# 11 Engineering Mathematics

## Practice Questions

**Q.1** If $P=\begin{bmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2}\\ -\frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix} \cdot A=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$ and $Q=PAP^T$. Then $P(Q^{2005})P^T$ equal to

(A) $\begin{bmatrix}1 & 2005\\ 0 & 1\end{bmatrix}$ (B) $\begin{bmatrix}\frac{\sqrt{3}}{2} & 2005\\ 1 & 0\end{bmatrix}$

(C) $\begin{bmatrix}1 & 2005\\ \frac{\sqrt{3}}{2} & 1\end{bmatrix}$ (D) $\begin{bmatrix}1 & \frac{\sqrt{3}}{2}\\ 0 & 2005\end{bmatrix}$

**Q.2** The system of linear equation

$$x+y+z=2$$
$$2x+3y+2z=5$$
$$2x+3y+(a^2-1)z=a+1$$

(A) Has infinitely many solution for $a=4$

(B) Is inconsistent when $|a|=\sqrt{3}$

(C) Is inconsistent when $a=4$

(D) Has a unique solution for $|a|=\sqrt{3}$

**Q.3** For $3\times 3$ matrices $M$ and $N$, which of the following statement(s) is/are not correct **[MSQ]**

(A) $N^T MN$ is symmetric or skew symmetric, according as $M$ is symmetric or skew symmetric

(B) $MN-NM$ is skew symmetric for all symmetric matrices $M$ and $N$

(C) $MN$ is symmetric for all symmetric matrices $M$ and $N$

(D) $(adj\ M)(adj\ N)=adj(MN)$ for all invertible matrices $M$ and $N$

**Q.4** Let $P=[a_{ij}]$ be a $3\times 3$ matrix and let $Q=[b_{ij}]$, where $b_{ij}=2^{i+j}a_{ij}$ for $1\le i, j\le 3$. If determinant of $P$ is 2, then the determinant of the matrix $Q$ is

(A) $2^{10}$ (B) $2^{11}$

(C) $2^{12}$ (D) $2^{13}$

**Q.5** $X_1, X_2, X_3$ and $X_4$ are vectors of length.

$$X_1=[a_1,a_2,a_3,a_4]$$
$$X_2=[b_1,b_2,b_3,b_4]$$
$$X_3=[c_1,c_2,c_3,c_4]$$
$$X_4=[d_1,d_2,d_3,d_4]$$

It is known that $X_2$ is not a scalar multiple of $X_1$. Also, $X_3$ is linearly independent of $X_1$ and $X_2$. Further $X_4=3X_1+2X_2+X_3$. The rank of the matrix

$\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \\ c_1 & c_2 & c_3 & c_4 \\ d_1 & d_2 & d_3 & d_4 \end{bmatrix}$ is _____. (in integer)

**Q.6** Which of the following is the characteristic equation of

$$\begin{bmatrix} a & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & a & 0 \\ 0 & 0 & 0 & a \end{bmatrix}$$

(A) $\sum_{k=1}^{4}(-1)^k.{}^4C_k.a^{k-4}.\lambda^k = 0$

(B) $\sum_{k=0}^{4}{}^4C_k.a^{k-4}.\lambda^k = 0$

(C) $\sum_{k=1}^{4}(-1)^k.{}^4C_k.a^{4-k}.\lambda^k = 0$

(D) $\sum_{k=0}^{4}(-1)^k.{}^4C_k.a^{4-k}.\lambda^k = 0$

**Q.7** If the characteristic values of $A=\begin{bmatrix} 3 & -1 \\ 5 & 6 \end{bmatrix}$ are $\lambda_1$ and $\lambda_2$ and that of $B=\begin{bmatrix} 1 & 2 \\ -1 & 5 \end{bmatrix}$ are $\mu_1$ and $\mu_2$, the equation whose roots are $\frac{1}{\lambda_1}+\frac{1}{\lambda_2}$ and $\frac{1}{\mu_1}+\frac{1}{\mu_2}$ is

(A) $201x^2 - 161x + 54 = 0$

(B) $161x^2 - 201x + 54 = 0$

(C) $201x^2 + 161x - 54 = 0$

(D) $161x^2 + 201x - 54 = 0$

**Q.8** If $(1+3p)/3$, $(1-p)/4$ and $(1-2p)/2$ are the probabilities of three mutually exclusive events, then the set of all values of $p$ is

(A) $\frac{1}{3} \le p \le \frac{1}{2}$ (B) $\frac{1}{3} < p < \frac{1}{2}$

(C) $\frac{1}{2} \le p \le \frac{2}{3}$ (D) $\frac{1}{2} < p < \frac{2}{3}$

**Q.9** An unbiased coin is tossed. If the result is a head, a pair of unbiased dice is rolled and the number obtained by adding the numbers on the two faces is noted. If the result is a tail, a card from a well shuffled pack of eleven cards numbered 2, 3, 4, and 12 is picked and the number on the card is noted. The probability that the noted number is either 7 or 8, is

(A) 0.24 (B) 0.244

(C) 0.024 (D) None of these

**Q.10** In sampling a large number of parts manufactured by a machine, the mean number of defectives in a sample of 20 is 2. Out of 1000 such samples, how many would be expected to contain at least 3 defective parts

**Q.11** The value of $\lim_{y \to 0} \frac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4}$ is

(A) Exists and equals $\frac{1}{4\sqrt{2}}$

(B) Does not exist

(C) Exists and equals $\frac{1}{2\sqrt{2}}$

(D) Exists and equals $\frac{1}{2\sqrt{2}(\sqrt{2}+1)}$

**Q.12** The value of

$$\lim_{x \to 0} \frac{x+2\sin x}{\sqrt{x^2+2\sin x+1}-\sqrt{\sin^2 x - x+1}}$$ is

(A) 3 (B) 2

(C) 6 (D) 1

**Q.13** Let $\alpha$ and $\beta$ are roots of the quadratic equation $x^2+bx+c=0$. Then the value of $\lim_{x\to\alpha}\dfrac{1-\cos(x^2+bx+c)}{(x-\alpha)^2}$ is

(A) $b^2+4c$ (B) $b^2-4c$

(C) $\frac{1}{2}(b^2-4c)$ (D) None of these

**Q.14** Which of the following functions is differentiable at $x=0$

(A) $\cos(|x|)+|x|$ (B) $\cos(|x|)-|x|$

(C) $\sin(|x|)+|x|$ (D) $\sin(|x|)-|x|$

**Q.15** The value of

$$\lim_{x\to 0}\frac{x(1+a\cos x)-b\sin x}{x^3}=1$$

then, the value of $a+b$ is ________. (in integer)

**Q.16** $I=\lim_{x\to\frac{\pi}{4}}\dfrac{\int_2^{\sec^2 x} f(t)\,dt}{\left(x^2-\frac{\pi^2}{16}\right)}$ at $f(2)=\pi$ is

________. (in integer)

**Q.17** For $f(x)$, which of the following statements is/are True **[MSQ]**

$$f(x)=\begin{cases}0; & x=0\\ \frac{1}{2}-x; & 0<x<\frac{1}{2}\\ \frac{1}{2} & x=\frac{1}{2}\\ \frac{3}{2}-x; & \frac{1}{2}<x<1\\ 1: & x=1\end{cases}$$

(A) $f(x)$ is discontinuous at $x=0$.

(B) $f(x)$ is discontinuous at $x=\frac{1}{2}$

(C) $f(x)$ is discontinuous at $x=1$

(D) None of these

**Q.18** The value of $k$ and $m$ so that $f(x)$ is differentiable at $x=3$;

$$f(x)=\begin{cases}k\sqrt{x+1}; & 0\le x\le 3\\ mx+2; & 3<x\le 5\end{cases}$$

(A) $\frac{8}{5},\frac{2}{5}$ (B) $\frac{5}{8},\frac{5}{2}$

(C) $\frac{5}{8},\frac{5}{5}$ (D) $\frac{5}{8},\frac{4}{5}$

**Q.19** The total number of maxima and minima points of function $f(x)=\sin^4 x+\cos^4 x$ occur between interval $[0,2\pi]$ is ______. (in integer)

**Q.20** A book of 600 pages contain 40 printing mistakes. Let these errors are randomly distributed throughout the book and $r$ is the number of errors per page has a Poisson distribution. Then, the probability that 10 pages selected at random will be free from error is

(A) 0.50 (B) 0.49

(C) 0.97 (D) 0.51

**Q.21** Players A and B, playing the game by tossing a coin with a dice, one who gets head and 6 will win the game. If A start the game, probability of winning of A is _______. (rounded upto two decimal places)

[Note : They played it alternatively]

**Q.22** A bag contains 3 red and $n$ white balls. Miss A draws two balls together from the bag. The probability they have the same color is $\frac{1}{2}$. Miss B draws one ball from bag, notes its color and replace it. She then draws a second ball from bag and find both have same color with probability $\frac{5}{8}$. The possible value of $n$ is

(A) 9 (B) 6
(C) 5 (D) 1

**Q.23** If '$x$' is a zero mean, unit variance Gaussian random variable, then expected value $E(|5x|)$ is _______. (rounded upto two decimal places)

**Q.24** $f(x)=\begin{cases}\dfrac{x}{2}, & 0>x\le 1\\ \dfrac{1}{2}, & 1<x\le 2\\ \dfrac{3-x}{3}, & 2<x\le 3\end{cases}$

Let '$x$' be Random variable having probability density function $f(x)$, then the probability $P(1.5<x\le 2.5\,|\,x>1)$ is _______. (rounded upto one decimal place)

