Q.1) Max Marks: 1

The Game of Logic, has these two assumptions:
1. "Logic is difficult or not many students like logic."
2. "If mathematics is easy, then logic is not difficult."

By translating these assumptions into statements involving propositional variables and logical connectives, determine which of the following is a valid conclusion from the above assumptions:

A Mathematics is not easy, if many students like logic. Correct Option

Solution: (A)

INTERPRETATION SYMBOLS

Negation $\neg p$: not p

Disjunction $p \vee q$: p or q

Conjunction $p \wedge q$: p and q

Conditional statement $p \to q$: if p, then q

Biconditional statement $p \leftrightarrow q$: p if and only if q

Existential quantification $\exists x P(x)$: There exists an element x in the domain such that $P(x)$.

Universal quantification $\forall x P(x)$: $P(x)$ for all values of x in the domain.

LOGICAL EQUIVALENCES

$$p \to q \equiv \neg p \vee q \quad (I)$$

Commutative laws:

$$p \vee q \equiv q \vee p$$
$$p \wedge q \equiv q \wedge p$$

De Morgans laws:

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$
$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

RULES OF INFERENCE

Conjunction:

$$\begin{array}{rc} & p \\ & q \\ \hline \therefore & p \wedge q \end{array}$$

Resolution:

$$\begin{array}{rc} & p \vee q \\ & \neg p \vee r \\ \hline \therefore & q \vee r \end{array}$$

Let us assume:

p: Logic is difficult
q: Many students like logic
r: Mathematics is easy

We can translate the two given assumptions in mathematical symbols using the above interpretations.

	Step	Reason
1.	$p \vee \neg q$	*Premise*
2.	$r \to \neg p$	*Premise*
3.	$\neg r \vee \neg p$	Logical equivalence (I) from (2)
4.	$\neg(r \wedge p)$	De Morgan's law from (3)
5.	$\neg p \vee \neg r$	Commutative law from (3)
6.	$p \to \neg r$	Logical equivalence (I) from (5)
7.	$\neg q \vee \neg r$	Resolution from (1) and (3)
8.	$q \to \neg r$	Logical equivalence (I) from (7)
9.	$\neg r \vee \neg q$	Commutative law from (7)
10.	$r \to \neg q$	Logical equivalence (I) from (9)
11.	$(p \vee \neg q) \wedge (\neg r \vee \neg q)$	Conjunction from (1) and (9)
12.	$\neg q \vee (p \wedge \neg r)$	De Morgan's law from (11)
13.	$q \to (p \wedge \neg r)$	Logical equivalence (I) from (12)

(a) "Mathematics is not easy, if many students like logic" can be represented mathematically as $q \to \neg r$. We note that the proposition $q \to \neg r$ is mentioned in step (8), thus the conclusion is valid.

(b) "Not many students like logic, if mathematics is not easy" can be represented mathematically as $\neg r \to \neg q$. We note that the proposition $r \to \neg q$ is mentioned in step (10) and $r \to \neg q$ is not logically equivalent with $\neg r \to \neg q$, thus the conclusion is invalid.

(c) "Mathematics is not easy or logic is difficult" can be represented mathematically as $\neg r \vee p$. We note that the proposition $\neg r \vee \neg p$ is mentioned in step (3) and $\neg r \vee p$ is not logically equivalent with $\neg r \vee \neg p$, thus the conclusion is invalid.

Therefore, Option A is the answer.

B not many students like logic, if mathematics is not easy.

C mathematics is not easy or logic is difficult.

D None of the above

Q.2) Max Marks: 1

Which one of the following Boolean expressions is NOT a tautology?

A $((a \longrightarrow b) \wedge (b \longrightarrow c)) \longrightarrow (a \longrightarrow c)$

B $(a \leftrightarrow c) \longrightarrow (\sim b \longrightarrow (a \wedge c))$ Correct Option

Solution: (B)

Answer: B

Explanation:

Option A:

$((a \to b) \wedge (b \to c)) \to (a \to c)$

$\equiv ((\sim a \vee b) \wedge (\sim b \vee c)) \to (\sim a \vee c)$

$\equiv \sim ((\sim a \vee b) \wedge (\sim b \vee c)) \vee (\sim a \vee c)$

$\equiv ((a \wedge \sim b) \vee (b \wedge \sim c)) \vee (\sim a \vee c)$

$\equiv (\sim a \vee (a \wedge \sim b)) \vee ((b \wedge \sim c) \vee c)$

$\equiv ((\sim a \vee a) \wedge (\sim a \vee \sim b)) \vee ((b \vee c) \wedge (\sim c \vee c))$

