from Rs. 50/ USD to Rs. 60/USD.
i.e. value of Rupee reduced to 5/6 of original value.

$$\text{So, new GDP (in USD)} = 107 \times \frac{5}{6} = 89.16$$

Effective change is from 100 to 89.16
= 10.83% decrease ≃ 11% decrease

1.69 (c)

Ratio of male to female 2001 is given.
Let there be 100 males and 100 female in 2011.
Now number of female in 2012 is same as number of females in 2012 as given in question.
So female in 2012 is also 100.
Ratio of male of female in 2012 is 1.5

$$\frac{M}{F} = 1.5$$

$$\Rightarrow \frac{M}{F} = 1.5 \quad \Rightarrow \quad M = 150$$

So there are 150 males n 2012.
Ratio of male in 2012 to male in 2011

$$= \frac{150}{100} = \frac{1.5}{1}$$

1.70 (c)

$(7526) - (Y)_8 = (4364)_8$
So, $Y = (7526)_8 - (4364)_8$
$= (7526 - 4364)_8 = (31420)_8$

1.71 (a)

Didn't you buy any paper when you went shopping?

1.72 (c)

She enjoyed herself immensely at the party. The sentence with closest meaning is " she had a terrific time at the party".

1.73 (a)

Total number of ways of selecting two numbers
$= {}^4C_1 \times {}^5C_1 = 4 \times 5 = 20$
Favourable outcomes are {2, 14}, {3, 13}, {4, 12}, {5, 11}

$$\therefore \text{Probability} = \frac{4}{20} = \frac{1}{5} = 0.20$$

1.74 (a)

Statement 1:

$$\text{Number of steps} = \frac{9}{3/4} = 12 \text{ steps}$$

∴ Statement 1 is alone sufficient.
Statement 2:
Only width of step is given which is not sufficient to calculate number of steps.
∴ Option (a) is correct.

1.75 (b)

The incorrect combination is
Wheedle – Roundabout

1.76 (c)

Total number of students = 44
Marks obtained by answering 1st question correctly $\Rightarrow 2 \times 12 = 42$
From 2nd question answered correctly
$\Rightarrow 15 \times 3 = 45$
From 3rd question answered correctly
$\Rightarrow 11 \times 1 = 11$
From 4th question answered correctly
$\Rightarrow 2 \times 223 = 46$
From 5th question answered correctly
$\Rightarrow 5 \times 31 = 155$

Total = 299

$$\therefore \text{Average marks} = \frac{299}{44} = 6.795$$

1.77 (c)

The chain snatchers **took to their heels** when the police party arrived.
The alternative meaning for "took to their heels" is "**took to flight**".

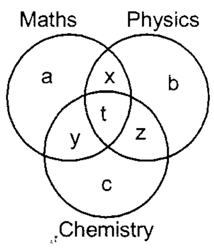
1.78 (a)

Only statement 1 and 2 follows.

1.79 Sol.

Let total number of students in Electrical be T_e.
Male students in Electrical is 40. So $5/9 \times T_e = 40 \Rightarrow T_e = 72$
20% are Electrical students. Therefore $20/100 \times$ Total $= 72 \Rightarrow$ Total $= 360$.
Number of females in Civil
$= 30\%$ of $360 \times 4/9 = 48$
Number of females in Mechanical
$= 10\%$ of $360 \times 4/9 = 16$
Different between the numbers of female students in the Civil department and the female students in the Mechanical department $= 48 - 16 = 32$.

1.80 (d)



$75\% = a + b + c + (x + y + z) + t$
$(x + y + z) + t = 50\%$
$x + y + z = 40\% \quad \Rightarrow \quad t = 10\%$
$\therefore$ Statement 3 is correct.
$a + b + c = 25$
Also Maths + Physics + Chemistry
$= (a + x + y + t)+(b + x + z + t)+(c + y + z +t)$
$= (a + b + c) + 2\ (x + y + z) + 3t$
$= 25 + 2 \times 40 + 3 \times 10 = 135\%$
$= 27/20$
$\therefore$ Statement 1 is correct.

1.81 (c)

We **completely forgot** our friend's birthday and we **just don't know** how to make it up to him.

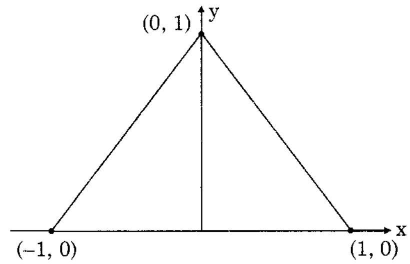
1.82 (c)

All **personnel** are being given the day off.

1.83 (c)

$f(x) = 1 - |x|$ on $-1 \le x \le 1$
$|x| = -x$ on $x < 0$
$= x$ on $x \ge 0$
Substituting in the given function we get
$f(x) = 1 - (-x)$ on $-1 \le x \le 0$
$= 1 - x$ on $0 \le x \le 1$

Which is same as
$f(x) = 1 + x$ on $-1 \le x \le 0$
$= 1 - x$ on $0 \le x \le 1$
The graph of this function is shown below



From the graph we can easily see that the function attains the maximum at $x = 0$ and the maximum value of the function is 1.

1.84 (d)

Apparel include cloth items such as skirt, trousers, shirts, etc.

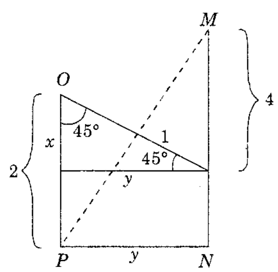
1.85 (c)

Statements Either 1 or 2 alone is sufficient.
$1/4^{th}$ of weight of 1 pole = 5 kg $\Rightarrow$ Weight of 1 pole = 20 kg
$\therefore$ Statement 1 is alone sufficient.
(weight of 10 poles) – (weight of 2 poles) = 160 kg
$\Rightarrow$ Weight of 8 poles = 160 kg
$\Rightarrow$ Weight of 1 pole = 20 kg
$\therefore$ Statement 2 is alone sufficient.

1.86 (b)

Adding by a constant, subtracting by a constant, and multiplying by a constant to the Arithmatic sequence do not destroy the property of Arithmatic progression.

1.87 (a)



$\frac{1}{x} = \sin 45°, \frac{1}{y} = \sin 45°$

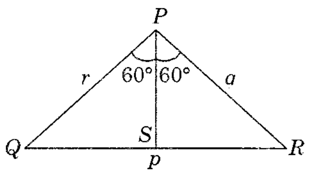
$x = \frac{1}{\sqrt{2}} \qquad y = \frac{1}{\sqrt{2}}$

$$PM^2 = y^2 + \left(4 + 2 - \frac{1}{\sqrt{2}}\right)^2$$

$$= \left(\frac{1}{\sqrt{2}}\right)^2 + \left(6 - \frac{1}{\sqrt{2}}\right)^2$$

$$PM = \sqrt{\frac{1}{2} + 28.01} = 5.34$$

1.88 (b)



$$\text{Area of } \Delta PQS = \frac{1}{2} \times PS \times r \times \sin 60°$$

$$\text{Area of } \Delta PRS = \frac{1}{2} \times PS \times q \times \sin 60°$$

$$\text{Area of } \Delta PQR = \frac{1}{2} PS \frac{\sqrt{3}}{2}(q + r) \quad \text{...(1)}$$

$$\text{Also } \Delta PQR = \frac{1}{2} \times r \times q \times \sin 120° \quad \text{...(2)}$$

$$\text{Equating (1) and (2)} PS = \frac{qr}{r+q}$$

1.89 Sol.

h(2, 5, 7, 8) = h(p, q, r, s)

(r × s) > (p × q)

Therefore $h(2, 5, 7, 3)$ returns remainder of

$$\frac{7 \times 3}{2 \times 5} = 1$$

Now $f_g(1, 4, 6, 8) = f(1, 4, 6, 8) \times g(1, 4, 6, 8)$
$= 8 \times 1 = 8$

1.90 (a)

Since the report lacked needed information, it was of no use to them.

1.91 (b)

R O A D
↓ ↓ ↓ ↓
U R D G

Replace with 3rd letter in alphabetical order.

Therefore SWAN → VZDQ

1.92 (c)

The Tamil version of **the** John Abraham-starrer *Madras Cafe* **was** cleared by the Censor Board with no cuts last week, but the film's distributors **found** no takers among the exhibitors for a release in Tamil Nadu **on** this Friday.

1.93 (b)

Extreme focus on syllabus and studying for tests has become such a dominant concern of Indian students that they close their minds to anything **extraneous** to the requirements of the exam.

1.94 (b)

Females and Gynaecologist relationship is similar.

1.95 (c)

$f(x) = ax + b$ is the linear form
$f(-2) = -2a + b = 29$
$f(3) = 3a + b = 39$
$\Rightarrow \quad 5a = 10$
$\Rightarrow \quad a = 2$
$\Rightarrow \quad b = 33$
$f(x) = ax + b \Rightarrow f(5) = 2 \times 5 + 33 = 43$

1.96 (d)

Alexander turned his attention towards India, since he had conquered Persia.
The following statement is logically valid and can be inferred from the above sentence is: "Since Alexander's kingdom extended to Indian borders after the conquest of Persia, he was keen to move further".

1.97 (b)

R-Home, S-Power, P-Defense, Q-Telecom, T-Finance.

1.98 (b)

From the given graph at x = 2, y = –1
Put this in each of the four options and test
Option (a) $2 = -1 - |-1| = -2$ (False)
Option (b) $2 = -(-1 - |-1|) = 2$ (True)
Option (c) $2 = -1 + |-1| = 0$ (False)
Option (d) $2 = -(-1 + |-1|) = 0$ (False)
So option (b) is the right answer.

1.99 (b)

The following statements are logically valid and can be inferred from the given passage.

(ii) He has to improve his temperament in order to become a great batsman.

(iii) He failed to make many of his good starts count.

1.100 Sol.

Year	2000	2001	2002	2003	2004	2005	2006	2007
Sum of Imports and Exports	40 + 50 = 90	60 + 50 = 110	70 + 60 = 130	70 + 60 = 130	70 + 80 = 150	70 + 90 = 160	100 + 120 = 220	120 + 110 = 230
Change		20	20	0	20	30	60	10

Therefore combined percentage increases is highest in 2006.

1.105 (d)

Original surface area = 6 $(4)^2$ = 96

If corner cubes are removed, three exposed surfaces are removed which will create 3 new surfaces in original large cube. So surface area will remain unchanged, i.e. 96

1.106 (b)

Elegance:

(27300 + 25222 + 28976 + 21012) × 48 = A

Executive:

(999 + 8942 + 10234 + 10234 + 10109) ×173 = B

Smooth:

(20009 + 9392 + 22429 + 18229) × 63 = C

Soft

(17602 +18445 + 19544 + 16595) × 78 = D

Which is highest for B (executive).

1.108 (d)

From conditions I, II and IV, following inference can be drawn

P always beats Q

Q always beats R

and R always beats S

Thus, arranging players from best performing to worst performing is

P – Q – R – S

Now, condition III states that S loses to P only sometimes. That means S most of the times beats the otherwise best performing player P.

Thus, statement (i) P is likely to beat all the other three players is FALSE, As P is likely to lose from S.

Statement (ii) S is absolutely the worst player in the set is also FALSE as S beats P most of the times.

1.109 (b)

$$f(x) = 2x^7 + 3x - 5$$

Use option (b), if $(x - 1)$ will be a factor then on putting $x - 1 = 0$ i.e. $(x = 1)$ in $f(x)$.

$$f(1) = 2(1)^7 + 3 - 5 = 5 - 5 = 0$$

1.14 (d)

Same pattern follows in all options A, B, C only (d) doesn't follow this pattern.

Hence odd number out (d).

1.15 (a)

$$\frac{\alpha^n + \beta^n}{\alpha^{-n} + \beta^{-n}} = \frac{\alpha^n + \beta^n}{\frac{1}{\alpha^n} + \frac{1}{\beta^n}} = \frac{\alpha^n + \beta^n}{\left(\frac{\beta^n + \alpha^n}{\alpha^n \beta^n}\right)} = (\alpha\beta)^n$$

Given $\alpha \times \beta = 4$

Hence $(\alpha\beta)^n = 4^n$

1.16 (a)

Following venn diagram can be drawn as

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

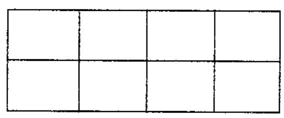
$$120 = 55 + 85 - n(A \cap B)$$

$$n(A \cap B) = 20$$

So only facebook 35.

1.19 (c)

Number of rectangules will be



$^5C_2 \times {}^3C_2$

$10 \times 3 = 30$

As whenever any two horizontal and any two vertical lines are choosen their intersection will produce a rectangle.

Conventional Questions & Solutions

Contents

Unit 1 Engineering Mathematics

1. Mathematical Logic

Q.1 Uses Modus ponens (A, A $\rightarrow$ | = B) or resolution to show that the following set is inconsistent:

(1) $Q(x) \rightarrow P(x) \vee \sim R(a)$

(2) $R(a) \vee \sim Q(a)$

(3) $Q(a)$

(4) $\sim P(y)$

Where x and y are universally quantifies variables, a is a constant and P, Q, R are monadic predicates.

[1992 : 5 Marks]

Solution:

Given that

(i) $Q(x) \rightarrow P(x) \vee \sim R(a)$

(ii) $R(a) \vee \sim Q(a)$

(iii) $Q(a)$

(iv) $\sim P(y)$

From (i), by universal specification

$Q(a) \rightarrow P(a) \vee \sim R(a)$ ...(v)

From (iv), by universal specification

$\sim P(a)$ ...(vi)

From (ii) and (iii), by rule of disjunctive syllogism

$R(a)$ ...(vii)

From (iii) and (v), by modus ponens

$P(a) \vee \sim R(a)$ ...(viii)

From (vi) and (viii), by disjunctive syllogism

$\sim R(a)$ ...(ix)

However, (vii) and (ix) contradict each other

Therefore, the given set of premises are inconsistent.

Q.2 Show that proposition C is a logical consequence of the formula

$$A \wedge (A \rightarrow (B \vee C)) \wedge (B \rightarrow \sim A)$$

Using truth tables.

[1993 : 5 Marks]

Solution:

A	B	C	$B \vee C$	$A \rightarrow (B \vee C)$	$B \rightarrow \sim A$	$X = A \wedge (A \rightarrow B \vee C) \wedge (B \rightarrow \sim A)$	$X \rightarrow C$
T	T	T	T	T	F	F	T
T	T	F	T	T	F	F	T
T	F	T	T	T	T	T	T
T	F	F	F	F	T	F	T
F	T	T	T	T	T	F	T
F	T	F	T	T	T	F	T
F	F	T	T	T	T	F	T
F	F	F	F	T	T	F	T

So $(A \wedge (A \rightarrow (B \vee C)) \wedge (B \rightarrow \sim A)\ C \equiv T$, which is a tautology, Therefore proposition C is a logical consequence of the formula

$A \wedge (A \rightarrow (B \wedge C)) \wedge (B \rightarrow \sim A)$

Q.3 (a) Show that the formula $[\sim p \vee q) \Rightarrow (q \Rightarrow p)]$ is not a tautology

(b) Let A be a tautology and B be any other formula. Prove that $(A \vee B)$ is a tautology.

[1999 : 5 Marks]

Solution:

(a) $(\sim p \vee q) \rightarrow (q \rightarrow p)$

$\equiv (\sim p \vee q) \rightarrow (\sim q \vee p)$

$\equiv (\sim(\sim p \vee q) \vee (\sim q \vee p))$

$\equiv ((p \wedge \sim q) \vee (\sim q \vee p))$

$\equiv (\sim q \vee p)$

So $((\sim p \vee q) \rightarrow (q \rightarrow p))$ is a contingency not a tautology.

(b) $A \vee B$

Since A is a tautology

$\equiv T \vee B$

$\equiv T$

So $(A \vee B)$ is a tautology if A is a tautology and B any other formula.

