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TOPICWISE : DISCRETE MATHEMATICS-1 (GATE - 2020) - REPORTS

OVERALL ANALYSIS | COMPARISON REPORT | SOLUTION REPORT

ALL(17) CORRECT(0) INCORRECT(0) SKIPPED(17)

Q. 1

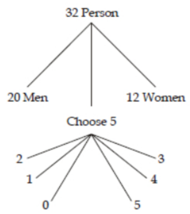
Solution Video | Have any Doubt ?

How many committees of five people can be chosen from 20 men and 12 women such that each committee contains atleast three women?

(A) 75240

(B) 52492 — Correct Option

Solution :
(b)
Given,



We must choose at least 3 women, so, we calculate 3 women, 4 women and 5 women and by addition rule add the results:

$$= {}^{12}C_3 \times {}^{20}C_2 + {}^{12}C_4 \times {}^{20}C_1 + {}^{12}C_5 \times {}^{20}C_0$$
$$= 220 \times 190 + 495 \times 20 + 792 \times 1 = 52492$$

(C) 41800

(D) 9900

QUESTION ANALYTICS +

Q. 2

Solution Video | Have any Doubt ?

Let $P(m, n)$ be the statement "m divides n" where the Universe of discourse for both the variables is the set of positive integers. Determine the truth values of the following propositions.

S_1: $\exists m \forall n\ P(m, n)$
S_2: $\forall n\ P(1, n)$
S_3: $\forall m \forall n\ P(m, n)$

(A) S_1: True; S_2: True; S_3: False — Correct Option

Solution :
(a)
Given $P(m, n)$ is "m divides n" in the set of positive integers. Therefore,
$S_1 : \exists m \forall n\ P(m, n)$, there is atleast one m which can divide all number in given set (it is true if $m = 1$)
$S_2 : \forall n\ (1, n)$, 1 divides n in given set. It is true.
$S_3 : \forall m, \forall n\ P(m, n)$, all m can divide all n in given set (it is false, if n is a prime numbers).

(B) S_1: True; S_2: False; S_3: False

(C) S_1: False; S_2: False; S_3: False

(D) S_1: True; S_2: True; S_3: True

QUESTION ANALYTICS +

Q. 3

Solution Video | Have any Doubt ?

Consider the following arguments:
A1: $(\exists x)\ [A(x) \wedge B(x)] \rightarrow [(\exists x) A(x)] \wedge [\exists x\ B(x)]$

A2: $(\exists x)\ (\forall y)\ Q(x, y) \rightarrow (\forall y)(\exists x)\ Q(x, y)$

Which of the above argument is valid?

(A) A1 only

(B) A2 only

C Both A1 and A2 — Correct Option

Solution :
(c)
$(\exists x)[A(x) \wedge B(x)]$ implies $[(\exists x)A(x)] \wedge [\exists x\ B(x)]$
$\exists x\ \forall y\ Q(x, y)$ implies $\forall y\, \exists x\, Q(x, y)$ but converse need not be true.
Both arguments are valid.

D Neither A1 nor A2

QUESTION ANALYTICS

Q. 4

Solution Video | Have any Doubt ?

Consider a function f from A to B such that $f: A \to B$ is bijective. f^{-1} represents inverse of f. Which of the following is incorrect?

A $f^{-1}: B \to A$ exist

B $f^{-1}: B \to A$ is unique

C $f^{-1}: B \to A$ is bijective

D None of these — Correct Option

Solution :
(d)
$f: A \to B$ is bijective.
$\Rightarrow f: A \to B$ is one-one (injective) f onto (surjective)
1. $f: A \to B$ is one-one $\Rightarrow f^{-1}: B \to A$ exists and it is unique.
$\Rightarrow f^{-1}$ is also one-one ...(1)
2. $f: A \to B$ is onto $\Rightarrow f(A) = B$
$\Rightarrow A = f^{-1}(B)$ or $f^{-1}(B) = A \Rightarrow f^{-1}: B \to A$ is also onto ...(2)
from (1) and (2) $f^{-1}: B \to A$ is bijective.

QUESTION ANALYTICS

Q. 5

Solution Video | Have any Doubt ?

Consider of the following is/are valid:
1. $((p \Rightarrow q) \wedge (r \Rightarrow s) \wedge (p \vee r)) \Rightarrow (q \vee s)$
2. $((p \Rightarrow q) \wedge (r \Rightarrow s) \wedge (\sim q \vee \sim s) \Rightarrow (\sim p \vee \sim r)$
3. $((p \Rightarrow q) \wedge (q \Rightarrow r)) \Rightarrow (p \Rightarrow r)$

A 1 and 2

B 2 and 3

C 1 and 3

D 1, 2 and 3 — Correct Option

Solution :
(d)
1. is valid by constructive dilemma.
2. is valid by destructive dilemma.
3. is valid by hypothetical syllogism.
All of the above are known rules of inference.

QUESTION ANALYTICS

Q. 6

Solution Video | Have any Doubt ?

Let f and $g: R \to R$ be defined on R (R is a set of real numbers) as: $f(x) = x + 2$, $g(x) = (1 + x^2)^{-1}$ Find the value of $f^{-1}g$ (3) ______?

-1.9 — Correct Option

Solution :
-1.9

$$\begin{aligned} f(x) &= x+2 & \text{Let } y = f(x) \\ y &= x+2 & \Rightarrow x = f^{-1}(y) \\ \Rightarrow \quad x &= y-2 & \\ \Rightarrow \quad f^{-1}(y) &= y-2 & \text{or } f^{-1}(x) = x-2 \end{aligned}$$

$$g(3) = (1+(3)^2)^{-1} = (1+9)^{-1} = \frac{1}{10}$$

$$f^{-1}g(3) = f^{-1}(g(3)) = g(3) - 2$$

$$\Rightarrow \quad f^{-1} g(3) = \frac{1}{10} - 2 = -1.9$$

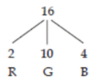
QUESTION ANALYTICS +

Q. 7 Solution Video | Have any Doubt ?

Number of ways to paint 16 offices such that 2 of them are red, 10 of them are green and remaining 4 are blue _______?

120120 Correct Option

Solution :
120120
This problem reduces to selecting 10 offices from 16, then 4 from remaining 6.



i.e., number of ways $= \dfrac{16!}{10!\,4!\,2!} = 120120$

QUESTION ANALYTICS +

Q. 8 Solution Video | Have any Doubt ?

In an examination of 9 papers a candidate has to pass in more papers than the number of papers in which he fails in order to be successful. The number of ways in which he can be unsuccessful is _______.

256 Correct Option

Solution :
256
The candidate is unsuccessful if he fails in 9 or 8 or 7or 6 or 5 papers.
$\therefore$ The number of ways to be unsuccessful

$$= {}^9C_9 + {}^9C_8 + {}^9C_7 + {}^9C_6 + {}^9C_5 = 256$$

QUESTION ANALYTICS +

Q. 9 Solution Video | Have any Doubt ?

Consider the set $S = \{a, b\}$ and 'L' be a binary relation such that L = {all binary relations except reflexive relation set S}. The number of relation which are symmetric _______.

6 Correct Option

Solution :
6
Set $S = \{a, b\}$

Total number of binary relation $= 2^{n^2} = 2^{2^2} = 2^4 = 16$
We need to count relation which are symmetric (S) but not reflexive (R) which is $n(S - R)$.

$$\begin{aligned} n(S - R) &= n(S) - n\,(S \cap R) \\ &= 2^{n(n+1)/2} - 2^{n(n-1)/2} \\ &= 2^{(2 \times 3)/2} - 2^{(2 \times 1)/2} \\ &= 2^3 - 2^1 = 6 \end{aligned}$$

So total 6 such relations will be there.

