Mission ISRO 2023

Computer Science

Discrete Mathematics

Lecture No. 1

By- SATISH YADAV SIR

TOPICS TO BE COVERED

01 Definition of Graph

02 Handshaking Lemma

03 Types of Graphs

04 No of Graphs

05 Simple Graphs theorem

Basics of Graph

Combinatorics:

- Sum Rule/product Rule.
- Combination with Reptn.
- Inclusion-Exclusion
- pigeonhole principle.
- Recurrence Relation
- Generating function.

Sum Rule :

$$\begin{tikzcd} & \text{Task } T \arrow[dl] \arrow[dr] & \\ T_1 & & T_2 \\ n_1 \text{ ways} \arrow[u] & & n_2 \text{ ways} \arrow[u] \end{tikzcd}$$

both the tasks are not happening simultaneously

$$\text{Total ways} = n_1 + n_2$$

Product Rule :

$$\begin{tikzcd} & \text{Task } T \arrow[dl] \arrow[dr] & \\ T_1 & & T_2 \\ n_1 \text{ ways} \arrow[u] & & n_2 \text{ ways} \arrow[u] \end{tikzcd}$$

both the tasks are happening simultaneously.

$$\text{Total ways} = n_1 \times n_2$$

Sum Rule:

$k = 0$

for $i = 1$ to n_1

$\quad k = k + 1$

for $j = 1$ to n_2.

$\quad k = k + 1$

$$K = n_1 + n_2$$

product Rule:

$k = 0$

T_1 for $i = 1 \longrightarrow n_1$

T_2 for $j = 1 — n_2$.

$\quad K = K + 1$

$K = ?$

$K = n_1 \times n_2$

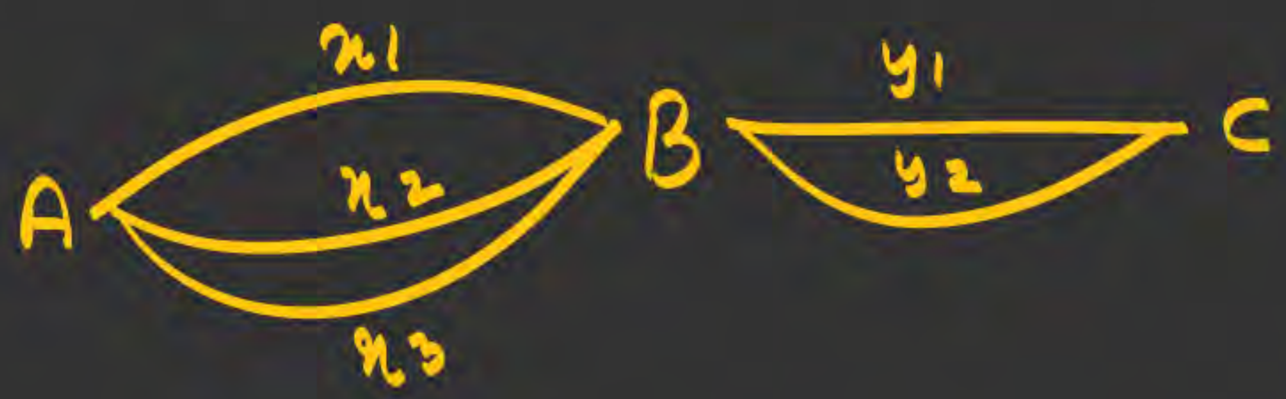
Sum Rule:

$A \longrightarrow C$.

via B — Direct.

$3 \times 2 + 2$.

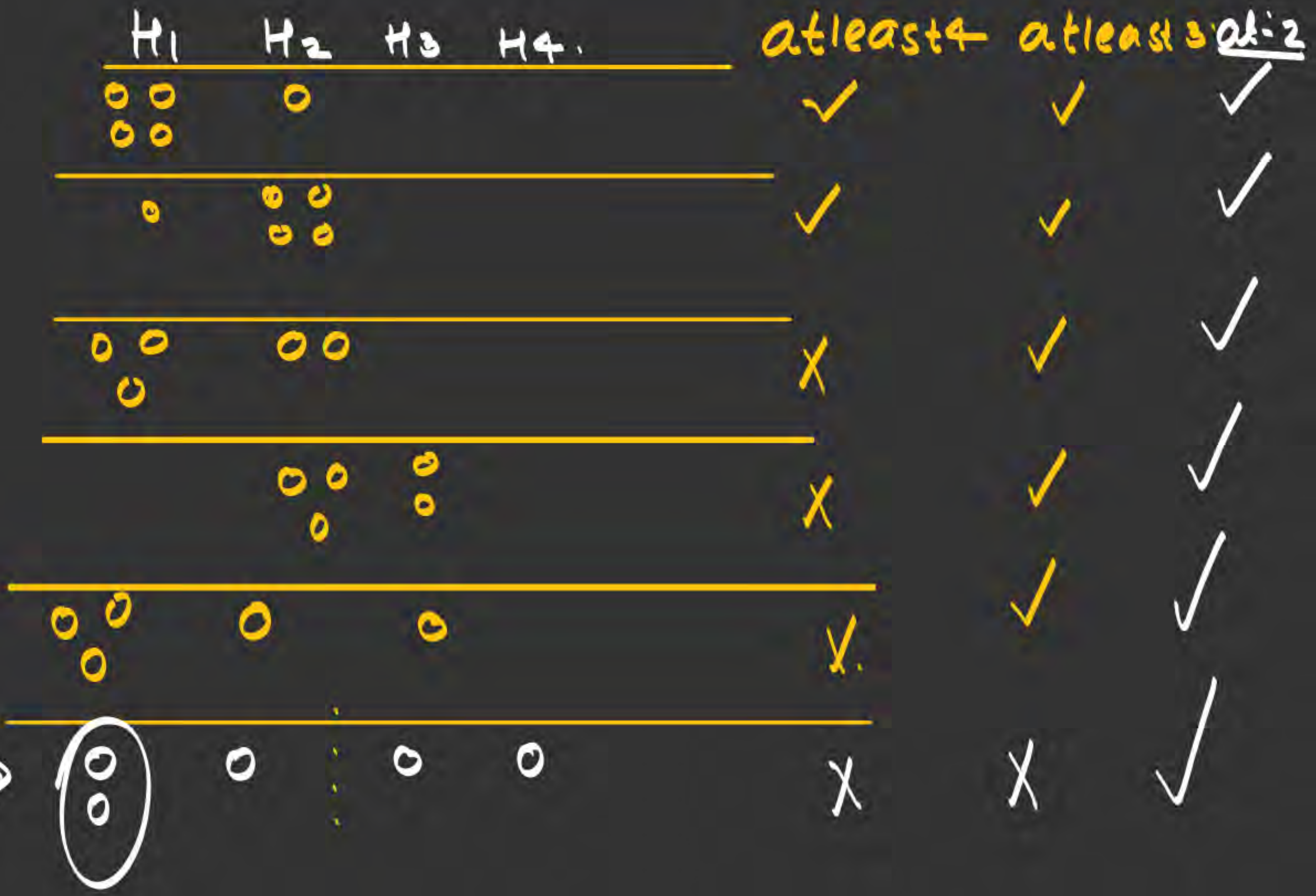
product Rule:



how many ways we can go from $A \to C$?

$3 \times 2 = 6$ ways.

Pigeonhole principle:



Some Holes contains atleast 4 . X

atleast 3 X ✓

atleast 2 ✓

Basic: if $n+1$ pigeons, we want to distribute to n holes then some holes contains atleast 2 pigeons.

Advance:

Basic → atleast 2

Advance → atleast 3.
atleast 4
at
⋮

In school system, we have 5 diff grade system. what will be min: no: of students requires such that atleast 6 belongs to same grade?

$5 \times 5 = 25$

$+1$

$= 26$

pigeon → 26

pigeonhole → 5

atleast → atleast 6

→ how many students will require such that atleast 3 will have today in same month?

P PH atleast 3.
? 12m.

atleast 3

m1	m2	.....	m12
o	o	.....	o
o	o	.....	o
o			

$12 \times 2 + 1 = 25$

(GATE)

what will be no.of cards we will widrawn, such that atleast 3 belongs to same suite?

H	S	D	C
o	o	o	o
o	o	o	o
o			

Ans: 9

Inclusion - Exclusion :

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→ no. of elements which are ÷ by 2 or 5 in a set {1 ...... 10} ?

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

÷ by 3

$= \left\lfloor \frac{10}{3} \right\rfloor = \underline{\underline{3}}$ 3.3

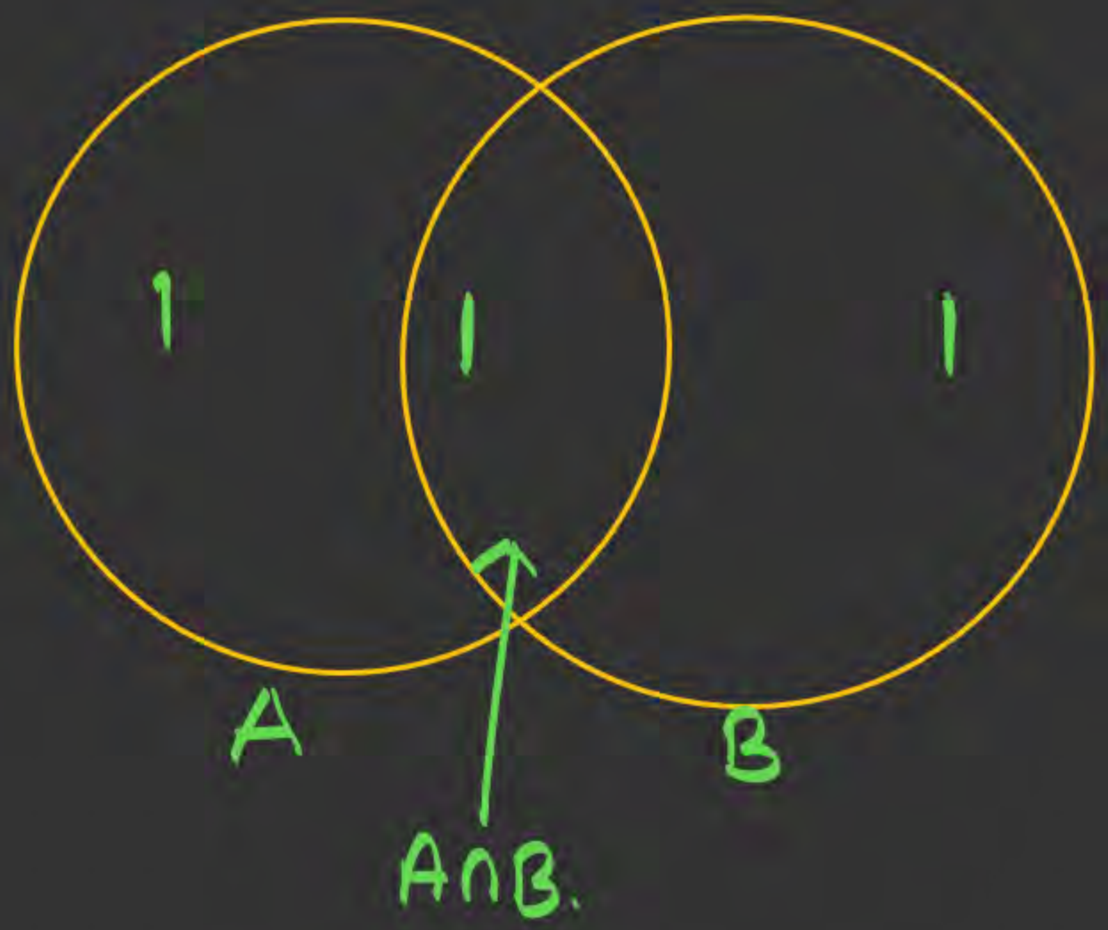
A: no. of elements ÷ by 2 = $\left\lfloor \frac{\text{total elements}}{2} \right\rfloor = \left\lfloor \frac{10}{2} \right\rfloor = 5$

B: no. of elements ÷ by 5 $= \left\lfloor \frac{10}{5} \right\rfloor = 2$

A∩B: no. of elements ÷ 2 ∧ 5 $= \left\lfloor \frac{10}{10} \right\rfloor = 1$

$A \cup B = A + B - A \cap B$

$= 5 + 2 - 1$



$$A \cup B = A + B - A \cap B.$$

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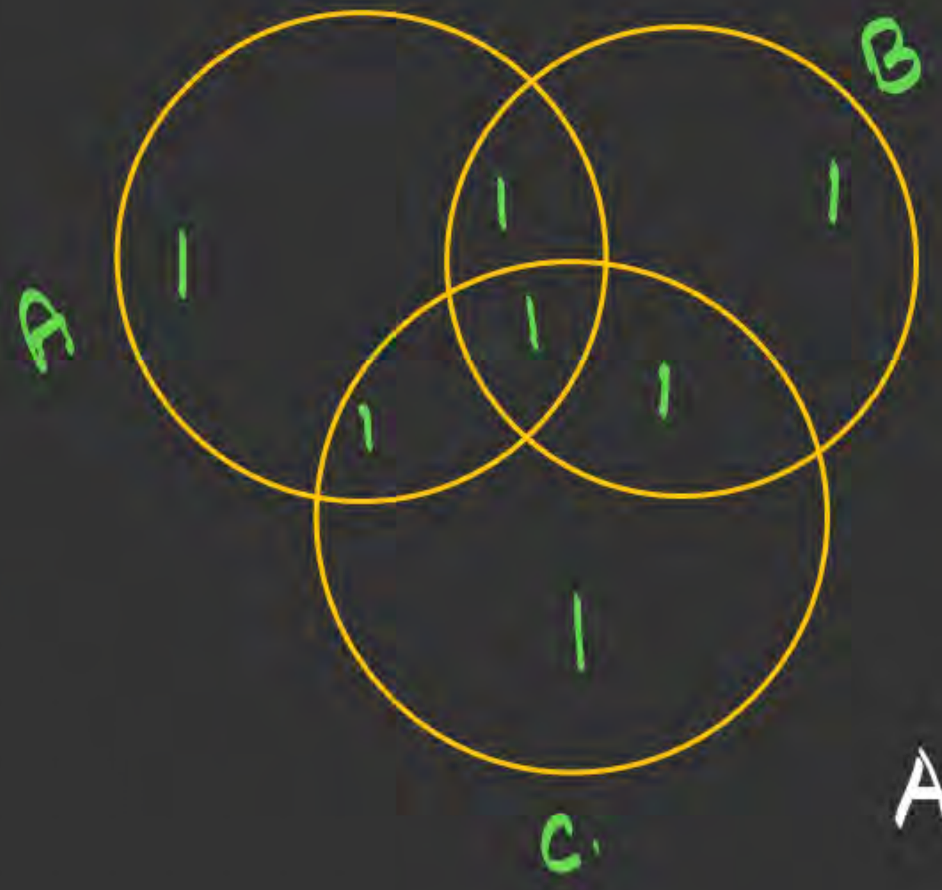


$$A \cup B \cup C = \underbrace{A + B + C} - \underbrace{A \cap B - B \cap C - A \cap C} + \underbrace{A \cap B \cap C}.$$

$$A_1 \cup A_2 \cup A_3 = A_1 + A_2 + A_3 - A_1 \cap A_2 - A_2 \cap A_3 - A_1 \cap A_3 + A_1 \cap A_2 \cap A_3.$$

$$= \Sigma A_i - \Sigma A_i \cap A_j + A_1 \cap A_2 \cap A_3. \checkmark$$

$$A_1 \cup A_2 \cup A_3 \cup A_4 = \Sigma A_i - \Sigma A_i \cap A_j + \Sigma A_i \cap A_j \cap A_K - A_1 \cap A_2 \cap A_3 \cap A_4$$

