



SOME IMPORTANT TRICKS USEFUL IN DATA INTERPRETATION

- If the data are given in text form, converting the data first into a simple table saves a lot of time and is easy to refer to in a shortage of time.
- If the quantity of the item is not mentioned in bar graphs or line graphs, etc., label the quantity beforehand so that you don't look back again and again at both axes.
- Use approximation if options are not close. ex: in question, you do not need to calculate $\frac{11}{9}$ first and then multiply it by 100 or in any other longer way. You can take an approximation of $\frac{100}{9}$ as 11.1 or 11, and thus

the answer can be calculated in one step as 11^2 .

Or,

In question $\frac{15600}{9400}$ could be approximated

$$\text{as } \frac{15000}{9000} \times 100 = \frac{10}{6} \times 100$$

Next, you know $\frac{1}{6}$ is 0.166%. Thus, $\frac{1000}{6} =$

166.6%, and $\frac{15600}{9400}$ will be near to 166.6%;

hence, one could ultimately choose 165.9% from the options.

- Write the calculated values at a separate place so that if any question refers to them again, you do not have to spend time again in calculating.

Chapter Summary



- If data are represented in degrees:

$$\text{Value of any sector} = \frac{\text{Angle of any sector}}{360^\circ} \times \text{Total Value}$$

- If data are represented in percentage:

$$\text{Value of any sector} = \frac{\text{Percent of any sector}}{360^\circ} \times \text{Total Value}$$

- Growth Rate = $\frac{B - A}{A} \times 100$

- Average = $\frac{\text{Sum of units}}{\text{Number of years}}$

PRACTICE QUESTIONS

Data: The table lists the share of students (in per cent) per total population who had at least one smartphone available at home and the share of students (in per cent) who bought a new smartphone after 2020 in the ASER survey of

the year 2020 and 2021, categorised according to the level of education of their parents. Given that the total population of India is 1.5 billion.

PARENTS' EDUCATION	TOTAL FAMILIES	ASER 2020		ASER 2021	
		At least one smartphone at home	Bought a new smartphone for children's education since March 2020	At least one smartphone at home	Bought a new smartphone for children's education since March 2020
Low	50 billion	45	5	50	25
Medium	30 billion	60	10	65	25
High	0.5 billion	80	10	80	30
Nil	20 billion	60	10	65	25

Source: The Hindu Data Point

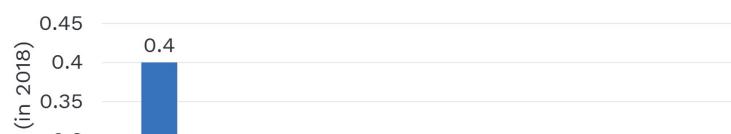
1. The total of people who had at least one smartphone at home in the 2020 survey is what per cent of total people who bought a new phone after march 2020 in the 2021 survey? (use approximate values)
 - A. 190%
 - B. 210%
 - C. 250%
 - D. 175%
2. The number of nil parental education people who bought a new phone after march 2020 in the 2021 survey are what per cent less than the same in 2020 survey?
 - A. 60%
 - B. 50%
 - C. 80%
 - D. 30%
3. What is the respective ratio of people who had at least one smartphone at home whose parents had low income in 2020 and people who didn't have even one smartphone at home in a high parental education home in the same year survey?
 - A. 1 : 225
 - B. 150 : 4
 - C. 225 : 1
 - D. 175 : 2
4. What is the difference between the total number of people who had at least one smartphone in 2021 and people who bought a new phone after march 2020 in a medium parental education home in the 2020 survey?
 - A. 56.7 billion
 - B. 48.5 billion
 - C. 52 billion
 - D. 54.9 billion
5. What is the difference between the total number of people who didn't even have at least one smartphone in 2020 and that in 2021 in a nil education family?
 - A. 1 billion
 - B. 1.5 billion
 - C. 3.5 billion
 - D. 2 billion

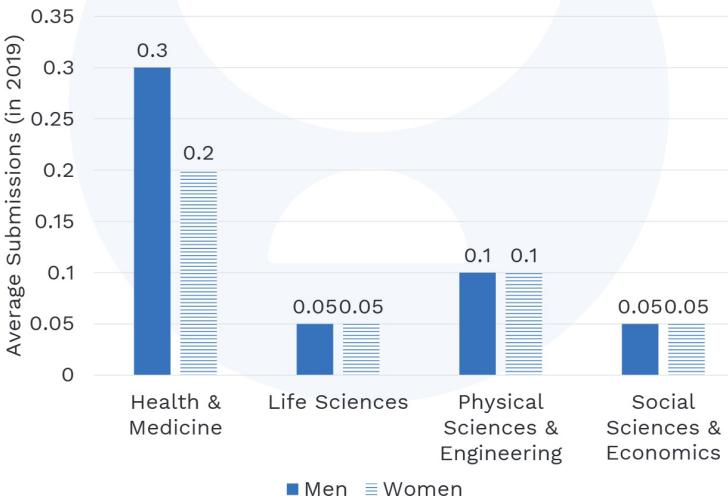
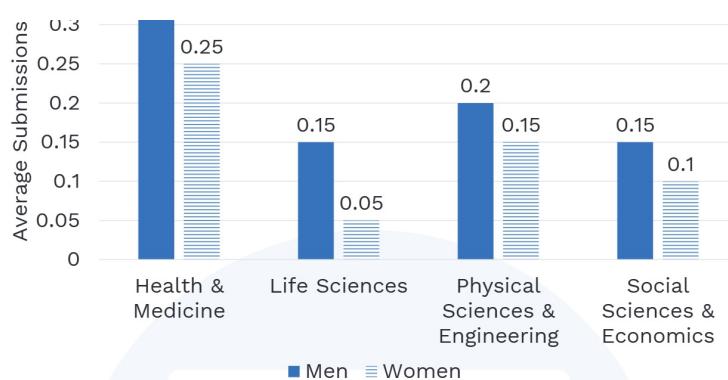
Data Interpretation

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Data: The chart shows the average change in scholarly article submissions of men compared to the average number of submissions of women in 2018 and 2019 across various research areas.

Source: The Hindu Data Point





6. What is the ratio of the number of times when average submissions of women in any research area are higher/equal than men in the same to the number of times when average submissions of women in any research area are lower than men in both years?
- 2 : 7
 - 3 : 4
 - 3 : 5
 - 4 : 3
7. What is the difference between the total average submissions of men and women in all the research areas in 2018?
- 0.45
 - 0.35
 - 0.50
 - 1.25

Data Interpretation

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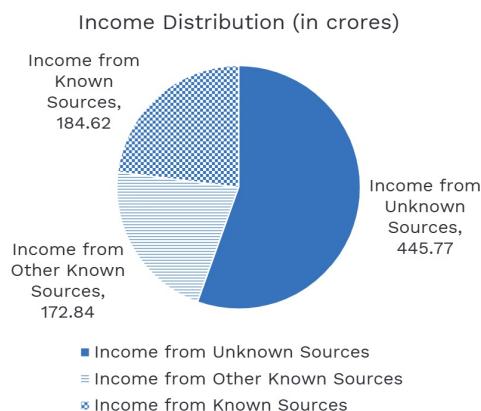
8. How many times did women have at least 50% of the average submissions of men in respective research areas?
- 4 times
 - 3 times
 - 7 times
 - 9 times
9. Sum Average submission of men in life science area in both the years is approximately what per cent of sum Average submission of men in social science and economics area in both years?
- 100%
 - 50%
 - 75%
 - 25%

PARTY	UNKNOWN INCOME (IN %)	TOTAL INCOME (IN CRORE)
ABC	25	10
BCD	45	15
CDE	55	20
DEF	90	20
EFG	20	50
FGH	70	60
GHI	55	90

- 10.** The area in which the percentage decrease of total average submission of men and women in 2019 was maximum from its preceding year?
- health and medicine
 - life science
 - physical sciences and engineering
 - social sciences and economics

Data: The pie chart shows the sources of income of select regional parties in FY20. The total income for the selected regional parties in FY20 amounted to ₹803.24 crores, of which 55% was from unknown sources. The table depicts the total income of select regional parties in FY20 against the share from unknown sources.

Source: The Hindu Data Point



HIJ	10	90
IJK	80	90
JKL	90	90
KLM	40	110
LMN	70	130

- 11.** The unknown income of ABC is less than the unknown income of GHU by (approximately)
- 80%
 - 95%
 - 85%
 - 70%
- 12.** What is the difference between the unknown income and the rest income of IJK?
- 54 Crores
 - 56 Crores
 - 52 Crores
 - 50 Crores
- 13.** The total income from other known sources is what per cent of total income from known sources? (approximately)
- 85%
 - 69%
 - 93%
 - 75%

Data Interpretation

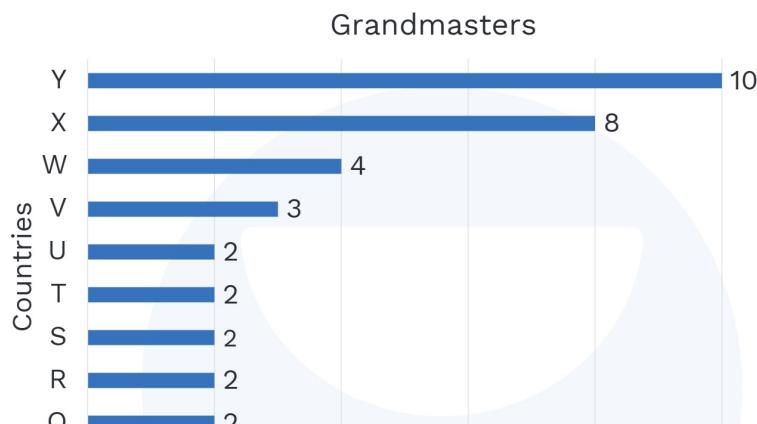
235

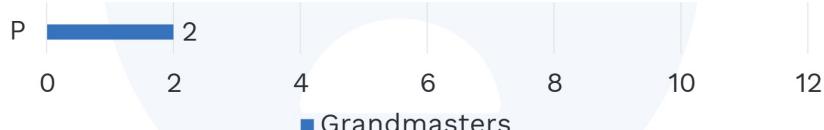
- 14.** What is the total income from unknown sources of KLM, ABC, and GHU together?
- 89
 - 98
 - 96
 - 81

- 15.** What is the difference between the income which is not unknown of LMN and that of JKL?
- 10 Crores
 - 20 Crores
 - 30 Crores
 - 40 Crores

Data: The Bar chart shows the number of Chess Grandmasters from different countries born after 2000.

Source: The Hindu Data Point





- 16.** How many times are the number of grandmasters from Y than the number of grandmasters from P?
- 2
 - 3
 - 4
 - 5
- 17.** The total grandmasters from T and V are what per cent of total grandmasters from the X and Q?
- 75%
 - 25%
 - 70%
 - 50%
- 18.** What is the per cent difference in Grandmasters from W than that from S?
- 75%
 - 100%
 - 50%
 - 25%
- 19.** What is the difference in the average grandmasters from P, Q, R, S and T taken together to the average of U, V, W, X, and Y?
- 35.5
 - 25.5
 - 20.5
 - 18.5
- 20.** If there are 23 grandmasters from P who were born before 2000, then what is the percentage difference in the number of grandmasters born after 2000 than the number of grandmasters born before 2000 in P? (approximately)
- 97%
 - 80%
 - 91%
 - 89%

Data Interpretation

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Data: 1.7 million migrants tried to illegally enter the US between FY 21. Of which, 0.15 million were unaccompanied minors. And 2500 of the 1.7 million were Indians. The line graph represents the share of individuals who were apprehended more than once by the Border Patrol out of total immigrants.

Source: The Hindu Data Point



- 21.** Which year has the highest per cent difference in repeated apprehenders as compared to the preceding year?
- FY 16
 - FY 21
 - FY 20
 - FY 17
- 22.** What is the difference in people who
- 9%
 - 9.8%
- 23.** If the number of repeated apprehenders in 2017 who were Indian were 2% of the total immigrants of the same year and were 100 less than in 2021, then what is the total number of illegal immigrants in FY 17?
- 0.13 million

crossed the US border in FY 19 and FY 21, its total immigrants in FY 19 were 90% of total immigrants in FY 21?

- A. 0.17 million
- B. 0.15 million
- C. 0.19 million
- D. 0.13 million

23. Unaccompanied minors are what per cent of illegal immigrants in FY 21?

- A. 10%
- B. 8.8%

- B. 0.14 million
- C. 0.12 million
- D. 0.11 million

25. For how many years, repeated apprehender immigrants per cent were more than the average per cent overall years?

- A. 2 years
- B. 3 years
- C. 4 years
- D. 5 years

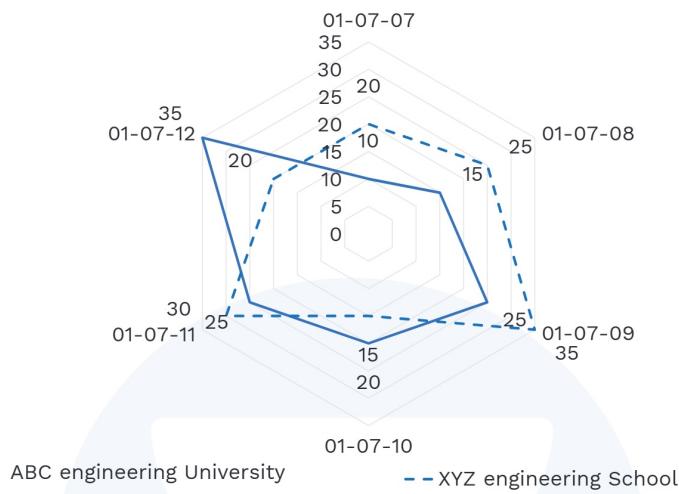
Data Interpretation

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Data: The radar graph below represents the number of students(in hundred) in ABC engineering university and XYZ engineering school in the starting of six different academic years.

Source: The Hindu Data Point



26. What was the difference between the number of students in ABC engineering school in the year 2010 and the number of students in XYZ engineering school in the year 2012?

- A. 500
- B. 1200
- C. 700
- D. 1000

- A. 1000
- B. 1100
- C. 1200
- D. 1300

27. What is the sum of the number of students in ABC engineering university in the year 2007 and the number of students in XYZ engineering school in the year 2011 together?

- A. 2500
- B. 2800
- C. 3500

29. What was the per cent increase in the number of students in ABC engineering university in the year 2011 as compared to the previous year?

- A. 33.3%
- B. 14.28%
- C. 75%
- D. 66.6%

30. In which year was the difference between the number of students in ABC engineering university and XYZ highest?

- A. 2008
- B. 2007

D. 3000

- 28.** If 20% of the students in XYZ in the year 2010 were females, what was the number of male students in XYZ in the same year?

C. 2010

D. 2012

Data: The table shows the average spending on drugs per patient for both forms of treatment in the 10 high TB-burden countries in 2020.

Source: The Hindu Data Point

COUNTRY	FIRST-LINE TREATMENT (IN DOLLAR)	DRUG- RESISTANT TB TREATMENT (IN DOLLAR)
A	93	20287
B	46	2188
C	24	1723
D	45	1600
E	47	1352
F	67	1339
G	45	913
H	26	864
I	63	713
J	33	700

- 31.** Among the given countries, which country spent the total most approximately to the average total spending of all countries?

- A. H
B. C
C. I
D. B

- 32.** What is the difference between the total spending of the top five countries and that of the bottom five countries?

- A. 23423 dollars
B. 22656 dollars
C. 34567 dollars
D. 43513 dollars

- 33.** How much will J spend next year if its total spending next year increases by 13%?

- A. 834.2 dollars
B. 849.6 dollars
C. 857.5 dollars
D. 828.2 dollars

- 34.** How much is the total spending of H more than that of C?

- A. 678 dollars
B. 756 dollars
C. 857 dollars
D. 934 dollars

- 35.** Which of the following countries has the lowest difference in respective spending over both types of treatment?

- A. J
B. I
C. H
D. B



SOLUTIONS

- 1.** **(B)** People who had at least one smartphone in 2020

$$\begin{aligned}
 &= \frac{45}{100} \times 50 + \frac{60}{100} \times 30 + \frac{80}{100} \times 0.5 + \frac{60}{100} + 20 \\
 &= 22.5 + 18 + 0.4 + 12 = 53 \text{ billion (approx.)}
 \end{aligned}$$

People who bought a new smartphone in 2021

$$\begin{aligned}
 &= \frac{25}{100} \times 50 + \frac{25}{100} \times 30 + \frac{30}{100} \times 0.5 + \frac{25}{100} \times 20 \\
 &= 12.5 + 7.5 + 0.15 + 5 = 25 \text{ billion (approx.)}
 \end{aligned}$$



$$\text{Required percentage} = \frac{53}{25} \times 100 = 210\%$$

2. (A) People who bought a new phone after march 2020 in 2021 = $\frac{10}{100} \times 20 = 2 \text{ billion}$

$$\text{People who bought a new phone after march 2020 in 2020} = \frac{25}{100} \times 20 = 5 \text{ billion}$$

$$\text{Required percentage} = \frac{5 - 2}{5} \times 100 = 60\%$$

3. (C) People who had at least one smartphone at home whose parents had low income in 2020 = $\frac{45}{100} \times 50 = \frac{45}{2} \text{ billion}$

$$\text{People who didn't have even one smartphone at home in high parental education home in 2020} = \frac{20}{100} \times 0.5 = \frac{1}{10} \text{ billion}$$

$$\text{Ratio} = \frac{45}{2} : \frac{1}{10} = \frac{225}{1} = 225:1$$

4. (D) Total number of people who had at least one smartphone in 2021

$$= \frac{50}{100} \times 50 + \frac{65}{100} \times 30 + \frac{80}{100} \times 0.5 + \frac{65}{100} \times 20$$

$$= 25+19.5+0.4+13 = 57.9 \text{ billion}$$

People who bought a new phone after march 2020 in medium parental education

$$\text{home} = \frac{10}{100} \times 30 = 3 \text{ billion}$$

$$\text{Required difference} = 57.9 - 3 = 54.9 \text{ billion}$$

5. (A) Total number of people who didn't even had at least one smartphone in 2020 =

$$\frac{40}{100} \times 20 = 8 \text{ billion}$$

Total number of people who didn't even had at least one smartphone in 2021 =

$$\frac{35}{100} \times 20 = 7 \text{ billion}$$

$$\text{Required difference} = 8 - 7 = 1 \text{ billion}$$

6. (C) Number of times when average submissions of women in any research area are higher than men = 3

Number of times when average submissions of women in any research area are lower than men = 5

$$\text{Required ratio} = 3:5$$

7. (B) Total average submissions of men = 0.4

$$+ 0.15 + 0.2 + 0.15 = 0.9$$

$$\text{Total average submissions of women} = 0.25$$

$$+ 0.05 + 0.15 + 0.1 = 0.55$$

$$\text{Required difference} = 0.9 - 0.55 = 0.35$$

8. (C) Self-explanatory. 7 times

9. (A) Sum average submission of men in life science area in both years = $0.15 + 0.05 = 2$

Sum average submission of men in social science and economics area in both years = $0.15 + 0.05 = 2$

$$\text{Required per cent} = \frac{2}{2} \times 100 = 100\%$$

10. (D) Percentage decrease of total average submission of men and women than its preceding year in one area

Total submission in 2019-

$$= \frac{\text{Total submission in 2018}}{\text{Total submission in 2018}} \times 100$$

$$\text{Percentage change in health and medicine area} = \frac{0.5 - 0.65}{0.65} \times 100 = 23\% \text{ decrease}$$

$$\text{Percentage change in life science area} = \frac{0.1 - 0.2}{0.2} \times 100 = 50\% \text{ decrease}$$

$$\text{Percentage change in physical sciences and engineering area} = \frac{0.2 - 0.35}{0.35} \times 100 = 42\% \text{ decrease}$$

$$\text{Percentage change in social sciences and economics area} = \frac{0.1 - 0.25}{0.25} \times 100 = 60\% \text{ decrease}$$

Thus, the maximum percentage decrease is in social sciences and economics.

11. (B) Unknown income of ABC = $\frac{25}{100} \times 10 = 2.5 \text{ crores approx.}$

$$\text{Unknown income of GHU} = \frac{55}{100} \times 90 = 50 \text{ crores approx.}$$

$$\text{Required per cent} = \frac{\text{Required value}}{\text{Total value}} \times 100 = 95\%$$

- 12. (A)** Unknown income of IJK = $\frac{80}{100} \times 90 = 72$ crores

$$\text{Rest income of IJK} = \frac{20}{100} \times 90 = 18 \text{ crores}$$

$$\text{Required difference} = 72 - 18 = 54 \text{ crores}$$

- 13. (C)** Income from other known sources = 172 crores

$$\text{Income from known sources} = 184 \text{ crores}$$

$$\text{Required per cent} = \frac{172}{184} \times 100 = 93\% \text{ approximately}$$

- 14. (C)** Income from unknown sources of KLM = $\frac{40}{100} \times 110 = 44 \text{ crores}$

$$\text{Income from unknown sources of ABC} = \frac{25}{100} \times 10 = 2.5 \text{ crores}$$

$$\text{Income from unknown sources of GHU} = \frac{55}{100} \times 90 = 49.5 \text{ crores}$$

$$\text{Total} = 44 + 2.5 + 49.5 = 96$$

- 15. (C)** Income that is not unknown of LMN = $\frac{30}{100} \times 130 = 39 \text{ crores}$

$$\text{Income which is not unknown of JKL} = \frac{10}{100} \times 90 = 9 \text{ crores}$$

$$\text{Required difference} = 30 \text{ crores}$$

- 16. (D)** Grandmasters from Y = 10
Grandmasters from P = 2
Self-explanatory 5 times.

- 17. (D)** Total grand masters from T and V = 2 + 3 = 5

$$\text{Total grand masters from X and Q} = 8 + 2 = 10$$

$$\text{Required per cent} = \frac{5}{10} \times 100 = 50\%$$

- 18. (B)** Grand masters from W = 4
Grand masters from S = 2

$$\text{Required per cent difference} = \frac{\text{Required value}}{2} \times 100$$

$$= 100\%$$

- 19. (B)** Average grandmasters from P, Q, R, S, and T = 2

$$\text{Average of U, V, W, X, and Y} = 27.5$$

$$\text{Required difference} = 25.5$$

- 20. (B)** Grandmasters from P who were born before 2000 = 23

$$\text{Number of grandmasters born after 2000 in P} = 2$$

$$\text{Required difference per cent} = \frac{21}{13} \times 100 = 91\%$$

- 21. (C)** Per cent difference in repeated apprehenders as compared to the preceding year:

$$\text{FY 16} = \frac{12 - 14}{14} \times 100 = 0.14\%$$

$$\text{FY 17} = \frac{10 - 12}{12} \times 100 = 16.6\%$$

$$\text{FY 18} = \frac{11 - 10}{10} \times 100 = 10\%$$

$$\text{FY 19} = \frac{6 - 11}{11} \times 100 = 45.4\%$$

$$\text{FY 20} = \frac{26 - 6}{6} \times 100 = 333.3\%$$

$$\text{FY 21} = \frac{27 - 26}{26} \times 100 = 3.8\%$$

Thus, highest in FY 20

- 22. (A)** People who crossed US border in the

$$\text{FY 19} = \frac{90}{100} \times 1.7 = 1.53 \text{ million}$$

People who crossed US border in the FY 21 = 1.7 million

Required difference = 0.17 million

- 23. (B)** Required per cent = $\frac{0.15}{1.7} \times 100 = 8.8\%$.

- 24. (C)** Number of Indian repeated apprehenders in 2017 = 2500 - 100

2400 = total immigrants of the same year $\times \frac{2}{100}$

Total immigrants of the same year = 0.12 million

Data Interpretation

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- 25. (A)** Average per cent overall years = $(14 + 12 + 10 + 11 + 6 + 26 + 27)/7 = \frac{106}{7} = 15\%$ approximately

Thus, repeated apprehender immigrants per cent were more than the average per cent overall years for 2 FY.

- 26. (A)** The number of students in ABC engineering school in the year 2010 = 2000

- 31. (D)** Average spending of all countries = $(20380 + 2234 + 1747 + 1645 + 1399 + 1406 + 958 + 890 + 958 + 890 + 776 + 733)/10 = 3216.8$ dollars

Thus, B spent total most approximate to the average total spending of all countries (i.e., 2234 dollars)

- 32. (B)** Total spending of top five countries = $20380 + 2234 + 1747 + 1645 + 1399 = 27412$

Number of students in XYZ engineering school in the year 2012 = 1500
Difference = 500

27. (C) Self-explanatory

28. (C) Female students in XYZ in the year 2010
 $= \frac{20}{100} \times 1500 = 300$

Male students = $1500 - 300 = 1200$

29. (D) Number of students in ABC engineering university in the year 2011 = 25

Number of students in ABC engineering university in the year 2010 = 15

Per cent increase = $\frac{25 - 15}{15 \times 100} \times 100 = 66.6\%$

30. (D) Year 2007 difference = 10

Year 2008 difference = 10

Year 2009 difference = 10

Year 2010 difference = 5

Year 2011 difference = 5

Year 2012 difference = 15

Thus, highest in 2012.

dollars

Total spending of bottom five countries =
 $1406 + 958 + 890 + 958 + 890 + 776 + 733 = 4756$ dollars

Required difference = 22656 dollars

33. (D) J's spending next year = $\frac{113}{100} \times 733 = 828.2$ dollars

34. (C) Total spending of H = 890 dollars

Total spending of C = 1747 dollars

Required difference = 857 dollars

35. (A) Difference in both spendings:

A = 20194

B = 2142

C = 1699

D = 1555

E = 1305

F = 1292

G = 868

H = 868

I = 838

J = 650

Thus, lowest in J.