QUESTION ANALYTICS +

Q. 10 Solution Video | Have any Doubt ?

Which of the following is/are not true?
S_1 : The set of negative integers is countable.
S_2 : The set of integers that are multiples of 7 is countable.
S_3 : The set of even integers is countable.
S_4 : The set of real numbers between 0 and $\frac{1}{2}$ is countable.

A. S_1 and S_3 only

B. S_2 and S_4 only

C. S_2 only

D S_4 only Correct Option

Solution :
(d)
A set is **countable** if: "(S_1) it is finite or (S_2) it has the same cardinality (size) as the set of natural numbers".

Equivalently, a set is countable if it has the same cardinality as the subset of the set of natural numbers, otherwise it is uncountable.

The set of real numbers between 0 and $\frac{1}{2}$ is uncountably infinite.

The set of all integers are countable.

QUESTION ANALYTICS +

TOPICWISE : DISCRETE MATHEMATICS-1 (GATE - 2020) - REPORTS

Q. 11 Solution Video Have any Doubt ?

Consider two sets A and B such that:
$A \cup B \subseteq A \cap B$
Then, which of the following is incorrect?

A $A = \{\}, B = \{\}$ always — Correct Option

Solution :
(a)
$A \cup B \subseteq A \cap B$ holds true when $A = B$. It is true for empty as well as nonempty sets.
$\Rightarrow |A| = |B|$ is true $|A| \geq 0$ eg. $A = B$ $\{a, b\}$
Hence $A = \{\ \}, B = \{\ \}$ "always" is false.

B ?A? = ?B?

C $A = B$

D None of these

QUESTION ANALYTICS +

Q. 12 Solution Video Have any Doubt ?

A binary relation R on Z × Z is defined as follows:
$(a, b)\ R\ (c, d)$ iff $a = c$ or $b = d$
Consider the following propositions:
1. R is reflexive.
2. R is symmetric.
3. R is antisymmetric.
Which one of the following statements is True?

A Both 1 and 2 are true — Correct Option

Solution :
(a)
R is reflexive: Since $(a, b)\ R\ (a, b)$ for all elements (a, b) because $a = a$ and $b = b$ are always true.
R is symmetric: Since $(a, b)\ R\ (c, d)$ and $a = c$ or $b = d$ which can be written as $c = a$ or $d = b$.
So, $(a, b)\ R\ (a, b)$ is true.
R is not antisymmetric: Since $(1, 2)\ R\ (1, 3)$ and $1 = 1$ or $2 = 3$ true b/c $1 = 1$.
So $(1, 3)\ R\ (1, 2)$ but here $2 \neq 3$ so $(1, 2) \neq (1, 3)$.
So, only statement 1 and 2 are correct.

B 1 is true and 2 is false

C 1 is false and 3 is true

D Both 2 and 3 are true

QUESTION ANALYTICS +

Q. 13 Solution Video Have any Doubt ?

The function f is mapped from natural numbers to integer numbers and $f(x) = x^2 - 2x + 3$.Consider $N = \{0, 1, 2, 3, \ldots\}$ and $Z = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$. What is the function f ?

A Injective function

B Surjective function

C Bijective function

D None of these — Correct Option

Solution :
(d)
$$f : N \to Z$$
$$f(0) = f(2) = 3$$
$\Rightarrow$ f is not injective
Clearly f is not surjective, all numbers in Z do not have preimages in N (example: 0 has no preimage)
f is function which is not injective and not surjective.

QUESTION ANALYTICS

Q. 14

Solution Video | Have any Doubt ?

Let satisfiable (x) be a predicate which denotes that x is satisfiable logic. Let Valid (x) be a predicate which denotes that x is valid logic. Which of the following first order logic sentences does not represents the statements:
"Not every satisfiable logic is Valid"

A. $\neg\forall x\,(\text{satisfiable }(x) \Rightarrow \text{Valid }(x))$

B. $\exists x\,(\text{satisfiable }(x) \wedge \neg\,\text{Valid }(x))$

C. $\neg\forall x\,(\neg\text{satisfiable }(x) \vee \text{Valid }(x))$

D. $\forall x\,(\text{satisfiable }(x) \Rightarrow \neg\,\text{Valid }(x))$

Correct Option

Solution :
(d)
"Not every satisfiable logic is Valid"

$$
\begin{aligned}
&= \text{Not (every satisfiable logic is Valid)} \\
&= \text{Not }(\forall x(\text{satisfiable }(x) \Rightarrow \text{Valid }(x))) && \text{option (a)} \\
&= \text{Not }(\forall x(\neg\text{satisfiable} \vee \text{Valid }(x))) && \text{option (c)} \\
&= \exists x(\text{satisfiable }(x) \wedge \neg\,\text{Valid }(x)) && \text{option (b)}
\end{aligned}
$$

Statement (d) says every satisfiable logic is invalid.
So option (d) is not represent given statement.

QUESTION ANALYTICS

Q. 15

Solution Video | Have any Doubt ?

Consider the set $S = \{1, 2\}$ and 'L' be a binary relation such that L = {all binary relations except relations which are neither reflexive nor irreflexive}. The number of such relations are ______.

8

Correct Option

Solution :
8

Set $\quad S = \{1, 2\}$

Total number of binary relation $= 2^{n^2} = 2^{2^2} = 2^4 = 16$

We need to count relations which are neither reflexive (R) nor irreflexive (IR) which is $n(R \cup IR)^c$

$$
\begin{aligned}
\text{Now,} \quad n(R \cup IR)^c &= 2^{n^2} - [n(R \cup IR)] \\
&= 2^{n^2} - [n(R) + n(IR) - n(R \cap IR)] \\
&= 2^{n^2} - \left(2^{n^2-n} + 2^{n^2-n} - 0\right) \\
&= 16 - (4 + 4) = 8
\end{aligned}
$$

QUESTION ANALYTICS

Q. 16

Solution Video | Have any Doubt ?

Consider the set S = {1, 2, 3, . . . , 25}. The number of subsets $T \subseteq S$ of size five such that T has at least one odd number in it is ______.

52338

Correct Option

Solution :
52338

Total number of subset of 5 element $= {}^{25}C_5$

$$= \frac{25\times24\times23\times22\times21}{5\times4\times3\times2\times1} = 23 \times 22 \times 21 \times 5 = 53130$$

T be a 5 element subset contain no odd number $= {}^{12}C_5$

$$= \frac{12\times11\times10\times9\times8}{5\times4\times3\times2\times1} = 792$$

So number of 5 element subset with atleast 1 odd number

$$
\begin{aligned}
T \subseteq S &= {}^{25}C_5 - {}^{12}C_5 \\
&= 53130 - 792 = 52338
\end{aligned}
$$

QUESTION ANALYTICS

Q. 17

Solution Video | Have any Doubt ?

The number of positive integer less than or equal to 1000 that are relatively prime to 15 are _____.

533

Correct Option

Solution :
533

A number is relatively prime to 15 iff it is not divisible by 3 and not divisible by 5.

