GATE PYQ SERIES
COMPUTER SCIENCE

PYQ Discussion

Theory of Computation

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Topics To Be Covered

7-10M

- Regular Expression
- DFA, NFA

$\approx$ 2-4 marks

- Context Free Grammar (DPDA, NPDA) $\Rightarrow$ 2 Marks $\approx$
- Turing Machine
- Undecidability

2-4 m

Regular Expression

Q 1

Let L = { w ∈ (0 + 1)* | w has $\boxed{\text{even number of 1s}}$}, i.e., L is the set of all bit strings with even number of 1s. Which one of the regular expressions below represents L? [2010: 2 Marks]

ε, 0, 00, 11, 000, 011, 101, 110, ...

~~(A)~~ $(0^*10^*1)^*$ → 0 ✗

~~(B)~~ $0^*(10^*10^*)^*$

~~(C)~~ $0^*(10^*1)^*0^*$ — 11011 ✗

~~(D)~~ $0^*1(10^*1)^*10^*$ → ε ✗

I) $\boxed{\text{even no. of 0's}}$

$1^*(1^*01^*01^*)^*1^*$

III) $\boxed{\text{odd no. of 0's}}$

$1^*(1^*01^*01^*)^*1^*01^*$

II) $\boxed{\text{even no. of 1's}}$

$0^*(0^*10^*10^*)^*0^*$

IV) $\boxed{\text{odd no. of 1's}}$

$0^*(0^*10^*10^*)^*0^*10^*$

even no. of 1s over $\Sigma = \{0, 1\}$

$$= 0^*(0^*1\,0^*1\,0^*)^*0^* \quad = (0^*10^*10^*)^* + 0^*$$

$$= 0^*(10^*10^*)^*$$

$$= (0^*10^*1)^*0^*$$

class of Reg$_s$ $\cap$ class of CFL$_s$ = class of Reg$_s$

Q 2

Let P be a regular language and Q be a context- free language such that Q $\subseteq$ P. (For example, let P be the language represented by the regular expression $p^* q^*$ and Q be $\{p^n q^n \mid n \in N\}$. Then which of the following is ALWAYS regular?

$a^*b^* \cap a^n b^n = a^n b^n$

[2011: 1 Mark]

(A) $P \cap Q$ $= Reg \cap CFL$ = Need not be Regular $\Rightarrow$ ~~CFL~~

(B) $\Sigma^* - P$ $= \Sigma^* - Reg = \overline{Reg} \Rightarrow$ Regular

P Q

Reg CFL

(C) $P - Q$ $= Reg - CFL = Reg \cap \overline{CFL} = Reg \cap CSL = CSL$

$Q \subseteq P$

(D) $\Sigma^* - Q$ $= \overline{Q} = \overline{CFL} \Rightarrow CSL$

$\overline{L} = \Sigma^* - L$

$\overline{Reg} = \Sigma^* - Reg$

Q3

Given the language L = {ab, aa, baa}, which of the following strings are in L*?

~~1.~~ abaabaaabaa $\in L^*$ 2. aaaabaaaa $\in L^*$

~~3.~~ baaaaabaaaa$\boxed{b}$ $\notin L^*$ 4. baaaaabaa $\in L^*$ [2012: 1 Mark]

$L^* = (ab+aa+baa)^*$

A. 1, 2 and 3

B. 2, 3 and 4

C. 1, 2 and 4

D. 1, 3 and 4

GATE

- Difficult only for those who don't prepare
- Why we make difficult eventhough we prepare?
  - 1st : We don't have patience
  - 2nd : Systematic preparation not done

When you know the things

⇓

Then it will be easy

Q4

Consider the languages $L_1 = \phi$ and $L_2 = \{a\}$. Which one of the following represents $L_1 L_2^* \cup L_1^*$?

[2013: 1 Mark]

(A) $\{\varepsilon\}$

(B) ϕ

(C) a^*

(D) (ε, a)

$\underbrace{\phi . \{a\}^*} \cup \phi^*$

$= \phi \cup \{\varepsilon\}$

$= \{\varepsilon\}$

$\phi . \text{Any} = \phi$

$\phi^* = \phi^0 \cup \underbrace{\phi^1 \cup \phi^2 \cup \cdots}$

$= \{\varepsilon\} \cup \phi$

$= \{\varepsilon\}$

$\phi^* = \varepsilon \qquad \phi^2 = \phi$

$\boxed{\phi^0 = \varepsilon} \qquad \phi^{100} = \phi$

Q5

The length of the shortest string NOT in the language (over $\Sigma = \{a, b\}$) of the following regular expression is $\underline{\quad = 3 \quad}$.

$$a^*b^*(ba)^*a^*$$

[2014-Set3: 1 Mark]

0 ε ✓ $= a^0 b^0 (\ )^0 a^0$

1 $\begin{cases} a ✓ = a^1 b^0 (\ )^0 a^0 \\ b ✓ = a^0 b^1 (\ )^0 a^0 \end{cases}$

2 $\begin{cases} aa ✓ \\ ab ✓ \\ ba ✓ \\ bb ✓ \end{cases}$

aaa ✓
aab ✓
aba ✓
abb ✓
baa ✓
$\boxed{bab}$ ✗
bba ✓
bbb ✓

How many 3 length Strings not generated?
$= 1$

Q6

Consider alphabet $\Sigma = \{0, 1\}$, the null/empty string λ and the sets of strings X_0, X_1 and X_2 generated by the corresponding non-terminals of a regular grammar. X_0, X_1 and X_2 are related as follows:

[2015-Set2: 2 Marks]

$$X_0 = 1X_1$$
$$X_1 = 0X_1 + 1X_2$$
$$X_2 = 0X_1 + \{\lambda\}$$

$(a+b)^* = (a^*b^*)^*$

$X_0 \to 1X_1$
$X_1 \to 0X_1 \mid 1X_2$
$X_2 \to 0X_1 \mid \varepsilon \Rightarrow \boxed{X_2 = 0X_1 + \varepsilon}$

Which one of the following choices precisely represents the strings in X_0?

(A) $\boxed{10}(0^* + (10)^*)1$

(B) $10(0^* + (10)^*)^*1$

(C) $1(0 + 10)^*1 = 1(0^*(10)^*)^*1$

(D) $10(0 + 10)^*1 + 110(0 + 10)^*1$

$X_1 \to 0X_1 \mid 10X_1 \mid 1$
$A \to aA \mid bA \mid c$
$\boxed{A = (a+b)^*c}$

$X_1 = (0+10)^*1$
$X_0 = 1X_1 = 1(0+10)^*1$

Q7

Which one of the following regular expressions represents the language: *the set of all binary strings having two consecutive 0s and two consecutive 1s?*

[2016-Set1: 1 Mark]

(A) $(0+1)^*\boxed{0011}(0+1)^* + (0+1)^*\boxed{1100}(0+1)^*$ ⇒ having 0011 or 1100

001011

(B) $(0+1)^*(00(0+1)^*11 + 11(0+1)^*00)(0+1)^*$

$X\,[00\,X\,11 + 11\,X\,00]\,X$

(C) $(0+1)^*\underline{00}(0+1)^* + (0+1)^*\underline{11}(0+1)^*$ having 00 or 11

(D) $\underline{00}(0+1)^*\underline{11} + \underline{11}\,(0+1)^*\underline{00}$

$00X11 + 11X00$

$X\,00\,X\,11\,X + X\,11\,X\,00\,X$

$X[00X11 + 11X00]X$

Q 8

Which one of the following regular expression represents the set of all binary strings with an odd number of 1's?

= {1, 01, 10, 100, 010, 001, ...}

[2020: 1 Mark]

(A) $(0^*10^*10^*)^*0^*\boxed{1}$ ends always

(B) $\boxed{1}0^*(0^*10^*10^*)^*$ starts always

(C) $((0 + 1)^* 1(0 + 1)^*1)^*\boxed{1}0^*$ must

01

(D) $(0^*10^*10^*)^*\boxed{1}0^*$ must

01

$0^*(0^*10^*10^*)^*0^*10^*$

$0^*10^*(0^*10^*10^*)^*0^*$

no option is correct

Q 9

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Which of the following regular expressions represents(s) the set of all binary numbers that are $\boxed{\text{divisible by three}}$? Assume that the strings ∈ is divisible by three. [2021-Set2-$\boxed{\text{MSQ}}$: 2 Marks]

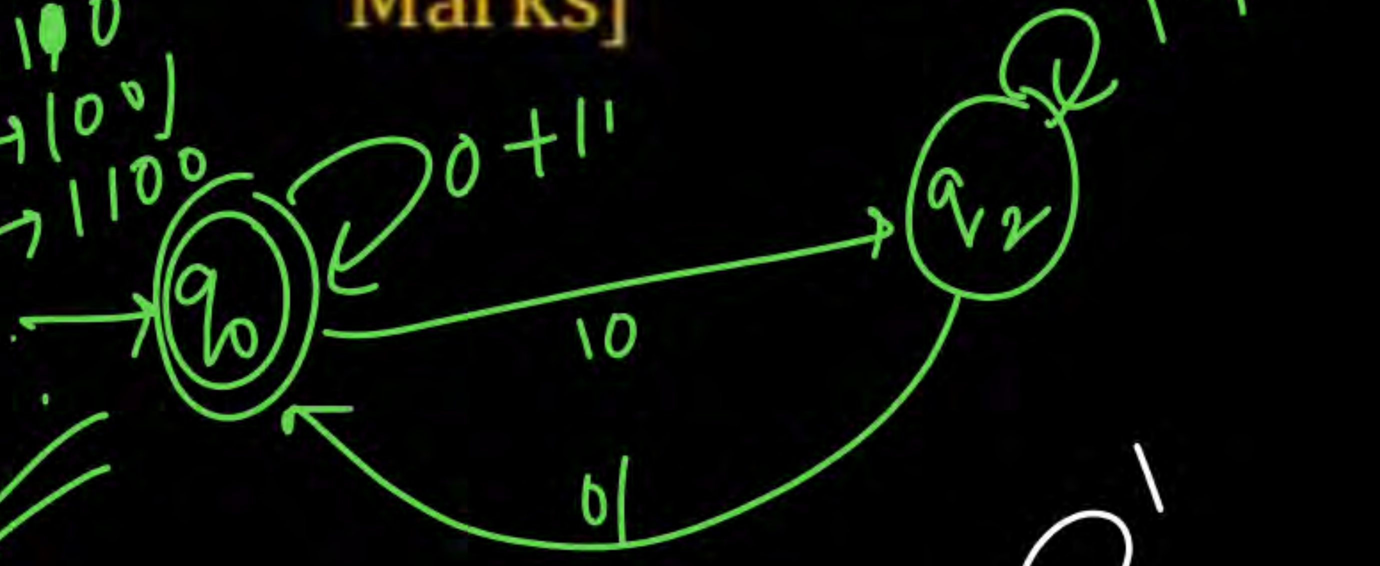
A. $(0^*(1(01^*0)^*1)^*)$

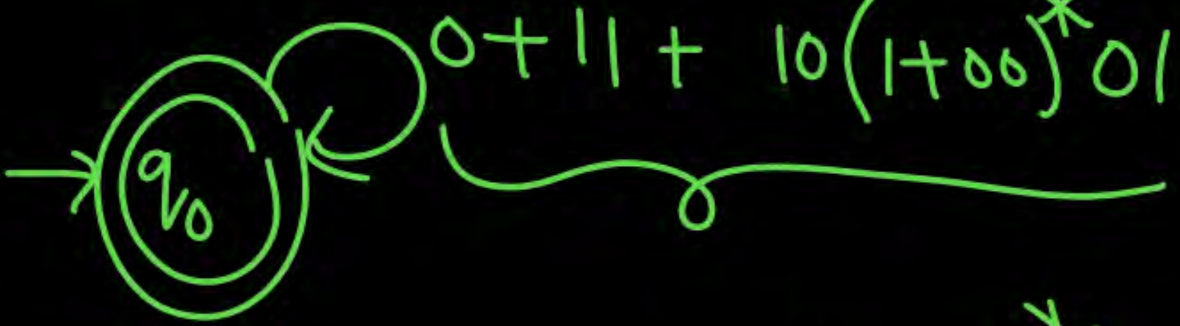
B. $(0 + 1(01^*0)^* 1)^*$ → why invalid? 1001 = 9

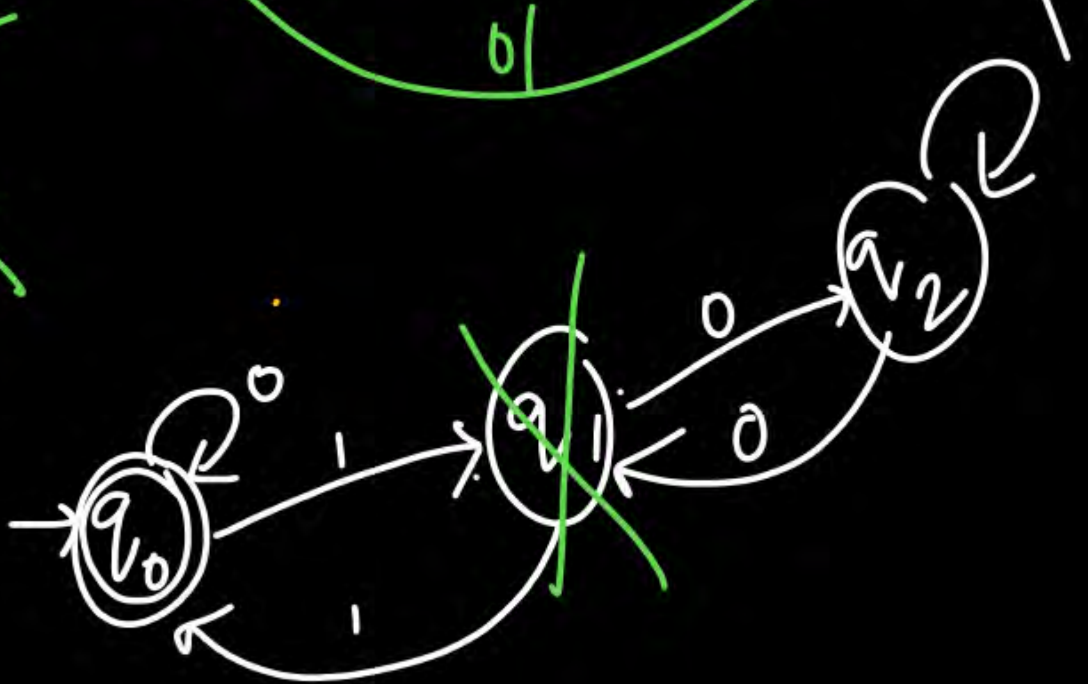
C. $(0 + 11 + \underbrace{10(1 + 00)^*01}))^*$

D. $(0 + 11 + 11(1 + 00)^*00)^*$ → 1001 ×

0 → 0
3 → 11
6 → 110
9 → 1001
12 → 1100







$= [0+11+10(1+00)^*01]^*$

DFA, NFA

Q 10

Let w be any string of length n in {0, 1}*. Let L be the set of all $\boxed{\text{substrings}}$ of w. What is the minimum number of states in a non-deterministic finite automaton that accepts L?

[2010: 2 Marks]

A) n-1

B) n

C) n+1

D) 2^{n-1}

Prefixes of ω ⇒ n+1 states

Suffixes of ω

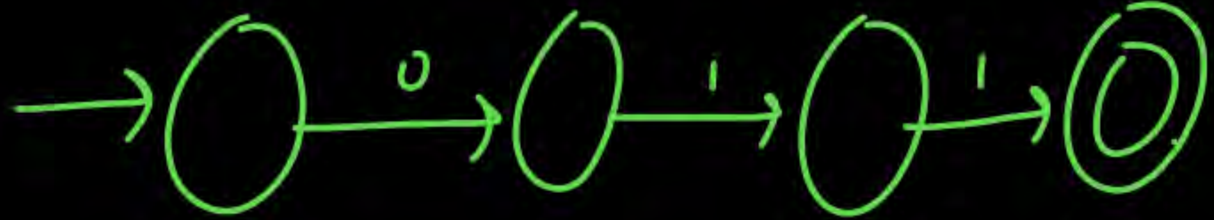
Substrings of ω

Subsequences of ω

$L = \{ x \mid x \text{ is substring of } \omega \}$

any string in $(0+1)^*$

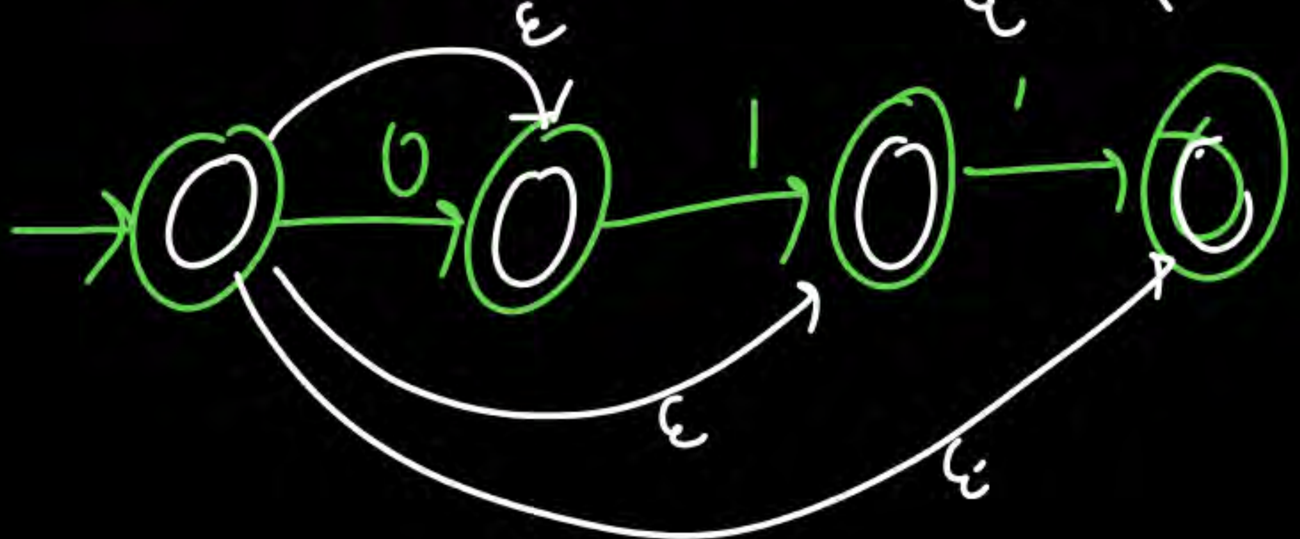
$W = \underbrace{011}_{3\ \text{len}}$



ε-NFA ⇒ NFA

k states   k states

Substrings of ω:



Q.11

The lexical analysis for a modern computer language such as Java needs the power of which one of the following machine models in a necessary and sufficient sense?

[2011: 1 Marks]

(A) Finite state automata

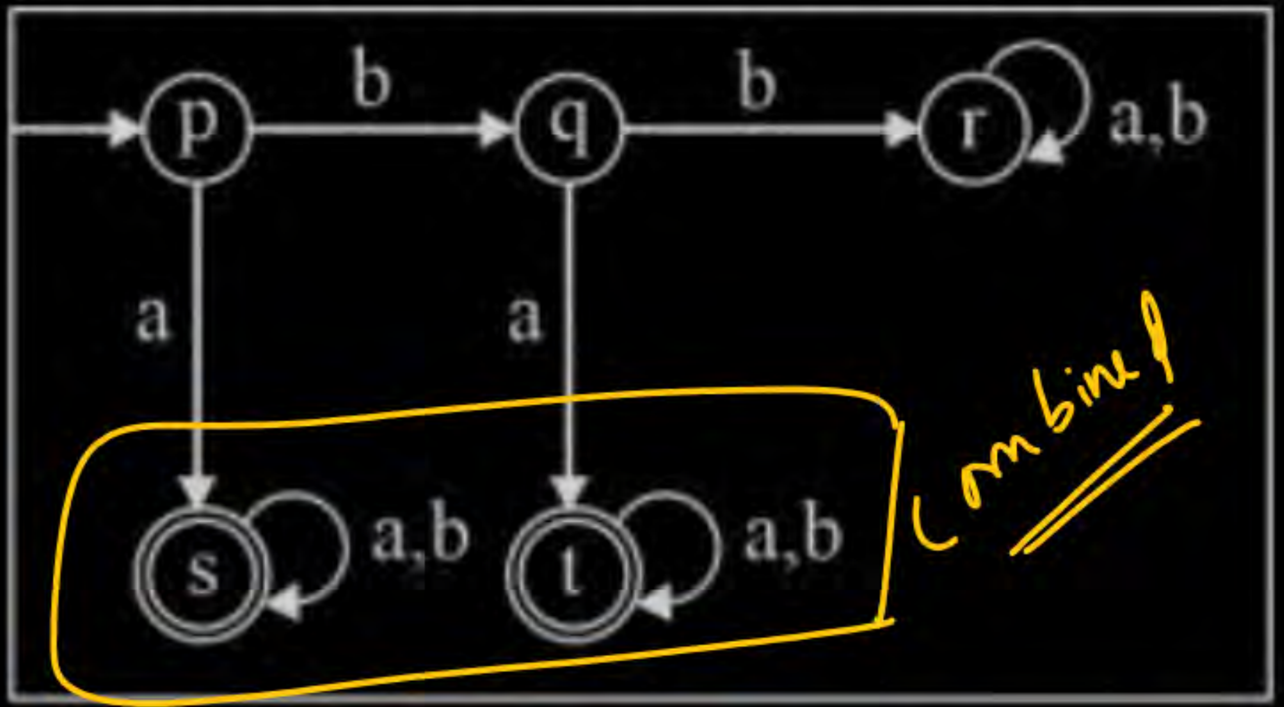
(B) Deterministic pushdown automata

(C) Non-deterministic pushdown automata

(D) Turing machine

Q 12

A deterministic finite automaton (DFA) D with alphabet $\Sigma = \{a, b\}$ is given below:

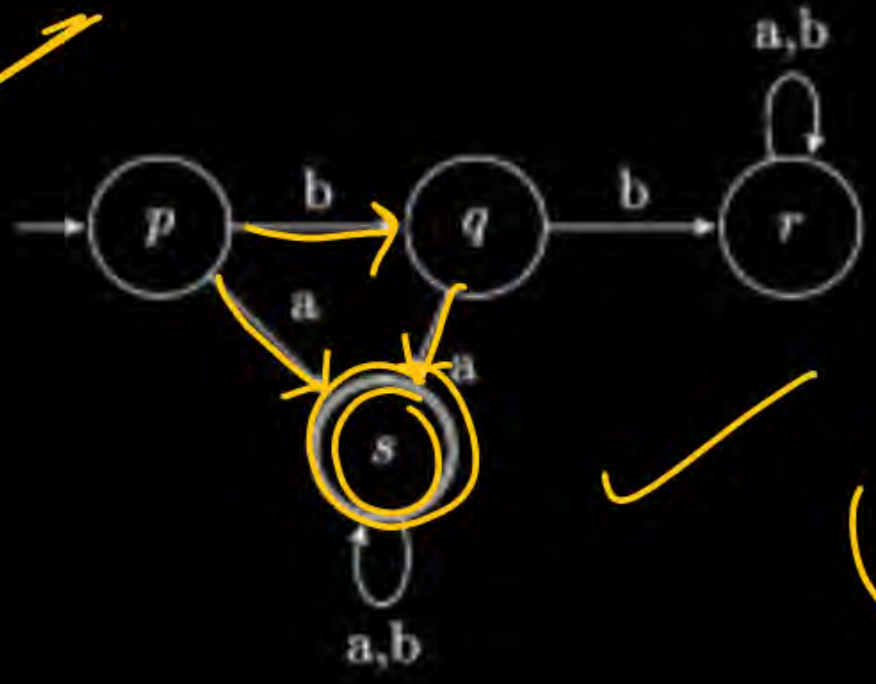


Which of the following finite state machines is a valid minimal DFA which accepts the same language as D?