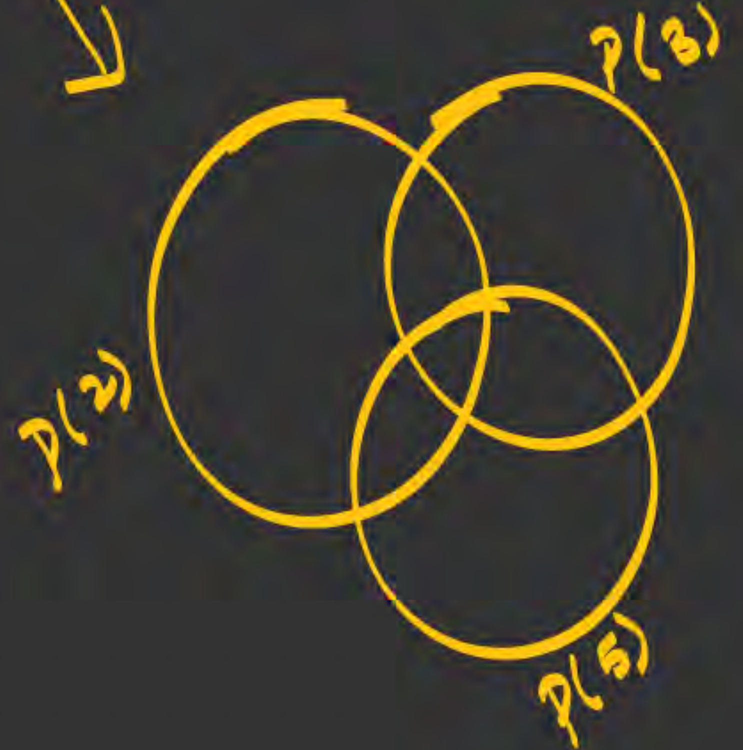
$$A_1 \cup A_2 \ldots A_n = \Sigma A_i - \Sigma A_i \cap A_j + \Sigma A_i \cap A_j \cap A_K \ldots \left((-1)^{n+1} A_1 \cap \ldots A_n\right)$$

How many elements are $\div$ by 2, 3 or 5 in a set $\{1 \ldots 123\}$. ISRO

90

How many elements are not $\div$ by 2, 3 or 5 in a set $\{1 \ldots 123\}$?

33.



$$P(2 \cup 3 \cup 5) = P(2) + P(3) + P(5) - P(2 \cap 3) - P(3 \cap 5) - P(2 \cap 5) + P(2 \cap 3 \cap 5)$$

$$= \left\lfloor \frac{123}{2} \right\rfloor + \left\lfloor \frac{123}{3} \right\rfloor + \left\lfloor \frac{123}{5} \right\rfloor - \left\lfloor \frac{123}{2 \cdot 3} \right\rfloor - \left\lfloor \frac{123}{3 \cdot 5} \right\rfloor - \left\lfloor \frac{123}{2 \cdot 5} \right\rfloor + \left\lfloor \frac{123}{2 \cdot 3 \cdot 5} \right\rfloor$$

$$= 90$$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Computer Science

Discrete Mathematics

Lecture No. 02

By- SATISH YADAV SIR

TOPICS

01 Homogenous equation

02 Non Homogenous equation

3 Exercise

Recurrence relation:

In a colony, at time 0, 5 bacterias were present. bacterias are increasing 2 times as the previous. What will be total bacterias at time 100?

a_0: at time 0, 5 bacterias.

a_1:

a_2:

a_{100}: at time 100 —— " —— ?

a_n = total bacterias at time n.

Given: Intial condition.

$a_0 \quad a_1 \quad a_2 \quad \ldots \quad a_{98} \quad a_{99} \quad a_{100}$

$a_0 \to 5$ $\qquad$ $a_{100} \to ?$

Soltn.

$$a_n = 2a_{n-1}$$

$a_0 = 5$

$a_1 = 2 \cdot a_0$

$a_2 = 2 \cdot a_1$

$= 2(2 \cdot a_0)$

$a_2 = 2^2 \cdot a_0$

$a_3 = 2^3 \cdot a_0$

$a_4 = 2^4 \cdot a_0$

$a_{100} = 2^{100} \cdot a_0 \leftarrow$

Recurrence/relation

$a_{99} \quad a_{100}$

$a_{98} \quad a_{99} \quad a_{100}$

Type-1 :

a_0 = initial condition.

$a_n = d \cdot a_{n-1}$.

Soltn : $\boxed{a_n = d^n \cdot a_0}$

eg:

$a_0 = 5$

$a_n = 2 \cdot a_{n-1}$.

$\downarrow$

$a_n = 2^n \cdot a_0$

$a_n = 2^n \cdot 5$

$$a_n = 5a_{n-1} - 6a_{n-2} \qquad a_0 = 2 \quad a_1 = 4.$$

$$1.\ a_n - 5a_{n-1} + 6a_{n-2} = 0$$

Characteristic eqtn,

$$a_n = (R_1)^n c_1 + (R_2)^n c_2.$$

$$c_1 = 0$$
$$c_2 = 2.$$

$$1.\ x^2 - 5x + 6 = 0$$

$$x^2 - 5x + 6 = 0$$

$$a_n = 3^n c_1 + 2^n c_2 \rightarrow a_n = 3^n \cdot 0 + 2^n \cdot 2.$$

$$a_n = 2 \cdot 2^n.$$

$$\boxed{a_n = 2^{n+1}.}$$

$$x^2 - 3x - 2x + 6 = 0$$

$$x(x-3) - 2(x-3) = 0$$

$$(x-3)(x-2) = 0$$

Roots: 3, 2.

$n = 0$

$$a_0 = 3^0 c_1 + 2^0 c_2.$$

$$2 = c_1 + c_2$$

$n = 1.$

$$a_1 = 3^1 c_1 + 2^1 c_2.$$

$$4 = 3c_1 + 2c_2.$$

$$a_{n+2} = 7a_{n+1} - 10a_n \qquad a_0 = 1 \qquad a_1 = 3.$$

$$x^2 = 7x - 10$$

$$x^2 - 7x + 10 = 0$$

Roots: 2, 5

CE: $a_n = (R_1)^n c_1 + (R_2)^n c_2.$

$$a_n = 2^n c_1 + 5^n c_2 \rightarrow \boxed{a_n = 2^n\left(\tfrac{2}{3}\right) + 5^n\left(\tfrac{1}{3}\right)}$$

$n = 0$

$$a_0 = 2^0 c_1 + 5^0 c_2$$

$$1 = c_1 + c_2$$

$$1 - c_2 = c_1$$

$n = 1$

$$a_1 = 2c_1 + 5c_2$$

$$3 = 2c_1 + 5c_2.$$

$$3 = 2(1 - c_2) + 5c_2$$

$$3 = 2 - 2c_2 + 5c_2$$

$$1 = 3c_2 \qquad c_2 = 1/3.$$

$$c_1 = 2/3.$$

$$a_n = a_{n-1} + a_{n-2} \quad a_0 = 0 \quad a_1 = 1.$$

$$x^2 = x + 1.$$

$$x^2 - x - 1 = 0$$

Roots: $\frac{1+\sqrt{5}}{2}, \frac{1-\sqrt{5}}{2}$

$$c_1 = \frac{1}{\sqrt{5}}$$

$$c_2 = -\frac{1}{\sqrt{5}}.$$

$$\text{CE}: a_n = \left(\frac{1+\sqrt{5}}{2}\right)^n c_1 + \left(\frac{1-\sqrt{5}}{2}\right)^n c_2$$

$$a_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n$$

$$x^2 - x - 1 = 0$$

$$ax^2 + bx + c = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 1$

$b = -1$

$c = -1.$

$$\frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{1 \pm \sqrt{5}}{2}$$

$$a_n = \left(\frac{1+\sqrt{5}}{2}\right)^n c_1 + \left(\frac{1-\sqrt{5}}{2}\right)^n c_2.$$

$n = 0.$

$$a_0 = (\quad)^0 c_1 + (\quad)^0 c_2$$

$$0 = c_1 + c_2$$

$$c_1 = -c_2$$

$n = 1.$

$$a_1 = \frac{1+\sqrt{5}}{2} c_1 + \frac{1-\sqrt{5}}{2} c_2$$

$$1 = \left(\frac{1+\sqrt{5}}{2}\right) c_1 + \left(\frac{1-\sqrt{5}}{2}\right) - c_1$$

$$= \left(\frac{\cancel{1}+\sqrt{5}-\cancel{1}+\sqrt{5}}{2}\right) c_1$$

$$1 = \frac{\cancel{2}\sqrt{5}}{\cancel{2}} \cdot c_1$$

$$c_1 = \frac{1}{\sqrt{5}}.$$

$$c_2 = -\frac{1}{\sqrt{5}}.$$

Type-2 :

Roots : R_1, R_2.

CE: $a_n = (R_1)^n c_1 + (R_2)^n c_2$

Type-3 :

Roots : (same) R, R.

CE: $a_n = R^n c_1 + n \cdot R^n c_2$.

$a_n = 6a_{n-1} - 9a_{n-2}$. $a_0 = 1$

$a_1 = 2$

$x^2 = 6x - 9$

$x^2 - 6x + 9 = 0$

Root: 3, 3.

CE: $a_n = 3^n c_1 + n \cdot 3^n c_2$ ←

$a_n = 3^n \cdot 1 + \left(-\frac{1}{3}\right) n \cdot 3^n$

n = 0

$a_0 = 3^0 c_1 + 0 \cdot 3^0 c_2$

$1 = c_1 + 0$

$c_1 = 1$

n = 1.

$a_1 = 3^1 c_1 + 1 \cdot 3^1 c_2$.

$2 = 3c_1 + 3c_2$

$2 = 3(1) + 3c_2$.

$c_2 = -1/3$.

Type-1:

$a_n = \alpha a_{n-1}$.

$a_0 =$ initial

Soltn: $a_n = \alpha^n a_{n-1}$.

Type-2: Roots: R_1, R_2.

CE: $a_n = (R_1)^n c_1 + (R_2)^n c_2$.

Type-3: Roots: R, R.

$a_n = (R)^n c_1 + n \cdot R^n c_2$.

$a_n = A a_{n-1} + B a_{n-2} + C a_{n-3}$.

Type-4:

Roots: R_1, R_2, R_3

CE: $a_n = (R_1)^n c_1 + (R_2)^n c_2 + (R_3)^n c_3$

Roots: R, R, R_1 $a_n = R^n c_1 + n \cdot R^n c_2 + (R_1)^n c_3$

Roots: R, R, R. $a_n = R^n c_1 + n \cdot R^n c_2 + n^2 R^n c_3$.

$$\frac{1}{1-ax} = 1 + ax + (ax)^2 + (ax)^3 + (ax)^4 + \dots\dots$$

$a = 1.$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \dots\dots$$

$a = 2$

$$\frac{1}{1-2x} = 1 + 2x + (2x)^2 + (2x)^3 + \dots\dots$$

$$\frac{1}{1+ax} = 1 - ax + (ax)^2 - (ax)^3 + (ax)^4 -$$

$a = 1.$

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 - x^5$$

$a = 2.$

$$\frac{1}{1+2x} = 1 - 2x + (2x)^2 - (2x)^3 + (2x)^4 \dots$$

Generating Functions :

$$\langle a_0, a_1, a_2, a_3 \ldots\ldots \rangle$$

$$G(x) = a_0 x^0 + a_1 x^1 + a_2 x^2 + a_3 x^3$$

$$\boxed{G(x) = \sum_{i=0}^{\infty} a_i * x^i}$$

$\rightarrow \langle \underset{a_0}{1}, \underset{a_1}{1}, \underset{a_2}{1}, \underset{a_3}{1}, \underset{a_4}{1} \ldots\ldots \rangle$

$$G(x) = a_0 x^0 + a_1 x^1 + a_2 x^2 + a_3 x^3 + a_4 x^4 + \ldots$$

$$= 1 \cdot x^0 + 1 \cdot x + 1 \cdot x^2 + 1 \cdot x^3 + 1 \cdot x^4$$

$$= 1 + x + x^2 + x^3 + x^4 + \ldots$$

$$G(x) = \frac{1}{1-x}$$

$\langle 0, 0, 0, 6, 6, 6, 6, 6 \ldots \rangle$

$a_0 \; a_1 \; a_2 \; a_3 \; a_4 \; a_5$

$G(x) = a_0x^0 + a_1x^1 + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + a_6x^6 + \ldots$

$= 0 \cdot x^0 + 0 \cdot x^1 + 0 \cdot x^2 + 6x^3 + 6x^4 + 6x^5 + 6x^6 + \ldots$

$G(x) = \dfrac{6x^3}{1-x}$

$= 6x^3 + 6x^4 + 6x^5 + 6x^6 + 6x^7 + \ldots$

$= 6x^3(1 + x + x^2 + x^3 + x^4 + \ldots)$

$G(x) = \dfrac{6x^3}{1-x}$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + x^5$$

$$\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}.$$

$$\frac{d}{dx}\left(\frac{1}{1-x}\right)$$

$$\frac{1}{(1-x)^2} = 0 + 1 + 2x + 3x^2 + 4x^3 + 5x^4 + 6x^5$$

$$\frac{-1}{(1-x)^2}\frac{d(1-x)}{dx}.$$

$$\boxed{\frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + 4x^3 + 5x^4 + 6x^5 + \cdots}$$

$$\frac{1}{(1-x)^2}$$

3. If a_n, $n \geq 0$, is the unique solution of the recurrence relation $a_{n+1} - da_n = 0$, and $a_3 = 153/49$, $a_5 = 1377/2401$, what is d?