29 Number System



SYNOPSIS

- Introduction
- Types of numbers
- Factors
- HCF
- LCM
- Simplification
- Fractions
- Types of fractions
- Surds and indices
- Divisibility rule

INTRODUCTION

Number system basically comprises ten digits, i.e., **0, 1, 2, 3, 4, 5, 6, 7, 8, and 9**. When these digits are combined together, they form a particular number. And when we form a number using these digits, then each digit takes a particular place, and that particular place of the digits we call the *place value* of that particular digit in that particular number. But the value of these

particular digits remains the same wherever they are placed in a number and called as *face value* of that particular number.

Example:

Let us form a number using some of the digits given above, 23789 is an example of a number that we formed using digits 2, 3, 7, 8, and 9.

Now, let us see the place value of these digits in this number, so here, the place value of each number is:

PLACE VALUE	TEN THOUSAND	THOUSANDS	HUNDREDS	TENS	ONES/UNITS
Digit	2	3	7	8	9

We use a number system in our day-to-day lives, for example, while playing cricket we count runs scored by a team, or how many wickets are taken by a team or by a particular bowler and so on.

After understanding the basic concept of the number system now comes the types of numbers in the number system.

TYPE OF NUMBERS

Natural numbers

Natural numbers are all positive counting numbers that begin with 1 and can be counted

indefinitely. Because natural numbers can never be negative, these numbers are also known as positive integers.

Example:

1, 2, 3, 4,... 267,...4999,... ∞ (infinity)

Whole numbers

If we add the number zero to the collection of natural numbers, we get the collection of whole numbers. Whole numbers are counting numbers that can be counted starting from digit 0 to infinity. Whole numbers include natural numbers that begin from 1 onwards.

Number System

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Whole numbers include positive integers along with 0.

Example:

0, 1, 2, 3, 4, 5,...556, 721,... ∞ (infinity)

Even numbers

Even numbers are basically integers, positive integers as well as negative integers, which can be divided exactly or evenly by digit 2. If an integer is exactly divided by 2, it implies that the number in question has a remainder of 0 upon the division of 2.

Example:

2, 4, 6, 8,... 498, 8888,...

Odd numbers

Odd numbers are integers, positive integers as well as negative integers, which cannot be divided exactly or evenly by digit 2. If an integer is not exactly divisible by 2, it implies that the number in question has a remainder of 1 upon the division of 2.

Example:

1, 3, 5, 7, ...379, 689,...

Prime numbers

no common factors between these two numbers.

Example:

5 and 24, 6 and 37, etc.

Composite numbers

Composite numbers are those numbers that have more than 2 factors excluding the number itself. Also, a composite number is a positive integer that can be formed by multiplying two smaller positive integers. Equivalently, it is a positive integer that has at least one divisor other than 1 and itself.

Example:

4, 6, 8, 9, 10,... 656, 7864,...

Integers

From negative infinity to positive infinity, integers are all positive and negative counting numbers. Integers are a type of number that consists of both positive and negative numbers. Integers, like whole numbers, do not include the fractional portion. Integers, on the other hand, are numbers that can be positive, negative or zero but not a fraction.

Zero is also an integer.

Prime numbers are those positive integers that have only two factors, 1 and the number itself. They are not divisible by any other numbers.

Example:

2, 3, 5, 11,...71,...1381,...

Note

- There are only 25 prime numbers up to 100, i.e., 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, and 97.
- 2 is the only even prime number.

Co-prime numbers

Co-prime numbers is basically a set of two numbers that have HCF as 1, i.e., there are

Example:

...-23, -498,... 890, 4567,...

Positive integers

Positive integers are all whole numbers that are larger than zero and do not include fractions or decimals. The positive integers lie on the right side of 0 on a number line.

Example:

1, 2, 3, 4,... 666, 879,...

Negative integers

Negative integers are numbers that have a value less than zero. They do not include fractions or decimals. The negative integers lie on the left side of 0 on a number line.

Example:

...-37, -44, -990, -8769...



Note

- Zero is defined as neither negative nor positive. The ordering of integers is compatible with the algebraic operations in the following way: if $a < b$ and $c < d$, then $a + c < b + d$. If $a < b$ and $0 < c$, then $ac < bc$.

Rational numbers

Rational numbers are those numbers that can be written in the form $\frac{p}{q}$, where $q \neq 0$. Also, rational numbers include all integers, zero, or fractions.

Example:

$\frac{5}{2}, \frac{11}{13}, 0.7777, \frac{-1}{2}$, etc.

Irrational numbers

Irrational numbers are those numbers that cannot be expressed in the form $\frac{p}{q}$.

Example:

$\pi, \sqrt{7}, 0.12122122212222\ldots$

Real numbers

The collection of all rational numbers and irrational numbers together makes up what we call the collection of real numbers. All the concepts that we have discussed above, about different types of numbers, fall under the category of real numbers. So, all of the above-given examples are examples of real numbers also.

given number exactly with 0. For example, the number 15 is divisible by 1, 3, 5, and 15.

HCF

The Highest Common Factor is the full form of HCF. HCF is also known as Greatest Common Factor (GCF) or Greatest Common Divisor (GCD). The largest positive integer that divides all the provided integers with zero remainders is the greatest common factor (GCF, GCD, or HCF) for a subset of whole numbers.

Example: Consider two numbers 15 and 18.

Factors of 15 = 1, 3, 5, and 15

Factors of 18 = 1, 2, 3, 6, 9, and 18

The common factors of both 15 and 18 are 1 and 3. Here, 3 is the highest common factor of both numbers. Hence, 3 is the HCF of 15 and 18.

LCM

LCM stands for Lowest or Least Common Multiple in its full form. The smallest positive integer divisible by all the provided numbers is the LCM of two or more numbers.

Example:

Consider two numbers: 7 and 21.

The multiples of 7 are:

$$7 \times 1 = 7,$$

$$7 \times 2 = 14,$$

$$7 \times 3 = 21,$$

$7 \times 4 = 28$, and so on...

The multiples of 21 are:

$$21 \times 1 = 21,$$

$$21 \times 2 = 42,$$

$$21 \times 3 = 63,$$

Complex numbers

Complex numbers are those numbers that can be written in the form $a + bi$ where a and b are real numbers, and i is the square root of -1 .

Example:

$\dots, -7 + 2i, 0, 1 + 3i \dots$

FACTORS

Factors of a number are the exact divisors of the provided numbers because they divide the

$21 \times 4 = 84$, and so on...

21 is the lowest common multiple from all the above multiples of 7 and 21. So, 21 is the LCM of 7 and 21.

SOME OTHER IMPORTANT CONCEPTS

Simplification

As we know that in GATE and other engineering entrance examinations, fast and quick calculations are very important, and the simplification concept is one of the concepts

of quantitative aptitude, which helps you in solving questions fast and quick. Every question in the math section of your exam can be solved quickly with the help of this concept, and this makes this concept a very important one.

Finding an answer to a difficult calculation including division, multiplication, square roots, and cube roots, plus, and minus is known as simplification. When a calculation is given, it is sometimes observed that one of the numbers is missing. We must either approximate the given values or do basic operations to determine the missing number. We have to simplify the calculation when all the numbers are given with some operations between them.

Now, let's discuss some of the basic rules related to simplification:

1. Rule 1: Always replace 'of' by multiplication and '/' by division.

Example:

Find $\frac{1}{8}$ of 72.

Solution:

$$\frac{1}{8} \times 72 \text{ or } \frac{72}{8}$$

Therefore, 9 is the answer.

2. Rule 2: Always apply the rule of BODMAS in every number operation.

B	Bracket
O	Open, of
D	Division
M	Multiplication
A	Addition
S	Subtraction

So, whenever in any question when there is

Example:

$$2 + 6 - (1 \times 4) \div 4 \times 5$$

Solution:

By applying the rule of BODMAS here, First, we will solve what is inside the bracket, i.e. $1 \times 4 = 4$

So, now the operation is $2 + 6 - 4 \div 4 \times 5$

Now, we will divide the number, i.e., $-4 \div 4 = -1$

Then, $2 + 6 - 1 \times 5$

Now, we will multiply the numbers, i.e. $-1 \times 5 = -5$

So, $2 + 6 - 5$

The next step is to add the number, i.e. $2 + 6 = 8$

So, $8 - 5$

Therefore, the answer is 3.

Fraction

Fractions are portions of a whole or a collection that are equal in size. Each part of a whole divided into equal pieces is a fraction of the total. Fractions are numbers that represent a piece of a larger whole. When an object or a set of objects is divided into equal pieces, each individual part becomes a fraction. Like most cases, a fraction is written as $\frac{1}{21}, \frac{50}{17}, \frac{7}{19}$, and so on. The numerator represents the total number of equal parts into which the whole is divided, and the denominator represents the total number of equal parts into which the whole is divided. The total number of equal pieces removed is the numerator. For example,

in the fraction $\frac{37}{4}$, the numerator is 37, and the denominator is 4. We can also convert decimal numbers into fractions by putting 1 and the number of zeroes equal to the number of digits after the decimal, in the denominator.

Examples

If you divide a pizza into two equal portions,

So, whenever in any question when there is more than one type of calculation then we have to apply the rule of BODMAS while solving the question.

each half equals half of the total pizza,
 $0.2 = \frac{2}{10}, 1.879 = \frac{1879}{1000}$, etc.

Example:

What is the greatest fraction among:

$$\frac{11}{16}, \frac{7}{8}, \frac{13}{20}, \text{ and } \frac{31}{40}$$

Solution:

If the denominator of some fractions is the same then the fraction with the largest numerator will be the largest fraction. So, to make the denominator equal to each fraction take LCM of 16, 8, 20, and 40 which is 80.

$$\text{Now, } \frac{80}{16} = 5 = \frac{11}{16} \times \frac{5}{5} = \frac{55}{80}$$

$$\frac{80}{8} = 10 = \frac{7}{8} \times \frac{10}{10} = \frac{70}{80}$$

$$\frac{80}{20} = 4 = \frac{13}{20} \times \frac{4}{4} = \frac{52}{80}$$

$$\frac{80}{40} = 2 = \frac{31}{40} \times \frac{2}{2} = \frac{62}{80}$$

$$70 > 62 > 55 > 52 \Rightarrow \frac{7}{8} > \frac{31}{40} > \frac{11}{16} > \frac{13}{20}$$

Therefore, the greatest fraction is $\frac{7}{8}$.

Types of fraction**1. Unit fraction:**

In a fraction, the numerator with 1 is called a unit fraction. For example, $\frac{1}{2}, \frac{1}{3}, \frac{1}{99}$, etc.

2. Proper fraction:

If a numerator's value is less than the denominator's value, it is called a proper fraction. For example, $\frac{2}{7}, \frac{8}{11}, \frac{73}{79}$, etc.

3. Improper fraction:

If a numerator's value is greater than the denominator's value, then it is called an improper fraction. For example, $\frac{6}{5}, \frac{3}{2a}, \frac{97}{95}$, etc.

4. Mixed fraction:

If a fraction consists of a whole number with a proper fraction, it is called a mixed fraction. For example, $5\frac{3}{4}, 10\frac{1}{2}, 22\frac{1}{3}$, etc.

5. Like fractions:

The fractions with the same denominator are called like fractions. For example, $\frac{3}{2}, \frac{5}{2}, \frac{99}{2}$, etc.

6. Unlike fractions

The fractions with different denominators are called unlike fractions. For example, $\frac{3}{2}, \frac{4}{7}, \frac{67}{3}$, etc.

7. Equivalent fractions:

If two fractions result in the same value, after simplification, then they are equivalent to each other. For example, $\frac{2}{3}, \frac{4}{6}$ (when we simplify it, it will become $\frac{2}{3}$), $\frac{6}{9}$ (when we simplify it, it will become $\frac{2}{3}$), etc.

Surds and indices

Surds are the square roots ($\sqrt{}$) of numbers that cannot be reduced to a single whole or rational number. It is impossible to portray it accurately in a fraction. Surds are also the root values that cannot be expressed in whole numbers. Indices are a value's power or exponent.

Examples:

$\sqrt{2} \approx 1.414$, and therefore, we leave it as a surd $\sqrt{2}$.

There are some rules for surds and indices, which are:

1. Rules for indices:

- a. $X^0 = 1$
- b. $X^a \times X^b = X^{a+b}$
- c. $\frac{X^a}{X^b} = X^{a-b}$
- d. $(X^a)^b = X^{ab}$
- e. $(X^y)^a = X^{ay}$
$$\left(\frac{X}{Y}\right)^a = \frac{X^a}{Y^a}$$

2. Rules for surds:

- a. $\sqrt[x]{a} = a^{\frac{1}{x}}$
- b. $\sqrt[x]{ab} = \sqrt[x]{a} \times \sqrt[x]{b}$
- c. $\sqrt[x]{\frac{a}{b}} = \frac{\sqrt[x]{a}}{\sqrt[x]{b}}$
- d. $(\sqrt[x]{a})^x = a$



$$\begin{aligned} \text{e. } & \sqrt[3]{\sqrt[3]{a}} = \sqrt[3]{a} \\ \text{f. } & (\sqrt[3]{a})^y = \sqrt[3]{a^y} \end{aligned}$$

Example:

Find the value of $(\sqrt{64})^{\frac{1}{3}}$.

Solution:

Removing the root from the base value and putting it in the form of power, we get,

$$\Rightarrow \left(64^{\frac{1}{2}}\right)^{\frac{1}{3}} = (64)^{\frac{1}{6}}$$

$$(4^3)^{\frac{1}{6}} = 4^{\frac{1}{2}} = \sqrt{4} = 2$$

Therefore, the answer is 2.

Divisibility rule

Divisibility rule is when a number is divided by another number or digit without leaving a remainder, it is said to be divisible. To make division simple we use this rule, and here are some of the rules of few numbers:

1. **For 2:** If the unit or ones digit of any number is 0, 2, 4, 6, and 8, then that number is divisible by 2. For example, 2378, 666, and 3675344. As we see here the last digits of the numbers are 8, 6, and 4, respectively, so these numbers are divisible by 2.
2. **For 3:** A number is only divisible by 3 when the sum of all the digits of the number is divisible by 3. For example, 333 ($3 + 3 + 3 = 9$), 33455 ($3 + 3 + 4 + 5 + 5 = 20$). As we see here the sum of the digits of the first number is 9, which is divisible by 3, so 333 is also divisible by 3, but the sum of the digits of the second number is 20, which is not divisible by 3, so the number 33455 will also not be divisible by 3.
3. **For 4:** If the last digit of a number is 0, or the last two digits are divisible by 4, then that number is divisible by 4. For example, 2508, and 2506. As we see here, the last digits of the number 2508 are 08. Since 08 is divisible by 4, then the number 2508 is also divisible

by 4. But 2506 is not divisible by 4 because the last two digits, 06, are not divisible by 4.

4. **For 5:** If a number has 0 or 5 at its unit's place then it is divisible by 5. For example, 2700, 23675, and 34583805. As we see here the last digit of the numbers are 0, 5, and 5 respectively, so these numbers are divisible by 5.
5. **For 6:** If a number is divisible by 2 and 3 both are also divisible by 6. For example, 36, 216, and 1296. As we see here, all the three numbers are divisible by 2 as well as 3. Hence, the numbers are divisible by 6 also.
6. **For 7:** The difference between twice the unit digit of the given number and the remaining part of the given number should be a multiple of 7, or it should be equal to 0. Let us learn this using an example: 161. The first step would be to double the number at units place, i.e., $1 \times 2 = 2$. Now subtract this from the remaining number $16 - 2 = 14$. Check if the reduced number is divisible by 7 or not. If yes then number 161 is also divisible by 7.
7. **For 8:** A number is only divisible by 8 when its last 3 digits, i.e., digits at hundred's, ten's, and unit's place, are divisible by 8, or if the last 3 digits are zero. For example, 4608, 36864, 294912. As we see here the last three digits of all the three numbers are divisible by 8. So, all three numbers are completely divisible by 8.
8. **For 9:** A number is only divisible by 9 if the sum of all its digits is divisible by 9. For example, 999 ($9 + 9 + 9 = 27$), 89919 ($8 + 9 + 9 + 1 + 9 = 36$). As we see here the sum of the digits of the first number is 27, which is divisible by 9, so 999 is also divisible by 9 likewise the sum of the digits of the second number is 36, which is also divisible by 9, so the number 89919 will also be divisible by 9.
9. **For 10:** If the last digit of any number is 0, then the number is divisible by 10. For example,



10. For 11: If the difference between the sum of digits at odd places and the sum of digits at even places is either 0 or multiple of 11, then the number is divisible by 11. For example, 1331 ($1 + 3 = 4$; $3 + 1 = 4$, difference

0). As we see here, the difference between the sum of digits at odd places and the sum of digits at even places of the numbers is 0; hence, these numbers are divisible by 11.

PRACTICE QUESTIONS

1. $93 + 3 - 7 \div 43 \times 29 = ?$
 - A. 90.44
 - B. 91.27
 - C. 100
 - D. 98.77
2. $213 + 711 \times 322 + 23 - 9 = ?$
 - A. 229769
 - B. 229169
 - C. 229669
 - D. 229969
3. $1.8 \times 7 + 5 - 14 \div 15 + 3 = ?$
 - A. 11.99
 - B. 12
 - C. 11.27
 - D. 12.50
4. $121 + 517 + 47 - 17 \div 17 \times 41 = ?$
 - A. 600
 - B. 650
 - C. 666
 - D. 644
5. $2 - 7 + 3 + 4 - 2 \times 19 \div 12 = ?$
 - A. 1.166
 - B. 2.679
 - C. -2.679
 - D. -1.166
6. $\frac{1}{3 + \frac{8}{4 + \frac{3}{5 + \frac{1}{4}}}} = ?$
 - A. $\frac{4}{33}$
 - B. $\frac{4}{67}$
7. If $4x - 1 \times 6 \times \frac{82x}{2x} \times 3x - 1 = 12$, then the value of x is:
 - A. $\frac{1}{3}$
 - B. $\frac{1}{2}$
 - C. $\frac{1}{4}$
 - D. $\frac{1}{5}$
8. If $Y = \sqrt{2} + 1$, then the value of $Y + \frac{1}{Y}$ is:
 - A. $2\sqrt{2}$
 - B. $\sqrt{2}$
 - C. $2\sqrt[3]{2}$
 - D. 2
9. Which among the following is the greatest fraction? $\frac{2}{5}, \frac{3}{5}, \frac{1}{5}, \frac{7}{15}, \frac{4}{5}$
 - A. $\frac{2}{5}$
 - B. $\frac{4}{5}$
 - C. $\frac{3}{5}$
 - D. $\frac{7}{15}$

- 10.** The sum of the first squares of the first ten natural numbers is:
 - A. 333
 - B. 388
 - C. 385
 - D. 365
- 11.** A pile of coconuts is divided into groups of 2, 3 and 5, with one coconut left out each time. What is the smallest number of
- 16.** What is the smallest value that must be provided to * in order for $63576*2$ to be divisible by 8?
 - A. 1
 - B. 3
 - C. 5
 - D. 2
- 17.** Which of the following is divisible by 99 exactly?
 - A. 111111111111
 - B. 1111111111111111
 - C. 11111111111111111111
 - D. 111111111111111111111111

coconuts in the heap?

- A. 62
- B. 30
- C. 35
- D. 31

12. The remainder is 3 when n is divided by 4. When $2n$ is divided by 4, what is the remainder?

- A. 2
- B. 3
- C. 4
- D. 1

13. What is the unit digit in the number $(211)^{111}$?

- A. 1
- B. 2
- C. 3
- D. 4

14. What is the sum of the first 45 natural numbers?

- A. 1000
- B. 1022
- C. 1035
- D. 1099

15. What is the smallest value that * must have in order for the number 451*603 to be divisible by 9 exactly?

- A. 9
- B. 6
- C. 7
- D. 8

- A. 114345
- B. 114344
- C. 114343
- D. 114342

18. What is the greatest five-digit number that can be divided by 99?

- A. 99990
- B. 90909
- C. 99099
- D. 90999

19. When a number is divided by 338, the remainder is 68; nevertheless, when the same amount is divided by 13, the remainder is?

- A. 3
- B. 5
- C. 1
- D. 7

20. Between 200 and 600, how many numbers are divisible by 4, 5 and 6?

- A. 6
- B. 8
- C. 10
- D. 12

21. What is the closest number to 99547 that is divisible by 687 exactly?

- A. 99617
- B. 99615
- C. 99619
- D. 99614

Number System

250

22. Six bells began to toll in sync, at 3, 6, 9, 12, 15 and 18 seconds intervals, respectively. How many times did they toll collectively in 30 minutes?

- A. 9
- B. 10
- C. 11
- D. 12

- C. 736
- D. 737

23. What is the smallest five-digit number that can be divided by 11?

- A. 10000
- B. 10001
- C. 10009
- D. 10010

28. Between 500 and 600, how many numbers include the number 9 only once?

- A. 18
- B. 15
- C. 8
- D. 9

24. The sum of a two-digit number's digits is 9. If the digits are flipped, the number falls by 45, thus what is the original number?

29. When $(1923^{1924^{1925}})$ is divided by 1924, what is the remainder?

- A. 0
- B. 2
- C. 1
- D. 11

30. A chain smoker had used up all of his cash. He couldn't afford to buy cigarettes

- A. 72
- B. 63
- C. 90
- D. 54

25. Find the smallest integer that leaves a remainder of 2, 3 or 4 when divided by 3, 4 or 5.

- A. 56
- B. 57
- C. 58
- D. 59

26. When a number is divided by 943, the remainder is 76. When you divide the same number by 23, what is the remainder?

- A. 4
- B. 6
- C. 9
- D. None of the above

27. 2093 is the sum of two numbers. The quotient is 2 when the larger number is divided by the difference in the numbers, and the remainder is 118. What is the smaller of the two numbers?

- A. 734
- B. 735

because he didn't have any money left. As a result, he joined the stubs and smoked them together. To make a single cigarette, he needed four stubs. How many cigarettes could he smoke in total if he received a pack of ten cigarettes as a gift?

- A. 12
- B. 13
- C. 11
- D. 14

31. How many of the numbers given below are divisible by 132?

264, 396, 462, 792, 968, 2178, 5184, 6336

- A. 7
- B. 6
- C. 5
- D. 4

32. Which of the following numbers is divisible by 11 exactly?

- A. 415624
- B. 415625
- C. 415626
- D. 415627

33. In total, how many three-digit numbers are divisible by six?

Number System

251

- A. 144
- B. 150
- C. 160
- D. 164

- A. 7
- B. 6
- C. 5
- D. 4

34. What is the smallest prime number?

- A. 0
- B. 1
- C. 2
- D. 3

40. When we divide a number by 357, the remainder is 39. What is the remainder when you divide the same number by 17?

- A. 3
- B. 4
- C. 5
- D. 6

35. How many prime numbers are less than 50?

- A. 12
- B. 13
- C. 14
- D. 15

41. We receive 3 as a remainder when we divide a number by 5. When the square of this number is divided by 5, what is the remainder?

- A. 4
- B. 5
- C. 6
- D. 7

36. What is the smallest three-digit prime number?

- A. 333
- B. 497
- C. 459
- D. None of the above

42. 1365 is the difference between the two numbers. We obtain 6 as the quotient and 15 as the remainder when we divide the larger number by the smaller. What is the smaller of the two numbers?

37. Which of the following is even if a and b are odd numbers?

- A. $a + b + 1$

- B. $a + 3b$
 C. $a + b$
 D. $a + \frac{b}{3}$

38. We obtain 269 as the quotient and 0 as the remainder when we divide an integer by 68. What will be the remainder if you divide the same number by 67?

- A. 3
 B. 2
 C. 1
 D. 0

39. When we divide a number by 56, the remainder is 29. What is the remainder when you divide the same number by 8?

- A. 278
 B. 270
 C. 275
 D. 289

43. The remainder in a division sum is 0. When a pupil multiplied the divisor by 12 instead of 21, the quotient was 35. What is the right quotient to use?

- A. 12
 B. 21
 C. 20
 D. 35

44. The product of two numbers is 35, while the sum of two numbers is 12. What is the sum of these numbers' reciprocals?

A. $\frac{35}{12}$

- B. $\frac{1}{12}$
 C. $\frac{1}{35}$
 D. $\frac{12}{35}$

45. A positive proper fraction and its reciprocal have a difference of $\frac{9}{20}$. What is the fraction?

- A. $\frac{5}{4}$
 B. $\frac{4}{5}$
 C. $\frac{20}{9}$
 D. $\frac{1}{5}$

46. We receive the same remainder when we divide 2272 by 3 digit number X as we do when we divide 875 by 3 digit number X . What is the sum of X 's digits?