$\equiv (T \wedge (\sim a \vee \sim b)) \vee ((b \vee c) \wedge T)$

$\equiv \sim a \vee (\sim b \vee b) \vee c$

$\equiv \sim a \vee T \vee c$

$\equiv T$

Option B:

$(a \leftrightarrow c) \to (\sim b \to (a \wedge c))$

$\equiv ((a \to c) \wedge (c \to a)) \to ((\sim b \to (a \wedge c))$

$\equiv \sim ((\sim a \vee c) \wedge (\sim c \vee a)) \vee ((b \vee (a \wedge c))$

$\equiv \sim ((a \wedge c)(\sim a \wedge \sim c)) \vee ((b \vee (a \wedge c))$

$\equiv ((a \wedge \sim c) \vee (\sim a \wedge c)) \vee ((b \vee (a \wedge c))$

$\equiv ((a \wedge \sim c) \vee c(\sim a \wedge a)) \vee b$

$\equiv ((a \wedge \sim c) \vee c \vee b$

$\equiv a \vee b \vee c$

Option C:

$(a \wedge b \wedge c) \to (c \vee a)$

$\equiv \sim (a \wedge b \wedge c) \vee (c \vee a)$

$\equiv \sim a \sim b \sim c \vee c \vee a$

$\equiv (a \vee \sim a) \vee \sim b \vee (\sim c \vee c)$

$\equiv T \vee \sim b \vee T$

$\equiv T$

Option D:

$a \to (b \to a)$

$\equiv \sim a \vee (\sim b \vee a)$

$\equiv (\sim a \vee a) \vee \sim b$

$\equiv T \vee \sim b$

$\equiv T$

Hence, Option (B) is the correct choice.

C $(a \wedge b \wedge c) \longrightarrow (c \vee a)$

D $a \longrightarrow (b \longrightarrow a)$

Q.3) Max Marks: 1

Let P and Q be two propositions

$\sim (p \leftrightarrow Q)$ is equivalent to :

(1) $P \leftrightarrow \sim Q$

(2) $\sim P \leftrightarrow Q$

(3) $\sim P \leftrightarrow \sim Q$

(4) $Q \to P$

A **1 and 2** Correct Option

Solution: (A)
Answer: A

Explanation:

P	Q	$\sim p$	$\sim Q$	$Q \to P$	$\sim (P \leftrightarrow Q)$	$P \leftrightarrow \sim Q$	$\sim P \leftrightarrow Q$	$\sim P \leftrightarrow \sim Q$
0	0	1	1	1	0	0	0	1
0	1	1	0	0	1	1	1	0
1	0	0	1	1	1	1	1	0
1	1	0	0	1	0	0	0	1

B
1 only

C
2 only

D
None of the above

Q.4)

Max Marks: 1

The notation "⇒" denotes "implies" and "∧" denotes "and" in the following formulae.

Let X denote the formula:
$(b \Rightarrow a) \Rightarrow (a \Rightarrow b)$

Let Y denote the formula:
$(a \Rightarrow b) \land b$

Which of the following is TRUE ?

A
X is satisfiable and Y is not satisfiable.

B
X is satisfiable and Y is tautology.

C
X is not tautology and Y is not satisfiable.

D
X is not tautology and Y is satisfiable.

Correct Option

Solution: (D)
Since we only have to deal with 2 variables (a and b), solving this with truth table is feasible.

Truth table for X will be:

a	b	$b \Rightarrow a$	$a \Rightarrow b$	$(b \Rightarrow a) \Rightarrow (a \Rightarrow b)$
0	0	1	1	1
0	1	0	1	1
1	0	1	0	0
1	1	1	1	1

So, clearly, X is satisfiable but not a tautology.

Truth table for Y will be:

a	b	$a \Rightarrow b$	$(a \Rightarrow b) \land b$
0	0	1	0
0	1	1	1
1	0	0	0
1	1	1	1

So, Y is also satisfiable but not a tautology.

This gives the **OPTION (D)** as the correct answer.

Q.5)

Max Marks: 1

Given hypotheses:

H1: “If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on,”

H2 : “If the sailing race is held, then the trophy will be awarded,”

H3 : “The trophy was not awarded”

Which conclusion is correct?

