①. Profit and Loss:-

① Cost Price (CP) = కొన్న ధర

② Selling Price (SP) = అమ్మకపు ధర.

Profit (or) Gain = SP−CP i.e. Profit = SP > CP

Loss = CP−SP i.e. Loss = SP < CP.

③ Profit % = $\frac{SP-CP}{CP} \times 100$ (or) $\frac{Profit}{CP} \times 100$

④ Loss % = $\frac{CP-SP}{CP} \times 100$ (or) $\frac{Loss}{CP} \times 100$

⑤ When shopkeeper earns Profit.

$$CP = \left(\frac{100}{100+P\%}\right) \times SP$$

$$SP = \left(\frac{100+P\%}{100}\right) \times CP$$

⑥ When shopkeeper incurs loss.

$$CP = \left(\frac{100}{100-L\%}\right) \times SP$$

$$SP = \left(\frac{100-L\%}{100}\right) \times CP$$

⑥ If a Profit earned by selling an article is 25% then SP = 125% of CP.

⑦ If an article is sold at a loss of 30% then SP = 70% of CP.

⑧ A shopkeeper sells two similar items A and B. If A is sold at a gain (or) Profit of $x\%$ and B is sold at a loss of $x\%$ then

$$\text{loss } \% = \left[\frac{\text{common loss and gain } \%}{10}\right]^2$$

$$= \left(\frac{x}{10}\right)^2 \text{ (or) } \left(\frac{x^2}{100}\right)$$

Scanned with CamScanner

(9) A trader sells goods at cost Price but uses a weight of x kg instead of y kg (false weights) and makes Profit. This Profit can be calculated using the formula:

True weight − False weight = Errors.

$$\text{Gain (or) Profit \%} = \left[\frac{\text{Error}}{(\text{True weight} - \text{Error})} \times 100\right]\%$$

(10) If CP of X articles is equal to SP of Y articles then Profit can be calculated using the formula:

(a) CP of X = SP of Y

(b) No. of X articles > No. of Y articles

$$\text{Profit \%} = \frac{\text{No. of X articles} - \text{No. of Y articles}}{\text{No. of Y-articles}} \times 100.$$

(11) If a seller makes x% above CP and offers a discount of y% then Profit % (or) loss % can be calculated using the formula:

$$\boxed{\text{Profit (or) Loss \%} = (X - Y) - \frac{XY}{100}}$$

Discount:-

(1) $$\text{Discount \%} = \frac{\text{Discount}}{\text{Marked Price}} \times 100$$

(2) If "a" and "b" are two successive discount %, then single equivalent discount % is given as:

$$\boxed{\text{Single equivalent discount \%} = \left[(a+b) - \frac{ab}{100}\right]}$$

Scanned with CamScanner

②. Number System & Divisibility:-

① Geometric Progression (GP):-

a, ar, ar^2, ar^3---- are said to be G.P.

Here, a is first term

r is common ratio

* n^{th} term $= ar^{(n-1)}$
* If $r<1$, Sum of n terms $= \frac{a(1-r^n)}{(1-r)}$
* If $r>1$, Sum of n terms $= \frac{a(r^n-1)}{(r-1)}$

② Arithmetic Progression (AP):-

$a, a+d, a+2d, a+3d$, --- are said to be in AP.

Here, a is first term

d is common difference

$\boxed{l \text{ is last term}}$

① n^{th} term $= a+(n-1)d$

② Sum of n terms $= \frac{n}{2}[2a+(n-1)d]$ (or) $\frac{n}{2}[a+l]$

* $1+2+3+\cdots+n = \frac{n(n+1)}{2}$
* $(1^2+2^2+3^2+\cdots+n^2) = \frac{n(n+1)(2n+1)}{6}$
* $(1^3+2^3+3^3+\cdots+n^3) = \left[\frac{n(n+1)}{2}\right]^2$

Basic Formulaes:-

① $(a+b)^2 = a^2+b^2+2ab$

② $(a-b)^2 = a^2+b^2-2ab$

③ $(a+b)(a-b) = (a^2-b^2)$

④ $(a^3+b^3) = (a+b)(a^2+b^2+ab)$

⑤ $(a^3-b^3) = (a-b)(a^2+b^2-ab)$

⑥ $(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$

⑦ $(a^3+b^3+c^3-3abc) = (a+b+c)(a^2+b^2+c^2-ab-bc-ac)$

Scanned with CamScanner

Difference between AP and GP:-

AP:- It is the sequence of numbers in which each term after first is obtained by adding a constant to preceding term. The constant term is called common difference.

GP:- It is a sequence of non-zero numbers. The ratio of any term and its preceding term is always constant.

Divisibility of Numbers:-

① **No. divisible by 2:**
Unit digits = 0, 2, 4, 6, 8
Ex:- 42, 66, 98, 1124

② **No. divisible by 3:**
Sum of digits divisible by 3.
Ex:- 267 = (2+6+7) = 15
∴ 15 is divisible by 3.

③ **No. divisible by 4:**
No. formed by last 2 digits is divisible by 4.
Ex:- 832
The last 2 digits is divisible by 4. Hence 832 is divisible by 4

④ **No. divisible by 5:**
Unit digit is either 0 (or) 5
Ex:- 50, 20, 55, 65 etc

⑤ **No. divisible by 6:**
* The Number is divisible by both 2 and 3

Ex:- 168.
Last digit 8 → 8 is divisible by 2
Sum of digits = 1+6+8 = 15
∴ 15 is divisible by 3
Hence, 168 is divisible by 6.

⑥ **No. divisible by 12:**
No. is divisible by 3 and 4.
Ex:- 1932
Sum of digits = 1+9+3+2 = 15
∴ 15 is divisible by 3
Last 2 digits 32. 32 is divisible by 4.
∴ 1932 is divisible by 12

Scanned with CamScanner

7 No. divisible by 11:-

If the difference b/w sum of digits at even places and odd places is either 0 or divisible by 11.

Ex:- 4527039

Digits on even places = 4+2+0+9
= 15

Digits on odd places = 5+7+3 = 15

Difference b/w even & odd = 15-15
= 0.

∴ Number is divisible by 11

Types of No.'s	Definition	Example	Points to Remember
① Natural Numbers	Numbers used for counting and ordering	1,2,3,4,---	
② Whole numbers	All counting numbers along with zero form a set of whole numbers	0,1,2,3,4,---	① Any natural no. is a whole number. ② Zero is a whole no. which is not a natural number
③ Integers	Counting Numbers + Negative Counting Num + Zero, all are integers	-2,-1, 0,1, 2,3,----	
④ Even Numbers	No. divisible by 2	0,2,4,6,8---	
⑤ odd Numbers	No. not divisible by 2	1,3,5,7,9---	
⑥ Prime Numbers	A number having exactly 2 factors i.e. 1 & itself	2,3,5,7,11,--	
⑦ Composite Numbers	Natural numbers which are not Prime numbers	4,6,8,9,10---	
⑧ Co Primes	Any two natural numbers x and y are co-prime if their HCF is 1.	(4,5), (7,8), (10,11)---	

Quick Tips and Tricks:-

(1) HCF of 2 numbers is 1, then the numbers are said to be Co-Prime.

(2) Sum of first n odd numbers $= \boxed{n^2}$.

(3) Sum of first n even numbers $= \boxed{n(n+1)}$.

(4) Even numbers divisible by 2 can be expressed as $\boxed{2n}$, where n is an integer otherthan zero.

(5) Odd numbers which are not divisible by 2 can be expressed as $\boxed{2(n+1)}$, where n is an integer

(6) Dividend $= [(\text{Divisor} + \text{Quotient})] + \text{Remainder}$.

(7) If dividend $= a^n + b^n$ (or) $a^n - b^n$

(i) If n is even : $a^n - b^n$ is divisible by $(a+b)$.

(ii) If n is odd : $a^n + b^n$ is divisible by $(a+b)$.

(iii) $a^n - b^n$ is always divisible by $(a-b)$.

(8) To find the unit digit of number which is in the form a^b (Ex:- 7^{105}, 9^{125}).

(9) If b is not divisible by 4:

(i) Divide by 4, if it is not divisible then find the remainder of b, when divided by 4.

(ii) Units digit $= a^r$ where r is the remainder.

(10) If b is multiple of 4:

Unit digit is 6: When even numbers 2,4,6,8 are raised to multiple of 4.

Unit digit is 1: When odd numbers 3,7 and 9 are raised to multiple of 4.

Scanned with CamScanner

Unit digits Simple Method:-

Numbers	Powers 1	2	3	4	Cyclic
2	2	4	8	6	4
3	3	9	7	1	4
4	4	6	4	6	2
5	5	5	5	5	1
6	6	6	6	6	1
7	7	9	3	1	4
8	8	4	2	6	4
9	9	1	9	1	2

$\boxed{l^x}$ l = last number

$l = 2,4,6,8$ UD → 6

$l = 3,7,9$ UD → 0

Rough work:-

$2^1 = 2$
$2^2 = 4$
$2^3 = 8$
$2^4 = 16 \Rightarrow UD = 6$

$2^5 = 2^4 \times 2 = 32 \Rightarrow UD = 2$
$2^6 = 2^5 \times 2 = 64 \Rightarrow UD = 4$
$2^7 = 2^6 \times 2 = 128 \Rightarrow UD = 8$
$2^8 = 2^7 \times 2 = 256 \rightarrow UD = 6$
Again Repeated

$3^1 = 3$
$3^2 = 9$
$3^3 = 27 \Rightarrow UD = 7$
$3^4 = 3^3 \times 3 = 81 \Rightarrow UD = 1$

$3^5 = 3^4 \times 3 = 3$
$3^6 = 9$
$3^7 = 7$
$3^8 = 1$
Again Repeated

$4^1 = 4$, $4^2 = 16 \Rightarrow UD = 6$

$4^3 = 4^2 \times 4 = UD = 4$
$4^4 = 4^3 \times 4 = 256 \Rightarrow UD = 6$

$5^1 = 5$
$5^2 = 25 \Rightarrow UD = 5$

$6^1 = 6$
$6^2 = 36 \Rightarrow UD = 6$

$7^1 = 7$
$7^2 = 49 \Rightarrow UD = 9$
$7^3 = 343 \Rightarrow UD = 3$
$7^4 = 7^3 \times 7 \Rightarrow UD = 1$

$7^5 = 7^4 \times 7 \Rightarrow UD = 7$
$7^6 = 7^5 \times 7 \Rightarrow UD = 9$
$7^7 = 7^6 \times 7 \Rightarrow UD = 3$
$7^8 = 7^7 \times 7 \Rightarrow UD = 1$
Again Repeated

$8^1 = 8$
$8^2 = 64 \Rightarrow UD = 4$
$8^3 = 8^2 \times 8 \Rightarrow UD = 2$
$8^4 = 8^3 \times 8 \Rightarrow UD = 6$

$8^5 = 8^4 \times 8 \Rightarrow UD = 8$
$8^6 = 8^5 \times 8 \Rightarrow UD = 4$
$8^7 = 8^6 \times 8 \Rightarrow UD = 2$
$8^8 = 8^7 \times 8 \Rightarrow UD = 6$
Again Repeated

$9^1 = 9$
$9^2 = 81 \Rightarrow UD = 1$

$9^3 = 9^2 \times 9 \Rightarrow UD = 9$
$9^4 = 9^3 \times 9 \Rightarrow UD = 1$

Scanned with CamScanner

3. PERCENTAGES:-

1. $y\%$ is expressed as $\frac{y}{100}$
2. To find Percent of $\frac{x}{y} = \left(\frac{x}{y} \times 100\right)\%$
3. x is what % of $y = \frac{x}{y} \times 100$

   What % of x is $y = \frac{y}{x} \times 100$
4. x is what % more (or) less (exceed) than $y = \frac{x-y}{y} \times 100$

   if answer comes $-ve$ it is less

   $+ve$ it is more

---

1 Prices of Goods:-

1. If the Price of goods increased by R% then the reduction in consumption so as not to increase the expenditure can be calculated using

$$\boxed{\left[\frac{R}{(100+R)} \times 100\right]\%}$$

2. If the Price goods decreases by R% then the increase in consumption so as not to decrease the expenditure can be calculated using

$$\boxed{\left[\frac{R}{(100-R)} \times 100\right]\%}$$

2 Numerical on Population:-

Population of a city at Present is P and it increases at the rate of R% Per annum.

(i) To find Population after n years $= \boxed{P\left[1+\frac{R}{100}\right]^n}$

(ii) To find Population n years ago $= \boxed{\dfrac{P}{\left[1+\frac{R}{100}\right]^n}}$

Scanned with CamScanner

(iii) After n years Population increases, then $\boxed{\left[1+\frac{R}{100}\right]}$ is used

③ Numerical on Depreciation:-

Present Value of machine is M, if it depreciates at the rate of R% Per annum.

(i) To find Value of machine after n yrs $= \boxed{P\left[1-\frac{R}{100}\right]^n}$

(ii) To find Value of machine n years ago $= \boxed{\dfrac{P}{\left[1-\frac{R}{100}\right]^n}}$

(iii) After n years the value of machine decreases, then $\boxed{\left[1-\frac{R}{100}\right]}$ is used.

*** ** $\boxed{x\% \text{ of } y = y\% \text{ of } x}$

Eg:- 36% of 50 = ?

Interchange

$x\%$ of y = 50% of 36

$\Rightarrow$ 36% of 50 = 50% of 36

$\Rightarrow \frac{36}{100} \times 50 = \frac{50}{100} \times 36$

$\Rightarrow 18 = 18.$

So, condition satisfied

① Problems on Based on Salary:-

Base Value : 100% (or) Rs.100

Savings = Subtract

Spends = Addition

② % Based on Election (or) Votes:-

① Candidate-1 got 5000 Votes and won the election

② Candidate-1 won the election by a $\boxed{\text{majority}}$ of 5000 votes.

Note:- Majority } Winner − Losser.
By

* Total Votes Enrolled = 100%.

Scanned with CamScanner

4. ALLIGATIONS & MIXTURES:-

Alligation:- Ratio of alligation enables us to find the ratio in which two (or) more ingredients at a given Price must be mixed to Produce a resultant mixture of desired Price.

Mean Price:- It is the cost of unit quantity of mixture

Mixture:- Mixture is formed by mixing 2 (or) more quantities. It can be expressed in the form of % (or) ratio.

→ 10% of sugar in water.

→ A Solution of water and sugar is 12:20, which means water : Sugar = 12:20.

According to the rule of Alligation:

When two ingredients are mixed,

$$\frac{\text{Cheaper Quantity }(x)}{(\text{Quantity of } y)} = \frac{(\text{CP of } y) - (\text{Mean Price})}{(\text{Mean Price}) - (\text{CP of } x)}$$

① If a vessel contains A litres of milk and if B litres of milk is withdrawn and replaced by water, and again if B litres of mixture is withdrawn and this operation is replaced n times in all then

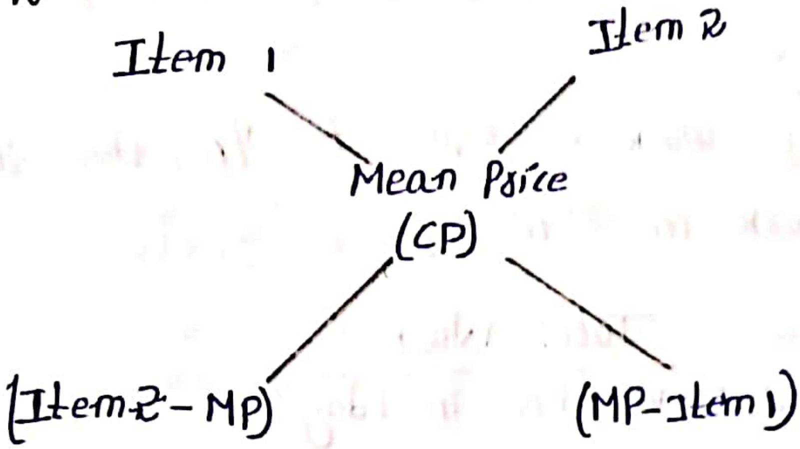
$$\frac{(\text{Quantity of milk left after } n\text{th operation})}{(\text{Initial quantity of milk})} = \left[\frac{(A-B)}{A}\right]^n$$

Scanned with CamScanner

② Quantity of milk left after nth operation $= A \times \left[\frac{A\left(1-\frac{A}{n}\right)}{A}\right]$

Simplified formula to calculate quantity of milk left after nth operation $= A\left[\left(1-\left(\frac{B}{A}\right)\right)^n\right]$

Rule of Alligation:-



Ratio = $\frac{(\text{Item 2} - MP)}{(MP - \text{Item 1})}$

*** Item 1 should be the lowest value (or) quantity and Item 2 should be the highest quantity.

* Mean Price should cannot contain Profit (or) loss Value.

* Mean Price (MP) is the final cost Price (CP).

Problems on Removed & Replaced :-

$$\frac{\text{Final Quantity}}{\text{Initial Quantity}} = \left(1-\frac{x}{c}\right)^t$$

where, c = Capacity of Tank

x = how much of mixture will be removed

t = No. of times.

Scanned with CamScanner

5. TIME & WORK :-

① **Work from days :**

If a Person can do a work in "n" days then Person's 1 day work $= \frac{1}{n}$.

② **Days from Work :**

If a Person's 1 day work is equal to $1/n$, then the Person can finish the work in "n" days.

③ Numbers of days $= \dfrac{\text{Total work}}{\text{Workdone in 1 day}}$.

① **Ratio** :- If 'A' is the 'x' times as good as workman as 'B' then :

(a) Ratio of workdone by A and B in equal time $= x:1$

(b) Ratio of time taken by A & B to complete the work $= 1:x$

This means that A takes $\frac{1}{x}$th time as that of B to finish same amount of work.

Eg :- If A is twice good a workman as B, then it means that :

(a) A does twice as much work as done by B in equal time (i.e) $A:B = 2:1$

(b) A finishes his work in half the time as B.

Scanned with CamScanner

② Combined Work:-

(i) If 'A' and 'B' can finish the work in 'x' & 'y' days respectively then.

* A's 1 day work $= \frac{1}{x}$, B's 1day work $= \frac{1}{y}$

(A+B)'s 1day work $= \frac{1}{x} + \frac{1}{y} = \left(\frac{x+y}{xy}\right)$

Together they finish the work in $\frac{xy}{(x+y)}$ days

(ii) If A, B and C complete the work in x, y, z days respectively then:

(A+B+C)'s 1day work $= \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{(xy+yz+zx)}{xyz}$

Together they complete the work $\frac{xyz}{(xy+yz+zx)}$ days

(iii) If A can do a work in "x" days and if the same amount of work is done by A and B together in "y" days then

* A's 1day work $= \frac{1}{x}$

B's 1 day work $= \frac{1}{y} - \frac{1}{x} \Rightarrow \frac{x-y}{xy}$

(A+B)'s 1day work $= \frac{1}{y}$

B alone will take $\frac{xy}{x-y}$ days

(iv) If A and B together performs some part of work in 'x' days, B and C together perform it in 'y' days and C, A together perform it in 'z' days respectively then:

(A+B)'s 1 day work $= \frac{1}{x}$

(B+C)'s 1day work $= \frac{1}{y}$

(C+A)'s 1day work $= \frac{1}{z}$

$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 2$ (A+B+C)'s 1 day work

$\Rightarrow$ (A+B+C)'s 1 day work $= \frac{\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)}{2}$

(A+B+C) will together complete work in $\frac{2}{\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)}$ days

Scanned with CamScanner

(e) If A works alone then deduct A's work from the total work of B and C to find the time taken by A alone.

For A working alone, time required = A's work − (A+B+C)'s combine work

$$= \frac{2}{\left(\frac{1}{x}-\frac{1}{y}-\frac{1}{z}\right)} \Rightarrow \frac{2xyz}{(xy+yz-zx)} \text{ days}$$

Similarly,

If B works alone, then req time $= \frac{2xyz}{(-xy+yz+zx)}$

If C works alone, then req time $= \frac{2xyz}{(xy-yz+zx)}$

③ Man – Work – Hour related Problems:–

$\frac{MDH}{W}$ = Constant

$$\boxed{M_1 D_1 T_1 W_2 = M_2 D_2 T_2 W_1}$$

where, M = Number of Men | H = Number of Hours
D = No. of days | W = Amount of Workdone

② If men are fixed work is Proportional to time. If work is fixed, time is inversely Proportional to men. Thus,

$$\boxed{\frac{M_1 \times T_1}{W_1} = \frac{M_2 \times T_2}{W_2}}$$

(a) Work and time are directly Proportional to each other.
(b) No. of men and time are invesely Proportional to each other
(c) Work can be divided into equal Parts i.e., if a tasks is finished in 10days, in 1 day you will finish $\left(\frac{1}{10}\right)^{th}$ Part of work.