- A. 9
 B. 10
 C. 11
 D. 13

47. On multiplying a number by 7 the product is a number each of whose digits is 3. The smallest such number is:

- A. 47617
 B. 47619
 C. 47666
 D. 47615

48. Which of the following integers is divisible by the difference of the squares of two successive even integers?

- A. 3
 B. 4
 C. 5
 D. 6

49. Which of the following integers is divisible by the difference of the squares of two successive odd integers?

- A. 8
 B. 9
 C. 13
 D. 17

50. If n is a natural number, then $(6n^2+6n)$ is always divisible by:

- A. 6
 B. 12
 C. Both 6 and 12
 D. None of the above

SOLUTIONS

- (B)** As there is no bracket in this question, first we will solve the division part of the sum, then the multiplication part, then the
- (B)** As there is no bracket in this question, first we will solve the division part of the sum, then the multiplication part, then the

addition part and lastly the subtraction part.

$$\begin{aligned} \text{So, } 93 + 3 - \frac{7}{43} \times 29 \\ = 93 + 3 - 0.16 \times 29 \\ = 93 + 3 - 4.72 \\ = 96 - 4.72 \\ = 91.279069767442. \end{aligned}$$

Therefore, the correct option is B.



sum, then the multiplication part, then the addition part and finally the subtraction part.

$$\begin{aligned} &= 1.8 \times 7 + 5 - \frac{14}{1.5} + 3 \\ &= 12.6 + 5 - \frac{14}{1.5} + 3 \\ &= 12.6 + 5 - 9.3 + 3 \\ &= 17.6 - 9.3 + 3 \\ &= 8.27 + 3 \\ &= 11.27 \end{aligned}$$

Therefore, the correct option is C.

4. (D) As there is no bracket in this question, first we will solve the division part of the sum, then the multiplication part, then the addition part and finally the subtraction part.

$$\begin{aligned} &= 121 + 517 + 47 - \frac{17}{17} \times 41 \\ &= 121 + 517 + 47 - 1 \times 41 \\ &= 121 + 517 + 47 - 41 \\ &= 638 + 47 - 41 \\ &= 685 - 41 = 644 \end{aligned}$$

Therefore, the correct option is D.

5. (D) As there is no bracket in this question, first we will solve the division part of the sum, then the multiplication part, then the addition part and finally the subtraction part.

$$\begin{aligned} &= 2 - 7 + 3 + 4 - 2 \times \frac{19}{12} \\ &= 2 - 7 + 3 + 4 - \frac{38}{12} \\ &= 2 - 7 + 3 + 4 - 3.16 \\ &= -5 + 3 + 4 - 3.16 \\ &= -2 + 4 - 3.16 \\ &= 2 - 3.166666666667 \\ &= -1.166 \end{aligned}$$

Therefore, the correct option is D.

6. (C) Solving the sum,

$$= \frac{1}{3} + \frac{168}{96}$$

addition part and finally the subtraction part.

$$\begin{aligned} &= 213 + 711 \times 322 + 23 - 9 \\ &= 213 + 228942 + 23 - 9 \\ &= 229155 + 23 - 9 \\ &= 229178 - 9 = 229169 \end{aligned}$$

Therefore, the correct option is B.

3. (C) As there is no bracket in this question, first we will solve the division part of the

Therefore, the correct option is C.

$$\begin{aligned} 7. (B) & 2^{2x-2} \times 2^x \times 3^x \times \frac{2^{6x}}{2^x} \times 3^x - 1 \\ &= 2^2 \times 3^1 \\ & 2^{2x-2+x+x+6x-x} \times 3^{x-x+1} = 2^2 \times 3 \\ & X = \frac{1}{2} \end{aligned}$$

Therefore, the correct option is B

$$8. (A) Y + \frac{1}{Y} = \frac{Y^2 + 1}{Y}$$

Now put the value of $Y = \sqrt{2} + 1$

$$\begin{aligned} & \left[(\sqrt{2} + 1)^2 + 1 \right] \\ & \quad \sqrt{2} + 1 \\ &= 2\sqrt{2} \end{aligned}$$

Therefore, the correct option is A.

9. (B) If the denominator of some fractions is the same then the fraction with the largest numerator will be the largest fraction. So, to make the denominator equal to each fraction take LCM of 5, 5, 5, 15 and 5 which is 15.

$$\text{Now, } \frac{15}{5} = 3 \Rightarrow \frac{2}{5} \times \frac{3}{3} = \frac{6}{15}$$

$$\frac{15}{5} = 3 \Rightarrow \frac{3}{5} \times \frac{3}{3} = \frac{9}{15}$$

$$\frac{15}{5} = 3 \Rightarrow \frac{1}{5} \times \frac{3}{3} = \frac{3}{15}$$

$$\frac{15}{15} = 1 \Rightarrow \frac{7}{15} \times \frac{1}{1} = \frac{7}{15}$$

$$\frac{15}{5} = 3 \Rightarrow \frac{4}{5} \times \frac{3}{3} = \frac{12}{15}$$

$$12 > 9 > 7 > 6 > 3 \frac{4}{5} > \frac{3}{5} > \frac{7}{15} > \frac{2}{5} > \frac{1}{5}$$

Hence, the greatest fraction is $\frac{4}{5}$.

Therefore, the correct option is B.

10. (C) $1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2$
 $= 1 + 4 + 9 + 16 + 25 + 36 + 49 + 64 + 81 + 100$
 $= 385$

$$= \frac{4}{19}$$

Therefore, the correct option is C.

11. (D) Take the LCM of 2, 3, 5 which is 30

So, number of coconuts = $30 + 1 = 31$,
Therefore, the correct option is D.

12. (A) $N = 4 \times Q + 3$ where Q is quotient

$$2n = 2 \times 4 \times Q + 6$$

$$2n = 4(2Q+1)+2$$

Hence, when $2n$ is divided by 4, the remainder is 2.

Therefore, the correct option is A.

13. (A) Any power of 1 will give 1 as a unit digit.
Therefore, the correct option is A.

14. (C) Add all the numbers starting from 1 to 45.
Therefore, the correct option is C.

15. (D) Since the sum of all the digits of the number must be divisible by 9, as per the divisibility rule of 9. So, the smallest value of * is 8.

Therefore, the correct option is D.

16. (B) Since the sum of the last three digits of the number must be divisible by 8, as per the divisibility rule of 8. So, the smallest value of * is 3.

Therefore, the correct option is B.

17. (A) For a number to be divisible by 99, it should have to be divisible by both 9 and 11. So, applying the divisibility rule of both 9 and 11 we find that only 114345 is divisible by 99.

Therefore, the correct option is A.

18. (A) For a number to be divisible by 99, it should have to be divisible by both 9 and 11. So, applying the divisibility rule of both 9 and 11 we find that only 99990 is divisible by 99.

Therefore, the correct option is A.

19. (A) Let the number

$$N = 338X + 68 = 13 \times 26X + 13 \times 5 + 3$$

$$\Rightarrow 13(26X + 5) + 3$$

Hence, when the number is divided by 13, then the remainder is 13.

Therefore, the correct option is A.

20. (A) Every such number must be divisible by the LCM of 4, 5 and 6 which is 60. Such numbers are 240, 300, 360, 420, 480 and 540.

Hence, there are only 6 such numbers.

Therefore, the correct option is A.

21. (B) When 99547 is divided by 687, remainder is 619.

Hence, the nearest number = $99547 + 68 = 99615$

Therefore, the correct option is B.

22. (C) LCM of 3, 6, 9, 12, 15 and 18 is 180.

So, the bell will toll together after every 3 minutes.

And in 30 minutes, they will together $10 + 1$ (in the starting, i.e. at 0 minutes) = 11 times.
Therefore, the correct option is C.

23. (D) Apply the divisibility rule of 11 and eliminate three options from the options.

Therefore, the correct option is D.

24. (A) Let the unit digit be X and tens digit be Y
Then, number = $10Y + X$

Now, according to the question,

$$X + Y = 9 \quad \dots \quad (1)$$

$$\text{And } 10X + Y = 10Y + X - 45$$

$$\Rightarrow 9X - 9Y = -45$$

$$\Rightarrow Y - X = 5 \quad \dots \quad (2)$$

Adding equations (1) and (2)

$$2Y = 14 \text{ or } Y = 7, \text{ then, } X = 2$$

Hence, the number is 72.

Therefore, the correct option is A.

25. (D) The smallest number divisible by 3, 4 and 5 is 60. Here, we see the difference between the divisor and remainder is the same for every divisor that is 1. So 59 is the answer.

Therefore, the correct option is D

26. (D) Let the number be N

$$\text{So, } N = 943Q + 76$$

$$\Rightarrow 23 \times 41Q + 3 \times 23 + 7$$

$$\Rightarrow 23(41Q + 3) + 7$$

Hence, the remainder is 7.



Therefore, the correct option is D.

- 27. (D)** Let the numbers be X and Y

So, $X + Y = 2093$...Equation (1)

According to the question

$$\Rightarrow \frac{X}{X} - Y = 2 + \frac{118}{X} - Y$$

$$\Rightarrow 2Y - X = 11 \dots \text{Equation (2)}$$

Solving equations (1) and (2) we get $Y = 737$.
Therefore, the correct option is D.

- 28. (D)** All numbers between 500 and 600 in

which 9 occurs only once are: 509, 519, 529, 539, 549, 559, 569, 579, 589. Hence there are 9 such numbers.

Therefore, the correct option is D.

- 29. (C)** The remainder is 1.

Therefore, the correct option is C.

- 30. (B)** 10 cigarettes give 10 stubs. From 10

stubs 3 more cigarettes can be made. 13 is the answer.

Therefore, the correct option is B.

- 31. (D)** For a number to be divisible by 132 it

should have to be divisible by 3, 4 and 11 also. So, applying the divisibility rules of these three numbers on each of the given numbers, we find that there are only 4 such numbers (264, 396, 792 and 6336). Therefore, the correct option is D.

- 32. (A)** Apply the divisibility rule of 11 on each option.

Therefore, the correct option is A.

- 33. (B)** There are 150 such numbers.

Therefore, the correct option is B.

- 34. (C)** 2 is the smallest prime number.

Therefore, the correct option is C.

- 35. (D)** There are 15 prime numbers less than

50 (2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, and 47).

Therefore, the correct option is D.

- 36. (D)** None of the given numbers is the smallest prime number.

Therefore, the correct option is D.

- 37. (C)** The sum of two odd numbers is always even.

Therefore, the correct option is C.

- 38. (C)** Dividend = $68 \times 269 + 0 = 18292$

Dividing 18292 by 67 we get 1 as remainder.

Therefore, the correct option is C.

- 39. (C)** Let's assume the number is $56 + 29 = 85$

$$85 = 56 \times 1 + 29$$

When it is divided by 8,

$$\Rightarrow \frac{85}{8} = 8 \times 10 + 5 \text{ or } 29 = 3 \times 8 + 5$$

Hence, the remainder is 5.

Therefore, the correct option is C.

- 40. (C)** Let x be the number and y be the quotient.

$$\text{Then, } x = 357y + 39 =$$

$$\Rightarrow (17 \times 21 \times y) + (17 \times 2) + 5 =$$

$$\Rightarrow 17 \times (21y + 2) + 5$$

Required number = 5.

Therefore, the correct option is C.

- 41. (A)** 4 will be the remainder in this case due

to the square condition applied here.

Therefore, the correct option is A.

- 42. (B)** Let the smaller number be x . Then the

larger number = $(x + 1365)$.

$$\Rightarrow x + 1365 = 6x + 15$$

$$\Rightarrow 5x = 1350$$

$$x = 270$$

Smaller number = 270.

Therefore, the correct option is B.

- 43. (C)** Number = (12×35)

$$\text{Correct quotient} = 420 \div 21 = 20$$

Therefore, the correct option is C.

- 44. (D)** $\frac{12}{35}$ is reciprocal.

Therefore, the correct option is D.

- 45. (B)** Let the required fraction be X .

$$\text{Then, } X - \frac{1}{X} = \frac{9}{20}$$

$$\text{So, } 1 - \frac{x^2}{X} = \frac{9}{20}$$

$$\therefore \quad \checkmark \quad 4$$

$$(2n+2)^2 - (2n)^2 = (2n+2+2n)(2n+2-2n)$$

$$\Rightarrow 2(4n+2)$$

$\Rightarrow 4(2n+1)$. which is divisible by 4.



$$\Rightarrow x = \frac{1}{5}$$

Therefore, the correct option is B.

- 46. (B)** Clearly, $(2272 - 875) = 1397$, is exactly

divisible by X.

$$Now, 1397 = 11 \times 127$$

The required 3-digit number is 127, the sum of whose digits is 10.

Therefore, the correct option is B.

- 47. (B)** 47619 is such a number.

Therefore, the correct option is B.

- 48. (B)** Let the two consecutive even integers be $2n$ and $(2n+2)$.

Then,

Therefore, the correct option is B.

- 49. (A)** Let the two consecutive odd integers be $(2n+1)$ and $(2n+3)$.

$$\text{Then, } (2n+3)^2 - (2n+1)^2$$

$$\Rightarrow (2n+3 + 2n+1)(2n+3 - 2n-1)$$

$$\Rightarrow (4n+4) \times 2$$

$$8(n+1), \text{ which is divisible by 8.}$$

Therefore, the correct option is A.

- 50. (C)** $(6n^2 + 6n) = 6n(n+1)$, which is always

divisible by 6 and 12 both, since $n(n+1)$ is always even.

Therefore, the correct option is C.

30 Percentages



SYNOPSIS

- Introduction
- Percentage chart
- Percentage difference formula
- Important percentage formulas
- Summary

INTRODUCTION

The percentage concept is used to calculate the proportion of a value in relation to its original value. Percentages are often used in business to calculate a company's profit or loss percentage. It is also used in schools and universities to indicate the grades that

Some other understanding-based examples of percentages are:

10% is equal to $\frac{1}{10}$ fraction.

20% is equal to $\frac{1}{5}$ fraction.

25% is equal to $\frac{1}{4}$ fraction.

students have obtained. A percentage formula can be used to assess the students' grades. In general, a percentage is defined as a ratio of any value to the total value multiplied then by 100. The percentage symbol % is used to denote the percentage.

Percentage is a relative figure that represents one-hundredth of a quantity. One per cent (1%) equals one hundredth; two per cent (2%) equals two hundredths, and so on.

There are no dimensions to the %. It means they are integers with no dimensions. When we say 80% of a number, we mean 80 percentage of the whole. It can also be stated as a decimal or a fraction, for example, 0.76% or 0.17%. The proportion of marks earned by students in any subject is calculated during the examination by using the concept of percentage.

Example:

A student received a 78% on his exam. So, this percentage is derived by dividing a student's total marks into all subjects by the total maximum marks and then multiplying the result by 100.

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50% is equal to $\frac{1}{2}$ fraction.

75% is equal to $\frac{3}{4}$ fraction.

90% is equal to $\frac{9}{10}$ fraction.

Example:

Find the 40% of 300.

Solution:

$$\frac{40}{100} \times 300 = 40 \times 3 = 120$$

Hence, 120 is 40% of 300.

Example:

In a class of 72 students, 87.5% of the students cleared the GATE examination. How many did not clear it?

Solution:

The number of students who did not clear the

$$\text{GATE exam} = 12.5 \times \frac{72}{100} = 9.$$



Example:

The population of a town increases at the rate of 3.7% each year. It is 31,110 now. What was it like last year?

Solution:

Let's assume last year it was x

$$\text{So, } 31,110 = x \left(1 + \frac{3.7}{100}\right)$$

Hence, $x = 30,000$.

Example:

Kyler needed Rs. 800 to cover her fees. Her brother provided 20% of the funding, and her mother provided 30% of the remaining funds. She had Rs. 200 in her bank account. How much extra (in Rs.) does she require?

Solution:

Required amount = 800

$$\text{From her brother she got} = 800 \times \frac{20}{100} = 160$$

$$\text{From her mother she got} = (800 - 160) \times \frac{30}{100} =$$

$$640 \times \frac{30}{100} = 192$$

From the bank she got Rs. 200

$$\text{Now, she needs } 800 - (160 + 192 + 200) = 800 - 552 = \text{Rs. 248 more}$$

Example:

Allen, the barber, shaved 40% of his customers

FRACTIONS	PERCENTAGE
$\frac{1}{2}$	50%
$\frac{1}{3}$	33.33%
$\frac{1}{4}$	25%
$\frac{1}{5}$	20%
$\frac{1}{6}$	16.66%
$\frac{1}{7}$	14.28%
$\frac{1}{8}$	12.50%
$\frac{1}{9}$	11.11%
$\frac{1}{10}$	10%
$\frac{1}{11}$	9.09%
$\frac{1}{12}$	8.33%
$\frac{1}{13}$	7.69%

and gave a haircut to 80% of his customers. He charged Rs. 7 for a shave and Rs. 5 for a haircut. If 20% of customers who opted for a shave also had a haircut, what were Allen's earnings if he had 75 customers (in Rs.)?

Solution:

Total customers = 75

$$\text{Numbers of customers shaved} = 75 \times \frac{40}{100} = 30$$

Number of customers who got hair cut

$$= 75 \times \frac{80}{100} = 60$$

$$\text{Hence, his total income} = (30 \times 7) + (60 \times 5) = 210 + 300 = \text{Rs. } 510.$$

Percentage chart

The percentage chart is given here for fractions converted into percentages.

$\frac{1}{14}$	7.14%
$\frac{1}{15}$	6.66%
$\frac{1}{16}$	6.25%
$\frac{1}{17}$	5.88%
$\frac{1}{18}$	5.55%
$\frac{1}{19}$	5.26%
$\frac{1}{20}$	5%

Percentages

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Percentage difference formula

If we are given two numbers suppose X and Y , and the question asked us to find the percentage difference between the two numbers, then we will apply the below-given formula to find the percentage difference quickly:

$$\text{Percentage difference} = \frac{|X - Y|}{\left(\frac{X + Y}{2}\right)} \times 100$$

Example:

If two numbers are given suppose 20 and 30, and then the percentage difference between these two numbers is:

Solution:

$$\text{Percentage difference} = \frac{|20 - 30|}{\left(\frac{20 + 30}{2}\right)} \times 100$$

$$\text{On solving the above equation, } \frac{10}{25} \times 100 = 40$$

Therefore, the percentage difference between 20 and 30 is 40%.

Some other important percentage formulas

1. Percentage formula in case of fraction:

$$\frac{\text{Numerator}}{\text{Denominator}} \times 100$$

Example:

Convert $\frac{3}{16}$ into a percentage.

Solution:

Using above given formula,

$$\frac{3}{16} \times 100 = 3 \times \frac{25}{4}$$

$$\Rightarrow \frac{75}{4} \% \text{ or } 18.75\%$$

2. Percentage change formula:

$$\frac{\text{New value} - \text{Original value}}{\text{Original value}} \times 100$$

Example:

Annie works in a supermarket for \$10.00 per hour. If her pay is increased to \$12.00, then what is her percentage increase in pay?

Solution:

Here, original value = 10, and new value = 12

$$\frac{12 - 10}{10} \times 100$$

$$\frac{2}{10} \times 100 = 20$$

- In general, a percentage is defined as a ratio of any value to the total value multiplied then by 100.
- The percentage symbol % is used to denote the percentage.

- Percentage difference = $\frac{|X - Y|}{\left(\frac{X + Y}{2}\right)} \times 100$

- Percentage formula in case of fraction:

Chapter Summary

$$\frac{\text{Numerator}}{\text{Denominator}} \times 100$$

- Percentage change formula:

$$\frac{\text{New value} - \text{Original value}}{\text{Original value}} \times 100$$

PRACTICE QUESTIONS

1. Aaron receives 35% of the maximum marks in an exam, while John receives 50% of the maximum marks. Aaron would have received the same percentage as John if he had received 45 more marks.

- What is the exam's maximum score?
- A. 300
 - B. 400
 - C. 350
 - D. 450



2. Peter received 20% of the maximum marks in an exam, but he failed by 30 marks. Peter received 40% of the same and was passed by ten marks. What were the minimum passing marks for the exam?
- A. 70
 - B. 80
 - C. 120
 - D. 60
3. Tom's pay is 12% more than Jerry's. Jerry's pay is lesser than Tom's by what percentage?
- A. 10.71%
 - B. 10%
 - C. 9.71%
 - D. 11.71%
4. Benett is 25% older than Cyril, his younger brother. Cyril is younger than Benett by what percentage?
- A. 20%
 - B. 25%
 - C. 30%
 - D. 16.66%
5. Patrick's pay is reduced by 16% initially and increased by 16% in the next month. Patrick's current income is approximately how much less than his starting salary?
- A. 2% less
 - B. 2.16% less
 - C. 2.56% less
 - D. 2.009% less
6. James's salary is boosted by 20% for the first month and then reduced by 20% for the second month. What is James's current salary in relation to his starting salary?
- A. 2% more
 - B. 3% less
 - C. 4% less
 - D. 5% more
7. A commission of 5% on the first Rs. 5000 and 2.5% on the rest of the selling price is charged by an agent. What was his commission if the selling price was Rs. 30,600?
- A. 850
 - B. 1015
 - C. 890
 - D. 1020
8. In a two-candidate election, 6% of voters did not cast their ballots. Won by 3000 votes, the winning candidate received 48% of the total votes cast. How many people cast ballots in the election?
- A. 12000
 - B. 10000
 - C. 15000
 - D. 20000
9. To be eligible for a scholarship, you must get at least 80% in an examination. Tony received 1005 marks but was just 13% of the way to receiving the scholarship. What was the highest possible score?
- A. 2000
 - B. 2500
 - C. 3000
 - D. 2800
10. Marcos has a particular quantity of mangoes, with 15% of them being rotten. He offers his friend 60% of the remaining mangoes, leaving him with 102 mangoes. How many mangoes did he have when he first started?
- A. 200
 - B. 300
 - C. 400
 - D. 500
11. In a two-candidate election, one candidate received 55% of the total valid votes, while 20% of the total votes cast were invalid.



- received by the other candidate if the total votes cast were 7500?
- A. 2500
B. 2700
C. 3000
D. 3200
- 12.** A person's daily income has been boosted by 75%, and he now receives Rs. 75. How much did he earn on a daily basis before the raise?
- A. 41
B. 42.85
C. 45.85
D. 45
- 13.** What is 66% of 66.67?
- A. 22
B. 11
C. 33
D. 44
- 14.** When Albert's income is 25% more than Pinto's, how much less does Pinto's income have to be?
- A. 10%
B. 15%
C. 18%
D. 20%
- 15.** In an exam, a candidate receives 20% of the maximum score and fails by ten marks. Another candidate receives 42% of the maximum possible marks, which are 12 marks more than the passing grade. What is the exam's maximum score?
- A. 80
B. 90
C. 110
D. 100
- 16.** In a poll between A, B, and C, a total of 600 votes were cast. A received 30% of the vote, B received 360 votes, with the remaining votes going to C. In relation to his nearest rival, what percentage of the overall votes did the winner receive?
- A. 100%
B. 500%
C. 300%
D. 200%
- 17.** When two candidates, A and B, are running for office election. A received 60% of the total valid votes, 15% of the total 5,00,000 ballots cast was considered illegitimate. What is the total number of votes that B has received?
- A. 100000
B. 150000
C. 170000
D. 190000
- 18.** An accountant charges a fee of 14% of the transaction amount. How much did Rory pay the accountant if he completed a transaction of Rs. 14,00,000 for him?
- A. 200000
B. 196000
C. 190000
D. 193000
- 19.** A carpenter manufactured 50 chairs, 14 of which are defective. What is the percentage of chairs that are in good condition?
- A. 50%
B. 75%
C. 80%
D. 72%
- 20.** In an election between A and B, A received 65% of the vote and won the election by a margin of 2748 votes. What is the total number of votes cast if no vote is declared invalid?
- A. 9160
B. 9000
C. 9169
D. 9167
- 21.** In a contest between Kurian and Pablo, Kurian received 25% of the vote but was defeated. Pablo received 14000 votes more



than Kurian in the election. What is the total number of votes cast?

- A. 25000
- B. 27000
- C. 26000
- D. 28000

22. Nathan is 22% more productive than Lyon. In comparison to Nathan, how inefficient is Lyon?

- A. 17.03%
- B. 18.03%
- C. 19.03%
- D. 20.03%

23. Simon's total salary was reduced by 35%. He spent 45% of the money left over, leaving him with Rs. 2500. What was his overall salary?

- A. 6991
- B. 6992
- C. 6993
- D. 6994

24. If A's salary is 25% more than B's salary, what percentage of B's salary is less than A's salary?

- A. 10%
- B. 20%
- C. 15%
- D. 18%

25. Only 75% of the total number of eligible voters voted in a presidential election. The victorious candidate received 58% of all votes cast and won by a margin of 12000 votes. What was the total number of voters who were eligible to vote?

- A. 125000
- B. 100000
- C. 150000
- D. 175000

26. Stark's pay is set at 80% of that of his boss. If he spends 40% of his salary on rent and 20% on food and is left with Rs. 2400, what is Stark's boss's salary?

- A. 6200
- B. 6250
- C. 6225
- D. 6275

27. In the market, the price of rice increases by 14%. A family's rice intake must be reduced by what percentage in order to avoid an increase in rice expenditure?

- A. 12.28%
- B. 12.50%
- C. 12.47%
- D. 12.98%

28. Walter's pay has been boosted by 10%. At this point, his pay is 10% less than his boss's. What percentage of Walter's pay is smaller than his boss's salary if his salary is now lowered by 10%?

- A. 17%
- B. 18%
- C. 19%
- D. 20%

29. Jessi's salary is reduced by 40%, then boosted by 20%, and lastly increased by 20%. What percentage of his beginning pay does he earn now?

- A. 12.6% less
- B. 11.6% less
- C. 14.6% less
- D. 13.6% less

30. Robert, Sandy and Phil ran for office election in the same election. Robert received 40% of the vote and so won the election. Sandy and Phil received 35% and 25% of the overall vote, respectively. What is the amount of votes Robert received in the election if Sandy received 2000 more than Phil?

