Kunal Jha
Course: GATE
Computer Science Engineering(CS)

DISCRETE MATHEMATICS-1: (GATE - 2021) - REPORTS

OVERALL ANALYSIS | COMPARISON REPORT | SOLUTION REPORT

ALL(17) | CORRECT(0) | INCORRECT(0) | SKIPPED(17)

Q. 1 FAQ | Solution Video | Have any Doubt ?

Which of the formula is correct for given sentence.
"No students are allowed to carry book"

A $\forall_x$ (student $(x) \rightarrow \neg$ carry book (x)) — Correct Option

Solution :
(a)
No students are allowed to carry book
OR
None of the students are allowed to carry book
OR
There exists no student that are allowed to carry book
i.e.
$\neg \exists_x$ (student $(x) \wedge$ carry book (x))
$\Rightarrow \forall_x$ ($\neg$ student $(x) \vee \neg$ carry book (x))
$\Rightarrow \forall_x$ (student $(x) \rightarrow \neg$ carry book (x))
Hence, option (a) is correct.

B $\forall_x$ ($\neg$ student $(x) \rightarrow \neg$ carry book (x))

C $\forall_x$ (student $(x) \rightarrow$ carry book (x))

D $\exists_x$ ($\neg$ student $(x) \rightarrow \neg$ carry book (x))

QUESTION ANALYTICS +

Q. 2 Solution Video | Have any Doubt ?

Consider a mapping $f: n \rightarrow N$, where N is the set of natural numbers is defined as

$$f(n) = \begin{cases} n^2, & \text{for } n \text{ odd} \\ 2n+1, & \text{for } n \text{ even} \end{cases}$$

for $n \in$ N, which of the following is true about f?

A Bijective

B Surjective but not injective

C Injective but not surjective

D Neither surjective nor injective — Correct Option

Solution :
(d)
'N' is given as {1, 2, 3, ... }

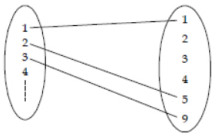
$$f(n) = \begin{cases} n^2, & \text{for } n \text{ odd} \\ 2n+1, & \text{for } n \text{ even} \end{cases}$$

- **Check for injective:**

$f(3) = n^2 = 9$
$f(4) = 2n + 1 = 2 \times 4 + 1 = 9$

Since both $f(3)$, $f(4)$ maps to same element 9.

- **Check for surjective:**



Since, co-domain elements 2, 3 and 4 do not have any pre-image. Hence, cannot be surjective.

QUESTION ANALYTICS +

Q. 3 FAQ | Solution Video | Have any Doubt ?

Consider the relation 'R' on the power set $P(A)$ of a set A as,
$\forall a, b \in P(A)$ $\{(a, b) \in R \leftrightarrow a \cap b \neq \phi\}$
Which of the following is true?

A *R* is not reflexive, not transitive but symmetric — Correct Option

Solution :
(a)

- R is not reflexive because ϕ is an element of power set, and any subset of A and $\phi \cap \phi = \phi$ which belongs to R.
- R is symmetric because intersection ϕ is commutative thus $a \cap b \neq \phi$ then $b \cap a \neq \phi$.
- R is transitive because $a \cap b \neq \phi$ and $b \cap c \neq \phi$ does not assure $a \cap c \neq \phi$

e.g. $a = \{1, 2\}, b = \{2, 3\}, c = \{3, 4\}$
So, $\{1, 2\} \cap \{2, 3\} \neq \phi$
$\{2, 3\} \cap \{3, 4\} \neq \phi$
but $\{1, 2\} \cap \{3, 4\} = \phi$ so it is fail.

B *R* is reflexive, transitive but not symmetric

C *R* is reflexive, symmetric and transitive

D *R* is reflexive, not symmetric, not transitive

QUESTION ANALYTICS +

Q. 4 — FAQ | Solution Video | Have any Doubt ?

Suppose for natural numbers x and y, $x\ R\ y$ iff 11 divides $x - y$. Then which of the following is false?

A *R* is reflexive

B *R* is symmetric

C *R* is transitive

D None of these — Correct Option

Solution :
(d)
$R = x\ R\ y$ iff 11 divides $x - y$

- R is reflexive
  $x - y = 0$ divided by 11
- R is symmetric
  $x - y \equiv y - x$
- R is transitive
  $x - y$ && $y - z \Rightarrow x - z$ divided by 11
- Hence none of these option is false.

NOTE : R is not anti-symmetric.

QUESTION ANALYTICS +

Q. 5 — FAQ | Solution Video | Have any Doubt ?

The domain of x is a set of students and the domain of y is a set of courses. If P(x, y) is "Student x is in course y". Which of the following paraphrases the sentence: "No course is without students"?

A $\exists_y \forall_x P(x, y)$

B $\exists_x \forall_y P(x, y)$

C $\forall_y \exists_x P(x, y)$ — Correct Option

Solution :
(c)
No course is without students means every course has a student.
$\Rightarrow \forall_y \exists_x P(x, y)$

D $\forall_x \exists_y P(x, y)$

QUESTION ANALYTICS +

Q. 6 — FAQ | Have any Doubt ?

Consider the following logical sequence:
(i) $\exists !\ x\ P(x) \rightarrow \exists x\ P(x)$
(ii) $\forall x\ P(x) \rightarrow \exists !\ x\ P(x)$
(iii) $\exists !\ x\ P(x) \rightarrow \neg\ \forall x\ P(x)$

The number of the above logical sequence which is/are logically valid _____. (Assume the domain of x has more than one element).

2 — Correct Option

Solution :
2
Only (i) and (iii) are valid.
$\exists!$ implies unique element or exactly one element (uniqueness identifier).
Let's see how (ii) is incorrect. "It says if all x are true then exactly one x is true which itself is wrong".

QUESTION ANALYTICS +

Q. 7 ? FAQ | Solution Video | Have any Doubt ?

Consider the following statements:
I. The set of all rational numbers in {0, 1}.
II. The set of all ordered pairs of rational number in {0, 1}.
III. The set of all real numbers in {0, 1}.
IV. The set of all ordered pairs of real numbers in {0, 1}.
The number of statements uncountable is/are _____.

2 — Correct Option

Solution :
2
Only statement III and IV are uncountable.

QUESTION ANALYTICS +

Q. 8 ? FAQ | Solution Video | Have any Doubt ?

Consider the relation xRy between times of day such that x and y are at most twenty minutes apart. Then which of the following property is/are true about the given relation?

A Reflexive — Correct Option
B Transitive
C Irreflexive
D Symmetric — Correct Option

YOUR ANSWER - NA | CORRECT ANSWER - a,d | STATUS - SKIPPED

Solution :
(a, d)
This relation is reflexive and symmetric. It is not transitive;
Consider the counter-example: 1 : 00R 1 : 15, 1 : 15R 1 : 22 but ¬1 : 00R 1 : 22.

QUESTION ANALYTICS +

Q. 9 ? FAQ | Solution Video | Have any Doubt ?

Consider the relation between vertices in an undirected graph: Two vertices v, and w related iff w is reachable from v. Then which of the following property is/are true about the given relation?

A Reflexive — Correct Option
B Transitive — Correct Option
C Irreflexive
D Symmetric — Correct Option

YOUR ANSWER - NA | CORRECT ANSWER - a,b,d | STATUS - SKIPPED

Solution :
(a, b, d)
In fact, the connected components of the graph are the equivalence classes of this relation.

QUESTION ANALYTICS +

Q. 10 ? FAQ | Solution Video | Have any Doubt ?

Consider two well formed formulas in propositional logic:

Consider two well formed formulas in propositional logic.

$F_1 : (pV \neg q) \wedge (\neg pV\, q) \wedge (\neg pV \neg q)$

$F_2 : (p \leftrightarrow q) \wedge (\neg p \leftrightarrow q)$

Which of the following is correct?