* A is m times as efficient as B and takes "D" day less than B, then the time required to complete the job together is given by $\boxed{T = m \times \frac{D^2}{(m^2-1)}}$

Scanned with CamScanner

6. TIME, SPEED and Distance:-

① Speed is defined as the distance travelled per unit time

$$\boxed{Speed = \frac{Distance}{Time}}$$

$\Rightarrow T = \frac{D}{S} \quad \Rightarrow D = S \times T.$

② If same distance "x" is travelled at two different speeds S_1 and S_2 then average speed (S_a) is

$$\boxed{S_a = \frac{2S_1S_2}{(S_1+S_2)}}$$

③ Two bodies A and B move between two points P & Q one starts from P and goes to Q while the other starts from Q and goes to P. They meet on the way and reach their destinations in time t_a & t_b respectively after meeting. Their speeds S_a & S_b are given by : $\boxed{\frac{S_a}{S_b} = \frac{\sqrt{t_b}}{\sqrt{t_a}}}$

④ When two bodies moves in opposite direction, their speeds are added to find the relative speed (S_R).

$$\boxed{S_R = S_1 + S_2}$$

⑤ When two bodies moves in same direction, their speeds are subtracted to find relative speed (S_R) : $\boxed{S_R = S_1 - S_2}$

⑥ If two bodies start moving towards each other at the same time from points A & B on crossing each other, if they take x and y hours in reaching B & A respectively then: $\boxed{\text{Speed of A : Speed of B} = \sqrt{y} : \sqrt{x}.}$

Conversion of Units:-

① Converting km/hr into m/s

$\frac{km}{hr} = \frac{1000m}{60 \times 60\, sec} = \frac{5}{18}\, m/s$

② Converting m/s into km/hr

$\frac{m}{sec} = \frac{18}{5}\, km/hr$

* To convert minutes into seconds, multiply by 60

To convert hours into seconds, multiply by 60 × 60

Scanned with CamScanner

7. BOATS & STREAMS :-

1. Stream :- The moving water of river is called Stream.

2. Still water :- The water which is not moving.

3. Upstream :- In water, the direction against (or) opposite the stream is known as upstream.

4. Downstream :- In water, the direction along (or) same the stream is known as downstream.

** If there's no mention of stream speed in the question, assume it to be speed of boat in still water.

5. Problems similar to boats and streams may also occur which include:

(a) Cyclist and Wind (where cyclist is related to boat while wind is related to stream)

(b) Swimmer and Stream (where swimmer is related to boat)

Formulas:-

1. If the speed of boat in still water is x km/hr and the speed of stream is y km/hr, the downstream (S_d) is $\boxed{(x+y) \text{ km/hr}}$

2. While going downstream, boat moves in the direction of flow of river. So, water stream increases the speed of boat. Hence, speeds are added while going downstream.

$\boxed{\text{Upstream Speed } (S_u) = (x-y) \text{ km/hr}}$

3. If boat takes "t" hours to move a certain place and come back again then

$$\boxed{\text{Distance b/w Places} = \frac{[t(x^2-y^2)]}{2x} \text{ Km}}$$

Scanned with CamScanner

4. While going upstream, boat moves in the direction of opposite to the flow of river. So, water stream decreases the speed of boat due to opposite flow. Hence, speeds are subtracted while going upstream.

In still water,

$\boxed{\text{Speed of boat}(x) = \frac{1}{2}[S_d + S_u]}$.

* If we see the above formula closely, we see that adding upstream and downstream speeds gives us $2x$, which is twice the speed of boat i.e. $[x+\cancel{y}+x-\cancel{y}] = 2x$

5. If you are given speed of boat upstream and downstream and are required to find out the speed of the current, the following concept is very useful:

$$\boxed{S_d - S_u = 2(\text{Speed of current})}$$

$$x+y-(x-y) = 2y.$$

i.e. $\boxed{\text{Speed of stream}(y) = \frac{1}{2}[S_d - S_u]}$

6. If a boat moves at x km/hr speed and covers the same distance up and down in a stream of speed y km/hr then average speed of boat is calculated by:

$$\text{Avg Speed} = \frac{S_d \times S_u}{\text{Speed in still water}} = \frac{(x+y)(x-y)}{x} \text{ km/hr}$$

7. If a boat takes time "t" hrs more than going upstream and than move downstream for same distance, then distance is given by:

$$\boxed{\text{Distance} = \frac{[(x^2-y^2)(t)]}{2y}} \text{ km}$$

8. If a boat moves to a certain distance downstream in "t_1" hrs and returns the same distance upstream in "t_2" hrs then

$$\boxed{\text{Speed of boat in still water} = y\frac{(t_2+t_1)}{(t_2-t_1)} \text{ km.}}$$

Scanned with CamScanner

8. PROBLEMS ON TRAINS:-

1. If the length of one train is P and the length of 2nd train is Q, the total distance to be covered is $\boxed{\text{"}(P+Q)\text{"}}$.

2. Finding Relative Speed:-

(i) Objects moving in same direction. $\xrightarrow{V_1}$ $\xrightarrow{V_2}$

Relative Speed = $V_1 - V_2$

(ii) Objects moving in opposite direction. $\xrightarrow{V_1}$ $\xleftarrow{V_2}$

Relative Speed = $V_1 + V_2$

3. If two trains of lengths P and Q move in opposite direction at V_1 m/s and V_2 m/s, then time taken by the trains to cross each other, can be calculated by

$$\boxed{\text{Time taken} = \frac{(P+Q)}{V_1+V_2}}$$

4. If two trains of lengths P and Q move in same direction, at V_1 m/s and V_2 m/s then time taken by the trains to cross each other, can be calculated by

$$\boxed{\text{Time taken} = \frac{(P+Q)}{V_1-V_2}}$$

Conversion of Units:-

1. Km/hr into m/s.

$1 \text{ Km/hr} = \frac{5}{18} \text{ m/s}$.

2. m/s into Km/hr

$1 \text{ m/s} = \frac{18}{5} \text{ Kmph}$

1. $\text{Speed} = \frac{\text{Distance}}{\text{Time}}$

2. Distance = Speed × Time

3. $\text{Time} = \frac{\text{Distance}}{\text{Speed}}$

Scanned with CamScanner

Quick Tips and Tricks:-

(1) Time taken by a train of length L meter to pass a signal post (or) standing man = Time taken by the train to cover L meter.

$$\boxed{\text{Time} = \frac{L}{\text{Speed}}}$$

A signal post (or) a standing man is considered to be the point object.

(2) The time taken by a train of length L_1 meter to pass a stationary object of length L_2 is basically the time taken by the train to cover (L_1+L_2) meters.

$$\boxed{\text{Time} = \frac{(L_1+L_2)}{\text{Speed}}}$$

(3) The time taken by a train of length L meter to pass a moving object of length L_2 is determined by considering the relative speed between moving objects

$$\boxed{\text{Time} = \frac{(L_1+L_2)}{R_s}}$$

where, R_s is relative speed b/w moving objects in same (or) opposite direction.

L_1 is the length of train.

L_2 is the length of moving object other than train

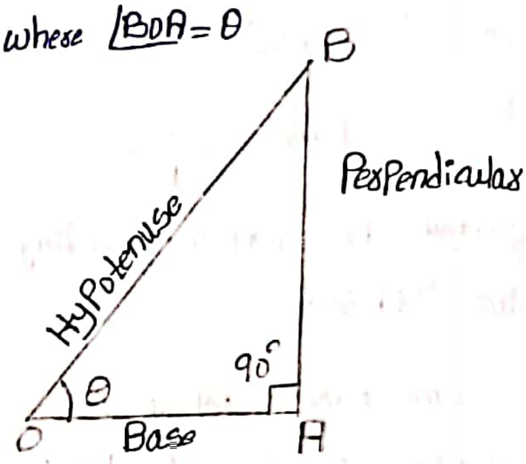
(4) Two trains start from two points P and Q at the same time and move towards each other. These trains take p and q seconds to reach points Q and P respectively, the relation b/w them is given by

$$\frac{(P\text{'s Speed})}{(Q\text{'s Speed})} = \frac{\sqrt{q}}{\sqrt{p}}$$

⑨. HEIGHTS AND DISTANCES:-

① Trigonometry:-

In a right angled ΔOAB, where $\angle BOA = \theta$



(i) $\sin\theta = \dfrac{AB}{OB} = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}}$

(ii) $\cos\theta = \dfrac{OA}{OB} = \dfrac{\text{Base}}{\text{Hypotenuse}}$

(iii) $\tan\theta = \dfrac{AB}{OA} = \dfrac{\text{Perpendicular}}{\text{Base}}$

(iv) $\text{cosec}\,\theta = \dfrac{OB}{AB} = \dfrac{1}{\sin\theta}$

(v) $\sec\theta = \dfrac{OA}{OB} = \dfrac{1}{\cos\theta}$

(vi) $\cot\theta = \dfrac{AB}{OA} = \dfrac{1}{\tan\theta}$

② Trigonometrical Identities:-

① $\sin^2\theta + \cos^2\theta = 1$

② $1 + \tan^2\theta = \sec^2\theta$

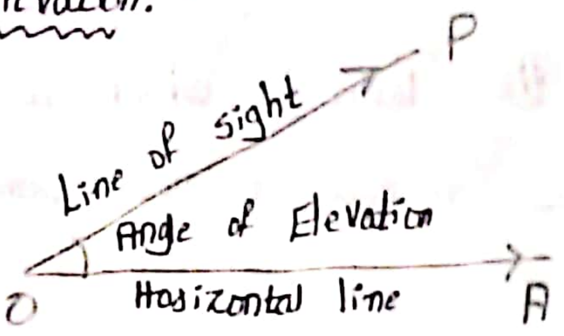
③ $1 + \cot^2\theta = \text{cosec}^2\theta$

③ Values of Trigonometric ratios:-

θ	$0°$	$\pi/6$ $(30°)$	$\pi/4$ $(45°)$	$\pi/3$ $(60°)$	$\pi/2$ $(90°)$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not define

Scanned with CamScanner

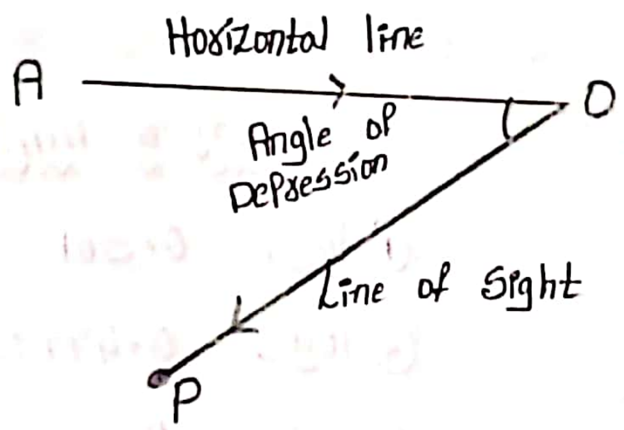
(4) Angle of Elevation:-



Suppose a man from a Point O looks up at an object P, placed above the level of his eye. Then the angle with the line of sight makes with the horizontal through O is called angle of elevation of P as seen from O.

$\therefore$ Angle of elevation of P from O = $\underline{|AOP}$.

(5) Angle of Depression:-



Suppose a man from a Point O looks down an object P, placed below the level of his eye then the angle which the line of sight makes with horizontal through O is called angle of depression of P as seen from O.

Scanned with CamScanner

(10). LOGARITHMS:-

Logarithm:- It is the Power to which a number must be raised in order to get some other number.

Common Logarithm:- Logarithms with base 10 are called common logarithms.

* Common logarithms are written as $\log_{10} x$ and if any expression is not indicated with a base, then base 10 is considered.

Natural logarithm:- Logarithms with base "e".

* Natural logarithms are written as $\log_e x$ and denoted as $\ln(x)$.

Properties of logarithm:-

① $\log_x x = 1$

② $\log_a 1 = 0$

③ $a^{\log_a x} = x$

④ $\log_a x = \dfrac{1}{\log_x a}$

⑤ $\log_a (x^P) = P(\log_a x)$

⑥ $\log_a x = \dfrac{\log_b x}{\log_b a} = \dfrac{\log x}{\log a}$

⑦ $\log_a \left[\dfrac{x}{y}\right] = \log_a x - \log_a y$

⑧ $\log_a (xy) = \log_a x + \log_a y$

Log of numbers (2-10):-

① $\log 2 = 0.301$

② $\log 3 = 0.477 \simeq 0.48$

③ $\log 4 = 0.60$

④ $\log 5 = 0.698 \simeq 0.7$

⑤ $\log 6 = 0.778 \simeq 0.78$

⑥ $\log 7 = 0.845 \simeq 0.85$

⑦ $\log 8 = 0.90$

⑧ $\log 9 = 0.954 \simeq 0.96$

⑨ $\log 10 = 1$

Scanned with CamScanner

* Logarithm of a number contains 2 Parts:

① characteristic ② Mantissa

① **Characteristic:**— It is an integral Part of logarithm.

Case-1:— If number is greater than 1.

In this condition, characteristic is considered as one less than the number of digits in the left of deciminal Point in the given number.

Ex:— 256.23

No. of digits to the left of decimal Point are 3. Hence, the characteristic is one less than no. of digits before decimal Points i.e. 2

Case-2:— If number is less than 1.

In this condition, characteristic is considered as one more than the number of zeros between decimal Point and first digit of number. It is negative and is denoted as $\bar{1}$ (or) $\bar{2}$

Ex:— 0.00735

No. of Zeros between decimal Point and first significant digit 7 are 2. Hence the characteristic is one more than number of Zeroes. i.e. $\bar{3}$.

② **Mantissa:**— It is the decimal Part of logarithm. Log table is used to find the mantissa

① Logarithms are opposite to exponentials which means logs are inverses of exponentials.

② Given logarithmic form can be converted into exponential form.

base = b, y = exponent, x = answer

Equals "x"

$$\log_b(x) = y$$

b raised to "y"

Exponential form : $b^y = x$

Logarithmic form : $\log_b(x) = y$

Scanned with CamScanner

Finding "log" without using calculator:-

① You should know all the Prime factors of given numbers

② Method-2:- Log of 4 to 9 can be easily determined if only the Value of log 2 & log 3 is remembered.

Method-1:- Log of composite number (x) = Sum of logarithms of its Prime factors.

Ex:- log of 9 = ?

Sol:- $\log 3 = 0.477$

log 9 = Sum of logarithms of its Prime factors

$\log 9 = \log 3 + \log 3$

$= 0.477 + 0.477$

log of $9 = 0.9542$

Method-2:-

$\log 2 = 0.301$

$\log 3 = 0.477$

$\log 4 = 2 \times \log 2 = 0.60206$

$\log 5 = 1 - \log 2 = 0.6989$

$\log 6 = \log 2 + \log 3 = 0.778$

$\log 7 = 0.84510 \rightarrow$ (Remember)

$\log 8 = 3 \times \log 2 = 0.9030$

$\log 9 = 2 \times \log 3 = 0.9542$

Scanned with CamScanner

(11). PERMUTATIONS & COMBINATIONS:-

Permutation:- The various ways of arranging a given number of things by taking some of all at a time.

* Permutation includes word formation, number formation, circular permutation etc.

* In permutation, objects are to be arranged in particular order. It is denoted by ^nP_r (or) $P(n,r)$

Ex:- Arrange the given 3 numbers 1,2,3 by taking two at a time.

Now these numbers can be arranged in 6 different ways (12, 23, 31, 13, 21, 32)

Here, 12 & 21, 13 & 31 and 23 & 32 donot mean the same because here order of numbers is important.

Combination:-

Each of different groups or selections formed by taking some or all number of objects.

* Combinations is used in different cases which include team / group / committee.

* In combination, objects are selected randomly and here order of objects doesn't matter. It is denoted by ^nC_r (or) $C(n,r)$.

Ex:- If we have to select 2 girls out of 3 girls x, y, z then find the number of combinations possible.

Now only 2 girls are to be selected and arranged. Hence, this is possible in 3 different ways (xy, yz, zx).

Here you cannot make a combination as xy and yx, because these combinations mean the same.

Scanned with CamScanner

Quick Tips and Tricks:-

① **Factorial $n!$** :- It is the Product of all Positive integer less than or equal to n.

Ex:- $4! = 4\times3\times2\times1 = 24$.

Theorem of Counting:-

① **Rule of Addition**:- If a first task is Performed in x ways and second task is Performed in y ways then either of two Operations can Performed in $(x+y)$ ways.

② **Rule of Multiplication**:-

If a first task is Performed in x ways and Second task is Performed in y ways, then both of two Operations can Performed in $(x\times y)$ ways.

* Suppose n different cakes are done in $a_1, a_2, ---- a_n$ different ways respectively independent of each other then

(i) Anyone of them can be done in $a_1+a_2+--+a_n$ ways (a_1 way or a_2 way $+---+ a_n$ ways)

(ii) All of them can be done in $a_1\times a_2\times --- a_n$ ways (a_1 way and a_2 way $+ --- + a_n$ ways)

All about Permutation:-

① $${}^nP_r = \frac{n!}{(n-r)!}$$

Here, "n" values must be higher than "r" value.

Ex:- ① ✓ ${}^4P_3$. Here $n>r$. $n=4, r=3$

② ✗ ${}^3P_4$. Here $n<r$.

Scanned with CamScanner

(2) Number of Permutations of n things, taken r at a time is given as follows:

$${}^{n}P_r = n(n-1)(n-2)(n-3)\cdots(n-r+1) = \frac{n!}{(n-r)!}$$

(3) If there are N balls and out of these B_1 balls are alike (different), B_2 balls are alike, B_3 balls are alike and so on B_r are alike of rth kind, such that
$(B_1 \text{ balls} + B_2 \text{ balls} + B_3 \text{ balls} + \cdots + B_r \text{ balls}) = N$ balls.

In such condition,

$$\text{No. of Permutation of N balls} = \frac{N!}{(B_1)! \times (B_2)! \times (B_3)! \cdots (B_r)!}$$

(4) If number of Permutations of n objects are all taken at a time, then ${}^{n}P_r = \frac{n!}{0!} = n!$

(5) If N different objects are to be arranged, then they can be arranged in <u>N! ways</u>.

(6) N number of objects are can be arranged around a circle in <u>(N-1)! ways</u>.

(7) Sometimes we have to solve Problems on Permutation considering the condition of Repetition.

Repetition:-

Number of Permutation of N objects taken "r" at a time when each selected object can be replaced any number of times is given as:

No. of Permutations $= n^r$.

Scanned with CamScanner

Restricted Permutation:-

The number of Permutations of "n" objects taken "r" at a time in which if k Particular objects are:

(a) Never included : $(n-k)P_r$

k are the number of objects not included.

(b) Always included : $(n-k)C_{r-k} \times r!$

K are the number of objects always included.

All about combinations:-

No. of combinations of n objects, taken "r" at a time is given as follows:

$$^nC_r = \frac{n!}{(n-r)!(r!)} = \frac{n(n-1)(n-2) \text{ --- } r \text{ factors}}{r!}$$

Important Formulaes:-

① $^nC_n = {^nC_0} = 1$

② $^nC_{n-1} = {^nC_1} = n$

③ $^nC_r = {^nC_{n-r}}$

④ $0! = 1$

⑤ $n! = n(n-1)!$

⑥ $^nP_r = \dfrac{n!}{(n-r)!}$

⑦ $^nC_r = \dfrac{^nP_r}{r!}$

Scanned with CamScanner

12. PROBABILITY:-

1. $\text{Probability} = \dfrac{\text{Sum of observations}}{\text{Possibility}}$

2. AND / OR
   $\times$ / $(-)$

Whereever we find "AND" in question we multiply and "OR" we subtract.

