5 Algorithm

Practice Questions

Q.1 Consider a job scheduling problem with 6 jobs J_1, J_2, J_3, J_4, J_5 and J_6 with corresponding deadlines $(d_1, d_2, d_3, d_4, d_5, d_6) = (6, 4, 3, 5, 4, 3)$.

Which of the following schedules are feasible without violating any job schedule? **[MSQ]**

(A) $J_1, J_2, J_3, J_4, J_5, J_6$

(B) $J_3, J_6, J_2, J_5, J_4, J_1$

(C) $J_6, J_5, J_3, J_2, J_4, J_1$

(D) $J_6, J_5, J_4, J_3, J_2, J_1$

Q.2 A edTech Company uses a compression technique to encode the message before transmitting over the network. Suppose the message contains the following string. Message $=$ "gate academy"

Each character in input message takes 1 byte. If the compression technique used is Huffman coding. The number of bits saved in the message is________.

Q.3 Which of the following pair of algorithms are Greedy algorithms? **[MSQ]**

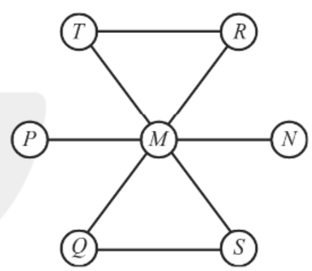
(A) Prism's and Kruskal's

(B) Huffman coding and optimal merge pattern.

(C) Dijkshtra and Bellman ford

(D) Dijkshtra and floyd warshall.

Q.4 Breath First Search (BFS) has been implemented using queue data structure. **[MSQ]**



Which of the following is/are not possible sequence of BFS.

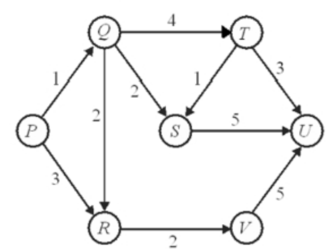
(A) M, N, P, R, T, Q, S

(B) N, M, P, Q, S, T, R

(C) Q, S, N, M, P, T, R

(D) R, T, Q, P, N, M, S

Q.5 Suppose we run Dijkstra's single source shortest-path algorithm on the following edge weighted directed graph with Vertex. P as the source. In what order do the nodes get included into the set of vertices to reach 'U' Vetex. For which the shortest path distances are finalized?



(A) P, Q, S, T, U

(B) P, Q, S, U

(C) P, Q, T, U

(D) P, Q, R, V, U

Q.6 Consider the above question the shortest path selected by the Dijkstra algorithm having minimum cost of _____.

Q.7 Consider the string *pqrrrqqpsppeerep* each letter in the string must be assigned a binary code.

Satisfying the following properties:

I. For any two letters, the code assigned to one letter must not be prefix of the code assigned to the other letter.

II. For any two letter of the same frequency, the letter which occurs earlier in the dictionary order is assigned a code whose length is at most the length of the code assigned to the other letter

The minimum length of encoded string "peers" is ________.

Q.8 Consider the question 7, which of the following is the binary encoded code for "Peers". **[MSQ]**

(A) 11 101 101 01 100

(B) 11 100 100 101 01

(C) 00 100 100 11 01

(D) 00 0100 10 100 11

Q.9 Consider the following two sequences :

$P = <M, N, O, N, R, M, N>$

$Q = <N, R, O, M, N, M>$

The length of longest common subsequence of P and Q is _____.

Q.10 Consider the following two sequences.

$P = < M, N, O, N, R, M, N >$

$Q = < N, R, O, M, N, M >$

The number of largest common length subsequence is 'a',

The number of smallest common two length subsequences is 'b'.

Then $b + a =$ ____________.

Q.11 What is the time complexity f following code

```
gateacademy()
{
   int i = 1, s =1
  while (s <= n)
{
  i++j
  s = s + i;
  printf ("gate academy");
}
}
```

Q.12 What is complexity of following recurrence relation?

$$T(n) = \sqrt{n}T\left(\sqrt{n}\right) + 100n$$

(A) $\theta(n \log \log n)$

(B) $\theta(n \log n \log n)$

(C) $\theta(n)$

(D) $\theta(n \log n)$

Q.13 Gate Academy contains 500 employees whose monthly salary is stored in the form of array and assume every employees salary is based on performance means each employee has distinct salary. Now Gate Academy wants to find min and max salary crediting to employee for which minimum number of salary comparisions needed is ________.

Q.14 A array contains elements which can be changed dynamically (periodic) then which of the following algorithm gives surety that time complexity of sorting array element will never change?

(A) Insertion sort (B) Bubble sort

(C) Merge sort (D) Quick sort

Q.15 Let $f(n)=n^{1+\cos x}$ and $g(n)=n^{1+\sin x}$ where n is a positive number. Which of the following is/are correct

(I) $f(n)=0(g(n))$

(II) $f(n)=\Omega(g(n))$

(III) $f(n)=0(g(n))$

(IV) $f(n)=\Omega(g(n))$

(A) I and II

(B) III and IV

(C) All are correct

(D) All are incorrect

Q.16 Consider the equality **[MSQ]**

$X=\sum_{i=0}^{n^2} i^2+i$ and the choices for X are,

I. $0(n^6)$ II. $0(n^5)$

III. $\Omega(n^4)$ IV. $\Omega(n^6)$

(A) I (B) II

(C) III (D) IV

Q.17 An unordered list contains n distinct elements which is stored in array. The number of minimum comparisions required to find element in array is not a minimax is_________.

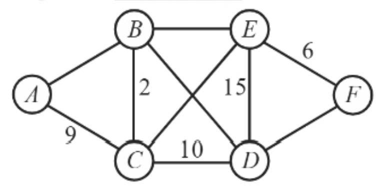
(A) $\theta(\log n)$ (B) $\theta(n \log n)$

(C) $\theta(n)$ (D) $\theta(1)$

Q.18 The graph shown below 8 edges with distinct integer edge weight. The minimum spanning tree (MST) is of weight 42 and contains the edges $\{(A,C),(B,C),(C,D),(D,E),(E,F)\}$.

The edge weights of only those edges which are in the MST are given in the figure shown.

The minimum possible sum of weights of remaining edges (other than MST edges) is _________.



Q.19 A BST contains n elements in the tree if further n elements are inserted in the BST then worst case complexity is _____

(A) $\theta(n\log n)$ (B) $0(n^2)$

(C) $0(n)$ (D) $0(n^2 \log n)$

Q.20 Which one of the following is the recurrence equation for the worst case time complexity of Quick sort algorithm for sorting n elements. Consider pivoted element is last element of array.

(A) $T(n)=2T\left(\frac{n}{2}\right)+C\cdot n$

(B) $T(n)=2T(n-2)+C\cdot n$

(C) $T(n)=T\left(\frac{n}{2}\right)+C\cdot n$

(D) $T(n)=T(n-1)+C\cdot n$

Q.21 The minimum number of scalar multiplication required for parenthesized of a matrix chain product whose sequence of dimensions for four matrices is :

$<5,7,4,10,12>$

Q.22 For the above given matrix chain multiplication which of the following parenthesis is selected for the minimum number of scalar multiplications?

(A) $(A(B(CD)))$ (B) $(((AB)C)D)$

(C) $((AB)(CD))$ (D) $(A(BC)D)$

Q.23 Let $T(n)$ be the function defined by $T(1)=1,\ T(n)=2T(\lfloor n/2 \rfloor)+\sqrt{n}$ for $n \geq 2$.

Which of the following statements is true?

(A) $T(n)=O(\sqrt{n})$ (B) $T(n)=O(n)$

(C) $T(n)=O(\log n)$ (D) None of these

Q.24 Consider a given array, 8, 3, 4, 6, 7, 2, 1, 9,5 performed quick sort to sort array by considering last element as pivot. Which of the following will be the correct array after performing pivoting on 5?

(A) 1, 2, 3, 4, 5, 6, 7, 8, 9

(B) 1, 2, 3, 4, 5, 6, 9, 8, 7

(C) 3, 4, 2, 1, 5, 8, 6, 9, 7

(D) 1, 3, 2, 4, 5, 6, 7, 9, 8

Q.25 Which of the following sorting algorithm is used to sort largest element of the unsorted array or to find next largest element in each iteration?

Note : We need to arrange elements in increasing order.

(A) Bubble sort (B) Selection sort

(C) Insertion sort (D) Quick sort

Q.26 Which of the following is the correct pair of the time complexity with algorithm in undirected graph with given weight to find shortest path.

Algorithm	**Time Complexity**
1. Bellman ford	a. $E\log V$
2. Floyd warshall	b. EV
3. Dijkshtra	c. V^3
4. Breadth first search	d. V^2
	e. $(E+V)$

(A) 1-b, 2-c, 3-a, 4-d

(B) 1-d, 2-c, 3-a, 4-e

(C) 1-c, 2-b, 3-a, 4-e

(D) 1-a, 2-c, 3-b, 4-d

Q.27 Optimal merge pattern algorithm used then minimum number of comparisons is______________.

Files	a	b	c	d	e	f
Records	30	5	10	8	15	20

Q.28 Consider the 15×15 upper triangular matrix whose elements are stored in an array having starting index '0'. The position of array (7, 9) is ______.

Q.29 Match the following

A. Prims

B. Quick sort

C. Bellman ford

D. Subset sum problem

1. Divide and conquer
2. Dynamic programming
3. Recursion
4. Greedy algorithm
5. Backtracking

(A) A-4, B-1, C-2, D-4

(B) A-4, B-1, C-2, D-2

(C) A-3, B-1, C-4, D-4

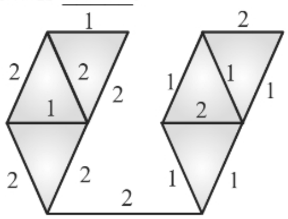
(D) A-2, B-3, C-4, D-2

Q.30 Let G be connected undirected graph of 50 vertices and 150 edges. The weight of a minimum spanning tree of G is 250. When the weight of each edge of G is increased by five, the weight of a minimum spanning tree becomes ______.

Q.31 Consider a complete undirected graph with vertex set $\{1, 2, 3, 4, 5\}$. Entry W_{ij} in the matrix W below is the weight of the edge $\{i, j\}$. The minimum possible weight of a spanning tree T in this graph such that vertex 1 is a leaf node in the tree T is ____________.

$$W = \begin{bmatrix} 0 & 1 & 6 & 1 & 3 \\ 1 & 0 & 10 & 4 & 7 \\ 6 & 10 & 0 & 5 & 2 \\ 1 & 4 & 5 & 0 & 2 \\ 4 & 7 & 3 & 2 & 0 \end{bmatrix}$$

Q.32 The number of distinct minimum spanning tree for the weighted graph below is ______.



Q.33 An undirected graph G has n nodes. Its adjacency matrix is given by an $n \times n$ square matrix whose (i) diagonal elements are 0's and (ii) non-diagonal elements are distinct. Let e_{max} be the edge with maximum weight and e_{min} be the edge with minimum weight. Which of the following is/are correct?

[MSQ]

(A) Graph G has multiple minimum spanning trees of same cost $(n-1)$

(B) Every minimum spanning tree of G must contain e_{min}.

(C) If e_{max} is in a minimum spanning tree, then its removal must disconnect G.

(D) No minimum spanning tree contains e_{max}.

Q.34 $G = (V, E)$ is an undirected graph with distinct positive edge weights. If every edge weight is increased by same value, and e be the particular edge of G which of the following statement is/are True regarding minimum spanning tree.

[MSQ]

(A) Minimum spanning tree of G does not change

(B) If e is the minimum weighted edge of some cycle in G then every MST of G includes e.

(C) Shortest path between any pair of vertices does not change.

(D) If e is the maximum weighted edge of some cycle in G then every MST of G excludes e.

