

3.  $\varepsilon + 0(10 * 1 + 10) * 1$   
 4.  $\varepsilon + 0(10 * 1 + 10) * 10 *$

(A)  $P - 2, Q - 1, R - 3, S - 4$   
 (B)  $P - 1, Q - 3, R - 2, S - 4$   
 (C)  $P - 1, Q - 2, R - 3, S - 4$   
 (D)  $P - 3, Q - 2, R - 1, S - 4$

[GATE 2008 : IISc Bangalore]

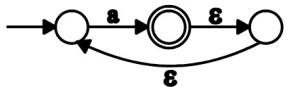
- Q. 12** Let  $L = \{w \in (0+1)^* | w \text{ has even number of 1's}\}$ , i.e. L is the set of all bit strings with even number of 1's. Which one of the regular expressions below represents L?

(A)  $(0^*10^*)^*$       (B)  $0^*(10^*10^*)^*$   
 (C)  $0^*(10^*1)^*0^*$       (D)  $0^*1(10^*1)^*10^*$

**[GATE 2010 : IIT Guwahati]**

- Q.13** What is the complement of the language accepted by the NFA shown below?

Assume  $\Sigma = \{a\}$  and  $\varepsilon$  is the empty string.






## Theory of Computation

11

corresponding to the regular expression  $(0+1)^*(10)$  is \_\_\_\_.

[GATE 2015 : IIT Kanpur]

- Q.20** Consider the alphabet  $\Sigma = \{0,1\}$ , the null/empty string  $\lambda$  and the set of strings  $X_0$ ,  $X_1$ , and  $X_2$  generated by the corresponding non-terminals of a regular grammar  $X_0, X_1$ , and  $X_2$  are related as follows.

$$X_0 = 1 X_1$$

$$X_1 = 0 X_1 + 1 X_2$$

$$X_2 \equiv 0 \, X_1 + \{\lambda\}$$

Which one of the following choices precisely represents the strings in  $X$ ?

- (A)  $10(0^* + (10)^*)1$
  - (B)  $10(0^*+(10^*))1$
  - (C)  $1(0+10)^*1$
  - (D)  $10(0+10)^*1+110(0+10)^*1$

[GATE 2015 : IIT Kharagpur]

- [GATE 2015 : IIT Kanpur]

5.  $(\cup + 1)^{\wedge 1}$   
(A) 1 and 2 only      (B) 1 and 3 only  
(C) 2 and 3 only      (D) 1, 2, and 3

[GATE 2014 : IIT Kharagpur]

- Q. 17** Let  $L_1 = \{w \in \{0,1\}^* | w \text{ has at least as many occurrences of } (110) \text{'s as } (011) \text{'s}\}$ .  
Let  $L_2 = \{w \in \{0,1\}^* | w \text{ has at least as many occurrences of } (000) \text{'s as } (111) \text{'s}\}$ .  
Which one of the following is TRUE?

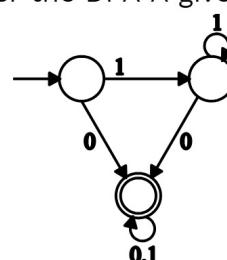
- (A)  $L_1$  is regular but not  $L_2$
  - (B)  $L_2$  is regular but not  $L_1$
  - (C) Both  $L_1$  and  $L_2$  are regular.
  - (D) Neither  $L_1$  nor  $L_2$  are regular

[GATE 2014 : IIT Kharagpur]

- Q.18** Let  $L$  be the language represented by the regular expression  $\Sigma^*0011\Sigma^*$  where  $\Sigma = \{0,1\}$ . What is the minimum number of states in a DFA that recognizes  $\bar{L}$  (complement of  $L$ )?



**Q.19** The number of states in the minimal deterministic finite automaton



Which of the following are FALSE?

1. Complement of  $L(A)$  is context - free
  2.  $L(A) = L((11^*0+0)(0+1)^*0^*1^*)$
  3. For the language accepted by  $A$ ,  $A$  is the minimal DFA.
  4.  $A$  accepts all strings over  $\{0, 1\}$  of length at least 2.

(A) 1 and 3 only    (B) 2 and 4 only  
(C) 2 and 3 only    (D) 3 and 4 only

[GATE 2013 : IIT Bombay]

- Q. 25** Consider the following two statements:

  - If all states of an NFA are accepting states then the language accepted by the NFA is  $\Sigma^*$ .
  - There exists a regular language A

the set of all binary strings having two consecutive 0's and two consecutive 1's?

- (A)  $(0+1)^*0011(0+1)^*$   
 $+ (0+1)^*1100(0+1)^*$
- (B)  $(0+1)^*(00(0+1)^*11$   
 $+11(0+1)^*00)(1+1)^*$
- (C)  $(0+1)^*00(0+1)^*+(0+1)^*11(0+1)^*$
- (D)  $00(0+1)^*11+11(0+1)^*00$

**[GATE 2016 : IISc Bangalore]**

- Q.22** The number of states in the minimum sized DFA that accepts the language defined by the regular expression  $(0+1)^*(0+1)(0+1)^*$  is \_\_\_\_\_.

**[GATE 2016 : IISc Bangalore]**

- Q.23** Consider the language L given by the regular expression  $(a+b)^*b(a+b)$  over the alphabet {a, b}. The smallest number of states needed in a deterministic finite-state automaton (DFA) accepting L is \_\_\_\_\_.

**[GATE 2017 : IIT Roorkee]**

- (A) I only
- (B) II only
- (C) Both I and II
- (D) Neither I nor II

**[GATE 2020 : IIT Delhi]**

- Q.28** Which one of the following regular expressions represents the set of all binary strings with an odd number of 1's?

- (A)  $((0+1)^*1(0+1)^*1)^*10^*$
- (B)  $(0^*10^*10^*)^*0^*1$
- (C)  $10^*(0^*10^*10^*)^*$
- (D)  $(0^*10^*10^*)^*10^*$

**[GATE 2020 : IIT Delhi]**

- Q.29** Which of the following regular expression identities are true?

- (A)  $r^{\ast\ast} = r^{\ast}$
- (B)  $(r^{\ast}s^{\ast})^{\ast} = (r + s)^{\ast}$
- (C)  $(r + s)^{\ast} = r^{\ast} + s^{\ast}$
- (D)  $r^{\ast}s^{\ast} = r^{\ast} + s^{\ast}$

**[GATE 1992 : IIT Delhi]**

such that for all languages B,  $A \cap B$  is regular.

Which one of the following is CORRECT?

- (A) Only I is true
- (B) Only II is true
- (C) Both I and II are true
- (D) Both I and II are false

**[GATE 2016 : IISc Bangalore]**

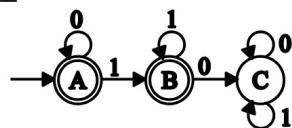
- Q.26** For  $\Sigma = \{a, b\}$ , let us consider the regular language  $L = \{x \mid x = a^{2+3k} \text{ or } x = b^{10+12k}, k \geq 0\}$ . Which one of the following can be a pumping length (the constant guaranteed by the pumping lemma) for L?

- (A) 3
- (B) 5
- (C) 9
- (D) 24

**[GATE 2019 : IIT Madras]**

- Q.27** Consider the following statements.
- I. If  $L_1 \cup L_2$  is regular, then both  $L_1$  and  $L_2$  must be regular.
  - II. The class of regular languages is closed under infinite union.
- Which of the above statements is/are TRUE?

- Q.30** The regular expression for the language recognized by the finite state automation of the below figure is \_\_\_\_\_.



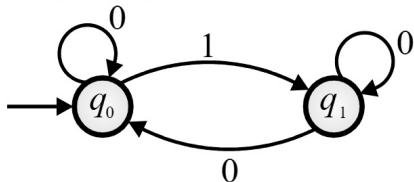
**[GATE 2002 : IISc Bangalore]**

- Q.31** Which one of the following languages over the alphabet {0, 1} is described by the regular expression  $(0 + 1)^* 0(0+1)^* 0(0+1)^*$

- (A) The set of all strings containing the substring 00
- (B) The set of all strings containing at most two 0's
- (C) The set of all strings containing at least two 0's
- (D) The set of all strings that begin and end with either 0 or 1

**[GATE 2009 : IIT Roorkee]**

**Q.1** Which of the following options represents regular expression accepted by above DFA ' $M$ '?



- (A)  $(0+1)^*$
- (B)  $0^*1(0+00^*1)^*$
- (C)  $(0+10^*0)^*10^*$
- (D)  $1(0+1)^*$

**Q.2** Which of the following language is/are NOT regular?

- (A)  $L = \{WxW^R \mid W, x \in \{0,1\}^*\}$
- (B)  $L = \{WxW \mid W, x \in \{0,1\}^*\}$
- (C)  $L = \{WxW \mid W, x \in \{0,1\}^+\}$
- (D)  $L = \{WW \mid W \in \{0,1\}^*\}$

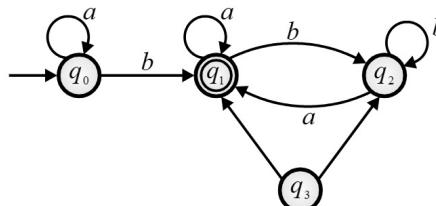
**Q.3** R.E. =  $(111+1111)^*$

Which of the following regular language represents above regular expression?

- (A)  $L = \{1^n \mid n \geq 0\}$
- (B)  $L = \{1^n \mid n \geq 0 \text{ and } n \neq 1, 2\}$
- (C)  $L = \{1^n \mid n \geq 0 \text{ and } n \neq 1, 2, 5, 7\}$
- (D)  $L = \{1^n \mid n \geq 0 \text{ and } n \neq 3, 4\}$

**Q.4** Regular expression  $= (0+1)^*(0+1)(0+1)^*$  what is the minimum no. of states in DFA that accept above R.E. \_\_\_\_\_?

**Q.5** The language accepted by above automaton is given by regular expression



- (A)  $a^*b(a+b)^*$

### Theory of Computation

13

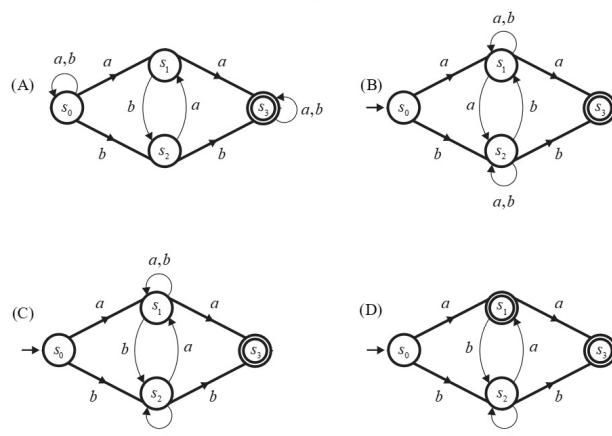
- (B)  $a^*ba^*bb^*aa^*$
- (C)  $a^*b(a+bb^*a)^*$
- (D)  $a^*b(bb+aa)^*$

**Q.6** Which of the following two regular expression is/are equivalent?

- (i)  $(00)^*(0+\epsilon)$       (ii)  $(0+1)^*1$
- (iii)  $(0^*1^*)^*$       (iv)  $(0+1)^*$
- (v)  $0^*1(1+00^*1)^*$       (vi)  $(0^*0^*)^*$
- (A) (iii) and (iv)      (B) (i) and (vi)
- (C) (ii) and (v)      (D) None

**Q.7** Consider the regular expression.  $R = (a+b)^*(aa+bb)(a+b)^*$ . Which of the following non-deterministic finite automata recognizes the language

defined by the regular expression  $R$ ? Edges labeled  $\lambda$  denote transitions on the empty string.



### Answer Key

#### Classroom Practice Questions

1.	A	2.	C	3.	D	4.	D	5.	C,D
6.	C	7.	A	8.	D	9.	C	10.	B
11.	C	12.	B	13.	B	14.	C	15.	3
16.	B	17.	B	18.	B	19.	3	20.	C
21.	B	22.	2	23.	4	24.	D	25.	C

<b>26.</b>	<b>D</b>	<b>27.</b>	<b>D</b>	<b>28.</b>	<b>D</b>	<b>29.</b>	<b>B</b>	<b>30.</b>	<b>0*1*</b>
<b>31.</b>	<b>C</b>								
<b>Practice Questions</b>									
<b>1.</b>	<b>A, B</b>	<b>2.</b>	<b>C, D</b>	<b>3.</b>	<b>B</b>	<b>4.</b>	<b>2</b>	<b>5.</b>	<b>C</b>
<b>6.</b>	<b>A, B, C</b>	<b>7.</b>	<b>A</b>						

1

3

# **Grammar, Language and it's Application**



## **Classroom Questions**



Which one of the following statements is **FALSE**?

- (A)  $L_2$  is context-free
  - (B)  $L_1 \cap L_2$  is context-free
  - (C) Complement of  $L_2$  is recursive
  - (D) Complement of  $L_1$  is context-free  
but not regular

[GATE 2013 : IIT Bombay]

- Q.6** Which of the following languages is /are regular?

$L_1 : \{wxw^R \mid w, x \in \{a, b\}^*, |w|, |x| > 0, w^R \text{ is the reverse of string } w\}$

$$L_2 : \{a^n b^m \mid m \neq n \text{ and } m, n \geq 0\}$$

$$J_2 : \{a^p b^q c^r \mid p, q, r \geq 0\}$$

- (A)  $L_1$  and  $L_3$  only (B)  $L_2$  only  
 (C)  $L_1$  and  $L_2$  only (D)  $L_1$  only

[GATE 2015 • IIT Kharagpur]

- (A)  $L = \emptyset$   
 (B) L is regular but not  $\emptyset$   
 (C) L is context free but not regular  
 (D) L is not context free

[GATE 2000 : IIT Kharagpur]

- Q.4** Let  $L = L_1 \cap L_2$ , where  $L_1$  and  $L_2$  are languages defined as follows,

$$L_1 = \{a^m b^n c a^n b^m \mid m, n \geq 0\}$$

$$L_2 = \{a^i b^j c^k \mid i, j, k \geq 0\}$$

Then L is

- (A) Not recursive  
 (B) Regular  
 (C) Context free but not regular  
 (D) Recursively enumerable but not context free

[GATE 2009 : IIT Roorkee]

- Q.5** Consider the following languages

$$L_1 = \{0^p 1^q 0^r \mid p, q, r \geq 0\}$$

$$L_2 = \{0^p 1^q 0^r \mid p, q, r \geq 0, p \neq r\}$$

Which of the following languages is generated by the given grammar?

$$S \rightarrow aS|bS|\epsilon$$

- (A)  $\{a^n b^m \mid n, m \geq 0\}$   
 (B)  $\{w \in \{a, b\}^* \mid w \text{ has equal number of } a's \text{ and } b's\}$   
 (C)  $\{a^n \mid n \geq 0\} \cup \{b^n \mid n \geq 0\} \cup \{a^n b^n \mid n \geq 0\}$   
 (D)  $\{a, b\}^*$

[GATE 2016 : IISc Bangalore]

- Q.8** Language  $L_1$  is defined by the grammar:

$$S_1 \rightarrow aS_1b|\epsilon$$

Language  $L_2$  is defined by the grammar:

$$S_2 \rightarrow abS_2|\epsilon$$

Consider the following statements :

$$P: L_1 \text{ is regular}$$

$$Q: L_2 \text{ is regular}$$

Which one of the following is TRUE?

- (A) Both P and Q are true

15

Theory of Computation

- (B) P is true and Q is false  
 (C) P is false and Q is true  
 (D) Both P and Q are false

[GATE 2016 : IISc Bangalore]

- Q.9** A context-free grammar is ambiguous if :

- (A) The grammar contains useless non-terminals.  
 (B) It produces more than one parse tree for same sentence.  
 (C) Some production has two non-terminals side by side on the right-hand side.  
 (D) None of the above.

[GATE 1987 : IIT Bombay]

- Q.10** FORTRAN is a :

- (A) Regular language.  
 (B) Context free language.  
 (C) Context sensitive language.  
 (D) None of the above.

[GATE 1987 : IIT Bombay]

- Q.11** Consider the following grammar G :

$$S \rightarrow bS|aa|b$$

$$A \rightarrow bA|aB$$

$$B \rightarrow bB|aS|a$$

Let  $N_a(w)$  and  $N_b(w)$  denote the number of a's and b's in a string w respectively. The language  $L(G) \subseteq \{a, b\}^*$  generated

terminals and Y is a non-terminal), is always regular.

4. The derivation trees of strings generated by a context-free grammar in Chomsky Normal form are always binary trees.

- (A) 1, 2, 3 and 4  
 (B) 2, 3 and 4 only  
 (C) 1, 3 and 4 only  
 (D) 1, 2 and 4 only

[GATE 2008 : IISc Bangalore]

- Q.13** Match the following **List-I** with **List-II**

**List-I**

- (E) Checking that identifiers are declared before their use  
 (F) Number of formal parameters in the declaration of a function agrees with the number of actual parameters in a use of that function  
 (G) Arithmetic expressions with matched pairs of parentheses  
 (H) Palindromes

**List-II**

- (P)  $L = \{a^n b^m c^n d^m \mid n \geq 1, m \geq 1\}$   
 (Q)  $X \rightarrow XbX \mid XcX \mid dXf \mid g$   
 (R)  $L = \{wcw \mid w \in (a|b)^*\}$  (S)  
 $X \rightarrow bXb \mid cXc \mid \epsilon$

by G is

- (A)  $\{w \mid N_a(w) > 3N_b(w)\}$
- (B)  $\{w \mid N_b(w) > 3N_a(w)\}$
- (C)  $\{w \mid N_a(w) = 3k, k \in \{0, 1, 2, \dots\}\}$
- (D)  $\{w \mid N_b(w) = 3k, k \in \{0, 1, 2, \dots\}\}$

**[GATE 2004 : IIT Delhi]**

**Q. 12** Which of the following statements are true?

1. Every left-recursive grammar can be converted to a right-recursive grammar and vice-versa.
2. All  $\epsilon$ -productions can be removed from any context-free grammar by suitable transformations.
3. The language generated by a context-free grammar all of whose productions are of the form  $X \rightarrow w$  or  $X \rightarrow wY$  (where, w is a string of

**Codes :**

- (A)  $E - P, F - R, G - Q, H - S$
- (B)  $E - R, F - P, G - S, H - Q$
- (C)  $E - R, F - P, G - Q, H - S$
- (D)  $E - P, F - R, G - S, H - Q$

**[GATE 2008 : IISc Bangalore]**

**Q. 14**  $S \rightarrow aSa \mid bSb \mid a \mid b$

The language generated by the above grammar over the alphabets {a, b} is the set of

- (A) All palindromes
- (B) All odd length palindromes
- (C) Strings that begin and end with the same symbol
- (D) All even length palindromes

**[GATE 2009 : IIT Roorkee]**

**Q. 15** Consider the languages

$$L_1 = \{0^i 1^j \mid i \neq j\}, L_2 = \{0^i 1^j \mid i = j\},$$

$$L_3 = \{0^i 1^j \mid i = 2j + 1\},$$

$$L_4 = \{0^i 1^j \mid i \neq 2j\},$$

Which one of the following statement is true?

- (A) Only  $L_2$  is context free
- (B) Only  $L_2$  and  $L_3$  are context free
- (C) Only  $L_1$  and  $L_2$  are context free
- (D) All are context free

**[GATE 2010 : IIT Guwahati]**

**Q. 16** Which of the following languages are context-free?

$$L_1 = \{a^m b^n a^n b^m \mid m, n \geq 1\}$$

$$L_2 = \{a^m b^n a^m b^n \mid m, n \geq 1\}$$

$$L_3 = \{a^m b^n \mid m = 2n + 1\}$$

- (A)  $L_1$  and  $L_2$  only (B)  $L_1$  and  $L_3$  only
- (C)  $L_2$  and  $L_3$  only (D)  $L_3$  only

**[GATE 2015 : IIT Kanpur]**

**Q. 17** Consider the following context-free grammars :

$$G_1 : S \rightarrow aS \mid B, B \rightarrow b \mid bB$$

$$G_2 : S \rightarrow aA \mid bB, A \rightarrow aA \mid B \mid \epsilon, B \rightarrow bB \mid \epsilon$$

Which one of the following pairs of languages is generated by  $G_1$  and  $G_2$ , respectively?

- (A)  $\{a^m b^n \mid m > 0 \text{ or } n > 0\}$  and

$$\{a^m b^n \mid m > 0 \text{ and } n > 0\}$$

$$(A) \{(ab)^n (cb)^n \mid n \geq 1\}$$

$$(B) \{(ab)^n cb^{m_1} cb^{m_2} \dots cb^{m_n} \mid \dots, m_n \geq 1\}$$

$$(C) \{(ab)^n (cb^m)^n \mid m, n \geq 1\}$$

$$(D) \{(ab)^n (cb^n)^m \mid m, n \geq 1\}$$

**[GATE 2017 : IIT Roorkee]**

**Q. 19** Identify the language generated by the following grammar, where S is the start variable.

$$S \rightarrow XY$$

$$X \rightarrow aX \mid a$$

$$Y \rightarrow aYb \mid \epsilon$$

$$(A) \{a^m b^n \mid m \geq n, n > 0\}$$

$$(B) \{a^m b^n \mid m \geq n, n \geq 0\}$$

$$(C) \{a^m b^n \mid m > n, n \geq 0\}$$

$$(D) \{a^m b^n \mid m > n, n > 0\}$$

**[GATE 2017 : IIT Roorkee]**

**Q. 20** If G is a grammar with productions

$$S \rightarrow SaS \mid aSb \mid bSa \mid SS \mid \epsilon$$

Where S is the start variable, then which one of the following string is not generated by G?

- (A) abab (B) aaab
- (C) abbaa (D) babba

**[GATE 2017 : IIT Roorkee]**

**Q. 21** Consider the context-free grammar



**Practice Questions**

**Q.1**  $G : S \rightarrow aS | bA$

$A \rightarrow aA | bC | \epsilon$

$C \rightarrow aC | bS$

Language accepted by above grammar is

(A)  $L(G) = (a+b)^*$

(B)  $L(G) = \{W \in \{a,b\}^* \mid \text{Number of } b's \text{ are odd}\}$

(C)  $L(G) = \{W \in \{a,b\}^* \mid \#_b(W) \bmod 3 = 0\}$

(D)  $L(G) = \{W \in \{a,b\}^* \mid \#_b(W) \bmod 3 = 1\}$

**Q.2**  $G : S \rightarrow AB | BC$

$B \rightarrow aAb | \epsilon$

$A \rightarrow aA | a$

$C \rightarrow bC | a$

Language accepted by above grammar is

(A)  $L(G) = \{a^n b^n \mid n \geq 1\}$

18

Theory of Computation

(B)  $L(G) = \{a^n b^m \mid n > m\}$

(C)  $L(G) = \{a^n b^m \mid m > n\}$

(D)  $L(G) = \{a^n b^m \mid n \neq m\}$

**Q.3** Which of the following CFLS is represented by following grammar?

$S \rightarrow AB / CD$

$C \rightarrow 0C / \epsilon$

$A \rightarrow 0A1 / \epsilon$

$D \rightarrow 1D2 / \epsilon$

$B \rightarrow 0B / \epsilon$

(A)  $\{0^i 1^j 2^k \mid i = j < k \text{ OR } i > j = k\}$

(B)  $\{0^i 1^j 2^k \mid i = j \text{ OR } j = k\}$

(C)  $\{0^i 1^j 2^k \mid i = j = k \text{ OR } i > j = k\}$

(D) None.

**Q.4** Consider the following grammar.

$G_1 : S \rightarrow bA | aB$

$A \rightarrow aS | bAA | a$

$B \rightarrow bS | bBB | b$

$G_2 : S \rightarrow aSbS | bSaS | \epsilon$

Which one is /are TRUE?

(A)  $L(G_1) = L(G_2)$

(B)  $L(G_1) \subseteq L(G_2)$

(C)  $L(G_1) \cap L(G_2) = \emptyset$

(D)  $L(G_2) \cup \{\epsilon\} = L(G_1)$

**Q.5** Consider following grammar:-

$G : \{ S \rightarrow aA,$

$A \rightarrow aA | bA$

(C) It is necessarily non- regular

(D) None of the above

**Common Data for Q.7 & Q.8 Questions**

Consider the context-free grammar

$E \rightarrow E + E$

$E \rightarrow (E * E)$

$E \rightarrow id$

Where E is the starting symbol, the set of terminals is  $\{id, (,), +, *\}$ , and the set of non-terminals is {E}.

**Q.7** Which of the following terminal strings has more than one parse tree when parsed according to the above grammar?

(A)  $id + id + id + id$

(B)  $id + (id * (id * id))$

(C)  $(id * (id * id)) + id$

(D)  $((id * id + id) * id)$

**Q.8** For the terminal string with more than one parse tree obtained as solution to Q. 8A. How many parse trees are possible?

(A) 5 (B) 4

(C) 3 (D) 2

**Q.9** Let L be a context-free language and M a regular language. Then the language  $L \cap M$  is

(A) Always regular

(B) Never regular

(C) Always a deterministic context-free language

(D) Always a context-free language

**Q.10** Consider an ambiguous grammar G

$$B \rightarrow bB \mid cC \mid c$$

$$C \rightarrow cC \mid c \quad \}$$

The  $L(G_1)$ ?

- (A)  $a^*b^*c^*$       (B)  $aa^*b^*c^*$   
(C)  $aa^*bb^*cc^*$     (D)  $(abc)^*$

- Q.6** Let L be a regular language and M be a context free language, both over the alphabet  $\Sigma$ . Let  $L^c$  and  $M^c$  denote the complements of L and M respectively. Which of the following statements about the language  $L^c \cup M^c$  is TRUE?  
(A) It is necessarily regular but not necessarily context free  
(B) It is necessarily context free

Theory of Computation

19



Which of the following choices describes the properties satisfied by the strings in these languages?

- (A) G1: No y appears before any x  
G2: Every x is followed by at least one y  
(B) G1: No y appears before any x  
G2: No x appears before any y  
(C) G1: No y appears after any x  
G2: Every x is followed by at least one y  
(D) G1: No y appears after any x  
G2: Every y is followed by at least one x

Q.10

Consider an ambiguous grammar G and its disambiguated version D. Let the language recognized by the two grammars be denoted by  $L(G)$  and  $L(D)$  respectively.

- (A)  $L(D) \subset L(G)$     (B)  $L(D) \supset L(G)$   
(C)  $L(D) = L(G)$     (D)  $L(D)$  is empty

Q.11

The two grammars given below generate a language over the alphabet  $\{x, y, z\}$

$$\begin{aligned} G1: S &\rightarrow x \mid z \mid xS \mid zS \mid yB \\ B &\rightarrow y \mid z \mid yB \mid zB \\ G1: S &\rightarrow y \mid z \mid yS \mid zS \mid xB \\ B &\rightarrow y \mid yS \end{aligned}$$

Q.12

Consider the grammar

$$\begin{aligned} S &\rightarrow ABCc \mid bc \\ BA &\rightarrow AB \\ Bb &\rightarrow bb \\ Ab &\rightarrow ab \\ Aa &\rightarrow aa \end{aligned}$$

Which of the following sentence can be derived by this grammar?

- (A) abc                         (B) aab  
(C) abcc                        (D) abbc

### Answer Key

Classroom Practice Questions									
1.	C	2.	A	3.	B	4.	C	5.	D
6.	A	7.	D	8.	C	9.	B	10.	B
11.	C	12.	C	13.	C	14.	B	15.	D
16.	B	17.	D	18.	B	19.	C	20.	C
21.	B	22.	D	23.	31	24.	5	25.	B
26.	C	27.	C	28.	A				
Self - Practice Questions									
1.	D	2.	D	3.	A	4.	B	5.	C
6.	A	7.	A	8.	A	9.	D	10.	C
11.	A	12.	A						



**4****Push Down Automata****Classroom Questions**

**Q.1** Context free languages and regular languages are both closed under the operation(s) of :

- (A) Union
- (B) Intersection
- (C) Concatenation
- (D) Complementation

**[GATE 1989 : IIT Kanpur]**

**Q.2** Context-free languages are

- (A) Closed under union
- (B) Closed under complementation
- (C) Closed under intersection
- (D) Closed under Kleene closure.

**[GATE 1992 : IIT Delhi]**

**Q.3** If  $L_1$  and  $L_2$  are context free languages and  $R$  is a regular set, one of the languages below is not necessarily a context free language. Which one?

- (A)  $L_1L_2$
- (B)  $L_1 \cap L_2$
- (C)  $L_1 \cap R$
- (D)  $L_1 \cup L_2$

**[GATE 1996 : IISc Bangalore]**

**Q.4** Which of the following languages over  $\{a, b, c\}$  is accepted by a deterministic push down automata?

- (A)  $\{wcw^R \mid w \in \{a, b\}^*\}$
- (B)  $\{ww^R \mid w \in \{a, b, c\}^*\}$
- (C)  $\{a^n b^n c^n \mid n \geq 0\}$

$L_E$  the set of all languages accepted by empty stack. Which of the following is true?

- (A)  $L_D = L_E$
- (B)  $L_D \supset L_E$
- (C)  $L_D \subset L_E$
- (D) None of the above

**[GATE 1999 : IIT Bombay]**

**Q.7** If  $L_1$  is a context free language and  $L_2$  is a regular language which of the following is/are false?

- (A)  $L_1 - L_2$  is not context free
- (B)  $L_1 \cap L_2$  is context free
- (C)  $\sim L_1$  is context free
- (D)  $\sim L_2$  is regular

**[GATE 1999 : IIT Bombay]**

**Q.8** Which of the following statement is true?

- (A) If a language is context free it can always be accepted by deterministic pushdown automaton
- (B) The union of two context free languages is context free
- (C) The intersection of two context free languages is context free
- (D) The complement of a context free

- (D)  $\{w|w \text{ is a palindrome over } \{a, b, c\}\}$

**[GATE 1997 : IIT Madras]**

**Q.5** Context free languages are closed under :

- (A) Union, intersection
- (B) Union, Kleene closure
- (C) Intersection, complement
- (D) Complement, Kleene Closure

**[GATE 1999 : IIT Bombay]**

**Q.6** Let  $L_D$  be the set of all languages accepted by a PDA by final state and



**Q.10** Let  $G = (\{S\}, \{a, b\}, R, S)$  be a context free grammar where the rule set R is

$$S \rightarrow aSb | SS | \epsilon$$

Which of the following statements is true?

- (A) G is not ambiguous
- (B) There exist  $x, y \in L(G)$  such that  $xy \notin L(G)$
- (C) There is a deterministic pushdown automaton that accepts  $L(G)$
- (D) We can find a deterministic finite state automaton that accepts  $L(G)$

**[GATE 2003 : IIT Madras]**

**Q.11** The language  $\{a^m b^n c^{m+n} | m, n \geq 1\}$  is

- (A) Regular
- (B) Context free but not regular
- (C) Context sensitive but not context free
- (D) Type-0 but not context sensitive

**[GATE 2004 : IIT Delhi]**

**Q.12** Let  $M = (K, \Sigma, F, \Delta, S, F)$  be a pushdown automaton.

Where,  $K = \{s, f\}$ ,  $F = \{f\}$ ,  $\Sigma = \{a, b\}$ ,  $F = \{a\}$  and

$$\begin{aligned} \Delta = & \{((s, a, \epsilon), (s, a)), \\ & ((s, b, \epsilon), (s, a)), ((s, a, \epsilon), \\ & (f, \epsilon)), ((f, a, a), (f, \epsilon)), \\ & ((f, b, a), (f, \epsilon))\}. \end{aligned}$$

Which one of the following strings is not a member of  $L(M)$  ?

- (A) aaa
- (B) aabab
- (C) baaba
- (D) bab

**[GATE 2004 : IIT Delhi]**

language is context free

**[GATE 2001 : IIT Kanpur]**

**Q.9** The language accepted by a Pushdown Automation in which the stack is limited to 10 items is best described as

- (A) Context free
- (B) Regular
- (C) Deterministic Context Free
- (D) Recursive

**[GATE 2002 : IISc Bangalore]**

- (B)  $D_f \subset N_f$  and  $D_p = N_p$

- (C)  $D_f = N_f$  and  $D_p = N_p$

- (D)  $D_f = N_f$  and  $D_p \subset N_p$

**[GATE 2005 : IIT Bombay]**

**Q.14** Consider the languages :

$$L_1 = \{a^n b^n c^m | n, m > 0\} \text{ and}$$

$$L_2 = \{a^n b^n c^m | n, m > 0\}$$

Which one of the following statement is FALSE?

- (A)  $L_1 \cap L_2$  is a context-free language
- (B)  $L_1 \cup L_2$  is a context-free language
- (C)  $L_1$  and  $L_2$  are context-free languages
- (D)  $L_1 \cap L_2$  is a context sensitive language

**[GATE 2005 : IIT Bombay]**

**Q.15** Consider the languages :

$$L_1 = \{ww^R | w \in \{0, 1\}^*\}$$

$L_2 = \{w\#w^R | w \in \{0, 1\}^*\}$  where # is a special symbol

$$L_3 = \{ww | w \in \{0, 1\}^*\}$$

Which one of the following is TRUE?

- (A)  $L_1$  is a deterministic CFL
- (B)  $L_2$  is a deterministic CFL
- (C)  $L_3$  is a CFL, but not a deterministic CFL
- (D)  $L_3$  is a deterministic CFL

**[GATE 2005 : IIT Bombay]**

**Q.16** Let  $L_1 = \{0^{n+m} 1^n 0^m | n, m \geq 0\}$ ,

$$L_2 = \{0^{n+m} 1^{n+m} 0^m | n, m \geq 0\}, \text{ and}$$

$$L_3 = \{0^{n+m} 1^{n+m} 0^{n+m} | n, m \geq 0\}$$

**Q.13** Let  $N_f$  and  $N_p$  denote the classes of languages accepted by non-deterministic finite automata and non-deterministic push-down automata, respectively. Let  $D_f$  and  $D_p$  denote the classes of languages accepted by deterministic finite automata and deterministic push-down automata, respectively. Which one of the following is TRUE?

- (A)  $D_f \subset N_f$  and  $D_p \subset N_p$

22

3. G can be accepted by a deterministic PDA.

Which combination below expresses all the true statements about G?

- (A) 1 only      (B) 1 and 3 only  
 (C) 2 and 3 only      (D) 1, 2 and 3

[GATE 2006 : IIT Kharagpur]

**Q.18** The language  $L = \{0^i 2^{j_1} i \geq 0\}$  over the alphabet {0, 1, 2} is

- (A) Not recursive.  
 (B) Is recursive and is a deterministic CFL.  
 (C) Is a regular language.  
 (D) None

[GATE 2007 : IIT Kanpur]

**Q. 19** Which one of the following is FALSE?

- (A) There is a unique minimal DFA for every regular language  
 (B) Every NFA can be converted to an equivalent PDA  
 (C) Complement of every context free language is recursive  
 (D) Every non deterministic PDA can be converted to an equivalent deterministic PDA

[GATE 2009 : IIT Roorkee]

**Q.20** Consider the languages  $L_1, L_2$  and  $L_3$  as given below.

$$L_1 = \{0^p 1^q \mid p, q \in N\}$$

$$L_2 = \{0^p 1^q \mid p, q \in N \text{ and } p = q\}$$

$$L_3 = \{0^p 1^q 0^r \mid p, q, r \in N \text{ and } p = q = r\}$$

Which of the following statements is not TRUE?