3. $a_{n+1} - da_n = 0$, $n \geq 0$, so $a_n = d^n a_0$. $153/49 = a_3 = d^3 a_0$, $1377/2401 = a_5 = d^5 a_0 \Longrightarrow a_5/a_3 = d^2 = 9/49$ and $d = \pm 3/7$.

1. Solve the following recurrence relations. (No final answer should involve complex numbers.)

a) $a_n = 5a_{n-1} + 6a_{n-2}$, $n \geq 2$, $a_0 = 1$, $a_1 = 3$

b) $2a_{n+2} - 11a_{n+1} + 5a_n = 0$, $n \geq 0$, $a_0 = 2$, $a_1 = -8$

c) $a_{n+2} + a_n = 0$, $n \geq 0$, $a_0 = 0$, $a_1 = 3$

d) $a_n - 6a_{n-1} + 9a_{n-2} = 0$, $n \geq 2$, $a_0 = 5$, $a_1 = 12$

1. (a) $a_n = 5a_{n-1} + 6a_{n-2}$, $n \geq 2$, $a_0 = 1$, $a_1 = 3$.
Let $a_n = cr^n$, $c, r \neq 0$. Then the characteristic equation is $r^2 - 5r - 6 = 0 = (r-6)(r+1)$, so $r = -1, 6$ are the characteristic roots.
$a_n = A(-1)^n + B(6)^n$
$1 = a_0 = A + B$
$3 = a_1 = -A + 6B$, so $B = 4/7$ and $A = 3/7$.
$a_n = (3/7)(-1)^n + (4/7)(6)^n$, $n \geq 0$.

(b) $a_n = 4(1/2)^n - 2(5)^n$, $n \geq 0$.

(c) $a_{n+2} + a_n = 0$, $n \geq 0$, $a_0 = 0$, $a_1 = 3$.
With $a_n = cr^n$, $c, r \neq 0$, the characteristic equation $r^2 + 1 = 0$ yields the characteristic roots $\pm i$. Hence $a_n = A(i)^n + B(-i)^n = A(\cos(\pi/2) + i\sin(\pi/2))^n + B(\cos(\pi/2) + i\sin(-\pi/2))^n = C\cos(n\pi/2) + D\sin(n\pi/2)$.
$0 = a_0 = C$, $3 = a_1 = D\sin(\pi/2) = D$, so $a_n = 3\sin(n\pi/2)$, $n \geq 0$.

(d) $a_n - 6a_{n-1} + 9a_{n-2} = 0$, $n \geq 2$, $a_0 = 5$, $a_1 = 12$.
Let $a_n = cr^n$, $c, r \neq 0$. Then $r^2 - 6r + 9 = 0 = (r-3)^2$, so the characteristic roots are 3,3 and $a_n = A(3^n) + Bn(3^n)$.
$5 = a_0 = A$; $12 = a_1 = 3A + 3B = 15 + 3B$, $B = -1$.
$a_n = 5(3^n) - n(3^n) = (5 - n)(3^n)$, $n \geq 0$.

3. If $a_0 = 0$, $a_1 = 1$, $a_2 = 4$, and $a_3 = 37$ satisfy the recurrence relation $a_{n+2} + ba_{n+1} + ca_n = 0$, where $n \geq 0$ and b, c are constants, determine b, c and solve for a_n.

3. $(n = 0):\ a_2 + ba_1 + ca_0 = 0 = 4 + b(1) + c(0)$, so $b = -4$.
$(n = 1):\ a_3 - 4a_2 + ca_1 = 0 = 37 - 4(4) + c$, so $c = -21$.
$a_{n+2} - 4a_{n+1} - 21a_n = 0$
$r^2 - 4r - 21 = 0 = (r-7)(r+3)$, $r = 7, -3$
$a_n = A(7)^n + B(-3)^n$
$0 = a_0 = A + B \Longrightarrow B = -A$
$1 = a_1 = 7A - 3B = 10A$, so $A = 1/10$, $B = -1/10$ and $a_n = (1/10)[(7)^n - (-3)^n]$, $n \geq 0$.

Find the coefficient of x^{16} in the expansion of $\left(2x^2 - \frac{x}{2}\right)^{12}$.

$$\binom{12}{k}(2x^2)^{12-k}\left(-\frac{x}{2}\right)^k = \binom{12}{k}2^{12-k}\left(-\frac{1}{2}\right)^k x^{24-k}.$$

We want $24 - k = 16$; thus, $k = 8$. The coefficient is $\binom{12}{8}2^4(-\frac{1}{2})^8 = \frac{1}{16}\binom{12}{8}$ $\frac{495}{16}$.

Find generating functions for the following sequences.

a) $\binom{8}{0}, \binom{8}{1}, \binom{8}{2}, \ldots, \binom{8}{8}$

b) $\binom{8}{1}, 2\binom{8}{2}, 3\binom{8}{3}, \ldots, 8\binom{8}{8}$

c) $1, -1, 1, -1, 1, -1, \ldots$

d) $0, 0, 0, 6, -6, 6, -6, 6, \ldots$

e) $1, 0, 1, 0, 1, 0, 1, \ldots$

f) $0, 0, 1, a, a^2, a^3, \ldots, a \neq 0$

(a) $(1+x)^8$ **(b)** $8(1+x)^7$ **(c)** $(1+x)^{-1}$
(d) $6x^3/(1+x)$ **(e)** $(1-x^2)^{-1}$ **(f)** $x^2/(1-ax)$

Determine the sequence generated by each of the following generating functions.

a) $f(x) = (2x-3)^3$ **b)** $f(x) = x^4/(1-x)$

c) $f(x) = x^3/(1-x^2)$ **d)** $f(x) = 1/(1+3x)$

e) $f(x) = 1/(3-x)$

f) $f(x) = 1/(1-x) + 3x^7 - 11$


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Computer Science

Discrete Mathematics

Lecture No. 03

By- SATISH YADAV SIR

TOPICS TO BE COVERED

01 Basics operations in sets

02 set theory laws

03 Different operations thm

04 Infinite union

05 Union on intervals

Set:

$$\begin{cases} A \cup A = A \\ A \cap A = A \end{cases}$$

$$\begin{cases} A \cup U = U \\ A \cap U = A \end{cases}$$

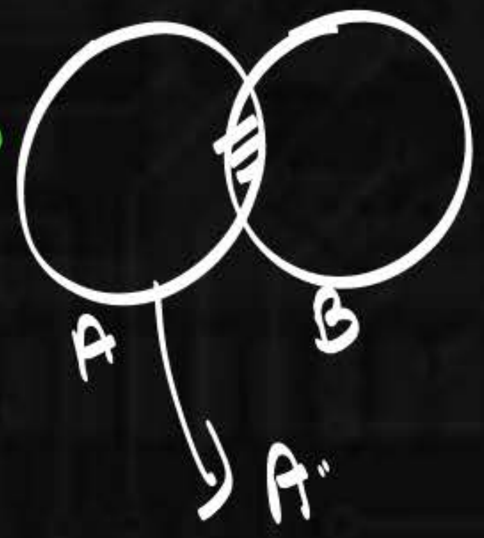
$$\begin{cases} A \cup B = B \cup A \\ A \cap B = B \cap A \end{cases}$$

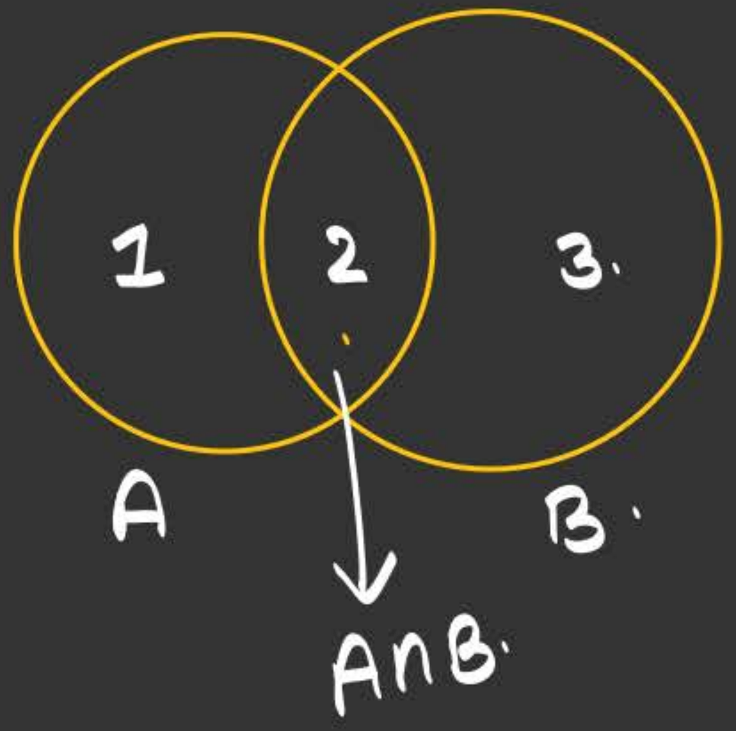
$$\begin{cases} A \cup (B \cup C) = (A \cup B) \cup C \\ A \cap (B \cap C) = (A \cap B) \cap C. \end{cases}$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$\begin{cases} A \cup (A \cap B) = A \\ A \cap (A \cup B) = A. \end{cases}$$





$A = 1, 2.$
$B = 2, 3.$

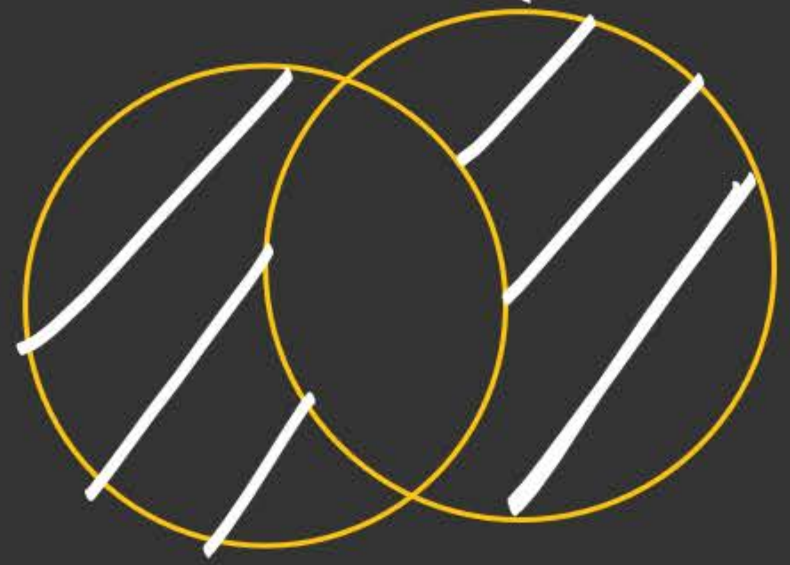
$\Delta / \oplus$

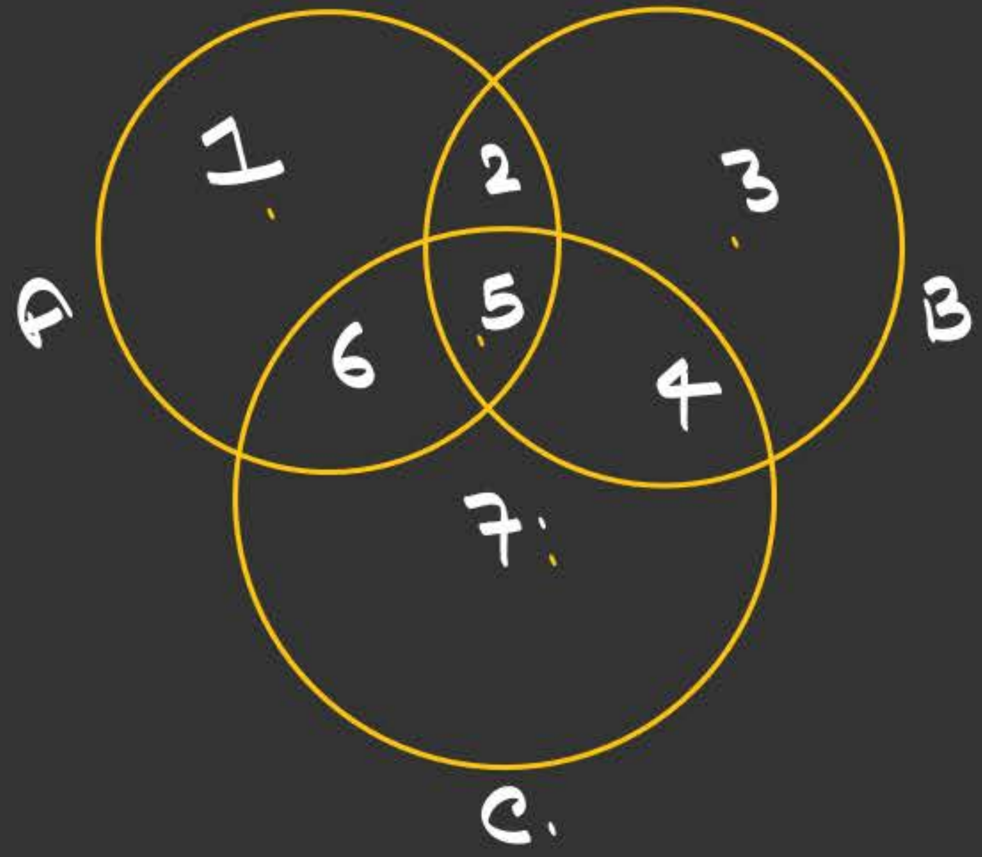
A or in B
but not in both.