Q.35 If algorithm X is having better asymptotic time complexity than algorithm Y, then which of the following is/are TRUE? (Mark all choices which are CORRECT)

(A) X will outperform Y for all inputs

(B) X will outperform Y for all small inputs

(C) X will outperform Y for all large inputs

(D) X will outperform Y for all inputs above 1 million in size

Q.36 Suppose you are given the following set of keys to insert into a hash table that holds exactly 10 values: 13,107,49,50,64,98,16,33. Which of the following best demonstrates the contents of the hash table after all the keys have been inserted using linear probing and mod 10 as hash function

(A) 50….13 64 33 16 107 98 49

(B) 50 33….13 64 ….. 16 107 98 49

(C) 50 …… 33 13 64 …… 16 107 98 49

(D) 50…….33 64 13 16 107 98 49

Q.37 An array of 8 integers is being sorted by the heapsort algorithm. After the initial phase of the algorithm (constructing the heap), which of the following is a

possible ordering for the array? (Mark all the appropriate choices) **[MSQ]**
(A) 2 6 11 13 21 41 43 92
(B) 2 13 6 43 92 11 21 41
(C) 2 6 11 41 92 13 21 43
(D) 2 6 11 21 11 41 43 13

Q.38 Which of the following is/are TRUE about Huffman Coding? (Mark all the appropriate choices) **[MSQ]**
(A) Huffman coding may become lossy in some cases
(B) Huffman Coding does not always have an exact solution
(C) In Huffman coding, no code is prefix of any other code
(D) There exists a greedy algorithm to do Huffman coding

Q.39 Consider the following functions **[MSQ]**

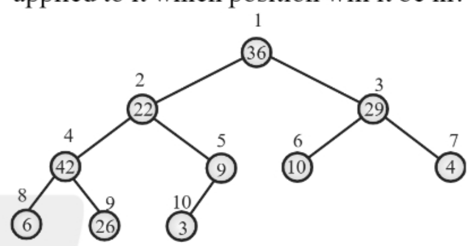
$\log_{10} n, \log_2 n, \lceil \log_2 n \rceil, \lfloor \log_2 n \rfloor$

Which of the following is/are TRUE? (Mark all the appropriate choices)
(A) $\log_{10} n = O(\log_2 n)$
(B) $\log_2 n = \Omega \lfloor \log_2 n \rfloor$
(C) $\lceil \log_2 n \rceil = O(\lfloor \log_2 n \rfloor)$
(D) $\lceil \log_2 n \rceil = \Omega(\lfloor \log_2 n \rfloor)$

Q.40 Consider a min heap implemented as an array with the elements 1, 2, 3, 6,7,8, 9,10. The level (starting from 1) in which 7 is stored is _______.

Q.41 It is required to sort a large number large number of records of employee details based on the employee IDs. Assuming the records are already sorted on the basis of employee names and it is required to maintain name as the secondary sort order (similar to order by id name in SQL), which of the following sorting algorithms is/are the most appropriate for this?
(A) Merge Sort
(B) Selection Sort
(C) Quick Sort
(D) Heap Sort

Q.42 Consider the binary tree shown below, which is an almost max-heap with the node 22 violating the max -heap property. Once heapify procedure is applied to it which position will it be in?



Q.43 In a hash table with 20 slots 30 records are inserted with collisions being resolved by chaining what is the expected number of key comparisons in an unsuccessful search assuming uniform hashing?

Q.44 Which of the following is/are the possible array contents after second pass of Quick Sort for the following initial ordering assuming first element is taken as pivot?

34, 8, 64, 51, 32, 21
(A) 8, 21, 32, 34, 51, 64
(B) 8, 8, 32, 34, 51, 64, 21
(C) 8, 34, 51, 64, 32, 21
(D) 8, 34, 64, 51, 32, 21

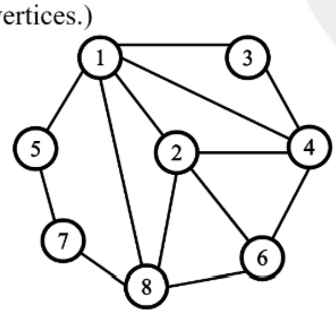
Q.45 Given an array of n elements, you have to design an algorithm to find the k smallest elements in sorted order. The time complexity of the best algorithm assuming comparison based sorting will be
(A) $O(k \log n)$
(B) $\theta(n + k \log k)$
(C) $\theta(n^2)$
(D) $\theta(n \log k)$

Q.46 Suppose that you store 6 records in a hash table of size 8 by chaining, and suppose that you have a good hash function so that the probability that a key is hashed into any of the 8 slots is 1/8. For a particular slat in the hash table, what is the probability that this slot is empty, that is, none of the 6 keys hashes into this slot?

(A) 0.125^6 (B) 0.875^6

(C) 0.166^8 (D) 0.833^8

Q.47 Consider the following graph. If BFS is implemented on the following graph then which of the following set of the nodes are present in the queue after performing 5th dequeue operation? (Root vertex is 1 and consider descending order while visiting the vertices.)



(A) 2, 7, 6 (B) 2, 3, 4

(C) 3, 4, 6 (D) 8, 7, 6

Q.48 Consider the following C function and an input array to it as [1, 4, 10, 12] and len as 4. The return value of the function will be

```
int foo (int arr [], int len)
{
int result = 0 ;
for (int i = 0; i < len; ++)
{
result + = arr [i] * (i+1)* (len – i);
}
return result:
```

Q.49 You are given an empty hash table of size 6 that uses open addressing. The following sequence of keys is to be inserted: **[MSQ]**

32 7 14 22 29 3

If linear probing with $h(x) = x\%6;$ is used, which all elements will cause a collision? (Mark all choices which are CORRECT)

(A) 3 (B) 14

(C) 29 (D) 22

Q.50 Consider the following C function

```
int findme (int n)
{
if (n < = 0) return 1;
return findme (n/2) + findme (n-1)
}
```

What will be the output if the above function is called with argument 12 ?

Q.51 For quick sort pivot selection you are provided with an buggy implementation of Median select which can return any element between (n/3)rd smallest and (2n/3)rd smallest in O(n) time. With this function to choose the pivot, which of the following is/are TRUE for the Quicksort implementation? (Mark all CORRECT choices) **[MSQ]**

(A) In best ease it can run in $O(n)$ time

(B) In worst case it will take $\theta(n^2)$ time

(C) In worst case it will take $\theta(n\log n)$ time

(D) In best case, It will run in $\Omega(n\log n)$ time

Q.52 What is the time complexity of following code if foo(n) takes $\theta(\log n)$ time to execute?

```
int i , j;
for (i = n / 2; I < = n; i++)
{
```

```
for (j = 1; j <= n; j = j * 2)
{
foo (j);
}
}
```

(A) $\theta(n)$ (B) $\theta(n\lg n)^2$

(C) $\theta(n(\lg n)^2)$ (D) $\theta(n^2 \lg n)$

Q.53 The return value, time complexity and space complexity of the following code are

```
int foo (int n)
{
    int a = 0
    while (n > 0)
{
    a + = n;
    n/=2;
}
    return a;
}
```

(A) $\theta(\log n,), \theta(\log n), \theta(\log n)$

(B) $\theta(n), \theta(\log n), O(1)$

(C) $\theta(\log n), \theta(\log n), O(1)$

(D) $\theta(n^2), O(n), O(1)$

Q.54 Suppose you are having 5 notes each with denominations of 2000, 500,300 and 50 and you are asked to make a sum of Rs 4650. For each note you choose you are asked to pay a service charge of Rs. 2.5 which is irrespective of the denomination of the note, what will he the minimum amount in rupees you'll have to pay as service charge?

Q.55 Which of the following statements is/are TRUE? (Mark all CORRECT choices)

(A) The expected number of comparisons in a successful linear search is n

(B) The expected number of comparisons in a successful binary search is $O(\log n)$

(C) The best case runtime of linear search is asymptotically better than binary search

(D) In worst case binary search can lead to $\Omega(n)$ comparisons

Q.56 Suppose the letters *a, b, c, d* have probabilities 1/3, 1/6, 1/6, 3/10 respectively. What is the average length of Huffman codes (correct to 1 decimal point)?

Q.57 You are given an empty hash table of size 7 that uses open addressing. The following sequence of keys is to be inserted:

13 18 25 11 20 29

If hash function $h_1(x)\%7$; and double hashing with $h_2(x) = x/7+1$ is followed, which of the inserted elements causes a collision? (Mark all the appropriate choices) **[MSQ]**

(A) 25 (B) 11

(C) 20 (D) 29

Q.58 Which of the following statements is/are TRUE? (Mark all the appropriate choices) **[MSQ]**

(A) The worst case runtime for searching in a binary tree can be O(n).

(B) The minimum number of comparisons to sort 5 elements is 4.

(C) There can be at most $\left\lceil \frac{n}{2^{h-1}} \right\rceil$ nodes of height h (h starting from 1) in any n element heap.

(D) The time to find the maximal element in a min-heap of $2^n - 1$ elements is $\Omega(2^n)$

Q.59 Four matrices M_1, M_2, M_3 and M_4 of dimensions $p \times q, q \times r, r \times s$ and $s \times t$ respectively can be multiplied is several ways with different number of total scalar multination's. For example, when multiplied as $((M_1 \times M_2) \times (M_2 \times M_4))$, the total number of multiplications is $pqr + rst + prt$. When multiplied as $(((M_1 \times M_2) \times M_3) \times M_4)$, the total number of scalar multiplications is $pqr + prs + pst$.

If $p = 20, q = 90, r = 10, s = 15$ and t = 40, then the minimum number of scalar multiplications needed is

Q.60 Maximum Subarray Sum problem is to find the subarray with maximum sum. For example, given an array {12, –13, –5, 25, –20, 30, 10}, the maximum subarray sum is 45. The best possible algorithm to compute the maximum subarray sum will run in

(Mark all the appropriate choices)

(A) $O(n)$ (B) $\Omega(n \log n)$

(C) $O(\log n)$ (D) $O(n^2)$

Q.61 During solution of $T(n)$ recurrence relation we find the following series, solve it and find the value of $T(n)$.

$$T(n) = 1.2^1 + 2.2^2 + 3.2^2$$
$$+4.2^4 + 5.2^5 + 6.2^6$$
$$+7.2^7 + \ldots n.2^n$$

(A) $O(2^n)$ (B) $O(2^n)$

(C) $O(n.2^n)$ (D) $O(n^2.2^n)$

Q.62 Consider the code given below, which runs insertion sort;

```
void insertion Sort (int arr [], int size)
{
 int i, j, value
for (i = 1; i < size; i++)
{
value = arr [i];
j = i;
while (________)
{
arr [j] = arr [j - 1];
J = j – 1;
}
arr [j] = value
}
}
```

Which condition will correctly implement the which loop?

(A) (j > 0) || (arr [j – 1] > value)

(B) (j > 0) && (arr [j – 1] > value)

(C) (j>0) && (arr[j+1]) > value)

(D) (j>0)&&(arr[j+1]<value)

Q.63 Consider the following C function for arguments m, n > 1

```
int foo (int n, int m)
{
    While (m! = n)
{
    if (m>n)
    m = m – n;
    else
    n = n – m;
}
    return n;
}
```

Which of the following is/are true about the above function? (Mark all CORRECT choices)

(A) If m = 9 , n = 13 the function returns 1.

(B) The asymptotic time complexity of the function is O(log(max(m,n)))

(C) For any positive integers m, n the function returns the least common multiple of m, n.

(D) The space complexity of the function is $\Omega(1\text{og})(\min(m,n))$

Q.64 Which of the following statements is/are TRUE? (Mark all CORRECT choices}

(A) When 90% of the input is already in sorted order, insertion sort will do less number of comparisons than merge sort for all sufficiently large inputs

(B) If the number of inversions in an array is O(n), then insertion sort will be having fewer comparisons than quick sort for all inputs

(C) If the number of inversions in an array is O(n), bubble sort will be having fewer number of comparisons than merge sort for all inputs

(D) If the number of inversions in an array is O(n), heap sort will be having fewer number of comparisons than insertion sort for all inputs

Q.65 Consider the following recurrence relation which is applicable on two arrays x and y, x_i and y_i, are the i^{th} elements of x and y array respectively.

$$c(i,j)=\begin{cases} 0 \\ c(i-1,j-1)+1 \\ \min(c(i-1,j),c(i,j-1)) \end{cases}$$

if $i=0$ or $j=0$

if $i,j>0$ and $x_i=y_i$

if $i,j>0$ and $x_i\neq y_i$

Suppose each array is of n size and time complexity to compute c (n,n) using dynamic programming is $O(n^a(\log n)^b)$ then what will be value of a + b?

Q.66 You're working on a dynamic programming problem that has a recurrence elation $A(i,j)=F(A(\lfloor i/2\rfloor,j)A,(i,\lfloor j/2\rfloor))$ where F is a known function that can be evaluated in constant time and $A(i,j)=0$ when $i=0$ or $j=0$. To compute $A(m,n)$ for some m and n, you can use either a top – down or a bottom – up method. Which one is more efficient in solving this problem?

(A) Top Down will take asymptotically lesser time than bottom up

(B) Bottom-up will take asymptotically lesser time than Top Down

(C) Both will take the same time

(D) None of them

Q.67 In bottom-up dynamic programming, we need an order to fill m the solution cells m a table, such that all needed subproblems are solved before solving a subproblem. For the following relation, give a valid traversal order.

$A(i,j)=F(A(i,j-1),A(i-1,j-1)$

$A(i-1,j+1))$

$A(0,j)=0$

$A(i,0)=0$

Where F(.) is a function.

(A) Fill values of $A(i,\ j)$ column-wise, i.e., fill the first column then the second column, and so on.

(B) Fill the last column of $A(i,j)$. then the second last column, and so on.

(C) Fill values of $A(i,j)$ row-wise i.e. fill the first row then the second row, and so on.

(D) No order is possible as there Is cyclic dependency in subproblems.