- A. 8000
- B. 8100
- C. 8200
- D. 8500

31. A total of 45,000 individuals took part in the exam, with 40% of them being female. What was the percentage of passed students if 70% of the boys and 75% of the

- A. 0.50
- B. 0.73
- C. 0.93
- D. 1



girls passed the exam?

- A. 70%
- B. 71%
- C. 72%
- D. 73%

32. A mine's lead ore provides 60% metal, with 0.75% of that being silver. How much silver can be extracted from 8000 kg of ore?

- A. 34 grams
- B. 35 grams
- C. 36 grams
- D. 37 grams

33. What percentages will the volume of a cone rise if the height and base radius of the cone are both increased by 50%?

- A. 237%
- B. 237.5%
- C. 238%
- D. 238.5%

34. When 80% of a number is added to 80, the outcome is the same number. Which of the following numbers is the correct answer?

- A. 400
- B. 500
- C. 600
- D. 700

35. What percentage does the volume of a cone rise if the radius and height of the cone are increased by 200% and 100%, respectively?

- A. 1700%
- B. 1800%
- C. 2000%
- D. 2500%

36. What percentage of 800 is $\frac{2}{3}$ less than 0.2%?

37. Rehan's pay is equal to half of Mariyam's. Rehan's pay is equivalent to 80% of Denver. What is Mariyam's monthly income if the total compensation of the three for a month is Rs. 22000?

- A. 2000
- B. 3000
- C. 4000
- D. 5000

38. By how much is $\frac{2}{3}$ less than 0.2% of 800?

- A. 72%
- B. 73%
- C. 74%
- D. 75%

39. What is the increase in the area of a rectangle if the length and breadth of the rectangle are raised by 20% and decreased by 10%, respectively?

- A. 5%
- B. 6%
- C. 7%
- D. 8%

40. What is the number if 41% of a number is less than 76% of the same number by 105?

- A. 200
- B. 300
- C. 400
- D. 500

41. What is the increase in the area of a square if the sides of a square are increased by 20%?

- A. 33%
- B. 44%
- C. 55%
- D. 66%

42. A total of 4000 people applied for an engineering entrance exam. One-fourth of these were girls. What is the number of successful candidates if 80% of the girls and 95% of the boys did not make it through?

- A. 350
- B. 450
- C. 550
- D. 650

43. From a 6-litre sugar solution containing 4% sugar, 1 litre of water is evaporated.

His scale reads 1 metre for 95 centimetres. What is his approximate % gain or loss?

- A. 5.25% gain
- B. 5% gain
- C. 5.50% gain
- D. 5.75% gain

47. A 20% increase in bus fares resulted in a 10% decrease in passenger numbers. Despite this, the bus depot's daily collection climbed by Rs. 160. Before the rise, how much money did they gather on a daily basis?

What is the sugar content of the remaining solution?

- A. 4%
- B. 4.8%
- C. 5%
- D. 5.8%

44. A fruit dealer has some bad oranges, about 2% of which are rotten. He sells 95% of the remaining oranges, leaving him with only

49. How many oranges did he start with?

- A. 100
- B. 10000
- C. 1000
- D. 100000

45. There are two papers in an examination. A student receives a 30% grade on the first paper out of 180. How much should he score in the second paper if he has to earn 50% overall and the second paper is worth 150 marks?

- A. 222
- B. 333
- C. 444
- D. 111

46. A fabric dealer claims to sell cloth at cost price, but his measurements are inaccurate.

- A. 1000

- B. 2000

- C. 3000

- D. 4000

48. A 25% reduction in the price of rice allows a buyer to get 3 kg more for Rs. 189. What was the original per-kilogram price of rice?

- A. 19 per kg
- B. 20 per kg
- C. 21 per kg
- D. 22 per kg

49. At EPS, 35 boys and 25 girls took part in the annual sports day. 60% of the students obtained certificates. What is the percentage of girls who earned certificates if the total number of guys who received certificates was 12?

- A. 50%
- B. 33.33%
- C. 25%
- D. 66.67%

50. The diameter of a circle is expanded by 15%. What is the area's percentage increase?

- A. 32.25%
- B. 32%
- C. 32.50%
- D. 32.75%

SOLUTIONS

1. (A) Let maximum marks be X

$$\text{Difference} = 15\%$$

$$\text{So, } 15\% \text{ of } X = 45$$

$$\frac{15}{100} \times X = 45$$

$$X = 300$$

Therefore, option A is correct.

2. (A) Let total marks be X

$$\text{So, } \frac{20X}{100} + 30 = \frac{40X}{100} - 10$$

$$\frac{20X}{100} = 40$$

$$X = 200$$

Hence, passing marks = 70

Therefore, option A is correct.

3. (A) Let Jerry's salary be Rs. 100

So, Tom's salary be Rs. 112

$$112 - \frac{100}{112} \times 100$$

Therefore, option C is correct.

8. (C) Voters who voted = $100 - 6 = 94\%$

So the losing candidate got = $94 - 48 = 46\%$

Difference = 2%

Let the total voters be X

2% of $X = 3000$

On solving

$$X = 150,000$$

Therefore, option C is correct.

9. (B) Difference = $80 - 13 = 67\%$

So, let's assume max marks be X

67% of $X = 1005$

$$X = 1500$$

Therefore, option B is correct.

10. (B) Let's assume Marcos have 100 mangoes at the beginning

So, the number of rotten mangoes = 15

And number of mangoes given to friend =

$$85 \times 0.60 = 51$$

= 10.71%

Therefore, option A is correct.

4. (A) $\frac{25}{125} \times 100$

= 20%

Therefore, option A is correct.

5. (C) Let Patrick's pay be Rs. 100

So, after 16% reduction = 84

And after 16% increment = 84×1.16 = Rs.

97.44

Hence, the difference = $100 - 97.44$

= Rs. 2.56

Therefore, option C is correct.

6. (C) Let James's starting salary be 100

So, after 20% increment it becomes 120

and after 20% reduction = 120×0.80

= Rs. 96

Hence, difference $100 - 96 = 4$

Therefore, option C is correct.

7. (C) Commission due = $\frac{5}{100} \times 5000 + \frac{2.5}{100}$

$\times (30600 - 5000)$

= Rs. 890

Remaining mangoes = $85 - 51 = 34$

According to question,

$34X = 102$

So, $100X = \frac{102}{34} \times 100$

= 300

Therefore, option B is correct.

11. (B) Valid votes = $7500 - \frac{20}{100} \times 7500$

= 6000

So, 55% of 6000 = 3300

Other candidate got = $6000 - 3300$

= 2700

Therefore, option B is correct.

12. (B) Let his daily wage before increase be X

$X + \frac{75x}{100} = 75$

$x = 42.85$

Therefore, option B is correct.

13. (D) $\frac{200}{3} \times 66\%$

= 44

Therefore, option D is correct.

Percentages

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14. (D) Let Pinto's income be 100

So, Albert's income is 125

Hence, it has to be 20% less

Therefore, option D is correct.

15. (D) Let the max marks be X

$\frac{20X}{100} + 10 = \frac{42X}{100} - 12$

$X = 100$

Therefore, option D is correct.

16. (D) $A = \frac{30}{100} \times 600 = 180$ votes

B = 360 votes

So, C = 60 votes.

Hence, winner B received 200% votes as compared to his rival A

Therefore, option D is correct.

17. (C) Invalid votes = $\frac{15}{100} \times 500000 = 75000$

So, valid votes = 425000

$A = \frac{60}{100} \times 425000$

= 255000

So, B = 170000

Therefore, option C is correct.

18. (B) Fee paid = $\frac{14}{100} \times 1400000$

= 196000

Therefore, option B is correct.

19. (D) Good chairs = $50 - 14 = 36$

22. (B) $\frac{22}{122} \times 100$

= 18.03%

Therefore, option B is correct.

23. (C) Remaining salary after reduction = 65%

45% of the remaining = $65 \times 0.45 = 29.25\%$

Remaining salary in hand = 35.75% which is equal to 2500

So, total salary = 6993

Therefore, option C is correct.

24. (A) Let B's salary be 100

So, A's salary be 125

$125 - \frac{100}{125} \times 100$

= 20%

Therefore, option A is correct.

25. (B) Difference between votes = $58 - 42 = 16\%$ which is equal to 12000

So, 100% = 75000, i.e., voters voted, i.e., 75%

Hence, total eligible voters = 100000

Therefore, option B is correct.

26. (B) Let's assume Stark's salary be 100

So, rent = 40

Food = 12

Remaining salary = 48 which is equal to 2400

So, Stark's total salary = 5000

Hence, his boss's salary = 6250

Therefore, option B is correct.

$$\begin{aligned}\text{Percentage} &= \frac{36}{50} \times 100 \\ &= 72\%\end{aligned}$$

Therefore, option D is correct.

20. (A) A votes = 65%
 B votes = $100 - 65 = 35\%$
 Difference = $65 - 35 = 30\%$
 So, $30\% = 2748$
 So, $100\% = 9160$
 Therefore, option A is correct.

21. (D) Let total votes be X
 Pablo votes = 75%
 Kurian votes = 25%
 So, 50% of $X = 14000$
 $X = 28000$
 Therefore, option D is correct.

Percentages

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30. (A) Sandy got 2000 votes more than Phil
 So $35 - 25\% = 2000$
 Let total votes be X
 So, 10% of $X = 2000$
 $X = 20000$
 So, votes received by Robert = 8000
 Therefore, option A is correct.

31. (C) Total students = 45000
 So, female = 18,000
 Then, males = $45000 - 18000 = 27000$
 70% of the boys = 18900
 75% of the girls = 13500
 So, total passed students = $18900 + 13500 = 32400$
 And, 32400 is 72% of 45000.
 Therefore, option C is correct.

32. (C) Silver obtained = $\frac{0.75}{100} \times \frac{60}{100} \times 8000$
 $= 36$ grams
 Therefore, option C is correct.

33. (B) New volume = $\frac{1}{3}\pi \left(\frac{3}{2}R\right)^2 \times \frac{3}{2}H$
 $= \frac{9}{8}\pi R^2 H$
 Difference = $(\frac{9}{8} - \frac{1}{3})\pi R^2 H$
 $= \frac{19}{24}\pi R^2 H$
 So, increase in volume = 237.5%
 Therefore, option B is correct.

34. (A) Let the number be X
 $\frac{80X}{100} + 80 = X$
 $X = 400$
 Therefore, option A is correct.

35. (A) The new radius and height are $3r$ and $2h$ respectively.

27. (A) $\frac{\frac{17}{114}}{114} \times 100$

$= 12.28\%$

Therefore, option A is correct.

28. (C) Let's take Walter's salary to be 100.
 So, his new salary = 110
 Percentage smaller as compared with boss
 $= 19\%$
 Therefore, option C is correct.

29. (D) Let's take salary be 100
 After reduction = 60
 After increment = 72
 After another increment = 86.4
 Difference = $100 - 86.4 = 13.6$
 Therefore, option D is correct.

36. (C) 0.2% of 800 = $\frac{8}{5}$
 Difference = $\frac{8}{5} - \frac{2}{3}$
 $= 0.93$
 Therefore, option C is correct.

37. (C) Let Denver's salary be 100
 Rehab's salary = 80
 Mariyam's salary = 40
 Total = $100 + 80 + 40 = 220 = 22000$
 Hence, Mariyam's salary = 4000
 Therefore, option C is correct.

38. (D) 0.2% of 800 = $\frac{8}{5}$
 Now, difference = $\frac{8}{5} - \frac{2}{3} = \frac{14}{15}$
 And, this difference is 75%
 Therefore, option D is correct.

39. (D) New area = $1.2L \times 0.9B$
 $= 1.08LB$
 Change in area = $\frac{1.08LB}{LB} \times 100$
 $= 8\%$
 Therefore, option D is correct.

40. (B) Difference = $76 - 41 = 35$
 35% of $X = 105$
 $X = 300$
 Therefore, option B is correct.

41. (B) Let the original side be X , then area = X^2
 Now, new area = $(1.2X)^2 = 1.44X^2$
 Percentage change = $1.44\left(\frac{X}{X}\right)^2 = 1.44$
 OR 44%
 Therefore, option B is correct.

42. (A) Number of girls who gave the exam =
 1000 ($\frac{1}{4}$ of 4000)
 Number of boys = $1000 - 1000 = 3000$

So, the new volume will become = $6\pi r^2 h$

Now, increase in volume

$$= 6\pi r^2 h - \frac{\frac{1}{3}\pi r^2 h}{3} \times 100$$

$$17 \times 100 = 1700\%$$

Therefore, option A is correct.

Number of boys = 4000 - 1000 = 3000

Now, as given 80% girls failed then 20% succeeded

Similarly, 95% boys failed then 5% succeeded

So, girls who got through = 20% of 1000 = 200

Boys who got through = 5% of 3000 = 150



Hence, total successful candidates = 200 + 150 = 350

Therefore, option A is correct.

43. (B) Sugar = $\frac{4}{100} \times 6$
= 0.24L

$$\% \text{ of sugar in remaining} = \frac{0.24}{5} \times 100
= 4.8\%$$

Therefore, option B is correct.

44. (C) Let he had X oranges

$$\text{Now, according to the question, } \frac{2}{100} X + \frac{95}{100} \left(X - \frac{2}{100} X \right) + 49 = X$$
$$\frac{1}{50} X + \frac{19}{20} X - \frac{19}{1000} X - X = -49$$

$$20X + 950X - 19X - 1000X = -49 \times 1000$$

$$X = 1000$$

Therefore, option C is correct.

45. (D) Total marks = 180 + 150 = 330

$$50\% \text{ of } 330 = 165$$

$$30\% \text{ of } 180 = 54$$

$$\text{He should score} = 165 - 54 = 111$$

Therefore, option D is correct.

46. (A) On every 95 cm, the trader gains 5 cm

So, on a trade of Rs. 95 his gain is Rs. 5

$$\text{So, gain \%} = \frac{5}{95} \times 100 = 5.25\%$$

Therefore, option A is correct.

47. (B) Let the bus fare be X and number of passenger be Y

So, total collection = XY

Now, after the hike, total collection,
$$\left(X + \frac{20}{100} X \right) \left(Y - \frac{10}{100} Y \right) = 1.08 XY$$

Increase in collection = $1.08XY - XY = 0.08XY$

According to the question, $0.08XY = 160$

$$So, XY = 2000$$

Therefore, option B is correct.

48. (C) New price = $\frac{3}{4}$ Initial price and total amount is constant that is Rs. 189

$$\text{New quantity} = \frac{4}{3} \text{ Initial quantity}$$

New quantity is 3 kg more than original quantity. Hence, original quantity was 9 kg

$$\text{Then, original price} = \frac{189}{9} = \text{Rs. 21 per kg.}$$

Therefore, option C is correct.

49. (D) Total students participating = $35 + 25 = 60$

Total students receiving certificates = $60 \times 0.6 = 36$

Total girls receiving certificates = $36 - 12 = 24$

Percentage of girls receiving certificates = $\frac{24}{36} \times 100 = 66.66\%$

Therefore, option D is correct.

50. (A) $\pi \times r^2 = \pi \times (1.15r)^2 = \pi \times (1.3225)$

Hence, percentage increase = 32.25%

Therefore, option A is correct.

PRACTICE QUESTIONS

Questions 1–5: Vishesh is living in Delhi and works in a reputed company. The details of the monthly expenditure of Vishesh are given here. He spends 19% of the total monthly expenditure on Rent and 5% of the total monthly expenditure on Transport. He spends 21% of the total monthly expenditure on Food and 15% of the total monthly expenditure on Education. He spends 30% of the total monthly expenditure on other expenses and he saves 10% of his total expenditure.

1. If Vishesh is earning Rs. 50,000 per month, then what's his total expenditure on rent, transport and education?

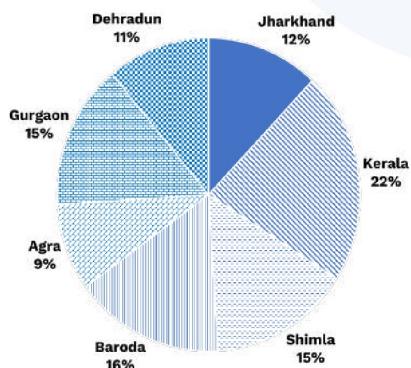
- A. 20000 B. 19000
C. 20500 D. 19500

2. If Vishesh is earning Rs. 20,000 per month, then how much more does he spend on food than on education and transportation put together?

- A. 300 B. 200
C. 400 D. 250

3. In 'others,' Vishesh spends 40% on the accessories of his car which amounts to Rs. 3000. What's his total expenditure on transportation?
- 1050
 - 1200
 - 1200
 - 1250
4. If a pie chart is drawn by taking the savings as one part and total expenses as the second part, then what is the angle made by total expenses?
- 330°
 - 280°
 - 324°
 - 320°
5. Due to an increase in the price of fuel, Vishesh has to spend Rs. 300 more on transportation. This is 20% more than usual. If other expenses remain constant, find his new savings.
- 2700
 - 2600
 - 2500
 - 2400

Questions 6–10: The following pie gives the data of the total number of passengers travelling from Delhi to different districts = 12000



6. The number of passengers travelling from Delhi to Baroda is what percentage more than the number of passengers travelling to Jharkhand?
- 20%
 - 33%
 - $33 \frac{1}{3}\%$
 - 30%
7. The number of passengers travelling from Delhi to Gurgaon is what percentage of the total number of passengers travelling from Delhi to Dehradun and Agra together?
- 70%
 - 75%
 - 72%
 - 68%
8. What is the average number of passengers travelling from Delhi to Shimla, Kerala and Baroda together?
- 2120
 - 2020
 - 2000
 - 2100
9. What will be the central angle for passengers travelling from Delhi to Baroda if it is put in the form of a pie-chart?
- 45%
 - 57%
 - 57.6%
 - 50%
10. Among the passengers who are travelling from Delhi to Kerala, 42% are women and 38% are men. What is the number of children?
- 428
 - 528
 - 280
 - 410

SOLUTIONS

1. (D) Total percentage of salary spent on rent, transport and education = $\{19 + 5 + 15\} = 39\%$

Amount spent on rent, transport and education = 39% of 50,000 = Rs. 19500
Therefore, option D is correct.
Therefore, option B is correct.



2. (B) The total percentage of salary spent on education and transportation together is $(15 + 5) = 20\%$
His expenditure is 21% on food.
Vishesh spends $21 - 20 = 1\%$ more on food.
Amount = 1% of 20,000 = Rs. 200
3. (D) Expenditure of Vishesh on car = 40% of $30\% = 12\%$
Given $12\% = 3000$
(Transportation) $5\% = x$
 $x = (5 \times \frac{3000}{12}) = \text{Rs. } 1250$
Therefore, option D is correct.
4. (C) Vishesh spends 90% of his income
Angle made by this division = $\frac{90 \times 360}{100} = 324^\circ$
Therefore, option C is correct.
5. (A) Vishesh spends 5% on transportation
His new expenditure on transportation = 120% of $5\% = 6\%$
Increase in expenditure = 1%
His new savings = $10\% - 1\% = 9\%$
 $= 1\% = 300$
 $9\% = x$
 $x = 9(300) = \text{Rs. } 2700$
Therefore, option A is correct.
6. (C) The number of passengers from Delhi to Baroda = 16%
Number of passengers from Delhi to Jharkhand = 12%

$$\text{Required \%} = \frac{16 - 12}{12} \times 100\% = \frac{4 \times 100}{12}$$

$$= 33 \frac{1}{3}\%$$

Therefore, option C is correct.

7. (B) Number of passengers travelling from Delhi to Gurgaon = 15%
Number of passengers travelling from Delhi to Dehradun and Agra = $11\% + 9\% = 20\%$
Required % = $\frac{15}{20} \times 100 = 75\%$
Therefore, option B is correct.
8. (A) Total percentage of Shimla, Kerala and Baroda = $(15\% + 22\% + 16\%) = 53\%$
Therefore, 53% of 12000
 $= (50\% \text{ of } 12000 + 3\% \text{ of } 12000) = 6000 + 360 = 6360$
Required average = $\frac{6360}{3} = 2120$
Therefore, option A is correct.
9. (C) Central angle = $\frac{16}{100} \times 360^\circ = 57.6^\circ$
Therefore, option C is correct.
10. (B) Number of children travelling from Delhi to Kerala = $100\% - (42 + 38)\% = 100\% - 80\% = 20\%$
Total number of children travelling from Delhi to Kerala = 20% of 22% of 12000
 $= \frac{20}{100} \times \frac{22}{100} \times 12000 = 528$
Therefore, option B is correct.



- Introduction
- Average
- Properties of average
- Weighted average or grouped average
- Common types of questions asked
- Summary

INTRODUCTION

The word average is quite often used in our day-to-day life. The average salary in a company, the average score of a team, the average marks of a class, and many other similar phrases are some examples of average that we encounter in our daily routine.

Questions based on the concepts of average are being frequently asked in GATE, JEE Main, and other engineering entrance examinations. This topic is easy to understand, which makes it easy for the test takers to score more in these exams.

Average

The average is a mean or middle value of a set of data, which can be calculated by dividing the total sum of all values by the total number of values in the given data.

The formula for calculating average:

$$\text{Average} = \frac{\text{Total sum of values}}{\text{Number of values}}$$

Example:

If A, B, C, and D are the partners in a firm, the firm gains a profit of Rs. 1 lakh, which is to be divided equally among the partners. Then the average profit gained by a partner can be calculated as:

Solution:

The average profit of the partner is given by

$$\text{Average} = \frac{\text{Total sum of values}}{\text{Number of values}}$$

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 Let us assume if there are ' n ' groups with ' $A_1, A_2, A_3, A_4, A_5, \dots, A_n$ ' as the number of elements or values in these groups and ' $P_1, P_2, P_3, P_4, P_5, \dots, P_n$ ' be the averages of these groups, respectively. Then the combined average of all these groups taken together can be calculated by:

weighted average

$$= \frac{A_1P_1 + A_2P_2 + A_3P_3 + A_4P_4 + A_5P_5 + \dots + A_nP_n}{P_1 + P_2 + P_3 + P_4 + P_5 + \dots + P_n}$$

Example:

The XYZ International School has two divisions in standard A, P and Q. The number of students

$$\text{Average profit} = \frac{1,00,000}{4} = \text{Rs. } 25,000$$

Therefore, each partner would get an average profit of Rs. 25,000.

Properties of average

1. In a given set of data, the average always lies between its maximum and minimum value.
2. If a constant is added or subtracted to each value in the data, then the average also increases or decreases respectively by the same constant.
3. If each value in the given set of data is multiplied or divided by a constant, then the average also gets multiplied or divided by the same constant.

Weighted average or grouped average

When the average of two or more groups with the same or the different number of values is given, and we have to find the average of the values of all the groups taken together, then the combined average of the groups is known as the weighted average or grouped average.

If there are two groups with ' A_1 ' and ' A_2 ' as the number of elements or values in them, let ' P_1 ' and ' P_2 ' be their averages, respectively. Then the combined average of these two groups taken together can be calculated by: weighted

$$\text{average} = \frac{A_1P_1 + A_2P_2}{A_1 + A_2}$$

Solution:

The new person's weight, if the average is increased, is given by

New person's weight = weight of person who left + (increase in the average \times total number of people)

$$\text{New person's weight} = 56 + (2.5 \times 8) = 76 \text{ kg}$$

2. If the average is decreased:

The following formula will apply:

New person's weight = weight of person who left + (decrease in the average \times total number of people)

Example:

in divisions P and Q is 42 and 35, respectively. The average weight of students in division P is 63 and that of students in division Q is 52. What is the average weight of standard A of XYZ International School?

Solution:

The average weight of the two divisions taken together is given by

$$\text{Weighted average} = \frac{A_1 P_1 + A_2 P_2}{A_1 + A_2}$$

$$\text{Weighted average} = \frac{42 \times 63 + 36 \times 52}{35 + 42}$$

Therefore, the average weight of standard A is 58.

Common types of questions asked

Case I

When a person is replaced by another person in a group, for example in cases of age, weight, height, etc. related problems.

1. If the average is increased:

The following formula will apply:

New person's weight = weight of person who left + (increase in the average × total number of people)

Example:

The average weight of 8 persons is increased by 2.5 kg when one of them whose weight is 56 is replaced by a new man, the weight of the new man is:

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Age of a new student is = $12 + (1 \times 11) = 23$ years

2. If there is an increase in average, the following formula will apply:

New person's weight = earlier average - (decrease in the average × total number of people)

Example:

The average weight of 19 students is 15 kg. By the admission of a new student, the average weight is reduced to 14.8 kg. The weight of the new student is

Solution:

The average of new person's weight if there is a decrease in average

New person's weight = earlier average - (decrease in the average × total number of people)

$$\text{New student weight} = 15 - (0.2 \times 20) = 11\text{kg}$$

Case III

For calculating average speed in speed, time, and distance the following formula will apply

The average weight of 10 persons is decreased by 2 kg when one of them whose weight is 50 is replaced by a new man; the weight of the new man is:

Solution:

The new person's weight if the average is decreased is given by

New person's weight = weight of person who left + (decrease in the average × total number of people)

$$\text{New man's weight} = 50 + (-2 \times 10) = 30\text{kg}$$

Case II

When a person joins a group then there may be two cases:

1. If there is an increase in average, the following formula will apply:

New person's weight = earlier average + (increase in the average × total number of people)

Example:

The average age of 10 students is 12 years. By the admission of a new student, the average age is increased to 13 years. The age of the new student is

Solution:

The average of new person's weight if there is an increase in average

New person's weight = earlier average + (increase in the average × total number of people)

Solution:

The average speed is given by

$$\text{Average speed} = \frac{2xy}{x+y}$$

$$\text{Average speed} = \frac{2(40 \times 24)}{40 + 24} = \frac{1920}{64} = 30 \text{ km/h}$$

Case IV

Average when arithmetic progression (AP) series is odd:

The average of the odd AP series will be the term that is in the middle of the series.

