EC/EE/CS & IT/IN

Digital Electronics

Boolean algebra:

K MAP

LECTURE NO. 3

Chandan Jha Sir (CJ Sir)

लहरों से डरकर नौका पार नहीं होती, कोशिश करने वालों की कभी हार नहीं होती।
नन्हीं चींटी, जब दाना लेकर चलती है, चढ़ती दीवारों पर, सौ बार फिसलती है।
मन का विश्वास, रंगो में साहस भरता है, चढ़कर गिरना, गिरकर चढ़ना, कभी ना अखरता है।
आखिर उसकी मेहनत बेकार नहीं होती, कोशिश करने वालों की, कभी हार नहीं होती।
असफलता एक चुनौती है, स्वीकार करो, क्या कमी रह गयी, देखो और सुधार करो।
जब सफल न हो, नींद चैन को त्यागो तुम, संघर्षो का मैदान छोड़, कभी मत भागो तुम,
कुछ किये बिना ही, जय-जयकार नहीं होती, कोशिश करने वालों की, कभी हार नही होती।
~ हरिवंश राय बच्चन की पंक्तियाँ

ABOUT ME

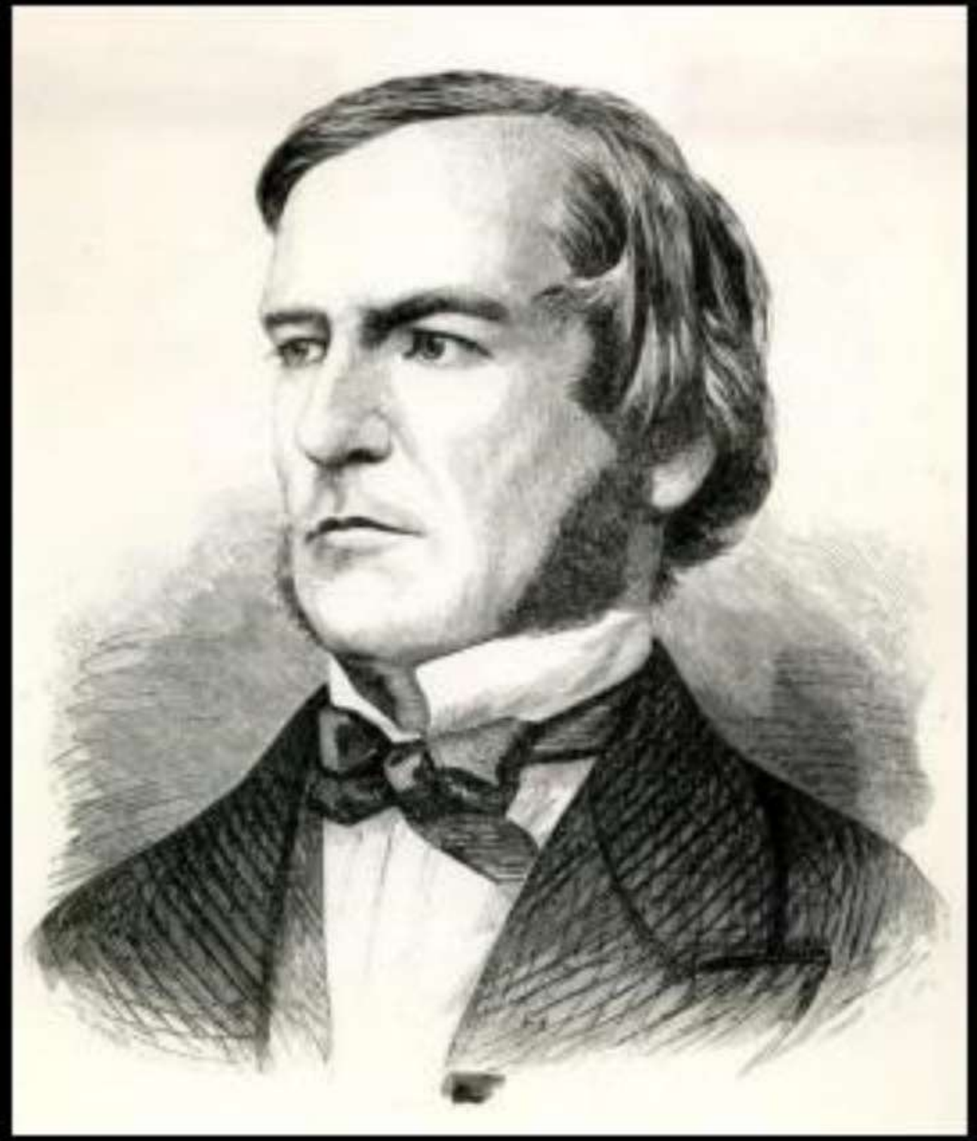
- Cleared Gate Multiple times with double Digit Rank (AIR 23, AIR 26)
- Qualified ISRO Exam
- Mentored More then 1 Lakhs+ Students (Offline & Online)
- More then 250+ Motivational Seminar in various Engineering College including NITs & Some of IITs

GATE 2023

BSc →

Chandan Jha

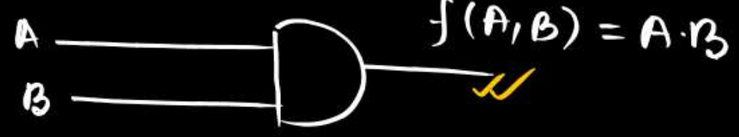
1854- George Boole



"An Investigation of Law of Thoughts"

Variable

Ex.1. $f(A,B) = AB$

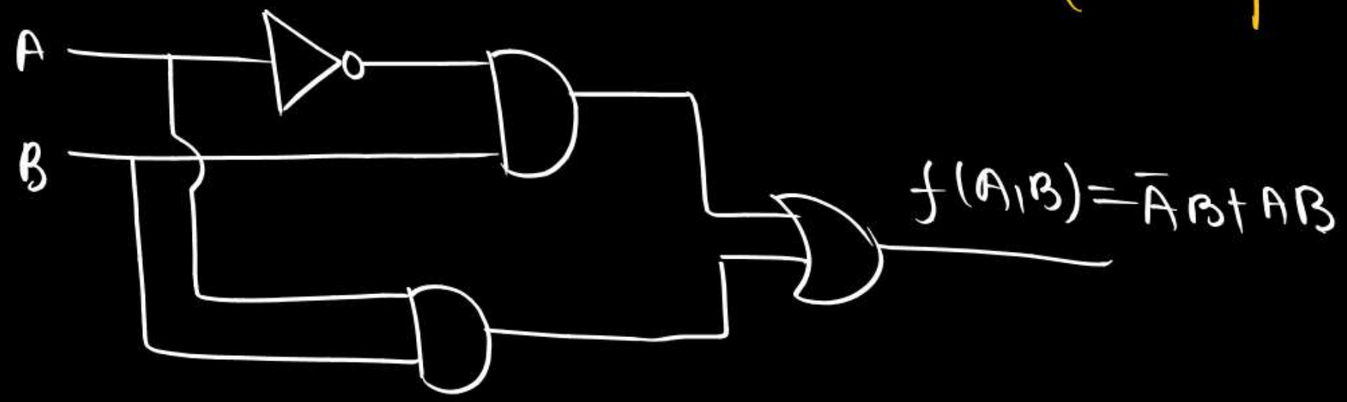


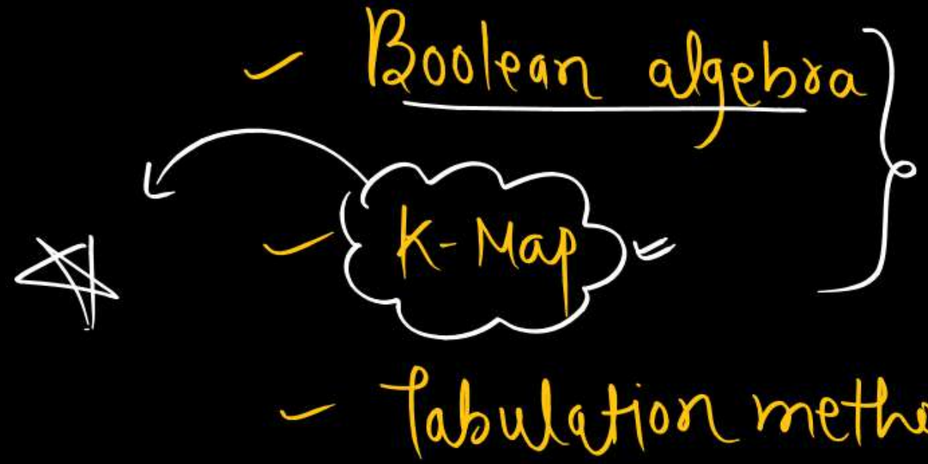
Variable

Ex.2. $f(A,B) = \bar{A}B + AB$

minimize

{ Costly
Delay
Complex ↓





	8	4	2	1
0 →	0	0	0	0
1 →	0	0	0	1
2 →	0	0	1	0
3 →	0	0	1	1
4 →	0	1	0	0
5 →	0	1	0	1

Example

Decimal	A B C	Min term	Max term	Function
0	0 0 0	$\bar{A}\bar{B}\bar{C}$	$A+B+C$	1
1	0 0 1	$\bar{A}\bar{B}C$	$A+B+\bar{C}$	0
2	0 1 0	$\bar{A}B\bar{C}$	$A+\bar{B}+C$	0
3	0 1 1	$\bar{A}BC$	$A+\bar{B}+\bar{C}$	1
4	1 0 0	$A\bar{B}\bar{C}$	$\bar{A}+B+C$	1
5	1 0 1	$A\bar{B}C$	$\bar{A}+B+\bar{C}$	0
6	1 1 0	$AB\bar{C}$	$\bar{A}+\bar{B}+C$	0
7	1 1 1	ABC	$\bar{A}+\bar{B}+\bar{C}$	1

Standard Canonical SOP form :→

$$f(A,B,C) = \bar{A}\bar{B}\bar{C} + \bar{A}BC + A\bar{B}\bar{C} + ABC$$

3 Variables

$$= m_0 + m_3 + m_4 + m_7$$

$$= \Sigma m(0,3,4,7)$$

$$= \Sigma(0,3,4,7)$$

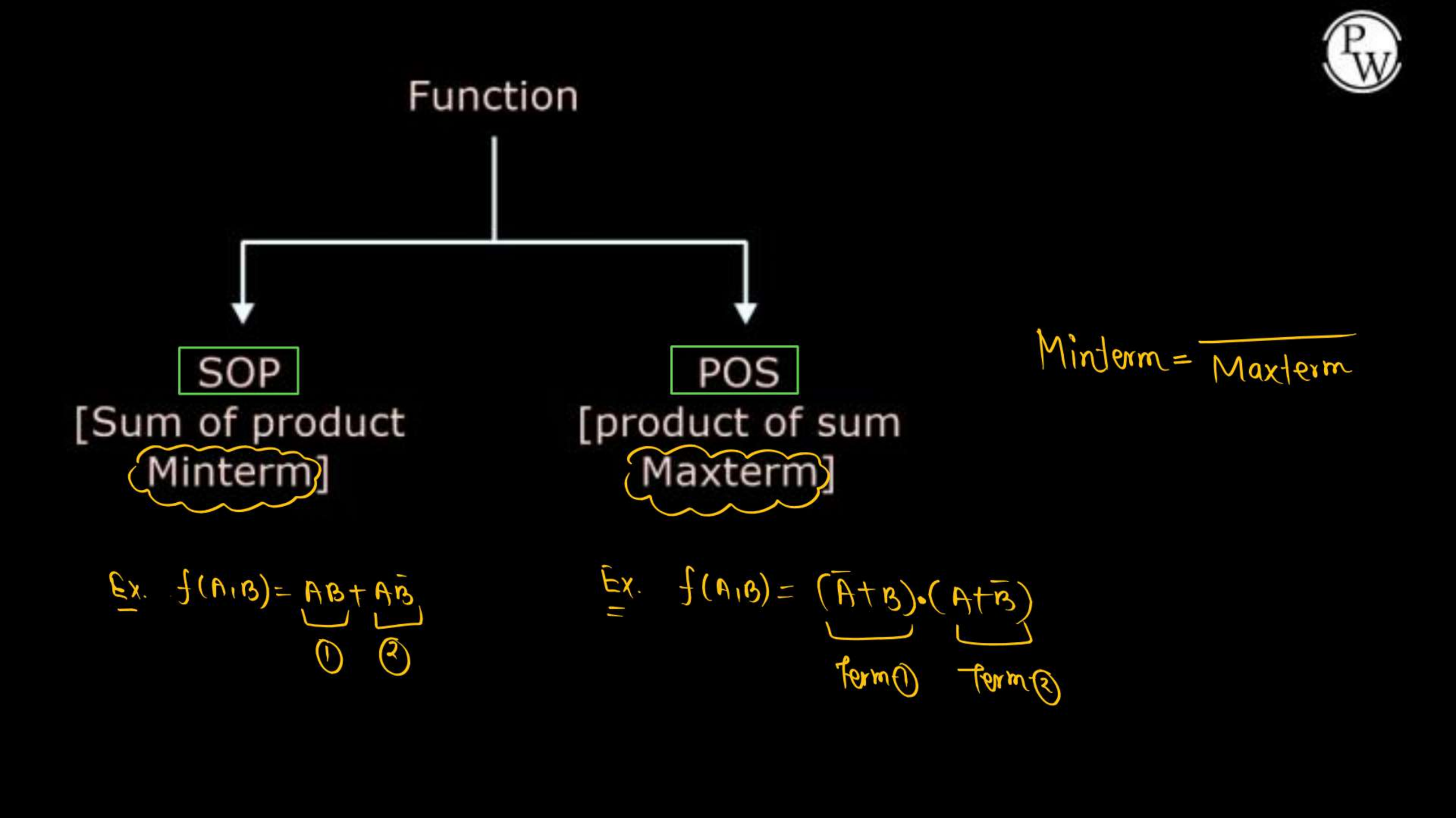
Standard Canonical POS form :-

$$F(A,B,C) = (A+B+\bar{C}) \cdot (A+\bar{B}+C) \cdot (\bar{A}+B+\bar{C}) \cdot (\bar{A}+\bar{B}+C)$$

$$= M_1 \cdot M_2 \cdot M_5 \cdot M_6$$

$$= \Pi M(1,2,5,6)$$

$$= \Pi(1,2,5,6)$$


Function
SOP
[Sum of product
Minterm]
POS
[product of sum
Maxterm]
Minterm = $\overline{\text{Maxterm}}$
Ex. $f(A,B) = AB + A\bar{B}$
1 2
Ex. $f(A,B) = (\bar{A}+B)\cdot(A+\bar{B})$
term 1 term 2

