FORMULA SERIES

ONE SHOT FOR COMPLETE CS

NEXT-CN

SUBJECT

Engineering Mathematics

Probability
Calculus (Single)
Linear Algebra

5 marks

Probl

1

5 marks

Important Topics

conditional Prob / Bayes Theorem

Uniform / Binomial Distribution

Guassian Distribution

CALCULUS

2

Limit

Improper Integrals

Max / min

Continuity

Basic Integration

Linear algebra

Basics

Rank of matrix / System of Eqn

eigen values / eigen vector

(1) CALCULUS (Single variable)

Limits

$$\lim_{x\to 0} \frac{e^x - 1}{x} = \lim_{x\to 0} \frac{\sin x}{x} = \lim_{x\to 0} \frac{\tan x}{x} = \lim_{x\to 0} \frac{x}{\sin x}$$

$$= \lim_{x\to 0} \frac{\sin^{-1} x}{x} = \lim_{x\to 0} \frac{\tan^{-1} x}{x} = 1$$

Imp

L-Hospital Rule

$\frac{\to 0}{\to 0}$ or $\frac{\to \infty}{\to \infty}$ form

Indeterminate form

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to 0} \frac{f'(x)}{g'(x)} = \lim_{x\to 0} \frac{f''(x)}{g''(x)}$$

remove It $\frac{\to 0}{\to 0}$ or $\frac{\to \infty}{\to \infty}$

$\Rightarrow$ Plug in

Ans

1) $\Rightarrow$ $L = \lim_{x \to a} \left[f(x)\right]^{g(x)}$ Taking log both sides

$$\log L = \lim_{x \to a} g(x) \log f(x)$$

existence = LHL = RHL = finite

$$L = e^{\lim_{x \to a} g(x) \ln f(x)}$$

<u>Continuity</u>

<u>Continuous</u>

$$LHL = RHL = f(a)$$

$$LHL = \lim_{h \to 0} f(a-h) \qquad RHL = \lim_{h \to 0} f(a+h)$$

Condition for Discontinuity: $LHL \neq RHL$

A) Removable Discontinuity $LHL = RHL \neq f(a)$

B) Jump Discontinuity $LHL \neq RHL$

C) Infinite Discontinuity $\left.\begin{matrix} LHL \\ RHL \end{matrix}\right] \pm \infty \Big]$ Infinite

Condition for Differentiability

$LHD \neq RHD$

$$LHD = \lim_{h \to 0} \frac{f(a-h) - f(a)}{-h}$$

$$RHL = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Imp

Max/Min (one variable)

Interval Not given / open Interval

$y = f(x)$

step ① Calculate dy/dx to get The stationary Point x_0

Step ② find $\frac{d^2y}{dx^2}$

Step ③ $\left.\frac{d^2y}{dx^2}\right|_{x=x_0} < 0$ Max

$\left(\frac{d^2y}{dx^2}\right) > 0$ min

LOCAL MAX/min

Relative max/min

$\frac{d^2y}{dx^2} = 0$

Point of Inflection

Global max/min or Absolute max/min

"Closed Interval"

Global max/min

absolute max/min

step ① $f'(x) = 0$ to get the stationary Pt

Step ② Put the value x_0

$$\text{Max global} = \Big\{ \underbrace{f(x_0), f(x_1), f(x_2) \ldots f(x_n)}_{\text{max}} \Big\}$$

$$\text{min global} = \Big\{ \underbrace{f(x_0), f(x_1), f(x_2) \ldots f(x_n)}_{\text{min value}} \Big\}$$

Improper Integral

Basic formula copy

A) Range of Interval is Infinite

$$\int_a^\infty f(x)\,dx = \lim_{t\to\infty} \int_a^t f(x)\,dx = \text{Finite / convergent / bounded}$$
$$\text{Infinite / Divergent / unbounded}$$

$$\int_{-\infty}^a f(x)\,dx = \lim_{t\to\infty} \int_t^a f(x)\,dx = \text{finite / convergent / bounded}$$
$$\text{Infinite / Divergent / unbounded}$$

2) function is Not defined

$\int_a^b f(x)\,dx$

If $x = a$ (Not defined)

$$\lim_{h \to 0} \int_{a+h}^{b} f(x)\,dx \longrightarrow \text{RHL}$$

If $x = b$ (Not defined)

$$\text{LHL} \longleftarrow \lim_{h \to 0} \int_{a}^{b-h} f(x)\,dx$$

Basic Integral :

Master | Killer

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

DEfault Property

Imp

$$\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$$

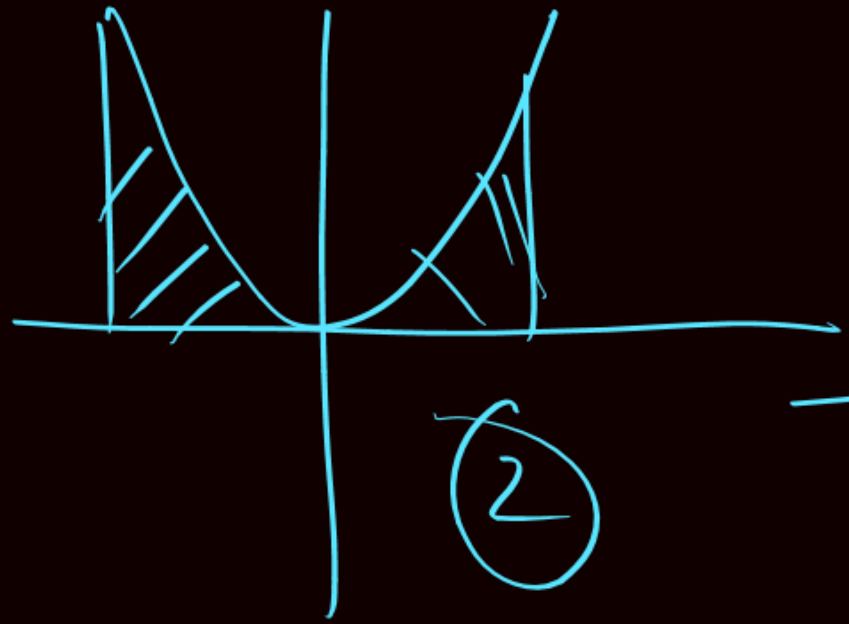
DEfault

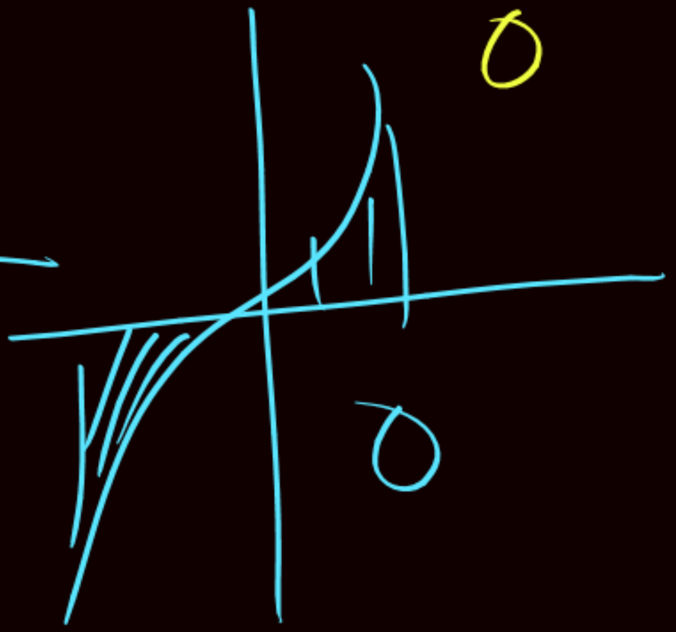
even function

$f(-x) = f(x)$

$$0$$

$f(-x) = -f(x)$





(2) Probability And Statistics:

(A) $P(E) = \frac{n(E)}{n(S)}$ 2) $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$ conditional Prob

3) Dependent events / without replacement

$$P(A \cap B) = P(A)\, P\left(\frac{B}{A}\right)$$

c) Independent events

$$P(A \cap B) = P(A)\, P(B)$$

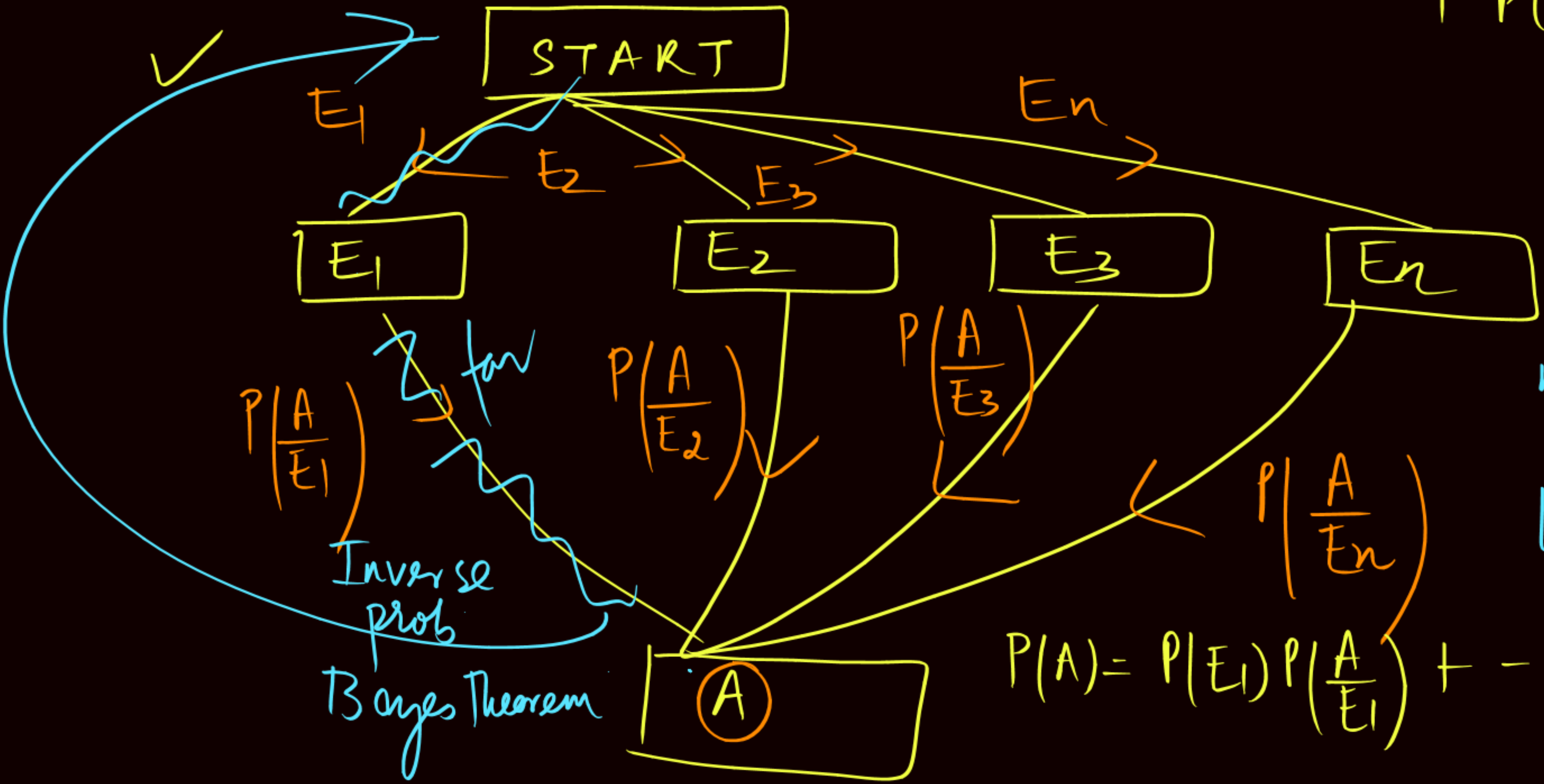
A and B Are Indep events

c) $P(\text{only } A) = P(A \cap \bar{B}) = P(A)\, P(\bar{B}) = P(A)\left[1 - P(B)\right]$