- (A) Pushdown automata (PDA) can be used to recognize  $L_1$  and  $L_2$   
 (B)  $L_1$  is a regular language  
 (C) All the three languages are context free

Which of these languages are NOT context free?

- (A)  $L_1$  only      (B)  $L_3$  only  
 (C)  $L_1$  and  $L_2$       (D)  $L_2$  and  $L_3$

[GATE 2006 : IIT Kharagpur]

**Q.17** Consider the following statements about the context-free grammar  $G = \{S \rightarrow SS, S \rightarrow ab, S \rightarrow ba, S \rightarrow \epsilon\}$

1. G is ambiguous
2. G produces all strings with equal number of a's and b's

Theory of Computation



Here,  $w^r$  is the reverse of the string w. Which of these language are deterministic context-free languages?

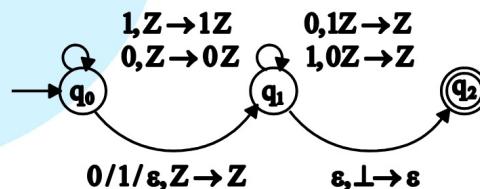
- (A) None of the languages  
 (B) Only  $L_1$   
 (C) Only  $L_1$  and  $L_2$   
 (D) All the three languages.

[GATE 2011 : IIT Madras]

**Q.22** Consider the NPDA

$$(Q = \{q_0, q_1, q_2\}, \Sigma = \{0, 1\}, \Gamma = \{0, 1, \perp\}, \delta, q_0, \perp, F = \{q_2\}),$$

where (as per usual convention) Q is the set of states,  $\Sigma$  is the input alphabet,  $\Gamma$  is the stack alphabet,  $\delta$  is the state transition function,  $q_0$  is the initial state,  $\perp$  is the initial stack symbol, and F is the set of accepting states. The state transition is as follows :



$0/1/Z \rightarrow Z$        $0/Z \rightarrow 0Z$        $1/Z \rightarrow Z$        $1,0Z \rightarrow Z$

Which one of the following sequences must follow the string 101100 so that the overall string is accepted by the automaton?

- (A) 10110      (B) 10010  
 (C) 01010      (D) 01001

[GATE 2015 : IIT Kanpur]

**Q.23** Consider the transition diagram of a PDA given below with input alphabet  $\Sigma = \{a, b\}$  and stack alphabet  $\Gamma = \{X, Z\}$ . Z is the initial stack symbol. Let L denote the language

- (D) Turing machines can be used to recognize all the languages.

**[GATE 2011 : IIT Madras]**

- Q.21** Consider the following languages over the alphabet  $\Sigma = \{0,1,c\}$ :

$$\begin{aligned}L_1 &= \{0^n 1^n \mid n \geq 0\} \\L_2 &= \{wcw^r \mid w \in \{0,1\}^*\} \\L_3 &= \{ww^r \mid w \in \{0,1\}^*\}\end{aligned}$$

- (B)  $L = \{a^n \mid n \geq 0\} \cup \{a^n b^n \mid n \geq 0\}$  and is not accepted by any deterministic PDA  
 (C) L is not accepted by any Turing machine that halts on every input  
 (D)  $L = \{a^n \mid n \geq 0\} \cup \{a^n b^n \mid n \geq 0\}$  and is deterministic context-free

**[GATE 2016 : IISc Bangalore]**

- Q.24** Consider the following languages :

$$\begin{aligned}L_1 &= \{a^n b^m c^{n+m} \mid m, n \geq 1\} \\L_2 &= \{a^n b^n c^{2n} \mid n \geq 1\}\end{aligned}$$

Which one of the following is TRUE?

- (A) Both  $L_1$  and  $L_2$  are context free.  
 (B)  $L_1$  is context free while  $L_2$  is not context free.  
 (C)  $L_2$  is context free while  $L_1$  is not context free.  
 (D) Neither  $L_1$  nor  $L_2$  is context free.

**[GATE 2016 : IISc Bangalore]**

- Q.25** Let  $L_1, L_2$  be any two context-free languages and R be any regular language. Then which of the following is/are CORRECT?

- (i)  $L_1 \cup L_2$  is context free  
 (ii)  $\bar{L}_1$  is context free  
 (iii)  $L_1 - R$  is context free  
 (iv)  $L_1 \cap L_2$  is context free  
 (A) (i), (ii) and (iv) only  
 (B) (i) and (iii) only  
 (C) (ii) and (iv) only  
 (D) (i) only

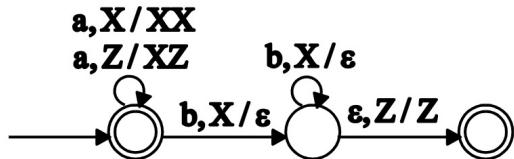
**[GATE 2017 : IIT Roorkee]**

- Q.26** Consider the following language over the alphabet  $\Sigma = \{a,b,c\}$ .

$$\text{Let } L_1 = \{a^n b^n c^m \mid m, n \geq 0\} \text{ and}$$

$$L_2 = \{a^m b^n c^n \mid m, n \geq 0\}.$$

accepted by the PDA.



Which one of the following is TRUE?

- (A)  $L = \{a^n b^n \mid n \geq 0\}$  and is not accepted by any finite automata

$$\begin{aligned}L_1 &= \{a^p \mid p \text{ is a prime number}\} \\L_2 &= \{a^n b^m c^{2m} \mid n \geq 0, m \geq 0\} \\L_3 &= \{a^n b^n c^{2n} \mid n \geq 0\} \\L_4 &= \{a^n b^n \mid n \geq 1\}\end{aligned}$$

Which of the following are CORRECT?

- I.  $L_1$  is context free but not regular  
 II.  $L_2$  is not context -free.  
 III.  $L_3$  is not context-free but recursive  
 IV.  $L_4$  is deterministic context-free  
 (A) I, II and IV only  
 (B) II and III only  
 (C) I and IV only  
 (D) III and IV only

**[GATE 2017 : IIT Roorkee]**

- Q.28** Consider the following languages :

- I.  $\left\{ a^m b^n c^p d^q \mid m + p = n + q, \right. \\ \left. \text{where } m, n, p, q \geq 0 \right\}$   
 II.  $\left\{ a^m b^n c^p d^q \mid m = n \text{ and } p = q, \right. \\ \left. \text{where } m, n, p, q \geq 0 \right\}$   
 III.  $\left\{ a^m b^n c^p d^q \mid m = n = p \text{ and } p \neq q, \right. \\ \left. \text{where } m, n, p, q \geq 0 \right\}$   
 IV.  $\left\{ a^m b^n c^p d^q \mid mn = p + q, \right. \\ \left. \text{where } m, n, p, q \geq 0 \right\}$

Which of the languages above are context-free?

- (A) I and IV only (B) I and II only  
 (C) II and III only (D) II and IV only

**[GATE 2018 : IIT Guwahati]**

- Q.29** Which one of the following languages over  $\Sigma = \{a, b\}$  is NOT context-free?

- (A)  $\{ww^R \mid w \in \{a, b\}^*\}$   
 (B)  $\{wa^n b^n w^R \mid w \in \{a, b\}^*, n \geq 0\}$   
 (C)  $\{wa^n w^R b^n \mid w \in \{a, b\}^*, n \geq 0\}$

Which of the following are context-free languages?

- |                   |                      |
|-------------------|----------------------|
| I. $L_1 \cup L_2$ | II. $L_1 \cap L_2$   |
| (A) I only        | (B) II only          |
| (C) I and II      | (D) Neither I nor II |

[GATE 2017 : IIT Roorkee]

**Q.27** Consider the following languages.

24

Theory of Computation

- (A)  $L_1$  is regular and  $L_2$  is context-free.  
 (B)  $L_1$  is context-free but not regular and  $L_2$  is context-free.

- (C) Neither  $L_1$  nor  $L_2$  is context-free.  
 (D)  $L_1$  is context-free but  $L_2$  is not context-free.

[GATE 2020 : IIT Delhi]

### Practice Questions

**Q.1** Which of the following language can't be solved by deterministic PDA?

- (A)  $L = \{a^n b^n \mid n \geq 1\}$   
 (B)  $L = \{a^n b^m c^m \mid n \geq 0 \text{ and } m \geq 1\}$   
 (C)  $L = \{a^n b^n \mid n \geq 0\} \cup \{a^n b^{2n} \mid n \geq 0\}$   
 (D)  $L = \{a^n b^m \mid n \neq m\}$

**Q.2** Which of the following options OR statement is/ are correct/

- S1:  $\{a^n b^n c^m \mid n \leq m \leq 2n\}$  is CFL  
 S1:  $\{a^i b^j \mid i = j \text{ OR } i = 2j\}$  is DCFL.  
 (A) S1 only      (B) S2 only  
 (C) Both      (D) None.

**Q.3** Consider the following grammar:-

$$G_1 : S \rightarrow aSa \mid a$$

$$G_2 : S \rightarrow aSa \mid b$$

Which of the options is / are TRUE

- (A)  $L(G_1)$  and  $L(G_2)$  are regular  
 (B)  $L(G_1)$  is regular but  $L(G_2)$  is CFL  
 (C) Both  $L(G_1)$  and  $L(G_2)$  are CFL  
 (D)  $L(G_2)$  is regular but  $L(G_1)$  is CFL

**Q.4** Equivalent CFL for

$$G = \{S \rightarrow aS \mid aSbS \mid \epsilon\}$$

- (A)  $L = \{X \mid X \text{ is a palindromes}\}$   
 (B)  $L = \{X \mid X = a^n b^n \mid n \geq 0\}$   
 (C)  $L = \{X \mid \text{each prefix of } X \text{ has atleast as many a's as b's}\}$   
 (D)  $L = \{X \mid X \text{ has equal no. of a's and b's}\}$

- (D)  $\{a^n b^i \mid i \in \{n, 3n, 5n\}^*, n \geq 0\}$

[GATE 2019 : IIT Madras]

**Q.30** Consider the following languages.

$$L_1 = \{wxyx \mid w, x, y \in (0+1)^+\}$$

$$L_2 = \{xy \mid x, y \in (a+b)^*, |x| = |y|, x \neq y\}$$

Which one of the following is TRUE?



- (C) Neither  $L_1$  nor  $L_2$  is context-free.

- (D)  $L_1$  is context-free but  $L_2$  is not context-free.

### Practice Questions

**Q.6** Which of the following options is/are TRUE?

- (A)  $L = \{a^i b^j \mid i \geq 0, j = n^2\}$  is not CFL  
 (B) For grammar  $G = \{S \rightarrow aSb \mid SS \mid \epsilon\}$  The  $L(G)$  can be solved by DPDA.  
 (C)  $L = \{wx^n w^R y^n \mid w \in \{x, y\}^*\}$  is CRL.  
 (D) For  $L = \{ww \mid w \in \{a, b\}^*\}$ ,  $\bar{L}$  is not CFL.

**Q.7** Consider the regular grammar below  
 $S \rightarrow bS \mid aA \mid \epsilon$

$$A \rightarrow aS \mid bA$$

The Myhill- Nerode equivalence classes for the language generated by the grammar are

- (A)  $\left\{ \begin{array}{l} \{w\epsilon(a+b)^* \mid \#_a(w) \text{ is even}\} \text{ and} \\ \{w\epsilon(a+b)^* \mid \#_a(w) \text{ is odd}\} \end{array} \right\}$   
 (B)  $\left\{ \begin{array}{l} \{w\epsilon(a+b)^* \mid \#_b(w) \text{ is even}\} \text{ and} \\ \{w\epsilon(a+b)^* \mid \#_b(w) \text{ is odd}\} \end{array} \right\}$   
 (C)  $\left\{ \begin{array}{l} \{w \in (a+b)^* \mid \#_a(w) = \#_b(w)\} \\ \text{and } (w \in (a+b)^* \mid \#_a(w) \text{ is even}) \text{ and} \\ \{w\epsilon(a+b)^* \mid \#_a(w) \neq \#_b(w)\} \end{array} \right\}$   
 (D)  $\{\epsilon\}, \left\{ \begin{array}{l} wa \mid w \in (a+b)^* \text{ and} \\ wb \mid w \in (a+b)^* \end{array} \right\}$

**Q.8** Consider the pushdown automaton (PDA) below which runs over the input alphabet (a, b, c). It has the stack alphabet  $\{Z_0, X\}$  where  $Z_0$  is the bottom-of-stack marker. The set of states of the PDA is {s, t, u, f} where s is the start and f is the final state. The

Which of the following options is/are CFL?

- (A)  $L = \{ww \mid w \in \{a,b\}^*\}$
- (B)  $L = \{w_1 w_2 \mid w_1 \neq w_2 \text{ and } w_1, w_2 \in \{a,b\}^*\}$
- (C)  $L = \{a^n b^m c^n d^m \mid n, m \geq 0\}$
- (D)  $L = \{x w y w \mid a, y, w \in \{a,b\}^+\}$



read b from the input and move to state t after popping the top of stack and pushing the symbols  $Z_0$  and X (in that order) on the stack.

$$(s, a, Z_0) \rightarrow (s, XXZ_0)$$

$$(s, \epsilon, Z_0) \rightarrow (f, \epsilon)$$

$$(s, a, X) \rightarrow (s, XXX)$$

$$(s, b, X) \rightarrow (t, \epsilon)$$

$$(t, b, X) \rightarrow (t, \epsilon)$$

$$(t, c, X) \rightarrow (u, \epsilon)$$

$$(u, c, X) \rightarrow (u, \epsilon)$$

$$(u, \epsilon, Z_0) \rightarrow (f, \epsilon)$$

The language accepted by the PDA is

- (A)  $\{a^\ell b^m c^n \mid \ell = m = n\}$
- (B)  $\{a^\ell b^m c^n \mid \ell = m\}$
- (C)  $\{a^\ell b^m c^n \mid 2\ell = m + n\}$
- (D)  $\{a^\ell b^m c^n \mid m = n\}$

In the context-free grammar below, S is the start symbol, a and b are terminals, and  $\epsilon$  denotes the empty string.

$$S \rightarrow aSAb \mid \epsilon$$

$$A \rightarrow bA \mid \epsilon$$

The grammar generates the language

- (A)  $((a+b)^* b)^*$
- (B)  $\{a^m b^n \mid m \leq n\}$
- (C)  $\{a^m b^n \mid m = n\}$
- (D)  $a^* b^*$

PDA accepts by final state. The transitions of the PDA given below are depicted in a standard manner. For example, the transition  $(s, b, X) \rightarrow (t, XZ_0)$  means that if the PDA is in state a and the symbol on the top of the stack is X, then it can

**Q.10** Which of the following languages is accepted by a non-deterministic pushdown automaton (PDA) but NOT by a deterministic PDA?

- (A)  $\{a^n b^n c^n \mid n \geq 0\}$
- (B)  $\{a^\ell b^m c^n \mid \ell \neq m \text{ or } m \neq n\}$
- (C)  $\{a^n b^n \mid n \geq 0\}$
- (D)  $\{a^m b^n \mid m, n \geq 0\}$

**Q.11** Consider the following languages.

$$L_1 = \{a^i b^j c^k \mid i = j, k \geq 1\}$$

$$L_2 = \{a^i b^j \mid j = 2i, i \geq 0\}$$

Which of the following is true?

- (A)  $L_1$  is not a CFL but  $L_2$  is
- (B)  $L_1 \cap L_2 = \emptyset$  and  $L_1$  is non-regular
- (C)  $L_1 \cup L_2$  is not a CFL but  $L_2$  is
- (D) There is a 4-state PDA that accepts  $L_1$ , but there is no DPDA that accepts  $L_2$

**Q.12** Which of the following languages is (are) non-regular?

$$L_1 = \{0^m 1^n \mid 0 \leq m \leq n \leq 10000\}$$

$$L_2 = \{w \mid w \text{ reads the same forward and backward}\}$$

$$L_3 = \{w \in \{0,1\}^* \mid w \text{ contains an even number of 0's and an even number of 1's}\}$$

- (A)  $L_2$  and  $L_3$  only
- (B)  $L_1$  and  $L_2$  only
- (C)  $L_3$  only
- (D)  $L_2$  only

#### Classroom Practice Questions

1.	A, C	2.	A, D	3.	B	4.	A	5.	B
6.	A	7.	A	8.	B	9.	B	10.	C
11.	B	12.	B	13.	D	14.	A	15.	B
16.	D	17.	B	18.	D	19.	D	20.	D
21.	C	22.	B	23.	D	24.	B	25.	B
26.	A	27.	D	28.	B	29.	C	30.	A

<b>1.</b>	<b>C</b>	<b>2.</b>	<b>D</b>	<b>3.</b>	<b>B</b>	<b>4.</b>	<b>C</b>	<b>5.</b>	<b>B, D</b>
<b>6.</b>	<b>A, C</b>	<b>7.</b>	<b>A</b>	<b>8.</b>	<b>C</b>	<b>9.</b>	<b>B</b>	<b>10.</b>	<b>B</b>
<b>11.</b>	<b>B</b>	<b>12.</b>	<b>D</b>						

□□□

# 5

# TM and Un-Decidability



## Classroom Questions

- Q.1** Let  $L_1$  be a recursive language. Let  $L_2$  and  $L_3$  be languages that are recursively enumerable but not recursive. Which of the following statement is not necessarily true?
- (A)  $L_2 - L_1$  is recursively enumerable
  - (B)  $L_1 - L_3$  is recursively enumerable.
  - (C)  $L_2 \cap L_3$  is recursively enumerable.
  - (D)  $L_2 \cup L_3$  is recursively enumerable.

**[GATE 2010 : IIT Guwahati]**

- Q.2** Consider the following decision problems :

$P_1$  : Does a given finite state machine accept a given string.

$P_2$  : Does a given context free grammar generate an infinite number of strings. Which of the following statement is true?

- (A) Both  $(P_1)$  and  $(P_2)$  are decidable
- (B) Neither  $(P_1)$  nor  $(P_2)$  are decidable
- (C) Only  $(P_1)$  is decidable
- (D) Only  $(P_2)$  is decidable

**[GATE 2000 : IIT Kharagpur]**

- Q. 3** Which of the following statement is false?

- (A) Every NFA can be converted to an equivalent DFA.
- (B) Every non-deterministic Turing machine can be converted to an equivalent deterministic Turing machine.
- (C) Every regular language is also a context free language.
- (D) Every subset of a recursively enumerable set is recursive.

- (B) Always recognizable by pushdown automata.

- (C) Also called type (0) languages.

- (D) Recognizable by Turing machines.

**[GATE 1990 : IISc Bangalore]**

- Q.5** In which of the cases stated below is the following statement true?  
“For every nondeterministic machine  $M_1$  there exists an equivalent deterministic machine  $M_2$  recognizing the same language”.

- (A)  $M_1$  is nondeterministic finite automation
- (B)  $M_1$  is a nondeterministic PDA
- (C)  $M_1$  is a nondeterministic Turing machine
- (D) For no machine  $M_1$  use the above statement true

**[GATE 1992 : IIT Delhi]**

- Q.6** Which of the following conversions is not possible (algorithmically)?

- (A) Regular grammar to context-free grammar
- (B) Non-deterministic FSA to deterministic FSA
- (C) Non-deterministic PDA to deterministic PDA
- (D) Non-deterministic Turing machine to deterministic Turing machine.

**[GATE 1994 : IIT Kharagpur]**

- Q.7** Which one of the following is not decidable?

- (A) Given a Turing machine  $M$ , a string  $s$  and an integer  $k$ ,  $M$  accepts  $s$  within  $k$  steps
- (B) Equivalence of two given Turing machines
- (C) Language accepted by a given finite state machine is non empty

**Q.4** Recursive languages are :

- (A) A proper superset of context free languages.



**Q.8** Regarding the power of recognition of languages, which of the following statement is false?

- (A) The non-deterministic finite-state automata are equivalent to deterministic finite-state automata.
- (B) Non-deterministic Push-down automata are equivalent to deterministic Push-down automata.
- (C) Non-deterministic Turing machines are equivalent to deterministic Turing machines.
- (D) Multi-tape Turing machines are equivalent to single-tape Turing machines.

**[GATE 1998 : IIT Delhi]**

**Q.9** Which of the following is true?

- (A) The complement of a recursive language is recursive.
- (B) The complement of a recursively enumerable language is recursively enumerable.
- (C) The complement of a recursive language is either recursive or recursively enumerable.
- (D) The complement of a context-free language is context-free.

**[GATE 2002 : IISc Bangalore]**

**Q.10** If the strings of a language L can be effectively enumerated in lexicographic (i.e. alphabetic) order, which of the following statements is true?

- (A) L is necessarily finite
- (B) L is regular but not necessarily finite
- (C) L is context free but not necessarily regular
- (D) L is recursive but not necessarily context free

**[GATE 2003 : IIT Madras]**

**Q.11** A single tape Turing Machine M has two states  $q_0$  and  $q_1$  of which  $q_0$  is the starting state. The tape alphabet of M is  $\{0, 1, B\}$  and its input alphabet is  $\{0, 1\}$ . The symbol B is the blank symbol

(D) Language generated by a context free grammar is non empty

**[GATE 1997 : IIT Madras]**

	0	1	B
$q_0$	$q_1, 1, R$	$q_1, 1, R$	Halt
$q_1$	$q_1, 1, R$	$q_1, 1, R$	$q_0, B, L$

The table is interpreted as illustrated below.

The entry  $(q_1, 1, R)$  in row  $q_0$  and column 1 signifies that if M is in state  $q_0$  and reads 1 on the current tape square, then it writes 1 on the same tape square, moves its tape head one position to the right and transitions to state  $q_1$ . Which of the following statement is true about M?

- (A) M does not halt on any string in  $(0+1)^+$
- (B) M does not halt on any string in  $(00 + 1)^*$
- (C) M halts on all string ending in a 0
- (D) M halts on all string ending in a 1

**[GATE 2003 : IIT Madras]**

**Q.12** Let  $L_1$  be a recursive language, and let  $L_2$  be a recursive enumerable but not a recursive language. Which one of the following is TRUE?

- (A)  $\bar{L}_1$  is recursive and  $\bar{L}_2$  is recursively enumerable
- (B)  $\bar{L}_1$  is recursive and  $\bar{L}_2$  is not recursively enumerable
- (C)  $\bar{L}_1$  and  $\bar{L}_2$  are recursively enumerable
- (D)  $\bar{L}_1$  is recursively enumerable and  $\bar{L}_2$  is recursive

**[GATE 2005 : IIT Bombay]**

**Q.13** For  $s \in (0+1)^*$ , let  $d(s)$  denote the decimal value of s (e. g.  $d(101) = 5$ ) Let  $L = \{s \in (0+1)^* \mid d(s) \bmod 5 = 2 \text{ and } d(s) \bmod 7 \neq 4\}$

Which one of the following statement is true?

- (A) L is recursively enumerable, but not recursive
- (B) L is recursive, but not context free

used to indicate end of an input string. The transition function of M is described in the following table.

28

- (C) L is context free, but not regular  
(D) L is regular

[GATE 2006 : IIT Kharagpur]

Theory of Computation



- Q.14** Let  $L_1$  be a regular language,  $L_2$  be a deterministic context-free language and  $L_3$  be a recursively enumerable, but not recursive language. Which one of the following statement is false?
- (A)  $L_1 \cap L_2$  is a deterministic CFL  
(B)  $L_3 \cap L_1$  is recursive  
(C)  $L_1 \cup L_2$  is context free  
(D)  $L_1 \cap L_2 \cap L_3$  is recursively enumerable

[GATE 2006 : IIT Kharagpur]

- Q.15** Which of the following is true for the language  $\{a^p \mid p \text{ is prime}\}$ ?
- (A) It is not accepted by a Turing machine  
(B) It is regular but not context-free  
(C) It is context-free but not regular  
(D) It is neither regular nor context-free, but accepted by a Turing machine

[GATE 2008 : IISc Bangalore]

- Q.16** If L and  $\bar{L}$  are recursively enumerable then L is
- (A) Regular  
(B) Context free  
(C) Context sensitive  
(D) Recursive

[GATE 2008 : IISc Bangalore]

- Q.17** Which of the following statement is/are FALSE?
- For every non-deterministic Turing machine, there exists an equivalent deterministic Turing machine
  - Turing recognizable languages are closed under union and complementation
  - Turing decidable languages are closed under intersection and complementation
  - Turing recognizable languages are closed under union and intersection
- (A) 1 and 4 only    (B) 1 and 3 only  
(C) 2 only            (D) 3 only

[GATE 2013 : IIT Bombay]

- Q.18** Let L be a language and  $\bar{L}$  be its complement. Which one of the following is NOT a viable possibility?
- (A) Neither L nor  $\bar{L}$  is recursively enumerable (r.e.).  
(B) One of L and  $\bar{L}$  is r.e. but not recursive; the other is not r.e.  
(C) Both L and  $\bar{L}$  are r.e. but not recursive.  
(D) Both L and  $\bar{L}$  are recursive.

[GATE 2014 : IIT Kharagpur]

- Q.19** Let  $\langle M \rangle$  be the encoding of a Turing machines as a string over  $\Sigma = \{0,1\}$ . Let L =  $\{\langle M \rangle \mid M \text{ is a Turing machine that accepts a string of length } \leq 2014\}$ . Then, L is
- (A) Decidable and recursively enumerable  
(B) Un-decidable but recursively enumerable  
(C) Un-decidable and not recursively enumerable  
(D) Decidable but not recursively enumerable

[GATE 2014 : IIT Kharagpur]

- Q.20** For any two languages  $L_1$  and  $L_2$  such that  $L_1$  is context-free and  $L_2$  is recursively enumerable but not recursive, which of the following is / are necessarily true?
- $\bar{L}_1$  (complement of  $L_1$ ) is recursive
  - $\bar{L}_2$  (complement of  $L_2$ ) is recursive
  - $\bar{L}_1$  is context-free
  - $L_1 \cup L_2$  is recursively enumerable
- (A) I only  
(B) III only  
(C) III and IV only  
(D) I and IV only

[GATE 2015 : IIT Kanpur]

- Q.21** Consider the following types of languages :
- $L_1$  : Regular,  $L_2$  : Context-free,  
 $L$  : Recursive.

$L_4$  : Recursively enumerable.

Which of the following is/are TRUE?

- I.  $\bar{L}_3 \cup L_4$  is recursively enumerable
  - II.  $\bar{L}_2 \cup L_3$  is recursive
  - III.  $L_1^* \cap L_2$  is context-free
  - IV.  $L_1 \cup \bar{L}_2$  is context-free
- (A) I only  
 (B) I and III only  
 (C) I and IV only  
 (D) I, II and III only

**[GATE 2016 : IISc Bangalore]**

**Q.22** Consider the following languages.

$L_1 = \{ \langle M \rangle \mid M \text{ takes at least } 2016 \text{ steps on some input} \}$ ,

$L_2 = \{ \langle M \rangle \mid M \text{ takes at least } 2016 \text{ steps on all inputs} \}$  and

$L_3 = \{ \langle M \rangle \mid M \text{ accepts } \epsilon \}$ ,

Where for each Turing machine  $M$ ,  $\langle M \rangle$  denotes a specific encoding of  $M$ .

Which one of the following is TRUE?

- (A)  $L_1$  is recursive and  $L_2, L_3$  are not recursive  
 (B)  $L_2$  is recursive and  $L_1, L_3$  are not recursive  
 (C)  $L_1, L_2$  are recursive and  $L_3$  is not recursive  
 (D)  $L_1, L_2, L_3$  are recursive

**[GATE 2016 : IISc Bangalore]**

**Q.23** Let  $L(R)$  be the language represented by regular expression  $R$ . Let  $L(G)$  be the language generated by a context free grammar  $G$ . Let  $L(M)$  be the language accepted by a Turing machine  $M$ .

Which of the following decision problems are undecidable?

- I. Given a regular expression  $R$  and a string  $w$ , is  $w \in L(R)$ ?
  - II. Given a context free grammar  $G$ , is  $L(G) = \emptyset$ ?
  - III. Given a context free grammar  $G$ , is  $L(G) = \Sigma^*$  for some alphabet  $\Sigma$ ?
  - IV. Given a Turing machine  $M$  and a string  $w$ , is  $w \in L(M)$ ?
- (A) I and IV only

(B) II and III only  
 (C) II, III and IV only  
 (D) III and IV only

**[GATE 2017 : IIT Roorkee]**

**Q.24** The set of all recursively enumerable languages is

- (A) closed under complementation.
- (B) closed under intersection.
- (C) a subset of the set of all recursive languages.
- (D) an uncountable set.

**[GATE 2018 : IIT Guwahati]**

**Q.25** Consider the following problems.  $L(G)$  denotes the language generated by a grammar  $G$ .  $L(M)$  denotes the language accepted by a machine  $M$ .

- (I) For an unrestricted grammar  $G$  and a string  $w$ , whether  $w \in L(G)$
- (II) Given a Turing machine  $M$ , whether  $L(M)$  is regular
- (III) Given two grammars  $G_1$  and  $G_2$ , whether  $L(G_1) = L(G_2)$
- (IV) Given an NFA  $N$ , whether there is a deterministic PDA  $P$  such that  $N$  and  $P$  accept the same language.

Which one of the following statements is correct?

- (A) Only I and II are undecidable
- (B) Only III is undecidable
- (C) Only II and IV are undecidable
- (D) Only I, II and III are undecidable

**[GATE 2018 : IIT Guwahati]**

**Q.26** Which of the following problems are un-decidable?

- (A) Membership problem in context-free languages.
- (B) Whether a given context-free language is regular.
- (C) Whether a finite state automation halts on all inputs.
- (D) Membership problem for type 0 languages

**[GATE 1989 : IIT Kanpur]**

**Q.27** It is undecidable whether :

- (A) An arbitrary Turing machine halts after 100 steps.
- (B) A Turing machine prints a specific letter.

- (C) A Turing machine computes the product of two numbers.  
(D) None of the above.

**[GATE 1990 : IISc Bangalore]**

**Q.28** Which one of the following is the strongest correct statement about a finite language over some finite alphabet  $\Sigma$ ?

- (A) It could be un-decidable  
(B) It is Turing-machine recognizable  
(C) It is a regular language  
(D) It is a context-sensitive language

**[GATE 1991 : IIT Madras]**

**Q.29** Which of the following statements is false?

- (A) The halting problem for Turing machines is un-decidable  
(B) Determining whether a context free grammar is un-decidable  
(C) Given two arbitrary context free grammars  $G_1$  and  $G_2$ , it is undecidable whether  $L(G_1) = L(G_2)$   
(D) Given two regular grammars  $G_1$  and  $G_2$ , it is un-decidable whether  $L(G_1) = L(G_2)$

**[GATE 1996 : IISc Bangalore]**

**Q.30** Which of the following are decidable?

1. Whether the intersection of two regular language is infinite
  2. Whether a given context-free language is regular
  3. Whether two push-down automata accept the same language
  4. Whether a given grammar is context-free
- (A) 1 and 2      (B) 1 and 4  
(C) 2 and 3      (D) 2 and 4

**[GATE 2008 : IISc Bangalore]**

**Q.31** Which of the following problems are decidable?

1. Does a given program ever produce an output?
2. If  $L$  is a context-free language, then, is  $\bar{L}$  also context-free?
3. If  $L$  is a regular language, then, is  $\bar{L}$  also regular?



4. If  $L$  is a recursive language, then is  $\bar{L}$  also recursive?  
(A) 1,2,3,4      (B) 1,2  
(C) 2,3,4      (D) 3,4

**[GATE 2012 : IIT Delhi]**

**Q.32** Which of the following is/are undecidable?

1.  $G$  is a CFG. Is  $L(G) = \Phi$ ?
2.  $G$  is a CFG. Is  $L(G) = \Sigma^*$ ?
3.  $M$  is a Turing machines. Is  $L(M)$  regular?
4.  $A$  is a DFA and  $N$  is an NFA. Is  $L(A) = L(N)$ ?  
(A) 3 only      (B) 3 and 4 only  
(C) 1, 2 and 3 only      (D) 2 and 3 only

**[GATE 2013 : IIT Bombay]**

**Q.33** Which one of the following problems is un-decidable?

- (A) Deciding if a given context-free grammar is ambiguous.
- (B) Deciding if a given string is generated by a given context-free grammar.
- (C) Deciding if the language generated by a given context-free grammar is empty.
- (D) Deciding if the language generated by a given context-free grammar is finite.

**[GATE 2014 : IIT Kharagpur]**

**Q.34** Which of the following decision problems are undecidable?

- I. Given NFAs  $N_1$  and  $N_2$ , is  $L(N_1) \cap L(N_2) = \emptyset$ ?
- II. Given a CFG  $G = (N, \Sigma, P, S)$  and a string  $x \in \Sigma^*$ , does  $x \in L(G)$ ?
- III. Given CFGs  $G_1$  and  $G_2$  is  $L(G_1) = L(G_2)$ ?
- IV. Given a TM  $M$ , is  $L(M) = \emptyset$ ?  
(A) I and IV only  
(B) II and III only  
(C) III and IV only  
(D) II and IV only

**[GATE 2016 : IISc Bangalore]**

**Q.35** Define languages  $L_0$  and  $L_1$  as follows

$$L_0 = \{ \langle M, w, 0 \rangle \mid M \text{ halts on } w \}$$



$L_1 = \{<M, w, i> | M \text{ does not halts on } w\}$

Here  $<M, w, i>$  is a triplet, whose first component,  $M$  is an encoding of a Turing machine, second component,  $w$ , is a string, and third component,  $i$ , is a bit.

Let  $L = L_0 \cup L_1$ . Which of the following is true?

- (A)  $L$  is recursively enumerable, but  $\bar{L}$  is not
- (B)  $\bar{L}$  is recursively enumerable, but  $L$  is not
- (C) Both  $L$  and  $\bar{L}$  are recursive
- (D) Neither  $L$  nor  $\bar{L}$  is recursively enumerable

**[GATE 2003 : IIT Madras]**

**Q.36**  $L_1$  is recursively enumerable language over  $\Sigma$ . An algorithm  $A$  effectively enumerates its words as  $w_1, w_2, w_3, \dots$

Define another language  $L_2$  over  $\Sigma \cup \{\#\}$  as

$\{w_i \# w_j : w_i, w_j \in L_1, i < j\}$ . Here  $\#$  is a new symbol. Consider the following assertions :

$S_1 : L_1$  is recursive implies  $L_2$  is recursive

$S_2 : L_2$  is recursive implies  $L_1$  is recursive

Which of the following statements is TRUE?

- (A) Both  $S_1$  and  $S_2$  are true
- (B)  $S_1$  is true but  $S_2$  is not necessarily true
- (C)  $S_2$  is true but  $S_1$  is not necessarily true
- (D) Neither is necessarily true

**[GATE 2004 : IIT Delhi]**

**Q.37** Let  $A \leq_m B$  denotes that language  $A$  is mapping reducible (also known as many-to-one reducible) to language  $B$ . Which one of the following is FALSE?

- (A) If  $A \leq_m B$  and  $B$  is recursive then  $A$  is recursive.

- (B) If  $A \leq_m B$  and  $A$  is undecidable then  $B$  is un-decidable.
- (C) If  $A \leq_m B$  and  $B$  is recursively enumerable then  $A$  is recursively enumerable.
- (D) If  $A \leq_m B$  and  $B$  is not recursively enumerable then  $A$  is not recursively enumerable.

**[GATE 2004 : IIT Delhi]**

**Q.38** Let  $X$  be a recursive language and  $Y$  be a recursively enumerable but not recursive language. Let  $W$  and  $Z$  be two languages such that  $\bar{Y}$  reduce to  $W$  and  $Z$  reduces to  $\bar{X}$  (reduction means the standard many-one reduction). Which one of the following statements is TRUE?