A F_1 is satisfiable, F_2 is a satisfiable

B F_1 is satisfiable, F_2 is a un-satisfiable — Correct Option

Solution :
(b)

$F_1 : (pV \neg q) \wedge (\neg pV\, q) \wedge (\neg pV \neg q)$

$= (p + q')\,(p' + q)\,(p' + q')$

$= (p + q')\,(p' + qq')$

$= (p + q')p'$

$= p'q'$ which is not valid but satisfiable

$F_2 : (p \leftrightarrow q) \wedge (\neg p \leftrightarrow q)$

$\neg(p \leftrightarrow q) \equiv (\neg p \leftrightarrow q)$

So if $(p \leftrightarrow q) \equiv A$

the $A \wedge A' = 0$ means un-satisfiable.

C F_1 and F_2 is satisfiable

D F_1 is satisfiable, F_2 is valid

QUESTION ANALYTICS +

Kunal Jha
Course: GATE
Computer Science Engineering(CS)

DISCRETE MATHEMATICS-1: (GATE - 2021) - REPORTS

OVERALL ANALYSIS | COMPARISON REPORT | SOLUTION REPORT

ALL(17) | CORRECT(0) | INCORRECT(0) | SKIPPED(17)

Q. 11 FAQ | Solution Video | Have any Doubt ?

Consider the following statements:
S_1 : The relation $R = \Phi$ on empty set is symmetric and transitive but not reflexive.
S_2 : The relation R is defined as xRy if $xy \geq 1$ on the set of real numbers is symmetric and transitive.
Which of the above statements is/are true?

A Only S_1

B Only S_2

C Both S_1 and S_2

D None of these — Correct Option

Solution :
(d)

- On empty set ϕ is an equivalence relation therefore S_1 is false.
- In S_2 and relation is not transitive.
  e.g.

$(2, 5) \in R$ as $10 \geq 1$

$\left(5, \frac{1}{4}\right) \in R$ as $\frac{5}{4} \geq 1$

but $\left(2, \frac{1}{4}\right) \notin R$ as $\frac{2}{4} < 1 \Rightarrow$ Not transitive

So, both S_1 and S_2 are false.

QUESTION ANALYTICS +

Q. 12 Solution Video | Have any Doubt ?

Let $f: A \to B$ and $g: B \to C$ denote two functions. If the function $g \circ f: A \to C$ is a surjection and g is an injection then function f is

A Injection

B Surjection — Correct Option

Solution :
(b)
$f : A \to B$
$g : B \to C$ is injection $\forall b \in B$, $g(b) = C$ distinct images inc
$gof : A \to C$ is surjection

$$g(f(a)) = C$$
$$g(f(a)) = g(b)$$
$$\exists a \in A$$
$$f(a) = b$$

So, $A \to B$ is surjection.

C Bijection

D None of these

QUESTION ANALYTICS +

Q. 13 FAQ | Solution Video | Have any Doubt ?

Let X and Y be two sets. Then $(X - Y) \cup (X \cap Y) \cup (\overline{X} \cap Y)$ is equal to

A X

B Y

C $X \cap Y$

D $X \cup Y$ — Correct Option

Solution :
(d)
First lets simplify the expression a bit

$$= (X - Y) \cup (X \cap Y) \cup (\bar{X} \cap Y)$$
$$= X\bar{Y} + XY + \bar{X}Y$$
$$= X + \bar{X}Y = X + Y$$

QUESTION ANALYTICS +

Q. 14 Solution Video | Have any Doubt ?

The number of one-one functions possible from a set having 5 elements to a set having 10 elements is _____.

30240 — Correct Option

Solution :
30240
The number of one-one function from n element set to m element set = ${}^{m}P_{n}$

$$m = 10$$
$$n = 5$$
$${}^{10}P_{5} = \frac{10!}{5!} = 10 \times 9 \times 8 \times 7 \times 6 = 30240$$

QUESTION ANALYTICS +

Q. 15 Solution Video | Have any Doubt ?

Consider the following sentence:
"Golden eggs are tasty"
$G(x)$: x is golden
$E(x)$: x is an egg
$T(x)$: x is tasty
Now consider the following first order statements.
I. $\forall_x(E(x) \Rightarrow (G(x) \Rightarrow T(x))$
II. $\forall_x(\sim T(x) \Rightarrow \sim G(x) \vee \sim E(x))$
III. $\forall_x(G(x) \Rightarrow ((E(x) \Rightarrow T(x)))$
The number of correct predicate translations for the sentence given above is _____.

3 — Correct Option

Solution :
3
- In (I), we first check if it is an egg and then see if it is golden, so both I and III mean the same thing.
- (III) is contra positive of $(G(n) \wedge E(n) \Rightarrow T(n))$ and hence it also true.

QUESTION ANALYTICS +

Q. 16 FAQ | Solution Video | Have any Doubt ?

Let the domain be all the objects in a room. Which of the following is a correct translation of "There is a red bouncing ball in the room".
$R(x)$: x is Red
$BO(x)$: x is bouncing
$B(x)$: x is a ball

A $\exists x\ (R(x) \wedge BO(x) \wedge B(x))$ — Correct Option

B $\exists x\ R(x) \wedge \exists x\ BO(x) \wedge \exists x\ B(x)$

C $\exists x\ (BO(x) \wedge R(x) \wedge B(x))$ — Correct Option

D None of these

YOUR ANSWER - NA | CORRECT ANSWER - a,c | STATUS - SKIPPED

Solution :
(a, c)

QUESTION ANALYTICS +

Q. 17 FAQ | Solution Video | Have any Doubt ?

Which of the following is/are true?

A The set of negative integers is countable. Correct Option

B The set of integers that are multiples of 7 is countable. Correct Option

C The set of even integers is countable. Correct Option

D The set of real numbers between 0 and $\frac{1}{2}$ is countable.

YOUR ANSWER - NA | CORRECT ANSWER - a,b,c | STATUS - SKIPPED

Solution :

(a, b, c)

- The set of negative integers is countable.
- The set of integers that are multiples of 7 is countable.
- The set of even integers is countable.
- The set of real numbers between 0 and $\frac{1}{2}$ is countable. This is not true because we can not count set of real numbers.

QUESTION ANALYTICS +

Kunal Jha
Course: GATE
Computer Science Engineering(CS)

HOME
MY TEST
BOOKMARKS
MY PROFILE
REPORTS
BUY PACKAGE
NEWS
TEST SCHEDULE

DISCRETE MATHEMATICS- : (GATE - 2021) - REPORTS

OVERALL ANALYSIS | COMPARISON REPORT | SOLUTION REPORT

ALL(17) | CORRECT(0) | INCORRECT(0) | SKIPPED(17)

Q. 1 FAQ | Solution Video | Have any Doubt ?

The number of ways of splitting a set of n elements into two parts is

A 2^n

B 2^{n-1}

C $2^{n-1}-1$ — Correct Option

Solution :
(c)

D n^2

QUESTION ANALYTICS +

Q. 2 Solution Video | Have any Doubt ?

Which of the following algebraic structures form a group?
I. The set of integers under subtraction.
II. The set of integers under addition.
III. The set of rational numbers under multiplication.
IV. The set of irrational number under multiplication.

A I, III and IV only

B II, III and IV only

C II and III only

D None of these — Correct Option

Solution :
(d)
To form a group it should satisfy the following:
(i) Closure
(ii) Associative
(iii) Identify
(iv) Inverse
Only (II) i.e. the set of integers under addition form a group and satisfies all the mentioned properties.

QUESTION ANALYTICS +

Q. 3 FAQ | Solution Video | Have any Doubt ?

Let G be a planar connected graph in which every cycle contains at least 6 edges. If G has 10 vertices, what is the maximum number of edges that can be there in G?