1st part
element which
are present in
A but not in B.

$= A - (A \cap B)$
$= \{1, 2\} - \{2\}.$
$= \{1\}.$

$A \Delta B$
$= (A \cup B) - (A \cap B)$
$= (A - B) \cup (B - A)$





parts contains the elements:

$$A \triangle (B \triangle C) = (A \triangle B) \triangle C.$$

$$\{2, 3, \cancel{4}, \cancel{5}\} \triangle \{\cancel{4}, \cancel{5}, 6, 7.\}$$

$$\{1, \cancel{2}, 5, \cancel{6}.\} \triangle \{\cancel{2}, 3, \cancel{6}, 7.\}$$

1537

$$\{1, \cancel{2}, \cancel{5}, 6\} \triangle \{\cancel{2}, 3, \cancel{5}, 4.\}$$

$$\{1, \cancel{6}, 3, \cancel{4}.\} \triangle. \{\cancel{4}, 5, \cancel{6}, 7.\}$$

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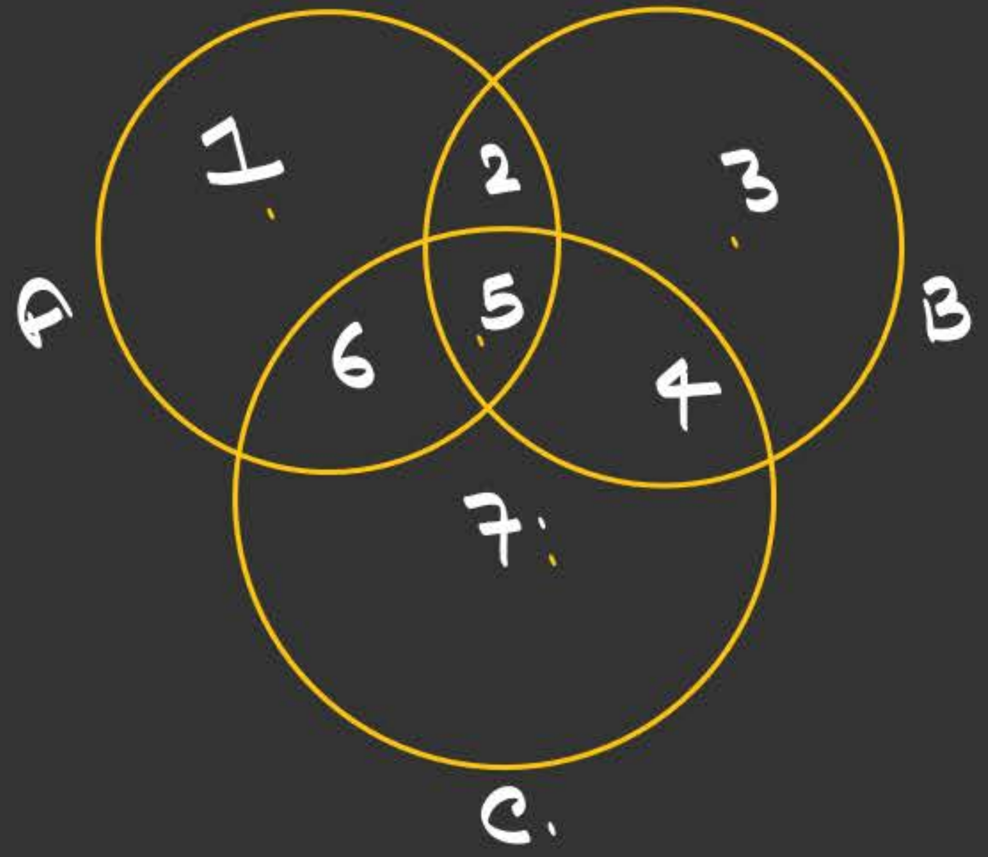
$$A-(B\cup C)=(A-B)\cup(A-C)$$



parts contains the elements:

$A-(B\cup C)$

$\begin{matrix} 1 \\ \cancel{2} \\ \cancel{5} \\ \cancel{6} \end{matrix} - \begin{matrix} \cancel{2} \\ 3 \\ 4 \\ \cancel{5} \\ \cancel{6} \\ 7 \end{matrix}$

$(A-B)\cup(A-C)$

$\begin{matrix} 1 \\ \cancel{2} \\ \cancel{5} \\ 6 \end{matrix} - \begin{matrix} \cancel{2} \\ 3 \\ \cancel{5} \\ 4 \end{matrix}$

$\left\{ \begin{matrix} 1 \\ 6 \end{matrix} \right\} \cup$

$A-(B\cup C) \neq (A-B)\cup(A-C) = \{1\}$

Set difference:

$A = \{1,2\} \quad B = \{2,4\}$

$A-B = \begin{matrix} 1 \\ \cancel{2} \end{matrix} - \begin{matrix} \cancel{2} \\ 4 \end{matrix}$

$= \{1\}$

$\boxed{A-B \neq B-A}$

$B-A = \begin{matrix} \cancel{2} \\ 4 \end{matrix} - \begin{matrix} 1 \\ \cancel{2} \end{matrix}$

$= \{4\}$

Relation :

$(1,2) \neq (2,1)$

$A = \{1, 2\}$ $B = \{1, 2\}$

order pair.

$A \times B = \{(1,1)\ (1,2)\ (2,1)\ (2,2)\}$

Relation: subset of crossproduct of sets.

$\boxed{a = b}$

Relation:

$R = \{(1,1)\ (2,2)\}$

$A = \{1, 2, 3\}$ $B = \{1, 2\}$.

$|A| = 3$ $|B| = 2$.

$|A \times B| = 3 \cdot 2 = 6$.

Total no. of relations = Total no. of subsets

$= 2^6$

$|A| = m$ $|B| = n$.

$|A \times B| = m \cdot n$.

Total no. of relations $= 2^{m \cdot n}$

$|A| = n$

$|A \times A| = n^2$.

Total no. of relations $= 2^{n^2}$

$A = \{1, 2, 3\}$

Reflexive: $\forall a \in A \ (a,a) \in R$.

$R_1 = \{ (1,1) \ (1,2) \}$ not reflexive.

$R_2 = \{ (1,1) (1,2) (2,2) (3,3) \}$ Reflexive ✓

$A = \{1, 2, 3\}$

$A \times A = \{(1,1) (1,2) (1,3)$
$(2,1) (2,2) (2,3)$
$(3,1) (3,2) (3,3)\}$

Total no. of relations $= 2^9$

Symmetric:

$(a,b) \to H$
$(b,a) \to W$

if $(a,b) \in R$ then $(b,a) \in R$.

$(a,b) \in R \to (b,a) \in R$.

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Symmetric ✓

$R_1 = \{ \quad \}$

$\underbrace{(a,b) \in R}_{F} \to (b,a) \in R$.

True.

not symmetric.

$R_2 = \{(1,2)\}$

$(a,b) \in R \to (b,a) \in R$.

$\underbrace{(1,2) \in R}_{T} \to \underbrace{(2,1) \in R}_{F}$.

$a=1$
$b=2$

False.

$R_3 = \{(2,3), (3,2)\}$.

✓

Antisymmetric :

$(a,b)\in R \wedge (b,a)\in R \longrightarrow a=b$

Flip → same.

Anti ✓

$R_1 = \{(1,1)\}$

$(a,b)\in R \wedge (b,a)\in R \rightarrow a=b$

$\underbrace{(1,1)\in R}_{T} \wedge \underbrace{(1,1)\in R}_{T} \rightarrow 1=1$

$a=1$ T $\wedge$ T $\longrightarrow$ T.

$b=1$

$R_2 = \{(1,2)\}$ ✓

$(a,b)\in R \wedge (b,a)\in R \rightarrow a=b$

$\underbrace{(1,2)\in R}_{T} \wedge \underbrace{(2,1)\in R}_{F} \rightarrow 1=2$

$a=1$ T $\wedge$ F $\rightarrow$

$b=2$

False $\longrightarrow$

True.

Transitive Relation :

$$(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R.$$

$R_1 = \{ \}$ ✓

$$\underbrace{\underbrace{\underbrace{(a,b) \in R}_{F} \wedge (b,c) \in R}_{F} \rightarrow (a,c) \in R}_{\text{True}}$$

$R_2 = \{(1 2)(2 4)(1 4)\}$.

$R_3 = \{(1 2), (2 1), (1 1)\}$. not transitive.

$$(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R.$$

$$\underbrace{(2,1) \in R}_{T} \wedge \underbrace{(1,2) \in R}_{T} \rightarrow \underbrace{(2,2) \in R}_{F}.$$

Equivalence Relation :

- Reflexive.
- Symmetric.
- Transitive.

$R = \{ (1\,1)\ (2\,2)\ (3\,3)\ \boxed{(1\,2)\ (2\,1)} \}$

R : $(a, a) \in R$.

Sym : does not have an issue.
with same elements.

→ Demands flipping.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Computer Science

Discrete Mathematics

Lecture No. 4

By- SATISH YADAV SIR

TOPICS TO BE COVERED

01 ONTO Function

02 BIJECTIVE FUNCTION

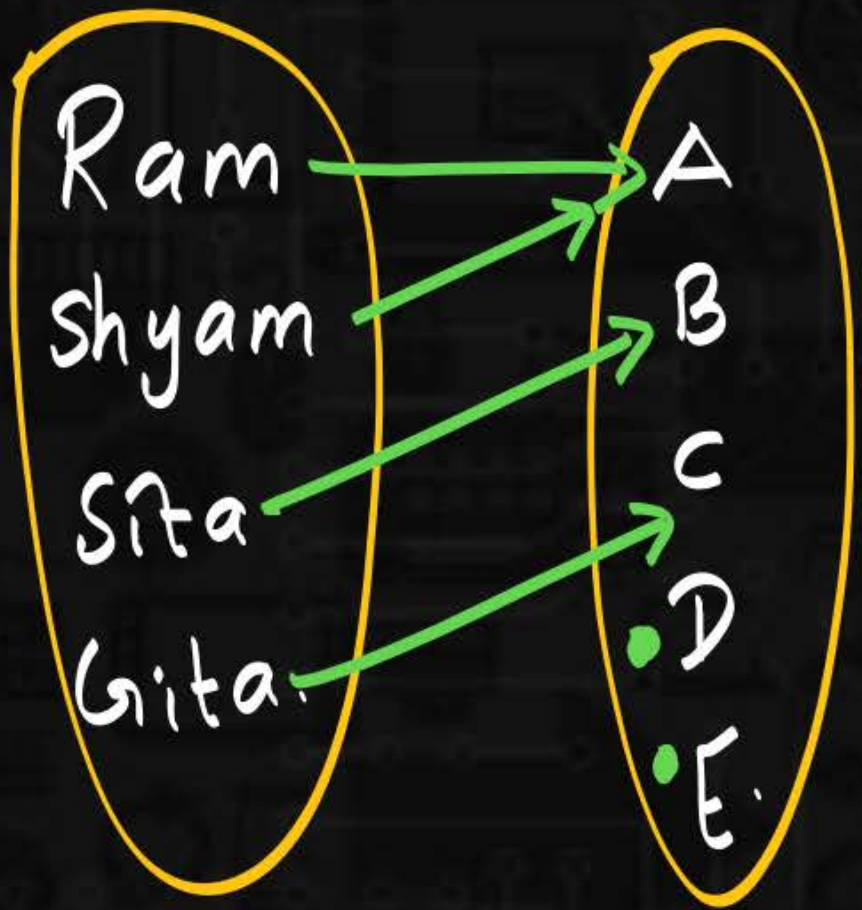
03 INVERTIBLE FUNCTION

04 TOTAL NUMBER OF BIJECTIVE FUNCTIONS

05 IDENTITY FUNCTION

Functions

function/mapping/transformation/assignment:



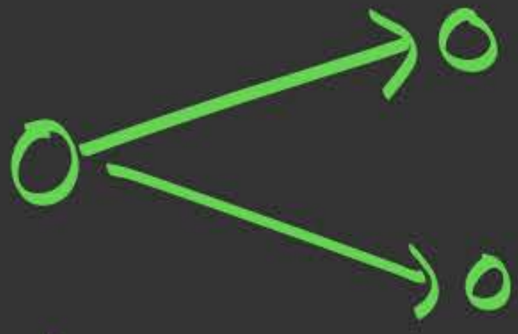
$f: \text{Name} \rightarrow \text{Grade}.$

$f: \text{set}_1 \rightarrow \text{set}_2.$

valid.

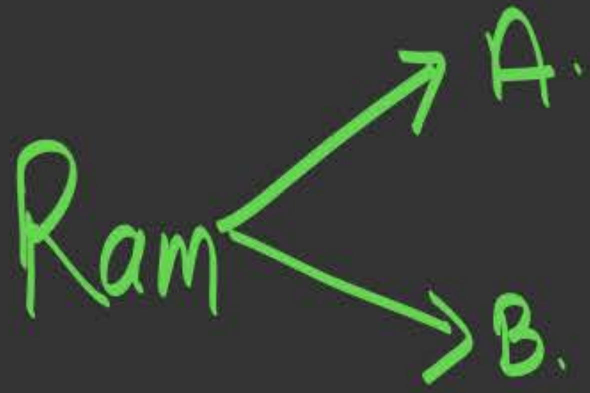
valid.

valid.



Invalid.

