Q.1) Max Marks: 1

The number of perfect matching in a complete graph with 4 vertices is _____

Correct Answer

Solution: (3)

Answer: 3

Explanation:

Number of perfect matching in complete graph $K_{2n} = \frac{2n!}{2^n n!} = (2n-1)(2n-3)(2n-5)\ldots 1$

For $K_4 = \frac{4!}{2^2 2!} = 3$

Q.2) Max Marks: 1

Let G be an undirected graph with n vertices. If G is isomorphic to its own complement G', then how many edges must G have?

A n(n+1)/2

B n(n-1)/2

C n(n-1)/4 — Correct Option

Solution: (C)

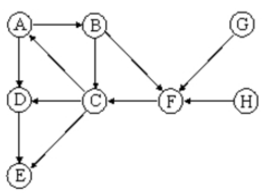
Let e be the number of edges of G.
Since G and $\overline{G}$ are isomorphic, it follows that the number of edges of $\overline{G}$ is also e.
On the other hand, the sum of the number of edges of G and $\overline{G}$ is equal to the number of edges of K_n, which is $\binom{n}{2} = \frac{n(n-1)}{2}$. Therefore, we have

$$2e = \frac{n(n-1)}{2} \Rightarrow e = \frac{n(n-1)}{4}$$

D n(n+1)/4

Q.3) Max Marks: 1

Consider the following graph (G):



How many strongly connected components are there in the above graph?

A 2

B 5 — Correct Option

Solution: (B)

Answer: B
A directed graph is strongly connected if there is a path between all pair of vertices.

The strongly connected components are:

1. A-B-F-C
2. E
3. D
4. G
5. H

So in total, 5 strongly connected component.

C 3

D 1

Q.4)

Max Marks: 1

A simple graph is one in which there are no self-loops and each pair of distinct vertices is connected by at most one edge.

Let graph G be a simple graph on 8 vertices such that there is a vertex of degree 1, a vertex of degree 2, a vertex of degree 3, a vertex of degree 4, a vertex of degree 5, a vertex of degree 6 and a vertex of degree 7.

Which of the following can be the degree of the last vertex?

A 3

B 0

C 5

D 4 Correct Option

Solution: (D)

Answer: D

Explanation:

Sum of degrees of all vertices in a graph = 2×no. of edges

Let number of edges be E and the degree of last vertex be x.

Then, $1+2+3+4+5+6+7+x=2\times E$.

$28+x=2\times E$.

Now, putting in options we get answer (B)0 or (D)4

But one vertex of degree 7 means it is connected to all other vertices making degree 0 impossible.

So, degree must be (D) 4.

Q.5)

Max Marks: 1

Given a simple undirected graph 'X' with 10 vertices. Now, if 'X' has 5 equally sized connected components, then the maximum number of edges in graph 'X' is ________.

Correct Answer

Solution: (5)

Answer: 5

Explanation:

We have 'n' vertices and 'k' equally sized connected components.
Then each components has (n/k) vertices . And now we will check maximum edges in a particular components using formula of complete graph(nC2) . So here, maximum edges in 1 component = [n/k] [(n/k)-1] / 2 = x(say it)

And total edges will be -- = number of components* maximum edges in each component= k * x.

You can simplify more and you will get, maximum number of edges = [n *(n-k)] /2k.

Now put n=10, k=5
maximum edges= 10*5/2*5 = 5

Q.6)

Max Marks: 1

If a graph G has 15 edges and G' has 13 edges. Number of Vertices G has is ______ ?

Correct Answer

Solution: (8)

Answer: 8

Explanation:

We know that :

G U G' is Complete Graph **Kn**

Therefore G+G' = 15+13 = 28

Total number of edges in the complete graph is **nC2**

nC2 = (n(n-1))/2 = 28

By solving the equation, we get n = 8 or -7

Therefore, the number of vertices, **n = 8**

Q.7)

For a simple graph G with 9 vertices and two components, the maximum number of edges possible is _____.