3. Assume that we have 3 balls. Our target is to pick 2 balls. How many chance we can able to pick 2 balls here.

① ② ③ → Formula

1-2
2-3
3-1 } 3 Possibilities

$$^3C_2 = \frac{3 \times \cancel{2}}{\cancel{2} \times 1} = 3$$

* $^{12}C_{10} = {}^{12}C_2$

  $^6C_4 = {}^6C_2$

  $^{14}C_{11} = {}^{14}C_3$

Here, we have $^{12}C_{10}$. Subtract $12-10=2$
So, $^{12}C_{10} = {}^{12}C_2$

$^6C_4$. Subtract $6-4=2$. So, $^6C_4 = {}^6C_2$

$^{14}C_{11}$. Subtract $14-11=3$. So, $^{14}C_{11} = {}^{14}C_3$

Scanned with CamScanner

(1) **COINS:-**

(1) 1 Coin : $\{H, T\} = 2 = 2^1$

(2) 2 Coin : $\{HH, HT, TH, TT\} = 4 = 2^2$

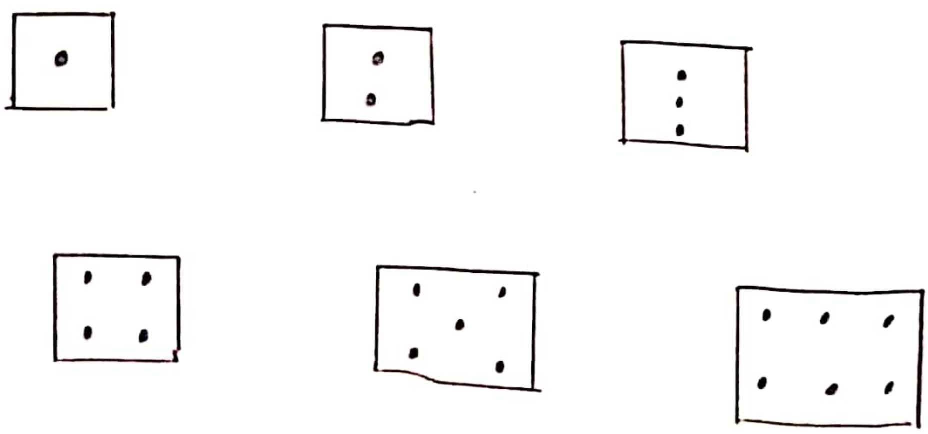
(3) 3 Coins : $\{HHH$ TTT
HHT TTH
HTH THT $= 8 = 2^3$
THH $HTT\}$

(4) 4 Coins : $\{HHHH$ $TTTT$
$HHHT$ $TTTH$
$HHTH$ $TTHT$
$HTHH$ $THTT$
$THHH$ $HTTT$
$HHTT$ $THHT$ $= 16 = 2^4$
$HTTH$ $HTHT$
$TTHH$ $THTH\}$

** 2 is common because we have only 2 chances while we are tossing i.e. (H, T). Power denotes the how many coins tossed.

---

(2) **DICE:-**



1 Dice : $\{1, 2, 3, 4, 5, 6\} = 6^1 = 6$

Scanned with CamScanner

2 Dice :

(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

$= 6^2$

$= 36$

* 6 is common because we have 6 chances while we are dicing i.e. {1, 2, 3, 4, 5, 6}. Power denotes the how many dices throwed.

③ CARDS:-

There are 52 playing cards.

* Two types in black colour and other 2 types in Red colou

club Heart Spade Diamond.

* In the above 4 types, we will have 13 cards in each type.

① club : 13 cards

② Heart ; 13 cards

③ Spade : 13 cards

④ Diamond: 13 cards

⑤ No. of Black cards : 26

⑥ No. of Red cards : 26

⑦ No. of Ace cards (named as "A") : 4

⑧ No. of Queen cards (named as "Q") : 4

⑨ No. of Jack cards (named as "J") : 4

Scanned with CamScanner

(10) No. of King cards (named as "K"): 4

(11) No. of Face cards (named as "J, Q and K"): 12

* Probability = $\frac{\text{Sum of observations}}{\text{Possibility}}$ (or) $P(A) = \frac{n(A)}{n(S)}$

Here, $\boxed{n(S) = 52}$ Because there are 52 playing cards total.

Atleast : Minimum to Maximum

Atmost : Maximum to Minimum + None.

Results on Probability :-

(1) $P(S) = 1$

(2) $0 \leq P(E) \leq 1$

(3) $P(\phi) = 0$

(4) For any events A and B we have :

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

(5) If $\bar{A}$ denotes (not A), then $P(\bar{A}) = 1 - P(A)$

Scanned with CamScanner

13. AVERAGES & AGES:-

Averages:- Average is defined as ratio of sum of all terms in a group to the number of items in the group.

$$\text{Average} = \frac{\text{Sum of observations}}{\text{No. of observations}}$$

(2) **Average Speed :-**

A Person travels a distance at a speed of V_1 Km/hr and same distance at a speed of V_2 Km/hr. His average speed in the whole journey can be determined using the formula:

$$\boxed{\text{Average Speed} = \frac{2V_1V_2}{V_1+V_2}}$$

1. Average of "n" natural numbers $= \frac{(n+1)}{2}$
2. Average of even numbers $= (n+1)$
3. Average of odd numbers $= n$
4. Average of 'n' multiples of any num $=$ Num $\times \frac{(n+1)}{2}$
5. Average of squares of first 'n' natural num $= \frac{(n+1)(2n+1)}{6}$
6. Average of cubes of first 'n' natural num $= \frac{n(n+1)^2}{4}$
7. Average of even number in 1 to n $= \frac{\text{Last even num} + 2}{2}$
8. Average of odd number in 1 to n $= \frac{\text{Last odd num} + 1}{2}$
9. Average of Consecutive num $= \frac{\text{First num} + \text{Last num}}{2}$

Scanned with CamScanner

Consecutive Num:- $x, x+1, x+2, x+3, x+4, \ldots$

Eg:- ~~1, 3, 5, 7, 9~~ 101, 102, 103, 104, 105

Consecutive odd Numbers:- $x, x+2, x+4, x+6, x+8 \ldots$

Eg:- 26, 28, 30, 32, 34.

Consecutive Even Numbers:- $x, x+2, x+4, x+6, x+8 \ldots$

Eg:- 52, 54, 56, 58.

*** ** If they given number they ask to find average means first we have to check the given number is Consecutive num (or) odd num (or) Even number and the middle Value is the average of that Particular number.

Eg:- ① 1, 3, 5, 7, 9 → This is consecutive odd Num. Middle Value is the avg of given num. So, avg is "5".

② 90, 91, 92, 93, 94 → Consecutive Number. Middle Value is the avg of given num. So, avg is 92

③ 52, 54, 56, 58 → Consecutive Even Num. Middle value is the avg of given number. So, avg is 55

Ages:-

① If the current age is x, then n times the age is nx

② If the current age is x, then age n years later (or) hence $= x+n$.

③ If the current age is x, then age n yrs ago $= x-n$

④ The ages in a ratio $a:b$ will be ax and bx

⑤ If the current age is x, then $\frac{1}{n}$ of the age is $\frac{x}{n}$.

Scanned with CamScanner

(14). RATIOS AND PROPORTIONS:-

(1) **Ratio:-** The ratio of two quantities a and b in the same units, is the fraction $\frac{a}{b}$ and we write it as $a:b$.

* In the ratio $a:b$, we call "a" as the first term (or) antecedent and "b" the 2nd term (or) consequent.

Eg:- The ratio $5:9$ represents $\frac{5}{9}$ with 1st term $=5$ and 2nd term $=9$

Rule:- The multiplication (or) division of each term of a ratio by the same non-zero number does not affect the ratio.

Eg:- $4:5 = 8:10 = 12:15$. Also, $4:6 = 2:3$

(2) **Proportion:-**

The equality of two ratios is called proportion.

If $a:b = c:d$, we write $a:b::c:d$ and we say that a, b, c, d are in proportion.

Here, a and d are called extremes

b & c are called Mean terms.

Product of means = Product of extremes

Thus, $a:b::c:d \Leftrightarrow (b \times c) = (a \times d)$

* If a number "a" is divided in the ratio $x:y$ then

1st Part $= \frac{ax}{(x+y)}$

2nd Part $= \frac{ay}{(x+y)}$.

(3) **Third Proportional:-**

$a:b = c:d$, then c is called third proportional to a & b.

Fourth Proportional:-

$a:b = c:d$, then d is called fourth proportional to a, b, c

Mean Proportional:-

Mean Proportional between a and b is $\sqrt{ab}$.

④ **Comparison of Ratios:-**

We say that $(a:b) > (c:d) \Leftrightarrow \frac{a}{b} > \frac{c}{d}$.

Compounded Ratio:-

The compounded ratio of the ratios $(a:b), (c:d), (e,f)$ is $(ace : bdf)$

⑤ **Duplicate Ratios:-**

① Duplicate ratio of $a:b$ is $(a^2 : b^2)$

② Sub-duplicate ratio of $a:b$ is $(\sqrt{a} : \sqrt{b})$

③ Triplicate ratio of $a:b$ is $(a^3 : b^3)$

④ Sub-triplicate ratio of $a:b$ is $(a^{1/3} : b^{1/3})$

* **Componendo and dividendo:-**

If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$.

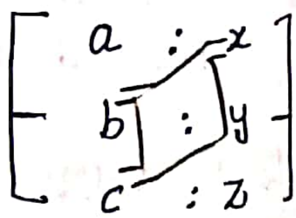
⑥ **Variations:-** If $a = kb$ for some constant k, then we can say that "a" is directly proportional to "b". (i.e) $a \propto b$.

* If $ba = k$ for some constant k, then we can say that "a" is inversely proportional to "b". (i.e) $a \propto \frac{1}{b}$

* If ratio between 1st & 2nd quantity $m:n = a:x$, 2nd & 3rd quantity $n:p = b:y$; 4th & 5th quantity $p:q = c:z$ then $m:n:p:q$ can be solved by using the trick shown below

$abc : xbc : xyc : xyz$

$m:n:p:q = abc : xbc : xyc : xyz$



Scanned with CamScanner

(15). AREAS, SHAPES & PERIMETERS (OR) MENSURATION :-

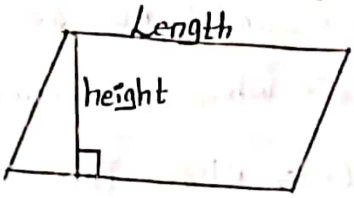
① Area of rectangle (A) = length (l) x breadth (b) $\Rightarrow lb$

② Perimeter of rectangle (P) = 2 x length x breadth $\Rightarrow 2lb$

③ Area of Square (A) = length x length $\Rightarrow l^2$

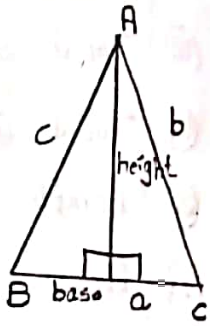
④ Perimeter of square (P) = 4 x length $\Rightarrow 4l$

⑤ Area of Parallelogram (A) = length x height $\Rightarrow lh$.



⑥ Perimeter of Parallelogram (P) = 2 x length x breadth $\Rightarrow 2lb$

⑦ Area of triangle (A) = $\frac{1}{2}$ x base x height $\Rightarrow \frac{1}{2}bh$



** And for a triangle with sides measuring "a, b & c".

Perimeter = $a+b+c$.

Semi Perimeter (S) = $\frac{\text{Perimeter}}{2} \Rightarrow \frac{a+b+c}{2}$

Area of Δ^{le} (A) = $\sqrt{s(s-a)(s-b)(s-c)}$

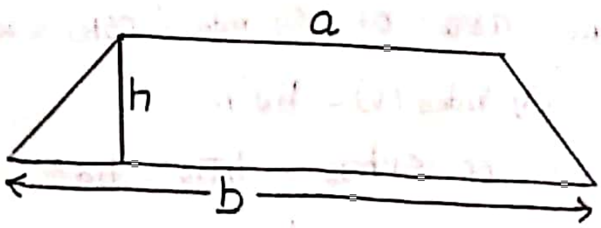
This formula is also known as "Hero's formula".

⑧ Area of isosceles triangle (A) = $\frac{b}{4}\sqrt{a^2-b^2}$

where, a = length of two equal sides

b = length of base of isosceles triangle

⑨ Area of trapezium (A) = $\frac{a+b}{2}$



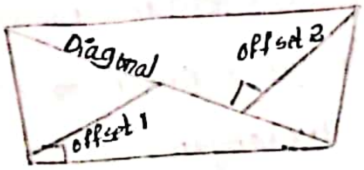
⑩ Perimeter of trapezium (P) = Sum of all sides

Scanned with CamScanner

(11) Area of Rhombus (A) = $\frac{1}{2}$ x Product of diagonals

(12) Perimeter of Rhombus (P) = 4 x length $\Rightarrow 4l$

(13) Area of quadrilateral (A) = $\frac{1}{2}$ x diagonal x (Sum of off sets)



(14) Area of kite (A) = $\frac{1}{2}$ x Product of it's diagonal.

(15) Perimeter of kite (P) = 2 x sum of non-adjacent sides.

(16) Area of Circle (A) = πr^2. Where r is radius, $\pi = \frac{22}{7}$ (or) 3.14

(17) Circumference of circle (c) = $2\pi r = \pi d$.
where, d is a diameter of circle

(18) Total Surface area of cuboid (TSA) = $2(lb+bh+hl)$

(19) Total Surface area of cube (TSA) = $6l^2$

(20) Length of diagonal of Cuboid = $\sqrt{l^2+b^2+h^2}$

(21) Length of diagonal of Cube = $\sqrt{3}l$.

(22) Volume of cuboid = lbh

(23) Volume of cube = l^3

(24) Area of base of cone = πr^2

(25) Curved Surface Area of cone (CSA) = πrl

(26) Total Surface Area of cone (TSA) = $\pi r(r+l)$

(27) Volume of cone (V) = $\frac{1}{3}\pi r^2 h$

(28) Curved Surface area of Cylinder (CSA) = $2\pi rh$

(29) Total Surface area of Cylinder (TSA) = $2\pi r(r+h)$

(30) Volume of Cylinder (V) = $\pi r^2 h$

(31) Surface area of Sphere = $4\pi r^2 = \pi d^2$

(32) Volume of Sphere (V) = $\frac{4}{3}\pi r^3 = \frac{1}{6}\pi d^3$

(33) Volume of hallow cylinder (V) = $\pi rh(R^2-r^2)$

Scanned with CamScanner

(34) Surface Area of Triangular Prism = (P x height) + (2 x Area of Δle)
where, P = Perimeter of base.

(35) Surface area of Polygonal Prism = (Perimeter of base x height) + (Area of Polygonal base x 2)

(36) Lateral Surface area of Prism (LSA) = Perimeter of base x height

(37) Volume of Triangular Prism (V) = Area of Δle base x height

(38) Surface area of right square Pyramid = $a\sqrt{4b^2 - a^2}$
where, a = length of base
b = length of equal side

(39) Volume of right square Pyramid = $\frac{1}{2}$ x base area x height

(40) Volume of hemisphere (V) = $\frac{2}{3}\pi r^3 = \frac{1}{12}\pi d^3$

(41) Total Surface area of Hemisphere (TSA) = $3\pi r^2$

(42) Curved Surface area of Hemisphere (CSA) = $2\pi r^2$

(43) Area of regular hexagon = $\frac{3\sqrt{3}a^2}{2}$

(44) Area of equilateral triangle = $\frac{3}{4}a^2$

(45) Area of Sector of circle = $\frac{\theta r^2 \pi}{360}$
where, θ is the measure of angle of sector.

(46) Curved Surface Area of Frustums (CSA) = $\pi h(r_1 + r_2)$

(47) Total Surface Area of Frustums (TSA) = $\pi(r_1^2 + h(r_1 + r_2) + r_2^2)$

---

change in it's surface area = $\left[a + b + \frac{ab}{100}\right]\%$

Scanned with CamScanner

(16). LCM & HCF :-

(1) **Factors :-** Factor is a number which exactly divides other number.

Ex:- 3 and 5 are factors of 15.

(2) **Multiple :-** A number is said to be multiple of another number, when it is exactly divisible by other number.

Ex:- 15 is a multiple of 3 and 5.

(3) **Common Multiple :-** A common multiple of two or more numbers is a number which is exactly divisible by each of them.

Ex:- 18 is a common multiple of 2, 3, 6 and 9.

(4) **HCF | GCF :-** HCF of 2 (or) more numbers is greatest number which divides each number exactly.

(5) **LCM :-** The least number exactly divisible by each one of the given numbers is called LCM.

---

(1) **Prime Numbers :-** Number 17 is a Prime number.
This number has no factors except $17 \times 1 = 17$
2, 3, 5, 7, 11, 13 are all Prime numbers.

(2) **Composite Numbers :-** Number 18 has many factors : 2, 3, 6, 9, 18.
Such numbers are called composite Numbers

(3) **Co-Prime Numbers :-** These numbers donot have common factors between given numbers. Ex:- 6 and 7.
** Two numbers are said to be co-Primes, if their H.C.F is 1.

Scanned with CamScanner

① HCF (Highest Common Factor):-

① It is the greatest number which exactly divides all the given numbers.

② The HCF of 2 (or) more numbers is smaller than (or) equal to the smallest number of given numbers.

③ HCF of given numbers divides their L.C.M.

④ Two numbers are said to be co-prime if their HCF is 1.

Methods to find HCF of given Numbers:-

① Prime Factorization Method:-

① Express the given numbers as product of their prime factors

② check for common prime factors and find least index of each common prime factor.

③ The product of all common prime factors with the respective least indexes is HCF of given numbers.

Ex:- HCF of 12, 36, 48.

Prime factors of 12, 36, 48

$12 = 2 \times 3 \times 2 = 3 \times 2^2$

$36 = 2 \times 2 \times 3 \times 3 = 3^2 \times 2^2$

$48 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3$

2 and 3 are common factors.

2^2 & 3 have least indices

HCF of 12, 36, 48 = Product of common prime factor with least indices.

HCF of 12, 36, 48 $= 2^2 \times 3 = 12$

$\therefore$ HCF of 12, 36, 48 = 12

② Pairing Prime factors:-

① List the factors of given numbers.

② Find the common factors.

③ Product of the common factors is the HCF of given numbers.

Ex:- HCF of 12, 36, 48

Factors of $12 = \boxed{2} \times \boxed{2} \times \boxed{3}$

Factors of $36 = \boxed{2} \times \boxed{2} \times \boxed{3} \times 3$

Factors of $48 = \boxed{2} \times \boxed{2} \times \boxed{3} \times 2 \times 2$

HCF of 12, 36, 48 =

$2 \times 2 \times 3 = 12$.

Scanned with CamScanner

③ Division Method:-

① Draw a table as shown and arrange the given numbers horizontally.

② Divide the numbers with their common factors.

③ Divide till the given numbers have no common factors.

④ Finally multiply the common factors on left hand side of the table to find the HCF.

Ex:- HCF of 12, 36, 48

2	12	36	48
2	6	18	24
3	3	9	12
	1	3	4

HCF = 2 × 2 × 3 = 12

② LCM (Least Common Multiple):-

* To find LCM of 2 co-prime numbers, just multiply the given two numbers.
* The LCM of 2 (or) more numbers is greater than (or) equal to greatest number of given numbers.

Methods to find L.C.M of given numbers:-

① Prime Factorization Method:-

① Express the given numbers as product of their prime factors

② check for common prime factors and find highest index of each common prime factor.

③ The product of all common prime factors with the respective highest indexes is LCM of given numbers.

Ex:- LCM of 14, 36, 42

14 = 2 × 7

36 = 2 × 2 × 3 × 3 = $2^2 \times 3^2$

42 = 2 × 3 × 7

Scanned with CamScanner

2, 7 and 3 are common factors. Highest index are 2, 2, 1

LCM of 14, 36, 42 = Product of common Prime factors with highest indices.

LCM of 14, 36, 42 = $2^2 \times 3^2 \times 7 = 252$

② Pairing Prime Factors:-

① List the factors of given numbers.

② Find the common factors.

③ Product of all factors is the LCM of given numbers.

Ex:- LCM of 12, 36, 48.

Factors of 12 = $\boxed{2} \times \boxed{2} \times \boxed{3}$

Factors of 36 = $\boxed{2} \times \boxed{2} \times \boxed{3} \times 3$

Factors of 48 = $\boxed{2} \times \boxed{2} \times \boxed{3} \times 2 \times 2$

LCM = $2 \times 2 \times 3 \times 3 \times 2 \times 2$

$= 144$

③ Division Method:-

① Draw a table as shown and arrange the given numbers horizontally.

② Divide the numbers with their common factors.

③ Divide till the given numbers have no common factors.

④ Finally multiply the common factors and the remainders to obtain LCM of given numbers.

Ex:- LCM of 12, 36, 48

2	12	36	48
2	6	18	24
3	3	9	12
	1	3	4

LCM of 12, 36, 48 = $2^2 \times 3^2 \times 4$

$= 144$

Scanned with CamScanner

Quick Tips and Tricks:-

① HCF and LCM of fractions:

(a) $HCF = \dfrac{\text{HCF of Numerator}}{\text{LCM of denominator}}$

Ex:- HCF of $\dfrac{l}{a}, \dfrac{m}{b}, \dfrac{n}{c} = \dfrac{\text{HCF of }(l, m, n)}{\text{LCM of }(a, b, c)}$

(b) $LCM = \dfrac{\text{LCM of Numerator}}{\text{HCF of denominator}}$

Ex:- LCM of $\dfrac{l}{a}, \dfrac{m}{b}, \dfrac{n}{c} = \dfrac{\text{LCM of }(l, m, n)}{\text{HCF of }(a, b, c)}$

② Product of 2 numbers = Product of their HCF and LCM.