- (A)  $W$  can be recursively enumerable and  $Z$  is recursive.
- (B)  $W$  can be recursive and  $Z$  is recursively enumerable.
- (C)  $W$  is not recursively enumerable and  $Z$  is recursive.
- (D)  $W$  is not recursively enumerable and  $Z$  is not recursive.

**[GATE 2016 : IISc Bangalore]**

**Q.39** Let  $A$  and  $B$  be finite alphabets and let  $\#$  be a symbol outside both  $A$  and  $B$ . Let  $f$  be a total function from  $A^*$  to  $B^*$ . We say  $f$  is computable if there exists a Turing machine  $M$  which given an input  $x$  in  $A^*$ , always halts with  $f(x)$  on its tape. Let  $L_f$  denote the languages  $\{x\#f(x) | x \in A^*\}$ .

Which of the following statement is TRUE:

- (A)  $f$  is computable if and only if  $L_f$  is recursive
- (B)  $f$  is computable if and only if  $L_f$  is recursively enumerable
- (C) If  $f$  is computable then if  $L_f$  is recursive, but not conversely
- (D) If  $f$  is computable then if  $L_f$  is recursive, but not conversely

**[GATE 2017 : IIT Roorkee]**



**Q.40** Consider the following sets:

- S1. Set of all recursively enumerable languages over the alphabet {0, 1}
- S2. Set of all synthetically valid C programs
- S3. Set of all languages over the alphabet {0, 1}
- S4. Set of all non – regular languages over the alphabet {0, 1}

Which of the above sets are uncountable?

- (A) S1 and S2
- (B) S3 and S4
- (C) S2 and S3
- (D) S1 and S4

**[GATE 2019 : IIT Madras]**

**Q.41** Consider the following problem  $X$ . "Given a Turing machine  $M$  over the input alphabet  $\Sigma$ , any state  $q$  of  $M$  and a word  $w \in \Sigma^*$ , does the computation of  $M$  on  $w$  visit the state  $q$ "

Which of the following statements about  $X$  is correct?

- (A)  $X$  is decidable
- (B)  $X$  is un-decidable but partially decidable
- (C)  $X$  is un-decidable and not even partially decidable
- (D)  $X$  is not a decision problem

**[GATE 2001 : IIT Kanpur]**

**Q.42** Consider three decision problems  $P_1$ ,  $P_2$  and  $P_3$ . It is known that  $P_1$  is decidable and  $P_2$  is un-decidable. Which of the following is TRUE?

- (A)  $P_3$  is decidable if  $P_1$  is reducible to  $P_3$

- (B)  $P_3$  is un-decidable if  $P_3$  is reducible to  $P_2$
- (C)  $P_3$  is un-decidable if  $P_2$  is reducible to  $P_3$
- (D)  $P_3$  is decidable if  $P_3$  is reducible to  $P_2$ 's complement

**[GATE 2005 : IIT Bombay]**

**Q.43** Let  $\Sigma$  be a finite non-empty alphabet and let  $2^{\Sigma^*}$  be the power set of  $\Sigma^*$ . Which one of the following is TRUE?

- (A) Both  $2^{\Sigma^*}$  and  $\Sigma^*$  are countable
- (B)  $2^{\Sigma^*}$  is countable and  $\Sigma^*$  is uncountable
- (C)  $2^{\Sigma^*}$  is uncountable and  $\Sigma^*$  is countable
- (D) Both  $2^{\Sigma^*}$  and  $\Sigma^*$  are uncountable

**[GATE 2014 : IIT Kharagpur]**

**Q.44** Consider two decision problems  $Q_1$ ,  $Q_2$  such that  $Q_1$  reduces in polynomial time to 3-SAT and 3-SAT reduces in polynomial time to  $Q_2$ . Then which one of the following is consistent with the above statement?

- (A)  $Q_1$  is in NP,  $Q_2$  is NP hard.
- (B)  $Q_2$  is in NP,  $Q_1$  is NP hard.
- (C) Both  $Q_1$  and  $Q_2$  in NP.
- (D) Both  $Q_1$  and  $Q_2$  are NP hard.

**[GATE 2015 : IIT Kanpur]**

### Practice Questions

**Q.1** Which of the following statements is false?

- (A) Every context-sensitive language is recursive
- (B) The set of all languages that are not recursively enumerable is countable.
- (C) The family of recursively enumerable languages is closed under union.

- (D) The families of recursively enumerable and recursive languages are closed under reversal.

**Q.2** Which of the following problems is undecidable?

- (A) To determine if two finite automata are equivalent
- (B) To determine if two finite automata are equivalent



- (C) Finiteness problem for finite automata
- (D) Ambiguity problem for context free grammar

**Q.3** Let  $S$  be an NP-complete problem.  $Q$  and  $R$  are other two problems not known to be NP.  $Q$  is polynomial time reducible to  $S$  and  $S$  is polynomial time reducible to  $R$ . Which of the following statements is true?

- (A)  $R$  is NP-complete
- (B)  $R$  is NP-hard
- (C)  $Q$  is NP-complete
- (D)  $Q$  is NP-hard

**Q.4** Let  $L(R)$  be the language represented by regular expression  $R$ . Let  $L(G)$  be the language generated by a context free grammar  $G$ . Let  $L(M)$  be the language accepted by a Turing machine  $M$ . Which of the following decision problems are decidable? I. Whether  $L(G)$  is deterministic context free language? II. Whether  $L(G_1) \cap L(G_2)$  is a context free language, where  $G_1$  and  $G_2$  are deterministic grammar? III. Given a context-free grammar  $G$ , is  $L(G) = \Sigma^*$  for some alphabet  $\Sigma$ ? IV. Given a Turing machine  $M$  and a string  $w$ , is  $w \in L(M)$ ?

- (A) III and IV only
- (B) II and IV only
- (C) I and II only
- (D) None of the above

**Q.5** Which of the following statement is false?

- (A) Checking the ambiguity of CFL is decidable.
- (B) Checking whether a given context free language is regular is decidable.
- (C) Checking whether a given context free language is empty is decidable.
- (D) Both A and B

**Q.6** Which of the following statement is false?

- (A) Checking the ambiguity of CFL is decidable.

- (B) Checking whether a given context free language is regular is decidable.

- (C) Checking whether a given context free language is empty is decidable.
- (D) Both A and B

**Q.7** Given the following two statements:  
 $S_1$ : If  $L_1$  and  $L_2$  are recursively enumerable languages over  $\Sigma$ , then  $L_1 \cup L_2$  and  $L_1 \cap L_2$  are also recursively enumerable.  
 $S_2$ : The set of recursively enumerable languages is countable.  
Which of the following is correct ?

- (A)  $S_1$  is correct and  $S_2$  is not correct
- (B)  $S_1$  is not correct and  $S_2$  is correct
- (C) Both  $S_1$  and  $S_2$  are not correct.
- (D) Both  $S_1$  and  $S_2$  are correct.

**Q.8** A problem whose language is recursive is called?

- (A) Unified problem
- (B) Boolean function
- (C) Recursive problem
- (D) Decidable

**Q.9** Which of the following is FALSE with respect to possible outcomes of executing a Turing Machine over a given input?

- (A) it may halt and accept the input
- (B) it may halt by changing the input
- (C) it may halt and reject the input
- (D) it may never halt

**Q.10** Which of the following pairs have different expressive power?

- (A) Single-tape-turing machine and multi-dimensional turing machine.
- (B) Multi-tape turing machine and multi-dimensional turing machine.
- (C) Deterministic push down automata and non-deterministic pushdown automata.
- (D) Deterministic finite automata and Non-deterministic finite automata

**Q.11** What is the highest type number that can be assigned to the following grammar?

$$\begin{aligned} S &\rightarrow Aa \\ A &\rightarrow Ba \end{aligned}$$



B → abc



**Q.12** If  $L$  and  $P$  are two recursively enumerable languages, then they are not closed under

- (A) Kleene Star  $L^*$  of  $L$
  - (B) Intersection  $L \cap P$
  - (C) Union  $L \cup P$
  - (D) Set Difference

**Q.13** Which of the following statements is not correct?

- (A) Every recursive language is recursively enumerable.
  - (B)  $L = \{0^n 1^n 0^n \mid n=1, 2, 3, \dots\}$  is recursively enumerable.
  - (C) Recursive languages are closed under intersection.
  - (D) Recursive languages are not closed under intersection.

**Q.14** Let  $L = \{a^p \mid p \text{ is a prime}\}$ . Then which of the following is true?

- (A) It is not accepted by a Turing Machine
  - (B) It is regular but not context free
  - (C) It is context free but not regular
  - (D) It is neither regular nor context free, but accepted by a Turing Machine

**Q.15** A language L is called Turing-decidable (or just decidable), if there exists a Turing Machine M such that on input x, M accepts if  $x \in L$ , and M rejects otherwise. L is called undecidable if it is not decidable. Which of following option is false?

- (A) The class of decidable languages is closed under complement.
  - (B) The class of decidable languages is closed under union
  - (C) The class of decidable languages is closed under intersection
  - (D) None of these

## Answer Key

## **Classroom Practice Questions**

1.	B	2.	C	3.	D	4.	D	5.	A, C
6.	C	7.	B	8.	B	9.	A, C	10.	D
11.	A	12.	B	13.	D	14.	B	15.	D
16.	D	17.	D	18.	B	19.	B	20.	D
21.	D	22.	C	23.	D	24.	B	25.	D
26.	B, D	27.	B	28.	C	29.	D	30.	B
31.	D	32.	D	33.	A	34.	C	35.	D
36.	A	37.	B	38.	C	39.	A	40.	B
41.	B	42.	C	43.	C	44.	A		

## Practice Questions

1.	B	2.	D	3.	B	4.	D	5.	D
6.	D	7.	D	8.	D	9.	B	10.	C
11.	D	12.	D	13.	D	14.	D	15.	D

□ □ □



## **Classroom Questions**

- Q.1** A finite state machine with the following state table has a single input X and a single output Z.

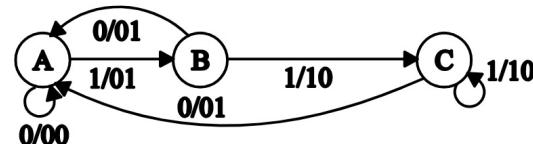
Present state	Next state Z	
	X = 1	X = 0
A	D,0	B,0
B	B,1	C,1
C	B,0	D,1
D	B,1	C,0

If the initial state is unknown, then the shortest input sequence to reach the final state C is



[GATE 1995 : IIT Kanpur]

- Q.2** The finite state machine described by the following state diagram with A as starting state, where an arc label is  $x/y$  and x stands for 1-bit input and y stands for 2-bit output



- (A) Outputs the sum of the present and the previous bits of the input
  - (B) Outputs 01 whenever the input sequence contains 11
  - (C) Outputs 00 whenever the input sequence contains 10
  - (D) None of the above

[GATE 2002 : IISc Bangalore]

## Practice Questions

- Q.1** Given the following state table of an FSM with two states A and B, one input and one output :

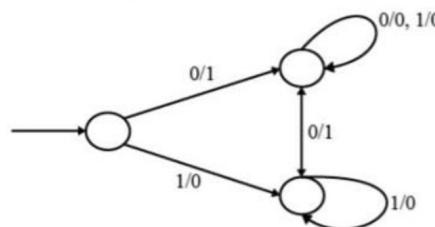
Present state A	Present state B	Input	Next state A	Next state B	Output
0	0	0	0	0	1
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	1	0	0
0	0	1	0	1	0
0	1	1	0	0	1
1	0	1	0	1	1
1	1	1	0	0	1

If the initial state is  $A = 0, B = 0$ , what is the minimum length of an input string

- which will take the machine to the state  $A=0, B=1$  with Output = 1?



- Q.2** The FSM (finite state machine) machine pictured in the figure above



- (A) Complements a given bit pattern
  - (B) Finds 2's complement of a given bit pattern
  - (C) Increments a given bit pattern by 1
  - (D) Changes the sign bit

### Classroom Practice Questions

1.	B	2.	A							
<b>Practice Questions</b>										
1.	A	2.	C							

□□□



## General Aptitude Part

### Q.1 to Q.5 Carry ONE Mark Each

#### Question 1

The \_\_\_\_\_ is too high for it to be considered \_\_\_\_\_.

- (A) Fare/Fair      (B) Fair/ Fare  
 (C) Faer/ Fair      (D) Fare/fare

**Ans. A**

**Sol.** Fare : The amount of money you pay to travel by Bus, Train, Taxi, etc.

Fair : appropriate and acceptable in a particular situation.

The **fare** is too high for it to be considered **fair**.

Hence, the correct option is (A).

#### Question 2

A palindrome is a word that reads the same forward and backward. In a game of words, a player has the following two plates painted with letters



From the additional plates given in the options, which one of the combinations of additional plates would allow the players to construct in five letter palindrome. The players should use all five plates exactly once. The plates can be rotated in their plane.

- (A)  R     V     R    (B)  Z     E     D  
 C)  I     T     Y    (D)  D     I     T

**Ans. A**

**Sol. Given :**

A palindrome is a word that reads the same forward and backward.

In a game of words, a player has the two plates painted with letters



The player need to construct a five letter palindrome by using those given two letter plates.

The player should use all five plates exactly one and the plates can be rotated in their plane.

According to option A,



Can be used by rotating and combining with given letter plates.

The palindrome in a word will look like



That word will give same meaning either reads forward or backwards.

Hence, the correct option is (A).

#### Question 3

Let  $r$  be a root of the expression

$$x^2 + 2x + 6 = 0$$

Then the value of the expression

$$(r+2)(r+3)(r+4)(r+5)$$

- (A) - 51      (B) 126

- (C) - 126      (D) 51

**Ans. C**

**Sol.** Given :  $r$  is a root of the expression

$$x^2 + 2x + 6 = 0$$

$$r^2 + 2r + 6 = 0, r^2 + 2r = -6$$

$$= (r+2)(r+3)(r+4)(r+5)$$

$$= (r^2 + 5r + 6)(r^2 + 9r + 20)$$

$$= (r^2 + 2r + 3r + 6)(r^2 + 2r + 7r + 20)$$

$$r^2 + 2r = -6,$$

$$= (-6 + 3r + 6)(-6 + 7r + 20)$$

$$= 3r(7r + 14)$$

$$= 21r(r + 2)$$

$$= 21(r^2 + 2r) = 21 \times (-6) = -126$$

Hence, the correct option is (C).

#### Question 4

A function  $y(x)$  is defined in the interval  $[0,1]$  on the  $x$ -axis as

Hence, the correct option is (A).

#### Question 5

Given below are four statements

Statement 1 : All students are inquisitive

$$y(x) = \begin{cases} 2 & \text{if } 0 \leq x \leq \frac{1}{3} \\ 3 & \text{if } \frac{1}{3} \leq x \leq \frac{3}{4} \end{cases}$$

$$\begin{cases} 5 & 4 \\ 1 \text{ if } \frac{3}{4} \leq x \leq 1 \end{cases}$$

Which one of the following is the area under the curve for the interval  $[0, 1]$  on the  $x$ -axis?

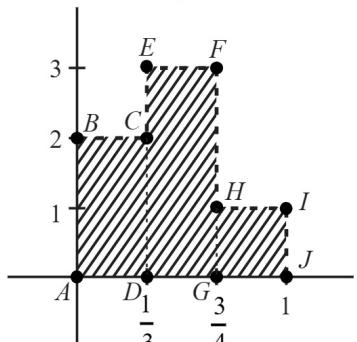
- |                    |                   |
|--------------------|-------------------|
| (A) $\frac{13}{6}$ | (B) $\frac{6}{5}$ |
| (C) $\frac{6}{13}$ | (D) $\frac{5}{6}$ |

**Ans. A**

**Sol.** Given : A function  $y(x)$  is defined in the interval  $[0, 1]$  on the  $x$ -axis as

$$y(x) = \begin{cases} 2 & \text{if } 0 \leq x \leq \frac{1}{3} \\ 3 & \text{if } \frac{1}{3} \leq x \leq \frac{3}{4} \\ 1 & \text{if } \frac{3}{4} \leq x \leq 1 \end{cases}$$

Graph of  $y(x)$  vs  $x$ , is drawn as shown below figure,



The Area under the curve for the interval  $[0, 1]$  on the  $x$ -axis will be define as

$$\begin{aligned} &= \text{Area of } ABCD + \text{Area of } DEFG \\ &\quad + \text{Area of } GHIJ \\ \text{Area of the rectangle} &= \text{Length} \times \text{Breadth} \\ &= 2 \times \frac{1}{3} + 3 \times \left( \frac{3}{4} - \frac{1}{3} \right) + 1 \left( 1 - \frac{3}{4} \right) \\ &= \frac{2}{3} + \frac{15}{12} + \frac{1}{4} = \frac{2}{3} + \frac{3}{2} = \frac{13}{6} \end{aligned}$$

inquisitive

Statement 2 : Some students are inquisitive

Statement 3 : No students are inquisitive

Statement 4 : Some students are not inquisitive

For the given four statements, find the statements what cannot be true, simultaneously, assuming that there is at least one student in the class

- (A) Statement 1 and Statement 3
- (B) Statement 3 and Statement 4
- (C) Statement 2 and Statement 4
- (D) Statement 1 and Statement 2

**Ans. A**

**Sol.** Given :

Statement 1 : All student are inquisitive.

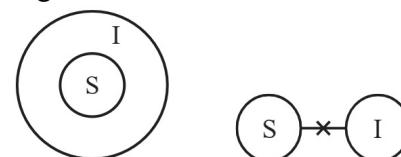
Statement 2 : some student are inquisitive

Statement 3 : No student are inquisitive.

Statement 4 : Some student are not inquisitive

For finding the statement which cannot be true, simultaneously, we will go through with all options, According to option (A).

Statement 1 and statement 3 can not be true simultaneously by the venn diagram of the statement



We can say that statement 1 and statement 3 cannot be true simultaneously.

According to option (B)

Statement 3 and Statement 4 can not be true simultaneously. By the venn diagram of the statements



We can say that both the statement can be true simultaneously. As when no student is inquisitive some students are also not inquisitive.



getting an orange ball in the next drawn?

- (A)  $\frac{23}{50}$
- (B)  $\frac{19}{50}$
- (C)  $\frac{8}{25}$
- (D)  $\frac{1}{2}$

According to option (C).

Statement 2 and statement 4 can not be true simultaneously by the venn diagram of the statements



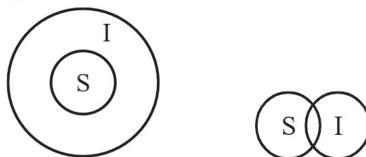
We can say that both the statement can be true simultaneously by that diagram.



By this diagram it is possible of the same time that some students are inquisitive and some students are not inquisitive.

According to option (D).

Statement 1 and statement 2 can be true simultaneously by the venn diagram of the statement.



By this diagram we can say that both the statement can be true simultaneously.

As, when all students are inquisitive then some students are already inquisitive.

Hence, the correct option is (A).

#### Question 6

A box contains five balls of same size and shape. Three of them are green coloured balls and two of them are orange coloured balls. Balls are drawn from the box one at a time. If a green ball is drawn it is not replaced. If an orange ball is drawn it is replaced with another orange ball. First ball is drawn. What is the probability of

**Ans. A**

**Sol. Given :**

A box contains five balls of same size and shape.

Three of them are green coloured balls and two of them are orange coloured balls.

Balls are drawn from the box one at a time.

If a green ball is drawn it is not replaced.

If an orange ball is drawn it is replaced with another orange ball.

For the probability of getting an orange ball in the next drawn we have two different cases.

Case I : One Green and one Orange

$$P(E) = \frac{3}{5} \times \frac{2}{4} = \frac{6}{20}$$

As we can't replace ball after getting green ball so when we are drawing orange ball as that time only 4 balls are remaining in box.

Case II : One Orange and one orange

$$P(E) = \frac{2}{5} \times \frac{2}{5} = \frac{4}{25}$$

As with orange we can replace ball so when we are drawing record orange ball at time 5 balls are present in box.

$$P(E) = \text{case 1} + \text{case 2}$$

$$P(E) = \frac{6}{20} + \frac{4}{25} = \frac{23}{50}$$

Hence, the correct option is (A).

#### Question 7

Some people believe that "what gets measured, improves". Some other believe that "what gets measured, gets gamed". One possible reason for the difference in the beliefs is the work culture in organization. In

organizations with good work culture, metrics help improve outcomes. However the same metrics are counterproductive in organizations with poor work culture.

Which one of the following is the correct logical inference based on the information in the above passage?

Which one of the following statements is correct based on the above information?

- (A) P cannot be placed at a corner
- (B) U cannot be placed at a mid point
- (C) S cannot be placed at a corner
- (D) R cannot be placed at a corner

- (A) Metrics are useful in organization with poor work culture  
 (B) Metrics are always counterproductive in organizations with good work culture  
 (C) Metrics are never useful in organization with good work culture  
 (D) Metrics are useful in organizations with good work culture

**Ans. D**

**Sol.** According to the given information,  
 Option (A); can not be inferred as metrics are counterproductive in organization with poor work culture.  
 Option (B); can not be inferred as metrics are counterproductive in organization with poor work culture not good work culture.  
 Option (C); can not be inferred as metrics help improve outcomes, in organization with good work culture.  
 Option (D); can be inferred as metrics are useful in organization with good work culture, is mentioned.

Hence, the correct option is (D).

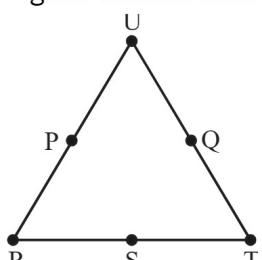
#### Question 8

The corners and the mid point of a triangle are name using distinct letters P, Q, R, S, T and U but not necessarily in the same order. Consider the following statements

- The line joining P and R is parallel to the line joining Q and S
- P is placed on the side opposite to the corner T
- S and U cannot be placed in the same side

4

R cannot be placed at a corner by but by the figure shown below,



**Ans. C**

**Sol. Given :**

The corners and midpoint of a triangle are name using distinct P, Q, R, S, T and U.

The line joining P and R is parallel to the line joining Q and S.

P is placed on the side opposite to the care T.

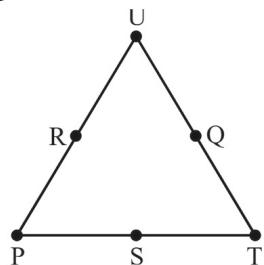
S and U cannot be placed in the same side.

with this given information we can draw multiple diagram which satisfy all the given conditions.

To prove given statement in the options which one is correct, we will draw different diagrams for each option.

According to option A,

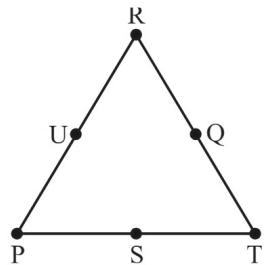
P cannot be placed at a corner but by the figure shown below,



P can be placed at a corner.

According to option B,

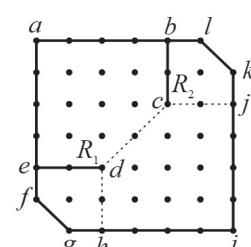
U cannot be placed at a mid point but by the figure shown below,



U can be placed at a midpoint.

According to option D,

GATE CSE-2022

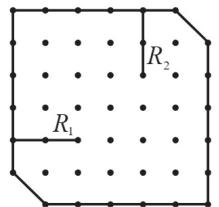


In the figure shown above we can clearly see the least number of additional straight ropes shown by

R can be placed at a corner.  
According to option C,  
S cannot be placed at a corner  
By all the possible diagram only that condition is not possible.  
Hence, the correct option is (C).

## Question 9

A plot of land must be divided between four families. They instant their individual plots to be similar in shape, not necessarily in area. The land has equally spaced placed marked as dots on the below figure. Two ropes  $R_1$  and  $R_2$  are already present and cannot be moved. What is the least number of additional straight ropes needed to needs to divided plot? A straight rope can pass through three poles that are aligned in a straight line






**Ans. B**

**Sol.** Given :

A plot of land must be divided between four families.

Divided plots should be similar in shape not necessarily in area.

Two ropes  $R_1$  and  $R_2$  are already present in the plot and cannot be moved.

A straight rope can pass through three poles that are in a straight line.

dotted line needed to divided plot in four parts.

The plots are abcde, bcjkl, cdhij and efghd.

Which are same in shape but not in size.

Hence, the correct option is (B).

## Question 10

In a currently conducted National Entrance Test, Boys constituted 65% of those who appeared for the test. Girls constituted the remaining candidates and they accounted for 60% of the qualified candidates. Which one of the following is the correct logical inference based on the information provided in the above passage?

- (A) Equal number of boys and girls appeared for the test
  - (B) The number of boys who qualified the test is less than the number of girls who qualified.
  - (C) The number of boys who appeared for the test is less than the number of girls who appeared.
  - (D) Equal number of boys and girls qualified.

**Ans. B**

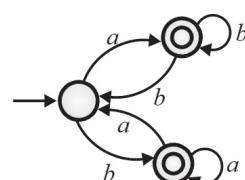
**Sol.** **Given :** In a currently conducted National Entrance Test, Percentage of boys who appeared for the test is 65% Percentage of girls who appeared for the test is  $100\% - 65\% = 35\%$  Percentage of qualified girls is 60% of appeared girls

Accounted percentage of total qualified boys and girls is equals to appeared percentage of girls.

Let, the number of students is  $x$ ,  
 Number of boys appeared for the test  
 is  $0.65x$

Number of girls appeared for the test  
is  $0.35x$

$$\text{Number of qualified girls} = 0.35x \times \frac{3}{5}$$



- (A)  $ab^*bab^* + ba^*aba^*$
  - (B)  $(ab^*b)^*ab^* + (ba^*a)^*ba^*$
  - (C)  $(ab^*b + ba^*a)^*(a^* + b^*)$
  - (D)  $(ba^*a + ab^*b)^*(ab^* + ba^*)$

Ans D

$= 0.21x$   
 Number of qualified boys  
 $= 0.35x - 0.21x = 0.14x$   
 So, the number of boys who qualified the test is less than the number of girls who qualified.  
 Hence, the correct option is (B).

### Technical Part

#### Q.11 to Q.35 Carry ONE Mark Each

##### Question 11                    698808206

Which one of the following statement is TRUE for all positive function  $f(n)$ ?

- (A)  $f(n^2) = \Theta(f(n)^2)$ , when  $f(n)$  is a polynomial
- (B)  $f(n^2) = o(f(n)^2)$
- (C)  $f(n^2) = O(f(n^2))$ , when  $f(n)$  is an exponential function
- (D)  $f(n^2) = \Omega(f(n)^2)$

**Ans.** A

**Sol.** A : It need not be true for a function which is decreasing.

B : An exponential function may be increasing or decreasing, so this condition may not always be true.

C : It always holds because if we square the input variable, then the highest order in the polynomial will also get squared.

D : It is not true in cases when  $f(n)$  is a polynomial function.

##### Question 12                    698808207

Which one of the following regular expression correctly represents the language of the finite automaton given below?

Where Q is the regular expression for set of all strings that end up at the state Q R is the regular expression for set of all strings.

$$Q = P_{ab}^*$$

$$R = Pba$$

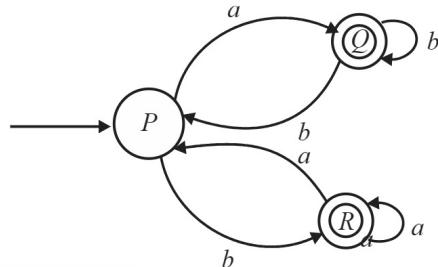
$$\text{So, } R \in (iv) = Q + R$$

$$= Pab^* + Pba^*$$

$$= P(ab^* + ba^*)$$

$$R \in (iv) = [(ab^*b) + (ba^*a)]^* (ab^* + ba^*)$$

**Sol. Given : From**



Finite automata

We have two final state  $Q, R$

First, find the regular expression for set of all strings that end up at the initial state P when running the given NFA.

Lets call P.

Note that, P is the initial state so NFA execution starts at P. Now to end up on state P at the end of the string we can do any of the following, in any order, any number of times:

1. Read "a" go to state Q then read any number of b's, then read b to come back to state P. so,  $1 ab^*b$
2. Read "b" go to state R, them read any number of a's, then read a to come back

$$\text{So, } P = (1+2)^*$$

$$= [(ab^*b) + (ba^*a)]^*$$

Because we can do 1 or 2 in any order, any number of times:

Now, we want to find language of the given NFA if, Whose final states are Q,R

So,

$$R \in (iv) = Q + R$$



**Statement C:** Data flow analysis is necessary for run time memory management.

**False**, data flow analysis is used for the optimization of codes in control flow graph.

**Statement D:** LR (1) parsing is sufficient for Deterministic Context Free Language.

**True**, LR (1) parsing is sufficient for Deterministic Context Free Language. Hence, the correct option is (D).

$$\Rightarrow (ab^m b + ba^m a)(ab^n + ba^n)$$

Hence, the correct option is (D).

### Question 13

**698808208**

Which of the following statements is TRUE?

- (A) The LALR (1) parser for a grammar G cannot have reduce – reduce conflict if the LR (1) parser for G does not have reduce – reduce conflict
- (B) Symbol table is accessed only during the lexical analysis phase.
- (C) Data flow analysis is necessary for run time memory management.
- (D) LR (1) parsing is sufficient for Deterministic Context Free Language.

**Ans. D**

**Sol. Given statements are as follows :**

**Statement A:** The LALR (1) parser for a grammar G cannot have reduce – reduce conflict if the LR (1) parser for G does not have reduce – reduce conflict.

**False,** because the LALR (1) parser for a grammar G can have reduce-reduce conflict, even though LR (1) parser for G does not have reduce-reduce conflict.

**Statement B:** Symbol table is accessed only during the lexical analysis phase.

**False,** symbol table can be accessed by most of the phases of a compiler, like beginning with lexical analysis, and continuing through optimization.

GATE CSE-2022

7

**False,** Let relation R with attributes (A, B, C) with functional dependency set

$$\{AB \rightarrow C, C \rightarrow A\}$$

In this relation all attributes are prime.

**Statement D:** If all attributes of a relation are prime attributes, then relation is in BCNF.

**False,** If all attributes of a relation are prime attributes, then relation is in 3NF, not in BCNF.

Hence, the correct option is (A).

### Question 15

**698808210**

### Question 14

**698808209**

In relational data model which is true.

- (A) A relation with only two attributes is always in BCNF
- (B) BCNF decompositions preserve functional dependencies
- (C) Every relation has at least one non prime attribute
- (D) If all attributes of a relation are prime attributes, then relation is in BCNF.

**Ans. A**

**Sol. Given :**

In relational data model,

Following statements are as follows:

**Statement A :** Relation with only two attributes is always in BCNF

**True,**

$$R(XY)$$

$$\{X \rightarrow Y\} \Rightarrow BCNF$$

$$\{Y \rightarrow X\} \Rightarrow BCNF$$

$$\{X \rightarrow Y, Y \rightarrow X\} \Rightarrow BCNF$$

{no non trivial functional dependency}

$$\Rightarrow BCNF$$

∴ A relation with two attributes is always in BCNF.

**Statement B :** BCNF decompositions preserve functional dependencies

**False,** BCNF can be used to obtain a lossless join decomposition into 3NF but does not ensure dependency preservation.

**Statement C:** Every relation has at least one non prime attribute

So, Best algorithm for the problem takes  $\theta(n)$  time in worst case.

Hence, the correct option is (A).

### Question 16

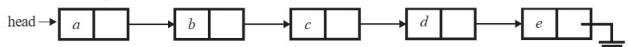
**698808211**

Suppose we are given n keys, m hash table slots and two simple uniform hash functions  $h_1$  and  $h_2$ . Further suppose our hashing scheme uses  $h_1$  for the odd keys and  $h_2$  for the even keys. What is the expected no. of keys in a slot?

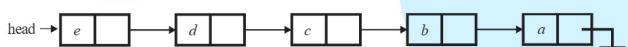
$$(A) \frac{n}{m}$$

$$(B) \frac{m}{n}$$

Consider the problem of reversing a singly linked list. To take an example, given the linked list below,



the reversed linked list should look like



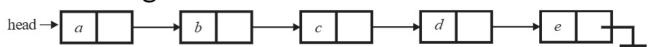
Which one of the following statements is TRUE about the time complexity of algorithms that solve the above problem in  $O(1)$  space?

- (A) The best algorithm for the problem takes  $\Theta(n)$  time in the worst case.
- (B) The best algorithm for the problem takes  $\Theta(n \log n)$  time in the worst case.
- (C) The best algorithm for the problem takes  $\Theta(n^2)$  time in the worst case.
- (D) It is not possible to reverse a singly linked list in  $O(1)$  space.

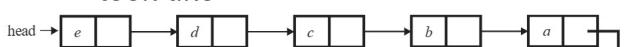
**Ans. A**

**Sol. Given :**

Single linked list



And the reversed linked list should look like



So, From given statements, statement A is suitable for the time complexity of algorithms that solve the given problem in  $O(1)$  space.

Since  $h$  is uniform, we can say it will distribute all keys uniformly. This means if there are 50 keys and 10 slots then each slot will get 5 keys. i.e.,  $n/m$  is answer.

But we can do this using probability. Let  $X$  = the Number of items in slot 1. (Note that I am. only talking about some random slot, say slot 1)

$$X_i = \{1 \text{ if } i^{\text{th}} \text{ item maps to slot 1; otherwise, } 0\}$$

$$\text{So, we can say } X = X_1 + X_2 + \dots + X_n$$

They are asking  $E[X]$

$$E[X] = E[X_1 + X_2 + \dots + X_n]$$

$$E[X] = E[X_1] + E[X_2] + \dots + E[X_n]$$

$m$

$$(C) \frac{n}{2m}$$

$n$

$$(D) \frac{2n}{m}$$

**Ans. A**

**Sol.** **Given :**  $n$  keys,  $m$  hash table slots, and two simple uniform hash functions  $h_1$  and  $h_2$ .

Hashing scheme uses  $h_1$  for the odd keys and  $h_2$  for the even keys.

Uniform hash function definition:

$$Pr[h(x)=i] = 1/m$$

which means for every  $x$ , we have an equal probability of mapping to any of slot  $i$ .

The hash function given in the question is Uniform:

Take any slot  $i$  and calculate the probability of mapping some arbitrary  $x$  to  $i$

$$i.e. Pr[h(x)=i] = ?$$

Let the probability of choosing  $h_1$  is  $p$  and choosing  $h_2$  is  $1-p$  then

$$Pr[h(x)=i] = pPr[h_1(x)=i]$$

$$+ (1-p)Pr[h_2(x)=i]$$

$$\Rightarrow Pr[h(x)=i] = p/m + 1 - p/m \quad (\text{since } h_1 \text{ and } h_2 \text{ both are uniform hash functions}) = 1/m$$

As you see, value of  $p$  does not matter but we can calculate  $p$  as = number of even keys / Total keys



$$(D) R1 = 1001 \text{ and } R2 = 1111$$

**Ans. B**

**Sol. Given :**

$R1$  and  $R2$  are two 4 – bit registers and they stored numbers in 2's complement form.

We know that the range of 2's complement numbers representable with 4 bits are  $-8$  to  $+7$ .