**Q.25** If $A=\begin{bmatrix}1 & 2 & -3\\ 0 & 4 & 5\\ 0 & 0 & 6\end{bmatrix}$, then determinant of $A^3+A^2+2A$ is _______. (in integer)

**Q.26** The value integral

$I=\int_1^3 e^{3x}\left(\log x+\frac{1}{x}\right)dx$ is

(A) $e^9\log 3$ (B) $e^9\log 2$
(C) $e^9\log 4$ (D) $e^9\log 5$

**Q.27** The number of linearly independent solution in matrix $A=\begin{bmatrix}1 & 2 & 3 & 2\\ 2 & 3 & 5 & 1\\ 1 & 3 & 4 & 5\end{bmatrix}_{3\times 4}$ is _______. (in integer)

**Q.28** If $x,y$ are independent binomial random variables $\in\left\{3,\frac{1}{3}\right\}$. The probability that the matrix $P=\begin{bmatrix}\frac{x}{\sqrt{2}} & \frac{y}{\sqrt{2}}\\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{bmatrix}$ is orthogonal is _______. (rounded upto three decimal places)

**Q.29** Consider the matrices $X_{4\times 3}$, $Y_{4\times 3}$ and $P_{2\times 3}$. The order of $(P(X^TY)^{-1}P^T)^T$ will be $p\times q$, then $p-q$ is _______. (in integer)

**Q.30** If $|A|_{3\times 3}=5$ and $B=\text{adj}(5\text{A})$ then the value of $\sqrt[8]{|B|_{3\times 3}}$ is _______. (in integer)

**Q.31** If $\lim\limits_{x\to 0}\dfrac{a\sin x+b\cos x+cx}{x^3}=\dfrac{-1}{6}$ then $\dfrac{a+b}{c}$ is _______. (in integer)

**Q.32** The function $f(x,y)=x^2+y^2+4x+8$ **[MSQ]**

(A) Has minimum value at point $(-2,0)$
(B) Has maximum value at point $(-2,0)$
(C) Minimum value of function is 4
(D) Maximum value of function is 4

**Q.33** Matrix $P=\begin{bmatrix}1 & -3 & 4\\ 0 & -4 & 8\\ 0 & 0 & 5\end{bmatrix}$ then **[MSQ]**

(A) Eigen value of $P^{-1}$ are $\frac{1}{2},\frac{-1}{8},\frac{1}{25}$
(B) Determinant of $5P^T$ is $-2500$
(C) $P$ is an orthogonal matrix
(D) Eigen value of $P^{2022}$ are 1, $2^{4044}$, $5^{2022}$

**Q.34** If '$P$' is a non-singular matrix, then which of the following is/are not true about its Eigen value **[MSQ]**

(A) All the Eigen value of '$P$' are non-zero.
(B) The Eigen value of '$P$' may or may not be zero.
(C) Only one Eigen value can be zero and above should be negative.
(D) Nothing can be said about their Eigen value.

**Q.35** For the function $f(x)=\int_{x^2}^{x^3} t\,dt$ **[MSQ]**

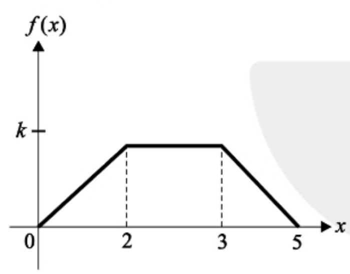
(A) Total number of extremum points are '3'.

(B) Total number of extremum points are '5'.

(C) Point of minimum value is $\sqrt{\frac{2}{3}}$.

(D) Point of inflection is at $x=0$.

**Q.36** If '$x$' is a Random variable then the expected value of $f(x)$, for the their given graph is _______. (rounded upto one decimal place)




**Q.37** The value of $\lim\limits_{x\to\infty}\dfrac{\ln(x^2-4x+8)}{\ln(x^{12}+x^6+6)}$ is _______. (rounded upto three decimal places)

**Q.38** If $A=\begin{bmatrix}\frac{1}{2} & \frac{1}{2} & 4\\ \frac{-7}{2} & 2 & -2\\ \frac{\beta}{2} & 4 & \frac{\alpha}{2}\end{bmatrix}$, then '$A$' is an orthogonal matrix for **[MSQ]**

(A) $\alpha=1$ (B) $\beta=\dfrac{8}{9}$

(C) $\alpha=\dfrac{1}{9}$ (D) $\beta=\dfrac{27}{8}$

**Q.39** Person on a trip has a choice between private car and public transport. The probability of using a private car is 0.45. While using public transport, further choice available are bus and metro. Out of which the probability of commuting by a bus is 0.55. In such a situation, the probability of using a car, bus and metro respectively would be

(A) 0.45, 0.30 and 0.25

(B) 0.45, 0.25 and 0.30

(C) 0.45, 0.55 and 0

(D) 0.45, 0.35 and 0.20

**Q.40** A husband and wife appear in an interview for two vacancies for same post. The probability of husband getting selected is $\dfrac{1}{5}$ while the probability of wife getting selected is $\dfrac{1}{7}$. Then the probability that anyone of them getting selected is _______. (rounded upto three decimal places)

**Q.41** The value of $\int_0^\infty e^{-y^3}\,y^{\frac{1}{2}}\,dy$ is

(A) $\dfrac{1}{2}\sqrt{\pi}$ (B) $\dfrac{1}{3}\sqrt{\pi}$

(C) $\dfrac{\sqrt{\pi}}{2}$ (D) $3\sqrt{\pi}$

**Q.42** The value of the following definite integral $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\dfrac{\sin 2x}{1+\cos x}\,dx$ is,

(A) $-2\ln 2$ (B) 2

(C) 0 (D) $(\ln 2)^2$

**Q.43** $\int_0^{\frac{\pi}{4}}\left(\dfrac{1-\tan x}{1+\tan x}\right)dx$ evaluates to

(A) 0 (B) 1

(C) $\ln 2$ (D) $\dfrac{1}{2}\ln 2$

**Q.44** Let $X$ and $Y$ be two independent random variables. Which one of the relations between expectation ($E$), variance (Var) and covariance (Cov) given below is False?

(A) $E(XY) = E(X)\,E(Y)$

(B) $\text{Cov}(X, Y) = 0$

(C) $\text{Var}(X+Y) = \text{Var}(X) + \text{Var}(Y)$

(D) $E(X^2Y^2) = \left(E(X)\right)^2\left(E(Y)\right)^2$

**Q.45** Let $A$ be $n \times n$ real matrix such that $A^2 = I$ and $y$ be an $n$-dimensional vector.

Then the linear system of equations $Ax = y$ has

(A) no solution

(B) a unique solution

(C) more than one but finitely many independent solutions

(D) infinitely many independent solutions.

**Q.46** $P_x(x) = Me^{-2|x|} + Ne^{-3|x|}$ is the probability density function for the real random variable $X$ over the entire $x$ axis. $M$ and $N$ are both positive real numbers. The equation relating $M$ and $N$ is

(A) $M + \dfrac{2}{3}N = 1$

(B) $2M + \dfrac{1}{3}N = 1$

(C) $M + N = 1$

(D) $M + N = 3$

**Q.47** Real matrices $[A]_{3\times1}$, $[B]_{3\times3}$, $[C]_{3\times5}$, $[D]_{5\times3}$, $[E]_{5\times5}$ and $[F]_{5\times1}$ are given. Matrices $[B]$ and $[E]$ are symmetric. Following statements are made with respect to these matrices.

I. Matrix product $[F]^T[C]^T[B][C][F]$ is a scalar.

II. Matrix product $[D]^T[F][D]$ is always symmetric.

With reference to above statements, which of the following applies?

(A) Statement I is true but II is false.

(B) Statement I is false but II is true.

(C) Both the statement are true.

(D) Both the statements are false.

**Q.48** If $A = \begin{bmatrix} 1 & 1 \\ 0 & i \end{bmatrix}$ and $A^{2024} = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$ then $a+d$ is _______. (rounded upto one decimal place)

**Q.49** The integral $I = \int \dfrac{dx}{|6x|\sqrt{36x^2 - 36}}$ is $f(x)$ then value of $f(\sqrt{2})$ if $f(1) = 0$ is _______.

**Q.50** For two independent events $A, B$;

$$P(B) = \frac{3}{4},\ P(A \cup B^C) = \frac{1}{2}$$

then $P(A)$ is _______.

**Q.51** $I = \lim\limits_{x \to \frac{\pi}{4}} \dfrac{\int_{2}^{\sec^2 x} f(t)\,dt}{\left(x^2 - \dfrac{\pi^2}{16}\right)}$ at $f(2) = \pi$ is _______.

**Q.52** For events $A$, $B$ and $C$

$P$ (exactly one of $A$ or $B$) = $P$ (Exactly one of $B$ or $C$) $= P$(Exactly one of $C$ or $A$) $= \dfrac{1}{4}$; $P$ (all events) $= \dfrac{1}{16}$; $P$ (at least one event) is _______.

**Q.53** In the matrix equation $PX = Q$ which of the following is a necessary condition for the existence of at least one solution for the unknown vector $X$

(A) Augmented matrix $[P:Q]$ must have the same rank as matrix $P$.
(B) Vector $Q$ must have only non-zero elements.
(C) Matrix $P$ must be singular.
(D) Matrix $P$ must be square.

**Q.54** Consider a $3\times3$ real symmetric matrix $S$ such that two of its Eigen values are $a\neq0$, $b\neq0$ with respective Eigen vectors $\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix},\begin{bmatrix}y_1\\y_2\\y_3\end{bmatrix}$.