[2011:2 Mark]

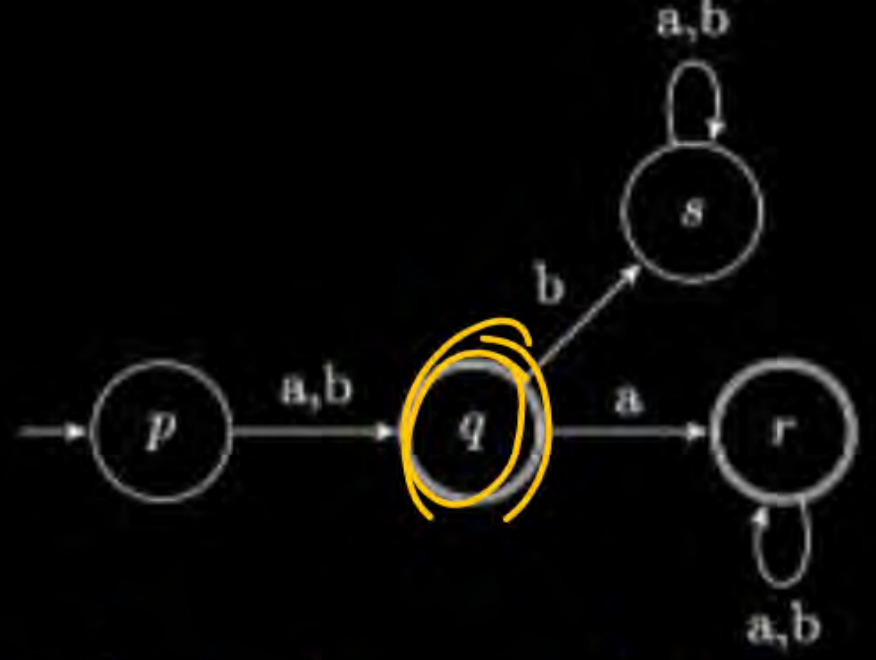
$$a(a+b)^* + ba(a+b)^* = (a+ba)(a+b)^*$$

A



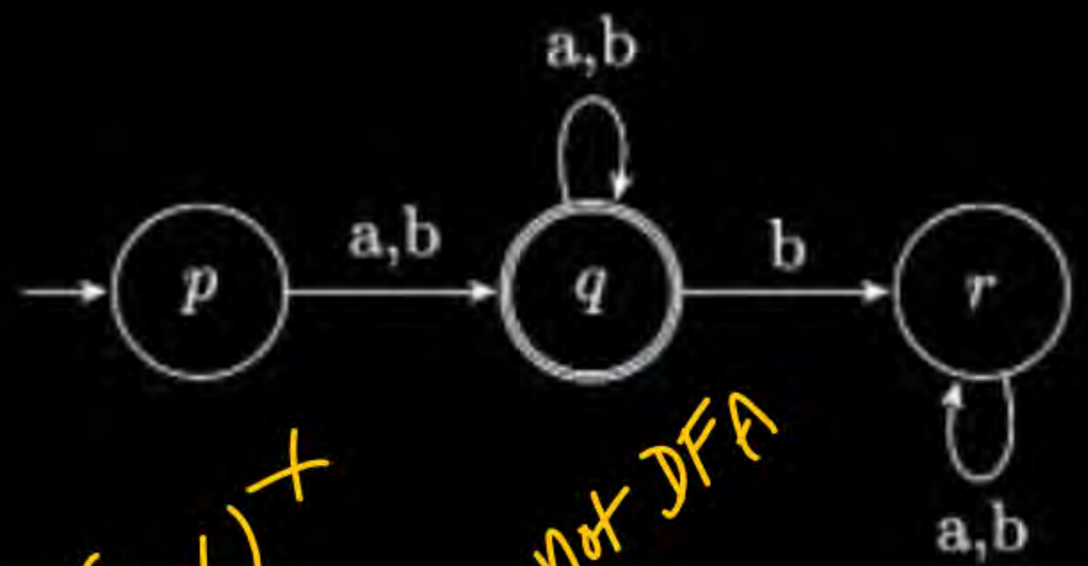
$(a+ba)X$

B



$a+b+(aa+ba)X$

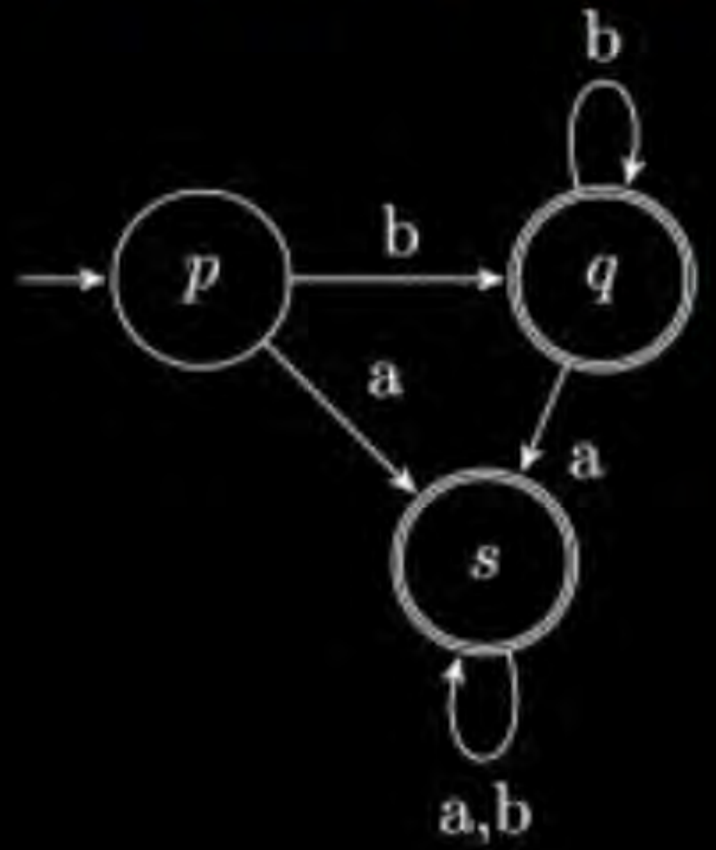
C



$L=(a+b)^+$

not DFA

D



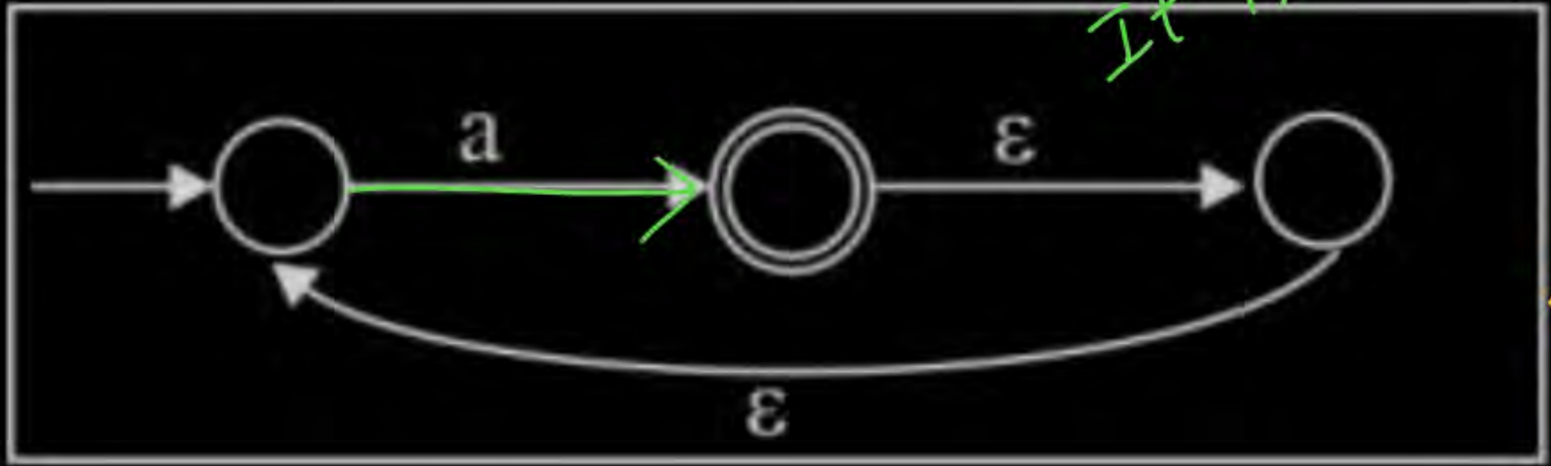
$$\boxed{aX + bb^* + bb^*a(a+b)^*}$$

Q 13

What is the complement of the language accepted by the NFA shown below? Assume $\Sigma = \{a\}$ and ε is the empty string.

[2012: 1 Mark]

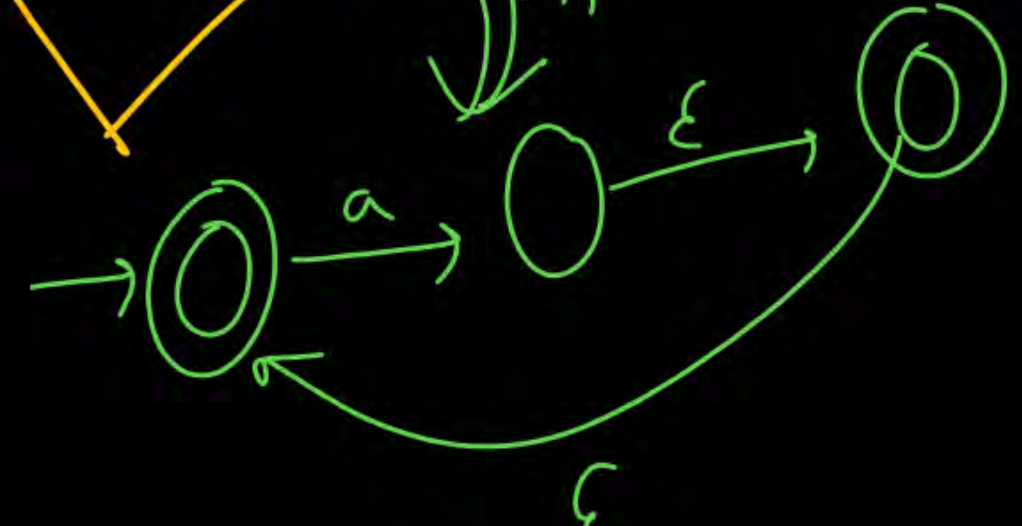
It is NFA



This is not DFA

Don't interchange final & non-final

If you do



$L = a^*$

(A) $\varnothing$

(B) $\{\varepsilon\}$

(C) a^*

(D) $\{a, \varepsilon\}$

$L = a^+$

$\bar{L} = \{\varepsilon\}$

Q 14

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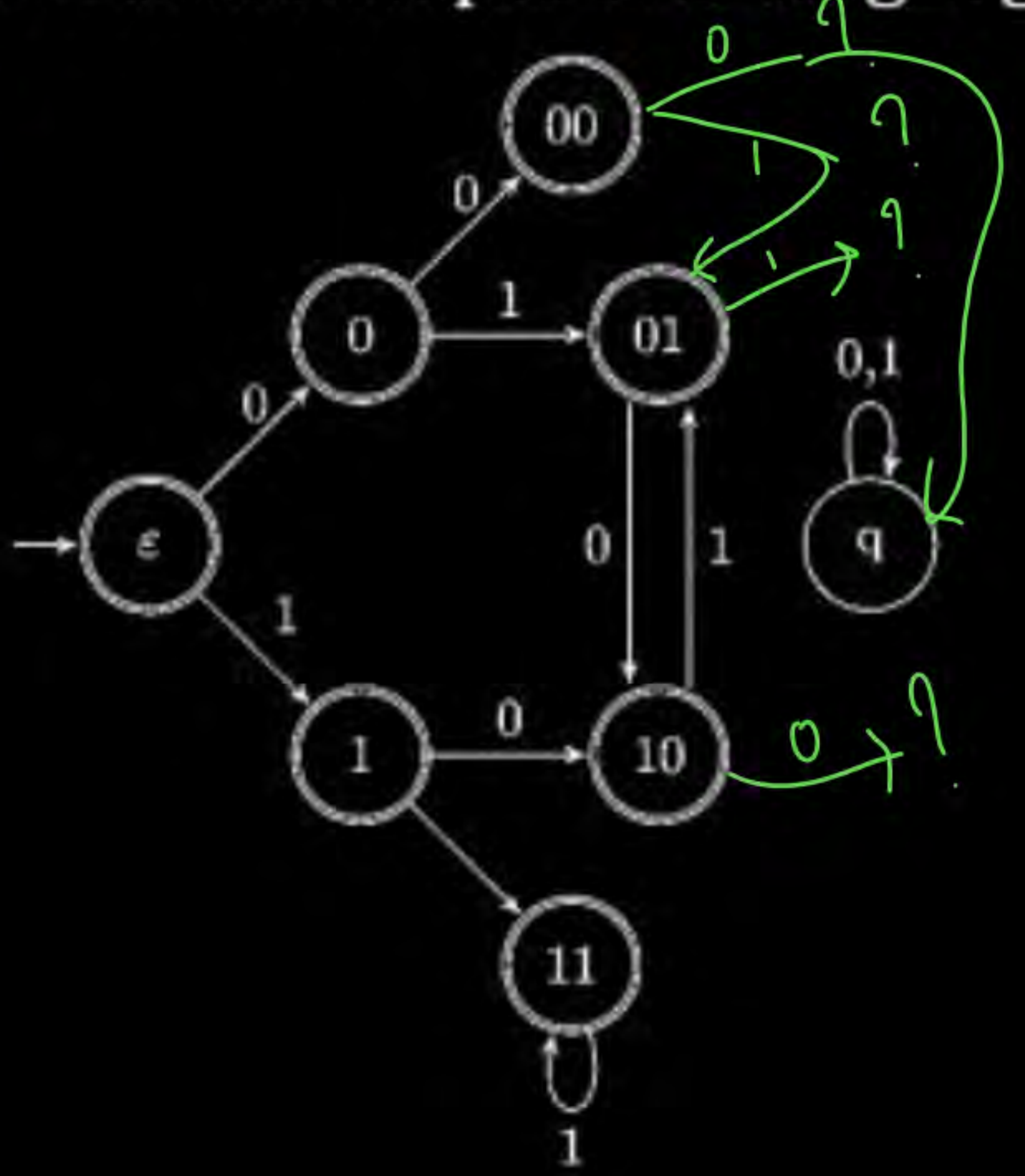
Tree Approach

Consider the set of strings on {0, 1} in which, every substring of 3 symbols has at most two zeros. For example, 001110 and 011001 are in the language, but 100010 is not. All strings of length less than 3 are also in the language. A partially complete DFA that accepts this language is shown below.



The missing arcs in the DFA are?

[2012: 2 Marks]

$\delta(00, 0) = q$ (state, input)

$\delta(00, 1) = 01$

$\delta(01, 1) = 11$

$\delta(10, 0) = 00$

A

	00	01	10	11	q
00	1	0			
01				1	
10	0				
11			0		

B

	00	01	10	11	q
00		0			1
01		1			
10				0	
11		0			

011

C

	00	01	10	11	q
00		1			0
01		1			
10			0		
11		0			

001 011

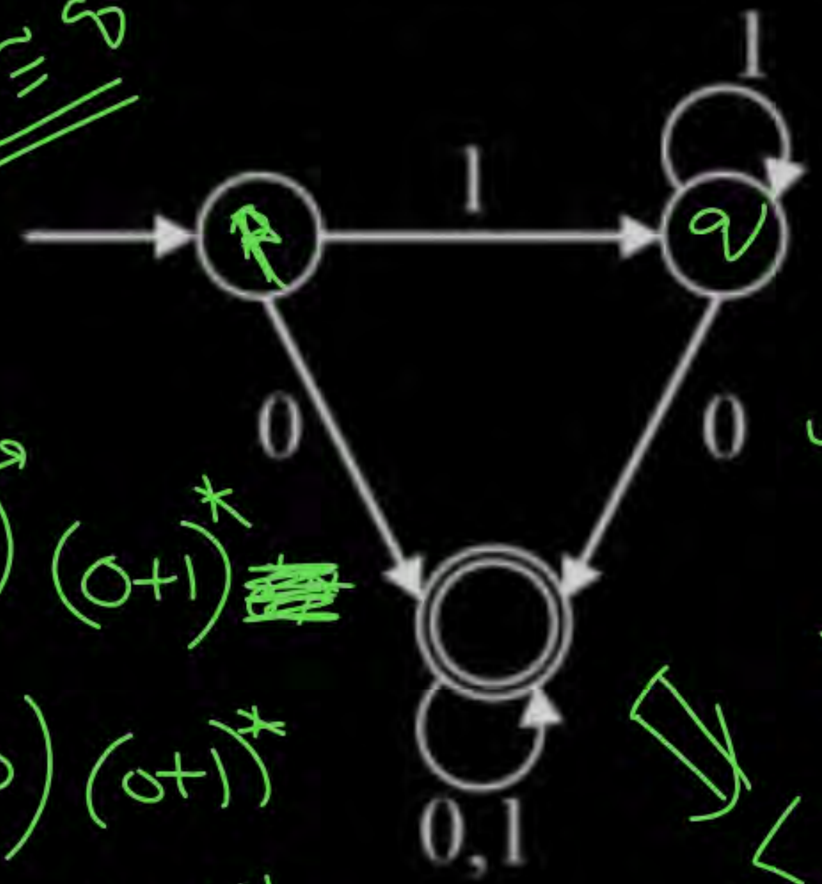
D

	00	01	10	11	q
00		1			0
01				1	
10	0				
11			0		

Q 15

Consider the DFA A given below:

P=8



$(0+11^*0)(0+1)^*$

$((\varepsilon+11^*)0)(0+1)^*$

$= 1^*0(0+1)^*$ contains 0

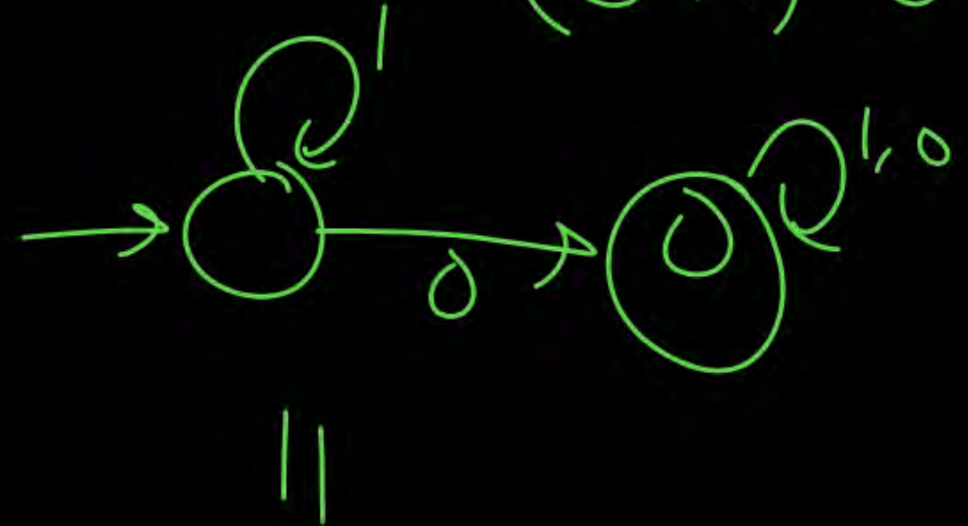
$L(A)$ Reg

Which of the following are **FALSE**?