A It didn't rain

B It rained **Correct Option**

Solution: (B)

r: "It rains" f: "It is foggy" s: "The sailing race will be held" d: "The lifesaving demonstration goes on" t: "The trophy will be awarded."	Assign statements to propositional variables
1. (¬r ∨ ¬f) → (s ∧ d)	Premise 1
2. s → t	Premise 2
3. ¬t	Premise 3
4. ¬s	Modus Tollens with Premise 2 and Premise 3
5. ¬(s ∧ d) → ¬(¬r ∨ ¬f)	Use equivalency p → q ≡ ¬q → ¬p with Premise 1
6. (¬s ∨ ¬d) → (r ∧ f)	De Morgan's Laws with 5
7. (¬s ∨ ¬d)	Addition with 4
8. (r ∧ f)	Modus Ponens with 6 and 7
9. r	Simplifiation of 8
Thus, it rained.	

C It was not foggy.

D None of the above

Q.6) **Max Marks: 1**

Given Statement :

You cannot edit a protected Wikipedia entry unless you are an administrator.

Express your answer in terms of -

e: "You can edit a protected Wikipedia entry"

a: "You are an administrator."

Which of the following translation is correct?

A a —> e

B e —> a **Correct Option**

Solution: (B)
Given:

e: "You can edit a protected Wikipedia entry"
a: "You are an administrator"

INTERPRETATION SYMBOLS

Negation $\neg p$: not p

Disjunction $p \vee q$: p or q

Conjunction $p \wedge q$: p and q

Conditional statement $p \rightarrow q$: if p, then q

Biconditional statement $p \leftrightarrow q$: p if and only if q

"q unless $\neg p$" can be rewritten as "if p, then q".

"You cannot edit a protected Wikipedia entry unless you are an administrator" can then be rewritten as "if not a, then not e", or rewritten using the above symbols:

$$\neg a \rightarrow \neg e$$

Note that, ~a —> ~e is logically equivalent to e —> a.

C ~a —> e

D ~e —> a

Q.7)

Max Marks: 1

Give the first order predicate calculus of the following statement:
"Some boys like every girl"

A ∀(x) [girl(x) -> ∃ (y) [boy(y) ∧ likes(y,x)]]

B ∀(x) [girl(x) ∧ ∃ (y) [boy(y) -> likes(y,x)]

C ∀(x) [girl(x) -> ∃ (y) [boy(y) -> likes(y,x)]]

D **None of these** Correct Option

Solution: (D)

Answer: D

Explanation:
A - Every girl is liked by some boy.

B - Everything is a girl, and is either liked by some boy, or something is not a boy.

C - Every girl is liked by some boy, or there is something which is not a boy.

Thus, D is the answer

Q.8)

Max Marks: 1

Which of the following is a valid first order formula?

A $(\exists y)(\forall x)P(x,y) \rightarrow (\forall x)(\exists y)P(x,y)$

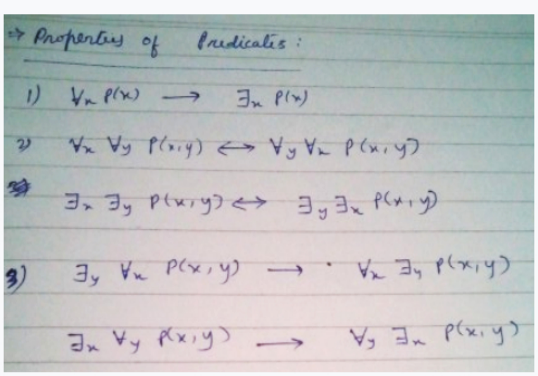
B $(\forall y)(\exists x)P(x,y) \rightarrow (\exists x)(\exists y)P(x,y)$

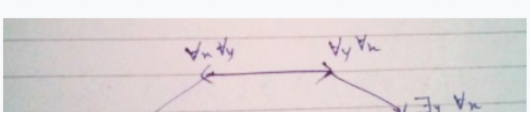
C $(\exists x)(\exists y)P(x,y) \leftrightarrow (\exists y)(\exists x)P(x,y)$

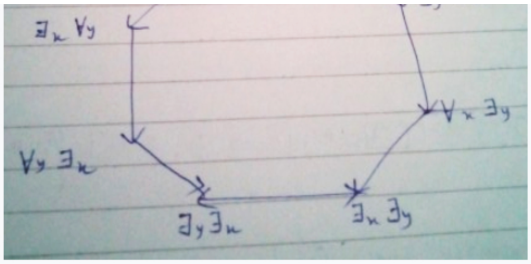
D **All of the above** Correct Option

Solution: (D)
Answer: D

Explanation: All are correct. So Option D should be the correct option.