Q.68 Consider the following code snippet :

```
int i = 0;
while (i < n)
{
    printf ("computer science batch1");
    while (i < n)
{
    printf ("computer network");
    while (i < n)
{
    printf ("compiler design");
    i++;
}
    i++;
}
    i++;
}
```

What will be the time complexity of above code snippet?

(A) $\theta(n^5)$ (B) $\theta(n^6)$

(C) $\theta(n^4)$ (D) $\theta(n^3)$

Q.69 While walking on the beach one day you find a treasure trove. Inside there are n treasures with weights w_1.., w_n and values v_1....v_n. Unfortunately, you have a knapsack that only holds a total weight M. Fortunately, there is a knife handy so that you can cut treasure if necessary to take home treasure of maximal value: a cut treasure retains its fractional value (so, for example a third of treasure I has weight w_i / 3 and value v_i/3). What is the maximum value that can take home if we have 3 treasures with weights (18, 15, 10) and values (25, 24, 15) and also a knapsack of total weight 20?

Q.70 Consider G (V, E) be a complete undirected graph with 6 edges having a distinct weight from 1, 3, 9, 27, 81 and 243. Which of the following will not be the weight of the minimum spanning tree of G? **[MSQ]**

(A) 121 (B) 13

(C) 40 (D) 31

Q.71 Consider the following 0 -1 knapsack problem with the item's weight and value given in the table

Item	Weight	Value
1	3	Rs 25
2	2	Rs 20
3	1	Rs 15
4	4	Rs 40
5	5	Rs 50

Capacity W = 6

If the capacity of the knapsack (W), is 6 then how many optimal solutions (number of set of items) are possible?

(A) 1 (B) 2

(C) 3 (D) 4

Q.72 For a given sequence of integers $a_1, a_2, ..., a_n$ decreasing subsequence is one for which every integer is strictly smaller than the previous one.

The longest decreasing subsequence is the longest among all decreasing subsequences of a given sequence.

For example, decreasing subsequences of 5, 2, 4, 8,3, 40 are .

- 5, 2
- 5, 4
- 4, 3
- 5, 4, 3 etc

And longest among all possible decreasing subsequences is 5, 4, 3.

For a given sequence of integers $a1, a2, ...a_n$ let L(j) be length of the longest decreasing subsequence that starts with a_j.

$L(n)=1$

$$L(j)=\begin{cases}1 & \text{if } a_j=\min(a_j,a_{j+1},a_{j+2},..,a_n)\\ ? & \text{otherwise}\end{cases}$$

Complete the above recurrence relation with the appropriate value of ?

(A) 1 + min{L(k):j < k ≤ n and $a_j > a_k$ }

(B) 1 + max {L(k) : j < k ≤ n and $a_j > a_k$}

(C) 1 + max {L(k) : j < k ≤ n and $a_j < a_k$}

(D) 1+ min {L(k): j < k ≤ n and $a_j > a_k$ }

Q.73 Consider a DAG G = (V, E) which has topological ordering of vertices $v_1, v_2, ..., v_n$. We want to count number of paths from vertex v_1 to vertex v_n. Let Paths(i) represents total number of paths from i to n.

Complete the following recurrence for Paths (i).

$$\text{Paths(i)}=\begin{cases}1 & \text{if } i=n\\ ? & \text{otherwise}\end{cases}$$

In options, $\Sigma_{j:(i,j)\in E}$ Paths (j) is a quantity which sums Paths (j) for all outgoing edges from i i.e., for edges $i \to j$

(A) $\Sigma_{j:(i,j)\in E}$ (1+Paths (j))

(B) $\Sigma_{j:(i,j)\in E}$ Paths (j)

(C) $1+\Sigma_{j:(i,j)\in E}$ Paths (j)

(D) 1+Paths (j) for some edge $i \to j$

Q.74 Find total number of scalar multiplications of a matrix-chain product of 6 matrices whose sequence of dimensions is 5,10,3, 12,5,50 and 6. That is 5×10 is dimension of first matrix, 10×3 is dimension of second matrix and so on.

Q.75 The number of longest common subsequences for "bacb" and 'abcabc' are:

(A) 2 (B) 3

(C) 4 (D) 5

Q.76 For the given recurrences calculate the tightest bound on time and space that would be required by a dynamic programming algorithm to compute OPT(n).

$$\text{OPT}(i)=\min_{1\le j<i}\{\text{OPT}(j)/j+w(j)\}$$

where OPT(l) = 1

(A) Time = $O(n^2)$, space = $O(n)$

(B) Time = $O(n^3)$, Space = $O(n^2)$

(C) Time = $O(n^2)$, Space = $O(n^2)$

(D) Time = $O(n)$, Space = $O(n)$.

Q.77 Suppose you have a row of coins with values that are positive integers $c_1, ..., c_n$. These values might not be distinct. Your task is to pick up coins that have as much total value as possible, subject to the constraint that you don't ever pick up two coins that lie beside each other. For example - Given c_1 to c_6 as follows- (5, 1, 2, 10, 6, 2), the maximum value is 17 and using coins $\{c_1=5, c_1=10, c_6=2\}$

Let $f(n)$ be maximum total value with n coins with $f(0)=0$ and $f(1)=c_1$ then what will be correct recurrence for $f(n)$?

(A) $f(n)=\max(c_n+f(n-2), f(n-1))$

(B) $f(n) = \max(c_{n-1} + f(n-2), f(n-1))$

(C) $f(n) = \max(c_{n-1} + f(n-1), f(n-2))$

(D) $f(n) = \max(c_n + f(n-1), f(n-2))$

Q.78 Consider the following code snippet. It accepts a positive integer n as input,

```
int i = 0, j = 0, val = 1;
for (i = 1, i < = n; i++){
        j = n;
      if (i % 2 = = 0){
      while (j >1) {
        val = val * j;
      j = j/2
                    }
    }
}
```

What is the worst-case time complexity of the algorithm?

(A) $O(n)$ (B) $O(n\log n)$

(C) $O(n^2)$ (D) $O(n^2 \log n)$

Q.79 The time required by an efficient algorithm to determine whether an arbitrary array of size n is min-heap or not is

(A) $O(\log n)$ (B) $O(n)$

(C) $O(n\log n)$ (D) None of these

Q.80 What is time complexity of matrix multiplication (Strassen's) using divide and conquer approach?

(A) $O(n^{\log_2 5})$ (B) $O(n^{\log_2 6})$

(C) $O(n^{\log_2 7})$ (D) None of these

Q.81 Consider the following steps :

S_1 : Characterize the structure of an optimal solution.

S_2 : Compute the value of an optimal solution in bottom-up fashion.

Which of the following step (s) is/are common to both dynamic programming and Greedy algorithm?

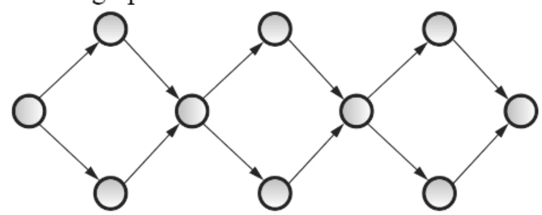
(A) Only S_1

(B) Only S_2

(C) Both S_1 and S_2

(D) Neither S_1 or S_2

Q.82 Consider the directed acyclic graph given below. How many topological ordering are possible for the given graph?



Q.83 Match the following and choose the correct solution for the order A,B,C,D

A	Strassen matrix multiplication	p	Decrease and Conquer
B	Insertion sort	q	Dynamic programming
C	Gaussian Elimination	r	Divide and conquer
D	Floyd shortest path algorithm	s	Transform and Conquer

(A) r,s,p,q (B) r,p,s,q

(C) q,p,s,q (D) s,p,q,r

Q.84 Consider the following functions:

$f(n) = 2^n, g(n) = n!, h(n) = n^{\log n}$

Which of the following statements about the asymptotic behavior of $f(n)$, $g(n)$ and $h(n)$ is true?

(A) $f(n) = O(g(n)); g(n) = O(h(n))$

(B) $f(n) = O(g(n)); g(n) = O(h(n))$

(C) $g(n) = O(f(n)); h(n) = O(f(n))$

(D) $h(n) = O(f(n)); f(n) = O(g(n))$

Q.85 The weight of a sequence $a_0, a_1, \ldots, a_{n-1}$ of real numbers is defined as $a_0+a_1/2+\ldots+a_{a-1}/2^{n-1}$. A subsequence of a sequence is obtained by deleting some elements from the sequence, keeping the order of the remaining elements the same. Let X denote the maximum possible weight of a subsequence of $a_0, a_1, \ldots, a_{n-1}$ and Y the maximum possible weight of a subsequence of $a_1, a_2, \ldots, a_{n-1}$. Then X is equal to

(A) max(Y, a_0+Y)

(B) max(Y, a_0+Y/2)

(C) max(Y, a_0+2Y)

(D) a_0+Y/2

Q.86 Consider the following C code segment:

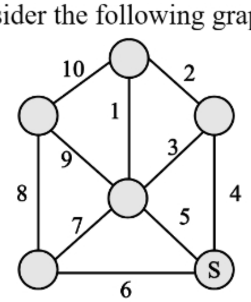
```
int f(int x)
{
    if (x < 1) return 1;
    else return (f(x-1) + g(x));
}
int g(int x)
{
  if (x < 2) return 2;
  else return (f(x-1) + g(x/2));
}
```

Of the following, which best describes the growth of $f(x)$ as a function of x?

(A) K_1x+K_2 (B) K^x

(C) Kx^2 (D) Kx^3

Q.87 Consider the following graph.



Assume that node 'S' is the starting vertex for prim's algorithm. Which of the following can be the correct order of edges in which they are added to construct the minimal spanning tree?

(A) 4, 2, 1, 6, 8 (B) 4, 1, 2, 6, 8

(C) 4, 2, 1, 7, 10 (D) None of these

Q.88 The minimum number of comparisons required to find maximum element in a min-heap of n elements (assume $n>1$)?

(A) $n-1$ (B) $\left\lfloor \frac{n}{2} \right\rfloor -1$

(C) $\left\lceil \frac{n}{2} \right\rceil -1$ (D) n

Q.89 The running time of an algorithm is given by $T(n)=T(n-1)+T(n-2)-T(n-3), \text{if } n>3$, T(n)= n otherwise, then what should be the relation between T(1), T(2) and T(3), so that the order of the algorithm is constant?

(A) T(1) = T(2) = T(3)

(B) T(1) + T(3) = 2*T(2)

(C) T(1) – T(3) = T(2)

(D) T(1) + T(2) = T(3)

Q.90 What is the time complexity for the following C module? Assume that n>0

```
int module (int n)
{
if (n = = 1)
return 1;
else
return (n + module (n-1));
}
```

(A) O(n) (B) O(log n)

(C) $O(n^2)$ (D) O(n!)

Q.91 Match the following:

P.	$T(n)=3T(\lfloor n/4 \rfloor)+O(n^2)$	1.	$O(n\log n)$
Q.	$T(n)=T(n/3)+T(2n/3)+O(n)$	2.	$O(n^2)$
R.	$T(n)=\sum_{n=1}^{\log n}\left(\frac{n}{2}\right)$	3.	$O(n^3)$
S		4.	$O(n)$

Codes:

(A) P-3, Q-2, R-1 (B) P-3, Q-2, R-2

(C) P-2, Q-1, R-1 (D) P-2, Q-1, R-4

Q.92 Let $T(n)=T(n/5)+T(7n/10)+c.n$ where c is a constant. Find running time of $T(n)$.

(A) $\Theta(n)$ (B) $\Theta(n\log n)$

(C) $\Theta(n^2)$ (D) None of these

Q.93 Consider the following two functions :

$$g_1(n)=\begin{cases} n^3 \text{ for } 0 \le n \le 10{,}000 \\ n^2 \text{ for } n \ge 10{,}000 \end{cases}$$

$$g_2(n)=\begin{cases} n \text{ for } 0 \le n \le 100 \\ n^3 \text{ for } n > 100 \end{cases}$$

Which of the following is true?

(A) $g_1(n)$ is $O(g_2(n))$

(B) $g_1(n)$ is $O(n^{\frac{1}{2}})$

(C) $g_2(n)$ is $O(g_1(n))$

(D) $g_2(n)$ is $O(n)$

Q.94 A Priority-Queue is implemented as a Max- Heap. Initially, it has 5 elements. The level- order traversal of the heap is given below:

10, 8, 5, 3, 2

Two new elements '1' and '7' are inserted in the heap in that order. The level- order traversal of the heap after the insertion of the elements is:

(A) 10, 8, 7, 5, 3, 2, 1

(B) 10, 8, 7, 2, 3, 1, 5

(C) 10, 8, 7, 1, 2, 3, 5

(D) 10, 8, 7, 3, 2, 1, 5

Q.95 The characters a to f have set of frequencies as

a : 5, b : 9, c : 12, d : 13, e : 16, f : 55

Huffman coding is used to encode the character what is the average number of bits required for character__________.