Distribution Theorem

$(A+B)(A+C)$

$\Rightarrow A\cdot A + AB + AC + BC$

$\Rightarrow A + AB + AC + BC$

$\Rightarrow A[1+B+C] + BC$

$\Rightarrow A\cdot 1 + BC$

$\Rightarrow A + BC$

$A+BC = (A+B)\cdot(A+C)$

$A+BCD = (A+B)(A+C)(A+D)$

$1+A = 1$

$1+A+B = 1$

$1 + \text{anything} = 1$

$\bar{A}+A = 1$

$\bar{B}+B = 1$

Ex $f(A,B) = \bar{A} + AB$

$= (\bar{A}+A)(\bar{A}+B)$

$= 1\cdot(\bar{A}+B)$

$= \bar{A}+B$

Ans

Ex

$A + \bar{A}B$

$(A + \bar{A})(A + B)$

$1 \cdot (A + B)$

$A + B$

$A + BC = (A + B)(A + C)$

Consensus Theorem

$\bar{A} + A = 1$

$f(A,B,C) = AB + \bar{A}C + BC$

$= AB + \bar{A}C + (\bar{A}+A)BC$

$= AB + \bar{A}C + \bar{A}BC + ABC$

$= AB[1+C] + \bar{A}C[1+B]$

$= AB + \bar{A}C$

$AB + \bar{A}C + BC = AB + \bar{A}C$

→ Redundant

- ✓ 3 terms
- ✓ each term having Two Variable
- ✓ each variable repeated Two times but one repeated in the form of complement

$$f(A,B,C) = AB + \bar{A}C + BC$$

$$= AB + \bar{A}C$$

Redundant term

Ex. $f(A,B) = A\bar{B} + BC + AC$

$$= A\bar{B} + BC$$

Ans

$$\begin{cases} A + B\bar{C} = (A+B)(A+C) \\ AB + \bar{A}C + BC = AB + \bar{A}C \end{cases}$$

Transpose Theorem

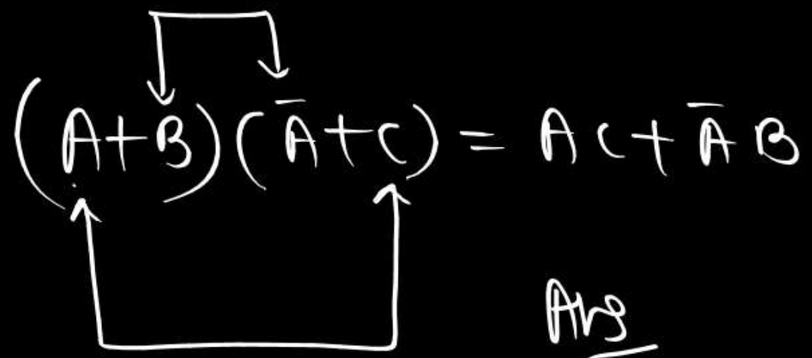
$A \cdot \bar{A} = 0$

$(A+B)(\bar{A}+C)$

$\Rightarrow A\cdot\bar{A} + \bar{A}B + AC + BC$

$\Rightarrow \bar{A}B + AC + BC$ / Concensus

$\Rightarrow \bar{A}B + AC$



Ans

Ex $(A+B)(\bar{A}+\bar{B}) = A\bar{B} + \bar{A}B$

$= A \oplus B$

Ex. $(A+\bar{B})(\bar{A}+B) = AB + \bar{A}\bar{B}$

$= A \odot B$

D-Morgans Law

$$\overline{A \cdot B \cdot C} = \bar{A} + \bar{B} + \bar{C}$$

$$\overline{A + B + C} = \bar{A} \cdot \bar{B} \cdot \bar{C}$$

- $A + BC = (A+B)(A+C)$
- $AB + \bar{A}C + BC = AB + \bar{A}C$
- $(A+B)(\bar{A}+C) = AC + \bar{A}B$
- $\overline{ABC} = \bar{A} + \bar{B} + \bar{C}$

  $\overline{A+B+C} = \bar{A} \cdot \bar{B} \cdot \bar{C}$

Ex. 1. Minimize the expression.

$$f(A, B) = \bar{A} + AB$$

$$= (\bar{A} + A)(\bar{A} + B)$$

$$= \bar{A} + B$$

Ans

Ex. 2. Minimize the expression.

$f(A, B) = \bar{A}\bar{B} + \bar{A}B + AB$

$\bar{A}\bar{B} + \bar{A}B + AB$

$\bar{A}[\bar{B} + B] + AB$

$\bar{A} + AB$

$(\bar{A} + A)(\bar{A} + B)$

$\bar{A} + B$

Ex. 3. Minimize the expression.

$$f(A, B) = \bar{A}\bar{B} + \bar{A}B + A\bar{B} + AB$$

$$= \bar{A}[\bar{B} + B] + A[\bar{B} + B]$$

$$= \bar{A} + A = 1$$

$$\left\{ \bar{A}\bar{B},\ \bar{A}B,\ A\bar{B},\ AB \right\}$$

A

$h = 1$

$\bar{A}$, A, 0, 1 } 4

A, B

$h = 2$

$(A+\bar{A})\cdot(\bar{B}+B)$

$1 \cdot 1 = 1$

$A\cdot\bar{A} + \bar{B}\cdot B = 0$

$\bar{A}\bar{B}$	$A+B$	A	0
$\bar{A}B$	$A+\bar{B}$	$\bar{A}$	1
$A\bar{B}$	$\bar{A}+B$	B	$\bar{A}B + A\bar{B}$
AB	$\bar{A}+\bar{B}$	$\bar{B}$	$\bar{A}\bar{B} + AB$

16

minterm Maxterm

"n" Variables → 2^{2^n} Different Expression

$2^n \rightarrow$ minterm

$2^n \rightarrow$ Maxterm

Ex. 4. Minimize the expression.

$$f(A, B) = AB + \bar{A}C + BC$$

(A) $AB + \bar{A}C$

(B) $AB + BC$

(C) BC

(D) None

$f(A,B) = AB + \bar{A}C + BC$ (1) (2) (3)

2 Variables

$f(\ )$ Wrong format

argument

Ex. 5. Minimize the expression.

$$f(A, B, C) = \bar{A}\bar{B} + \bar{A}C + B\bar{C}$$

$$= \bar{A}C + \bar{B}\bar{C}$$

Ans

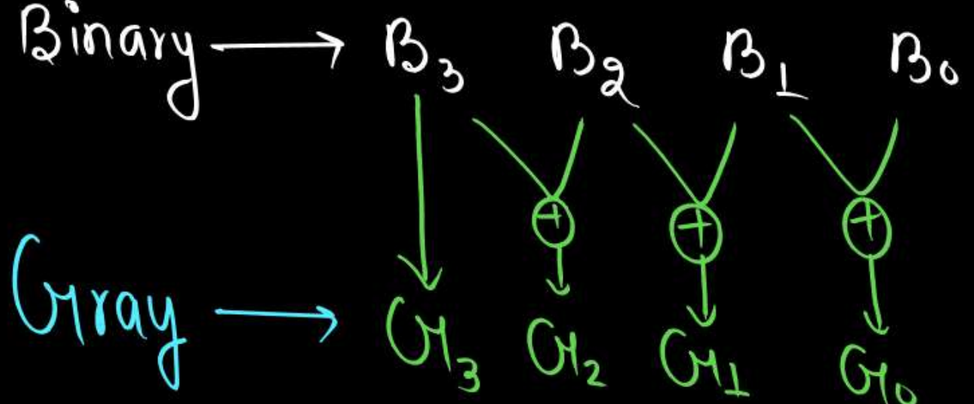
Modified Veitched Diagram

↳ Also known as K-MAP

K-MAP

→ Based on gray code.

"Gray Code"



Ex

Binary → 1 1 0 0 1 0

Gray → 1 0 1 0 1 1

Ex.

Binary → 1 1 0 0 0 1 1

Gray → 1 0 1 0 0 1 0

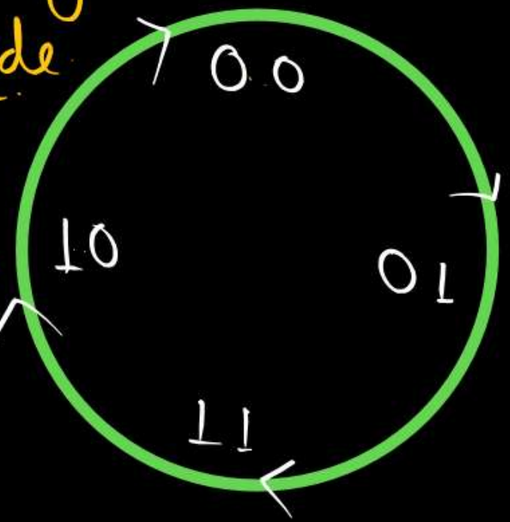
Gray code

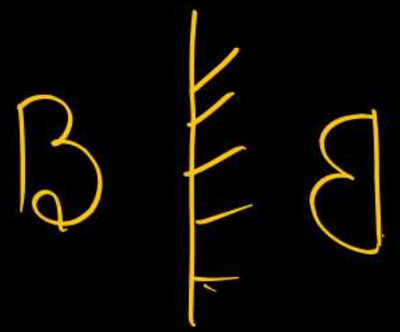
Decimal	Binary	Gray code
0	0 0	0 0
1	0 1	0 1
2	1 0	1 1
3	1 1	1 0

Unity haming distance code

Cyclic code

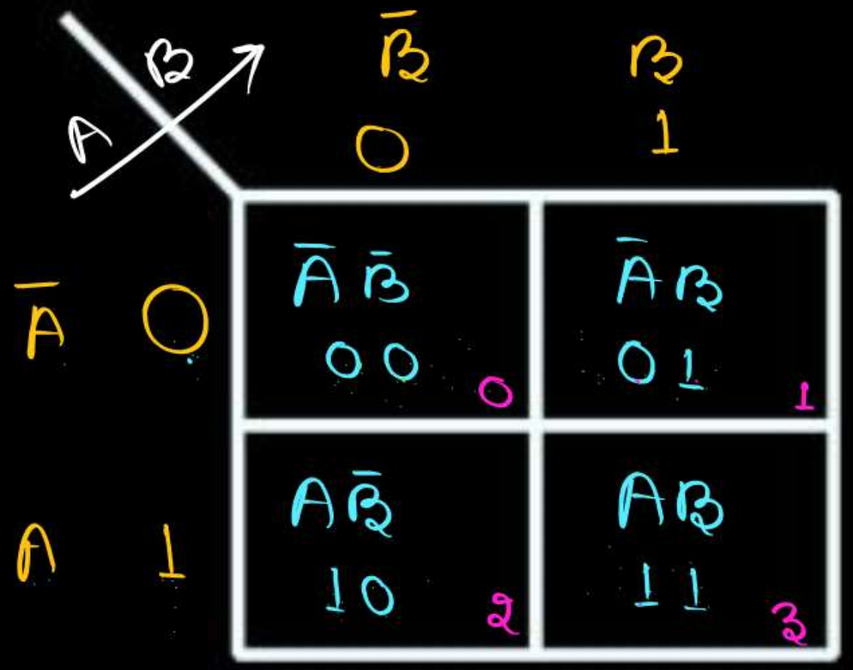
Reflecting Code





Gray code

Decimal	Binary	Gray code
0	0 0 0	0 0 0
1	0 0 1	0 0 1
2	0 1 0	0 1 1
3	0 1 1	0 1 0
4	1 0 0	1 1 0
5	1 0 1	1 1 1
6	1 1 0	1 0 1
7	1 1 1	1 0 0