D) $P(\text{only } B) = P(B \cap \bar{A}) = P(B)\, P(\bar{A}) = P(B)\left[1 - P(A)\right]$

E) $P(A \cap B) = \frac{n(A \cap B)}{n(S)}$ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$



$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)\,P\left(\frac{A}{E_1}\right)}{\sum_{i=1}^{n} P(E_i)\,P\left(\frac{A}{E_i}\right)}$$

$$P(A) = P(E_1)\,P\left(\frac{A}{E_1}\right) + \cdots + P(E_n)\,P\left(\frac{A}{E_n}\right)$$

Cumulative Distribution

$$\boxed{F_X(x_i) = P(X \le x_i)}$$

cdf

pdf $$\boxed{F_X(x) = \int_{\text{region}} f(x)\,dx}$$

Relation between cdf and pdf

$$\boxed{\frac{d}{dx} F_X(x) = f(x)}$$

Expected Value $E[X] = \sum_{i=1}^{n} x_i p_i$ (discrete)

$E[X] = \int_a^b x f(x)\,dx$ [conti]

$$\boxed{\begin{aligned} &\# \; E[X+Y] = E[X] + E[Y] \\ &\# \; E[aX] = a\,E[X] \end{aligned}}$$

\# $E[ax+by] = a\,E[x] + b\,E[y]$

variance $V(X) = E[X^2] - \{E[X]\}^2$

Continuous $E[X^2] = \int_a^b x^2 f(x)\,dx$

discrete $E[X^2] = \sum_{i=1}^{n} x_i^2 p_i$

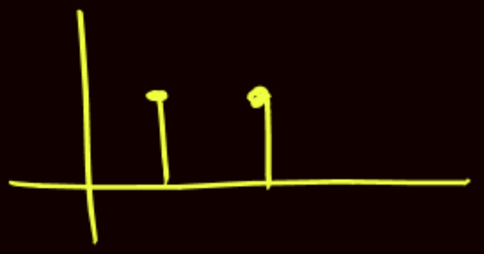
\# standard deviation $= \sqrt{\text{variance}}$

Imp

$$\boxed{\begin{aligned} &var(-X) = (-1)^2 var(X) \\ &var(ax+by) = a^2 var(X) + b^2 var(y) \end{aligned}}$$

Important SECTION

Distribution PER

Binomial	Poisson	(Normal Guassian	Uniform
Discrete (n, p)	Discrete (λ)	continuous (μ, σ)	continuous $[a, b]$
$P(X=r) = {}^nC_r \underset{S}{p}^r \underset{F}{q}^{n-r}$	$P(X=r) = \frac{e^{-\lambda}\lambda^r}{r!}$ $\lambda =$ mean	$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	$\frac{1}{b-a} = f(x)$
MEAN $= np$ variance $= npq$ S.D $= \sqrt{npq}$	mean $= \lambda$ variance $= \lambda$ S.D $= \sqrt{\lambda}$	mean $= \mu$ S.D $= \sigma$ variance $= \sigma^2$	$E[X] = \frac{a+b}{2}$ $V(X) = \frac{(b-a)^2}{12}$ S.D $= \sqrt{V(X)}$
	 Average	 $-3\ -2\ -1\ 1\ 2\ 3$	 a b

Guassian Distribution

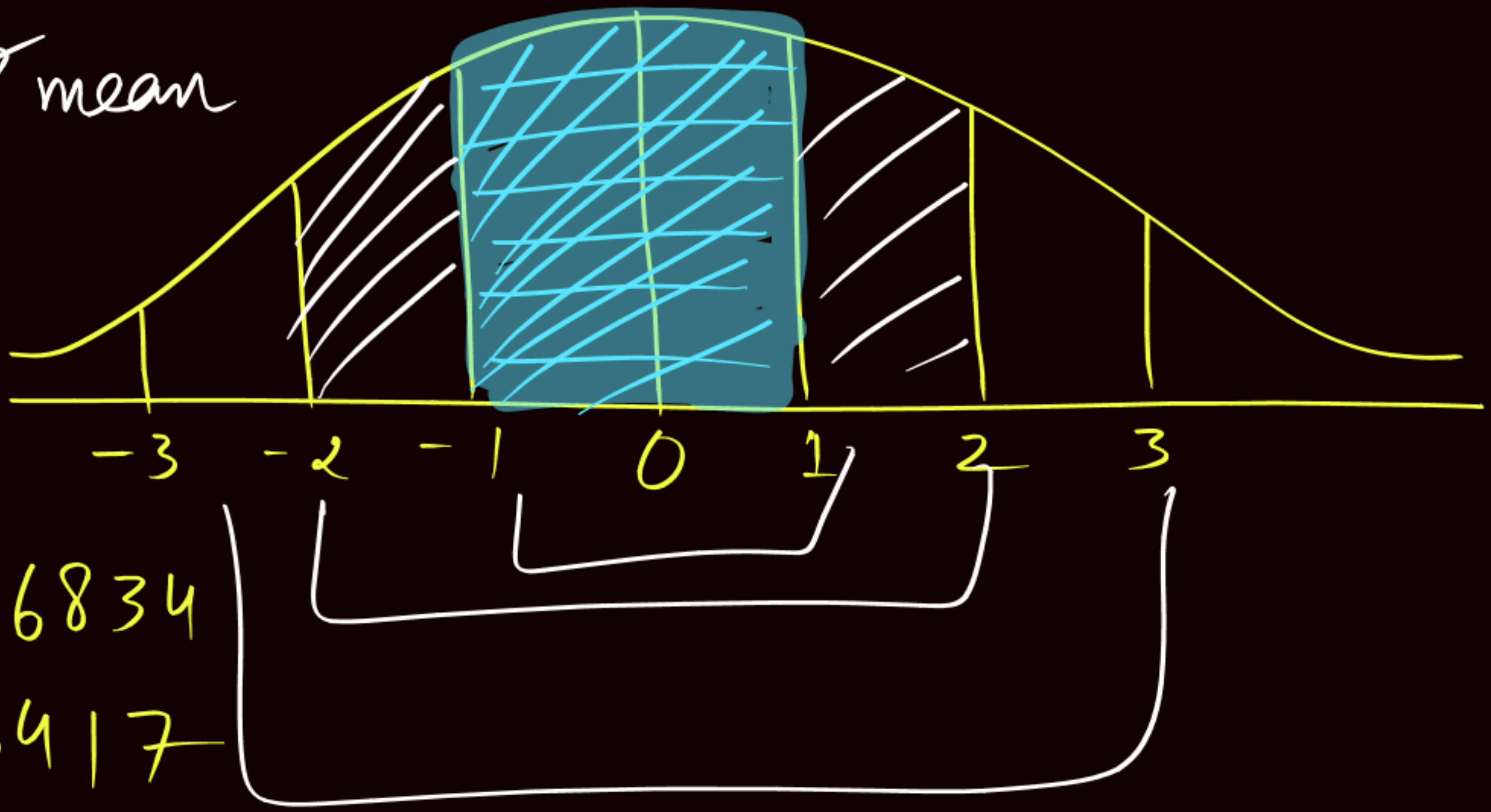
Class

Normal Dist.

$$Z\text{-SCORE} = \frac{X - \mu}{\sigma}$$

μ → mean

σ → S.D



$P(-1 \le Z \le 1) = 0.6834$

$P(0 \le Z \le 1) = 0.3417$

$P(-2 \le Z \le 2) = 0.9545$

$P(0 \le Z < 2) = 0.4774$

$P(-3 \le Z \le 3) = 0.9971$

$P(0 < Z < 3) = \frac{0.9971}{2}$

Linear algebra

Basics $\longrightarrow$ Inverse / adjoint / Determinant / Type of matrices

System of Equations

Eigen Values or eigen Vectors

System of Equations

1) Rank of matrix
= No. of linearly Indep $\boxed{\text{Row}}$ / columns.

2) No. of Non ZERO rows
(Non Square matrix) — Using elimination

$AX = 0$ (Homogenous)

$AX = B$ (Non-Homogenous System)

$\det A \neq 0 \quad r(A) = 3.$

$3\times3 \quad \det A = 0 \quad r(A)$
- 2 all subminors Are Not ZERO
- 1 all subminors Are ZERO

$\boxed{AX = 0}$ Homogeneous

$\boxed{\text{Unique}}$ $\boxed{\text{Infinite}}$

$\det A \neq 0$

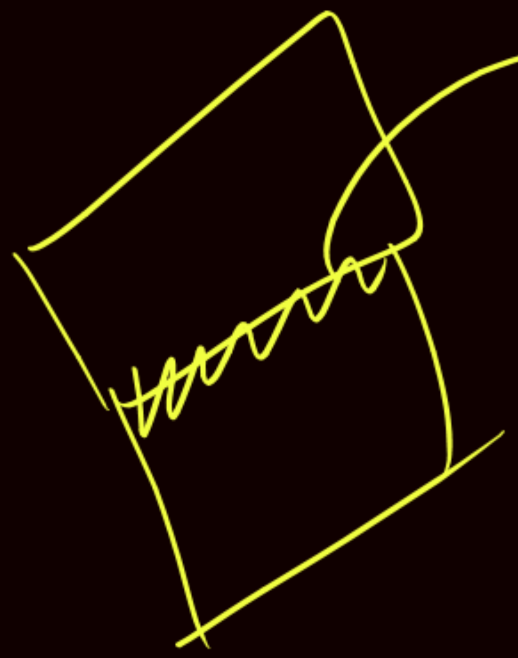
rank of matrix = No. of variables

Unique / Trivial sol

$\det A = 0$ (Non Trivial sol^n)

rank of matrix < No. of variables

Infinite



Infinite solution

Consistent

$AX = B$ (Non-Homogeneous Equ^n)

1) If rank A = rank C = no. of variables = (Unique solution)

$$C = [A \,;\, B] = \underbrace{\left[\begin{array}{c|c} \underbrace{\begin{matrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{matrix}}_{A} & \begin{matrix} b_1 \\ b_2 \\ b_3 \end{matrix} \end{array}\right]}_{C}$$

2) rank = rank $C < n$ = (Infinite solution)

Inconsistent 3) $r(A) \neq r(C)$ (No solution)

Eigen Values / eigen Vectors

$$AX = \lambda X$$

A = matrix (square)

X = eigen Vector

λ = eigen value (scalar quantity)

1) $|A - I\lambda|_{n \times n} = 0$

eigen value · characteristic equ^n

$A \to \lambda_1, \lambda_2, \lambda_3 \,--\, \lambda_n$	$A^m = \lambda_1^m, \lambda_2^m, \lambda_3^m \,--\, \lambda_n^m$
$A \to \lambda_1, \lambda_2, \lambda_3 - \lambda_n$	$KA = K\lambda_1, K\lambda_2, K\lambda_3 - K\lambda_n$
$A \to \lambda_1, \lambda_2 \,--\, \lambda_n$	$A^{-1} = \frac{1}{\lambda_1}, \frac{1}{\lambda_2} \,--\, \frac{1}{\lambda_n}$
$A \to \lambda_1, \lambda_2, - \lambda_n$	$\operatorname{adj} A = \frac{\lvert A\rvert}{\lambda_1}, \frac{\lvert A\rvert}{\lambda_2} - \frac{\lvert A\rvert}{\lambda_n}$
✓ If one eigen value is atleast ZERO	det $A = 0$
✓ $A^n + a_0 A^{n-1} + \cdots + a_n$ $A \longrightarrow \lambda_1, \lambda_2 - \lambda_n$	Satisfies the equⁿ $A = \lambda_1, \lambda_2, \lambda_3$ $= \lambda_1^n + a_0 \lambda_1^{n-1} + \cdots$ $\lambda_2^n + a_0 \lambda_2^{n-1} + \,--$