* This condition is only true for two given numbers. If HCF and LCM of 3 (or) more numbers are given, then this rule is not applicable.

*** If A and B are 2 numbers,

$\boxed{A \times B = \text{HCF of A and B} \times \text{LCM of A and B}}$

③ HCF and LCM of decimal fractions:-

To find HCF (or) LCM of 0.5, 0.6, 1.60 etc. consider these numbers without decimal point i.e. 5, 6, 60 and solve using normal methods used to determine LCM (or) HCF.

* In the result, mark off as many decimal places as there are in each of given number.

④ Comparision of Fractions:-

Find LCM of denominators of given fractions. Convert each fraction into equivalent fraction with LCM as denominator, by multiplying both Numerator and denominator by same number. The resultant fraction with the greatest numerator is greatest.

Scanned with CamScanner

17. SIMPLE INTEREST & COMPOUND INTEREST:-

Simple Interest (SI):-

① **Principal:-** The money borrowed (or) lent out for a certain Period is called Principal (or) sum.

② **Interest:-** Extra money Paid for using other's money is called Interest.

③ **Simple Interest (SI):-** If the interest on a sum borrowed for certain Period is reckoned (calculated) uniformly, then it is called Simple Interest.

(i) $\boxed{\text{Simple Interest (SI)} = \frac{PTR}{100}}$

(ii) $P = \frac{100 \times SI}{RT}$

(iii) $R = \frac{100 \times SI}{PT}$

(iv) $T = \frac{100 \times SI}{PR}$

(v) $\boxed{\text{Amount (A)} = P\left[1 + \frac{RT}{100}\right]}$

$A = P + I$

where, P = Principal.

R = Rate of interest per annum.

T = Time in years.

A = Total Amount.

I = Interest.

① Simple Interest on a $\boxed{\text{certain Value}}$ is Rs. 20
R

② $\underbrace{\text{Rs. 2000}}_{\text{Principal}}$ is invested

③ The amount becomes 1800
↳ P+I=A= Total amount

④ $\underbrace{\text{Maturity}}_{\text{A= Total amount}}$ amount is Rs. 1000

COMPOUND INTEREST :-

1) Compound Interest (CI) = $P\left\{\left[1+\frac{R}{100}\right]^{n}-1\right\}$

$$A = P\left(1+\frac{R}{100}\right)^{n}$$

Maturity amount
Total amount
Amount becomes
} = A

where, n = No. of years.

② Half-yearly (6 months) :-

$$\boxed{A = P\left[1+\frac{R/2}{100}\right]^{2n}}$$

* n should be in years.

Eg:- (1) 1 year

6 months   6 months

(2) 18 months

6 months   6 months   6 months

③ Quarterly (4 months) :-

$$\boxed{A = P\left[1+\frac{R/4}{100}\right]^{4n}}$$

Eg:- 1 year

3 months   3 months   3 months   3 months

④ Half / Quart Yearly :-

Quart yearly :- $\boxed{A = P\left[1+\frac{R}{100}\right]^{n}\left[1+\frac{R/4}{100}\right]^{4n}}$

T = 27 months = 2 1/4 yrs.

1 yr   1 yr   3 months

Half yearly :- $\boxed{A = P\left[1+\frac{R}{100}\right]^{n}\left[1+\frac{R/2}{100}\right]^{2n}}$

T = 30 months = 2 1/2 yrs

1 yr   1 yr   6 months

⑤ Population Increase (or) Decrease :-

① After "n" years $A = P\left[1+\frac{R}{100}\right]^{n}$

② "n" years ago $A = \dfrac{P}{\left[1+\frac{R}{100}\right]^{n}}$

Difference $= P\left(\frac{R}{100}\right)^{2} = \frac{SI \times R}{200}$

Scanned with CamScanner

(18). ALGEBRA :-

1. $(a+b)^2 = a^2+b^2+2ab$
2. $(a-b)^2 = a^2+b^2-2ab$
3. $a^2+b^2 = (a-b)^2+2ab$
4. $a^2-b^2 = (a-b)(a+b)$
5. $(a+b+c)^2 = a^2+b^2+c^2+2ab+2ac+2bc$
6. $(a+b+c)^3 = a^3+b^3+c^3+3(a+b)(b+c)(c+a)$
7. $a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$
8. $(a-b-c)^2 = a^2+b^2+c^2-2ab-2ac+2bc$
9. $(a+b)^3 = a^3+b^3+3a^2b+3ab^2$ (or) $a^3+b^3+3ab(a+b)$
10. $(a-b)^3 = a^3-b^3+3ab^2-3a^2b$
11. $a^3-b^3 = (a-b)(a^2+b^2+ab)$
12. $a^3+b^3 = (a+b)(a^2+b^2-ab)$
13. $(a+b)^4 = a^4+b^4+4a^3b+6a^2b^2+4ab^3$
14. $(a-b)^4 = a^4+b^4-4a^3b+6a^2b^2-4ab^3$
15. $a^4-b^4 = (a-b)(a+b)(a^2+b^2)$
16. $a^5-b^5 = (a-b)(a^4+b^4+a^3b+a^2b^2+ab^3)$
17. If $a+b+c=0$, then $a^3+b^3+c^3=0$
18. $x^2+x(a+b)+ab = (x+a)(x+b)$
19. $ab(a+b)+bc(c+b)+ca(c+a) = (a+b)(b+c)(c+a)$
20. $a^2(b+c)+b^2(c+a)+c^2(a+b)+3abc = (a+b+c)(ab+bc+ca)$
21. $a^2(b-c)+b^2(c-a)+c^2(a-b) = (a-b)(b-c)(c-a)$
22. $(a+b+c+----)^2 = a^2+b^2+c^2+---+2(ab+ac+bc+----)$

Scanned with CamScanner

(23) If n is a natural number,

$$a^n - b^n = (a-b)(a^{n-1} + a^{n-2}b + \cdots + b^{n-2}a + b^{n-1})$$

(24) If n is even $(n = 2k)$,

$$a^n + b^n = (a+b)(a^{n-1} - a^{n-2}b + \cdots + b^{n-2}a - b^{n-1})$$

(25) If n is odd $(n = 2k+1)$

$$a^n + b^n = (a+b)(a^{n-1} - a^{n-2}b + \cdots - b^{n-2}a + b^{n-1})$$

Laws of Exponents:-

(1) $(a^m)(a^n) = a^{m+n}$

(2) $(ab)^m = a^m b^m$

(3) $(a^m)^n = a^{mn}$

Scanned with CamScanner

(19). SIMPLIFICATIONS:-

(1) BODMAS Rule :-

This rule depicts the correct sequence in which the operation are to be executed, so as to find out the value of given expression.

Here, B = Bracket | A = Addition
O = of | S = Subtraction
D = Division
M = Multiplication

Thus, in simplifying an expression, first of all the brackets must be removed, strictly in the order ( ), { } and | |.

* After removing the brackets, we must use the following operations strictly in the order:

(i) OF
(ii) Division
(iii) Multiplication
(iv) Addition
(v) Subtraction

(2) Modulus of a Real Number:-

Modulus of a real number "a" is defined as:

$$|a| = \begin{cases} a, & \text{if } a > 0 \\ -a, & \text{if } a < 0 \end{cases}$$

Thus, $|5| = 5$ and $|-5| = -(-5) = 5$.

(3) Virnaculum (or Bar):-

When an expression contains Virnaculum, before applying the "BODMAS" rule, we simplify the expression under the Virnaculum.

Scanned with CamScanner

(10). PIPES AND CISTERN :-

(1) **Inlet :-** A pipe connected with a tank or cistern or reservoir, that fills it is called Inlet.

Outlet :- A pipe connected with a tank or cistern or reservoir, emptying it is called outlet.

(2) If a pipe can fill a tank in x hours then:
Part filled in 1 hour $= \frac{1}{x}$.

(3) If a pipe can empty a tank in y hours then:
Part emptied in 1 hour $= \frac{1}{y}$.

(4) If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours (where $y > x$), then on opening both the pipes, then the net part filled in 1 hour $= \left[\frac{1}{x} - \frac{1}{y}\right]$.

(5) If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours (where $x > y$) then on opening both the pipes, then the net part ~~filled~~ ~~ir~~ emptied in 1 hour $= \left[\frac{1}{y} - \frac{1}{x}\right]$.

Scanned with CamScanner

(21). PARTNERSHIP :-

① **Partnership :-**

When two (or) more than two Persons run a business Jointly, then they are called Partners and the deal is known as Partnership.

② **Ratio of Divisions of Gains :-**

① When investments of all Partners are for the same time, the gain (or) loss is distributed among the Partners in the ratio of their investments.

* Suppose P and Q invest Rs. x and Rs. y respectively for a year in a business, then at end of year:

$$\boxed{(\text{P's share of Profit}) : (\text{Q's share of Profit}) = x : y}$$

② When investments are for different time Periods, then equivalent capitals are calculated for a unit of time by taking (Capital × no. of units of time). Now gain (or) loss is divided in the ratio of these capitals.

* Suppose P invests Rs. x for A months and Q invests Rs. y for B months then

$$\boxed{(\text{P's share of Profit}) : (\text{Q's share of Profit}) = xA : yB}$$

③ **Working and Sleeping Partners :-** A Partner who manages the business is known as working Partner and the one who simply invests the money is known as sleeping Partner.

---

① Ratio of investment × Time = Rate of Profit.

$\therefore$ (A's investment × Time) : (B's investment × Time) = Profit of A : Profit of B

② Ratio of investment × Ratio of utilization = Ratio of Profit

③ Ratio of time = Ratio of Profit → When investment is same

④ Time ratio is found by $= \left(\frac{P_1}{I_1}\right) : \left(\frac{P_2}{I_2}\right) : \left(\frac{P_3}{I_3}\right)$

Scanned with CamScanner

(22) DECIMAL FRACTIONS:-

(1) **Decimal Fractions:-** Fractions in which denominators are Powers of 10 are known as decimal Fractions.

Thus, $\frac{1}{10} = 0.1$ | $\frac{7}{1000} = 0.007$

$\frac{1}{100} = 0.01$

$\frac{99}{100} = 0.99$

(2) **Conversion of Decimal into Vulgar Fraction:-**

Put 1 in the denominator under the decimal Point and annex with it as many zeros as the number of digits after the decimal Point.

* Now, remove the decimal Point and reduce the fraction to its lowest terms.

Thus, $0.25 = \frac{25}{100} = \frac{1}{4}$; $2.008 = \frac{2008}{1000} = \frac{251}{125}$.

(3) **Annexing zeros and Removing Decimal Signs:-**

Annexing Zeros to the extreme right of a decimal fraction does not change its value.

Thus, $0.8 = 0.80 = 0.800$ ---- etc.

* If Numerator and denominator of a fraction contain the same number of decimal Places then we remove decimal sign.

Thus, $\frac{1.84}{2.99} = \frac{184}{299} = \frac{8}{13}$

(4) **Operations on Decimal Fractions:-**

(i) **"+" and "−" of Decimal Fractions:-**

The given numbers are so placed under each other that the decimal Points lie on one column. The numbers so arranged can be added (or) Subtracted in the usual way.

Scanned with CamScanner

(ii) Multiplication of Decimal Fraction by Power of 10:-

Shift the decimal point to the right by as many places as it the power of 10.

Thus, $5.9632 \times 100 = 596.32$

$0.073 \times 100 = 7.30$

(iii) Multiplication of Decimal Fraction:-

Multiply the given numbers considering them without decimal point. Now, in the product the decimal point is marked off to obtain as many places of decimal as is the sum of number of decimal places in given numbers.

* Suppose we have to find the product $(0.2 \times 0.02 \times 0.002)$.

Now, $2 \times 2 \times 2 = 8$. Sum of decimal places $= (1+2+3) = 6$.

$\therefore\ 0.2 \times 0.02 \times 0.002 = 0.000008$.

(iv) Dividing a decimal fraction by Counting Number:-

Divide the given number without considering the decimal point by the given counting number. Now, in the quotient put the decimal point to give as many places of decimals as there are in dividend.

* Suppose we have to find quotient of $(0.0204 \div 17)$

Now, $204 \div 17 = 12$

Dividend contains 4 places of decimal. So, $0.0204 \div 17 = 0.0012$.

(v) Dividing a decimal Fraction by decimal Fraction:-

Multiply both the dividend and divisor by a suitable power of 10 to make divisor a whole number.

Thus, $\dfrac{0.00066}{0.11} = \dfrac{0.00066 \times 100}{0.11 \times 100}$

$= \dfrac{0.066}{11} = 0.006$

Scanned with CamScanner

(5) Comparision of Fractions:-

Suppose some fractions are to be arranged in ascending (or) descending order of magnitude, then convert each one of the given fractions in decimal form and arrange them accordingly.

* Let us arrange fractions $\frac{3}{5}$, $\frac{6}{7}$ and $\frac{7}{9}$ in descending order.

Now, $\frac{3}{5} = 0.6$; $\frac{6}{7} = 0.857$; $\frac{7}{9} = 0.777$

Since, $0.857 > 0.777 > 0.6$. So, $\frac{6}{7} > \frac{7}{9} > \frac{3}{5}$

(6) Some basic Formulaes:-

(1) $(a+b)(a-b) = (a^2-b^2)$

(2) $(a+b)^2 = a^2+b^2+2ab.$

(3) $(a-b)^2 = a^2+b^2-2ab$

(4) $(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$

(5) $(a+b)^3 = (a+b)(a^2+b^2-ab)$

(6) $(a^3-b^3) = (a-b)(a^2+b^2+ab)$

(7) $a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

(8) When $a+b+c=0$, then $a^3+b^3+c^3 = 3abc$

(7) Recurring Decimal:-

In a decimal fraction, a figure (or) a set of figures is repeated continuously then such a number is called Recurring decimal.

* In a recurring decimal, if a single figure is repeated. then it is expressed by putting a dot on it. If a set of figures is repeated, it is expressed by putting a bar on set.

Thus, $\frac{1}{3} = 0.333---- = 0.\bar{3}$

$\frac{22}{7} = 3.142857--- = 3.\overline{142857}.$

Scanned with CamScanner

Pure Recurring Decimal:- A decimal fraction, in which all the figures after the decimal point are repeated.

Converting Pure Recurring Decimal into Vulgar Fraction:-
Write the repeated figures only once in the numerator and take as many nines in the denominator as is the number of repeating figures.

Thus, $0.\overline{5} = \frac{5}{9}$; $0.\overline{53} = \frac{53}{99}$; $0.\overline{067} = \frac{67}{999}$ etc

Mixed Recurring Decimal:-
A decimal fraction in which some figures do not repeat and some of them are repeated, is called Mixed Recurring decimal.

Eg:- $0.17333333 = 0.17\overline{3}$

Converting Mixed Recurring decimal into Vulgar Fraction:-
In the numerator, take the difference between the number ~~found~~ formed by all the digits after decimal points (taking repeated digits only once) and that formed by the digits which are not repeated.

* In denominator, take the number formed by as many nines as there are repeating digits followed by as many zeros as is the number of non-repeating digits.

Thus, $0.1\overline{6} = \frac{16-1}{90} = \frac{15}{90} = \frac{1}{6}$

$0.22\overline{73} = \frac{2273-22}{9900} = \frac{2251}{9900}$

Scanned with CamScanner

23. QUADRATIC EQUATIONS:-

First and foremost, Relation between X and Y is establi-shed only when the relationship is defined for all Solutions.

① **Linear Equations:-** In linear equations, both X and Y have only one value. So, relation can be established easily

Ex:- $4x+3y=18$, $7x+5y=12$

$(4x+3y=18)\times 5$, $(7x+5y=12)\times 3$

$= 20x+15y = 90 \rightarrow ①$

$21x+15y = 36 \rightarrow ②$

$(-) \quad (-) \quad (-)$

$-x = 54 \Rightarrow \boxed{x=-54}$

$20(-54)+15y=90$

$-1080+15y=90$

$-1080-90=-15y$

$\cancel{-}1170 = \cancel{-}15y \Rightarrow \boxed{y=78}$

Hence, $y>x$.

② **Squares:-** In this, Solutions have both negative and Positive value.

$x^2=1600$ and $y^2=3600$

$x=\pm 40$ and $y=\pm 60$

+60 is greater than both −40 and +40, but −60 is less than both −40 and +40. So, the answer will be cannot be determined.

Trick:- Whenever both equations are given in square form our ANSWER will be "Can't be determined".

$$\boxed{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$$

Scanned with CamScanner

(3) Squares and Square root case :-

$x^2 = 1600$ and $y = \sqrt{3600}$

We know that square root always gives a positive values.
So, y will have only +60 Not −60.
+60 is greater than both +40 and −40
Hence $y > x$.

(4) Cubes Case:-

(1) If $x^3 = 1331$, $y^3 = 729$
then $x = 11$ and $y = 9$
x is greater than y, so relation is $x > y$.

(2) If $x^3 = -1331$ and $y^3 = 729$
then $x = -11$ and $y = 9$.
y is greater than x, so relation is $x < y$.

TRICK:- When both equations are in cube form. If $x^3 > y^3$, then $x > y$ and $x^3 < y^3$ then $x < y$.

(5) Square and Cube cases:-

(i) If $x^2 = 16$ and $y^3 = 64$
then $x = +4, -4$ and $y = 4$
So, $y = 4$ is equal to $x = 4$ and $y = 4$ is greater than $x = -4$
So, $y \geq x$

(ii) If $x^2 = 25$ and $y^3 = 64$
then $x = +5, -5$ and $y = 4$
So, $y = 4$ is greater than $x = -5$ and less than $x = +5$.
So, relation can't be determined

Scanned with CamScanner

100 SHORTCUTS

to crack

QUANTITATIVE APTITUDE

CN Pragadeeswara Prabhu

IBPS	SBI	UPSC	RBI	RRB	SSC
BSNL	NTPC	BHEL	TNEB	NLC	CBI
TANCET	GATE	CAT	MAT	XAT	SAT
POLICE	ARMY	GMAT	GRE	POST	IPM

All Central and State Government Exams
Campus Recruitment Tests

Scanned with CamScanner

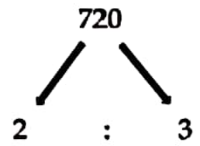
Table of Contents

Scanned with CamScanner

<u>Shortcut # 1 - Ratio and Proportion</u>

Splitting a number in the given ratio.

1. Divide 720 in the ratio 2: 3.

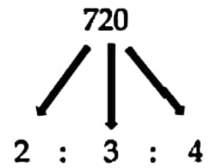


Solution 1:
Total parts = 2 + 3 = 5
First number = (2/5) 720 = 288
Second number = (3/5) 720 = 432

Solution 2:
$2x + 3x = 720$; $5x = 720$; $x = 144$
$2x = 2(144) = 288$; $3x = 3(144) = 432$

---

2. Divide 720 in the ratio 2: 3: 4.



Solution 1:
Total parts = 2 + 3 + 4 = 9
First number = (2/9) 720 = 160; Second number = (3/9) 720 = 240; Third number = (4/9)720 = 320

Solution 2:
$2x + 3x + 4x = 720$; $9x = 720$; $x = 80$
$2x = 2(80) = 160$; $3x = 3(80) = 240$; $4x = 4(80) = 320$

Scanned with CamScanner

Shortcut # 2 – Ratio and Proportion

Direct Proportion.

When two parameters are in direct proportion if one parameter increases the other one will also increase and if one parameter decreases the other one will also decrease.

If the parameters A and B are in direct proportion they will satisfy the below equation.

$$\frac{A_1}{A_2} = \frac{B_1}{B_2}$$

Question:

Price of diamond is directly proportional to its weight. If 2 grams of diamond costs $ 45,000 then what is the price of a diamond that weighs 6 grams?

Answer:

Weight of diamond in first case, $A_1 = 2$
Weight of diamond in second case, $A_2 = 6$
Price of diamond in first case, $B_1 = 45000$
Price of diamond in second case, $B_2 = ?$

According to the equation,

$2/6 = 45000/B_2$

$B_2 = 45000 \text{ x } (6/2)$

$B_2 = 135{,}000$

Scanned with CamScanner

Shortcut # 3 – Ratio and Proportion

Inverse Proportion.

When two parameters are in inverse proportion if one parameter increases the other one will decrease and if one parameter decreases the other one will decrease.