If $a\neq b$ then $x_1y_1+x_2y_2+x_3y_3$ equals

(A) $a$ (B) $b$
(C) $ab$ (D) 0

**Q.55** Consider the matrix $A=\begin{bmatrix}2&1&1\\2&3&4\\-1&-1&-2\end{bmatrix}$

whose Eigen values are $1,-1$ and $3$.

The trace of $(A^3-3A^2)$ is ________.

**Q.56** A box has a 8 red balls and 8 green balls. Two balls are drawn randomly in succession from the box without replacement. The probability that the first ball drawn is red and the second ball drawn is green is

(A) $\frac{4}{15}$ (B) $\frac{7}{16}$
(C) $\frac{1}{2}$ (D) $\frac{8}{15}$

**Q.57** If each element of a matrix is either 0 or 1 then the number of such different $n\times n$ symmetric matrix will be possible for **[MSQ]**

(A) $2^{10}$ for $n=4$ (B) $2^6$ for $n=3$
(C) $2^{36}$ for $n=8$ (D) $2^{21}$ for $n=6$

**Q.58** A real $n\times n$ matrix $A=[a_{ij}]$ is defined as follows :

$a_{ij}=i$ ; if $i=j$

$=0$ ; otherwise

The summation of all elements of $A$ is

(A) $n\frac{(n+1)}{2}$ (B) $n\frac{(n-1)}{2}$
(C) $\frac{n(n+1)(2n+1)}{6}$ (D) $n^2$

**Q.59** Given that $X$ is a random variable in the range $[0,\infty)$ with a probability density function $\frac{e^{\frac{-x}{3}}}{G}$, the value of the constant $G$ is ________.

**Q.60** Given a real-valued continuous function $f(y)$ defined over $[0,1]$,

$$\lim_{y\to0}\frac{1}{y}\int_0^y f(x)\,dx$$

(A) $\infty$ (B) 0
(C) $f(1)$ (D) $f(0)$

| Answers | Engineering Mathematics |
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|---|---|---|---|---|---|---|---|---|---|
| 1. | A | 2. | B | 3. | C, D | 4. | D | 5. | 3 |
| 6. | D | 7. | B | 8. | A | 9. | B | 10. | 323 |
| 11. | A | 12. | B | 13. | C | 14. | D | 15. | – 4 |
| 16. | 8 | 17. | A, B, C | 18. | A | 19. | 8 | 20. | D |
| 21. | 0.52 | 22. | D | 23. | 3.99 | 24. | 0.5 | 25. | 92928 |
| 26. | A | 27. | 1 | 28. | 0.197 | 29. | 0 | 30. | 4 |
| 31. | – 1 | 32. | A, C | 33. | B, D | 34. | B, C, D | 35. | B, C, D |
| 36. | 2.5 | 37. | 0.167 | 38. | A, D | 39. | A | 40. | 0.314 |
| 41. | B | 42. | C | 43. | D | 44. | D | 45. | B |
| 46. | A | 47. | A | 48. | 2 | 49. | 0.021 | 50. | 0.33 |
| 51. | 8 | 52. | 0.44 | 53. | A | 54. | D | 55. | – 6 |
| 56. | A | 57. | A,B,C,D | 58. | A | 59. | 3 | 60. | D |

## Explanations — Engineering Mathematics

**1. (A)**

**Given :** $Q = PAP^T$

and $X = P^T Q^{2005} P$

where $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}_{2\times2}$ and $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}_{2\times2}$

$$P^T = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}_{2\times2}$$

$$P^T P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 1$$

Thus, $P^T P = I$

We begin our analysis with $Q = PAP^T$

Then, $Q^2 = Q.Q = (PAP^T)(PAP^T)$

$$= PA(P^T P)AP^T$$

$$= PA(I)AP^T$$

$$Q^2 = PA^2P^T$$

Similarly, we can prove $Q^3 = PA^3P^T$

$$Q^{2005} = PA^{2005}P^T$$

Similarly, $A = \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}$

Thus, $A^2 = A.A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

Similarly, $A^3 = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

So, $A^{2005} = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$

$$Q^{2005} = PA^{2005}P^T$$

So, $X = P^T Q^{2005} P = P^T P A^{2005} P^T P$

$$= IA^{2005} I = A^{2005} = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$$

Hence, the correct option is (A).

**2.** **(B)**

**Given :** Augmented matrix

$$C = [A : B]$$

$$C = \begin{bmatrix} 1 & 1 & 1 & : & 2 \\ 2 & 3 & 2 & : & 5 \\ 2 & 3 & (a^2-1) & : & a+1 \end{bmatrix}$$

$R_3 \to R_3 - R_2$,

$$C = \begin{bmatrix} 1 & 1 & 1 & : & 2 \\ 2 & 3 & 2 & : & 5 \\ 0 & 0 & (a^2-3) & : & a-4 \end{bmatrix}$$

From option (A), $a = 4$

$$C = \begin{bmatrix} 1 & 1 & 1 & : & 2 \\ 2 & 3 & 2 & : & 5 \\ 0 & 0 & 13 & : & 0 \end{bmatrix}$$

$P(A) = P(A:B) = 3$ Unique solution

From option (B), $|a| = \sqrt{3}$

$$C = \begin{bmatrix} 1 & 1 & 1 & : & 2 \\ 2 & 3 & 2 & : & 5 \\ 0 & 0 & 0 & : & \sqrt{3}-4 \end{bmatrix}$$

$P(A) = 2,\ P(A:B) = 3$

$P(A) \neq P(A:B)$

Inconsistent at $|a| = \sqrt{3}$

Hence, the correct option is (B).

**3.** **(C), (D)**

**Given :** $3\times3$ matrices $M$ and $N$

Checking from options :

(A) $(N^T MN)^T = N^T M^T N$ is symmetric if $M$ is symmetric and skew-symmetric if $M$ is skew-symmetric.

(B) $(MN - NM)^T = (MN)^T - (NM)^T$

$= NM - MN$

$= -(MN - NM)$

Skew symmetric

(C) $(MN)^T = N^T M^T$

$= NM$

$\neq MN$ hence NOT correct

(D) Standard result is

$$adj(MN) = [(adjN)(adjM)]$$

$$\neq (adjM)(adjN)$$

Hence, the correct options are (C) and (D).

**4.** **(D)**

**Given :** $P = [a_{ij}]_{3\times3}$, $b_{ij} = 2^{i+j} a_{ij}$, $Q = [b_{ij}]_{3\times3}$

$$P = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$Q = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

$$Q = \begin{bmatrix} 4a_{11} & 8a_{12} & 16a_{13} \\ 8a_{21} & 16a_{22} & 32a_{23} \\ 16a_{31} & 32a_{32} & 64a_{33} \end{bmatrix}$$

$$\text{Determinant of } Q = \begin{vmatrix} 4a_{11} & 8a_{12} & 16a_{13} \\ 8a_{21} & 16a_{22} & 32a_{23} \\ 16a_{31} & 32a_{32} & 64a_{33} \end{vmatrix}$$

$$= 4\times8\times16 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ 2a_{21} & 2a_{22} & 2a_{23} \\ 4a_{31} & 4a_{32} & 4a_{33} \end{vmatrix}$$

$$= 4\times8\times16\times2\times4 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$= 2^2.2^3.2^4.2^1.2^2.2^1 = 2^{13}$$

Hence, the correct option is (D).

**5.** **3**

**Given :** $X_2, X_3$ are linearly independent of $X_1$

$X_4$ is linearly dependent of $X_1, X_2, X_3$

Number of linearly independent vectors = 3

Rank of matrix = Number of linearly independent vectors = 3

Hence, the correct answer is 3.

**6.** **(D)**

**Given :** $A = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & a & 0 \\ 0 & 0 & 0 & a \end{bmatrix}$

Characteristic equation is $|A - \lambda I| = 0$

$$\begin{vmatrix} (a-\lambda) & 0 & 0 & 0 \\ 0 & (a-\lambda) & 0 & 0 \\ 0 & 0 & (a-\lambda) & 0 \\ 0 & 0 & 0 & (a-\lambda) \end{vmatrix} = 0$$

$$(a-\lambda)\left[(a-\lambda)(a-\lambda)^2\right] = 0$$

$$(a-\lambda)^4 = 0$$

$$a^4 - 4a\lambda^3 + 6a^2\lambda^2 - 4a^3\lambda + \lambda^4 = 0$$

$$\sum_{k=0}^{4} (-1)^k \cdot {}^4C_k \cdot a^{4-k} \cdot \lambda^k = 0$$

Hence, the correct option is (D).

**7.** **(B)**

**Given :** $A = \begin{bmatrix} 3 & -1 \\ 5 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ -1 & 5 \end{bmatrix}$

We have,

$$\lambda_1 + \lambda_2 = \text{trace of } A = 9$$

$$\lambda_1 \lambda_2 = |A| = 18 + 5 = 23$$

$$\frac{1}{\lambda_1} + \frac{1}{\lambda_2} = \frac{9}{23}$$

Again, $\mu_1 + \mu_2 = 6$, $\mu_1 \mu_2 = |B| = 7$

$$\frac{1}{\mu_1} + \frac{1}{\mu_2} = \frac{6}{7}$$

Sum of the roots $= \frac{9}{23} + \frac{6}{7} = \frac{201}{7 \times 23}$

Product of the roots $= \frac{54}{7 \times 23}$

Required equation is,

$$x^2 - \frac{201}{7 \times 23} x + \frac{54}{7 \times 23} = 0$$

$$161x^2 - 201x + 54 = 0$$

Hence, the correct option is (B).