Set of integer from 1 to 1000 divisible by $3 = \left\lfloor \frac{1000}{3} \right\rfloor = 333.$

Set of integer from 1 to 1000 divisible by $5 = \left\lfloor \frac{1000}{5} \right\rfloor = 200.$

So, number of integer not relatively prime to 15 are

$$|A \cup B| = |A| + |B| - |A \cap B|$$

$$= \left\lfloor \frac{1000}{3} \right\rfloor + \left\lfloor \frac{1000}{5} \right\rfloor - \left\lfloor \frac{1000}{15} \right\rfloor$$

$$= 333 + 200 - 66 = 467$$

So, number of integer relatively prime to 15 are

$$|\overline{A \cup B}| = 1000 - 467 = 533$$

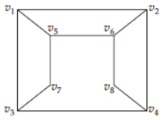
QUESTION ANALYTICS +

TOPICWISE : DISCRETE MATHEMATICS-2 (GATE - 2020) - REPORTS

Q. 1

Solution Video | Have any Doubt ?

Consider the graph given below:



The two distinct sets of vertices, which make the graph bipartite are:

A. $(v_1, v_4, v_6); (v_2, v_3, v_5, v_7, v_8)$

B. $(v_1, v_7, v_8); (v_2, v_3, v_5, v_6)$

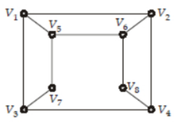
C. $(v_1, v_4, v_6, v_7); (v_2, v_3, v_5, v_8)$ — Correct Option

Solution :
(c)

A **bipartite graph** is a graph whose vertices can be divided into two disjoint sets U and V such that every edge connects a vertex in U to one in V. Vertex sets U and V are usually called the parts of the graph. Equivalently, a bipartite graph is a graph that does not contain any odd length cycles. It is also 2-colorable and the degree sum formula for a bipartite graph states that

$$\sum_{v \in V} \deg(V) = \sum_{u \in U} \deg(U) = |E|$$

Given graph:



(a) $(3 + 3 + 3) \neq (3 + 3 + 3 + 2 + 2)$
(b) $(3 + 2 + 2) \neq (3 + 3 + 3 + 3)$
(c) $(3 + 3 + 3 + 2) = (3+3+3+2) = 11 = |E|$
(d) $(3 + 3 + 3 + 2 + 2) \neq (3 + 3 + 3)$

D. $(v_1, v_4, v_6, v_7, v_8); (v_2, v_3, v_5)$

QUESTION ANALYTICS +

Q. 2

Solution Video | Have any Doubt ?

Which of the following statement is incorrect?

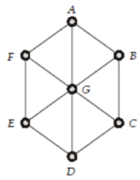
A. A graph of 6 vertices can be 1-chromatic.

B. Every tree with 2 or more vertices is 2-chromatic.

C. A wheel graph of n-vertices is $\left(\left\lfloor \frac{n}{2} \right\rfloor + 1\right)$ chromatic. — Correct Option

Solution :
(c)

Consider each options:
(a) Null graph of 6 vertices is 1-chromatic so it is correct.
(b) It is correct because tree with 2 or more vertices is always bichromatic.
(c) It is incorrect. Consider a wheel graph of 7 vertices.



The chromatic number of graph is 3.

- Color 1 for G
- Color 2 for A, E, C
- Color 3 for F, B, D
- A wheel graph is 3-chromatic when n-vertices are odd and 4-chromatic when n-vertices is even.
- So here $n = 7$, $\left(\left\lfloor \frac{n}{2} \right\rfloor + 1\right) = \left(\left\lfloor \frac{7}{2} \right\rfloor + 1\right) = 4$ which is incorrect because only 3 colors are required to color the above wheel graph

(d) This statement is correct because graph without odd length cycle having atleast 1 edge is bichromatic.

All other statements are true except option (c).

D A graph which has no circuit of odd-length and has atleast 1 edge is 2-chromatic.

QUESTION ANALYTICS +

Q. 3

Solution Video | Have any Doubt ?

Which recurrence relation satisfy the sequence: 2, 3, 4, ..., for $n \geq 1$.

A $a_n = 2a_{n-1} - a_{n-2}$ Correct Option

Solution :
(a)

Clearly, $a_n = n + 1$

$\Rightarrow$ $a_{n-1} = n$

$\Rightarrow$ $a_{n-2} = n - 1$

$\Rightarrow$ $a_n = 2a_{n-1} - a_{n-2}$ $[\because\ 2(n) - (n-1) = n + 1]$

B $a_n = a_{n-1} + a_{n-2}$

C $a_n = n$

D None of these

QUESTION ANALYTICS +

Q. 4

Solution Video | Have any Doubt ?

Consider statements:

S_1 : A finite lattice is always bounded.

S_2 : Complemented lattice is a proper subset of bounded lattice.

S_3 : A bounded and complemented lattice may or may not be a distributive lattice.

Which of the following is/are true?

A S_1 and S_2 only

B S_2 and S_3 only

C S_1 and S_3 only

D S_1, S_2 and S_3 Correct Option

Solution :
(d)

- A lattice is bounded iff the lattice has a greatest and a least element.
  $\therefore$ A finite lattice is always bounded.
- Complemented lattice is defined only for bounded lattice. A bounded lattice is complemented iff atleast one complement of every element exist in lattice. An element should one or more complements.
- A complemented lattice is distributive iff every element has a unique complement.

QUESTION ANALYTICS +

Q. 5

Solution Video | Have any Doubt ?

Which of the following statements are equivalent for graph with atleast 1 edge?

1. G is bipartite.
2. A graph G is 2 colourable.
3. Graph G has a Hamiltonian circuit.
4. Every cycle of G is of even length.

A 1 and 3

B 1, 2 and 3

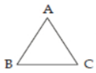
C 1, 2 and 4 Correct Option

Solution :
(c)

Considering each statements:

- If a graph is bipartite, then its two colourable. Because a bipartite graph can be represented as two groups of vertices such that vertices in same graph are not adjacent.
  Similarly, statement 2 is equivalent to statement 1.
- If a bipartite graph has a cycle, then it has to be of even length. Graph G is bipartite iff no odd

length cycle.

- 

This graph has a Hamiltonian circuit, but the cycle is of odd length and not bipartite.

$\therefore \quad 3 \not\equiv 4$

- 

This graph is bipartite and 2 colorable but does not have Hamiltonian circuit.

So 1, 2 and 4 are equivalent statements.

D 3 and 4

QUESTION ANALYTICS +

Q. 6

Solution Video | Have any Doubt ?

Consider complete graphs K_5 and K_6. Let X_5 and X_6 are number of perfect matching of K_5 and K_6 respectively. Then $X_5 + X_6$ = _______.

15 — Correct Option

Solution :
15

Number of perfect matching in a complete graph with even number of vertices,

$$X_n = 1 \times 3 \times 5 \times \ldots (n-1)$$

$$\therefore \quad X_6 = 1 \times 3 \times 5 = 15$$

Number of perfect matching in a complete graph of odd number of vertices = 0.

$$\therefore \quad X_5 = 0$$

$$\therefore \quad X_5 + X_6 = 15$$

QUESTION ANALYTICS +

Q. 7

Solution Video | Have any Doubt ?

Assumed undirected graph G is connected. G has 6-vertices and 10 edges. Find the minimum number of edges whose deletion from graph G is always guarantee that it will become disconnected.

6 — Correct Option

Solution :
6

Let there be n vertices and m edges. Therefore atleast $(n-1)$ edges are required to connect the graph.

$(n-2)$ edges will lead to disconnected graph therefore we need to delete $m - (n-2) = (m - n + 2)$ edges to get the disconnected graph.