A 12 — Correct Option

Solution :
(a)

$$r = e - n + 2$$
$$e \geq \frac{6r}{2}$$
$$e \geq 3r$$
$$e \geq 3(e - n + 2)$$
$$e \leq \frac{3n+6}{2}$$
$$e \leq \frac{3\times 10+6}{2} = 12$$

B 13

C 11

D 14

QUESTION ANALYTICS +

Q. 4

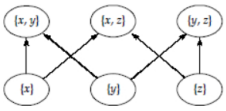
FAQ | Solution Video | Have any Doubt ?

Consider the following statements:
S_1 : Every finite lattice has a least element.
S_2 : Every poset has a greatest element.
Which of the following is true?

A Only S_1 — Correct Option

Solution :
(a)

- S_1 is true. Suppose elements of lattice are a_1, a_2, a_3 ..... then $a_1 \wedge a_2 \wedge a_3$ ..... a_n is the least element.
- S_2 is false.
  Poset $(R, \leq)$ has no greatest element.



No greatest element as 3 sets are non comparable.

B Only S_2

C Both S_1 and S_2

D None of these

QUESTION ANALYTICS +

Q. 5

Solution Video | Have any Doubt ?

Which of the following is true?

A Chromatic number of complete graph with n vertices is $n - 1$.

B Two graphs G_1 and G_2 are isomorphic then their complements may or may not be isomorphic.

C If any simple graph with n nodes with $n > 1$, there are atleast two vertices of same degree. — Correct Option

Solution :
(c)
• Chromatic number of complete graph with n vertices is n.
• If two graph G_1 and G_2 are isomorphic, then their complements will always be isomorphic.
• Option (c) is correct.

D None of these

QUESTION ANALYTICS +

Q. 6

Solution Video | Have any Doubt ?

The maximum number of edges present in a disconnected graph with 6 vertices is _____.

10 — Correct Option

Solution :
10
Maximum number of edges in connected graph

$$^{n}C_2 = \frac{n(n-1)}{2}$$

So, B_2 disconnected one vertex from it, we get

$$^{n-1}C_2 = \frac{(n-1)(n-2)}{2}$$

So, for 6 vertices, maximum

$$\text{Edges} = \frac{(6-1)(6-2)}{2} = \frac{5 \times 4}{2} = 10$$

QUESTION ANALYTICS +

Q. 7

FAQ | Solution Video | Have any Doubt ?

The number of ways in which we can place 3 white pawns and 3 black pawns on 3 × 3 chess board is equal to _____.

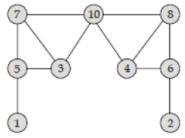
1680 Correct Option

Solution :
1680
The required number of ways,
$= {}^{9}C_{6} \times {}^{6}C_{3} \times {}^{3}C_{3} = 1680$

QUESTION ANALYTICS +

Q. 8 ? FAQ ▶ Solution Video Have any Doubt ?

Which of the following is/are correct with respect to the graph given below?



A It is bipartite.

B It has an Euler circuit.

C The diameter of the graph would be 6. Correct Option

D Chromatic number of the graph is 3. Correct Option

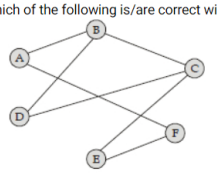
YOUR ANSWER - NA | CORRECT ANSWER - c,d | STATUS - SKIPPED

Solution :
(c, d)
No graph is not bipartite since there is an odd-length cycle.
This graph does not have an Euler circuit because there are vertices with odd degree.
Yes the shortest distance between two terminal point is 6.
Chromatic number is 3 so true.

QUESTION ANALYTICS +

Q. 9 ? FAQ ▶ Solution Video Have any Doubt ?

Which of the following is/are correct with respect to the graph given below?



A Diameter of the above graph is 2.

B There exists atleast one possible Hamiltonian Cycle on G. Correct Option

C Chromatic number of G would be 2.

D Graph G does not have any euler cycle. Correct Option

YOUR ANSWER - NA | CORRECT ANSWER - b,d | STATUS - SKIPPED

Solution :
(b, d)
Recall that the diameter is the maximum of all shortest path lengths between pairs of vertices. Note that the shortest path length between D and F is 3, and all other pairs of non-adjacent vertices share a neighbor. One possible solution is (A, F, E, C, D, B, A). This cycle and its reverse should constitute all possible solutions. One possible 3-coloring is: {A, D, E} red; {B, F} green; C blue. Because there exists an odd-length
cycle (e.g. (B, D, C)), no 2-coloring exists. No. This follows from the fact that there exist vertices with odd degree.

QUESTION ANALYTICS +

Q. 10 ? FAQ ▶ Solution Video Have any Doubt ?

Consider the recurrence relation $a_k = -8a_{k-1} - 15a_{k-2}$ with initial conditions $a_0 = 0$ and $a_1 = 2$. Which of the following is an explicit solution to this recurrence relation?

A $k(-3)^k - k(-5)^k$

B $k(-5)^k - k(-3)^k$

C $(-3)^k - (-5)^k$ Correct Option

Solution :
(c)

$$a_k = -8a_{k-1} - 15a_{k-2}$$
$$k-2 = 1$$
$$k-1 = n$$
and $$k = n^2$$
Using characteristics equation
$$n^2 + 8 + 15 = 0$$
$$n = -3$$
and $$n = -5$$
So, $$a_k = (-3)^k C_1 + (-5)^k C_2$$
$$= (-3)^0 C_1 + (-5)^0 C_2 = 0$$
$$C_1 + C_2 = 0 \quad \text{...(i)}$$
$$a_1 = (-3)^1 C_1 + (-5)^1 C_2$$
$$= -3C_1 + (-5)C_2 = 2 \quad \text{...(ii)}$$
Solving equation (i) and (ii), we get,
$$C_1 = 1$$
and $$C_2 = -1$$
then $$a_n = (-3)^k - (-5)^k$$

D $(-5)^k - (-3)^k$

QUESTION ANALYTICS +

Kunal Jha
Course: GATE
Computer Science Engineering(CS)

DISCRETE MATHEMATICS- : (GATE - 2021) - REPORTS

OVERALL ANALYSIS | COMPARISON REPORT | SOLUTION REPORT

ALL(17) | CORRECT(0) | INCORRECT(0) | SKIPPED(17)

Q. 11 FAQ | Solution Video | Have any Doubt ?

Consider the following POSETs:
I. ({1, 2, 3, 6, 14, 21, 42}, /)
II. ({1, 2, 3, 6, 11, 22, 33, 66}, /)
III. ({1, 2, 5, 7, 10, 14, 35, 70}, ≤)
Which of the above POSETs are isomorphic to (P(s), ⊆) , where S = {a, b, c}?

A I and II only

B II only — Correct Option

Solution :
(b)

- I is not D_{42} because the divisor 7 is missing. So, there is no way for I to be isomorphic to $(P\{a,b,c\},\subseteq)$ as it needs to have 8 divisors but right now it has only 7.
- II is D_{66} a well known boolean algebra and has 8 vertices and its masses diagram will be isomorphic $(P(\{a,b,c\}),\subseteq)$.
- III is not isomorphic even though it looks like D_{70}, it is on the relation ≤, resulting in a chain, which won't be boolean algebra.

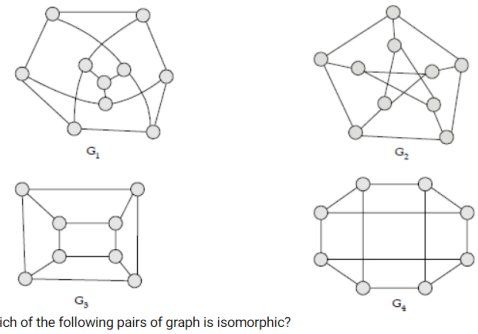
C II and III only

D III only

QUESTION ANALYTICS +

Q. 12 FAQ | Solution Video | Have any Doubt ?

Consider the graph shown below:



Which of the following pairs of graph is isomorphic?