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ 2×2

Characteristic eqn

$$\lambda^2 - (\text{Trace})\lambda + \det A = 0$$

3×3 $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

$$\lambda^3 - (\text{Trace})\lambda^2 + (A_{11} + A_{22} + A_{33})\lambda - \det A = 0$$

Properties of eigen Vector :

A	A^m	SAME
A	A^{-1}	SAME
A	A^T	does Not SAME

A, $A^n + a_0 A^{n-1} + \dots + a_n$

$= \text{SAME}$

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1. IP ADDRESSING, SUBNETTING & SUPERNETTING

Introduction to Addressing

IPv4 Address = 32 bit

Total number of IP addresses = 2^{32} = 4,294,967,296

IP Addressing

1. Class A : 0 → (1 - 126), No. of IP Addresses = 2^{31}
2. Class B : 10 → (128 - 191), No. of IP Addresses = 2^{30}
3. Class C : 110 → (192 - 223), No. of IP Addresses = 2^{29}
4. Class D : 1110 → (224 - 239), No. of IP Addresses = 2^{28}
5. Class E : 1111 → (240 - 255), No. of IP Addresses = 2^{28}

Default subnet Mask

1. Class A : 255.0.0.0
2. Class B : 255.255.0.0
3. Class C : 255.255.255.0

Private IP Address Range

1. 10.0.0.0 to 10.255.255.255 → 1 class A Network.
2. 172.16.0.0 to 172.31.255.255 → 16 class B Network.
3. 192.168.0.0 to 192.168.255.255 → 256 class C Network

Class	Number of Networks	Number of hosts per Network
Class A	$2^7 - 2 = 126$	$2^{24} - 2 = 1,67,77,214$ hosts
Class B	$2^{14} = 16,384$	$2^{16} - 2 = 65,534$ hosts
Class C	$2^{21} = 20,97,125$	$2^8 - 2 = 254$ hosts
Class D	No NID and HID, all 28 remaining bits are used to define multicast address	
Class E	No NID and HID, it is meant for research and future purpose	

NOTE:

The IP address 127.x.y.z is known as loop back address and it is used to check the connectivity.

	NID	**HID**		
1.	Valid	0's	→	Network ID
2.	Valid	1's	→	Direct Broadcast Address (DBA) ✓
3.	1's	1's	→	Limited Broadcast Address (LBA)
.	1's	0's	→	Network Mask or Subnet Mask

Subnet Mask

It is a 32 bit number used to indicate no. of bits borrowed from host –id and there positions based on the following rules

Rule1: No of 1's in the subnet mask indicate $\boxed{\text{NID+SID}}$

Rule2: No of 0's in the subnet mask indicate $\boxed{\text{HID part}}$

Subnet Mask	Supernet Mask
(1) No. of 1's in the subnet Mask either equal to NID bits or more than NID bits	(1) No. of 1's in the supernet mask always less than NID bits
(2) Subnetting is applicable for single n/w	(2) Supernetting is applicable for two or more network
(3) In subnetting we borrowed from Host ID	(3) In supernetting we borrowed from network-ID

Address	Class A	Class B	Class-C
255.0.0.0 → 8 → 1's	Subnet mask ✓	Supernet mask ✓	Supernet Mask ✓
255.255.252 → 22 → 1's	Subnet mask ✓	Subnet mask ✓	Supernet Mask ✓
255.255.255.0 → 24 → 1's	Subnet mask ✓	Subnet mask ✓	Subnet Mask ✓

2. ERROR CONTROL

NOTE:

- The number of corrupted bits or affected bits depends on the data rate and duration of noise.
- The number of corrupted bits or affected bits = $\boxed{\text{Data rate * Noise duration.}}$
- Burst error is more likely to occur than a single bit error.
- Error correction is more difficult than error detection.

Minimum Hamming Distance for Error detection

To detect ‘d’ bit error minimum hamming distance required $\boxed{=d+1}$

Minimum Hamming Distance for Error Correction

To Correct ‘d’ bit error minimum hamming distance required $\boxed{=2d+1}$

CRC

Length of the dataword = n

Length of the divisor = k

Append (k-1) Zero's to the original message

Perform modulo 2 division

Remainder of division = CRC

Codeword = dataword with Appended (k-1) Zero's + CRC

NOTE:

1. CRC must be (k-1) bits.
2. If the generator has more than one term and coefficient of x^0 is 1, all single bit error can be detected.
3. If a generator can't divide $\boxed{x^t + 1}$ (t between 0 and n – 1) then all isolated Double error can be detected.
4. The generator that contains a Factor of x + 1 can detect all odd numbered errors.

A good polynomial generator needs to have the following characteristics:

1. It should have at least two terms.
2. The coefficient of the term x^0 should be 1.
3. It should not divide $\boxed{x^t + 1,}$ for t between 2 and $n - 1$.
4. It should have the factor $\boxed{x + 1.}$

Hamming Code

1. Hamming code is used for error correction.
2. Hamming code can correct 1 bit error only.
3. Hamming code can detect upto 2 bit error.

m = Message bits

r = redundant bits or Check bits or parity bits or extra bits

n = m + r (n = codeword)

According to the hamming code, number of redundant bits

$$\boxed{m + r + 1 \leq 2^r}$$

where r = lower limit

3. FLOW CONTROL

Transmission Delay

Amount of time taken to transfer a packet on to the outgoing link is called as Transmission delay.

$$\boxed{\text{Transmission delay}(T_d) = \frac{\text{Length of Packet}}{\text{Bandwidth}}}$$

$$\boxed{T_d = \frac{L}{B}}$$

Propagation Delay

Amount of time taken to reach a packet from one (sender) point to another (receiver) point is called as propagation delay.

$$\boxed{\text{Propagation Delay (Pd)} = \frac{\text{Distance}}{\text{Velocity}}}$$

$$\boxed{P_d = \frac{d}{v}}$$

Stop and Wait Control

Efficiency OR Line utilization OR Link utilization OR Sender utilization:

$$\text{Efficiency} = \frac{\text{Useful Time}}{\text{Total Time}}$$

$$\text{Efficiency} = \frac{\mathrm{T_d}(frame)}{\mathrm{T_d}(frame) + 2 * \mathrm{Pd} + \mathrm{Qd} + \mathrm{Prd} + \mathrm{Td}(ACK)}$$

Throughput Or Effective Bandwidth Or Bandwidth utilization Or Maximum Data Rate Possible:

$$\boxed{\text{Throughput} = \frac{\text{Length of the Frame}}{\text{Total Time}}}$$

$$\boxed{\text{Throughput} = \frac{L}{T_d(\text{frame}) + 2 * Pd + Qd + Prd + Td(ACK)}}$$

Or

$$\boxed{\text{Throughput} = \eta * B}$$

Efficiency of GBN

$$\text{Efficiency} = \frac{\text{Useful Time}}{\text{Total}}$$

$$\text{Efficiency} = \frac{\mathrm{N}*\mathrm{Td}(frame)}{\mathrm{T_d}(frame)+2*\mathrm{Pd}+\mathrm{Qd}+\mathrm{Prd}+\mathrm{Td}(ACK)}$$

Throughput in GBN

$$\text{Throughput} = \frac{\text{N*Length of Frame}}{\text{Total time}}$$

$$\text{Throughput} = \frac{\text{N*Length of Frame}}{T_d(\text{frame})+2*\text{Pd}+\text{Qd}+\text{Prd}+\text{Td(ACK)}}$$

Or

$$\text{Throughput} = \eta * B$$

Efficiency of Selective repeat (SR) Protocol

$$\text{Efficiency} = \frac{\text{Useful time}}{\text{Total time}}$$

$$\text{Efficiency} = \frac{\text{Ws}\times\text{Td}(frame)}{\text{T}_{\text{d}}(\text{frame})+2*\text{Pd}+\text{Qd}+\text{Prd}+\text{Td}(ACK)}$$

Throughput of Selective report (SR) Protocol

$$\text{Throughout} = \frac{W_s * \text{Length of the frame}}{\text{Total time}}$$

$$\text{Throughput} = \frac{W_s * \text{Length of the frame}}{T_d(frame) + 2 * Pd + Qd + Prd + Td(ACK)}$$

OR

$$\boxed{\text{Throughput} = \eta * B}$$

Comparison among stop and wait, GBN and SR protocols:

	Stop & Wait	GBN	SR
Efficiency	$\eta = \frac{\text{Useful Time}}{\text{Total Time}}$ or $\eta = \frac{\text{Td(frame)}}{\text{Total Time}}$	$\eta = \frac{\text{Useful time}}{\text{Total time}}$ or $\eta = \frac{\text{N*Td(frame)}}{\text{Total time}}$	$\eta = \frac{\text{Useful time}}{\text{Total time}}$ or $\eta = \frac{\text{Ws*Td(frame)}}{\text{Total time}}$
Throughput	$\frac{\text{Length of frame}}{\text{Total Time}}$ or $\eta * B$	$\frac{\text{N*Length of the frame}}{\text{Total time}}$ Or $\eta * B$	$\frac{\text{Ws*Length of the frame}}{\text{Total time}}$ Or $\eta * B$
Buffer	1 + 1	N + 1	N + N
Seq No.	2 (0 & 1)	N + 1	2N
Seq. No. = K bit		$W_s = 2^K - 1$ $W_R = 1$	$W_s = 2^{K-1}$ $W_R = 2^{K-1}$

RTT or Total time = T_d(frame) + 2*P_d + T_d(ACK) + P_{rd} + Q_d

4. IPv4 HEADER

IPv4 Header

<table>
<tr><td>VER (4 bits)</td><td>HL (4 bits)</td><td>Services (8 bits)</td><td>Total Length (16 bits)</td></tr>
<tr><td colspan="2">Identification number (16 bits)</td><td>Flags (3 bits)</td><td>Fragment offset (13 bits)</td></tr>
<tr><td colspan="2">Time to Live (8 bits)</td><td>Protocol (8 bits)</td><td>Header checksum (16 bits)</td></tr>
<tr><td colspan="4">Source IP Address (32 bits)</td></tr>
<tr><td colspan="4">Destination IP Address (32 bits)</td></tr>
<tr><td colspan="4">Option (0-40 bytes)</td></tr>
</table>

5. TCP & UDP

Introduction

Transport Layer can be connection oriented or connection less.