Q.4 Determine whether each of the following is a tautology, a contradiction, or neither ("$\vee$") is a disjuction, "$\wedge$" is conjunction, "$\rightarrow$" is implication, "$\neg$" in negation, and "$\leftrightarrow$" is biconditional (if and only if).

(i) $A \leftrightarrow (A \vee A)$

(ii) $(A \vee B) \rightarrow B$

(iii) $A \wedge (\neg (A \vee B))$

[2002 : 5 Marks]

Solution:

(i) $A \leftrightarrow (A \vee A)$

$\equiv A \leftrightarrow A$

$\equiv T$

So $A \leftrightarrow (A \vee A)$ is a tautology.

(ii) $(A \vee B) \rightarrow B$

$\equiv \sim(A \vee B) \vee B$

$\equiv (\sim A \vee \sim B) \vee B$

$\equiv \sim A \vee B$

So $((A \vee B) \rightarrow B)$ is neither a tautology nor a contradiction, however, it is a contingency.

(iii) $A \wedge (\sim(A \vee B))$

$\equiv A \wedge (\sim A \wedge \sim B)$

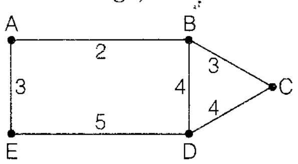
$\equiv (A \wedge \sim A) \wedge \sim B$

$\equiv F \wedge \sim B \equiv F$

So $(A \wedge (\sim(A \vee B))$ is a contradiction.

2. Graph Theory

Q.5 How many minimum spanning trees does the following graph have? Draw them. (Weights are assigned to the edge).

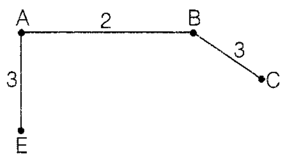


[1995 : 5 Marks]

Solution:

Using Kruskal's algorithm

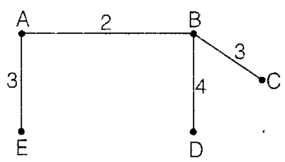
Adding edges AB, AE, BC



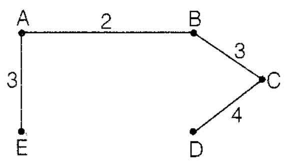
Now there are two options for next min weight edge BD and DC

So there are two different min-spanning trees.

(i)

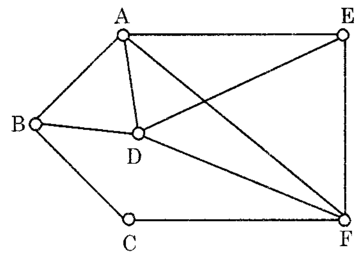


(ii)



Therefore, there are two minimum spanning trees.

Q.6 Let G be a connected, undirected graph. A cut in G is a set of edges whose removal results in G being broken into two or more components, which are not connected with each other. The size of a cut is called its cardinality. A min-cut of G is a cut in G of minimum cardinality. Consider the following graph.



(a) Which of the following sets of edges is a cut?
   (i) {(A, B), (E, F), (B, D), (A, E), (A, D)}
   (ii) {(B, D), (C, F), (A,B)}

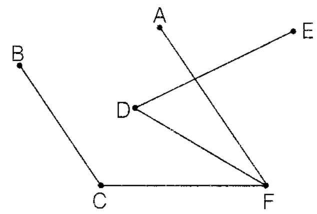
(b) What is the cardinality of min-cut in this graph?

(c) Prove that if a connected undirected graph G with n vertices has a min-cut of cardinality k, then G has at least (nk/2) edges.

[1999 : 5 Marks]

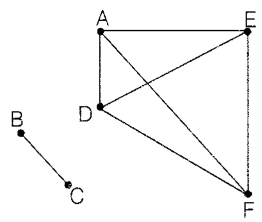
Solution:

(a) (i) The set of edges {(A, B), (E, F), (B, D), (A, E), (A, D)} following is the graph after removing the set of edges.



Above graph is connected. Therefore set {(A, B), (E, F), (B, D), (A, E), (A, D)} is not a cut.

(ii) Following is the graph after removing the set of edges {(B, D), (C, F), (A, B)}



Above graph is disconnected, therefore the set {(B, D), (C, F), (A, B)} is a cut.

(b) A cut -edges is a edge of graph G whose removal disconnect the graph. The given graph doesn't contain any cut-edges. So miminum cardinality of min-cut in given graph should be greater than 1.

By removing the two edges {(B, C), (C, F)}, the graph becomes disconnected.

Therefore, the cardinality of min-cut = 2.

(c) If G has a min-cut of cardinality k, then degree of each vertex is $\geq$ k.
By Handshaking theorem
Sum of all degrees = 2e

$\therefore \quad nk \leq 2e$

$\therefore \quad e \geq \frac{nk}{2}$

$\therefore \quad$ G has atleast $\left(\frac{nk}{2}\right)$ edges.

3. Set Theory and Algebra

Q.7 (a) If G is a group of even order, then show that there exists an element a $\neq$ e, the identity in g, such that $a^2 = e$

(b) Consider the set of integers {1, 2, 3, 4, 6, 8, 12, 24} together with the two binary operations LCM (lowest common multiple) and GCD (greatest common divisor).

Which of the following algebraic structures does this represent?
(i) group (ii) ring (iii) field (iv) lattice
Justify your answer

[1992 : 5 Marks]

Solution:

Let G be a group of even order. Assume every x not equal to e in G has an inverse not equal to x. Then for every x in G there exists some x^{-1} not equal to x. Then (since for every x there exists x^{-1}) there are 2n elements not equal to e in G, where n is a positive integer, and every group has a unique identity e so this makes 2n + 1 elements in G, which is contradiction (since there are only 2n elements in G)

So there must be one element $a = a^{-1}$ not equal to e in G.

So, there is an element such that $a = a^{-1}$

So; $\quad a^2 = a.a^{-1}$

$\quad a^2 = e$

Therefore, there exists an element a $\neq$ e such that $a^2 = e$.

(b) (iv) The given algebraic structures represent Lattice because the set is closed under LCM and GCD operations.

Q.8 Let S be the set of all integers and let n > 1 be a fixed integer. Define for a, b $\in$ S, a R b if a–b is a multiple of n. Show that R is an equivalence relation and find its equivalence classes for n = 5.

[1992 : 5 Marks]

Solution:

Given for a, b $\in$ S, aRb iff a – b is a multiple of n.
means
aRb iff (a – b) = nk
where n > 1 and k is an integer

(i) Relation R is reflexive
$(a - a) = 0 = n \times 0$
So, aRa, means that relation Ris reflexive

(ii) Relation R is Symmetic
If aRb that means a – b = nk.
Then bRa; $b - a = -(a - b) = -nk$ is also related.
Therefore, relation is symmetric.

(iii) Transitive
If aRb and bRc means
$a - b = nk_1$ where k_1 and k_2 are integers
$b - c = nk_2$
Then adding above two equation
$a - c = n(k_1 + k_2)$
$a - c = nk$ where k is integer

Therefore, if aRb and bRc then aRc this means relation R is transitive.
Since relation R is reflexive, symmetric, and transitive, R is equivalence relation. This relation R is also known as "congurent modulus n".
The equivalence classes for n = 5 are
$[0]_5 = \{..., -10, -5, 0, 5, 10, ...\}$
$[1]_5 = \{..., -9, -4, 1, 6, 11, ...\}$
$[2]_5 = \{..., -8, -3, 2, 7, 12, ...\}$
$[3]_5 = \{..., -7, -2, 3, 8, 13, ...\}$
$[4]_5 = \{..., -6, -1, 4, 9, 14, ...\}$

Q.9 Let ({p,q},*) be a semi group where
p * p = q. Show that:
(a) p * q = q * p and
(b) q * q = q

[1993 : 5 Marks]

Solution:

Given, p * p = q

(dp, q) *) is semigroup

Hence, it is closed and satisfies associative operation

(a) p * q = q * p

LHS = p • q = p * (p * p) [∵ p*p = q]

= (p * p) * p [∵ associative]

= q * p

= RHS

Q.10 Let G_1 and G_2 be subgroups of a group G.

(a) Show that $G_1 \cap G_2$ is also a subgroup of G.

(b) Is $G_1 \cup G_2$ always a subgroup of G?

[1995 : 5 Marks]

Solution:

(a) Suppose for any elements

$x \in (G_1 \cap G_2)$ and

$y \in (G_1 \cap G_2)$

$\Rightarrow x \in G_1, y \in G_1, x \in G_2$ and $y \in G_2$

Since G_1 is a subgroup of G and

$x \in G_1, y \in G_1$

$\Rightarrow xy^{-1} \in G_1$

Similarly G_2 is a subgroup of G and

$x \in G_1, y \in G_2$

$\Rightarrow xy^{-1} \in G_2$

So $xy^{-1} \in (G_1 \cap G_2)$

This proofs that $(G_1 \cap G_2)$ is a subgroup of G.

(b) $(G_1 \cup G_2)$ is a subgroup of G iff either G_1 is contained in G_2 or G_2 is contained in G_1. Therefore, $(G_1 \cup G_2)$ is not always a subgroup of G.

Q.11 Let S = {0, 1, 2, 3, 4, 5, 6, 7} and ⊗ denote mutliplication moduls 8, that is x ⊗ y = (xy) mod 8.

(a) Prove that ({0, 1}, ⊗) is not a group

(b) Write 3 distinct groups (G, ⊗) where $G \subset S$ and G has 2 elements.

[2000 : 5 Marks]

Solution:

(a) A algebraic structure is a group iff it follows property of closure, associativity, identity and inverse. For the algebraic structure {{0, 1}, ⊗} inverse for element 0 doesn't exist. Therefore, {{0, 1}, ⊗} is not a group.

(b) {{1, 3}, ⊗}, {{1, 5}, ⊗}, {{1, 7}, ⊗} are group and also subset of S.

Q.12 A multiset is an unordered collection of elements where elements may repeat any number of times. The size of a multiset is the number of elements in it counting repetitions.

(a) What is the number of multisets of size 4 that can be constructed from n distinct elements so that at least one element occurs exactly twice?

(b) How many multisets can be constructed from n distinct elements?

[2000 : 5 Marks]

Solution:

(a) There are exactly four places to be filled in the multiset using n distinct elements. Atleast one element has to occur exactly twice. That would leave 2 more places in the multiset. This means, atmost two elements can occur exactly twice. So dividing this into 2 mutually exclusive cases as follows:

Case-1: Exaclty one element occurs exactly twice:

Selecting the element which occurs exactly twice in ${}^nC_1$ = n ways. Fill up the remaining two positions using 2 distinct elements from the remaining (n – 1) elements in ${}^{n-1}C_2$ ways.

Total ways for this case

$= n \times {}^{n-1}C_2$

$= \dfrac{n \times (n-1)(n-2)}{2}$ ways

Case-2: Exactly two elements that occur twice each: These two will fill up the multiset. So we have to select only two elements out of n elements in

${}^nC_2 = \dfrac{n(n-1)}{2}$ ways

Since these two cases are mutually exclusive, the total number of ways to form the multiset is

$$\frac{n(n-1)(n-2)}{2} + \frac{n(n-1)}{2} = \frac{n(n-1)^2}{2}$$

Therefore the required number of multisets are

$\dfrac{n(n-1)^2}{2}$ for $n \geq 2$.

(b) Infinite multisets can be constructed from n distinct elements because an element can be repeated any number of times.

Q.13 Let A be a set of n(> 0) elements. Let N_r be the number of binary relations on A and Let N_f be the number of functions from A to A.
(a) Give the expression for N_r in terms of n.
(b) Give the expression for N_f in terms of n.
(c) Which is larger for all possible n, N_r or N_f?
[2002 : 5 Marks]

Solution:
(a) Number of binary relations on set A with n elements, $N_r = 2^{n^2}$.
(b) Number of functions from set A with n elements onto itself, $N_f = n^n$.
(c) Since every function is a relation however, every relation need not be a function, the number of relations is larger than number of functions therefore
$N_r > N_f$ for all possible n.

Q.14 (a) S = {<1, 2>, <2, 1>} is binary relation on set A = {1, 2, 3}. Is it irreflexive? Add the minimum number of ordered pairs to S to make it an equivalence relation.
Give the modified S.
(b) Let S = {a, b} and let $\mathcal{P}$(S) be the power set of S. Consider the binary relation '⊆(set inclusion)' on $\mathcal{P}$(S). Draw the Hasse diagram corresponding to the lattice ($\mathcal{P}$(S), ⊆)
[2002 : 5 Marks]

Solution:
(a) Relation S = {(1, 2), (2, 1)} on set A = {1, 2, 3}
Since (1, 1), (2, 2) and (3, 3) all are not present in relation S, the relation S is irreflexive.
relation S is already symmetric, transitive, however it is not reflexive. To make relation S as reflexive.

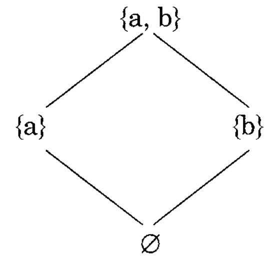
$$\Rightarrow S \cup \{(x, x) \mid \forall x \in A\}$$

$$\Rightarrow S \cup \{(1, 1), (2, 2), (3, 3)\}$$

$$\Rightarrow \{(1, 1), (1, 2), (2, 2), (2, 1), (3, 3)\}$$

So the modified S,
S' = {(1, 1,), (2, 2), (3, 3), (1, 2), (2, 1)}
Since S' is reflexive, symmetric and transitive, the relation S' is equivalence relation.
(b) S = {a, b}
The power set of S, $\mathcal{P}$(S) is given as
$\mathcal{P}$ (S) = {∅, {a}, {b}, {a, b}}

The Hasse diagram corresponding to the lattice ($\mathcal{P}$(S), ⊆) is given as follows:



Hasse diagram of ($\mathcal{P}$ (S), ⊆)

4. Calculus

Q.15 (a) Find the points of local maxima and minima, if any of the following function defined $0 \le x \le 6$.
$x^3 - 6x^2 + 9x + 15$

(b) Integrate $\int_{-\pi}^{\pi} x \cos x \, dx$

[1998 : 5 Marks]

Solution:
(a) $f(x) = x^3 - 6x^2 + 9x + 15$
To find local maxima and minima put $f'(x) = 0$

$$f'(x) = 0$$
$$f'(x) = 3x^2 - 12x + 9 = 0$$
$$(x-1)(x-3) = 0$$
$$x = 1, 3$$
$$f''(x) = 6x - 12$$
$$f''(1) = -6 < 0$$

(So x = 1 is a point of local maximum)

$$f''(3) = 6 > 0$$

(So x = 3 is a point of local minimum)

So x = 1 is a point of local maximum
and x = 3 is a point of local minimum

(b)
$$f(x) = x \cos x$$
$$f(-x) = (-x)\cos(-x)$$
$$= +x \cos x$$
$$= +f(x)$$

Since f(–x) = + f(x), then f(x) is odd function.

$$\int_{-a}^{a} f(x) = 0 \quad \text{If f(x) is odd function}$$

So, $\int_{-\pi}^{\pi} x \cos x dx = 0$

because xcosx is odd function

■■■■

Unit 2 Theory of Computation

1. Finite Automata : RL, RE and RG

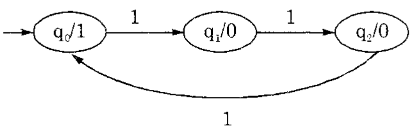
Q.1 Give minimal DFA that perfomrs as a Mod-3 1's counter, i.e., outputs a 1 each time the number of 1's in the input sequence is a multiple of 3.

[1987 : 5 Marks]

Solution:

The minimal DFA for given language contain 3 states

Let the alphabets $\Sigma = \{1\}$



Q.2 Give the regular expression over {0, 1} to denote the set of proper non-null substrings of the string 0110.