Example:

The average for series 3, 6, 9, 12, 15, 18, 21, 24, and 27 is:

Solution:

The middle term of the series is 15. Therefore, its average will be 15.

Case V

Average when AP series is even:

If the AP series is even, then the average can

and distance, the following formula will apply for calculating average speed:

$$\text{Average speed} = \frac{2xy}{x+y} \quad (\text{wherein, } x \text{ and } y \text{ denote}$$

for the different average speeds)

Example:

A covers a journey from Delhi to Jaipur by car at an average speed of 40 km/h and returns by scooter with an average speed of 24 km/h. What is his average speed during the whole journey?

be calculated from the average of two middle terms.

Example:

The average for series 2, 4, 6, 8, 10, and 12 is:

Solution:

The two middle terms of the series are 6 and 8. By calculating the average of both terms, i.e., $\frac{6+8}{2}=7$

Therefore, the average of the given series is 7.

Chapter Summary



- Average is an arithmetic mean of a given set of data.
- Average also changes accordingly if each value in the given set of data multiplies or divides by a constant.
- The average of two or more groups can be calculated by the weighted average.



PRACTICE QUESTIONS

1. The population of 5 cities is 35560, 27500, 30000, 25000, and 25600 people. What is the average population of a city?
 - 28742
 - 28562
 - 28732
 - 28730
2. The average weight of 48 employees is 70 kg. If the weight of two new employees is included, the average increases by 800 grams. What is the average weight of new employees?
 - 55 kg
 - 50 kg
 - 45 kg
 - 40 kg
3. The average of the first nine multiples of 5 is:
 - 23
 - 24
 - 25
 - 26
4. The average weight of 8 workers in a truck is increased by 1 kg when one of the workers, whose weight is 60 kg, is replaced by a new worker. What is the weight of the new worker?
5. The average of 40 numbers is 25. If two numbers 24 and 26 are removed, then the average of the remaining numbers will be:
 - 27
 - 24
6. The average of 0.60204, 0.60203, 0.60202, and 0.60201 is:
 - 0.60203
 - 0.60204
 - 0.602025
 - 0.60201
7. The average mark of 40 students in a class is 60, and the average mark of 60 students in a different class is 40. What is the average of all the tests?
 - 48
 - 50
 - 46
 - 40
8. The average weight of 30 boys in a class of 40 is 50, and the average of remaining boys is 46. What is the average weight of the whole class?
 - 48
 - 50
 - 49
 - 40
9. The average of 40 numbers is 25. If two numbers 24 and 26 are removed, then the average of the remaining numbers will be:
 - 27
 - 24

- A. 58 kg
- B. 68 kg
- C. 70 kg
- D. 64 kg

5. Raj travels from Delhi to Gurgaon by train at an average speed of 80 km/h and returns by car with an average speed of 48 km/h. What is his average speed during the whole journey?
- A. 60 km/h
 - B. 58 km/h
 - C. 54 km/h
 - D. 62 km/h

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four days is 0.3 inch. What is the average rainfall for the whole week?

- A. 0.7 inch
- B. 0.6 inch
- C. 0.4 inch
- D. 0.5 inch

12. The average collection of a shop in the first three days is Rs. 2700, and that of the next three is Rs. 2900. If the average collection of the whole week is Rs. 2850, then what is the collection on the last day of the week?
- A. Rs. 3150
 - B. Rs. 3050
 - C. Rs. 2950
 - D. Rs. 3000

13. The average of 3 numbers is 63. The first is twice the second and four times the third. Find the second number.
- A. 48
 - B. 54
 - C. 56
 - D. 50

14. The average weight of 19 bags is 30 kg, and by adding one more bag, the average weight is reduced to 29.6 kg. What is the weight of the new bag?
- A. 28 kg
 - B. 20 kg
 - C. 26 kg
 - D. 22 kg

15. The average weight of 8 students in a class is increased by 2.5 kg when one of them whose weight is 28 kg is replaced by the new student, what is the weight of the new student?

- C. 20
- D. 25

10. The average number of persons crossing a toll plaza on the first four days of a week is 1163 and that on the last three days is 1660. What is the daily average for the whole week?

- A. 1370
- B. 1350
- C. 1376
- D. 1440

11. The average rainfall of the first three days of a week is 1 inch, and that of the next

16. The average weight of 40 members in a joint family is 40 kg. If the weight of grandfather is included, the average weight increases by 500 grams. What is the weight of the grandfather?

- A. 58 kg
- B. 60.5 kg
- C. 60 kg
- D. 55.5 kg

17. The average mark of four siblings is 12. If the gap between their marks obtained is 4, then what are the least marks amongst all?

- A. 8
- B. 5
- C. 6
- D. 4

18. The average daily wage of 20 workers in a factory is Rs. 190 per day. If the daily wage of a new worker is added, the average daily wage becomes Rs. 200 per day. What is the daily wage of a newly added worker?

- A. 480
- B. 500
- C. 460
- D. 400

19. The average age of P, Q, R, and S ten years ago was 40 years. By including A, the present average age of all is 55 years. Find the present age of A.

- A. 75 years
- B. 70 years
- C. 76 years
- D. 74 years

20. A street vendor earned Rs. 1008 in 12 days. His average earnings for the first four days

- A. 48
- B. 50
- C. 46
- D. 40

were Rs. 80 a day. What is his average earning for the remaining days?

- A. Rs. 88
- B. Rs. 80
- C. Rs. 86
- D. Rs. 74



- 21.** The average height of 24 pillars in a class is 10 feet. If the height of the new pillar is included, the average height increases by 1 foot. The height of the new pillar is:
- A. 38
 - B. 35
 - C. 36
 - D. 40
- 22.** The average income of 4 employees in the office is Rs. 7350. One of the employees left the office, and therefore the average income was reduced to Rs. 6500. What is the income of the person who left the job?
- A. 9900
 - B. 9000
 - C. 9600
 - D. 9400
- 23.** Find the average of the given series: 4, 8, 12, 16, 20, 24, 28, 32, 36, and 40.
- A. 24
 - B. 25
 - C. 26
 - D. 22
- 24.** The average weight of 12 students in a class is 20 kg. If the teacher's weight is included, the average weight increases by 2 kg. What is the weight of the teacher?
- A. 48 kg
 - B. 44 kg
 - C. 46 kg
 - D. 40 kg
- 25.** The average height of 5 persons is 1.2 m. If the height of a new person is included, the average height increases by 100 cm. What is the height of the newly added person?
- A. 1.8 m
 - B. 1.4 m
 - C. 1.6 m
 - D. 1.4 m
- 26.** Find the average of the first six multiples of 9.
- A. 28
 - B. 31.5
 - C. 29
 - D. 30
- 27.** Arpit travelled from Kolkata to Jaipur by car at an average speed of 60 km/h. He returned to Kolkata by motorcycle at an average speed of 40 km/h. What is the average speed of the whole journey?
- A. 48
 - B. 50
 - C. 46
 - D. 40
- 28.** The average of 40 values is 20, and the average of 20 different values is 40. What is the average of all the values?
- A. 22
 - B. 24
 - C. 26
 - D. 20
- 29.** The average score of the first six batsmen is 15, and that of the next three batsmen is 18. Find the average score of nine batsmen.
- A. 14
 - B. 15
 - C. 16
 - D. 14
- 30.** The average daily expenditure on four vehicles is Rs. 100. If one of the vehicles gets sold, the average daily expenditure becomes Rs. 90. What is the daily expenditure on the vehicle which gets sold?
- A. 130
 - B. 160
 - C. 150
 - D. 140


SOLUTIONS

1. **(D)** Average

$$= \frac{35560 + 27500 + 30000 + 25000 + 25600}{5}$$

$$= \frac{143660}{5}$$

$$= 28732$$

Therefore, the correct option is D.

2. **(A)** Total weight of 48 employees

$$= 48 \times 70 = 3430 \text{ kg}$$

Total weight of all the employees including 2 new employees = $50 \times 70.8 = 3540 \text{ kg}$

Average weight of new employees

$$= (3540 - 3430) \text{ kg} = 110$$

$$= \frac{110}{2} = 55 \text{ kg}$$

The average weight of new employees is 55 kg.

Therefore, option A is correct.

3. **(C)** The first nine multiples of 5 are: 5, 10, 15, 20, 25, 30, 35, 40, and 45

The given numbers or multiples of 5 form an AP series and the middle term of the series is 25, therefore the average of the first nine multiples of 5 is 25.

Therefore, option C is correct.

4. **(B)** Weight of new crew member = Weight of person who left + (Increase in the average \times Total number of people)

$$\text{Weight of new worker} = 60 + (1 \times 8) = 68 \text{ kg}$$

The weight of the new worker is 68 kg.

Therefore, option B is correct.

5. **(A)** The average speed of the whole journey

$$= \frac{2xy}{x+y}$$

$$= \frac{2(80 \times 48)}{80+48}$$

$$= \frac{7680}{128} = 60 \text{ km/h}$$

The average speed of the whole journey is

60 km/h.

Therefore, option A is correct.

6. **(C)** Average

$$= \frac{0.60204 + 0.60203 + 0.60202 + 0.60201}{4}$$

$$= \frac{2.4081}{4} = 0.602025$$

The average of given four values is 0.60205.

Therefore, option C is correct.

7. **(A)** Average marks of all the students can

$$\text{be calculated by } = \frac{A1P1 + A2P2}{A1 + A2}$$

$$= \frac{(40 \times 60) + (60 \times 40)}{60 + 40}$$

$$= \frac{4800}{100} = 48$$

The average marks of all the students are 48.

Therefore, option A is correct.

8. **(C)** Average weight of whole class =

$$\frac{A1P1 + A2P2}{A1 + A2} = \frac{(30 \times 50) + (46 \times 10)}{30 + 10}$$

$$= \frac{1960}{40} = 49 \text{ kg}$$

The average weight of the whole class is 49 kg.

Therefore, option C is correct.

9. **(D)** Total sum of all numbers = $40 \times 25 = 1000$

Total sum of remaining numbers

$$= 1000 - (24 + 26) = 950$$

$$\text{Average of remaining numbers} = \frac{950}{38} = 25$$

The average of remaining numbers will be 25.



Therefore, option D is correct.

- 10. (C)** Daily average for whole week

$$= \frac{A1P1 + A2P2}{A1 + A2}$$

$$= \frac{(1163 \times 4) + (1660 \times 3)}{4 + 3}$$

$$= \frac{(4652 + 4980)}{7} = \frac{9632}{7}$$

$$= 1376$$

The daily average for the whole week is 1376.

Therefore, option C is correct.

- 11. (B)** Average rainfall of whole week

$$= \frac{A1P1 + A2P2}{A1 + A2}$$

$$= \frac{(1 \times 3) + (0.3 \times 4)}{4 + 3}$$

$$= \frac{4.2}{7} = 0.6 \text{ inch}$$

The average rainfall of the whole week is 0.6 inch.

Therefore, option B is correct.

- 12. (A)** Total collection of first three days

$$= 2700 \times 3 = 8100$$

Total collection of next three days

$$= 2900 \times 3 = 8700$$

Total collection of first six days

$$= 8100 + 8700 = 16800$$

Collection of the last day = Total collection in a week – Total collection in first six days

$$\text{Collection of the last day} = (2850 \times 7) - 16800$$

$$= \text{Rs. } 3150.$$

The collection of the last day of a shop is Rs. 3150.

Therefore, option A is correct.

- 13. (B)** If the third number is x , the first number is $4x$ and the second is $2x$.

The total sum of the numbers is $= 63 \times 3 = 189$

$$= x + 2x + 4x = 7x = 189$$

$$x = \frac{189}{7} = 27$$

The numbers are

$$= (27 \times 4), (27 \times 2), 27 = 108, 54, 27$$

Therefore, option B is correct.

- 14. (D)** The weight of new bag = Earlier average

– (Decrease in average \times Total number of bags)

$$\text{Weight of new bag} = 30 - (0.4 \times 20)$$

$$= 30 - 8 = 22 \text{ kg}$$

The weight of the new bag is 22 kg.

Therefore, option D is correct.

- 15. (A)** Weight of new student = Weight of student who left + (Increase in average \times Total number of students)

$$\text{Weight of new student} = 28 + (2.5 \times 8)$$

$$= 48 \text{ kg}$$

The weight of the new student is 48 kg.

Therefore, option A is correct.

- 16. (B)** Weight of grandfather = Earlier average + (Increase in average \times Total number of people)

The average weight of joint family = 40 kg

$$\text{Weight of grandfather} = 40 + (0.5 \times 41)$$

$$= 60.5 \text{ kg}$$

The weight of grandfather is 60.5 kg.

Therefore, option B is correct.

- 17. (C)** Let the minimum marks obtained be x
Total marks = $12 \times 4 = 48$

Then from the question,

$$x + (x + 4) + (x + 4 + 4) + (x + 4 + 4 + 4) = 48$$

$$\text{So, } 4x + 24 = 48, x = 6$$

The least mark obtained is 6.

Therefore, option C is correct.

- 18. (D)** The daily wage of newly added worker = Earlier average + (Increase in average \times Total number of people)

$$= 190 + (10 \times 21)$$

$$= \text{Rs. } 400$$

The daily wage of a newly added worker is Rs. 400 per day.

Therefore, option D is correct.

- 19. (A)** The total sum of ages of P, Q, R, and S ten years ago = $40 \times 4 = 160$

Total sum of ages of P, Q, R, and S at present

$$= 160 + 40 = 200$$

Total sum of ages of P, Q, R, S, and A = $55 \times 5 = 275$

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Present age of A = $275 - 200 = 75$ years

The present age of A is 75 years.

Therefore, option A is correct.

- 20. (C)** Total earnings = 1008

Total earnings in first four days = $80 \times 4 = 320$

Total earnings for remaining days

- 25. (A)** Height of new person = earlier average + (increase in average \times total number of people)

$$= 1.2 + (0.1 \times 6)$$

$$= 1.8 \text{ m}$$

The height of the newly added person is 1.8 m.

Therefore, option A is correct.

$$= 1008 - 320 = 688$$

$$\text{Average for remaining days} = \frac{688}{8} = \text{Rs. } 86$$

The average earning of the remaining days is Rs. 86.

Therefore, option C is correct.

- 21. (B)** Height of new pillar = Earlier average + (Increase in average × Total number of pillars)

$$= 10 + (1 \times 25)$$

$$= 35 \text{ feet}$$

The height of the new pillar is 35 feet.

Therefore, option B is correct.

- 22. (A)** Total income of 4 employees

$$= 7350 \times 4 = 29400$$

The total income of 3 employees

$$= 6500 \times 3 = 19500$$

Income of the person who left

$$= 29400 - 19500 = \text{Rs. } 9900$$

The income of the person who left is Rs. 9900.

Therefore, option A is correct.

- 23. (D)** The two middle terms of the given series are 20 and 24

Average of two middle terms

$$= \frac{20 + 24}{2} = 22$$

The average of the given series is 22.

Therefore, option D is correct.

- 24. (B)** Weight of teacher = earlier average + (increase in average × total number of people)

$$= 20 + (2 \times 12)$$

$$= 44 \text{ kg}$$

The weight of the teacher is 44 kg.

Therefore, option B is correct.

- 26. (B)** The first six multiples of 9 are: 9, 18, 27,

36, 45, and 54.

The middle terms of the formed series are 27 and 36.

$$\text{The average of the series is } = \frac{27 + 36}{2}$$

$$= 31.5$$

The average of the first six multiples of nine is 31.5.

Therefore, option B is correct.

- 27. (A)** The average speed of the whole journey

$$= \frac{2xy}{x+y}$$

$$= \frac{2(60 \times 40)}{60+40} = \frac{4800}{100}$$

$$= 48 \text{ km/h}$$

The average speed of the whole journey is 48 km/h.

Therefore, option A is correct.

- 28. (D)** Grouped average of all values

$$= \frac{A1P1 + A2P2}{A1 + A2}$$

$$= \frac{(40 \times 20) + (20 \times 40)}{20 + 40}$$

$$= 20$$

The average of all the values is 20.

Therefore, option D is correct.

- 29. (C)** Average score of nine batsman

$$= \frac{A1P1 + A2P2}{A1 + A2}$$

$$= \frac{(15 \times 6) + (18 \times 3)}{6 + 3} = \frac{144}{9}$$

$$= 16 \text{ runs}$$

The average score of nine batsmen is 16 runs.

Therefore, option C is correct.

- 30. (A)** Total expenditure on four vehicles =

$$100 \times 4 = \text{Rs. } 400$$

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Total daily expenditure after one of the vehicles gets sold = $90 \times 3 = \text{Rs. } 270$

Daily expenditure of vehicle which gets sold = $400 - 270 = \text{Rs. } 130$

The daily expenditure of a vehicle that gets sold is Rs. 130.

Therefore, option A is correct.

PRACTICE QUESTIONS

Questions 1–5: The following table shows the data of sales of cars of different companies from 2015 to 2020.

COMPANY	2015	2016	2017	2018	2019	2020
A	440	480	470	500	520	510
B	400	410	415	415	420	430

C	380	390	390	400	420	495
D	360	380	400	415	440	500
E	480	440	440	420	425	435

1. What is the average sale of cars in company C from the year 2016–2020?
A. 420
B. 419
C. 425
D. 415
2. In which year the average sale of cars is the highest?
A. 2020
B. 2019
C. 2015
D. 2018
3. If the average sale of cars of company D in 2015–2017 is 320 and that of 2018–2020 is 340. Find the average sale of cars of company D from 2015 to 2021.
A. 320
B. 319
C. 335
D. 330
4. What is the average sale of cars in company B from 2016 to 2019?
- A. 420
B. 419
C. 425
D. 415
5. What is the average sale of cars in company A from 2015 to 2019?
A. 470
B. 480
C. 482
D. 450
- Questions 6–10:** There are five sections in the 12th standard which are A, B, C, and D. The number of students in section A is 40 and the average weight of the class is 45 kg. The total sum of weights of 30 students of section C is 1200 kg. The total number of students in section B is 20. The average weight of 35 students of section D is 30 kg. The average weight of section B is 30 kg.
6. If the weight of the teacher is added, the average weight is 46 kg. Find the weight of the teacher.

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- A. 86 kg
B. 79 kg
C. 75 kg
D. 85 kg
7. Find the average weight of students in section C.
A. 42 kg
B. 41 kg
C. 40 kg
D. 38 kg
8. If a student weighing 25 kg is replaced by a new student, the average weight of section D increases by 500 grams. What is the weight of the new student?
A. 42.5 kg
B. 42 kg
- C. 41 kg
D. 40.5 kg
9. What is the average weight of the 12th standard?
A. 37
B. 37.2
C. 37.5
D. 35
10. What will be the average weight of section B, if a student weighs 30 kg removed from the class?
A. 30
B. 31
C. 32
D. 31.5

Questions 11–15: The following table represents the data of marks obtained by students in different subjects.

NAME OF STUDENTS	ENGLISH	HINDI	MATHS	SCIENCE
—	—	—	—	—

P	30	32	38	40
Q	36	32	30	36
R	42	36	34	30
S	40	40	42	26

11. What are the average marks obtained by P?
A. 37
B. 36
C. 37
D. 35
12. What are the average marks obtained by students in science subjects?
A. 32
B. 31
C. 33
D. 34
13. What is the combined average of marks obtained by students in Hindi and Maths?
A. 32
B. 35
C. 34
D. 33
14. What are the average marks obtained by students in English and Science?
A. 33
B. 35
C. 34
D. 33
15. Which student has scored the highest average marks?
A. R B. S
C. Q D. P

SOLUTIONS

1. **(B)** Average sale of C from 2016 to 2020

$$= \frac{390 + 390 + 400 + 420 + 495}{5}$$

$$= \frac{2095}{5} = 419$$

The average sale of company C from 2016 to 2020 is 419.
Therefore, option B is correct.
2. **(A)** Average sales for year 2015

$$= \frac{440 + 400 + 380 + 360 + 480}{5}$$

$$= \frac{2060}{5} = 412$$

Average sales for year 2016

$$= \frac{480 + 410 + 390 + 380 + 440}{5}$$

$$= \frac{2100}{5} = 420$$

Average sales for year 2017

$$= \frac{470 + 415 + 390 + 400 + 440}{5}$$

$$= \frac{2115}{5} = 423$$

Average sale for year 2018

$$= \frac{500 + 415 + 400 + 415 + 420}{5}$$

$$= \frac{2150}{5} = 430$$

Average sale for year 2019

$$= \frac{500 + 400 + 400 + 410 + 405}{5}$$
3. **(D)** Average sale of company D from 2015 to 2020

$$= \frac{A1P1 + A2P2}{A1 + A2}$$

$$= \frac{(320 \times 3) + (340 \times 3)}{3 + 3} = \frac{1980}{6}$$

$$= 330$$

The average sale of company D from 2015 to 2020 is 330.
Therefore, option D is correct.
4. **(D)** The average sale of cars in B from 2016 to 2019

$$= \frac{410 + 415 + 415 + 420}{4}$$

$$= \frac{1660}{4} = 415$$

The average sale of cars in company B from 2016 to 2019 is 415.
Therefore, option D is correct.
5. **(C)** Average sale of cars in A

$$= \frac{440 + 480 + 470 + 500 + 520}{5}$$

$$= \frac{2410}{5} = 482$$

The average sale of cars in company A from 2015 to 2019 is 482.
Therefore, option C is correct.
6. **(A)** Weight of teacher = earlier average + (increase in average \times total number of persons)

$$= 45 + (1 \times 41)$$

$$= \frac{520 + 420 + 420 + 440 + 420}{5}$$

$$= \frac{2225}{5} = 445$$

Average sale for year 2020

$$= \frac{510 + 430 + 495 + 500 + 435}{5}$$

$$= \frac{2370}{5} = 474$$

The highest average sale of cars was in 2020.

Therefore, option A is correct.

= 86 kg

The weight of the teacher is 86 kg.

Therefore, option A is correct.

- 7. (C)** The average weight of students in

$$\text{section C} = \frac{1200}{30}$$

$$= 40 \text{ kg}$$

The average weight of students in section C is 40 kg.

Therefore, option C is correct.

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- 8. (A)** Weight of new student = weight of person who left + (increase in average × total number of people)
 $= 25 + (0.5 \times 35)$

$$= 25 + 17.5 = 42.5 \text{ kg}$$

The weight of the new student is 42.5 kg.
 Therefore, option A is correct.

- 9. (B)** Average weight of 12th standard

$$\begin{aligned} &= \frac{A1P1 + A2P2 + A3P3 + A4P4 + A5P5 \dots + AnPn}{P1 + P2 + P3 + P4 + P5 \dots + Pn} \\ &= \frac{(40 \times 45) + 1200 + (35 \times 30) + (20 \times 30)}{40 + 30 + 35 + 20} \\ &= \frac{4650}{125} = 37.2 \text{ kg} \end{aligned}$$

The total average of the 12th standard is 37.2 kg.
 Therefore, option B is correct.

- 10. (A)** Total sum of weights of section B

$$= 20 \times 30 = 600 \text{ kg}$$

Total weight after removal of student
 $= 600 - 30 = 570 \text{ kg}$

$$\text{New average weight} = \frac{570}{19} = 30 \text{ kg}$$

The average of section B will be the same, i.e., 30 kg.
 Therefore, option A is correct.

- 11. (D)** Average marks of P = $\frac{30 + 32 + 38 + 40}{4}$

$$= \frac{140}{4} = 35$$

The average marks obtained by P are 35.
 Therefore, option D is correct.

- 12. (C)** Average marks in science

$$= \frac{40 + 36 + 30 + 26}{4}$$

$$= \frac{132}{4} = 33$$

- 13. (D)** Average marks in Hindi

$$= \frac{32 + 32 + 36 + 40}{4}$$

$$\text{Average marks in Hindi} = \frac{140}{4} = 35$$

$$\text{Average marks in Maths} = \frac{38 + 30 + 34 + 42}{4}$$

$$= \frac{144}{4} = 36$$

$$\text{Combined average} = \frac{35 + 36}{2} = 35.5$$

The combined average is 35.5.
 Therefore, option D is correct.

- 14. (B)** Average marks in English

$$\begin{aligned} &= \frac{30 + 36 + 42 + 40}{4} \\ &= \frac{148}{4} = 37 \end{aligned}$$

$$\text{Average marks in Science} = \frac{40 + 36 + 30 + 26}{4}$$

$$= \frac{132}{4} = 33$$

$$\text{Combined average} = \frac{37 + 33}{2} = 35$$

Average marks obtained by students in English and Science is 35, therefore, option B is correct.

- 15. (B)** Average marks of P = $\frac{30 + 32 + 38 + 40}{4}$

$$= \frac{140}{4} = 35$$

$$\text{Average marks of Q} = \frac{36 + 32 + 30 + 36}{4}$$

$$= \frac{144}{4} = 36$$

$$\text{Average marks of R} = \frac{42 + 36 + 34 + 30}{4}$$

$$= \frac{142}{4} = 35.5$$

$$\text{Average marks of S} = \frac{40 + 40 + 42 + 26}{4}$$

The average marks of students in science subjects are 33.

Therefore, option C is correct.

$$= \frac{148}{4} = 37$$

S has scored the highest average marks.
Therefore, option B is correct.