Here $\quad m = 10$ and $n = 6$

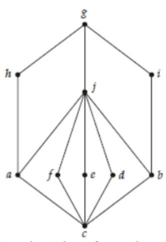
Therefore $\quad 10 - 6 + 2 = 6$ edges need to delete.

QUESTION ANALYTICS +

Q. 8

Solution Video | Have any Doubt ?

Consider the following lattice:



Total number of complements for the element 'b' is _______.

1 — Correct Option

Solution :
1

LUB = g, GLB = c

$b \vee h = g$ and $b \wedge h = c$

$\therefore$ b has only 1 complement [complement of $a = h$]

[Note: x and y are complement to each other iff $x \vee y = 1$ (Greatest element) and $x \wedge y = 0$ (least element)]

Q. 9

Solution Video | Have any Doubt ?

The coefficient of x^3 in the expression of $(1 + x)^3 (2 + x^2)^{10}$ is _______.

16384 — Correct Option

Solution :
16384

Given equation

$$= (1 + x)^3 (2 + x^2)^{10}$$
$$= (1 + x^3 + 3x^2 + 3x)\,({}^{10}C_0\, 2^0 (x^2)^{10} + {}^{10}C_1\, 2^1 (x^2)^9 + \ldots\ldots\ {}^{10}C_{10}\, 2^{10} (x^2)^0)$$

Coefficient of x^3 are

$$= {}^{10}C_{10}\, 2^{10} + {}^{10}C_9\, 2^9 \times 3$$
$$= 1.2^{10} + 10.2^9 \times 3$$
$$= 2^9 (30 + 2)$$
$$= 2^9 (2^5)$$
$$= 2^{14} = 16384$$

QUESTION ANALYTICS +

Q. 10

Solution Video | Have any Doubt ?

Define a recursive function such that : $f(n) = 10^{2^n}, n \geq 0$.

A. $f(n) = f(n-1) * f(n-2)$

B. $f(n) = f(n-1) + f(n-2)$

C. $f(n) = (f(n-1))^2$ — Correct Option

Solution :
(c)

$$f(n) = 10^{2^n},\ f(n-1) = 10^{2^{n-1}}$$

$$\Rightarrow \quad \frac{f(n)}{f(n-1)} = \frac{(10)^{2^n}}{(10^{2^{n-1}})}$$

$$\Rightarrow \quad f(n) = f(n-1)\left[10^{2^n - 2^{n-1}}\right]$$

$$\Rightarrow \quad f(n) = f(n-1)\left[10^{2^{n-1}(2-1)}\right]$$

$$\Rightarrow \quad f(n) = f(n-1)\left[10^{2^{n-1}}\right] \Rightarrow f(n) = \left[f(n-1)^2\right]$$

D. $f(n) = 100\, f(n-1)$

QUESTION ANALYTICS +

TOPICWISE : DISCRETE MATHEMATICS-2 (GATE - 2020) - REPORTS

Q. 11

Solution Video | Have any Doubt ?

Let {p, q, r, s} be the set. A binary operation $*$ is defined on the set and is given by the following table

$*$	p	q	r	s
p	p	r	s	p
q	p	q	r	s
r	p	q	p	r
s	p	q	q	q

Which of the following is true about the binary operation?

A. It is commutative but not associative

B. It is associative but not commutative

C. It is both associative and commutative

D. It is neither associative nor commutative — Correct Option

Solution :
(d)
The operation is not commutative as since upper and lower triangle is not same.
$q * p = p$ and $p * q = r$
The operation is not associative as $p * (q * r) \neq (p * q) * r$
LHS $p * r = s$
RHS $r * r = p$

QUESTION ANALYTICS +

Q. 12

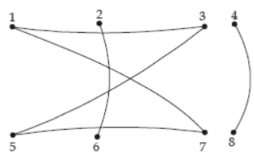
Solution Video | Have any Doubt ?

Let G be a graph with $V(G) = \{i \mid 1 \le i \le 4n, n \ge 1\}$ where $V(G)$ is the set of vertices of G. Such that two numbers x and y in $V(G)$ are adjacent if and only if $(x + y)$ is a multiple of 4. Assume there are k components each component C_k has m_k vertices, then what is the maximum value of m_k in the graph G?

A. n

B. $2n$ — Correct Option

Solution :
(b)
Let $n = 2 \Rightarrow$ # vertices = 8 [$\because$ Number of vertices in $G = 4n$]



$\Rightarrow$ 3 components [Note: For any n, the #components in G = 3]

$$\left.\begin{array}{l} V(C_1) = \{1, 3, 5, 7\} \Rightarrow m_1 = 4 \\ V(C_2) = \{2, 6\} \Rightarrow m_2 = 2 \\ V(C_3) = \{4, 8\} \Rightarrow m_3 = 2 \end{array}\right\} \max = 4$$

C. $3n$

D. None of these

QUESTION ANALYTICS +

Q. 13

Solution Video | Have any Doubt ?

Solve the following recurrence relation:
$T(n) = 9T(n-1) - 20T(n-2)$, $T(0) = -3$, $T(1) = -10$

A. $2.5^n - 5.4^n$ — Correct Option

Solution :
(a)

$T(n) - 9T(n-1) + 20T(n-2) = 0$

Let $a_n = T(n)$

$\Rightarrow a_n - 9a_{n-1} + 20a_{n-2} = 0$

$$\begin{aligned} t^2 - 9t + 20 &= 0 \\ t^2 + 5t - 4t + 20 &= 0 \\ t(t-5) - 4(t-5) &= 0 \\ (t-4)(t-5) &= 0 \\ t &= 4, 5 \end{aligned}$$

Homogenous equation become

$$a_n = c_1 . 5^n + c_2 . 4^n \quad ...(1)$$

Put $n = 0$ in equation (1)

$$\begin{aligned} a_0 &= c_1 . 5^0 + c_2 . 4^0 \\ -3 &= c_1 + c_2 \quad ...(2) \end{aligned}$$

Put n = 1 in equation (1)

$$\begin{aligned} a_1 &= c_1 . 5^1 + c_2 . 4^1 \\ -10 &= 5c_1 + 4c_2 \quad ...(3) \end{aligned}$$

Solving equation (2) and (3) and get c_1 and c_2

$$\begin{aligned} (c_1 + c_2 &= -3) \times 5 \\ 5c_1 + 4c_2 &= -10 \end{aligned}$$

$$\begin{aligned} 5c_1 + 5c_2 &= -15 \\ 5c_1 + 4c_2 &= -10 \\ \hline c_2 = -5 \quad &\text{and} \quad c_1 = 2 \end{aligned}$$

Put value of c_1 and c_2 in eq. (1)

$$a_n = 2.5^n - 5.4^n$$

B $3.5^n - 4.3^n$

C $3.4^n - 2.5^n$

D $4.5^n - 2.3^n$

QUESTION ANALYTICS +

Q. 14

Solution Video | Have any Doubt ?

Which of the following statement(s) is/are false?

S_1 : A connected multigraph has an Euler Circuit if and only if each of its vertices has even degree.

S_2 : A connected multigraph has an Euler Path but not an Euler Circuit if and only if it has exactly two vertices of odd degree.

S_3 : A complete graph (K_n) has a Hamilton Circuit whenever n ≥ 3.

S_4 : A cycle over six vertices (C_6) is not a bipartite graph but a complete graph over 3 vertices is bipartite.

