OVERALL ANALYSIS | Solution Report

All | Correct Answers | Wrong Answers | Not Attempted Questions

Q.1) Max Marks: 1

The rank of Matrix A is ____

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 3 & 4 & 7 \\ 234 & 343 & 723 \end{bmatrix}$$

Correct Answer

Solution: (3)

There are 2 ways to determine the rank, one way is by elementary transformations and other is by calculating the determinant.

|A|=-106 which is non zero, therefore, rank is 3.

Q.2) Max Marks: 1

Let A and B be 2 non zero matrices such that, If AB=A and BA=B then which of the following statements are true.

I. $A^2=A$
II. $B^2=B$
III. A=I
IV. B=I

A **I and II** Correct Option

Solution: (A)

Solution Option B

We have
$A= AB=A(BA)=(AB)A=A^2$.
$B=BA=B(AB)=(BA)B=B^2$.
I and II are true.

B **I and III**

C **II and III**

D **II and IV**

Q.3) Max Marks: 1

A unimodular matrix is a matrix if its determinant=1 If A and B are unimodular matrices then which of the following are unimodular matrices.

I A+B
II A-B
III AB
IV $B^{-1}A^{-1}$

A **I and II**

B **II and III**

C **III and IV** Correct Option

Solution: (C)

Solution C

| AB|= |A|*|B|
If |A|=1 and |B|=1 then |AB| also also 1.
Similarly

$|A^{-1}|=|A|^{-1}$ therefore $|B^{-1}|=1$ also $|A^{-1}|=1$ and therefore $|B^{-1}A^{-1}|=1$.

D **III only**

Q.4)

Max Marks: 1

If all the elements of a square matrix are multiplied by a constant c.
I. The determinant is also multiplied by the same constant c.
II. The Eigenvalues also get multiplied by the same constant c.

A **Both I and II are true.**

B **Only I is true.**

C **Only II is true.** Correct Option

Solution: (C)

If a matrix is multiplied by a scalar its determinant is multiplied by c^n where n is the order of the square matrix, therefore statement I is incorrect.
If a matrix is multiplied by a scalar its eigenvalues are also multiplied by the same factor c, therefore the statement II is correct.

D **Neither I nor II is true.**

Q.5)

Max Marks: 1

For the following matrix A, if A is invertible

$$A=\begin{bmatrix} a & b & b \\ a & a & b \\ a & a & a \end{bmatrix}.$$

Which of the following statements are true

Statement I: a ≠0
Statement II: a=b

A **I only** Correct Option

Solution: (A)

Solution: Option A

On applying elementary row operations R_1->R_1-R_3 and R_1->R_1-R_3
We will get the following matrix

$$\begin{bmatrix} 0 & b-a & b-a \\ 0 & 0 & b-a \\ a & a & a \end{bmatrix}$$

The determinant of the above matrix is given by

$a(b-a)^2 \neq 0$
Which means a≠0 and b-a≠0
I.e. a≠b.
Which means statement I is only true.

B **II only**

C **I and II**

D **Neither I nor II.**

Q.6)

Max Marks: 1

If A is an invertable matrix then the x cannot take the value.

$$A=\begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}$$

A -1

B 2

C 3

D None of the above. Correct Option

Solution: (D)

If the matrix A is not invertible then |A|=0

I.e.

4(x+1)-(2x-3)(x+2)=0

= $2x^2 - 3x - 10 = 0$

If we solve for the roots

$x = \frac{3 \pm \sqrt{89}}{4}$ x=3.1 or -1.6 approximately therefore none of the options are correct

Q.7) Max Marks: 1

Given the matrix A as below

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & 1 & 1 \end{bmatrix}$$

Find the sum of all the values of λ for which there exists a column vector X≠0 such that

AX=λX.

Correct Answer

Solution: (3)

Solution 3.

The given condition is of the eigenvector X.

|A-λI|=0

The characteristic equation is $\lambda^3-3\lambda^2+\lambda+7=0$

Sum of solutions of the equation $ax^3+bx^2+cx+d=0$ is -b/a

Therefore the sum of all eigenvalues is 3.

Q.8) Max Marks: 1

Given that the following system of linear equations does not have a solution then

2x + 2y +3z=23

ax + by +3z=3

2x +2y +4z=2

A a+b=0

B a-b=0 Correct Option

Solution: (B)

If there is no solution for the system of equations then |A|=0

[2 2 3

a b 3

2 2 4]

Applying row transformations R_3->R_3-R_1

[2 2 3

a b 3

0 0 4]

|A| = 4(2b-2a)=0

a-b=0

C a-2b=0

D None of the above.