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Ratio and Proportion



SYNOPSIS

- Introduction
- Ratio
- Calculating ratio
- Important concepts relating to ratio
- Properties of ratio
- Tips and tricks
- Proportion
- Continued proportion
- Third proportion
- Fourth proportion
- Direct proportion
- Indirect proportion
- Points to remember
- Summary

INTRODUCTION

This topic covers concepts related to ratio and proportion, and it is considered as a very important topic in Mathematics. In daily life also, the concept of ratio and proportion is frequently used. Questions related to ratio and proportion are often asked in GATE, JEE Main, and other engineering entrance examinations; therefore, test takers must be cleared with the concepts related to this topic.

Ratio

Basically, a ratio is a comparison of two quantities or numbers. It refers to the relationship between the two quantities and represents how many times one quantity is equal to the other quantity. Ratios are represented by the symbol of colon, i.e. (:).

The ratio of a to b is written as,

$$a : b = \frac{a}{b} = a \div b$$

In the ratio $a : b$, a and b are called the terms of the ratio, wherein ' a ' is the first term or known

as an antecedent, and ' b ' is the second term or known as consequent.

Example:

In the ratio $3 : 4$, 3 is the first term or antecedent, and 4 is the second term or consequent.

Calculating ratio

For calculating the ratio of two quantities of the same units, we can use the steps quoted hereunder.

Example:

If 30 kg of soil and 40 kg of cement are used to make a wall, let us calculate the ratio of soil and cement used to make a wall.

Solution:

Step 1: First find the quantities for which we have to determine the ratio, and in this case, it is 30 and 40.

Step 2: Write both the quantities in the fraction form, such as $\frac{30}{40}$.

Step 3: Thereafter, simplify the fraction, if possible. The simplified fraction will give the final ratio. Here, $\frac{30}{40}$ can be simplified into $\frac{3}{4}$.

Step 4: Therefore, the ratio of soil to cement is $\frac{3}{4}$.

Important concepts relating to ratio

- To find the ratio of any two quantities, the quantities must be expressed in the same units.

Example:

4 m : 200 cm, therefore, for finding its ratio, both the units must be converted into the same. By converting any of the two units, we can find its ratio. 4 m = 400 cm; therefore the ratio will be 400 cm : 200 cm = 2 : 1

- The ratio would remain same if the antecedent and the consequent in a ratio are multiplied or divided by a constant.

Example:

$$a : b = \frac{a}{b} = \frac{a \times y}{b \times y} = \frac{\underline{a}}{\underline{b}} \text{, wherein } y = \text{constant}$$

- If two different ratios are expressed in different units, then to find out the combined ratio of the two ratios, we have to compound the ratios. This is known as compounding.

The compound of $a : b$ and $p : q$ is $\frac{a \times p}{b \times q}$

Example:

To complete a task, 1 man works for 8 hours a day for 10 days, and 1 boy works for 12 hours a day for 20 days. How many boys can complete the same task that 1 man can do?

Solution:

To know how many boys are required to complete the same task that 1 man can do, we need to combine two different units, i.e., hours and days.

The ratio of days is 10 : 20, the ratio of hours is 8 : 12.

The compound ratio of days can be calculated as, $\frac{a \times p}{b \times q}$

$$= \frac{10 \times 8}{20 \times 12} = \frac{80}{240} = \frac{1}{3} = 1 : 3$$

The inverse ratios of two equal ratios are equal. This property of ratio is called **invertendo**.

- In this law, if $a : b :: c : d$, then $a : c :: b : d$. In this law, the property of continued proportion is used. Suppose you are given two ratios, and the values in both the ratios are proportional to each other. In this law,

This ratio signifies the ratio of men to boys. Therefore, 3 boys are required to complete the same task that one man can do in a day.

- There should be significant order of terms in the ratio.
- If there are two quantities in the ratio $x : y$, then the first quantity is $\frac{x}{x+y}$ times the total and the second quantity is $\frac{y}{x+y}$ times the total of both the quantities.

Properties of ratio

To solve the questions of ratio and proportion, you need to remember the different sets of rules. Particularly while solving questions on proportion, you are required to remember the following set of rules. They are invertendo, alternando, componendo, dividendo, and componendo-dividendo.

- This is the property of the ratio that if $a : b = c : d$, then the overall ratio of the series will be the sum of antecedent to the sum of all the consequents.

$$\text{If } \frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{g}{h} = \dots,$$

$$\text{Ratio} = \frac{\text{Sum of numerators}}{\text{Sum of denominators}}$$

$$= \frac{a+c+e+g+\dots}{b+d+f+h+\dots}$$

- In invertendo, if $a : b = c : d$, then $b : a = d : c$. In this law, you need to remember that if the ratio is in proportion then the inverse of that ratio is also the proportion to each other.

$$\frac{a}{b} = \frac{c}{d}$$

$$a \times b = c \times d$$

$$\frac{a \times d}{a \times c} = \frac{c \times b}{a \times c} \text{ (dividing both sides by } a \times c)$$

$$\frac{d}{c} = \frac{b}{a}$$

$$\text{If, } \frac{a}{b} = \frac{c}{d}, \text{ then } \frac{b}{a} = \frac{d}{c}$$

denominator from the numerator in both ratios. Rest everything in this law is similar to the componendo.

$$\frac{a}{b} = \frac{c}{d}$$

$$\frac{a-1}{b-1} = \frac{c}{d} - 1 \text{ (subtracting 1 from both sides)}$$

$$\underline{a-b} \quad \underline{c-d}$$

when you replace the denominator of the first ratio with the numerator of the second ratio, the two ratios remain proportional to each other.

$$\frac{a}{b} = \frac{c}{d}$$

$$a \times d = b \times c$$

$$\frac{a \times d}{c \times d} = \frac{b \times c}{c \times d} \text{ (dividing both sides by } c \times d)$$

$$\frac{a}{c} = \frac{b}{d}$$

$$\text{If } \frac{a}{b} = \frac{c}{d}, \text{ then } \frac{a}{c} = \frac{b}{d}$$

The ratios of the antecedents and the consequent of two equal ratios are equal. This property of ratio is called **alternando**.

- 4.** In componendo, the basic rule that you need to remember is if $a : b :: c : d$, then $(a + b) : b :: (c + d) : d$. Thus, in componendo as you can see, you need to add the denominator to the numerator given in the ratios and then equate them. If you are using the rule on the left-hand side, then you also need to use the rule on the right-hand side.

$$\frac{a}{b} = \frac{c}{d}$$

$$\frac{a}{b} + 1 = \frac{c}{d} + 1 \text{ (adding 1 to both sides)}$$

$$\frac{a+b}{b} = \frac{c+d}{d}$$

$$\text{If } \frac{a}{b} = \frac{c}{d}, \text{ then } \frac{a+b}{b} = \frac{c+d}{d}$$

Therefore, this property of ratio is known as **componendo**.

- 5.** In the law of dividendo, if $a : b :: c : d$, then $(a - b) : b :: (c - d) : d$. In dividendo, instead of addition, you are required to subtract the

$$\frac{b}{d} - \frac{a}{d}$$

$$\text{If } \frac{a}{b} = \frac{c}{d}, \text{ then } \frac{a-b}{b} = \frac{c-d}{d}$$

Therefore, this property of ratio is called **dividendo**.

- 6.** If the ratio of any two numbers is equal to the ratio of another two numbers, then the ratios of the sum of numerator and denominator to the difference of numerator and denominator of both rational numbers are equal. This property of ratio is called **componendo-dividendo**.

$$\frac{a}{b} = \frac{c}{d}$$

$$\text{From componendo, } \frac{a+b}{b} = \frac{c+d}{d} \quad \dots (1)$$

$$\text{From dividendo, } \frac{a-b}{b} = \frac{c-d}{d} \quad \dots (2)$$

By dividing (1) by (2)

$$\text{We get, } \frac{a+b}{a-b} = \frac{c+d}{c-d}$$

$$\text{If } \frac{a}{b} = \frac{c}{d}, \text{ then } \frac{a+b}{a-b} = \frac{c+d}{c-d}$$

Therefore, this property of ratio is called **componendo-dividendo**.

Example:

$$\text{If } \frac{3a^3 + 5b^3}{3a^3 - 5b^3} = \frac{109}{19}, \text{ find the value of } \frac{5a + 2b}{5a - 2b}.$$

Solution:

$$\frac{3a^3 + 5b^3}{3a^3 - 5b^3} = \frac{109}{19}$$

By applying componendo-dividendo,

$$\begin{aligned} &= \frac{\left[(3a^3 + 5b^3) + (3a^3 - 5b^3) \right]}{\left[(3a^3 + 5b^3) - (3a^3 - 5b^3) \right]} = \frac{109 + 19}{109 - 19} \\ &= \frac{6a^3}{10b^3} = \frac{128}{90} \end{aligned}$$

Ratio and Proportion

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$$\begin{aligned} &= \frac{a^3}{b^3} = \frac{64}{27} \\ &= \frac{a}{b} = \frac{4}{3} \text{ (by taking cube root)} \\ &= \frac{5a}{2b} = \frac{20}{6} = \frac{10}{3} \end{aligned}$$

By applying componendo-dividendo,

$$\frac{5a+2b}{5a-2b} = \frac{13}{7}$$

Tips and Tricks

- In the ratio of $a : b$, if a and b are equal, then $a : b = 1$.
- In ratio $a : b$, if $a > b$, then $a : b > 1$.
- In ratio $a : b$, if $a < b$, then $a : b < 1$.
- To determine the ratio of two quantities, we divide one quantity by the other.

When quantities such as a , b , and c are in continued proportion, b is known as the geometric mean or mean proportion between a and c .

Example:

If $\frac{3}{x} = \frac{4}{14}$ are in continued proportion, then find the value of x .

Solution:

$$\text{In continued proportion } \frac{a}{b} = \frac{b}{c}$$

Therefore, $x = 4$

2. Third proportion:

If a , b , and c are three quantities or numbers such that $a : b = b : c$, then c is called the third proportion to a and b .

Illustration: The third proportion of 6 and

- To determine the ratio, quantities must be of similar units.

Proportion

When the ratio of two terms is equal to the ratio of two other terms, then these four terms are said to be in proportion. In simple words, the ratios are said to be in proportion when the two ratios are equal. If the two sets of given numbers are increasing or decreasing proportionally, then the ratios are said to be in proportion with each other.

To exemplify, if $\frac{a}{b} = \frac{c}{d}$ or $a : b = c : d$, then a , b , c , and d are in proportion.

Here, a and d are known as extremes, while b and c are known as means.

Example:

If $\frac{2}{3} = \frac{7}{14}$, then $2 : 3 = 7 : 14$.

1. Continued proportion:

The three quantities a , b , and c are said to be in continued proportion if the ratio between a and b is equal to the ratio between b and c .

Therefore, if $\frac{a}{b} = \frac{b}{c}$, then $b^2 = ac$, or we can say $b = \sqrt{ac}$

$$15 \text{ is } 'x' \text{ such that } \frac{6}{15} = \frac{15}{x}$$

So, $6x = 225$; therefore, $x = 37.5$.

3. Fourth Proportion:

If a , b , c , and d are four quantities or numbers such that $a : b = c : d$, then d is called the fourth proportion of a , b , and c .

Example:

The fourth proportion of 10, 18, and 30 is a number ' y ' such that $\frac{10}{18} = \frac{30}{y}$

So, $10y = 540$, therefore, $y = 54$.

4. Direct proportion:

If two quantities are related to each other in such a manner that an increase or decrease in one quantity will create a proportionate increase or decrease in the other quantity, then the two quantities are said to be in direct proportion. In simple words, if one quantity increases, the other quantity will increase proportionately, and if one quantity decreases, then the other quantity will also decrease proportionately.

Example:

The area of a circle increases with its increase in radius and decreases with its decrease in radius; therefore, the radius of a circle is directly proportional to its area.

If a and b are directly proportional to each other, then they will be denoted as $a \propto b$.

$a = kb$, where ' k ' is a non-zero constant, which is known as the constant of proportionality.

If a and b are directly proportional to each other, then $\frac{a_1}{b_1} = \frac{a_2}{b_2}$

Example:

The cost of salt varies directly as its weight. 12 kg of salt costs Rs. 138. Find the cost of 5 kg of salt.

Solution:

Let us take the cost of salt to be ' a ', and its weight is ' b '.

Therefore, we have $a \propto b$

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{138}{12} = \frac{a_2}{5}$$

$$a_2 = \frac{138 \times 5}{12}$$

$$a_2 = \text{Rs. } 57.5$$

The cost of 5 kg of salt is Rs. 57.5

5. Indirect proportion:

If two quantities are related to each

$$a = \frac{k}{b}, \text{ where } k \text{ is a non-zero constant,}$$

which is known as the constant of proportionality.

If a and b are inversely proportional to each other, then $a_1b_1 = a_2b_2$

Example:

The number of days required to finish a work is inversely proportional to the number of workers employed. 15 men finish the work in 8 days. Find the number of men to be employed if the same job is to be completed in 6 days.

Solution:

Let us take the number of days to finish the work as ' x ' and the number of men employed as ' y ', then $x \propto \frac{1}{y}$

$$x_1 y_1 = x_2 y_2$$

$$8 \times 15 = 6 \times y_2$$

$$y_2 = \frac{8 \times 15}{6} = 20$$

20 men should be employed to finish the work in 6 days.

other in such a manner that increases in one quantity will create a proportionate decrease in another quantity, and a decrease in one quantity will create a proportionate decrease in another quantity, then the two quantities are said to be in indirect proportion. In simple words, if one quantity increases, the other quantity will decrease proportionately, and if one quantity decreases, then the other quantity will increase proportionately.

If a and b are inversely proportional to each other, then they will be denoted as $a \propto \frac{1}{b}$

Some brief points to remember

1. If $A \propto B$ and $B \propto C$, then $A \propto C$.
2. If $A \propto C$ and $B \propto C$, then $A + B \propto C$ or $A - B \propto C$.
3. If $A \propto BC$, then $B \propto \frac{A}{C}$ and $C \propto \frac{A}{B}$.
4. If $A \propto B$ and $C \propto D$, then $AC \propto BD$.
5. If $A \propto B$, then $A^n \propto B^n$.
6. If $A \propto B$ and $A \propto C$, then $A \propto (B-C)$ and $A \propto (B+C)$.
7. If $A \propto B$, then $AX \propto BX$ where X is any constant, variable or quantity.



Chapter Summary

- The ratio is used to compare two quantities of the same kind, and when two or more ratios are equal, then they are said to be in proportion.
- Comparison can only be done between the quantities of the same units.
- Two ratios are said to be in proportion if they are equal.
- The ratio will remain the same after multiplication and dividing each term of the ratio by a constant.
- We can denote ratio and proportion in two ways, either by $\frac{a}{b} = \frac{c}{d}$ or by $a : b = c : d$.

PRACTICE QUESTIONS

1. What is the third proportion of 32 and 48?
 - 32
 - 48
 - 72
 - None of the above
2. What must be subtracted from the terms of the ratio 3 : 7 to make it 2 : 6?
 - 3
 - 5
 - 6
 - Data insufficient
3. The sum of the present ages of Abhay, Bhanu, and Chetan are 96 years. Eight years ago, their ages were in the ratio 1 : 2 : 3, what is the present age of Chetan?
 - 44
 - 45
4. $\frac{1}{8} : \frac{1}{10} : \frac{1}{12}$. What is the minimum number of chocolates before the distribution?
 - 52
 - 56
 - 57
 - 60
5. How many factors are there of 120?
 - 16
 - 12
 - 10
 - 18
6. If $6x = 8y$ and $5y = 9z$, then find the value of $\frac{z}{x}$.
 - $\frac{20}{50}$

- C. 48
D. 40

4. What is the fourth proportion of 12, 28, and

- 40?
A. 28
B. 56
C. 12
D. 40

5. A person distributes chocolates among four children A, B, C, and D in the ratio $\frac{1}{6} :$

- B. $\frac{20}{38}$
C. $\frac{20}{40}$
D. $\frac{20}{48}$

8. If $a : b = \frac{3}{5}$, find $\frac{2a + 3b}{4a - b}$.

- A. 3 : 1
B. 1 : 2
C. 3 : 2
D. 4 : 3

9. The ratio between the three angles in the triangle is 3 : 4 : 5. What is the difference between the smallest and the largest angle?

- A. 25°
B. 32°
C. 30°
D. 28°

10. If A has 10% less money than that of B who has 20% less than that of C. If C has Rs. 900, then what is the total amount of money A, B, and C have together?

- A. 2896
B. 2268
C. 3000
D. 2800

11. Rs. 9600 is distributed among Suresh, Ramesh, and Mahesh in the ratio of 6 : 5 : 4. What is the difference in the shares of Suresh and Mahesh?

- A. 1280
B. 1250
C. 1200
D. 1300

12. If two friends Vineet and Karan earn in the ratio of 5 : 6. If both of them earn more Rs. 100, then their earnings will be in the ratio 7 : 8. What were Vineet's initial earnings?

- A. 260
B. 240
C. 250
D. 248

13. A person distributed papers among Kartik, Ajay, Vijay, and Deepak in the ratio of 5 : 2 : 4 : 3. If Vijay got 100 papers more than Deepak, then how many papers did Kartik

14. A handbag contains coins of 25 paise, 50 paise, and Re. 1 in the ratio of 5 : 6 : 4. If Rs. 33 is in the handbag, then how many 1 rupee coins are there in the bag?

- A. 16
B. 12
C. 10
D. 18

15. A person distributed Rs. 770 among Ravi, Deepak, and Amit. If Ravi has received two-ninth of what Deepak and Amit together receive, then how much did Ravi receive?

- A. 160
B. 120
C. 140
D. 180

16. Chocolates are distributed among children A, B, and C. For every 10 chocolates that A gets, B gets 6 chocolates and for every 4 chocolates that A gets, C gets 3 chocolates. If a total number of 141 chocolates are distributed, then how many chocolates does B get?

- A. 36
B. 32
C. 30
D. 38

17. An office consists of 70 employees. The ratio of women to men in the office is 2 : 3. How many more men should join the office so that the number of women is half the number of men in the office?

- A. 16
B. 12
C. 10
D. 14

18. A sum of Rs. 1248 was divided among A, B, and C in the ratio of $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$, respectively.

and Ajay together get?

- A. 760
- B. 750
- C. 650
- D. 700

What is the total share of B?

- A. 360
- B. 372
- C. 380
- D. 384



- 19.** A person has distributed a sum of money to Arpit, Bhim, and Chetan in such a manner that for each rupee Arpit gets, Bhim gets 65 paise and Chetan gets 35 paise. If Chetan's share is Rs. 560, then what is the total sum?
- A. 3600
 - B. 3200
 - C. 3100
 - D. 3800
- 20.** The ratio of the number of male employees and female employees in a company of 720 employees is 7 : 5. How many more female employees should join to make the ratio 1 : 1?
- A. 160
 - B. 120
 - C. 130
 - D. 180
- 21.** The prices of a laptop and its accessories are in the ratio 9 : 5. If the laptop costs Rs. 6800 more than its accessories, then what is the price of the laptop?
- A. 16000
 - B. 15200
 - C. 15300
 - D. 15800
- 22.** After the addition of 7 in both the numerator and denominator, a fraction becomes $\frac{3}{4}$. What is the original fraction?
- A. $\frac{5}{12}$
 - B. $\frac{7}{9}$
 - C. $\frac{2}{5}$
 - D. $\frac{8}{9}$
- 23.** If the salaries of three friends A, B, and C are in the ratio of 5 : 7 : 8. If there is an increase in the salary of all of them by 40%, 50%, and 75%, respectively, what will be the ratio of increased salaries?
- 24.** The ratio of pens and pencils in a box is 7 : 8. If the percentage increases in pens and pencils by 20% and 10% respectively, then what will be the new ratio?
- A. 21 : 22
 - B. 12 : 13
 - C. 20 : 21
 - D. 22 : 21
- 25.** In the ratio of 5 : 8, the antecedent is 40, then the consequent is-
- A. 86
 - B. 62
 - C. 64
 - D. 60
- 26.** The population of two cities A and B are in the ratio 2 : 3. If the population is increased by 2000, then the new ratio of the population would become 40 : 57. What is B's population?
- A. 16000
 - B. 12000
 - C. 19000
 - D. 14000
- 27.** If the weights of A, B, and C are in the ratio 2 : 3 : 5. If there is an increase in weight by 15%, 10%, and 20%, then what will be the ratio of their weights?
- A. 23 : 33 : 60
 - B. 12 : 13 : 14
 - C. 22 : 21 : 40
 - D. 33 : 23 : 60
- 28.** Ramesh and Vijay invested in a joint venture in the ratio of 3 : 2. If 5% of the total gains or profits are donated to the trust, and Ramesh's share is Rs. 1710, what is the total gain?
- A. 2500
 - B. 3000
 - C. 2000
 - D. 2800

SOLUTIONS



1. **(C)** If A : B and B : C, then C is the third proportion.
As per the given options, 32 is A, hence cannot be the third proportion.
48 is B, hence cannot be the third proportion.
Therefore, option C, i.e., 72 can be the third proportion.
2. **(D)** The data given in this question are not sufficient to answer the same.
Therefore, option D is correct.
3. **(A)** Total sum of their ages before 8 years = $96 - (3 \times 8) = 72$
Let the age in ratio be x , therefore, eight years ago, ages of Abhay, Bhanu and Chetan be x , $2x$ and $3x$
 $x + 2x + 3x = 72$ years
 $6x = 72$, $x = 12$ years
Age of Chetan eight years ago = $3x = 3(12) = 36$ years
Present age of Chetan = $36 + 8 = 44$ years
The present age of Chetan is 44 years.
Therefore, option A is correct.
4. **(B)** If A : B = C : D, then D is the fourth proportion.
Options A, C and D are the quantities already present in the ratio proportion; thus, they cannot be the fourth proportion.
Therefore, only option B being the different quantity, can be the fourth proportion.
5. **(C)** The ratio among the children A, B, C, and D = $\frac{1}{6} : \frac{1}{8} : \frac{1}{10} : \frac{1}{12}$
On rearranging the ratio = $\frac{120}{6}, \frac{120}{8}, \frac{120}{10}, \frac{120}{12} = 20 : 15 : 12 : 10$

Hence, the minimum number of chocolates can be when the common ratio is 1.
Thus, minimum number of chocolates = $20 + 15 + 12 + 10 = 57$.
Therefore, option C is correct.

6. **(A)** The factors of 120 are the integers that divide 120 without leaving any remainder.
Therefore, such factors of 120 are, 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, and 120.
Hence, there are a total 16 factors of 120.
Therefore, option A is correct.
7. **(D)** $6x = 8y$ (1)
 $9z = 5y$ (2)
Dividing (2) by (1), we get
$$\frac{9z}{6x} = \frac{5y}{8y}$$

$$\frac{z}{x} = \frac{5}{12} = \frac{20}{48}$$

Therefore, option D is correct.
8. **(A)** Let $a = 3x$ and $b = 5x$
$$\frac{2a + 3b}{4a - b} = \frac{2(3x) + 3(5x)}{4(3x) - 5x}$$

$$= \frac{21}{7} = 3 : 1$$

Therefore, option A is correct.
9. **(C)** Let the angles in the triangle be $3x$, $4x$, and $5x$
Sum of angles in a triangle = 180
Smallest angle = $\frac{3}{12} \times 180 = 45^\circ$
Largest angle = $\frac{5}{12} \times 180 = 75^\circ$
Difference between smallest and largest = $75 - 45 = 30^\circ$.
Therefore, option C is correct.
10. **(B)** Given that C has Rs. 900

$$D = 900 \times \frac{1}{100} = \text{Rs. } 120$$

$$C = 720 \times \frac{90}{100} = \text{Rs. } 648$$

Total amount A, B, and C have together
 $= 900 + 720 + 648 = \text{Rs. } 2268$
 Therefore, option B is correct.

11. (A) Let the shares be $6x$, $5x$ and $4x$

$$\text{So, } 6x + 5x + 4x = 9600$$

$$15x = 9600, x = 640$$

$$\text{Share of Suresh} = 6x = 6(640) = 3840$$

$$\text{Share of Mahesh} = 4x = 4(640) = 2560$$

$$\text{Difference} = 3840 - 2560 = \text{Rs. } 1280$$

Therefore, option A is correct.

12. (C) Let Vineet and Karan's earnings be $5x$ and $6x$ then

$$= \frac{5x + 100}{6x + 100} = \frac{7}{8}$$

$$= 40x + 800 = 42x + 700$$

$$x = 50$$

$$\text{Vineet's earnings} = 5(50) = \text{Rs. } 250.$$

Therefore, option C is correct.

13. (D) Let the shares of papers among Kartik, Ajay, Vijay, and Deepak be $5x$, $2x$, $4x$, $3x$ respectively

$$\text{Therefore, } 4x - 3x = 100$$

$$x = 100$$

$$\text{Kartik and Ajay share} = 5(100) + 2(100) = 700$$

Therefore, option D is correct.

14. (A) Let the number of coins of 25 paise, 50 paise and Re. 1 be $5x$, $6x$, and $4x$, respectively.

$$\text{Then the net value} = 0.25(5x) + 0.50(6x) + 1(4x) = 33$$

$$1.25x + 3x + 4x = 33$$

$$8.25x = 33, x = 4$$

$$\text{Number of Re. 1 coins} = 4 \times 4 = 16.$$

Therefore, option A is correct.