[1987 : 5 Marks]

Solution:

The proper non-null substrings of the string 0110 are : {0, 1, 01, 11, 10, 011, 110, 0110}.

So the regular expression of proper non-null substrings of the string 0110 is

RE = 0 + 1 + 01 + 11 + 10 + 011 + 110 + 0110

Q.3 Is the class of regular sets closed under infinite union? Explain.

[1989 : 5 Marks]

Solution:

No, the class of regular sets is not closed under infinite union.

If regular sets is closed under infinite union then $L = \{a^n b^n / n \geq 0\}$ must be regular because $a^n b^n$ is the infinite union of regular sets as

$L_0 = \epsilon$
$L_1 = ab$
$L_2 = aabb$
$L_3 = aaa\,bbb$
$\vdots$
$L = \{a^n b^n\}$

$= (\epsilon) \cup (ab) \cup (aabb) \cup (aaabbb) \cup \ldots$

However

$L = \{a^n b^n / n \geq 0\}$

is not regular language, it is context free language.

Q.4 Given that language L_1 is regular and that the language $L_1 \cap L_2$ is regular. Is the language L_2 is always regular?

[1994 : 5 Marks]

Solution:

No, L_2 need not to be regular always

$L_1 = \{abc\}$ which is regular language

$L_2 = \{a^n b^n c^n / n \geq 0\}$ which is context sensitive language (CSL)

$L_1 \cap L_2 = \{abc\}$ which is regular language

So L_2 need not to be regular always.

2. Push Down Automata: CFL & DCFL

Q.5 State whether the following statements are true/ false.

(a) regularity is preserved under the operation of string reversal.
(b) all subsets of regular sets are regular
(c) a minimal DFA that is equivalent to an NDFA with n nodes has always 2^n states
(d) the intersection of two CFL's is also CFL
(e) A is recursive if both A and its complement are accepted by Turing Machine M accepts
(f) the problem as to whether a turing machine m accepts input w is undecidable.

[1987 : 5 Marks]

Solution:

(a) True, because regular languages are closed under string reversal.
(b) False, some subsets of regular sets are regular but not all.
(c) False, a minimal DFA that is equivalent to an NDFA with n nodes has atmost 2^n states, not always.

(d) False, CFL's are not closed under intersection.

(e) False, A is recursive if every w in A is accepted by turing machine M and every w which is not in A (i.e., complement of A) is rejected by turing machine M.

(f) True, membership in turing machine M is undecidable.

3. Turing Machine: RS & RES

Q.6 Which of the following statements is/or true?

(a) Union of two recursive languages is recursive

(b) The lanugage $\{0^n \mid n \text{ is prime}\}$ is not regular

(c) Regular languages are closed under infinite union

[2002 : 5 Marks]

Solution:

(a) Since recursive languages are closed under union so union of two recursive languages is recursive.

(b) The language $L = \{O^n/n \text{ is prime}\}$ is context sensitive language. So L is not regular.

(c) No language is closed under infinite union. If regular languages are closed under infinite union then a^nb^n which is infinite union of following regular languages should be regular.

$a^nb^n = \{ab\} \cup \{aabb\} \cup \{aaabbb\}....$

However a^nb^n is CFL

■■■■

Unit 3 Digital Logic

1. Logic Functions & Minimization

Q.1 Find the minimum product of sums of the following expression :

$$f = ABC + \bar{A}\bar{B}\bar{C}$$

[1990 : 5 Marks]

Solution:

Given SOP of function is

$$f = ABC + \bar{A}\bar{B}\bar{C}$$

A \ BC	00	01	11	10
0	1	0	0	0
1	0	0	1	0

The required minimal POS form is

$$f = (\bar{B} + C)(A + \bar{C})(\bar{A} + B)$$

Q.2 Show with the help of a block diagram represent Boolean function:

$f = AB + BC + CA$

can be realised using only 4 : 1 multiplexer.

[1990 : 5 Marks]

Solution:

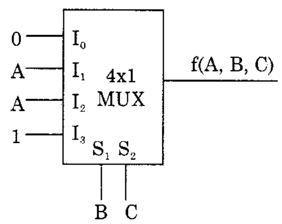
$$f = B + BC + CA$$

A \ BC	00	01	11	10
0	$_0$	$_1$	1 $_3$	$_2$
1	$_4$	1 $_5$	1 $_7$	1 $_6$

Using MSB method

	I_0	I_1	I_2	I_3
$\bar{A}$	0	1	2	3
A	4	5	6	7
	0	A	A	1

Implementing f(A, B, C) using 4 × 1 MUX



2. Combinational Circuits

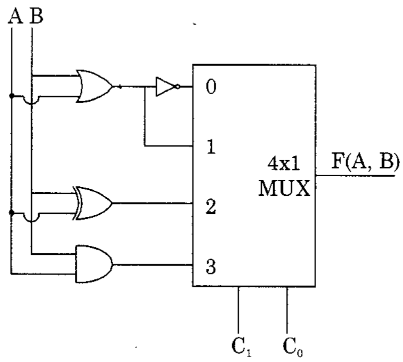
Q.3 A logic network has two data inputs A and B, and two control inputs C_0 and C_1. It implements the function F according to the following table.

C_1	C_0	F
0	0	$\overline{A+B}$
0	1	A + B
1	0	$A \oplus B$
1	1	AB

Implement the circuit using one 4 to 1 Multiplexer, one 2-input Exclusive OR gate, one 2-input AND gate, one 2-input OR gate and one Inverter.

[1996 : 5 Marks]

Solution:



Q.4 Find the minimum sum of products form of the logic function.

$f(A, B, C, D) = \Sigma m(0, 2, 8, 10, 15) + \Sigma d(3, 11, 12, 14)$

Where m and d denote the min-terms and don't cares respectively.

[1991 : 5 Marks]

Solution:

$f(A, B, C, D) = \Sigma m(0, 2, 8, 10, 15) + \Sigma d(3, 11, 12, 14)$

AB\CD	00	01	11	10
00	1 $_{0}$	$_{1}$	x $_{3}$	1 $_{2}$
01	$_{4}$	$_{5}$	$_{7}$	$_{6}$
11	x $_{12}$	$_{13}$	1 $_{15}$	x $_{14}$
10	1 $_{8}$	$_{9}$	x $_{11}$	1 $_{10}$

The minimum SOP form of the logic function is given as: f(A, B, C, D) = B'D' + AC.

Q.5 Express the function $f(x, y, z) = xy' + yz'$ with only one complement operation and one or more AND/OR operations. Draw the logic circuit implementing the expression obtained, using a single NOT gate and one or more AND/OR gates.

[2002 : 5 Marks]

Solution:

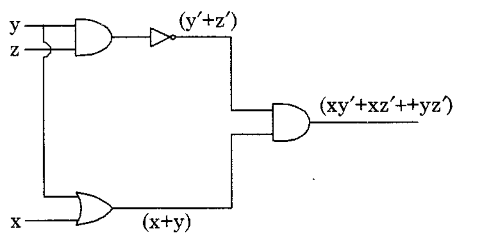
To use only single NOT gate, express it with only one complementation and one or more AND/OR operations.

$$f(x, y, z) = xy' + yz'$$

Add reduced term to f

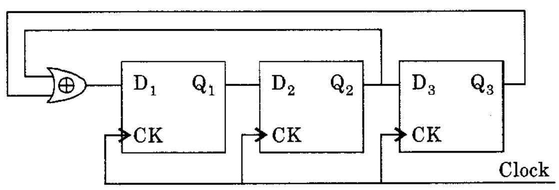
$$f(x, y, z) = xy' + yz' + xz'$$
$$= yz' + x(y' + z')$$
$$= y(y' + z') + x(y' + z')$$

Logic circuit implementing the expression obtained is given as below:



3. Sequential Circuits

Q.6 Consider the synchronous sequential circuit in figure.



(a) Draw a state diagram which is implemented by the circuit. Use the following names for the states corresponding to the values of flip-flops as given below.

Q1	Q2	Q3	State
0	0	0	S_0
0	0	1	S_1
⋮	⋮	⋮	⋮
1	1	1	S_7

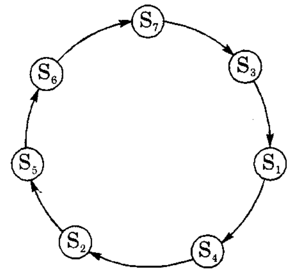
(b) Given that the initial state of the circuit is S_4, identify the set of states which are not reachable.

[1996 : 5 Marks]

Solution:

$D_1(Q_2 \pm Q_3)$	$D_2(Q_1)$	$D_3(Q_2)$	Q_1	Q_2	Q_3	State
Initially			1	1	1	(S_7)
0	1	1	0	1	1	(S_3)
0	0	1	0	0	1	(S_1)
1	0	0	1	0	0	(S_4)
0	1	0	0	1	0	(S_2)
1	0	1	1	0	1	(S_5)
1	1	0	1	1	0	(S_6)
1	1	1	1	1	1	(S_7)

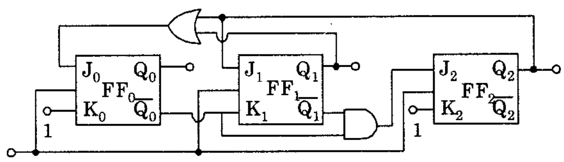
(a) The state diagram which is implemented by the circuit is as follows:



(b) If the initial state of the circuit is S_4, then the state which is not reachable is S_0

$$S_4 \rightarrow S_2 \rightarrow S_5 \rightarrow S_6 \rightarrow S_7 \rightarrow S_3 \rightarrow S_1$$

Q.7 For the synchronous counter shown in figure write the truth table of Q_0, Q_1 and Q_2 after each pulse starting from $Q_0 = Q_1 = Q_2 = 0$ and determine the counting sequence and also the modulus of the counter.



What is the modulus of the counter with initial state $Q_2\, Q_1\, Q_0 = 000$

(a) 3
(b) 4
(c) 5
(d) 6

[1990 : 5 Marks]

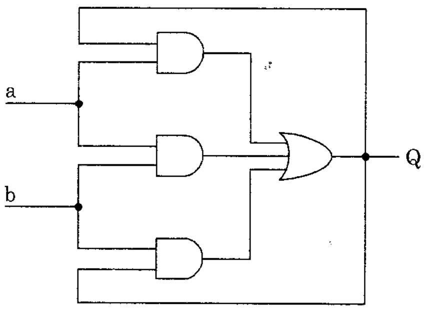
Solution: (c)

J_2 $(\bar{Q}_0\bar{Q}_1)$	K_2 (1)	J_1 (Q_2)	K_1 $(\bar{Q}_0)$	J_0 (Q_1+Q_2)	K_0 (1)	Q_2	Q_1	Q_0
Initially						0	0	0
1	1	0	1	0	1	1	0	0
1	1	1	1	1	1	0	1	1
0	1	0	0	1	1	0	1	0
0	1	0	1	1	1	0	0	1
0	1	0	0	0	1	0	0	0

There are 5 different states in the synchronous counter.
Therefore, the modulus of the counter is 5.

Q.8 Analyse the circuit in figure and complete the following table:

a	b	Q_n
0	0	
0	1	
1	0	
1	1	



[1991 : 5 Marks]

Solution:

The output of the circuit is given as

$$Q_n = aQ_{n-1} + ab + bQ_{n-1}$$
$$Q_n = Q_{n-1}(a+b) + ab$$

a	b	Q_n
0	0	0
0	1	Q_{n-1}
1	0	Q_{n-1}
1	1	1

■■■■

Unit 4 Computer Organization & Architecture

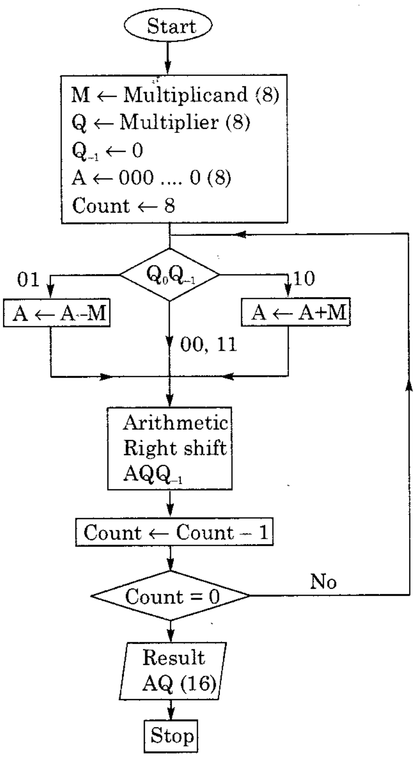
1. Cache and Main Memory

Q.1 State the Booth's algorithm for multi-plication of two numbers, Draw a block diagram for the implementation of the Booth's algorihtm for determining the product of two 8-bit signed numbers.

[1990 : 5 Marks]

Solution:

The Booth algorithm generates a 2n-bit product and treats both positive and negative 2's - complement n-bit operands uniformly. Booth algorithm is fast multiplication algorithm.



Booth algorithm has two attractive features:

(i) First, it handles both positive and negative multipliers uniformly.

(ii) Second, it achieves some efficiency in thenumber of additions required when the multiplier has a few large blocks of 1s.

Block diagram for multiplication of two 8-bit signed numbers.

Q.2 A computer system has a three level memory hierarchy, with access time and hit ratios as shown below:

Level 1 (cache memory)
Access time = 50 nsec/byte

Size	Hit ratio
8 M byte	0.80
16 M byte	0.90
64 M byte	0.95

Level 2 (main memory)
Access time = 200 nsec/byte

Size	Hit ratio
4 M byte	0.98
16 M byte	0.99
64 M byte	0.995

Level 3 (secondary memory)
Access time = 5 μsec/byte

Size	Hit ratio
260 Mbyte	1.0

(i) What should be the minimum sizes of level 1 and 2 memories to achieve an average access time of less than 100 nsec?

(ii) What is the average access time achieved using the chosen sizes of level 1 and level 2 memories?

[1996 : 5 Marks]

Solution:

Taking level 1 memory of size 8 MB
Level 2 memory of size 4 MB and
Level 3 memory of size 260 MB
Average access time,

T_{avg} = 0.80(50) + 0.2 × 0.98 × 250 + 0.2 × 0.02 × (5000 + 50 + 250) ns

T_{avg} = 110.2 ns > 100 nsec

So taking level 1 memory of size 16 MB
Level 2 memory of size 4 MB and
Level 3 memory of size 260 MB

T_{avg} = 0.90(50) + 0.1 × 0.98 × 250 + 0.2 × 0.02 × (5000 + 50 + 250)
= 90.7 nsec < 100 nsec

(i) The miminum sizes of level 1 and level 2 memories are 16 MB, 4 MB respectively.

(ii) The average access time achieved is 90.7 nsec.

Q.3 A CPU has 32-bit memory address and a 256 KB cache memory. The cache is organized as a 4-way set associative cache with cache block size of 16 bytes.

(a) What is the number of sets in the cache?

(b) What is the size (in bits) of the tag field per cache block?

(c) What is the number and size of comparators required for tag matching?

(d) How many address bits are required to find the byte offset within a cache block?

(e) What is the total amount of extra memory (in bytes) required for the tag bits?

[2001 : 5 Marks]

Solution:

Main memory address size = 32-bit

Cache size = 256 kB

Block size = 16 byte

(a) $\# \text{ of sets} = \dfrac{\#\text{of blocks in cache}}{\#\text{ of blocks in a set}}$

$$= \frac{256 \text{ kB}/16\text{B}}{4} = 4096$$

(b)

32 bits

TAG	SET OFFSET	WORD OFFSET
16 bits	12 bits	4 bits

Size of the tag field per cache block is 16 bits.

(c) 4 comparators of 16 bit size.

(d) Address bits for byte offeset (word offset) is 4 bits

(e) # of blocks in cahce X# of bits in tag field

$$= \frac{256\text{kB}}{16\text{B}} \times 16\text{bits} = 256 \text{ k bits} = 32 \text{ kB}$$

2. Instructions : Pipelining and Addressing Modes

Q.4 An instruction pipeline consists of 4 stages – Fetch (F), Decode field (D). Execute (E) and Result Write (W). The 5 instructions in a certain instruction sequence need these stages for the different number of clock cycles as shown by the table below.