A G_1 and G_2

B G_2 and G_3

C G_3 and G_4 — Correct Option

Solution :
(c)
• G_1 has 6 items and G_2 has 5 cycles. Hence it can not be isomorphic.
• G_3 and G_4 are also isomorphic.
• Hence the option (c) is correct.

D Both (a) and (c)

QUESTION ANALYTICS +

Q. 13 FAQ | Solution Video | Have any Doubt ?

Which of the following is not a subgroup of (z, +)? (Assume 'O' and 'E' denote the sets odd integers and even integers respectively.

A (O, +) — Correct Option

Solution :
(a)

'O' is not a subgroup of z, as (odd + odd = even) which does not belong to be set 0.
Therefore, it is not closed. So, option (a) is correct.

B (E, +)

C ({3 k | $k \in z$}, +)

D ({− 4 k | $k \in z$}, +)

QUESTION ANALYTICS +

Q. 14 FAQ | Solution Video | Have any Doubt ?

A group G having O(G) = 100 is known to be cyclic with g being one of its generators. It is given that $O(g^x) = 50$ for some $x \in z$. The value of x is ____.

2 Correct Option

Solution :
2

$$O(g^x) = 50$$

If g is a generator of G then

$$O(g^x) = \frac{n}{gcd(x, n)} \qquad [n = O(g) = 100]$$

$$50 = \frac{100}{gcd(x, n)}$$

$$gcd(x, 100) = 2$$

So, $x = 2$ will satisfied.

QUESTION ANALYTICS +

Q. 15 FAQ | Solution Video | Have any Doubt ?

Consider an equivalence relation R on the positive integers = {2, 3, 4, 5, 6, 7, 8, ... 13, 14, 15, 16} defined as 'mRn' if the largest prime divisor of 'm' is the same as the largest prime divisor of 'n'. The number of equivalence classes of R is _____.

6 Correct Option

Solution :
6
"mRn" (largest prime divisor of m = largest prime divisor of n)
So, equivalence classes are
1. 2 R {2, 4, 8, 16}
2. 3 R {3, 6, 9, 12}
3. 5 R {5, 10, 15}
4. 7 R {7, 14}
5. 11 R {11}
6. 13 R {13}
Number of equivalence classes = 6.

QUESTION ANALYTICS +

Q. 16 Solution Video | Have any Doubt ?

Which one of the following is/are necessarily a property of a Group?

A Commutativity

B Associativity Correct Option

C Existence of inverse for every element Correct Option

D Existence of identity Correct Option

YOUR ANSWER - NA | CORRECT ANSWER - b,c,d | STATUS - SKIPPED

Solution :
(b, c, d)
A group is a set, G, together with an operation • (called the group law of G) that combines any two elements a and b to form another element, denoted $a \bullet b$ or ab. To qualify as a group, the set and operation, (G, •), must satisfy four requirements known as the group axioms:
Closure
For all a, b in G, the result of the operation, $a \bullet b$, is also in G.b
Associativity
For all a, b and c in G, $(a \bullet b) \bullet c = a \bullet (b \bullet c)$.
Identity element
There exists an element e in G, such that for every element a in G, the equation $e \bullet a = a \bullet e = a$ holds. Such an element is unique (see below), and thus one speaks of the identity element.
Inverse element
For each a in G, there exists an element b in G such that $a \bullet b = b \bullet a = e$, where e is the identity

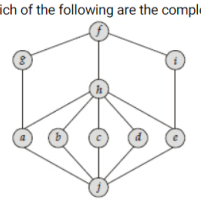
element.
The result of an operation may depend on the order of the operands. In other words, the result of combining element a with element b need not yield the same result as combining element b with element a; the equation.

QUESTION ANALYTICS +

Q. 17

Solution Video | Have any Doubt ?

Which of the following are the complements for the elements "e" for the graph given below:



A f

B j

C g Correct Option

D i

YOUR ANSWER - NA | CORRECT ANSWER - c | STATUS - SKIPPED

Solution :

(c)
The least upper bound of 'a' and 'x' should be the upper bound of the lattice which is 'f' here.
The greatest lower bound of 'a' and 'x' should be the lower bound of the lattice which is 'j' here.
Only element 'g' satisfied this.
Option (c) is true.

QUESTION ANALYTICS +

Kunal Jha
Course: GATE
Computer Science Engineering(CS)

HOME
MY TEST
BOOKMARKS
MY PROFILE
REPORTS
BUY PACKAGE
NEWS
TEST SCHEDULE

DISCRETE MATHEMATICS (GATE - 2021) - REPORTS

OVERALL ANALYSIS | COMPARISON REPORT | SOLUTION REPORT

ALL(33) | CORRECT(0) | INCORRECT(0) | SKIPPED(33)

Q. 1 Have any Doubt ?

How many factors of 1080 are perfect squares?

A 4 — Correct Option

Solution :
(a)

$$1080 = 2^3 \times 3^3 \times 5$$

For any perfect square, all the powers of the primes have to be even numbers.
So, if the factor is of the form $2^a \times 3^b \times 5^c$.
The values 'a' can take are 0 and 2, b can take are 0 and 2 and c can take the value 0.
Totally there are 4 possibilities. 1, 4, 9, and 36.
Option (a) is correct.

B 6

C 8

D 5

QUESTION ANALYTICS +

Q. 2 Have any Doubt ?

Which of the following biconditionals are true?

A 2 + 2 = 4 iff 1 + 3 = 8

B 2 + 8 = 10 iff 1 + 2 = 5

C 0 > 1 iff 4 > 3

D 1 + 1 = 3 iff tigers can fly — Correct Option

Solution :
(d)
(a) This is T ↔ F which is false.
(b) This is T ↔ F which is also false.
(c) This is F ↔ T which is false.
(b) This is F ↔ F which is true.

QUESTION ANALYTICS +

Q. 3 FAQ | Have any Doubt ?

Let $f(x)$ is a function from the set of integers. Find which of the following function is neither oneone nor onto function.

A $f(x) = x + 2$

B $f(x) = 3x - 5$

C $f(x) = x^3 + 3$

D $f(x) = x^2 + 1$ — Correct Option

Solution :
(d)
To check function is one-one:

$$f(x_1) = f(x_2)$$
$$f(x) = x^2 + 1$$
$$x_1^2 + 1 = x_2^2 + 1$$

$\Rightarrow x_1 = \pm 1$ here x_1 has two images so. It is not one function.
To check for onto:

$$f(x) = x^2 + 1$$
$$y = x^2 + 1$$
$$x = \sqrt{y-2}$$

So, range = $|y|$ for $y > 1 \neq Z$

So it is not onto.

QUESTION ANALYTICS +

Q. 4 Have any Doubt ?

A simple graph is one in which there are no self loops and each pair of distinct vertices is connected by at most one edge. Let G be a simple graph on 8 vertices such that there is a vertex of degree 1, a vertex of degree 2, a vertex of degree 3, a vertex of degree 4, a vertex of degree 5, a vertex of degree 6 and a vertex of degree 7. Which of the following can be the degree of the last vertex?

A 3

B 0

C 5

D 4 Correct Option

Solution :
(d)
The number of odd degree vertices in any graph is even. Since we already have four vertices of odd degree, the degree of the last vertex cannot be 3 or 5. It cannot be 0 either, since there is one vertex with degree 7, which means that it is a neighbour to all the other vertices, which implies that there is no isolated vertex in the graph. Thus the only possible degree of the last vertex is 4.