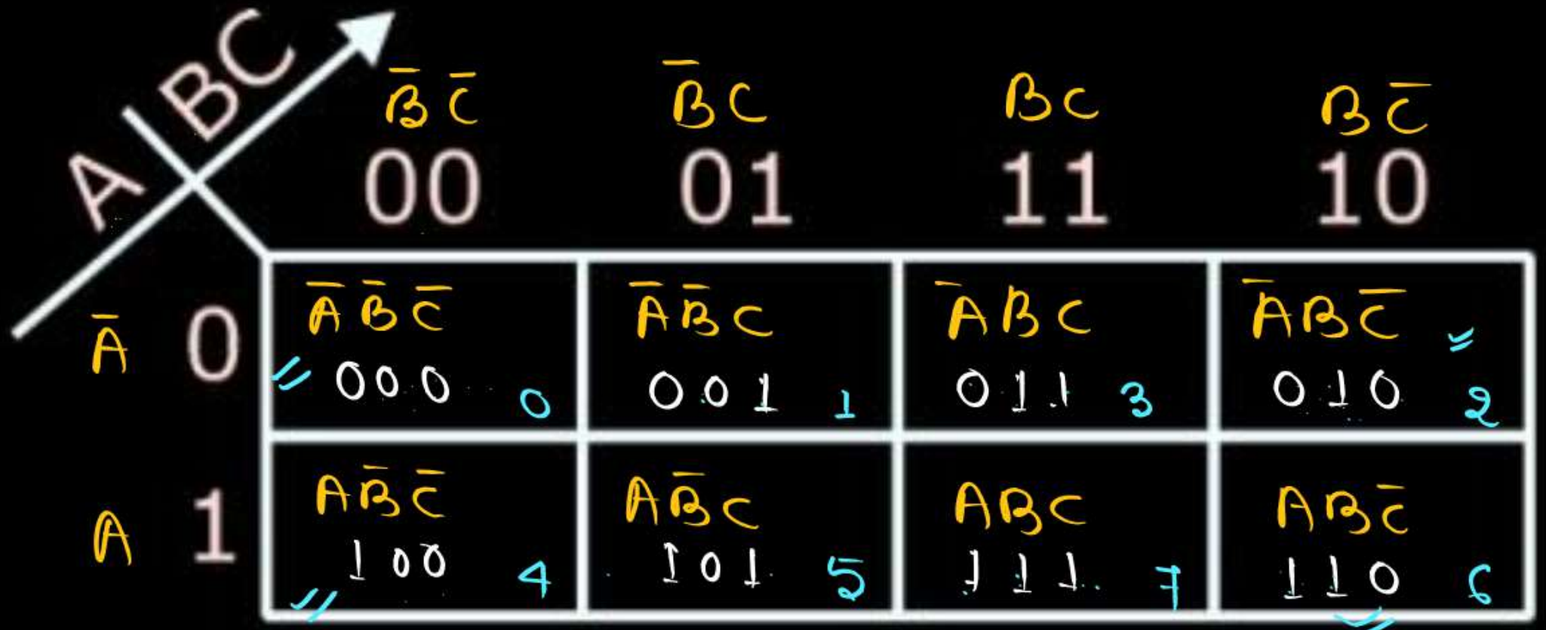


$f(A, B)$

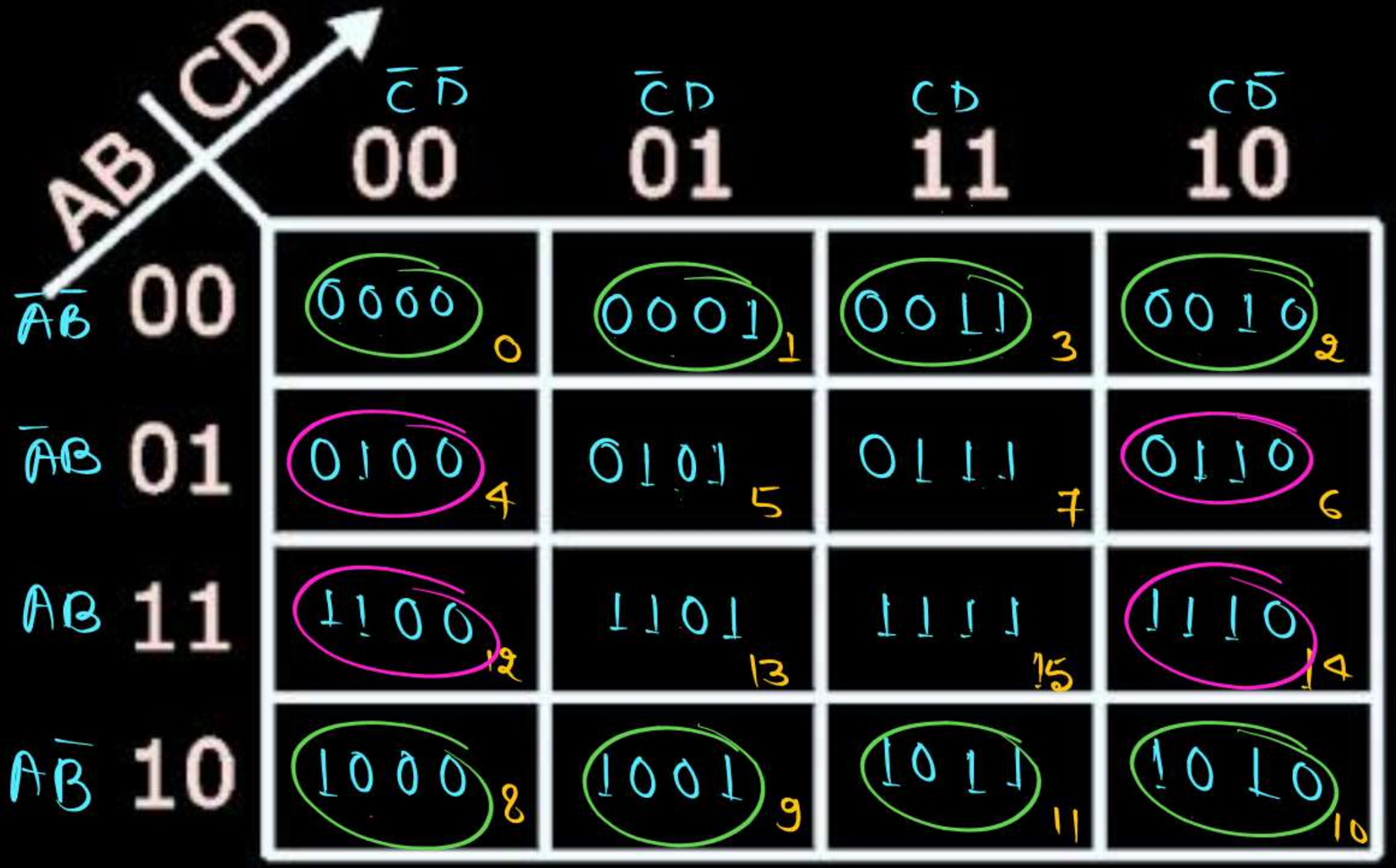
$f(A, B)$ → MSB, LSB

Most significant Bit

$(100\ 10\ 1)_2 = (5\ 0\ 1)_2$


A
BC
$\bar{B}\bar{C}$
$\bar{B}C$
BC
$B\bar{C}$
00
01
11
10
$\bar{A}$ 0
A 1
$\bar{A}\bar{B}\bar{C}$ 000 0
$\bar{A}\bar{B}C$ 001 1
$\bar{A}BC$ 011 3
$\bar{A}B\bar{C}$ 010 2
$A\bar{B}\bar{C}$ 100 4
$A\bar{B}C$ 101 5
ABC 111 7
$AB\bar{C}$ 110 6

$f(A,B,C)$


AB \ CD
$\bar{C}\bar{D}$ 00
$\bar{C}D$ 01
CD 11
$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00
$\bar{A}B$ 01
AB 11
$A\bar{B}$ 10
0000 0
0001 1
0011 3
0010 2
0100 4
0101 5
0111 7
0110 6
1100 12
1101 13
1111 15
1110 14
1000 8
1001 9
1011 11
1010 10

$f(A,B,C,D)$

Formation Of Groups

16 group = 2^4 → 4 Variables minimize.

8 group = 2^3 → 3 Variables minimize.

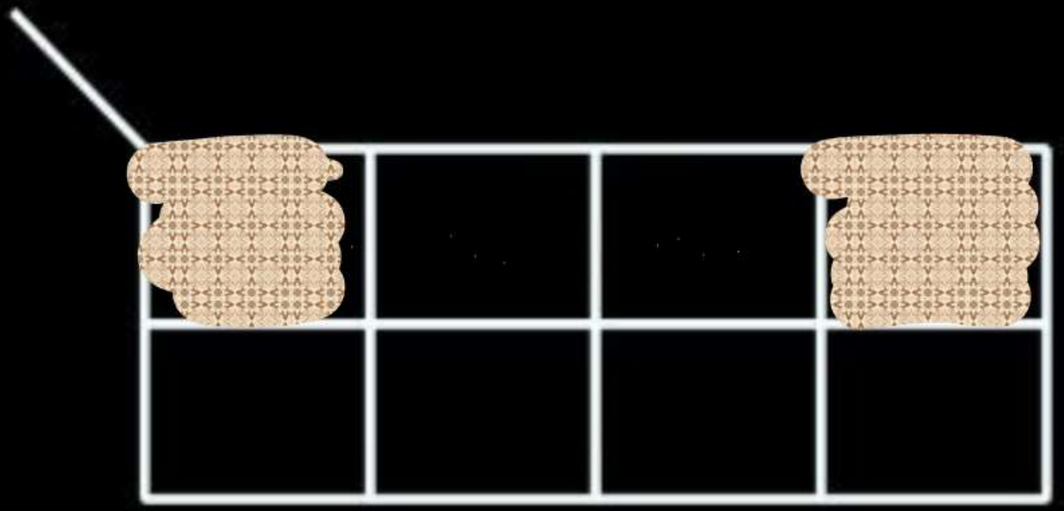
4 group = 2^2 → 2 Variables minimize.

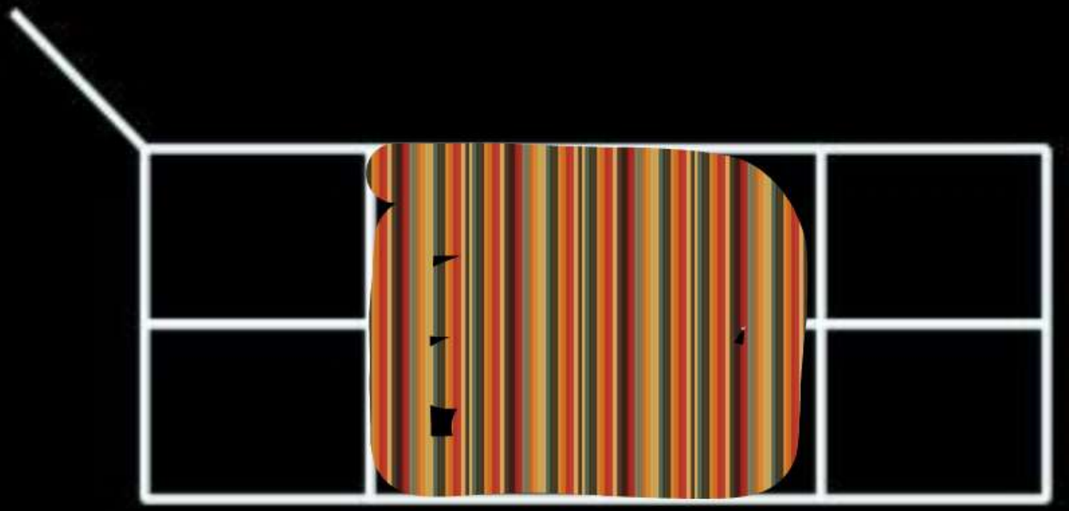
2 group = 2^1 → 1 Variable minimize.

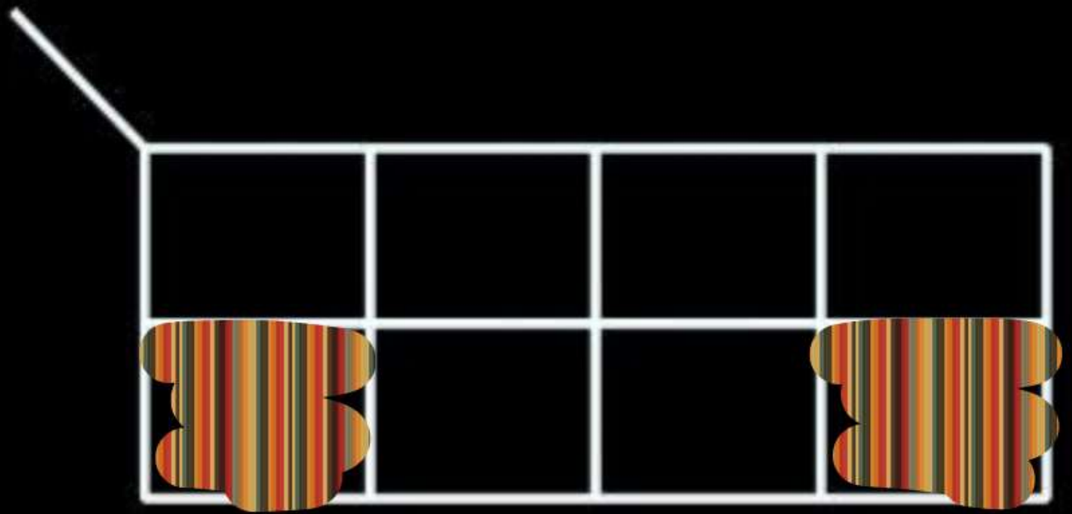
1 group = 2^0 → 0 Variable minimize.