Then, if the result of the operation  $R1 + R2$  is out of the range  $-8$  to  $7$ , then it is overflow

So, from given options,

**Option (A) :**

$$R1 = 1011 \text{ and } R2 = 1110$$

$P(X_i=1) = 1/m$  because whatever happens within the black box ( $h_1$  or  $h_2$ ), the overall hash function will be uniform.

$$E[X_i] = 1P(X_i=1) + 0P(X_i=0) = 1/m$$

$$\text{So, } E[X] = n/m.$$

Hence, the correct option is (A).

### Question 17

**698808212**

Which one of the following facilitates transfer of bulk data from hard disk to main memory with the highest throughput?

- (A) DMA based I/O transfer
- (B) Interrupt driven I/O transfer
- (C) Polling based I/O transfer
- (D) Programmed I/O transfer

**Ans. A**

**Sol.** from the given options, option (A) DMA based input output transfer facilitates transfer of bulk data from secondary memory or hard disk to main memory without the use of CPU.

Hence, the correct option is (A).

### Question 18

**698808213**

Let R1 and R2 be two 4-bit registers that store numbers in 2's complement form. For the operation R1+R2, which one of the following values of R1 and R2 gives an arithmetic overflow?

- (A) R1 = 1011 and R2 = 1110
- (B) R1 = 1100 and R2 = 1010
- (C) R1 = 0011 and R2 = 0100

$$R1 = -1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = -7$$

$$R2 = -1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = -1$$

$$R1 + R2 = -8 \text{ (no overflow)}$$

**Option (B) :**

$$R1 = 1100 \text{ and } R2 = 1010$$

$$R1 = -1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 0 \times 2^0 = -4$$

$$R2 = -1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = -6$$

$$R1 + R2 = -10 \text{ (over flow)}$$

**Option (C) :**

$$R1 = 0011 \text{ and } R2 = 0100$$

$$R1 = -1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = -5$$

$$R2 = -1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = -2$$

$$R1 + R2 = -7 \text{ ( no overflow)}$$

**Option (D) :**

$$R1 = 1001 \text{ and } R2 = 1111$$

$$R1 = 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 3$$

$$R2 = 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 0 \times 2^0 = 4$$

$$R1 + R2 = 7 \text{ (no overflow)}$$

Hence, the correct option is (B).

### Question 19

**698808214**

Consider the following threads,  $T_1, T_2$ , and  $T_3$  executing on a single processor, synchronized using three binary semaphore variables,  $S_1, S_2$  and  $S_3$ , operated upon using standard wait() and signal(). The threads can be context switched in any order and at any time.

$T_1$	$T_2$	$T_3$
while (true)	while (true)	while (true)
{	{	{
wait( $S_3$ );	wait( $S_1$ );	wait( $S_2$ );
print ("C");	print ("B");	print ("A");
signal ( $S_2$ );	signal ( $S_3$ );	signal ( $S_1$ );
}	}	}

Which initialization of the semaphores would print the sequence BCABCABC...?

- (A)  $S_1 = 1; S_2 = 1; S_3 = 1$
- (B)  $S_1 = 1; S_2 = 1; S_3 = 0$
- (C)  $S_1 = 1; S_2 = 0; S_3 = 0$
- (D)  $S_1 = 0; S_2 = 1; S_3 = 1$

After this Process  $P_3$  can successfully execute wait( $S_2$ ): and then it executes print ("A");, after which it executes signal( $S_1$ ): and then gets stuck at wait ( $S_2$ ):

Here A gets printed in this process.

After this Process  $P_2$  can execute wait ( $S_1$ ): successfully.

The process thus keeps repeating and the pattern printed is BCABCABC...

Hence, the correct option is (C).

### Question 20

**698808215**

Consider the following two statements with respect to matrices

**Ans. C****Sol. Given :**

A single processor in which three threads  $T_1$ ,  $T_2$  and  $T_3$  are executing.

And they are synchronized by using three binary semaphore variables such that  $S_1$ ,  $S_2$  and  $S_3$ .

They are operated by using standard wait() & signal() operation.

So, for checking that which initialization of semaphores would print the sequence of BCABCABCA...

Then, initially if  $S_1 = 1, S_2 = 0, S_3 = 0$ ,

Process  $P_2$  can successfully execute wait( $S_1$ ); while  $P_1$  and  $P_3$  remain stuck at wait( $S_3$ ); and wait( $S_2$ ); respectively.

After process  $P_2$  prints B it executes signal( $S_3$ ); and gets stuck at wait( $S_1$ ); Here B gets printed in this process.

After this Process  $P_1$  can successfully execute wait( $S_3$ ); and then it executes print("C");, after which it executes signal( $S_2$ ); and then gets stuck at wait( $S_3$ );

Here C gets printed in this process.

Since, eigen values are the same, thus the sum of eigen values (i.e. the trace) is also the same.

Hence, the correct option is (C).

**Question 21****698808216**

What is printed by ANSI C program?

```
#include <stdio.h>
int main (int argc, char *argv [])
{
    int x=1, z[2]={10,11};
    int *p = NULL;
    p= &x;
    *p = 10;
    p = &z[1];
    *(z [0]+1) +=3;
    printf("%d, %d, %d\n", x , z [0], z [1]);
}
```

 $A_{m \times n}, B_{n \times m}, C_{n \times n}$  and  $D_{n \times n}$ Statement 1 :  $\text{Tr}(AB) = \text{Tr}(BA)$ Statement 2 :  $\text{Tr}(CD) = \text{Tr}(DC)$ 

$\text{Tr}$  represents the trace of matrix, which one of the following holds

- (A) S1 is wrong and S2 is correct
- (B) S1 is correct and S2 is wrong
- (C) Both statements are correct
- (D) Neither of the statements correct

**Ans.****Sol.**

**Given :** Two statements with respect to matrices  $A_{m \times n}, B_{n \times m}, C_{n \times n}$  and  $D_{n \times n}$

Statement 1 :  $\text{Tr}(AB) = \text{Tr}(BA)$ Statement 2 :  $\text{Tr}(CD) = \text{Tr}(DC)$ 

Then, from question.

The eigen values (counting multiplicity) of  $AB$  are the same as those of  $BA$ .

This is a corollary of theorem in second edition of matrix analysis by Horn and Johnson.

Paraphrasing from the cited theorem. If  $A$  is an  $m$  by  $n$  matrix and  $B$  is an  $n$  by  $m$  matrix with  $n > m$  then the characteristic polynomial  $P_{BA}$  of  $BA$  is related to the characteristic polynomial  $P_{AB}$  of  $AB$ .



$$z[2] = \{10, 11\}; \quad \begin{array}{c|cc} x & z \\ \hline 1 & 10 & 11 \\ 100 & 200 & 204 \end{array}$$

$$p = \&x;$$

$$\boxed{100}$$

$$300$$

$$\begin{array}{c} x \\ \hline \boxed{10} \\ 100 \end{array}$$

$$p = \&z[1];$$

$$\boxed{100204}$$

$$300$$

$$*(&z[0]+1) +=3;$$

$\Rightarrow$  \*(address of 1<sup>st</sup> element of array)

$$+=3;$$

$\Rightarrow$  i.e. add 3 to 1<sup>st</sup> element of array, therefore second elements becomes 14.

Hence, the correct option is (A).

**Question 22****698808217**

```
    return 0;  
}  
(A) 10, 10, 14          (B) 1, 10, 11  
(C) 1, 10, 14          (D) 10, 14, 11
```

**Ans. A**

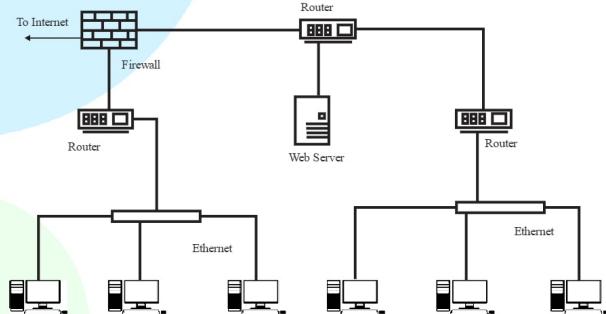
**Sol.** Given :

## **ANSI C program :**

```
#include <stdio.h>
int main (int argc, char *argv [])
{
    int x = 1, z[2]={10,11};
    int *p = NULL;
    p= &x;
    *p = 10;

    p = &z[1];
    *(&z [0]+1) +=3;
    printf("%d, %d, %d\n", x , z [0], z
[1]);
    return 0;
}
```

Consider an enterprise network with two Ethernet segments, a web server and a firewall, connected via three routers shown below :



What is the number of subnets in the enterprise network?



**Ans.** C

**Sol.** **Given :** A network which has Two Ethernet segments, a web server and a firewall.

And they are connected through three routers.

So, from given figure,

This is just like non equal sub netting where router 2 has the half of the addresses, Router 1 has other half, which is further divided into two subnets which is Web server and router 3, So total of 3 subnets possible.



Hence, the correct option is (C).

### **Question 23**

698808218

Which of the following statements is/are true?

- (A) Every subset of recursively enumerable Language is recursive
  - (B) If Language  $L$  & its complement  $\bar{L}$  are both recursively enumerable then  $L$  must be recursive
  - (C) Complement of context free language must be recursive.
  - (D) If  $L_1$  and  $L_2$  are regular, then  $L_1 \cap L_2$  must be Deterministic Context Free Language.

**Ans. B, C, D**

**Sol.** Given :

**Statement A :** Every subset of recursively enumerable language is recursive.

**True,** If  $L_1$  and  $L_2$  are regular then  
 $L_1 \cap L_2$  is Regular thus also  
deterministic context free Language.  
Hence, the correct options are (B), (C),  
(D).

### **Question 24**

698808219

Let WB and WT be 2 set associative cache organizations that use LRU algorithm for cache block replacement. WB is Write Back cache and WT is Write through cache. Which of the following statements are false?

- or the following statements are false:

  - (A) Each cache block in WB and WT has a dirty bit
  - (B) Every write hit in WB leads to a data transfer from cache to main memory
  - (C) Eviction of a block from WT will not lead to data transfer from

**False,** Since,  $\Sigma^*$  is a recursively enumerable Language but there are many languages which are subset of  $\Sigma^*$  which are not Recursive.

**Statement B :** If Language L & its complement  $\bar{L}$  are both recursively enumerable then L must be recursive  
**True,** If L is recursively enumerable Language, then for all member strings of L, the TM of L will halt within finite time. If  $\bar{L}$  is recursively enumerable Language, then for all nonmember strings of L. The TM of  $\bar{L}$  will halt within finite time. Thus for both member and nonmember strings of L and  $\bar{L}$  we have a TM which halts within finite time. Thus L is a Recursive Language.

**Statement C :** Complement of context free language must be recursive.

**True,** Complement of CFL is a CSL in the worst case which is Recursive.

**Statement D :** If  $L_1$  and  $L_2$  are regular, then  $L_1 \cap L_2$  must be Deterministic Context Free Language.

not lead to data transfer from cache to main memory

(D) A read miss in WB will never lead to eviction of a dirty block from WB

**Ans. A, B, D**

**Sol. Given :**

WB and WT is two set associative cache organization,

Where WB belongs to write back cache and WT belongs to write through cache.

LRU algorithm used for cache block replacement.

So, from given statements:

**Statement A :** Each cache block in WB and WT has a dirty bit

**Statement B :** Every write hit in WB leads to a data transfer from cache to main memory. False, for the hit operation no need to fetch the data from the main memory.

**Statement C :** Eviction of a block from WT will not lead to data transfer from cache to main memory

**Statement D :** A read miss in WB will never lead to eviction of a dirty block from WB

Hence, the correct options are (A), (B), (D).

### Question 25

698808220

Consider the following three relations in a relational database.

Employee (eId, Name)

Brand (bId, bName)

Own (eId, bId)

Which of the following relational algebra expressions return the set of eIds who own all the brands?

- (A)  $\pi_{eId}(\pi_{eId, bId}(Own) / \pi_{bId}(Brand))$
- (B)  $\pi_{eId}(\pi_{eId, bId}(Own) / \pi_{bId}(Own))$
- (C)  $\pi_{eId}(Own) - \pi_{eId} / \pi_{eId}(Own) \times \pi_{bId}(Brand) - \pi_{eId, bId}(Own))$
- (D)  $\pi_{eId}(\pi_{eId}(Own) \times \pi_{bId}(Own)) / \pi_{bId}(Brand)$

**Ans. A, C**

**Sol. Given :**

Three relations in relational database

assignment edge, then the system is not in deadlock state.

(C) Circular wait is a necessary condition for the formation of deadlock

(D) If the current allocation of resources to processes leads the system to unsafe state then deadlock will necessarily occur.

**Ans. B, C**

**Sol. Given :**

**Statement A :** In a system where each resource has more than one instance, a cycle in its wait-for-graph indicates the presence of a deadlock.

**False,** A cycle exist in the system but since  $P_1$  does not need any more resources it will finish after some time and  $P_3$  will get resource  $R_1$  this even in the presence of cycle the deadlock

are as follows:

Employee (eId, ename)

Brand (bId, bname)

Own (eId, bId)

To find set of eIds who own all the brand we need to divide the table own by table brand like.

(A)  $\pi_{eId}(\pi_{eId, bId}(Own) / \pi_{bId}(Brand))$  and

the division operation can be represented using  $(\times, -)$  operations like

(C)  $\pi_{eId}(Own) - \pi_{eId} / \pi_{eId}(Own)$

$\times \pi_{bId}(Brand) - \pi_{eId, bId}(Own))$

Hence, the correct options are (A), (C)

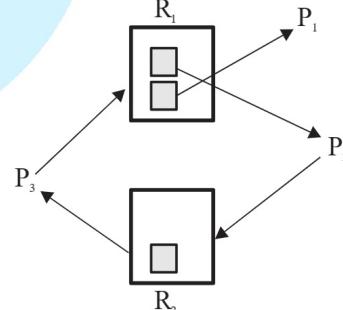
### Question 26

**698808221**

Which of the following statements is/are TRUE with respect to deadlocks?

- (A) In a system where each resource has more than one instance, a cycle in its wait-for-graph indicates the presence of a deadlock.  
(B) In the resource-allocation graph of a system, if every edge is an

does not exist here



**Statement B :** In the resource-allocation graph of a system, if every edge is an assignment edge, then the system is not in deadlock state.

**True,** When every edge is the assignment edge, that is, no process needs any more resources than the ones that are already allocated, thus deadlock doesn't exist here.

**Statement C:** Circular wait is a necessary condition for the formation of deadlock

**True,** Circular wait is a necessary condition for Deadlock but it is not sufficient. There are following 4 necessary conditions for the occurrence of deadlock

1. Mutual exclusion

- 2. Hold and wait
- 3. No preemption
- 4. Circular wait

**Statement D :** If the current allocation of resources to processes leads the system to unsafe state then deadlock will necessarily occur.

**False,** Even if the allocation results in an unsafe state, some processes still may release the resources that are allocated to them for a while which may lead to elimination of deadlock from the system.

Hence, the correct options are (B), (C).

### Question 27

**698808222**

Which of the following is /are true for group G?

(A) If for all  $x, y \in G$ ,  $(xy)^2 = x^2y^2$ , then G is commutative.

(B) If for all  $x \in G$ ,  $x^2 = 1$ , then G is commutative, here 1 is identity

**Theorem :** All groups with less than 6 elements are abelian.

**D. False**

**Theorem :** Every subgroup of an abelian group has to be abelian.

### Question 28

**698808223**

Suppose binary search tree with 1000 distinct elements is also complete binary tree. The tree is sorted using array representation of Binary Heap Tree. Assuming that the array indices start with 0, the 3<sup>rd</sup> largest element of tree is stored at index \_\_\_\_?

**Ans. 509**

**Sol. Given :**

Binary search tree which has 1000 distinct elements & it also work as complete binary tree.

Array representation of binary heap tree is used for sorting of tree,

So, the largest element in the BST is the right most element with index

- element of G
- (C) If the order of G is 2, then G is commutative
- (D) If G is commutative then a subgroup of G need not to be commutative

**Ans. A, B, C**

**Sol. A. True**

$$\text{Since } (ab)^2 = a^2b^2 \rightarrow (ab)(ab) = (aa)(bb)$$

We know that in a group, left and right cancellation is allowed, so

$ba = ab$ , Hence Commutative

**B. True**

Key Point:  $x^2 = 1$  means each element is inverse of itself.

Let a,b be two elements in group G. Consider the element ab.

$$\text{Now, } (ab)^2 = 1 \rightarrow ab.ab = 1$$

Now, multiply both sides on the right with b,

$$ababb = 1b \rightarrow aba = b$$

Now, multiply both sides on the right with a,

$$abaa = ba \rightarrow ab = ba$$

Hence, G is abelian.

**C. True**

$510(2^9 - 2)$  as the indexing of the array is starting with 0) its parent is the second-largest element and its left child is the third-largest element which is of the index  $510 - 1 = 509$ . Hence, the correct Answer is 509.

**Question 29**

**698808224**

Consider the augmented grammar with  $\{+, *, (,), id\}$  as the set of terminals.

$$\begin{aligned} S' &\rightarrow S \\ S &\rightarrow S + R \mid R \\ R &\rightarrow R * P \mid P \\ P &\rightarrow (S) \mid id \end{aligned}$$

If  $I_0$  is the set of two LR(0) items

$$\{[S' \rightarrow S], [S \rightarrow S + R]\},$$

then go to  $(\text{Closure}(I_0), +)$  contains exactly \_\_\_\_\_ items.

**Ans. 5**

**Sol. Given :**

Augmented grammar

$$\begin{aligned} S' &\rightarrow S \\ S &\rightarrow S + R \mid R \end{aligned}$$

GATE CSE-2022

14

$$R \rightarrow R * P \mid P$$

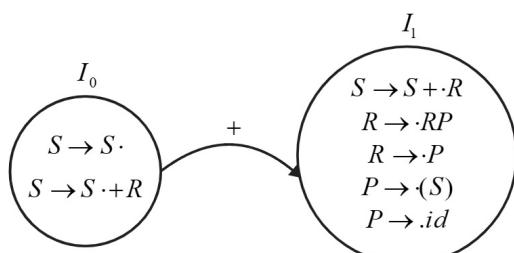
$$P \rightarrow (S) \mid id$$

And set of terminals =  $\{+, *, (,), id\}$

And there is a condition if  $I_0$  is the set of two LR (0) Items such that

$$\{[S' \rightarrow S], [S \rightarrow S + R]\},$$

Then go to  $(\text{Closure}(I_0), +)$  contains :



$\therefore 5$  item

Hence, the correct answer is 5.

**Question 30**

**698808225**

Consider a simple undirected graph of 10 vertices if the graph is disconnected then the maximum number of edges it can have is

Consider a Relation R (A, B, C, D, E) with the following three functional dependencies.

$$AB \rightarrow C, BC \rightarrow D, C \rightarrow E$$

The number of super keys in the relation R is \_\_\_\_\_.

**Ans. 8**

**Sol. Given :**

Relation R with attributes (A, B, C, D, E) and

There are three functional dependencies such that

$$AB \rightarrow C$$

$$BC \rightarrow D$$

$$C \rightarrow E$$

So, from these functional dependencies,

Candidate key = AB

$\therefore$  Number of super keys  $2^{5-2} = 2^3 = 8$  super keys

Hence, the correct answer is 8.

**Question 32**

**698808227**

**Ans.** **36**

**Sol.** **Given :**

Undirected graph which has 10 vertices and we have to find the maximum number of edges when the graph is disconnected.

So, suppose we have 1 vertex on one side and other n-1 vertices on another side. To make it connected maximum possible edges (if consider it as complete graph) is

$$C_2^{n-1} \text{ which is } \frac{(n-1)(n-2)}{2}$$

Thus to make it a disconnected graph we have 1 separate vertex on another side which is not connected. Thus the maximum possible edges is

$$C_2^{n-1} = {}^9C_2 = \frac{9 \times 8}{2} = 36$$

Hence, the correct answer is 36.

**Question 31**

**698808226**

GATE CSE-2022

15

not affected. The minimum hit rate (rounded off to decimal two places) needed after the optimization such that it should not increase the average memory access time is \_\_\_\_\_.

**Ans.** **0.85**

**Sol.** **Given :**

a cache

which has,

Hit rate = 0.8

∴ miss rate = 0.2

Access latency = 10 ns

Miss penalty = 100 ns

$$\therefore \text{AMAT}_{old} = 0.8(10) + 0.2(100)$$

$$= 8 + 20 = 28$$

Where, AMAT is average memory access time.

For Optimized cache.

where access latency = 15 ns = hit time

Let x be the cache hit rate after optimization.

$$\therefore \text{AMAT}_{new} = x(15) + (1-x)100$$

$$= 15x + 100 - 100x = 100 - 85x$$

The number of arrangement of 6 identical ball in 3 identical bins \_\_\_\_\_.

**Ans.** **7**

**Sol.** **Given :**

6 identical balls

For which we have to find out the number of arrangement for these balls into three identical bins

Now, it is the case of distribution with identical objects and identical boxes.

We simply need to partition number 6 in to maximum 3 parts and count the partition.

(6,0,0), (5,1,0), (4,2,0), (4,1,1), (3,2,0), (3,1,1), (2,2,2).

Hence, the correct answer is 7.

**Question 33**

**698808228**

A cache memory that has a hit rate of 0.8 has an access latency 10 ns and miss penalty 100 ns. An optimization is done on the cache to reduce the miss rate. However the optimization results in an increase of cache access latency to 15 ns, whereas the miss penalty is

$$\begin{aligned} &= \lim_{x \rightarrow 0^+} \left[ \frac{1}{-e^{2\sqrt{x}}(2)} \right] = \lim_{x \rightarrow 0^+} \left[ \frac{1}{-e^{2\sqrt{0}}(2)} \right] \\ &= \frac{-1}{2} = -0.5 \end{aligned}$$

**Question 35**

**698808230**

Consider the resolution of domain name www.gate.org.in by a DNS resolver. Assume that no resource records are cached anywhere across the DNS servers and that query resolution mechanism is used in the resolution. The number of DNS query response pairs involved in completely resolving the domain name is \_\_\_\_\_.

**Ans.** **4**

**Sol.** **Given :** Domain name [www.gate.Org.in](http://www.gate.Org.in)

There is no resource records are cached any where across the DNS servers,

In the iterative query the DNS resolver go to the these three servers which is root server, TLD DNS server, authoritative server. So there will be three pairs of request and response

AMAT<sub>Old</sub> ≥ AMAT<sub>new</sub>

$$\rightarrow 100 - 85x \leq 28$$

$$\rightarrow 72 = 85x$$

$$\rightarrow x = \frac{72}{85} = 0.85$$

The required hit rate = 0.85

Hence, the correct answer is 0.85.

**Question 34****698808229**

The value of following limit is \_\_\_\_\_.

$$\lim_{x \rightarrow 0^+} \frac{\sqrt{x}}{1 - e^{2\sqrt{x}}}$$

**Ans. – 0.5****Sol. Given :**

$\lim_{x \rightarrow 0^+} \frac{\sqrt{x}}{1 - e^{2\sqrt{x}}}$  is in the form of  $\frac{0}{0}$  form at

If we apply L'Hopital's rule then

$$= \lim_{x \rightarrow 0^+} \left[ \frac{\frac{d}{dx}(\sqrt{x})}{\frac{d}{dx}(1 - e^{2\sqrt{x}})} \right] = \lim_{x \rightarrow 0^+} \left[ \frac{\frac{1}{2\sqrt{x}}}{0 - e^{2\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}} \right]$$

$$\text{So, } G(x) = \sum_{r=0}^{\infty} a_r x^r$$

$$G(x) = \sum_{r=0 \text{ (even)}}^{\infty} a_r x^r + \sum_{r=1 \text{ (odd)}}^{\infty} a_r x^r$$

$$G(x) = \sum_{r=0 \text{ even}}^{\infty} x^r + \sum_{r=1 \text{ odd}}^{\infty} (r+1)x^r$$

As we know,

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + x^5 + \dots$$

$$\frac{1}{1-x^2} = 1 + x^2 + x^4 + x^6 + x^8 + x^{10} + \dots$$

... (i)

$$\frac{2x}{(1-x^2)^2} = 2x + 4x^3 + 6x^5 + 8x^7 + 10x^9 \dots$$

... (ii)

Add equation (i) and (ii)

$$\begin{aligned} \frac{1}{1-x^2} + \frac{2x}{(1-x^2)^2} &= 1 + 2x + x^2 + 4x^3 \\ &\quad + x^4 + 6x^5 + x^6 + \dots \end{aligned}$$

$$\frac{1+x-x}{1-x^2} + \frac{2x}{(1-x^2)^2}$$

here.

Hence, the correct answer is 4.

**Q.36 to Q.65 Carry TWO Marks Each****Question 36****698808231**Which one of the following is closed form for the generating function of the sequence  $\{a_n\}_{n \geq 0}$  defined below?

$$a_n = \begin{cases} n+1, & n = \text{odd} \\ 1, & \text{Otherwise} \end{cases}$$

- (A)  $\frac{x(1+x^2)}{(1-x^2)^2} + \frac{1}{1-x}$     (B)  $\frac{x(3-x^2)}{(1-x^2)^2} + \frac{1}{1-x}$   
 (C)  $\frac{2x}{(1-x^2)^2} + \frac{1}{1-x}$     (D)  $\frac{x}{(1-x^2)^2} + \frac{1}{1-x}$

**Ans. A****Sol. Given :**

Generating function

$$a_n = \begin{cases} n+1, & n = \text{odd} \\ 1, & \text{otherwise} \end{cases}$$

Of the sequence  $\{a_n\}_{n \geq 0}$ 

$$(C) \frac{A^3}{3} \quad (D) \frac{A^3}{6}$$

**Ans. D****Sol. Given :**

An undirected unweighted graph which has atleast 3 vertices

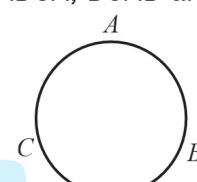
And A is adjacency matrix of graph.

So, as we know,

Diagonal element of  $A^3$  gives number of paths of length 3,

From any vertex to itself (cycle of 3).

For each participating vertex, each cycle will be counted thrice. As given below (ABCA, BCAB and CABC).



Furthermore, since the graph is undirected, every cycle will be counted twice. So overall every cycle of length 3 in  $A^3$  will be counted 6 times, so we divide by 6 also. Therefore, the number of Cycle is

$$\begin{aligned}
&= \sum_{r=0(\text{even})} x^r + \sum_{r=1(\text{odd})} (r+1)x^r \\
\Rightarrow & \frac{1+x}{1-x^2} - \frac{x}{1-x^2} + \frac{2x}{(1-x^2)^2} = G(x) \\
\Rightarrow & G(x) = \frac{1}{1-x} + \frac{x}{1-x^2} \left( -1 + \frac{2}{1-x^2} \right) \\
\Rightarrow & \frac{1}{1-x} + \frac{x}{1-x^2} \left( \frac{-1+x^2+2}{1-x^2} \right) \\
\Rightarrow & \frac{1}{1-x} + \frac{x(1+x^2)}{(1-x^2)^2}
\end{aligned}$$

Hence, the correct option is (A).

**Question 37** **698808232**

Consider simple undirected unweighted graph with at least 3 vertices. If A is adjacency matrix of graph then the number of 3-cycle in the graph is given by trace of:

- (A)  $A^3$       (B)  $\frac{A^3}{2}$

GATE CSE-2022

17

- (D) The memory access time using a given inverted page table is always same for all incoming virtual addresses.

**Ans. D**

**Sol. Given :**

**Statement A :** The TLB performs an associative search in parallel on all its valid entries using page number of incoming virtual address.

**True,** TLB Lookups:

1. Sequential search of the TLB
2. Direct mapping: assigns each virtual page to a specific slot in the TLB e.g., use upper bits of VPA to index TLB (Translation lookaside buffer)
3. Set associativity: use N TLB banks to perform lookups in parallel
4. Fully associative cache: allows looking up all TLB entries in parallel
5. Typically :
  - a. TLBs are small and fully associative
  - b. Hardware caches use direct mapped or set-associative cache

trace of  $\frac{A^3}{6}$ .

Hence, the correct option is (D).

**Question 38**

**698808233**

Which one of the following statements is False?

- (A) The TLB performs an associative search in parallel on all its valid entries using page number of incoming virtual address
- (B) If the virtual address of a word given by CPU has a TLB hit, but the subsequent search for the word results in cache miss, then the word will always be present in the main memory.
- (C) In a system that uses hashed page tables, if two distinct virtual address V1 and V2 map to the same value while hashing, then the memory access time of these addresses will not be the same

**True,** a hashed page table lookup may require many memory references to search the desired virtual address and its corresponding frame number because there is no guarantee on the number of entries in the linked list.

**Statement D :** The memory access time using a given inverted page table is always same for all incoming virtual addresses.

**False,** when a memory reference takes place, this virtual address is matched by the memory-mapping unit and the Inverted Page table is searched for a match and the corresponding frame number is obtained.

If the match is found at the it entry then the physical address of the process is sent as the real address otherwise if no match is found then Segmentation Fault is generated. Finding a match requires searching the entire table. Depending on the match, the memory access time will vary.

Hence, the correct option is (D).

**Question 39**

**698808234**

Let  $R_i(z)$  and  $W_i(z)$  denote read and write operations on data element  $z$  hv

**Statement B :** If the virtual address of a word given by CPU has a TLB hit, but the subsequent search for the word results in cache miss, then the word will always be present in the main memory.

**True,** a cache stores a copy of data from memory in a fast storage near the CPU. In case of TLB hit, we got the physical address (in main memory). We look into cache before accessing main memory. In case of a cache miss, we will definitely find the word in main memory as there was a TLB hit.

**Statement C :** In a system that uses hashed page tables, if two distinct virtual address V1 and V2 map to the same value while hashing, then the memory access time of these addresses will not be the same

Write operations on data element z by a transaction  $T_i$ , consider schedule S with four transactions

$S : R_4(x)R_2(x)R_3(x)R_1(y)W_1(y)W_2(x)W_3(y)R_4(y)$

Which one of the following serial schedules is conflict equivalent to S?

- (A)  $T_4 \rightarrow T_1 \rightarrow T_3 \rightarrow T_2$
- (B)  $T_1 \rightarrow T_4 \rightarrow T_3 \rightarrow T_2$
- (C)  $T_3 \rightarrow T_1 \rightarrow T_4 \rightarrow T_2$
- (D)  $T_1 \rightarrow T_3 \rightarrow T_4 \rightarrow T_2$

**Ans. D**

**Sol.** Given: A data element z.

In which transactions  $T_i$  occur by the operations read and write

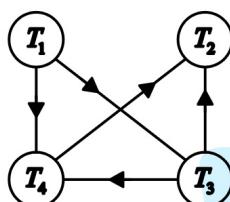
Where Read is denoted by  $R_i(z)$  & write is denoted by  $W_i(z)$

& there is schedule S for 4 transactions i.e.,

$S : R_4(x)R_2(x)R_3(x)R_1(y)W_1(y)W_2(x)W_3(y)R_4(y)$

So, from given schedule :

$T_1$	$T_2$	$T_3$	$T_4$
			$R(x)$
	$R(x)$		
		$R(x)$	
$R(x)$			
	$W(y)$		
		$W(x)$	
			$W(y)$
			$R(y)$



Now, Apply topological sort then the sequence is

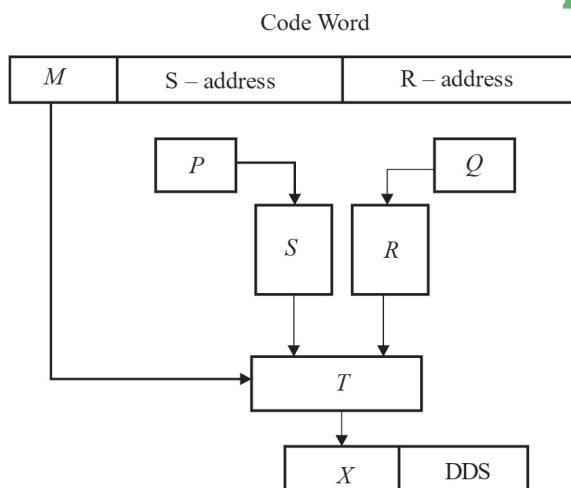
$T_1T_3T_4T_2$

Hence, the correct option is (D).

**Question 40**

**698808235**

Consider a digital display system (DDS) shown in the figure that displays the contents of register X. A 16-bit



(A)	P is 10: $2^{10}$ decoder,	Q is 5: $2^5$ decoder,	T is 2:1 Multiplexer
(B)	P is 10: $2^{10}$ decoder,	Q is 5: $2^5$ decoder,	T is 2:1 encoder
(C)	P is 10: 1 Multiplexer,	Q is 5: 1 Multiplexer,	T is 2:1 Multiplexer
(D)	P is 1:10 de-multiplexer	Q is 1:5 de-multiplexer	T is 2:1 Multiplexer

**Ans. A**

**Sol.** Given: Digital display system (DDS).

Where, it show the contents of register X and length of data in register X = 16 bit

Where, S is 1024 word memory

code word is used to load a word in X, either from S or from R. S is a 1024-word memory segment and R is a 32-word register file. Based on the value of mode bit M, T selects an input word to load in X. P and Q interface with the corresponding bits in the code word to choose the addressed word. Which one of the following represents the functionality of P, Q, and T?

segment and R is 32 word register file.

Based on the value of M, T selects an input for loading in register X.

P & Q work as interface for choosing time addressed word.

So, based on the Address provided to P, one of the words of the memory unit S has to be selected. This function is performed by a Decoder r To be able to address 1024 words, we need 10 bits, and to select one out of  $2^{10}$  addresses, we need  $1G:2^{10}$  Decoder which is unit P.

Similarly to decoder Address R we need  $5:2^5$  decoder.

Now one of the two inputs has to be selected to be loaded into X, selecting one input among many to

place in the output. This function is performed by a Multiplexer here we need a  $2 \times 1$  MUX.

Hence, the correct answer is (A).

**Question 41** **698808236**

Consider three floating point numbers A, B and C stored in registers  $R_A$ ,  $R_B$  and  $R_C$ , respectively as per IEEE-754 single precision floating point format. The 32-bit content stored in these registers (in hexadecimal form) are as follows.

$$R_A = 0xC1400000 \quad R_B = 0x42100000 \quad R_C = 0x41400000$$

Which one of the following is false?

- (A)  $A+C=0$                          (B)  $B=3C$   
 (C)  $C=A+B$                          (D)  $(B-C)>0$

**Ans. C**

**Sol. Given :**

Three registers which store three floating point numbers A, B & C in IEEE-754 single precision floating point format.

Size of register = 32 bit

$$R_A = 0xC1400000 \quad R_B = 0x42100000 \quad R_C = 0x41400000$$

$$A=1100\ 0001\ 0100\ 0000$$

$$\text{Biased Exponent} = 130,$$

$$\text{Exponent} = 130 - 127 = 3$$

$$\text{Decimal} = -1.1 \times 2^3 = -1100 = -12$$

$$B=0100\ 0010\ 0001\ 0000$$

units. The processes arrive in the order  $P, Q, R, S$  all at time  $t=0$ . There is exactly one context switch from  $S$  to  $Q$ , exactly one context switch from  $R$  to  $Q$ . Exactly two context switches from  $Q$  to  $R$ . There is no context switch from  $S$  to  $P$ . Switching to a ready process after the termination of another process is also considered a context switch. Which one of the following is NOT possible a CPU burst time (in time units) of these process?