Q.96 Find the average number of comparisons in a binary search on sorted array of 7 elements _________.

Q.97 Consider 5 items along their respective weights and values

$I=[I, I_2, I_3, I_4, I_5]$

$W=[5,10,20,30,40]$

$V=[30,20,100,90,160]$

The capacity of knapsack $W=60$. Find the maximum value using fractional knapsack is ____

Q.98 Consider the program

```
gate academy (int n)
{
    int i, j, c = 0
for (i = 1; i <= n/2; i++)
{
for (j = 1; j <= n; j = j*2)
{
c++;
 }
 }
}
```

The complexity of the program is?

(A) $O(\log n)$

(B) $O(n^2)$

(C) $O(n^2 \log n)$

(D) $O(n\log n)$

Answers	Algorithm

1.	A,B	2.	56	3.	A,B	4.	C,D	5.	B
6.	8	7.	13	8.	A.D	9.	4	10.	13
11.	$0\sqrt{n}$	12.	A	13.	748	14.	C	15.	D
16.	C,D	17.	D	18.	61	19.	B	20.	D
21.	860	22.	C	23.	B	24.	C	25.	A
26.	C	27.	207	28.	77	29.	B	30	495
31.	9	32.	24	33.	B,C	34.	A,D	35.	C
36.	A	37.	A,C	38.	C,D	39.	A,B,C,D	40	3
41.	A	42.	9	43.	1.5	44.	A	45.	B
46.	B	47.	A	48.	136	49.	A,B	50.	94
51.	C,D	52.	C	53.	B	54.	12.5	55.	B
56.	2	57.	A,B,C,D	58.	A,D	59.	32000	60.	A
61.	C	62.	B	63.	A	64.	B	65.	2
66.	A	67.	C	68.	D	69.	28	70.	A,C
71.	A	72.	B	73.	B	74.	1860	75.	C
76.	A	77.	A	78.	B	79.	B	80.	C
81.	A	82.	8	83.	B	84.	D	85.	B
86.	B	87.	D	88.	C	89.	A	90.	A
91.	C	92.	A	93.	A	94.	D	95.	2.12
96.	2.428	97.	270	98.	D				

Explanations Algorithm

1. (A,B)

(A)

1	2	3	4	5	6
J_1	J_2	J_3	J_4	J_5	

Invalid

(B)

1	2	3	4	5	6
J_3	J_6	J_2	J_5	J_4	J_1

Valid

(C)

1	2	3	4	5	6
J_6	J_5	J_3	J_2	J_4	J_1

Valid

(D)

1	2	3	4	5	6
J_6	J_5	J_4	J_3		

Invalid

Hence, the correct option are (A,B).

2. 56

$g-1=1110$

$a-3=10$

$t-1=1111$

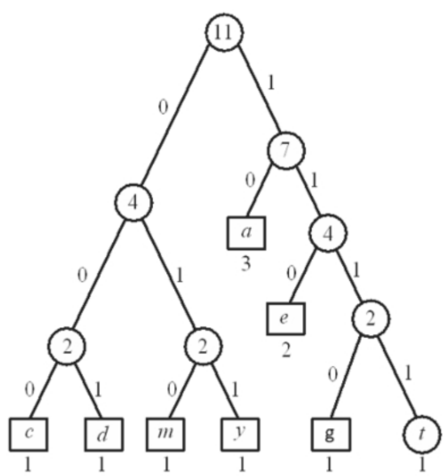
$e-2=110$

$c-1=000$

$d-1=001$

$m-1=010$

$y-1=011$



Total bits without compression $=11\times 8=88$ bits

Number of bits

$$=(1\times 4)+(3\times 2)+(1\times 4)+(2\times 3)+(1\times 3)+(1\times 3)+(1\times 3)+(1\times 3)$$

$$=4+6+4+6+3+3+3+3=32 \text{ bits}$$

Number of hits saved $=88-32=56$ bits

Hence, the correct answer is 56.

3. (A,B)

Dijkshtra algorithm is greedy algorithm but Bellman ford and floyd warshall are dynamic programming algorithms.

Hence, the correct option are (A,B).

4. (C,D)

(A) Starting from M which is connected all nodes so any sequence $\Rightarrow$ all valid

(B) N $\rightarrow$ M $\Rightarrow$ all valid

(C) $Q\rightarrow \underbrace{S\rightarrow N}_{\text{Invalid}}$

(D) $R\rightarrow \underbrace{T\rightarrow Q}_{\text{Invalid}}$

Hence, the correct option are (C,D).

5. (B)

P	Q	R
0	1	3

Q	R	S	T
1	3	3	5

R	S	T	V
3	3	5	5

S	T	V	U
3	5	5	8

T	V	U
5	5	8

V	U
5	8

U	-
8	-

node	π	distance
P	–	–
Q	1	P
R	3	P
S	Q	3
T	Q	5
U	S	8
V	R	5

Selected shortest path by Dijkstra

$$P \xrightarrow{1} Q \xrightarrow{2} S \xrightarrow{5} U$$

Note : $P \xrightarrow{1} Q \xrightarrow{4} T \xrightarrow{3} U$. This is also shortest path but it will not be selected by Dijkstra.

Hence, the correct option is (B).

6. 8

Selected path by Dijkstra's algorithm,

$$P \xrightarrow{1} Q \xrightarrow{2} S \xrightarrow{5} U$$

Cost $= 1+2+5 = 8$

Hence, the correct answer is 8.

7. 13

Frequency

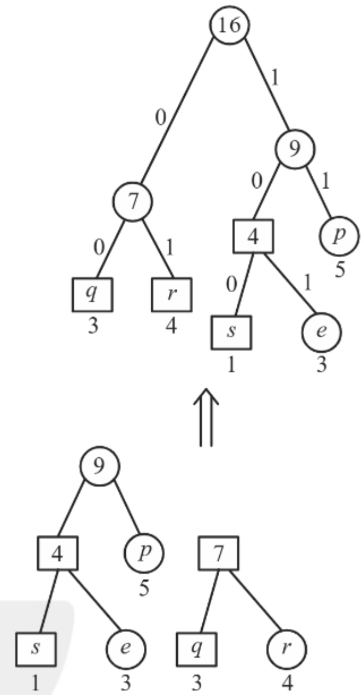
$p-5 \Rightarrow 11$

$q-3 \Rightarrow 00$

$r-4 \Rightarrow 01$

$s-1 \Rightarrow 100$

$e-3 \Rightarrow 101$



Peers $\Rightarrow$ $p+e+e+r+s$

$$\downarrow$$

$$2+3+3+2+3=13$$

Hence, the correct answer is 13.

8. (A,D)

Frequency

$p-5 \Rightarrow 11$

$q-3 \Rightarrow 00$

$r-4 \Rightarrow 01$

$s-1 \Rightarrow 100$

$e-3 \Rightarrow 101$

$$\begin{array}{ccccc} p & e & e & r & s \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 11 & 101 & 101 & 01 & 100 \end{array}$$

binary code =

$$\underline{11\ 101\ 101\ 01\ 100}$$

$$\Big\Downarrow \text{ 1's complement}$$

$\underline{0001001010011} \Rightarrow$ This is also a binary code (valid)

Hence, the correct option are (A,D).

9. **4**

$$P = M, N, O, N, R, M, N$$
$$Q = N, R, O, M, N, M$$

1. $MNM = 3$
2. $NRMN = 4$
3. $RMN = 3$
4. $NONM = 4$
5. $NONM = 4$

Hence, the correct answer is 4.

10. **13**

Longest common length subsequences :

1. $NRMN$
2. $NONM$
3. $NOMN$

$a = 3$

Smallest common length subsequences :

1. MN
2. NM
3. RM
4. RN
5. NR
6. NO
7. ON
8. OM
9. NN
10. MM

$b = 10$

$b + a = 10 + 3 = 13$

Hence, the correct answer is 13.

11. $0\sqrt{n}$

$i = 1, s = 1$

i	1	2	3	4	5
s	1	3	6	10	15

We see that s is the sum of n terms function is

$$s = \frac{k(k+1)}{2}$$

But $s \le n$ must be true

So, the maximum value of s is

$$\frac{k(k+1)}{2} = n \Rightarrow \frac{k^2 + k}{2} = n$$

$$k^2 = \sqrt{2n}$$

$$k = 0\sqrt{n}$$

Hence, the correct answer is $0\sqrt{n}$.

12. **(A)**

$$T(x) = \sqrt{n}T\left(\sqrt{n}\right) + 100n$$

Let, $n = 2^k \Rightarrow k = \log n$

$$T(2k) = (2)^{k/2}T(2^{k/2}) + 200 \times 2^k$$

(dividing by 2^k)

$$\frac{T(2^k)}{2^k} = \frac{2^{k/2}T(2^{k/2})}{2^k} + 100$$

$$\frac{T(2^k)}{2^k} = \frac{T(2^{k/2})}{2^{k/2}} + 100$$

Let, $y(k) = \frac{T(2^k)}{2^k}$, then …(i)

$$y(k) = y\left(\frac{k}{2}\right) + 100$$

Now, applying master theorem $[a = b^k = 1]$

$$y(k) = \log k$$

From (i) we also know that $T(2^k) = 2^k y(k)$, then

$$T(2^k) = 2k\log k = T(n)$$

$= n\log\log n$ (because $n = 2^k$ and $k\log n$)

Finally, $T(n) = \theta(n\log\log n)$

Hence, the correct option is (A).

13. **748**

$n = 500$ (array)

I. linear comparision :

Total comparisons

$= 2n = 2 \times 500 = 1000$

II. Using Divide and Conquer approach :

Total comparisons $= \frac{3n}{2} - 2$ (go through minmax algorithm)

$$= \frac{3 \times 500}{2} - 2 = 748$$

Min (748, 1000) $= 748$

Hence, the correct answer is 748.

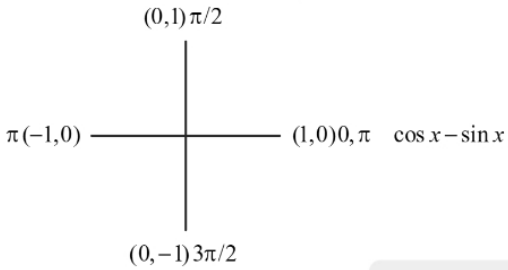
14. **(C)**

Merge sort does not depend on input data but Selection, Bubble and Quick sort depends on input data based on that their time complexity varies.

Hence, the correct option is (C).

15. **(D)**

Note : $\cos x$ and $\sin x$ are periodic functions.



Range of $\cos x$ & $\sin x$ is $[-1, 1]$

$\Rightarrow \quad f(n) = n^{1+\cos x}$

$\Rightarrow \quad \left(0(1), 0(n), 0(n^2)\right)$ possible values

$\quad g(n) = \left(0(1), 0(n), 0(n^2)\right)$

So, we can't predict or conclude exact relation between $f(n)$ and $g(n)$.

Hence, the correct option is (D).

16. **(C, D)**

$$X = \frac{n^2(n^2+1)(2n^2+1)}{6} + \frac{n^2(n^2+1)}{2}$$

$$= n^2\left[n^4 + n^2 + 1 + n^2 + 1\right]$$

$$= n^6 + n^4 + n^4 + n^2 + n^4 + n^2$$

$$= n^6 + 3n^4 + 2n^2$$

(lower terms neglected)

$$= 0(n^6)$$

Hence, the correct option are (C,D).

17. **(D)**

To find minimax algorithm we can use divide and conquer algorithm with $\frac{3n}{2} - 2$ minimum comparisons. But here we have to find the number which is not minimax so, simply we can pic 3 number and compare them the middle element of three of will be not an minimax, so always required only 2 comparisions

So, $\theta(1)$ time complexity.

Hence, the correct option is (D).

18. **61**

edge $AB = 10$ ($AC = 9$ selected)

$BC = 2$

edge $BE = 16$ ($DE = 15$)

$CE = 17$

edge $BD = 11$ ($CD = 10$)

edge $DF = 7$ ($EF = 6$)

Minimum sum $= AB + BD + BE + CE + DF$

$= 10 + 11 + 16 + 17 + 7 = 61$

Hence, the correct answer is 61.

19. **(B)**

If we consider Left skewed or Right skewed then in worst case we need to traverse n nodes so, its complexity becomes $0(n)$. For 1 insertion, we have to insert n elements,

Time Complexity $= n \cdot 0(n)$

$= 0(n^2)$

Hence, the correct option is (B).