1. Complement of L(A) is context-free. TRUE
2. $L(A) = L((11^*0 + 0)(0 + 1)^*0^*1^*)$ TRUE
3. For the language accepted by A, A is the minimal DFA. False
4. A accepts all strings over {0, 1} of length at least 2. False [2013: 1 Mark]

$11 \times$

$(0+1)^*0^*1^* = (0+1)^*$



A 1 and 3 only

B 2 and 4 only

C 2 and 3 only

D 3 and 4 only

Q16 Which one of the following is $\boxed{\text{TRUE?}}$ [2014-Set1: 1 Mark]

(A) The language $L = \{a^n b^n \mid n \geq 0\}$ is regular. ✗

(B) The language $L = \{a^n \mid n \text{ is prime}\}$ is regular. ✗

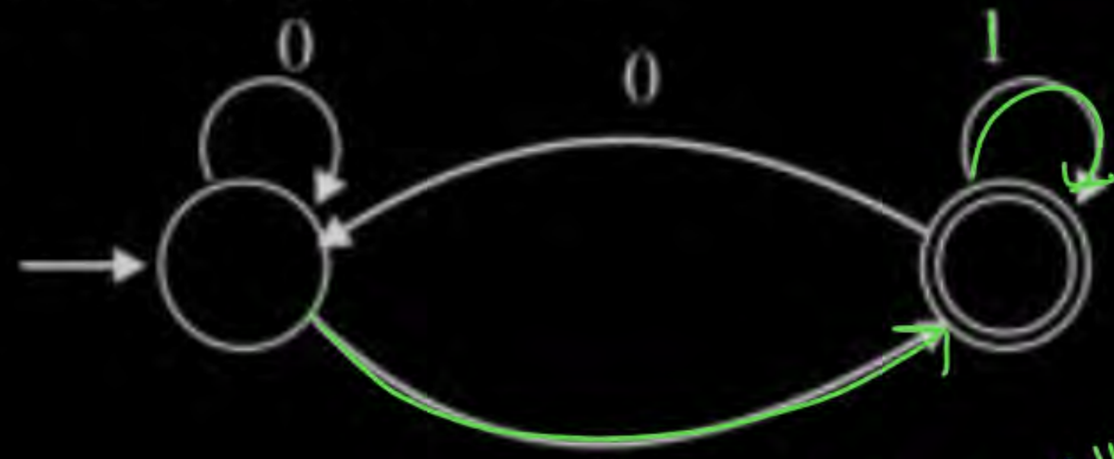
(C) The language $L = \{w \mid w$ has $3k + 1$ b's for some $k \in \mathbb{N}$ with $\Sigma = \{a, b\}\}$ is regular. ✓

$\boxed{\# b\text{'s} \,\%\, 3 = 1}$

(D) The language $L = \{ww \mid w \in \Sigma^*$ with $\Sigma = \{0, 1\}\}$ is $\boxed{\text{regular}}$. ✗

Q 17

Which of the regular expressions given below represent the following DFA?



Ending with 1

I. $0^*1(1 + 00^*1)^* = 0^*1\left[(\varepsilon + 0^+)1\right]^* = 0^*1(0^*1)^* = (0^*1)^+$

II. $0^*1^*1 + 11^*0^*1^*$ (110)

III. $(0 + 1)^*1$

[2014-Set1: 2 Mark]

$(0+1)^*1 = (0^*1)^+$

(A) I and II only

(B) I and III only

(C) II and III only

(D) I, II, and III

Q 18

If $L_1 = \{a^n \mid n \geq 0\}$ and $L_2 = \{b^n \mid n \geq 0\}$, consider

I. $L_1 \cdot L_2$ is a regular language

II. $L_1 \cdot L_2 = \{a^n b^n \mid n \geq 0\}$

Which one of the following is CORRECT?

(A) Only I

(B) Only II

(C) Both I and II

(D) Neither I nor II

[2014-Set2: 1 Mark]

$L_1 = a^n = a^*$

$L_2 = b^n = b^*$

$\boxed{L_1 L_2 = a^* b^*} \neq a^n b^n$

$= a^m b^n$

Q 19

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Let $L_1 = \{\omega \in \{0,1\}^* \mid \omega$ has at least as many occurrences of (110)'s as (011)'s$\}$. Let $L_2 = \{\omega \in \{0,1\}^* \mid \omega$, has at least as many occurrences of (000)'s as (111)'s$\}$. Which one of the following is TRUE?

[2014-Set2: 2 Marks]

(A) ~~L_1 is regular but not L_2~~

(B) L_2 is regular but not L_1

(C) Both L_1 and L_2 are regular

(D) Neither L_1 nor L_2 ~~are~~ regular

$n_{01}(\omega) \geq n_{10}(\omega)$

$n_{001}(\omega) = n_{100}(\omega)$

pattern based

$L_1 = \{\omega \mid \omega \in (0+1)^*, n_{110}(\omega) \geq n_{011}(\omega)\}$

$L_2 = \{\omega \mid \omega \in \{0,1\}^*, n_{000}(\omega) \geq n_{111}(\omega)\}$

not reg

$n_{110} = n_{011}$

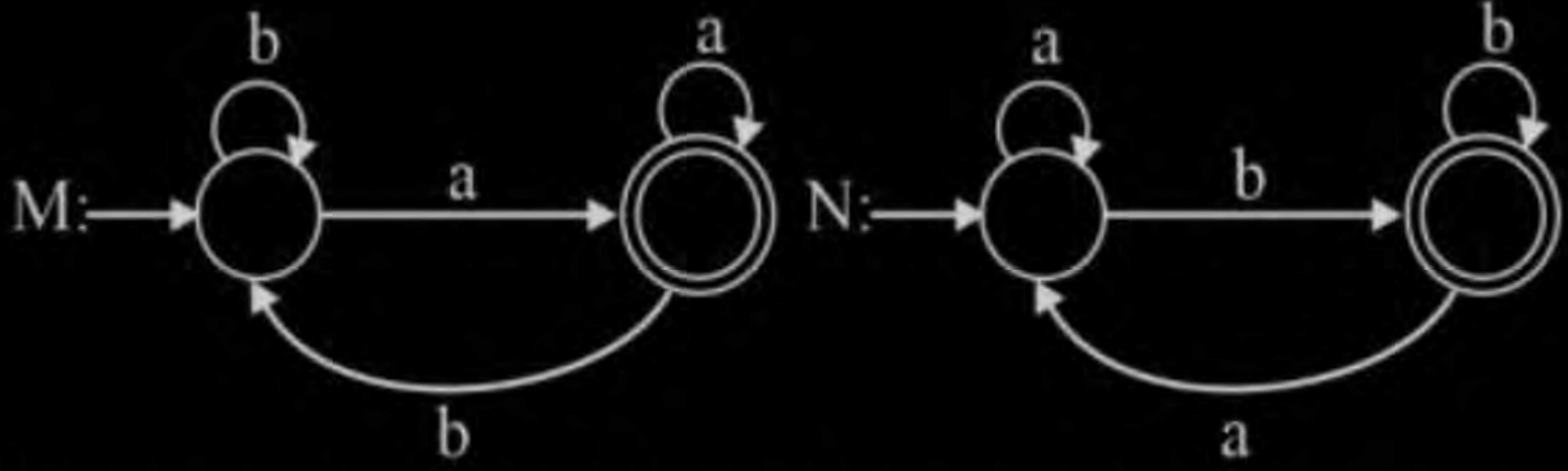
$n_{110} = n_{011} + 1$

$n_{011} = n_{110} + 1$

$P_1 = rev(P_2)$ regular

$n_{P_1}(\omega) \; = \; \neq \; \leq \; \geq \; n_{P_2}(\omega)$

Q.20



$= 1$

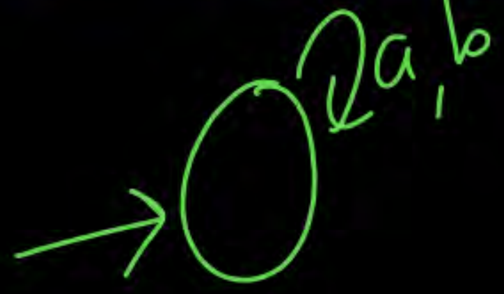
Consider the DFAs M and N given above. The number of states in a minimal DFA that accepts the language $L(M) \cap L(N)$ is

[2015-Set1: 2 Marks]

$L(M) = Xa$

$L(N) = Xb$

$L(M) \cap L(N) = \phi$



Q 21

The number of states in the minimal deterministic finite automaton corresponding the regular expression $(0 + 1)^*(10)$ is______.

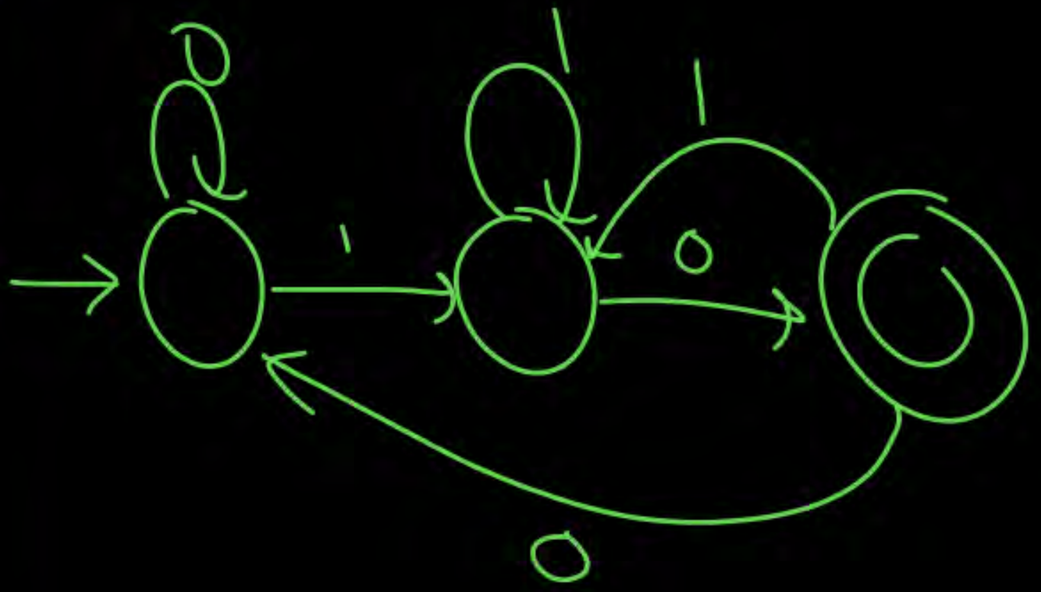
[2015-Set2: 2 Marks]

$X_{00} \Rightarrow$ 3 states

$X_{000} \Rightarrow$ 4 states

X_{010}

$X_{111000} \Rightarrow$ 7 states



Ends with 10

$\Downarrow$

min = 10

2 len

$\Downarrow$

3 states

9:30 PM

Resume

Q 22

Let L be the language represented by the regular expression $\Sigma^*0011\Sigma^*$ where $\Sigma = \{0, 1\}$.
What is the minimum number of states in a DFA that recognizes L'(complement of L)? [2015-Set3: 1 Mark]

(A) 4

(B) 5

(C) 6

(D) 8

$\Sigma^*0\Sigma^* \longrightarrow 2$ state

$\Sigma^*1\Sigma^* \longrightarrow$

$\Sigma^*00\Sigma^* \longrightarrow 3$ states

$\Sigma^*01\Sigma^*$

$\Sigma^*011\Sigma^* \longrightarrow 4$ states

$L = \Sigma^*\underbrace{0011}\Sigma^*$

f $\longleftrightarrow$ nf

min = 0011 $\Downarrow$ 5 states

$\bar{L} \Rightarrow 5$ states

Q 23

Which of the following languages is/are regular?

L_1: $\{wxw^R \mid w, x \in \{a, b\}^*$ and $\underbrace{|w|, |x| > 0}_{0}\}$ w^R is the reverse of string $w\}$ $\rightarrow$ Regular

L_2: $\{a^n b^m \mid \underbrace{m \neq n}$ and $m, n \geq 0\}$ $\rightarrow$ non reg

L_3: $\{a^p b^q c^r \mid p, q, r \geq 0\}$ $= a^* b^* c^* \rightarrow$ reg

[2015-Set2: 2 Marks]

(A) L_1 and L_3 only

(B) L_2 only

(C) L_2 and L_3 only

(D) L_3 only

$L_1 = \{wxw^R \mid w, x \in \{a, b\}^+\}$

$= \{wxw^R \mid \underbrace{w, x \in \{a, b\}^*, |w|, |x| > 0}_{}\}$

$= axa + bxb$

$= a(a+b)^+a + b(a+b)^+b$

wxw^R
ww^Rx
xww^R

$w, x \in \{a,b\}^*$ $\Longrightarrow$ all are regular

$= (a+b)^*$

put $w = \epsilon$

$wxw^R = \epsilon x \epsilon = x$

$= (a+b)^*$

$\cancel{w, x \in \{a,b\}}$

$wxw^R \longrightarrow$ reg

ww^Rx
xww^R $\longrightarrow$ non reg

Q 24

Which of the following languages is generated by the given grammar? [2016-Set1: 1 Mark]

$S \rightarrow aS | bS | \varepsilon$

MCQ

$S \rightarrow aS | bS | \varepsilon$

(A) $\{a^n b^m | n, m \geq 0\}$

(B) $\{w \in \{a, b\}^* \; w \text{ has equal number of a's and b's}\}$

OR

(C) $\{a^n | n \geq 0\} \cup \{b^n | n \geq 0\} \cup \{a^n b^n | n \geq 0\}$

$S \rightarrow Sa | Sb | \varepsilon$

(D) $\{a, b\}^*$

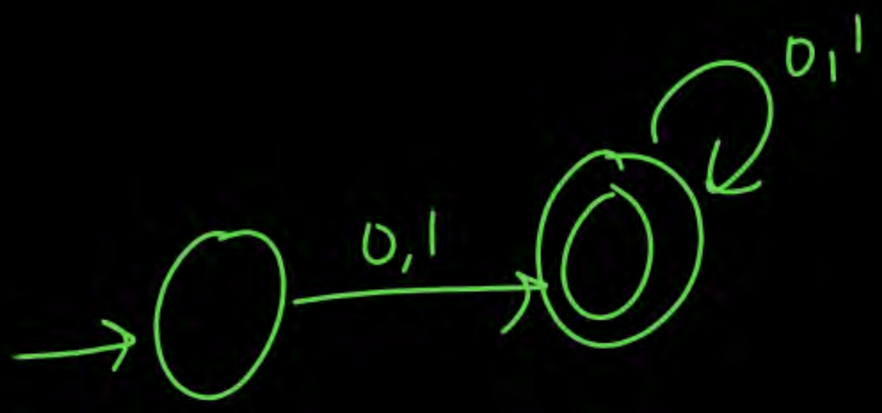
$\boxed{L = (a+b)^*}$

Q 25 The number of states in the minimum sized DFA that accepts the language defined by the regular expression $(0 + 1)^* (0 + 1) (0 + 1)^*$ is __2__. **[2016-Set2: 1 Mark]**

$x^* x x^* = x^+$

$= (0+1)^+$

$|w| \geq 1$



Q26

Consider the following two statements:

I. If all states of an $\boxed{\text{NFA}}$ are accepting (final) states then the language accepted by the NFA is $\boxed{\Sigma^*.}$ False

II. There exists a regular language $\boxed{A}$ such that for all language B, $A \cap B$ is regular. TRUE

[2016-Set2: 2 Marks]

Which one of the following is CORRECT

(A) Only I is true

(B) Only II is true

(C) Both I and II are true

(D) Both I and II are false

$\forall B \quad \phi \cap B \Rightarrow reg$

Q 27

If G is a grammar with productions more a's

$S \rightarrow \boxed{SaS} \mid aSb \mid bSa \mid SS \mid \in$

Where S is the start variable, then which one of the following strings is $\boxed{\text{not generated}}$ by G?

[2017-Set1: 2 Marks]

(A) abab $\in L(G)$

(B) aaab $\in L(G)$

(C) abbaa $\in L(G)$

(D) babba $\notin L(G)$

$$S \rightarrow SS \rightarrow SSbSa \rightarrow SaSbbSa$$

$$aSb \rightarrow aSaSb \rightarrow aSaSaSb$$

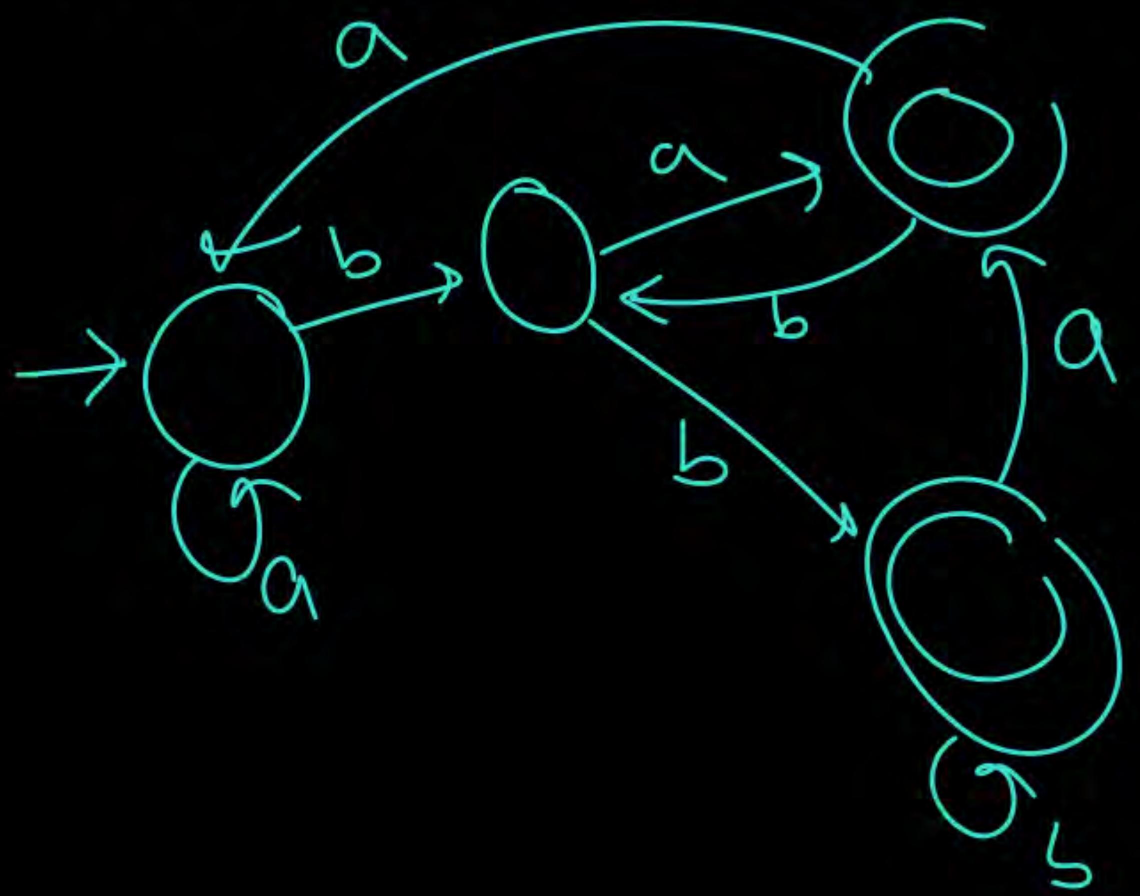
$$SaS \rightarrow SabSa \rightarrow SabbSaa$$

Q 28

Consider the language L given by the regular expression $(a + b)^*b(a + b)$ over the alphabet $\{a, b\}$. The smallest number of states needed in a deterministic finite-state automaton (DFA) accepting L is $= 4$.

baa

[2017-Set1: 2 Marks]



(k^{th}) 2^{nd} symbol from ending is 'b'

$\Downarrow$

$2^k = 2^2 = 4$ states

Q29

The minimum possible number of states of a deterministic finite automaton that accepts the regular language $L = \{w_1\, a w_2 \mid w_1, w_2 \in \{a, b\}^*, |w_1| = 2, |w_2| \geq 3\}$ is $\underline{= 8}$.