Transport Layer

- TCP (Transmission Control Protocol)
  Connection oriented
- UDP (User Datagram Protocol)
  Connection less

TCP Header

<table>
<tr><td colspan="4">Source Port (16 bits)</td><td colspan="5">Destination Port (16 bits)</td></tr>
<tr><td colspan="9">Sequence number (32 bits)</td></tr>
<tr><td colspan="9">Acknowledgement number (32 bits)</td></tr>
<tr><td>Header Length (4 bits)</td><td>Reserved bits (6 bits)</td><td>U R G</td><td>A C K</td><td>P S H</td><td>R S T</td><td>S Y N</td><td>F I N</td><td>Window Size (Advertisement Window) (16 bits)</td></tr>
<tr><td colspan="8">Check sum (16 bits)</td><td>Urgent Pointer (16 bits)</td></tr>
<tr><td colspan="9">Options
(0 - 40 bytes)</td></tr>
</table>

20B fixed

Variable (40B)

TCP Header

Port Number	Name
0 – 1023	Well known Port No.
1024 – 49151	Registered Port No. OR Resurved Port No
49152 – 65535	Dynamic Port No.

$$\boxed{\text{Wrap Around Time (WAT)} = \frac{\text{Total Sequence Number}}{\text{Bandwidth [Bytes/Sec]}}}$$

- Minimum sequence number required to Avoid wrap around time with in Life time = $\boxed{B \times LT}$
- Minimum number of bits required to Avoid wrap Around time with in Life time = $\boxed{\lceil \log_2 B * LT \rceil}$

SYN = 1 → Consume one sequence number.

Ack = 1 → Consume zero sequence number.

FIN = 1 → Consume one sequence number.

1 Data byte → Consume one sequence number.

SYN	Ack	Meaning
1	0	request
1	1	reply
0	1	Ack
0	0	Data

TCP Header

Time out timer in TCP

Basic Algorithm	Jacobson's Algorithm
Time Out Timer = $\boxed{2 * RTT}$ Next Round Trip Time (NRTT) $\boxed{= \alpha(IRTT) + (1 - \alpha)ARTT}$ $\boxed{0 \le \alpha \le 1}$	Time Out Timer = $\boxed{4 * ID + RTT}$ Next Round Trip Time (NRTT) $\boxed{= \alpha\ (IRTT) + (1 - \alpha)\ ARTT}$ $0 \le \alpha \le 1$ Actual Deviation (AD) = $\boxed{\lvert IRTT - ARTT \rvert}$ Next Deviation $\boxed{(ND) = \alpha\ (ID) + (1 - \alpha)\ AD}$

Congestion Control

$$\boxed{W_s = \min(W_c, W_R)}$$

Slow start	Congestion Avoidance	Congestion Detection
1. If ACK Arrives $\boxed{Wc = Wc + 1}$ OR	1. IF ACK Arrives $\boxed{Wc = Wc + 1/Wc}$ OR	1. Time out ✓
2. After one RTT $\boxed{Wc = 2 * Wc}$	2. After one RTT $\boxed{Wc = Wc + 1}$	2. 3 duplicate ACK ✓

Token Bucket

t

Maximum number of packet = $\boxed{C + rt}$

Maximum average rate for token bucket $\boxed{M = \frac{C+rt}{t}}$

$Mt = C + rt \Rightarrow Mt - rt = C \Rightarrow (M - r)t = C$

$$\boxed{T = \frac{C}{M-r}}$$

$C \rightarrow$ token bucket capacity

$r \rightarrow$ token arrival rate

UDP

UDP Header

Source port (16 bit)	Destination port (16 bit) → 4B
Total Length (16 bit)	Checksum (16 bit) → 4B

(8Byte)

6. MEDIUM ACCESS CONTROL[MAC]

Aloha

Pure Aloha	Slotted Aloha
Any station transmits the data at any time.	Any station can transmit the data at the beginning of any time slot.
Vulnerable time in which collision may occur $\boxed{= 2 * T_f}$ (T_f - Transmission time for single frame)	Vulnerable time in which collision may occur $\boxed{= T_f}$
Throughput of pure aloha = $\boxed{G*e^{-2G}}$	Throughput of slotted Aloha = $\boxed{G * e^{-G}}$
Maximum throughput s_{max} $\boxed{= 18.4\ \%}$ (When G = 1/2)	Maximum throughput smax = 36.8 % (When G = 1)
The main advantage of pure aloha is it simplicity in implementation	The main advantage of slotted aloha is that it reduces the number of collisions to Half and double the throughput of pure aloha

CSMA/CD (Carrier Sense Multiple access/Collision Detection):

1. Minimum size of frame to detect the collision in Ethernet (CSMA/CD)

$$\boxed{T_{d(frame)} \geq 2 * P_d + T_d(\text{Jam signal})}$$

2. Backoff Algorithm

Waiting time = $\boxed{K * \text{Slot duration}}$

= $\boxed{K * RTT}$

= $K * 2 * Pd$ ✓

K is any random number in between 0 to $2^n - 1$

n is collision number (Collision number is respect to data packet).

CSMA/CD (Carrier Sense Multiple access/Collision Detection):

3. Efficiency in Ethernet (CSMA/CD)

$$\boxed{\eta = \frac{1}{1+6.44\,a}} \text{ or } \frac{\text{Useful time}}{\text{Total time}} = \frac{\text{Td}}{\text{Collison time+Td+Pd}}$$

4. $\boxed{P(1-P)^{N-1}}$ → Probability of success for single station(Throughput of Host)

5. $\boxed{NP(1-P)^{N-1}}$ → Probability of success for any station among all stations [Throughput of channel]

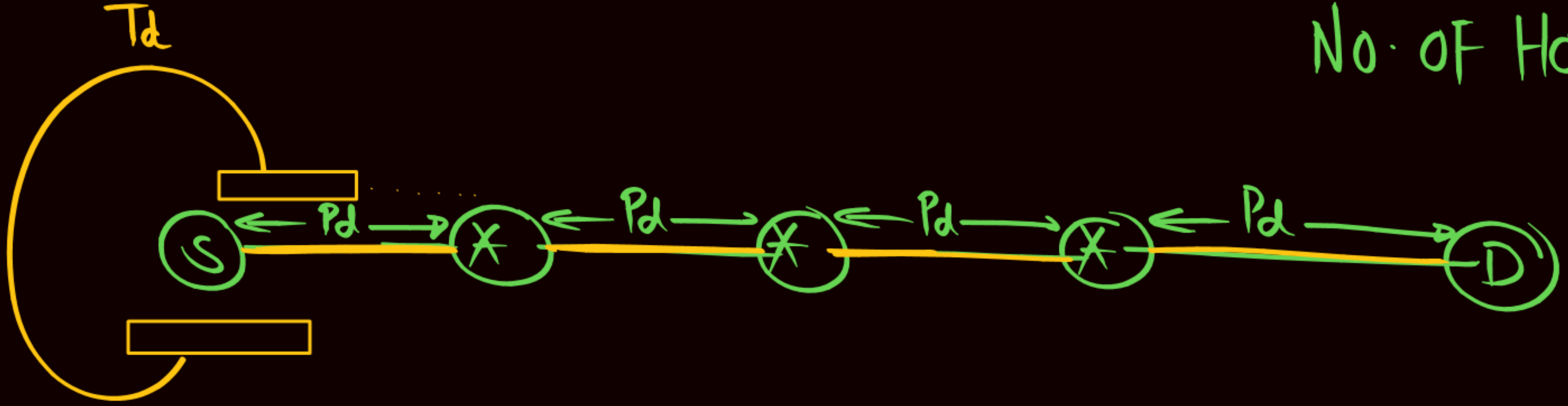
7. SWITCHING

Switching

Circuit Switching	Packet switching
(6) No store and forward transmission	Supports store and forward transmission
(7) Congestion can happen during connection establishment phase	Congestion can happen during data transfer phase
(8) It is reliable	Not reliable
(9) Better for sending large messages	Better for sending small messages
(10) Not fault tolerant technique	Fault tolerant technique
(11) Circuit switching is implemented at physical layer.	Packet switching is implemented at network layer
Total time = Setup time + T_d + P_d + tear down time $TT = S + \frac{L}{B} + X.\frac{d}{V} + T$	For X Hop and N packet Total time $= X[Td + P_d] + X - 1[P_{rd} + D_q] + N - 1(T_d)$

circuit switching

No. of Hop = X



Total time = Setup time + Td + Pd + Tear down time

$$= S + \frac{L}{B} + X \cdot \frac{d}{v} + T$$

Packet switching

For X → Hop & 'N' Packet

$$\boxed{\text{Total time} = X[T_d + P_d] + X-1[Q_d + Pr_d] + N-1(T_d)}$$

Generalized Formula for optimal packet size (P)

M = Message size

h = Header size

p = Payload/Packet data size

No. of Hops = X

$$\boxed{p = \sqrt{\frac{Mh}{X-1}}}$$

So optimum packet size $\boxed{P = p + h}$

No. of Packets $= \frac{M}{p}$

8. APPLICATION LAYER PROTOCOL

Important table

Application	Port Number.	Transport Protocol
DNS	53	UDP
HTTP	80	TCP
FTP	20 (Data connection) 21(Control connection)	TCP
SMTP	25	TCP
POP	110	TCP
SNMP	161, 162	UDP
TFTP	69	UDP
IMAP	143	TCP
Telnet	23	TCP
DHCP	67 (DHCP Server) 68 (DHCP Client)	UDP

TCP	UDP
SMTP HTTP FTP POP IMAP Telnet	DNS SNMP TFTP DHCP All real time & Multimedia Protocol

Important table

SHORT TRICK	DNS	HTTP	SMTP	POP	IMAP	FTP
Stateful/ Stateless	Stateless	Stateless	Stateless	Stateful	Stateful	Stateful
Transport Protocol Used	UDP	TCP	TCP	TCP	TCP	TCP
Connectionless/ Connection oriented	Connection less	Connection less	Connection oriented	Connection oriented	Connection oriented	Connection oriented
Persistent/Non-persistent	Non-persistent	HTTP 1.0 is non persistent HTTP 1.1 is persistent.	Persistent	Persistent	Persistent	Control connection is persistent. Data connection is non-persistent.
Push/Pull	-	-	Push	Pull	Pull	Can't
Port Number Used	53	80	25	110	143	20 for data connection. 21 for control connection.
In band/ Out-of-band	In band	In band	In band	In band	In band	Out-of- band

Stateless	Stateful
DNS HTTP SMTP	POP IMAP FTP

Commands

HTTP	FTP	SMTP
GET	USER	HELO
HEAD	PASS	MAIL FROM
PUT	ACCT	RCPT TO
POST	CWD	DATA
TRACE	REIN	QUIT
DELETE	QUIT	RSET
CONNECT	PORT*	VRFY
OPTIONS	PASV	NOOP
	TYPE	TURN
	MODE	EXPN
	PROMPT	HELP
	STRU	SEND FROM
		SMOL FROM
		SMAL FROM

FTP Command shortcut

तुम फिर से U P आ रहे क्या फिर चलेंगे पीने

T M P S U P A R Q P C P

T	Type
M	Mode
P	Part
S	STRU
U	USER
P	PASV
A	ACCT
R	REIN
Q	Quit
P	PROMPT
C	CWD
P	PASS

SMTP Commands

Vijay	VRFY
Sir	SEND FROM
Teaches	TURN
Hard	HELO
Subject	SMOL FROM
iN	NOOP
Easy	EXPN
Manner	MAIL FROM
Difficult	DATA
Subject	SMAL FROM
Re Quires	RSET, QUIT
Ha Rdwork	HELP, RCPTTO

If there are n objects with n_1 indistinguishable objects of a first type, n_2 indistinguishable objects of a second type, $\ldots$, and n_r indistinguishable objects of an rth type, where $n_1 + n_2 + \cdots + n_r = n$, then there are $\dfrac{n!}{n_1!\,n_2!\cdots n_r!}$ (linear) arrangements of the given n objects.

$$\frac{n!}{n_1! * n_2! * n_3! \ldots}$$

$$\frac{(n+r-1)!}{r!(n-1)!} = \binom{n+r-1}{r}.$$

Consequently, the number of combinations of n objects taken r at a time, *with repetition*, is $C(n+r-1, r)$.