- (A)  $P=3, Q=7, R=7, S=3$   
 (B)  $P=4, Q=10, R=6, S=2$   
 (C)  $P=4, Q=12, R=5, S=4$   
 (D)  $P=2, Q=9, R=5, S=1$

**Ans. A**

**Sol. Given :**

Four process P,Q,R,S which are scheduled on a CPU by round robin algorithm

Where, time quantum = 4 units,  
 Process arrived in the order P, Q, R, S at time  $t=0$ .

So, from given conditions we can draw gantt chart according to given options.  
 Valid Required Contexts switches are  
 1 – S to Q, 1 – R to Q, 2 – Q to R and  
 no S to P.

**Option (A)**

B = 0100 0010 0001 0000

Biased Exponent = 132,

Exponent =  $132 - 127 = 5$

Decimal =  $+1.001 \times 2^5 = 100100 = 36$

C = 0100 0001 0100 0000

Biased Exponent =  $128 + 2 = 130$ ,

Exponent =  $130 - 122 = 3$

Decimal =  $+1.1 \times 2^3 = 1100 = +12$

A = -12

B = 36

C = +12

Hence, the correct option is (C).

#### Question 42

698808237

Consider four processes P, Q, R and S scheduled on a CPU as per round robin algorithm with a time quantum of 4

20

GATE CSE-2022

Contexts switches are 1 – P to Q, 2 – Q to R, 1 – R to S, 1 S to Q and 1 – R to Q

#### Option (D) :

D :

P	Q	R	S	Q	R	Q
0	2	6	10	11	15	16

Contexts switches are 1 – P to Q, 2 – Q to R, 1 – R to S, 1 S to Q and 1 – R to Q

So, from option B, C, D it satisfy all the condition given in the question,

So, the answer is option (A).

Hence, the correct option is (A).

#### Question 43

698808238

What is printed by ANSI C programme

```
#include<stdio.h>
int main (int argc, char * argv [])
{
    int a [3][3][3]={{1, 2, 3, 4, 5, 6, 7, 8,
                      9}, {10, 11, 12, 13, 14, 15, 16, 17, 18},
                      {19, 20, 21, 22, 23, 24, 25, 26, 27}};
    int i= 0, j= 0, k=0;
    for (i = 0; i< 3; i++) {
        for(k = 0; k<3; k++)
            printf("%d", a [i][j][k]);
        printf("\n");
    }
    return 0;
}
```

(A) 1 2 3

(B) 1 4 7

#### Option (A) :

A :

P	Q	R	S	Q	R
0	3	7	11	14	17

Contexts switches are 1 – P to Q, 2 – Q to R, 1 – R to S and 1 – S to Q

#### Option (B) :

B :

P	Q	R	S	Q	R	Q
0	4	8	12	14	18	20

Contexts switches are 1 – P to Q, 2 – Q to R, 1 – R to S, 1 S to Q and 1 – R to Q

#### Option (C) :

C :

P	Q	R	S	Q	R	Q
0	4	8	12	16	20	21

$$j = k = i = 0$$

a[3][3][3] = {{1, 2, 3,.....9}, → a[0]

{10, 11,.....18}, → a[1]

{19,.....27} } → a[2]

In short, there are total three 2-D arrays, and in the code we need to find first row of each 2D array i.e.

1, 2, 3 First row of first 2-D array

10, 11, 12 First row of second 2-D array

19, 20, 21 First row of third 2-D array

So, the output of the code is

1 2 3

10 11 12

19 20 21

Hence, the correct option is (A).

#### Question 44

698808239

What is printed by the following ANSI C program?

```
#include<stdio.h>
int main(int argc, char *argv[])
{
    char a='P';
    char b='x';
    char c=(a&b)+'*';
    char d=(a|b)- '-';
    char e=(a^b)+'+';
    printf ("%c%c%c%c\n",c,d,e);
    return 0;
}
```

- |           |           |
|-----------|-----------|
| 10 11 12  | 10 13 16  |
| 19 20 21  | 19 22 25  |
| (C) 1 2 3 | (D) 1 2 3 |
| 4 5 6     | 13 14 15  |
| 7 8 9     | 25 26 27  |

**Ans. A**

**Sol.** Given :

**Code**, in which

```
int a [3][3][3] = {{1, 2, 3, 4, 5, 6, 7, 8, 9},
{10, 11, 12, 13, 14, 15, 16, 17, 18},
{19, 20, 21, 22, 23, 24, 25, 26, 27}};
int i= 0, j= 0, k=0;
for (i = 0; i< 3; i++)
{
    for(k = 0; k<3; k++)
        So, from given code
```

}

ASCII encoding for relevant characters are given below :

A	B	C	...	Z
65	66	67	...	90
a	b	c	...	z
97	98	99	...	122

*	+	-
42	43	45

- (A) z K S      (B) 122 75 83  
(C) \* - +      (D) P x +

**Ans. A**

**Sol.** Given :

**ANSI C program :**

```
#include<stdio.h>
int main(int argc, char *argv[])
{
    char a='P';
    char b='x';
    char c=(a&b)+'*';
    char d=(a|b)- '-';
    char e=(a^b)+'+';
    printf("%c%c%c\n",c,d,e);
    return 0;
}
```

ASCII value of P = 80 and x=120

$$c = \frac{a \& b}{\downarrow} + \frac{'*' }{\downarrow}$$

$$80 \quad \quad 42 \rightarrow 122 \Rightarrow z$$

$$d = \frac{a/b}{\downarrow} - \frac{'-' }{\downarrow}$$

$$120 \quad \quad 45 \rightarrow 75 \Rightarrow K$$

$$e = \frac{a^b}{\downarrow} + \frac{'+' }{\downarrow}$$

$$40 \quad \quad 43 \rightarrow 83 \Rightarrow S$$

Hence, the correct option is (A).

**Question 45**

**698808240**

Consider solving the following system of simultaneous equations using LU decomposition.

$$x_1 + x_2 - 2x_3 = 4, \quad x_1 + 3x_2 - x_3 = 7 \quad \text{and}$$

$$2x_1 + x_2 - 5x_3 = 7$$

where L and U are denoted as

$$\begin{pmatrix} L_{11} & 0 & 0 \end{pmatrix}$$

$$(C) L_{32} = -\frac{1}{2}, U_{33} = 2, x_1 = 0$$

$$(D) L_{32} = -\frac{1}{2}, U_{33} = -\frac{1}{2}, x_1 = 0$$

**Ans. D**

**Sol.** Given :

Simultaneous equations :

$$x_1 + x_2 - 2x_3 = 4$$

$$x_1 + 3x_2 - x_3 = 7$$

$$2x_1 + x_2 - 5x_3 = 7$$

and L and U Which is denoted by

$$L = \begin{pmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{pmatrix}$$

$$\text{and } U = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{pmatrix}.$$

So,  $LU = A$

$$\begin{bmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix} = \begin{bmatrix} 1 & 1 & -2 \\ 1 & 3 & -1 \\ 2 & 1 & -5 \end{bmatrix}$$

$$\begin{bmatrix} U_{11} & U_{12} & U_{13} \\ L_{21}U_{11} & L_{21}U_{12} + U_{22} & L_{21}U_{13} + U_{23} \\ L_{31}U_{11} & L_{31}U_{12} + L_{32}U_{22} & L_{31}U_{13} + L_{32}U_{23} + U_{33} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & -2 \\ 1 & 3 & -1 \\ 2 & 1 & -5 \end{bmatrix}$$

$$L = \begin{pmatrix} L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{pmatrix},$$

$$U = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{pmatrix}$$

Which one of the following is the correct combination of values for  $L_{32}, U_{33}$ , and  $x_1$ ?

- (A)  $L_{32} = 2, U_{33} = -\frac{1}{2}, x_1 = -1$   
 (B)  $L_{32} = 2, U_{33} = 2, x_1 = -1$

22

GATE CSE-2022

Putting value of  $L_{21}$  and  $U_{12}$

$$1 \times 1 + U_{22} = 3$$

$$U_{22} = 2$$

Comparing  $A_{23}$

$$L_{21}U_{13} + U_{23} = -1$$

Putting value of  $L_{21}$  and  $U_{13}$

$$1 \times -2 + U_{23} = -1$$

$$U_{23} = 1$$

Comparing  $A_{31}$

$$L_{31}U_{11} = 2$$

Putting value of  $U_{11}$

$$L_{31} \times 1 = 2$$

$$L_{31} = 2$$

Comparing  $A_{32}$

$$L_{31}U_{12} + L_{32}U_{22} = 1$$

Putting value of  $L_{31}, U_{12}$  and  $U_{22}$

$$2 \times 1 + L_{32} \times 2 = 1$$

$$L_{32} \times 2 = -1$$

$$L_{32} = \frac{-1}{2}$$

Comparing  $A_{33}$

$$L_{31}U_{13} + L_{32}U_{23} + U_{33} = -5$$

$$2 \times -2 + \frac{-1}{2} \times 1 + U_{33} = -5$$

$$-4 - \frac{1}{2} + U_{33} = -5$$

$$\boxed{U_{33} = -\frac{1}{2}}$$

Comparing  $A_{11}, A_{12}$  and  $A_{13}$

$$U_{11} = 1$$

$$U_{12} = 1$$

$$U_{13} = -2$$

Comparing  $A_{21}$ ,

$$L_{21}U_{11} = 1$$

Putting value of  $U_{11}$

$$L_{21} \times 1 = 1$$

$$L_{21} = 1$$

Comparing  $A_{22}$

$$L_{21}U_{12} + U_{22} = 3$$



(D) Given a Turing machine, M decide if M takes more than 1073 steps on every string.

**Ans.** A, C, B

**Sol.** For a given Turing machine, M decide that m takes more than 1073 steps on every string is decidable only.

Remaining are undecidable

(A), (C) and (B) are undecidable.

Hence, the correct options are (A), (C) & (B).

**Question 47**

**698808242**

Consider language

$$L_1 = \{a^n \omega a^n \mid \omega \in \{a, b\}^*\}$$

$$L_2 = \{\omega x \omega^R \mid \omega, x \in \{a, b\}^*, |\omega|, |x| > 0\}$$

Note  $\omega^R$  is reversal of string  $\omega$  which one of the following is true?

(A)  $L_1, L_2$  are regular

(B)  $L_1$  and  $L_2$  are context free language

(C)  $L_1$  is regular and  $L_2$  is context free language

(D)  $L_1$  and  $L_2$  are context free language but not regular

**Ans.** A, B, C

**Sol.** Given :

Languages  $L_1$  &  $L_2$

and  $w^R$  is reversal of string  $w$

$L_1 = \{a^n \omega a^n \mid \omega \in (a, b)^*\}$  is regular

$L_2 = \{wxw^R \mid w, x \in (a, b)^*, |w|, |x| > 0\}$  is

Hence, the correct option is (D).

#### Question 46

**698808241**

- Which of the following statements is / are undecidable?
- Given two Turing machines  $M_1$  and  $M_2$  decide of  $L(M_1) = L(M_2)$ .
  - Given a Turing machine M, decide if  $L(M)$  is regular.
  - Given a Turing machine M, decide if M accepts all strings.

GATE CSE-2022

23

the expression which is regular such that  $a(a+b)^+a + b(a+b)^*b$ , which covers all the other string which can be obtained by putting w as "aa", "ab", "bb", "ba".

$\therefore$  Both  $L_1$  and  $L_2$  are regular

Any language which is regular is context free language as well.

Hence, the correct options are (A), (B) & (C).

#### Question 48

**698808243**

Consider the following languages

$$L_1 = \{ww \mid w \in (a, b)^*\},$$

$$L_2 = \{a^n b^n c^m \mid m, n \geq 0\}$$

$$\text{and } L_3 = \{a^m b^n c^n \mid m, n \geq 0\}$$

Which of the following statements are false?

- $L_1$  is not Context Free Language but  $L_2$  and  $L_3$  are Deterministic Context Free Language.
- Neither  $L_1$  nor  $L_2$  is Context Free Language.
- $L_2, L_3$  and  $L_2 \cap L_3$  all are Context Free Language.
- Neither  $L_1$  nor its complement is Context Free Language.

**Ans. B, C, D**

**Sol. Given :**

Languages

$$L_1 = \{ww \mid w \in (a, b)^*\}$$

$$L_2 = \{a^n b^n c^m \mid m, n \geq 0\}$$

also regular

So, now from language  $L_1$

$L_1$  is regular, because when we put the value of n is 0, then it will create a subset  $\{w \mid w \in \{ab\}^*\}$

Which contains all possible strings.

So, if the subset of  $L_1$  is  $(a+b)^*$  then,  $L_1 = (a+b)^*$ .

Now, from language  $L_2$ ,  $L_2$  is also regular, because when we put "a" and "b" in place of w, then we get

$L_2$  &  $L_3$  they both are CFL because they both have only 1 comparison.

$L_2 \cap L_3$  is not CFL.

$\therefore$  from these conditions,

(B), (C) and (D) are false

Hence, the correct options are (B), (C), (D)

#### Question 49

**698808244**

Consider a simple undirected weighted graph G, all of whose edge weights are distinct. Which of the following statements about the Minimum Spanning Tree of G is/are true?

- Suppose  $S \subseteq V$  be such that  $S \neq \emptyset$  and  $S \neq V$ . Consider the edge with minimum weight such that one of its vertices is in S and the other in  $V \setminus S$ . Such an edge will always be part of any Minimum Spanning Tree of G.
- G can have multiple Minimum Spanning Tree.
- One or both of the edges with the third smallest and the fourth-smallest weight are part of any MST of G.
- The edge with the second-smallest weight is always part of any Minimum Spanning Tree of G.

**Ans. A, C, D**

**Sol. Given :**

Undirected weighted graph G

Where, edge weights are distinct.

So, from given statements,

$$L_3 = \{a^m b^n c^n \mid m, n \geq 0\}$$

Now, from given languages:

$L_1 = \{ww \mid w \in \{a, b\}^*\}$  this language is not CFL, because it has the strings which has straight order.

but complement of  $\bar{L}_1$  is CFL.

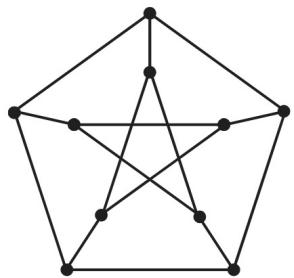
$$L_2 = \{a^n b^n c^m \mid nm \geq 0\} \text{ is CFL}$$

$$L_3 = \{a^n b^m c^m \mid n, m \geq 0\} \text{ is CFL}$$

24

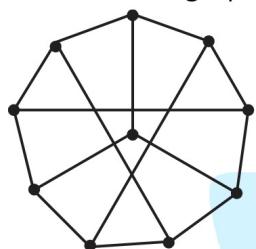
698808245

GATE CSE-2022



Which of the following statements is/are TRUE?

- (A) The chromatic number of the graph is 3.
- (B) The graph has a Hamiltonian path.
- (C) The following graph is isomorphic to the Peterson graph.



- (D) The size of the largest independent set of the given graph is 3. (A subset of vertices of a graph form an independent set if no two vertices of the subset are adjacent.)

**Ans. A, B, C**

**Sol. Given :**

Undirected graph which is referred as the Peterson graph, then from

**Statement A :** The chromatic number of the graph is 3.

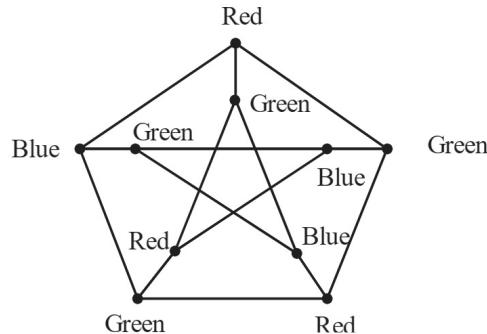
**True,**



The smallest edge is always part of the MST. The graph does not have multiple spanning trees, as all the edge weights are unique. The second and third-smallest edge will be part of the MST if the number of vertices are greater than  $n > 3$  and 4 respectively. Hence the correct options are (A, C, D).

### Question 50

The following simple undirected graph is referred to as the Peterson graph.



**Statement C :** The size of the largest independent set of the given graph is 3. (A subset of vertices of a graph form an independent set if no two vertices of the subset are adjacent.)

**False,**

A set of vertices  $I$  is called an independent set if no two vertices in set  $I$  are adjacent to each other or in other words the set of non-adjacent vertices is called an independent set. It is 4. From option B by simply keeping all the green vertices together we can simply observe that largest independent set contains 4 vertices.

**Statement D :** The following graph is isomorphic to the Peterson graph.

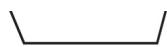
**True,**

We can say given graphs are isomorphic if they have :

1. Equal number of vertices
2. Equal number of edges
3. Same degree sequence
4. Same number of circuit of particular length

**Note :** In most graphs checking the first three conditions is enough.

Hence, the correct options are (A), (B) & (D).



**Statement B :** The graph has a Hamiltonian path.

**True,**

GATE CSE-2022

**698808246**

**Question 51**

Consider the following recurrence :

$$f(1) = 1$$

$$f(2n) = 2f(n) - 1, \text{ for } n \geq 1;$$

$$f(2n+1) = 2f(n) + 1, \text{ for } n \geq 1;$$

Then, which of the following statement is/are TRUE?

$$(A) f(2^n + 1) = 2^n + 1$$

$$(B) f(5 \cdot 2^n) = 2^{n+1} + 1$$

25

$$(C) f(2^n - 1) = 2^n - 1$$

$$(D) f(2^n) = 1$$

**Ans. B, C, D**

**Sol. Given :**

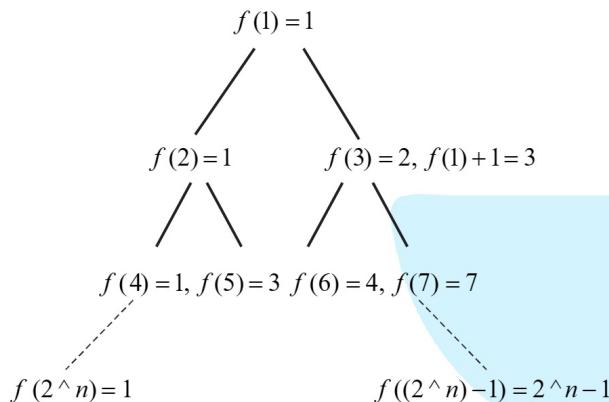
Recurrence function :

$$f(1) = 1$$

$f(2n) = 2f(n) - 1$ , for  $n \geq 1$ ; {i.e. for even input to function  $f$ }

$f(2n+1) = 2f(n) + 1$ , for  $n \geq 1$ ; {i.e. for odd input to function  $f$ }

Options (C) and (D) are true as given in the following diagram.



**For option (A) :**

$2^n + 1$  definitely odd

$$\therefore f(2^n + 1) = 2f(2^{n-1}) + 1 = 2(1) + 1$$

$$\left\{ \text{As we know } f(2^{n-1}) = 1 \right\}$$

$$= 3$$

Therefore, option (A) is false.

**For option (B) :**

$5 \cdot 2^n$  = Definitely even number for all  $n \geq 1$

$$f(5 \cdot 2^n) = 2f(5 \cdot 2^{n-1}) - 1$$

$$= 2(2f(5 \cdot 2^{n-2}) - 1) - 1$$

$$= 2^2 f(5 \cdot 2^{n-2}) - 2 - 1$$

$$= 2^{n+1} + 2^n - 2^n + 1$$

$$= 2^{n+1} + 1$$

Therefore, option (B) is false.

Hence, the correct options are (B), (C) & (D).

**Question 52**

**698808247**

Which of the properties hold for the adjacency matrix  $A$  of a simple undirected unweighted graph having  $n$  vertices?

(A) The diagonal entries of  $A^2$  are the degrees of the vertices of the graph

(B) If the graph is connected then none of the entries of  $A^{n-1} + I_n$  can be zero

(C) If the sum of all the elements of  $A$  is at most  $2(n-1)$  then the graph must be acyclic

(D) If there is at least a 1 in each of  $A$ 's rows and columns, then the graph must be connected

**Ans. A**

**Sol. Option (A) : True.**

Let's think about what  $(A^2)_{i,i}$ , the  $i$ -th term on the diagonal is. We have

$$(A^2)_{ii} = (A \times A)_{ii} = \sum j A_{i,j} A_{j,i}$$

But  $A_{ij} = A_{ji}$ , assuming that the graph is undirected, and  $A_{ji} = 1$  ( $i \sim j$ ), i.e. is 1 if  $i$  and  $j$  are adjacent and 0 otherwise.

Thus  $A_{ij} A_{ji} = A_{i,j}^2 = A_{i,j} = 1$  ( $i \sim j$ ). So the sum is just the number of  $j$  such that  $i \sim j$ , which precisely the degree is.

This work if vertices in your graph may

$$\begin{aligned}
&= 2^2(2f(5 \cdot 2^{n-3}) - 1) - 2 - 1 \\
&= 2^3 f(5 \cdot 2^{n-3}) - 2^2 - 2 - 1 \\
&= 2^n f(5 \cdot 2^0) - 2^{n-1} - 2^{n-2} - 2^{n-3} \dots - 2 - 1 \\
&= 2^n (f(5)) - (1 + 2 + 2^2 + \dots + 2^{n-2} + 2^{n-1}) \\
&= 2^n (3) - (2^n - 1) \\
&= 2^n (2 + 1) - (2^n - 1)
\end{aligned}$$

26

GATE CSE-2022

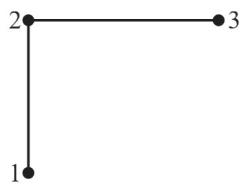
It does not work more generally, however, as  $A_{ii}^2 \neq A_{i,j}$  if  $A_{i,j} > 1$ .

$$(A^T A)_{i,j} = \sum_i A_{i,j}^2 = \deg_{\text{out}}(i),$$

even for an undirected graph, provided the graph is simple.

### Option (B) : False.

Take following connected graph with  $n=3$ ,



Adjacency matrix,  $(A) = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

$$\begin{aligned}
A^{n-1} &= A^3 = A^2 \\
&= \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix}
\end{aligned}$$

$$A^{n-1} + I_n = A^2 + I_3$$

$$\begin{aligned}
&= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix}
\end{aligned}$$

We can see that  $A^{n-1} + I_n$  have some entries as zero. So option (B) false.

### Option (C) : False.

A cyclic graph is a graph containing at least one graph cycle. Consider a graph with 10 vertices where only three vertices form a cycle while rest are isolated vertices (that is a disconnected graph).

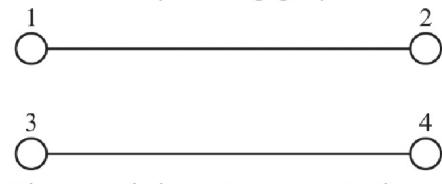
In such a case, sum of all the elements of A is  $(1+1+1)=3$  and 3 is less than  $2(10-1)=2*9=18$ . But the graph is

have a single self-loop, provided you count that as 1 (not 2) for the degree. Indeed, the term in the sum when  $j=i$  is just  $A_{i,i}^2$ , but you need this to be equal to  $A_{i,i}$ . Thus it needs to be 0 or 1.



$$\begin{array}{l}
1 \ 2 \ 3 \ 4 \\
\hline
1 | 0 \ 1 \ 0 \ 0 \\
2 | 1 \ 0 \ 0 \ 0 \\
3 | 0 \ 0 \ 0 \ 1 \\
4 | 0 \ 0 \ 1 \ 0
\end{array}$$

The corresponding graph is :



The graph is not connected.

Hence, the correct option is (A).

### Question 53

698808248

Which is /are eigen vector for given

matrix  $\begin{pmatrix} -9 & -6 & -2 & -4 \\ -8 & -6 & -3 & -1 \\ 20 & 15 & 8 & 5 \\ 32 & 21 & 7 & 12 \end{pmatrix}$

(A)  $\begin{pmatrix} -1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$  (B)  $\begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$

(C)  $\begin{pmatrix} -1 \\ 0 \\ 2 \\ 2 \end{pmatrix}$  (D)  $\begin{pmatrix} 0 \\ 1 \\ -3 \\ 0 \end{pmatrix}$

**Ans. A, C, D**

**Sol. Given :**

Matrix  $\begin{pmatrix} -9 & -6 & -2 & -4 \\ -8 & -6 & -3 & -1 \\ 20 & 15 & 8 & 5 \\ 32 & 21 & 7 & 12 \end{pmatrix}$

Then from given options :

**Option (A) :**

$$[-9 \ -6 \ -2 \ -4] [-1] \ [-3] \ [-1]$$

still cyclic.

**Option (D) : False.**

Considering the following adjacency matrix :

$$\begin{bmatrix} -8 & -6 & -3 & -1 \\ 20 & 15 & 8 & 5 \\ 32 & 21 & 7 & 12 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ -3 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ -9 \end{bmatrix} = 3 \begin{bmatrix} 0 \\ 1 \\ -3 \\ 0 \end{bmatrix}$$

**Option (B) :**

$$\begin{bmatrix} -9 & -6 & -2 & -4 \\ -8 & -6 & -3 & -1 \\ 20 & 15 & 8 & 5 \\ 32 & 21 & 7 & 12 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ -3 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ -9 \\ 0 \end{bmatrix} = 3 \begin{bmatrix} 0 \\ 1 \\ -3 \\ 0 \end{bmatrix}$$

**Option (C) :**

$$\begin{bmatrix} -9 & -6 & -2 & -4 \\ -8 & -6 & -3 & -1 \\ 20 & 15 & 8 & 5 \\ 32 & 21 & 7 & 12 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix} = 1 \begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

**Option (D) :**

$$\begin{bmatrix} -9 & -6 & -2 & -4 \\ -8 & -6 & -3 & -1 \\ 20 & 15 & 8 & 5 \\ 32 & 21 & 7 & 12 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} -7 \\ -5 \\ 12 \\ 25 \end{bmatrix}$$

Hence the correct options are (A, C, D).

**Question 54**

**698808249**

Consider a system with 2 KB direct mapped data cache with a block size of 64 bytes. The system has a physical address space of 64 KB and a word length of 16 bits. During the execution of a program, four data words P, Q, R, and S are accessed in that order 10 times (i.e., PQRSPQRS...). Hence, there are 40 accesses to data cache altogether. Assume that the data cache is initially empty and no other data words are accessed by the program. The addresses of the first bytes of P, Q, R, and S are 0xA248, 0xC28A, 0xCA8A, and 0xA262, respectively. For the execution of the above program, which of the following statements is/are TRUE with respect to the data cache?

- (A) Every access to S is a hit.
- (B) Once P is brought to the cache it is never evicted.

Direct Cache memory

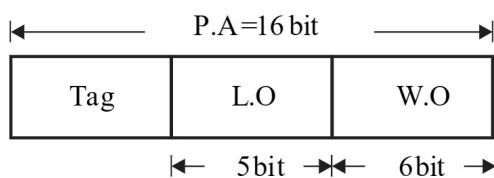
Where, cache memory size = 2 kB, block size = 64 Bytes, physical address space = 64 kB, word length = 16 bits.

$$\therefore \text{Cache line } \frac{2\text{kB}}{64\text{B}} = 32$$

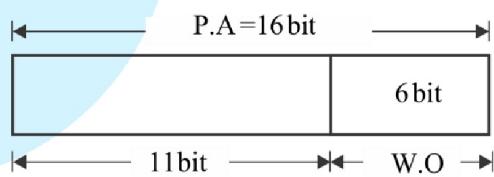
$\therefore$  Line offset = 5 bit

Bits in Physical address =  $\log_2$  (Main memory size =  $\log_2$  (64 kB) = 16 bit)

Physical address format for direct mapped cache



Physical address format for main memory



Tag in main memory		Tag in cache		LO	W.O
P = A 2 4 8	= 1 0 1 0, 0	0 1 0, 0 1	0 0, 1 0 0 0 0		
Q = C A 8 A	= 1 1 0 0, 1	0 1 0, 1 0	0 0, 1 0 1 0		
R = C 2 8 A	= 1 1 0 0, 0	0 1 0, 1 0	0 0, 1 0 1 0		
S = A 2 6 2	= 1 0 1 0, 0	0 1 0, 0 1	1 0, 0 0 1 0		

- P and S are in same block of memory physical address
- Q and S are different block of main memory but mapped to same cache line.

Hence, the correct options are (A) (B) & (D).

**Question 55**

**698808250**

Consider routing table of an organization's router shown below :

Subnet number	Subnet mask	Next hop

- (C) At the end of the execution only R and S reside in the cache.  
(D) Every access to R evicts Q from the cache.

**Ans. A, B, D**

**Sol. Given :**

12.20.164.0	255.255.252.0	$R_1$
12.20.170.0	255.255.254.0	$R_2$
12.20.168.0	255.255.254.0	Interface 0
12.20.166.0	255.255.254.0	Interface 1
(default)		$R_3$

28

GATE CSE-2022



Which of the following prefixes in CIDR notation can be collectively used to correctly aggregate all of the subnets in the routing table?

- (A) 12.20.164.0/20 (B) 12.20.164.0/22  
(C) 12.20.164.0/21 (D) 12.20.168.0/22

**Ans. A**

**Sol. Given :** Routing table of an organization's router shown below :

Subnet number	Subnet mask	Next hop
12.20.164.0	255.255.252.0	$R_1$
12.20.170.0	255.255.254.0	$R_2$
12.20.168.0	255.255.254.0	Interface 0
12.20.166.0	255.255.254.0	Interface 1
(default)		$R_3$

- (A) 12.20.1010 0100.00000000, here the network address is 12.20.1010 0100.00000000 and this network covers all the required IP addresses and hence the required answer.  
(B) 12.20.10100 100.00000000, here the network address is 12.20.10100 100.00000000, but this network does not have IP address 12.20.170.0, hence this cannot be the answer.  
(C) 12.20.101010 00.00000000, here in this network address we do not have IP address 12.20.164.0, hence this cannot be the answer.  
(D) 12.20.101001 00.00000000, here in this network we do not have IP address 12.20.170.0, hence this cannot be the answer.

So, 12.20.164.0/20 will be the network – ID of aggregated route.

Hence the correct option is (A).

**Question 56**

**698808251**

Consider relational database with the four schemas and their respective

Register (*sNo*, dNo)

**Student**

sNo	sName	dNo
S1	James	D01
S2	Rocky	D01
S3	Jackson	D02
S4	Jane	D01
S5	Milli	D03

**Department**

dNo	dName
D01	CSE
D02	EEE

**Course**

cNo	cName	dNo
$C_{11}$	DS	D01
$C_{12}$	DS	D01
$C_{21}$	DE	D02
$C_{22}$	PT	D02
$C_{23}$	CV	D03

**Register**

sNo	cNo
S01	$C_{17}$
S01	$C_{12}$
S02	$C_{11}$
S03	$C_{21}$
S03	$C_{22}$
S03	$C_{23}$
S04	$C_{11}$
S04	$C_{12}$
S05	$C_{11}$
S05	$C_{21}$

SELECT \* FROM Student AS S WHERE NOT EXIST

(SELECT cNo FROM Course WHERE dNo = "D01")

EXCEPT

SELECT cNo FROM Register WHERE

instances :

Student (sNo, sName, dNo)

Department (dNo, dName)

Course (cNo, cName, dNo)

SNO = S.SNO)

The number of rows returned by the above SQL query is \_\_\_\_.

**Ans. 2**

**Sol. Given :**

### Relational database

Where, four schemas and their respective instances are as follows :

Student (sNo, sName, dNo)

Department (dNo, dName)

Course (cNo, cName, dNo)

Register (sNo, dNo)

So from given query,

Select \* from student S where not exist

((Select cNo from course where dNo = D01 except))

(Select cNo from register where sNo = S.sno)

The above query is co-related sub query so we need to execute inner query for every row of outer table.

Hence it gives 2 rows in output

S1	J	D01
S4	M	D03

Hence the correct answer is 2.

### Question 57

**698808252**

Consider a network with three routers P, Q, R shown in the figure below. All the links have cost of unity.



The routers exchange distance vector routing information and have converged on the routing tables, after which the link Q-R fails. Assume that P and Q send out routing updates at random times, each at the same average rate. The probability of a routing loop formation (rounded off to one decimal place) between P and Q, leading to count-to-infinity problem, is \_\_\_\_\_.

**Ans. 0.5**

**Sol. Given :** Network with three routers P, Q, R as shown in the figure.



Once Q-R fails then Q will immediately update its distance to R to  $\infty$ . But P

Now it depends on P and Q who is sending distance vector first.

If Q sends then system becomes stable immediately but if P sends first then it will be count to infinity.

Since it is given in question that both have same average rate hence

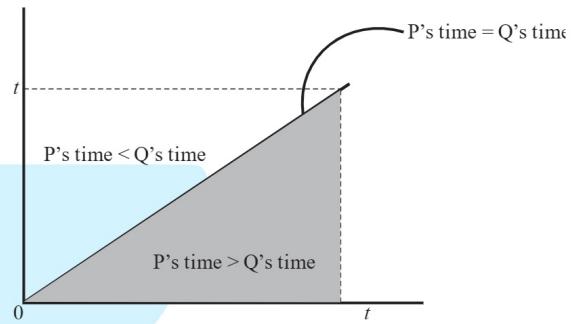
probability is also  $\frac{1}{2} \rightarrow$  OR 0.5 that P

sends first than Q.

Hence the correct answer is 0.5.

OR

We can also calculate the answer more mathematically considering that time is continuous variable.



Considering the probability represented by shaded area :

$$P = \frac{\text{Area of Triangle}}{\text{Area of Square}} = \frac{\frac{1}{2}t^2}{t^2} = \frac{1}{2} = 0.5$$

Hence, the correct answer is 0.5.

### Question 58

**698808253**

Let G (V,E) be a directed graph  $V = \{1,2,3,4,5\}$  is the set of vertices and E is the set of directed edges, as defined by the following adjacency matrix A :

$$A[i][j] = \begin{cases} 1, & 1 \leq j \leq i \leq 5 \\ 0, & \text{otherwise} \end{cases}$$

$A[i][j]=1$  indicates a directed edge from node i to node j. A directed spanning tree of G, rooted at  $r \in V$ , is defined as a subgraph T of G such that the undirected version of T is a tree, and T contains a directed path from r to every other vertex in V. The number of such directed spanning trees rooted at r is \_\_\_\_\_.

update its distance to  $\pi$  to  $\omega$ , but  $\pi$  will still be having same finite value (which is 2).

**30**

at vertex 5 is \_\_\_\_\_.

**Ans. 24**

**Sol. Given :**

GATE CSE-2022

A directed graph  $G$  with  $V$  vertices and  $E$  edges

And it is defined by adjacency matrix  $A$ :

$$A[i][j] = \begin{cases} 1, & 1 \leq j \leq i \leq 5 \\ 0, & \text{otherwise} \end{cases}$$

$A[i][j]=1$  indicates a directed edge from node  $i$  to node  $j$

So, when we consider a graph of two elements, we get only 1 possible MST ( $2 \rightarrow 1$ ). When we consider a graph of three elements, we get 2 possible MSTs ( $3 \rightarrow 1, 3 \rightarrow 2$  or  $2 \rightarrow 1, 3 \rightarrow 2$ ). Similarly, When we consider a graph of four elements, we get only  $3 \times 2 \times 1$  possible MSTs. Similarly. When we consider a graph of five elements, we get only  $4 \times 3 \times 2 \times 1 = 24$  possible MSTs.

Hence the correct answer is 24.