No. of clock cycles needed for

Instruction	F	D	E	W
1	1	2	1	1
2	1	2	2	1
3	2	1	3	2
4	1	3	2	1
5	1	2	1	2

Find the number of clock cycles needed to perform the 5 instructions.

[1999 : 5 Marks]

Solution:

Stages of pipeline	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
W					I_1	–	–	I_2	–	–	I_3	I_3	I_4	I_5	I_5	
E				I_1	–	I_2	I_2	I_3	I_3	I_3	I_4	I_4	I_5			
D		I_1	I_1	I_2	I_2	I_3	–	I_4	I_4	I_4	I_5	I_5				
F	I_1	I_2	–	I_3	I_3	I_4	–	I_5	–	–						

No. of clock cycles

The number of clock cycles needed to perform the 5 instructions are 15 cycles.

Q.5 An instruction pipeline has five stages where each stage takes 2 nanoseconds and all instructions use all five stages. Branch instructions, are not overlapped, i.e., the instruction after the branch is not fetched till the branch instruction is completed. Under ideal conditions.

(a) Calculate the average instruction execution time assuming that 20% of all instruction executed are branch instructions. Ignore the fact that some branch instructions may be conditional.

(b) If a branch instruction is a conditional branch instruction, the branch need not be taken. If the branch is not taken, the following instructions can be overlapped. When 80% of all branch instructions are conditional branch instructions, and 50% of the conditional branch instructions are such that the branch is taken, calculate the average instruction execution time.

[2000 : 5 Marks]

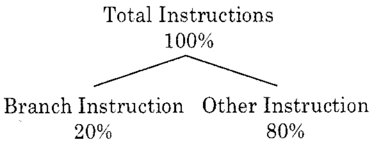
Solution:

$T_{Clock} = 2$ nsec

Since the instruction after the branch is not fetched till the branch instruction is completed, therefore, stall cycles = 5 – 1 = 4 cycle.

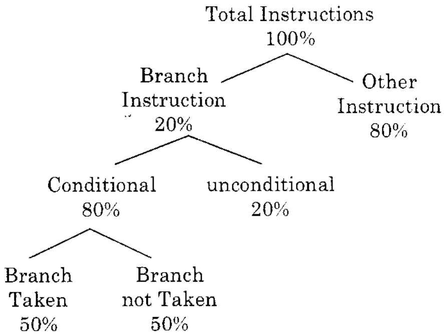
$T_{avg} = (1 + \text{stall frequency} * \text{stall cycle}) * T_{clock}$

(a)



$T_{avg} = (1 + 0.2 \times 4) * 2$ nsec
$= 3.6$ nsec

(b)



$T_{avg} = (1 + ((0.2 \times 0.8 \times 0.5) + (0.2 \times 0.2)) \times 4) \times 2$ nsec
$= 2.96$ nsec

3. Secondary Memory & DMA

Q.6 Hard disk is connected to a 50 MHz processor through a DMA controller. Assume that the initial set-up of a DMA transfer takes 1000 clock cycles for the processor, and assume that the handling of the interrupt at DMA completion requires 500 clock cycles for the processor. The hard disk has a transfer rate of 2000 Kbytes/sec and average block size transferred is 4 K bytes. What fraction of the processor time is consumed by the disk, if the disk is actively transferring 100% of the time?

[1996 : 5 Marks]

Solution:

Cycle time of processor

$$= \frac{1}{50\text{ MHz}} = 20\text{ ns}$$

DMA transfer requires
1000 + 500 = 1500 cycle
Processor time

$$= 1500 * 20\text{ ns} = 30\ \mu\text{sec}$$

Transfer time of hard disk

$$= \frac{4\text{kB}}{2000\text{kB}} = 2\text{ ms}$$

% of processor time consumed by the disk

$$= \frac{30\mu\text{sec} \times 100}{2\text{ms}} = 1.5\%$$

Q.7 Consider the following program fragment in the assembly language of a certain hypothetical processor. The processor has three general purpose registers R1, R2 and R3. The meanings of the instructions are shown by comments (starting with;) after the instructions.

```
X : CMP R1,0   ; Compare R1 and 0, set flags
                 appropriately in status register
    JZZ        ; Jump if zero to target Z.
    MOV R2, R1; Copy contents of R1 to R2
    SHR R1     ; Shift right R1 by 1 bit
    SHL R1     ; Shift left R1 by 1 bit
    CMP R2, R1; Compare R2 and R1 and set
                 flag in status register
    JZY        ; Jump if zero to target Y
    INC R3     ; Increment R3 by 1;
Y : SHR R1     ; Shift right R1 by 1 bit
    JMP X      ; Jump to target X
Z :...
```

(a) Initially R1, R2 and R3 contain the values 5,0 and 0 respectively, what are the final values of R1 and R3 when control reaches Z?

(b) In general, if R1, R2 and R3 initially contain the values n, 0, and 0 respectively. What is the final value of R3 when control reaches Z?

[1999 : 5 Marks]

Solution:

(a) The instructions performs
$R_1 = 0$, $R_3 = 3$ and $R_2 = 5$

(b) $R_1 = 0$, $R_2 = n$ and $R_3 = \log_2 N$

Q.8 Consider the following 8085 program segment, where registers B and C contain BCD values:

```
S1: MVI    A, 99H
    MVI    D, 00H
    SUB    C
    ADD    B
    DAA
```

S2:	JC	S3
	MOV	E, A
	MVI	A, 99H
	SUB	E
	MOV	E, A
	JZ	S4
	MVI	D, FFH
	JMP	S4
S3:	INC	A
	DAA	
	MOV	E, A
S4:	....	

(a) For the two pairs (B = 44, C = 25) and (B = 33, C = 46) at S1,

  (i) Find the values in register A when control reaches S2.

  (ii) Find the values in registers D and E when control reaches S4.

(b) What, in general, is the value of D and E as a function of B and C when control reaches S4.

[2000 : 5 Marks]

Solution:

(a) The program basically performs BCD subtraction (99 – C) + B

  (i) B = 44 and C = 25

  (ii) B = 33 and C = 46

(b) (i) 19,00

  (ii) 86,00

Q.9 Consider a disk with following specifications: 20 surface, 1000 tracks/surface, 16 sectors/track, data density 1 KB/sector, rotation speed 3000 rpm. The operating system initiates the transfer between the disk and the memory sector-wise. Once the head has been placed on the right track, the disk reads a sector in a single scan. It reads bits from the sector while the head is passing over the sector. The read bits are formed into bytes in a serial-in-parallel-out buffer and each byte is then transferred to memory. The disk writing is exactly a complementary process.

For parts (c) and (d) below, assume memory read-write time = 0.1 microsecond/ byte, interrupt driven transfer has an interrupt overhead = 0.4 microseconds, the DMA initialization and termination overhead is negligible compared to the total sector transfer time. DMA requests are always granted.

(a) What is the total capacity of the disk?

(b) What is the data transfer rate?

(c) What is the percentage of time the CPU is required for this disk I/O for bytewise interrupts driven transfer?

(d) What is the maximum percentage of time the CPU is held up for this disk I/O for cycle-stealing DMA transfer?

[2001 : 5 Marks]

Solution:

\# of surface = 20 surface

\# of tracks per surface = 1000 tracks/surface

\# of sectors per track = 16 sectors/track

\# of bytes per sector = 1 kB/sector

Rotational speed = 3000 rpm

(a) Total capacity of the disk

$$= 20 \text{ surface} \times 1000 \frac{\text{tracks}}{\text{surface}} \times \frac{16 \text{ sectors}}{\text{tracks}} \times \frac{1 \text{kB}}{\text{sectors}}$$

$= 312.5$ MB

(b) Data transfer rate

In 20 ms $\rightarrow$ 16 kB

$$1 \text{ sec} \rightarrow \frac{16}{20 \times 10^{-3}}$$

1 sec $\rightarrow$ 800 kbps

Therefore data transfer rate is 800 kbps

(c) $\% \text{ CPU time} = \frac{0.1}{0.5} \times 100 = 20\%$

(d) Max % time CPU blocked in cycle mode = 5%

■■■■

Unit 5 Programming & Data Structures

1. Programming Language Concepts

Q.1 Consider the following pseudo-code (all data items are of type integer):

```
procedure P(a, b, c);
    a : = 2;
    c : = a + b;
end {P}
begin
    x : = 1;
    y : = 5;
    z : = 100;
    P(x, x*y, z);
    Write ('x=',x, 'z=',z)
end
```

Determine its output, if the parameters are passed to the procedure P by
(i) value, (ii) reference and (iii) name.

[1991 : 5 Marks]

Solution:

(i) Pass by value : 1, 100
(ii) Pass by reference : 2, 7
(iii) Pass by name: 2, 7

Q.2 For the following pseudo-code, indicate the output, if (i) static scope rules and (ii) dynamic scope rules are used

```
Var, a, b : integer;
Procedure P;
            a:=5; b:=10
end {P};
procedure Q;
var a, b : integer;
P;
end {Q};
begin
a:=1; b:=2;
Q;
Write ('a =', a, 'b=',b)
end.
```

[1991 : 5 Marks]

Solution:

(i) 5, 10 (ii) 1, 2

Q.3 State whether the following statements are True or False with reasons for your answer:

(a) Coroutine is just another name for a subroutine.
(b) A two pass assembler uses its machine opcode table in the first pass of assembly.

[1994 : 5 Marks]

Solution:

(a) False

Coroutines are well suited for implementing more familiar program component such as cooperative task, iterators, infinie lists, while subroutines to allow multiple points for suspending and reassuming execution at certain condition.

(b) False

In first pass assembler generate the storage for tokens.

Q.4 State whether the following statements are True or False with reasons for your answer:

(a) A subroutine cannot always be used to replace a macro in an assembly language program.
(b) A symbol declared as 'external' in assembly language is assigned an address outside the program by the assembler itself.

[1994 : 5 Marks]

Solution:

(a) True
(b) True

Q.5 Consider the program below:

```
Program main;
    var r:integer;
    procedure two;
    begin write (r) end;
    procedure one;
    var r:integer;
    begin r:=5; two; end
    begin r:=2;
    two; one; two;
    end.
```

What is printed by the above program if
(i) Static scoping is assumed for all variables;
(ii) Dynamic scoping is assumed for all variables.
Give reasons for your answer.

[1994 : 5 Marks]

Solution:

(i) 2, 2, 2,
(ii) 2, 5, 2

In static scoping all write open performed on global variable value, while dynamic scoping one time it uses its parent variable value.

Q.6 Consider the following recursive function:

```
function fib (1:integer);integer;
begin
if (n=0) or (n=1) then fib:=1
else fib:=fib(n – 1) + fib(n – 2)
end;
```

The above function is run on a computer with a stack of 64 bytes. Assuming that only return address and parameters are passed on the stack, and that an integer value and an address takes 2 bytes each, estimate the maximum value of n for which the stack will not overflow. Give reasons for your answer.

[1994 : 5 Marks]

Solution:

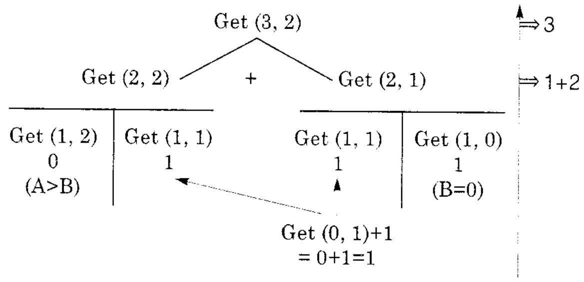
The running time of fibonacci series is '2^n'
So runtime stack maximum value can be
$64 = 2^n$
$2^6 = 2^n$
$n = 6$

Q.7 Consider the following Pascal function where A and B are non-zero positive integers. What is the value of GET(3,2)?

```
function GET(A,B:integer);integer;
begin
      if B = 0 then
      GET:=1
else if A < B then
      GET:=0
else
      GET:=GET(A-1,B)+GET(A-1,B-1)
end ;
```

[1994 : 5 Marks]

Solution:



Value of GET(3, 2) = 3.

Q.8 What will be the output of the following program assuming that parameter passing is
(i) call by value
(ii) call by reference
(iii) call by copy restore

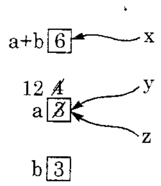
```
    procedure P{x, y, z};
              begin y: =y + 1; z: =x + x end;
    begin
              a: = : b : = 3;
              P(a + b, a, a);
              Print (a)
    end.
```

[1999 : 5 Marks]

Solution:

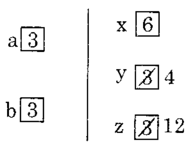
(i) Call by value: Value will not change so it prints "3" as value of a .

(ii) Call by reference = 12



It prints a = 12.

(iii) Using call by copy restore



Here, y and z are going to store in same variable 'a' hence the value depends on the order of restore. So it prints either '4' or '12'.

Q.9 Consider the following program is pseudo-Pascal syntax.

```
program main;
        var x: integer;
        procedure Q (z:integer);
        begin
              z: =z + x;
              writeln(z)
        end;
procedure P (y:integer);
              var x: integer;
        begin
              x: =y + 2;
              Q(x);
              writeln(x)
        end;
        begin
        x:=5;
        P(x);
        Q(x);
        writeln(x)
end.
```

What is the output of the program, when

(a) The parameter passing mechanism is call-by-value and the scope rule is static scoping?

(b) The parameter passing mechanism is call-by-reference and the scope rule is dynamic scoping?

[2000 : 5 Marks]

Solution:

(a) 12, 7, 10, 5 (b) 14, 14, 10, 10

Q.10 Consider the following C program :

```
void abc(char*s)
{ if(s[0] = ='\0')return;
  abc(s+1);
  abc(s+1);
  printf("%c", s[0]);
}
main()
{
  abc("123")
}
```

(a) What will be the output of the program?

(b) If abc(s) is called with a null-terminated string s of length n characters (not counting the null('\0') character), how many characters will be printed by abc(s)?

[2001 : 5 Marks]

Solution:

```
main ()
{
    abc ("123")
{
```

abc(123)
1.Check if condition false
2.abc (23)

abc(23)
1.Check if condition false

2. | abc(3) |
|---|
| 1.Check if condition false |

2. | abc(10) |
|---|
| 1.Check if True return |

3. | abc(10) |
|---|
| 1.Check if True return |

4. printf("%c", S[0]) = 3

Now again go to

abc(23)
3. abc(3)

Due to 3rd line we again get abc (3).
So same o/p i.e., = 3.
Hence, o/p is 123233

(b) For 'n' characters it produces o/p: '2n' characters.

2. Arrays

Q.11 The following Pascal program segments finds the largest number in a two dimensional integer array A[0 ... n – 1, 0 ... n – 1] using a single loop. Fill up the boxes to complete the program and write against $\boxed{A}$, $\boxed{B}$, $\boxed{C}$ and $\boxed{D}$ in your answer book. Assume that max is a variable to store the largest value and i,j are the indices to the array.

begin

  max:= $\boxed{A}$, i:=0,j:=0;

  while $\boxed{B}$ do

    begin

    if A[i,j]>max then max:=A[i,j]

    if $\boxed{C}$ then j:=j+1

    else begin

j:= 0;

i:= $\boxed{D}$;

end

end

end.