Q.9)

Max Marks: 1

Which of the following translation of the given statement into propositional logic using the propositions provided is correct ?

Statement: You can see the movie only if you are over 18 years old or you have the permission of a parent.

Express your answer in terms of :

m: “You can see the movie,”

e: “You are over 18 years old,” and

p: “You have the permission of a parent.”

A (e V p) --> m

B e V (p --> m)

C m --> (e V p) Correct Option

Solution: (C)

"p only if q" can be rewritten as "if p, then q".

"You can see the movie only if you are over 18 years old or you have the permission of a parent" can then be rewritten as "if m, then (e or p)", or rewritten using the above symbols:

$$m \rightarrow (e \vee p)$$

D (m --> e) V p

Q.10)

Max Marks: 1

Assume the following predicate and constant symbols:
W(x,y): x wrote y
L(x,y): x is longer than y
N(x): x is a novel
a: Amit h: Harshal
Which of the following predicate logic formula represents the sentence:

"Harshal wrote a novel which is longer than any of the Amit's novels"

A $\forall x\ \exists y\ (L(x,y) \longrightarrow W(x,y) \wedge W(a,x))$

B $\forall x\ \forall y\ (W(h,x) \wedge W(a,y) \implies L(x,y))$

C $\exists x\ \forall y\ (N(x) \wedge W(h,x) \implies N(y) \wedge W(a,y) \wedge L(x,y))$

D $\exists x\ (N(x) \wedge W(h,x) \wedge \forall y\ (N(y) \wedge W(a,y) \implies L(x,y)))$ Correct Option

Solution: (D)

Answer: D

Explanation:
(A) For every book if there exists a shorter book, then harshal has written the shorter one and amit the longer one.

(B) Every book written by Harshal is longer than every book written by Amit.

(C) There exists an x such that if x is a novel written by Harshal, then all novels written by Amit are shorter than x.

(D) There exists an x such that x is a novel written by Harshal and all novels written by Amit are shorter than x.

So, (D) is the answer

Q.11)

Max Marks: 2

Which of the following is principal conjunctive normal form for $[(p \vee q) \wedge \sim p \rightarrow \sim q]$?

A p V ~q

Correct Option

Solution: (A)

Answer: A

Explanation:
Conjunctive Normal Form of a given well formed formulae (wff) is an equivalent wff consisting of product of elementary sum terms.

Principal Conjunctive Normal Form of a given wff is an equivalent wff consisting of product of sums where each sum term consists of all variables used in the formulae in negated or non-negated form.

B p V q

C ~p V q

D ~p V ~q

Q.12)

Max Marks: 2

Consider the following propositions:
p: n is a prime number
q: n mod 30 is prime number
Which of the following implications is true?

A **p implies q**

B **q implies p**

C **p implies q or q implies p**

Correct Option

Solution: (C)
Answer: C
Explanation:
p: n is a prime number
q: n mod 30 is a prime number

Option A -
$p \rightarrow q$
Statement - If n is a prime number then n mod 30 is a prime number
`Counter Example` - let n = 31
If 31 is a prime number then 31 mod 30 is a prime number
31 mod 30 = 1
1 is not a prime number.
$p \rightarrow q = T \rightarrow F$ = **F**

Options B and D are same.
B is $q \rightarrow p$
Option D-
$(p \rightarrow q) \rightarrow (q \rightarrow p)$
$= \neg(\neg p \vee q) \vee (\neg q \vee p)$
$=(p \wedge \neg q) \vee (\neg q \vee p)$
$=((p \wedge \neg q) \vee \neg q) \vee ((p \wedge \neg q) \vee p)$
$=\neg q \vee p$
$= q \rightarrow p$
Statement - If n mod 30 is a prime number then is a prime number
`Counter-example` for options B and D -
Take n = 32
n mod 30 = 2 which is a prime number but n is not a prime number.
$q \rightarrow p = T \rightarrow F$ = F

Option C -
$(p \rightarrow q) \vee (q \rightarrow p)$
Now see.
If $p \rightarrow q$ is false then $q \rightarrow p$ must be true and vice versa.
$(p \rightarrow q) \vee (q \rightarrow p)$
$(\neg p \vee q) \vee (\neg q \vee p)$
T
`It's a tautology so it will always be true irrespective the value of n.`

D **If p implies q then q implies p**

Q.13)