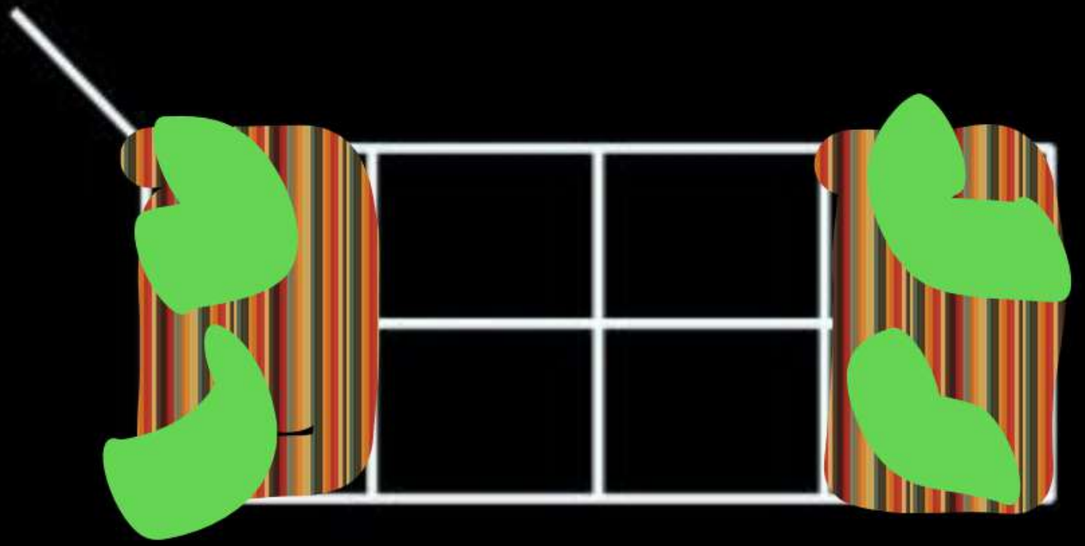
Formation Of Groups

Quad

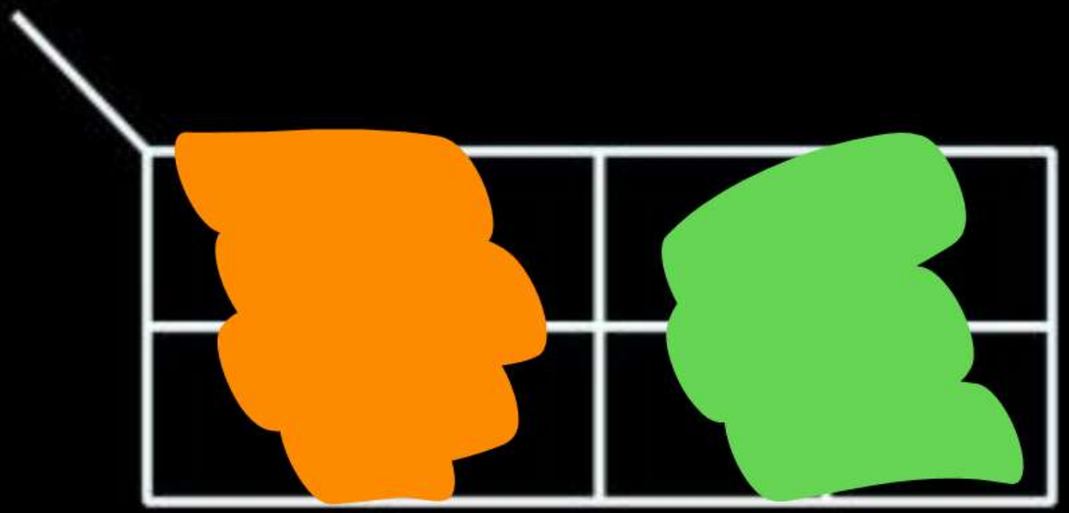






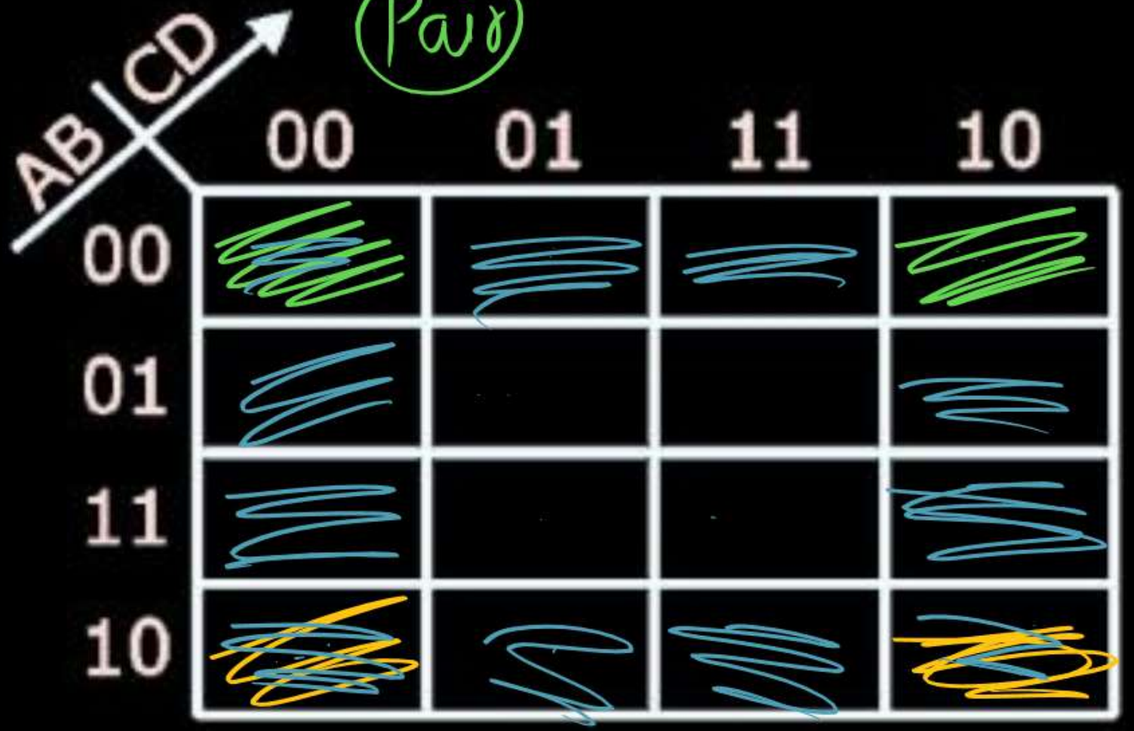


Formation Of Groups

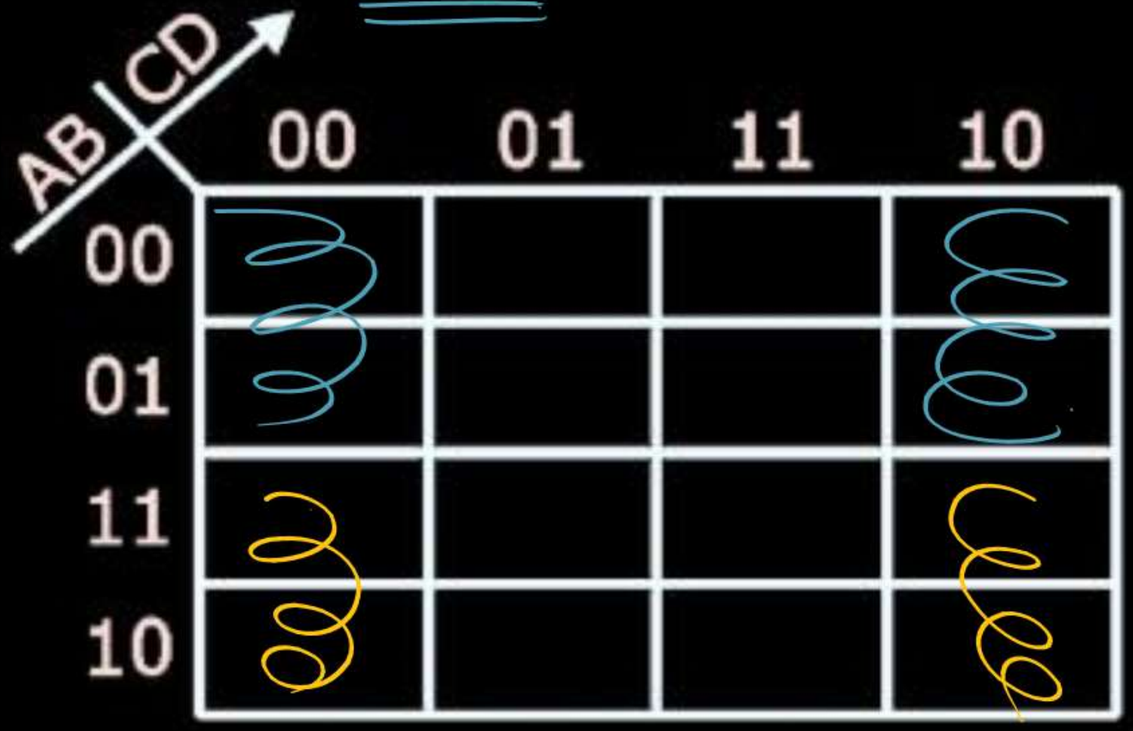


Formation Of Groups

Pair

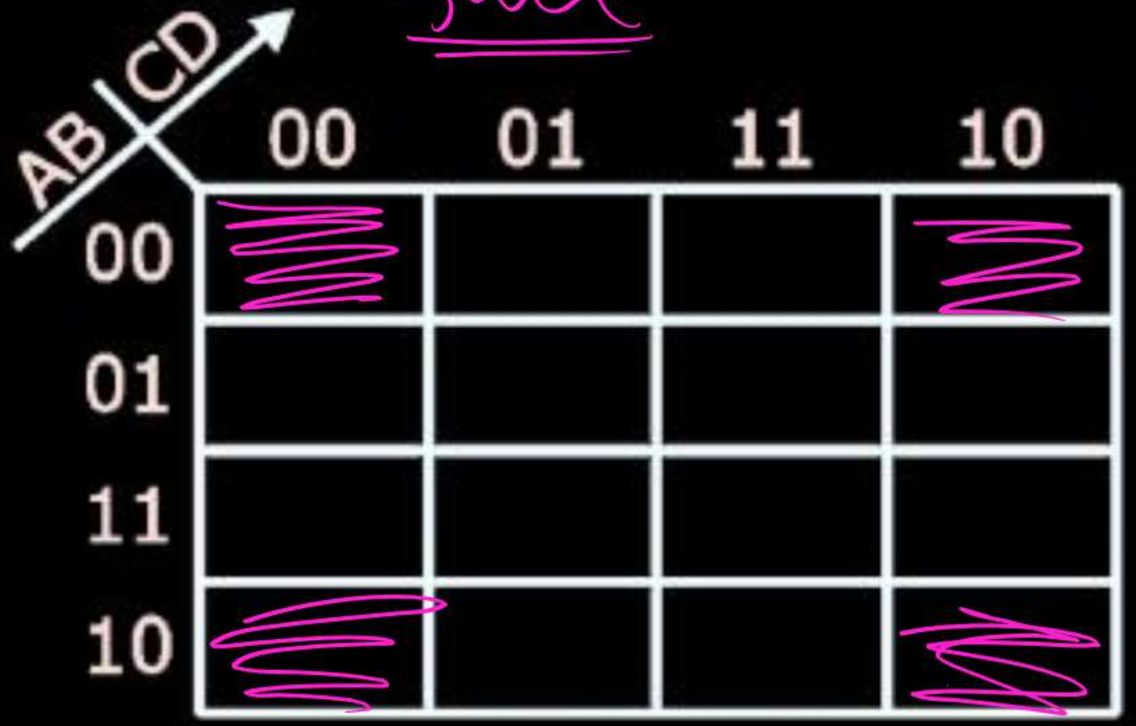


Quad

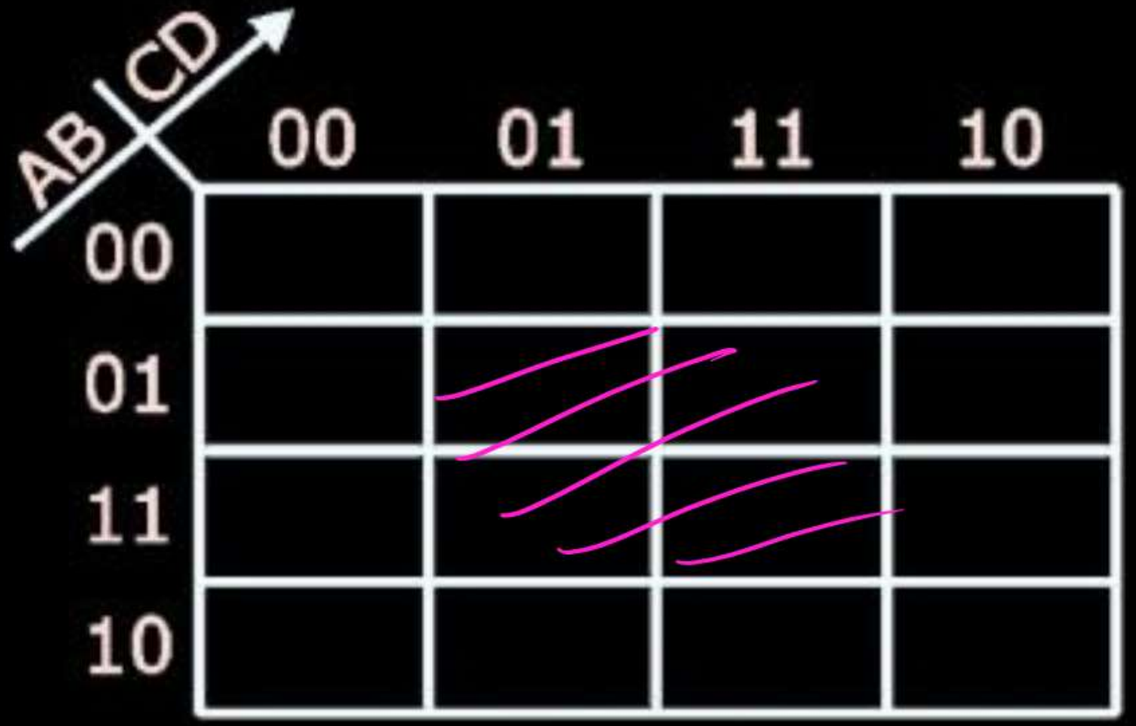


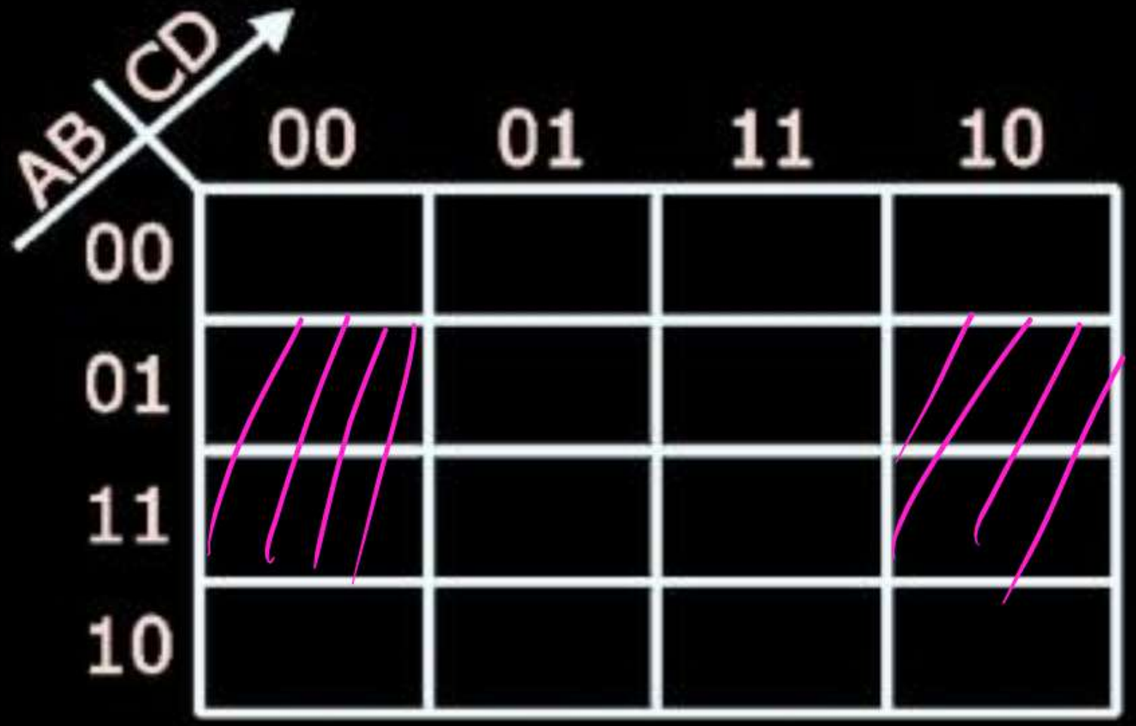
Formation Of Groups

Quad



Formation Of Groups