15. (C) Let the amount received by Ravi, Deepak and Amit be x , y , and z .

$$x + y + z = 770$$

$$\frac{2(z+y)}{9} + y + z = 770$$

$$11x + 11y = 6930$$

Ratio and Proportion

To make the ratio $1 : 1$, $420 - 300 = 120$ more female employees should join the company.

Therefore, option B is correct.

21. (C) Let the price of the laptop and its accessories be $9x$ and $5x$, respectively.

$$\text{Given that } 9x - 5x = \text{Rs. } 6800$$

$$4x = \text{Rs. } 6800, x = 1700$$

$$\text{Price of laptop} = 9(1700) = \text{Rs. } 15300.$$

and Amit = Rs. 630

Amount received by Ravi = $770 - 630 = \text{Rs. } 140$

Therefore, option C is correct.

16. (A) $A : B = 10 : 6 = 5 : 3$ (1)

$$A : C = 4 : 3 \quad \dots\dots(2)$$

Multiplying (1) by 4 and (2) by 5 to get a common value of A

$$A : B = 20 : 12; A : C = 20 : 15$$

$$A : B : C = 20 : 12 : 15$$

$$\text{Share of B} = \frac{12}{47} \times 141 = 36.$$

Therefore, option A is correct.

17. (D) Number of women = $\frac{2}{5} \times 70 = 28$

$$\text{Number of men} = \frac{3}{5} \times 70 = 42$$

Let x be the more men should join so that

$$\frac{28}{42+x} = \frac{1}{2}$$

$$56 = 42 + x, x = 14.$$

Therefore, option D is correct.

18. (D) Share of A : B : C = $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$

$$= \frac{6}{12} : \frac{4}{12} : \frac{3}{12} = 6 : 4 : 3$$

$$\text{Share of B} = \frac{4}{13} \times 1248 = \text{Rs. } 384.$$

Therefore, option D is correct.

19. (B) Share of Arpit = $\frac{560}{35} \times 100 = \text{Rs. } 1600$

(Rs. 1 = 100 paisa)

$$\text{Share of Bhim} = \frac{540}{35} \times 65 = \text{Rs. } 1040$$

$$\text{Total sum} = 1600 + 1040 + 560 = \text{Rs. } 3200.$$

Therefore, option B is correct.

20. (B) Let the number of male employees and female employees be $7x$ and $5x$, respectively.

$$7x + 5x = 720$$

$$12x = 720, x = 60$$

Number of male employees = 420, number of male employees = 300

Therefore, option A is correct.

25. (C) Let the antecedent and consequent be $5x$ and $8x$, respectively.

$$5x = 40, x = 8$$

$$\text{Consequent} = 8(8) = 64$$

Therefore, option C is correct.

26. (C) Let $2x$ and $3x$ be the original population of A and B, respectively.

$$2x + 2000 = 40$$

Therefore, option C is correct.

22. (C) Option A $\frac{5+7}{12+7} = \frac{12}{19}$

Option B $\frac{7+7}{9+7} = \frac{14}{16} = \frac{7}{8}$

Option C $\frac{2+7}{5+7} = \frac{9}{12} = \frac{3}{4}$

Therefore, option C is correct.

23. (D) Let the salaries of A, B and C be $5x$, $7x$ and $8x$ respectively

Increment in salary (140% of $5x$), (150% of $7x$), (175% of $8x$)

$$\left(\frac{140}{100} \times 5x\right), \left(\frac{150}{100} \times 7x\right), \left(\frac{175}{100} \times 8x\right)$$

$$7x, \frac{21x}{2} \text{ and } 14x$$

Thus, the required ratio will be $14x : 21x : 14x = 2 : 3 : 4$.

Therefore, option D is correct.

24. (A) Let the pens and pencils in the box be $7x$ and $8x$ respectively.

Increased number is (120% of $7x$) and (110% of $8x$)

$$\left(\frac{120}{100} \times 7x\right) \text{ and } \left(\frac{110}{100} \times 8x\right)$$

$$\text{So, } \frac{42x}{5} \text{ and } \frac{44x}{5}$$

Thus, the required ratio would be $42 : 44 = 21 : 22$.

Then, $\frac{3x + 2000}{3x + 2000} = \frac{57}{57}$

$$57(2x + 2000) = 40(3x + 2000)$$

$$6x = 34000$$

$$3x = 17000$$

$$\text{B's present population} = 17000 + 2000 = 19000$$

Therefore, option C is correct.

27. (A) Let the weights of A, B and C be $2x$, $3x$ and $5x$ respectively.

$$\text{A's weight} = \frac{115}{100} \times 2x = \frac{23x}{10}$$

$$\text{B's weight} = \frac{110}{100} \times 3x = \frac{33x}{10}$$

$$\text{C's weight} = \frac{120}{100} \times 5x = \frac{6x}{10}$$

$$\text{New required ratio} = 23 : 33 : 60$$

Therefore, option A is correct.

28. (B) Let the profit be Rs. 100

After donating to the trust, Ramesh's share

$$\text{is Rs. } (95 \times \frac{3}{5}) = \text{Rs. } 57$$

If Ramesh's share is Rs. 57, total profit = Rs. 100

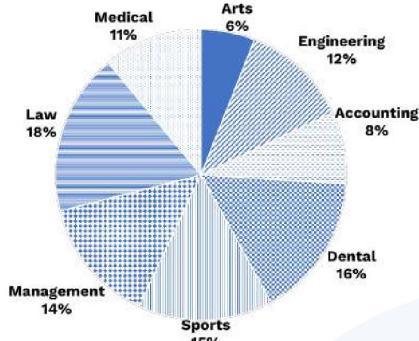
$$\text{If Ramesh's share is Rs. 1710, the total profit} \\ = \text{Rs. } \left(\frac{1710 \times 100}{57}\right) = \text{Rs. } 3000$$

Therefore, option B is correct.

PRACTICE QUESTIONS

Questions 1–5: Analyse the given data and answer the following questions:

Total number of students = 2500



The ratio of boys to girls:

C. 75

D. 100

3. What is the ratio between the total number of boys in Dental and Sports and the total number of girls in these two departments, respectively?

A. 13 : 17

B. 17 : 14

C. 13 : 14

D. 14 : 13

4. How many girls are pursuing accounts?

A. 130

B. 120

C. 125

D. 110

DEPARTMENT	BOYS	GIRLS
Engineering	7	5
Accounting	2	3
Dental	5	3
Sports	7	8
Management	3	4
Law	5	4
Medical	6	5
Arts	2	1

- What is the ratio between boys and girls students in engineering and arts respectively?
 - 7 : 4
 - 4 : 7
 - 3 : 4
 - 2 : 3
- What is the difference between the total number of students in management and law?
 - 80
 - 150

Ratio and Proportion

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working females of region P is how much per cent less than that working females of region Q?

- $\frac{58}{11}\%$
- $\frac{67}{11}\%$
- $\frac{100}{11}\%$
- $\frac{89}{11}\%$
- Total female citizens of region P and R together is what per cent of total citizens of these two regions together?
 - 28.18%
 - 30.18%
 - 32.18%
 - 38.18%

- What is the difference between boys and girls in the medical department?
 - 25
 - 75
 - 100
 - 40

Questions 6–10: Analyse the given data and answer the following questions:

Total citizens of Konoha are 8000 and there are 3 regions, i.e., P, Q and R in it. Ratio of citizens of these 3 regions (P : Q : R) is 8 : 5 : 3. Number of males in region P is 1900 more than that of females in region R. The ratio of number of females in region P to that of males in region R is 5 : 3. The number of females in region Q is 30% of the number of males in region P.

- What is the average of the number of males in region P and Q together is how much more than the number of females in region P?
 - 650
 - 625
 - 675
 - 610
- 60% and 88% of females of region P and Q, respectively, are working, then find non-working females of region P and Q together.



- What is the ratio of the total number of females in all regions to that of the total number of males in all regions?
 - $\frac{42}{89}$
 - $\frac{47}{139}$
 - $\frac{39}{55}$
 - $\frac{57}{103}$
- The average of male citizens of region Q and R is how much more than the average of female citizens of these two regions?
 - 650
 - 600
 - 550
 - 500

SOLUTIONS

- (A) Boys in engineering = $\frac{7}{12}$ of 12% of total students

$$\text{Boys in arts} = \frac{2}{15} \text{ of } 6\% \text{ of total students}$$

$$\text{Total students in sports} = \frac{7}{15} \times 375 = 175$$

$$\text{Girls in sports} = 375 - 175 = 200$$

$$\text{Required ration} = 425 : 350 = 17 : 14$$

$$\begin{aligned} & \text{3} \\ & \text{Ratio} = \frac{7}{12} \text{ of } 12\% \times \text{total students} \\ & \quad = \frac{2}{3} \times 6\% \times \text{total students} \\ & \quad = \frac{14}{12} \times \frac{3}{2} = 7 : 4 \end{aligned}$$

Therefore, option A is correct.

$$\begin{aligned} 2. \quad (\mathbf{D}) \text{ Required difference} &= 2500 \times (18-14)\% \\ &= \frac{2500 \times 4}{100} = 100 \end{aligned}$$

Therefore, D is correct.

$$\begin{aligned} 3. \quad (\mathbf{B}) \text{ Total students in dental} &= 16\% \text{ of total students} \\ &= \frac{16}{100} \times 2500 = 400 \text{ students in dental} \\ \text{Girls in dental} &= 400 - 250 = 150 \end{aligned}$$

Therefore, option B is correct.

$$\begin{aligned} 4. \quad (\mathbf{B}) \text{ Total girls} &= \frac{3}{5} \text{ of } 8\% \text{ of total students} \\ &= \frac{3}{5} \times \frac{8}{100} \times 2500 = 120 \\ \text{Therefore, option B is correct.} \end{aligned}$$

$$\begin{aligned} 5. \quad (\mathbf{A}) \text{ Difference} &= \frac{6}{11} \times \frac{11}{100} \times 2500 - \frac{5}{11} \times \frac{11}{100} \times 2500 \\ &= 150 - 125 = 25 \\ \text{Therefore, option A is correct.} \end{aligned}$$

Solutions 6–10:

Let the number of males in region R be $3x$, the number of females in region P will be $5x$.
Ratio of the citizens in three regions P : Q : R
 $= 8 : 5 : 3$



$$\text{Citizens in P} = 8 \times \frac{8000}{16} = 4000$$

$$\text{Citizens in Q} = 5 \times \frac{8000}{16} = 2500$$

$$\text{Citizens in R} = 3 \times \frac{8000}{16} = 1500$$

Number of males in region P is 1900 more than that of females in region R.

The given information can be tabulated as:

P(4000)		Q(2500)		R(1500)	
MALES	FEMALES	MALES	FEMALES	MALES	FEMALES
1500 – $3x + 1900$	$5x$		$30 \frac{\{1500 - 3x + 1900\}}{100}$	$3x$	$1500 - 3x$

The total citizens in region P = 4000

$$(1500 - 3x + 1900) + 5x = 4000$$

$$3400 + 2x = 4000$$

$$x = 300$$

Now, the table will be

P(4000)		Q(2500)		R(1500)	
MALES	FEMALES	MALES	FEMALES	MALES	FEMALES
2500	1500	1750	750	900	600

6. **(B)** Males in P = 2500, males in Q = 1750

$$\text{Average} = \frac{2500 + 1750}{2} = 2125$$

$$\text{Females in P} = 1500$$

$$\text{Required difference} = 2125 - 1500 = 625$$

Therefore, option B is correct.

7. **(C)** Females in P = 1500, working females in P = 60% of 1500 non-working females in P

Therefore, option D is correct.

9. **(D)** Total number of females in all regions = $1500 + 750 + 600 = 2850$

Total number of males in all regions = $2500 + 1750 + 900 = 5150$

$$\text{Required ratio} = \frac{2850}{5150} = \frac{57}{103}$$

r = 60% of 1500, non-working females in R

$$= 40\% \text{ of } 1500 = 600$$

Females in Q = 750, working females in Q = 88% of 750 = 660

$$\text{Required \%} = (660 - 600) \times \frac{100}{660} = \frac{100}{11} \%$$

Therefore, option C is correct.

8. (D) Females in P and R = 1500 + 600 = 2100

Total citizens of P and R = 4000 + 1500 = 5500

$$\text{Required \%} = 2100 \times \frac{100}{5500} = 38.18\%$$

Therefore, option D is correct.

10. (A) Males in Q = 1750

Males in R = 900

$$\text{Average} = \frac{1750 + 900}{2} = 1325$$

Females in Q = 750

Females in R = 600

$$\text{Average} = \frac{750 + 600}{2} = 675$$

$$\text{Required difference} = 1325 - 675 = 650$$

Therefore, option A is correct.

33 Mixtures and Alligations



SYNOPSIS

- Mixtures
- Mean price
- Alligation
- Rule of alligation
- Criss-cross method
- Common types of questions asked
- Summary

MIXTURE

A mixture is the combination of two or more quantities.

Mean price

The cost price of the unit quantity of a mixture is called the mean price.

When a person buys x kg of the first kind of an item and y kg of the second kind of an item, the total amount he spends is $\text{Rs.}(px + qy)$ and the total amount of mixture is $(x + y)$ kg.

But the ratio in which these two ingredients at the given price must be mixed to produce a mixture of a different desired price is calculated using the rule of alligation.

ALLIGATION

Rule of alligation is used to find the ratio in which two or more ingredients must be mixed at a given price to produce the mixture at a desired price.

Rule of alligation

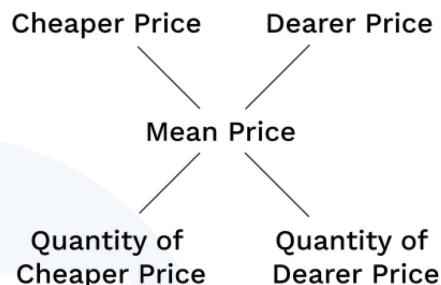
If two items are mixed, then

$$\left(\frac{\text{quantity of cheaper item}}{\text{quantity of dearer item}} \right) = \frac{(\text{CP of dearer}) - (\text{mean price})}{(\text{mean price}) - (\text{CP of cheaper})}$$

$$\text{cheaper quantity : dearer quantity} = (d - m) : (m - c)$$

Where, d , m , and c are dear price, mean price, and cheap price respectively.

Criss-cross method



The difference between mean price and cheaper price in the place of 'quantity of dearer price' and the difference between mean price and dearer price in the place of 'quantity of cheaper price.'

Example:

In what ratio were two different kinds of cereals costing Rs. 45 per kg and 60 per kg, mixed to get a mixture costing Rs. 50 per kg.

Solution:

Applying the Criss-Cross method:

$$\text{Ratio} = \frac{(60 - 50)}{50 - 45} = \frac{10}{5}$$

Therefore, the ratio of the two types of cereals = 2 : 1.

COMMON TYPES OF QUESTIONS ASKED

1. The ratio of items required

In this type of questions, the price and quantity of the mixture along with the price of items

Or,

are provided, and the students are required to calculate the ratio in which the items are



to be mixed to get the required amount of mixture, using the given price and quantity of the mixture along with the price of items.

Example:

In what proportion must a shopkeeper mix rice type 1 at Rs. 4.08 per kg and rice type 2 at Rs. 5.76 per kg to make a mixture worth Rs. 5.04 per kg?

Solution:

$$\text{Cheap price} = \text{Rs. } 4.08$$

$$\text{Mean price} = \text{Rs. } 5.04$$

$$\text{Dear price} = \text{Rs. } 5.76$$

Thus, according to the alligation formula, cheap quantity : dear quantity = $(d - m) : (m - c)$

$$\begin{aligned}\text{Required proportion} &= (5.76 - 5.04) : (5.04 - 4.08) \\ &= 0.72 :: 0.96 = 3 : 4\end{aligned}$$

2. The final quantity of the original item

In these types of questions, the original quantity of a liquid is provided along with the amount of liquid that is being taken out and replaced by some other liquid two or three times, and the students have to calculate the final quantity of the original liquid left.

Example:

A container has 20 L milk. 4 L of milk is replaced with an equal quantity of water. What will be the final quantity of milk left in the container if this process is repeated once again?

Solution:

$$\text{Original quantity of milk} = 20 \text{ L}$$

After taking out 4 L and replacing it with water, milk left = 16 L

When repeating the process, after taking out 4 L of this new mixture

$$\text{Quantity of milk taken out} = 4 \times \frac{16}{20} = 3.2 \text{ L}$$

Thus, the required quantity of milk left = $32 - 3.2 \text{ L} = 28.8 \text{ L}$

3. The ratio of two liquids in the third container

In these types of questions, the ratio of two liquids in two different containers is given, and the students are asked to calculate the new ratio of these two liquids if both the containers are poured out in a third container together.

Example:

Two jars have oil and milk in the ratio 4 : 1 and 5 : 1, respectively. If both jars are emptied into a third jar, then what will be the ratio of oil and water in this third jar?

Solution:

$$\text{Ratio of oil and milk in 1st jar} = 4 : 1$$

$$\text{Quantity of oil in this 1st jar} = \frac{4}{5}$$

$$\text{Quantity of milk in this 1st jar} = \frac{1}{5}$$

$$\text{The ratio of oil and milk in 2nd jar} = 5 : 1$$

$$\text{Quantity of oil in this 2nd jar} = \frac{5}{6}$$

$$\text{Quantity of milk in this 2nd jar} = \frac{1}{6}$$

$$\text{In the third jar, the quantity of milk} = \frac{4}{5} + \frac{5}{6} = \frac{49}{30}$$

$$\text{Quantity of water} = \frac{1}{5} + \frac{1}{6} = \frac{11}{30}$$

$$\text{Therefore, required ratio} = \frac{49}{30} : \frac{11}{30} = 49 : 11$$

Note

- If not given, assume the cost of water to be Rs. 0.



Chapter Summary



- Rule of alligation: of two items are mixed, then

$$\left(\frac{\text{quantity of cheaper item}}{\text{quantity of dearer item}} \right) = \frac{(\text{CP of dearer}) - (\text{mean price})}{(\text{mean price}) - (\text{CP of cheaper})}$$

Or,

$$\text{cheaper quantity : dearer quantity} = (d - m) : (m - c)$$

- Where, d , m , and c are dearer price, mean price, and cheap price, respectively.

PRACTICE QUESTIONS

- If 50 L of oil and water mixture comprises of 50% oil, then how much more oil should be mixed to make it 70% in the mixture?
 - 33.33 L
 - 28.14 L
 - 16.66 L
 - 11.11 L
- A mixture of x mL milk and 8 mL water costs Rs. 90 per mL. If the cost price of milk is Rs. 108 mL, then how much milk is there in the mixture?
 - 40 mL
 - 32 mL
 - 36 mL
 - 44 mL
- In a container, liquid A and liquid B are in their ratio of 7 : 5. If 45 L more of liquid B is mixed in the mixture, the ratio of liquid B and A becomes 8 : 7. Find the amount of liquid B in the old mixture.
 - 65 L
 - 70 L
 - 75 L
 - 80 L
- A mixture of Rooh Afza and 16 L of water costs rupees 95 per litre. If pure Rooh Afza costs Rs.114 per litre, then what is the amount of Rooh Afza in the original mixture?
 - 7 mg
 - 10 mg
 - 15 mg
 - 12 mg
- A shopkeeper sells a mixture of 45 L milk and 9 L water at Rs. 66 per kg, what is the cost price of pure milk?
 - 86.4 L
 - 76.5 L
 - 79.2 L
 - 84.7 L
- In a mixture of 15 mL, ratio of two liquids is 4 : 2. What amount of liquid 2 should be mixed in this to make the ratio of the two liquids 1 : 2.
 - 5 mL
 - 8 mL
 - 15 mL
 - 18 mL
- 150 mg of sugar solution has 20% sugar in it. What quantity of sugar should be added in the sugar water solution to make sugar 25% in the solution?
 - 7 mg
 - 10 mg
 - 15 mg
 - 12 mg
- 5 kg of alloy X is mixed with 25 kg of alloy Y. If the amount of copper and zinc in alloy
 - Mixtures and Alligations



X is in the ratio 1 : 2, and the amount of copper and zinc in alloy Y is in the ratio of

- If selling price of milk is Rs. 100. In what ratio should a salesman mix water in that

2 : 3, then what is the ratio of copper to tin in the resulting mixture?

- A. 1 : 3
- B. 3 : 5
- C. 5 : 7
- D. 7 : 11

9. A deodorant contains butane and benzyl alcohol in the ratio 5 : 3. If 8 L of the mixture is replaced with the same quantity of benzyl alcohol, the ratio becomes 3 : 5. What is the total quantity of the container?
- A. 5 L
 - B. 10 L
 - C. 15 L
 - D. 20 L

10. 27 litres of milk and water mixture in the ratio of 4 : 5 is added to a bottle containing x litre pure milk. If the new mixture formed contains 70% milk, what was the original quantity of pure milk in the bottle?
- A. 21 L
 - B. 23 L
 - C. 25 L
 - D. 27 L

11. A mixture of 10 g of wheat and maida contains 20% maida. How much maida should be added to this mixture of wheat and maida to get 33.33% maida?
- A. 6 g
 - B. 2 g
 - C. 4 g
 - D. 5 g

12. A jewellery shop has two qualities of gold, 24 carats, and 22 carats purity gold. In what proportion should he mix both qualities of gold to make an ornament having 20 carats purity?
- A. 1 : 2
 - B. 5 : 3
 - C. 3 : 4
 - D. 4 : 5

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- C. 6.5 mL
- D. 5 mL

18. 4 mL of liquid from an oil bottle is replaced with water. And then this action is repeated three times more, until the ratio of oil to water in the container equals 16 : 65. How much oil was in the container at the start?
- A. 16 mL
 - B. 12 mL
 - C. 15 mL

milk, to sell the water-milk mixture at Rs. 80, and obtain a profit of 50%?

- A. 7 : 8
- B. 8 : 9
- C. 9 : 11
- D. 11 : 13

14. How much amount of butter worth 25 per kg should be blended with 30 kg butter that costs 30 per kg to gain 10% and sell the mixture at 30 per kg?

- A. 36 kg
- B. 35 kg
- C. 40 kg
- D. 38 kg

15. There are two different types of plastic available in market, one having the ratio of components A and B as 2 : 11 and other having same, 5 : 21. In what proportion should a manufacturer combine both the components to get A and B in 7 : 32 ratio?

- A. 1 : 2
- B. 3 : 4
- C. 2 : 3
- D. 5 : 6

16. When 70% of the juice and 30% of the water were removed from a full vessel, it gets emptied by 55%, leaving only 160 L of mixture. What is the remaining amount of juice and water in the mixture?

- A. Water = 60 L, juice = 100 L
- B. Water = 90 L, juice = 70 L
- C. Water = 80 L, juice = 80 L
- D. Water = 100 L, juice = 60 L

17. The ratio of acid to ethanol in a 6 mL liquid is 2 : 1. If the acid-to-ethanol ratio has to be 1 : 2, the amount of ethanol to be added to the mixture is

- A. 5.5 mL
- B. 6 mL

buttermilk in the ratio 5 : 8, calculate the ratio in which mixture from these two jars must be mixed.

- A. 3 : 5
- B. 3 : 4
- C. 1 : 5
- D. 7 : 9

23. A jar full of soda water contains 40% soda. A part of this soda water is replaced by another mixture containing 19% soda,

D. 13 mL

19. A container is filled with pulses, 6 parts of which are type 1 and 10 part type 2. How much of the mixture must be replaced with type 2 so that the mixture contains half type 1 and half type 2?

A. $\frac{2}{3}$
B. $\frac{3}{7}$
C. $\frac{1}{5}$
D. $\frac{1}{8}$

20. A mixture of 48 mL contains H_2SO_4 and HCl in the ratio of 3 : 5. How much HCl must be added to this mixture to reverse the ratio?

A. 10 mL
B. 14 mL
C. 8 mL
D. 16 mL

21. How many litres of refined oil costing Rs. 9 per L must be mixed with 27 L of refined oil costing Rs. 7 per L to get a gain of 10% by selling the mixture at Rs. 9.24 L.

A. 61 L
B. 58 L
C. 65 L
D. 63 L

22. Two jars B and A have water and buttermilk mixed in the ratio 2 : 5 and 6 : 7. If a new mixture in vessel C must have water and

and now the jar contains 26% soda. The quantity of soda water replaced is:

A. $\frac{2}{5}$
B. $\frac{1}{5}$
C. $\frac{2}{3}$
D. $\frac{3}{5}$

24. A juice shop has two big jars of juice. First one contains 25% water and 75% fruit pulp. Second one contains 50% water and 50% fruit pulp. How much juice should the shop mix from each of the containers to get 12 litres of fruit pulp, and ratio of water and pulp being 3 : 5?

A. 2 L
B. 4 L
C. 6 L
D. 8 L

25. The cost of onion A is Rs. 15 per kg and onion B is Rs. 20 per kg. If both types A and B are mixed in the ratio of 2 : 3, then the price per kg of the mixed variety of onion is:

A. 13 per kg
B. 18 kg
C. 16 kg
D. 12 kg

26. In what proportion water must be added to ethanol to gain 20% by selling resulting mixture at the cost price?

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A. 4 : 3
B. 3 : 4
C. 5 : 6
D. 1 : 5

27. In a pot, a mixture of curd and water is in the ratio of 4 : 5. When it is added an additional 8 g of curd, the pot gets full, and the ratio of curd and water becomes 6 : 5. What is the capacity of the pot?