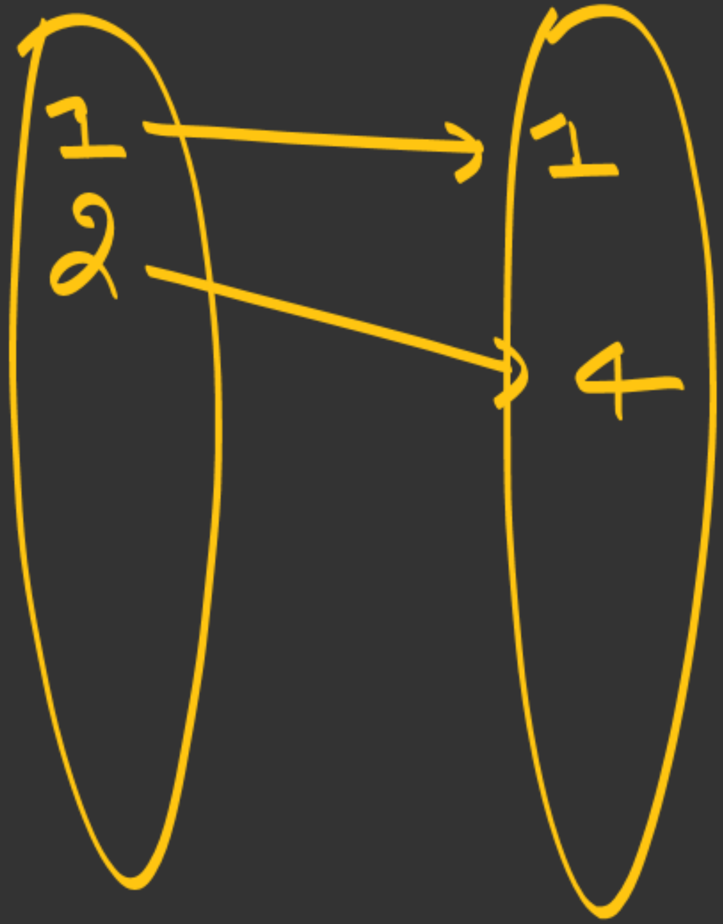
Invalid.



$f(x) = x^2$

$f(x) = \log x$

$f(x) = x^2 + 3x$



$f(x) = x^2$

$f : \mathbb{Z} \to \mathbb{Z}$ ✓

input:

set: $\mathbb{Z}$.

$f(x) = x^2$

$$\begin{cases} f(x) = x^2 \\ f: \mathbb{Z} \to \mathbb{Z}. \end{cases}$$

O/P: output: $\mathbb{Z}$.

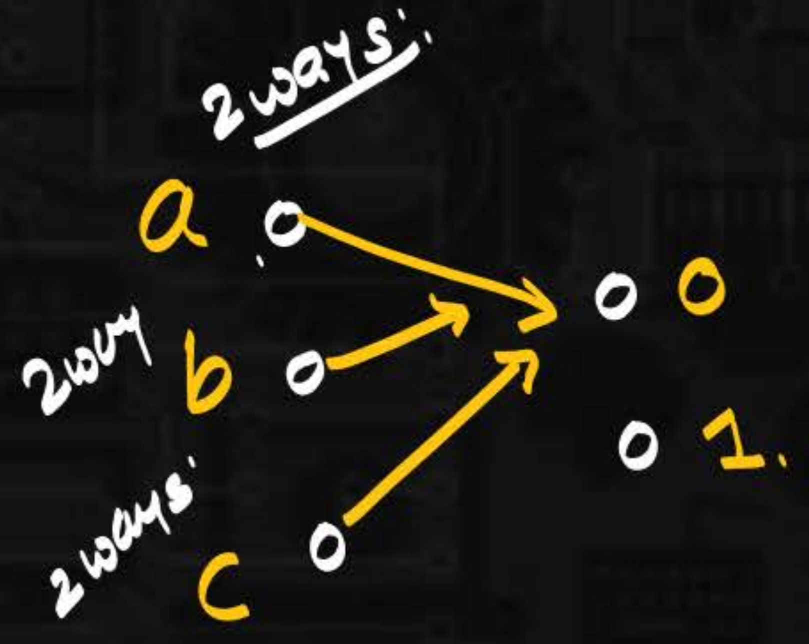
Functions

$f: A \to B$

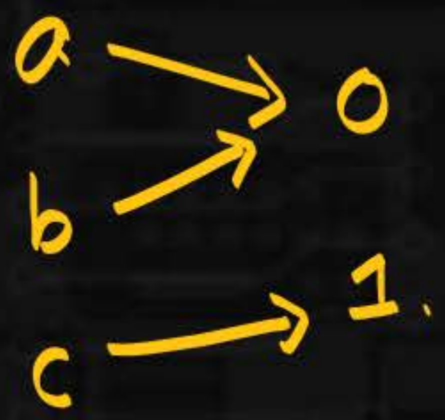
$|A| = 3 \quad |B| = 2.$

Total functions = Tota diff arrows representn.

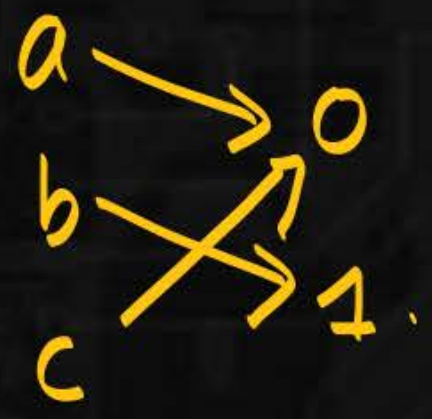
$$= 2^3 = (R.S)^{I.S.}$$



$$\frac{a\ b\ c}{0\ 0\ 0}$$

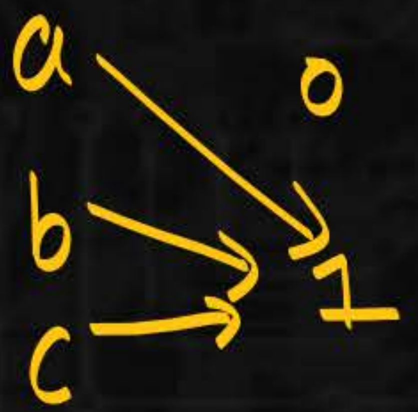


$$\frac{a\ b\ c}{0\ 0\ 1}$$



$$\frac{a\ b\ c}{0\ 1\ 0}$$

. . . . . .



$$\frac{a\ b\ c}{1\ 1\ 1}$$

Functions

$p \to q \equiv \neg q \to \neg p$.

Function

- one-to-one (1:1)
- onto
- 1:1 correspondance.

1:1 Function

$(f(a) = f(b) \longrightarrow a = b)$

OR.

$a \neq b \to f(a) \neq f(b)$

$f(x) = x+1$. $f: \mathbb{Z} \to \mathbb{Z}$.

1:1 ? ✓

$$f(a) = f(b) \rightarrow a = b.$$

$$a+1 = b+1$$

$$\boxed{a = b}$$

$f: \dfrac{4x}{2x-1}$ $\left(x \neq \frac{1}{2}\right)$

1:1 ✓

$$f(a) = f(b) \longrightarrow a = b. ✓$$

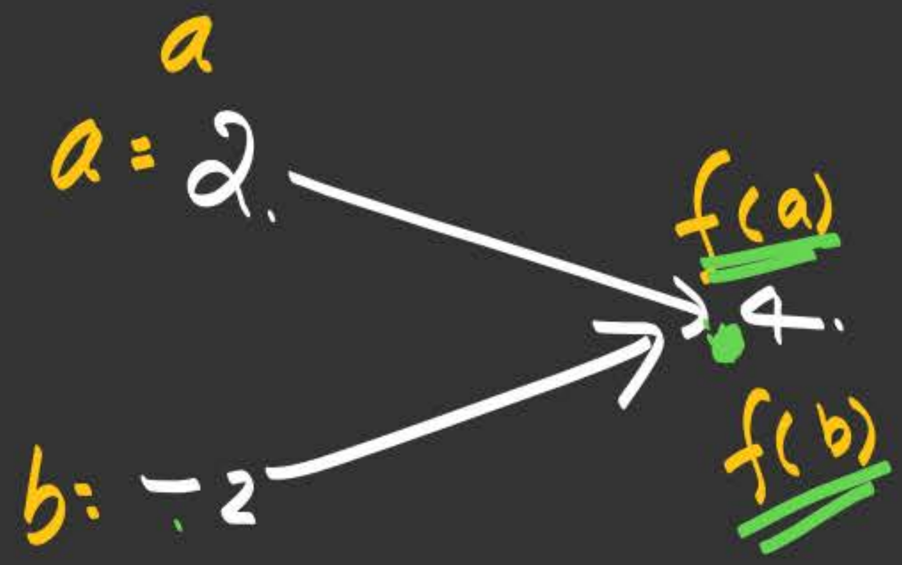
$$\frac{\cancel{4}a}{2a-1} = \frac{\cancel{4}b}{2b-1}$$

$$a(2b-1) = b(2a-1)$$

$$\cancel{2}\cancel{a}b - a = \cancel{2}ab - b.$$

$$a = b.$$

$f(x) = x^2$. $\quad f: Z \to Z$.



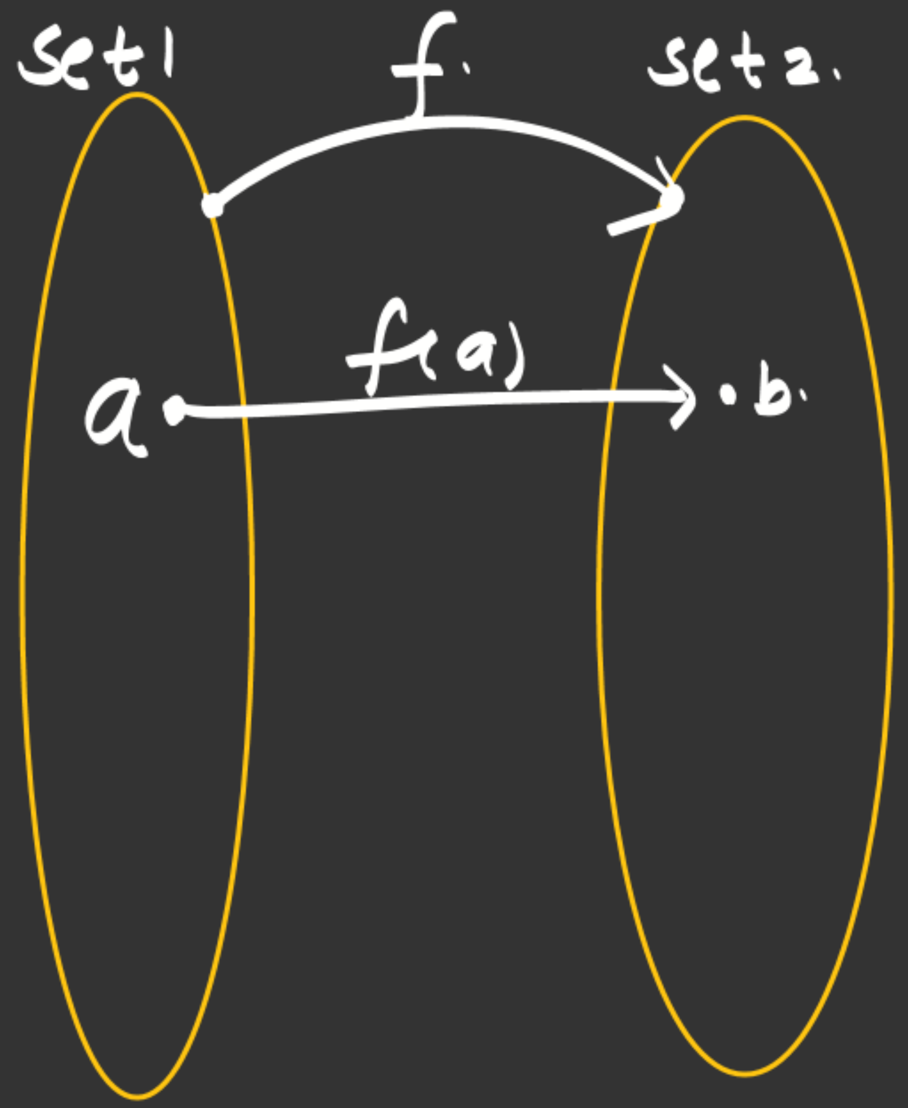
True $\qquad\qquad$ True

$f(a) = f(b) \longrightarrow a = b$.

$\underbrace{f(2) = f(-2)}_{T} \longrightarrow \underbrace{2 = -2}_{F}$.

not 1:1 function.