Max Marks: 1

Correct Answer

Solution: (28)

Answer: 28

Explanation:

No. of Vertices in component 1	No. of ~~Components~~ vertices in Comp-2	Max Possible edges
8	1	$E(K_8) + 0 = 28$
7	2	$E(K_7) + E(K_2) = 22$
6	3	$E(K_6) + E(K_3) = 18$
5	4	$E(K_5) + E(K_4) = 16$

A simple graph with n vertices and k components has at most (n-k)*(n-k+1)/2 edges. So the given graph can have at most (9-2)*(9-2+1)/2=28 edges.

Q.8)

K_n is bipartite when n is ______

Max Marks: 1

Correct Answer

Solution: (2)

Answer: 2

Explanation:
Let G be a complete graph and G' is corresponding bipartite graph.
In bipartite graph there are two groups of vertices. Let us call that group as group A, B respectively.
Let a random edge 'e' connect two vertices V1 and V2.

Bipartite property says there is no edge 'e' for which V1 and V2 belongs to the same group of vertices i.e. no edge in graph for which both V1 and V2 belongs to A or both belongs to B.

Obviously in complete graph there is an edge which connects every pair of vertices.
So a complete graph is bipartite only if it is complete graph of two vertices.

Q.9)

For a forest with v vertices and k components, what is the number of edges?

Max Marks: 1

A (v+1)−k

B (v+1)/2−k

C v−k

Correct Option

Solution: (c)

Answer: C

Explanation:

Forest is nothing but disjoint union of trees.

So no of edges in 'k' components of 'v' vertices is nothing but $\sum_{i=1}^{k} v_i - 1$,

where v_i is the no. of vertices in the i^{th} component.

$\sum_{i=1}^{k} v_i = v$.

So, $\sum_{i=1}^{k} v_i - 1 = v - k$

D v+k

Q.10)

Max Marks: 1

A sequence d is graphic if there is a simple undirected graph with degree sequence d.
Which one of the following sequences is graphic?

A (2, 3, 3, 4, 4, 5)

B (1, 3, 3, 3)

C (2, 3, 3, 4, 5, 6, 7)

D (2, 3, 3, 3, 3) Correct Option

Solution: (D)

Answer: D

Explanation:

Since degree of vertices is given let us first calculate Sum of degree of vertices.

Option A)
$\sum d(2,3,3,4,4,5)=21$

Option B)
$\sum d(1,3,3,3)=10$

Option C)
$\sum d(2,3,3,4,5,6,7)=30$

Option D)
$\sum d(2,3,3,3,3)=14$

Option A is ruled out sum of degree of vertices must be even

Option C ruled out Maximum degree cannot be 7 in 7 vertex simple graph.

Arrange B in Non decreasing order d(3,3,3,1)

Now if we remove one vertices degree of other will decrease by 1

d(2,2,0)

Similarly, d(1,-1) degree cannot be negative so option C is also not possible.

D) Arranging it in non decreasing order d(3,3,3,3,2)

d(2,2,2,2)

d(1,1,2)=d(2,1,1)

d(0,0)

Option D is answer

Q.11)

Max Marks: 2

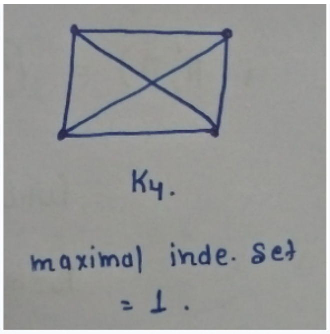
Consider the following statements:
S1: Maximal Independence Number is the cardinality of maximal independence set
S2: Independence set V of graph G is set in which no vertex of the set have a direct edge between them.
Which of the following is correct?

A Maximal Independence Number of a complete graph is n-1

B Maximal Independence Number of a complete bipartite graph is n / 2

C Maximal Independence Number of a complete graph is 1 Correct Option

Solution: (C)
Answer: C
Explanation:
The maximal Independence Number of a complete graph is 1
In complete graph, all vertex have a direct edge between them. so maximal independent set must be 1.
Pick any vertex v from complete simple graph and then no other vertex can be part of the set because v is connected to all other vertices.



D Maximal Independence Number of complete graph is ≥ $n/2$

Q.12) Max Marks: 2

Consider an undirected graph with n vertices, vertex 1 has degree 1, while each vertex 2,3......, n – 1 has degree 4. The degree of vertex n is unknown. Which of the following statement must be TRUE?