A S_1 only

B S_2 and S_3

C S_3 only

D S_4 only — Correct Option

Solution :
(d)

A connected graph has a euler path only if it has number of odd degree vertices is either 0 or 2.

A connected graph has a euler circuit only if it has number of odd degree vertices is 0.

Dirac's theorem: If every vertex of an n-vertex simple graph G has degree ≥ $n/2$, then G is hamiltonian graph.

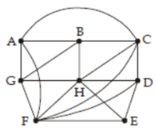
Cycle graphs have even number of vertices are bipartite.

QUESTION ANALYTICS +

Q. 15

Solution Video | Have any Doubt ?

Consider the following graph:



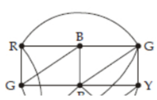
The chromatic number of the above graph is _______.

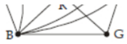
4 — Correct Option

Solution :
4

$d(i)$ = Degree of node i. $d(A) = 4$, $d(B) = 4$, $d(C) = 5$ $d(D) = 4$, $d(E) = 3$, $d(F) = 6$, $d(G) = 4$, $d(H) = 6$

using Welsh-powell's algorithm.





Chromatic number = 4

QUESTION ANALYTICS

Q. 16

Solution Video | Have any Doubt ?

Consider the collection of all undirected graph with 10 nodes and 6 edges. If a graph has no self loops and their is atmost one edge between any pair of node. The maximum number of connected components is _______.

7 — Correct Option

Solution :
7

Maximum and minimum number of component given by:

$$n - K \le e \le \frac{(n-K+1)(n-K)}{2}$$

1.
$$n - K \le e$$
$$n - e \le K$$
$$10 - 6 \le K \quad (\because \text{ Minimum number of component})$$

2.
$$e \le \frac{(n-K+1)(n-K)}{2}$$
$$6 \le \frac{(10-K+1)(10-K)}{2}$$
$$2 \times 6 \le (11 - K)(10 - K)$$
$$12 \le (10 - K)(11 - K)$$
$$12 \le K^2 + 110 - 21K$$
$$0 \le K^2 + 98 - 21K$$
$$K^2 + 98 - 21K = 0$$
$$K = 14, 7$$

Maximum value of K is 7 because number of components never be larger than nodes.

QUESTION ANALYTICS

Q. 17

Solution Video | Have any Doubt ?

Consider the undirected graph G defined as follows. The vertices are bit string of length 5. We have an edge between vertex "*a*" and vertex "*b*" iff "*a*" and "*b*" differ only in one bit position (i.e., hamming distance 1).

If the ratio of chromatic number of G to the diameter of G is $\frac{X}{Y}$ then $(Y - X)$ is _______.

3 — Correct Option

Solution :
3

Since bit are '0' and '1' form. The hamming distance relation on bit has a digraph which will be always an 5-cube where 5 is the number of bits.

- Chromatic number of n-cube = 2 (Since n-cube is always bipartite)
  So chromatic number of 5-cube = 2

i.e., '0' = One color
'1' = Second color

- Diameter of n-cube = n
  Diameter of 5 cube = 5

i.e., maximum length between any two vertex.

So ratio $$\frac{2}{5} = \frac{X}{Y}$$
$$Y - X = 5 - 2 = 3$$

QUESTION ANALYTICS

SINGLE SUBJECT : DISCRETE MATHEMATICS (GATE - 2020) - REPORTS

Q. 1 Solution Video | Have any Doubt ?

Let A be a finite set of size m. The number of elements in the power set of A × A × A × A × A is

(A) 2^m

(B) m^5

(C) 2^{m^5} — Correct Option

Solution :
(c)
Total number of element in $A \times A \times A \times A \times A = m^5$
Power set of $A \times A \times A \times A \times A = 2^{m^5}$

(D) 2^{2^m}

QUESTION ANALYTICS +

Q. 2 Solution Video | Have any Doubt ?

Let a graph (G) has 5 vertices and here are 3 vertices of degree 2, one vertex of degree 1 and the remaining vertex of degree 3. The complement of G is

(A) Connected

(B) Disconnected

(C) Either (a) or (b) — Correct Option

Solution :
(c)

Maximum number of edges with 5 vertices $= \frac{5(5-1)}{2} = 10$

Now, G has 3 vertices of degree 2, 1 vertex of degree 1 and 1 vertex of degree 3.

Then $3 \times 2 + 1 \times 1 + 1 \times 3 = 2|E|$

$10 = 2|E|$

$|E| = 5$

- G has 5 edges
- $\bar{G}$ has $10 - 5 = 5$ edges

With 5 vertices and 5 edges, the graph may or may not be connected.
Hence option (c) is correct.

(D) None of these

QUESTION ANALYTICS +

Q. 3 Solution Video | Have any Doubt ?

Which of the following is not correct?

(A) $\neg\forall x\, \exists y\, M(x, y) \equiv \exists x\, \forall y\, [\neg M(x, y)]$

(B) $\neg\forall x\, M(x) \equiv \exists x\, [\neg M(x)]$

(C) $\neg\exists x\, \forall y\, [M(x, y) \vee Q(x, y)] \equiv \forall x\, \exists y\, [\neg M(x, y) \wedge \neg Q(x, y)]$

(D) None of these — Correct Option

Solution :
(d)
All logical equivalents are correct.

QUESTION ANALYTICS +

Q. 4

Solution Video | Have any Doubt ?

Let f : A → B and g : B → C denote two functions. If the function g$o$$f$: A → C is a surjection then

A. f is onto but g need not to be onto

B. g is onto but f need not to be onto — Correct Option

Solution :
(d)
gof : A → C is onto then g must be onto but f need not be onto.

C. Both have to be onto

D. Both need not to be onto

QUESTION ANALYTICS +

Q. 5

Solution Video | Have any Doubt ?

Consider (S, ×) is a semi group where S = {a, b, c, d}. Assume $a \times b = c$, $b \times b$ = a and $d \times a = b$. Then $d \times c$ =

A. a — Correct Option

Solution :
(a)

$$\begin{aligned} d \times c &= d \times (a \times b) \quad [\text{Given, } c = a \times b] \\ &= (d \times a) \times b \end{aligned}$$

[Associative holds in semi group]

$$\begin{aligned} &= b \times b \quad [\text{Given, } b \times b = a] \\ &= a \end{aligned}$$

B. b

C. c

D. d

QUESTION ANALYTICS +

Q. 6

Solution Video | Have any Doubt ?

Assume a undirected connected graph G has n-vertices and m edges. What is the minimum number of edges whose deletion from G is always guarantee that is will become disconnected

A. $m - n + 2$ — Correct Option

Solution :
(a)
Atleast $(n - 1)$ edges are required to make the graph connected.
To make it disconnected graph should contains at most $n - 1 - 1 = n - 2$ edges.
The graph has m edges, so to make it disconnected $m - (n - 2) = m - n + 2$ edges must be deleted.

- $(m - n + 2)$ edges deletion always guarantee that any graph will be become disconnected.

B. ${}^{n}C_2 + 1 + m$

C. ${}^{n}C_2 - 1 + m$

D. ${}^{n-1}C_2 + 1 - n$

QUESTION ANALYTICS +

Q. 7

Solution Video | Have any Doubt ?

Consider $a_n = -4a_{n-1} + 12a_{n-2}$. Then which of the following represent a_n?