Arrow represent^n is not there in 1:1.



$$f: \text{set1} \to \text{set2}$$

$$f(a) = b$$

Functions

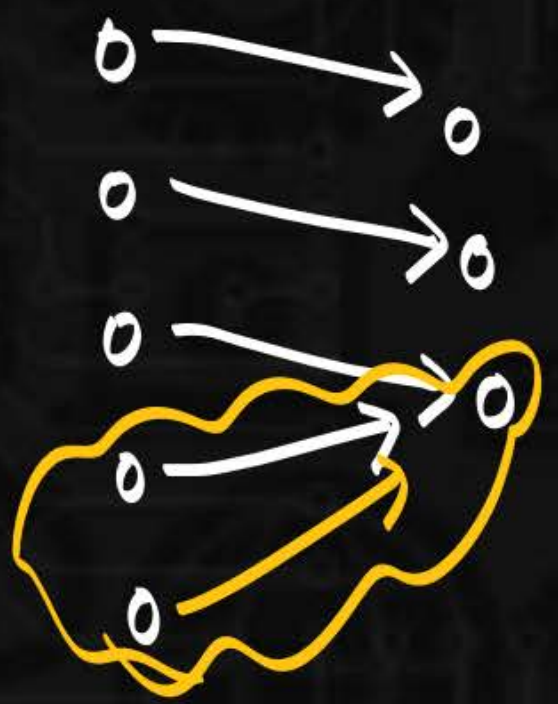
$f: A \to B$

$|A| = 5 \qquad |B| = 3$

L.S > R.S

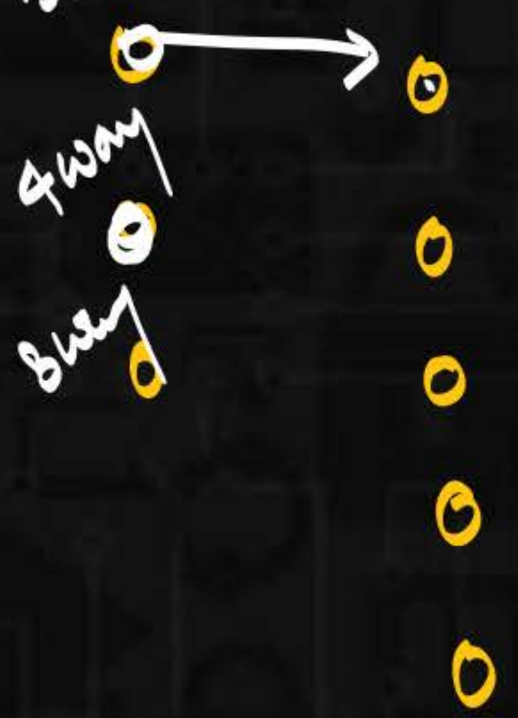
not 1:1

Function



$f: A \to B$

$|A| = 3 \qquad |B| = 5$



${}^{R.S}P_{L.S}$

Total Functions

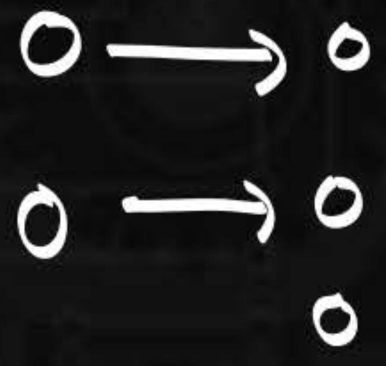
$= 5 \cdot 4 \cdot 3 \left(\frac{2!}{2!} \right)$

$= \frac{5!}{2!}$

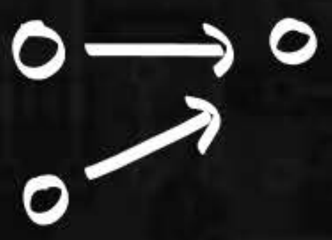
$= \frac{5!}{(5-3)!} = {}^{5}P_{3} = {}^{R.S}P_{L.S}$

Functions

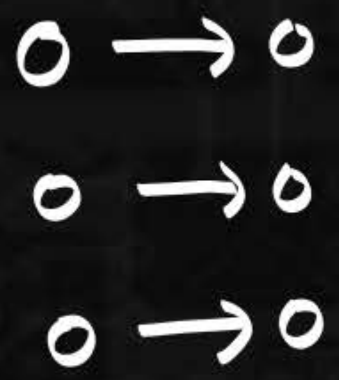
onto (Right side must be full)



not onto



onto



onto ✓

Functions

$f(x) = x+1$

$f: Z \to Z$

$1 \longrightarrow 2$

$2 \longrightarrow 3$

$3 \longrightarrow 4$

$4 \longrightarrow 5$

$5 \longrightarrow 6$

$\vdots \qquad \vdots$

$f(x) = x^2$

$0 \longrightarrow 0$

$1 \longrightarrow 1$

$\cdot\, 2$

$\cdot\, 3$

$2 \longrightarrow 4$

not onto

$0 \longrightarrow 0$

$0 \longrightarrow 0$

0

1:1 ✓

onto ✗

$f : A \to B \quad (m \geq n)$

$|A| = m \quad |B| = n$

$$\sum_{i=0}^{n} (-1)^i \, {}^nC_i \, (n-i)^m$$

7 different guest, we have to adjust in 4 diff rooms, such that none of the rooms should be empty?

$A = 7 \quad B = 4.$

$m = 7 \quad n = 4.$

0 0 0 0 0 0 0

0 0 0 0

Functions

$m = 7 \quad n = 4$

alternate +/-

$$\sum_{i=0}^{n} (-1)^i \, {}^nC_i \, (n-i)^m$$

$n = 4 \quad m = 7.$

$${}^4C_0 (4-0)^7 - {}^4C_1 (4-1)^7 + {}^4C_2 (4-2)^7 - {}^4C_3 (4-3)^7$$

$$+ {}^4C_4 (4-4)^7$$

Functions

1:1 correspondance : 1:1 + onto.

bijective : injective $\wedge$ surjective.

: 1:1 $\wedge$ onto.

$L.S \leq R.S \wedge L.S \geq R.S \longrightarrow L.S = R.S$

$\because$

Functions

$f(x) = x + 1$. $f : \mathbb{Z} \to \mathbb{Z}$.

$1 \longrightarrow 2$

$2 \longrightarrow 3$

$0 \longrightarrow 0$

$0 \longrightarrow 0$

$0 \longrightarrow 0$

$\vdots \qquad \vdots$

$\left\{ \begin{array}{l} 1:1 \checkmark \\ \text{onto} \checkmark \end{array} \right.$

1 : 1 correspondance.

$f : A \to B \qquad |A| = |B| = n$.

Total 1 : 1 correspondance

$= n!$

Functions

Combination with Reptn :

Combination : $\rightarrow$ order is not important.

$\{ \dots\dots \}$

$\hookrightarrow \{ \dots\dots \}$

How many ways to select 6 students in a class of 10 ?

$\{ S_1, S_2 \dots\dots S_{10} \}$

$\hookrightarrow {}^{10}C_6$

In a set of 3 elements
how many ways to select 1?

$\{a, b, c\}$

no. of ways to select 1 element
is same as shifting 1 line in a place of 3.

$\{a\} \rightarrow \{\ |\ -\ -\ \}$

$\{b\} \rightarrow \{\ -\ |\ -\ \}$

$\{c\} \rightarrow \{\ -\ -\ |\ \}$

$^3C_1 = \text{Places } C \text{ line.}$

Functions

How many ways to select 4 fruits from a mall having containers of Apple, papaya, orange. ?

A	O	P

15

4A
4O
4P

3A 1O
3A 1P
3O 1A
3O 1P
3P 1A
3P 1O

2A 2P
2A 2O
2O 2P

2A 1O 1P
2O 1A 1P
2P 1A 1O

Functions

$\rightarrow$ no. of ways to shift 2 lines will give one of possible outcome.

$$2A\,1O\,1P \rightarrow \left\{ \begin{array}{l} \overset{1\ 2\ 3\ 4\ 5\ 6}{A\,A \mid O \mid P} \end{array} \right\}$$

$$\{A \mid OO \mid P\} \Rightarrow 2O\,1A\,1P$$

Places C_{line}

$$\{\mid OOO \mid P\} \Rightarrow 3O\,1P$$

$$6C_2 = \frac{6 \cdot 5}{2 \cdot 1} = 15$$

Functions

How many nonnegative sol^n are possible ?

$x_1 + x_2 + x_3 = 10 \quad x_i \geq 0$. $\quad x_1 = 1 \quad x_2 = 3 \quad x_3 = 6$.

Places C line

$^{12}C_2$.

x_1 | x_2 | x_3

0 | 0 0 0 | 0 0 0 0 0 0

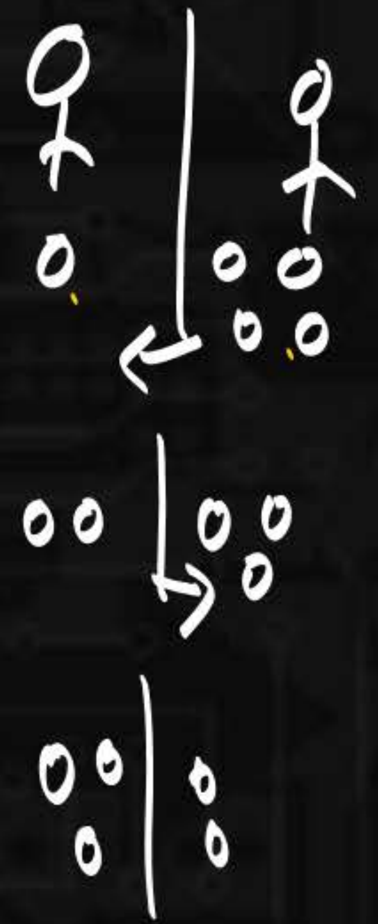
0 | 0 0 | 0 0 0 0 0 0 0

Functions

how many ways to distribute 5 coins among 2 students ?

0	5
5	0
1	4
4	1
2	3
3	2.

Ans: 6



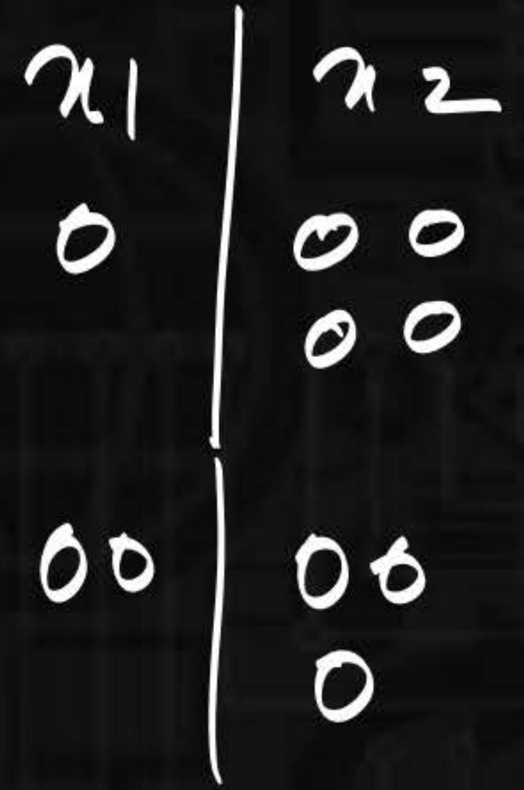
places C_{line}

$^{5+1}C_1$.

$^{6}C_1 = 6$

how many nonnegative soltn are ?

$x_1 + x_2 = 5 \cdot \; x_i \geq 0$

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Computer Science

Discrete Mathematics

Lecture No. 05

By- SATISH YADAV SIR

TOPICS

01 Connectives

02 Type 1

03 Type 2

$p \wedge p \equiv p$
$p \vee p \equiv p$

$p \wedge T \equiv p$
$p \vee F \equiv p$

$p \vee T \equiv T$
$p \wedge F \equiv F$

$p \vee q = q \vee p$
$p \wedge q = q \wedge p$

$p \vee (q \vee R) \equiv (p \vee q) \vee R$
$p \wedge (q \wedge R) \equiv (p \wedge q) \wedge R$

$p \vee (q \wedge R) \equiv (p \vee q) \wedge (p \vee R)$
$p \wedge (q \vee R) \equiv (p \wedge q) \vee (p \wedge R)$

absorption law:

$$\begin{cases} a \wedge (a \vee b) \equiv a \\ a \vee (a \wedge b) \equiv a \end{cases}$$

$\wedge$	$\cap$	$\cdot$
$\vee$	$\cup$	$+$
T	U	1
F	ϕ	0

Demorgan's law:

$\neg(a \vee b) \equiv \neg a \wedge \neg b$
$\neg(a \wedge b) \equiv \neg a \vee \neg b$

$$a \vee (b \wedge c) \equiv \boxed{(a \vee b) \wedge (a \vee c)}$$

$$(a \vee b) \wedge (a \vee c)$$

$$a \vee (b \wedge c)$$

$p \to q \equiv \neg p \vee q$

$\neg(p \to q) \equiv \neg(\neg p \vee q)$

$\equiv p \wedge \neg q$

$\neg(p \to q) \equiv p \wedge \neg q$

$p \to q \equiv \neg q \to \neg p$

$(p \to q) \wedge (p \to R) \equiv p \to (Q \wedge R)$

$(p \to R) \wedge (Q \to R) \equiv (p \vee Q) \to R$

$(p \to Q) \vee (p \to R) \equiv p \to (Q \vee R)$

$(p \to R) \vee (Q \to R) \equiv (p \wedge Q) \to R$

1st is same operator does not changes

when 2nd is same operator changes

when First P is same.

$(P \to Q) \wedge (P \to R) \equiv P \to (Q \wedge R)$

$(\boxed{\neg P \vee} Q) \wedge (\boxed{\neg P \vee} R)$

$\neg P \vee (Q \wedge R)$

$\downarrow$

$P \to (Q \wedge R)$

when last is same.

$(P \to R) \wedge (Q \to R) \equiv (P \vee Q) \to R.$

$(\neg P \boxed{\vee R}) \wedge (\neg Q \boxed{\vee R})$

$(\neg P \wedge \neg Q) \vee R$

$\neg(P \vee Q) \vee R.$

$(P \vee Q) \to R.$

$$(p \leftrightarrow q) \equiv (p \to q) \wedge (q \to p) \equiv (\neg q \to \neg p) \wedge (q \to p)$$

$$\equiv (\neg p \vee q) \wedge (\neg q \vee p)$$

$$\equiv (p \to q) \wedge (\neg p \to \neg q)$$

$$\equiv (\neg q \to \neg p) \wedge (\neg p \to \neg q)$$

$$\boxed{\begin{aligned} p \to q &\equiv \neg q \to \neg p \\ p \to q &\equiv \neg p \vee q \end{aligned}}$$

open stmt:

$P(x)$: x is even no.

$P(0)$: 0 is even no. (T)

$P(1)$: 1 is even no (F)

open stmt
Domain
predicate
variable

Domain: set of possible choices for open stmt.

Universe.

Input1

$x=0$

$x=1$

open stmt:

x is even no.

we can define truth value after putting the input.

→ simple propositional

→ S_2

eg: $x^2 \leq 4$

$x^2+y^2 \leq 9$

$x^2+y^2+z^2 \leq 9$

O.S: x is even no.

Domain:

1 F 2 T 3 F

All →
some →

1 . . . . . $\mathbb{Z}$

1 10

quantity

Truth value

↓

Tool

↓

Quantifier

All → universal quantifier

Some → Existential quantifier

Universal quantifier ($\forall x$)

↓

Tool → Truth value.