20. **(D)**

Quick sort algorithm Recurrence Relation is,

$$T(n) = T(k) + T(n-k-1) + 0(n)$$

Worst case : $k = 0$ (last element is sorted in its position only).

$$T(n) = T(0) + T(n-0-1) + 0(n)$$

$$T(n) = T(n-1) + 0(n)$$

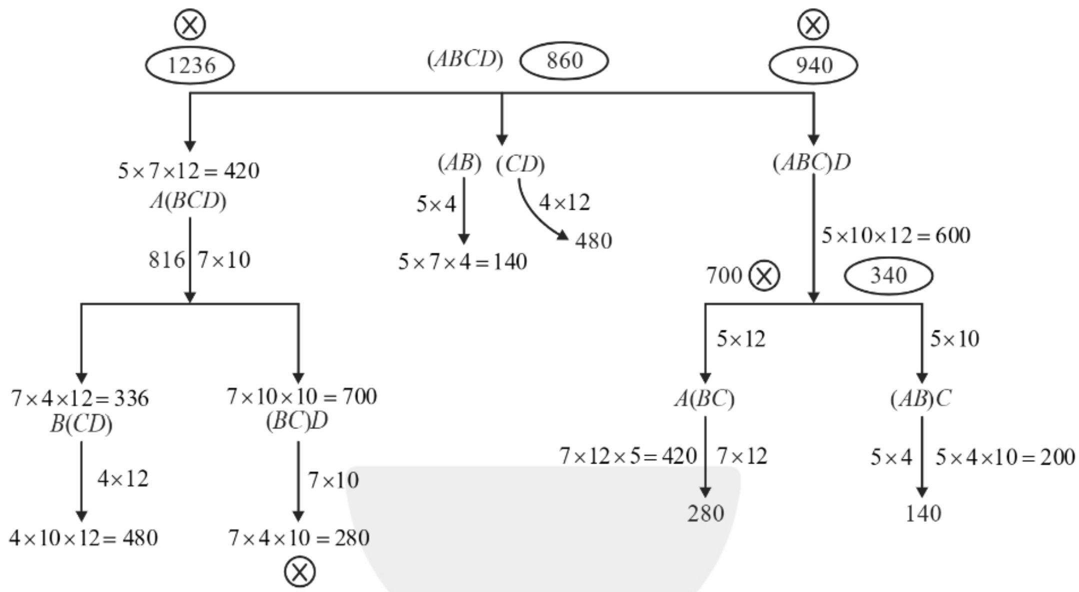
$$T(n) = 1$$

Hence, the correct option is (D).

21. **860**

Dimensions of four matrices $ABCD$.

$A = 5\times 7, B = 7\times 4, C = 4\times 10, D = 10\times 12$



Hence, the correct answer is 860.

22. **(C)**

Settle above multiplication tree $((AB)(CD))$ gives minimal scalar products as 860 with dimension 5×12

Hence, the correct option is (C).

23. **(B)**

Given : $T(n) = 2T\left(\frac{n}{2}\right) + \sqrt{n}$

As we know by Masters theorem :

For $\quad T(n) = aT\left(\frac{n}{b}\right) + \theta(n^k \log^p n)$

$$a > b^k \Rightarrow T(n) = \theta(n^{\log} b^a)$$

$$a = 2,\ b = 2,\ k = \frac{1}{2},\ p = 0$$

$$2 > 2^{1/2} \qquad \{a > b^k\}$$

$$T(n) = \theta(n)$$

$$= \theta(n^{\log_b a}) = \theta(n^1) = \theta(n)$$

Hence, the correct option is (B).

24. **(C)**

8	3	4	6	7	2	1	9	5
$A[1]$	$A[2]$	$A[3]$	$A[4]$	$A[5]$	$A[6]$	$A[7]$	$A[8]$	$A[9]$

5 as pivot

So i is 1 and j is 0 [check $A[i] < 5$]

Check $A[j] < 5$ false $i = i + 1$

Check $A[2] < 5$ true

$j \leftarrow j + 1$ and exchange $A[i] \leftrightarrow A[j]$

So, $A[1]$ and $A[2]$ are exchange then $i \leftarrow i + 1$

3	8	4	6	7	2	1	9	5
$A[1]$	$A[2]$	$A[3]$	$A[4]$	$A[5]$	$A[6]$	$A[7]$	$A[8]$	$A[9]$

Check $A[3] < 5$ true $j \leftarrow j + 1$

So, $A[2]$ and $A[3]$ are exchange, $i \leftarrow i + 1$

3	4	8	6	7	2	1	9	5
$A[1]$	$A[2]$	$A[3]$	$A[4]$	$A[5]$	$A[6]$	$A[7]$	$A[8]$	$A[9]$

Check $A[4] < 5$ false, $i \leftarrow i+1$

Check $A[5] < 5$ false, $i \leftarrow i+1$

Check $A[6] < 5$ true

So $j \leftarrow j+1$ and exchange $A[i] \leftrightarrow A[j]$

So, $A[3]$ and $A[6]$ are changed $50 i \leftarrow i+1$

3	4	2	6	7	8	1	9	5
$A[1]$	$A[2]$	$A[3]$	$A[4]$	$A[5]$	$A[6]$	$A[7]$	$A[8]$	$A[9]$

Check $A[7] < 5$ true

So $j \leftarrow j+1$ and exchange $A[i] \leftrightarrow A[i]$

So, $A[4]$ and $A[7]$ are exchanged, $i < i$

3	4	2	1	7	8	6	9	5
$A[1]$	$A[2]$	$A[3]$	$A[4]$	$A[5]$	$A[6]$	$A[7]$	$A[8]$	$A[9]$

Check $A[8] < 5$ false

So, $j < j+1$ and exchange $A[j+1]$ with our data (5)

3	4	2	1	5	8	6	9	7
$A[1]$	$A[2]$	$A[3]$	$A[4]$	$A[5]$	$A[6]$	$A[7]$	$A[8]$	$A[9]$

The answer is 3 4 2 1 5 8 6 9 7.

Hence, the correct option is (C).

25. (A)

Bubble sort is the sorting algorithm in which every iteration it eliminates largest element form the given array and reduces the array size for the next iteration.

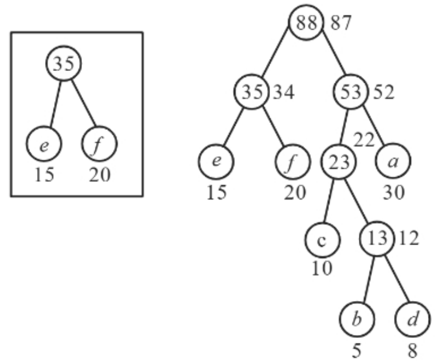
Hence, the correct option is (A).

26. (C)

1. Dijkshtra $\Rightarrow 0(E+V)\log V$ or $E \log V$
2. Floyd warshall $\Rightarrow n^3$ or V^3
3. Bellman $\Rightarrow n^3$ or V^3
4. BFS $\Rightarrow 0(E+V)$

Hence, the correct option is (C).

27. 207



Minimum record movements

$$= 12 + 22 + 34 + 52 + 87 = 207$$

Hence, the correct answer is 207.

28. 77

Index position in upper triangular matrix is,

$$= n(i-1) - \frac{(i-1)\times(i-2)}{2} + j - i$$

$$= 15(7-1) - \frac{(7-1)\times(7-2)}{2} + 9 - 7$$

$$= 15\times 6 - \frac{6\times 5}{2} + 2$$

$$= 90 - 15 + 2 = 77$$

Hence, the correct answer is 77.

29. (B)

$A \rightarrow$ Prim's algorithm $\Rightarrow$ Greedy algorithm

$B \rightarrow$ Quick sort $\Rightarrow$ Divide and conquer

$C \rightarrow$ Bellman ford $\Rightarrow$ D.P.

$D \rightarrow$ Subset sum problem $\Rightarrow$ D.P.

Hence, the correct option is (B).

30. 495

Number of vertices $(n) = 50$

Number of edges in minimum spanning tree

$$= (n-1)$$

$$= (50-1) = (49)$$

Weight of every edge increased by 5 $= 49 * 5$

$$= 245$$

New weight $= 250 + 245 = 495$

Hence, the correct answer is 495.

31. **9**

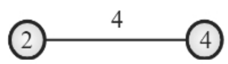
We will use Kruskal's algorithm (For MST)

	1	2	3	4	5
1	0	1	6	1	3
2	1	0	10	4	7
3	6	10	0	5	2
4	1	4	5	0	2
5	4	7	3	2	0

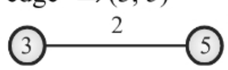
node 1 : Minimum weight edge (1, 2) & (1, 4)

Note : node 1 is a leaf node so out of (1, 2) or (1, 4) only 1 can be considered.

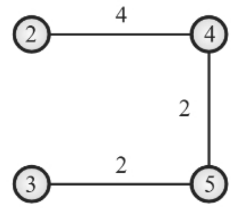
Node 2 :



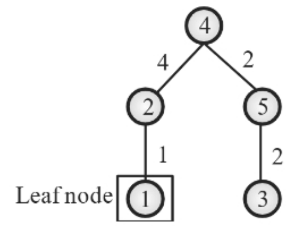
Node 3 : min edge $\Rightarrow (3, 5)$



Node 4 : min edge $\Rightarrow (4, 5)$



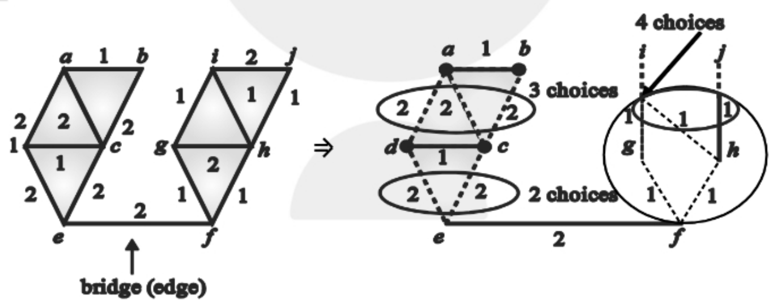
Final tree with node 1 as leaf node



Total weight of MST $= 4+2+1+2 = 9$

Hence, the correct answer is 9.

32. **24**



Total MST possible $= 3 \times 2 \times 4 = 24$

Hence, the correct answer is 24.

33. **(B,C)**

(A) False : Due to distinct weight there will be unique MST.

(B) True : e_{min} will always be part of MST due to distinct weight

(C) True : If e_{max} is a part of MST it means it is bridge whose removal disconnects graph.

(D) False : Option C is the reason

Hence, the correct option are (B,C).

34. **(A,D)**

(A) True :

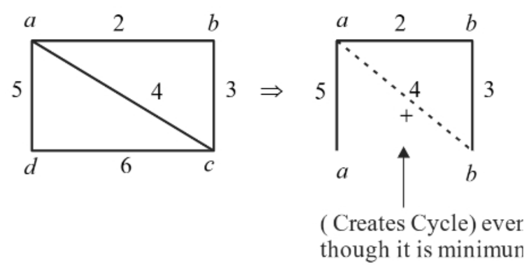
(C) False : Shortest path may differ.

Ex : 5 edges with shortest path 15 and other path with 2 edges with weight 25 but now 10 cost increased then,

$5*10 = 50+15 = 65$

$2*10 = 20+25 = 45$ this becomes now shortest path.

(B) False :



(D) True :

Hence, the correct option are (A,D).

35. (C)

Asymptotically better means for sufficiently large inputs, X is better than Y. i.e., beyond a large input size N, time taken by X is lower than that taken by Y. Only option C is TRUE.

Option D is wrong because "1 million" is not a limit to be "large" - this limit can be any value - 1 million +100,1 billion or even larger value. All we need is to ensure such a limit exists, however large that may be.

Hence, the correct option is (C).

36. (A)

Given that, the values: 13, 107, 49, 50, 64, 98, 16, 33

- 13 mod 10 = 3 – No collision
- 107 mod 10 = 7 – No collision
- 49 mod 10 = 9 – No collision
- 50 mod 10 = 0 – No collision
- 64 mod 10 = 4 – No collision
- 98 mod 10 = 8 – No collision
- 16 mod 10 = 6 – No collision
- 33 mod 10 = 3 – There is collision, so 33 will go to next free slot.

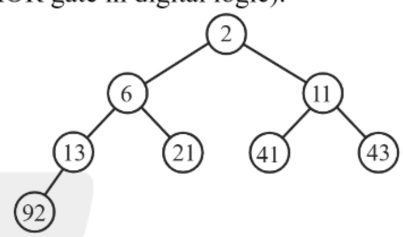
Hash Table

Index	Values
0	50
1	
2	
3	13
4	64
5	33
6	16
7	107
8	98
9	49

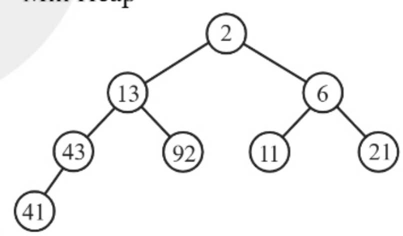
Hence, the correct option is (A).