A. $A(-6)^n + B \cdot 2^n$ — Correct Option

Solution :
(a)

$$\begin{aligned} a_n &= -4a_{n-1} + 12a_{n-2} \\ a_n + 4a_{n-1} - 12a_{n-2} &= 0 \\ x^2 + 4x - 12 &= 0 \\ (x + 6)(x - 2) &= 0 \\ x &= -6, x = 2 \\ a_n &= A(-6)^n + B \cdot 2^n \end{aligned}$$

So, option (a) is correct.

B $A \cdot 3^n + B(-2)^n$

C $A \cdot 6^n + B \cdot 3^n$

D $A(-3)^n + B \cdot 2^n$

QUESTION ANALYTICS +

Q. 8

Solution Video Have any Doubt ?

Consider the following predicates with the domain as set of integers for all variables.
$R(m) : m^2 \geq 0$
$S(m, n) : m^2 = n$
$T(m) : m^2 - 3m + 2 = 0$
Which of the following is true?

A $\forall m\ [R(m) \wedge T(m)]$

B $\forall m\ R(m) \wedge \exists m\ \forall n\ S(m, n)$

C $\forall m\ R(m) \wedge \exists n\ T(n)$ Correct Option

Solution :
(c)

$$\underline{\forall m\, R(m)} \wedge \underline{\exists n\, T(n)}$$

$\downarrow$ Square of every integer is always greater than zero so it is true

$\downarrow$ It is also true because there exist a solution i.e. $m = 1, 2$

Hence option (c) is correct.

D $\forall n\, \forall m\ [R(m) \rightarrow S(m, n)]$

QUESTION ANALYTICS +

Q. 9

Solution Video Have any Doubt ?

Consider the following two statements:
S_1 : x R y iff $(x + y)$ is odd over the set of integers is an equivalence relation.
S_2 : If a relation is symmetric and transitive both then it will always be reflexive.
Which of the following statement is true with respect to above relations?

A Both S_1 and S_2 are correct

B S_1 is correct, but not S_2

C S_2 is correct, but not S_1

D None of S_1 or S_2 is correct Correct Option

Solution :
(d)
S_1 : x R y if $f(x + y)$ is odd
Reflexivity : Let $x = 3$, $3 \not R 3$
Hence, the relation is not reflexive and, Hence it's not an equivalence relation.
S_2 : Let relation R is $\{(1, 2)\ (2, 1)\ (1, 1), (2, 2)\}$ over domain $\{1, 2, 3\}$ R is symmetric as well as transitive both, but the relation is still not reflexive it's irreflexive since $(3, 3)$ is not included in the relation.
So, both the statements are incorrect.

QUESTION ANALYTICS +

Q. 10

Solution Video Have any Doubt ?

Assume g is an element of the group G. Consider the following conditions of g with e as identity element.
(i) $g^8 = e$
(ii) $g^2 \neq e$
(iii) Order of g is not 8.
Find the order of g.

A 1

B 2

C 4 Correct Option

Solution :
(c)

$$g^8 = e$$

$$\Rightarrow \quad O(g) \div 8$$

$$O(g) = 1, 2, 4, 8$$

Now it is given that $O(g) \neq 8$

$$O(g) = 1, 2, 4$$

Since $g^2 \neq e$, hence $O(g) \neq 2$

$$O(g) = 1 \text{ or } 4$$

Now $O(g) \neq 1$ because if $O(g) = 1$ then $g = e$, $g^2 = e^2 = e$, but given that $g^2 \neq e$

$$\therefore \quad O(g) = 4$$

D 6

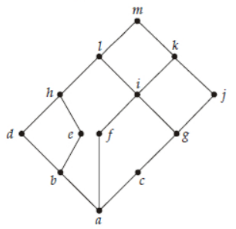
QUESTION ANALYTICS +

SINGLE SUBJECT : DISCRETE MATHEMATICS (GATE - 2020) - REPORTS

Q. 11

Solution Video | Have any Doubt ?

Consider the following lattice.



The number of complements of *e* is/are _______.

2 — Correct Option

Solution :
2

$$\left[\begin{array}{c} x \text{ and } y \text{ are complement iff} \\ x \vee y = \text{Greatest element} = m \\ x \wedge y \ = \text{Least element} = a \end{array}\right]$$

$$e^c = k, j$$

QUESTION ANALYTICS +

Q. 12

Solution Video | Have any Doubt ?

The coefficient of x^{10} in $(1 + x + x^2 + ...)^2$ is _______.

11 — Correct Option

Solution :
11

$$= (1 + x + x^2 + x^3 + ...)^2$$

$$= \left(\frac{1}{1-x}\right)^2 = (1-x)^{-2}$$

$$= \sum_{r=0}^{\infty} {}^{2-1+r}C_r x^r$$

The co-efficient of x^{10} is equal to

$$= ({}^{2-1+10}C_{10}) = {}^{11}C_{10}$$

$$= \frac{11!}{10!1!} = 11$$

QUESTION ANALYTICS +

Q. 13

Solution Video | Have any Doubt ?

A person assigns 7 different tasks to 4 other persons with atleast 1 task per person. The number of ways the tasks can be assigned is _______.

8400 — Correct Option

Solution :
8400

The assignment is similar to onto function, from 7 element set to 4 element set

$$= 4^7 - {}^4C_1(3)^7 + {}^4C_2(2)^7 - {}^4C_3(1)^7$$

$$= 4^7 - 4 \times 3^7 + \frac{4 \times 3}{2}\, 2^7 - 4 \times 1^7$$

$$= 16384 - 8748 + 768 - 4 = 8400$$

QUESTION ANALYTICS +

Q. 14

Solution Video | Have any Doubt ?

Order of graph *G* is 6 and the number of edges in the same graph are 7. The number of edges in complement of *G* is _______.

8 Correct Option

Solution :
8

$$e(\bar{G}) = \frac{n(n-1)}{2} - e(g)$$
$$= \frac{6(6-1)}{2} - 7 = 8$$

QUESTION ANALYTICS +

Q. 15 Solution Video Have any Doubt ?

What is the number of symmetric binary relation on a Set $A = \{a_1, a_2, a_3, a_4\}$ is ______.

1024 Correct Option

Solution :
1024

Total number of symmetric relations $= 2^n \times 2^{\frac{n(n-1)}{2}} = 2^{\frac{n(n+1)}{2}}$

Here in the given question $n = 4$.

So total number of relations will be $2^{4\times5/2} = 2^{10} = 1024$.

QUESTION ANALYTICS +

Q. 16 Solution Video Have any Doubt ?

Suppose X is a finite set with 60 elements. The number of elements in largest equivalence relation of X is ________.

3600 Correct Option

Solution :
3600

For X be finite set with n element

Largest equivalence relation contain n^2 elements i.e. cross product of

$$|X| \times |X| = n \times n = n^2$$

So, for 60 element $= 60 \times 60 = 3600$

QUESTION ANALYTICS +

Q. 17 Solution Video Have any Doubt ?

Let f is a function from set of integers to the set of integers. Find which of the following function is neither one-to-one nor onto function.

A $f(x) = x + 300$

B $f(x) = 8x - 300$

C $f(x) = x^3 + 300$

D $f(x) = x^2 + 300$ Correct Option

Solution :
(d)

To check function is one-to-one

$$f(x) = x^2 + 300$$
$$f(x_1) = f(x_2)$$
$$x_1^2 + 300 = x_2^2 + 300$$

$\Rightarrow x_2 = \pm x_1$, x_1 has 2 image so, it is not one-to-one function.