A **Vertex n has degree 1.**

B **Graph is connected.**

C **There is a path from vertex 1 to vertex n.** Correct Option

Solution: (C)

Answer: C

Explanation:
A) False.
By handshaking lemma you can argue that degree of n must be odd but it doesn't mean it will be 1 always.
For example 1,4,4,4,4,3 is a valid degree sequence.

B) False.
Suppose the graph has 2 components as degree sequence G1 = 1,4,4,4,4,3 and G2= 4,4,4,4,4, which means, it is not always true that graph is connected.

C) True
Vertex 1 and vertex n should be a vertex of same component of graph otherwise it will not follow handshaking lemma. Graph is undirected and vertex 1 and n are connected so, there must be a path from 1 to n.

D) False
Not every spanning tree will contain the edge joining 1 to n.

D **Spanning tree will include the edge connecting vertex 1 and n.**

Q.13) Max Marks: 2

Consider a complete bipartite graph with 'm' and 'n' vertices. If m = 4, n = 4, then the number of spanning trees of graph are ________.

Correct Answer

Solution: (4096)

Answer: 4096

Explanation:

If G is complete Bipartite Graph $K_{p,q}$, then number of spanning tree in G are
$t(G) = p^{q-1}q^{p-1}$
m=4,n=4 hence $4^3 * 4^3 => 4096$

Q.14) Max Marks: 2

The maximum number of edges in an n node undirected graph with self-loops is?

A **n(n-1)/2**

B **n/2**

C **(n+1)/2**

D **n(n+1)/2** Correct Option

Solution: (D)

Answer: D

Explanation:

If the graph is without self-loop,

then to determine the maximum no,. of edge

We need to select 2 node from n nodes i.e. ${}^nC_2$ or $\frac{n(n-1)}{2}$

But, The question tells us to find out the maximum no. of edge of a graph without self-loop

Now, Every vertex is having $(n+1)$ edge in an undirected graph having $n-$ nodes & with self-loop

(Assuming every vertex is having a self-loop)

If we take a universal vertex which is having a degree $(n-1)$ & we know that a self-loop is contributing $+2$ (both in-degree & out-degree) degree in a universal or dominating vertex.

As the graph has n- vertex, $\therefore$ there can be maximum n-loops.

$\therefore$ Maximum no. of edges $= n + {}^nC_2$

$$= n + \left\{\frac{n!}{2! \times (n-2)!}\right\}$$

$$= n + \left\{\frac{n \times (n-1) \times (n-2) \times \ldots\ldots}{2! \times (n-2)!}\right\}$$

$$= n + \frac{n(n-1)}{2}$$

$$= \frac{n(n-1)+2n}{2}$$

$$= \frac{n(n-1+2)}{2}$$

$$= \frac{n(n+1)}{2}$$

Q.15)

Max Marks: 2

The largest possible number of vertices in a graph G with 35 edges and all vertices are of degree at least 3 is :

A 24

B 25

C 23

Correct Option

Solution: (C)

Answer: C

Explanation:

The Handshaking theorem states that,

$$\sum_{v \in V} deg(v_i) = 2 \times |E|$$

Which means, Each edge contributes twice to the sum of the degrees of all vertices.

Sum of degree of vertices $= 2 \times Edges$

Now, the graph has 35 Edges.

$\therefore 2 \times Edges = 2 \times 35 = 70$ is the Maximum Sum of degree of the vertices.

We don't know the number of vertices.

So, we assumed the total number of vertices will be n

Condition given is "All vertices are of degree at least 3"

Sum of the degree of vertices $= 3 \times n$ [$\because$ All vertices are of at least degree 3]

$\therefore$ Minimum sum of degree of vertices will be $3n$ which will be less than or equal to 70 because 70 is the Maximum Sum of degree of vertices.

$\therefore 3 \times n \leq 2 \times (35)$

Or, $n \leq \frac{70}{3}$

Or, $n \leq 23.33$

$\therefore$ Largest possible number of vertices in the graph must be 23
as we can't take the fractions & will take only the whole number.

D 26