QUESTION ANALYTICS +

Q. 5 FAQ Solution Video Have any Doubt ?

A college prepares its timetable by grouping courses in slots A, B, C, ... All courses in a slot meet at the same time, and courses in different slots have disjoint timings. Course registration has been completed and the administration now knows which students are registered for each course. If the same student is registered for two courses, the courses must be assigned different slots. The administration is trying to compute the minimum number of slots required to prepare the timetable.
The administration decides to model this as a graph where the nodes are the courses and edges represent pairs of courses with an overlapping audience. In this setting, the graph theoretic question to be answered is:

A Find a spanning tree with minimum number of edges.

B Find a minimal colouring. Correct Option

Solution :
(b)
If we represent each slot by a colour, then a colouring of the graph is an assignment of courses to slots, such that courses with overlapping audiences are in different slots. A minimal colouring minimizes the number of slots needed?

C Find a minimum size vertex cover.

D Find a maximum size independent set.

QUESTION ANALYTICS +

Q. 6 Have any Doubt ?

An undirected graph has 10 vertices labelled {1, 2, ..., 10} and 37 edges. Vertices 1, 3, 5, 7, 9 have degree 8 and vertices 2, 4, 6, 8 have degree 7. What is the degree of vertex 10?

A 5

B 6 Correct Option

Solution :
(b)
The sum of the degrees is twice the number of edges, which is 74 in this case.
The degrees of the vertices {1, 2, ..., 9} add up to 68. Hence 6 is the correct answer for vertex 10.

C 7

D 8

QUESTION ANALYTICS +

Q. 7 FAQ Have any Doubt ?

For what value of n, W_n is bipartite

A 1

B 2

C 4

D Never be bipartite — Correct Option

Solution :
(d)
Wheel can never be bipartite. Since the center vertex is adjacent to every other vertex in the graph.

QUESTION ANALYTICS +

Q. 8 FAQ | Have any Doubt ?

There are 7 switches on a switchboard, some of which are on and some of which are off. In one move, you pick any 2 switches and toggle each of them-if the switch you pick is currently off, you turn it on, if it is on, you turn it off. Your aim is to execute a sequence of moves and turn all 7 switches on. For which of the following initial configurations is this not possible? Each configuration lists the initial positions of the 7 switches in sequence, from switch 1 to switch 7.

A (off, on, off, on, off, off, on)

B (on, off, on, on, on, on, on) — Correct Option

Solution :
(b)
The parity of switches in each position is unchanged after each move. If all 7 switches are on at the end, the final parity of "off" is even and of "on" is odd. So we can only achieve this if we start with a configuration where the number of "off" switches is even and the number of "on" switches is odd. The exact order does not matter since we can pick any two to toggle at each step

C (off, on, on, on, on, on, off)

D (off, off, off, off, off, on, off)

QUESTION ANALYTICS +

Q. 9 FAQ | Have any Doubt ?

10 oranges are to be placed in 5 distinct boxes labeled U, V, W, X, Y. A box may contain any number of oranges including no oranges or all the oranges. What is the number of ways to distribute the oranges so that exactly two of the boxes contain exactly two oranges each?

150 — Correct Option

Solution :
150
From the five distinct boxes, there are 10 ways to pick the two boxes that will have 2 oranges each. We need to distribute the remaining 6 oranges in the remaining three boxes such that none of the three boxes gets exactly 2 oranges. The possible distributions are 6 + 0 + 0 (which can be done in 3 ways) or 5 + 1 + 0 (6 ways) or 4 + 1 + 1 (3 ways) or 3 + 3 + 0 (3 ways).
Thus the required answer is 10 × (3 + 6 + 3 + 3) = 150.

QUESTION ANALYTICS +

Q. 10 FAQ | Have any Doubt ?

Positive integers a and b, possibly equal, are chosen randomly from among the divisors of 400. The numbers a, b are chosen independently, each divisor being equally likely to be chosen. Find the probability that gcd(a, b) = 1 and $lcm(a, b)$ = 400 is _____. (Upto 3 decimal places)

0.017 (0.016 - 0.018) — Correct Option

Solution :
0.017 (0.016 - 0.018)
$400 = 5^2 \times 2^4$ has $(2 + 1) \times (4 + 1) = 15$ factors, so total number of pairs (a, b) is $15 \times 15 = 225$. For a, b to be coprime, they should have no prime factor in common and then their lcm is just their product, which is required to be 400.
So there are only four allowed pairs: (1, 400), (400, 1), (25, 16) and (16, 25).

QUESTION ANALYTICS +

Kunal Jha
Course: GATE
Computer Science Engineering(CS)

HOME
MY TEST
BOOKMARKS
MY PROFILE
REPORTS
BUY PACKAGE
NEWS
TEST SCHEDULE

DISCRETE MATHEMATICS (GATE - 2021) - REPORTS

OVERALL ANALYSIS | COMPARISON REPORT | SOLUTION REPORT

ALL(33) | CORRECT(0) | INCORRECT(0) | SKIPPED(33)

Q. 11 Have any Doubt ?

How many rows appear in a truth table for the given compound proposition? _____
$(p \vee \neg t) \wedge (p \vee \neg s)$

8 — Correct Option

Solution :
8
A truth table will need 2^n rows if there are n variables.
So, $2^3 = 8$ is the answer.

QUESTION ANALYTICS +

Q. 12 FAQ | Have any Doubt ?

Consider a Hamiltonian graph G with no loops as parallel edges and with $|V(G)| = n \geq 3$. Given following statements. Find number of statements which are correct _____.

I. $\deg(V) \geq \frac{n}{2}$ for each vertex V.

II. $|E(G)| \geq \frac{1}{2}(n-1)(n-2) + 2$.

III. deg(V) + deg(W) ≥ n whenever V and W are not connected by an edge.

3 — Correct Option

Solution :
3

- Statement I is true. If G = (V, E) has $n \geq 3$ and every vertex has degree $\geq \frac{n}{2}$ then G has a Hamiltonian circuit.
- Statement II is also true. Any graph with n vertices and atleast $\frac{1}{2}(n-1)(n-2) + 2$ edges must be Hamiltonian.
- Statement III is also true using the Ore's theorem.

QUESTION ANALYTICS +

Q. 13 Have any Doubt ?

Which of these collections of subsets are partition of {−3, −2, −1, 0, 1, 2, 3}? Count the number of such collections _____.
I. {−3, −1, 1, 3}, {−2, 0, 2}
II. {−3, −2, −1, 0}, {0, 1, 2, 3}
III. {−3, 3}, {−2, 2}, {−1, 1}, {0}
IV. {−3, −2, 2, 3}, {−1, 1}

2 — Correct Option

Solution :
2
I. A partition.
II. Not a partition 0 is there in both.
III. A partition.
IV. Not a partition 0 is not there in any partition.

QUESTION ANALYTICS +

Q. 14 FAQ | Have any Doubt ?

For sets A and B, let f : A ? B and g : B? A be functions such that $f(g(x)) = x$ for each x. Which of the following option(s) is/are true?

A. The function f must be one-to-one.

B. The function f must be onto. — Correct Option

C. The function g must be one-to-one. — Correct Option

D. The function g must be onto.

YOUR ANSWER - NA | CORRECT ANSWER - b,c | STATUS - SKIPPED

Solution :

(b, c)

If $g(x_1) = g(x_2)$, then $x_1 = f(g(x_1)) = f(g(x_2)) = x_2$, so g is one-to-one. Also f is onto because each x? B is in the image of f, namely $x = f(g(x))$. The other two statements are false, e.g. by constructing an example in which A is a larger finite set than B.