To check function is onto

$$y = x^2 + 300$$
$$x = \sqrt{y - 300}$$

So, for many integer $\sqrt{y-300}$ is not an integer so, Range $\neq z$.

QUESTION ANALYTICS +

Q. 18 Solution Video Have any Doubt ?

Match **List-I** with **List-II** and select the correct answer using the codes given below:

Assume [Loves (x, y) : x loves y]

List-I		List-II
A. Everybody loves Qasadilla	1.	$\neg(\exists x \forall y \neg \text{Loves}(x, y))$
B. Everybody loves somebody	2.	$\exists y \forall x \text{ Loves}(x, y)$
C. There is somebody whom no one loves	3.	$\neg(\forall y \exists x \text{ Loves}(x, y)$
D. There is somebody whom every body loves	4.	$\neg(\exists x \neg(\text{Loves}(x, \text{Qasadilla}))$

Codes:

	A	B	C	D
(a)	2	3	1	2
(b)	2	1	3	2
(c)	4	3	1	2
(d)	4	1	3	2

A a

B b

C c

D d Correct Option

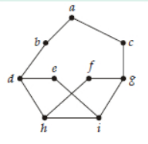
Solution :
(d)

- Everybody loves Qasadilla : $\forall x$ Loves $(x, \text{Qasadilla}) \equiv \neg(\exists x \neg(\text{Loves}(x, \text{Qasadilla}))$
- Everybody loves somebody : $\forall x \exists y$ Loves $(x, y) \equiv \neg(\exists x \forall y \neg \text{Loves}(x, y))$
- There is somebody whom no one loves : $\exists y \forall x (\neg \text{Loves}(x, y) \equiv \neg(\forall y \exists x \text{ Loves}(x, y))$
- There is somebody whom every body loves : $\exists y \forall y$ Loves (x, y)

QUESTION ANALYTICS +

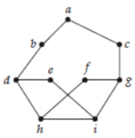
Q. 19 Solution Video Have any Doubt ?

Which of the following graphs have Hamiltonian cycle?

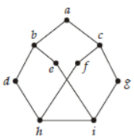
A 

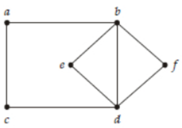
Correct Option

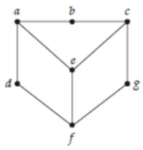
Solution :
(a)



$(f - g - c - a - b - d - e - i - h - f)$ sequence covers all the vertices without repetition of vertex and forming cycle.

B 

C 

D 

QUESTION ANALYTICS +

Q. 20 Solution Video Have any Doubt ?

A relation "congruence modulo m" over the set of integers is given by $R = \{<x, y> \mid x \equiv y \bmod n\}$ then relation R is

A Equivalence relation

Correct Option

Solution :
(a)

- $R = \{<x, y> \mid x \equiv y \bmod n\}$ when $x = y$ then $x \equiv x \bmod n$ satisfies. Hence it is reflexive.
- $(x - y) \bmod m \equiv (y - x) \bmod m$, so it is symmetric.
- If $(x - y) \bmod m \equiv (y - z) \bmod m$ which is equal to $(x - z) \bmod m$. Hence it is transitive.

So R is reflexive, symmetric, transitive, hence it is equivalence relation.

B Reflexive and symmetric relation

C Reflexive and transitive relation

D Symmetric and transitive relation

QUESTION ANALYTICS +

Kunal Jha
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OVERALL ANALYSIS | COMPARISON REPORT | SOLUTION REPORT
ALL(33) CORRECT(0) INCORRECT(0) SKIPPED(33)

Q. 21 Solution Video | Have any Doubt ?

Consider the following statements:
S_1 : A simple graph G having n-vertices and k-components is a forest if and only if G has $(n - k)$ edges.
S_2 : A graph G with n-vertices and depth traversal of G contains m edges then there is $n - m$ connected components in G.
Which of the following is correct?

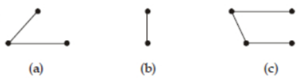
A. S_1 only

B. S_2 only

C. Both S_1 and S_2 — Correct Option

Solution :
(c)

S_1 :

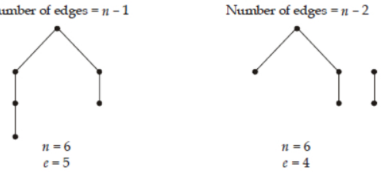


Here a, b, c are trees.
"A forest is an undirected graph, all of whose connected components are trees".
$(n - k)$ edges since $n = 9$
$k = 3$
$|E| = 6$

S_2 : DFT is a tree with n vertex

$k = 1$
Number of edges $= n - 1$

$k = 2$
Number of edges $= n - 2$



Hence both statements are true.

D. None of these

QUESTION ANALYTICS +

Q. 22 Solution Video | Have any Doubt ?

Consider the undirected graph G defined as follows. The vertices are bit string of length 5. We have an edge between vertex "a" and vertex "b" iff "a" and "b" differ only in one bit (i.e., hamming distance1). What is the ratio of chromatic number of G to the diameter of G?

A. $\frac{2}{5}$ — Correct Option

Solution :
(a)
Since bit are '0' and '1' form. The hamming distance relation on bit has a digraph which will be always an 5-cube where 5 is the number of bits.
So chromatic number of 5-cube = 2 (Since all n-cube are bipartite)
Diameter of 5 cube = 5. (Diameter of n-cube is n)

So, ratio $= \frac{2}{5}$

B. 2

C. $\frac{5}{2}$

D. $\frac{4}{5}$

QUESTION ANALYTICS +

Q. 23 Solution Video | Have any Doubt ?

Let G be a directed graph where vertex set is the set of numbers from 1 to 100. There is an edge from a vertex i to a vertex j iff either $j = i + 1$ or $j = 3i$. The minimum number of edges in a path in G from vertex 1 to vertex 100 is:

A 4

B 7 — Correct Option

Solution :
(b)
There is an edge from a vertex i to a vertex j iff either $j = i + 1$ or $j = 3i$.
So, possible set of edge are:

$$1 \to 2 \to 3 \to 4 \to 5 \to 6 \to 7 \to 8 \to 9 \to \ldots\ldots \to 100$$

So, minimum number of edges in a path or from vertex 1 to vertex 100 is

$$\underbrace{1 \to 3 \to 9 \to 10 \to 11}_{4 \text{ edges}} \to \underbrace{33 \to \ldots\ldots \to 99 \to 100}_{3 \text{ edges}}$$

So, total $= 4 + 3 = 7$

C 23

D 99

QUESTION ANALYTICS +

Q. 24

Solution Video | Have any Doubt ?

If G is a bipartite planar with n-vertices, then find the maximum number of edges in G.

A $2n - 4$ — Correct Option

Solution :
(a)
Since graph is bipartite so it not contain odd length cycle.
So, minimum length cycle = 4 length
Every region is bounded by 4 edges and each edge is counted twice.

$$e \geq \frac{4r}{2}$$

$$e \geq 2r \qquad \ldots(i)$$

we know that $r = e - n + 2$ (Euler formula)
by putting euler formula in equation (i)

$$e \geq 2(e - n + 2)$$

$$e \leq 2n - 4$$

B $3n - 2$

C $n - 2$

D n

QUESTION ANALYTICS +

Q. 25

Solution Video | Have any Doubt ?

Let P, Q and R be three sets. Then $(P - Q) - R =$ ________.