Rule of minimization

CJ Baba Rule.

↳ Less number of groups and bigger group.

No. of Terms Reduce

No. of Variables Reduce.

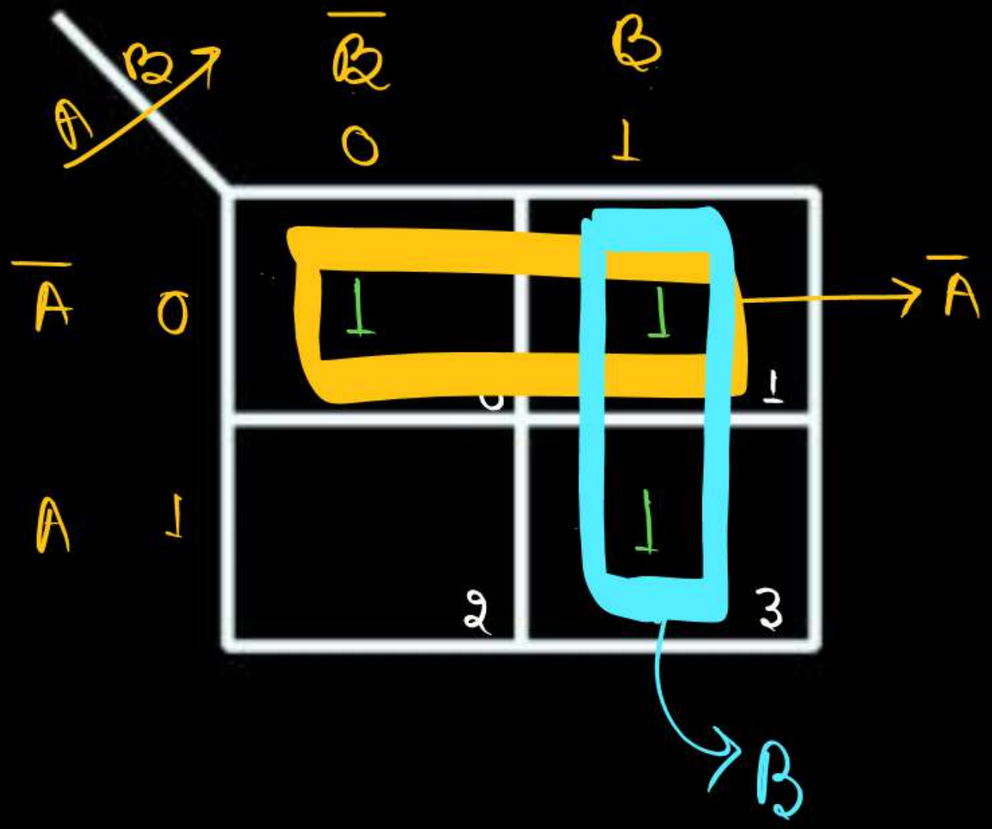
कम से कम group बनाना है and बड़े से बड़ा group बनाना है।

मूर्तिकार और पत्थर

Ex.1. $f(A,B) = \underset{0\,0}{\bar{A}\bar{B}} + \underset{0\,1}{\bar{A}B} + \underset{1\,1}{AB} = \sum m(0,1,3)$

$\underset{0}{\bar{A}\bar{B}} + \underset{1}{\bar{A}B} + \underset{3}{AB} = \sum m(0,1,3)$