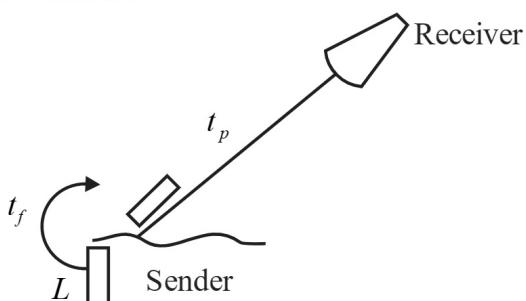
**Question 59**

**698808254**

Consider a 100 Mbps link between an earth station (sender) and a satellite (receiver) at an altitude of 2100 km. The signal propagates at a speed of  $3 \times 10^8$  m/s. The time taken (in milliseconds runoff to 2 decimal places) for the receiver to completely receive a packet of 1000 bytes transmitted by the sender is \_\_\_\_\_.

**Ans. 7.08**

**Sol. Given :**



A sender (earth station) & a receiver (satellite)

Where, Distance = 2100 km,

Speed =  $3 \times 10^8$  m/s

Bandwidth =  $10^8$  m/s,

Now,  $t_p = \frac{d}{s} = \frac{2100 \text{ km}}{3 \times 10^8 \text{ m/s}} = \frac{2100 \times 10^3 \text{ m}}{3 \times 10^8 \text{ m/s}}$   
 $= 7 \times 10^{-3} \text{ sec} = 7 \text{ ms}$

$$t_t = \frac{L}{R} = \frac{1000 \text{ bytes}}{100 \text{ Mbps}} = \frac{1000 \times 8 \text{ bits}}{100 \times 10^6 \text{ bits/sec}}$$
  
 $= \frac{8}{100} \times 10^{-3} \text{ sec} = 0.08 \text{ ms}$

$$T = t_t + t_p = 0.08 + 7 \text{ ms} = 7.08 \text{ ms}$$

Hence the correct answer is 7.08 ms.

**Question 60**

**698808255**

Consider the data transfer using TCP over a one Gbps link. Assuming that the maximum segment life time (MSL) is set to 60 second, the minimum number of bits required for the sequence number field of the TCP header, to prevent the sequence number space from wrapping around during the MSL is \_\_\_\_\_.

**Ans. 33**

**Sol. Given :** TCP connection

Where, Maximum segment lifetime = 60 second, Bandwidth = 1 Gbps.

$$4 \times 3 \times 2 \times 1 = 24,$$

$n$  = Sequence Number bits

$$\Rightarrow 60 \text{ sec} = \frac{2^n \times 8 \text{ bits}}{10^9 \text{ bits/sec}}$$

$$\Rightarrow 60 \times 10^9 = 2^{n+3}$$

$$\Rightarrow \log_2(60 \times 10^9) = n+3$$

$$\Rightarrow \log 60 + 9 \times \log 10 = n+3$$

$$\Rightarrow 5.9 + 29.87 = n+3$$

$$\Rightarrow 35.79 = n+3$$

$$\Rightarrow n+3 = 36$$

$$\Rightarrow n = 33$$

Hence the correct answer is 33.

**Question 61**

**698808256**

A processor  $X_1$  operating at 2 GHz has a standard 5-stage RISC instruction pipeline having a base CPI (cycles per instruction) of one without any pipeline hazards. For a given program  $P$  that has 30% branch instructions, control hazards incur 2 cycles stall for

processor  $X_2$  operating at same clock frequency has an additional branch predictor unit (BPU) that completely eliminates stalls for correctly predicted branches. There is neither any savings nor any additional stalls for wrong predictions. There are no structural hazards and data hazards for  $X_1$  and  $X_2$ . If the BPU has a prediction accuracy of 80%, the speed up (rounded off to two decimal places) obtained by  $X_2$  over  $X_1$  in executing P is \_\_\_\_\_.

**Ans. 1.42**

**Sol. Given :**

2 processors  $X_1$  and  $X_2$

Where, Cycle time = 0.5 ns and Prediction accuracy of BPU = 80%.

Speed Up  $x_2$  over  $x_1$  =

$$\frac{\text{Execution time using } x_1}{\text{Execution time using } x_2}$$

$$= \frac{(\text{Avg CPI})_{x_1} \times \text{Cycle time}}{(\text{Avg CPI})_{x_2} \times \text{Cycle time}}$$

$$= \frac{(1+0.3 \times 2) \times t_p}{(1+0.3 \times (0.8 \times 0 + 0.2 \times 2)) \times t_p}$$

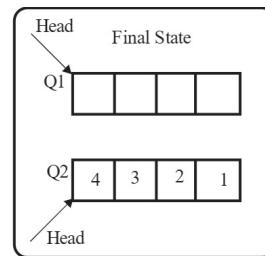
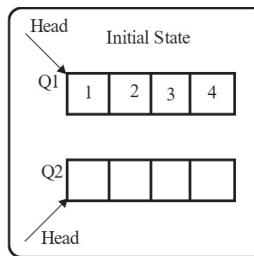
$$= \frac{1.6}{1.12} = 1.42$$

Hence the correct answer is 1.42.

**Question 62**

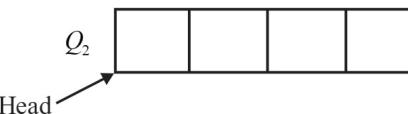
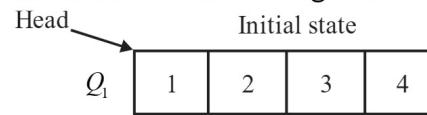
**698808257**

Consider the queues  $Q_1$  containing four elements and  $Q_2$  containing none (shown as the Initial State in the figure). The only operations allowed on these two queues are Enqueue ( $Q, \text{element}$ ) and Dequeue( $Q$ ). The minimum number of Enqueue operations on  $Q_1$  required to place the elements of  $Q_1$  in  $Q_2$  in reverse order (shown as the Final State in the figure) without using any additional storage is \_\_\_\_\_.

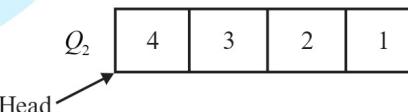
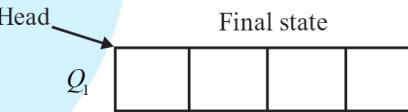


**Ans. 0**

**Sol. Given :** Queues  $Q_1$  containing four elements and  $Q_2$  containing none as shown below in the figure.



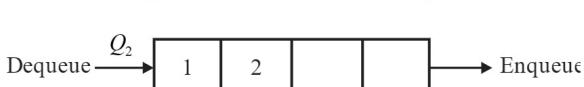
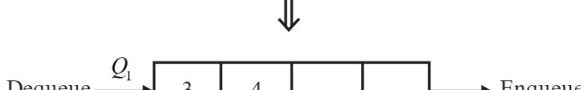
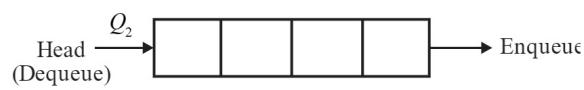
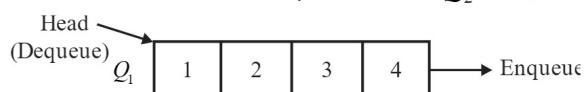
We have to obtain the following situation as shown below in the figure.



As we know that;

If queue is just having 2 elements then we can reverse a queue in place by just using simple enqueue and dequeue and if more than 2 elements we cannot reverse so, first divide 4 elements into 2-2 element each.

**Step 1 :** Enqueue 1 in  $Q_2$ , it is already sorted. Also Enqueue 2 in  $Q_2$  i.e 1, 2.

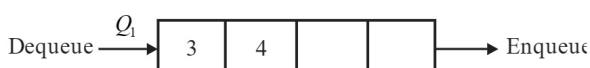
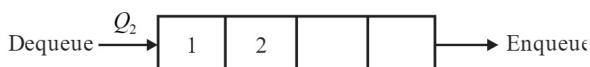
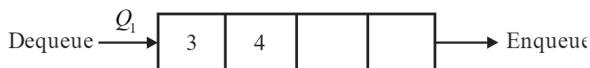




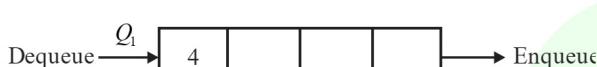
Till now, No enqueue to  $Q_1$ .

Now reverse  $Q_2$ . This doesn't cost any Enqueue to  $Q_1$ .

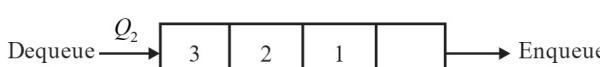
**Step 2 :** Perform 1 Dequeue and 1 Enqueue simultaneously for 1 time 2 in  $Q_2$  i.e. 2, 1 (head pointing at 2 now)



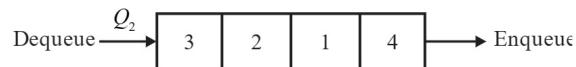
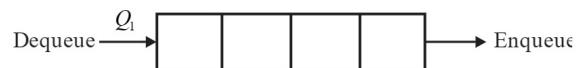
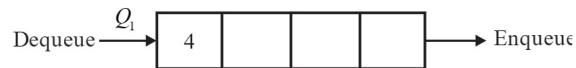
**Step 3 :** Enqueue 3 in  $Q_2$  i.e. 2, 1, 3



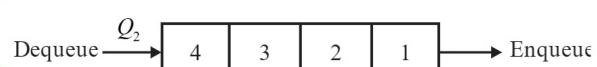
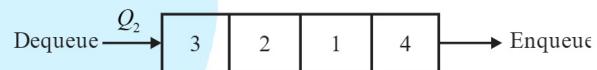
**Step 4 :** Perform 1 Dequeue and 1 Enqueue simultaneously for 2 times in  $Q_2$  i.e. 2, 1, 3  $\rightarrow$  1, 3, 2  $\rightarrow$  3, 2, 1 (head pointing at 3 now)



**Step 5 :** Enqueue 4 in  $Q_2$  i.e. 3, 2, 1, 4



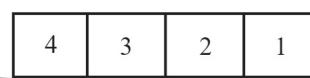
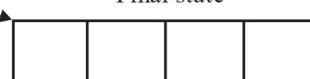
**Step 6 :** Perform 1 Dequeue and 1 Enqueue simultaneously for 3 times in  $Q_2$  i.e. 3, 2, 1, 4  $\rightarrow$  2, 1, 4, 3  $\rightarrow$  1, 4, 3, 2  $\rightarrow$  4, 3, 2, 1



So the number of enqueue operations in  $Q_1$  is 0.

Thus, we have obtain the final state as shown below

Head  $\rightarrow$  Final state



Hence, the correct answer is 0.

**Question 63**

**698808258**

Consider two file-systems A and B, that use contiguous allocation and linked allocation respectively. A file of size 100 blocks is already stored in A and also in B. Now, consider inserting a new block in the middle of the file (between 50<sup>th</sup> and 51<sup>st</sup> block), whose

data is already available in the memory. Assume that there are enough free blocks at the end of the file and that the file control blocks are already in memory. Let the number of disk accesses required to insert a block in the middle of the file in A and B are  $n_A$  and  $n_B$  respectively, then the value of  $n_A + n_B$  is \_\_\_\_\_.

**Ans. 153**

**Sol. Given :**

Two file systems A and B, Which uses contiguous allocation and linked allocation respectively, Data blocks stored in A and B = 100 So, for contiguous allocation, we have to store all the blocks in a sequence, if we have to store a block in the middle, we have to first shift all the blocks after that one place right, for which we need one disk read and one disk write for each block (2 disk accesses). Now since we are storing one block in contiguous allocation at 51<sup>st</sup> position, the blocks (51<sup>st</sup> to 100<sup>th</sup>) have to be moved one step right, for which 2\*50 disk accesses are required, then one access to store the new block, thus  $n_A = 100 + 1 = 101$ .

In Linked allocation, we must read 50 blocks to find the middle. Then we must write the new block somewhere with the next block pointing to the block after the 50<sup>th</sup> block. Then we must write the 50<sup>th</sup> block to point to this new block, thus

$$n_B = 50 + 1 + 1 = 52$$

Hence our required answer  $n_A + n_B$  is 153.

**Question 64**

**698808259**

Consider a demand paging system with four page frames (initially empty) and LRU page replacement policy. For the following page reference string

7, 2, 7, 3, 2, 5, 3, 4, 6, 7, 7, 1, 5, 6, 1  
the page fault rate, defined as the ratio of number of page faults to the

number of memory accesses. (2 decimal) is \_\_\_\_\_.

**Ans. 0.6**

**Sol. Given :**

Demand paging system with four page frames

Where initial frame is empty and LRU page replacement policy used in this. Page reference string are as follows :

7,2,7,3,2,5,3,4,6,7,7,1,5,6,1

7	4	5
2	6	
3	1	
5	7	

Total page faults = 9

Total Access = 15

Thus page fault rate = 9/15 = 0.6

Hence the correct answer is 0.6.

**Question 65**

**698808260**

Consider the following grammar along with translation rules:

$$\begin{array}{ll} S \rightarrow S_1 \# T & \{S_{.val} = S_{1.val} * T_{.val}\} \\ S \rightarrow T & \{S_{.val} = T_{.val}\} \\ T \rightarrow T_1 \% R & \{T_{.val} = T_{1.val} \div R_{.val}\} \\ T \rightarrow R & \{T_{.val} = R_{.val}\} \\ R \rightarrow id & \{R_{.val} = id_{.val}\} \end{array}$$

Here # and % are operators and id is a token that represents an integer and  $id_{.val}$  represents the corresponding integer value. The set of non-terminals is {S, T, R, P,} and a subscripted non-terminal indicates an instance of the non-terminal.

Using this translation scheme, the computed value of  $S_{.val}$  for root of the parse tree for the expression 20#10%5#8%2%2 is \_\_\_\_\_.

**Ans. 80**

**Sol. Given :**

Grammar with translation rules :

$$\begin{array}{ll} S \rightarrow S_1 \# T & \{S_{.val} = S_{1.val} * T_{.val}\} \\ S \rightarrow T & \{S_{.val} = T_{.val}\} \end{array}$$



$$T \rightarrow T_1 \% R \quad (T_{\text{val}} = T_{1,\text{val}} \div R_{,\text{val}})$$

$$T \rightarrow R \quad \{T_{\text{val}} = R_{,\text{val}}\}$$

$$R \rightarrow id \quad \{R_{,\text{val}} = id_{,\text{val}}\}$$

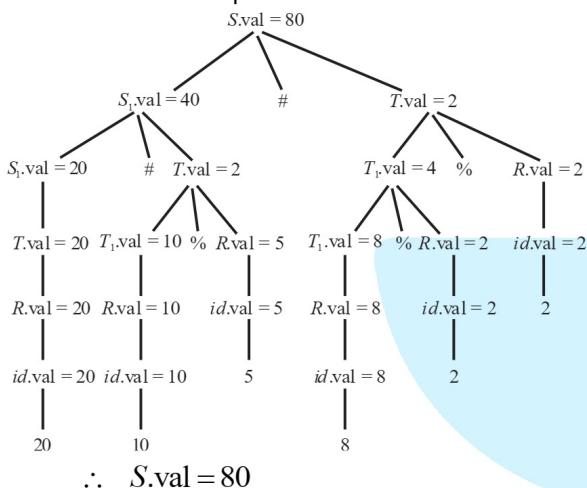
□□□

Where, # and % are operators & and *id* is a token which represents an integer &  $id_{,\text{val}}$  represents corresponding integer value.

Set of non terminals =  $\{S, T, R, P, \}$

So, from given conditions :

Annotated parse tree :



$\therefore S.\text{val} = 80$

Hence the correct answer is 80.





Hence, the correct option is (A).

#### Question 4

A survey of certain year found that 90% of pregnant women received medical care at least once before giving birth. Of these women, 60% received care from doctors, while 40% received from other healthcare providers.

Given this information, which one of the following statements can be inferred with certainty?

- (A) Less than half of pregnant women received medical care at least once from a doctor.
- (B) More than half of pregnant women received medical care at least once from a doctor.
- (C) Less than half of pregnant women received medical care at most once from a doctor.
- (D) More than half of pregnant women received medical care at most once from a doctor.

**Ans. (B)**

**Sol.** **Given :** A survey for a certain year found that 90% of pregnant women received medical care at least once before giving birth of these women, 60% received medical from doctors.

With this given data option (A) can be inferred with certainty, as half of 90% will be 45% and from these 90%, 60% pregnant women received medical care from doctor, and 60% of 90% is 54% which is more then the percentage of half of pregnant women.

Hence, the correct option is (B).

#### Question 5

Looking at the surface of a smooth 3-dimensional object from the outside, which one of the following options is TRUE?

- (A) The surface of the object may be concave in some places and convex in other places.
- (B) The surface of object must be concave everywhere.

(C) The surface of object must be convex everywhere.

(D) The object can have edges, but no corners.

**Ans. (A)**

**Sol.** **Given :** We can combine the convex lens and the concave lens and the combined lens is called a convexo-concave or concavo-convex lens for which one side is convex and other side is concave. Since, convex and concave lenses are 3-dimensional object because each one is formed from two spheres (a three-dimensional object) and so the combined object is also a 3-dimensional object.

Hence, (B) and (C) are eliminated and (A) is correct.

Now, option (D), if you consider the edge as an straight line then for a finite three-dimensional object, option (D) is wrong because where at least two lines or straight edges meet, it creates a corner and according to the definition of smoothness, it should not have a sudden rise or fall and so it will not be a smooth object and so if edge means straight edges.

Hence, the correct option is (A).

#### Q.6 to Q.10 Carry Two Marks Each

#### Question 6

The country of Zombieland is in distress since more than 75% of its working population is suffering from serious health issues. Studies conducted by competent health experts concluded that a complete lack of physical exercise among its working population was one of the leading causes of their health issues. As one of the measures to address the problem, the Government of Zombieland has decided to provide monetary incentives to those who ride bicycles to work.

Based only on the information provided above, which one of the following statements can be logically inferred with certainty?

- (A) All the working population of Zombieland will henceforth ride bicycles to work.
- (B) Riding bicycles will ensure that all of the working population of Zombieland is free of health issues.
- (C) The health experts suggested to the Government of Zombieland to declare riding bicycles as mandatory.
- (D) The Government of Zombieland believes that riding bicycles is a form of physical exercise.

**Ans. (D)**

**Sol.** **Given :** The country of Zombieland is in distress since more than 75% of its working population is suffering from serious health issues.

Studies conducted by competent health experts conducted that a complete lack of physical exercise among its working population was one of the leading cause of their health issues.

According to this information the government of Zombieland has to take action for the physical fitness of its working population.

So, they decided to provide monetary incentives to those who ride bicycles to work.

Therefore, we can say the government of Zombieland believes that riding bicycles is a form of physical exercise. Hence, the correct option is (D).

#### Question 7

Consider two functions of time ( $t$ ),

$$f(t) = 0.01t^2$$

$$g(t) = 4t$$

Where  $0 < t < \infty$ .

Now consider the following two statements :

- (i) For some  $t > 0$ ,  $g(t) > f(t)$
- (ii) There exists a  $T$ , such that  $f(t) > g(t)$  for all  $t > T$

Which one of the following options are TRUE?

- (A) only (ii) is correct
- (B) both (i) and (ii) are correct
- (C) neither (i) nor (ii) is correct
- (D) only (i) is correct

**Ans. (B)**

**Sol.** **Given :** Two functions of time ( $t$ ),

$$f(t) = 0.01t^2$$

$$g(t) = 4t$$

Where  $0 < t < \infty$ .

Statements (i), for some  $t > 0$ ,  $g(t) > f(t)$  is true.

For example, if  $t = 1$ ,  $g(t) = 4$ ,  $f(t) = 0.01$

Hence,  $g(t) > f(t)$  for some  $t = 1$ .

Statement (ii), there exists a  $T$ , such that  $f(t) > g(t)$  for all  $t > T$ . There exist  $T = 400$  such that  $f(t) > g(t) \forall t > 400$ , it is true.

Here, both statement (i) and (ii) are correct.

Hence, the correct option is (B).

#### Question 8

Which one of the following sentence sequences creates a coherent narrative?

- (i) Once on the terrace, on her way to her small room in the corner, she notices the man right away.
- (ii) She begins to pant by the time she has climbed all the stairs.
- (iii) Mina has bought vegetables and rice at the market, so her bags are heavy.
- (iv) He was leaning against the parapet, watching the traffic below.

- (A) (iv), (ii), (i), (iii)    (B) (iii), (ii), (i), (iv)  
 (C) (ii), (iii), (i), (iv)    (D) (i), (ii), (iv), (iii)

**Ans. (B)**

**Sol.** **Given :**

- (i) Once on the terrace, on her way to her small room in the corner, she notices the man right away.
- (ii) She begins to pant by the time she has climbed all the stairs.

at the market, so her bags are heavy.

- (iv) He was leaning against the parapet, watching the traffic below.

We will make pair of two sentence for the sequences which creates a coherent narrative.

The pairs will be (iii) and (ii) which gives a meaningful narrative in the same way (i) and (iv).

According to the options, sequence in option (B) will give best coherent meaning.

Hence, the correct option is (B).

### Question 9

$f(x)$  and  $g(y)$  are functions of  $x$  and  $y$ , respectively, and  $f(x)=g(y)$  for all real values of  $x$  and  $y$ . Which one of the following options is necessarily TRUE for all  $x$  and  $y$ ?

(A)  $f(x)=0$  and  $g(y)=0$

(B)  $f(x)=g(y)=\text{constant}$

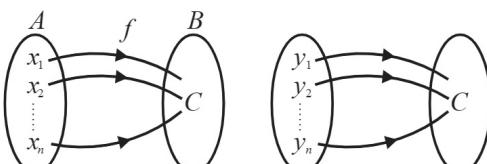
(C)  $f(x)\neq \text{constant}$  and  $g(y)\neq \text{constant}$

(D)  $f(x)+g(y)=f(x)-g(y)$

**Ans. (B)**

**Sol.** Given :  $f(x)$  and  $g(x)$  are functions of  $x$  and  $y$  and  $f(x)=g(y)$  for all real values of  $x$  and  $y$ .

Here, for all values of  $x$  and  $y$  it is necessary that image of ' $x$ ' under  $f$  is same as image at ' $y$ ' is same as image of ' $y$ ' using ' $g$ ' for all real values of  $x$  and  $y$  i.e.  $f(x)=g(y)=\text{Constant}$ .



$$f(x) = g(y) \forall y \in R$$

Hence, the correct option is (B).

### Question 10

Which one of the options best describes the transformation of the 2-

$R$  as shown?



- (A) **Operation 1:** A clockwise rotation by  $90^\circ$  about an axis perpendicular to the plane of the figure.

**Operation 2 :** A reflection along a vertical line.

- (B) **Operation 1:** A clockwise rotation by  $90^\circ$  about an axis perpendicular to the plane of the figure

**Operation 2 :** A reflection along a horizontal line

- (C) **Operation 1 :** A counter clockwise rotation by  $90^\circ$  about an axis perpendicular to the plane of the figure.

**Operation 2 :** A reflection along a horizontal line

- (D) **Operation 1 :** A counter clockwise rotation by  $180^\circ$  about an axis perpendicular to the plane of the figure.

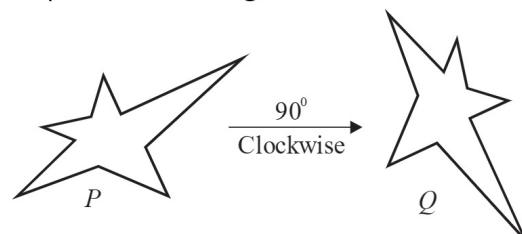
**Operation 2 :** A reflection along a vertical line.

**Ans. (B)**

**Sol.** Given : The transformation of the 2-dimensional figure  $P$  to  $Q$ , and then to  $R$  as shown in below figure,

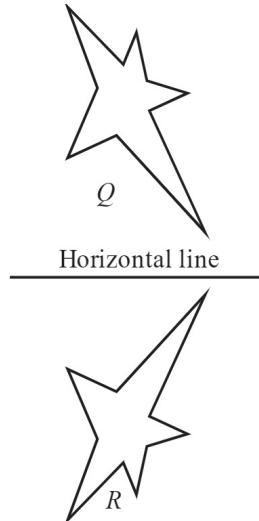


We can clearly see from  $P$  to  $Q$ , operation 1 is a clockwise rotation by  $90^\circ$  about an axis perpendicular to the plane of the figure.



From  $Q$  to  $R$ , operation 2 is a reflection along a horizontal line.

takes intermediate code and generates target code as output. It is dependent on target hardware. So, it is true.



Hence, the correct option is (B).

## Technical Part

### Q.1 to Q.25 Carry One Mark Each

#### Question 1

Consider the following statements regarding the front-end and back-end of a compiler. S1: The front-end includes phases that are independent of the target hardware.

S2: The back-end includes phases that are specific to the target hardware.

S3: The back-end includes phases that are specific to the programming language used in the source code.

Identify the CORRECT option

- (A) Only S1 is TRUE.
- (B) Only S1 and S2 are TRUE
- (C) S1, S2, and S3 are all TRUE
- (D) Only S1 and S3 are TRUE

**Ans. (B)**

**Sol.** S1: The front end or analysis phase consists of lexical, syntax and semantic analysis. It takes source language and produces intermediate code representation. It is independent of target hardware. So, S1 is true.

S2: The back-end or synthesis phase consists of code optimization and target code generation phases which

on target hardware. S2 is true.

S3: Back-end phase is independent of source program as its task is to convert the intermediate code to target code. S3 is false.

Hence, the correct option is (B).

#### Question 2

Which one of the following sequences when stored in an array at locations A[1],...,A[10] forms a max-heap?

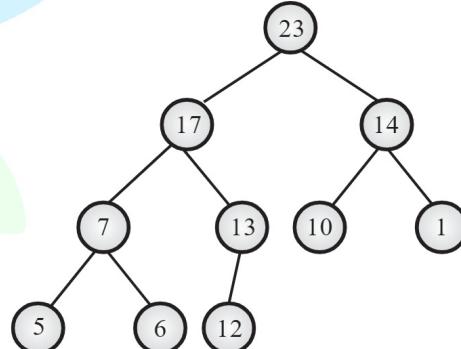
- (A) 23, 17, 10, 6, 13, 14, 1, 5, 7, 12
- (B) 23, 17, 14, 7, 13, 10, 1, 5, 6, 12
- (C) 23, 17, 14, 6, 13, 10, 1, 5, 7, 15
- (D) 23, 14, 17, 1, 10, 13, 16, 12, 7, 5

**Ans. (B)**

**Sol.** Here, wave to check all options for finding which one satisfies property of max-heap. i.e.

- (i) Heap is a complete binary Tree
- (ii) Parent element is always greater than child element value.

Upon checking only



Hence, the correct option is (B).

#### Question 3

Let SLLdel be a function that deletes a node in a singly-linked list given a pointer to the node and a pointer to the head of the list. Similarly, let DLLdel be another function that deletes a node in a doubly-linked list given a pointer to the node and a pointer to the head of the list.

Let  $n$  denote the number of nodes in each of the linked lists. Which one of the following choices is TRUE about the

worst-case time complexity of SLLdel and DLLdel?

- (A) SLLdel is  $O(1)$  and DLLdel is  $O(n)$
- (B) Both SLLdel and DLLdel are

**Ans. (C)**

**Sol.** The given F.A. generate strings like:

$$\Sigma = \{1, 10, 110, 100, 1011, 1000, \dots\}$$

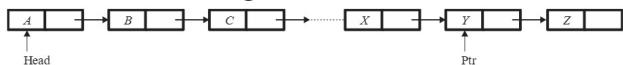
So start with 1 to reach final state from

$O(\log(n))$

- (C) Both SLLdel and DLLdel are  $O(1)$   
 (D) SLLdel is  $O(n)$  and DLLdel is  $O(1)$

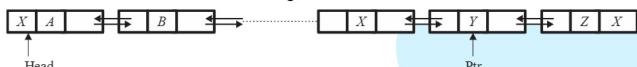
**Ans. (D)**

**Sol.** Given a single linked list SLL:



Here, we're given head pointer and ptr pointer of the node to be deleted (Here y)  
 y) We have to traverse from head node till node before the one pointed by ptr (here X) which take  $O(n)$  time in worst case.

Given a doubly linked list DLL:

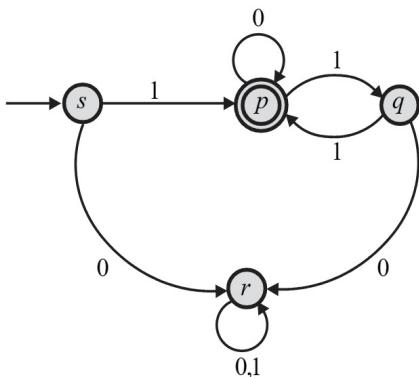


Simply we can do it as:

$\text{pt} \rightarrow \text{prev} \rightarrow \text{next} = \text{ptr} \rightarrow \text{next}$   
 $\text{ptr} \rightarrow \text{next} \rightarrow \text{prev} = \text{ptr} \rightarrow \text{prev}$   
 delete (ptr)  
 since it can be performed in  $O(1)$  time:  
 Hence, the correct option is (D).

**Question 4**

Consider the Deterministic Finite-state Automaton (DFA) A shown below. The DFA runs on the alphabet  $\{0, 1\}$ , and has the set of states  $\{s, p, q, r\}$ , with  $s$  being the start state and  $p$  being the only final state



Which one of the following regular expressions correctly describes the language accepted by A?

- (A)  $1(0*11)^*$       (B)  $0(0 + 1)^*$   
 (C)  $1(0 + 11)^*$       (D)  $1(110^*)^*$

So, start with 1 to reach final state from

where wave two choices os  $(0+11)^*$

Hence R.E. is  $1(0+11)^*$ .

Checking options:

- (a) Doesn't generate 10  
 (b) Doesn't generate any os it starts with 0.  
 (d) Doesn't generate 10.  
 Hence, the correct option is (C).

**Question 5**

The Lucas sequence  $L_n$  is defined by the recurrence relation :

$$L_n = L_{n-1} + L_{n-2}, \text{ for } n \geq 3,$$

with  $L_1 = 1$  and  $L_2 = 3$ .

Which one of the options given is TRUE?

- (A)  $L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n$   
 (B)  $L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{3}\right)^n$   
 (C)  $L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{3}\right)^n$   
 (D)  $L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n$

**Ans. (A)**

**Sol.** Lunar sequence

Put  $n = 1$  in option we will get

$$l_1 = \frac{1+\sqrt{5}}{2} + \frac{1-\sqrt{5}}{3} = \frac{2}{2} = 1$$

given that  $l_1 = 1$

Put  $n = 2$  in option we will get

$$\begin{aligned} l_2 &= \left(\frac{1+\sqrt{5}}{2}\right)^2 + \left(\frac{1-\sqrt{5}}{2}\right)^2 \\ &= 2\left(\frac{1}{4} + \frac{5}{4}\right) = 2\left(\frac{6}{4}\right) = 3 \end{aligned}$$

Given that  $l_2 = 3$

$$L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n$$

Hence, the correct option is (A).

**Question 6**

$$= \frac{L / Bw}{\frac{L}{Bw} + 2\left(\frac{D}{V}\right)} = \frac{L}{Bw\left(T_t + 2\frac{D}{V}\right)}$$

$$\text{So, Efficiency} \propto \frac{1}{D(\text{Link Length})}$$

Which one of the options given below refers to the degree (or arity) of a relation in relational database systems?

- (A) Number of attributes of its relation schema.
- (B) Number of tuples stored in the relation.
- (C) Number of entries in the relation.
- (D) Number of distinct domains of its relation schema.

**Ans. (A)**

**Sol.** By definition: “The degree of relation is the number of attributes it contains”. Hence, the correct option is (A).

### Question 7

Suppose two hosts are connected by a point-to-point link and they are configured to use Stop-and-Wait protocol for reliable data transfer. Identify in which one of the following scenarios, the utilization of the link is the lowest.

- (A) Longer link length and lower transmission rate
- (B) Longer link length and higher transmission rate
- (C) Shorter link length and lower transmission rate
- (D) Shorter link length and higher transmission rate

**Ans. (B)**

**Sol.** Transmission Time

$$(T_t) = \frac{\text{Length of Packet (L)}}{\text{Band width (B.W)}}$$

$$\text{Propagation Time } (T_p) = \frac{\text{Link Length (O)}}{\text{Speed (V)}}$$

$$\text{Link Utilisation} = \text{Efficiency} = \frac{T_t}{T_t + 2T_p}$$

digit  $\rightarrow [0-9]$   
 id  $\rightarrow \text{letter} (\text{letter/digit})^*$

Which one of the following Non-deterministic Finite-state Automata with  $\epsilon$ -transitions accepts the set of valid identifiers? (A double-circle

Also, Transmission rate = Band width

and Efficiency  $\propto \frac{1}{Bw}$

For low utilization, we need longer link length and higher transmission rate. Hence, the correct option is (B).

### Question 8

Let  $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \\ 3 & 4 & 1 & 2 \\ 2 & 3 & 4 & 1 \end{bmatrix}$  and

$B = \begin{bmatrix} 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{bmatrix}$

Let  $\det(A)$  and  $\det(B)$  denote the determinants of the matrices  $A$  and  $B$ , respectively.

Which one of the options given below is TRUE?

- (A)  $\det(A) = \det(B)$
- (B)  $\det(B) = -\det(A)$
- (C)  $\det(A)=0$
- (D)  $\det(AB) = \det(A) + \det(B)$

**Ans. (B)**

**Sol.**  $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \\ 3 & 4 & 1 & 2 \\ 2 & 3 & 4 & 1 \end{bmatrix}$   $B = \begin{bmatrix} 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{bmatrix}$

$$R_1 \leftrightarrow R_3$$

$$\det B = - \det A$$

Hence, the correct option is (B).

### Question 9

Consider the following definition of a lexical token id for an identifier in a programming language, using extended regular expressions :

$$\text{Letter} \rightarrow [A-Za-z]$$

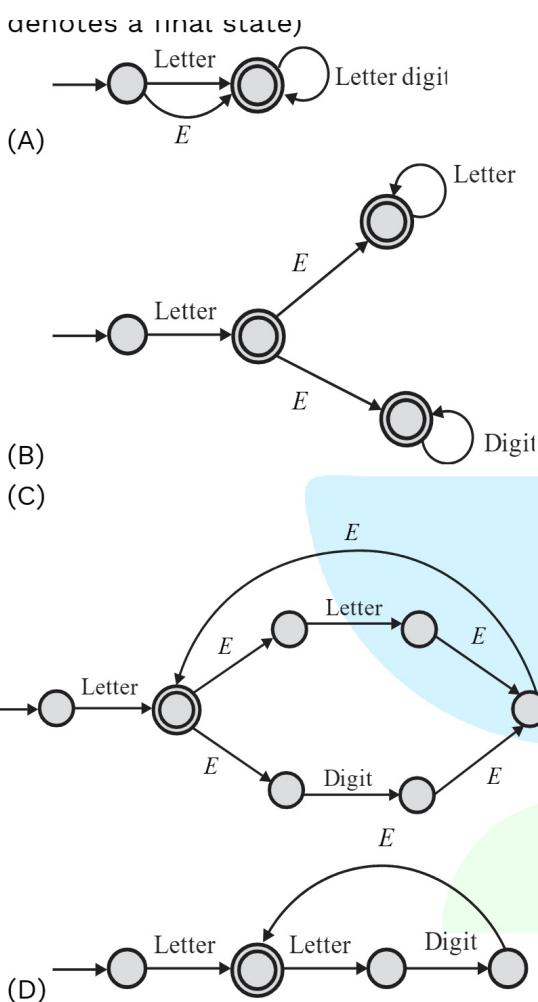


digit  $\rightarrow [0-9]$

id  $\rightarrow \text{letter} (\text{letter/digit})^*$

Which one of the following Non-deterministic Finite-state Automata with  $\epsilon$ -transitions accepts the set of valid identifiers? (A double-circle

An algorithm has to store several keys generated by an adversary in a hash table. The adversary is malicious who tries to maximize the number of collisions. Let  $k$  be the number of keys,  $m$  be the number of slots in the hash table, and  $k > m$ . Which one of the



**Ans. (C)**

**Sol.** The regular expression is:

Letter (letter+digit)\*.