37. (A,C)

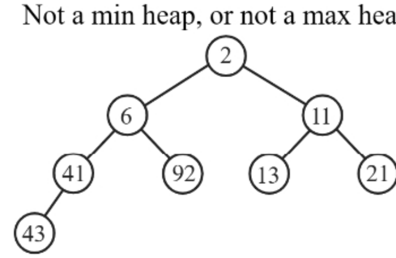
We know, that heap can be either min heap or max heap, but not both at the same time (Similar to EXOR gate in digital logic).



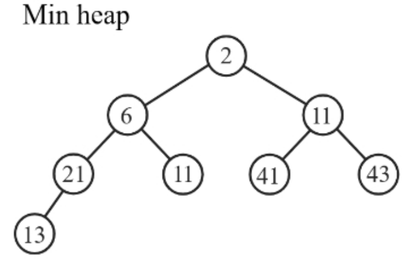
(A) Min Heap



(B) Not a min heap, or not a max heap



(C) Min heap



(D) Not a min heap, or not a max heap

Hence, the correct option are (A,C).

38. **(C,D)**

Huffman coding is a lossless coding and has an exact greedy algorithm. In this coding, the codes are represented by the binary paths from root to leaf and thus no code can be a prefix of another.

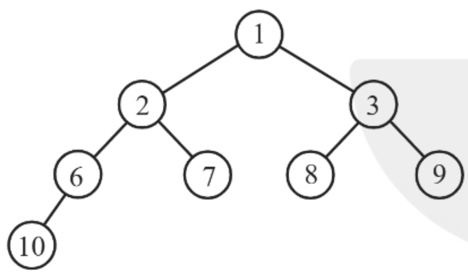
39. **(A,B,C,D)**

Ceil and floor do not affect the asymptotic growth. And the base of logarithm also does not affect the asymptotic growth. So, here,

$$\log_{10} n = \Theta(\log_2 n) = \Theta(\lceil \log_2 n \rceil) = \Theta(\lfloor \log_2 n \rfloor)$$

Hence, the correct option are (A,B,C,D).

40. **3**



7 is in level 3.

Hence, the correct answer is 3.

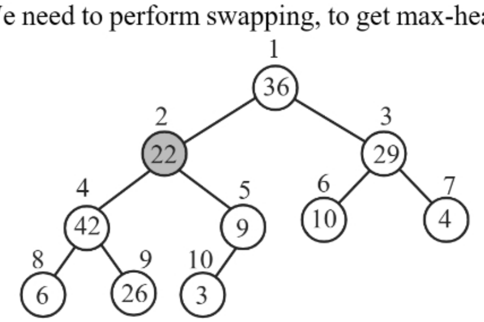
41. **(A)**

Question is implying a stable sort. Among the ones in options, only Merge Sort is stable.

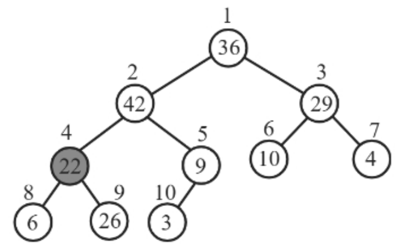
Hence, the correct option is (A).

42. **9**

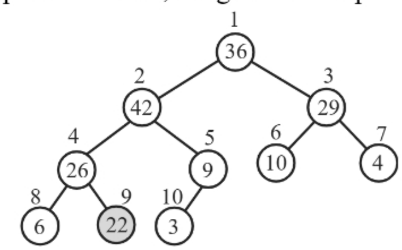
We need to perform swapping, to get max-heap.



Swapped 22 and 42, we get



Swapped 22 and 26, we get max-heap.



Hence, the correct answer is 9.

43. **1.5**

In a hash-table with chaining and uniform hashing, the expected number of comparisons in an unsuccessful search is given by α where $\alpha = \frac{m}{n}$ is the load factor.

Here, $\alpha = \frac{3}{2} = 1.5$. So, expected number of comparisons for an unsuccessful search $= 1.5$

Hence, the correct answer is 1.5.

44. **(A)**

In first pass 34 is the pivot. So, it'll get position as in the final sorted order which is 4th position. This is true only for option A.

Hence, the correct option is (A).

45. **(B)**

The best possible algorithm will find the k^{th} smallest element in $O(n)$ time and then move all elements smaller than this to its left - partition part of quick sort which again takes $O(n)$. Finally, these k elements can be sorted in 0(k log k) time.

Hence, the correct option is (B).

46. (B)

We want all the n keys to go to $m-1$ locations only. Probability for this will be

$$\frac{(m-1)^n}{m^n} \Rightarrow (1-0.125)^6 = 0.875^6$$

Hence, the correct option is (B).

47. (A)

Enqueue node 1 in the queue.

Queue: 1

Mark it as visited. Dequeue 1 and enqueue its non-visited adjacent nodes in ascending order.

Queue: 8 5 4 3 2

Mark 8 as visited. Dequeue 8 and enqueuer its non-visited adjacent nodes in ascending order.

Queue :5 4 3 2 7 6

Mark 5 as visited. Dequeue 5 and enqueue it non-visited adjacent nodes.

Queue :4 3 2 7 6

Mark 4 as visited. Dequeue 4 and enqueue it non-visited adjacent nodes.

Queue :3 2 7 6

Mark 3 as visited. Dequeue 3 and enqueue it non-visited adjacent nodes.

Queue: 2 7 6

Hence, the correct option is (A).

48. 136

Given that,

$arr[0]=1, arr[1]=4, arr[2]=10$

$arr[3]=12, \text{and } len=4$

initially, the value of result = 0

- When
  $i=0 \mid \text{result} = \text{result} + arr[0]*$
  $(0+1)*(4-0) = 0+1*1*4 = 4$
- When
  $i=1 \mid \text{result} = \text{result} + arr[1]*(1+1)$
  $*(4-1) = 4+4*2*3 = 28$
- When
  $i=2 \mid \text{result} = \text{result} + arr[2]*(2+1)*$
  $(4-2) = 28+10*3*2 = 88$
- When
  $i=3 \mid \text{result} = \text{result} + arr[3]*(3+1)*$
  $(4-1) = 88+12*4*1 = 136$

Hence, the correct answer is 136.

49. (A,B)

- $32 \bmod 6 = 2 -$ occupies slot 2
- $7 \bmod 6 = 1 -$ occupies slot 1
- $14 \bmod 6 = 2 -$ collide. Occupies next slot which is slot 3
- $22 \bmod 6 = 4 -$ occupies slot 1
- $29 \bmod 6 = 5 -$ occupies slot 5
- $3 \bmod 6 = 3 -$ collide on slot 3. Occupies next slot which is slot 0.

Hence, the correct option are (A,B).

50. 94

Solve in bottom-up fashion.

Find me (0) = 1

- $\text{findme}(1) = \text{findme}(0) + \text{findme}(0) = 2$
- $\text{findme}(2) = \text{findme}(1) + \text{findme}(1) = 4$
- $\text{findme}(3) = \text{findme}(1) + \text{findme}(2) = 6$
- $\text{findme}(4) = \text{findme}(1) + \text{findme}(3) = 10$
- $\text{findme}(5) = \text{findme}(2) + \text{findme}(4) = 14$
- $\text{findme}(6) = \text{findme}(3) + \text{findme}(5) = 20$
- $\text{findme}(7) = \text{findme}(3) + \text{findme}(6) = 26$
- $\text{findme}(8) = \text{findme}(4) + \text{findme}(7) = 36$
- $\text{findme}(9) = \text{findme}(4) + \text{findme}(8) = 46$
- $\text{findme}(10) = \text{findme}(5) + \text{findme}(9) = 60$
- $\text{findme}(11) = \text{findme}(5) + \text{findme}(10) = 74$
- $\text{findme}(12) = \text{findme}(6) + \text{findme}(11) = 94$

Hence, the correct answer is 94.

51. **(C,D)**

17 Worst case complexity of quick sort occurs in 2 cases, when smallest or largest element of array is chosen as pivot element of array. So both the case of worst cases are avoided and hence worst case can't be $O(n\hat{}2)$.

Hence, the correct option are (C,D).

52. **(C)**

The i loop iterates $n/2=\Omega(n)$ times

j loop does call to foo(j). So, enumerating the calls to foo(j) we get

$foo(1)+foo(2)+foo(2^2)+...+foo(2^{\lg n})$

Complexity of $foo(k)=\Theta(\log k)$

So, we get complexity of j loop as

$$\Theta(\lg 1)+\Theta(\lg 2)+\Theta(\lg 2^2)+...+\Theta(\lg 2^{\lg n})$$
$$=\Theta(1+2+3+...+\lg n)$$
$$=\Theta((\lg n)^2)$$

So, the time complexity of entire code

$$=\Theta((\lg n)^2).$$

Hence, the correct option is (C).

53. **(B)**

The while loop is iterating $\lg n$ times as in each iteration n is halved. So times complexity will be $\Theta(\lg n)$.

There is only constant amount of space required in the function. So, space complexity is $O(1)$.

α value is incremented as

$$n+n/2+n/4+...n/2^{\lg n}$$
$$=n[1+1/2+1/4+...1/n]$$
$$=n\frac{[1-1/2^{\lg n}]}{0.5}$$ (Sum to n terms of GP with $\alpha=1, r=0.5=\frac{\alpha.(1-r^n)}{1-r}$

$$=2n\left[\frac{n-1}{n}\right]=2(n-1)=\Theta(n).$$

Hence, the correct option is (B).

54. **12.5**

$$2\times 2000+2\times 300+1\times 50=4650.$$

So, 5 notes in minimum $\Rightarrow 5\times 2.5=12.5$

Hence, the correct answer is 12.5.

55. **(B)**

Only B option is true here.

The expected number of comparisons in a successful linear search is $[n/2]$ As the successful item is equally likely to be in any of the n location.

In best case both linear search and binary search can work in $\Omega(\log n)$ comparisons as after each search half the elements get eliminated.

Hence, the correct option is (B).

56. **2**

Arranging by the order of probability of occurrences we get

- $a:0.33$
- $d:0.3$
- $c:0.2$
- $b:0.16$

So, first we combine c and b nodes. Sum of their probabilities will be 0.36.

Next, we combine the two smallest probabilities which are for a and d. This will give sum as 0.63.

So, we get complete and full binary tree with 3 levels and so all codes are of length 2. So, expected length of the binary codes will be 2.

Hence, the correct answer is 2.

57. **A,B,C,D**

$h(x,i)=(h_1(x)+i\times h_2(x)) \bmod m.$

$13: h_1(x) = 6.$goes to slot 6

$18: h_1(x) = 4.$goes to slot 4

$25: h_1(x) = 4.$goes to slot 4, collision

$h_2(x) = 4, (2\times 4) \bmod 7 = 1 -$ goes to slot 1

$11: h_1(x) = 4.$goes to slot 4, collision-

$h_2(x) = 2, (4\times 2) \bmod 7 = 6 -$ collision

$(4+2\times 2) \bmod 7 = 1$ collision-

$(4+3\times 2) \bmod 7 = 3 -$ goes to slot 3.

$20: h_1(x) = 6 -$ goes to slot 6, collision-

$h_2(x) = 3, (6+3) \bmod 7 = 2 -$ goes to slot 2.

$29: h_1(x) = 1 -$ goes to slot 1. Collision.

Thus, 25,11,20,29 all collide.

Hence, the correct option are (A,B,C,D).

58. (A,D)

A is TRUE. Happens for a left-skewed or right-skewed binary tree.

B is FALSE. Minimum number of comparisons to sort 5 elements is 7

C is FALSE. There can be at most $\left\lceil \frac{n}{2^{h-1}} \right\rceil$ nodes of height h in any n element heap when h starts from 1 and this changes to $\left\lceil \frac{n}{2^{h+1}} \right\rceil$ when h starts from 0.

D is TRUE as maximum element in a min-heap will be at the last level and it can be any one of the possible $n/2$ elements in the last level.

Hence, the correct option are (A,D).

59. 32000

By a quick look we can see that 90 is by far the largest dimension. So, we must do $(M_1 \times M_2)$ to get rid of this dimension. This means the only orderings left are $(((M_1 \times M_2) \times M_3) \times M_4)$ and $(M_1 \times M_2) \times (M_3 \times M_4)$.

These two will give the total number of multiplications as

$$[20\times 90\times 10] + [20\times 10\times 15] +$$
$$[20\times 15\times 40] = 33000$$

And

$$[20\times 90\times 10] + [10\times 15\times 40] +$$
$$[20\times 10\times 40] = 32000$$

So, minimum number of multiplication required $= 32000$.

Hence, the correct answer is 32000.

60. (A)

Maximum subarray sum can be found in linear time using dynamic programming.

So, option A is TRUE.

Hence, the correct option is (A).