[1993 : 5 Marks]

Solution:

$\boxed{A} \Rightarrow 0$

$\boxed{B} \Rightarrow$ i< = n – 1 && j< = n – 1

$\boxed{C} \Rightarrow$ (j! = n – 1)

$\boxed{D} \Rightarrow$ i = i + 1

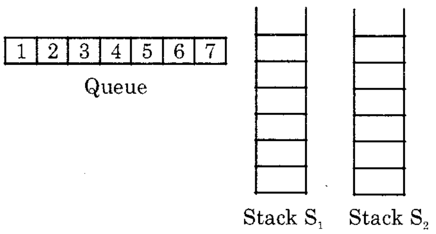
3. Stack & Queues

Q.12 A queue Q containing n items and an empty stack S are given. It is required to transfer all the items from the queue to the stack, so that the item at the front of queue is on the top of the stack, and the order of all other items is preserved. Show how this can be done in O(n) time using only a constant amount of additional storage. Note that the only operations which can be performed on the queue and stack are Delete, Insert, Push and Pop. Do not assume any implementation of the queue or stack.

[1994 : 5 Marks]

Solution:

To implement queue we use 2 stacks



1. Delete first element from queue and push into stack S_1.
2. Continue this way push all elements in S_1 untill queue become empty, so by this way we can implement queue 'enqueue operation.
3. For 'Dequeue operation' first POP all elements from S_1 andthen push into S_2 one by one then POP the elements from S_2.

Q.13 The Fibonacci sequence $\{f_1, f_2, f_3 \ldots f_n\}$ is defined by the following recurrence:
$f_{n+2} = f_{n+1} + f_n$, $n \geq 1$; $f_2 = 1$: $f_1 = 1$:
Prove by induction that every third element of the sequence is even.

[1996 : 5 Marks]

Solution:

Given: $f_{n+1} = f_{n+1} + f_n, n \geq 1$
$f_1 = 1, f_2 = 1$

Where c is any integer constant.

Note: Every third element is
$f_3, f_6, f_9, f_{12}, \ldots$

Prove: $f_{3n} = 2 * c$

Base Case:n = 1

$\Rightarrow \quad f_3 = f_2 + f_1$
$= 1 + 1 = 2 \Rightarrow$ Even

Induction: Assume $f_{3n} = 2c$ holds for any $n \geq 1$

Let n = k + 1.

$f_{3(k+1)} = f_{3k+3} = f_{3k+2} + f_{3k+1}$
$[\because f_{n+2} = f_{n+1} + f_n \Rightarrow f_x = f_{x-1} + f_{x-2}]$
$= (f_{3k+1} + f_{3k}) + f_{3k+1}$
$= 2f_{3k+1} + f_{3k}$
$= 2f_{3k+1} + 2c$
$= 2(f_{3k+1} + c)$
= Even

$f_{3(k+1)}$ is also holds

$\therefore$ we proved f_{3n} is even.

Q.14 Suppose a stack implementation supports, in addition to PUSH and POP, an operation REVERSE, which reverses the order of the elements on the stack.

(a) To implement a queue using the above stack implementation, show how to implement ENQUEUE using a single operation and DEQUEUE using a sequence of 3 operations.

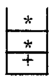
(b) The following postfix expression, containing single digit operands and arithmetic operators + and *, is evaluated using a stack.
5 2 * 3 4 + 5 2 * * +
Show the contents of the stack.
(i) After evaluating 5 2 * 3 4 +
(ii) After evaluating 5 2 * 3 4 + 5 2
(iii) At the end of evaluation.

[2000 : 5 Marks]

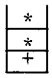
Solution:

(a) ENQUEUE is simple just odd one by one element into stack.
DEQUEUE: (1) Reverse stack element. (2) TOP = TOP-1 (3) reverse again all element.

(b) First convert it into infix expression
$5 * 2 + (3 + 4) * 5 * 2$
(i) Content of stack after 52 * 34 + = evaluating



(ii) Content of stack after 52 * 34 + 52 evaluating



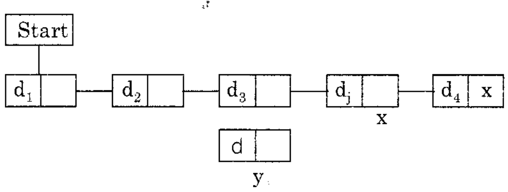
(iii) At end; empty stack

4. Linked List

Q.15 Consider a singly linked list having n nodes. The data items $d_1, d_2, ..., d_n$ are stored in these n nodes. Let X be a pointer to the j-th node ($1 \le j \le n$) in which d_j is stored. A new data item d stored in a node with address Y is to be inserted. Give an algorithm to insert d into the list to obtain a list having items $d_1, d_2, ... d_{j-1}, d, d_j, ...$, dn in that order without using the header.

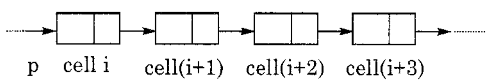
[1993 : 5 Marks]

Solution:




```
ptr = start;
while (ptr → next! = x)
{
ptr = ptr → next;
}
ptr → next = y;
y → next = x;
```

Q.16 Let p be a pointer as shown in the figure in a signly linked list.



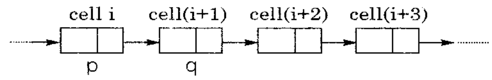
What do the following assignment statements achieve?

(i) q : = p → next
(ii) p → next : = q → next
(iii) q → next : = (q → next) → next
(iv) (p → next) → next : = q

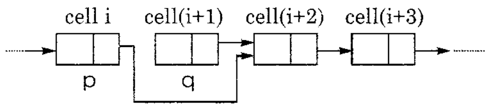
[1998 : 5 Marks]

Solution:

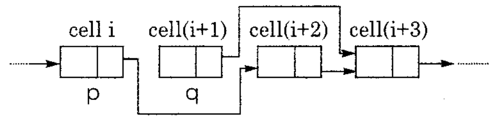
(i) q = p → next (q points to cell (i + 1)



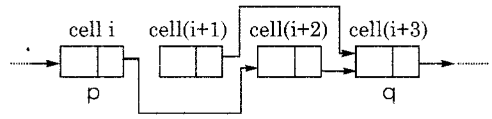
(ii) p → next = q → next (cell i is connected with cell (i + 2))



(iii) q → next : = (q → next) → next



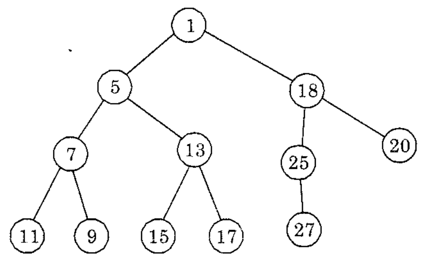
(iv) (p → next) → next : = q



(i + 1)st cell is deleted from the above four statements.

5. Trees

Q.17 Consider the binary tree in Figure:
(a) What structure is represented by the binary tree?

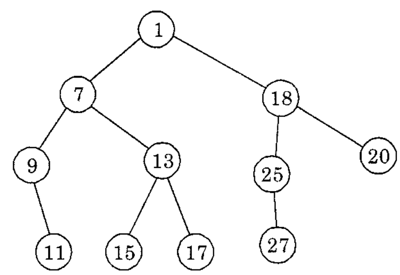


(b) Give the different steps for deleting the node with key 5 so that the structure is preserved.

[1991 : 5 Marks]

Solution:

(a) It's is min heap tree.
(b) The node has 2 children so after deleting it we replace it by either inorder successor or inorder predecessor.

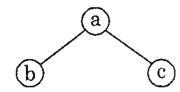


Q.18 Prove by the principal of mathematical induction that for any binary tree, in which every non-leaf node has 2 descendants, the number of leaves in the tree is one more than the number of non-leaf nodes.

[1993 : 5 Marks]

Solution:

Step-I: Binary tree with one root node and 2 descendants

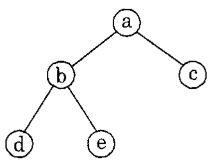


Here; non-leaf nodes = 1

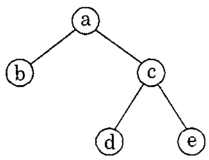
leaf nodes = 2

$\therefore$ leaf node = non-leaf nodes + 1.

Step-II: Binary tree with two nodes having 2 descendants

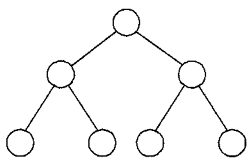


or

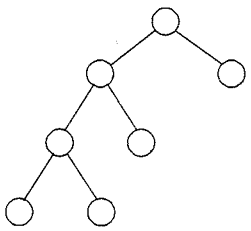


(2 – 1) = 1 leaf node added and 1 internal node added. For one internal node increase there (are) is only one leaf node increase.

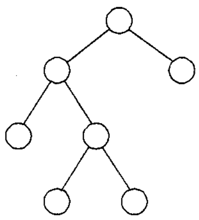
Similarly binary tree with 3 nodes having 2 descendants (leaf nodes = internal nodes + 1).



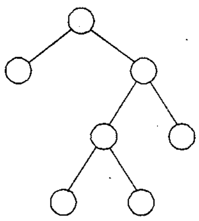
or



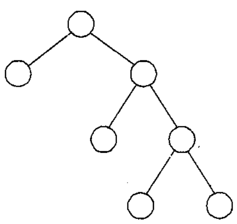
or



or



or



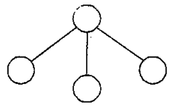
For one internal node increase there is only one leaf node increases by mathematical induction. By principle of induction for a binary tree with n internal nodes having 2 descendants there will be (n+1) leaf nodes.

Q.19 A 3-ary tree is a tree in which every internal node has exactly three children. Use induction to prove that the number of leaves in a 3-ary tree with n internal nodes is $2(n-1) + 3$.

[1994 : 5 Marks]

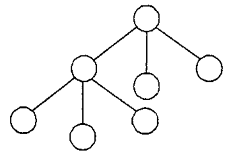
Solution:

Step-I: 3-ary tree with one internal node



= 3 leaf nodes

Step-II: 3-ary tree with 2 internal node



We lost one leaf but 3 new leaves added so as a whole change in number of leaves is $3 - 1 = 2$.

Step-III: Therefore, if we have (n – 1) internal nodes and l_{n-1} leaves. Then for n internal nodes we will get

$$I_n = I_{n-1} + 2 \text{ leaves. } I_1 = 3$$

So

$$I_n = I_{n-1} + 2$$
$$I_n = (I_{n-2} + 2) + 2$$
$$I_n = (I_{n-3} + 2) + 2 + 2$$
$$I_n = I_{n-i} + \frac{2+2+2\ldots+2}{i \text{ times}}$$
$$I_n = I_{n-i} + 2i$$

When $i = n - 1$

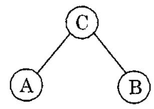
$$I_n = I_1 + 2(n-1)$$
$$I_n = I_1 + 2(n-1)$$
$$I_n = 3 + 2(n-1) \quad \text{Since, } I_1 = 3$$

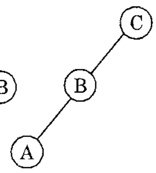
Q.20 What is the number of binary trees with 3 nodes when traversed in postorder give the sequence A, B, C? Draw all these binary trees.

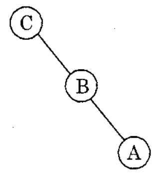
[1995 : 5 Marks]

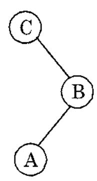
Solution:

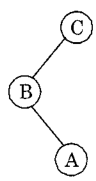
Number of binary trees with 3 nodes = 5











Q.21 A size-balanced binary tree is a binary tree in which for every node the difference between the number of nodes in the left and right subtree is at most 1. The distance of a node from the root is the length of the path from the root to the node. The height of a binary tree is the maximum distance of a leaf node from the root.

(a) Prove, by using induction on h, that a size-balance binary tree of height h contains at least 2^h nodes.

(b) In a size-balanced binary tree of height $h \geq 1$, how many nodes are at distance $h - 1$ from the root? Write only the answer without any explanations.

[1997 : 5 Marks]

Solution:

(a) **Base case:** $h = 0$

$\Rightarrow$ least no. of nodes $= 2^0 = 1$

Induction: Assume that it is true for $h = k$

minimum no. of nodes $= 2^k$

If we increase the height by 1 by adding a node. We must also add nodes to fill the level.

$h = k + 1 \Rightarrow$ least no. of nodes $= 2*2^k = 2^{k+1}$.

Hence it is Proved.

(b) 2^{h-1}

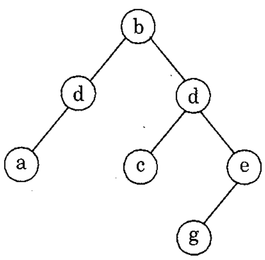
Q.22 Draw the binary tree with node labels a, b, c, d, e, f and g for which the inorder and postorder traversals result in the following sequences.

Inorder : a f b c d g e

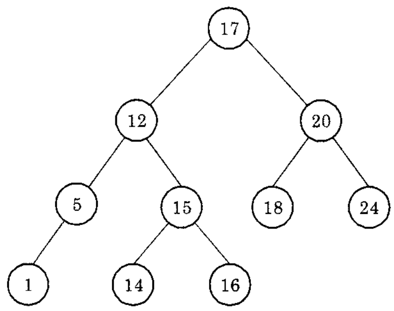
Postorder : a f c g e d b

[1998 : 5 Marks]

Solution:



Q.23 (i) Define the height of a binary tree or subtree and also define a height balanced (AVL) tree.



(ii) Mark the balance factor of each on the tree given in figure and state whether it is height-balanced.

(iii) Into the same tree given above, insert 13 and show the new balance factors that would arise if the tree is not rebalanced. Finally, carry out the required rebalancing of the tree and show the new tree with the balance factors on each node.

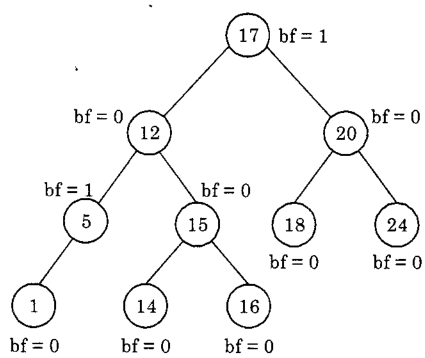
[1998 : 5 Marks]

Solution:

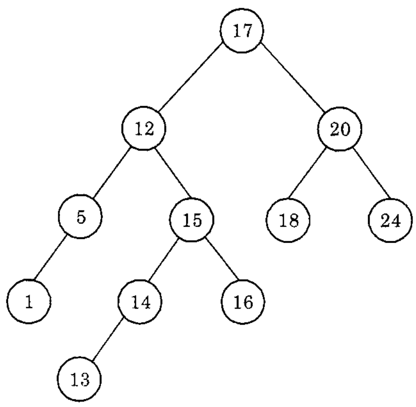
(i) The height of a tree or a subtree is defined as the length of the longest path from the root node to the leaf.

A tree is said to be height balanced if all the nodes are having a balance factor –1, 0 or 1. The balance factor is the height of the left subtree minus height of the right subtree.

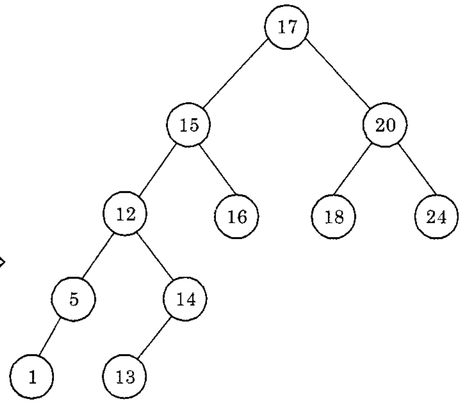
(ii)



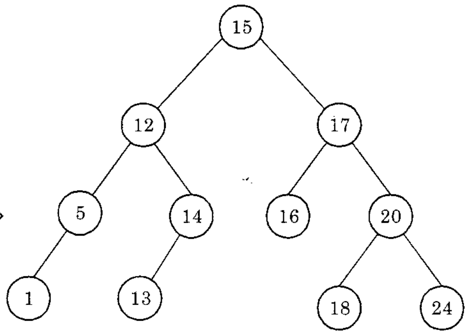
(iii) Insert 13:



⇒



⇒



Q.24 (a) In binary tree, a full node is defined to be a node with 2 children. Use induction on the height of the binary tree to prove that the number of full nodes plus one is equal to the number of leaves.