A $(P - Q) - (Q - R)$

B $(Q - R) - (P - R)$

C $(P - Q) - (P - R)$

D $(P - R) - (Q - R)$ — Correct Option

Solution :
(d)

$$A - B = AB'$$

So,

$$(P - Q) - R = PQ' - R = PQ'R'$$

Consider option (d)

$$\begin{aligned}(P - R) - (Q - R) &= PR' - QR' \\ &= PR'(QR')' \\ &= PR'(Q' + R) \\ &= PR'Q' + PR'R \\ &= PR'Q' \\ &= PQ'R'\end{aligned}$$

Both are equivalent.

QUESTION ANALYTICS

Q. 26

Solution Video | Have any Doubt ?

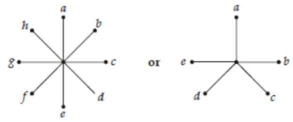
G be a connected graph in which only one node have degree > '1' and rest of nodes are of degree '1'. Add an edge between every two node of degree '1' in such a way that if a, b, c, d are node then a to b one edge, b to c one edge, c to d one edge and d to a one edge. The resultant graph is sure to be

A Regular

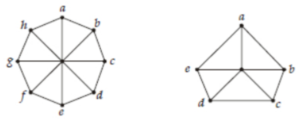
B Complete

C Hamiltonian — Correct Option

Solution :
(c)
According to question graph given is a star graph $k_{1,n}$.
Example:



Now, add edge between every two adjacent vertex which has degree 1.
So, resultant graph will be a wheel graph which always has a Hamiltonian cycle.



Wheel graph is not euler since the outer vertices has degree 3 (odd).

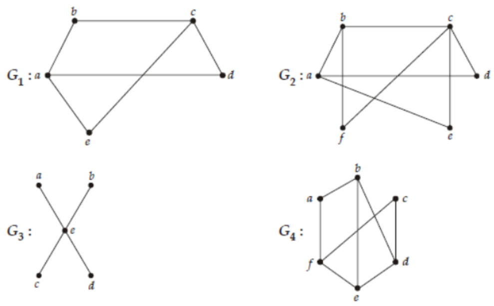
D Euler

QUESTION ANALYTICS

Q. 27

Solution Video | Have any Doubt ?

Consider the following graphs:



Which of the above graph are bipartite graphs?

A G_2, G_4

B G_1, G_2 and G_3

C Both G_1 and G_3 — Correct Option

Solution :
(c)
A graph is bipartite when there is no odd length cycle.
Clearly graph G_2 and G_4 contains odd length cycle but G_1 and G_3 does not contains any odd length cycle.
Hence option (c) is correct.

D G_2, G_3

QUESTION ANALYTICS

Q. 28

Solution Video | Have any Doubt ?

Consider the following predicates for the domain of real numbers:
$P(x, y) : x > y$
$Q(x, y) : x \leq y$
$R(x) : x - 7 = 2$
$S(x) : x > 9$
Which of the following proposition gives the False as the truth value?

A $\exists x\ (R(x)) \vee \forall y\ (\sim S(y))$

B $\forall x\ \exists y\ P(x,y)$

C $\forall x\ \forall y\ [P(x, y) \vee Q(x, y)]$

D None of these — Correct Option

Solution :
(d)

(a) $\exists x\ (R(x)) \vee \forall y\ (\sim S(y))$

For $x = 9$. $\exists x\ (R(x))$ evaluated to True.

$x - 7 = 2 \Rightarrow 9 - 7 = 2$

$\therefore$ True $\vee \forall y\ (\sim S(y))$ evaluated to True.

(b) $\forall x\ \exists y\ P(x, y)$

$\forall x\ \exists y\ x > y$ evaluated to True.

There exist y for every x in the real domain

(c) $\forall x\ \forall y\ [P(x,y) \vee Q(x, y)]$

$\forall x\ \forall y\ [(x > y) \vee (x \le y)]$ is True.

For every x and y, $x > y$ or $x \le y$ always satisfies.

$\therefore$ All statements are evaluates to True.

QUESTION ANALYTICS +

Q. 29 Solution Video | Have any Doubt ?

Consider set $R = \{(1, 1), (2, 1), (3, 2), (4, 3)\}$. $Q = R^1 \cup R^2 \cup R^3$, then cardinality of set Q is ______.
($R^2 = R^1$ o R^1 i.e. composition of R with R)

7 — Correct Option

Solution :
7

- $R^1 = R$
- $R^2 = R \circ R$

If $(a, b) \in R$ then $(a, c) \in R^2$ iff $(b, c) \in R$

$$R^2 = \{(1, 1), (2, 1), (3, 1), (4, 2)\}$$
$$R^3 = \{(1, 1), (2, 1), (3, 1), (4, 1)\}$$
$$Q = \{(1, 1), (2, 1), (3, 2), (4, 1), (4, 2), (4, 3), (3, 1)\}$$

Cardinality of $Q = 7$

QUESTION ANALYTICS +

Q. 30 Solution Video | Have any Doubt ?

Let G be a simple graph with 12 edges and $\bar{G}$ be a complement of graph G has 33 edges, then the number of vertices in graph G is ______.

10 — Correct Option

Solution :
10

Maximum number of edges in G with n vertex $= {}^nC_2$.

i.e., $${}^nC_2 = 12 + 33$$
$$\frac{n(n-1)}{2} = 45$$

So, $$n^2 - n = 90$$
$$n^2 - n - 90 = 0$$
$$n(n - 10) + 9(n - 10) = 0$$
$$(n + 9)(n - 10) = 0$$

But n cannot be less than 0, so $n = 10$.

QUESTION ANALYTICS +

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Q. 31 Solution Video | Have any Doubt ?

A simple undirected graph 'X' has 12 vertices. If 'X' has 6 equally sized connected components, the maximum number of edges in graph 'X' is ______.

6 — Correct Option

Solution :
6

$$\text{Number of vertices} = 12$$
$$\text{Number of components} = 6$$
$$\text{Number of vertices/component} = \frac{12}{6} = 2$$

With 2 vertices, only 1 edge is possible.
So, 6 edges are there in total.

QUESTION ANALYTICS +

Q. 32 Solution Video | Have any Doubt ?

The number of solutions are there to the equation $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 31$ are ______.
Where $x_1 \geq 1, x_2 \geq 2, x_3 \geq 2, x_4 \geq 4, x_5 \geq 6, x_6 \geq 5$?

4368 — Correct Option

Solution :
4368

To satisfy the least condition,

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 31 - (1 + 2 + 2 + 4 + 6 + 5) = 11$$
$$\text{Number of solutions} = {}^{n+r-1}C_r$$
$$n = 6, r = 11$$
$$\therefore \quad \text{Number of solutions} = {}^{6+11-1}C_{11} = {}^{16}C_{11} = 4368$$

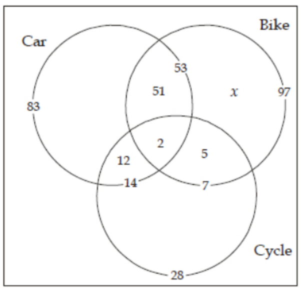
QUESTION ANALYTICS +

Q. 33 Solution Video | Have any Doubt ?

A survey of 150 college students reveals that 83 own cars, 97 own bikes, 28 own cycles, 53 own a car and a bike, 14 own car and a cycle, 7 own a bike and a cycle and 2 own all three. The number of students own a bike and nothing else are ______.

39 — Correct Option

Solution :
39



Students own a bike and nothing else $= x + 5 + 2 + 51 = 97$

$$x = 39$$

QUESTION ANALYTICS +