[2017-Set2: 1 Mark]

exactly 2 length — min 3 length

$\boxed{w_1}\; a\; \boxed{w_2}$

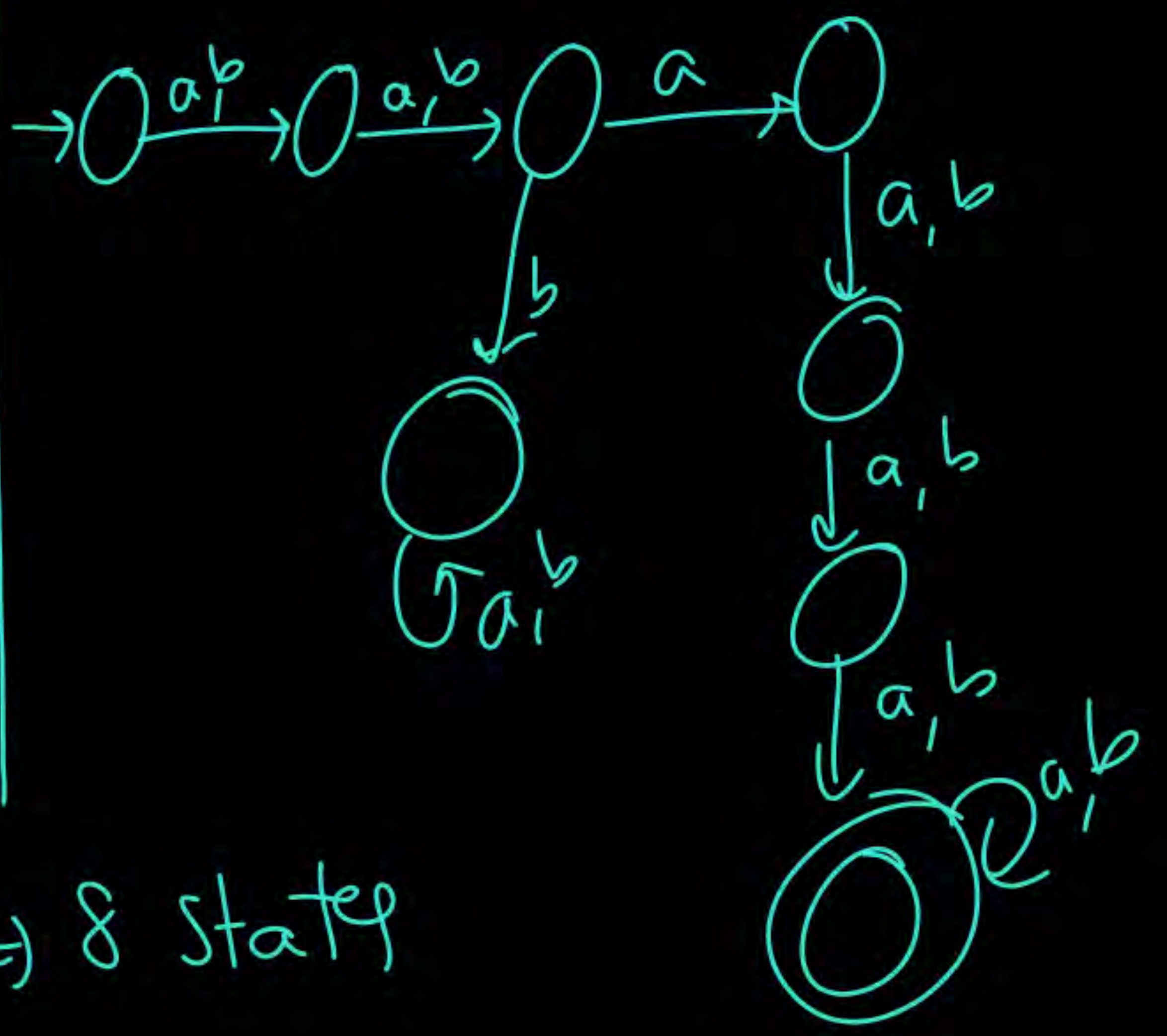
3rd pos

$\underbrace{\square\ \square\ a\ \square\ \square\ \square}$

to accept 6 length ⇒ 7 states

1 Dead

⇒ 8 states



Q 30

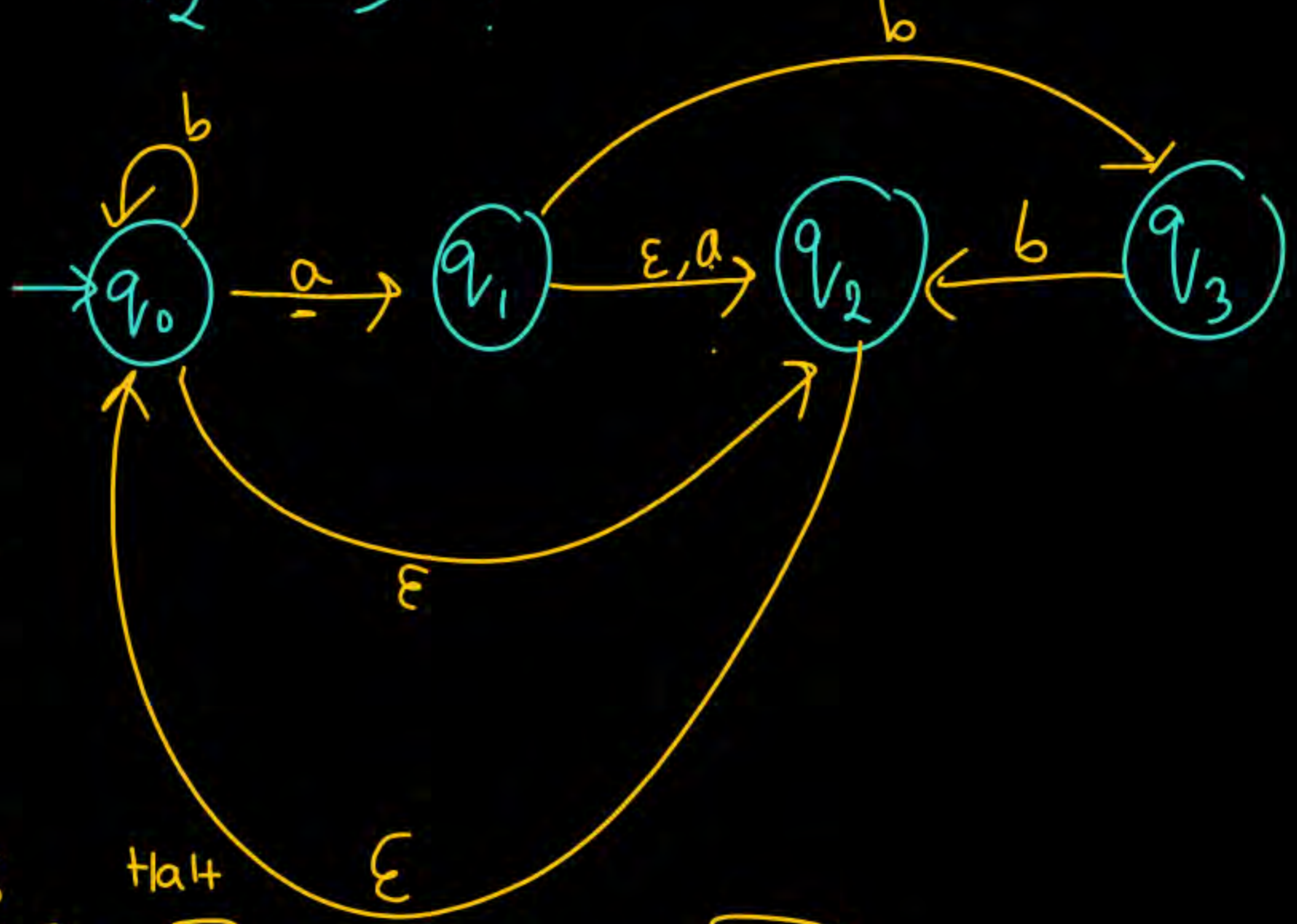
***

Let δ denote the transition function and $\hat{\delta}$ denote the extended transition function of the ε-NFA whose transition table is given below:

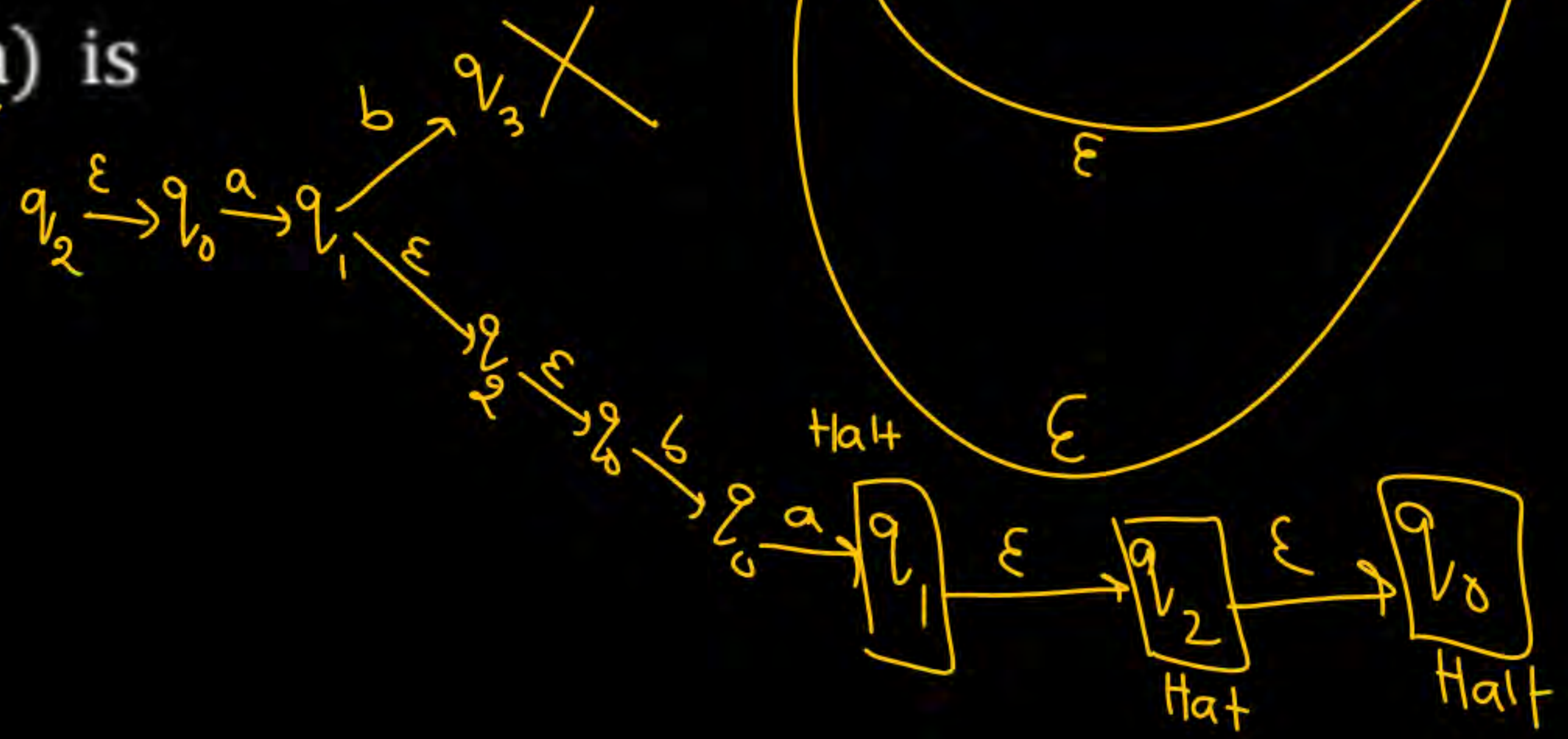
[2017-Set2: 2 Mark]

$q_2 \xrightarrow{aba} ?$

δ	ϵ	a	b
$\rightarrow q_0$	$\{q_2\}$	$\{q_1\}$	$\{q_0\}$
q_1	$\{q_2\}$	$\{q_2\}$	$\{q_3\}$
q_2	$\{q_0\}$	ϕ	ϕ
q_3	ϕ	ϕ	$\{q_2\}$



The $\hat{\delta}(q_2, aba)$ is



A. ϕ

B. $\{q_0, q_1, q_3\}$

C. $\{q_0, q_1, q_2\}$

D. $\{q_0, q_2, q_3\}$

Q 31

Let N be an NFA with n states. Let k be the number of states of a minimal DFA which is equivalent to N. Which one of the following is necessarily true? [2018: 1 Mark]

(A) $k \geq 2^n$

(B) $k \geq n$

(C) $k \leq n^2$

(D) $k \leq 2^n$

(N) NFA $\cong$ Min DFA

n states   K state

almost 2^n

$\boxed{\leq 2^n}$

NFA $\Rightarrow$ DFA

n $\leq 2^n$ state

K is $\leq 2^n$

Q 32

***

Algo & TOC

Given a language L, define L^i as follows:

$L^0 = \{\in\}$

$L^i = L^{i-1} \cdot L$ for all $i > 0$

The order of a language L is defined as the $\boxed{\text{smallest } k}$ such that $L^k = L^{k+1}$.

Consider the language L_1 (over alphabet 0) accepted by the following automaton

$L = \boxed{\varepsilon} + 0(00)^*$

$L^1 = L$

$L^2 = L^1 . L$

$= (\varepsilon + 0(00)^*)^2$

$= 0^*$

$L^3 = 0^* . L = 0^*$

$L^2 = L^3$

$O(L) = 2$

The order of L_1 is ____.

2

[2018: 2 Marks]

In Exam : $\rightarrow$ Do you need pen & paper for question ?

65 Q $\rightarrow$ min 10 to 15 Questions can be solved without pen & paper

Q 33

For $\Sigma = \{a, b\}$, let us consider the regular language $L = \{x \mid x = a^{2+3k} \text{ or } x = b^{10+12k}, k \geq 0\}$. Which one of the following can be a pumping length (the constant guaranteed by the pumping lemma) for L?

[2019: 1 Mark]

(A) 9

(B) 24

(C) 3

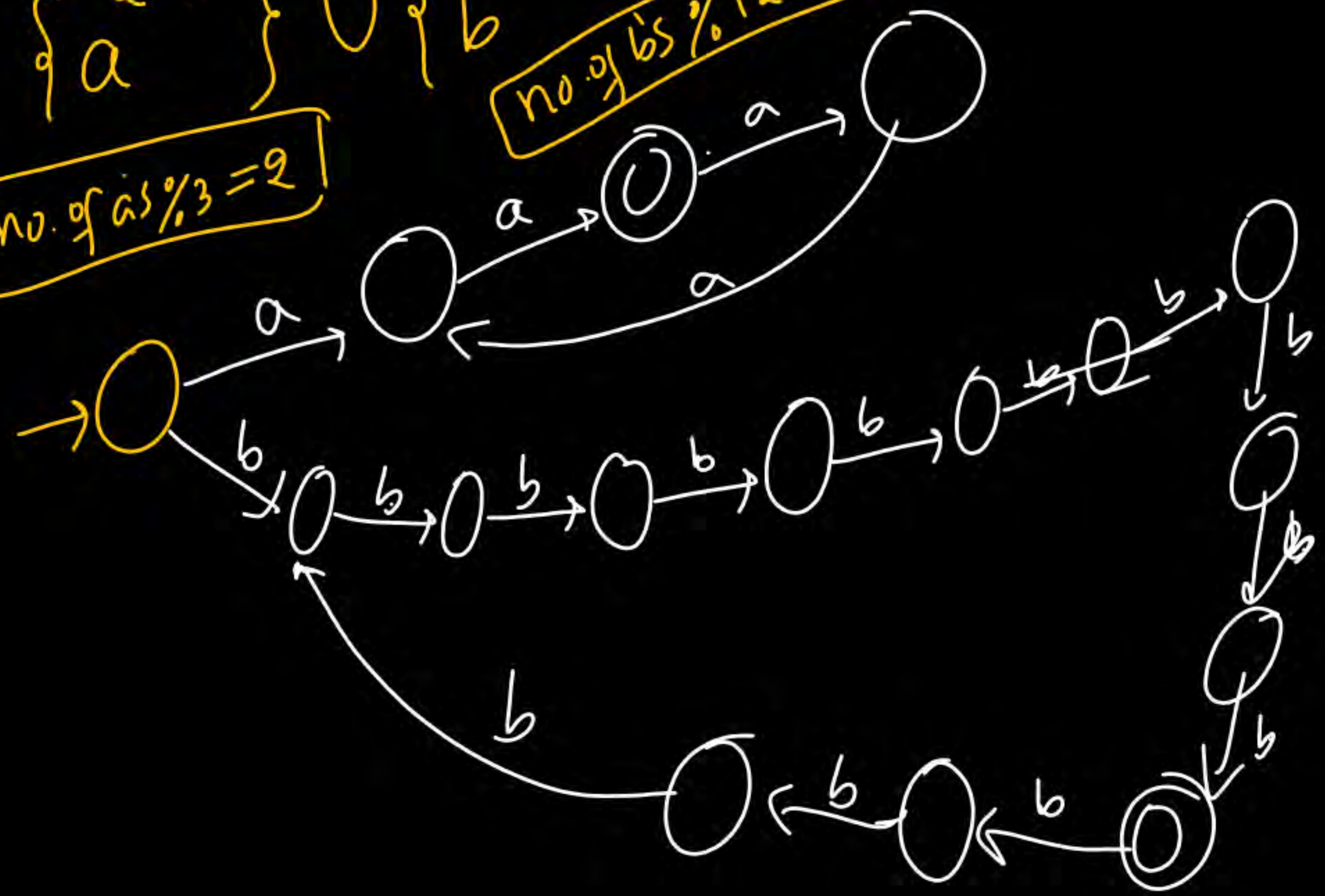
(D) 5

$L = \{a^{2+3k}\} \cup \{b^{10+12k}\}$

no. of a's % 3 = 2

no. of b's % 12 = 10

≥ 16



Q 34

If L is a regular language over $\Sigma = \{a, b\}$, which one of the following languages is NOT regular?

$\{\varepsilon\}$

[2019: 1 Mark]

A. $L \cdot L^R = \{xy \mid x \in L, y^R \in L\}$ → regular

B. Suffix (L) = $\{y \in \Sigma^* \mid \exists x \in \Sigma^*$ such that $xy \in L\}$ → reg

C. Prefix (L) = $\{x \in \Sigma^* \mid \exists y \in \Sigma^*$ such that $xy \in L\}$ → reg

D. $\{ww^R \mid w \in L\}$ → Always not regular

reg

→ Sometimes not regular

If $L = \{\varepsilon\}$ (reg)

$\{ww^R \mid w \in L\} = \{\varepsilon\}$ → reg

Q 35

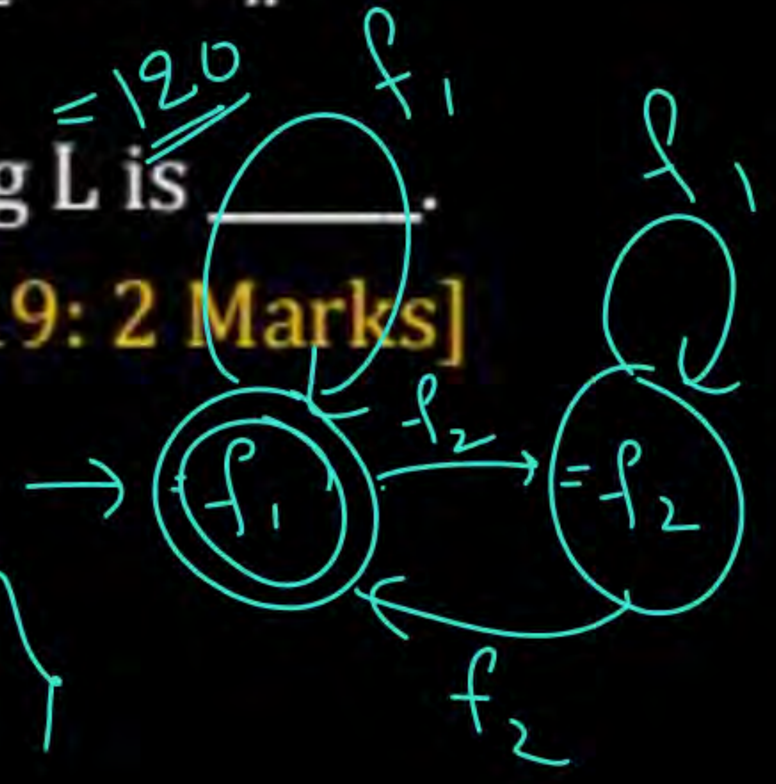
***

Maths & TOC

$f \circ I = f$

Let Σ be the set of all bijections from $\{1, \ldots, 5\}$ to $\{1, \ldots, 5\}$, where *id* denotes the identity function, i.e. $id(j) = j, \forall j$.
Let $\circ$ denote composition on functions.
For a string $x = x_1 x_2 \ldots x_n \in \Sigma^n$, $n \geq 0$, let $\pi(x) = x_1 \circ x_2 \circ \ldots \circ x_n$. → composition
Consider the language $L = \{x \in \Sigma^* \mid \pi(x) = id\}$.
The minimum number of states in any DFA accepting L is ____. $= 120$

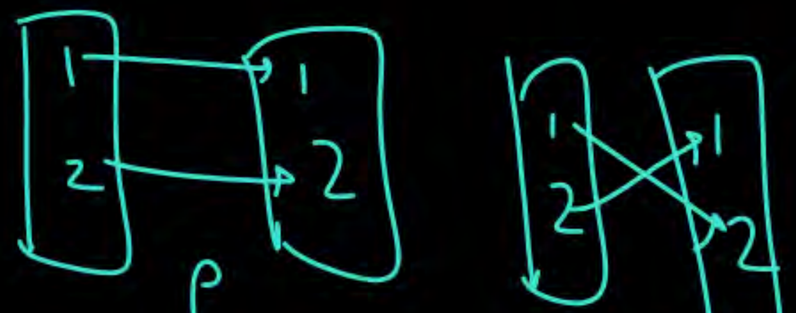
[2019: 2 Marks]



$f_1 \circ f_1 = f_1$

$f_1 \circ f_2 = f_2$

$\Sigma = \{f_1, f_2\} \Rightarrow \Sigma^* = \{\varepsilon, f_1, f_2, \underbrace{f_1 f_1}_{f_1}, \underbrace{f_1 f_2}_{f_2}, \underbrace{f_2 f_1}_{f_2}, \underbrace{f_2 f_2}_{f_1} \ldots\}$



f_1 f_2

only one identity

$L = \{\varepsilon, f_1, f_1 f_1, f_2 f_2, \ldots\}$

$f_1 f_1 f_1 = f_1$

$f_1 f_1 f_2 = f_2$

Q 36

Consider the following statements:

~~I.~~ If $L_1 \cup L_2$ is regular, then both L_1 and L_2 $\boxed{\text{must be}}$ regular.

~~II.~~ The class of regular languages is closed under infinite union.

Which of the above statements is/are TRUE?

(A) Neither I nor II

(B) II only

(C) I only

(D) Both I and II

[2020: 1 Mark]

If $L_1 \cup L_2$ is Reg then $\implies$ L_1 and L_2 are always reg

$$\underbrace{a^n b^n \cup \overline{a^n b^n} = (a+b)^*}$$

Q 37

Consider the following language:

L = {x∈{a, b}*| number of a's in x is divisible by 2 but not divisible by 3}

The minimum number of states in a DFA that accepts L is ____. $= 6$

[2020: 2 Marks]

#a's = ~~0~~, 2, 4, ~~6~~, 8, 10, ~~12~~, 14, 16, ~~18~~, ...

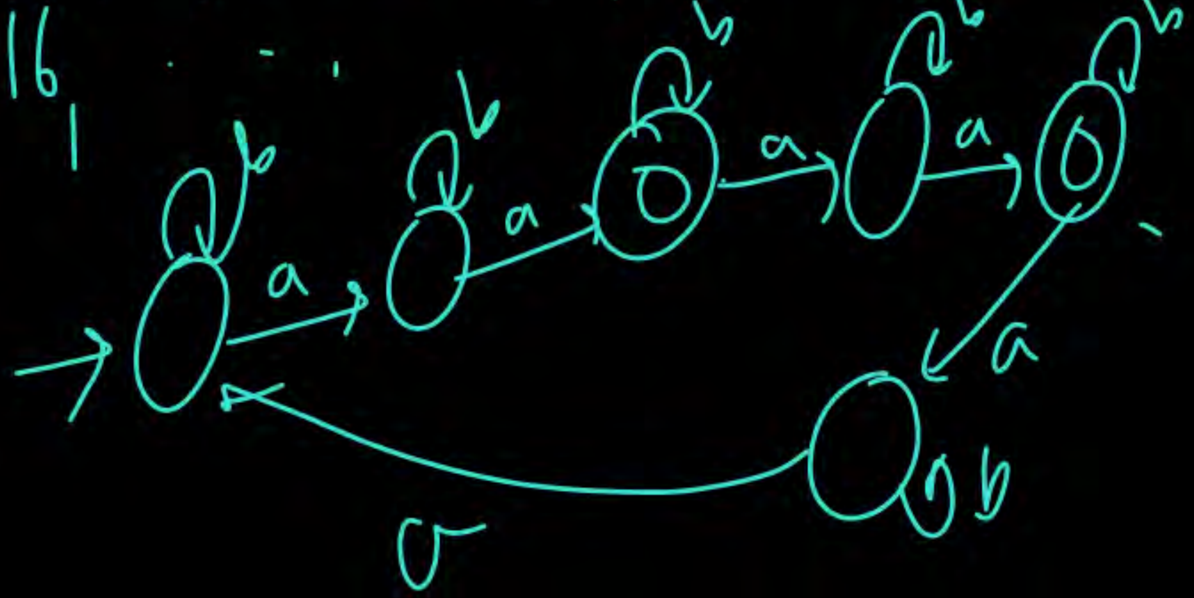
$L_1 = \{x \mid x \in \{a,b\}^*, n_a(x) \text{ is div by } 2\} = \{\cancel{\varepsilon}, \cancel{b}, aa, \cancel{bb}, \ldots\}$ ⇒ 2 states

#a's = 0, 3, 6, 9, 12, 15, ...