↓

ALL

$\forall x . p(x)$

for all value of x, x will satisfy $p(x)$

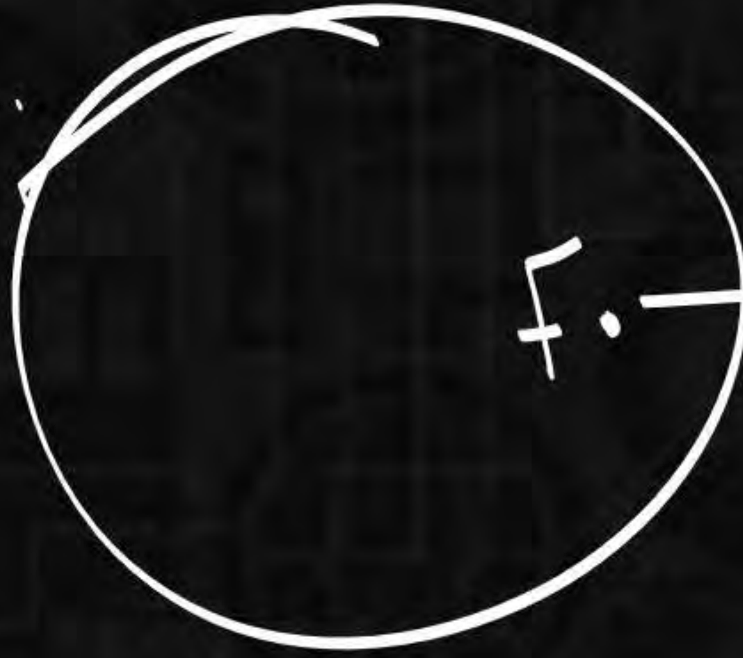
for all of x, such that $p(x)$

for every value of x such that $p(x)$

$\forall x p(x) \rightarrow$ True.

→ when all elements are in True in Domain.

$\forall x p(x) \rightarrow$ False.



→ atleast 1 element is false

$D: \{1, 2, 3\}$

$p(x): x^2 \le 9$

$\forall x\, p(x):$ → True

$x=1 \quad 1^2 \le 9\ (T)$

$x=2 \quad 2^2 \le 9\ (T)$

$x=3 \quad 3^2 \le 9\ (T)$

$D: Z$

$\forall x\, (x^2 \ge 0)$ → True

Existential quantifier: $(\exists x)$

↓

some / atleast 1 element

$\exists x \, p(x)$

There exist x such that $p(x)$
Some value of x such that $p(x)$
atleast 1 value of x such that $p(x)$.

$\exists x \, p(x)$

True.

→ atleast 1 element is True

$\exists x \, p(x) \rightarrow$ false.

→ when all elements are false

D: $\{1, 2, \ldots \infty\}$

O.S: $x^2 = 4$.

$\exists x\, (x^2 = 4)$

$\hookrightarrow$ True

$\exists x\, (x^2 \geq 1)$

$\hookrightarrow$ True

* $\exists x\,[P(x) \wedge Q(x)] \rightarrow \exists x P(x) \wedge \exists x Q(x)$

$\exists x\,[P(x) \vee Q(x)] \equiv \exists x P(x) \vee \exists x Q(x)$

$\forall x\,[P(x) \wedge Q(x)] \equiv \forall x P(x) \wedge \forall x Q(x)$

$\forall x\,[P(x) \vee Q(x)] \leftarrow \forall x P(x) \vee \forall x Q(x)$

$\forall x\,[P(x) \rightarrow Q(x)] \rightarrow \forall x P(x) \rightarrow \forall x Q(x)$

$\forall x\,[P(x) \Leftrightarrow Q(x)] \rightarrow \forall x P(x) \Leftrightarrow \forall x Q(x)$

$P(x): x+1=4$ $\quad Q(x): 2x+1=7$

$D: \{1, 2, 3\}$

$x = 3$ $\quad x = 3$

$$\exists x\,[P(x) \land Q(x)] \longrightarrow \exists x\, P(x) \land \exists x\, Q(x)$$

$$T \;\left\{\;\begin{array}{l} \boxed{\underset{F}{P(1)} \land \underset{F}{Q(1)}} \; F \\ \lor \\ \boxed{\overset{F}{P(2)} \land \overset{F}{Q(2)}} \; F \\ \lor \\ \boxed{\underset{T}{P(3)} \land \underset{T}{Q(3)}} \; T \end{array}\right.$$

$$\overset{T}{\boxed{\begin{array}{c} P(1) \\ \lor \\ P(2) \\ \lor \\ P(3) \; T \end{array}}} \;\land\; \overset{T}{\boxed{\begin{array}{c} Q(1) \\ \lor \\ Q(2) \\ \lor \\ Q(3) \; T \end{array}}}$$


THANK YOU GW
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Mission ISRO 2023

Computer Science

Discrete Mathematics

Lecture No. 06

By- SATISH YADAV SIR

TOPICS TO BE COVERED

01 Definition of Graph

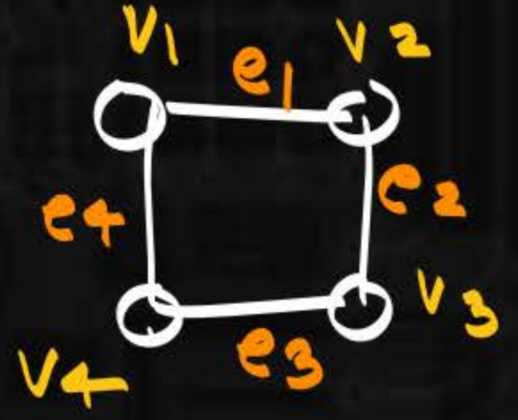
02 Handshaking Lemma

03 Types of Graphs

04 No of Graphs

05 Simple Graphs theorem

Basics of Graph



point / joint → vertex / vertices (V)

line / branch → edge / edges (E)

Graph = (set of points, set of lines)

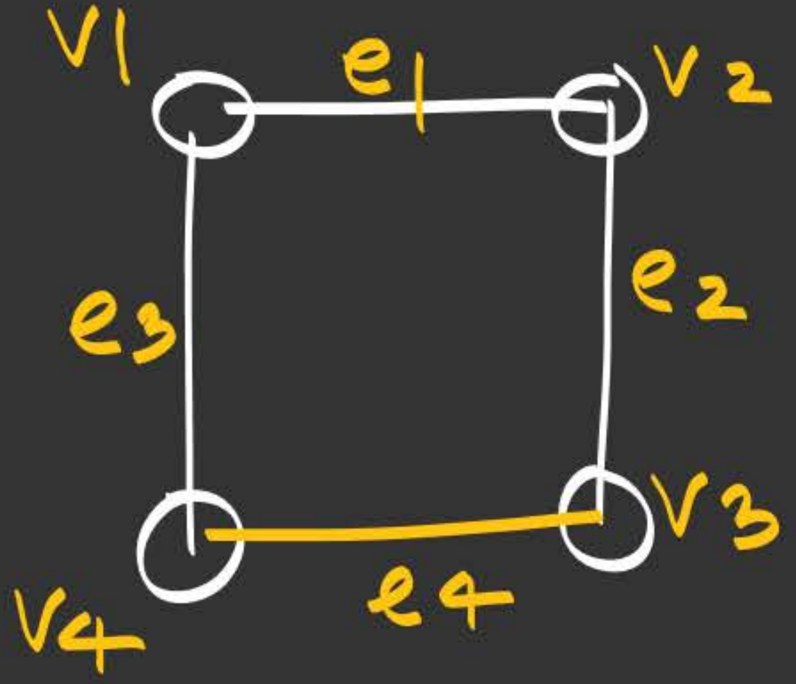
$G = (V, E)$

$V = \{v_1, v_2, v_3, v_4\}$

$E = \{e_1, e_2, e_3, e_4\}$

$G = (V, E)$

each edge must be associated with unordered pair of vertices.

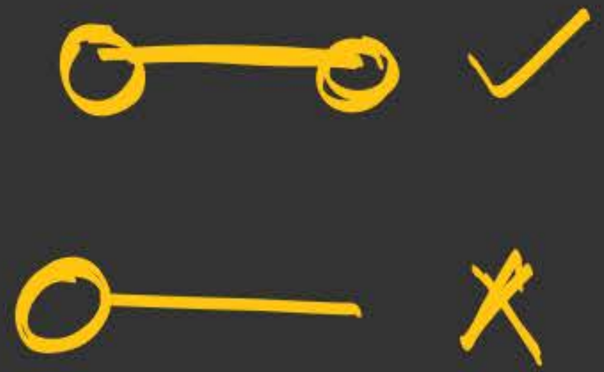


$G=(V,E)$

$V=\{v_1 \dots v_4\}$

$E=\{e_1 \dots e_4\}$

$e_1 \rightarrow (v_1, v_2) \mid (v_2, v_1)$



Basics of Graph

incident point : meeting point

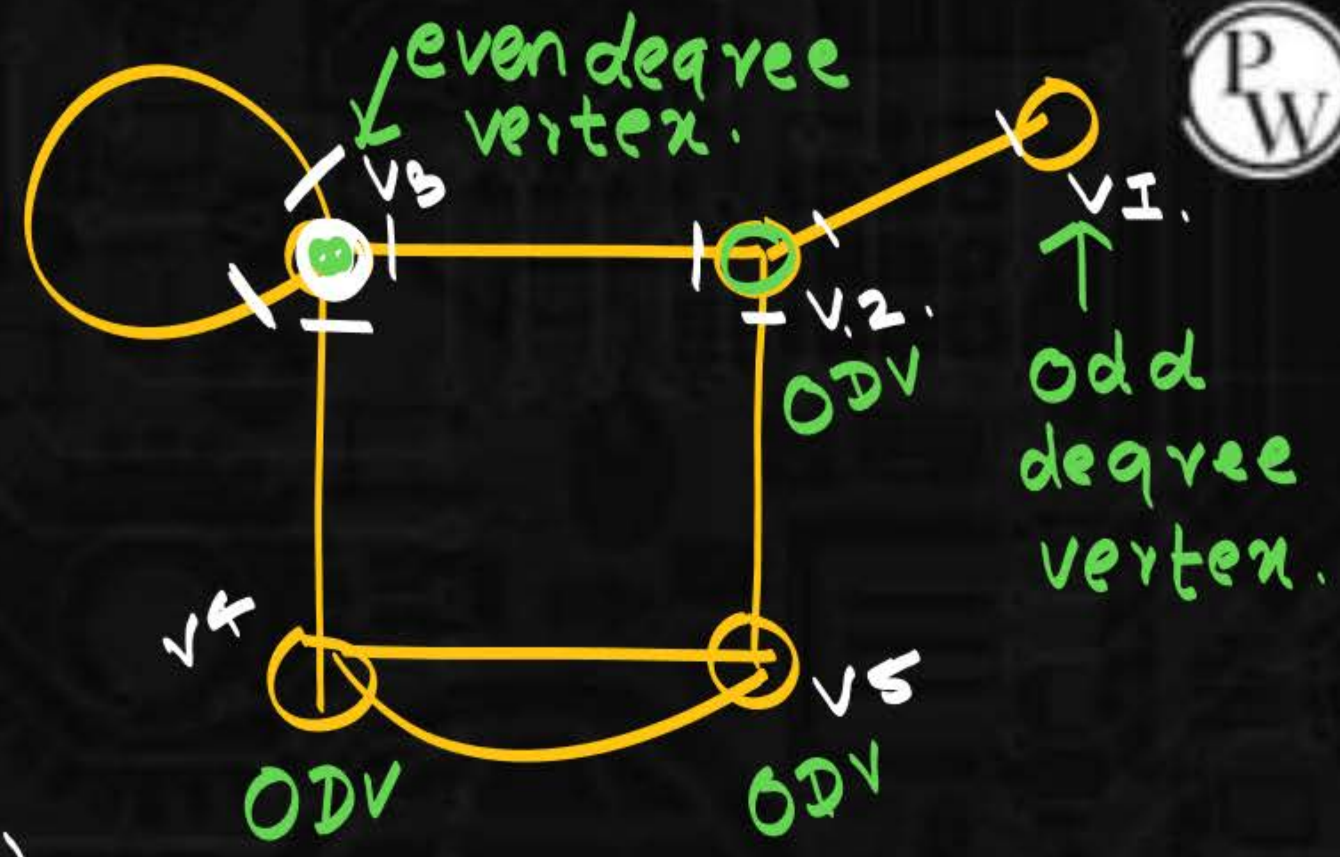
Degree/valency ($d(v_i)$)

no. of incident point.

Basics of Graph

$d(v_1) = 1 \quad d(v_2) = 3 \quad d(v_3) = 4$

$d(v_4) = 3 \quad d(v_5) = 3$



$d(v_1) + d(v_2) + d(v_3) + d(v_4) + d(v_5)$

$1 + 3 + 4 + 3 + 3 = 14 = 2 \cdot 7$

no. of edges.

Thm1: Sum of degrees of all vertices is equals to twice the no. of edges.