If the parameters A and B are in inverse proportion they will satisfy the below equation.

$$\frac{A_1}{A_2} = \frac{B_2}{B_1}$$

Question:

Mileage and Engine capacity are inversely proportional. A bike with 100cc engine capacity gives a mileage of 80 km. What will be the mileage given by a bike with 150cc engine capacity?

Answer:

Engine capacity of first bike,	A_1	= 100
Engine capacity of second bike,	A_2	= 150
Mileage of first bike,	B_1	= 80
Mileage of second bike,	B_2	=?

According to the equation,

$100/150 = B_2/80$

$B_2 = 100(80)/150$

$B_2 = 53.33$ km

Scanned with CamScanner

Shortcut # 4 – Ratio and Proportion
Finding A: C from A: B and B: C

$$\begin{array}{ccccc} A & : & B & & \\ & & B & : & C \end{array}$$

A x B : B x B : B x C

A : B : C

Question:

The ratio between salary of A and B is 4: 5. The ratio between salary of B and C is 3: 4. Find the salary of C if A is earning $3600.

Answer:

$$\begin{array}{ccccc} 4 & : & 5 & & \\ & & 3 & : & 4 \end{array}$$

4x3 : 5x3 : 5x4

12 : 15 : 20

A: C = 12: 20 = 3: 5

A/C = 3/5

3600/C = 3/5; C = 6000

Scanned with CamScanner

Shortcut # 5 – Ratio and Proportion

Finding A: D from A: B, B: C and C: D

A : B

B : C

A×B : B×B : B×C

A : B : C

C : D

A×C : B×C : C×C : C×D

A : B : C : D

Question:

Ratio between salary of A and B is 3: 5, B and C is 4: 5, C and D is 6: 7. If the salary of A is $ 7200, find the salary of D.

Answer:

3 : 5

4 : 5

3×4 : 5×4 : 5×5

12 : 20 : 25

6 : 7

12×6 : 20×6 : 25×6 : 25×7

72 : 120 : 150 : 175

A: D = 72: 175

7200/D = 72/175

D = 17500

Scanned with CamScanner

Shortcut # 6 – Ratio and Proportion

Usage of Common Factor x

Question:

Two numbers are in the ratio 4: 5. Sum of their squares is 1025. Find the numbers.

Answer:

Substitute the common factor x to the ratio 4: 5

Assume the actual numbers as 4x and 5x

Given,

$(4x)^2 + (5x)^2 = 1025$

$16x^2 + 25x^2 = 1025$; $41x^2 = 1025$

$x^2 = 25$; $x = 5$

Substitute x = 5 in 4x and 5x to find the numbers

The numbers are, 20 and 25.

Question:

Two numbers are in the ratio 3: 2. Cube of their difference is 125. Find the numbers.

Answer:

$(3x - 2x)^3 = 125$

$x^3 = 125$

$x = 5$

The numbers are 15 and 10.

Scanned with CamScanner

Shortcut # 7 – Ratio and Proportion

Finding number of Coins in a bag.

Quantity ratio ----- Q_1 : Q_2 : Q_3

Value ratio ----- V_1 : V_2 : V_3

$(Q_1 \times V_1) + (Q_2 \times V_2) + (Q_3 \times V_3) = T$

Common factor X = Total Amount/T

Quantity of each coin = Q_1X, Q_2X, Q_3X

Question:

A bag contains 5 paise, 10 paise and 20 paise coins in the ratio 2:4:5. Total amount in the bag is Rs. 4.50. How many coins are there in 20 paise?

Answer:

T = (2 x 5) + (4 x 10) + (5 x 20)

= 150 paise

= Rs. 1.50

X = 4.50/1.50

X = 3

Quantity of 20 paise coins

= 5 x 3

= 15 coins.

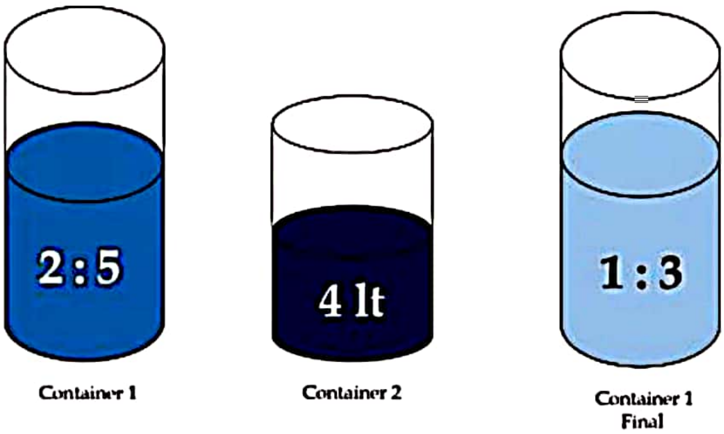
Scanned with CamScanner

Shortcut # 8 – Ratio and Proportion

Adding and removing quantities.

Question:

Container 1 has milk and water in the ratio 2 : 5. After adding 4 liters of pure water from container 2, the ratio between milk and water in container 1 became 1 : 3. Find the quantity of milk in the container.



Answer:

Let us assume the actual quantity of milk and water as 2x and 5x.

New quantity of water = 5x + 4

2x/(5x + 4) = 1/3

6x = 5x + 4

x = 4

Quantity of milk = 2x

= 2(4)

= 8 liters.

Scanned with CamScanner

Shortcut # 9 – Partnership

Finding ratio of the profit share.

Income (or) Profit share from a business is determined using two parameters – Investment and Duration.

Profit share is directly proportional to the duration of investment and the amount invested.

$P_1 : P_2 : P_3$ = Ratio between profits

T_1, T_2, T_3 = Respective time of investment

I_1, I_2, I_3 = Respective investment

Question:

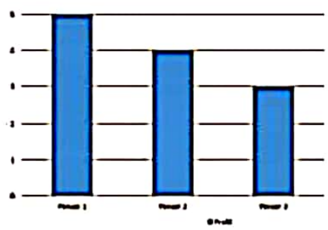
A started a business investing Rs. 5000. After 1 year B joined him investing Rs. 6000. After another year C joined them investing Rs. 9000. What is the ratio between their profit shares at the end of 3 years?

Answer:

$P_1 : P_2 : P_3 = 5000 \text{ x } 3 : 6000 \text{ x } 2 : 9000 \text{ x } 1$

$P_1 : P_2 : P_3 = 15000 : 12000 \text{ x } 9000$

$P_1 : P_2 : P_3 = 5 : 4 : 3$



Scanned with CamScanner

Shortcut # 10 – Mixtures and Allegation

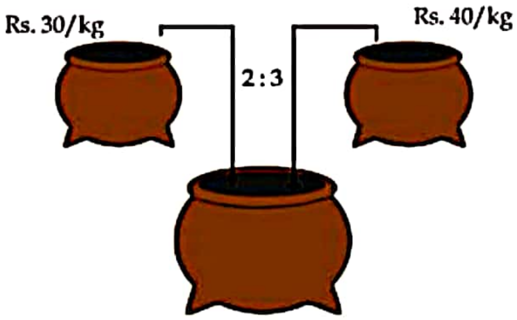
Finding average price of a mixture.

$$A_p = \frac{A_1N_1 + A_2N_2}{N_1 + N_2}$$

A_p = Average price of the mixture.
A_1 = Price of the first variety.
A_2 = Price of the second variety.
N_1 = Quantity of the first variety.
N_2 = quantity of the second variety.

Question:

Two varieties of rice with prices Rs. 30 per kg and Rs. 40 per kg are mixed in the ratio 2 : 3. Find the mean price.



Answer:

$N_1 = 2$; $N_2 = 3$

$A_1 = 30$; $A_2 = 40$

Substitute the values in the above equation.

Mean price = Rs. 36

Scanned with CamScanner

Shortcut # 11 – Mixtures and Allegation

Allegation Rule.

$$
\begin{array}{ccc}
A_1 & & A_2 \\
 & \searrow \quad \swarrow & \\
 & A_p & \\
 & \swarrow \quad \searrow & \\
(A_2 \sim A_p) & & (A_1 \sim A_p) \\
N_1 & : & N_2
\end{array}
$$

Question:

In what ratio two varieties of rice worth Rs. 40 per kg and Rs. 60 per kg should be mixed to give a variety worth Rs. 46 per kg?

Answer:

$$
\begin{array}{ccc}
40 & & 60 \\
 & \searrow \quad \swarrow & \\
 & 46 & \\
 & \swarrow \quad \searrow & \\
(60 \sim 46) & & (40 \sim 46) \\
14 & : & 6
\end{array}
$$

The two mixtures have to be mixed in the ratio 7 : 3 to get Rs. 46 mixture.

Scanned with CamScanner

Shortcut # 12 – Mixtures and Allegation

Successive removal and replacement type.

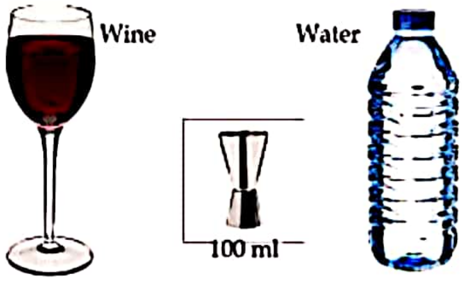
$$F = I[1 - (R/I)]^N$$

I – Initial quantity; R – Removal quantity;

F – Final quantity; N – Number of processes.

Question:

A container has 1000 liters of wine. 100 liters of wine is drawn from the container and filled with water. This process is done two more times. What is the ratio between wine and water in the final mixture?



Answer:

Number of draws n = 3

FQ = 1000 $[1-(100/1000)]^3 = 1000(0.9)^3 = 729$

Present quantity of wine = 729

Present quantity of water = 1000 – 729 = 271

Ratio between wine and water = 729 : 271

(Or)

1000 – 100 = 900
900 – 90 = 810
810 – 81 = 729

729 = final quantity of wine.

Scanned with CamScanner

Shortcut # 13 – Average

Common increase or decrease for all the elements.

If al the elements in a series are increased or decreased or multiplied or divided by a certain number, the old average should also be added or subtracted or multiplied or divided respectively to get the new average.

Question:

Average age of a family of four members is 34. What is the average age of the family after 4 years?

Answer:

Since every ones age is increased by four in 4 years, the average will also increase by 4

The new average = 34 + 4 = 38

Proof:

Total age of the family before 4 years = 34 x 4 = 136

Total increased age for four members = 4 x 4 = 16

New total age = 136 + 16 = 152

New average
= 152/4
= 38

Scanned with CamScanner

Shortcut # 14 – Average

Adding or removing or replacing an element.

$A_1N_1 \sim A_2N_2$

Question 1:

Average age of 5 students was 18. After adding teacher's age the average became 20. What is the age of the teacher?

Answer = (18 x 5) ~ (20 x 6) = 30

Question 2:

Average age of 5 students and a teacher was 21. After removing the teacher's age the average became 18. What is the age of the teacher?

Answer = (21 x 6) ~ (18 x 5) = 36

Question 3:

Average weight of 40 bags is 20kg. After removing 1 bag the average became 19.5. What is the weight of removed bag?

Answer = (20 x 40) ~ (19.5 x 39) = 39.5

Question 4:

Average weight of 40 bags is 20kg. After replacing a bag by another bag the average became 39. What is the difference between the weights of those two bags?

Answer = (20 x 40) ~ (20 x 39) = 20

Scanned with CamScanner

Shortcut # 15 – Average
Weighted Average

$$A_o = \frac{A_1N_1 + A_2N_2 + \ldots + A_nN_n}{N_1 + N_2 + \ldots + N_n}$$

Question:

Average marks scored by students in three classes are 40, 50, and 60. If there are 10, 20 and 30 students in the classes respectively, find the overall average of the three classes.

Class 1
Average = 40
Numbers = 10

Class 2
Average = 50
Numbers = 20

Class 3
Average = 60
Numbers = 30

Answer:

$A_1 = 40$; $A_2 = 50$; $A_3 = 60$

$N_1 = 10$; $N_2 = 20$; $N_3 = 30$

Substitute in the above formula. We get,

=[(40x10) + (50x20) + (60x30)]/[10+20+30]

=[400 + 1000 + 1800]/60

= 53.33

Scanned with CamScanner

Shortcut # 16 - Average

Finding the middle subject mark, if the middle subject overlaps.

$$M = A_1N_1 + A_2N_2 - A_oN_o$$

Question:

Average marks scored by Sai in 11 subjects is 60. Average marks scored by her in first 6 subjects is 50 and average marks scored by her in last 6 subjects is 62. Find the mark scored by Sai in 6^{th} subject.

Answer:

$A_o = 60$; $N_o = 11$

$A_1 = 50$; $N_1 = 6$

$A_2 = 62$; $N_2 = 6$

Substituting the values in the formula, we get

$M = 50x6 + 62x6 – 60x11$

$= 300 + 372 – 660$

$= 12$

Marks scored by Sai in 6^{th} subject = 12

Scanned with CamScanner

Shortcut # 17 - Average

Finding the middle subject mark if the middle subject is left out.

$$M = A_oN_o - A_1N_1 - A_2N_2$$

Question:

The average marks scored by Shradha in 9 subjects is 75. The average marks in first 4 subjects is 69 and average marks in last 4 subjects is 78. Find the marks scored by her in 5th subject.

Answer:

$A_o = 75; N_o = 9$

$A_1 = 69; N_1 = 4$

$A_2 = 78; N_1 = 4$

Substituting the above values in the formula, we get

$M = 75x9 – 69x4 – 78x4$

$= 675 – 276 – 312$

$= 87$

Marks scored by Sradha in 5th subject = 87

Scanned with CamScanner

Shortcut # 18 – Percentage

Percentage Calculation

$$\% = \frac{\text{Value to be compared}}{\text{Value to which it has to be compared}} \times 100$$

V_c = Value to be compared

V_b = Value to which it has to be compared, base value.

Question:

What is the percentage of marks scored by Sai in final exam, if she has scored 1102 out of 1200?

Answer:

$V_c = 1102$

$V_b = 1200$

Percentage = (1102/1200) x 100

= 91.83%

Question:

Out of 65000 population, men are 39000. What is the percentage of women?

Answer:

Women = 65000 – 39000 = 26000

Percentage of women = (26000/65000) x 100

= (2/5) 100 = 40 %

Scanned with CamScanner

Shortcut # 19 – Percentage

Percentage increase or decrease.

$$\text{Percentage change} = \frac{\text{From} \sim \text{To}}{\text{From}} \times 100$$

From – the value from which the change is happening
To – the value to which the change has happened.

If **From** value is less than **To** value, it is % increase

If **From** value is more than **To** value, it is % decrease.

Question:

Sradha's monthly salary is Rs. 40000. If an increment of Rs. 5000 is provided, what is the percentage increase in her salary?

Answer:

From = 40000; To = 45000.

Substituting the values in the above formula, we get

% increase = [(45000 – 40000)/40000] x 100

= (5000/40000)x 100

= 12.5 % increase

Scanned with CamScanner

Shortcut # 20 – Percentage

Net change in percentage when two changes are made.

$$\text{Net percentage change} = \pm a \pm b + \frac{(\pm a) \times (\pm b)}{100}$$

a = first percentage change

b = second percentage change

Question:

Salary of Sai was Rs. 20000. If the salary is decreased by 10 percent and then increased by 30 percent, what is the new salary of Sai?

Answer:

a = -10

b = 30

Net change = -10 + 30 + [(-10)(30)/100]

= 20 – 3 = +17% (positive value = % increase)

New salary of Sai

= 20000 + (17/100)20000

= 23400

Note:
If the net change value is negative, it means percentage decrease.

Scanned with CamScanner

Shortcut # 21 – Percentage

Final value after multiple percentage changes.

$$\text{Final Value} = \left[\frac{100 \pm a}{100}\right] \times \text{Initial Value}$$

$$\text{Final Value} = \left[\frac{100 \pm a}{100} \times \frac{100 \pm b}{100}\right] \times \text{Initial Value}$$

$$\text{Final Value} = \left[\frac{100 \pm a}{100} \times \frac{100 \pm b}{100} \times \frac{100 \pm c}{100}\right] \times \text{Initial Value}$$

(a, b and c) are the percentage changes.
The formula can be expanded or reduce according to the number of changes given in the question.
Substitute '+' for percentage increase.
Substitute '-'for percentage decrease.

Question:

Salary of Shradha is Rs. 30000. It is increased by 10%, then decreased by 20% and again increased by 30%. What is her final salary?

Answer:

a = +10; b = -20; c = +30

Substitute the values in the above formula, we get

Final Value

= (11/10)(8/10)(13/10) x 30000

= 34320

Scanned with CamScanner

Shortcut # 22 – Percentage

Net percentage change when (a %) increase and (a %) decrease.

$$\text{Net percentage change} = \frac{a^2}{100} \ \% \text{ decrease}$$

Question:

Price of a watch was Rs. 1000. The price is increased by 10% and then decreased by 10%. What is the price of the watch now?

Answer:

a = 10

Net change in percentage = $10^2/100$ = 1% decrease.

New price of watch = 1000 – (1/100)1000

= 1000 – 10

= 990

Question:

Price of a product increased by 20% and the sales was reduced by 20%. By what percentage the income has reduced?

Answer:

a = 20%

Percentage reduction in income

= $20^2/100$

= 4%

Scanned with CamScanner

Shortcut # 23 – Percentage

Steady percentage increase or decrease.

$$\text{Final Value} = \left[\frac{100 \pm a}{100}\right]^n \times \text{Initial Value}$$

a – steady rate of percentage increase or decrease.
Substitute '+' for percentage increase.
Substitute '-'for percentage decrease.
n – Number of times the percentage change occurs.

Question:

The population of a city is 200000. The population increases by 10% every year. What will be the population of the city after 3 years?

Answer:

a = + 10

Initial Value = 200000

Substitute the values in the above equation, we get

Final Value = $200000[1.1]^3$

Final Value = 266200

Population of the city after three years

= 266200

Scanned with CamScanner

Shortcut # 24 – Profit, Loss and Discount

Profit or Loss Percentage.

$$\text{Profit or Loss Percentage} = \frac{S.P \sim C.P}{C.P} \times 100$$

S.P – Selling Price.

C.P – Cost Price.

Question:

A bike worth Rs. 40000 is sold for Rs. 46000. What is the profit percentage?

Answer:

S.P = 46000; C.P = 40000

Substitute the values in the above equation, we get

P% = [(46000-40000)/40000]x100 = 15% profit

Question:

A mobile phone worth Rs. 9900 is sold for Rs. 9000. What is the percentage loss?

Answer:

S.P = 9000; C.P = 9900

Substitute the values in the above equation, we get

L% = [(9900-9000)/9900] x 100; L% = 9.09% loss

Scanned with CamScanner

Shortcut # 25 – Profit, Loss and Discount

Net Profit or Loss Percentage when two changes are made.

$$\text{Net Profit or Loss \%} = \pm a \pm b + \frac{(\pm a) \times (\pm b)}{100}$$

a = percentage profit or loss in the first case.

b = percentage profit or loss in the second case.

Substitute '+' for profit percentage.
Substitute '-' for loss percentage.

Question:
A product worth $ 3000 is sold for 10% profit and then for 20% loss. What is the overall profit or loss percentage in the sale?

Answer:
Net change = 10 – 20 + [(10)(-20)/100]

= -12

The value obtained is negative.

This indicates that a loss is incurred in the sale.

12% loss.

Scanned with CamScanner

Shortcut # 26 – Profit, Loss and Discount

Finding selling price after multiple changes.

$$S.P = \left[\frac{100 \pm a}{100}\right] \times C.P$$

$$S.P = \left[\frac{100 \pm a}{100} \times \frac{100 \pm b}{100}\right] \times C.P$$

$$S.P = \left[\frac{100 \pm a}{100} \times \frac{100 \pm b}{100} \times \frac{100 \pm c}{100}\right] \times C.P$$

(a, b and c) are the profit or loss percentages.
The formula can be expanded or reduce according to the number of changes given in the question.
Substitute '+' for profit percentage.
Substitute '-' for loss percentage.

Question:

A watch worth Rs. 5000 is sold by A to B at 10% loss. B sold it to C at 30% loss. C sold it to D at 40% profit. What is the price at which D bought the watch?

Answer:

a = -10; b = -30; c = 40

C.P = 5000

Substitute the values in the above equation, we get

S.P = (9/10)(7/10)(14/10) x 5000

S.P = 4410

Scanned with CamScanner

Shortcut # 27 – Profit, Loss and Discount

Steady Profit or Loss.

$$\text{Selling Price} = \left[\frac{100 \pm a}{100}\right]^{n} \times \text{Cost Price}$$

a – steady rate of profit or loss percentage.
Substitute '+' for profit percentage.
Substitute '-' for loss percentage.
n – Number of times the percentage change occurs.