QUESTION ANALYTICS +

Q. 15 FAQ | Have any Doubt ?

Consider the relation $R = \{(x, y) \mid R \times R : x - y \in Z\}$ on R. Which of the following property does this relation is/are posses?

A Reflexive — Correct Option

B Symmetric — Correct Option

C Transitive — Correct Option

D Irreflexive

YOUR ANSWER - NA | CORRECT ANSWER - a,b,c | STATUS - SKIPPED

Solution :

(a, b, c)

In this relation, xRy means $x - y \in Z$.

To see that R is reflexive, take any $x \in R$ and observe that $x - x = 0 \in Z$, so xRx. Therefore R is reflexive.

To see that R is symmetric, we need to prove $xRy \rightarrow yRx$ for all $x, y? R$. We use direct proof. Suppose $xR\ y$. This means $x - y \in Z$. Then it follows that $-(x - y) = y - x$ is also in Z. But $y - x \in Z$ means yRx. We've shown $xR\ y$ implies yRx, so R is symmetric.

To see that R is transitive, we need to prove $(xRy \rightarrow yRz) \rightarrow xRz$ is always true. We prove this conditional statement with direct proof. Suppose xRy and yRz. Since xRy, we know $x - y \in Z$. Since yRz, we know $y - z \in Z$. Thus $x - y$ and $y - z$ are both integers; by adding these integers we get another integer $(x - y) + (y - z) = x - z$. Thus $x - z \in Z$, and this means xRz. We've now shown that if xRy and yRz, then xRz. Therefore R is transitive.

QUESTION ANALYTICS +

Q. 16 FAQ | Have any Doubt ?

Which of the relations on set of all people are equivalence relation?

A $\{(x, y) \mid x$ and y are the same age$\}$ — Correct Option

B $\{(x, y) \mid x$ and y have the same parents$\}$ — Correct Option

C $\{(x, y) \mid x$ and y share a common parent$\}$

D $\{(x, y) \mid x$ and y have met$\}$

YOUR ANSWER - NA | CORRECT ANSWER - a,b | STATUS - SKIPPED

Solution :

(a, b)

- Option (a) is equivalence relation.
- Option (b) is equivalence relation:
  Reflexive: Obviously x has its same parents.
  Symmetric: If x has the same parent of y (suppose x and y are brothers) then y has same parent of x. So it is symmetric.
  Transitive: If x and y have same parents and y and z have same parents then x has same parent of z (x, y, z are all brother). Thus R is transitive.
- Option (c) is not transitive: x can have the same father of y, and y have the same mother of z but x and z may not have neither the same mother nor the same father.
- Option (d) is not transitive if x have met y and y have met z, it doesn't mean that x have met z.

So, option (a) and (b) are equivalence relation.

QUESTION ANALYTICS +

Q. 17 Have any Doubt ?

Consider the following two statements:
S_1 : A student in IITB has not read the standard books.
S_2 : Everyone in IITB has passed the semester exam.

Which of the following statements follows from S_1 and S_2 as per sound inference rules of logic?

A Everyone who passed the semester exam has read the standard books.

B Someone who passed the semester exam has not read the standard books. Correct Option

Solution :
(b)

Let, $S(x)$ = x is a student in IITB
$SB(x)$ = x has read standard books
$P(x)$ = x has passed the semester exam

$S_1 : \exists x\ (S(x) \wedge \neg\ SB(x))$
$S_2 : \forall x\ (S(x) \rightarrow P(x))$
Conclusion: $\exists x\ (P(x) \wedge \neg\ SB(x))$
i.e. someone who passed the semester exam has not read the standard books.

C Everyone who passed the semester exam has not read the standard books.

D Someone who passed the semester exam has read the standard books.

QUESTION ANALYTICS +

Q. 18 FAQ Have any Doubt ?

binary relation R on Z × Z is defined as follow:
(a, b) R(c, d) iff a > c and b > d
Which of the following is correct?

A Both reflexive and symmetric

B Both symmetric and transitive

C Both antisymmetric and transitive Correct Option

Solution :
(c)

Given relation is $(a, b)\ R(c, d)$ iff $a > c$ and $b > d$

- **Reflexivity:** $(a, b)\ R(a, b)$. Since $a > a$ and $b > b$ is not true so it is not reflexive.
- **Symmetric:** It is not symmetric.
- **Transitivity:** If $(a, b)\ R(c, d)$ and $(c, d)\ R(e, f)$ then $a > c$ and $b > d$ also $c > e$ and $d > f$ which implies $a > e$ and $b > f$ so, it is transitive.
- **Antisymmetric:** It is antisymmetric.

D None of these

QUESTION ANALYTICS +

Q. 19 FAQ Have any Doubt ?

City authorities are concerned about traffic accidents on major roads. They would like to have ambulances stationed at road intersections to quickly reach the scene of any accident along these roads. To minimize response time, ambulances are to be located at intersections with traffic lights so that any segment of road can be reached by at least one ambulance that does not have to pass through a traffic light to reach the scene of the accident. If we model the road network as a graph, where intersections with traffic lights are vertices and edges represent road segments between traffic lights, the graph theoretic question to be answered is:

A Find a spanning tree with minimum number of edges.

B Find a spanning tree with minimum cost.

C Find a minimal colouring.

D Find a minimum size vertex cover. Correct Option

Solution :
(d)
Each ambulance "covers" the adjacent roads, and all roads are covered in this way.

QUESTION ANALYTICS +

Q. 20 Have any Doubt ?

Let G be a graph with 14 edges and G' be a graph with 22 edges then the number of vertices in graph G is _____.

A 8

B 9 Correct Option

Solution :

(D)

Maximum number of edges in G with n vertex = ${}^{n}C_{2}$

i.e. ${}^{n}C_{2} = 14 + 22$

$$\frac{n(n-1)}{2} = 36$$

$$n^2 - n - 72 = 0$$

$$n^2 - 9n + 8n - 72 = 0$$

$$(n - 9)(n + 8) = 0$$

But n can't be less than 0.

So, $n = 9$

C 5

D 6

QUESTION ANALYTICS +

Kunal Jha
Course: GATE
Computer Science Engineering(CS)

DISCRETE MATHEMATICS (GATE - 2021) - REPORTS

OVERALL ANALYSIS | COMPARISON REPORT | SOLUTION REPORT

ALL(33) | CORRECT(0) | INCORRECT(0) | SKIPPED(33)

Q. 21 FAQ | Have any Doubt ?

In a poset (A, ≤), if there is no element $x \in A$ with $x < y$ then which of the following is true?

A. An element x exists for which $x = y$.

B. An element x is maximal in poset. — Correct Option

Solution :
(b)
By the definition, an element x exist in a poset $(A, \leq)$ is maximal iff there is no $x \in A$ with $x < y$.

C. A set with the same subset of poset.

D. An element x is minimal in poset.

QUESTION ANALYTICS +

Q. 22 FAQ | Have any Doubt ?

How many paths are there in the plane from (0, 0) to (m, n) ? N × N, if the possible steps from (i, j) are either $(i + 1, j)$ or $(i, j + 1)$?

A. ${}^{2m}C_n$

B. ${}^{m}C_n$

C. ${}^{m+n}C_n$ — Correct Option

Solution :
(c)
From a vertex (i, j) with either $i > m$ or $j > n$, it is not possible to reach (m, n). Therefore one cannot overshoot m on the x-axis, and n on the y-axis. The total number of steps required to reach (m, n) is $m + n$ (m steps on the x-axis and n on the y-axis). Among the $m + n$ steps, the m x-axis steps and n y-axis steps can be distributed in any manner. This is obtained by choosing n steps for the y-axis (the rest would go to the x-axis).