$L_2 = \{x \mid x \in \{a,b\}^*, n_a(x) \text{ is div by } 3\} = \{\varepsilon, b, bb, aaa, bbb, \ldots\}$ ⇒ 3 states

2, 4, 8, 10, 14, 16, ...

+2 +4 +2 +4



$L_1 - L_2 = \{aa,$

Q 38

Consider the following language:

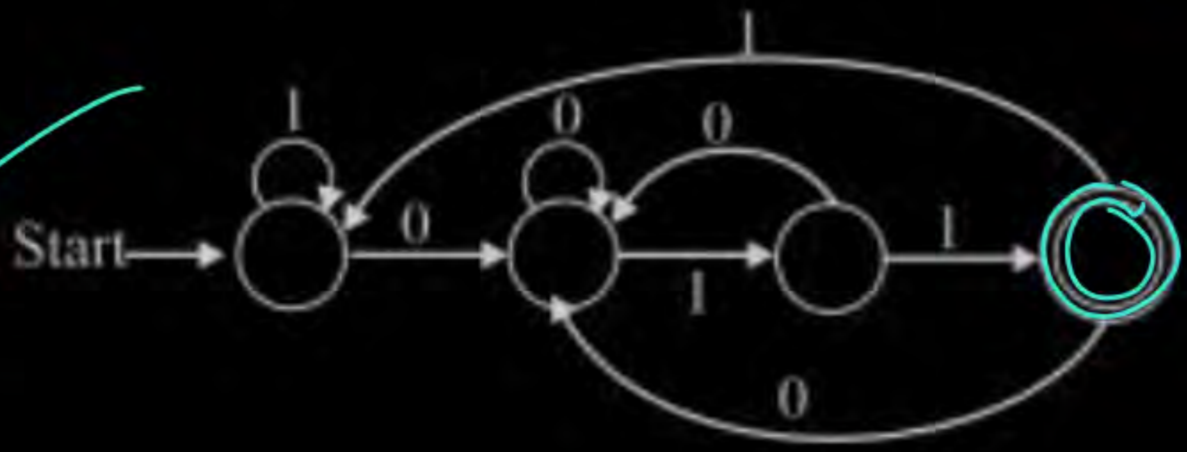
L = {w∈{0, 1}*| w ends with the substring 011}.

Which one of the following deterministic finite automata accepts L?

[2021-Set1: 2 Marks]

Look at last 3 symbols from final stat

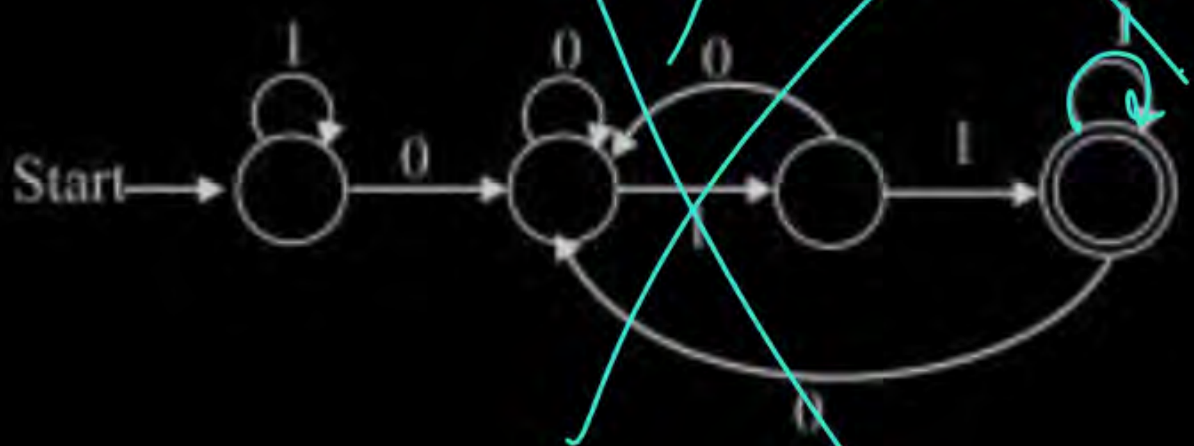
A



B

C

D



Q 39

Let $L \subseteq \{0, 1\}^*$ be an arbitrary regular language accepted by a minimal DFA with k states. Which one of the following languages must necessarily be accepted by a minimal DFA with k states?

[2021-Set2: 1 Marks]

(A) $\{0, 1\}^* - L$ $= \bar{L}$

(B) $L \cup \{01\}$

(C) $L \cdot L$

(D) $L - \{01\}$

L

k states in DFA

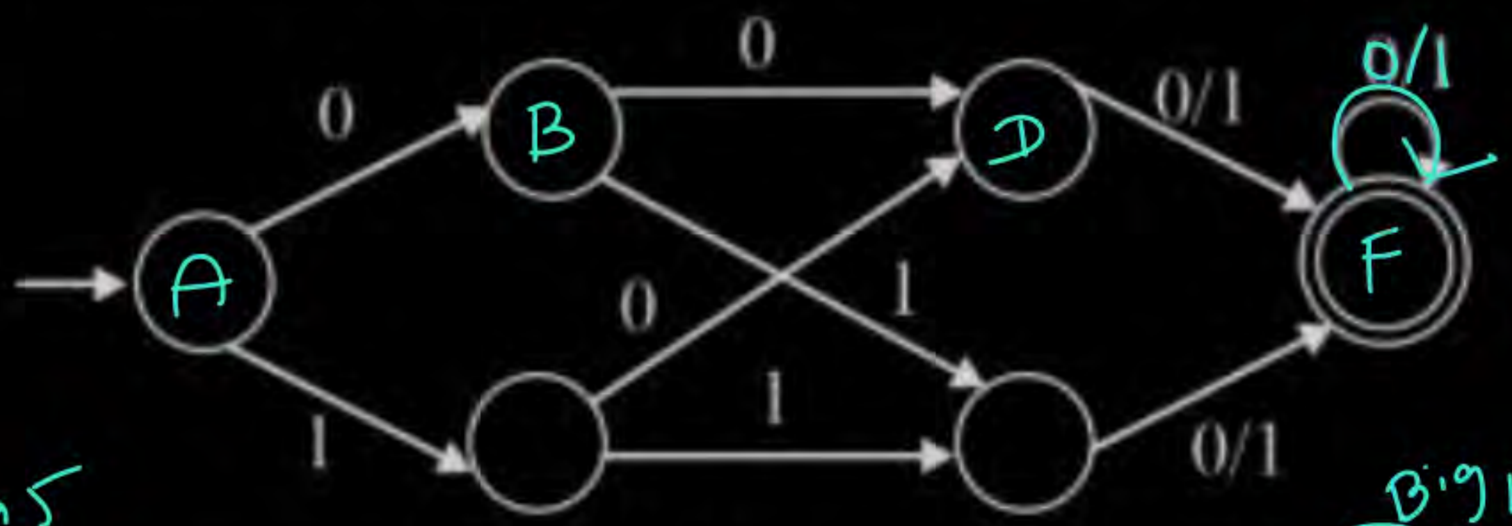
$\bar{L} = \Sigma^* - L$

$= \{0,1\}^* - L$

Q 40

***

Consider the following deterministic finite automaton (DFA).



Set of all 8 length strings = 256 strings

Big number

$2^8 = 256$ strings

How many accepted from 256 ?

5 → 5

5!

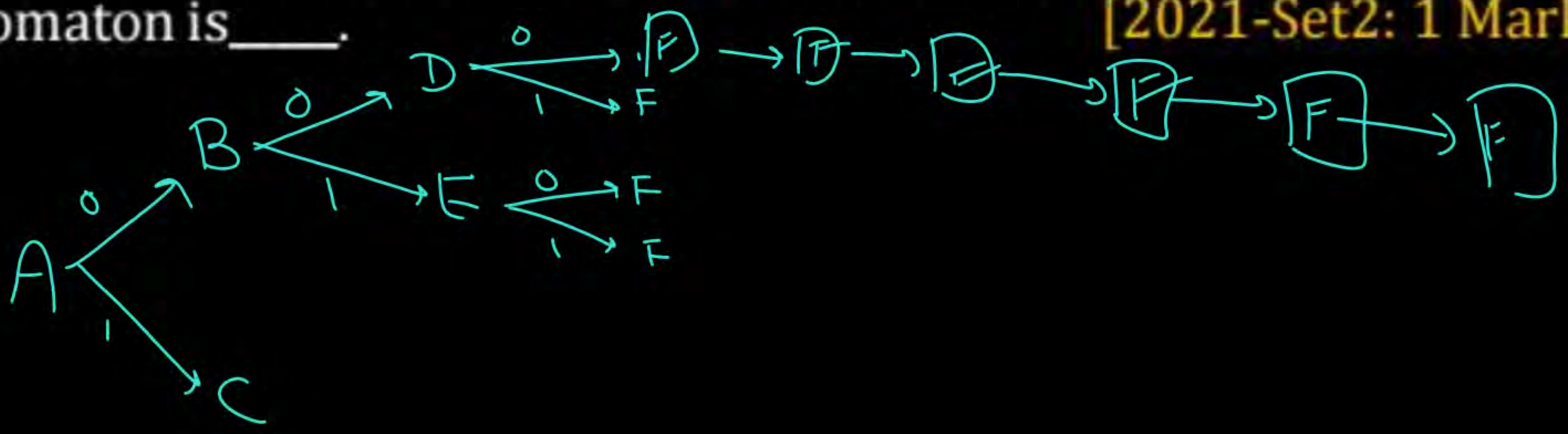
120

↓

120.

The number of strings of length 8 accepted by the above automaton is____.

[2021-Set2: 1 Marks]



Q 41

Suppose we want to design a synchronous circuit that processes a string of 0's and 1's. Given a string, it produces another string by replacing the first 1 in any subsequence of consecutive 1's by a 0. Consider the following example.

Input sequence: 00100011000011100

Output sequence: 00000001000001100

A *Mealy Machine* is a state machine where both the next state and the output are functions of the present state and the current input.

The above mentioned circuit can be designed as a two-state Mealy Machine. The states in the Mealy machine can be represented using Boolean values 0 and 1. We denote the current state, the next state, the next incoming bit, and the output bit of the Mealy machine by the variables s, t, b and y respectively.

Assume the initial state of the Mealy Machine is 0. What are the Boolean expressions corresponding to t and y in terms of s and b?

[2021(Set-2): 2 Marks]

Digital & TOC

TOC & Smalts

TOC & CD

TOC + Algo

11:20 PM Resume

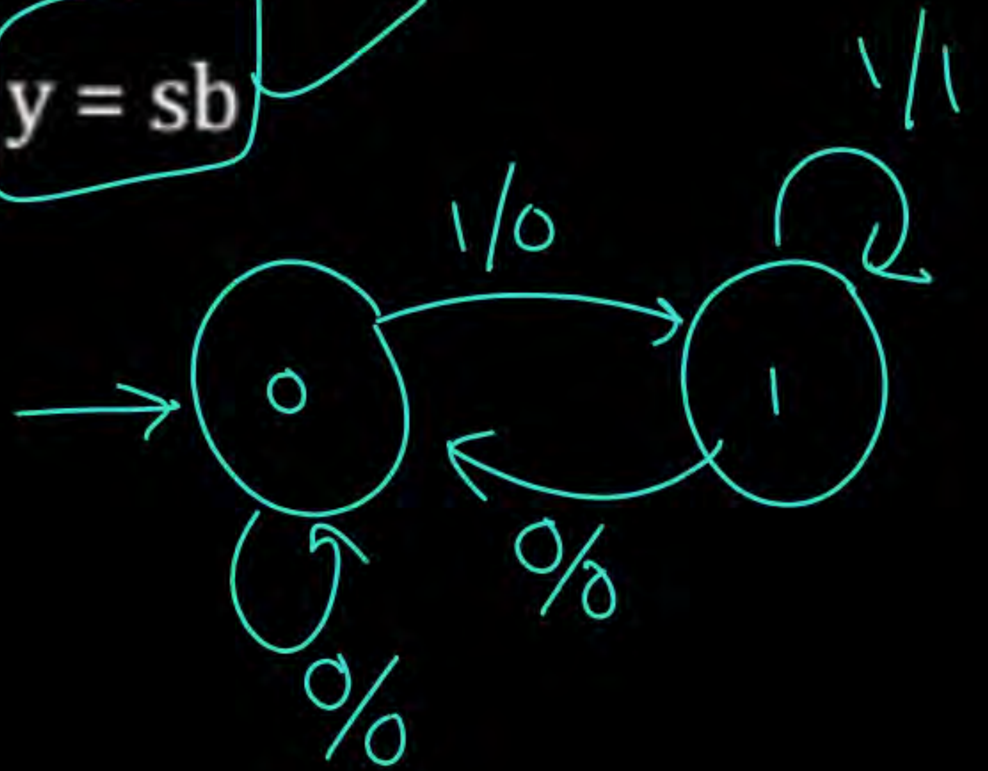
(A) $t = b, y = s\bar{b}$

(B) $t = s + b, y = sb$

(C) $t = s + b, y = s\bar{b}$

(D) $t = b, y = sb$

PS s	I/P b	O/P y	NS t
0	0	0	0
0	1	0	1
1	0	0	0
1	1	1	1



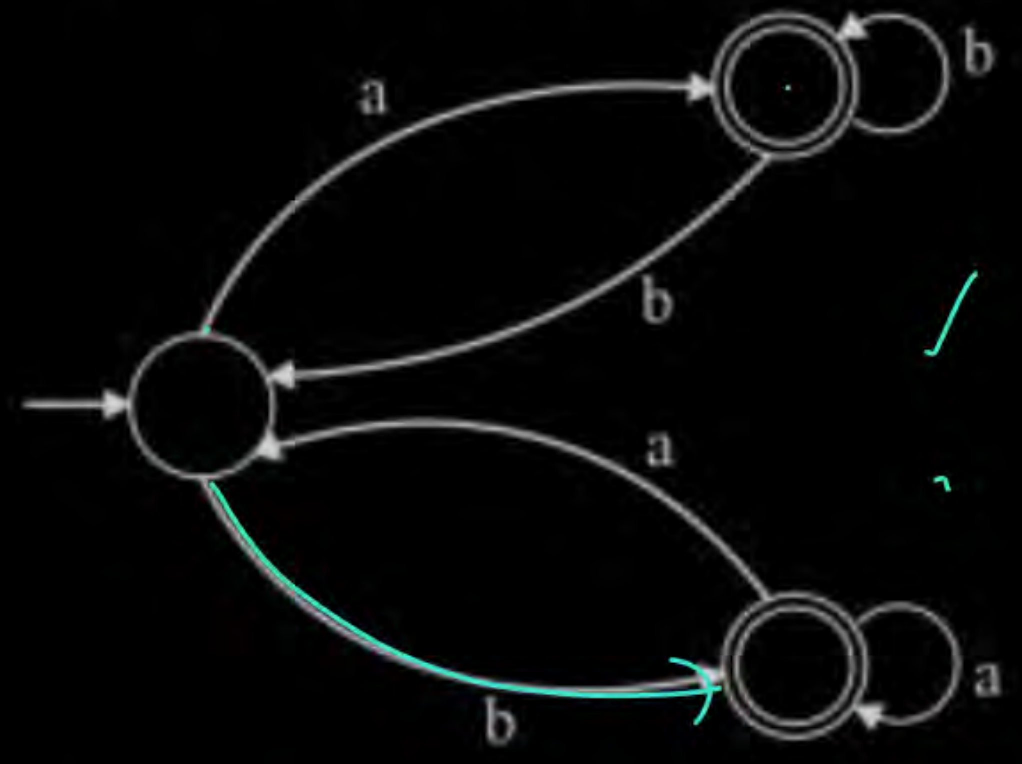
0 0 0 0 0 1 1 1 1 0

0 0 0 0 0 0 1 1 1 0

Q
42

Which one of the following regular expressions correctly represents the language of the finite automaton given below?

[2022: 1 Mark]



1st ⇓ Observe ≃ 1 min

2nd ⇓ options elimination ≃ 2-3 min

3rd ↓ General Approach (Using Algo) 3-10 min

(A) ab*bab* + ba*aba* → a ×

(B) (ab*b)*ab + (ba*a)*ba* → a ×

(C) (ab*b + ba*a)*(a* + b*) — generate aa ✓

(D) (ba*a + ab*b)*(ab* + ba*) — not aa ×

How to eliminate when we have only 2 options left

Context Free Grammar(DPDA, NPDA)

Q 43

Which of the following are decidable?

1. Whether the intersection of two regular languages is infinite IS $Reg_1 \cap Reg_2 = Inf$? $\Rightarrow$ IS $Reg = Inf$? $\Rightarrow$ Infiniteness

Algo exist

2. Whether a given context-free language is regular IS $\underbrace{CFL}_{given} = reg$?

3. Whether two push-down automata accept the same language IS $L(PDA_1) = L(PDA_2)$? $\Rightarrow$ Equivalence IS $L(PDA) = Reg$?

No algo

4. Whether a given $\underbrace{\text{grammar}}$ is $\underbrace{\text{context-free}}$

Algo exist IS $\underbrace{G}_{given}$ CFG ? $\boxed{LHS} \longrightarrow$ exactly ~~one~~ one non-t

[2008: 1 Marks]

A. 1 and 2

B. 1 and 4

C. 2 and 3

D. 2 and 4

Q44

Consider the language

$L1 = \{0^i1^j \mid i \neq j\}$, $\rightarrow$ DCFL

$L2 = \{0^i1^j \mid i = j\}$,

$L3 = \{0^i1^j \mid i = 2j + 1\}$, $= 0^{2j+1} 1^{j} \rightarrow$ DCFL

$L4 = \{0^i1^j \mid i \neq 2j\}$. $\rightarrow i < 2j$ or $i > 2j$

Which one of the following statements is true?

[2010: 2 Marks]

A. Only L2 is context free

B. Only L2 and L3 are context free

C. Only L1 and L2 are context free

D. All are context free

Q 45 Which one of the following grammars is free from *left recursion*?

[2010: 2 Marks]

(A)
$S \rightarrow AB$
$A \rightarrow Aa \mid b$
$B \rightarrow c$

(B)
$S \rightarrow Ab \mid Bb \mid c$
$A \rightarrow Bd \mid \varepsilon$
$B \rightarrow e$

(C)
$S \rightarrow Aa \mid B$
$A \rightarrow Bb \mid Sc \mid \varepsilon$
$B \rightarrow d$

(D)
$S \rightarrow Aa \mid Bb \mid c$
$A \rightarrow Bd \mid \varepsilon$
$B \rightarrow Ae \mid \varepsilon$

Q 46

Consider the languages L1, L2 and L3 as given below:

$L1 = \{0^p 1^q \mid p, q \in N\}$, → reg

$L2 = \{0^p 1^q \mid p, q \in N \text{ and } p = q\}$ and → DCFL

$L3 = \{0^p 1^q 0^r \mid p, q, r \in N \text{ and } p = q = r\}$ → CSL

$0^n 1^n 0^n$

Which of the following statements is NOT TRUE [2011: 2 Marks]

A. Push Down Automata (PDA) can be used to recognize L1 and L2. TRUE

B. L1 is a regular language. TRUE

C. All the three languages are context free. FALSE

D. Turing machines can be used to recognize all the languages. TRUE

Q.47

Consider the following languages:

I. $\{a^m b^n c^p d^q \mid m + p = n + q, \text{ where } m, n, p, q \geq 0\}$

II. $\{\boxed{a^m b^n}\, \boxed{c^p d^q} \mid m = n \text{ and } p = q, \text{ where } m, n, p, q \geq 0\}$

III. $\{a^m b^n c^p d^q \mid m = n = p \text{ and } p \neq q, \text{ where } m, n, p, q \geq 0\}$

IV. $\{\boxed{a^m b^n}\, c^p d^q \mid mn = p + q, \text{ where } m, n, p, q \geq 0\}$

Which of the language above are context-free?

CFL

DCFL

CFL

CSL

CSL

[2012: 2 Marks]

(A) I and IV only

(B) I and II only

(C) II and III only

(D) II and IV only

Q 48

Consider the following languages

$L1 = \{0^p 1^q 0^r \mid p, q, r \geq 0\}$ $= 0^* 1^* 0^* \longrightarrow$ reg

$L2 = \{0^p 1^q 0^r \mid p, q, r \geq 0, p \neq r\}$ $\longrightarrow$ DCFL

Which one of the following statements is FALSE?

[2013: 2 Marks]

A. L2 is context-free TRUE

B. $L1 \cap L2$ is context-free TRUE

C. Complement of L2 is recursive TRUE

D. Complement of L1 is context-free but not regular FALSE

Q 49

Consider the following languages over the alphabet $\Sigma = \{0, 1, c\}$:

$L_1 = \{0^n 1^n \mid n \geq 0\}$ → DCFL

$L_2 = \{wcw^r \mid w \in \{0, 1\}^*\}$ → DCFL

$\underbrace{w}_{\text{push}} c \underbrace{w^r}_{\text{match}}$

$L_3 = \{ww^r \mid w \in \{0, 1\}^*\}$ → CFL but not DCFL

Here, w^r is the reverse of the string w. Which of these languages are deterministic Context-free languages?