minterm



$\bar{A}+B$

Ans

$\bar{A}\bar{B} + \bar{A}B + AB$

$\bar{A}[\bar{B}+B] + AB$

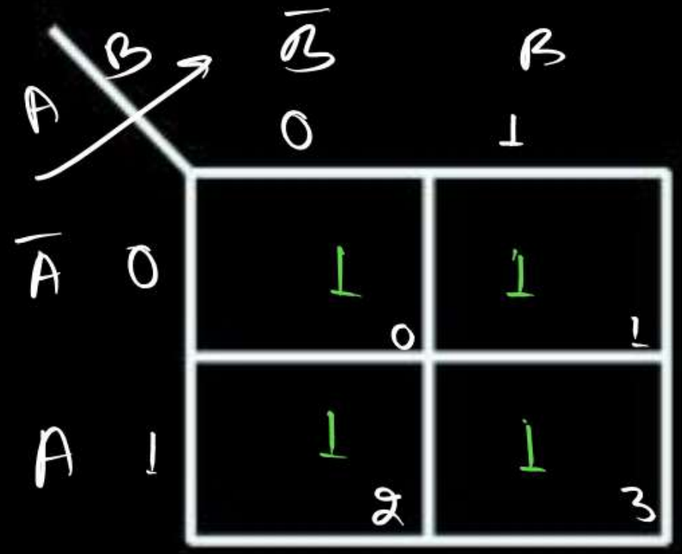
$\bar{A} + A\bar{B}$

$(\bar{A}+A)\cdot(\bar{A}+B)$

$\bar{A}+B$

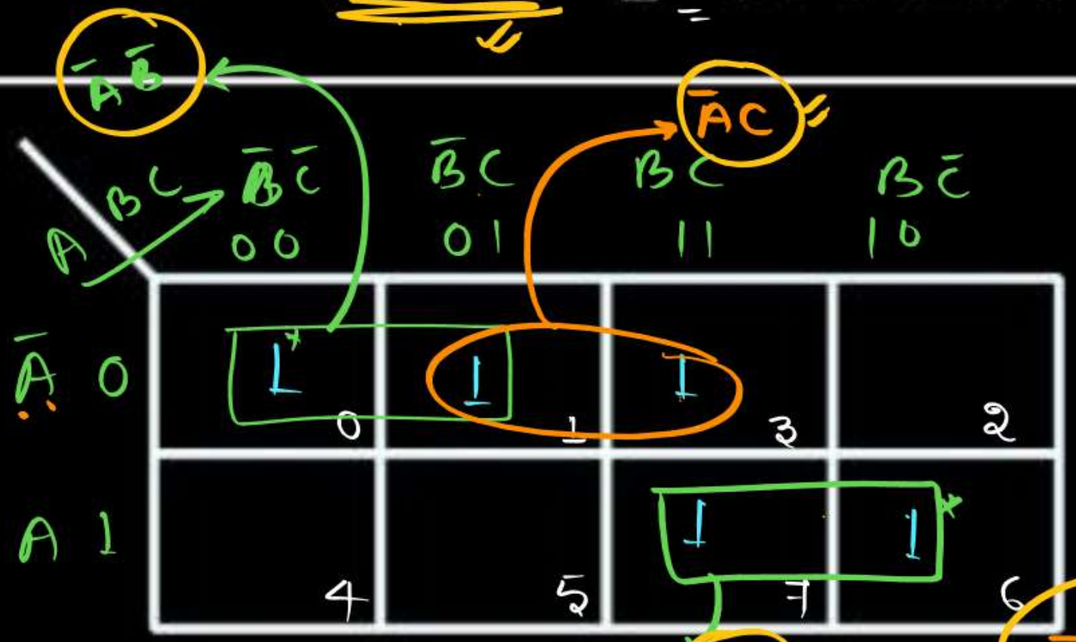
Ans

Ex.2. $f(A,B) = \bar{A}\bar{B} + \bar{A}B + A\bar{B} + AB = \Sigma m(0,1,2,3)$

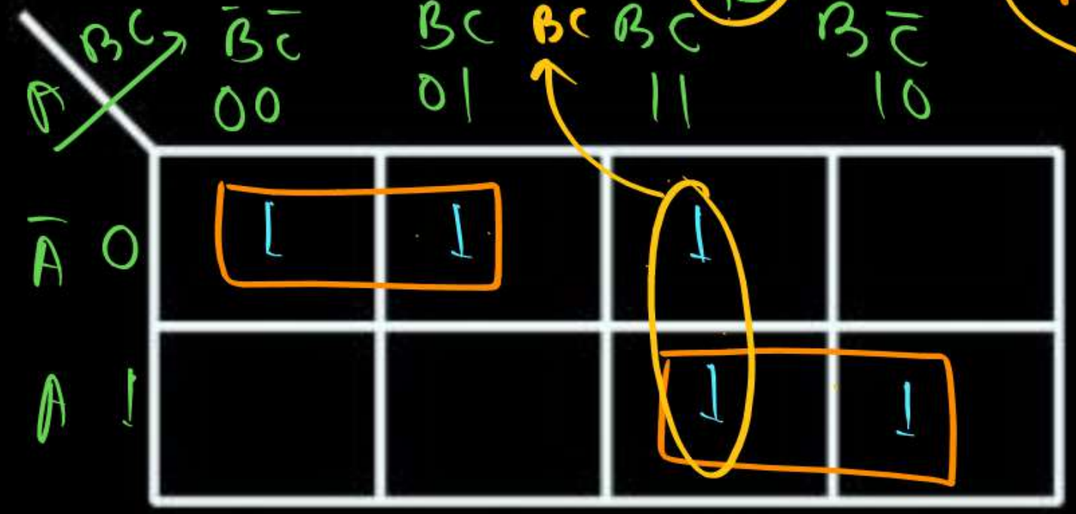


$= 1$

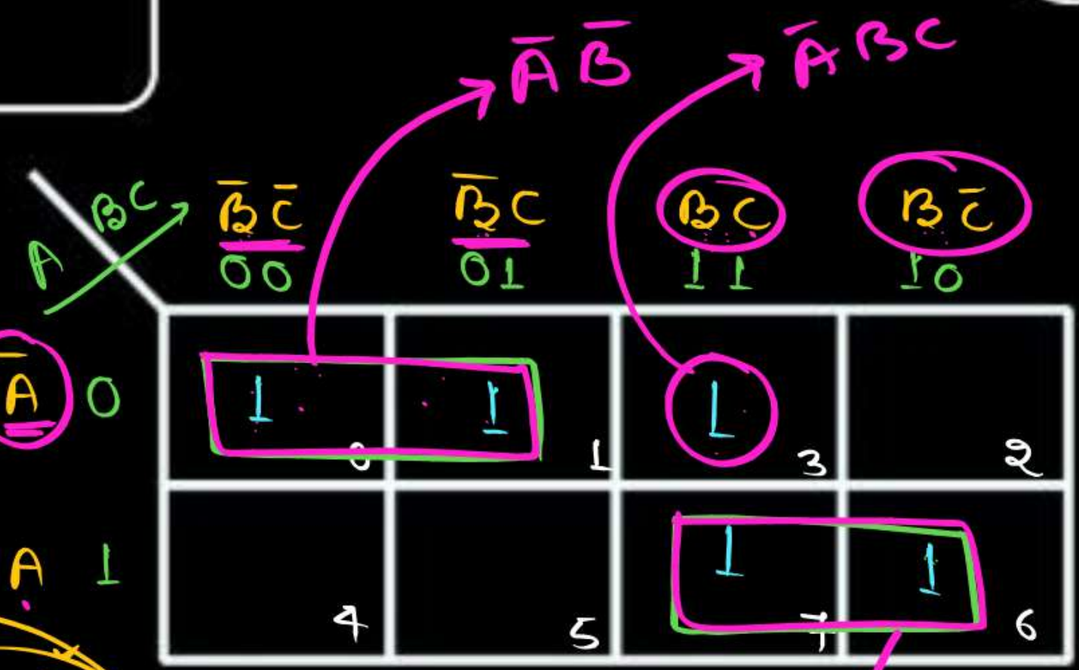
Ex. 3. $f(A, B, C) = \sum m(0, 1, 3, 6, 7)$



$= \bar{A}\bar{B} + AB + \bar{A}C$



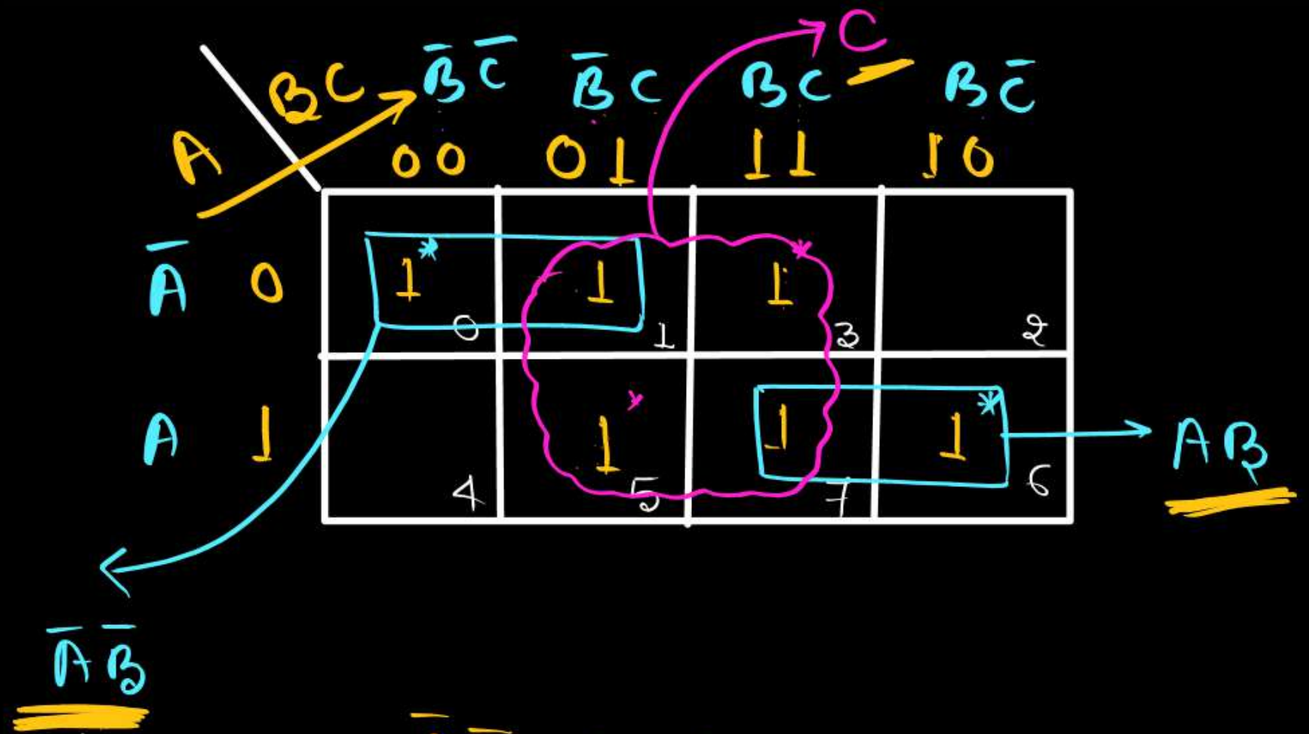
$= \bar{A}\bar{B} + AB + BC$



$\bar{A}\bar{B} + AB + \bar{A}BC$

→ Seminimized expression

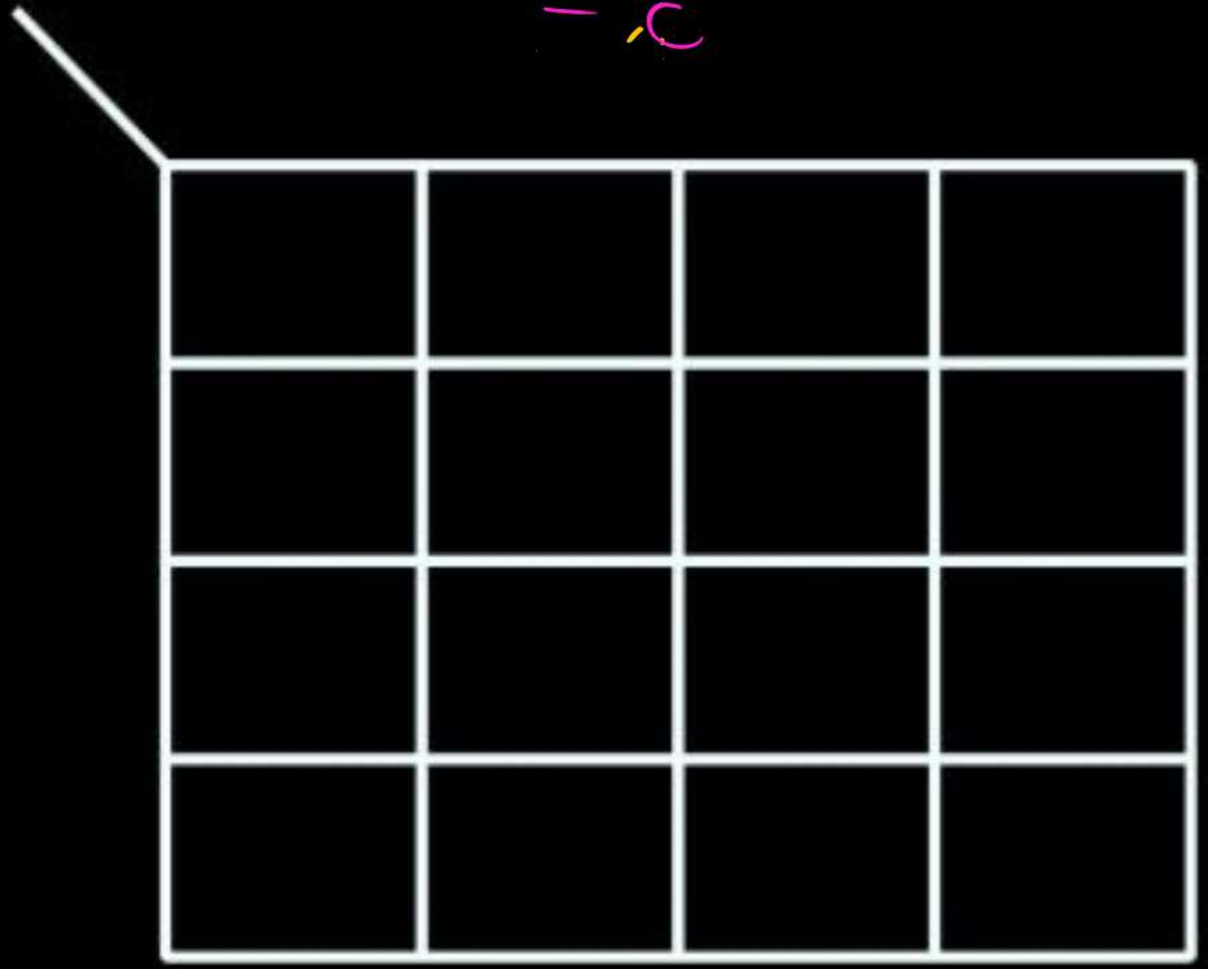
Ex. 4. $f(A, B, C) = \sum m\,(0, 1, 3, 5, 6, 7)$