A. 11 g
B. 22 g
C. 33 g
D. 44 g

28. The diluted hydrochloric acid contains 8 litres of HCl and rest water. How many litres of the mixture should be replaced with pure hydrochloric acid to get mixture

B. 70%
C. 75%
D. 62%

30. The ratio of ethanol and isopropanol in two different sanitisers is 2 : 3 and 4 : 5. In what ratio are we required to mix the mixtures of two sanitizers in order to get the new mixture in which the ratio of ethanol and isopropanol is 7 : 5?

A. 11 : 7
B. 7 : 5
C. 5 : 3
D. 3 : 1

31. A mixture contains diesel and oil in a ratio of 2 : 3. How much oil should be added to the 60 L mixture to make the ratio of diesel and mixture as 1 : 3?

- of 30% concentration if there were initially 32 litres of water in the mixture?
- 2 L
 - 3 L
 - 4 L
 - 5 L

- 29.** In a mixture of sand and water, the proportion of sand by weight was 75%. If in 60 grams of mixture, 15 gram sand was added, what would be the percentage of sand?
- A. 83%

- 16
- 12
- 10
- 18

- 32.** The ratio of water and dye is 5 : 2 in the mixture of 28 litres. If 2 litres of dye are added to the mixture, then what will be the ratio of water and dye in the new solution?
- 1 : 3
 - 1 : 2
 - 3 : 1
 - 2 : 5

SOLUTIONS

- 1. (A)** Initial quantity of oil = 25 L

Oil to be added = x L

Since water remains same,

$$30\% \text{ of } (50 + x) = 25 \text{ L}$$

$$150 + 3x = 250 \text{ L}$$

$$X = 33.33 \text{ L}$$

Therefore, option A is correct.

- 2. (A)** Water : Milk = $(108 - 90) : (90 - 0)$
 $= 18 : 90 = 1 : 5$

Thus, if the water in mixture is 8 mL, milk quantity will be $8 \times 5 = 40$ mL
 Therefore, option A is correct.

- 3. (C)** Liquid A : liquid B = 7 : 5
 Liquid B initially = $5x$
 Liquid B after addition = $5x + 45$
 New ratio = 7 : 8 = $7x : 5x + 45$
 $56x = 35x + 315$
 $21x = 315$

$$x = 15$$

Thus, original liquid B = 75 L

Therefore, option C is correct.

- 4. (D)** Water : Rooh Afza = $(114 - 95) : (95 - 0)$
 $= 19 : 95 = 1 : 5$

Thus, if water in mixture is 16 L, Rooh Afza quantity will be $16 \times 5 = 80$ L
 Therefore, option D is correct.

- 5. (C)** $\frac{\text{Quantity of water}}{\text{quantity of milk}} = \frac{x - 66}{66 - 0}$
 $\Rightarrow \frac{45}{9} = \frac{66}{x - 66}$
 $\Rightarrow 5x - 330 = 66$
 $\Rightarrow x = \frac{396}{5} = 79.2$

Therefore, option C is correct.

- 6. (C)** The initial quantity of liquid 1 = 10 mL
 The initial quantity of liquid 2 = 5 mL
 Since liquid 1 remains same,

$$\frac{1}{3} \times (15 + x) = 10 \text{ mL}$$

$$X = 15 \text{ mL}$$

To get the mixture in ratio 1 : 2, (i.e., liquid 2 double the amount of liquid 1)

Liquid 2 to be added = 15 mL

Therefore, option C is correct.

$$\text{Amount of zinc} = \frac{10}{3} + 15 = \frac{55}{3}$$

The ratio of copper and zinc in new mixture = 7 : 11
 Therefore, option C is correct.

- 9. (D)** Quantity of butane removed
 $= \frac{5}{5+3} \times 8 = 5 \text{ L}$

Quantity of benzyl alcohol removed
 $= \frac{3}{5+3} \times 8 = 3 \text{ L}$

$$\text{New ratio}, \quad 3 : 5 = \frac{5x - 5}{3x - 3 + 8}$$

$$9x + 15 = 25x - 25$$

$$x = 2.5$$

Total quantity of container = 20 L

Therefore, option D is correct.

- 10. (B)** Quantity of milk in 27 L mixture
 $= \frac{4}{4+5} \times 27 = 12 \text{ L}$

Quantity of water = 15 L

In new mixture,

$$\Rightarrow \frac{7}{3} = \frac{x + 12}{15}$$

$$3x + 36 = 15 \times 7 = 105$$

$$x = 23 \text{ L}$$

7. (B) Initial quantity of sugar = 30 g

Since water remains same,

$$75\% \text{ of } (150 + x) \text{ g} = 120 \text{ g}$$

$$x = 10 \text{ g}$$

Sugar to be added = 10 g

Therefore, option B is correct.

8. (C) Amount of copper in alloy $x = \left(\frac{1}{3}\right) \times 5 = \frac{5}{3}$

$$\text{Amount of zinc in alloy } x = \left(\frac{2}{3}\right) \times 5 = \frac{10}{3}$$

$$\text{Amount of copper in alloy } x = \left(\frac{2}{5}\right) \times 25 = 10 \text{ kg}$$

$$\text{Amount of zinc in alloy } y = \left(\frac{3}{5}\right) \times 25 = 15 \text{ kg}$$

When alloys x and y are mixed, then

$$\text{Amount of copper} = \frac{5}{3} + 10 = \frac{35}{3}$$

Therefore, option C is correct.

11. (B) In the mixture of 10 litres, maida is 20%, i.e., 2 kg.

Let x kg of maida be added.

According to the question,

$$\Rightarrow \frac{10+x}{2+x} = \frac{100}{33.33} = \frac{3}{1}$$

$$x = 2$$

Therefore, option B is correct.

12. (A) Proportion = $\frac{22 - 20}{20 - 24} = 1 : 2$

Therefore, option A is correct.

13. (A) The total price of milk is now $80 \times (1 + x)$

But, he gets a profit of 50%.

$$\text{So, } 80(1 + x) = 150$$

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$$1 + x = \frac{15}{8}$$

$$x = \frac{7}{8}$$

Therefore, option A is correct.

14. (A) $(25x + 30 \times 30) \times \frac{110}{100} = 30(30 + x)$

$$(275x + 9900) = (9000 + 300x)$$

$$\Rightarrow 300x - 275x = 900 = x = \frac{900}{25} = 36$$

Therefore, option A is correct.

15. (A) Quantity of component A in the first kind of plastic = 213

Quantity of component B in second kind plastic = 526

$$\text{Required ratio} = 178 : 139 = 1 : 2$$

Therefore, option A is correct.

16. (A) Juice : water = 5 : 3

Quantity of juice in the remaining mixture

$$= \frac{5}{8} \times 160 = 100 \text{ L}$$

Similarly, quantity of water

$$= \frac{3}{8} \times 160 = 60 \text{ L}$$

Therefore, option A is correct.

17. (B) Quantity of acid in the mixture = $\frac{2}{3} \times 6$

$$= 4 \text{ mL}$$

$$\text{Quantity of ethanol in the mixture} = \frac{1}{3} \times 6$$

$$= 2 \text{ mL}$$

Let the required quantity of ethanol be x L.

$$\Rightarrow \frac{4}{2} + x = \frac{1}{3}$$

$$\Rightarrow (x - 4x) = \frac{23}{3x} - 12$$

$$x = 12$$

Therefore, option B is correct.

19. (C) Let the container initially contain 16 kg of pulses.

Let a kg of pulses be compressing type 2 pulse.

Quantity of type 1 pulse in the new mixture
 $= \left(6 + \frac{6a}{16} + a\right)$

Quantity of type 2 pulse in the new mixture

$$= \left(10 - \frac{10a}{16}\right)$$

$$\Rightarrow 6 - \frac{6a}{16} + a = 10 - \frac{10a}{16}$$

$$\Rightarrow 96 - 6a + 16a = 160 - 10a$$

$$\Rightarrow 96 + 10a = 160 - 10a$$

$$\Rightarrow 20a = 64$$

$$\Rightarrow a = \frac{64}{20} = \frac{16}{5}$$

$$\text{Part of mixture replaced} = \left(\frac{1}{16}\right) \times \left(\frac{16}{5}\right) = \frac{1}{5}$$

Therefore, option C is correct.

20. (C) Initial quantity of HCl = $\left(\frac{3}{8}\right) \times 48$ litres

$$= 18 \text{ mL}$$

To get the ratio 5 : 3, new quantity of HCl

$$= \frac{5}{8} = \frac{18+x}{48+x}$$

$$= 120 + 5x = 144 + 8x$$

$$3x = 24$$

$$x = 8 \text{ mL}$$

Therefore, option C is correct.

$$8 = 2 + x$$

$$x = 6 \text{ mL}$$

Therefore, option B is correct.

18. (B) Quantity of oil in container = x mL.
Quantity of water left in cask = $[x(1 - 4x)4]$ mL
Thus, $\left[x \left(1 - \frac{4}{x} \right) 4x \right] = 1681$
 $\Rightarrow (1 - 4x)4 = (23)4$

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$$\Rightarrow x = 7 \times \frac{27}{3} = 63$$

Therefore, option D is correct.

22. (D) Buttermilk in 1-litre mix of A = $\frac{5}{7}$ litre

$$\text{Buttermilk in 1-litre mix of B} = \frac{7}{13} \text{ litre}$$

$$\text{Buttermilk in 1-litre mix of C} = \frac{8}{13} \text{ litre}$$

By rule of the alligation,

$$\text{Required ratio} = \frac{13}{1} : \frac{9}{19}$$

$$= 7 : 9$$

Therefore, option D is correct.

23. (C) Ratio of 1st and 2nd quantities = $7 : 14 = 1 : 2$

$$\text{Quantity of soda water replaced} = \frac{2}{3}$$

Therefore, option C is correct.

24. (C) Juice from the first container = x
Juice from the second container = $(12 - x)$
Juice pulp in x litres of the first container = $0.75x$
Water in x litres of the first container = $0.25x$
Juice pulp in $(12-x)$ litres of the second container = $0.5(12-x)$
Water in $(12-x)$ litres of the second container = $0.5(12-x)$
Ratio of water to juice pulp = $[0.25x + 0.5(12 - x)] : [0.75x + 0.5(12 - x)]$
= $3 : 5$
 $\Rightarrow \frac{(0.25x + 6 - 0.5x)}{(0.75x + 6 - 0.5x)} = \frac{3}{5}$
 $\Rightarrow \frac{6 - 0.25x}{6 + 0.25x} = \frac{3}{5} (6 - 0.25x)(0.25x + 6) = 35$
 $\Rightarrow 30 - 1.25x = 0.75x \cdot 18$
 $\Rightarrow 2x = 12$
 $\therefore x = 6$
Therefore, option C is correct.

25. (B) Price of the mixed variety = x per kg
According to alligation,

$$\frac{x - 15}{15} = \frac{3}{1}$$

21. (D) S = Rs. 9.24

Gain of 10%

$$C = \text{Rs.} \left(\frac{110}{100} \times 9.24 \right) = \text{Rs.} 8.40$$

According to alligation,

The ratio of 1st and 2nd kinds = $14 : 6 = 7 : 3$

Let x kg of butter of 1st be mixed with 27 L of 2nd kind

$$7 : 3 = x : 27$$

$$\Rightarrow 5x = 9$$

$$\Rightarrow x = 18$$

So, the price of the mixture is Rs. 18 per kg.

Therefore, option B is correct.

26. (D) $\frac{\text{Quantity of water}}{\text{Quantity of ethanol}} = \frac{\frac{5}{3}}{\frac{25}{3}} = \frac{1}{5}$

The ratio of water and ethanol = $1 : 5$

Therefore, option D is correct.

27. (D) Let the capacity of the pot be x g.

Quantity of curd in the mixture before adding more curd = $\left(\frac{4}{9} \right) \times (x - 8)$

After adding curd, the quantity of curd in the mixture = $\frac{6}{11}x$.

$$\Rightarrow \frac{6x}{11} - 8 = \left(\frac{4}{9} \right) \times (x - 8)$$

$$\Rightarrow 10x = 792 - 352 = x = 44.$$

Therefore, option D is correct.

28. (D) Original quantity of water = 32 L

Original quantity of hydrochloric acid = 8 L

Initial hydrochloric acid to water ratio = $1 : 4$

Required hydrochloric acid to water ratio = $3 : 7$

$$\text{Concentration ratio, } \frac{8+x}{40+x} = \frac{3}{10}$$

$$80 + 8x = 120$$

$$x = 5 \text{ L}$$

Therefore, option D is correct.

29. (A) Sand in 60 gram mixture = $60 \times \frac{75}{100} = 45$ gram and water = 15 gram.

After adding 15 gram of sand in mixture, total sand = $45 + 15 = 60$ gram and weight of a mixture = $60 + 15 = 75$ gram.

$$\text{So \% of sand} = 100 \times \frac{60}{75} = 80\%$$

Therefore, option A is correct.

30. (C) Let x be the quantity taken from the first container

$$\begin{aligned} \therefore 20 - x &= 2 \\ \Rightarrow 60 - 3x &= 2x - 30 \end{aligned}$$

In x litres, the volume of ethanol = $\frac{2x}{5}$



In x litres, the volume of isopropanol = $\frac{3x}{5}$

Let y be the quantity taken from the second container

In y litres, the volume of ethanol = $\frac{4y}{9}$

In y litres, the volume of isopropanol = $\frac{5y}{9}$

When x litres from the first container and y litre from the second container is mixed, quantity of ethanol in the new mixture

$$= \frac{2x}{5} + \frac{4y}{9}$$

Quantity of isopropanol in the new mixture

$$= \frac{3x}{5} + \frac{5y}{9}$$

According to the question, the ratio of isopropanol to ethanol in the new mixture should be 7 : 5

$$\Rightarrow \frac{3x}{5} + \frac{5y}{9} : \frac{2x}{5} + \frac{4y}{9} = 7 : 5$$

$$\Rightarrow 5\left(\frac{3x}{5} + \frac{5y}{9}\right) = 7\left(\frac{2x}{5} + \frac{4y}{9}\right)$$

$$\Rightarrow \frac{5(27x + 25y)}{45} = \frac{7(18x + 20y)}{45}$$

$$\Rightarrow 135x + 135y = 126x + 140y$$

$$\Rightarrow 9x = 15y$$

$$\Rightarrow \frac{x}{y} = \frac{15}{9}$$

$$\Rightarrow \frac{x}{y} = \frac{5}{3}$$

$$\Rightarrow x : y = 5 : 3$$

Therefore, option C is correct.

- 31. (B)** Let the ratio in the mixture be $2x, 3x$

$$2x + 3x = 60,$$

$$5x = 60, x = 12$$

Diesel in mixture = 24 L

To make the ratio 1 : 3, let the mixture be $3x$

$$\text{Thus, } 3 \times 24 = 72$$

Oil to be added in the mixture = $72 - 60 = 12$ L

Therefore, option B is correct.

- 32. (C)** Let the water and dye be $5x$ and $2x$, respectively

$$\text{So, } 5x + 2x = 28$$

$$7x = 28, x = 4$$

Dye in the new solution = $2(4) + 2 = 10$ litres

Then the new ratio will be $30 : 10 = 3 : 1$

Therefore, option C is correct.

PRACTICE QUESTIONS

Questions 1–5: A dairy house has 5000 L of pure milk. It sells 40% of the total milk to six different milk sellers and utilises remaining 60% of total milk in the dairy itself. Also, each seller, A, B, C, D, E, and F mixes water in their milk. The following table shows the sales of milk to six different sellers by dairy as a percentage of total sales and also the concentration of water in each seller's milk.

PERSON	MILK SOLD BY EACH PERSON	WATER ADDED BY EACH
A	24%	24%
B	10%	10%
C	12%	18%
D	7%	15%
E	28%	25%
F	19%	12%



1. Find the difference between total quantity of water added by seller A and C.
 - A. 105 litres
 - B. 91 litres
 - C. 99 litres
 - D. 95 litres

2. Calculate the respective ratio of quantity of water added by seller C and E.
 - A. 81 : 287
 - B. 3 : 11
 - C. 11 : 3
 - D. 287 : 81

3. Which seller added the least quantity of water (in litres)?
 - A. A
 - B. C
 - C. B
 - D. D

4. If sellers A, B, and C add their diluted milk in one can then what will be the concentration of pure milk in that can?
 - A. 70.75%
 - B. 80.25%
 - C. 90.50%
 - D. 60.25%

5. How many litres of water were added by all the sellers in total? (approximately)
 - A. 500 L
 - B. 450 L
 - C. 440 L
 - D. 490 L

Questions 6–10: For his photographic studies, a scientist buys a 50-litre mixture of two chemicals, H_2SO_4 and HCl, on a weekly basis at the rates shown in the line graph and the percentage composition shown in the pie chart. After thoroughly combining them, he sells them to a manufacturing company. Solve the questions below.

 6. The average per litre cost of the mixture for the 1st three weeks is:
 - A. $\frac{2863}{75}$

 7. What is the scientist's profit or loss percentage in the fourth week if he sells that week's mixture to the company for 3297?
 - A. 75% loss
 - B. 75% profit
 - C. 50% profit
 - D. 50% loss

 8. During the third week, 20% of the mixture from container P was replaced in container Q. Calculate the ratio of the sum of H_2SO_4 in P and HCl in Q and the sum of H_2SO_4 in P and HCl in Q.
 - A. $\frac{29}{52}$
 - B. $\frac{73}{52}$
 - C. $\frac{113}{52}$
 - D. $\frac{155}{52}$

 9. If the scientist had sold both H_2SO_4 and HCl separately for Rs. 35 and Rs. X per litre in the second week, he would have lost 7%, but if he had sold them at Rs. 28 and Rs. 35 per litre, he would have lost Y per cent. Then what is the product of X and Y ?
 - A. $\frac{4536}{17}$
 - B. $\frac{4536}{13}$
 - C. $\frac{4536}{15}$
 - D. $\frac{4536}{11}$

 10. If the scientist secretly converted some percentage of HCl to H_2SO_4 and then mixed them, the cost price difference of the mixture for the same volume increased by

Rs. $(\frac{248}{50})$ in the first week. What was the percentage of HCl that was transformed to H_2SO_4 ?

- B. $\frac{575}{14}$
- C. $\frac{577}{14}$

$$\begin{array}{r} 112 \\ \times 4 \\ \hline 557 \end{array}$$

$$\begin{array}{r} 775 \\ \times 14 \\ \hline \end{array}$$

SOLUTIONS

- 1. (C)** The total quantity of solution = $480 + a$ litre, if seller A adds 'a' litre of water

water = 24%

Thus, 24% of $(480 + a) = a$

$$100a = 24 \times 480 + 24a$$

$$76a = 24 \times 480$$

$$A = \frac{2880}{19} \text{ litres} = \text{approximately } 151.58$$

litres = Quantity of water added by Seller A
Similarly, let the seller C adds c litres of water then

8% of $(240 + c) = c$

$$82c = 240 \times 18$$

$$c = \frac{2160}{41} \text{ litres} = 52.68 \text{ litres approximately}$$

= quantity of water added by seller C

$$\text{required difference} = 151.58 - 52.68 = 99$$

litres approximately

Therefore, option C is correct.

- 2. (A)** $\frac{2160}{41}$ litres = quantity of water added

by C

Let the seller E adds e litres of water

Then, water = 25% of $(560 + e)$

$$75e = 560 \times 25$$

$$E = \frac{560}{3}$$

$$\text{Required ratio} = \frac{2160}{41} : \frac{560}{3} = 81 : 287$$

Therefore, option A is correct.

- 3. (C)** $\frac{2380}{19}$ litres = quantity of water added

by seller A

$\frac{2160}{41}$ litres = quantity of water added by seller C

$\frac{560}{3}$ litres = quantity of water added by seller E

Let the seller B adds b litres of water then,
10% of $(200 + b) = b$

$$\frac{200}{9} \text{ litres} = \text{approximately } 22.22 \text{ litres}$$

Let the seller D adds d litres of water

$$15\% \text{ of } (140 + d) = d$$

$$D = \frac{140 \times 15}{85} = 24.70 \text{ litres}$$

= quantity of water added by seller D

Let the seller F add f litres of water then,
12% of $(380 + f) = f$

$$88f = 380 \times 12$$

$$f = 51.82 \text{ litres}$$

= quantity of water added by seller F

B added the least quantity (in litres) of water

Therefore, option C is correct.

- 4. (B)** The total quantity of new solution = $920 + 226.36 = 1146.36$ litres

Quantity of milk in the new can = $480 + 200 + 240 = 920$ litres

The required concentration of milk in the new solution

$$= \frac{920 \times 100}{1146.36} = 80.25\% \text{ approximately}$$

Therefore, option B is correct.

- 5. (D)** Total quantity of water added by all the sellers together = $151.58 + 22.22 + 52.68$

+ 24.70 + 186.67 + 51.82 = approximately
489.67 litres = approximately 490 litres
Therefore, option D is correct.

- 6. (A)** Litres of HCl = 64% of 50 litres

$$= \left(\frac{64}{100} \right) \times 50 = \frac{64}{2}$$

Litres of HCl = 32 litres

Litres of H_2SO_4 = $50 - 32 = 18$ litres

$$\text{Average CP of } H_2SO_4 = \frac{49 + 28 + 42}{3} = \frac{\text{Rs. } 119}{3}$$

$$\text{Average CP of HCl} = \frac{35 + 42 + 35}{3} = \text{Rs. } \frac{112}{3}$$

$$= \frac{3.6 + 25.6}{6.4 + 14.4}$$

$$= \frac{29.2}{20.8} = \frac{73}{52}$$

Therefore, option B is correct.

- 9. (D)** In 2nd week, C P of H_2SO_4 = Rs. 28 and

SP = Rs. 35

C P of HCl = Rs. 42 and SP = Rs. X

Total C P = Rs. $(28 \times 18 + 42 \times 32) = \text{Rs. } (504 + 1344) = \text{Rs. } 1848 \dots (1)$

Total S P = Rs. $(35 \times 18 + X \times 32) = \text{Rs. } (630 + 32X)$

Let CP of per litre mixture be Rs. x .

$$\text{Then, } \left(\frac{119}{3} - x \right) 18 \text{ litres} = \left(x - \frac{112}{3} \right) 32 \text{ litres}$$

$$\Rightarrow \frac{119 \times 9}{3} + \frac{112 \times 16}{3} = 16x + 9x = 25$$

$$\Rightarrow \frac{1071 + 1792}{3} = 25x$$

$$x = \text{Rs. } \frac{2863}{75}$$

Therefore, option A is correct.

7. (C) CP for 4th week = Rs. x .

Then, $(49 - x) 32$ litres = $(x - 35) 18$ litres

$$x = \text{Rs. } \frac{2198}{50}$$

CP for 50 litres = Rs. 2198

Profit = Rs. $3297 - 2198 = \text{Rs. } 1099$

$$\text{Profit \%} = \left(\frac{1099}{2198} \right) \times 100 = 50\% \text{ profit}$$

Therefore, option C is correct.

8. (B) 20% of mixture = $\left(\frac{20}{100} \right) \times 50$ litres

= 10 litres (moved from P to Q)

H_2SO_4 in container P = 36% of 10 litres = 3.6 litres

HCl in container P = 64% of 10 litres = 6.4 litres

H_2SO_4 in container Q = 36% of 40 litres = 14.4 litres

HCl in container Q = 64% of 40 litres = 25.6 litres

$$\text{Ratio} = \frac{\text{H}_2\text{SO}_4 \text{ in P} + \text{HCl in Q}}{\text{H}_2\text{SO}_4 \text{ in P} + \text{HCl in Q}}$$

+ 32X)

Loss = Rs. $[1848 - (630 + 32X)] = \text{Rs. } (1218 - 32X)$

$$\text{Loss\%} = \left[\frac{1218 - 32X}{1848} \right] \times 100 = 7$$

$$(1218 - 32X) = \frac{7 \times 1848}{100}$$

$$32X = \frac{121800}{100} - \frac{12936}{100}$$

$$X = \frac{3402}{100}$$

Also,

Total SP = Rs. $(28 \times 18 + 35 \times 32) = \text{Rs. } (504 + 1120) = \text{Rs. } 1624$

Loss = Rs. $[1848 - 1624] = \text{Rs. } 224$

$$\text{Loss\%} = Y = \left(\frac{224}{1848} \right) \times 100 = \frac{400}{33}$$

$$\text{Then, } X \times Y = \left(\frac{3402}{100} \right) \times \left(\frac{400}{33} \right) = \frac{3402 \times 4}{33}$$

$$= \frac{4536}{11}$$

Therefore, option D is correct.

10. (D) New volume of HCl = $32 - s$, if s litres of HCl be converted.

New volume of $\text{H}_2\text{SO}_4 = 18 + s$

CP of mixture per litre for the 1st week = Rs. x .

Then, $[49 - x] (18 \text{ litres}) = [x - 35] (32 \text{ litres})$

$$882 - 18x = 32x - 1120$$

$$50x = 1120 + 882 = 2002$$

$$x = \text{Rs. } \frac{2002}{50}$$

Mixtures and Alligations

Then cost price of mixture will be

$$\frac{2002}{50} + \frac{248}{50} = \frac{2250}{50} = 45$$

$$(18 + s)(49 - 45) = (45 - 35)(32 - s)$$

$$4(18 + s) = 10(32 - s)$$

$$72 + 4s = 320 - 10s$$

$$14s = 320 - 72 = 248$$

$$s = \frac{248}{14} = \frac{124}{7}$$

$$\% \text{ of HCl} = \left[\frac{\frac{124}{7}}{32} \right] \times 100 = \frac{775}{14}$$

Therefore, option D is correct.