D. m^n

QUESTION ANALYTICS +

Q. 23 FAQ | Have any Doubt ?

Twin primes are pairs of numbers p and $p + 2$ such that both are primes-for instance, 5 and 7, 11 and 13, 41 and 43. The Twin Prime Conjecture says that there are infinitely many twin primes. Let TwinPrime(n) be a predicate that is true if n and $n + 2$ are twin primes. Which of the following formulas, interpreted over positive integers, expresses that there are only finitely many twin primes?

A. $\forall m.\ \forall n.m = n$ and not(TwinPrime(n))

B. $\exists m.\ \forall n.n = m$ implies TwinPrime(n)

C. $\forall m.\ \exists n.n = m$ and TwinPrime(n)

D. $\exists m.\ \exists n.$ TwinPrime(n) implies $n = m$ — Correct Option

Solution :
(d)
This says that there is a bound, m, such that any twin prime is below m. In other words, that there only finitely many twin primes.

QUESTION ANALYTICS +

Q. 24

FAQ | Have any Doubt ?

A company is due to send a shipment to a client and the CEO has resigned. To select a new CEO, some candidates have been interviewed. One of them will be chosen through a vote. If the workers union resort to a strike and the candidates have to be interviewed again, then the shipment deadline will be missed. If there are more abstainers than voters in the vote to choose the new CEO, then the candidates have to be interviewed again. Suppose that the shipment was sent on time. Which of the following is a valid conclusion?

A The workers union did not resort to a strike.

B The number of voters was more than the number of abstainers.

C (a) or (b).

D If the workers union resorted to a strike, then the number of voters was greater. Correct Option

Solution :
(d)
If the workers union resorted to a strike, then the number of voters was greater than or equal to the number of abstainers.
Let p, q, r, s denote the following assertions: p : abstainers > voters q : interview repeated r : workers strike s : shipment missed
Premises: p IMPLIES q, (q AND r) IMPLY s
Since it is given that shipment was sent on time, s is false.
By contra positive, (NOT q) OR (NOT r) and (NOT q) IMPLIES (NOT p) are valid conclusions.
(a) says (NOT r), which not valid since r could be true when s is false. (b) says voters > abstainers, which is not valid since even if voters = abstainers, s could be false.
(c) says (NOT r) OR (voters > abstainers), which not valid since we can have both r and voters = abstainers true and still have s false.
(d) says r IMPLIES (voters = abstainers). This is valid since (NOT q) OR (NOT r) is valid, which means r is false or q is false. If r is true, then q is false and hence p is false, which means abstainers = voters.

QUESTION ANALYTICS +

Q. 25

Have any Doubt ?

For the inter-hostel six-a-side football tournament, a team of 6 players is to be chosen from 11 players consisting of 5 forwards, 4 defenders and 2 goalkeepers. The team must include at least 2 forwards, at least 2 defenders and at least 1 goalkeeper. Then the number of different ways in which the team can be chosen _____.

260 Correct Option

Solution :
260
5 positions are fixed. The 6th can be forward/defender/goalkeeper.
Combination 1 : 3 forward, 2 defenders, 1 goal is : ${}^5C_3 * {}^4C_2 * {}^2C_1 = 120$
Combination 2 : 2 forward, 3 defenders, 1 goal is : ${}^5C_2 * {}^4C_3 * {}^2C_1 = 80$
Combination 3 : 2 forward, 2 defenders, 2 goal is : ${}^5C_2 * {}^4C_2 * {}^2C_2 = 60$
Add the three to get 260.

QUESTION ANALYTICS +

Q. 26

FAQ | Have any Doubt ?

Given recurrence relation:

$$a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$$

With the initial conditions $a_0 = 2$, $a_1 = 5$ and $a_2 = 15$.
If the solution is of form $a_n = C_1 \cdot r_1^n + C_2 \cdot r_2^n + C_3 \cdot r_3^n$
Find $C_1 + C_2 + C_3$ _____.

2 Correct Option

Solution :
2
The characteristics polynomial of this recurrence relation is $r^3 - 6r^2 + 11r - 6$.
Characteristics roots are $r = 1$, $r = 2$ and $r = 3$ because

$$r^3 - 6r^2 + 11r - 6 = (r-1)(r-2)(r-3)$$

So, solution of this recurrence relation is of the form.

$$a_n = C_1 \cdot 1^n + C_2 \cdot 2^n + C_3 \cdot 3^n$$

To find the constants we will use initial conditions.

$$a_0 = 2 = C_1 + C_2 + C_3$$
$$a_1 = 5 = C_1 + C_2 \cdot 2 + C_3 \cdot 3$$
$$a_3 = 15 = C_1 + C_2 \cdot 4 + C_3 \cdot 9$$

After solving these we get $C_1 = 1$, $C_2 = -1$ and $C_3 = 2$.
So, $C_1 + C_2 + C_3 = 1 - 1 + 2 = 2$

QUESTION ANALYTICS +

Q. 27

Have any Doubt ?

The 12 houses on one side of a street are numbered with even numbers starting at 2 and going up to 24. A free newspaper is delivered on Monday to 3 different houses chosen at random from these 12. Find the probability that at least 2 of these newspapers are delivered to houses with numbers strictly greater than 14 _____.
(Upto 2 decimal place)

0.36 (0.36 - 0.37) — Correct Option

Solution :
0.36 (0.36 - 0.37)

Total number of ways of choosing 3 houses : ${}^{12}C_3 = 220$

5 houses have number strictly greater than 14. So the number of good choices is

$${}^5C_3 + ({}^5C_2)({}^7C_1) = 80$$

So, required probability $= \frac{80}{220} = 0.36$

QUESTION ANALYTICS +

Q. 28

FAQ | Have any Doubt ? | Bookmark

Let G be a group. Suppose that the number of elements in G of order 5 is 28. Determine the number of distinct subgroups of G of order 5 _____.

7 — Correct Option

Solution :
7

Let g be an element in G of order 5.
i.e. $\langle g \rangle = \{e, g, g^2, g^3, g^4\}$ where e is the identity element in G.
Also if h is another element in G of order 5, then we have either $\langle g \rangle = \langle h \rangle$ or $\langle g \rangle \cap \langle h \rangle = \{e\}$.
This follows from the fact that the intersection $\langle g \rangle \cap \langle h \rangle$ is a subgroup of the order 5 group $\langle g \rangle$ and thus the order of $\langle g \rangle \cap \langle h \rangle$ is either 5 or 1.
On the other hand, if H is subgroup of G of order 5, then every non identity element in H has order 5.
So, according to these observation. Subgroup of order 5 contains exactly 4 elements of order 5 and each element of order 5 appears in exactly one of such subgroups.

As, there are 28 elements of order 5, there are $\frac{28}{4} = 7$ subgroups of order 5.

QUESTION ANALYTICS +

Q. 29

Have any Doubt ? | Bookmark

Given a positive integer n, let G = (V, E) be a graph where V = $\{0, 1\}^n$, i.e. V is the set of vertices which has one-to-one correspondence with the set of all n bit binary strings and $E = \{(u, v) \mid u, v$ belongs to V$\}$, u and v differ in exactly one bit position? Find the size of E if $n = 8$ _____.

1024 — Correct Option

Solution :
1024

Consider the vertex which is exactly the same bit pattern as v except first bit v_1, v_2 differs from v in only the 2nd bit and so on till v_n which is different form v in last bit.
Thus, there are exactly n vertices having a hamming distance of 1 from v.
Now there are total of 2^n vertices because each vertex corresponds to a bit string and for n bits 2^n bit strings.
Now, we know for directed graph
Sum of degrees of vertices = 2E, where E = edges
So, $2^n \times n = 2E$

Thus number of edges $E = \frac{n\,2^n}{2}$

So, for $n = 8$

$$E = \frac{8 \times 2^8}{2} = 2^{10} = 1024$$

Note: This is hyper cube graph and for hyper cube graph number of edges = $n \cdot 2^{n-1}$.