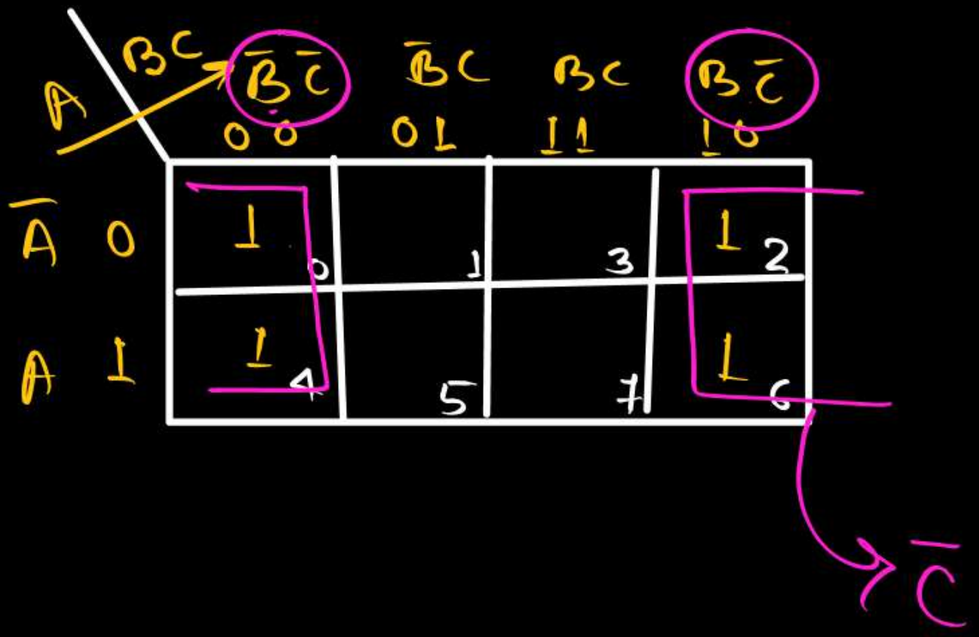


$\bar{A}\bar{B} + AB + C$

① ② ③

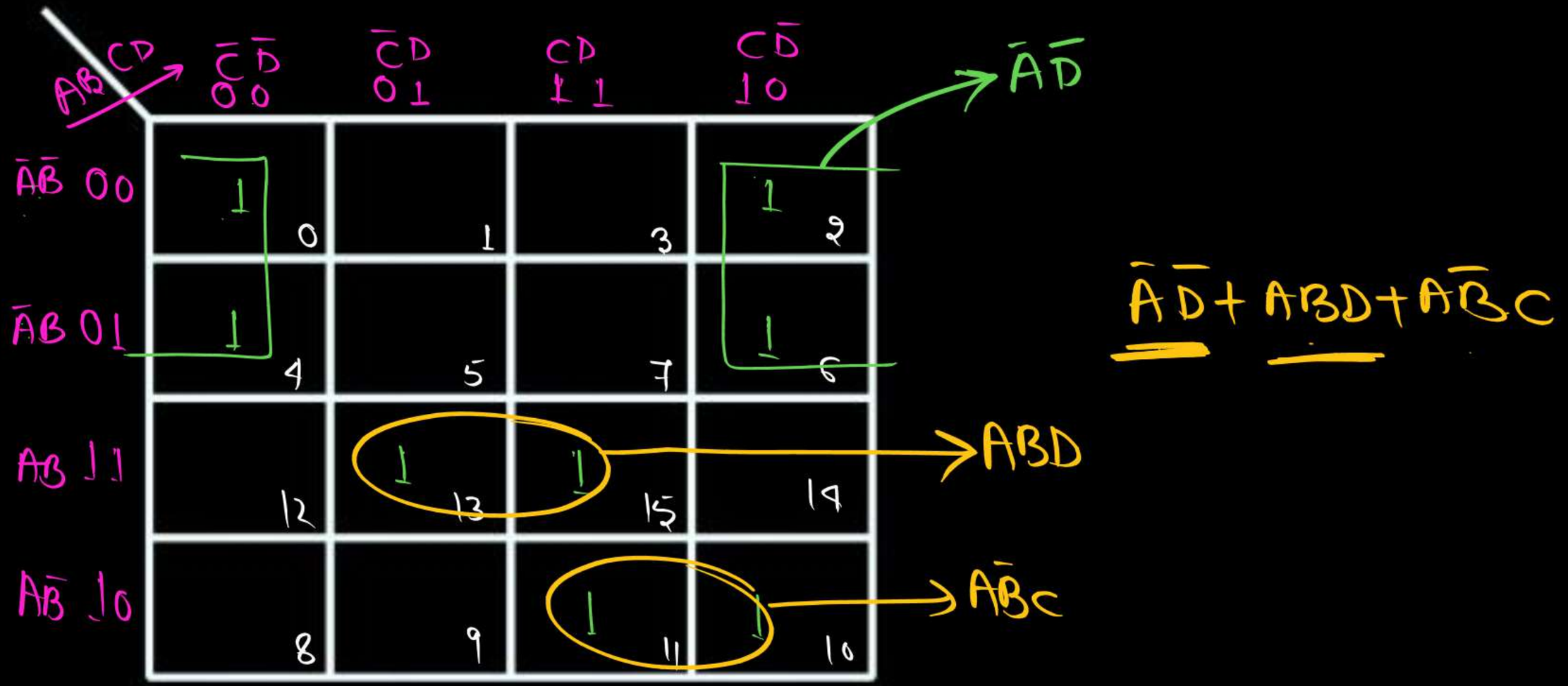
Ex. 5. $f(A, B, C) = \sum m(0, 2, 4, 6)$





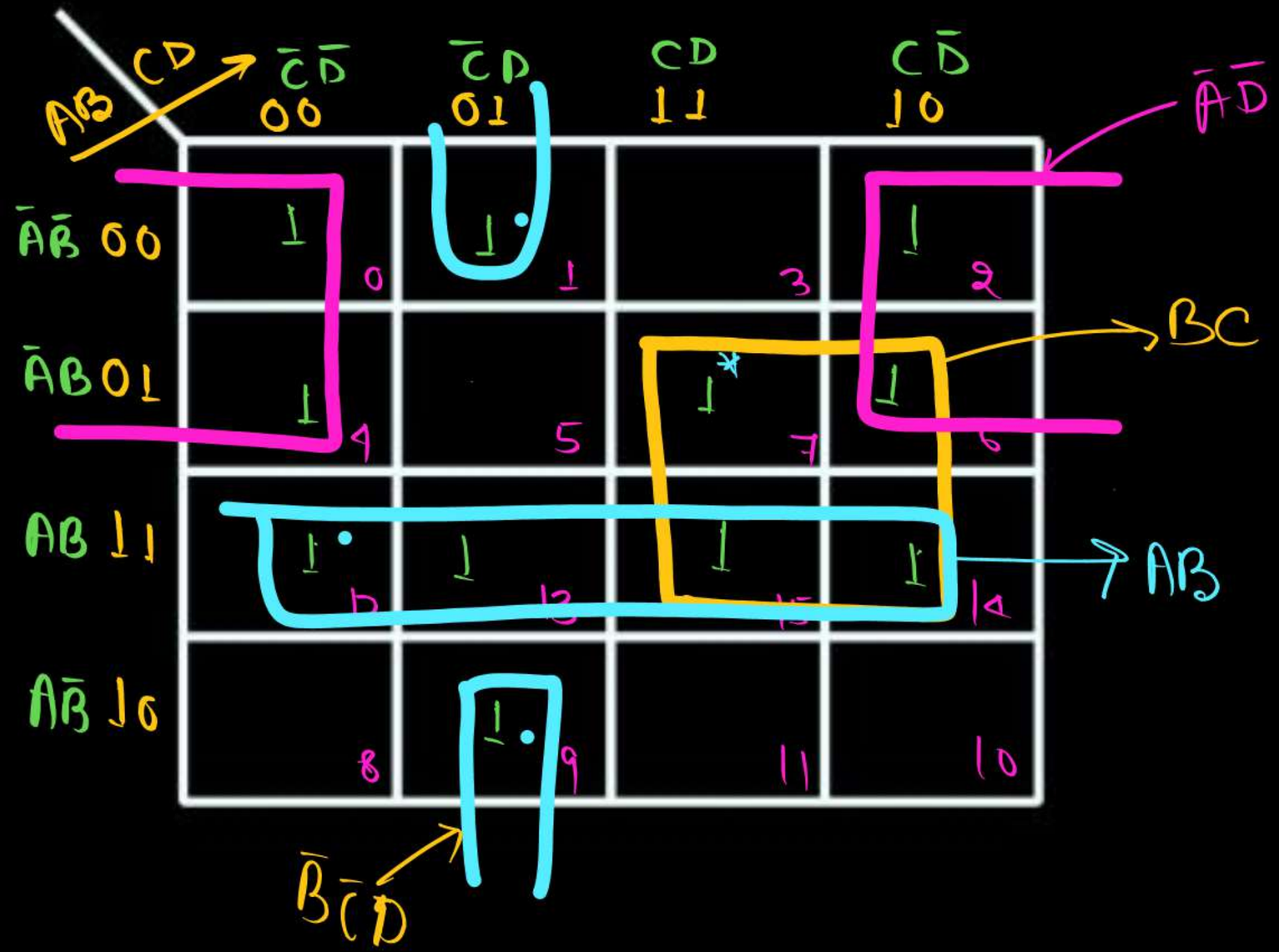
Ans = $\bar{C}$

Ex. 6. $f(A, B, C, D) = \sum m(0, 2, 4, 6, 10, 11, 13, 15)$



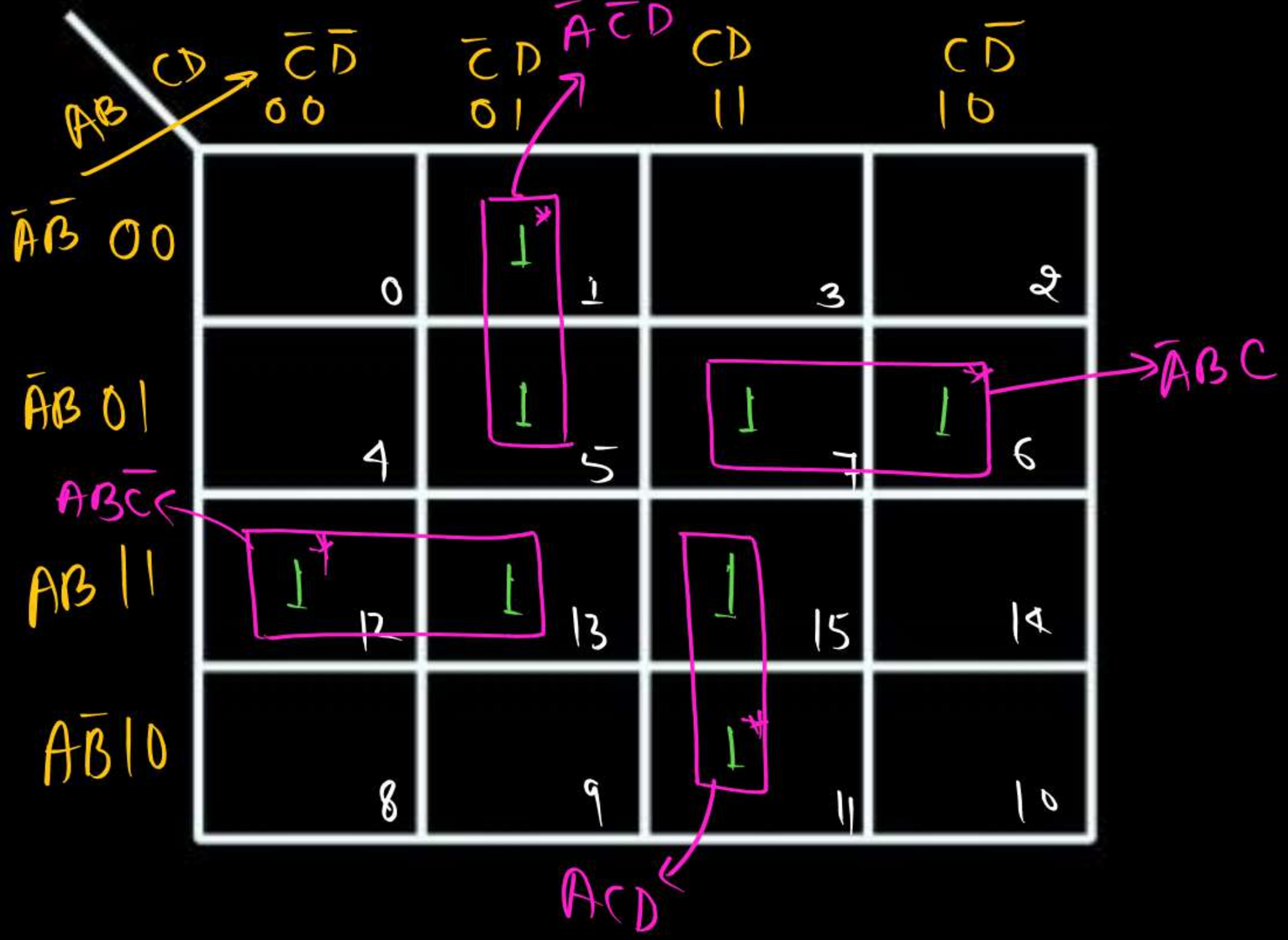
$\bar{A}\bar{D} + ABD + A\bar{B}C$

Ex. 7. $f(A, B, C, D) = \sum m\,(0, 1, 2, 4, 6, 7, 9, 12, 13, 14, 15)$



$\bar{A}\bar{D} + BC + AB + \bar{B}\bar{C}D$

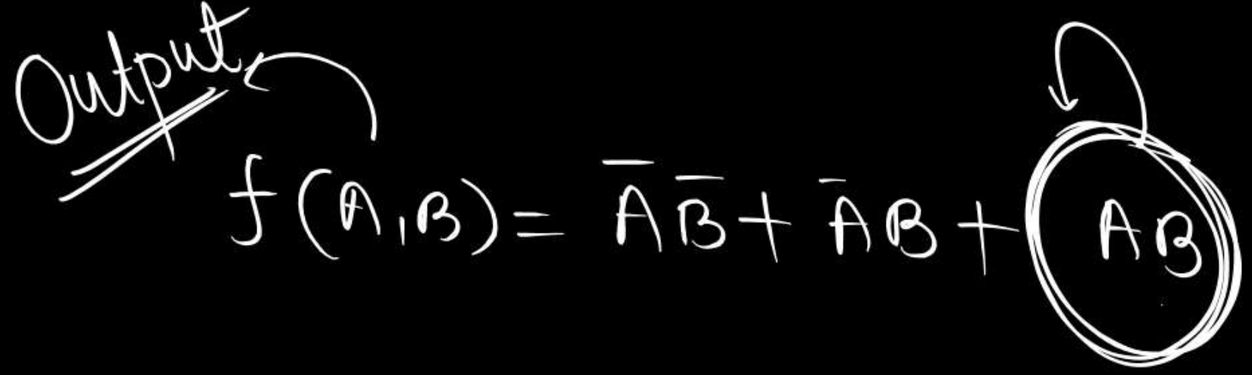
Ex. 8. $f(A, B, C, D) = \sum m\,(1, 5, 6, 7, 11, 12, 13, 15)$



$$= \bar{A}\bar{C}D + \bar{A}BC + AB\bar{C} + ACD$$

DON'T CARE CONDITION

Combination of inputs on which the output may or may not depends are called don't care condition.