(b) Draw the min-heap that results from insertion of the following elements in order into an initially empty min-heap: 7, 6, 5, 4, 3, 2, 1. Show the result after the deletion of the root of this heap.

[1999 : 5 Marks]

Solution:

(a) $h = 0 \Rightarrow$ [∵ #full_nodes = 0]

#leaves = #full_nodes + 1 = 0 + 1 = 1 [∵ root]

Assume at height h:

⇒ #leaves = #full_nodes

At height h + 1:

In the last level, assume there are n nodes.

Let m of the n nodes are full nodes.

Then (n – m) nodes are with one child.

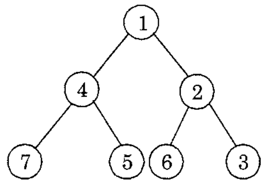
If we delete one full node from 'm' then one leaf is decreased from total.

If we delete a node from (n – m) which have one child then no effects on no. of leaves.

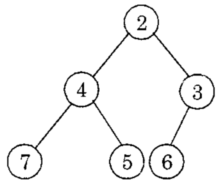
Binary tree with height h + 1:

⇒ #leaves = #full nodes + 1

(b) Minheaptree



Now delete the root then



6. Hashing

Q.25 Consider a hash table with n buckets, where external (overeflow) chaining is used to resolve collisions. The hash function is such that the probability that a key value is hashed to a particular buckets is $\frac{1}{n}$. The hash table is intially empty and K distinct values are inserted in the table.

(i) What is the probability that bucket number 1 is empty after the K^{th} insertion?

(ii) What is the probability that no collision has occurred in any of the K insertions?

(iii) What is the probability that the first collision occurs at the K^{th} insertion?

[1997 : 5 Marks]

Solution:

(i) Probability that a key goes to a bucket $= \frac{1}{n}$

Probability that a key doesnot goes to a bucket $= 1 - \frac{1}{n} = \frac{n-1}{n}$.

Probability that bucket number 1 is empty

$$= \left(\frac{n-1}{n}\right)\left(\frac{n-1}{n}\right)\left(\frac{n-1}{n}\right) \text{....k times}$$

$$= \left(\frac{n-1}{n}\right)^k$$

(ii) Probability that no collision has occurred in any of the k insertions is

$$\frac{n(n-1)(n-2)...(n-(k-1))}{n^k}$$

(iii) Probability that the first collision occurs at the k^{th} insertion is

$$\frac{n(n-1)(n-2)....(n-(k-2))(k-1)}{n^k}$$

■■■■

Unit 6

Algorithms

1. Algorithm Analysis and Asymptotic Notations

Q.1 Consider the following piece of 'C' code fragment that removes duplicate from an ordered list of integers.

```
struct node{
    int val;
    struct node *next;
};
    typedef struct node Node;
    Node *remove-duplicates (Node*heat, int *j)
{
    Node *t1, *t2;
    *j = 0
    t1 = head;
    if (t1! = NULL) t2 = t1 → next;
    else return head:
    *j = 1;
    if (t2 == NULL) return head;
    while t2! = NULL)
{
    if (t1.val! = t2.val) ⇒ (S1)
{
    (*j) ++; t1 → next = t2; t1 = t2 ⇒ (S2)
    }
    t2 = t2 → next;
    }
    t1 → next = NULL;
    return head;
    }
```

Assume the list contains n elements in the following questions.

(a) How many times is the comparison in statements S_1 made?

(b) What is the minimum and the maximum number of times statements marked S_2 get executed?

(c) What is the significance of the value of the integer pointed to by j when the fucntion completes?

[1997 : 5 Marks]

Solution:

(a) O(n)

S_1 statement every time check the value of all nodes of linked list. So it executes O(n) times. .

(b) O times, O(n)

(c) The integer value pointed by 'j' tells the different value nodes persent in linked list.

Q.2 Consider the following algorithms. Assume, procedure A and producdure B take O(1) and O(1/n) unit of time respectively. Derive the time complexity of the algorithm in O-notation.

```
algorithm what (n)
begin
    if n = 1 then call A
    else begin
    what (n – 1);
    call B(n)
  end
end.
```

[1999 : 5 Marks]

Solution:

O(n)

2. Divide and Conquer

Q.3 An input file has 10 records with keys as givne below:

25 7 34 2 70 9 61 16 49 19

This is to be sorted in non-decreasing order.

(i) Sort the input file using QUICKSORT by correctly positioning the first element of the file subfile. Show the subfiles obtained at all intermediate steps. Use square brackets to demarcate subfiles.

(ii) Sort the input file using 2-way MERGESORT showing all major intermediate steps. Use square brackets to demarcate subfiles.

[1989 : 5 Marks]

Solution:

(i) 25, 7, 34, 2, 70, 9, 61, 16, 49, 19

$\Downarrow$

Step-1: [9, 7, 19, 2, 16], 25 [61, 70, 49, 34]

Step-2: [2, 7], 9, [19, 16], 25, [49, 34], 61, [70]

Step-3: 2, [7], 9, [16], [19], 25, [34], 49, 61, 70

Step-4: 2, 7, 9, 16, 19, 25, 34, 49, 61, 70

Note: Circled element is a pivot which is one of the element in subfile of the previous step.

(ii) [7, 25], [2, 34], [9, 70], [16, 61], [19, 49]

$\Downarrow$

[2, 7, 25, 34], [9, 16, 61, 70], [19, 49]

[7, 9, 16, 25, 34, 61, 70], [19, 49]

[2, 7, 9, 16, 19, 25, 34, 49, 61, 70]

Q.4 Find a solution to the following recurrence equation:

$T(n) = \sqrt{n + T(n/2)}$

$T(1) = 1$

[1989 : 2 Marks]

Solution:

By using substitution method

We get $O\left(\sqrt{n \log_2 n}\right)$

Q.5 Give an optimal algorithm is pseudo-code for sorting a sequence of n numbers which has only k distinct numbers (k is not known a Priori. Give a brief analysis for the time-complexity of your algorithm).

[1991 : 2 Marks]

Solution:

Use Merge sort or Heap sort both having O(n log n) complexity.

3. Greedy Method

Q.6 A language uses an alphabet of six letters, {a, b, c, d, e, f}. The relative frequency of use of each letter of the alphabet in the language is as given below.

Letter	Relative frequency of use
a	0.19
b	0.05
c	0.17
d	0.08
e	0.40
f	0.11

Design a prefix binary code for the language which would minimize the average length of the encoded words of the language.

[1989 : 5 Marks]

Solution:

0.05 - 1010

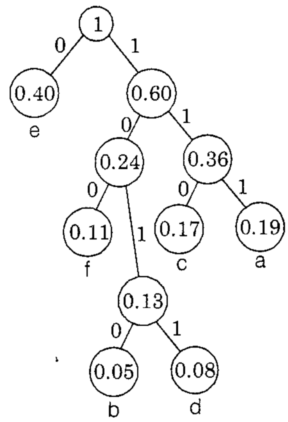
0.08 - 1011

3.17 - 110

0.19 - 111

0.11 - 100

0.40 - 0



a - 111	$0.19 \times 3 = 0.57$
b - 1010	$0.05 \times 4 = 0.20$
c - 110	$0.17 \times 3 = 0.51$
d - 1011	$0.08 \times 4 = 0.32$
e - 0	$0.40 \times 1 = 0.40$
d - 100	$0.11 \times 3 = 0.33$
	2.33

No. of bits $= \dfrac{2.33}{1} = 2.33$ b/msq

Q.7 An independent set in a graph is a subset of vertices such that no two vertices in the subset are connected by an edge. An incomplete scheme for a greedy algorithm to find a maximum independent set in a tree is given below:

V: Set of all vertices in the tree; I:=ϕ;

While V $\neq \phi$ do

begin

select a vertex u: $\in$ V such that

V:=V – {u};
if u is such that
then I:= I ∪ {u}
end;
output(I);

(a) Complete the algorithm by specifying the property of vertex u in each case
(b) What is the time complexity of the algorithm.

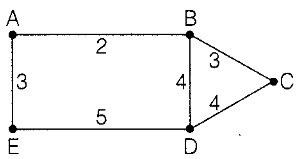
[1994 : 5 Marks]

Solution:

(a) Complete algorithm for u.
V: Set of all vertices in the tree
I: ϕ (initially)
While V $\neq \phi$ do
begin
Select a vertex u : ∈ V such
V = V – {u}
if (adjacent of (u) not present in {I})
then I: = I ∪ {u}
end
output (I);

(b) Time complexity
O(no. of vertices present in V)X
O(no. of vertices present in I)
$\cong O(|V|^2)$

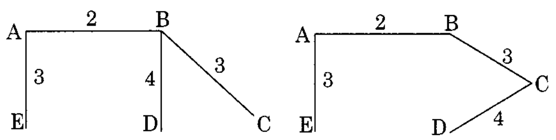
Q.8 How many minimum spanning trees does the following graph have? Draw them. (Weights are assigned to the edge).



[1995 : 5 Marks]

Solution:

2 min. spanning tree are possible



Q.9 A complete, undirected, weighted graph G is given on the vertex {0, 1 ....., n – 1} for any fixed 'n'. Draw the minimum spanning tree of G if

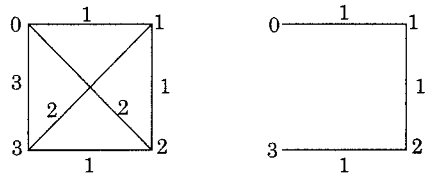
(a) the weight of the edge (u, v) is $|u - v|$
(b) the weight of the edge (u, v) is u + v

[1996 : 5 Marks]

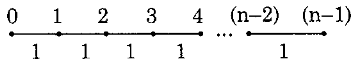
Solution:

Let n = 4

(a) Weight of the edge (u, v) is $|u - v|$

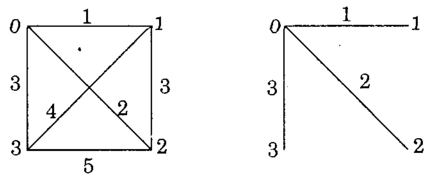


MST is:

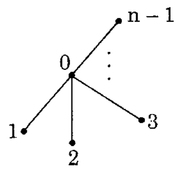


Weight of MST = 1 + 1 + ... (n – 1) times = n – 1

(b) Weight of the edge (u, v) is u + v

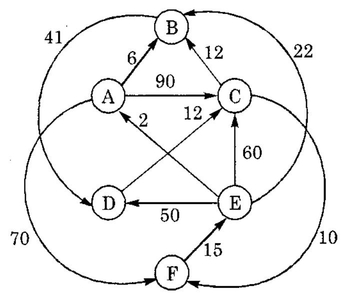


MST is:



Weight of MST is 1 + 2 + ... + n – 1 = n(n – 1)/2.

Q.10 Let G be the directed, weighted graph shown below in the given figure.



We are interested in the shortest paths from A.

(a) Output the sequence of vertices identified by the Dijkstra's algorithm for single source shortest path when the algorithm is started at node A.
(b) Write down sequence of vertices in the shortest path from A to E.
(c) What is the cost of the shortest path from A to E?

[1996 : 5 Marks]

Solution:

(a) Apply Dijkstra then output sequence is
A B D C F E

(b) Sequence of vertices between A to E
A B D C F E

(c) Cost = 84

Q.11 Consider a graph whose vertices are points in the plane with integer co-ordinates (x, y) such that $1 \le x \le n$ and $1 \le y \le n$, where $n \ge 2$ is an integer. Two vertices (x_1, y_1) and (x_2, y_2) are adjacent iff $|x_1 - x_2| \le 1$ and $|y_1 - y_2| \le 1$. The weight of an edge $\{(x_1, y_1), (x_2, y_2)\}$ is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

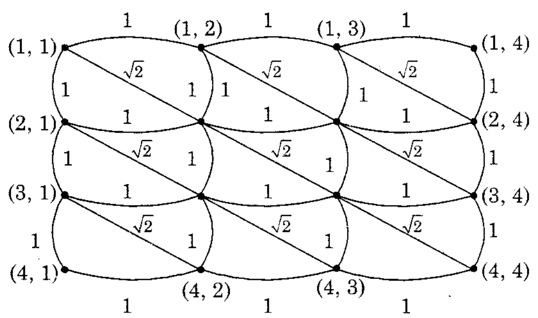
(a) What is the weight of a minimum weight spanning tree in this graph? Write only the answer without any explanations.

(b) What is the weight of a maximum weight-spanning tree in this graph? Write only the answer without any explanations?

[1997 : 5 Marks]

Solution:

For n = 4, the graph can be shown as



(a) Since the weight of all the edges is 1, and no. of edges in minimum spanning tree is (n – 1). Therefore, the weight of minimum spanning tree is (n – 1)

(b) If n = 2, the weight of maximum weight spanning tree is $\sqrt{2} + 2$

If n = 3, the weight is $2\sqrt{2} + 4 = 2(\sqrt{2} + 2)$

If n = 4, the weight is $3(\sqrt{2} + 6)$.

So the weight of maximum weight spanning tree is $(n-1)(\sqrt{2} + 2)$.

Q.12 Consider a weighted undirected graph with vertex set V = {n1, n2, n3, n4, n5, n6} and edge set E = {(n1, n2, 2), (n1, n3, 8), (n1, n6, 3), (n2, n4, 4), (n2, n5, 12), (n3, n4, 7), (n4, n5, 9), (n4, n6, 4)}. The third value in each tuple represents the weight of the edge specified in the tuple.

(a) List the edges of a minimum spanning tree of the graph.

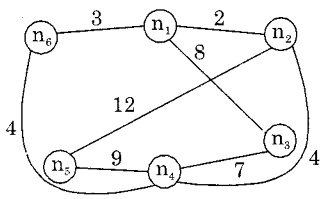
(b) How many distinct minimum spanning trees does this graph have?

(c) Is the minimum among the edge weights of a minimum spanning tree unique overall possible minimum spanning trees of a graph?

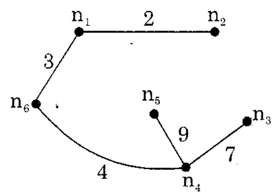
(d) Is the maximum among the edge weights of a minimum spanning tree unique over all possible minimum spanning trees of a graph?

Solution:

First draw the given graph according to question



(a) Min spanning tree



(b) No.of min. spanning trees = 2 [$(n_2, n_4, 4)$ is added and $(n_4, n_6, 4)$ is removed to get other MST from the above figure].

(c) Yes

(d) Yes

[2001 : 5 Marks]

4. Dynamic Programming

Q.13 Fill in the blanks in the following template of an algorithm to compute all pairs shortest path lengths in a directed graph G with n*n adjacency matrix A. A[i, j] equals 1 if there is an edge in G form i to j, and 0 otherwise. Your aim in filling in the blanks is to ensure that the algorithm is correct.

INITIALIZATION: For i = 1 ..... n
{For j = 1 ..... n
{if A[i, j] = 0 then P[i, j] = ... else
P[i, j] = ...;}
ALGORITHM: For i = 1 ..... n
{For j = 1 ..... n
{For k = 1 ..... n
{P[....., ...] = min (.....,.....);}
}
}

(a) Copy the complete line containing the blanks in the Initialization step and fill in the blanks.
(b) Copy the complete line containing the blanks in the Algorithm step and fill in the blanks.
(c) Fill in the blank: The running time of the Algorithm is O(_____).