Max Marks: 2

The formula $\{(p \to \sim q) \wedge (r \to q) \wedge r\} \to p$ is

A A tautology

B A contingency — Correct Option

Solution: (B)

Answer: B

Explanation:

By using Truth table. $z = [(p \to \sim q) \wedge (r \to q)]$

p	q	r	~q	p → ~q	r → q	z	z ∧ r	(z ∧ r) → p
T	T	T	F	F	T	F	F	T
T	T	F	F	F	T	F	F	T
T	F	T	T	T	F	F	F	T
T	F	F	T	T	T	T	F	T
F	T	T	F	T	T	T	T	F
F	T	F	F	T	T	T	F	T
F	F	T	T	T	F	F	F	T
F	F	F	T	T	T	T	F	T

From the last column, it's neither a tautology nor a contradiction, it's contingency.

C A contradiction

D None of these

Q.14)

Max Marks: 2

Determine which of the following is/are valid argument(s):

1. If x is a positive real number, then x^2 is a positive real number. Therefore, if a^2 is positive, where a is a real number, then a is a positive real number.

2. If $x^2 \neq 0$, where x is a real number, then $x \neq 0$. Let a be a real number with $a^2 \neq 0$; then $a \neq 0$.

A Only 1

B Only 2 — Correct Option

Solution: (B)

INTERPRETATION SYMBOLS

Negation $\neg p$: not p

Disjunction $p \vee q$: p or q

Conjunction $p \wedge q$: p and q

Conditional statement $p \to q$: if p, then q

Biconditional statement $p \leftrightarrow q$: p if and only if q

Existential quantification $\exists x P(x)$: There exists an element x in the domain such that $P(x)$.

Universal quantification $\forall x P(x)$: $P(x)$ for all values of x in the domain.

RULES OF INFERENCE

Modus ponens

$$\begin{array}{rl} & p \\ & p \to q \\ \hline \therefore & q \end{array}$$

Modus tollens

$$\begin{array}{rl} & \neg q \\ & p \to q \\ \hline \therefore & \neg p \end{array}$$

Conjunction:

$$\begin{array}{rl} & p \\ & q \\ \hline \therefore & p \wedge q \end{array}$$

Universal instantiation

$$\begin{array}{rl} & \forall x P(x) \\ \hline \therefore & P(c) \end{array}$$

Existential instantiation

$$\exists x P(x)$$

$$\therefore \frac{\exists x P(x)}{P(c) \text{ for some c}}$$

Existential generalization

$$\therefore \frac{P(c)}{\exists x P(x)}$$

(a) Let us assume:

$P(x) =$ "x is a positive real number"
$Q(x) =$ "x^2 is a positive real number"

We can then rewrite the given statements using the above interpretations.

	Step	Reason
1.	$\forall x(P(x) \to Q(x))$	*Premise*
2.	$P(a) \to Q(a)$	Universal instantiation from (1)

We cannot derive $Q(a) \to P(a)$ from $P(a) \to Q(a)$ by any rule of inference and thus the argument is invalid.

For example, let $a = -1$, then we obtain that $a^2 = (-1)^2 = 1$ is positive, while $a = -1$ is not positive and thus the statement "If a^2 is positive, then a is a positive real number" cannot be valid.

(b) Let us assume:

$P(x) =$ "$x^2 \neq 0$"
$Q(x) =$ "$x \neq 0$"

We can then rewrite the given statements using the above interpretations.

	Step	Reason
1.	$\forall x(P(x) \to Q(x))$	*Premise*
2.	$P(a) \to Q(a)$	Universal instantiation from (1)

The statement in step (2) means that if $a^2 \neq 0$, then $a \neq 0$ and thus the argument is valid.

C Both 1 and 2

D None of the above

Q.15) Max Marks: 2

If the proposition 'All thieves are poor' is false, which of the following propositions can be claimed certainly to be true?

A Some thieves are poor

B Some thieves are not poor — Correct Option

Solution: (B)

It can be easily proved.

Let $T(x)$ denotes x is a thief and $P(x)$ denotes x is poor, then `All thieves are poor` is $\forall x(P(x) \to T(x))$

Since given statement is false, take the negation

$\neg(\forall x(P(x) \to T(x))) = \exists x(\neg(P(x) \to T(x)))$ // Rewrite $A \to B$ as $A' \vee B$

$= \exists x(\neg(P(x)' \vee T(x)))$

$= \exists x(P(x) \wedge T(x)')$

Which means "`there are some thieves who are not poor`"

C No thief is poor

D No poor person is a thief

close