CBR.

a) The number of integer solutions of the equation

$$x_1 + x_2 + \cdots + x_n = r, \qquad x_i \geq 0, \qquad 1 \leq i \leq n.$$

b) The number of selections, with repetition, of size r from a collection of size n.

c) The number of ways r identical objects can be distributed among n distinct containers.

$x_1 + x_2 + x_3 = 10$

${}^{12}C_2$

Order Is Relevant	Repetitions Are Allowed	Type of Result	Formula
Yes	No	Permutation	$P(n, r) = n!/(n-r)!$, $0 \le r \le n$ P
Yes	Yes	Arrangement	n^r, $n, r \ge 0$ PR
No	No	Combination	$C(n, r) = n!/[r!(n-r)!] = \binom{n}{r}$, $0 \le r \le n$ C
No	Yes	Combination with repetition	$\binom{n+r-1}{r}$, $n, r \ge 0$ CR

Inclusion-Exclusion

$$|A \cup B| = |A| + |B| - |A \cap B|$$

$$\begin{aligned} |A \cup B \cup C| = {} & |A| + |B| + |C| \\ & - |A \cap B| - |A \cap C| - |B \cap C| \\ & + |A \cap B \cap C|. \end{aligned}$$

(Inclusion-Exclusion) Let $A_1, A_2, \ldots, A_n$ be finite sets. Then

$$\begin{aligned} |A_1 \cup A_2 \cup \cdots \cup A_n| = {} & |A_1| + |A_2| + \cdots + |A_n| \\ & - |A_1 \cap A_2| - |A_1 \cap A_3| - \cdots - |A_{n-1} \cap A_n| \\ & + |A_1 \cap A_2 \cap A_3| + |A_1 \cap A_2 \cap A_4| + \cdots \\ & + |A_{n-2} \cap A_{n-1} \cap A_n| \\ & - \cdots + \cdots\cdots\cdots \end{aligned}$$

$$D_n = n!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \cdots + \frac{(-1)^n}{n!}\right].$$

$$n!\left[\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{(-1)^n}{n!}\right]$$

$$\boxed{D_n = n! \times 0.36}$$

$n > 6.$

$$\phi(n) = n\left[1 - \frac{1}{P_1}\right]\left[1 - \frac{1}{P_2}\right]\cdots\left[1 - \frac{1}{P_k}\right]$$

$$\phi(n) = n.\frac{(P_1 - 1)}{P_1}\frac{(P_2 - 1)}{P_2}\frac{(P_3 - 1)}{P_3}$$

$D_1 = 0$
$D_2 = 1$
$D_3 = 2$
$D_4 = 9$
$D_5 = 44.$
$D_6 = 265$

Pigeon-hole principle.

If k is a positive integer and k + 1 or more objects are placed into k boxes, then there is at least one box containing two or more of the objects.

If N objects are placed into k boxes, then there is at least one box containing at least $\lceil N/k \rceil$ objects.

$$(x+y)^n = \binom{n}{0}x^0y^n + \binom{n}{1}x^1y^{n-1} + \binom{n}{2}x^2y^{n-2} + \cdots + \binom{n}{n-1}x^{n-1}y^1 + \binom{n}{n}x^ny^0 = \sum_{k=0}^{n}\binom{n}{k}x^ky^{n-k}.$$

For each integer $n > 0$,

a) $\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n} = 2^n$, and

b) $\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \cdots + (-1)^n\binom{n}{n} = 0$.

$$\binom{n}{0} \equiv {}^nC_0$$

$$(a+b)^n = {}^nC_0\, a^n b^0 + {}^nC_1\, a^{n-1} b^1 + {}^nC_2\, a^{n-2} b^2$$

$a \downarrow$

$b \uparrow$

$a=1 \quad b=1$

$$2^n = {}^nC_0 + {}^nC_1 + {}^nC_2 \ldots {}^nC_n$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

For positive integers n, t, the coefficient of $x_1^{n_1}x_2^{n_2}x_3^{n_3}\cdots x_t^{n_t}$ in the expansion of $(x_1+x_2+x_3+\cdots+x_t)^n$ is

$$\frac{n!}{n_1!\,n_2!\,n_3!\cdots n_t!},$$

In the expansion of $(x+y+z)^7$ it follows from the multinomial theorem that the coefficient of $x^2y^2z^3$ is $\binom{7}{2,2,3}=\frac{7!}{2!\,2!\,3!}=210$, while the coefficient of xyz^5 is $\binom{7}{1,1,5}=42$ and that of x^3z^4 is $\binom{7}{3,0,4}=\frac{7!}{3!\,0!\,4!}=35$.

$(x+y+z)^7$ $\quad x^2y^2z^3$ $\quad \frac{7!}{2!\cdot 2!\cdot 3!}$

Diff quests	Diff rooms	rooms empty	L.S = m R.S = n **Number of Distributions**
Yes	Yes	Yes	n^m → Function
Yes	Yes	No	$n!\,S(m, n)$ onto
Yes	No	Yes	$S(m, 1) + S(m, 2) + \cdots + S(m, n)$ ↳ bell no.
Yes	No	No	→ $S(m, n)$
No	Yes	Yes	$\binom{n+m-1}{m}$ CBR
No	Yes	No	$\binom{n+(m-n)-1}{(m-n)} = \binom{m-1}{m-n}$
			(atleast) $= \binom{m-1}{n-1}$

$f : A \to B$

→ $\dfrac{\text{onto}}{n!}$

eg: Diffquest Diffroom.

$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k = 1 + x + x^2 + \cdots$	1
$\frac{1}{1-ax} = \sum_{k=0}^{\infty} a^k x^k = 1 + ax + a^2x^2 + \cdots$	a^k
$\frac{1}{1-x^r} = \sum_{k=0}^{\infty} x^{rk} = 1 + x^r + x^{2r} + \cdots$	1 if $r \mid k$; 0 otherwise
$\frac{1}{(1-x)^2} = \sum_{k=0}^{\infty} (k+1)x^k = 1 + 2x + 3x^2 + \cdots$	$k+1$
$\frac{1}{(1-x)^n} = \sum_{k=0}^{\infty} C(n+k-1, k)x^k = 1 + C(n, 1)x + C(n+1, 2)x^2 + \cdots$	$C(n+k-1, k) = C(n+k-1, n-1)$
$\frac{1}{(1+x)^n} = \sum_{k=0}^{\infty} C(n+k-1, k)(-1)^k x^k = 1 - C(n, 1)x + C(n+1, 2)x^2 - \cdots$	$(-1)^k C(n+k-1, k) = (-1)^k C(n+k-1, n-1)$
$\frac{1}{(1-ax)^n} = \sum_{k=0}^{\infty} C(n+k-1, k)a^k x^k = 1 + C(n, 1)ax + C(n+1, 2)a^2x^2 + \cdots$	$C(n+k-1, k)a^k = C(n+k-1, n-1)a^k$

$\frac{1}{1-ax} =$

$1 + ax + (ax)^2 + (ax)^3 + (ax)^4$

$a = 1 \quad a = 2$

$$a_{n+2} - 10a_{n+1} + 21a_n = f(n), \qquad n \geq 0.$$

Here the homogeneous part of the solution is

$$a_n^{(h)} = c_1(3^n) + c_2(7^n).$$

Roots : 3, 7.

$f(n)$	$a_n^{(p)}$
5	A_0
$3n^2 - 2$	$A_3n^2 + A_2n + A_1$
$7(11^n)$	$A_4(11^n)$
$31(r^n), r \neq 3, 7$	$A_5(r^n)$
$6(3^n)$	A_6n3^n
$2(3^n) - 8(9^n)$	$A_7n3^n + A_8(9^n)$

Homogenous.

Roots: R_1, R_2.

CE: $a_n = (R_1)^n c_1 + (R_2)^n c_2$.

Roots: R, R.

CE: $a_n = (R)^n c_1 + n \cdot (R)^n c_2$.

The Laws of Logic

For any primitive statements p, q, r, any tautology T_0, and any contradiction F_0,

1) $\neg\neg p \Leftrightarrow p$	Law of *Double Negation*
2) $\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$ $\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$	*DeMorgan's* Laws
3) $p \vee q \Leftrightarrow q \vee p$ $p \wedge q \Leftrightarrow q \wedge p$	*Commutative* Laws
4) $p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r^{\dagger}$ $p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r$	*Associative* Laws
5) $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$	*Distributive* Laws
6) $p \vee p \Leftrightarrow p$ $p \wedge p \Leftrightarrow p$	*Idempotent* Laws
7) $p \vee F_0 \Leftrightarrow p$ $p \wedge T_0 \Leftrightarrow p$	*Identity* Laws
8) $p \vee \neg p \Leftrightarrow T_0$ $p \wedge \neg p \Leftrightarrow F_0$	*Inverse* Laws
9) $p \vee T_0 \Leftrightarrow T_0$ $p \wedge F_0 \Leftrightarrow F_0$	*Domination* Laws
10) $p \vee (p \wedge q) \Leftrightarrow p$ $p \wedge (p \vee q) \Leftrightarrow p$	*Absorption* Laws

Logic.

God's Rule.

$p \to q \equiv \neg p \vee q.$

$p \to q \equiv \neg q \to \neg p.$

$$\left\{\begin{array}{l}
(a \to b) \wedge (a \to c) \equiv a \to (b \wedge c) \\
(a \to b) \vee (a \to c) \equiv a \to (b \vee c) \\
(a \to c) \wedge (b \to c) \equiv (a \vee b) \to c. \\
(a \to c) \vee (b \to c) \equiv (a \wedge b) \to c.
\end{array}\right.$$

Rule of Inference	Related Logical Implication	Name of Rule
1) $\begin{array}{l} p \\ \underline{p \to q} \\ \therefore q \end{array}$	$[p \wedge (p \to q)] \to q$	Rule of Detachment (Modus Ponens)
2) $\begin{array}{l} p \to q \\ \underline{q \to r} \\ \therefore p \to r \end{array}$	$[(p \to q) \wedge (q \to r)] \to (p \to r)$	Law of the Syllogism
3) $\begin{array}{l} p \to q \\ \underline{\neg q} \\ \therefore \neg p \end{array}$	$[(p \to q) \wedge \neg q] \to \neg p$	Modus Tollens
4) $\begin{array}{l} p \\ \underline{q} \\ \therefore p \wedge q \end{array}$		Rule of Conjunction
5) $\begin{array}{l} p \vee q \\ \underline{\neg p} \\ \therefore q \end{array}$	$[(p \vee q) \wedge \neg p] \to q$	Rule of Disjunctive Syllogism
6) $\begin{array}{l} \underline{\neg p \to F_0} \\ \therefore p \end{array}$	$(\neg p \to F_0) \to p$	Rule of Contradiction
7) $\begin{array}{l} \underline{p \wedge q} \\ \therefore p \end{array}$	$(p \wedge q) \to p$	Rule of Conjunctive Simplification

1) $$\begin{array}{c} p \\ p \to q \\ \hline q \end{array}$$

2) $$\begin{array}{c} p \to q \\ \neg q \\ \hline \therefore \neg p \end{array}$$