61. (C)

Given : Series

$$T(n) = 1.2^1 + 2.2^2 + 3.2^3 + 4.2^4$$
$$+ 5.2^5 + 6.2^6 + 7.2^7 + \ldots n.2^n$$

Above series is bot A.P AND G.P series

Convert it into G.P. series by method given below

$$2.T(n) = 1.2^2 + 2.2^3 + 3.2^4 + 4.2^5$$
$$+ 5.2^6 + 6.2^7 + 7.2^8 + \ldots n.2^{n+1}$$
$$T(n) - 2.T(n) = 1.2^1 + (2-1).2^2 +$$
$$(3-2).2^3 + (4-3).2^4 + \ldots - n.2^{n+1}$$
$$-T(n) = 2^1 + 2^2 + 2^3 + 2^4 + 2^5$$
$$+ 2^6 + 2^7 + \ldots + 2^n - n.2^{n+1}$$

Now we get the G.P series

$$-T(n) = 2.((2^n - 1)/(2-1)) - n.2^{n+1}$$
$$T(n) = 2 + n.2^{n+1} - 2^{n+1}$$
$$T(n) = O(n.2^n)$$

Hence, the correct option is (C).

62. (B)

for ($j = 2$ to array. length)

key = array [j];

$i = j - 1;$

While ($i > 0$ && array $[i] >$ key)

array $[i+1]$ = array[i];

$i = i - 1;$

array $[i+1]$ = key;

above is the standard algorithm of insertion sort, from this we can easily infer that $(j > 0)$ && $(arr[j-1] > value)$ is the right option.

Hence, the correct option is (B).

63. (A)

Only Option A is correct

Option A: simple calculation or $GCD(9,13) = 1$

Option B: what if I take $m = 2$ the code will run for $n/2 +$ steps $= \Omega(n)$; So, Option B is wrong.

Option C: Given algorithm is doing GCD not LCM

Option D: space complexity is $0(1)$ for the given algorithm

Hence, the correct option is (A).

64. (B)

Even to sort 10% of the elements insertion sort in worst case can take $\Theta(n^2)$ comparisons.

Number of comparisons in insertion sort is $O(k)$, where k is the number of inversions in the input array.

For option C, consider the case where the smallest element is at the end of the array for ascending sort.

This element will come to the correct position only after $\Theta(n^2)$ comparisons in bubble sort.

Hence, the correct option is (B).

65. 2

This recurrence is same as LCS problem. In LCS we had "max" but here we have "min".

Time complexity of LCS is $O(mn)$.

Here it will be $O(n^2)$.$a = 2$, $b = 0$.

Hence, the correct answer is 2.

66. (A)

(This question aim to make you understand that top down is better if there are only few sub problems to be computed)

Bottom-up approach will fill $m \times n$ matrix, and each sub problem will take $\theta(1)$ so total time in bottom up $= \theta(mn)$

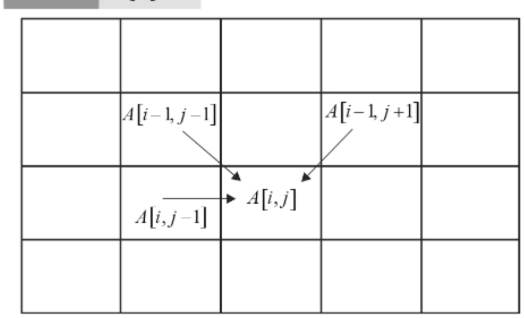
The top-down approach will start with a function call $A(n, n)$ and call sub problems recursively. And will only call required sub problems, not all sub problems.

Top-down method only calculates the needed sub problems, which are of the form $A\left(\left\lfloor \frac{m}{2^6} \right\rfloor, \left\lfloor \frac{m}{2^f} \right\rfloor\right)$. There are only $\Theta(\log\ m\ \log\ n)$ such sub problems, hence its time complexity is only $\Theta(\log\ m\ \log\ n)$.

Buttom-up method will require computing every sub problem, resulting in time complexity $\Theta(mn)$.

Hence, the correct option is (A).

67. (C)



Hence, the correct option is (C).

68.	(D)

1. Control enters into the first while $(i < n)$ (condition is true)
2. Control also enters into the inner while $(i < n)$ loop.
3. Control again enter into innermost while $(i < n^3)$ loop and then continues its execution until $i$is equal to n^3. Then the control cannot go inside any of the while loop because all the 3 conditions will result in false.

Hence total iteration is n^3 (inner most while) + 1 (inner while) + 1 (outer most while)

Hence time complexity $= O(n^3)$,

Hence, the correct option is (D).

69.	28

Capacity = 20

Weights are (18,15,10)

Profit values (25, 24, 15)

Find $\frac{\text{Weight}}{\text{value}} \Rightarrow \frac{18}{25}, \frac{15}{24}, \frac{10}{15}$

$\Rightarrow$ 0.72, 0.625, 0.666

Select the item with highest ratio.

After selection of the 1st item, remaining

Capacity = 2 and value = 25

Now select 1/5 part of 3rd item,

Remaining capacity = 0

Total value =25+(15/5)=28 .

Hence, the correct answer is 28.

70.	(A,C)

Concept : A minimum spanning tree (MST) or minimum weight spanning tree is a subset of the edges $(V-1)$ of a connected, edge-weighted undirected graph G (V,E) that connects all the vertices, without any cycles and with the minimum possible total edge weight.

Data:

Number of edges in $G = e = 6$

Number of vertices in $G = n$

Formula:

For a complete graph,

Degree of each vertex $= n-1$

From handshaking Lemma

$$2 \times 6 = n \times (n-1)$$

Calculation:

$$2 \times 6 = n^2 - n$$

$\therefore \quad n^2 - n - 12 = 0$

$$(n-4) \times (n+3) = 0$$

$\therefore \quad n = 4$ or $n = -3$

Since $\quad n \neq -3$

$\therefore \quad n = 4$

For a minimum spanning tree,

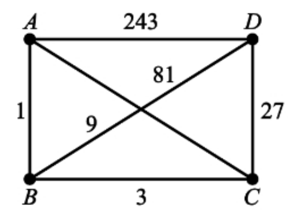
Number of edges needed $= 4 - 1 = 3$

One of the minimum spanning tree will defiantly be

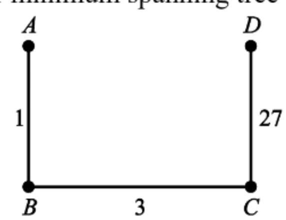
$1 \to 3 \to 9(13)$

Other possible

Graph: G



Weight for minimum spanning tree



Possible weight of a minimum spanning tree $= 1 + 3 + 27 = 31$

Only two MST is possible

Therefore option 1 and 3 are not possible

Hence, the correct option are (A,C).

71. **(A)**

For 0/1 knapsack problem,

1. Each item either can be chosen (or) cannot be chosen
2. Also, sum of the weights of the items in the knapsack should be less than (or) equal to W which is 6

Considering these two. Some of the possible solutions which satisfy the above two constraints.

Set 1: items (l, 2, 3), total value $=60$

Set 2: items (4,2), total value $=60$

Set 3: items (4, 3), total value $=55$

Set 4: items (5,3), total value $=65$

Set 5: items (l, 2), total value $=45$

Set 7: items (l, 3), total value $=40$

Set 8: items (2,3), total value $=35$

Out of these, clearly optimal solution is 65 and only Set 4 has this max value.

Hence, the correct option is (A).

72. **(B)**

a	5	2	4	8	3	40
	1	2	3	4	5	6

L						
	1	2	3	4	5	6

$a[j] \le \min(a[j+g]\ a[\text{n}])$

<u>Base Case :</u>

$L(6)=1$

L						1

a	5	2	4	8	3	40
	1	2	3	4	5	6
					$\uparrow$ j	$\uparrow$ k

$\min(a[6])=40$

$a[j]=3$

$a[j] \le \min(a[6])$

$\Rightarrow \quad L[s]=1$

So basically use are Checking that what hen the than minimum element. If so than mark 1 in $L[j]$.

L					1	1	$k=a_{j+1}$ to a_n
	1	2	3	4	5	6	

	1	2	3	4	5	6	
a	5	2	4	8	3	40	$a[j]=8$
				$\uparrow$ j	$\uparrow$ k		

L					1	1
	1	2	3	4	5	6

min $(3, 4\ 0)=3$

and $a[j] \ge \min(a[j+1],\ a[j+2]\ldots,a[n])$

So apply second case

i) $a[4] > \text{a}[5]$

$\text{max} = 1$

ii) $a[4] > \text{a}[5]$

So condition doesn't work

$1+\max(L[5])=2$

			2	1	1
1	2	3	4	5	6

Like this way before last step ;

a	5	2	4	8	3	40
	1	2	3	4	5	6

L		1	2	2	1	1
	1	2	3	4	5	6

e) Now analyze for $a[1]$

$k=a_{j+1}$ to a_n

	j $\downarrow$	k $\downarrow$				
a	5	2	4	8	3	40
	1	2	3	4	5	6

L		1	2	2	1	1
	1	2	3	4	5	6

We can see

$a[1] > \min(a[j+1]...a[n])$

$5 > 2,$ So apply second condition

So apply second case :

(i) $a[1] > a[2]$

$\max = 1$

(ii) $a[1] > a[3]$

$\max = 2$

(iii) $a[1] < a[4]$

$\max = 2$ [Conditional able so max remains some]

(iv) $a[1] > a[5]$

$\max = 2$

(v) $a[1] < a[6]$

$\max = 2$

So, $L[1] = 1 + \max$

$= 1 + 2 = 3$

L						
	3	1	2	2	1	1

So length of longest decreasing subsequence is 3

Another example :

a				
	1	2	3	4

L				
				1

→ Here use can see everytime condition 1 will apply

So,

L				
	1	1	1	1

So length of longest decreasing subsequence is 1.

Hence, the correct option is (B).

73. (B)

In the question we miss the point that we have to count the number of paths from vi to vn, not the count of nodes in that path.

Option C will return all nodes count in all the paths starting from i.

But we only want the count of paths which can be possible only when we return 1 from the end of any particular path & just take summation over that 1's which is what Option B is doing.

Hence, the correct option is (B).

74. 1860

The number of scalar multiplications required for multiplying matrices A_{mxn}, B_{nxp} is $m*n*p$.

Assuming we need to get least number of scalar multiplications, we eliminate the larger dimensions first.

Following are the 6 matrices and their dimensions

$A_{5\times10}, B_{10\times3}, C_{3\times12}, D_{12\times5}, E_{5\times50}, F_{50\times6}$

$\rightarrow \quad 5\times10, 10\times3, 3\times12, 12\times5, 5\times6$

(Multiplying matrices

$E_{5\times50}, F_{50\times6}, 50*5*6 = 1500$ multiplications)

$\rightarrow \quad 5\times10, 10\times3, 3\times5, 5\times6$

(Multiplying matrices

$C_{3\times12}, D_{12\times5}, 3*12*5 = 180$ multiplications)

$\rightarrow \quad 5\times3, 3\times5, 5\times6$

(Multiplying matrices with dimensions $3\times5, 5\times6, 3*5*6 = 90$ multiplications)

$\rightarrow \quad 5\times6$

(Multiplying matrices with dimensions $5\times3, 3\times6$ total $= 90$ multiplications)

Total scalar multiplications = 1860

Hence, the correct answer is 1860.

75. (C)

	0	1	2	3	4	5	6	
0	0	0	0	0	0	0	0	
1	0	0	1	1	1	1	1	b
2	0	1	1	1	2	2	2	a
3	0	1	1	2	2	2	3	c
4	0	0	2	2	2	3	3	b
		a	b	c	a	b	c	

There are four longest common subsequences : "bac" and "bab" and "bcb" and "acb"
Hence, the correct option is (C).

76. (A)

Given,

OPT $[1]=1$

For OPT [2] we need to look at OPT [1]

For OPT [3] we need to look at OPT [1] and OPT [2].

For OPT$[n]$ we need to look from OPT [1] to OPT $[n-1]$.

At first, for loop will run a single time, then two times, …… then $(n-1)$ times.

So the time complexity $T(n)$ will be:

$$T(n)=1+2+3+4+.........+(n-1)$$

$$T(n)=O(n^2)$$

Space complexity will be $O(n)$ because we are taking one extra array OPT which is a single-dimensional array.

Hence, the correct option is (A).

77. (A)

Option A– this is correct as this is satisfying the given constraint. For calculating $f(n)$ which is the maximum value with n coins we are either taking $(cn+f(n-2)$ or $f(n-1))$. So we are making sure that we are not taking coins that lie beside each other.

Option B, D – this is not correct as it is not satisfying the given constraint "don't ever pick up two coins that lie beside each other".

Option C – this is not correct as it is not taking the value of the current coin.

Row of coins with values

5	1	2	10	6	2
c_1	c_2	c_3	c_4	c_5	c_6

Given, $f(0)=0,\ f(1)=5$

$f[n]$ be maximum total value with n coins:

Index	0	1	2	3	4	5	6
	0	5					

So, $f[1]$ is the maximum value with 1 coin.

$f[2]$ is the maximum value with 2 coins:

→ We have two choices either taking c_1 or c_2. We can't take both coins due to the given constraint. So $\max(f[1],c_2+f[0])=5$. With two coins maximum value is 5.