**8.** **(A)**

**Given :** $\frac{(1+3p)}{3}, \frac{(1-p)}{4}$ and $\left(\frac{1-2p}{2}\right)$ are the probabilities of three events, we must have

$$0 \le \frac{1+3p}{3} \le 1,\ 0 \le \frac{1-p}{4} \le 1 \text{ and } 0 \le \frac{1-2p}{2} \le 1$$

$$-1 \le 3p \le 2,\ -3 \le p \le 1 \text{ and } -1 \le 2p \le 1$$

$$-\frac{1}{3} \le p \le \frac{2}{3}, -3 \le p \le 1 \text{ and } -\frac{1}{2} \le p \le \frac{1}{2}$$

Also as $\frac{1+3p}{3}, \frac{1-p}{4}$ and $\frac{1-2p}{2}$ are the probabilities of three mutually exclusive events

$$0 \le \frac{1+3p}{3} + \frac{1-p}{4} + \frac{1-2p}{2} \le 1$$

$$0 \le 4 + 12p + 3 - 3p + 6 - 12p \le 1$$

$$\frac{1}{3} \le p \le \frac{13}{3}$$

Thus, the required value of $p$ are such that

$$\text{Max}\left\{-\frac{1}{3}, -3, -\frac{1}{2}, \frac{1}{3}\right\} \le p \le \text{Min}\left\{\frac{2}{3}, 1, \frac{1}{2}, \frac{13}{3}\right\}$$

$$\frac{1}{3} \le p \le \frac{1}{2}$$

Hence, the correct option is (A).

**9.** **(B)**

**Given :** Let $E_1, E_2$ denote the events that the coin shows a head, tail and $A$ be the event that the noted number is either 7 or 8.

We have, $P(E_1) = \frac{1}{2}$ and $P(E_2) = \frac{1}{2}$

Now, $7 \rightarrow \{(1,6), (6,1), (2,5), (5,2), (3,4), (4,3)\}$

and $8 \rightarrow \{(2,6), (6,2), (3,5), (5,3), (4,4)\}$

Thus, $P(A/E_1) = \frac{11}{36}, P(A/E_2) = \frac{1}{11}$

Hence, the required probability,

$$P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2)$$

$$=\left(\frac{1}{2}\right)\left(\frac{11}{36}\right)+\left(\frac{1}{2}\right)\left(\frac{2}{11}\right)=\frac{193}{792}$$
$$=0.244$$

Hence, the correct option is (B).

**10. 323**

**Given :** Mean number of defectives $=2=np$

$$n=20$$

The probability of a defective part is,

$$p=2/20=0.1$$

And the probability of a non-defective part $=0.9$

The probability of at least three defectives in a sample of 20.

$=1$ – (Probability that either none, or one, or two are non-defective parts)

$$=1-\left[{}^{20}C_0(0.9)^{20}+{}^{20}C_1(0.1)(0.9)^{19}+{}^{20}C_2(0.1)^2(0.9)^{18}\right]$$

$$=1-(0.9^{18}\times 4.51)=0.323$$

Thus, the number of samples having at least three defective parts out of 1000 samples

$$=1000\times 0.323=323$$

Hence, the correct answer is 323.

**11. (A)**

**Given :** $\lim\limits_{y\to 0}\dfrac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4}$

$$=\lim_{y\to 0}\frac{1+\sqrt{1+y^4}-2}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)}$$

$$=\lim_{y\to 0}\frac{\left(\sqrt{1+y^4}-1\right)\left(\sqrt{1+y^4}+1\right)}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)\left(\sqrt{1+y^4}+1\right)}$$

$$=\lim_{y\to 0}\frac{1+y^4-1}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)\left(\sqrt{1+y^4}+1\right)}$$

$$=\lim_{y\to 0}\frac{1}{\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)\left(\sqrt{1+y^4}+1\right)}=\frac{1}{4\sqrt{2}}$$

Hence, the correct option is (A).

**12. (B)**

**Given :** $\lim\limits_{x\to 0}\dfrac{x+2\sin x}{\sqrt{x^2+2\sin x+1}-\sqrt{\sin^2 x-x+1}}$

$$\lim_{x\to 0}\frac{(x+2\sin x)(\sqrt{x^2+2\sin x+1}+\sqrt{\sin^2 x-x+1})}{x^2+2\sin x+1-\sin^2 x+x-1}$$

$$\lim_{x\to 0}\frac{(x+2\sin x)(2)}{x^2+2\sin x-\sin^2 x+x}\quad\left(\frac{0}{0}\text{ form}\right)$$

Using L' Hospital rule,

$$\lim_{x\to 0}\frac{(1+2\cos x)\times 2}{2x+2\cos x-2\sin x\cos x+1}=\frac{2\times 3}{(2+1)}=2$$

Hence, the correct option is (B).

**13. (C)**

**Given :** The equation $x^2+bx+c=0$ has roots $\alpha$ and $\beta$

$$\alpha+\beta=-b$$
$$\alpha\beta=c$$

So, $\quad x^2+bx+c=(x-\alpha)(x-\beta)$

$$\lim_{x\to\alpha}\frac{1-\cos(x^2+bx+c)}{(x-\alpha)^2}$$

$$=\lim_{x\to\alpha}\frac{2\sin^2\left[\frac{x^2+bx+c}{2}\right]}{(x-\alpha)^2}$$

$$=\lim_{x\to\alpha}\frac{2\sin^2\left[\frac{(x-\alpha)(x-\beta)}{2}\right]}{(x-\alpha)^2}$$

$$=2\lim_{x\to\alpha}\left[\frac{\sin\left[\frac{(x-\alpha)(x-\beta)}{2}\right]}{\frac{1}{2}(x-\alpha)(x-\beta)}\right]^2\times\frac{1}{4}(x-\beta)^2$$

$$=\frac{2}{4}(\alpha-\beta)^2=\frac{1}{2}\left[(\alpha+\beta)^2-4\alpha\beta\right]$$

$$=\frac{1}{2}\left[(-b)^2-4\times c\right]$$

$$=\frac{b^2-4c}{2}$$

Hence, the correct option is (C).

**14.** **(D)**

Checking from options :

**Option (A) :** $f(x)=\cos(|x|)+|x|=\cos x+|x|$

is not-differentiable at $x=0$ as $|x|$ is non-differentiable at $x=0$.

**Option (B) :**

Similarly, $f(x)=\cos(|x|)-|x|=\cos x-|x|$ is non-differentiable at $x=0$.

**Option (C) :**

$$f(x)=\sin|x|+|x|=\begin{cases}-\sin x-x, & x<0\\ +\sin x+x, & x\geq 0\end{cases}$$

$$f'(x)=\begin{cases}-\cos x-1, & x<0\\ +\cos x+1, & x>0\end{cases}$$

$$f'(0^-)=-2,\ f'(0^+)=2$$

Hence, $f(x)$ is not differential at $x=0$.

**Option (D) :**

$$f(x)=\sin|x|-|x|=\begin{cases}-\sin x+x, & x<0\\ +\sin x-x, & x\geq 0\end{cases}$$

$$f'(x)=\begin{cases}-\cos x+1, & x<0\\ +\cos x-1, & x>0\end{cases}$$

$$f'(0^-)=f'(0^+)=0$$

Therefore, $f$ is differentiable at $x=0$.

Hence, the correct option is (D).

**15.** **- 4**

**Given :** $\lim\limits_{x\to 0}\dfrac{x(1+a\cos x)-b\sin x}{x^3}=1$

$$\lim_{x\to 0}\frac{x+ax\cos x-b\sin x}{x^3}=1$$

$$\lim_{x\to 0}\frac{x+ax\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}+\ldots\right)-b\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}+\ldots\right)}{x^3}=1$$

$$\lim_{x\to 0}\frac{x+ax-\frac{ax^3}{2!}+\frac{ax^5}{4!}\ldots-bx+\frac{bx^3}{3!}-\frac{bx^5}{5!}}{x^3}=1$$

$$\lim_{x\to 0}\frac{x(1+a-b)+x^3\left(\frac{b}{3!}-\frac{a}{2!}\right)+x^5\left(\frac{a}{4!}-\frac{b}{5!}\right)}{x^3}=1$$

$$\lim_{x\to 0}\frac{1+a-b}{x^2}+\left(\frac{b}{3!}-\frac{a}{2!}\right)=1$$

By comparing, $1+a-b=0$ ...(i)

and $\dfrac{b}{3!}-\dfrac{a}{2!}=1$ ...(ii)

Solving equations (i) and (ii),

$$a=\frac{-5}{2},b=\frac{-3}{2}$$

$$a+b=-4$$

Hence, the correct answer is – 4.

**16.** **8**

**Given :** $I=\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\int_2^{\sec^2 x}f(t)\,dt}{\left(x^2-\frac{\pi^2}{16}\right)}$

The given limit can be solved by Leibnitz Rule,

$$I=\lim_{x\to\frac{\pi}{4}}\frac{\int_2^{\sec^2 x}f(t)\,dt}{\left(x^2-\frac{\pi^2}{16}\right)}=\frac{0}{0}\text{ form}$$

$$I=\lim_{x\to\frac{\pi}{4}}\frac{\frac{d}{dx}\left(\int_2^{\sec^2 x}f(t)\,dt\right)}{\frac{d}{dx}\left(x^2-\frac{\pi^2}{16}\right)}=\frac{0}{0}$$

$$=\lim_{x\to\frac{\pi}{4}}\frac{2\sec^2 x\tan x\times f(\sec^2 x)}{2x}$$

$$=\lim_{x\to\frac{\pi}{4}}\frac{\sec^2\frac{\pi}{4}\tan\frac{\pi}{4}\times f\left(\sec^2\frac{\pi}{4}\right)}{\frac{\pi}{4}}$$

$$=\lim_{x\to\frac{\pi}{4}}\frac{\sec^2\frac{\pi}{4}\tan\frac{\pi}{4}\times f\left(\sec^2\frac{\pi}{4}\right)}{\frac{\pi}{4}}$$

Hence, the correct answer is 8.