3) $$\begin{array}{c} p \to q \\ q \to R \\ \hline p \to R \end{array}$$

4) $$\begin{array}{c} p \vee q \\ \neg p \\ \hline \therefore q \end{array}$$

5) $$p \longrightarrow p \vee q \checkmark$$

6) $$p \wedge q \longrightarrow p$$

7) $$\begin{array}{c} p \vee q \\ \neg p \vee R \\ \hline q \vee R \end{array}$$

Invalid

$$\begin{array}{c} a \to b \\ b \\ \hline \therefore a \end{array}$$

8)	$\dfrac{p}{\therefore p \vee q}$	$p \rightarrow p \vee q$	Rule of Disjunctive Amplification
9)	$\dfrac{\begin{array}{l} p \wedge q \\ p \rightarrow (q \rightarrow r) \end{array}}{\therefore r}$	$[(p \wedge q) \wedge [p \rightarrow (q \rightarrow r)]] \rightarrow r$	Rule of Conditional Proof
10)	$\dfrac{\begin{array}{l} p \rightarrow r \\ q \rightarrow r \end{array}}{\therefore (p \vee q) \rightarrow r}$	$[(p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow [(p \vee q) \rightarrow r]$	Rule for Proof by Cases
11)	$\dfrac{\begin{array}{l} p \rightarrow q \\ r \rightarrow s \\ p \vee r \end{array}}{\therefore q \vee s}$	$[(p \rightarrow q) \wedge (r \rightarrow s) \wedge (p \vee r)] \rightarrow (q \vee s)$	Rule of the Constructive Dilemma
12)	$\dfrac{\begin{array}{l} p \rightarrow q \\ r \rightarrow s \\ \neg q \vee \neg s \end{array}}{\therefore \neg p \vee \neg r}$	$[(p \rightarrow q) \wedge (r \rightarrow s) \wedge (\neg q \vee \neg s)] \rightarrow (\neg p \vee \neg r)$	Rule of the Destructive Dilemma

$$\forall x\,[P(x) \wedge Q(x)] \equiv \forall x\,P(x) \wedge \forall x\,Q(x)$$

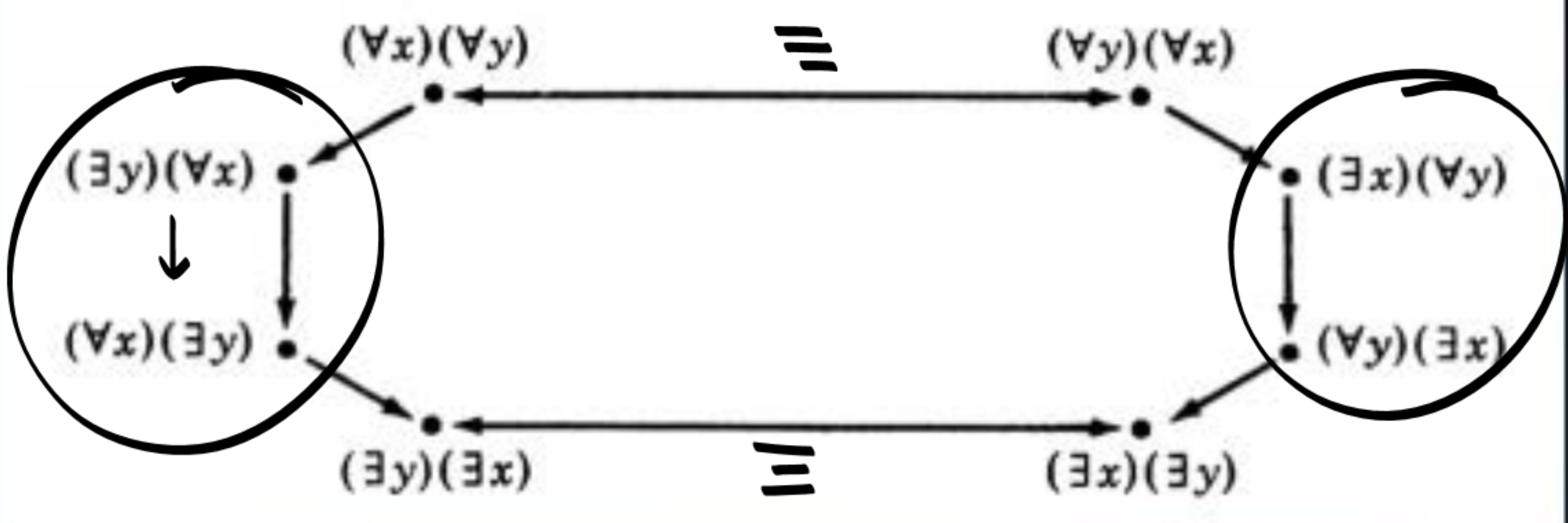
$$\forall x\,[P(x) \vee Q(x)] \longleftarrow \forall x\,P(x) \vee \forall x\,Q(x)$$

$$\exists x\,[P(x) \wedge Q(x)] \longrightarrow \exists x\,P(x) \wedge \exists x\,Q(x)$$

$$\exists x\,[P(x) \vee Q(x)] \equiv \exists x\,P(x) \vee \exists x\,Q(x)$$

$$\forall x\,[P(x) \rightarrow Q(x)] \longrightarrow (\forall x\,P(x) \rightarrow \forall x\,Q(x))$$

$$\forall x\,[P(x) \Leftrightarrow Q(x)] \longrightarrow (\forall x\,P(x) \Leftrightarrow \forall x\,Q(x))$$



$$(\forall x)\,(\forall y)\,P(x,y) \leftrightarrow (\forall y)\,(\forall x)\,P(x,y)$$
$$(\forall x)\,(\forall y)\,P(x,y) \rightarrow (\exists y)\,(\forall x)\,P(x,y)$$
$$(\forall y)\,(\forall x)\,P(x,y) \rightarrow (\exists x)\,(\forall y)\,P(x,y)$$
$$(\exists y)\,(\forall x)\,P(x,y) \rightarrow (\forall x)\,(\exists y)\,P(x,y)$$
$$(\exists x)\,(\forall y)\,P(x,y) \rightarrow (\forall y)\,(\exists x)\,P(x,y)$$
$$(\forall x)\,(\exists y)\,P(x,y) \rightarrow (\exists y)\,(\exists x)\,P(x,y)$$
$$(\forall y)\,(\exists x)\,P(x,y) \rightarrow (\exists x)\,(\exists y)\,P(x,y)$$
$$(\exists y)\,(\exists x)\,P(x,y) \leftrightarrow (\exists x)\,(\exists y)\,P(x,y)$$

All $\rightarrow$

some $\wedge$

not all $\neg\ \forall x[\ \rightarrow\ ]$

no/none $\forall x\ [\ \rightarrow \neg\ ]$

The Laws of Set Theory

For any sets A, B, and C taken from a universe $\mathcal{U}$

1) $\overline{\overline{A}} = A$ — Law of *Double Complement*

2) $\overline{A \cup B} = \overline{A} \cap \overline{B}$ — *DeMorgan's* Laws
   $\overline{A \cap B} = \overline{A} \cup \overline{B}$

3) $A \cup B = B \cup A$ — *Commutative* Laws
   $A \cap B = B \cap A$

4) $A \cup (B \cup C) = (A \cup B) \cup C$ — *Associative* Laws
   $A \cap (B \cap C) = (A \cap B) \cap C$

5) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ — *Distributive* Laws
   $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

6) $A \cup A = A$ — *Idempotent* Laws
   $A \cap A = A$

7) $A \cup \emptyset = A$ — *Identity* Laws
   $A \cap \mathcal{U} = A$

8) $A \cup \overline{A} = \mathcal{U}$ — *Inverse* Laws
   $A \cap \overline{A} = \emptyset$

9) $A \cup \mathcal{U} = \mathcal{U}$ — *Domination* Laws
   $A \cap \emptyset = \emptyset$

10) $A \cup (A \cap B) = A$ — *Absorption* Laws
    $A \cap (A \cup B) = A$

logic	set	Digital.
$\vee$	$\cup$	$+$
$\wedge$	$\cap$	$\cdot$
T	$\mathcal{U}$	1
F	$\emptyset$	0

Let $A, B, C \subseteq \mathcal{U}$.

a) If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

b) If $A \subset B$ and $B \subseteq C$, then $A \subset C$.

c) If $A \subseteq B$ and $B \subset C$, then $A \subset C$.

d) If $A \subset B$ and $B \subset C$, then $A \subset C$.

If $S, T \subseteq \mathcal{U}$, then S and T are disjoint if and only if $S \cup T = S \triangle T$.

For any sets $A, B, C \subseteq \mathcal{U}$:

a) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

b) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

c) $(A \cap B) \times C = (A \times C) \cap (B \times C)$

d) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

1) $A \subseteq B \wedge B \subseteq C \rightarrow A \subseteq C$.

2) $A \subset B \wedge B \subseteq C \rightarrow A \subset C$.

3) $A \subseteq B \wedge B \subset C \rightarrow A \subset C$.

4) $A \subset B \wedge B \subset C \rightarrow A \subset C$

Let $f: A \to B$, with $A_1, A_2 \subseteq A$. Then

a) $f(A_1 \cup A_2) = f(A_1) \cup f(A_2)$;

b) $f(A_1 \cap A_2) \subseteq f(A_1) \cap f(A_2)$;

c) $f(A_1 \cap A_2) = f(A_1) \cap f(A_2)$ when f is one-to-one.

$$\frac{1}{n!}\sum_{k=0}^{n}(-1)^k\binom{n}{n-k}(n-k)^m.$$

This will be denoted by $S(m, n)$ and is called a *Stirling number of the second kind.*

We note that for $|A| = m \geq n = |B|$, there are $n! \cdot S(m, n)$ onto functions from A to B.

$f: A \to B$ Total functions $= (R.S)^{L.S.}$

<u>1:1:</u>

$\forall a \forall b (f(a) = f(b) \to a = b)$ # $\to P(R.S, L.S)$

<u>onto:</u>

R.S must be full

$$\sum_{i=0}^{n} (-1)^i * {}^nC_i * (n-i)^m$$

<u>1:1 C</u>: $1:1 \wedge$ onto $\to n!$

Let $f: A \to B$ and $g: B \to C$.

a) If f and g are one-to-one, then $g \circ f$ is one-to-one.

b) If f and g are onto, then $g \circ f$ is onto.

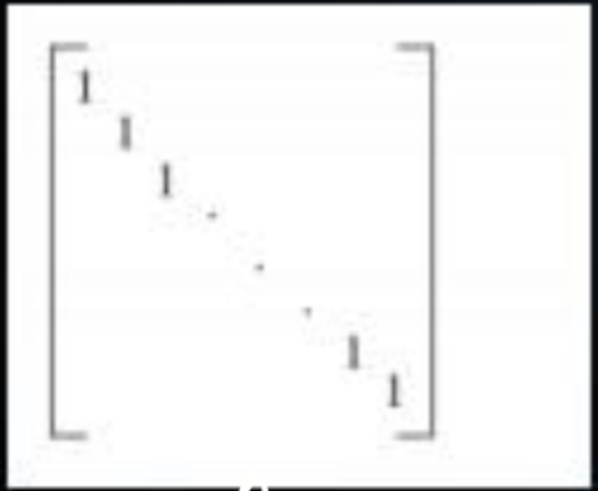
If $f: A \to B$, $g: B \to C$, and $h: C \to D$, then $(h \circ g) \circ f = h \circ (g \circ f)$. → Associative.