Question:

The price of a laptop decreases by 20% every year. It the laptop was bought for Rs. 45000, at what price will it be sold after 2 years?

Answer:

a = -20

C.P = 45000

Substitute the values in the above equation, we get

S.P = $45000[0.8]^2$

S.P = 28800

Selling price after two years

= Rs. 28800

Scanned with CamScanner

Shortcut # 28 – Profit, Loss and Discount
Net gain or loss percentage when (a %) profit and (a %) loss occurs.

$$\text{Net Loss Percentage} = \frac{a^2}{100}\ \%\ \text{loss}$$

a – percent loss and percentage gain.

Question:

Two laptops were sold for the same selling price. The first one is sold at 50% profit and the second one is sold at 50% loss. Due to this sale what is the profit or loss percentage incurred by the seller?

Answer:
Assume that the selling price of each laptop = Rs. 150
Cost price of first laptop:
150 = (150/100)CP
$C.P_1 = 100$
Cost price of second laptop:
150 = (50/100)CP
$C.P_2 = 300$
Total cost price = 100 + 300 = 400
Total selling price = 150 + 150 = 300
Loss % = (100/400)100 = 25%
(Or)
$(a^2/100)\%$ loss = $(50^2/100)\%$loss = 25% loss.

Note:
Shortcut applicable only when selling price of both products are equal.

Scanned with CamScanner

Shortcut # 29 – Profit, Loss and Discount

Discount, Marked Price and Selling Price.

$$S.P = \left[\frac{100 \pm d}{100}\right] \times M.P$$

d – Discount percentage

M.P = Marked price or Market price of the product

Question:

A product is marked Rs. 450 by the seller. If he sells at a price of Rs. 330, what is the discount provided in percentage?

Answer:

M.P = 450

S.P = 330

Substitute the values in the above equation, we get

D% = [(450-330)/450]x100

D% = 26.67%

Discount provided in percentage = 26.67%

Scanned with CamScanner

Shortcut # 30 – Problems on ages

One variable linear equation.

Question:

Age of father is twice that of son at present. After 5 years, sum of their ages will be 100. What is the present age of father?

Answer:

Assume age of son = S

Age of father = 2S

Age of father after 5 years = 2S + 5

Age of son after 5 years = S + 5

Given,

$(2S + 5) + (S + 5) = 100$

$3S + 10 = 100$

$S = 30$

Present age of father = 2S = 60

Scanned with CamScanner

Shortcut # 31 – Problems on ages

One variable quadratic equation.

Question:

Sum of the ages of father and son is 45. Product of their ages is 350. What is the difference between their ages?

Answer:

Age of father = F; Age of son = S

F + S = 45; F = 45 – S --- Equ (1)

F x S = 350 --- Equ (2)

Sub Equ (1) in (2)

(45 – S)S = 350

$45S - S^2 = 350$

$S^2 - 45S + 350 = 0$

By solving this quadratic equation, we get

S = 10, 35. (Neglect 35 since son's age cannot be greater than father's.)

From (1) Father's age = 35

Difference between their ages = 35 -10 = 25

Formula to find roots of a quadratic equation:

A – Coefficient of x^2; b - coefficient of x; c – constant

Scanned with CamScanner

Shortcut # 32 – Problems on ages
Simultaneous equations.

Question:

Sum of ages of father and son at present is 40. After 5 years the ratio between their ages become 7 : 3. Find the present ages of father and son.

Answer:

Assume present age of father = F

Assume present age of son = S

Given, F + S = 40 --- Equ (1)

Father's age after 5 years = F + 5

Son's age after 5 years = S + 5

Given, (F + 5)/(S + 5) = 7/3, which gives

3F + 15 = 7S + 35

3F – 7S = 20 --- Equ (2)

By solving Equ (1) and (2), we get

Age of son, S = 10

Age of father, F = 30

Scanned with CamScanner

Shortcut # 33 – Problems on ages

Solution using common factor 'x'.

$$A - C = d \left\{ \begin{matrix} A & : & B \\ C & : & D \end{matrix} \right\} B - D = d$$

$$\text{Common factor 'x'} = \frac{\text{Difference in years}}{d}$$

Question:

Age of MS and VK were in the ratio 4 : 3 five years back. Their ages will be in the ratio 5 : 4 in two years. Find their present ages.

Answer:

Difference between the two compared years = 7.

Difference in ratio = 1

x = 7/1 = 7

Age of MS and VK 5 years ago = 4x and 3x

4x = 4(7) = 28

3x = 3(7) = 21

Age of MS at present = 28 + 5 = 34

Age of VK at present = 21 + 5 = 26

Note:

This method is applicable only when 'd' is equal on both sides.

The ratios can be modified to get 'd' same in both sides.

Scanned with CamScanner

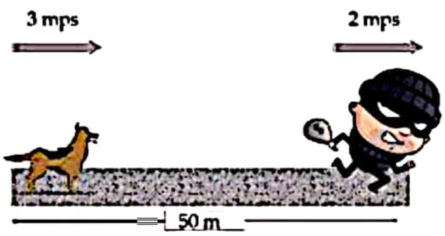
Shortcut # 34 – Time, Speed and Distance

Relative Speed – Same direction

When two objects are moving we have to take relative speed for the calculation.
Relative speed of two objects moving in same direction
$= S_1 \sim S_2$
Relative speed of two objects moving in opposite direction
$= S_1 + S_2$

$$\text{Time taken} = \frac{\text{Distance}}{\text{Relative Speed}}$$

Question:



A thief standing 50 away from a dog starts running away from it at a speed of 2 m/s. The dog suddenly starts chasing him at a speed of 3m/s. How long will the dog take to catch the thief?

Answer:

D = 50m; $S_1 = 3$; $S_2 = 2$

Time taken, T = 50/(3-2) = 50 seconds

Distance travelled by the thief = Speed x Time to catch

= 2 x 50 = 100 m

Distance travelled by the dog = 3 x 50 = 150 m

Scanned with CamScanner

Shortcut # 35 – Time, Speed and Distance

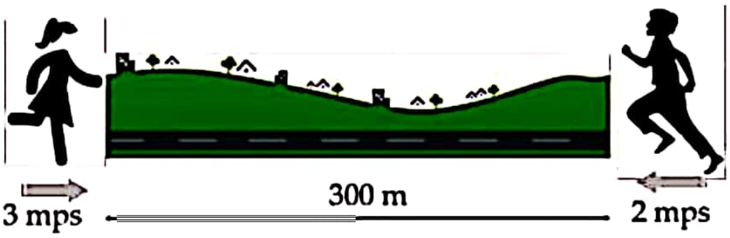
Relative Speed – Opposite direction

When two objects are moving we have to take relative speed for the calculation.
Relative speed of two objects moving in same direction
$= S_1 \sim S_2$
Relative speed of two objects moving in opposite direction
$= S_1 + S_2$

$$\text{Time taken} = \frac{\text{Distance}}{\text{Relative Speed}}$$

Question:

Two friends start from their home 300 m apart and walk towards each other at a speed of 2mps and 3mps. How long (in minutes) will they take to meet each other?



Answer:

D = 300m

$S_1 = 2$mps; $S_2 = 3$mps

T = 3000/(2+3) = 60 seconds

Time taken in minutes = 60/60 = 1 minute.

Scanned with CamScanner

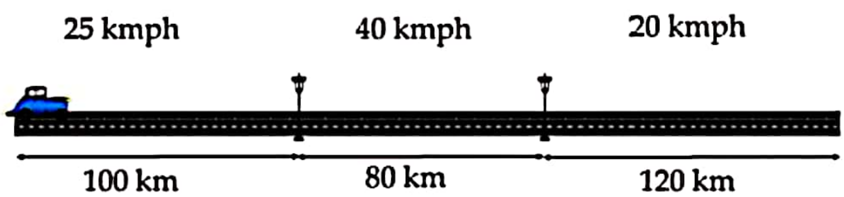
Shortcut # 36 – Time, Speed and Distance

Average Speed – Different distance and time

$$\text{Average Speed} = \frac{\text{Total Distance}}{\text{Total Time}} = \frac{D_1 + D_2 + \ldots\ldots + D_n}{\frac{D_1}{S_1} + \frac{D_2}{S_2} + \ldots + \frac{D_n}{S_n}}$$

Question:

A man travels from a point to other. The first 100 km he travels at a speed of 25 kmph. The next 80 km he travels at a speed of 40 kmph. The last 120 km he travels at a speed of 20 kmph. What is the average speed of his journey?



Answer:

$$= \frac{100 + 80 + 120}{\frac{100}{25} + \frac{80}{40} + \frac{120}{20}}$$

= 300/12

= 25 kmph

Scanned with CamScanner

Shortcut # 37 – Time, Speed and Distance

Average Speed – Distance is same.

General formula when distance is same in all the cases.

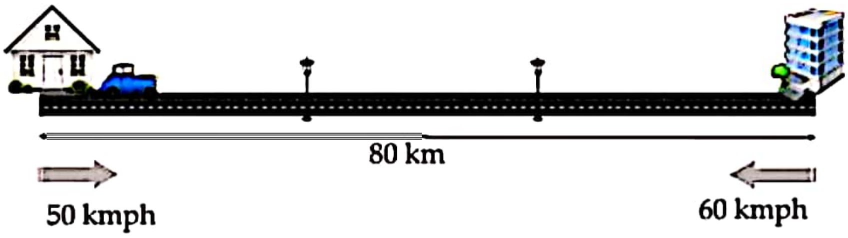
$$\frac{n}{\text{Average Speed}} = \frac{1}{S_1} + \frac{1}{S_2} + \dots + \frac{1}{S_n}$$

Formula for 2 cases with same distance.

$$\frac{2}{\text{Average Speed}} = \frac{1}{S_1} + \frac{1}{S_2} \text{ (Or) } \quad \text{Average Speed} = \frac{2\,S_1 S_2}{S_1 + S_2}$$

Question:

Sai travels from her house to office at a speed of 50kmph. She suddenly returns home at a speed of 60kmph. What is her average speed?



Answer:

$S_1 = 50$; $S_2 = 60$

$S_a = 2(50x60)/(50+60)$

$= 6000/110$ $= 54.54$ kmph

Note: Distance value is unnecessary.

Scanned with CamScanner

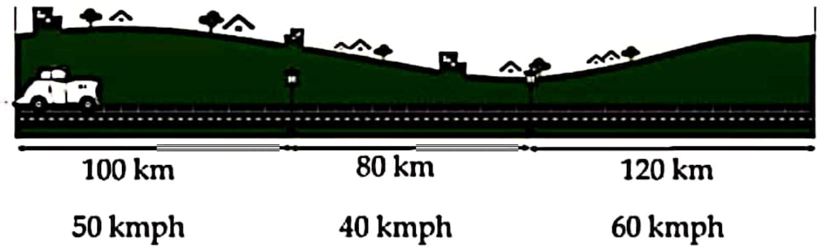
Shortcut # 38 – Time, Speed and Distance

Average Speed – Time taken is same in all the cases.

$$\text{Average Speed} = \frac{S_1 + S_2 + S_3}{3}$$

Question:

A man travels from a point to other. The first 100 km he travels at a speed of 50 kmph. The next 80 km he travels at a speed of 40 kmph. The last 120 km he travels at a speed of 60 kmph. What is the average speed of his journey?



Answer:
Time taken in the first case = 100/50 = 2 hours
Time taken in the second case = 80/40 = 2 hours
Time taken in the third case = 120/60 = 2 hours

Time taken is same in all the cases. So, average speed can be calculated by directly taking the average of the speeds in each case.

Average speed = (50 + 40 + 60)/3
= 50 kmph

Scanned with CamScanner

Shortcut # 39 – Time, Speed and Distance

Circular race – Time in which the runners meet at the starting point

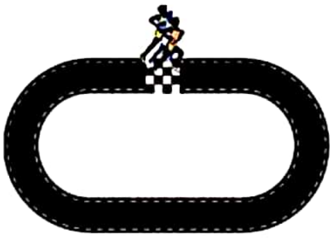
$$\text{LCM}\,(T_1, T_2, \ldots T_n) = \text{LCM}\,\left(\frac{D}{S_1}, \frac{D}{S_2}, \ldots, \frac{D}{S_n}\right)$$

D – Circumference of the race track.

$S_1, S_2, \ldots, S_n$ – Speed of the individual runners.

$T_1, T_2, \ldots, T_n$ – Time taken by the respective runners to complete one round the track.

Question:



Three persons participate in a race on a circular track of length 400m. They can run at a speed of 2mps, 4mps and 5mps respectively. How long will they take to meet in the starting point for the first time?

Answer:

Time taken by each person to complete one circle

Person 1 = 400/2 = 200 seconds = T_1

Person 2 = 400/4 = 100 seconds = T_2

Person 3 = 400/5 = 80 seconds = T_3

$\text{LCM}(T_1, T_2, T_3)$ = 400 seconds

Time taken by them to meet at starting point for the first time = 400 seconds.

Scanned with CamScanner

Shortcut # 40 – Trains

Train crossing a stationary object with negligible width.

$$\text{Time taken to cross} = \frac{L_t}{S_t}$$

L_t – Length of the train
S_t – Speed of the train

Conversion from Kmph to mps <|> kmph x (5/18) = mps						
Kmph	18	36	54	72	90	108
mps	5	10	15	20	25	30

Question:

A train of length 450m is travelling at a speed of 54kmph. How long will it take to cross a pole?

Answer:

Speed of train S_T in m/s = 54x(5/18)

= 15mps

Length of train = L_T = 450m

Time taken to cross = 450/15

= 30 seconds

Scanned with CamScanner

Shortcut # 41 – Trains

Train crossing a stationary object with considerable length.

$$\text{Time taken to cross} = \frac{L_t + L_p}{S_t}$$

L_t – Length of the train; L_p – Length of the platform

S_t – Speed of the train

Conversion from Kmph to mps <|> kmph x (5/18) = mps						
Kmph	18	36	54	72	90	108
mps	5	10	15	20	25	30

Question:

A train of length 200m crosses a platform of length 150m at a speed of 72kmph. How long will the train take to cross the platform?

Answer:

L_t= 200m

L_p= 150m

S_t= 72x(5/18) = 20mps

Time taken to cross the platform = (200+150)/20

= 17.5 seconds

Scanned with CamScanner

Shortcut # 42 – Trains

Two trains crossing in the opposite directions.

$$\text{Time taken to cross} = \frac{L_1 + L_2}{S_1 + S_2}$$

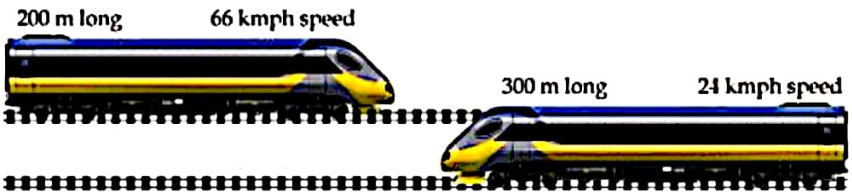
L_1 – Length of the train 1; L_2 – Length of train 2

S_1 – Speed of the train 1; S_2 – Speed of train 2

Conversion from Kmph to mps <|> kmph x (5/18) = mps						
Kmph	18	36	54	72	90	108
mps	5	10	15	20	25	30

Question:

Two trains A and B of length 200m and 300 are travelling at a speed of 54kmph and 36kmph respectively in opposite directions. How long will they take to cross each other?



Answer:

$L_1 = 200$; $L_2 = 300$

$S_1 + S_2 = 66 + 24 = 90$ kmph $= 25$ mps.

Substitute the values in the above equation, we get

T = (200+300)/(25)

= 20 seconds

Scanned with CamScanner

Shortcut # 43 – Trains

Train crossing another train from the same direction.

$$\text{Time taken to cross} = \frac{L_1 + L_2}{S_1 \sim S_2}$$

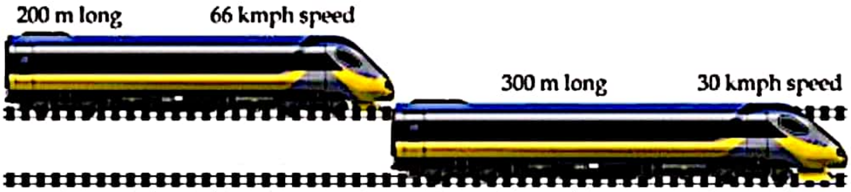
L_1 – Length of the train 1; L_2 – Length of train 2

S_1 – Speed of the train 1; S_2 – Speed of train 2

Conversion from Kmph to mps <|> kmph x (5/18) = mps						
Kmph	18	36	54	72	90	108
mps	5	10	15	20	25	30

Question:

Two trains A and B of length 200m and 300 are travelling at a speed of 66 kmph and 30 kmph respectively in same directions. How long will they take to cross each other?



Answer:

$L_1 = 200$; $L_2 = 300$

$S_1 \sim S_2 = 66 - 30 = 36$ kmph $= 10$ mps

Substitute the values in the above equation, we get

T = (200+300)/(10)

= 50 seconds

Scanned with CamScanner

Shortcut # 44 – Trains

Train crossing a man sitting on another train travelling in opposite direction.

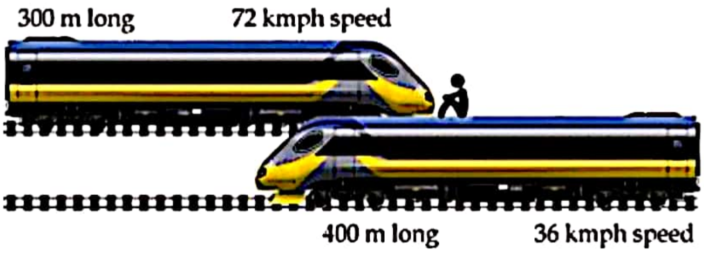
$$\text{Time taken to cross} = \frac{L_1}{S_1 + S_2}$$

L_1 – Length of the train which is crossing the man

S_1 – Speed of the train 1; S_2 – Speed of train 2

Question:

Two trains A and B of length 300m and 400 are travelling at a speed of 72kmph and 36kmph respectively in opposite directions. How long will train A take to cross a man travelling in train B?



Answer:

$L_1 = 300$, (Length of train crossing the man)

$S_1 + S_2 = 72 + 36 = 108$ kmph = 30 mps

Substitute the values in the above equation, we get

T = (300)/(30)

= 10 seconds

Scanned with CamScanner

Shortcut # 45 – Trains

Train crossing a man sitting on another train travelling in same direction.

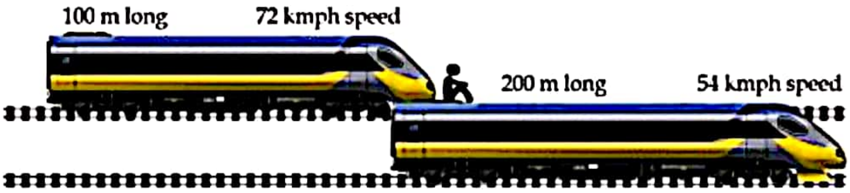
$$\text{Time taken to cross} = \frac{L_1}{S_1 \sim S_2}$$

L_1 – Length of the train which is crossing the man

S_1 – Speed of the train 1; S_2 – Speed of train 2

Question:

Two trains A and B of length 100m and 200 are travelling at a speed of 72kmph and 54kmph respectively in same directions. How long will train A take to cross a man travelling in train B?



Answer:

L_T = 100, (Length of train crossing the man)

S_1 = 72kmph = 72(5/18) mps = 20mps

S_2 = speed of the man travelling in train B

S_2 = 54kmph = 36(5/18) mps = 10mps

Substitute the values in the above formula, we get

T = (100)/(20-10)

= 10 seconds

Scanned with CamScanner

Shortcut # 46 – Boats

Time taken to row upstream.

$$\text{Time taken to row upstream} = \frac{D_{us}}{S_b - S_r}$$

D_{us} – Distance travelled upstream.

S_b – Speed of the boat in still water.

S_r – Speed of the river flow (or) stream (or) current.

Question:

The speed of boat in still water is 40kmph. How long will the boat take to row 175 km upstream in a river of speed 5kmph?

Answer:

D = 175km

S_b = 40kmph, (speed of boat in still water)

S_r = 5kmph, (speed of river water)

$S_b - S_r$ = 35kmph, (Upstream speed of boat)

Substitute the values in the above formula, we get

T = 175/35

= 5 hours

Scanned with CamScanner

Shortcut # 47 – Boats

Time taken to row downstream.

$$\text{Time taken to row downstream} = \frac{D_{ds}}{S_b + S_r}$$

D_{ds} – Distance travelled downstream.
S_b – Speed of the boat in still water.
S_r – Speed of the river flow (or) stream (or) current.

Question:

The speed of boat in still water is 30kmph. How long will the boat take to row 96 km downstreamin a river of speed 2kmph?