**17.** **(A), (B), (C)**

**Given :** $f(x)=\begin{cases} 0; & x=0 \\ \frac{1}{2}-x; & 0<x<\frac{1}{2} \\ \frac{1}{2} & x=\frac{1}{2} \\ \frac{3}{2}-x; & \frac{1}{2}<x<1 \\ 1: & x=1 \end{cases}$

For continuity,

$$f(x^-)=f(x^+)=f(0)$$

Lets check the point

At $x=0$,

$$f(0)\neq f(0^+)$$

$$0\neq\frac{1}{2}$$

So, discontinuous at $x=0$

At $x=\frac{1}{2}$,

$$f\left(\frac{1}{2^-}\right)=f\left(\frac{1}{2^+}\right)=f\left(\frac{1}{2}\right)$$

$$0\neq\frac{1}{2}=\frac{1}{2}$$

So, discontinuous at $x=\frac{1}{2}$

At $x=1$,

$$f(1^-)=f(1^+)=f(1)$$

$$\frac{1}{2}\neq 1$$

So, discontinuous at $x=1$

Hence, the correct options are (A), (B) and (C).

**18.** **(A)**

**Given :** $f(x)=\begin{cases} k\sqrt{x+1}; & 0\le x\le 3 \\ mx+2; & 3<x\le 5 \end{cases}$

For differentiable at $x=3$

So, $f(3^-)=f(3^+)=f(3)$

$$2k=3m+2 \quad \ldots\text{(i)}$$

For differentiability,

$$f'(x)=\begin{cases} \frac{k}{2\sqrt{x+1}}; & 0\le x\le 3 \\ m; & 3<x\le 5 \end{cases}$$

$$f'(3^+)=f'(3^-)$$

$$m=\frac{k}{4} \quad \ldots\text{(ii)}$$

From equation (i) and (ii),

$$m=\frac{2}{5} \text{ and } k=\frac{8}{5}$$

Hence, the correct option is (A).

**19.** **8**

**Given :** $f(x)=\sin^4 x+\cos^4 x$

Since, the function is smooth curve

So, $x\in R$

For maxima and minima $f'(x)=0$

$$f'(x)=4\sin^3 x\cdot\cos x+4\cos^3 x(-\sin x)$$

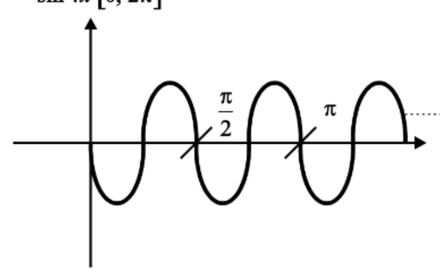
$$=4\sin x\cos x(\sin^2 x-\cos^2 x)$$

$$=-2(2\sin x\cos x)(\cos^2 x-\sin^2 x)$$

$$=-2(\sin 2x)(\cos 2x)$$

$$=-\sin 4x$$

$-\sin 4x$ $[0, 2\pi]$




$$f'(x)=-\sin 4x=0$$

$$\sin 4x=0$$

$$4x=n\pi$$

$$x=\frac{n\pi}{4}$$

Total 8 points of maxima and minima occur between $[0,2\pi]$.

Hence, the correct answer is 8.

**20. (D)**

**Given :** $p = \frac{40}{600} = \frac{1}{15}$ and $n = 10$

So, $\lambda = np = \frac{1}{15} \times 10 = \frac{2}{3}$

$$P(r) = \frac{e^{-\lambda}\lambda^r}{r!} = \frac{e^{-2/3} \times \left(\frac{2}{3}\right)^r}{r!}$$

$$P(0) = \frac{e^{-2/3} \times \left(\frac{2}{3}\right)^0}{0!} = e^{-2/3} = 0.51$$

Hence, the correct option is (D).

**21. 0.52**

Fair $\rightarrow$ Coin $\rightarrow$ $\{H, T\}$ $\rightarrow$ $\frac{1}{2} = P(H)$

Biased $\rightarrow$ Dice $\rightarrow$ $\{1, 2, 3, 4, 5, 6\} \rightarrow$ $\frac{1}{6} = P(6)$

Prob ($H$ and 6) $= P(H) \cdot P(6) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$

$$P(W) = \frac{1}{12}$$

$$P(L) = 1 - \frac{1}{12} = \frac{11}{12}$$

Now, $P$ (winning at A)

$$= P(W_A) + P(L_A) \times P(L_B) \times P(W_A) + P(L_A) \times P(L_B) \times P(L_A) \times P(L_B) \times P(W_A) + \ldots\ldots\infty$$

$$= \frac{1}{12} + \left(\frac{11}{12}\right)^2 \times \frac{1}{12} + \left(\frac{11}{12}\right)^4 \times \frac{1}{12} + \ldots\ldots\infty$$

$$= \frac{1}{12}\left[1 + \left(\frac{11}{12}\right)^2 + \left(\frac{11}{12}\right)^4 + \ldots\ldots\infty\right]$$

It is in G.P series $\left\{\because S_\infty = \frac{a}{1-r}\right\}$

$$\therefore \quad P(\text{Winning of A}) = \frac{1}{12}\left[\frac{1}{1 - \frac{121}{144}}\right] \approx 0.52$$

Hence, the correct answer is 0.52.

**22. (D)**

Miss A : P (2 balls same color)

$$= P(2\,\text{Red}) + P(2\,\text{White})$$

$$P(\text{Miss A}) = \frac{{}^3C_2 + {}^nC_2}{{}^{n+3}C_2} = \frac{1}{2} \quad \left[{}^nC_r = \frac{n!}{r!(n-r)!}\right]$$

$$n^2 - 7n + 6 = 0$$

$$n = 1, 6$$

$$P(\text{Miss B}) = \left[\frac{3}{n+3} \times \frac{3}{n+3}\right] + \left[\frac{n}{n+3} \times \frac{n}{n+3}\right]$$

$$= \frac{5}{8}$$

$$n^2 - 10n + 9 = 0$$

$$n = 9, 1$$

In both cases, common value of $n = 1$

Therefore, the possible of value of $n$ is 1

Hence, the correct option is (D).

**23. 3.99**

**Given :** Gaussian random variable function is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Mean $\mu = 0$

Variance $\sigma^2 = 1$

$$\text{So, } E(|5x|) = \int_{-\infty}^{\infty} |5x| f(x) dx$$

$$= \int_{-\infty}^{\infty} |5x| \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$$

$$= \int_0^{\infty} \frac{2}{\sqrt{2\pi}} \times 5x e^{-\frac{x^2}{2}} dx$$

$$= \frac{10}{\sqrt{2\pi}} \int_0^{\infty} e^{-\frac{x^2}{2}} .x dx = \frac{10}{\sqrt{2\pi}} = 3.99$$

Hence, the correct answer is 3.99.

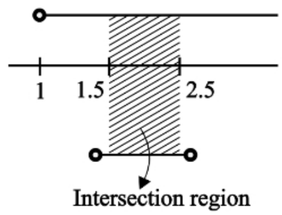
**24. 0.5**

**Given :** $P(A/B) = \frac{P(A \cap B)}{P(B)}$

$$P(1.5 < x \le 2.5 \mid x > 1)$$

$$= \frac{P\big((1.5 < x \le 2.5) \cap (x > 1)\big)}{P(x > 1)}$$

or $$\frac{P(1.5 < x \le 2.5)}{P(x > 1)}$$




$$= \frac{P(1.5 < x \le 2.5)}{1 - P(x \le 1)}$$

$$= \frac{\int_{1.5}^{2} \frac{1}{2} dx + \int_{2}^{2.5} \left(\frac{3-x}{3}\right) dx}{1 - \int_{0}^{1} \frac{x}{2} dx}$$

$$= \frac{\frac{1}{4} + \frac{1}{8}}{1 - \frac{1}{4}} = \frac{1}{2}$$

Hence, the correct answer is 0.5.

**25. 92928**

**Given :** $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$

Eigen value of a upper triangular matrix is equal to the principal diagonal element.

So, $\lambda(A) = 1, 4, 6$

$\lambda(A^3) = 1, 64, 216;\ \lambda(A^2) = 1, 16, 36$

$\lambda(A^3 + A^2 + 2A) = 1+1+2,\ 64+16+4\times 2,$

$216+36+12$

$= 4, 88, 264$

$\therefore \quad \text{Det}(A^3 + A^2 + 2A) = 4 \times 88 \times 264 = 92928$

Hence, the correct answer is 92928.

**26. (A)**

**Given :** $I = \int_{1}^{3} e^{3x} \left( \log x + \frac{1}{x} \right) dx$

Let $3x = t$

$$dx = \frac{dt}{3}$$

On substituting,

$$I = \int_{3}^{9} e^{t} \left( \log\left(\frac{t}{3}\right) + \frac{3}{t} \right) \frac{dt}{3} \qquad \ldots\text{(i)}$$

$$f(t) = \log\left(\frac{t}{3}\right)$$

If $\quad f'(t) = \frac{3}{t} \times \frac{1}{3}$

$$I = \int_{3}^{9} e^{t} [f(t) + 3f'(t)]\, dt \qquad \ldots\text{(ii)}$$

We know,

$$\int e^{x} f(x) + f'(x) dx = e^{x} f(x) + C \qquad \ldots\text{(iii)}$$

$$= [e^{t} f(t)]_3^9 = \left[ e^{t} \log\left(\frac{t}{3}\right) \right]_3^9$$

$$= e^{9} \log 3 - e^{3} \log 1 = e^{9} \log 3$$

Hence, the correct option is (A).