Index	0	1	2	3	4	5	6
	0	5	5				

$f[3]$ is the maximum value with 3 coins:

→ We have two choices either (taking $f[2]$ which is the maximum value with 2 coins) or ($c_3+f[1]$ which is the maximum value with 1 coin). So $\max(f[2],c_3+f[1])=7$.

Index	0	1	2	3	4	5	6
	0	5	5	7			

$f[4]$ is the maximum value with 4 coins:

→ We have two choices either (taking $f[3]$ which is the maximum value with 3 coins) or ($c_4+f[2]$ which is the maximum value with 2 coin). So, $\max(f[3],c_4+f[2])=15$.

Index	0	1	2	3	4	5	6
	0	5	5	7	15		

$f[5]$ is the maximum value with 5 coins:

→ We have two choices either (taking $f[4]$ which is the maximum value with 4 coins) or ($c_5+f[3]$ which is the maximum value with 3 coin). So $\max(f[4],c_5+f[3])=15$.

Index	0	1	2	3	4	5	6
	0	5	5	7	15	15	

$f[6]$ is the maximum value with 6 coins:

→ We have two choices either (taking $f[5]$ which is the maximum value with 5 coins) or $(c_6 + f[4]$ which is the maximum value with 4 coin).

So, $\max(f[5], c_6 + f[4]) = 17$.

Index	0	1	2	3	4	5	6
	0	5	5	7	15	15	17

So, maximum value with 6 coins is 17.

Hence, the correct option is (A).

78. (B)

Code Explanation:

int i = 0, j = 0, val = 1

for (i = 1; i < = n; i++)// n times Time complexity $= O(n)$

{

j = n;

if (i % 2 = =0)// out of n only n/2 times if will run

{

while (j < 1) {// log n times, since j = n}

val = val * j;

j = j/2; // j decrease in power of 2 leads to log n

}

}

}

Outer for loop $= O(n)$

Inner for loop $= O(\log n)$

Since nested loop

Time complexity

$$= O(n) \times O(\log n) = O(n \log n)$$

Hence, the correct option is (B).

79. (B)

Level - 1 is compared with root – 2 comparisons.

Level – 2 is compared with level – 1 elements – 4 comparisons.

$$2 + 4 + 8 + ... + 2^k \text{ where, } n = 2^k$$

$$\Rightarrow 2(1 + 2 + ... + 2^{k-1})$$

$$\Rightarrow 2(2^k - 1) = 2(n-1) = 2n - 2 = O(n)$$

Hence, the correct option is (B).

80. (C)

$$T(n) = 7T\left(\frac{n}{2}\right) + k \cdot n^2$$

Using master theorem $[n^{\log_2 7} > kn^2]$

$$T(n) = O\,(n^{\log_2 7})$$

Hence, the correct option is (C).

81. (A)

Both dynamic and Greedy algorithm find optimal substructure in the problem but only dynamic programming uses the bottom-up approach, whereas greedy algorithm uses top-down approach. So, S_1 is true so,

Hence, the correct option is (A).

82. 8

In the above graph, there are three stages with 2 vertices. Topological sort picks the element with zero in degree at any point of time. At each of two vertices stages, either the top vertex or the bottom vertex can be processed. So at each of these stages there will be two possibilities. Total number of possibilities $= 2 \times 2 \times 2 = 8$

Hence, the correct answer is 8.

83. (B)

Strassen matrix multiplication → Divide and conquer

Insertion sort → Decrease and Conquer

Gaussian Elimination $\rightarrow$ Transform and Conquer

Floyd shortest path algorithm $\rightarrow$ Dynamic programming

Hence, the correct option is (B).

84. (D)

$f(n) = 2^n \le g(n) = n!$ for $n \ge 4$

$f(n) = O(g(n))$

$h(n) = n^{\log n} \le f(n) = 2^n$ for $n \ge 2$

$h(n) = O(f(n))$

Hence, the correct option is (D).

85. (B)

$$S = (a_0, S_1)$$

$$S_1 = (a_1, a_2, a_3, ...a_{n-1})$$

Two possible cases arise :

1. a_0 is included in the max weight subsequence of S :

In this case,

$$X = \text{weight } (a_0, S_1) = a_0 + \frac{Y}{2}$$

2. a_0 is not included in the max weight subsequence of S :

In this case, $X = \text{weight } (S_1) = Y$

Since the value of a_0 can be anything (negative or $< \frac{Y}{2}$ in general) $\{\because a_i \in \mathbb{R}\}$, it is possible that $Y > a_0 + \frac{Y}{2}$.

The maximum possible weight of a subsequence of S is given by

$$X = \max\left(Y, a_0 + \frac{Y}{2}\right)$$

Hence, the correct option is (B).

86. (B)

$f(x)$ grows exponentially as a function of x

Hence, the correct option is (B).

87. (D)

In a connected graph, a vertex v is said to be an articulation point if by removing that vertex together with its edges the graph become disconnected. In the given graph there is no articulation point.

Hence, the correct option is (D).

88. (C)

There are $\left\lceil \frac{n}{2} \right\rceil$ number of leaf nodes. It requires $\left\lceil \frac{n}{2} \right\rceil - 1$ number of comparisons to find the maximum elements.

Hence, the correct option is (C).

89. (A)

$$T(n) = T(n-1) + T(n-2) - T(n-3)$$

Better way of solving is substitute

$$T(1) = 1$$

$$T(2) = 2$$

$$T(3) = 3$$

$$T(4) = T(3) + T(2) - T(1) = 4$$

$$T(5) = T(4) + T(3) - T(2) = 5$$

:

:

$$T(n) = T(n-1) + T(n-2) - T(n-3)$$

$$= n-1+n-2-n+3 = n$$

So order is O(n)

Hence, the correct option is (A).

90. (A)

Above code segment recurrence relation is

$$T(n) = T(n-1) + c$$

$$T(n) = T(n-1) + 1$$

$$T(0) = 1$$

$$T(n-1) = T(n-1-1) + 1$$

$$T(n) = [T(n-2)+1]+1 = T(n-2)+2$$

$$T(n-2) = T(n-2+1)+1$$
$$T(n) = [(T(n-3)+1)+1]+1$$
$$= T(n-3)+3$$
$$= T(n) = T(n-k)+k$$

Note: let k=n

Then $T(n) = T(0)+n = 1+n$

$\therefore$ O(n)

Hence, the correct option is (A).

91. **(C)**

P. $T(n) = 3T\left(\frac{n}{4}\right)+O(n^2)$

$$a = 3, b = 4, f(n) = cn^2$$

From case 3 of Master Method.

$$T(n) = \Theta(f(n)) = \Theta(n^2)$$

Q. $T(n) = T\left(\frac{n}{3}\right)+T\left(\frac{2n}{3}\right)+O(n)$

Use substitution with good guess

$$T(n) = n\log n$$

$$T\left(\frac{n}{3}\right)+T\left(\frac{2n}{3}\right)+\frac{1\text{n}}{3} = \frac{n}{3}\log\left(\frac{n}{3}\right)+$$
$$\frac{2n}{3}\log\left(\frac{2n}{3}\right)+\frac{1n}{3}$$

$$= \frac{n}{3}(\log n-\log 3)+\frac{2n}{3}(\log(2n)-\log 3)$$
$$+\frac{n}{3}$$

$$= \frac{n}{3}\log n-\frac{n}{3}\log 3+\frac{2n}{3}(\log 2n)$$
$$-\frac{2n}{3}\log 3+\frac{n}{3}$$

$$= \frac{n}{3}\log n+\frac{2n}{3}(\log 2+\log n)-\frac{n}{3}\log 3$$
$$-\frac{2n}{3}\log 3+\frac{n}{3}$$

$$= n\log n+\frac{2n}{3}-n\log 3+\frac{n}{3} = n\log n$$

R. $T(n) = \sum_{n=1}^{\log n}\left(\frac{n}{2}\right)$

$$= \frac{n}{2}\times\log_2 n = O(n\log n)$$

Assume $n = 2^k$

$\Rightarrow$ $\log n = k$

$$= \sum_{n=1}^{k}\left(\frac{n}{2}\right) = k\frac{n}{2}$$

Hence, the correct option is (C).

92. **(A)**

$$T(n) = T(n/5)+T(7n/10)+n$$

$$= \sum_{i=0}^{\log\frac{n}{5}} \cdot n = \Theta(n)$$

$$n = \left(\frac{n}{10}\right)$$

Hence, the correct option is (A).

93. **(A)**

Given : $g_1(n) = \begin{cases} n^3 \text{ for } 0 \le \text{n} \le 10{,}000 \\ n^2 \text{ for n} \ge 10{,}000 \end{cases}$

$$g_2(n) = \begin{cases} n \text{ for } 0 \le n \le 100 \\ n^3 \text{ for } n > 100 \end{cases}$$

For n> 10,000,

$$g_1(n) = 0(n^2)$$
$$g_2(n) = 0(n^3)$$

We can conclude

$$g_1(n) = 0(g_2(n))$$

Function $g_1(n)$ is said to be Big-Oh of $g_2(n)$ if

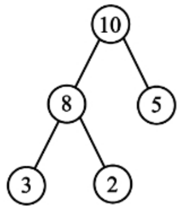
$g_1(n) \le cg_3(n)$, for $c > 0$ and

$$n \ge n_0$$
$$n_0 > 0$$

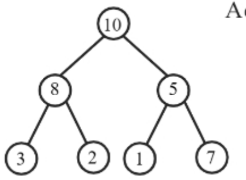
Hence, the correct option is (A).

94. **(D)**

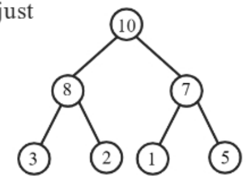
Max- heap using priority Queue



Insert 1 and 7



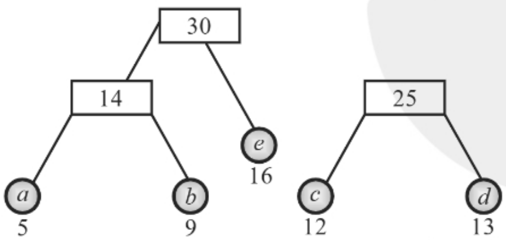
Adjust

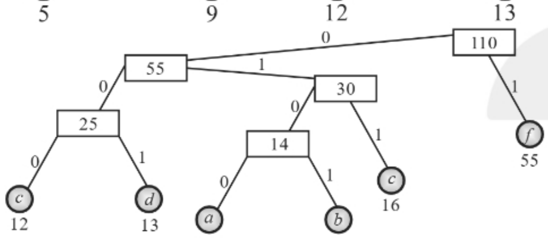


Hence, the correct option is (D).

95. **2.12**

a	*b*	*c*	*d*	*e*	*f*
5	9	12	13	16	55





Huffman Tree.

a	0100	(4 bit)
b	0101	(4 bit)
c	000	(3 bit)
d	001	(3 bit)
e	011	(3 bit)
f	1	(1 bit)

Average length

$$= \frac{5\times4+9\times4+12\times3+13\times3+16\times3+55\times1}{110}$$

$= 2.12$

Hence, the correct answer is 2.12.

96. **2.428**

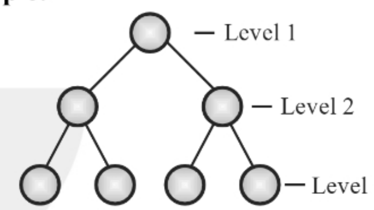
Average number of Comparison in Binary search with n keys in array can be calculated by a simple trick.

Make Almost complete BST, compute level wise comparisons for example, at level 1, one comparison on each element, at level 2, 2 comparisons on each element. At level 3, 3 comparisons on each element and so on.

Then Calculate Average by

$$\frac{\text{Total number of Comparisons}}{\text{Number of Elements}}$$

Example:-



Average number of comparisons

$$= \frac{1\times1+2\times2+3\times4}{7} = \frac{1+4+12}{7} = 2.428$$

Hence, the correct answer is 2.428.

97. **270**

First we draw table

Item	W_i	V_i
I_1	5	30
I_2	10	20
I_3	20	100
I_4	30	90
I_5	40	160

Taking value per weight ratio i.e., $P_i = V_i / W_i$

Item	W_i	V_i	$P_i = \frac{V_i}{W_i}$
I_1	5	30	6

I_2	10	20	2
I_3	20	100	5
I_4	30	90	3
I_5	40	160	4

$\rightarrow$Now arrange the value P_i in decreasing order

Item	W_i	V_i	P_i
I_1	5	30	6
I_3	20	100	5
I_5	40	160	4
I_4	30	90	3
I_2	10	20	2

I_1	I_3	$\frac{7}{8}\ I_5$	

$W = 60$ 55 35 0

i- is the outer loop & j–is the inner loop so the total time complexity is $= O(n * \log_2 n)$

Hence, the correct option is (D).

❖❖❖❖