Answer:

D = 96km

S_b = 30kmph, (speed of boat in still water)

S_r = 2kmph, (speed of river water)

$S_b + S_r$ = 32kmph, (Downstream speed of boat)

Substitute the values in the above formula, we get

T = 96/32

= 3 hours

Scanned with CamScanner

Shortcut # 48 – Boats

Finding speed of the boat when upstream and downstream speed are known.

$$\text{Speed of the boat, } S_b = \frac{S_{ds} + S_{us}}{2}$$

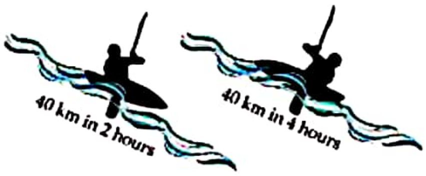
S_b – Speed of the boat in still water.

S_{us} - Upstream speed.

S_{ds} – Downstream speed.

Question:

Time taken by a boat to row 40km upstream in 4 hours. It took 2 hours to return back to the starting point. Find the speed of the boat.



Answer:

Step 1: Find the speed upstream and downstream.

$S_{us} = 40/4 = 10$kmph

$S_{ds} = 40/2 = 20$kmph

Step 2: Find the speed of boat.

$S_b = (10 + 20)/2 = 15$kmph

Scanned with CamScanner

Shortcut # 49 – Boats

Finding speed of the river when upstream and downstream speed are known.

$$\text{Speed of the boat, } S_b = \frac{S_{ds} - S_{us}}{2}$$

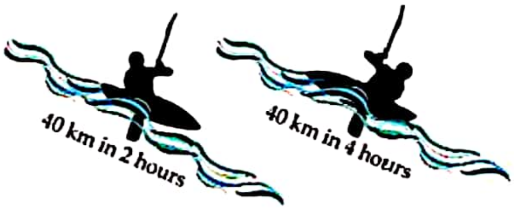
S_b – Speed of the boat in still water.

S_{us} - Upstream speed.

S_{ds} – Downstream speed.

Question:

Time taken by a boat to row 40km upstream in 4 hours. It took 2 hours to return back to the starting point. Find the speed the river.



Answer:

Step 1: Find the speed upstream and downstream

$S_{us} = 40/4 = 10$kmph

$S_{ds} = 40/2 = 20$kmph

Step 2: Find the speed of the river

$S_r = (20 – 10)/2 = 5$kmph

Scanned with CamScanner

Shortcut # 50 – Time and Work

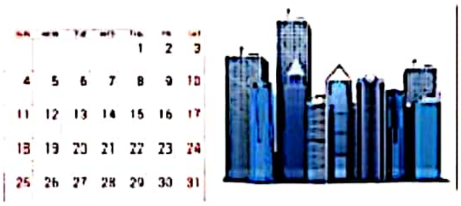
Work is directly proportional to time.

$$\frac{W_1}{W_2} = \frac{T_1}{T_2}$$

W_1 – Work done in first case.
T_1 – Time taken in first case.
W_2 – Work done in second case.
T_2 – Time taken in second case.

Question:

It took 40 days to build 10 buildings. How long will it take to build 50 buildings using same number of employees?



Answer:

$W_1 = 10$; $W_2 = 50$

$T_1 = 40$; $T_2 = ?$

Substitute the values in the above formula, we get

$10/50 = 40/T_2$

$T_2 = 200$ days

Scanned with CamScanner

Shortcut # 51 – Time and Work
Work is directly proportional to Resource.

$$\frac{W_1}{W_2} = \frac{R_1}{R_2}$$

Question:

40 men are required to complete 20 buildings. How many men are required to build 50 buildings in the same time?

Answer:

$R_1 = 40$; $R_2 = ?$

$W_1 = 20$; $W_2 = 50$

Substitute the values in the above formula, we get

$40/R_2 = 20/50$

$R_2 = 100$

Scanned with CamScanner

Shortcut # 52 – Time and Work

Time is inversely proportional to Resource.

$$\frac{T_1}{T_2} = \frac{R_2}{R_1}$$

Question:

If 20 men can do the work in 40 days, how many men are required to do the same work in 10 days?

Answer:

$R_1 = 20$; $R_2 = ?$

$T_1 = 40$; $T_2 = 10$

Substitute the values in the above formula, we get

$20/R_2 = 10/40$

$R_2 = 80$

Scanned with CamScanner

Shortcut # 53 – Time and Work

Comparison of Time, Work and Resource.

$$\frac{W_1}{W_2} = \frac{R_1 T_1}{R_2 T_2}$$

Question:

If 10 men can cut 40 trees in 4 days, how many trees can be cut by 40 men 6 days?

Answer:

$R_1 = 10$; $R_2 = 40$

$T_1 = 4$; $T_2 = 6$

$W_1 = 40$; $W_2 = ?$

Substitute the values in the above formula, we get

$40/W_2 = (10 \text{ x } 4)/(40 \text{ x} 6)$

$W_2 = 240$ trees

Scanned with CamScanner

Shortcut # 54 – Time and Work

When the number of working hours per day is included in the question.

$$\frac{W_1}{W_2} = \frac{R_1T_1H_1}{R_2T_2H_2}$$

Question:

30 men working 6 hours day for 50 days can make 2000 toys. How many hours per day should 40 men work for 60 days to make 3000 toys?

Answer:

$R_1 = 30$; $R_2 = 40$, (Resources in each case)

$T_1 = 50$; $T_2 = 60$, (Days in each case)

$W_1 = 2000$; $W_2 = 3000$, (Work in each case)

$H_1 = 6$; $H_2 = ?$, (Hours per day in each case)

Substitute the values in the above formula, we get

$2000/3000 = (30 \times 50 \times 6)/(40 \times 60 \times H_2)$

$H_2 = 5(5/8)$ hours = 5 hours 32 minutes 30 seconds

Scanned with CamScanner

Shortcut # 55 – Time and Work

Two resources working together.

$$T = \frac{AB}{A + B} \quad \text{or} \quad \frac{1}{T} = \frac{1}{A} + \frac{1}{B}$$

Question:

A can complete a work in 40 days. B can complete the same work in 60 days. How long will they take to complete the work if they are working together?

40 days

60 days

Answer:

A = 40

B = 60

Substitute the values in the above formula, we get

Time taken together = (40 x 60)/(40 + 60)

= 24 days

Scanned with CamScanner

Shortcut # 56 – Time and Work

Three resources working together.

$$T = \frac{ABC}{AB + BC + AC} \quad \text{or} \quad \frac{1}{T} = \frac{1}{A} + \frac{1}{B} + \frac{1}{C}$$

Question:

A can do a work in 40 days, B in 50 days and C in 60 days. If they work together, how long will they take to complete the work?

40 days 50 days 60 days

Answer:

A = 40

B = 50

C = 60

Substitute the values in the above formula, we get

Time taken together = (40 x 50 x 60)/(40 + 50 + 60)

= 16.21 days

Scanned with CamScanner

Shortcut # 57 – Time and Work

Work done.

Work done per day = 1/Time taken to do complete work

Work done = Number of days worked x Work done per day

Question:

A can do a piece of work in 40 days. What is the fraction of work done by him in 25 days?

Answer:

Per day work of A = 1/40

Number of days for which A worked = 25

Work done = (1/40) x 25

$= \frac{5}{8}$

Question:

A can do a piece of work in 30 days. What is the fraction of work done by him in 45 days?

Answer:

Per day work of A = 1/30

Number of days for which A worked = 45

Work done = (1/30) x 45

$= 1\frac{1}{2}$

Scanned with CamScanner

Shortcut # 58 – Time and Work
Remaining work.

Remaining Work = 1 – Work Done

Question:

A can do a work in 30 days, B in 40 days. A works for 9 days and the remaining work is done by B. What is the fraction of work done by B?

Answer:

Remaining work for B = 1 – Work done by A

Work done by A = (1/30) x 9 = 3/10

Remaining work for B = 1 – (3/10) = 7/10

Question:

A can do a work in 40 days, B in 60 days. A works for 24 days and the remaining work is done by B. What is the fraction of work done by B?

Answer:

Remaining work for B = 1 – Work done by A

Work done by A = (1/40) x 24 = 6/10

Remaining work for B = 1 – (6/10) = 4/10

= 2/5

Scanned with CamScanner

Shortcut # 59 – Time and Work

Time taken to do the remaining work.

Time taken to do remaining Work

= Remaining work x Time taken to do full work

Question:

A can do a work in 30 days, B in 40 days. A works for 9 days and the remaining work is done by B. What is the total number of days to complete the work?

Answer:

Time taken to complete the work

= Remaining work x time to finish full work

Remaining work for B = 1 – Work done by A

Work done by A = (1/30) x 9 = 3/10

Remaining work for B = 1 – 3/10 = 7/10

Time taken by B to complete the remaining work

= 7/10 x 40

= 28 days

Total time taken
= 28 + 9
= 37 days

Scanned with CamScanner

Shortcut # 60 – Time and Wok

Salary ratio of resources working together.

Salary of a resource is directly proportional to the per day work.

$$S_A : S_B = \frac{1}{A} : \frac{1}{B}$$

$$S_A : S_B : S_C = \frac{1}{A} : \frac{1}{B} : \frac{1}{C}$$

Question:

A can do a work in 20 days and B can do the same work in 40 days. They both work together and get a combined salary of Rs. 3000. What is the salary of A?

Answer:

S_A = Salary of A; S_B = Salary of B

A = 20; B =40

$S_A : S_B = 40 : 20 = 2 : 1$

$S_A = [2/(2+1)] \times 3000 = 2000$

Question:

A can do a work in 30 days B in 40 days and C in 50 days. If they all work together, what will the ratio between their salaries?

Answer:

A = 30; B = 40; C = 50

$S_A : S_B : S_C = (1/30) : (1/40) : (1/50) = 20 : 15 : 12$

Scanned with CamScanner

Shortcut # 61 – Pipes and Cisterns

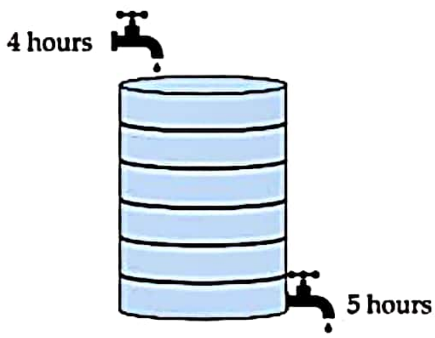
One pipe fills the tank and the other empties the tank.

$$\frac{1}{T} = \frac{1}{A} - \frac{1}{B}$$

The pipe which empties the tank is doing negative work. So we have to assign '-'symbol for the work done by that pipe.
T – Total time taken to fill the tank.

Question:

Pipe A can fill a tank in 4 hours and pipe B can empty the tank in 5 hours. If both pipes are opened together, how long will they take to fill a complete tank?



Answer:

A = 4; B = 5

Substitute the values in the above equation, we get

Time taken to fill the tank = (4 x 5)/(-4 + 5)

= 20 hours

Scanned with CamScanner

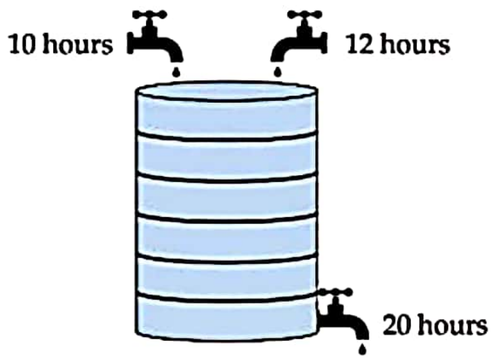
Shortcut # 62 – Pipes and Cisterns

Two pipes fill a tank and another pipes empties the tank.

$$\frac{1}{T} = \frac{1}{A} + \frac{1}{B} - \frac{1}{C}$$

Question:

Pipes A and B can fill a tank in 10 hours and 12 hours respectively. Pipe C can empty a tank in 20 hours. If all the pipes are opened together, how long will they take to fill a complete tank?



Answer:

A = 10; B = 12; C = 20

Substitute the values in the above equation, we get

Time taken to fill the tank

= (10 x 12 x 20)/[(-10 x 12) + (12 x 20) + (10 x 20)]

= 7. 5 hours = 7 hours 30 minutes

Scanned with CamScanner

Shortcut # 63 – Interest – Simple and Compound

Simple Interest

$$\text{Simple Interest} = \frac{PNR}{100}$$

P = Principle amount invested or borrowed

R = Rate of interest per term

N = Number of terms

Term = duration for which interest is calculated

Question:

A man invested Rs. 10000 at 4% per annum simple interest.

1. What is the interest he will get at the end of 3 years?
2. What is the total amount earned after 3 years?

Answer:

P = 10000; N = 3; R = 4

Substitute the values in the above equation, we get

SI = 10000 x 3 x 4/100

= 1200

Amount = Principle + Interest

Amount = 10000 + 1200 = 11200

Scanned with CamScanner

Shortcut # 64 – Interest – Simple and Compound

Compound Interest.

$$\text{Amount after } n^{th} \text{ term} = \frac{100 + R}{100} \text{ Amount after } (n-1)^{th} \text{ term}$$

Question:

What is the compound interest obtained for Rs. 16000 at 2% per annum for three years?

Answer:

Amount at the end of 1st year = $\frac{102}{100}$ x 16000 = 16320

Amount at the end of 2nd year = $\frac{102}{100}$ x 16320 = 16646.4

Amount at the end of 3rd year = $\frac{102}{100}$ x 16646.4 = 16979.328

Interest obtained at the end of three years:

Interest = Amount – Principle

Interest = 16979.328 – 16000 = 979.328

Note:
In simple interest, the interest is calculated only for the principle. In compound interest, the interest is calculated for the principle as well as the interest obtained till the last term.

Scanned with CamScanner

Shortcut # 65 – Number Systems

Divisibility.

Divisible by	Rule
2	Last digit of a number should be 0, 2, 4, 6 or 8
3	Sum of the digits should be divisible by 3
4	Last two digits of a number should be divisible by 4
5	Last digit of a number should be 0 or 5
6	Number should be divisible by 2 and 3
8	Last three digits of a number should be divisible by 8
9	Sum of the digits should be divisible by 9
10	Last digit of a number should be 0
11	Difference between sum of digits in odd places and sum of digits in even places should be o or 11

Divisibility rule for composite numbers:

If a number is a multiple of 'x' and 'y', then it is divisible by LCM of (x, y). 30 is the LCM of (2, 3 and 5). So all the multiples of 30 are divisible by 2, 3 and 5.

Question:

Which of the following numbers is divisible by both 5 and 9?

1115, 11115, 111115, 1111115

Answer:

11115 – It is divisible by 5 and 9.

Scanned with CamScanner

Shortcut # 66 – Number Systems

Finding largest four digit number that is divisible by certain numbers.

Step 1: LCM of the divisors
Step 2: Remainder of 10000/LCM
Step 3: 10000 – Remainder = Answer

Question:

Find the largest number that leaves

1. Remainder 0 when divided by 12, 15 and 20.
2. Remainder 2 when divided by 12, 15 and 20.
3. Remainder 9, 12 and 17 when divided by 12, 15 and 20 respectively.

Answer:

1. Remainder 0:

LCM(12, 15, 20) = 60
Remainder(10000/60) = 40
10000 – 40 = 9960.
The largest number that leaves remainder 0 = 9960

2. Remainder 2:

9960 + 2 = 9962 is the largest 4 digit number that leaves remainder 2 when divided by 12, 15 and 20.

3. Remainder 9, 12 and 17:

Difference between divisor and remainder = 3.
9960 – 3 = 9957 is the largest 4 digit number that leaves remainder 9, 12 and 17 when divided by 12, 15 and 20.

Scanned with CamScanner

Shortcut # 67 – Number Systems

Finding unit digit of a number with certain power.

The expression will be in the form $X^Y = Z$

Unit digit of X	Unit digit of Z
0	0
1	1
5	5
6	6

Power value

Unit place of X	Odd	Even
4	4	6
9	9	1

Unit place of Z

Remainder when power value is divided by 4

Unit place of X	1	2	3	0
2	2	4	8	6
3	3	9	7	1
7	7	9	3	1
8	8	4	2	6

Unit place of Z

Scanned with CamScanner

Shortcut # 68 – Number Systems
Remainder Concept.

$$\text{Remainder}\left(\frac{A \times B \times C}{K}\right) = R\left(\frac{A}{K}\right) \times R\left(\frac{B}{K}\right) \times R\left(\frac{C}{K}\right)$$

This concept is also applicable if the numbers in the numerator are added or subtracted.

Question:

Find the remainder when 212 x 313 x 414 is divided by 5.

Answer:

Using the above concept we can find the remainder for each term and then we can multiply.

R(212/5) = 2

R(313/5) = 3

R(414/5) = 4

2 x 3 x 4 = 24

The value we got is still greater than 5, so again divide it and find the remainder.

R(24/5) = 4

The remainder of the expression is = 4

Scanned with CamScanner

Shortcut # 69 – HCF and LCM

Finding both HCF and LCM using single L-Division.

While doing the L-Division method, initially divide only using the prime numbers which are common factors for all the numbers, then divide using the prime numbers which are not common.

Question:

Find LCM, and HCF for the numbers 240, 450 and 570.

2	240, 450, 570
5	120, 225, 285
3	24, 45, 57
19	8, 15, 19
5	8, 15, 1
3	8, 3, 1
2	8, 1, 1
2	4, 1 , 1
2	2, 1, 1
	1, 1, 1

The common factors for the three numbers are 2, 3 and 5.
Variation between common factors and the others is shown by using different font.

Multiply only the common factors to get the HCF

HCF = 2 x 3 x 5 = 30

Multiply all the factors to get LCM

LCM = 2 x 3 x 5 x 19 x 5 x 3 x 2 x 2 x 2

$= 2^4 \times 3^2 \times 5^2 \times 19^1$

= 68400

Scanned with CamScanner

Shortcut # 70 – HCF and LCM

Application problem using HCF.

There will be different sets of elements with different quantities in each. They have to be arranged based on the following conditions.

1. Every group should have equal number of elements.
2. Every group should have same kind of elements.

Under these conditions one must find out:

1. The maximum number of elements per group.
2. The minimum number of groups required.

HCF of the numbers of elements in all the groups will give the maximum number of elements per group.

Total number of elements divided by HCF will give the minimum number of groups required.

Question:

There are 40 apples and 32 oranges. They have to be packed in boxes such that each box will have one kind of fruit and every box will have equal number of fruits. Under this condition, what is the:

1. Maximum number of fruits per box?
2. Minimum number of boxes required?

Answer:

Maximum number of fruits per box = HCF(40, 32) = 8

Minimum number of boxes required = (40 + 32)/8 = 9

Scanned with CamScanner

Shortcut # 71 – HCF and LCM
Application problem using HCF.

For three or more numbers, the HCF of the difference between the successive numbers arranged in ascending order is the largest number that can divide the given numbers and leave same remainder.

Question:

What is the largest number that can divide 212, 254, 310 and 338 and leave the same remainder?

Answer:
The difference between the numbers = 42, 56 and 28.
HCF (42, 56, 28) = 14
Remainder (212/14) = 2
Remainder (254/14) = 2
Remainder (310/14) = 2
Remainder (338/14) = 2
14 is the largest number that can divide the given numbers and leave the same remainder.

Question:

What is the largest number that can divide 34, 52 and 88 and leave the same remainder?

Answer:
Difference between the numbers = 18 and 36
HCF (18, 36) = 18

18 is the largest number that can divide 34, 52 and 88 and leave same remainder.

Scanned with CamScanner

Shortcut # 72 – HCF and LCM

Application problem using LCM.

When different events occur in different intervals of their own and if all the events are started at the same time, at some point of time all the events will again occur together. To find the duration between two common occurrences, LCM of the intervals of each event has to be calculated.

Question:

There are three different bells. The first bell rings every 4 hour, the second bell rings every 6 hours and the third bell rings every 15 hours. If all the three bells are rung at same time, how long will it take for them to ring together again at the same time?

Answer:

Take LCM of 4, 6, 15

LCM (4,6,15) = 60

All the three bells will ring after 60 hours.

Question:

There are three different alarms. The first alarm rings every 2 minutes, the second alarm rings every 3 minutes and the third alarm rings every 5 minutes. If all the three alarms are rung at 6 am together, at what time the three alarms will ring together for the next time?

Answer:

LCM (2, 3, 5) = 30. After 30 minutes = 06:30 am.