The R.E. accepts strings starting with "letter" and followed by any number of "letter" "digits".

Checking all options:

- (a) is false as it also accepts empty string  $\in$ .
  - (b) is false as it doesn't accept letter. Digit.letter.
  - (d) is false as letter digit not accepted.
- Only option "c" is true.

Hence, the correct option is (C).

**Question 10**

8

$$V = \bar{S} I_0 + S.I_1$$

So, when  $S = 0$

$Y = I_0 = Q$ , the previous state value

When  $S = 1$

$Y = I_1$ , (Output = Input)

Thus, it is D, Latch

Hence, the correct option is (B).

following is the best hashing strategy to counteract the adversary?

- (A) Division method, i.e., use the hash function  $h(k) = k \bmod m$ .
- (B) Multiplication method, i.e., use the hash function  $h(k) = [m(kA - [kA])]$ , where  $A$  is a carefully chosen constant.
- (B) Universal hashing method
- (C) If  $k$  is a prime number, use Division method. Otherwise, use Multiplication method.

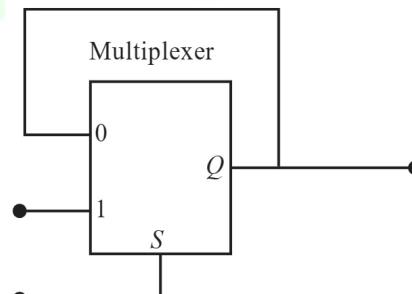
**Ans. (C)**

**Sol.** Here, the attacker is trying to maximize collision and to minimize it we have to use a method that randomly assigns keys to the slots. So option "C"- Universal hashing is best.

Hence, the correct option is (C).

**Question 11**

The output of a 2-input multiplexer is connected back to one of its inputs as shown in the figure.



Match the functional equivalence of this circuit to one of the following options

- (A) D Flip-flop
- (B) D Latch
- (C) Half-adder
- (D) Demultiplexer

**Ans. (B)**

**Sol.** The output equation of above  $2 \times 1$  mux is:

GATE CSE-2023



- System call guarantees that computer system will transition from user mode to kernel mode as using system calls a user requests services from OS and transition from user mode to kernel mode.
- Page fault occurs when program requests access of page not

### Question 12

Which one or more of the following need to be saved on a context switch from one thread (T1) of a process to another thread (T2) of the same process?

- (A) Page table base register
- (B) Stack pointer
- (C) Program counter
- (D) General purpose registers

**Ans.** **(B), (C), (D)**

**Sol.** (A) Page Table Base Register holds base address of page table for currently executing thread since, thread switch between same process so there's no need of updation:  
(B) Each thread has its own stack. So stack pointer needs to be saved.  
(C) PC register contains address of next instruction to be executed by current thread. So it needs to be saved when switch occurs.  
(D) These registers are used to store temporary data during thread execution and needs to be saved before thread switches.

Hence, the correct options are (B), (C) & (D).

### Question 13

Which one or more of the following options guarantee that a computer system will transition from user mode to kernel mode?

- (A) Function Call    (B) malloc Call
- (C) Page Fault      (D) System Call

**Ans.** **(C), (D)**

**Sol.** • Function calls and malloc calls do not necessarily result in transition to kernel mode.

currently in memory (Physical memory) so, OS needs to handle page fault and may need to allocate physical memory.

Hence, the correct options are (C) & (D).

### Question 14

Which of the following statements is/are CORRECT?

- (A) The intersection of two regular languages is regular.
- (B) The intersection of two context-free languages is context-free.
- (C) The intersection of two recursive languages is recursive.
- (D) The intersection of two recursively enumerable languages is recursively enumerable.

**Ans.** **(A), (C), (D)**

**Sol.** Intersection options are closed under Regular Language, Recursive Language and Recursively Enumerable Language. For Context Enumerable Language

Let  $L_1 = a^n b^n c^m / m, n \geq 0$  and

$L_2 = a^m b^n c^n / m, n \geq 0$

Both  $L_1$  and  $L_2$  are CFL languages.

But  $L_1 \cap L_2 = a^n b^n c^n / n \geq 0$  is a non-CFL language

So (B) is False

Hence, the correct options are (A), (C) & (D).

### Question 15

Which of the following statements is/are INCORRECT about the OSPF (Open Shortest Path First) routing protocol used in the Internet?

- (A) OSPF implements Bellman-Ford algorithm to find shortest paths.



- (B) OSPF uses Dijkstra's shortest path algorithm to implement least-cost path routing.
- (C) OSPF is used as an inter-domain routing protocol.
- (D) OSPF implements hierarchical routing.

**Ans.** **(A), (C)**

**Sol.** OSPF uses Dijkstra's algorithm to compute the shortest path tree for

$p(x) = p(c) = \text{True}$  and for all  $y \theta(6, y)$  is True.

Now, RHS :  $E = \forall x < \frac{p(x)}{A} \Rightarrow \frac{\exists y 2(x, y)}{B}$

$A = \text{True}$  for say  $x = 7$ , so  $p(7) = \text{True}$

For B, say there doesn't exist any y such that  $Q(7, y) = \text{True}$ . Hence  $A \Rightarrow B$

$T \Rightarrow F$  is false.

So its case of True  $\rightarrow$  False as LHS is

compute the shortest path tree for each route, the cost of a route is calculated by gathering link state information from available routers. Also, OSPF is hierarchical routing protocol, using are 0 (autonomous system) at top of hierarchy. So, A and C are False.

Hence, the correct options are (A) & (C).

### Question 16

Geetha has a conjecture about integers, which is of the form

$$\forall x(P(x) \Rightarrow \exists y Q(x, y)),$$

where  $P$  is a statement about integers, and  $Q$  is a statement about pairs of integers. Which of the following (one or more) option(s) would imply Geetha's conjecture?

- (A)  $\exists x(P(x)) \wedge \forall y Q(x, y)$
- (B)  $\forall_x \forall y Q(x, y)$
- (C)  $\exists y \forall x(P(x) \Rightarrow Q(x, y))$
- (D)  $\exists x(P(x) \wedge \exists y Q(x, y))$

**Ans. (B), (C)**

**Sol.** Here, domain is set of integers, So Elements

$$x, y \in \{-3, -2, -1, 0, 1, 2, \dots\}$$

$$\text{Expression } E = \forall x \{P(x) \Rightarrow \exists y Q(x, y)\}$$

Which says if  $x$  is  $P$  then ' always exists a  $y$  such such that  $Q(x, y)$ .

Now, checking options.

**For option (A) :**

$$\text{Is } \frac{\exists [P(x)] \wedge \forall_y \theta(x, y)}{\text{LHS}} \rightarrow \frac{E}{\text{RHS}} \text{ True?}$$

Here, for LHS to be true. Say there exists an  $x = 6$  for which

10

GATE CSE-2023

$$\frac{\exists x [P(x) \wedge \exists y Q(x, y)]}{\text{LHS}} \rightarrow \frac{E}{\text{RHS}} \text{ True?}$$

For LHS to be true, assume  $x = 6$  for which properly  $P(6)$  is true and there exists a  $y$  assume  $y = 2$  such that  $Q(6, 2)$  is true.

Now, RHS

$$= E = \forall x \left[ \frac{P(x)}{A} \Rightarrow \frac{\exists y Q(x, y)}{B} \right]$$

Say  $x = 3$  and  $P(x) = \text{True}$

So, its case of True  $\rightarrow$  False as LHS is

True and A

RHS is False

Therefore it becomes case of True  $\rightarrow$  False and eventually its False:

**For option (B) :**

$$\frac{\forall x \forall y Q(x, y)}{\text{LHS}} \rightarrow \frac{E}{\text{RHS}} \text{ True?}$$

Here, LHS is True for all values of  $x$  and  $y$ .

$$\text{Now Ans: } E = \forall x \left[ \frac{P(x)}{A} \Rightarrow \frac{\exists y Q(x, y)}{B} \right]$$

Now, since,

$$\forall x \forall y Q(x, y) = \text{True.}$$

$$\exists y Q(x, y) = \text{True too.}$$

So, For  $A \Rightarrow B$

$A \Rightarrow \text{True}$  is always True.

As RHS is True. Its case of True  $\rightarrow$  True which is true.

**For option (C) :**

$$\rightarrow \frac{\exists y \forall x [P(x) \Rightarrow Q(x, y)]}{\text{LHS}} \rightarrow \frac{E}{\text{RHS}} \text{ True?}$$

For LHS to be true, there exists some  $y$  say  $y = 2$  for which for all  $x$  which are satisfying property  $p$  implies property  $Q(x, y)$

$$\text{Now, RHS: } E = \forall x \left[ \frac{P(x)}{A} \Rightarrow \frac{\exists y Q(x, y)}{B} \right]$$

$B = \exists y Q(x, y)$  is always, true as there exists at least one  $y$  (we assumed  $y = 2$ ) such that  $\forall x Q(x, 2)$  is True. So B is true

The case becomes  $A \Rightarrow B$

$A \Rightarrow \text{True}$  which is true

So, L.H.S  $\rightarrow$  R.H.S. is true.

**For Option (D) :**



Hence, the correct options are (A), (C) & (D).

### Question 18

Let  $f(x) = x^3 + 15x^2 - 33x - 36$  be a real-valued function. Which of the following statements is/are TRUE?

- (A)  $f(x)$  does not have a local maximum.
- (B)  $f(x)$  has a local maximum.
- (C)  $f(x)$  does not have a local minimum.
- (D)  $f(x)$  has a local minimum

But there exists no  $y$  for  $Q(3, y)$  to be true.

Hence, it becomes  $A \Rightarrow B$

True  $\Rightarrow$  False so RHS is False

Also, it becomes case of True  $\rightarrow$  False which is false.

Hence, the correct options are (B) & (C).

### Question 17

Which one or more of the following CPU scheduling algorithms can potentially cause starvation?

- (A) First-in First-Out
- (B) Round Robin
- (C) Priority Scheduling
- (D) Shortest Job First

**Ans. (A), (C), (D) or (C), (D)**

**Sol.** • SJF and priority scheduling are prone to starvation as for SJF the shorter jobs might keep coming and longer burst time jobs have to keep waiting.  
 • Also, for priority scheduling the higher priority job might keep coming causing lower priority jobs to starve.  
 • Round Robin never cause starvation as every job gets a fixed time quantum to execute, which is finite and evry job get time for execution.  
 • For FCFs, in case of infinite loop like while (1);  
     then it cause starvation  
 so until there's special case of a task running forever there'll be no starvation.

$$\mathbf{Sol.} \quad f(x) = x^3 + 15x^2 - 55x - 56$$

$$f'(x) = 3x^2 + 30x - 33$$

$$f''(x) = 6x + 30$$

$$f'(x) = 0$$

$$3x^2 + 30x - 33 = 0$$

$$x^2 + 10x - 11 = 0$$

$$(x+11)(x-1) = 0$$

$$x = -11, x = 1$$

$$f''(x) = 6x + 30$$

$$\text{at } x = -11$$

$$f''(-11) = -66 + 30 = -36 < 0$$

Local maxima

$$\text{at } x = 1$$

$$f''(1) = 6 + 30 = 36 > 0$$

Local minima

$\therefore f(x)$  has a local maximum.

$\therefore f(x)$  has a local minimum.

### Question 19

Let  $f$  and  $g$  be functions of natural numbers given by  $f(n) = n$  and  $g(n) = n^2$ . Which of the following statements is/are TRUE?

- (A)  $f \in O(g)$
- (B)  $f \in \Omega(g)$
- (C)  $f \in o(g)$
- (D)  $f \in \Theta(g)$

**Ans. (A), (C)**

**Sol.** Given,  $f(n), n$  and  $g(n) = n^2$

$$(A) f \in O(g) \Rightarrow f(n) = O(g(n))$$

$f(n) = O(n^2)$ , is True

$$(B) f(n) = \Omega(g(n))$$

$f(n) = \Omega(n^2)$ , is False

$$(C) f \in o(g) \Rightarrow f(n) = o(g(n))$$

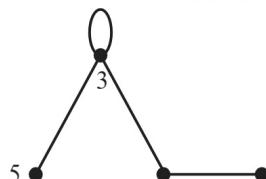
$f(n) = o(n^2)$  is True

- 
- (D)  $f \in O(g) \Rightarrow f(n) = \Theta(g(n))$   
 $f(n) = \Theta(n^2)$  is False

Hence, the correct options are (A) & (C).

### Question 20

Let  $A$  be the adjacency matrix of the graph with vertices  $\{1, 2, 3, 4, 5\}$ .



$$\int_{-3}^3 8x^2 \left( \int_{-2}^2 dy \right) dx$$

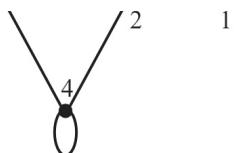
$$\int_{-3}^3 8x^2 (0) dx = 0$$

### Question 22

A particular number is written as 132 in radix-4 representation. The same number in radix-5 representation is\_\_\_\_\_.

**Ans. 110**

**Sol.** Given :  $(132)_4 = (1 \times 4^2 + 3 \times 4 + 2)_{10}$



Let  $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ , and  $\lambda_5$  be the five eigenvalues of  $A$ . Note that these eigenvalues need not be distinct. The value of  $\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 = \underline{\hspace{2cm}}$ .

**Sol.**

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 1 & 1 \\ 3 & 0 & 1 & 1 & 0 \\ 4 & 0 & 1 & 0 & 1 \\ 5 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Sum of Eigen values,

$$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 = T_r(A) \\ = 0 + 0 + 1 + 1 + 0 = 2$$

**Question 21**

The value of the definite integral

$$\int_{-3}^3 \int_{-2}^2 \int_{-1}^1 (4x^2y - z^3) dz dy dx = \underline{\hspace{2cm}}$$

(Rounded off to the nearest integer)

**Sol.**

$$\int_{-3}^3 \int_{-2}^2 \int_{-1}^1 (4x^2y - z^3) dz dy dx$$

$$\int_{-3}^3 \int_{-2}^2 \int_{-1}^1 (4x^2y dz) dy dx - \int_{-3}^3 \int_{-2}^2 \int_{-1}^1 (z^3 dz) dy dx$$

$$\int_{-3}^3 \int_{-2}^2 4x^2y(-1) dy dx = 0$$

$$\int_{-3}^3 \int_{-2}^2 8x^2y dy dx$$

12

it is determined after polling that a key has been pressed, the system consumes an additional  $200 \mu s$  to process the keystroke. Let  $T_1$  denote the fraction of a second spent in polling and processing a keystroke.

In an alternative implementation, the system uses interrupts instead of polling. An interrupt is raised for every keystroke. It takes a total of  $1 ms$  for servicing an interrupt and processing a keystroke. Let  $T_2$  denote the fraction of a second spent in servicing the interrupt and processing a keystroke.

$$= (30)_{10} = (25+5+0)_{10} \quad (\text{Power of 5})$$

$$= (110)_5$$

Hence, the correct answer is 110.

**Question 23**

Consider a 3-stage pipelined processor having a delay of  $10 ns$  (nanoseconds),  $20 ns$ , and  $14 ns$ , for the first, second, and the third stages, respectively. Assume that there is no other delay and the processor does not suffer from any pipeline hazards. Also assume that one instruction is fetched every cycle. The total execution time for executing 100 instructions on this processor is \_\_\_\_\_ ns.

**Ans.**

**2040**

**Sol.**

Given delays  $10ns, 20 ns, 14 ns$

There's no buffer delay or hazard and one instruction is fetched every cycles.

Total instruction ( $n$ ) = 100

Pipeline delay ( $T_p$ ) =  $m_a(10, 20, 14) = 20 ns$

Number of stages ( $k$ ) = 3

So, Total execution time,

$$T = [k + (n-1)] \times T_p$$

$$= (3 + 100 - 1) \times 20 = 2040 ns$$

Hence, the correct answer is 2040.

**Question 24**

A keyboard connected to a computer is used at a rate of 1 keystroke per second. The computer system polls the keyboard every  $10 ms$  (milli seconds) to check for a keystroke and consumes  $100 \mu s$  (micro seconds) for each poll. If

GATE CSE-2023



$$\text{Now, } \frac{T_1}{T_2} = \frac{10.2 ms}{1 ms} = 10.2$$

Hence, the correct answer is 10.2.

**Question 25**

The integer value printed by the ANSI-C program given below is \_\_\_\_\_.

```
#include<stdio.h>
```

```
int funcp(){
```

```
    static int x = 1;
```

```
    x++;
```

```
    return x;
```

```
}
```

```
int main(){
```

The ratio  $\frac{T_1}{T_2}$  is \_\_\_\_\_. (Rounded off to one decimal place)

**Ans. 10.2**

**Sol.** Computer system polls keyboard every 10 ms.

In one second, it polls

$$\frac{1\text{s}}{10\text{ms}} = \frac{1000\text{ms}}{10\text{ms}} = 100 \text{ times}$$

Each poll take  $100\mu\text{s}$

So, Total polling time

$$= 100 \times 100\mu\text{s} = 10 \times 10^3 \mu\text{s} = 10\text{ms}$$

Also, it takes  $200\mu\text{s}$  for processing keystroke i.e. 0.2 ms

Total time spent in polling

$$(T_1) = (10 + 0.2)\text{ms} = 10.2\text{ms}$$

In interrupt system, when there's keystroke CPU executes corresponding interrupt service routine i.e. ISR taking 1 ms. So,  $T_2 = 1\text{ms}$

```
int x,y;
x = funcp();
y = funcp() + x;
printf("%d\n", (x+y));
return 0;
```

}

**Ans. 7**

**Sol.** When  $x = \text{funcp}()$  is called, Static int  $x = 1$  and  $x++$  changes Static  $x$  to 2

In main  $x = 2$  is assigned.

Again  $y = \text{funcp}() + x = \text{funcp}() + 2$

In  $\text{funcp}()$  the static  $x = 2$  and  $x++$  makes static  $x = 3$

In main,  $y = 3 + 5 = 8$

As,  $x = 2$  and  $y = 8$

So,  $x + y = 15$  which is printed out

Hence, the correct answer is 7.



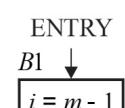
### Q.26 to Q.55 Carry Two Marks Each

#### Question 26

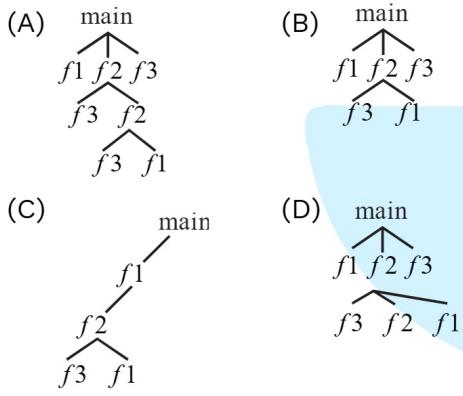
Consider the following program :

int main() {     f1();     f2(2);     f3();     return(0); }	int f1() {     return(1); }	int f2(int X) {     f3();     if (X==1)         return f1();     else         return (X*f2(X-1)); }	int f3() {     return(5); }
--	-----------------------------	---	-----------------------------

Which one of the following options represents the activation tree corresponding to the main function?

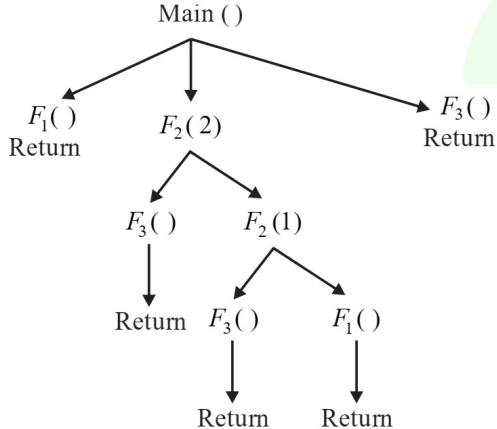


corresponding to the main function:



**Ans. (A)**

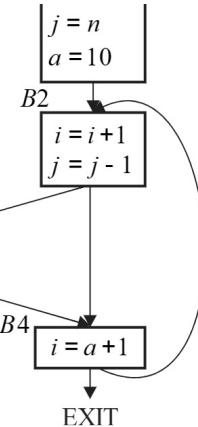
**Sol.** Following the execution sequence



Hence, the correct option is (A).

**Question 27**

Consider the control flow graph shown.



Which one of the following choices correctly lists the set of live variables at the exit point of each basic block?

- (A) B1: {}, B2: {a}, B3: {a}, B4: {a}
- (B) B1: {i, j}, B2: {a}, B3: {a}, B4: {i}
- (C) B1: {a, i, j}, B2: {a, i, j}, B3: {a, i}, B4: {a}
- (D) B1: {a, i, j}, B2: {a, j}, B3: {a, j}, B4: {a, i, j}

**Ans. (D)**

**Sol.** A variable 'V' is live (for statement b) if there exist a path from this statement to another statement is 'a' in CFG such that for each  $b \leq k < a$  and V is defined in any statement k in CFG.

For  $B_1$  : There's path to  $B_2, B_3$  and  $B_4$ .

In this path 'i' and 'J' are live as they're both used before modifying.

'a' is not live as it's used in  $B_3$  before being used in  $B_4$ .

Another path is  $B_2 \rightarrow B_4$

Where all 3 'a', 'i' and 'J' are live so live variable at exit of  $B_1 = \{a, i, J\}$ .

For  $B_2$  : Similarly for path  $B_3, B_4$  'J' is live for path  $B_4$  'a' and 'J' are live.

For  $B_3$  : Same as  $B_1$  as path for exit is same.

So, live variable at exits of  $B_4 = \{a, i, J\}$ .

Hence, the correct option is (D).

**Question 28**

Consider the two functions incr and decr shown below.

  
`incr(), decr(), incr(), decr(), incr(), decr()` which all make X as same value 10 and then two `incre()` making  $V_1 = 12$

For implementation  $I_2$  :

Counting semaphore  $s = 2$ .

So, one possible sequence is `decr()`

{

`wait(s); //s becomes '1'`  
`red X; //X becomes/reads and stores value 10.`

-----  
Run incr () 5 times  
-----

```

incr(){
    wait(s);
    X = X+1;
    signal(s);
}
decr(){
    wait(s);
    X = X-1;
    signal(s);
}

```

There are 5 threads each invoking *incr* once, and 3 threads each invoking *decr* once, on the same shared variable *X*. The initial value of *X* is 10.

Suppose there are two implementations of the semaphore *s*, as follows:

I-1: *s* is a binary semaphore initialized to 1.

I-2: *s* is a counting semaphore initialized to 2.

Let *V<sub>1</sub>*, *V<sub>2</sub>* be the values of *X* at the end of execution of all the threads with implementations I-1, I-2, respectively.

Which one of the following choices corresponds to the minimum possible values of *V<sub>1</sub>*, *V<sub>2</sub>*, respectively?

- |           |           |
|-----------|-----------|
| (A) 15, 7 | (B) 7, 7  |
| (C) 12, 7 | (D) 12, 8 |

**Ans. (C)**

**Sol.** For implementation *I<sub>1</sub>*:

Binary semaphore *s*=1 and *incr()* called 5 times and *decr()* called 3 times by threads

So, it alternate as sequence.

 Option D : It is false since the language generated has satisfied all conditions of being regular language.

Option B : It is true as it accepts all strings generated by  $a^*(a+b)b^*$ .

Hence, the correct option is (B).

### Question 30

Consider the pushdown automaton (PDA) *P* below, which runs on the input alpha-bet  $\{a,b\}$ , has stack alphabet  $(\perp, A)$ , and has three states  $\{s, p, q\}$ , with *s* being the start state. A transition from state *u* to state *v*, labelled *c/X/y*, where *c* is an input symbol or  $\epsilon$ , *X* is a stack symbol, and *y* is a string of stack symbols, represents the fact that in state *u*, the PDA can

*X*=9 //write *X*.

Two more *decr* ( ) so, *X*=7.

Value *V<sub>2</sub>*=7.

Hence, the correct option is (C).

### Question 29

Consider the context-free grammar *G* below

$$S \rightarrow aSb|X$$

$$X \rightarrow aX|Xb|a|b$$

where *S* and *X* are non-terminals, and *a* and *b* are terminal symbols. The starting non-terminal is *S*.

Which one of the following statements is CORRECT?

- (A) The language generated by *G* is  $(a+b)^*$
- (B) The language generated by *G* is  $a^*(a+b)b^*$
- (C) The language generated by *G* is  $a^*b^*(a+b)$
- (D) The language generated by *G* is not a regular language

**Ans. (B)**

**Sol.** Option A : Since *E* can't be generated by *G* so, option (A) is false, which accept *E*.

Option C : Since 'ba' *G* but is accepted by  $a^*b^*(a+b)$  10(C) s false.

**Sol.** Option B : This option is not true since *E* is not accepted by *P*.

Option C : This option is false since *E* is not accepted by *P*.

Option D : This option is false since *E* is not accepted by *P*.

Option A : It accept all strings generated by *P*.

Hence, the correct option is (A).

### Question 31

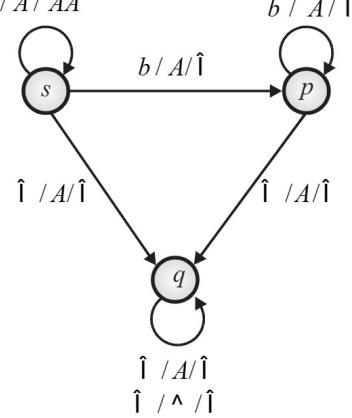
Consider the given C-code and its corresponding assembly code, with a few operands U1-U4 being unknown. Some useful information as well as the semantics of each unique assembly instruction is annotated as inline comments in the code. The memory is byte-addressable.

read  $c$  from the input, with  $X$  on the top of its stack, pop  $X$  from the stack, push in the string  $\gamma$  on the stack, and go to state  $v$ . In the initial configuration, the stack has only the symbol  $\perp$  in it.

The PDA accepts by empty stack.

$a / \wedge / A^\wedge$

$a / A / AA$



Which one of the following options correctly describes the language accepted by P?

- (A)  $\{a^m b^n \mid 1 \leq m \text{ and } n < m\}$
- (B)  $\{a^m b^n \mid 0 \leq n \leq m\}$
- (C)  $\{a^m b^n \mid 0 \leq m \text{ and } 0 \leq n\}$
- (D)  $\{a^m \mid 0 \leq m\} \cup \{b^n \mid 0 \leq n\}$

**Ans. (A)**

16

//C-code  
;assembly-code ( indicates comments)

;r1-r5 are 32-bit integer registers

;initialize r1=0, r2=10

;initialize r3, r4 with base address of a, b

int a[10], b[10], i;  
// int is 32-bit

L01: jeq r1, r2, end  
;if(r1==r2) goto end

L02: lw r5, 0(r4) ;r5 <- Memory[r4+0]

L03: shl r5, r5, U1  
;r5 <- r5 << U1

L04: sw r5, 0(r3) ;Memory[r3+0] <- r5

L05: add r3, r3, U2  
;r3 <- r3+U2

L06: add r4, r4, U3

L07: add r1, r1, 1

L08: jmp U4

;goto U4

L09: end

Which one of the following options is a CORRECT replacement for operands in the position (U1, U2, U3, U4) in the above assembly code?

GATE CSE-2023

- (A) (8, 4, 1, L02)      (B) (3, 4, 4, L01)
- (C) (8, 1, 1, L02)      (D) (3, 1, 1, L01)

**Ans. (B)**

**Sol.** Here after analyzing code we can observe that. We are to shift value of  $r_s$  left by  $u$  places. In code were multiplying element of  $b$  by 8. So,  $u_1 = 3$ , which is same as multiplying value by 8 or  $2^3$ .

Also,  $r_3$  and  $r_4$  stores storing address of next element of arrays  $a$  and  $v$ . Since it's 32 bit system and size of int is 4B so well increment by 4 so, value of  $u_2$  and  $u_3$  is 4.

We have to jump to short of code i.e.  $\angle 01$  so that for-loop can be run.

So,  $u_4$  is  $\angle 01$ .

Hence the correct option is (B).

- (B) (0, 1024, 2048, 3072)
- (C) (0, 8, 16, 24)
- (D) (0, 0, 0, 0)

**Ans. (C)**

**Sol.** The addresses are of length 12 bits. The 2:4 decoder with input  $IA_3$  and  $IA_4$  decides which chip is selected. Possible values of  $IA_4 IA_3$  are 4.

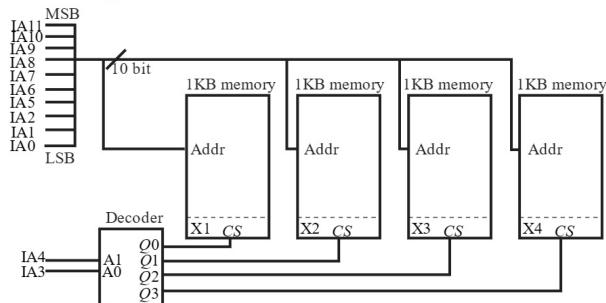
For starting address the value of  $I_{11} \dots I_5$  remains '0' and we're focused on value from  $I_4 \dots I_0$ .

So,

$I_4$	$I_2$	Resulting value /Starting address
0	0 ( $X_0$ )	0(00000)
0	1 ( $X_1$ )	8(01000)
1	0 ( $X_2$ )	16(10000)

**Question 32**

A 4 kilobyte (KB) byte-addressable memory is realized using four 1 KB memory blocks. Two input address lines (IA4 and IA3) are connected to the chip select (CS) port of these memory blocks through a decoder as shown in the figure. The remaining ten input address lines from IA11–IA0 are connected to the address port of these blocks. The chip select (CS) is active high.



The input memory addresses (IA11–IA0), in decimal, for the starting locations (Addr=0) of each block (indicated as X1, X2, X3, X4 in the figure) are among the options given below. Which one of the following options is CORRECT?

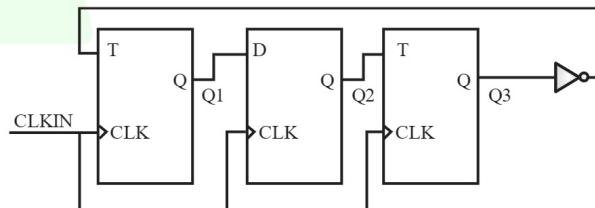
- (A) (0, 1, 2, 3)

1	$1 (X_3)$	24(11000)
---	-----------	-----------

Hence, the correct option is (C).

**Question 33**

Consider a sequential digital circuit consisting of T flip-flops and D flip-flops as shown in the figure. CLKIN is the clock input to the circuit. At the beginning, Q1, Q2 and Q3 have values 0, 1 and 1, respectively.



Which one of the given values of (Q1, Q2, Q3) can NEVER be obtained with this digital circuit?

- (A) (0, 0, 1)      (B) (1, 0, 0)  
 (C) (1, 0, 1)      (D) (1, 1, 1)

**Ans. (A)**

**Sol.** Given circuit is made from two  $T$  flip-flops and one  $D$  flop-flop.

$$\text{Here, } T_1 = \bar{Q}_3$$

$$\text{So, } Q_1^+ = T_1 \oplus Q_1 = \bar{Q}_3 \oplus Q_1 = Q_3 \odot Q_1$$

$$\text{Also, } D_2 = Q_1$$

$$\text{So, } Q_2^+ = Q_1$$

$$\text{Now, } T_3 = Q_2$$

$$Q_3^+ = T_3 \oplus Q_2 = Q_3 \oplus Q_2$$

The state table looks like :

$Q_1$	$Q_2$	$Q_3$	$Q_1^+$	$Q_2^+$	$Q_3^+$
0	1	1	0	0	0
0	0	0	1	0	0
1	0	0	0	1	0
0	1	0	1	0	1
1	0	1	1	1	1
1	1	1	1	1	0
1	1	0	0	1	1

So, the missing state is 001.

Hence, the correct option is (A).

**Question 34**

A Boolean digital circuit is composed using two 4-input multiplexers ( $M_1$  and  $M_2$ ) and one 2-input multiplexer ( $M_3$ ) as shown in the figure.  $X_0$ – $X_7$  are the

- (B) (1, 1, 0, 0, 1, 1, 0, 1)

- (C) (1, 1, 0, 1, 1, 1, 0, 0)

- (D) (0, 0, 1, 1, 0, 1, 1, 1)

**Ans. (C)**

**Sol. Given :**

$$F = \bar{A} + \bar{A} \cdot \bar{C} + A\bar{B}C.$$

Since final output is given by  $2 \times 1$  mux  $m_3$

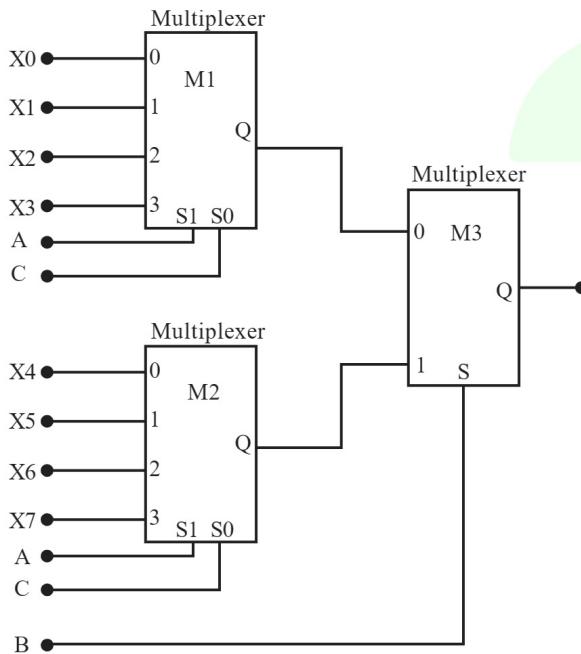
So, MSB (most significant bit) is selection line of  $m_3$  i.e.  $B$ .

Also we can observe from  $M_1$  and  $M_2$ , that the selection lines  $(S_1, S_0) = (A, C)$  so the function has  $C$  as LSB

The function  $F(B, A, C) = \bar{A} + \bar{A} \cdot \bar{C} + A\bar{B}C$  is implemented using inputs as :

	$B$	$A$	$C$	
$X_0$	0	0	0	1 → Due to $A'$
$X_1$	0	0	1	1 → Due to $A'$

as shown in the figure.  $X_0$  to  $X_7$  are the inputs of the multiplexers M1 and M2 and could be connected to either 0 or 1. The select lines of the multiplexers are connected to Boolean variables A, B and C as shown.



Which one of the following set of values of  $(X_0, X_1, X_2, X_3, X_4, X_5, X_6, X_7)$  will realise the Boolean function  $\bar{A} + \bar{A}\bar{C} + A\bar{B}C$ ?