**27. 1**

**Given :** $A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}_{3\times 4}$

On performing transformations,

$R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - R_1$,

$$A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & -1 & -3 \\ 0 & 1 & 1 & 3 \end{bmatrix}$$

$R_3 \to R_3 + R_2$,

$$\begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & -1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Above matrix is row echelon form we can say that $\rho(A)_{3\times 4} = 2$

Hence, number of independent solution

$$= \text{Number of row } - \rho(A)_{3\times 4}$$

$$= 3 - 2 = 1$$

Hence, the correct answer is 1.

**28. 0.197**

**Given :** $P = \begin{bmatrix} \frac{x}{\sqrt{2}} & \frac{y}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$

'$x$' and '$y$' follow binomial distribution with probability of success, $p = \frac{1}{3}$ and number of trials, $n = 3$

For $P$ to be orthogonal,

$$AA^T = I$$

$$= \begin{bmatrix} \frac{x}{\sqrt{2}} & \frac{y}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{x}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{y}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\frac{x^2}{2} + \frac{y^2}{2} = 1 \text{ and } \frac{-x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = 0$$

$$x^2 + y^2 = 2 \qquad \ldots\text{(i)}$$

and $x = y \qquad \ldots\text{(ii)}$

$\therefore \quad P(x=1, y=1) = P(x=1)P(y=1)$

($\because$ Independent variables)

$$= \left[{}^3C_1\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2\right]\left[{}^3C_1\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2\right] = \frac{16}{81}$$

Hence, the correct answer is 0.197.

**29. 0**

**Given :** $(P(X^TY)^{-1}P^T)^T = (P\times 3\times 3\times P^T)^T$

$$p - q = 2 - 2 = 0$$

Hence, the correct answer is 0.

**30. 5**

**Given :** $|A|_{3\times 3} = 5$

$$B = \text{adj }(5\text{A})$$

On taking determinant both sides,

$$|B| = |\text{adj}(5\text{A})| = |5^{3-1}\,\text{adj}(\text{A})|$$

$$|B| = |25\,\text{adj}(A)| = 25^3\,|\text{adj}(A)|$$

$$|B| = 25^3 \times |A|^2 = 5^6 \times 5^2 = 5^8$$

$\therefore \quad \sqrt[8]{|B|_{3\times 3}} = \sqrt[8]{5^8} = 5$

Hence, the correct answer is 5.

**31. – 1**

**Given :** On writing expansion,

$$\lim_{x\to 0} \frac{a\left(x - \frac{x^3}{3!} + \ldots\ldots\right) + b\left(1 - \frac{x^2}{2!} + \ldots\ldots\right) + cx}{x^3} = \frac{-1}{6}$$

For finite limit, $a = -c$, $b = 0$

$$\frac{a+b}{c} = \frac{-c+0}{c} = -1$$

Hence, the correct answer is – 1.

**32. (A), (C)**

**Given :** $f(x,y) = x^2 + y^2 + 4x + 8$

$$p = \frac{\partial f}{\partial x} = 2x + 4 = 0,\ x = -2$$

$$q = \frac{\partial f}{\partial y} = 2y = 0,\ y = 0$$

Point $p(-2, 0)$,

$$r = \frac{\partial^2 f}{\partial x^2} = 2;\ t = \frac{\partial^2 f}{\partial y^2} = 2;\ s = \frac{\partial^2 f}{\partial x \partial y} = 0$$

$$rt - s^2 = 2\times 2 - 0 = 4, \text{ i.e. } r > 0$$

Hence, $(-2, 0)$ is point of minimum

$$f_{(\min)} = (-2)^2 + 0 + 4(-2) + 8 = 4$$

Hence, the correct options are (A) and (C).

**33. (B), (D)**

**Given :** Matrix is an upper triangular matrix so, eigen value of $P$ are 1, $-4$ and 5.

(A) Then eigen value of $P^{-1}$ are 1, $\frac{-1}{4}$ and $\frac{1}{5}$.

(B) $|5P^T| = 5^3|P^T| = 125 \times |P|$

$= 125 \times$ Product of eigen values of '$P$'

$= 125 \times 1 \times -4 \times 5 = -2500$

(C) $\because$ $|P| \neq \pm 1$ hence can't be an orthogonal matrix

(D) Eigen values of $P$ are

$1^{2022}, (-4)^{2022}, 5^{2022} = 1, 2^{4044}, 5^{2022}$

Hence, the correct options are (B) and (D).

**34. (B), (C), (D)**

**Given :** '$P$' is non-singular.

$|P| \neq 0$

$|P - OI| \neq 0$

But for eigen values, $|P - \lambda I| = 0$

No Eigen value can be zero

Hence, only option (A) is correct.

Hence, the correct options are (B), (C) and (D).

**35. (B), (C), (D)**

**Given :** $f(x) = \int_{x^2}^{x^3} t\,dt$

$$f(x) = \int_{\phi(x)}^{g(x)} \psi(t)\,dt$$

$$f'(x) = g'(x)\psi g(x) - \phi'(x)\psi\phi(x)$$

$$f'(x) = 3x^2 \cdot x^3 - 2x \cdot x^2 = 3x^5 - 2x^3 = 0$$

$$x^3(3x^2 - 2) = 0$$

$$x = 0, 0, 0, \pm\sqrt{\frac{2}{3}}$$

Total 5 extremum points.

On finding $f''(x) = 15x^4 - 6x^2 = 3x^2(5x^2 - 2)$

$f''(0) = 0$

$x = 0$ is point of inflection

$$f''\left(\pm\sqrt{\frac{2}{3}}\right) = 3 \times \frac{2}{3}\left(5 \times \frac{2}{3} - 2\right) = 2\left(\frac{4}{3}\right) = \frac{8}{3}$$

Both at $x = \pm\sqrt{\frac{2}{3}}$, $f(x)$ has minimum value.

Hence, the correct options are (B), (C) and (D).

**36. 2.5**

**Given :** Total probability $= 1$

Area of $f(x) = 1$

$$\frac{1}{2}(1+5) \times K = 1$$

$$K = \frac{1}{3}$$

$f(x)$ can be written as,

$$f(x) = \begin{cases} \frac{x}{6}, & 0 \le x < 2 \\ \frac{1}{3}, & 2 \le x \le 3 \\ \frac{(5-x)}{6}, & 3 \le x < 5 \end{cases}$$

Then, $E(x) = \int_{-\infty}^{\infty} x\, f(x)\,dx$

$$= \int_0^2 \frac{x^2}{6}dx + \int_2^3 \frac{x}{3}dx + \int_3^5 \frac{x(5-x)}{6}dx$$

$$= \left(\frac{x^3}{18}\right)_0^2 + \left(\frac{x^2}{6}\right)_2^3 + \left(\frac{5x^2}{2} - \frac{x^3}{3}\right)_3^5 \times \frac{1}{6}$$

$$= \frac{8}{18} + \frac{5}{6} + (25-9)\frac{5}{12} - \left(\frac{125-27}{18}\right)$$

$$= 2.5$$

Hence, the correct answer is 2.5.

**37. 0.167**

**Given :** $L = \lim_{x \to \infty} \frac{\ln(x^2 - 4x + 8)}{\ln(x^{12} + x^6 + 6)}$

$$= \lim_{x\to\infty} \frac{\ln\left(x^2\left(1-\frac{4}{x}+\frac{8}{x^2}\right)\right)}{\ln\left(x^{12}\cdot\left(1+\frac{1}{x^6}+\frac{6}{x^{12}}\right)\right)}$$

$$= \lim_{x\to\infty} \frac{\ln x^2 + \ln\left(1-\frac{4}{x}+\frac{8}{x^2}\right)}{\ln x^{12} + \ln\left(1+\frac{1}{x^6}+\frac{6}{x^{12}}\right)}$$

$$= \lim_{x\to\infty} \frac{2\ln x}{12\ln x}\left[\frac{1+\frac{\ln\left(1-\frac{4}{x}+\frac{8}{x^2}\right)}{\ln x^2}}{1+\frac{\ln\left(1+\frac{1}{x^6}+\frac{6}{x^{12}}\right)}{\ln x^{12}}}\right]$$

$$= \frac{2}{12}\times 1\times 1 = \frac{1}{6} = 0.167$$

Hence, the correct answer is 0.167.

**38. (A), (D)**

**Given :** $A = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 4 \\ \frac{-7}{2} & 2 & -2 \\ \frac{\beta}{2} & 4 & \frac{\alpha}{2} \end{bmatrix}$

Since, $[A]$ is orthogonal rows $R_1, R_2, R_3$ and columns $C_1, C_2$ and $C_3$ are orthonormal.

$$C_1^T \cdot C_2 = 0$$

$$\begin{bmatrix} \frac{1}{2} & \frac{-7}{2} & \frac{\beta}{2} \end{bmatrix}\begin{bmatrix} \frac{1}{2} \\ 2 \\ 4 \end{bmatrix} = 0$$

$$\frac{1}{4} - 7 + 2\beta = 0$$

$$\beta = \frac{27}{8}$$

Similarly, $C_2^T \cdot C_3 = 0$

$$\begin{bmatrix} \frac{1}{2} & 2 & 4 \end{bmatrix}\begin{bmatrix} 4 \\ -2 \\ \frac{\alpha}{2} \end{bmatrix} = 0$$

$$2 - 4 + 2\alpha = 0$$

$$\alpha = 1$$

Hence, the correct options are (A) and (D).