QUESTION ANALYTICS +

Q. 30

Have any Doubt ? | Bookmark

In an office, there are 12 staffs namely: Jaya, Radha, Manorma, Priti, Leela, Anuradha, Akanksha, Gayatri, Pinki, Lalita, Babita and Arohi. All shake hands with each other. How many handshakes will there be altogether?

66 — Correct Option

Solution :
66

Total number of handshakes can be calculated by the formula = ${}^nC_2 = {}^{12}C_2 = \frac{12 * 11}{2} = 66$.

QUESTION ANALYTICS +

Item **21-30** of 33 « previous 1 2 3 4 next »

Kunal Jha
Course: GATE
Computer Science Engineering(CS)

HOME
MY TEST
BOOKMARKS
MY PROFILE
REPORTS
BUY PACKAGE
NEWS
TEST SCHEDULE

DISCRETE MATHEMATICS (GATE - 2021) - REPORTS

OVERALL ANALYSIS | COMPARISON REPORT | SOLUTION REPORT

ALL(33) | CORRECT(0) | INCORRECT(0) | SKIPPED(33)

Q. 31

Have any Doubt ?

Consider the set $S = \{1, w, w^2\}$, where w and w^2 are cube roots of unity. If * denotes the multiplication operation, the structure (S, *) form(s)

A. A Group — Correct Option
B. A Ring
C. An integral domain
D. A field

YOUR ANSWER - NA | CORRECT ANSWER - a | STATUS - SKIPPED

Solution :

(a)

We can directly answer this question as "A Group", because other three options require two operations over structure, but let us see whether (S, *) satisfies group properties or not.
Closure: If we multiply any two elements of S, we get one of three elements of S, so S is closed over *.
Associativity: multiplication operation is anyway associative.
Identity element: 1 is identity element of S.
Inverse element: inverse of 1 is 1 because 1 * 1 = 1, inverse of w is w^2, because $w*w^2 = 1$. Also inverse of w^2 is w, because $w^2 * w = 1$.
So S satisfies all 4 properties of group, so it is a group. Infact S is an abelian group, because it also satisfies commutative property.

QUESTION ANALYTICS +

Q. 32

Have any Doubt ?

Consider the following well-formed formulae:

(a) $\neg\forall x P(x)$ (b) $\neg\exists x(P(x))$

(c) $\forall x(P(x))$ (d) $\exists x(P'(x))$

Which of the above is/are equivalent?

A. a — Correct Option
B. b
C. c
D. d — Correct Option

YOUR ANSWER - NA | CORRECT ANSWER - a,d | STATUS - SKIPPED

Solution :

(a, d)

A formula $\neg\forall x P(x)$ is equivalent to formula $\exists x(P'(x))$.
So, $\neg\forall x P(x)$ is equivalent to $\exists x(P'(x))$.

QUESTION ANALYTICS +

Q. 33

Have any Doubt ?

For the composition table of a cyclic group shown below:

*	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

Which one of the following is/are generators?

A. a
B. b

C c Correct Option

D d Correct Option

YOUR ANSWER - NA | CORRECT ANSWER - c,d | STATUS - SKIPPED

Solution :

(c, d)

An element is a generator for a cyclic group if on repeated applications of it upon itself, it can generate all elements of group.

For example here : $a*a = a$, then $(a*a)*a = a*a = a$, and so on. Here we see that no matter how many times we apply a on itself, we can't generate any other element except a, so a is not a generator.

Now for b, $b*b = a$. Then $(b*b)*b = a*b = b$. Then $(b*b*b)*b = b*b = a$, and so on. Here again we see that we can only generate a and b on repeated application of b on itself. So it is not a generator.

Now for c, $c*c = b$. Then $(c*c)*c = b*c = d$. Then $(c*c*c)*c = d*c = a$. Then $(c*c*c*c)*c = a*c = c$.

So we see that we have generated all elements of group. So c is a generator.

For d, $d*d = b$. Then $(d*d)*d = b*d = c$. Then $(d*d*d)*d = c*d = a$. Then $(d*d*d*d)*d = a*d = d$.

So we have generated all elements of group from d, so d is a generator. So (c) and (d) are generators.

QUESTION ANALYTICS +

Kunal Jha
Course: GATE
Computer Science Engineering(CS)

HOME
MY TEST
BOOKMARKS
MY PROFILE
REPORTS
BUY PACKAGE
NEWS
TEST SCHEDULE

DISCRETE MATHEMATICS (GATE - 2021) - REPORTS

OVERALL ANALYSIS | COMPARISON REPORT | SOLUTION REPORT

ALL(33) | CORRECT(0) | INCORRECT(0) | SKIPPED(33)

Q. 31

Have any Doubt ?

Consider the set $S = \{1, w, w^2\}$, where w and w^2 are cube roots of unity. If * denotes the multiplication operation, the structure (S, *) form(s)

A. A Group — Correct Option
B. A Ring
C. An integral domain
D. A field

YOUR ANSWER - NA | CORRECT ANSWER - a | STATUS - SKIPPED

Solution :

(a)

We can directly answer this question as "A Group", because other three options require two operations over structure, but let us see whether (S, *) satisfies group properties or not.
Closure: If we multiply any two elements of S, we get one of three elements of S, so S is closed over *.
Associativity: multiplication operation is anyway associative.
Identity element: 1 is identity element of S.
Inverse element: inverse of 1 is 1 because 1 * 1 = 1, inverse of w is w^2, because $w*w^2 = 1$. Also inverse of w^2 is w, because $w^2 * w = 1$.
So S satisfies all 4 properties of group, so it is a group. Infact S is an abelian group, because it also satisfies commutative property.

QUESTION ANALYTICS +

Q. 32

Have any Doubt ?

Consider the following well-formed formulae:

(a) $\neg\forall x P(x)$ (b) $\neg\exists x(P(x))$

(c) $\forall x(P(x))$ (d) $\exists x(P'(x))$

Which of the above is/are equivalent?

A. a — Correct Option
B. b
C. c
D. d — Correct Option

YOUR ANSWER - NA | CORRECT ANSWER - a,d | STATUS - SKIPPED

Solution :

(a, d)

A formula $\neg\forall x P(x)$ is equivalent to formula $\exists x(P'(x))$.
So, $\neg\forall x P(x)$ is equivalent to $\exists x(P'(x))$.

QUESTION ANALYTICS +

Q. 33

Have any Doubt ?

For the composition table of a cyclic group shown below:

*	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

Which one of the following is/are generators?

A. a
B. b

C c Correct Option

D d Correct Option

YOUR ANSWER - NA | CORRECT ANSWER - c,d | STATUS - SKIPPED

Solution :

(c, d)

An element is a generator for a cyclic group if on repeated applications of it upon itself, it can generate all elements of group.

For example here : $a*a = a$, then $(a*a)*a = a*a = a$, and so on. Here we see that no matter how many times we apply a on itself, we can't generate any other element except a, so a is not a generator.

Now for b, $b*b = a$. Then $(b*b)*b = a*b = b$. Then $(b*b*b)*b = b*b = a$, and so on. Here again we see that we can only generate a and b on repeated application of b on itself. So it is not a generator.

Now for c, $c*c = b$. Then $(c*c)*c = b*c = d$. Then $(c*c*c)*c = d*c = a$. Then $(c*c*c*c)*c = a*c = c$.

So we see that we have generated all elements of group. So c is a generator.

For d, $d*d = b$. Then $(d*d)*d = b*d = c$. Then $(d*d*d)*d = c*d = a$. Then $(d*d*d*d)*d = a*d = d$.

So we have generated all elements of group from d, so d is a generator. So (c) and (d) are generators.

QUESTION ANALYTICS +