[2002 : 5 Marks]

Solution:

(a) Initialization:

```
for  i = 1 to n
{
    for j = 1 to n
    if A[i, j] = 0 then P[i, j] = φ
    else P[i, j] = 1;
```

(b) Algorithm

```
for  i = 1 to n
{
   for  j = 1 to n
   {
      for k = 1 to n
      {
         P[i, j] = min(P[i, k] + P[k, j], P[i, j])
      }
   }
}
```

(c) The running time of the algorithm is $O(n^3)$

5. Misc Topics

Q.14 Consider the function F(n) for which the pseudo code is given below:

```
Function F(n)
begin
F1 ← 1
if(n=1) then F ← 3
else for i = 1 to n do
    begin
    C ← 0
    for j = 1 to F(n – 1) do
    begin C ← C + 1 end
    F1 = F1 * C
    end
    F = F1
end
```

[n is a positive integer greater than zero]

(a) Derive a recurrence relation for F(n).
(b) Solve the recurrence relation for a closed form solutions of F(n).

[1992 : 5 Marks]

Solution:

(a) $T(n) = T(n-1) + n$
(b) $T(n) = O(n^2)$

Q.15 Consider the recursive algorithm given below:

```
procedure bubblesort (n);
var i,j: index; temp : item;
begin
    for i:=1 to n-1 do
    if A[i] > A [i+1] then
begin
    temp:= A[i];
    A[i]:=A[i+1];
    A[i+1]:=temp
    end;
  bubblesort (n -1 )
end
```

Let a_n be the number of times the 'if ..... then.....' Statement gets executed when the algorithm is run with value n. Set up the recurrence relation by defining a_n in terms of a_{n-1}, Solve for a_n.

[1993 : 5 Marks]

Solution:

Recurrence relation

$$T(n) = \begin{cases} \text{return } A[i] & \text{if } n = 1 \\ \text{swap } [a(i), a(i+1)] & A[i] > A(i+1) \end{cases}$$

Q.16 An array A contains n integers in non-decreasing order, $A[1] \leq A[2] \leq \ldots\ldots \leq A[n]$. Describe, using Pascal like pseudo code, a linear time algorithm to find i, j, such that A[i] + A[j] = a given integer M, if such i, j exist.

[1994 : 5 Marks]

Solution:

Take 2 variable : A - First, B - Last

100	200	300	400	500	800
A					B

if (A + B) = = 1000, return (A, B)
else
if (A + B > 1000)
B = B – 1;
else
A = A + 1;
O(n)

Q.17 A two dimensional array A[1 – n] [1 – n] of integers is partially sorted if
$\forall$ i, j$\in$ [1 ... n – 1] A[i][j] < A[i][j + 1] and
A[i][j] < A[i + 1][j]]

(a) The smallest item in the array is at A[i] [j] where i = ... and j = ... (1)

(b) The smallest item is deleted. Complete the following O(n) procedure to insert item x (which is guaranteed to be smaller than any item in the last row or column) still keeping A partially sorted.

```
procedure insert (x integer)
          var i j:integer;
          begin
(1)       i: =1; j:=A[i][j]:=x;
(2)       while (x > or x >) do
(3)       if A[ i + 1][j] > A[i]]j]
           then begin
(4)       A [i] [j]:= A[i +1] [j]; i:= i+1;
(5)       end
(6)       else begin
(7)
(8)       end
(9)       A [i] [j]:=
end
```

[1996 : 5 Marks]

Solution:

(a) i = 1, j = 1

(b) Line 2; while (x > A[i + j][j] or x > A[i] [j + 1])

line 7; A[i][j] = A[i][j + 1];
j = j + 1;
line 9; A[i][j] = x;

Q.18 An array A contains n ≥ 1 positive integers in the location A[1],A[2],.....,A[n]. the following program fragment prints the length of a shortest sequence of consecutive elements of A, A[i], A[i+1], ... , A[j] such that the sum of their values is ≥ M, a given positive number. It prints 'n+1' if no such sequence exists. Complete the program by filling the boxes. In each case use the simplest possible expression. Write only the line number and the contents of the box.

```
1.  begin
2.  i :+1;j:=1;
3.  Sum := [     ];
4.  min:=n; finish:= false;
5.  While not finish do
6.  If [     ] then
7.  if j = n then finish:=true.
8.  else
9.  begin
10. j:=j+1;
11. sum:= [   ]
12. end
13. else
14. begin
15. If(j – 1) < min then min:=j – 1;
16. sum:=sum – A[i];
17. i:= i+1;
18. end
19. writeln (min + 1);
20. end.
```

[1997 : 5 Marks]

Solution:

line 3; sum = A[j]
line 6; if (sum < M)
line 11; sum = sum + A[j]

■■■■

1. Parsing Techinques

Q.1 Consider the following grammar:

$S \to S$

$S \to SS \mid a \mid \varepsilon$

(a) Construct the collection of sets of LR(0) items for this-grammar and draw its goto graph.

(b) Indicate the shift-reduce and reduce-reduce conflicts (if any) in the various states of the LR(0) parser.

[1988 : 5 Marks]

Solution:

(a) **Step-1:** The augumented grammar is

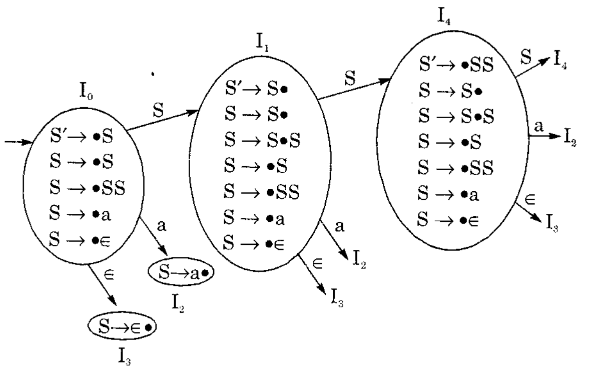
$S' \to \cdot S$

$S \to S \mid SS \mid a \mid \varepsilon$

Step-2: I_0 = closure $(S' \to \cdot S)$

$$I_0 = \begin{array}{l} S' \to \cdot S \\ S \to \cdot S \\ \quad\ \ \cdot SS \\ \quad\ \ \cdot a \\ \quad\ \ \cdot \epsilon \end{array}$$

Step-3: Using I_0 constructing goto graph



(b) Shift reduce conflicts are there in I_0, I_1, and I_4 but there are no reduce-reduce conflicts.

Unit 8

Operating System

1. Process Management-II

Q.1 Consider the following proposal to the "readers and writers problem" shared variablesa and semaphores:

```
aw, ar, rw, rr:integer;
mutex, reading, writing: semaphore:
ar = rr = aw = rw = 0
reading_value = writing_value = 0
mutex_value = 1.      process writer;
process reader;       begin
   begin              while true do
   repeat                begin
   P(mutex);             P(:mutex);
   ar: = ar + 1;         aw : = aw + 1;
   grantread;            grantwrite
   V (mutex);            V (mutex);
   P(reading);           P(Writing);
   Read;                 Write;
   P(mutex);             P(mutex);
   rr : = rr – 1;        rw : = rw – 1;
   ar : = ar – 1;        aw; = aw – 1;
   grantwrtie;           grantread;
   (mutex);              V(mutex);
   other-work            other-work
   until false           end
   end               end
Procedure grantread:
   begin
   If aw = 0
   then while (rr < ar)do
      begin rr: = rr + 1;
      V(reading)
      end
   end;
procedure grantwrite;
   begin
   if rr = 0
   then while (rw < aw) do
      begin rw : = rw + 1;
     V (writing)
      end
   end;
```

(a) Give the value of the shared variables and the states of semaphores when 12 readers are reading and 31 writers are waiting.

(b) Can a group of readers make waiting writers starve? Can writers starve readers?

(c) Explain in two sentences why the solution is incorrect.

[1987 : 5 Marks]

Solution:

(a) Value of shared variables 'ar' = 12 because one after another readers can get into the critical section and 'aw = 1' because once, one writer try to enter into CS, until all readers comes out it won't be happened that another writer comes into till grant write part. So 'aw' value will not increase.

(b) Yes readers can starve writers but writers can't starve readers.

(c) 'Progress' is not satisfied, because if one writer wants to enter into CS after just an another writer completed it's execution then it can't be possible.

Q.2 A system of four concurrent processes, P, Q, R and S use shared resources. A, B and C. The sequences in which processes P, Q, R and S request and release resources are as follows:

Process P:
1. P requests A
2. P requests B
3. P releases A
4. P releases B

Process Q:
1. Q requests C
2. Q requests A
3. Q releases C
4. Q releases A

Process R:
1. R requests B
2. R requests C
3. R releases B
4. R releases C

Process S:
1. S requests A
2. S requests C
3. S releases A
4. S releases C

If a resources is free, it is granted to a requesting process immediately. There is no preemption of granted resources. A resource is taken back from a process only when the process explicityly releases it.

(i) Can the system of four processes get into deadlock? If yes give a sequence (ordering) of operations (for requesting and releasing resources) of these processes which leads to a deadlock.

(ii) Will the processes always get into a deadlock? If your answer is no, give a sequence of these operations which leads to completion of all processes.

(iii) What strategiesa can be used to prevent deadlock in a system of concurrent processes using shared resources if preemption of granted resources is not allowed?

[1989 : 5 Marks]

Solution:

(i) Yes, system of 4 process get into deadlock as shown below:

Concurrently : P req. A
Q req. C
R req. B
S req. A

(ii) No, not always, given as below sequence is:

P	Q	R	S
req. A	req. C		
req. B	req. A		
release A		req. B	
release B	release C		req. A
	release A	req. C	
		release B	req. C
		release C	
			release A
			release C

(iii) The process should release all the resource before requesting the new one resource.

Q.3 Consider the following scheme for implementing a critical section in a situation with three processes P_i, P_j and P_k.

```
Pi;
repeat
flag [i]:=true;
while flag [j] or flag [k] do
case turn of
  j : if flag [j] then
      begin
      flag [i]:=false;
      while turn ≠ i do skip;
      flag [i] : true
      end;
  k : if flag [k] then
      begin
      flag [i]:=false,
      while turn ≠ i do skip;
      flag [i]:=true
      end
end
critical section
    if turn = i then turn:=j;
    flag [i]:=false
non-critical section
until false;
```

(a) Does the scheme ensure mutual exclusion in the critical section? Briefly explain.

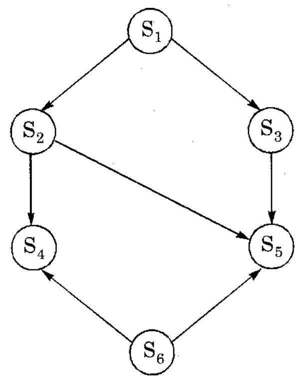
(b) Is there a situation in which a waiting process can never enter the critical section? If so, explain and suggest modifications to the code to solve this problem.

[1991 : 5 Marks]

Solution:

Yes, mutual exclusion satisfied, because entry into CS is depend on the value 'i' and value of 'j' has changed when any one flag has been chosen.

Q.4 Write a concurrent program using parbegin-parend and semaphores to represent the precedence constraints of the statements S_1 to S_6, as shown in figure below.



[1993 : 5 Marks]

Solution:

```
S1
par begin
S2
      par begin
      S4
      S5
      par end
S3
      begin
      S5
      end
par end
S6
par begin
      S4
      S5
par end
```

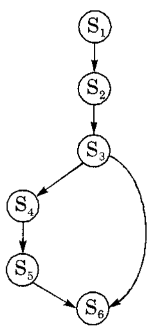
Q.5 (a) Draw a precedence graph for the following sequential code. The statements are numbered from S_1 to S_6

S_1 read n
S_2 i:=1
S_3 if i>n goto next
S_4 a(i):=i+1
S_5 i:=i+1
S_6 next : Write a(i)

(b) Can this graph be converted to a concurrent program using parbegin-parend construct only?

[1994 : 5 Marks]

Solution:



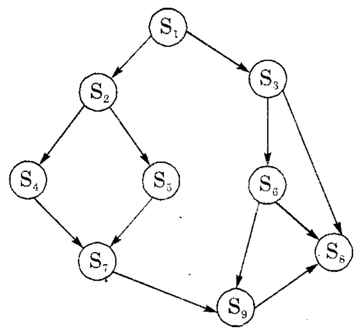

```
S1
begin
  S2
  begin
    S3
    begin
      S4
      begin
        S5
        begin
          S6
        end
      end
      S6
    end
  end
end
```

Q.6 Consider the following program segment for concurrent processing using semaphore operators P and V for synchronization. Draw the precedence graph for the statements S1 to S9.

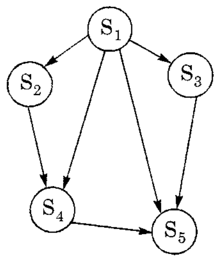
```
var
a, b, c, d, e, f, g, h, i, j, k : semaphore;
begin
cobegin
    begin S1; V(a); V(b) end;
    begin P(a); S2; V(c); V(d) end;
    begin P(c); S4; V(e) end;
    begin P(d); S5; V(f) end;
    begin P(e); P(f); S7; V(k) end;
    begin P(b); S3;V(g);V(h) end;
    begin P(g); S6; V(i) end;
    begin P(h); P(i); S8; V(j) end;
    begin P(i); P(j); P(k); S9 end;
coend
end;
```

[1995 : 5 Marks]

Solution:



Q.7 Write a concurrent program using par begin-par end to represent the procedure graph shown below.



[1998 : 5 Marks]

Solution:

```
S1
par begin
S2
begin
    S4
    end
```

S_3
begin
  S_5
  end
  par end
S_4
begin
  S_5
  end
  S_5

Q.8 (a) A certain processor provides a 'test and set' instruction that is used as follows.

TSET register, flag

This instruction atomically copies flag to register and sets flag to 1. Give pseudocode for implementing the entry and exit code to a critical region using this instruction.

(b) Consider the following solution to the producer-consumer problem using a buffer of size 1. Assume that the initial value of account is 0. Also assume that the testing of count and assignment to count are atomic operations.

```
Producer
Repeat
        Produce an item;
        if count = 1 then sleep;
        place item in buffer.
        Count =1:
        Wakeup (Consumer);
        Forever
Consumer:
        Repeat
        if count = 0 then sleep;
        Remove item from buffer;
        Count = 0
        Wakeup (producer);
        Consume an item;
        Forever:
```

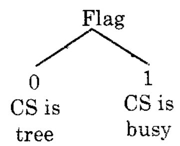
Show that in this solution it is possible that both the processes are sleeping at the same time.

[1999 : 5 Marks]

Solution:

(a) Entry section:

1. TSL R_i, M[J log]
2. cmp R_i, # 0
3. JNZ to step 1
4. $\boxed{CS}$
5. Store M[J log], # 0



(b) At the 3rd line in producer code the process preempt and consumer code is running so when it put count = 0 then after that producer process back and do count = 1, and from the next line of consumer wake up which is already count = 1, so it sleep again and consumer also sleep.

Q.9 Two concurrent processes P1 and P2 want to use two resources R1 and R2 in a mutually exclusive manner. Initially, R1 and R2 are free. The programs executed by the two processes are given below.

Program for P1:

S1: While (R1 is busy) do no-op;
S2: Set R1 ← busy;
S3: While (R2 is busy) do no-op;
S4: Set R2 ← busy;
S5: Use R1 and R2;
S6: Set R1 ← free;
S7: Set R2 ← free;

Program for P2:

Q1: While (R2 is busy) do no-op;
Q2: Set R2 ← busy;
Q3: While (R1 is busy) do no-op;
Q4: Set R1 ← busy;
Q5: Use R1 and R2;
Q6: Set R2 ← free;
Q7: Set R1 ← free;

(a) Is mutual exclusion guaranteed for R1 and R2? If not, show a possible interleaving of the statements of P1 and P2 such that mutual exclusion is violated (i.e., both P1 and P2 use R1 or R2 at the same time).

(b) Can deadlock occur in the above program? If yes, show a possible interleaving of the statements of P1 and P2 leading to deadlock.

(c) Exchange the statements Q1 and Q3 and statements Q2 and Q4. Is mutual exclusion guaranteed now? Can deadlock occur?

[2001 : 5 Marks]

Solution:

(a) Mutual exclusion guranteed.

(b) Deadlock occurs, when first 2 lines of both P_1 and P_2 are executed then after system can't do any progress.