Q.9)

$A=\begin{bmatrix}\alpha & 2\\ 2 & \alpha\end{bmatrix}$ and $|A^3|=125$, the α is equal to

A 2

B 5

C -3 Correct Option

Solution: (C)

Solution C

$|A^3|=|A|^3$
$|A|=5$

$\alpha^2-4=5$
$\alpha^2=9$
$\alpha=\pm3$
-3 is the most appropriate option.

D ±2

Q.10)

Given a 3 by 3 matrix A, Is A invertible if every row contains the numbers 0, 1, 2 in any order?

A **Yes**

B **No**

C **Cannot be determined.** Correct Option

Solution: (C)
Solution C
It depends on the order of the elements if the elements are arranged in such a way that the rows are linearly independent then it is invertible otherwise it is not invertible for example
0 1 2
0 2 1
1 2 0
The above matrix is invertible but if we have a matrix-like
0 1 2
0 1 2
2 1 0
Then the matrix is not invertible.

D **None of the Above.**

Q.11)

Which of the following is true with respect to solving a system of linear equations
Statement I. Elementary Row operations preserve the solution of the system of linear equations.
Statement II. Elementary Column operations also preserve the solution of the system of linear equations but they cannot be applied in conjunction with elementary row operations.

A **Both I and II are true**

B **Only I is true** Correct Option

Solution: (B)
Solution B
Only elementary row operations preserve the solution of a system of linear equations and elementary column operations do not preserve the solution of the system of linear equations.

C **Only II is true**

D **Neither I nor II is true.**

Q.12)

For which of the following matrices LU decomposition does not exist.

A $\begin{bmatrix} 2 & -3 & 0 \end{bmatrix}$

$$\begin{bmatrix} 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

B

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 1 & 3 & 4 \end{bmatrix}$$

Correct Option

Solution: (B)

To check if LU decomposition exists we need to check for the determinant of the leading principal minors of the matrix.

Option A- All $|A_1|\neq0$
$|A_2|\neq0$
$|A_3|\neq0$

Option B
$|A_1|\neq0$
$|A_2|=0$
Therefore the LU decomposition does not exist for this matrix.

C

$$\begin{bmatrix} 1 & 7 & 9 \\ 7 & 10 & 12 \\ 22 & 1 & 3 \end{bmatrix}$$

D **None of the above.**

Q.13)

Max Marks: 2

If A is a 2*2 matrix and its determinant is -221 and its trace is 4 then which of the following is true about A.

I. $A^3 = 237A + 884I$
II. $A^4 = 1382A + 52376I$

A **I Only**

Correct Option

Solution: (A)

Solution A.

The determinant of a matrix represents the product of the eigenvalues and the trace represents the sum of the Eigenvalues.

We will get $\lambda^2-4\lambda-221=0$
By Cayley Hamilton theorem we know that the matrix also satisfies the characteristic equation.
I.e.
$A^2 - 4A - 221I = 0$
To determine A^3 we need to multiply it with A.
$A^2 = 4A + 221I$
$A^3 = 4A^2 + 221A$
Replacing A^2 term with $4A + 221I$

$= 4(4A + 221I) + 221A$
$= 237A + 884I$ which means I is true
For A^4 multiplying the above equation with A
$= 237A^2 + 884A$
$= 237(4A + 221I) + 884A$
$= 1832A + 52377I$ Which means II is false.

B **II Only**

C **I and II**

D **Neither I nor II.**

Q.14)

Max Marks: 2

Given the following matrix A

$$\begin{bmatrix} 2 & -3 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

If $\lambda_1,\lambda_2,\lambda_3$ are the eigenvalues of the matrix A then the absolute value of $\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1$ is ______.

Correct Answer

Solution: (13)

We have to solve for the characteristic equation which is $|A-\lambda I|=0$

$$A - \lambda I = \begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix}$$

$(3-\lambda)((2-\lambda)(-5-\lambda)+6))=0$

On solving we get

$\lambda=3, 1, -4$

$\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1 = -13$

The absolute value of the above result=13.

Q.15)

Max Marks: 2

The solution for the system of linear equations is given by x,y,z,w then the absolute value of
xy+yz+zx=_______
2x+4y+3z+5w=140
9x+2y+7z+21w=390
3x+12y-21z+211w=2050
7x+22y-109z+11w=-690

Correct Answer

Solution: (300)

Solution: 300

There are two ways to solve the above system of given linear equations. If we observe the constant term is the sum of the coefficients multiplied by 10 for each of the equations.

Their fore solution is x=10 y=10 z=10 w=10

The other way to solve is to reduce the augmented matrix into echelon form using elementary row operations we will get the same solution.

The value of xy+yz+zx=10*10 + 10*10 +10*10= 300.

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