If $f: A \to B$, $g: B \to C$ are invertible functions, then $g \circ f: A \to C$ is invertible and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

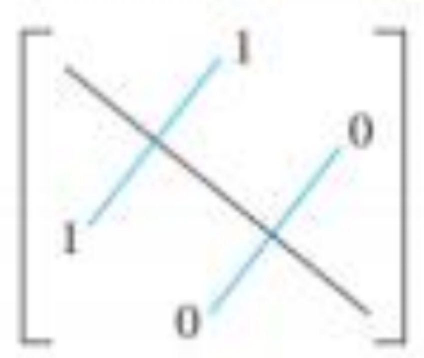
If $f: A \to B$ and $B_1, B_2 \subseteq B$, then (a) $f^{-1}(B_1 \cap B_2) = f^{-1}(B_1) \cap f^{-1}(B_2)$; (b) $f^{-1}(B_1 \cup B_2) = f^{-1}(B_1) \cup f^{-1}(B_2)$; and (c) $f^{-1}(\overline{B_1}) = \overline{f^{-1}(B_1)}$.

Let $f: A \to B$ for finite sets A and B, where $|A| = |B|$. Then the following statements are equivalent: (a) f is one-to-one; (b) f is onto; and (c) f is invertible.

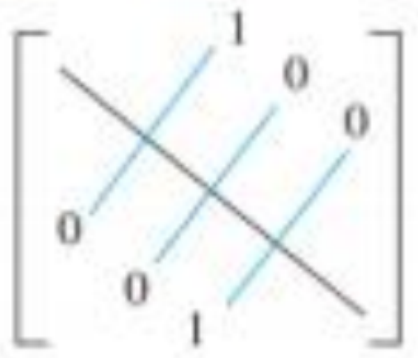
1. **Transitivity**	$\forall x,y,z,$	if $x \, R \, y$ and $y \, R \, z$, then $x \, R \, z$;
2. **Reflexivity**	$\forall x,$	$x \, R \, x$;
3. **Irreflexivity**	$\forall x,$	$x \not R \, x$;
4. **Symmetry**	$\forall x,y$	if $x \, R \, y$, then $y \, R \, x$;
5. **Antisymmetry**	$\forall x,y$	if $x \, R \, y$ and $y \, R \, x$, then $x = y$;
6. **Asymmetry**	$\forall x,y$	if $x \, R \, y$, then $y \not R \, x$.



Refle



(a) Symmetric



(b) Antisymmetric

Reflexive :

$aRa \rightarrow 2^{n^2-n}$

Symmetric :

$aRb \rightarrow bRa \quad 2^n \cdot 2^{\frac{n^2-n}{2}}$

Antisymmetric :

$aRb \wedge bRa \not\rightarrow a=b \quad 2^n \cdot 3^{\frac{n^2-n}{2}}$

Asymmetric :

$aRb \rightarrow b \not R a. \quad 3^{\frac{n^2-n}{2}}$

Transitive :

$aRb \wedge bRc \rightarrow aRc. \quad$ —

Irreflexive :

$a \not R a. \quad 2^{n^2-n}$

R_1, R_2	$\cup$	$\cap$
Reflexive	✓	✓
Sym	✓	✓
Anti	✗	✓
Asy	✗	✓
Trans	✗	✓

POR : EQR

POR		EQR
R		R
A		S
T.		T.
	$\cup$	$\cap$
R_1 R_2 POR	×	✓
R_1, R_2 EQR	×	✓

$[L, \vee, \wedge]$

$\vee \longrightarrow$ lub.

$\wedge \longrightarrow$ glb.

1) $a \vee a = a.$

$a \wedge a = a.$

2) $a \vee b = b \vee a.$

$a \wedge b = b \wedge c$

3) $a \vee (a \wedge b) = a.$

$a \wedge (a \vee b) = a.$

4) $a \vee (b \vee c) = (a \vee b) \vee c.$

$a \wedge (b \wedge c) = (a \wedge b) \wedge c.$

Lattice.

G.E.
L.E

bounded lattice

atleast 1 compleme

compleme.

D.L

boolean algebra.

closed Associative identity Inven.

Semigroup.

monoid.

Group.

Commutative → abelian Group.

1. $\sum d(v_i) = 2e$.

2. no. of odd degree vertices in a graph will always be even.

3. $\delta(G) \le \frac{2e}{n} \le \Delta(G) \le n-1$.

4. In Simple Graph atleast 2 vertices will have same degree.

5. $e \le \frac{n(n-1)}{2}$.

6. Degree $\le n-1$.

7. $2^{\frac{n(n-1)}{2}}$.

8. ${}^{\frac{n(n-1)}{2}}C_e$.

* $n-k \leq e \leq \frac{(n-k)(n-k+1)}{2}$.

* if $\delta(G) \geq \frac{n-1}{2}$ then G is connected.

* if G is Disconnected then $\overline{G}$ will be connected.

* if Graphs contains exactly 2 odd degree vertices then path will be available betn. those vertices

$$e(G) + e(\bar{G}) = \frac{n(n-1)}{2}$$

K_n : $n-1,\ n-1,\ n-1 \dots n-1$

$G \rightarrow d_1, d_2, d_3 \dots d_n$

$\bar{G}$: $n-1-d_1,\ n-1-d_2$

self complement :

$$e(G) = \frac{n(n-1)}{4}$$

G is Euler ✓

$L(G) \rightarrow$ Euler ✓

↳ Hamiltonian ✓

Degrees of each $L(K_n)$

$= 2(n-2)$

$$\boxed{n \equiv 0 \text{ OR } 1 \pmod 4}$$

maximum no of edges in bipartite Graph $e \le \left\lfloor \frac{n^2}{4} \right\rfloor$

$e(K_{m,n}) = m \times n$.

* bipartite Graph does not Contains odd length cycle.

Star Graph. $(K_{1,n-1})$

no of Hamiltonian cycle in $K_{n,n}$ $\frac{n!\,(n-1)!}{2}$.

no of Hamiltonian cycle in K_n $\frac{(n-1)!}{2}$.

$\chi(K_n) = n$

$\chi(C_n) = 2$ n - even

$\chi(C_n) = 3$ n → odd

$\chi(W_n) = 3$ n is odd

$\chi(W_n) = 4$ n is even

$\alpha(G) \leq \beta(G)$

$\beta(G) \geq \dfrac{n}{\chi(G)}$

$$M(C_n) = M(W_n) = M(K_n) = \left\lfloor \frac{n}{2} \right\rfloor$$

$$M(K_{m,n}) = \min(m,n)$$

<u>perfect matching</u>. $\Rightarrow$ if p.m exist then no. of vertices will be even.

total no. of perfect matching $\dfrac{(2n)!}{2^n\, n!}$

* $\boxed{n - e + f = 2}$

* if G is planar then $e \leq 3n - 6$

OR

$\boxed{\text{if } e > 3n - 6 \text{ then G is nonplanar}}$

$$K(G) \leq \lambda(G) \leq \delta(G) \leq 2e/n \leq \Delta(G) \leq n - 1$$

Number system

$$(a_3 a_2 a_1 a_0 . a_{-1} a_{-2})_r = (\quad)_{10}$$

$$\left(a_3 \times r^3 + a_2 \times r^2 + a_1 \times r^1 + a_0 \times r^0 + a_{-1} \times r^{-1} + a_{-2} \times r^{-2}\right)_{10}$$

$$\left(\underbrace{b_3 b_2 b_1 b_0}.\, b_{-1} b_{-2}\right)_{10} = (\quad)_r$$

r	$b_3 b_2 b_1 b_0$

Rem

x_0

x_1

x_2

$$0.b_{-1}b_{-2} \times r = y_0 . b_{-3} b_{-4} \qquad y_0$$

$$0.b_{-3}b_{-4} \times r = y_1 . b_{-5} b_{-6} \qquad y_1$$

$$(x_2 x_1 x_0 . y_0 y_1)_r$$

complement

0 1 0 1 0 $\longrightarrow$ 1's

$\hookrightarrow$ Decimal = 10

$$10101 \longrightarrow 1\text{'s}$$

$$01010$$

Decimal $= -10$

$=$ Ans

$\underline{\underline{0}}1011 \longrightarrow 2's$

Decimal = +11

$\underline{\underline{1}}01100 \longrightarrow 2's$

010100 → -20

unsigned $\rightarrow$ 0 to $2^n - 1$

Signed / 1's

Range $\Rightarrow -(2^{n-1} - 1)$ to $(2^{n-1} - 1)$

2's complement   Range $\Rightarrow -(2^{n-1})$ to $(2^{n-1} - 1)$

Logic GATE

$f = A \cdot B \cdot \overline{C} \cdot D \ldots$

$n \rightarrow$ no. of Variables

$k \rightarrow$ no. of complement

$NAND = (2n-2)+k$

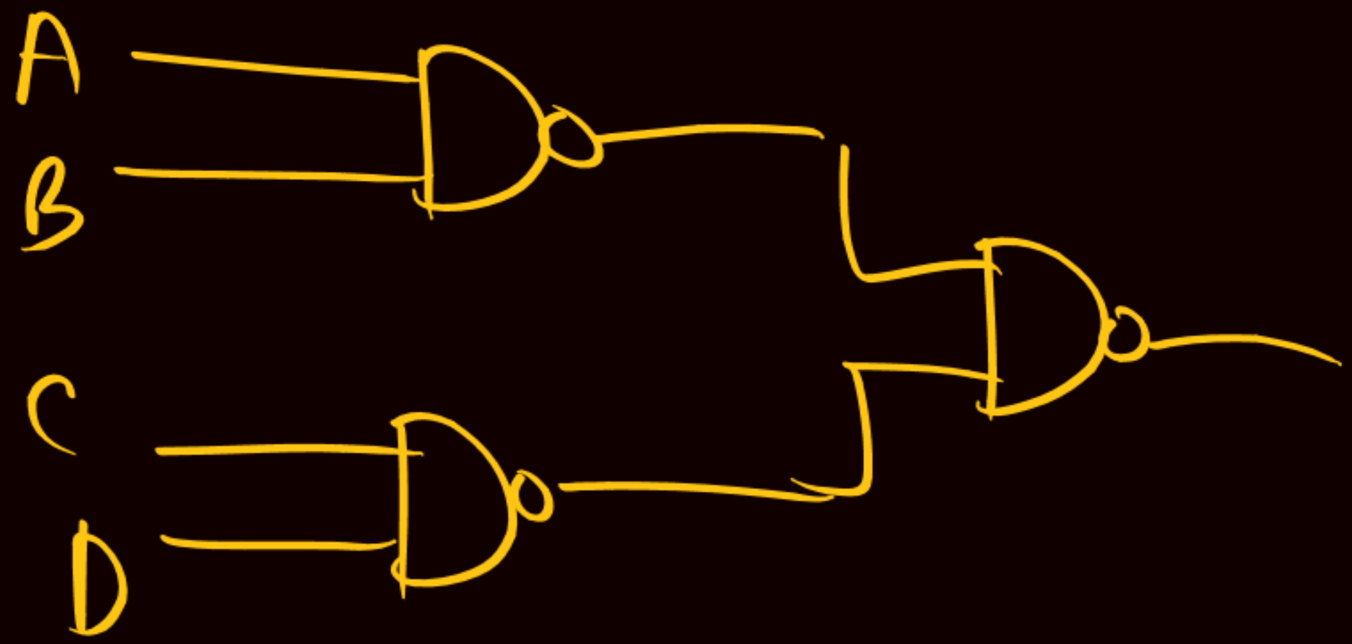
$NOR = (3n-3)-k$

$f = A + \overline{B} + C + \ldots$

$NAND = (3n-3)-k$

$NOR = (2n-2)+k$

$AB+CD$



$$\bar{A} \oplus B = A \oplus \bar{B} = A \odot B = \bar{A} \odot \bar{B}$$

$$\bar{A} \odot B = A \odot \bar{B} = A \oplus B = \bar{A} \oplus \bar{B}$$

$$A + BC = (A+B)(A+C)$$

$$AB + \bar{A}C + \overset{x}{BC} = AB + \bar{A}C$$

$$(A+B)(\bar{A}+C) = AC + \bar{A}B$$

$$\overline{ABC} = \bar{A} + \bar{B} + \bar{C}$$

$$\overline{A+B+C} = \bar{A} \cdot \bar{B} \cdot \bar{C}$$

comparator $\rightarrow$ n bit

Total combination $= 2^{2n}$

Equal combination $= 2^n$

unequal combinatio $= 2^{2n} - 2^n$

A > B or A < B $\Rightarrow \dfrac{2^{2n} - 2^n}{2}$

Q $4 \times 1 \text{ MUX} \xrightarrow{\frac{64}{4}+\frac{16}{4}+\frac{4}{4}} 64 \times 1 \text{ MUX}$