Scanned with CamScanner

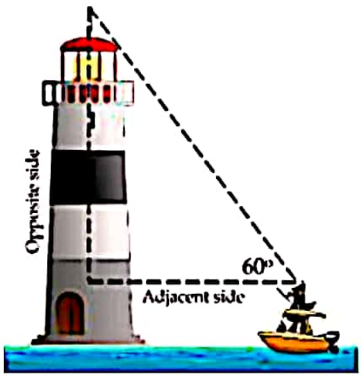
Shortcut # 73 – Heights and Distances

Solution using tan Θ .

θ	0	30	45	60	90
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

Question:

Jack Sparrow from a boat saw a lighthouse of height 100m at an angle of elevation, 60°. What is the distance between Jack Sparrow and the tower?



Answer:

$\tan \Theta = \frac{\text{opposite side}}{\text{adjacent side}}$

$\tan 60° = \frac{100}{x} =$ $\sqrt{3} = \frac{100}{x} =$ $x = 57.7$ m

Scanned with CamScanner

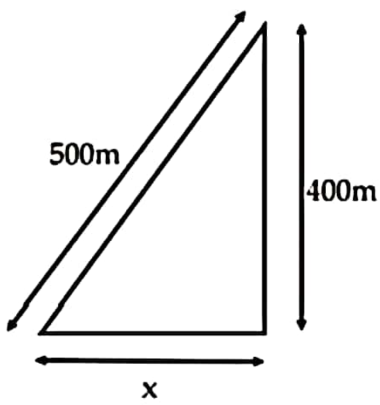
Shortcut # 74 – Heights and Distances

Solution using Pythagoras theorem.

Hypotenuse² = Oppsoite² + Adjacent²

Question:

A man looks at the top of a tower which is 400m height. The minimum distance between him and top of the tower is 500m. What is the distance between him and the base of the tower?



Answer:

$hyp^2 = opp^2 + adj^2$

$500^2 = 400^2 + x^2$

$x^2 = 90000$

$x = 300$ m

Scanned with CamScanner

Shortcut # 75 – Arithmetic Progression.

Finding nth term of an Arithmetic Progression.

$$t_n = a + (n - 1)d$$

Parameters:

a - first term of the series.

d - common difference between successive terms. $a_2 - a_1$

n - the number position of the term.

t_n - the term that has to found out.

Question:

What is the 100th term of the series?

1. 2, 7, 12, 17, 22, …
2. 501, 497, 493, 489, 485, …

Answer:

1. $a = 2$; $d = 5$; $n = 100$

   $t_n = 2 + (100 - 1)5$

   $t_n = 2 + 495 = 497$

   $t_{100} = 497$

2. $a = 501$; $d = - 4$; $n = 100$

   $t_n = 501 + (100 - 1)(- 4)$

   $t_n = 105$

   $t_{100} = 105$

Scanned with CamScanner

Shortcut # 76 – Arithmetic Progression

Number of terms in an Arithmetic Progression.

$$n = \frac{(l - a)}{d} + 1$$

l - Last term of the series.

Question:

Find the number of terms in the series:

1. 2, 7, 12, 17, 22, … 382
2. 501, 497, 493, 489, 485, … 201

Answer:

1. $n = \frac{(382 - 2)}{5} + 1$

   $n = 77$

2. $n = \frac{(201 - 501)}{-4} + 1$

   $n = 76$

Scanned with CamScanner

Shortcut # 77 – Arithmetic Progression

Sum of the terms in an Arithmetic Progression.

$$S_n = (a + l)\frac{n}{2}$$

l = Last term of the arithmetic series.

Question:

Find the sum of the series:

1. 2, 7, 12, 17, 22, … 497
2. 501, 497, 493, 489, 485, … 105

Answer:

1. Number of terms, $n = \frac{(497 - 2)}{5} + 1$

   n = 100

   Sum = (2 + 497)(100/2)
   S = 24950

3. Number of terms, $n = \frac{(105 - 501)}{-4} + 1$

   n = 100

   Sum = (501 + 105)(100/2)
   S = 30300

Scanned with CamScanner

Shortcut # 78 – Geometric Progression

Finding nth term of a Geometric Progression.

$$t_n = ar^{n-1}$$

t_n - n^{th} term.
a - first term of the series.
r - common ratio between the successive terms. (a_2/a_1)
n - number position of the term in the series.

Question:

Find 8^{th} term of the series 3, 6, 12, …

Answer:
n = 8; a = 3; r = (6/3) = 2
$t_8 = 3(2)^{8-1}$
$t_8 = 3(128) = 384$

Question:

Find 6^{th} term of the series 400, 200, 100, …

Answer:
n = 6; a = 400; r = (200/400) = 01/2
$t_6 = 400(1/2)^{6-1}$
$t_6 = 400(1/32) = 12.5$

Scanned with CamScanner

Shortcut # 79 – Geometric Progression

Sum of the terms of a finite series where r > 1.

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

S_n - Sum of n terms of the series.

Question:

Find the sum of the series 3, 6, 12, 24, … up to 10 terms.

Answer:

a = 3

n = 10

r = (6/3) = 2; here r > 1.

$$Sn = \frac{3(2^{10} - 1)}{2 - 1} = 3069$$

Question:

Find the sum of the series 10, 30, 90, 270, … up to 8 terms.

Answer:

a = 10

n = 8

r = (30/10) = 3; here r > 1.

$$Sn = \frac{10(3^{8} - 1)}{3 - 1} = 32800$$

Scanned with CamScanner

Shortcut # 80 – Geometric Progression

Sum of the terms of a finite series where r < 1.

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

Question:

Find the sum of the series 600, 300, 150, … up to 6 terms.

Answer:

a = 600
n = 6
r = (300/600) = 1/2;   here r < 1

$$Sn = \frac{600(1 - \frac{1}{2}^6)}{1 - \frac{1}{2}} = 1181.25$$

Question:

Find the sum of the series 128, 64, 32, … up to 10 terms.

Answer:

a = 128;   n = 10;   r = 64/128 = ½   (r < 1)

$$Sn = \frac{128(1 - \frac{1}{2}^{10})}{1 - \frac{1}{2}} = 255.75$$

Scanned with CamScanner

Shortcut # 81 – Geometric Progression

Sum of the terms of an infinite series where r < 1.

$$S_n = \frac{a}{1-r}$$

Question:

A ball is thrown up in the air for 400 m. every time the ball hits the ground it will bounce back one third of the height it went the last time. The ball repeats this bounce as long as it rests in the ground. What is the total distance covered by the ball?

Answer:

Distance travelled in the first throw = 450 + 450 = 900.
(450 + 450 is because the total distance is both up and down distance covered).

Distance covered in the first bounce = (1/3)900 = 300
Distance covered in second bounce = (1/3)300 = 100
The series is 900, 300, 100, … infinite terms.
Sum = 900/[1 – (1/3)]
Sum = 1350 m

Scanned with CamScanner

Shortcut # 82 – Calendar

Finding day of a date using a reference day.

Question:

Find the day of birth of MS Dhoni (7 July, 1981), if 31 Dec 1999 is Friday

Answer:

Step 1: Number of days after 7/7/1981 in 1981 = 177

Step 2: Number of years between 1981 and 1999 = 17

Step 3: Number of leap years between 1981 and 1999 = 4

Step 4: Number of days in 1999 till Dec 31 = 365

Step 5: 177 + 17 + 4 + 365 = 563

Step 6: Remainder of (563/7) = 3

Our reference day is Friday and the remainder is 3.

Step 7: 3 days before Friday is Tuesday.

MSD was born on Tuesday.

Note:
If there are no years in between the day in the question and reference day, step 2 and step 3 will be = 0
If a date in the future is asked, remainder number of days after the reference day will be the answer.
If remainder is 0, the day to be found out is the same day as the reference day.

Scanned with CamScanner

Shortcut # 83 – Clocks

Angle between the hour hand and the minute hand.

$$\text{Angle} = \left(30H + \frac{M}{2}\right) \sim 6M$$

H - Hour value
M - Minute value

Question:

What is the angle between minute hand and hour hand at 05:40?

Answer:

H = 5
M = 40
Angle = [30(5) + (40/2)] ~ 6(40)
= 170 ~ 240
= 70`

Note:
If the value is above 180`, subtract it from 360 to get refract angle.

Scanned with CamScanner

Shortcut # 84 – Clocks

Time at which the two hands overlap.

$$\text{Time} = \frac{12}{11}\,5H$$

H - Least hour between 'H' and 'H+1' hour

Question:

At what time between 3 o' clock and 4 o' clock the two hands of the clock will overlap?

Answer:

Time = [12/11][5(3)]
= 180/11 minutes past 3 'o' clock
= 16.36 minutes past 3 'o' clock
= 16 minutes 22 seconds past 3
= 03 : 16 : 22

Question:

At what time between 10 o' clock and 11 o' clock the two hands of the clock will overlap?

Answer:

Time = [12/11][5(10)]
= 600/11 minutes past 10 'o' clock
= 54.54 minutes past 10 'o' clock
= 54 minutes 32 seconds past 10
= 10 : 54 : 32

Scanned with CamScanner

Shortcut # 85 – Clocks

Time at which the two hands face opposite directions.

Case 1: $H < 6$

$$\text{Time} = \frac{12}{11}[5H + 30]$$

Question:

At what time between 1 o' clock and 2 o' clock the two hands will be facing opposite direction?

Answer:

Time = 12/11[5(1) + (180/6)]
= 12/11[5(1) + 30]
= 420/11 minutes past 1 'o' clock
= 38.18 minutes past 1 'o' clock
= 38 minutes 11 seconds past 1
= 01 : 38 : 11

Case 2: $H > 6$

$$\text{Time} = \frac{12}{11}[5H - 30]$$

Question:

At what time between 9 o' clock and 10 o' clock the two hands will be facing opposite directions?

Answer:

Time = 12/11[5(9) - (180/6)]
= 12/11[5(9) - 30]
= 180/11 minutes past 9 'o' clock
= 16.36 minutes past 9 'o' clock
= 16 minutes 22 seconds past 9
= 09 : 16 : 22

Scanned with CamScanner

Shortcut # 86 – Clocks

Time at which the two hands will be certain angle apart.

$$\text{Time} = \frac{12}{11}\left[5H \pm \frac{\theta}{6}\right]$$

There are two different time at which the hands will be certain degrees apart. T_1 and T_2.

Question:

At what time between 4 o' clock and 5 o' clock the two hands of the clock will be 60 degrees apart?

Answer:

T_1 = 12/11[5(4) + (60/6)]

= 360/11 minutes past 4 'o' clock

= 32.72 minutes past 4 'o' clock

= 32 minutes 44 seconds past 4

= 04 : 32 : 44

T_2 = 12/11[5(4) – (60/6)]

= 120/11 minutes past 4 'o' clock

= 10.9 minutes past 4 'o' clock

= 10 minutes 54 seconds past 4

= 04 : 10 : 54

Scanned with CamScanner

Shortcut # 87 – Clocks

Time at which the two hands will be certain minute spaces apart.

$$\text{Time} = \frac{12}{11}[5H \pm M]$$

There are two different time at which the hands will be certain degrees apart. T_1 and T_2.

Question:

At what time between 7 o' clock and 8 o' clock the two hands of the clock will be 15 minutes apart?

Answer:

T_1 = 12/11[5(7) + 15]

= 600/11 minutes past 7 'o' clock

= 54.54 minutes past 7 'o' clock

= 54 minutes 33 seconds past 7

= 07 : 54 : 33

T_2 = 12/11[5(7) – 15]

= 240/11 minutes past 7 'o' clock

= 21.81 minutes past 7 'o' clock

= 21 minutes 48 seconds past 7

= 07 : 21 : 48

Scanned with CamScanner

Shortcut # 88 – Permutation Combination and Probability

Dependent and Independent Events.

Dependent events – Multiplication; Key word – AND
Independent events - Addition; Key word – OR

Question:

There 3 busses from city A to city B and there are 5 busses from city B to city C. In how many ways a person can travel from city A to C through B?

Answer:

Choosing a bus from city B depends on choosing a bus from city A.

Number of ways = 3 x 5 = 15

Question:

There 3 busses from city A to city B and there are 5 busses from city A to city C. In how many ways a person can travel from city A to C or B?

Answer:

Choosing a bus to city B or C are not dependent.

Number of ways = 3 + 5 = 8

Scanned with CamScanner

Shortcut # 89 - Permutation Combination and Probability

Permutation – Arrangement with repetition.

n^n

Question:

In how many ways the letters of the word "ORANGE" can be arranged with repetition?

Answer:

n = 6 (n = total number of elements)

Since all the elements are taken,

Number of arrangements = 6^6

---

n^r

Question:

In how many ways three letters from the word "ORANGE" can be arranged with repetition?

Answer:

n = 6; r = 3 (r = number of elements taken for arrangement)

Number of arrangements = $6^3 = 216$

Scanned with CamScanner

Shortcut # 90 - Permutation Combination and Probability

Permutation – Arrangement without repetition.

$$n!$$

Question:

In how many ways the letters of the word “MANGO” can be arranged without repetition?

Answer:

n = 5

Since all the elements are taken for arrangement,

Number of elements = n! = 5! = 120

$$^{n}P_{r} = \frac{n!}{(n - r)!}$$

Question:

In how many ways any three letters of the word ‘MANGO” can be arranged without repetition?

Answer:

n= 5; r = 3

$^{n}P_{r}$ = 5!/(5 – 3)! = 120/2 = 60 ways

Scanned with CamScanner

Shortcut # 91 - Permutation Combination and Probability

Permutation – Elements occurring together.

If two elements occur together

2!(n – 1)!

If three elements occurtogther

3!(n – 2)!

If four elements occur together

4!(n – 3)!

and so on

Question:

In how many ways letters of the word "ORANGE" arranged so that the vowels will always occur together?

Answer:

n = 6

Three letters 'O, A and E' should occur together.

Since three letters occur together

3! x (6 – 2)! = 6 x 24 = 144

Note:

If there are 4 letters occurring together, the answer is

4!(6 – 3)!

If there are 5 letters occurring together, the answer is

5!(6 – 4)!

Scanned with CamScanner

Shortcut # 92 - Permutation Combination and Probability

Permutation – When similar kind of elements occur.

$$\frac{n!}{a! \times b!}$$

Question:

In how many ways the letters of the word "ENVIRONMENT" can be arranged?

Answer:

n = 11
Let, a = 2 (E is repeated twice)
Let, b = 3 (N is repeated thrice)
Number of arrangements = 11!/(2! x 3!)
= 277200

Question:

In how many ways the letters of the word "PIIZZZAAAA" can be rearranged?

Answer:

n = 10
a = 2 (I is repeated twice)
b = 3 (Z is repeated thrice)
c = 4 (A is repeated four times)
Number of arrangements = 10!/(2! x 3! x 4!)
= 12600

Scanned with CamScanner

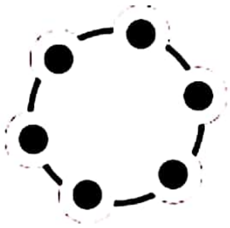
Shortcut # 93 - Permutation Combination and Probability

Permutation – Circular arrangement.

$(n - 1)!$

Question:

In how many ways 6 persons can be arranged in a circle?



Answer:

$n = 6$

Number of arrangements = $(6 - 1)! = 5! = 120$

Question:

In how many ways 8 persons can be arranged in a circle?

Answer:

$n = 8$

Number of arrangements = $(8 - 1)! = 7! = 5040$

Scanned with CamScanner

Shortcut # 94 - Permutation Combination and Probability

Permutation – Elements occurring together in a circle.

If two elements occur together

2!(n – 2)!

If three elements occur together

3!(n – 3)!

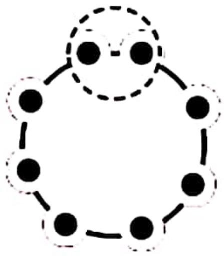
If four elements occur together

4!(n – 4)!

and so on

Question:

In how many ways 8 persons can be seated around a circular table with two persons always sitting together?



Answer:

2! x (8 – 2)! = 2 x 720 = 1440

Note:

If three persons are sitting together, the answer is

3! x (8 – 3)!

If four persons are sitting together, the answer is

4! x (8 – 4)!

Scanned with CamScanner

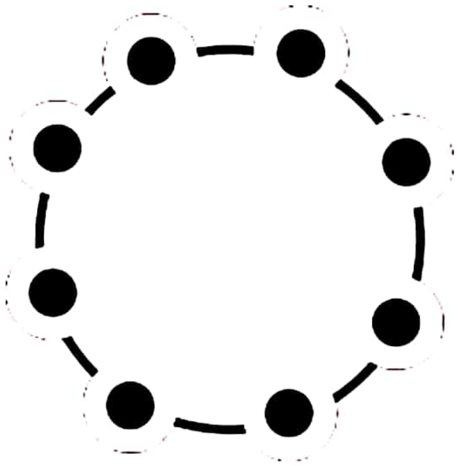
Shortcut # 95 - Permutation Combination and Probability

Permutation – Arrangement of Necklace.

$(n - 1)!/2$

Question:

In how many ways a necklace with 8 different colored beads can be arranged?



Answer:

$n = 8$

Number of arrangements = $(8 - 1)!/2 = 2520$

Scanned with CamScanner

Shortcut # 96 - Permutation Combination and Probability

Combination

$$^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

Question:

In how many ways 2 shirts and 3 pants can be selected from 5 shirts and 7 pants?

Answer:

Selecting 2 shirts out of 5 = $^5C_2$ = 5!/[2!(5 – 2)!] = 10

Selecting 3 shirts out of 7 = $^7C_3$ = 7!/[3!(7 – 3)!] = 35

Since the two events are dependent,

Total ways of selecting = 10 x 35 = 350

Question:

In how many ways 2 shirts or 3 pants can be selected from 5 shirts and 7 pants?

Answer:

Selecting 2 shirts out of 5 = $^5C_2$ = 5!/[2!(5 – 2)!] = 10

Selecting 3 shirts out of 7 = $^7C_3$ = 7!/[3!(7 – 3)!] = 35

Since the two events are independent,

Total ways of selecting = 10 + 35 = 45

Scanned with CamScanner

Shortcut # 97 - Permutation Combination and Probability

Probability.

$$\text{Probability} = \frac{\text{Expecting number of results}}{\text{Total number of results}}$$

Question:

What is the probability of selecting 2 spades from a pack of 52 cards?

Answer:

Total number of ways of selecting 2 cards from 52 = $^{52}C_2 = 1275$

Total ways of selecting 2 spades from 13 spades = $^{13}C_2 = 78$

Probability = 78/1275

Question:

What is the probability of selecting 2 students from a class of 10 boys and 8 girls, where both the selected students are boys?

Answer:

Total number of ways of selecting 2 students from 18

= $^{18}C_2 = 153$

Total ways of selecting 2 boys from 10 boys

= $^{10}C_2 = 45$

Probability = 45/153

Scanned with CamScanner

$$\text{Probability} = \frac{\text{Expecting number of results}}{\text{Total number of results}}$$

Question:

What is the probability of getting 4 heads when 7 coins are tossed?

Answer:

Total number of results = 2 x 2 x 2 x 2 x 2 x 2 x2 = 2^8 = 128

When the required results need 4 heads, the remaining three will be tails.
The results will be the different arrangement of the following:

" H H H H T T T "

The number of ways in which the above can be arranged is
= 7!/(4! x 3!)

= 5040/144

= 35

Probability = 35/128

Scanned with CamScanner

$$\text{Probability} = \frac{\text{Expecting number of results}}{\text{Total number of results}}$$

Question:

What is the probability of getting a sum which is equal to prime number from the results obtained by throwing two dice with six faces numbered 1 to 6 on the faces?

Answer:

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

The above table shows the different sums obtained from different results.
Total number of results = 36.
Expecting result = 15. (Highlighted with the shade).

Probability = 15/36

Scanned with CamScanner

$$\text{Probability} = \frac{\text{Expecting number of results}}{\text{Total number of results}}$$

Question:

What is the probability of selecting 4 balls from a bag that has 5 red balls and 6 black balls where the selection has at least 1 red ball?

Answer:

The different cases are:

1 red and 3 black	$= {}^5C_1 \times {}^6C_3$	= 100
2 red and 2 black	$= {}^5C_2 \times {}^6C_2$	= 150
3 red and 1 black	$= {}^5C_3 \times {}^6C_1$	= 60
4 red	$= {}^5C_4$	= 5

Number of ways of selecting 4 balls with at least 1 red ball:
= 100 + 150 + 60 + 5 = 315

Total ways of selecting 4 balls without any condition:
$= {}^{11}C_4 = 330$
Probability = 315/330

Alternate method:

Probability = 1 – probability of selecting no red balls
= 1 – probability of selecting 4 blackballs

Probability $= 1 - ({}^6C_4/{}^{11}C_4)$
= 1 – (15/330)
= 315/330

Scanned with CamScanner