**39. (A)**

**Given :** Probability of choosing a private car = 0.45

Probability of choosing a public transport

$$= 1 - 0.45 = 0.55$$

Among public transport,

Probability of choosing a bus (public transport)

$$= 0.55 \times 0.55$$

$$= 0.3$$

Probability of choosing metro (public transport)

$$= 0.55 - 0.3$$

$$= 0.25$$

Hence, the correct option is (A).

**40. 0.314**

**Given :** $P(H) = \frac{1}{5}$ and $P(W) = \frac{1}{7}$

Required probability $P(H \cup W)$

$$= P(H) + P(W) - P(H \cap W)$$

$$= \frac{1}{5} + \frac{1}{7} - \left(\frac{1}{5}\times\frac{1}{7}\right) = \frac{11}{35} = 0.314$$

Hence, the correct answer is 0.314.

**41. (B)**

**Given :** $I = \int_0^\infty e^{-y^3} y^{\frac{1}{2}}\, dy$

Putting $y^3 = t$

Differentiating both the sides with respect to $t$,

$$3y^2 dy = dt$$

$$y^{\frac{1}{2}} dy = \frac{1}{3} y^{\frac{-3}{2}} dt = \frac{1}{3} t^{\frac{-1}{2}} dt$$

$$I=\int_0^{\infty}e^{-t}\frac{1}{3}t^{\frac{-1}{2}}dt$$

Using the property of gamma function,

$$\int_0^{\infty}e^{-t}t^{n-1}dt=\Gamma n$$

Here, $n-1=\frac{-1}{2} \quad \Rightarrow \quad n=\frac{1}{2}$

$$I=\frac{1}{3}\Gamma n$$

$$I=\frac{1}{3}\Gamma\frac{1}{2}=\frac{1}{3}\sqrt{\pi}$$

Hence, the correct option is (B).

**42. (C)**

**Given :** $I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{\sin 2x}{1+\cos x}dx$

$$f(x)=\frac{\sin 2x}{1+\cos x}$$

$$f(-x)=\frac{\sin(2\times(-x))}{1+\cos(-x)}$$

$$f(-x)=\frac{\sin(-2x)}{1+\cos(-x)}=\frac{-\sin 2x}{1+\cos x}$$

$$\left[\begin{array}{ll}\text{Since,} & \sin(-\theta)=-\sin\theta \\ & \cos(-\theta)=\cos\theta\end{array}\right]$$

Since, $f(x)=-f(-x)$

Hence, it is a odd function.

$$\therefore \quad \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{\sin 2x}{1+\cos x}dx=0$$

Hence, the correct option is (C).

**43. (D)**

**Given :** $I=\int_0^{\frac{\pi}{4}}\left(\frac{1-\tan x}{1+\tan x}\right)dx$

**. Method 1:**

$$I=\int_0^{\frac{\pi}{4}}\left(\frac{1-\frac{\sin x}{\cos x}}{1+\frac{\sin x}{\cos x}}\right)dx$$

$$I=\int_0^{\frac{\pi}{4}}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right)dx$$

Let $\cos x+\sin x=t$, $(-\sin x+\cos x)dx=dt$

Changing the limits,

$$x=0\Rightarrow t=1$$

$$x=\frac{\pi}{4}\Rightarrow t=\sqrt{2}$$

Then, $I=\int_1^{\sqrt{2}}\frac{1}{t}dt$

$$I=[\ln t]_1^{\sqrt{2}}=\ln(\sqrt{2})-\ln(1)$$

$$I=\ln\sqrt{2}=\ln(2)^{\frac{1}{2}}=\frac{1}{2}\ln 2$$

Hence, the correct option is (D).

**. Method 2:**

$$I=\int_0^{\pi/4}\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\times\tan x}dx$$

$$I=\int_0^{\pi/4}\tan\left(\frac{\pi}{4}-x\right)dx$$

$$I=\int_0^{\pi/4}\tan x\,dx$$

$$I=[\log\sec x]_0^{\pi/4}$$

$$I=\log\sec\frac{\pi}{4}-\log\sec 0$$

$$I=\log\sqrt{2}-\log 1=\frac{1}{2}\ln 2$$

Hence, the correct option is (D).

**44. (D)**

For $X$ and $Y$ be two independent random variables.

(i) $E(XY)=E(X)E(Y)$ ...(i)

(ii) $\text{Cov}(X,Y)=E(XY)-E(X)E(Y)$

$$=E(X)E(Y)-E(X)E(Y)$$

[From equation (i)]

$$=0$$

(iii) $\text{Var}(X+Y) = \text{Var}(X) + \text{Var}(Y)$

(iv) $E(X^2Y^2) = E(X^2)E(Y^2)$

Therefore, relation in option (D) is False.
Hence, the correct option is (D).

**45. (B)**

**Given :** $A$ is an $n \times n$ real matrix and $A^2 = I$.
Determinant of $A^2$ is given by,

$$|A^2| = |I| = 1$$
$$|A| = \pm 1$$
$$|A| \neq 0 \quad \text{[Condition for unique solution]}$$

Therefore, $Ax = y$ is consistent and has unique solution given by $x = A^{-1}y$.

Hence, the correct option is (B).

**46. (A)**

**Given :** A probability density function is,

$$P_x(x) = Me^{-2|x|} + Ne^{-3|x|}$$

By the property of probability density function (P.D.F.),

$$\int_{-\infty}^{\infty} P_x(x)\,dx = 1$$

$$\int_{-\infty}^{\infty} (Me^{-2|x|} + Ne^{-3|x|})\,dx = 1$$

$P_x(x)$ is even function as $|x|$ is even function.

So, by the property of even function,

$$2\int_0^{\infty} (Me^{-2x} + Ne^{-3x})\,dx = 1$$

$$2M\left(\frac{e^{-2x}}{-2}\right)_0^{\infty} + 2N\left(\frac{e^{-3x}}{-3}\right)_0^{\infty} = 1$$

$$-M(e^{-\infty} - e^0) - \frac{2}{3}N(e^{-\infty} - e^0) = 1$$

$$-M(0-1) - \frac{2}{3}N(0-1) = 1$$

$$M + \frac{2}{3}N = 1$$

Hence, the correct option is (A).

**47. (A)**

**Given :**

(i) Real matrices are :
$[A]_{3\times1}, [B]_{3\times3}, [C]_{3\times5}, [D]_{5\times3}, [E]_{5\times5}$ and $[F]_{5\times1}$.

(ii) Matrices $[B]$ and $[E]$ are symmetric.

**Statement I :**

Matrix product $[F]^T[C]^T[B][C][F]$ is a scalar.

Product of $[F]^T$ and $[C]^T$ is given by,

$$[F]^T_{1\times5}[C]^T_{5\times3} = [P]_{1\times3}$$

Product of $[P]$ and $[B]$ is given by,

$$[P]_{1\times3}[B]_{3\times3} = [Q]_{1\times3}$$

Product of $[Q]$ and $[C]$ is given by,

$$[Q]_{1\times3}[C]_{3\times5} = [R]_{1\times5}$$

Product of $[R]$ and $[F]$ is given by,

$$[R]_{1\times5}[F]_{5\times1} = [S]_{1\times1}$$

Since, order of product of $[F]^T[C]^T[B][C][F]$ is $1\times1$ i.e. scalar quantity.

Hence, statement I is true.

**Statement II :**

Matrix product $[D]^T[F][D]$ is always symmetric.

Product of $[D]^T$ and $[F]$ is given by,

$$[D]^T_{3\times5}[F]_{5\times1} = [M]_{3\times1}$$

Product of $[M]_{3\times1}$ and $[D]_{5\times3}$ is not possible since number of columns of matrix $M$ is not equal to number of rows of matrix $D$.

Therefore, Matrix product $[D]^T[F][D]$ is not possible.

Hence, statement II is false.

Hence, the correct option is (A).

**48. 2**

**Given :** Matrix $A$ is upper triangular matrix

Eigen value of $A = 1, i$

Eigen value of $A^{2024} = 1$, $A^{2024} = 1, 1$

Hence, trace $A^{2024} = a + d = 1 + 1 = 2$

Hence, the correct answer is 2.

**49. 0.021**

**Given :** $I = \int \dfrac{dx}{6|x|\sqrt{x^2-1} \times 6} = \dfrac{1}{36}\int \dfrac{dx}{|x|\sqrt{x^2-1}}$

$$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$$

So, $\quad I = \dfrac{1}{36} \times \sec^{-1}(x) + C = f(x)$

At $x = 1$,

$$f(1) = \frac{\sec^{-1} x}{36} + C = 0$$

$$C = 0$$

So, $\quad f(x) = \dfrac{\sec^{-1}(x)}{36} = f(\sqrt{2}) = \dfrac{\sec^{-1}(\sqrt{2})}{36}$

$$= \frac{\pi/4}{36} = \frac{\pi}{144}$$

Hence, the correct answer is 0.021.

**50. 0.33**

**Given** :

$$P(A \cup B^C) = P(A) + P(B^C) - P(A \cap B^C)$$

$$\frac{1}{2} = P(A) + [1 - P(B)] - P(A).P(B^c)$$

$$\frac{1}{2} = P(A) + \left(1 - \frac{3}{4}\right) - P(A)\left(1 - \frac{3}{4}\right)$$

$$\frac{1}{2} = P(A) + \frac{1}{4} - \frac{1}{4}P(A)$$

$$\frac{1}{4} = \frac{3}{4}P(A)$$

$$P(A) = \frac{1}{3} = 0.33$$

Hence, the correct answer is 0.33.