$16+4+1=21$

Ripple carry adder

$$T = (n-1)T_{carry} + \mathrm{Max}\{T_{sum}, T_{carry}\}$$

$n \rightarrow$ Variable $\longrightarrow$ 2^{2^n} function

Self Dual $= 2^{2^{n-1}}$

minterm 2^n

Maxterm $= 2^n$

Half

NAND/NOR = 5

Full

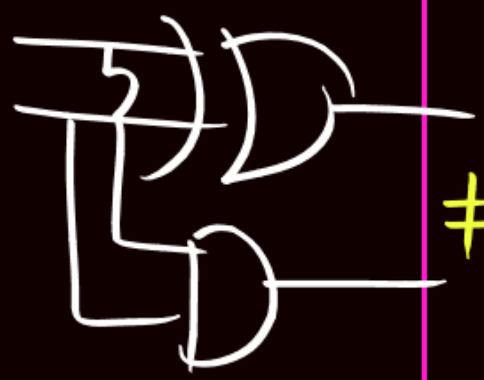
NAND/NOR = 9

Look ahead carry adder (LACA)

Carry Block

Total Delay = 4T

Carry Block = 2T



AND = $\frac{n(n+1)}{2}$

OR = n

FF

SR $\longrightarrow$ $S + \bar{R}Q_n$

JK $\longrightarrow$ $J\bar{Q}_n + \bar{K}Q_n$

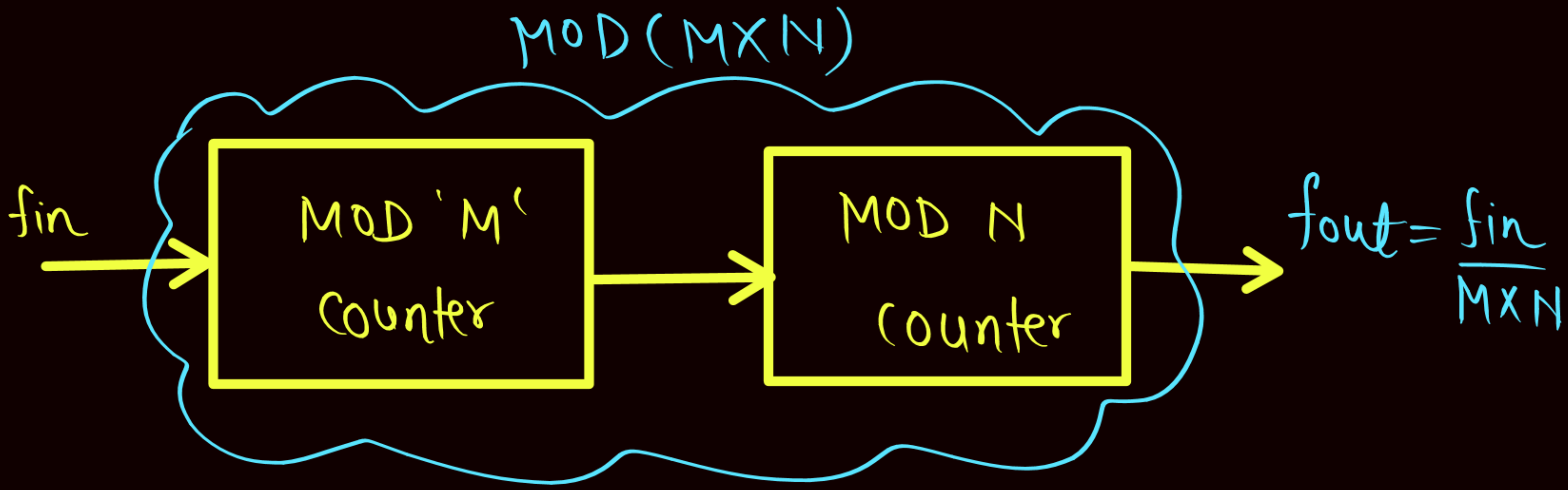
D $\longrightarrow$ D

T $\longrightarrow$ $T \oplus Q_n$

Excitation

Q_n	Q_{n+1}	S	R	J	K	D	T
0	0	0	X	0	X	0	0
0	1	1	0	1	X	1	1
1	0	0	1	X	1	0	1
1	1	X	0	X	0	1	0

	Store	Retrive	Total	
SISO	n	n-1	2n-1	$\rightarrow$ slow
SIPO	n	0	n	
PISO	1	n-1	n	
PIPO	1	0	1	$\rightarrow$ fast


MOD(MXN)
fin
MOD 'M' Counter
MOD N Counter
fout = fin / MXN

Synchronous

$$T_{clk} \geq \tau_{pdff}$$

$$f_{clk} \leq \frac{1}{\tau_{pdff}}$$

Asynchronous

$$T_{clk} \geq n \cdot \tau_{pdff}$$

$$f_{clk} \leq \frac{1}{n \cdot \tau_{pdff}}$$

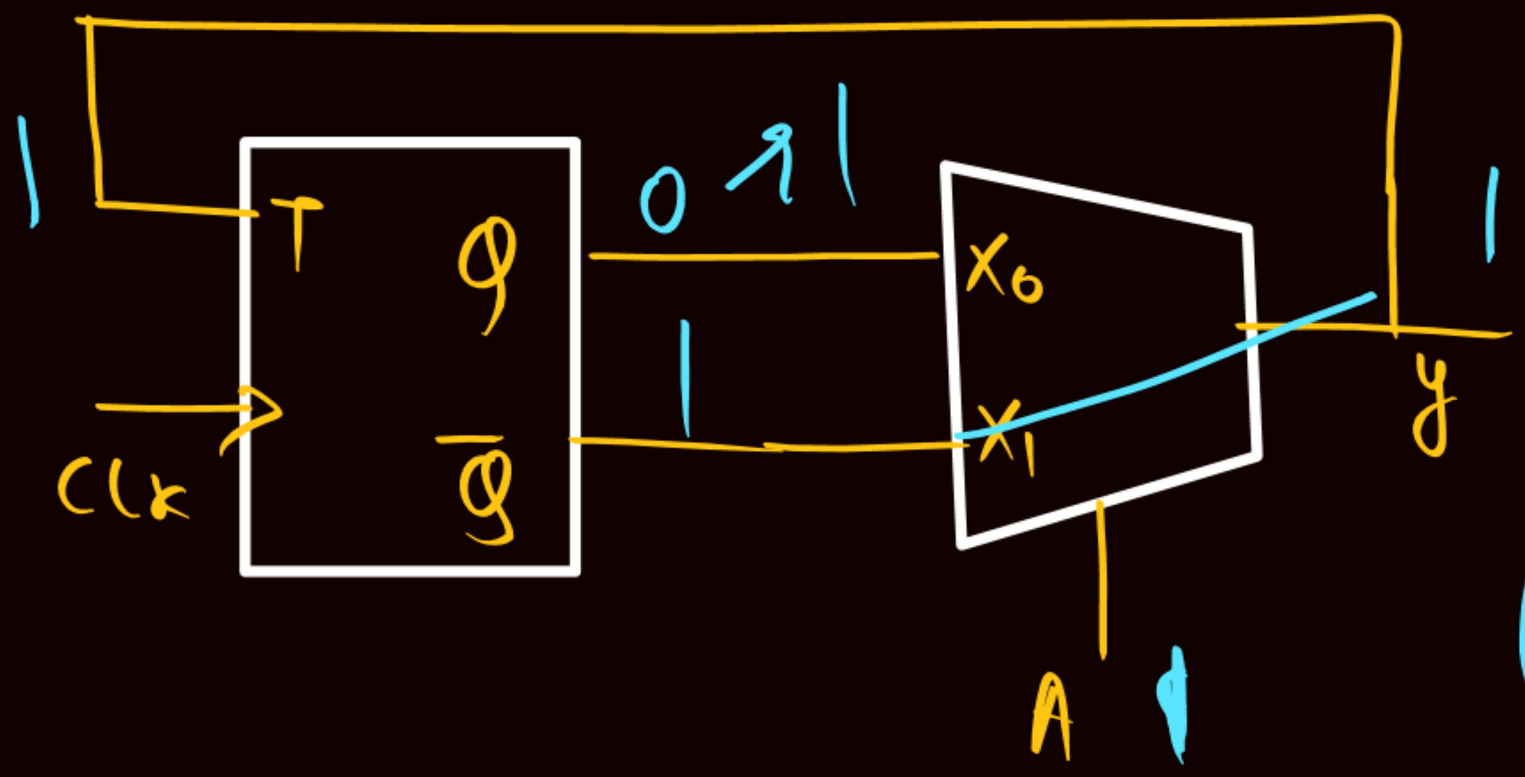
Series carry synchronous counter

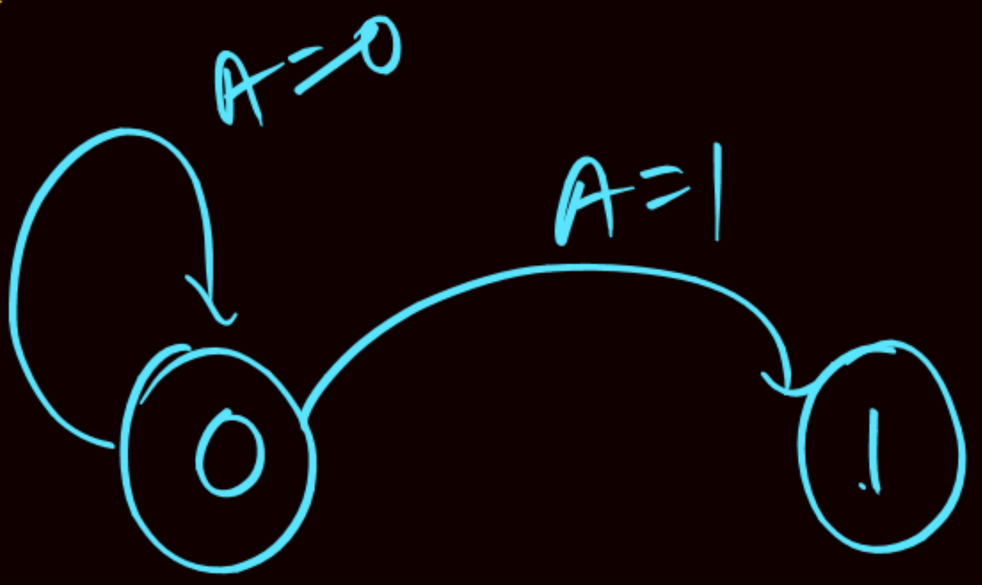
$$T_{clk} \geq \tau_{pd_{ff}} + (n-2)\,\tau_{pd_{AND}}$$

Parry carry synchronous counter

$$T_{clk} \geq \tau_{pd_{ff}} + \tau_{pd_{AND}}$$

2 PM





q	A	Q^+
0	0 ✓	0
0	1	1
1	0	
1	1	