[2014-Set3: 2 Marks]

A. None of the languages

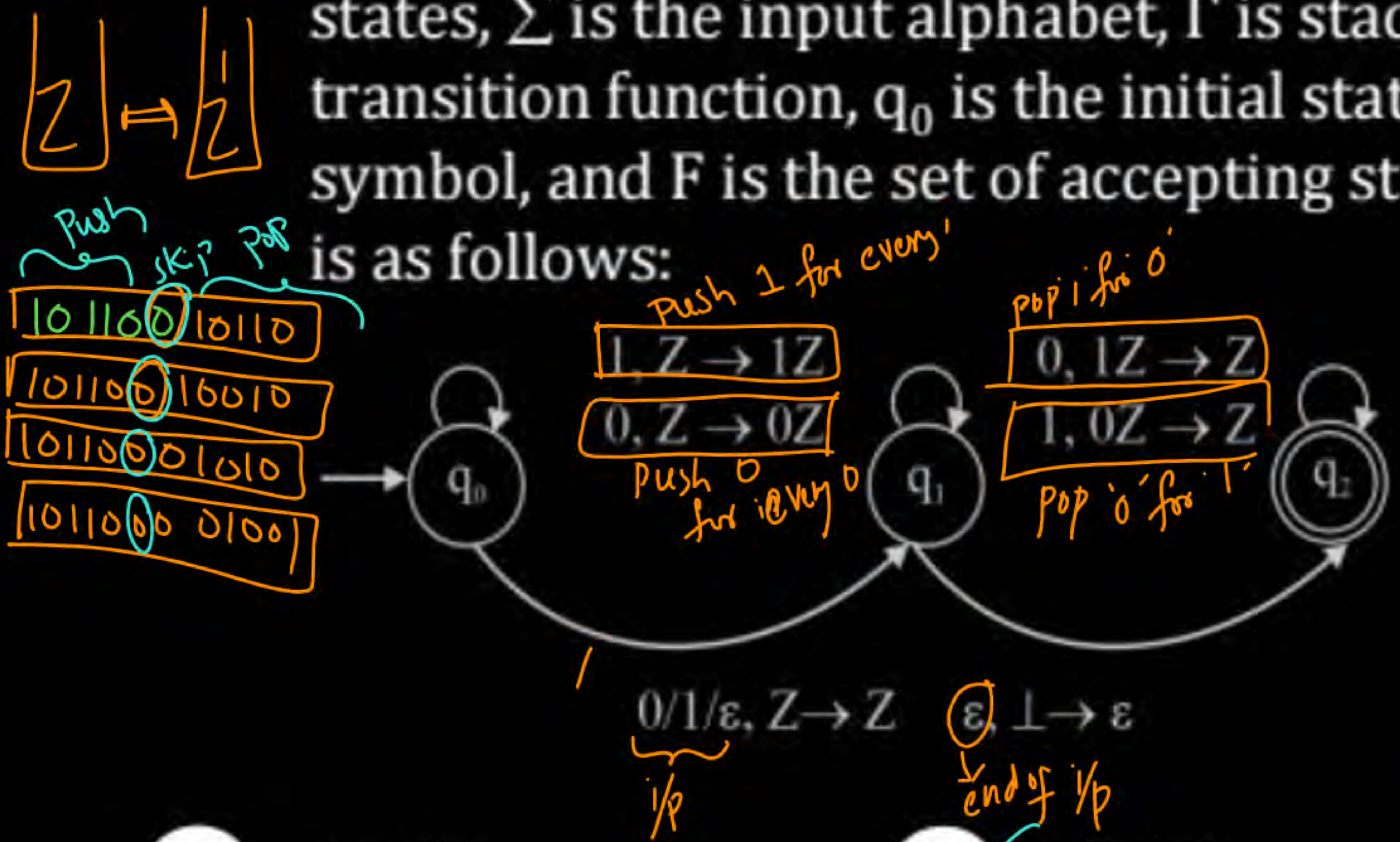
B. Only L_1

C. Only L_1 and L_2 ✓

D. All the three languages

Q 50

Consider the NPDA $\langle Q = \{q_0, q_1, q_2\}, \Sigma = \{0, 1\}, \Gamma = \{0, 1, \perp\}, \delta, q_0, \perp, F = \{q_2\} \rangle$, where (as per usual convention) Q is the set of states, Σ is the input alphabet, Γ is stack alphabet, δ is the state transition function, q_0 is the initial state, $\perp$ is the initial stack symbol, and F is the set of accepting states, The state transition is as follows:



Which of the following sequences must follow the string 101100 so that the overall string is accepted by the automaton?

[2015-Set1: 2 Marks]

(A) 10110

(B) 10010

(C) 01010

(D) 01001

$\varepsilon, -/-$

only at the end of i/p, it can

Q 51

Which of the following languages are context-free?

$L_1 = \{a^m b^n a^n b^m \mid m, n \geq 1\}$ → DCFL

$L_2 = \{a^m b^n a^m b^n \mid m, n \geq 1\}$ → not CFL

$L_3 = \{a^m b^n \mid m = 2n + 1\}$ → DCFL

[2015(Set-3): 1 Marks]

(A) L_1 and L_2 only

(B) L_1 and L_3 only

(C) L_2 and L_3 only

(D) L_3 only

$a^{2n+1} b^n = a\, a^{2n} b^n$

a → skip

a^{2n} → every 2 a's push 1 a

b^n → every b, pop 1 a

a a

skip push

Q 52

Consider the following context-free grammars:

$G_1: S \to aS \mid B,\ B \to b \mid bB$

$G_2: S \to aA \mid bB,\ A \to aA \mid B \mid \varepsilon,\ B \to bB \mid \varepsilon$

Which one of the following pairs of languages is generated by G_1 and G_2, respectively?

[2016(Set-1): 2 Marks]

(A) $\{a^m b^n \mid m > 0 \text{ or } n > 0\}$ and $\{a^m b^n \mid m > 0 \text{ and } n > 0\}$.

(B) $\{a^m b^n \mid m > 0 \text{ and } n > 0\}$ and $\{a^m b^n \mid m > 0 \text{ or } n \geq 0\}$.

(C) $\{a^m b^n \mid m \geq 0 \text{ or } n > 0\}$ and $\{a^m b^n \mid m > 0 \text{ and } n > 0\}$.

(D) $\{a^m b^n \mid m \geq 0 \text{ and } n > 0\}$ and $\{a^m b^n \mid m > 0 \text{ or } n > 0\}$.

$G_1: S \to aS \mid B$ ($B \to b^+$)

$B \to b \mid bB \to b^+$

$L_1 = a^* b^+$

$= a^+ b^+$

$\Downarrow$ $a^* b^+$

$a^+ b^* + a^* b^+$

$a^* b^+ \to b^+,\ a^+ b^*$

$G_2: S \to aA \mid bB$ ($\to a a^* b^*$, $\to b b^*$)

$A \to aA \mid B \mid \varepsilon \Rightarrow a^* b^*$

$B \to bB \mid \varepsilon \Rightarrow b^*$

$L_2 = a a^* b^* + b b^*$

$= a^+ b^* + b^+$

Q 53

Consider the transition diagram of a PDA given below with input alphabet $\Sigma = \{a, b\}$ and stack alphabet $\Gamma = \{X, Z\}$. Z is the initial stack symbol. Let L denote the language accepted by the PDA.

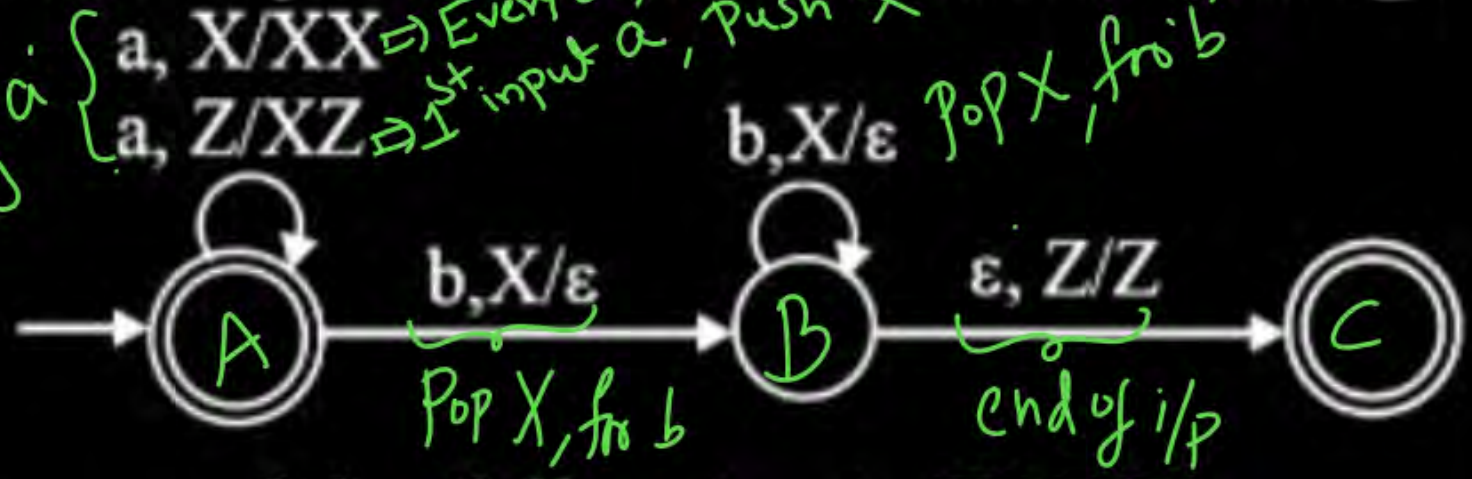
$\bot, Z_0$

$A = a^*$ ✓

$C = \{a^n b^n \mid n > 0\}$

$A \cup C = a^* \cup \{a^n b^n\} = a^*$

Push X for every 'a'



Which one of the following is TRUE?

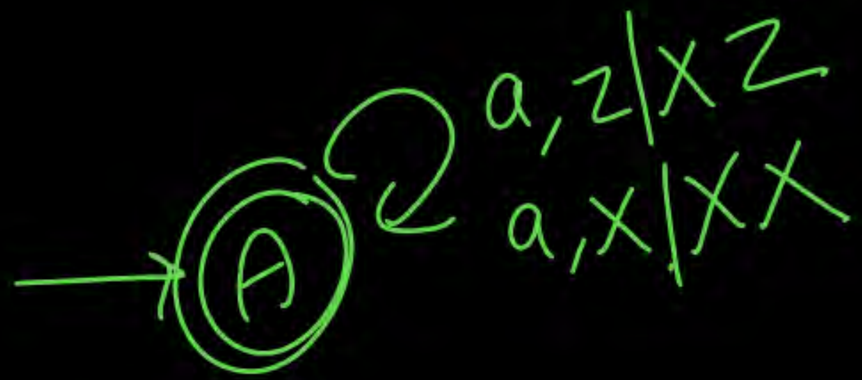
[2016(Set-1): 2 Marks]

(A) $L = \{a^n b^n \mid n \geq 0\}$ and is not accepted by any finite automata.

(B) $L = \{a^n \mid n \geq 0\} \cup \{a^n b^n \mid n \geq 0\}$ and is not accepted by any deterministic PDA.

(C) L is not accepted by any Turing machine that halts on every input.

(D) $L = \{a^n \mid n \geq 0\} \cup \{a^n b^n \mid n \geq 0\}$ and is deterministic context-free.



$L = a^*$

ε : A

a : $A \xrightarrow{a} A$ Halts at final

aa : $A \xrightarrow{a} A \xrightarrow{a} A$ Halt

Q 54

Consider the following languages:

$L_1 = \{a^n b^m c^{n+m} : m, n \geq 1\}$ → DCFL

$L_2 = \{a^n b^n c^{2n} : n \geq 1\}$ → CSL

Which one of the following is TRUE?

$a^n b^m c^{n+m}$

$X^n \; X^m$ Pop $n+m$ X's

[2016(Set-2): 2 Marks]

A. Both L_1 and L_2 are context-free.

B. L_1 is context-free while L_2 is not context-free

C. L_2 is context-free while L_1 is not context-free

D. Neither L_1 nor L_2 is context-free

$a^n b^n c^n$

$a^n b^{n+1} c^{n+2}$

$a^n b^{2n} c^{3n}$

$a^n b^{n^2} c^{n^3}$

not CFLs

Q. 55

Language L_1 is defined by the grammar: $S_1 \rightarrow aS_1b \mid \varepsilon$ $\Rightarrow a^n b^n$

Language L_2 is defined by the grammar: $S_2 \rightarrow abS_2 \mid \varepsilon$ $\Rightarrow (ab)^*$

Consider the following statements:

P: L_1 is regular

Q: L_2 is regular

Which one of the following is TRUE?

[2016(Set-2): 1 Marks]

(A) Both P and Q are true

(B) P is true and Q is false

(C) P is false and Q is true

(D) Both P and Q are false

Q 56

Consider the following context-free grammar over the alphabet $\Sigma = \{a, b, c\}$ with S as the start symbol

$S \rightarrow abScT \mid abcT$

$\boxed{T \rightarrow bT \mid b} \Rightarrow b^+$

$S \rightarrow abScb^+ \mid abcb^+ \Rightarrow L = (ab)^n abcb^+ (cb^+)^n \mid n \geq 0$

$= p^n p q q^n \Rightarrow p^n q^n, \; n \geq 1, \; n > 0$

Which one of the following represents the language generated by the above grammar?

$S \rightarrow xSy \mid z$

$\boxed{x^n z y^n}$

[2017(Set-1): 1 Marks]

(A) $\{(ab)^n (cb)^n \mid n \geq 1\}$

(B) $\{(ab)^n \underbrace{cb^{m_1} cb^{m_2} \ldots cb^{m_n}} \mid n, m_1, m_2, \ldots, m_n \geq 1\}$

$L = \{(ab)^n (cb^+)^n \mid n \geq 1\}$

(C) $\{(ab)^n (cb^m)^n \mid m, n \geq 1\}$ → $n=10$, $m=5$

$(ab)^{10} (cb^5)^{10}$

$cb^5 cb^5 cb^5 cb^5 \ldots$ 10 times

$n=10$: $(ab)^{10} (cb^+)^{10}$

$cb\,cb^2\,cb^3 \ldots cb^{100}$

(D) $\{(ab)^n (cb^n)^m \mid m, n \geq 1\}$

$$cb^+ = \{cb^m \mid m \geq 1\}$$

$$(cb^+)^n \neq \left\{ (cb^m)^n \;\middle|\; \begin{matrix} m \geq 1 \\ n \geq 1 \end{matrix} \right\}$$

$$= \left(\{cb^m \mid m \geq 1\}^n \;\middle|\; n \geq 1 \right)$$

$$\{cb^+\}^n$$

Q 57

Consider the following language over the alphabet $\Sigma = \{a, b, c\}$.

Let $L_1 = \{a^n b^n c^m \mid m, n \geq 0\}$ and $\rightarrow a^n b^n c^* \rightarrow$ DCFL

$L_2 = \{a^m b^n c^n \mid m, n \geq 0\}$. $\rightarrow a^* b^n c^n \rightarrow$ DCFL

Which of the following are context-free languages?

I. $L_1 \cup L_2 \rightarrow$ CFL ✓

II. $L_1 \cap L_2$ $= a^n b^n c^* \cap a^* b^n c^n = a^n b^n c^n$ not CFL

[2017(Set-1): 2 Marks]

(A) I only

(B) II only

(C) I and II

(D) Neither I nor II

Q58

Consider the context-free grammars over the alphabet $\{a, b, c\}$ given below. S and T are non-terminals.

$\boxed{G_1: S \rightarrow aSb \mid T, \boxed{T \rightarrow cT \mid \epsilon}} \rightarrow c^*$ $\Longrightarrow \{a^n c^* b^n \mid n \geq 0\}$

$G_2: S \rightarrow bSa \mid T, T \rightarrow cT \mid \epsilon \rightarrow c^*$ $\Longrightarrow \{b^n c^* a^n \mid n \geq 0\}$

The language $L(G_1) \cap L(G_2)$ is

[2017-Set1: 1 Mark]

(A) ~~Finite~~

(B) Not finite but regular ✓

(C) Context-free but not regular ✗

(D) Recursive but not context-free ✗

$a^n c^* b^n$

$b^n c^* a^n$

c^*

$n=0$ in both $\Rightarrow c^*$

$n>0$ in both $\Rightarrow \phi$

Q 59 Identify the language generated by the following grammar, where S is the start variable.

$S \to XY$

$X \to aX \mid a \longrightarrow a^+$

$Y \to aYb \mid \epsilon \longrightarrow a^n b^n$

$a^+ . a^n b^n \mid n \geq 0$

$\{a^m b^n \mid m > n\}$

[2017(Set-2): 1 Marks]

$a^* a^n b^n$

$a^m b^n \mid m \geq n$

$a^+ a^n b^n$

$a^m b^n \mid m > n$

(A) $\{a^m b^n \mid m \geq n, n > 0\}$

(B) $\{a^m b^n \mid m \geq n, n \geq 0\}$

(C) $\{a^m b^n \mid m > n, n \geq 0\}$

(D) $\{a^m b^n \mid m > n, n > 0\}$

Q.60 Let L_1, L_2 be any two context-free languages and R be any regular language. Then which of the following is/are CORRECT?

I. $L_1 \cup L_2$ is context-free ✓

II. $\overline{L_1}$ is context-free ✗

III. $L_1 - R$ is context-free ✓

IV. $L_1 \cap L_2$ is context-free ✗

[2017(Set-2): 1 Marks]

CFL ∪ reg
CFL ∩ reg
CFL − reg
CFL / reg
} = CFL

$CFL - Reg = CFL \cap \overline{Reg}$
$= CFL \cap Reg$
$= CFL$

(A) I, II and IV only

(B) I and III only

(C) II and IV only

(D) I only

$CFL \cup CSL \Rightarrow CSL$

$CFL \cup Reg \Rightarrow CFL$

$CFL \cup DCFL \Rightarrow CFL$

$CFL \cup Fin \Rightarrow CFL$

$CFL \cup Inf \Rightarrow Inf$

$CFL \cup Rec \Rightarrow Rec$

$CFL \cup REL \Rightarrow REL$

$CFL \cup \underbrace{L}_{\text{unknown}} \Rightarrow \text{Unknown}$

Q 61

Consider the following languages:

$L_1 = \{a^p \mid p \text{ is a prime number}\}$ → CSL

$L_2 = \{a^n b^m c^{2m} \mid n \geq 0, m \geq 0\}$ → DCFL

$L_3 = \{a^n b^n c^{2n} \mid n \geq 0\}$ → CSL

$L_4 = \{a^n b^n \mid n \geq 1\}$ → DCFL

Which of the following are CORRECT?

I. ~~L_1 is context-free but not regular.~~

II. ~~L_2 is not context-free.~~

III. L_3 is not context-free but recursive. ✓

IV. L_4 is deterministic context-free. ✓

[2017(Set-2): 2 Marks]

(A) I, II and IV only

(B) II and III only

(C) I and IV only

(D) III and IV only ✓

Q 62 Which one of the following languages over $\Sigma = \{a, b\}$ is NOT context-free?

[2019: 2 Marks]

A) $\{a^n b^i \mid i \in \{n, 3n, 5n\}, n \geq 0\}$ → CFL $= a^n b^i \mid i = n$ OR $i = 3n$ OR $i = 5n$

B) $\{w a^n w^R b^n \mid w \in \{a, b\}^*, n \geq 0\}$ → not CFL

C) $\{w w^R \mid w \in \{a, b\}^*\}$ → CFL

D) $\{w a^n b^n w^R \mid w \in \{a, b\}^*, n \geq 0\}$ → CFL

$S \to A \mid B \mid C$

$A \to aAb \mid \varepsilon$

$B \to aBbbb \mid \varepsilon$

$C \to aBbbbbb \mid \varepsilon$

Q.63 Consider the language $L = \{a^n \mid n \geq 0\} \cup \{a^n b^n \mid n \geq 0\}$ and the following statements.

I. L is deterministic context-free → TRUE

II. L is context-free but not deterministic context-free → FALSE

III. $\boxed{\text{L is not LL}(k) \text{ for any } k}$ TRUE

Which of the above statements is/are TRUE?

[2020: 1 Marks]

(A) I and III only

(B) III only

(C) I only

(D) II only

LL(1)
LL(2)
LL(3)
LL(k)

You always have common prefix

$S \to A \mid B$

$A \to aA \mid \varepsilon$

$B \to aBb \mid \varepsilon$

$a^* \cup a^n b^n \Rightarrow$ no LL(1) grammar

DCFL

common prefix

Q 64

Consider the following languages:

$L_1 = \{w\boxed{x}y\boxed{x} \mid w, x, y \in (0 + 1)^+\}$ put min x $= w0y0 + w1y1$ $= (0+1)^+0(0+1)^+0 + w1y1$

$\boxed{L_2 = \{xy \mid x, y \in (a + b)^*, |x| = |y|, x \neq y\}}$ → CFL but not DCFL

reg CFL

$= \{ab, ba, \ldots\}$ even length but not in ww form

Which of the following is TRUE

[2020: 2 Marks]

$\{ww \mid w \in \{a,b\}^*\}$ ⇓ CFL ✓

All odd length

$\{ww \mid w \in \{a,b\}^*\}$ → not CFL

(A) L_1 is regular and L_2 is context-free. ✓

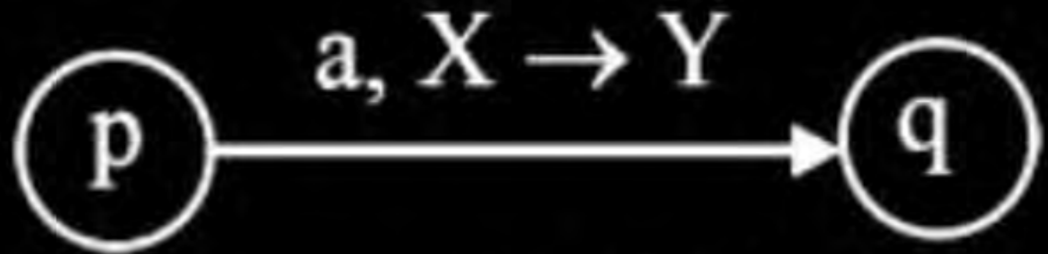
(B) L_1 is context-free but L_2 is not context-free.