(c) No, now mutual exclusion also not guaranteed. Deadlock can't occur at this situation.

Q.10 (a) Fill in the boxes below to get a solution for the readers-writers problem, using a single binary semphore, mutex (initialized to 1) and busy waiting. Write the box numbers (1,2 and 3), and their contents in your answer book.

```
    int R = 0, W = 0;
    Reader () {
L1: wait (mutex);
    If (W ==0) {
    R = R +1;
          [     ] ________(1)
    }
    else {
          [     ] ________(2)
    goto L1;
    }
    .../* do the read */
    wait (mutex)
    R = R – 1;
    signal (mutex);
    }
    Writer () {
L2: wait(mutex);
    If ([     ]) { ____(3)
          signal (mutex);
          goto L2;
    }
    W=1;
    signal (mutex);
    .../*do the write*/
    wait(mutex)
    W = 0;
    signal (mutex);
```

(b) Can the above solution lead to starvation of writers?

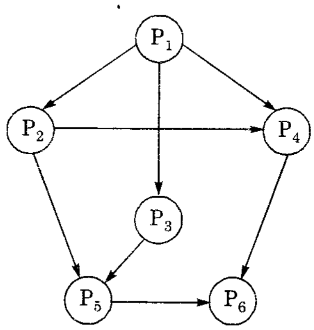
[2002 : 5 Marks]

Solution:

(a) 1. signal (mutex)
2. signal (mutex)
3. $w == 1$ or $R \geq 1$

(b) No, above solution can not starve the writers because whenever reader value reaches to '1' it put the w = 1 into code.

Q.11 Consider the following precedence graph (fig.) of processes where a node denotes a process and a directed edges from node P_i to node P_j implies a that P_i must complete before P_j commences. Implement of the graph using FORK and JOIN constructs. The actual computation done by a process may be indicated by a comment line.



[2002 : 5 Marks]

Solution:

Let initially : N = 2

Fork <level> creates new process

Join <lavel> (variable) which decrements the specified variable and terminates the variable if new value is not zero.

S = P;

Fork L_1

Fork L_2

Fork L_3

Fork L_4

S_1

L_1: S_2, S_3

L_2: Joint N

S_5

L_3 : S_4

goto L_2

L_4 : S_6

Next;

Q.12 The functionality of atomic TEST-AND-SET assembly language instruction is given by the following C function.

```
int TEST-AND-SET (int *x)
{
    int y;
    A1: y = *x;
    A2: * x = 1;
    A3: return y;
}
```

(i) Complete the following C functions for implementing code for entering and leaving critical sections based on the above TEST-AND-SET instruction.

```
int mutex = 0;
void enter-cs( )
{
while (.................................);
}
void leave-cs( )
{
.................................. ;
}
```

(ii) Is the above solution to the critical section problem deadlock free and starvation-free?

(iii) For the above solution, show by an example that mutual exclusing is not ensured if TEST-AND-SET instruction is not atomic.

[2002 : 5 Marks]

Solution:

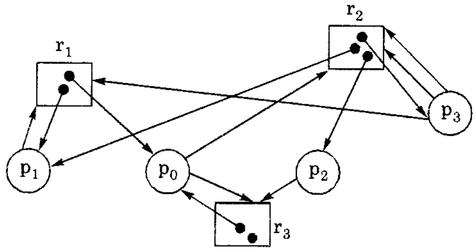
```
void enter_cs( )
{
while (Test-and-set (mutex));
}
void leave-cs ( )
{
    mutext= 0;
}
```

(i) Solutionis deadlock free but not starvation free.

(ii) If process gets preempted after all, then mutual exclusion is not ensured. The solution degenerates to lock variable which do not guarantee mutual exclusion.

2. Deadlock

Q.13 Consider the resource allocation graph given in the figure.



(a) Find if the system is in a deadlock state.

(b) Otherwise, find a safe sequence.

[1994 : 5 Marks]

Solution:

(a) No deadlock, every process request satisfy.

(b) P_2 P_0 P_1 P_3

Q.14 A computer system uses the Banker's Algorithm to deal with deadlocks. Its current state is shown in the tables below, where P0, P1, P2 are processes, and R0, R1, R2 are resources types.

	Maximum Need				Current Allocation		
	R0	R1	R2		R0	R1	R2
P0	4	1	2	P0	1	0	2
P1	1	5	1	P1	0	3	1
P2	1	2	3	P2	1	0	2

Available

R0	R1	R2
2	2	2

(a) Show that the system can be in safe state.

(b) What will system do on a request by process P0 for one unit of resource type R1?

[1996 : 5 Marks]

Solution:

(a) 1. First find, remaining need = maximum need – current allocation.

2. Then try to satisfy the remaining need with the help of available value of every process, if it is satisfied then safe state otherwise not safe sequence = P_2 P_1 P_0
P_1 P_2 P_0

(b) When P_0 requiest one unit of resource R_1 then first check available list of R_1, if it is ok then give that to P_0.

3. Memory Management and Virtual Memory

Q.15 Let the page reference and the working set window be c c d b c e c e a d and 4, respectively. The initial working set at time t = 0 contains the pages {a, d, e}, where a was referenced at time t = 0, d was referenced at time t = –1, and e was referenced at time t = –2. determine the total number of page faults and the average number of page frames used by computing the working set at each reference.

[1992 : 5 Marks]

Solution:

	b
e	e
d	d
a	a
Initially	1 page fault

Next referenc = e

So no change

d
e
b
d
1 page fault

Now next references are c d b c e c e fill no change.

Now a came

a	a
c	c
e	e
b	d
1 page fault	1 page fault

Total page faults = 4

Avg. no. of page frames are used 4.

Q.16 The following page addresses, in the given sequence, were generated by a program:

1 2 3 4 1 3 5 2 1 5 4 3 2 3

This program is run on a demand paged virtual memory system, with main memory size equal to 4 pages. Indicate the page references for which page faults occurs for the following page replacement algorithms.

(a) LRU (b) FIFO

Assume that the main memory is empty initially.

[1993 : 5 Marks]

Solution:

(a) LRU: Initially 4 references are page faults.

4	5	2	4	3	2
3	4	5	2	4	3
2	3	3	5	5	4
1	1	1	1	1	5
4 P.F.	1 P.F.	1 P.F.	1 P.F.	1 P.F.	1 P.F.

= 9 page faults

(b) FIFO

4	4	4	4	3
3	5	3	2	2
2	3	1	1	1
1	5	5	5	5
4 P.F.	1 P.F.	1 P.F.	1 P.F.	1 P.F.

= 8 page faults.

Q.17 A computer installation has 1000k of main memory. The jobs arrive and finish in the following sequences.

Job 1 requiring 200k arrives
Job 2 requiring 350k arrives
Job 3 requiring 300k arrives
Job 1 finishes
Job 4 requiring 120k arrives
Job 5 requiring 150k arrives
Job 6 requiring 80k arrives

(a) Draw the memory allocation table using Best Fit and First fit algorithms.
(b) Which algorithm performs better for this sequence?

[1995 : 5 Marks]

Solution:

(a) **Using Best Fit**

			150
200	350	300	

Now '1' finished

	50				30
150		350	300	120	

80 k job 6 can't get into it.

Using First Fit

200	350	300	150

Job '1' finished

120	80	350	300	150

All job easily get memory.

(b) First fit algorithm is best for this sequence.

Q.18 A demand paged virtual memory system uses 16 bit virtual address, page size of 256 bytes, and has 1 Kbyte of main memory. LRU page replacement is implemented using list, whose current status (page numbers in decimal) is

17	1	63

LRU page

For each hexadecimal address in the address sequence given below,

00FF, 010D, 10FF 11B0

indicate,

(i) the new status of the list

(ii) page faults, if any, and

(iii) page replacement, if any

[1996 : 5 Marks]

Solution:

Main member = 2^{10}

Page size = 2^8

No. of frames = $\frac{2^{10}}{2^8} = 2^2 = 4$

Address generated by CPU

16 bit

p	d
bit required to addressing the page table	bits required to define page size

Currently page no. 17, 1 & 63 are shown on the list, now, 1st address required; i.e.,

00FF convert the page no. into decimal = 255

So put 255 on the list.

Replace the page using LRU

So

(i) The new status of the list is

255	13	10

(ii) No. of page fault = 3

(iii) No. of page replacement = 3

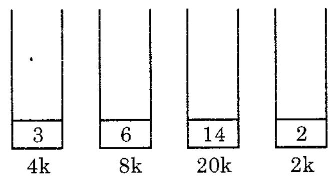
Q.19 In a computer system where the 'best-fit' algorithm is used for allocating 'jobs' to 'memory partitions', the following situation was encountered:

Partitions size in KB	4K 8K 20K 2K
Jobs sizes in KB	2K 14K 3K 6K 10K 20K 2K
Time for execution	4 10 2 4 1 8 6

When will the 20 K job complete?

[1998 : 5 Marks]

Solution:



Now 10 k have to wait fill 14 k not completed which take 10 unit of time.

After this 10k get 20k block and fill it's completion i.e., 1 unit, then at last 20k gets 20k block and execute fill completion, i.e., 8 unit.

So total time = 19 unit.

Q.20 A certain computer system has the segmented paging architecture for virtual memory. The memory is byte addressable. Both virutal and physical address spaces contain 216 bytes each. The virutal address space is divided in a 8 non-overlapping equal size segments. The Memory Management Unit (MMU) has a hardware segment table, each entry of which contains the physical address of the page table for the segment. Page tables are stored in the main memory and consist of 2 byte page table entries.

(a) What is the minimum page size in bytes so that the page table for a segment requires at most one page to store it? Assume that the page size can only be a power of 2.

(b) Now suppose that the pages size is 512 bytes. It is proposed to provide a TLB (Translation look-aside buffer) for speeding up address translation. The proposed TLB will be capable of storing page table entries for 16 recently referenced virtual pages, in a fast cache that will use the direct mapping scheme. What is the number of tag bits that will need to be associated with each cache entry?

(c) Assume that each page table entry contains (besides other information) 1 valid bit, 3 bits for page protection and 1 dirty bit. How many bits are available in page table entry for storing the paging information for the page? Assume that the page size is 512 bytes.

[1999 : 5 Marks]

Solution:

(a) Minimum page size = 128 bytes

Let page size = 2^k bytes

Page table size = $2^{13} = 2^{13-k} \times 2$ bytes

$= 2^{14-k}$ bytes

By given condition

$2^{14-k} = 2^k$

Then k = 7 and hence page size

$= 2^7 = 128$ bytes

(c)

1	3	1	x	
Valid	Page protection	Dirty bit	Page table entry	Page

Page table entry structure

f = frame no.

$= 128 = 2^7 = 7$ bits

So x = 7 bits

Then bits available in page table entry for storing paging information is 4 bits.

4. File System and Device Management

Q.21 A certain moving arm disk-storage device has the following specifications:

Number of tracks per surface=4004

Track storage capacity = 130030 bytes

Disk speed = 3600 rpm

Average seek time = 30 m secs.

Estimate the average latency the disk storage capacity and the data transfer rate.

[1990 : 5 Marks]

Solution:

Disk storage capacity = Track storage capacity × No. of tracks

i.e., 4004 × 130030 bytes

Data transfer rate

3600 R — 60 sec

$$1\text{ R} = \frac{60}{3600} = \frac{1}{60}\text{sec}$$

$$\text{Rotational latency} = \frac{1}{120} = 0.008$$

$$\frac{1}{60}\text{ sec} \xrightarrow{\text{Transfer}} 130030\text{ B}$$

$$1\text{ sec} \longrightarrow 60 \times 130030\text{ B}$$

$= 60 \times 130030$ bytes/sec

Q.22 Assuming the current disk cylinder to be 50 and the sequence for the cylinders to be 1, 36, 49, 65, 53, 1, 2, 3, 20, 55, 16, 65 and 78 find the sequences of servicing using

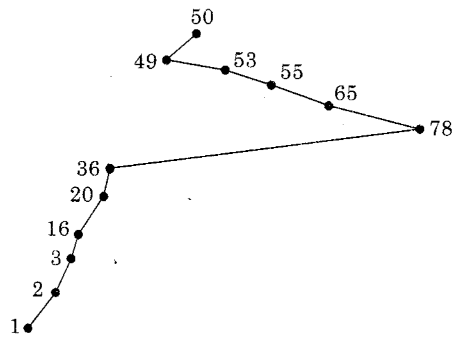
(i) shortest-seek time first (SSTF)

(ii) elevator disk scheduling polices.

[1990 : 5 Marks]

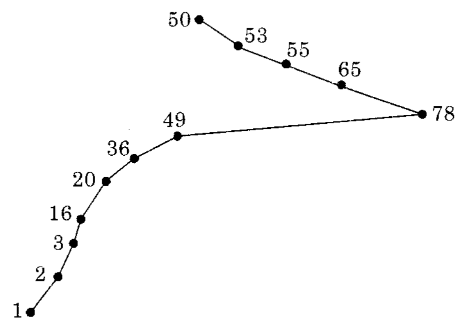
Solution:

(i) SSTF



Sequence is: 50, 49, 53, 55, 65, 78,36, 20, 16, 3, 2, 1

(ii) Elevator disk scheduling: Another name is "scan'



Sequence is 50, 53, 55, 65, 78, 49, 36, 20, 16, 3, 2, 1

Q.23 If the overhead for formatting a disk is 96 bytes for 40000 bytes sector,
Compuer the unformatted capacity of the disk for the following parameters:
Number of surfaces: 8
Outer diamter of the disk : 12 cm
Inner diameter of the disk : 4 cm
Inter track space: 0.1 mm
Number of sectors per track : 20

[1995 : 5 Marks]

Solution:

$$\text{No. of tracks} = \left(\frac{12-4}{2}\right)\text{cm}$$

$$= 4\text{cm} = \frac{4\text{cm}}{0.1\text{mm}}$$

$$= 400$$

Surface capacity

$$= 400 \times 20 \times 4096$$

$$\text{Disk capacity} = 8 \times 400 \times 20 \times 4096 \text{ bytes}$$

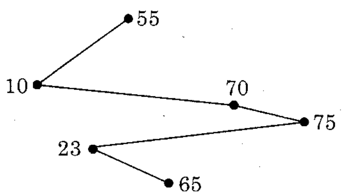
$$= 250\text{ MB}$$

Q.24 The head of a moving head disk with 100 tracks numbered 0 to 99 is currently serving a request at track 55. If the queue of requests kept in FIFO order is 10, 70, 75, 23, 65.
Which of two disk scheduling algorihtms FCFS (First come First serve) and SSTF (Shortest seek time First) will require less head movement? Find the total head movement for each of the algorithms.

[1995 : 5 Marks]

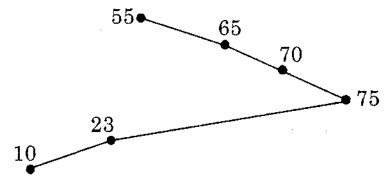
Solution:

FIFO:



Total number of head movements = 4

SSTF:



Total number of head movements = 2

Q.25 A file system with a one-level directory structure is implemented on a disk with disk block size of 4 K bytes. The disk is used as follows:

Disk-block 0: File allocation table, consisting of one 8-bit entry per data block, representing the data block address of the next data block in the file

Disk block 1: Directory, with one 32 bit entry per file.

Disk block 2: Data block 1;

Disk block 3: Data block 2; etc

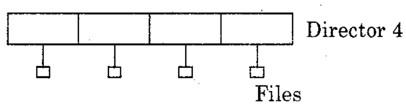
(a) What is the maximum possible number of files?

(b) What is the maximum possible file size in blocks?

[2002 : 5 Marks]

Solution:

Disk block size = 4 kB



(i) Maximum possible no. of files

$$= \frac{\text{Disk block size}}{\text{File size}} = \frac{2^{12}}{2^2} = 1024$$

(ii) Maximum possible file size
= Disk block size = 2^{12}

■■■■