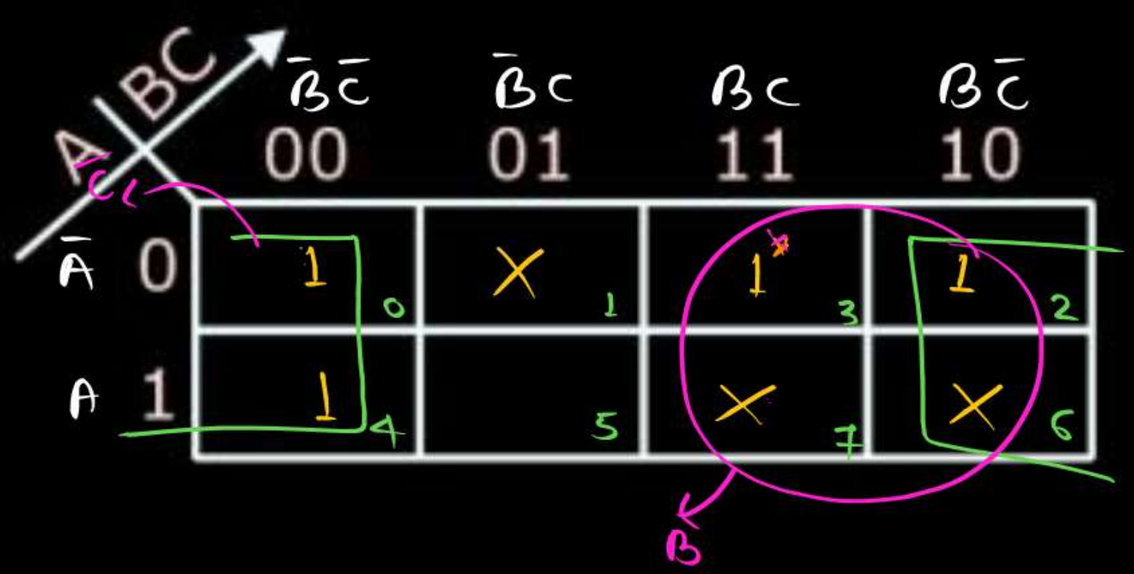


A, B

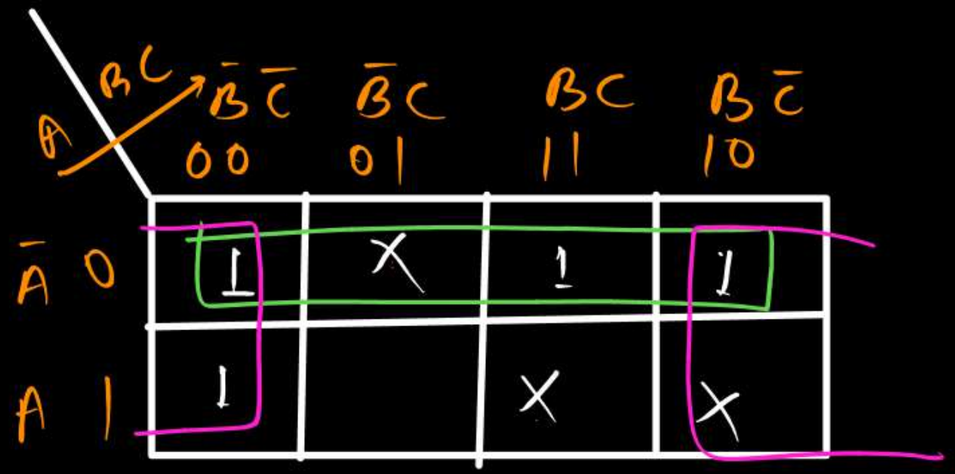
↳ inputs

Ex. 9. Find the minimized Boolean expression for the function given as

$f(A, B, C) = \sum m(0, 2, 3, 4) + \sum d(1, 6, 7)$



$B + \bar{C}$

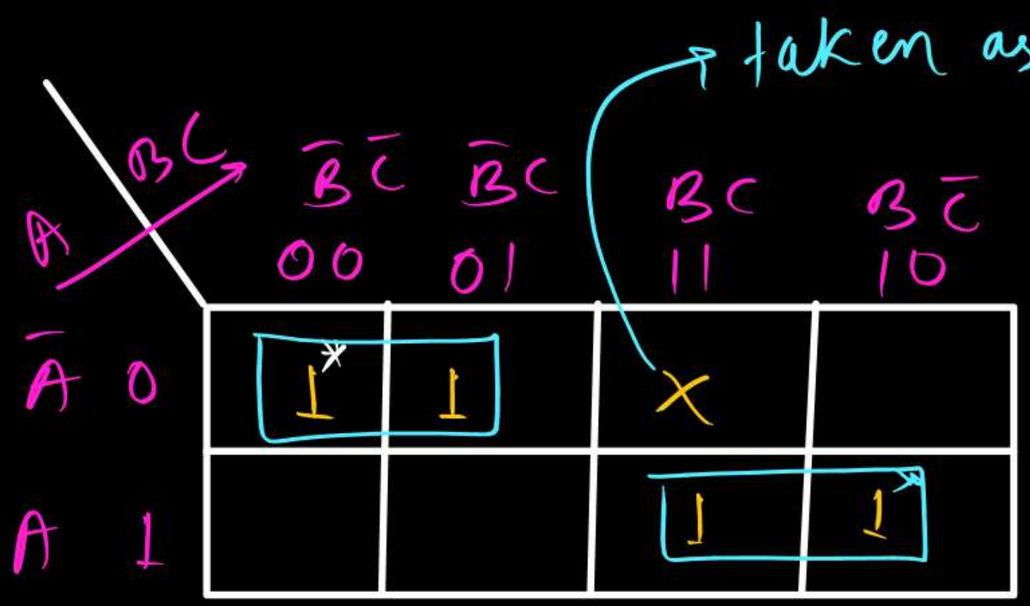


$\bar{A} + \bar{C}$

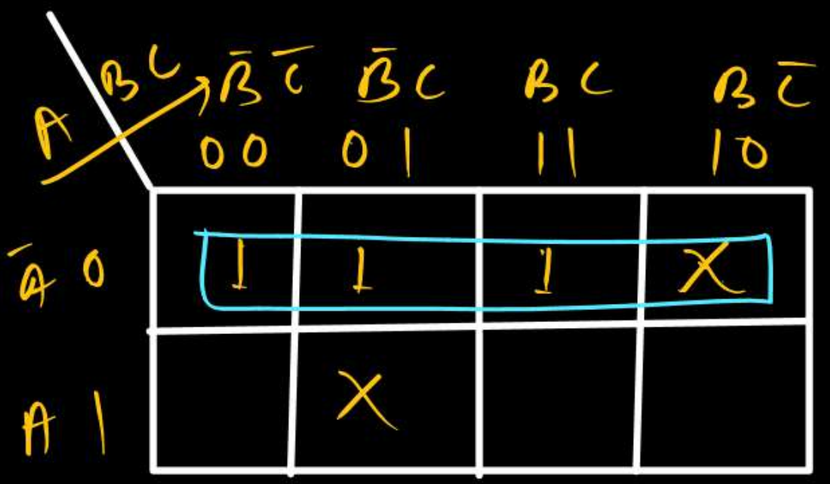
Ans

=

Q.



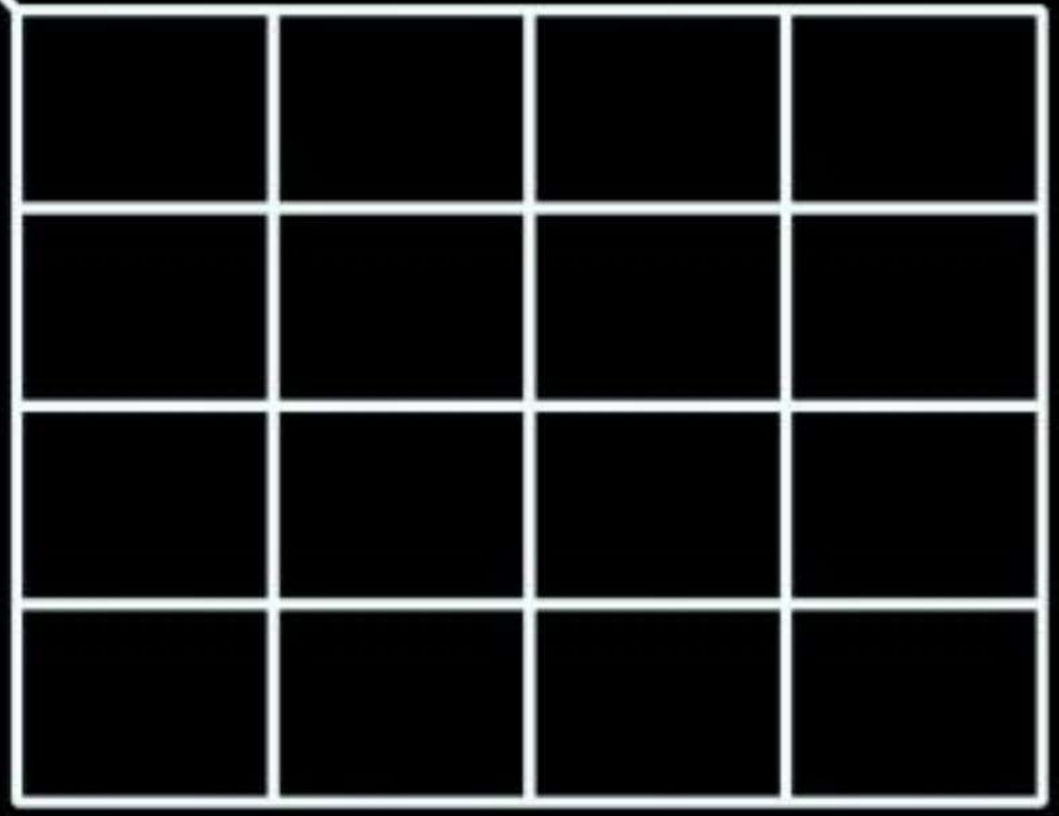
$\bar{A}\bar{B}+AB$



$\bar{A}$

Ex. 10. Find the minimized Boolean expression for the function given as

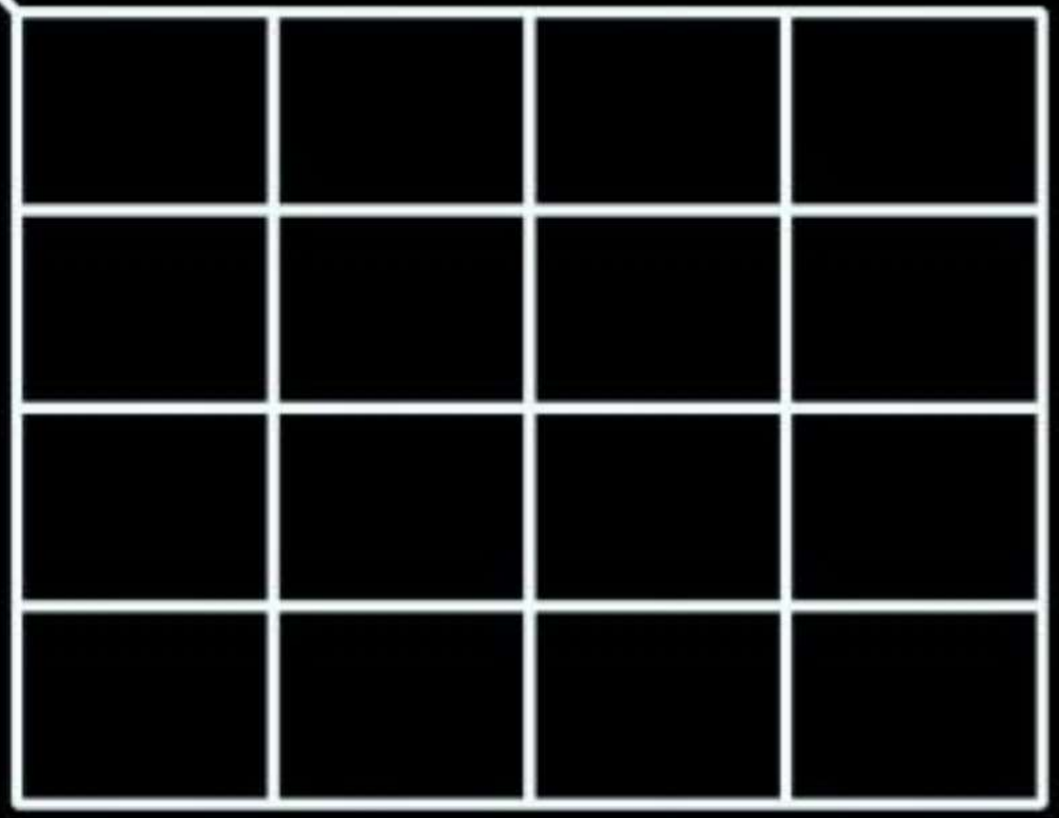
$f(A, B, C, D) = \sum m\,(0, 2, 4, 6, 7, 8, 10, 11, 12, 14, 15) + \sum d(1,3)$



Comment Box

Ex. 11 Find the minimized Boolean expression for the function given as $f(A, B, C, D) = \sum m(0,3,6,7,9,14) + \sum d(1,4,5,11,13,15)$

HW



100 kg

10 kg