(C) Neither L_1 nor L_2 is context-free.

(D) L_1 is context-free but not regular and L_2 context-free.

Q 65

In a pushdown automaton $P = (Q, \Sigma, \Gamma, \delta, q_0, F)$, a transition of the form,

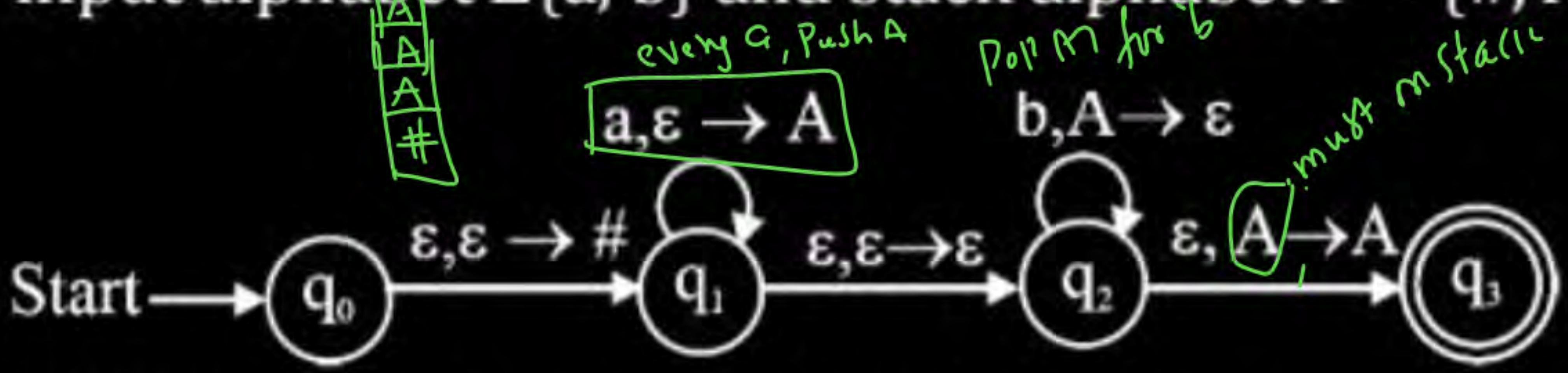


Where $p, q, \in Q$, $a \in \Sigma \cup \{\epsilon\}$, and $X, Y \in \Gamma \cup \{\epsilon\}$ represents

$$(q, Y) \in \delta(p, a, X)$$

$\delta(p, a, X) = (q, Y)$

Consider the following pushdown automaton over the input alphabet $\Sigma\{a, b\}$ and stack alphabet $\Gamma = \{\#, A\}$.



The number of strings of length 100 accepted by the above pushdown automaton is ______. $=50$

5x

$\{a^{100}, a^{99}b^{1}, \ldots, a^{51}b^{49}\}$ 50 strings having 100 length

$\{a^m b^n \mid m > n\}$

m	n
100	0
99	1
98	2
97	3
⋮	⋮
51	49
50	50 ✗

~~$a^{50}b^{50}$ $m=n$~~

[2021(Set-1): 2 Marks]

Q 66

Suppose that L_1 is a regular language and L_2 is a context-free language. Which one of the following languages is NOT necessarily context-free?

[2021(Set-1): 2 Marks]

$S \to A \cdot B$
reg . CFL

$S \to A \mid B$
reg ∪ CFL

(A) $L_1 \cdot L_2$ ⇒ CFL

(B) $L_1 \cup L_2$ ⇒ CFL

(C) $L_1 - L_2$

(D) $L_1 \cap L_2$ -

Reg − CFL $= a^*b^*c^* - \overline{a^nb^nc^n}$ → CFL

reg ∩ CFL ⇒ CFL

Reg ∩ $\overline{\text{CFL}}$ $a^*b^*c^* \cap \boxed{a^nb^nc^n}$ → not CFL

Reg ∩ CSL ⇒ CSL $a^*b^*c^* \cap a^nb^nc^n = \underbrace{a^nb^nc^n}_{\text{not CFL}}$

$\overline{a^nb^nc^n}$ → CFL

CFL − Reg ⇒ CFL

Reg − CFL ⇒ CSL

Q 67 For a string w, we define w^R to be the reverse of w. For example, if w = 01101 then w^R = 10110. Which of the following languages is/are context-free?

[2021(Set-2): 2 Marks]

(A) $\{wxw^Rx^R \mid w, x \in \{0, 1\}^*\}$

(B) $\{wxw^R \mid w, x \in \{0, 1\}^*\}$ → Reg $= (0+1)^*$

(C) $\{ww^Rxx^R \mid w, x \in \{0, 1\}^*\}$ → CFL

(D) $\{wxx^Rw^R \mid w, x \in \{0, 1\}^*\}$ → CFL

wAw^R

$$S \to aSa \mid bSb \mid A$$
$$A \to aAa \mid bAb \mid \epsilon$$

Q 68

Let L_1 be a regular language and L_2 be a context-free language. Which of the following languages is/are context-free?

[2021(Set-2)MSQ: 1 Marks]

(A) $L_1 \cap \overline{L_2} = R \cap \overline{C} = R \cap CSL \Rightarrow CSL$

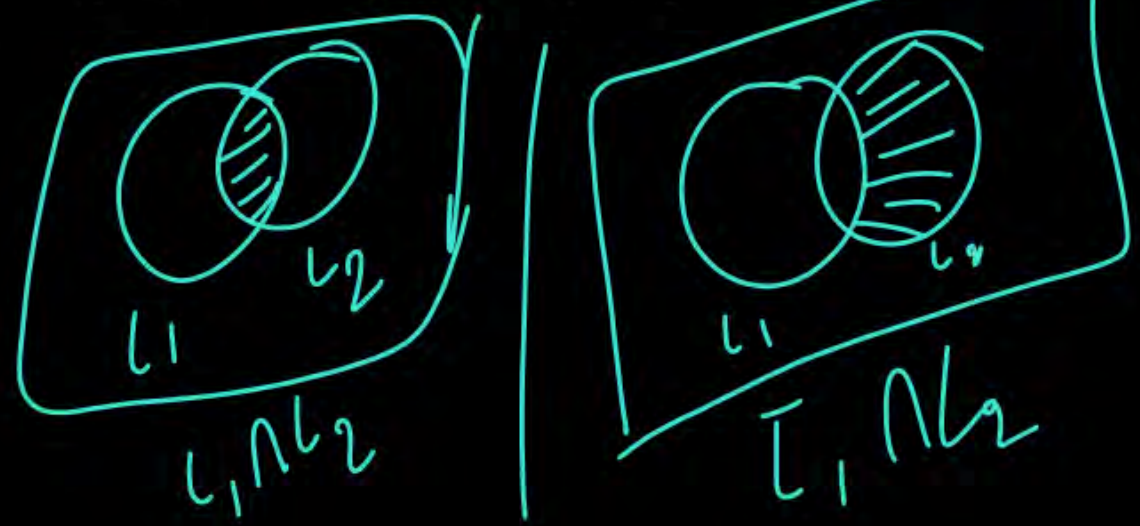
(B) $\overline{\overline{L_1} \cup \overline{L_2}} = L_1 \cap L_2 = \text{Reg} \cap CFL \Rightarrow CFL$

why D is CFL ?

(C) $L_1 \cup \underbrace{(L_2 \cup \overline{L_2})}_{\Sigma^*} = \Sigma^*$

$L_1 \cup \Sigma^*$

(D) $(L_1 \cap L_2) \cup (\overline{L}_1 \cap L_2) = L_2$



$= L_2 \Rightarrow CFL$

Q.69

Consider the following languages:

$L_1 = \{a^n w a^n \mid w \in \{a, b\}^*\}$ $\xrightarrow{\text{put } n=0}$ $L_1 = w = (a+b)^* \rightarrow$ reg

$L_2 = \{wxw^R \mid w, x \in \{a, b\}^*\}, |w|, |x| > 0\}$ $\rightarrow$ Reg

Note that w^R is the reversal of the string w. Which of the following is/are TRUE?

[2022: MSQ: 2 Marks]

(A) L_1 and L_2 are regular.

(B) L_1 and L_2 are context-free.

(C) L_1 is regular and L_2 is context-free.

(D) L_1 and L_2 are context-free but not regular.

Q. 70

Consider the following languages:

$L_1 = \{ww \mid w \in \{a, b\}^*\}$ → not CFL

$L_2 = \{a^n b^n c^m \mid m, n \geq 0\}$ → DCFL

$L_3 = \{a^m b^n c^n \mid m, n \geq 0\}$ → DCFL

$L_2 \cap L_3 = a^n b^n c^n$

Which of the following statements is/are FALSE?

BCD

[2022: 2 Marks]

(A) TRUE L_1 is not context-free but L_2 and L_3 are deterministic context-free.

(B) Neither L_1 nor L_2 is context-free. FALSE

(C) L_2, L_3 and $L_2 \cap L_3$ all are context-free. FALSE

(D) Neither L_1 nor its complement is context-free FALSE

Turing Machine

Q 71 Which of the following pairs have $\boxed{\text{DIFFERENT}}$ expressive power?

[2008: 1 Marks]

A. Deterministic finite automata (DFA) and Non-deterministic finite automata (NFA) $\equiv$

$L_s(DPDA) \subset L_s(PDA)$

$DPDA < NPDA$

B. Deterministic push down automata (DPDA) and Non-deterministic push down automata (NPDA) $\not\equiv$

$DPDA \not\equiv NPDA$

C. Deterministic single-tape Turing machine and Non-deterministic single-tape Turing machine $\equiv$

D. Single-tape Turing machine and multi-tape Turing machine $\equiv$

Q.72

Which of the following statements is/are FALSE?

TRUE 1. For every non-deterministic Turing machine, there exists an equivalent deterministic Turing machine. NTM ⇒ DTM

FALSE 2. $\underbrace{\text{Turing recognizable languages}}_{\text{RELs}}$ are closed under $\underbrace{\text{union}}_{✓}$ and $\underbrace{\text{complementation}}_{\times}$.

TRUE 3. $\underbrace{\text{Turing decidable languages}}_{\text{Recursives}}$ are closed under $\underbrace{\text{intersection}}_{✓}$ and $\underbrace{\text{complementation}}_{✓}$.

TRUE 4. Turing recognizable languages are closed under $\underbrace{\text{union}}$ and $\underbrace{\text{intersection}}$

[2013: 1 Marks]

A. 1 and 4 only

B. 1 and 3 only

C. 2 only

D. 3 only

$\overline{\text{Rec}} \Rightarrow \text{Rec}$

$\overline{\text{REL}}$ ⇒ Need not be REL

⇒ Either recursive OR Not REL

⇒ Never be "RE but not rec"

DFA ≅ NFA ≅ FA

DPDA ≇ PDA

DPDA ≇ NPDA

PDA & NPDA same

TM ≅ DTM ≅ NTM

≅ Single tape TM

≅ Multi tape TM

≅ Universal TM

≅ 2 Stack PDA

	$\cup$	$\cap$	$\overline{L}$
for RELs	✓	✓	✗
for Recursives	✓	✓	✓

Q 73 Let <M> be the encoding of a Turing machine as a string over $\Sigma = \{0,1\}$. Let L={<M>|*M is a Turing machine that accepts a string of length 2014*}. Then, L is

[2014(Set-2): 2 Marks]

(A) ~~decidable and recursively enumerable~~

(B) undecidable but recursively enumerable

(C) ~~undecidable and not recursively enumerable~~

(D) $\boxed{\text{decidable but not recursively enumerable}}$

$\boxed{\text{P: Whether M accepts string of length 2014}}$

YES, → Logic exist: TM halts at final for string of length 2014

NO → Logic not exist

Is Tm accepting 2014 length string!
what exist ?

$M \rightarrow$

Tm only 9.

HTm 9.

No Tm 9.

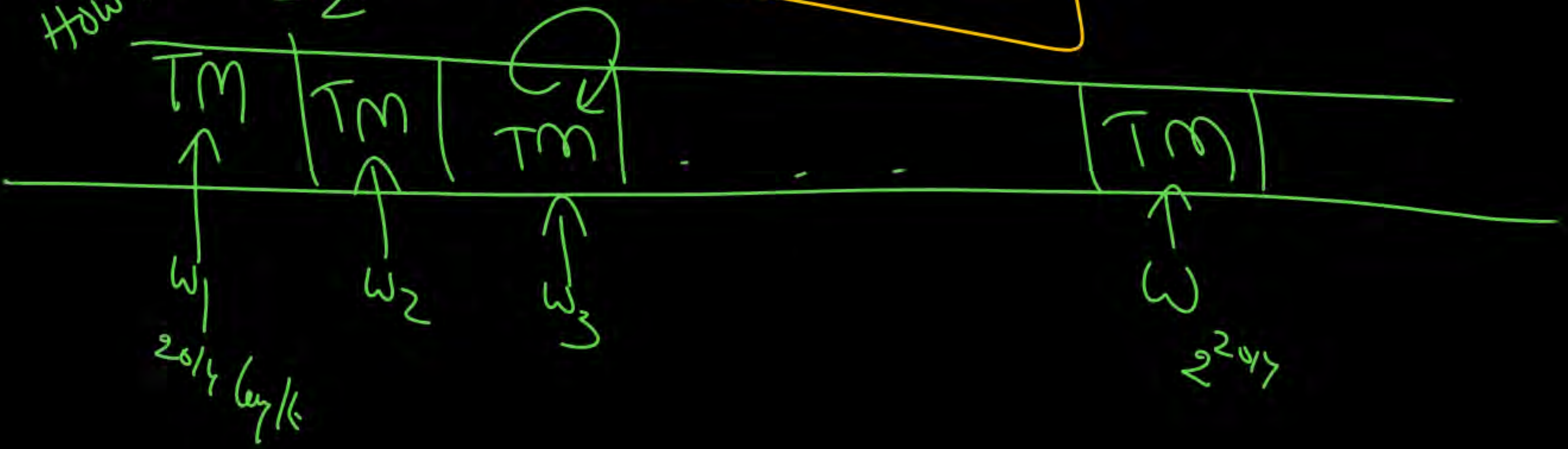
Yes : If any thread halts at final,

No $\Rightarrow$ 9

$\Sigma = \{a, b\}$

How many 2014 length?

$= 2^{2014}$



L

Decidable (Recursive)

Yes ⇒ } Both have logic
No ⇒ }

RE (SD)

Undecidable

RE but not rec (SD & UD)

Yes ⇒ logic exist
No ⇒ logic not exist

Only Yes has logic

Not RE (UD but not SD)

Yes ⇒ logic not exist

Q 74

Consider the following types of languages L_1: Regular, L_2: Context-free, L_3: Recursive, L_4: Recursively enumerable. Which of the following is/are TRUE?

I. $\overline{L_3} \cup L_4$ is recursively enumerable. ✓ $\overline{Rec} \cup RE \Rightarrow Rec \cup RE \Rightarrow RE$

II. $\overline{L_2} \cup L_3$ is recursive.

III. $L_1^* \cap L_2$ is context-free.

IV. $L_1 \cup \boxed{\overline{L_2}}$ is $\underbrace{\text{context-free}}_{\times}$.

[2016(Set-2): 1 Marks]

(A) I only

(B) I and III only

(C) I and IV only

(D) I, II and III only

Q 75

Let L(R) be the language represented by regular expression R. Let L(G) be the language generated by a context free grammar G. Let L(M) be the language accepted by a Turing machine M. Which of the following decision problems are $\boxed{\text{undecidable}}$?

D I. Given a regular expression R and a string w, is $w \in L(R)$? ⇒ membership

D II. Given a context-free grammar G, is $L(G) = \phi$? Emptiness

UD III. Given a context-free grammar G, is $L(G) = \Sigma^*$ for some alphabet Σ? Totality (completeness)

UD IV. Given a Turing machine M and a string w, is $w \in L(M)$? membership for TM

[2017(Set-2): 2 Marks]

(A) I and IV only

(B) II and III only

(C) II, III and IV only

(D) III and IV only

Q 76

Let ⟨M⟩ denote an encoding of an automaton M. Suppose that Σ{0,1}. Which of the following languages is/are $\boxed{\text{NOT recursive}}$?

not Decidable ⇓ Undecidable

[2021(set-1) 1 Marks]

totality for FA

D (A) $L = \{\langle M\rangle \mid \underbrace{M \text{ is a DFA such that } L(M) = \Sigma^*}_{D}\}$

UD ~~(B)~~ $L = \{\langle M\rangle \mid \underbrace{M \text{ is a PDA such that } L(M) = \Sigma^*}_{\text{Totality for PDA}}\}$

D (C) $L = \{\langle M\rangle \mid M \text{ is a PDA such that } \underbrace{L(M) = \phi}_{\text{Emptiness}}\}$

Emptiness for PDA

D (D) $L = \{\langle M\rangle \mid M \text{ is a DFA such that } \underbrace{L(M) = \phi}_{\text{Emptiness}}\}$

	FA	DPDA	PDA	LBA & HTM	TM
H	✓	✓	✓	✓	✗
M	✓	✓	✓	✓	✗
E	✓	✓	✓		✗
F	✓	✓	✓		✗
T	✓	✓			✗
E_q	✓	✓			✗
D	✓				✗
S	✓				✗

Undecidability

Q 77

Let L_1 be a recursive language. Let L_2 and L_3 be languages that are recursively enumerable but not recursive. Which of the following statements is not necessarily true?

[2010: 1 Marks]

A. $L_2 - L_1$ is recursively enumerable TRUE

B. $L_1 - L_3$ is recursively enumerable FALSE

C. $L_2 \cap L_3$ is recursively enumerable TRUE

D. $L_2 \cup L_3$ is recursively enumerable TRUE

$L_1 \rightarrow Rec$

$L_2 \rightarrow$ RE but not Rec

Rec − RE but not Rec

Rec $\cap$ $\overline{\text{RE but not Rec}}$

Rec $\cap$ Not REL = ?

RE but not Rec − Rec

RE but not Rec $\cap$ $\overline{\text{Rec}}$

RE but not Rec $\cap$ Rec $\Rightarrow$ RE

Q 78

Which of the following problems are decidable?

1. ~~Does a given program ever produce an output?~~ (TM above "program")
   - Yes ⇒ logic exist
   - No ⇒ logic not exist
2. ~~If L is a context-free language, then, is $\overline{L}$ also context-free?~~
   - Yes ⇒ logic exist
   - no ⇒ logic not exist
3. If L is a regular language, then, is $\overline{L}$ also regular? D
4. If L is a recursive language, then, is $\overline{L}$ also recursive? D

[2012: 1 Marks]

(A) 1, 2, 3, 4

(B) 1,2

(C) 2,3,4

(D) 3,4

Q 79

Which of the following is/are undecidable?

1. G is CFG. Is $L(G) = \phi$? Emptiness (simplification Algo) $\Rightarrow$ Decidable
2. G is a CFG. Is $L(G) = \Sigma^*$? Totality $\Rightarrow$ UD
3. M is a Turing machine. Is L(M) regular? $\Rightarrow$ UD
4. A is a DFA and N is an NFA. Is $L(A) = L(N)$? $\Rightarrow$ D

min DFA min DFA

[2013: 2 Marks]

(A) 3 only

(B) 3 and 4 only

(C) 1, 2 and 3 only

(D) 2 and 3 only

Q80 Let L be a language and $\bar{L}$ be its complement. Which one of the following is NOT a viable possibility?

[2014(Set-1): 2 Marks]

(A) Neither L nor $\bar{L}$ is recursively enumerable (r.e.). → possible

(B) One of L and $\bar{L}$ is r.e. but not recursive; the other is not r.e. → possible

(C) Both L and $\bar{L}$ are r.e. but not recursive. Impossible

(D) Both L and $\bar{L}$ are recursive. → possible

$L_B = \{TM \mid TM \text{ accepts } w\} \Rightarrow$ RE but not Rec

$\bar{L}_B = \{TM \mid TM \text{ does not accepts } w\} \Rightarrow$ Not RE

$L_A = \{TM \mid L(TM) = \text{finite}\} \rightarrow$ not RE

$\bar{L}_A = \{TM \mid L(TM) \neq \text{finite}\}$

Q 81

Let $A \leq_m B$ denotes that language A is mapping reducible (also known as many-to-one reducible) to language B. Which one of the following is FALSE?

[2014(Set-2): 1 Marks]

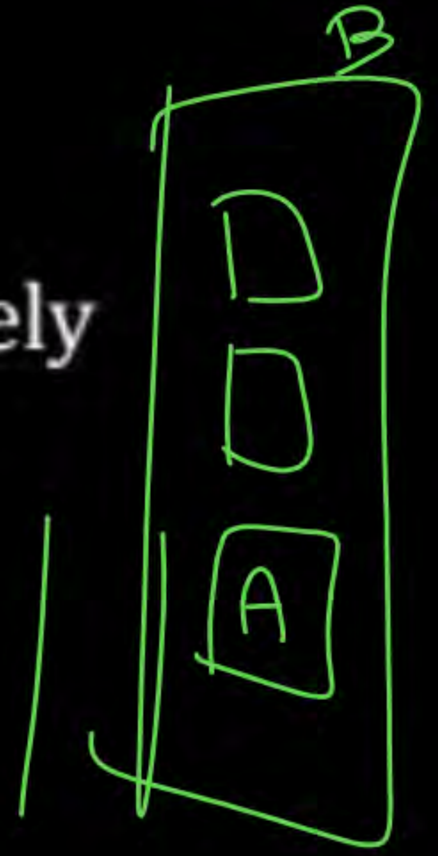
$A \leq B$

A. If $A \leq_m B$ and B is recursive then A is recursive. TRUE

B. If $A \leq_m B$ and A is undecidable then B is undecidable. TRUE

C. If $A \leq_m B$ and B is recursively enumerable then A is recursively enumerable. TRUE

D. If $A \leq_m B$ and B is not recursively enumerable then A is not recursively enumerable. FALSE



Recursive

$\cong$

Decidable

$\cong$

HTM

$\cong$

Algo

$\cong$

Halting problem

$\cong$

L has TM and $\bar{L}$ has TM

$\cong$

L is REL and $\bar{L}$ is REL

Q 82 Let Σ be a finite non-empty alphabet and let 2^{Σ^*} be the power set of Σ^*. Which one of the following is TRUE?