- (A) (1, 1, 0, 0, 1, 1, 1, 0)

18

GATE CSE-2023

$X_2$	0	1	0	0
$X_3$	0	1	1	1 → Due to $AB'C$
$X_4$	1	0	0	1 → Due to $A'$
$X_5$	1	0	1	1 → Due to $A'$
$X_6$	1	1	0	0
$X_7$	1	1	1	0

So,  $(X_0, X_1, \dots, X_7) = (11011100)$

Hence, the correct option is (C).

### Question 35

Consider the IEEE-754 single precision floating point numbers  $P=0xC1800000$  and  $Q=0x3F5C2EF4$ . Which one of the following corresponds to the product of these numbers (i.e.,  $P \times Q$ ), represented in the IEEE-754 single precision format?

- (A) 0x404C2EF4    (B) 0x405C2EF4  
 (C) 0xC15C2EF4    (D) 0xC14C2EF4

**Ans.** (C)

**Sol.** Here,  $P=0 \times 1800000 = 1100\ 0001\ 1000\ 0000\ 0000\ 0000\ 0000$   
 In IEEE 754 single precision format.

$(S) \rightarrow$  sign bit = 1

Biased exponent = 131

Actual exponent = 131 - 127 = 4

$(m) \rightarrow$  Mantissa = 0000 0000 0000 0000 0000 0000

The number =  $(-1)^s \times (1.m) \times 2^e$

$$=(-1)^1 \times 1.0 \times 2^4 = -16$$

Similarity,  $Q=0 \times 3F5C2EE4 = 0011\ 1111\ 0101\ 1100\ 0010\ 1110\ 1111\ 0100$

sign = 0

Biased exponent = 126

Actual exponent = 126 - 127 = -1

So,

$$Q = 1.10111000010111011110100 \times 2^{-1}$$

$$P * Q = -1.10111000010111011110100 \times 2^{-1} \times 2^4$$

$$P * Q = -1.10111000010111011110100 \times 2^3$$

sign = 1

Biased exponent = 127 + 3 = 130 = 10000010.

The number in IEEE 754 format is :

(C) Extract-Max( $A$ ) runs in  $O(1)$  whereas Insert( $A$ , key) runs in  $O(n)$ .

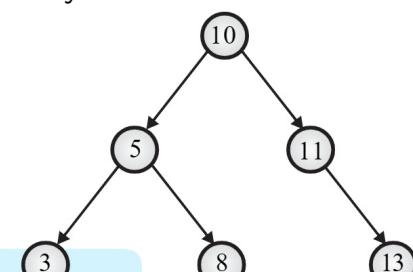
(D) Extract-Max( $A$ ) runs in  $O(1)$  whereas Insert( $A$ , key) runs in  $O(\log(n))$ .

**Ans.** (B)

**Sol.** Since both extract-max ( $A$ ) and Insert ( $A$ ) needs to perform heapify () operation, both take  $O(\log(n))$  time.  
 Hence, the correct option is (B).

### Question 37

Consider the C function foo and the binary tree shown.



typedef struct node {

int val;

struct node \*left \*right;

1	1000 0010	1011 1000 0101 1101 1110 100
↓ Sign bit	↓ Exponent	↓ Mostissa

0xC15C2EF4

Hence, the correct option is (C).

### Question 36

Let A be a priority queue for maintaining a set of elements. Suppose A is implemented using a max-heap data structure. The operation Extract-Max(A) extracts and deletes the maximum element from A. The operation Insert(A, key) inserts a new element key in A. The properties of a max-heap are preserved at the end of each of these operations.

When A contains n elements, which one of the following statements about the worst case running time of these two operations is TRUE?

- (A) Both Extract-Max(A) and Insert(A, key) run in O(1).
- (B) Both Extract-Max(A) and Insert(A, key) run in  $O(\log(n))$ .

```
struct node {int, right, left};  
} node;
```

```
int foo(node * p) {  
    int retval;  
    if (p == NULL)  
        return 0;  
    else {  
        retval = p->val +  
            foo(p->left) +  
            foo(p->right);  
        printf("%d ",  
            retval);  
        return retval;  
    }  
}
```

When foo is called with a pointer to the root node of the given binary tree, what will it print?

- (A) 3 8 5 13 11 10
- (B) 3 5 8 10 11 13
- (C) 3 8 16 13 24 50
- (D) 3 16 8 50 24 13

**Ans.**

**(C)**

**Sol.**

**Given :** Retval

$$= P \rightarrow val + f00(P \rightarrow left) + f00(P \rightarrow right)$$

Now, considering given tree with root 10.

Retval

$$= 10 + f00(P \rightarrow left) + f00(P \rightarrow right)$$

So, until we execute leaf node we won't get to return to root node.

Also child-Nodes 3, 8 and 13 will return value of leaf nodes, i.e. 3,8 and 13 only.

So, 5 node will return  $5+3+8=16$ .

11 node will return  $11+13=24$

10 node will return  $10+16+24=50$

Since there's no rule about evaluation order of parameters of '+' but considering/assuming left right rule by default we get output as : 3,8,16,13,24,50.

Hence, the correct option is (C).

### Question 38

Let  $U = \{1, 2, \dots, n\}$ , where  $n$  is a large positive integer  $U$  with  $|A|=|B|=k$  and  $A \cap B = \emptyset$ . We say that a permutation of  $U$  separates  $A$  from  $B$  if one of the following is true

$$A \cap B = \emptyset$$

Permutation of  $\cup$  separates of A from B

$$2 \binom{n}{2k} (n-2k)! (k!)^2$$

Hence, the correct option is (D).

### Question 39

Let  $f: A \rightarrow B$  be an onto (or surjective) function, where  $A$  and  $B$  are nonempty sets. Define an equivalence relation  $\sim$  on the set  $A$  as

$$a_1 \sim a_2 \text{ if } f(a_1) = f(a_2),$$

Where  $a_1, a_2 \in A$ . Let  $\varepsilon = \{[x] : x \in A\}$  be the set of all the equivalence classes under  $\sim$ . Define a new mapping  $F: \varepsilon \rightarrow B$  as

$$F([x]) = f(x), \text{ for all the equivalence classes } [x] \text{ in } \varepsilon.$$

Which of the following statements is/are TRUE?

- (A)  $F$  is NOT well-defined.
- (B)  $F$  is an onto (or surjective) function.

following is true.

- All members of  $A$  appear in the permutation before any of the members of  $B$ .
- All members of  $B$  appear in the permutation before any of the members of  $A$ .

How many permutations of  $U$  separate  $A$  from  $B$ ?

(A)  $n!$

(B)  $\binom{n}{2k}(n-2k)!$

(C)  $\binom{n}{2k}(n-2k)!(k!)^2$

(D)  $2\binom{n}{2k}(n-2k)!(k!)^2$

**Ans. (D)**

**Sol.**  $U = \{1, 2, \dots, n\}$

$$k < n < 1000$$

$$|A| = k$$

$$|B| = k$$

(C)  $F$  is a one-to-one (or injective) function.

(D)  $F$  is a bijective function.

**Ans. (B), (C), (D)**

**Sol.** The equivalence relation on set  $A$  is defined as :

$$a_1 \sim a_2 \text{ if } f(a_1) = f(a_2)$$

Where  $a_1, a_2 \in A$ .

Consider  $a_i, b_i, c_i, \dots \in A$  and

$\alpha, \beta, r, \dots \in B$  and the mapping  $a_i \rightarrow$ :

$$a_1 \rightarrow \alpha, a_2 \rightarrow \alpha, a_3 \rightarrow \alpha, \dots, a_n \rightarrow \alpha$$

Similarly,

$b_1 \rightarrow \beta, b_2 \rightarrow \beta, b_3 \rightarrow \beta, \dots, b_n \rightarrow \beta$  and so on.....

According to equivalent, equivalent loss is :

$$[a_1] = [a_2] = [a_3] = \dots = [a_m]$$

$[b_1] = [b_2] = [b_3] = \dots = [b_n]$  and so on.

So, set of equivalence classes under relation is :

20

GATE CSE-2023

$$\varepsilon = \{[a_1], [b_1], [c_1], \dots\}$$

Now, given new mapping  $F = \varepsilon \rightarrow B$  as :

$$F([x]) = F(x) \text{ for all } [x] \in \varepsilon$$

It means mapping will be :

$a_1 \rightarrow \alpha, b_1 \rightarrow \beta, c_1 \rightarrow r, \dots$  and so on.

Since, all distinct  $a_1, b_1, c_1, \dots$  maps to different element of set  $B$ . SO  $F$  is injective.

We've considered  $\{a_1, b_1, c_1, \dots\}$  as leaders of their equivalent class.

We can also consider  $\{a_2, b_2, c_2, \dots\}$

Also, its cleared from mapping of  $F$  that all the elements of set  $B$  are So,  $F$  is subjective.

Since  $F$  is both injective and subjective so  $F$  is bijective.

Also, we can observe that is well-defined function since its bijective.

Hence, the correct options are (B), (C), (D).

**Question 40**

$$\geq 30.48$$

$$\geq 31 \text{ bits}$$

Hence, the correct options are (B), (C) & (D).

**Question 41**

Let  $X$  be a set  $2^X$  and denote the powerset of  $X$ .

Define a binary operation  $\Delta$  on  $2^X$  as follows :

$$A \Delta B = (A - B) \cup (B - A).$$

Let  $H = (2^X, \Delta)$ . Which of the following statements about  $H$  is/are correct?

(A)  $H$  is a group.

(B) Every element in  $H$  has an inverse, but  $H$  is NOT a group.

(C) For every  $A \in 2^X$ , the inverse of  $A$  is the complement of  $A$ .

(D) For every  $A \in 2^X$ , the inverse of  $A$  is  $A$ .

**Ans. (A), (D)**

**Sol.** The symmetric difference is similar to EXOR operation in digital logic.

Now left check it for following

Suppose you are asked to design a new reliable byte-stream transport protocol like TCP. This protocol, named myTCP, runs over a 100 Mbps network with Round Trip Time of 150 milliseconds and the maximum segment lifetime of 2 minutes.

Which of the following is/are valid lengths of the Sequence Number field in the myTCP header?

- (A) 30 bits              (B) 32 bits
- (C) 34 bits              (D) 36 bits

**Ans.** **(B), (C) (D)**

**Sol.** Given bandwidth (BW) = 100 Mbps

So, in 1 second we can send  $100 \times 10^6$  bits =  $125 \times 10^3 B$

$$\begin{aligned} \text{In 120 seconds} &= 120 \times 125 \times 10^3 B \\ &= 15000 \times 10^5 B \end{aligned}$$

Since n lifetime of 120 seconds  $15 \times 10^8$  bytes are generated, so

$$\text{Sequence number bits} = \log_2(15 \times 10^8)$$

It's possible  $(A \cup B) = (A \cap B)$

If  $A = B$  then its possible.

Hence, every element of  $2^X$  is it's own inverse and  $H$  is a group.

Hence, the correct options are (A) & (D).

#### Question 42

Suppose in a web browser, you click on the www.gate-2023.in URL. The browser cache is empty. The IP address for this URL is not cached in your local host, so a DNS lookup is triggered (by the local DNS server deployed on your local host) over the 3-tier DNS hierarchy in an iterative mode. No resource records are cached anywhere across all DNS servers.

Let RTT denote the round trip time between your local host and DNS servers in the DNS hierarchy. The round trip time between the local host and the web server hosting www.gate-2023.in is also equal to RTT. The HTML file associated with the URL is small enough to have negligible transmission time and negligible rendering time by your web browser, which references 10 equally small objects on the same web

properties :

1. Colored : Operator  $\Delta$  is defined as  

$$\begin{aligned} A\Delta B &= (A - B) \cup (B - A) \\ &= (A \cup B) - (A \cap B) \end{aligned}$$

$\because 2^X$  is power set of  $X$ , so it contains all subset of  $X$ .

So,  $A\Delta B \in 2^X \forall A, B \in 2^X$ .
2. Associativity : It's similar to XOR operation which is associative always.
3. Identity :  $s_{ay}$   $C \in 2^X$  is identity element.  
 $\text{So, } A\Delta C = C\Delta A = A \text{ for } C \in 2^X$   
 $A\Delta C = (A \cup C) - (A \cap C) = A$  which is possible when  $C = \Phi$ .  
 So, identity exists.
4. Inverse :  $A\Delta B = B\Delta A = \Phi$  then  $A\Delta B$  are inverse  
 01 each option given  $A, B \in 2^X$ .  
 So,  $(A \cup B) - (A \cap B) = \Phi$

**Pipelined** : New HTTP request can be sent to server without receiving acknowledgement of previous.

$s_{ay}$  a client Alfiya request a page from server, following steps are followed in order before Alfiya gets all data needed :

- (i) There's 3-tier DNS hierarchy, 100 up is done in iterative mode taking 3 RTT.
  - (ii) 1 RTT is used for TCP connection establishment.
  - (iii) 1 RTT is used to fetch HTML base file.
  - (iv) 1 RTT is for all other 10 objects.
- So, n total 6 RTT's are used.

**Case-II** : Non persistent HTTP with 5 parallel connections :

**Non persistent** : TCP connection is made for each HTTP request and closed.

**5-parallel connections** : 5 objects could be sent parallel at same time.

So, in this case the client Alfiya have to wait for request to be fulfilled as following steps need to be completed.

equally small objects on the same web server.

Which of the following statements is/are CORRECT about the minimum elapsed time between clicking on the URL and your browser fully rendering it?

- (A) 7 RTTs, in case of non-persistent HTTP with 5 parallel TCP connections.
- (B) 5 RTTs, in case of persistent HTTP with pipelining.
- (C) 9 RTTs, in case of non-persistent HTTP with 5 parallel TCP connections.
- (D) 6 RTTs, in case of persistent HTTP with pipelining.

**Ans.** (C), (D)

**Sol.** **Case I :**

**Persistent HTTP :** TCP connection is established once and multiple files are transmitted in single connection.

22

GATE CSE-2023

second throw. Which of the following statements is/are TRUE?

- (A)  $A$  and  $B$  are independent.
- (B)  $A$  and  $C$  are independent.
- (C)  $B$  and  $C$  are independent.
- (D)  $\text{Prob}(B|C) = \text{Prob}(B)$

**Ans.** (C, D)

**Sol.** A : Head on both  $HH$

B : Head on 1<sup>st</sup>

$HT$

$HH$

C : Head on second

$HH$

$TH$

$$P(A) = \frac{1}{4}$$

$$P(B \cap C) = \frac{1}{4}$$

$$P(B).P(C) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

$$P(B \cap C) = P(B).P(C)$$

(∴ B and C are independent)

$$P(B) = \frac{1}{2}$$

$$P(A \cap B) = \frac{1}{4}$$

(i) 3 RTT for DNS resolution.

(ii) 1 RTT for TCP connection establishment.

(iii) 1 RTT for fetch base HTML page.

(iv) 1 RTT for TCP connection establishment.

(v) 1 RTT to get 5 objects parallelly (5 still test)

(vi) 1 RTT for TCP connection establishment

(vii) 1 RTT for getting remaining 5 objects.

In total it takes RTT's.

Hence, the correct options are (C) & (D).

### Question 43

Consider a random experiment where two fair coins are tossed. Let  $A$  be the event that denotes HEAD on both the throws,  $B$  be the event that denotes HEAD on the first throw, and  $C$  be the event that denotes HEAD on the



Consider functions Function 1 and Function 2 expressed in pseudocode as follows :

<b>Function_1</b> while $n > 1$ do for $i = 1$ to $n$ do for $i = 1$ to $n$ do $x = x + 1;$ do $x = x + 1;$ end for $n = \lfloor n/2 \rfloor;$ end while	<b>Function_2</b> for $i = 1$ to $100 * n$ do $x = x + 1;$ end for
---	---

Let  $f_1(n)$  and  $f_2(n)$  denote the number of times the statement " $x = x + 1$ " is executed in **Function\_1** and **Function\_2**, respectively.

Which of the following statements is/are TRUE?

- (A)  $f_1(n) \in \Theta(f_2(n))$  (B)  $f_1(n) \in o(f_2(n))$
- (C)  $f_1(n) \in \omega(f_2(n))$  (D)  $f_1(n) \in O(n)$

**Ans.** (A), (D)

**Sol.** Analysing function 1 first, we observe the number of times inner loop runs is halved every iteration

So, Number of times

$$P(A) \cdot P(B) = \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$$

$$P(A \cap B) \neq P(A) \cdot P(B)$$

(∴ A and B are not independent)

$$P(C) = \frac{1}{2}$$

$$P(A \cap C) = \frac{1}{4}$$

$$P(A) \cdot P(C) = \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$$

$$P(A \cap C) \neq P(A) \cdot P(C)$$

(∴ A and C are not independent)

$$P\left(\frac{B}{C}\right) = \frac{P(B \cap C)}{P(C)} = \frac{1/4}{1/2} = \frac{1}{2} = P(B)$$

Hence, the correct option is (C, D).

#### Question 44

GATE CSE-2023

23

$$n + \frac{1}{2} + \frac{1}{4} + \frac{1}{3} + \dots + 1$$

Inner loop runs

$$A(n) = n \left( 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{3} + \dots + \frac{1}{n} \right)$$

Say, it takes p timer, so

$$A(n) = n \left\{ \frac{1 - \left( \frac{1}{2} \right)^p}{1 - \frac{1}{2}} \right\}$$

Since, for large p, we can say

$$1 - \left( \frac{1}{2} \right)^p \approx 1$$

$$\text{So, } A(n) = \frac{n}{1} = 2n$$

$$\text{So, } f_1(n) = O(n)$$

Now, Function 2 runs for 100 n times.

 So,  $f_2(n) = O(100n)$   
 $= O(n)$

So,  $f_1(n) \in \Theta(f_2(n))$

Also,  $f_1(n) \in O(f_2(n))$  is true.

Hence, the correct options are (A) & (D).

#### Question 45

Let G be a simple, finite, undirected graph with vertex set  $\{v_1, \dots, v_n\}$ . Let  $\Delta(G)$  denote the maximum degree of G and let  $\mathbb{N} = \{1, 2, \dots\}$  denote the set of all possible colors. Color the vertices of G using the following greedy strategy :

for  $i = 1, \dots, n$

color

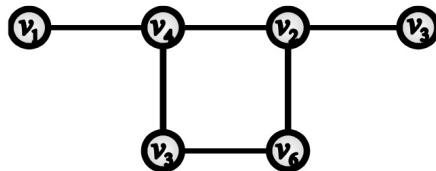
$$(v_i) \leftarrow \min \{j \in \mathbb{N} : \text{no neighbour of } v_i \text{ is colored } j\}$$

Which of the following statements is/are TRUE?

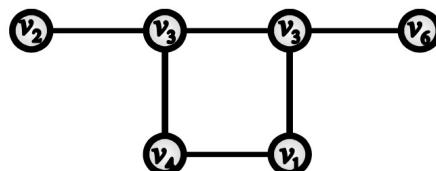
- (A) This procedure results in a proper vertex coloring of G.
- (B) The number of colors used is at most  $\Delta(G) + 1$ .
- (C) The number of colors used is at most  $\Delta(G)$ .
- (D) The number of colors used is equal to the chromatic number of G.

**For Option (C)** It's False as wave explained above.

**For Option (D)** It is not always the case as sometimes we might see that greedy coloring might not be giving optimal result. Consider example.



Uses 3 color and



Uses 3 color

When coloured greedily in order

$(v_1, v_2, \dots, v_6)$ . 90, number of color used  
 $\neq$  Chromatic number of graph

Hence, the correct options are (A) & (B).

#### Question 46

Let  $U = \{1, 2, 3\}$ . Let  $2^U$  denote the powerset of U. Consider an undirected graph G whose vertex set is  $2^U$ . For any  $A, B \in 2^U$ ,  $(A, B)$  is an edge in G if and only if (i)  $A \neq B$ , and (ii) either  $A \subseteq B$  or

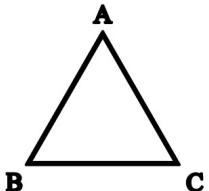
**Sol.** **For Option (A) :** Its, true as

"Color (vi)  $\leftarrow \min \{JEN = \text{No neighbor of } vi \text{ is colored } J\}$ "

So, it ensures proper coloring.

**For Option (B) :** We can take example of a cycle of length 3.

Here,  $\Delta G = 2$



But we need 3 colours to color it. Also, number of neighbor's can't be more than the degree, i.e.,  $\Delta G$ .

So, at most  $\Delta G + 1$  colours needed.

This option is true.

24

GATE CSE-2023

$B \subseteq A$ . For any vertex  $A$  in  $G$ , the set of all possible orderings in which the vertices of  $G$  can be visited in a Breadth First Search (BFS) starting from  $A$  is denoted by  $B(A)$ .

If  $\emptyset$  denotes the empty set, then the cardinality of  $B(\emptyset)$  is \_\_\_\_\_.

**Ans.** **5040**

**Sol.** Here, given that  $u = \{1, 2, 3\}$

Vertex set = Power set of  $U = 2^u = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$

So, number of vertices = 8

Now, there's edge between  $A$  and  $B$  iff either of them is proper subset of another.

Since,  $\emptyset$  is proper subset of all other vertices except itself, so its'

connected to all 7 vertices. Since it can be visited in any order.

So, cardinality of  $B(\emptyset) = 7! = 5040$

Hence, the correct answer is 5040.

#### Question 47

Consider the following two-dimensional array  $D$  in the C programming language, which is stored in row-major order :

int D[128][128];

Demand paging is used for allocating memory and each physical page frame holds 512 elements of the array  $D$ . The Least Recently Used (LRU) page-replacement policy is used by the operating system. A total of 30 physical page frames are allocated to a process which executes the following code snippet :

```
for (int i = 0; i < 128; i++)
    for (int j = 0; j < 128; j++)
        D[i][j] *= 10;
```

The number of page faults generated during the execution of this code snippet is \_\_\_\_\_.

**Ans.** **4096**

**Sol.** Given array  $D[128][128]$  is stored in Row - major Order.

Number of physical frames available = 30

Number of elements in 1 frame = 512

Consider a computer system with 57-bit virtual addressing using multi-level tree-structured page tables with  $L$  levels for virtual to physical address translation. The page size is 4 KB (1 KB = 1024 B) and a page table entry at any of the levels occupies 8 bytes.

The value of  $L$  is \_\_\_\_\_.

**Ans.** **5**

**Sol.** Virtual address is 57 bits, page size is 4 kB =  $2^{12}$  B

$$\text{Number of page} = \frac{2^{57}}{2^{12}} = 2^{45}$$

Page Table Entry = 8 kB

So, Each page can contain  $\frac{4kB}{8B} = 2^9$

page entries

So, We need 9 bits to index page table.

$$\text{So, number of levels} = \left\lceil \frac{45}{9} \right\rceil = 5$$

Hence, the correct answer is 5.

#### Question 49

Consider a sequence  $a$  of elements  $a_0 = 1, a_1 = 5, a_2 = 7, a_3 = 8, a_4 = 9$ , and  $a_5 = 2$ . The following operations are performed on a stack  $S$  and a queue  $Q$ , both of which are initially empty.

I : push the elements of  $a$  from  $a_0$  to

So, number of pages to accommodate all element of array  $D = \frac{123 \times 128}{512} = 32$ .

Since we need 32 frames and were given only 30 so, collision will occur. Also number of rows per frame

$$= \frac{512}{128} = 4$$

So, in 30 frames we can store 120 rows

Thus in 1<sup>st</sup> iteration, it's cause 32 page faults.

For 128 iterations it's cause  $128 \times 32 = 4096$  faults.

Hence, the correct answer is 4096.

### Question 48

GATE CSE-2023

25

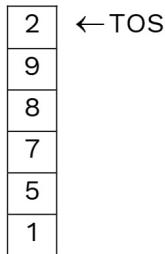


**Sol.**

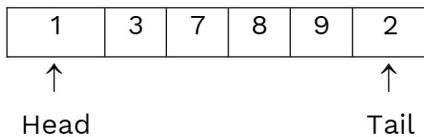
Given Elements

$$(a_0, a_1, \dots, a_4) = (11, 5, 7, 8, 9, 2)$$

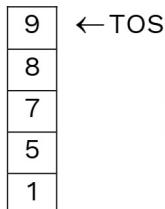
**Step - 1 :**



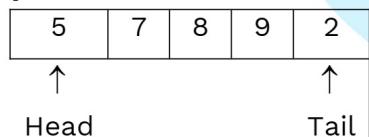
**Step - 2 :**



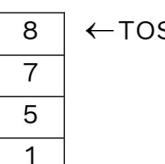
**Step - 3 :**



**Step - 4 :**



**Step - 5 :**



$a_5$  in that order into  $S$ .

II : enqueue the elements of  $a$  from  $a_0$  to  $a_5$  in that order into  $Q$ .

III : pop an element from  $S$ .

IV : dequeue an element from  $Q$ .

V : pop an element from  $S$ .

VI : dequeue an element from  $Q$ .

VII : dequeue an element from  $Q$  and push the same element into  $S$ .

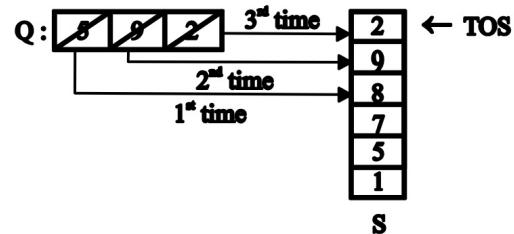
VIII : Repeat operation VII three times.

IX : pop an element from  $S$ .

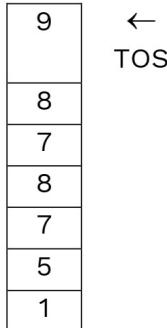
X : pop an element from  $S$ .

The top element of  $S$  after executing the above operations is \_\_\_\_\_.

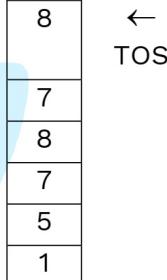
**Ans. 8**



**Step - 9 :**



**Step - 10 :**

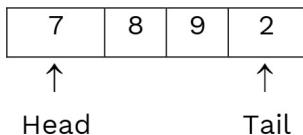
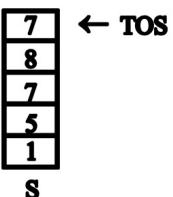
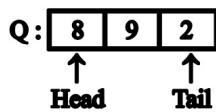


∴ '8' in Top of stack So answer is 8.

Hence, the correct option is 8.

### Question 50

Consider the syntax directed translation given by the following grammar and semantic rules. Here  $N$ ,  $I$ ,  $F$  and  $B$  are non-terminals.  $N$  is the starting non-terminal, and  $\#$ ,  $0$  and  $1$

**Step - 6 :****Step - 7 :****Step-8 :**

are lexical tokens corresponding to input letters “#”, “0” and “1”, respectively.  $X.val$  denotes the synthesized attribute (a numeric value) associated with a non-terminal  $X$ .  $I_1$  and  $F_1$  denote occurrences of  $I$  and  $F$  on the right hand side of a production, respectively. For the tokens **0** and **1**, **0.val** = 0 and **1.val** = 1.

$$N \rightarrow I \# F$$

$$N.val \rightarrow I.val + F.val$$

$$I \rightarrow I_1 B$$

$$I.val \rightarrow (2I_1.val) + B.val$$

$$I \rightarrow B$$

$$I.val = B.val$$

$$F \rightarrow BF_1$$

$$F.val = \frac{1}{2}(B.val + F_1.val)$$

$$F \rightarrow B$$

$$F.val = \frac{1}{2}B.val$$

$$B \rightarrow 0$$

$$B.val = 0.val$$

$$B \rightarrow 1$$

$$B.val = 1.val$$

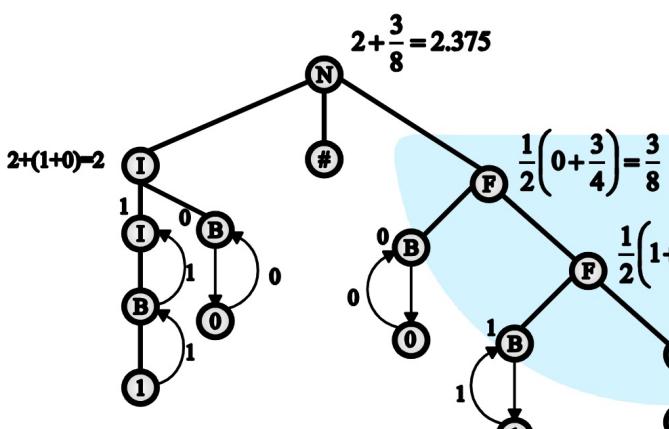
The value computed by the translation scheme for the input string

10 # 011

is \_\_\_\_\_. (Rounded off to three decimal places)

**Ans.** 2.375

**Sol.**



Hence, the correct answer is 2.375.

**Question 51**

Consider the following table named Student in a relational database. The primary key of this table is rollNum.

The number of rows returned by the query is \_\_\_\_\_.

**Ans.** 2

**Sol.** Were to return female students with marks greater than 65.

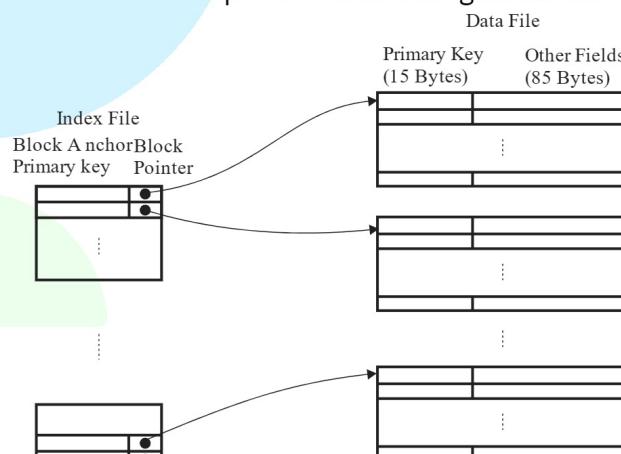
Output is :

Roll	Name	Gender	Marks
2	Aliya	F	70
3	Aliya	F	80

Hence, the correct answer is 2.

**Question 52**

Consider a database of fixed-length records, stored as an ordered file. The database has 25,000 records, with each record being 100 bytes, of which the primary key occupies 15 bytes. The data file is block-aligned in that each data record is fully contained within a block. The database is indexed by a primary index file, which is also stored as a block-aligned ordered file. The figure below depicts this indexing scheme.



**Student :**

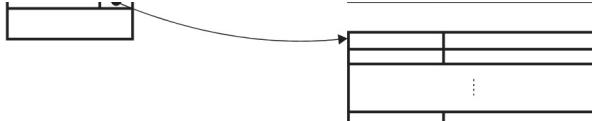
rollNum	Name	gender	marks
1	Naman	M	62
2	Aliya	F	70
3	Aliya	F	80
4	James	M	82
5	Swati	F	65

The SQL query below is executed on this database.

```
SELECT *
FROM Student
WHERE gender = 'F' AND
marks > 65;
```

GATE CSE-2023

27



Suppose the block size of the file system is 1024 bytes, and a pointer to a block occupies 5 bytes. The system uses binary search on the index file to search for a record with a given key. You may assume that a binary search on an index file of  $b$  blocks takes  $\lceil \log_2 b \rceil$  block accesses in the worst case.

Given a key, the number of block accesses required to identify the block

in the data file that may contain a record with the key, in the worst case, is \_\_\_\_\_.

**Ans. 6**

**Sol.** Given database is stored as ordered file and indexed by primary index file  
There're 25,000 records

100 B	records size
15 B	Primary key size
5 B	Pointer size
1024 B	Block size

It's stored in unspanned organization.

So, Number of records per block

$$= \left\lfloor \frac{\text{Block size}}{\text{Record size}} \right\rfloor = \left\lfloor \frac{1024 \text{ B}}{100 \text{ B}} \right\rfloor = 10$$

Number of data blocks needed

$$= \left\lceil \frac{\text{Number of records}}{\text{Number of records per block}} \right\rceil = \left\lceil \frac{25000}{10} \right\rceil = 2500$$

No. of Index records per block

$$= \left\lceil \frac{\text{Block size}}{\text{primary key size} + \text{Pointer size}} \right\rceil = \left\lceil \frac{1024}{15 + 5} \right\rceil = \left\lceil \frac{1024}{20} \right\rceil = 51$$

Number of index block needed

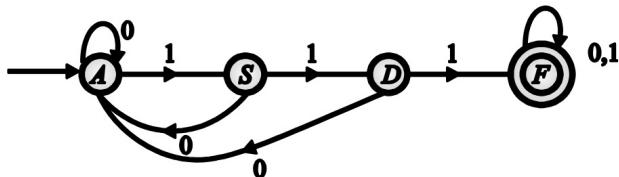
$$= \left\lceil \frac{2500}{51} \right\rceil = 50$$

Applying binary search,  $\lceil \log_2(50) \rceil = 6$

Hence, the correct answer is 6.

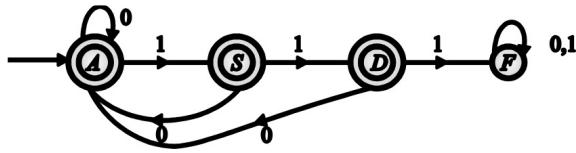
**Question 53**

Consider the language  $L$  over the



I: set of strings not containing 3 consecutive  
So, no. of state remains same i.e. 4

MDFA For  $I \rightarrow$



Hence, the correct answer is 4.

**Question 54**

An 8-way set associative cache of size 64 KB (1 KB = 1024 bytes) is used in a system with 32-bit address. The address is sub-divided into TAG, INDEX, and BLOCK OFFSET.

The number of bits in the TAG is \_\_\_\_\_.

**Ans. 19**

**Sol.** Given : Cache Size  $CS = 64 \times B = 2^{16} \text{ B}$

System use 32 bit address (A)

$$\text{So, TAG} = A : - \log_2(CS) + \log_2(P)$$

$$= 32 - \log_2(2^{16}) + \log_2(8)$$

$$= 32 - 16 + 8 = 19$$

Hence, the correct option is 19.

**Question 55**

The forwarding table of a router is shown below.

Subnet Number	Subnet Mask	Interface ID

Consider the language  $L$  over the alphabet  $\{0, 1\}$ , given below :

$L = \{\omega \in \{0, 1\}^* \mid \omega \text{ does not contain three or more consecutive } 1's\}$ .

The minimum number of states in a Deterministic Finite-State Automaton (DFA) for  $L$  is \_\_\_\_\_.

**Ans. 4**

**Sol.** Say  $L$  : set of strings containing 3 consecutive 1's.

The MDFA for  $L \rightarrow$

200.150.0.0	255.255.0.0	1
200.150.64.0	255.255.224.0	2
200.150.68.0	255.255.255.0	3
200.150.68.64	255.255.255.224	4
Default		0

A packet addressed to a destination address 200.150.68.118 arrives at the router. It will be forwarded to the interface with ID \_\_\_\_\_.

**Ans. 3**

**Sol.** We will perform AND operation between IP and Subnet mask and see it

28

GATE CSE-2023



we get same subnet -ID or not and well do longest prefix match.

So, checking subnet - 4.

IP  $\wedge$  subnet = (200.150.68.118)  $\wedge$  (255.255.255.224) = (200.150.68.96)

Subnet ID didn't match.

Checking subnet 3

(200.150.68.118)  $\wedge$  (255.255.255.0).(200.150.68.0)

Subnet ID matches.

If ll be forwarded to 3.

Hence, the correct answer is 3.

□□□