**51. 8**

**Given :** The given Limit can be solved by Leibnitz Rule

$$I = \lim_{x \to \frac{\pi}{4}} \frac{\int_2^{\sec^2 x} f(t)\,dt}{\left(x^2 - \frac{\pi^2}{16}\right)} = \frac{0}{0} \text{ form}$$

$$I = \lim_{x \to \frac{\pi}{4}} \frac{\frac{d}{dx}\left(\int_2^{\sec^2 x} f(t)\,dt\right)}{\frac{d}{dx}\left(x^2 - \frac{\pi^2}{16}\right)} = \frac{0}{0}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{2\sec^2 x \tan x \times f(\sec^2 x)}{2x}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{\sec^2 \frac{\pi}{4} \tan \frac{\pi}{4} \times f\left(\sec^2 \frac{\pi}{4}\right)}{\frac{\pi}{4}}$$

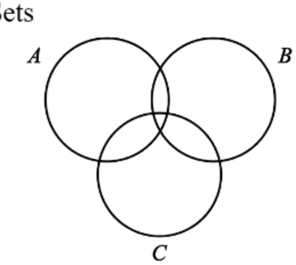
$$= \lim_{x \to \frac{\pi}{4}} \frac{\sec^2 \frac{\pi}{4} \tan \frac{\pi}{4} \times f\left(\sec^2 \frac{\pi}{4}\right)}{\frac{\pi}{4}}$$

$$= \frac{2f(2)}{\frac{\pi}{4}} = \frac{8f(2)}{\pi} = 8$$

Hence, the correct answer is 8.

**52. 0.44**

**Given :** Sets




$P$ (exactly one $A$ or $B$)

$$= P(A \cup B) - P(A \cap B)$$

$$= P(A) + \mathrm{P}(B) - 2P(A \cap B)$$

According to question,

$$P(A) + P(B) - 2P(A \cap B) = \frac{1}{4} \qquad \ldots\text{(i)}$$

$$P(B)+P(C)-2P(B\cap C)=\frac{1}{4} \quad \ldots\text{(ii)}$$

$$P(C)+P(A)-2P(A\cap C)=\frac{1}{4} \quad \ldots\text{(iii)}$$

On adding (i), (ii) and (iii)

$$2\begin{bmatrix} P(A)+P(B)+P(C)-P(A\cap B) \\ -P(B\cap C)-P(C\cap A) \end{bmatrix}=\frac{3}{4}$$

Also, $P(A\cap B\cap C)=\frac{1}{16}$

So, $P$ (at least one)

$$P(A\cup B\cup C)=\Sigma P(A)-\Sigma P(A\cap B) + P(A\cap B\cap C)$$

$$=\frac{3}{8}+\frac{1}{16}=\frac{7}{16}$$

Hence, the correct answer is 0.44.

**53. (A)**

**Given :** $PX=Q$

It is a non-homogenous equation.

So, for existence of at least one solution, the augmented matrix $[P:Q]$ must have the same rank as matrix $P$.

Hence, the correct option is (A).

**54. (D)**

**Given :** A $3\times 3$ real symmetric matrix $S$.

Two Eigen values and respective Eigen vector are :

$$\lambda_1=a\neq 0,\ X_1=\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\lambda_2=b\neq 0,\ X_2=\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

By the properties of real symmetric matrices,

$$[X_1]^T[X_2]=0$$

$$[x_1 \quad x_2 \quad x_3]\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}=0$$

$$x_1y_1+x_2y_2+x_3y_3=0$$

Hence, the correct option is (D).

**55. - 6**

**Given :** $A=\begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 4 \\ -1 & -1 & -2 \end{bmatrix}$

Eigen values are $\lambda_1=1$, $\lambda_2=-1$ and $\lambda_3=3$.

By Cayley Hamilton theorem, every square matrix satisfies its own characteristics equation.

The characteristic equation is given by,

$$|A-\lambda I|=0$$

$$AI=\lambda I \Rightarrow A=\lambda$$

The above expression shows that the values of $\lambda$ can be put in any expression of the matrix $A$.

For $\lambda_1=1$,

Eigen value of $A^3-3A^2$ is given by,

$$A^3-3A^2=1^3-3\times 1^2=-2$$

For $\lambda_2=-1$,

$$A^3-3A^2=(-1)^3-3\times(-1)^2=-4$$

For $\lambda_3=3$,

$$A^3-3A^2=3^3-3\times 3^2=0$$

So, trace of matrix $(A^3-3A^2)$

$$=\text{Sum of Eigen values}$$

$$=(-2)+(-4)+0=-6$$

Hence, the trace of $(A^3-3A^2)$ is – 6.

**56. (A)**

**Given :** Box contains 8 Red balls and 8 Green balls.

Two balls are drawn randomly in succession without replacement.

$\therefore$ Probability of first ball red and second ball green is

$$\frac{8}{16}\times\frac{8}{15}=\frac{4}{15}$$

Hence, the correct option is (A).

**57.** **(A), (B), (C), (D)**

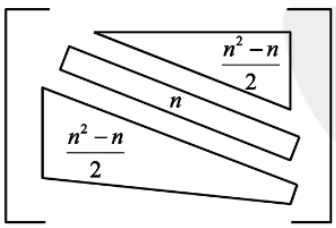
**Given :** Matrix $A_{2\times 2} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

Since $A$ is symmetric i.e. $a_{ij} = a_{ji}$

So, $a_{12} = a_{21}$, we have three possible places positions containing either 0 or 1 and it can be filled as,

$$[a_{11}] \quad [a_{21} \ \& \ a_{12}] \quad [a_{22}]$$
$$(0 \text{ or } 1) \quad (0 \text{ or } 1) \quad (0 \text{ or } 1)$$
$$2 \text{ ways} \times 2 \text{ ways} \times 2 \text{ ways} = 2^3 \text{ ways}$$

So, the total number of distinct symmetric matrix of order $2\times 2$ with each element being 0 or $= 2^3 = 8$ ways for $n\times n$ matrix




Total number of elements $= n^2$

Total number of elements $= n$

For symmetry, total possible positives $= \dfrac{n^2-n}{2}$

Total number of possible positions to be filled by either 0 or 1 $= \dfrac{n^2-n}{2} + n = \dfrac{n^2+n}{2}$

So, total number of ways to fill these positions with 0 or 1

$$= 2\times 2\times 2\times .........\times 2\left\{\frac{n^2-n}{2} \text{ times}\right\}$$

$$= (2)^{\frac{n^2+n}{2}}$$

For $n = 4 : 2^{\frac{16+4}{2}} = 2^{10}$

For $n = 8 : 2^{\frac{64+8}{2}} = 2^{36}$

For $n = 3 : 2^{\frac{9+3}{2}} = 2^{6}$

For $n = 6 : 2^{\frac{36+6}{2}} = 2^{21}$

Hence, the correct options are (A), (B), (C) and (D).

**58.** **(A)**

**Given :** A matrix is defined as, $A = [a_{ij}]_{n\times n}$

where, $a_{ij} = \begin{cases} i, & i = j \\ 0, & \text{Otherwise} \end{cases}$

Thus, all the elements except diagonal are zero and diagonal elements are given by,

$$a_{11} = 1, a_{22} = 2, a_{33} = 3, ....., a_{nn} = n$$

$$A = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 2 & 0 & \cdots & 0 \\ 0 & 0 & 3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & n \end{bmatrix}_{n\times n}$$

The sum of all elements is given by the sum of its main diagonal elements.

$$= (1 + 2 + 3 + ..... + n) = \frac{n(n+1)}{2}$$

Hence, the correct option is (A).

**59.** **3**

**Given :** A probability density function is as given below,

$$f(x) = \frac{e^{-\frac{x}{3}}}{G} \text{ and } x \in [0, \infty)$$

By the property of probability density function,

$$\int_{-\infty}^{\infty} f(x)\,dx = 1 \quad \Rightarrow \quad \int_0^{\infty} \frac{e^{\frac{-x}{3}}}{G}\,dx = 1$$

$$\frac{1}{G}\int_0^{\infty} e^{\frac{-x}{3}}\,dx = 1$$

$$-3\left[e^{\frac{-x}{3}}\right]_0^{\infty} = G$$

$$-3(e^{-\infty} - e^0) = G$$

$$K = 3$$

Hence, the value of constant $G$ is **3**.

**60.** **(D)**

**Given :** $f(y)=\lim_{y\to 0}\frac{1}{y}\int_0^y f(x)\,dx$

Let, $g(y)=\int_0^y f(x)dx$

Differentiating $g(y)$ with respect to $y$,

$$\frac{d}{dy}g(y)=f(y) \qquad \ldots\text{(i)}$$

Then, $f(y)=\lim_{y\to 0}\frac{g(y)}{y} \qquad \ldots\text{(ii)}$

$$f(y)=\frac{g(0)}{0}$$

where, $g(0)=g(y)\big|_{y=0}$

$$g(0)=\int_0^{y=0} f(x)dx=0$$

So, $f(y)=\frac{0}{0}$

It is in the form of $\left(\frac{0}{0}\right)$, so applying L-Hospital's rule,

$$f(y)=\lim_{y\to 0}\frac{\frac{d}{dy}g(y)}{\frac{d}{dy}(y)}$$

From equation (i),

$$f(\mathrm{y})=\lim_{y\to 0}\frac{f(y)}{1}$$

$$f(y)=f(0)$$

Hence, the correct option is (D).

❖❖❖❖