[2014(Set-3): 1 Marks]

(A) Both $\cancel{2^{\Sigma^*}}$ and Σ^* are $\boxed{\text{countable}}$

(B) 2^{Σ^*} is countable and Σ^* is uncountable

(C) 2^{Σ^*} is uncountable and Σ^* is countable

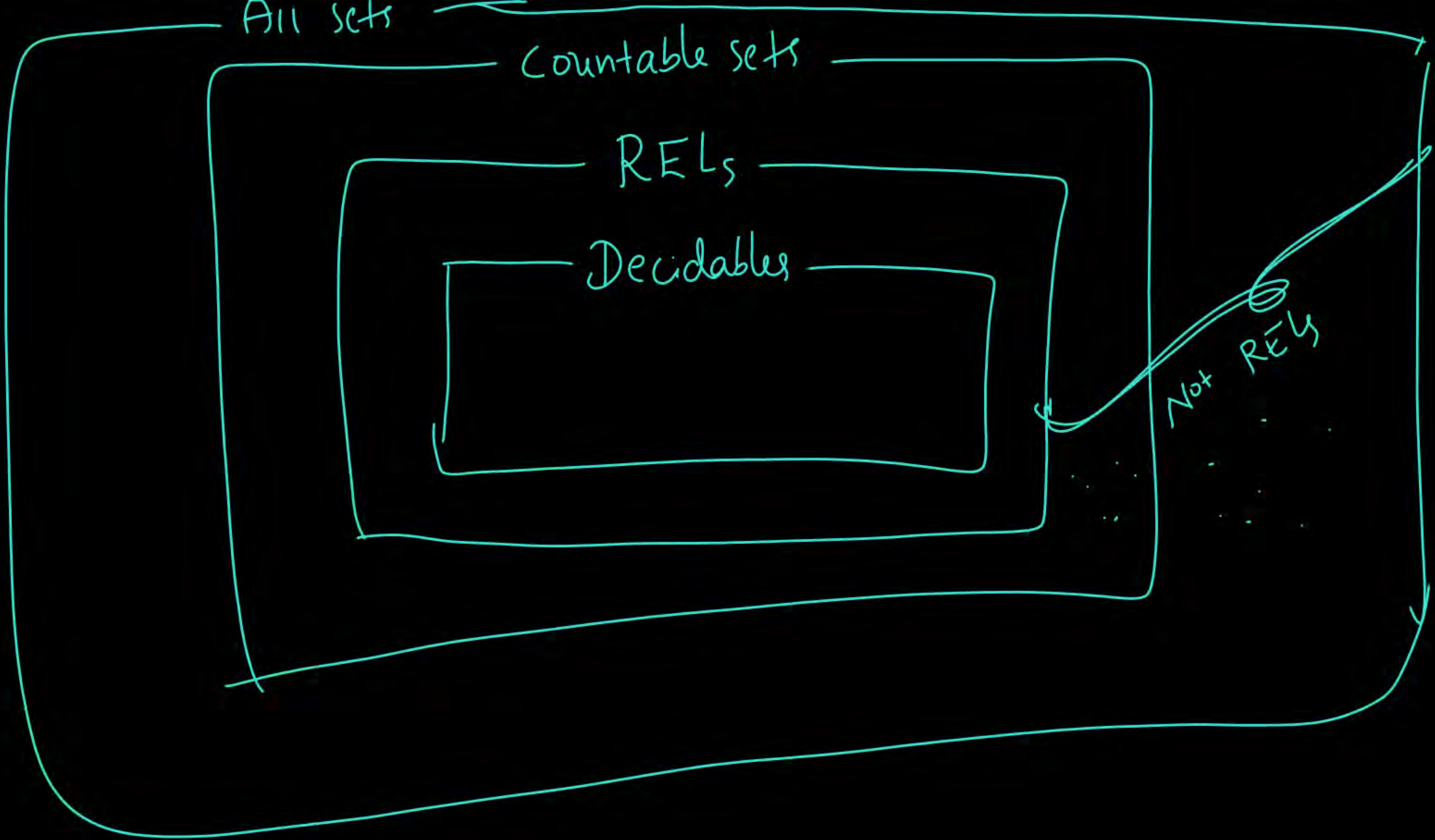
(D) Both 2^{Σ^*} and Σ^* are uncountable

$\{\phi, \{\varepsilon\}, \{a\}, \{b\}, \{aa\}, \ldots, \{\varepsilon, a\}, \{\varepsilon, b\}, \ldots\}$

$\Sigma \Rightarrow$ Set of symbols

$\Sigma^* \Rightarrow$ Set of all strings $\Rightarrow$ Regular

$2^{\Sigma^*} \Rightarrow$ Set of all languages



All sets
Countable sets
RELs
Decidables
Not RELs

Q. 83 Which one of the following problems is undecidable?

[2014(Set-3): 2 Marks]

(A) Deciding if a given context-free grammar is ambiguous. ⇒ UD

(B) Deciding if a given string is generated by a given context-free grammar. (membership) ⇒ Decidable

(C) Deciding if the language generated by a given context-free grammar is empty. (emptiness) ⇒ Decidable

(D) Deciding if the language generated by a given context-free grammar is finite. (Finiteness) ⇒ Decidable

Dependency graph

Q 84

For any two languages L_1 and L_2 such that L_1 is context free and L_2 is recursively enumerable but not recursive, which of the following is/are necessarily true?

1. $\overline{L}_1$ (complement of L_1) is recursive ✓ $\overline{L} \Rightarrow CSL \Rightarrow$ Recursive
2. $\overline{L}_2$ (complement of L_2) is recursive $\overline{L_2} \Rightarrow$ Not REL
3. $\overline{L}_1$ is context-free
4. $\overline{L}_1 \cup L_2$ is recursively enumerable ✓

[2015(Set-1): 1 Marks]

(A) 1 only

(B) 3 only

(C) 3 and 4 only

(D) 1 and 4 only

Q 85

Consider the following statements:

1. The complement of every Turing decidable language is Turing decidable.

   (Decidable lang)

2. There exists some language which is in NP but is not Turing decidable.

3. If L is a language in NP, L is Turing decidable.

Which of the above statements is/are True?

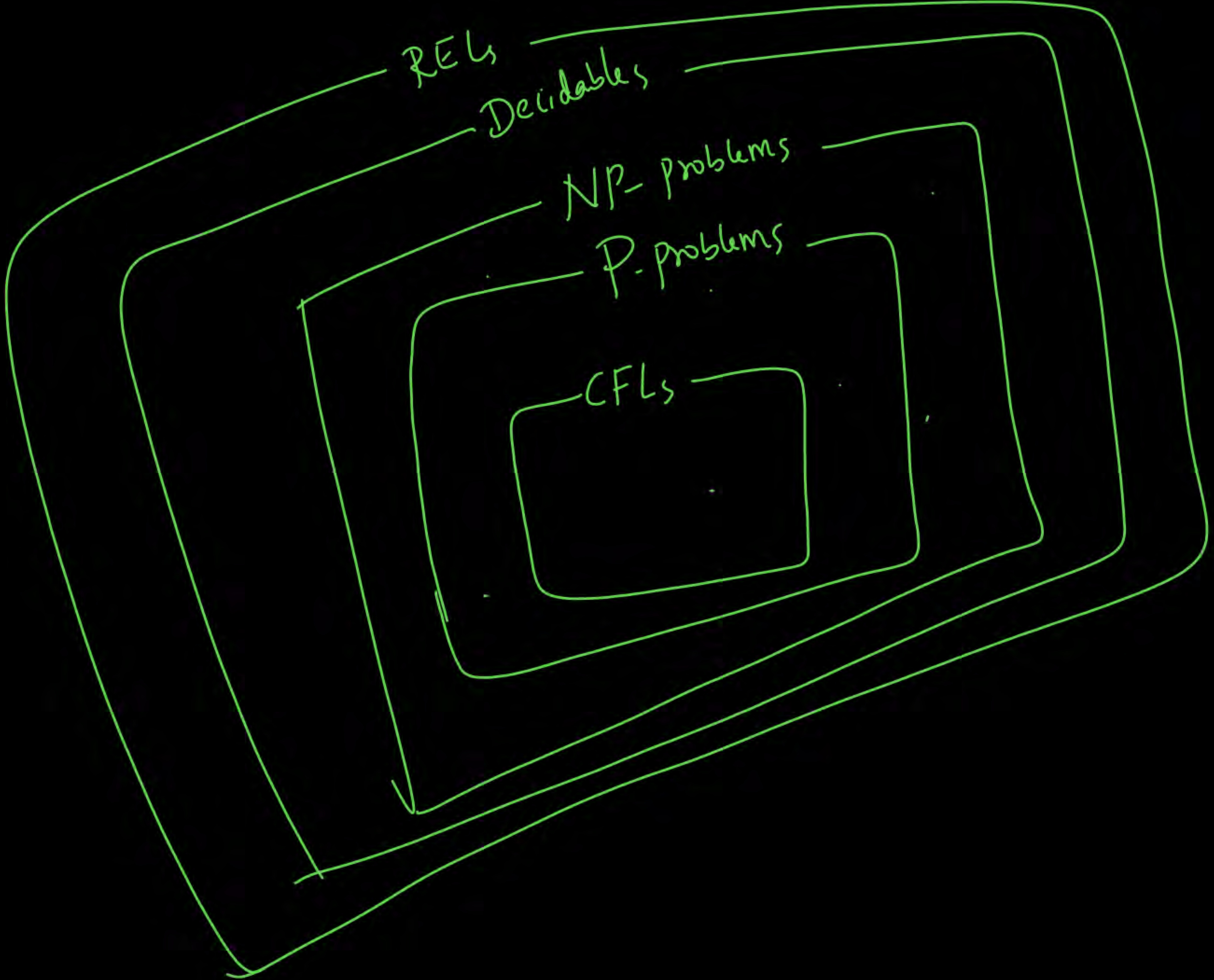
[2015(Set-2): 1 Marks]

(A) Only 2

(B) Only 3

(C) Only 1 and 2

(D) Only 1 and 3


RELs
Decidables
NP-problems
P-problems
CFLs

Q.86

Let X be a recursive language and Y be a recursively enumerable but not recursive language. Let W and Z be two languages such that $\overline{Y}$ reduces to W, and Z reduces to $\overline{X}$ (reduction means the standard many-one reduction). Which one of the following statements is TRUE?

[2016(Set-1): 2 Marks]

$\overline{Y} \leq W$ | $Z \leq \overline{X}$

Not RE $\leq W$ | $Z \leq$ Rec

(A) W can be recursively enumerable and Z is recursive.

Not RE   Rec

(B) W can be recursive and Z is recursively enumerable.

(C) W is not recursively enumerable and Z is recursive.

(D) W is not recursively enumerable and Z is not recursive.

Q87

Which of the following decision problems are undecidable?

D I. Given NFAs N_1 and N_2, is $L(N_1) \cap L(N_2) = \Phi$?

D II. Given a CFG $G = (N, \Sigma, P, S)$ and a string $x \in \Sigma^*$, does $x \in L(G)$?

III. Given CFGs G_1 and G_2, is $L(G_1) = L(G_2)$? $\Rightarrow$ UD

IV. Given a TM M, is $L(M) = \Phi$? $\Rightarrow$ UD

[2016-Set1: 1 Mark]

(A) I and IV only

(B) II and III only

(C) III and IV only

(D) II and IV only

Q.88 ***

We need to understand $|w| \le 2016$

Consider the following languages:

- $L_1 = \{\langle M \rangle | M$ takes at least 2016 steps on some input$\}$ → Yes : / → No :
- $L_2 = \{\langle M \rangle | M$ takes at least 2016 steps on all inputs$\}$ and → Yes : / → No :
- $L_3 = \{\langle M \rangle | M$ accepts $\epsilon\}$ ⇒ UD

Recursive

membership

if we verify upto 2016 length it means indirectly all inputs verified

Where for each turning machine M, $\langle M \rangle$ denotes a specific encoding of M. Which one of the following is TRUE?

2011 3000 length

[2016(Set-2): 1 Marks]

(A) L_1 is recursive and L_2, L_3 are not recursive

(B) L_2 is recursive and L_1, L_3 are not recursive

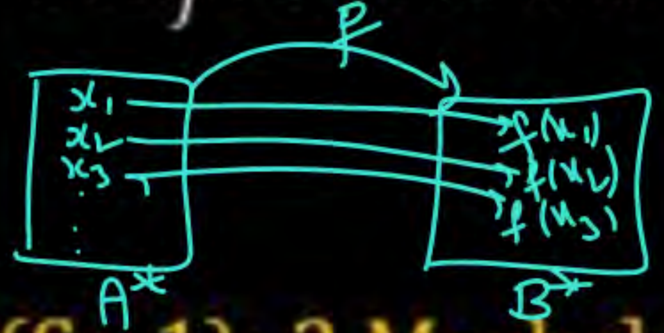
(C) L_1, L_2 is recursive and L_3 is not recursive

(D) L_1, L_2, L_3 are recursive

Q 89

Let A and B be finite alphabets and let # be a symbol outside both A and B. Let f be a $\boxed{\text{total function}}$ from A^* to B^*. We say f is computable if there exists a Turing machine M which given an input x in A^*, always halts with $f(x)$ on its tape. Let L_f denote the language $\{x\# f(x) \mid x \in A^*\}$.

Which of the following statements is true:



[2017(Se-1): 2 Marks]

$$L_f = \{ \underbrace{x_1 \# f(x_1)}, \underbrace{x_2 \# f(x_2)}_{\in L_f}, \ldots \}$$

(A) f is computable if and only if L_f is recursive.

(B) f is computable if and only if L_f is recursively enumerable.

(C) If f is computable then L_f is recursive, but not conversely.

(D) If f is computable then L_f is recursively enumerable, but not conversely.

Q .90 The set of all recursively enumerable languages is

[2018: 1 Marks]

A closed under complementation

B closed under intersection

C a subset of the set of all recursive languages

D an uncountable set

Set of RELs $\not\subseteq$ Set of Recs

$\supseteq$

Q.91

Consider the following problems. L(G) denotes the language generated by a grammar G. L(M) denotes the language accepted by a machine M.

I. For an unrestricted grammar G and a string w, whether $w \in L(G)$.

II. Given a Turing Machine M, whether L(M) is regular.

III. Given two grammars G_1 and G_2, whether $L(G_1) = L(G_2)$.

IV. Given an NFA N, whether there is a deterministic PDA P such that N and P accept the same languages.

Which one of the following statements is correct?

[2018: 2 Marks]

A. Only I and II are undecidable

B. Only III is undecidable

C. Only II and IV are undecidable

D. Only I, II and III are undecidable

H.W.

Q 92

Consider the following sets:

S_1: Set of all recursively enumerable languages over the alphabet {0,1}.

S_2: Set of all syntactically valid C programs. = Set of all CFLs

S_3: Set of all languages over the alphabet {0,1}. = 2^{Σ^*}

S_4: Set of all non-regular languages over the alphabet {0,1}. ⇒ uncount

Which of the above sets are uncountable?

[2019: 2 Marks]

(A) S_1 and S_2

(B) S_3 and S_4

(C) S_1 and S_4

(D) S_2 and S_3

H.W.

Q 93

Which of the following languages are undecidable? Note that ⟨M⟩ indicates encoding of the Turing machine M.

L_1: $\{\langle M\rangle | L(M)=\phi\}$

L_2: $\{\langle M, w, q\rangle |$ M on input w reaches state q in exactly 100 steps$\}$

L_3: $\{\langle M\rangle | L(M)$ is not recursive$)$

L_4: $\{\langle M\rangle | L(M)$ contains at least 21 members$)$

$|L(M)| \geq 21$

[2020: 2 Marks]

(A) L_2, L_3 and L_4 only

(B) L_1, L_3 and L_4 only

(C) L_1 and L_3 only

(D) L_2 and L_3 only

Q. 94

For the Turing machine M, $\langle M \rangle$ denotes an encoding of M. Consider the following two languages:

L_1: $\{\langle M \rangle | (M)$ takes more than 2021 steps on all inputs$\}$

L_2: $\{\langle M \rangle | (M)$ takes more than 2021 steps on some input$\}$

Which one of the following options is correct?

[2021(Set-1): 2 Marks]

(A) Both L_1 and L_2 are decidable.

(B) L_1 is undecidable and L_2 is decidable.

(C) Both L_1 and L_2 are undecidable.

(D) L_1 is decidable and L_2 is undecidable.

Q 95

Consider the following two statements about regular languages:

S_1: Every infinite regular language contains an undecidable language as a subset.

S_2: Every finite language is regular

Which of the following choices are correct?

[2021(Set-2): 2 Marks]

(A) Both S_1 and S_2 are true.

(B) Only S_2 is true.

(C) Only S_1 is true.

(D) Neither S_1 nor S_2 is true

Q 96

Which of the following statements is/are TRUE?

[2022: 1 Marks]

A. Every subset of a recursively enumerable language is recursive.

B. If a language L and its complement $\overline{L}$ are both recursively enumerable, then L must be recursive.

C. Complement of a context-free language must be recursive.

D. If L_1 and L_2 are regular, then $L_1 \cap L_2$ must be deterministic context-free

Q 97

Which of the following is/are undecidable?

[2022: 2 Marks]

(A) Given two Turing machines M_1 and M_2, decide if $L(M_1) = L(M_2)$.

(B) Given a Turing machine M, decide if L(M) is regular.

(C) Given a Turing machine M, decide if M accepts all strings

(D) Given a Turing machine M, decide if M takes more than 1073 steps on every string.

DCFL also called as LR(k) language
$k \geq 1$

DCFL also called as LR(1) language

Set of all DCFLs

=

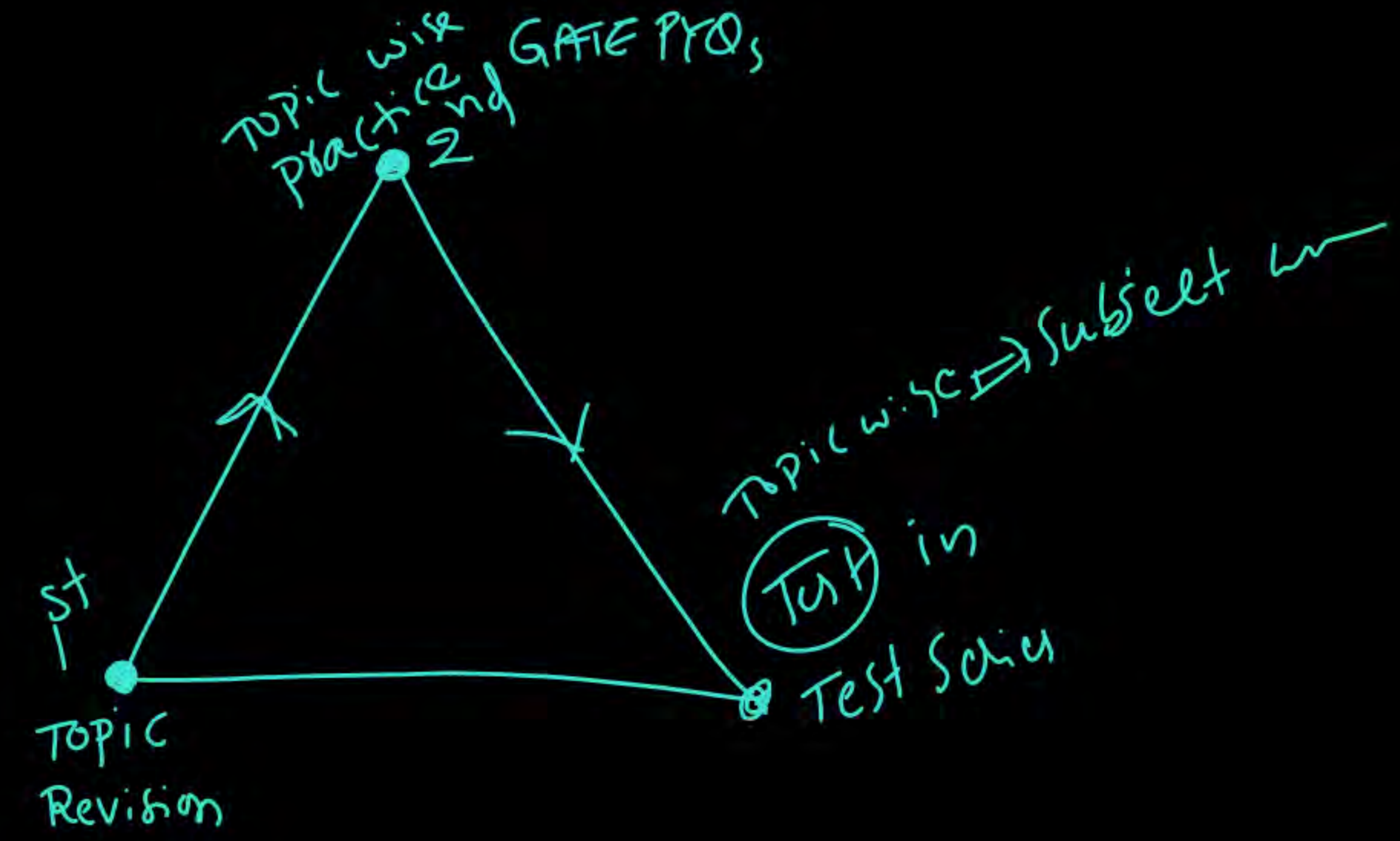
Set of all LR(1) languages

=

set of all LR(2) languages

=

Set of all LR(k) languages
$k \geq 1$



Topic wise Practice
2nd
GATE PYQs
1st
Topic Revision
Topic wise ⇒ Subject
Test in
Test Series

THANK YOU!