Basics of Graph

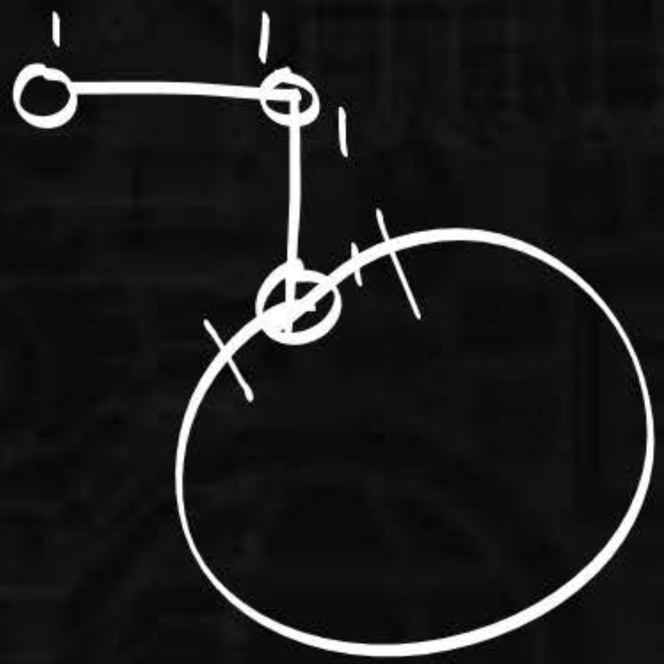
L.H.S   R.H.S

Degrees   edges.

$$2 = 2(1)$$

$$2+2 = 2(1+1)$$

$$2+2+2 = 2(1+1+1)$$



Thm1: $\sum d(v_i) = 2e.$

Basics of Graph

$\sum d(v_i) = 2e$

$\sum d(v_i) = \text{even}$

$d_1 \; d_2 \; d_3 \ldots d_n = \text{even}$

$\boxed{o \; o \; o \; o} + \boxed{e \; e \; e} = \text{even}$

$\text{even} + \text{even} = \text{even}$

$\dfrac{\text{even} + \text{even}}{\text{even}} = \text{even}$

$\boxed{e + e + e + e}$

$\downarrow$

even

$\boxed{o + o} = \text{even} \quad 1 + 3 = 4$

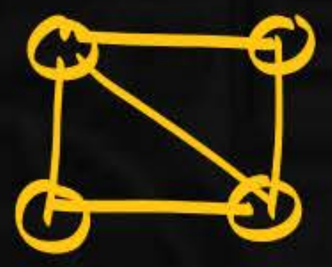
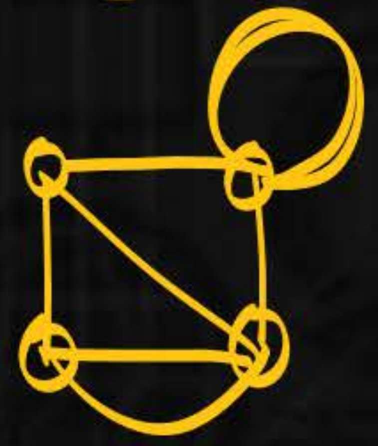
$\boxed{o + o + o} = \text{odd} \quad 1 + 3 + 5 = 9$

Basics of Graph

<u>Thm 2</u>: no. of odd degree vertices in a graph will always be even.

Basics of Graph

Types of Graphs :

	loop	$\parallel$ edges	
Simple Graph	✗	✗	
multigraph	✗	✓	 ← $\parallel$ edges
Pseudograph	✓	✓	

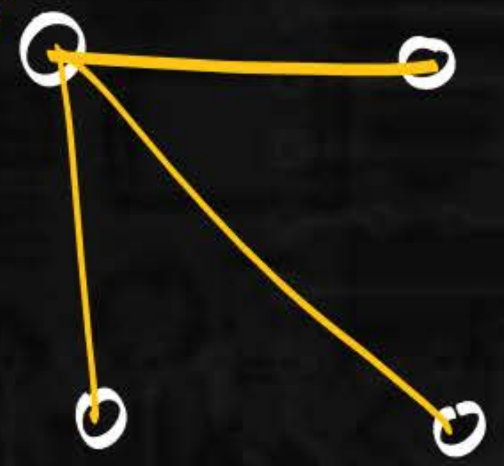
Basics of Graph

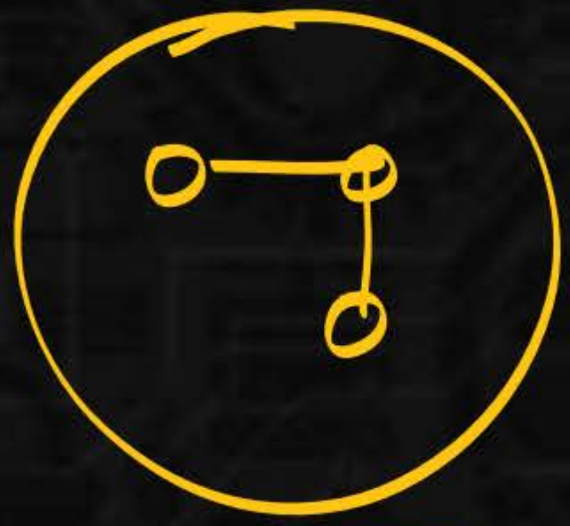
Thm3: In simple maximum degree $\leq n-1$.

Total vertices $= n$.

$n = 4$.

$x(n-1)$





$n = 4$.

max. degree $= 2$

$n = 4$

max. degree $= 3$.

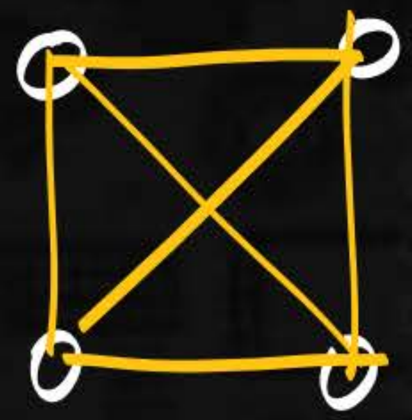
Basics of Graph

Thm 4: max no. of edges in simple Graph $\leq \frac{n(n-1)}{2}$.

Total vertices $= n$.

Degree of each vertex is $(n-1)$

$n = 4$



Degree of each vertex is $(n-1)$.

$\sum d(v_i) = 2e$

$n(n-1) = 2e$

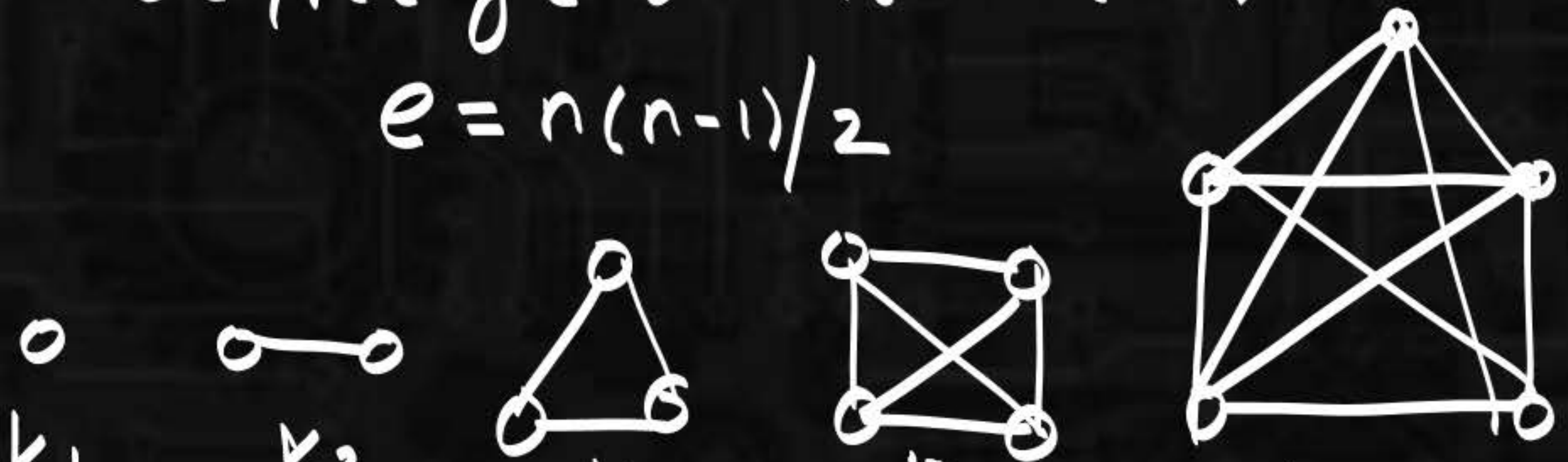
$\frac{n(n-1)}{2} = e$

Basics of Graph

Complete Graph (K_n)

Degree of each vertex is $(n-1)$.

$$e = n(n-1)/2$$



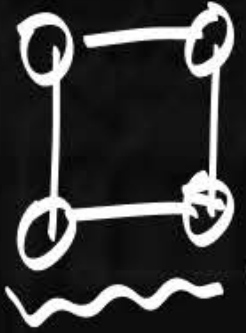
K_1 K_2 K_3 K_4 K_5

$K_n \rightarrow$ Regular (viceversa is not true)

Regular Graph :

Degrees of all vertices are same.

Regular

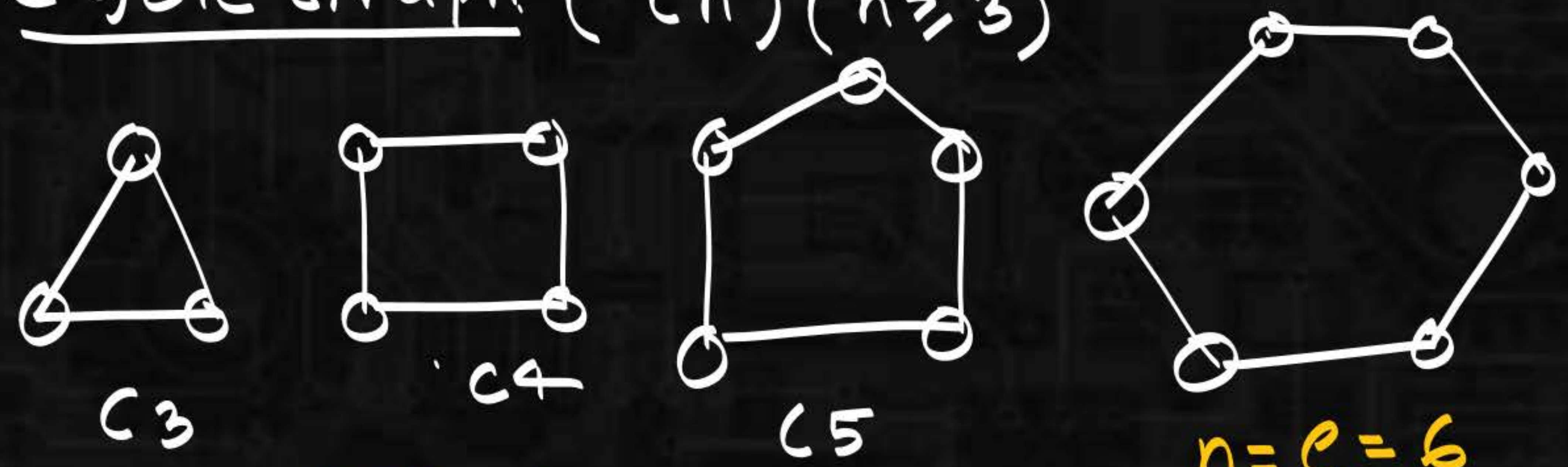


Regular

null graph

Basics of Graph

Cycle Graph (C_n) ($n \geq 3$)



$n = e = 4$ $n = e = 5$ $n = e = 6$

In C_n Graph all vertices will have degree 2.

$\sum d(v_i) = 2e$

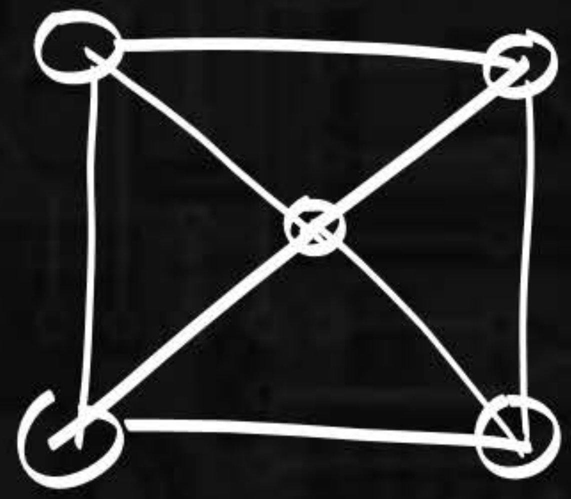
$n \times 2 = 2e$

$n = e.$

Basics of Graph

Wheel Graph (W_n) ($n \geq 4$)

W_4

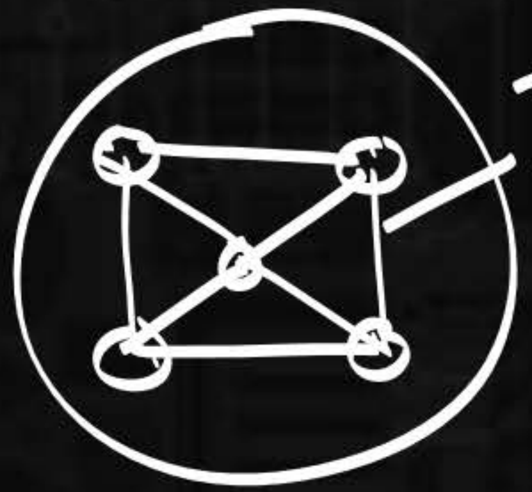


W_5

$$e(W_n) = n-1+n-1$$

$$= 2(n-1)$$

W_5



Total vertices = 5

$C_4 \rightarrow$ 4 edges $\quad n-1$

$\rightarrow$ 4 edges $\quad n-1$

Basics of Graph

Bipartite Graph :

2 partitions

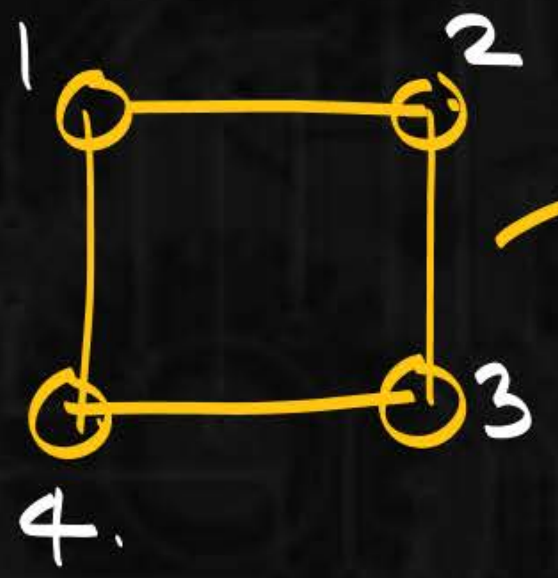
$G = (V, E)$

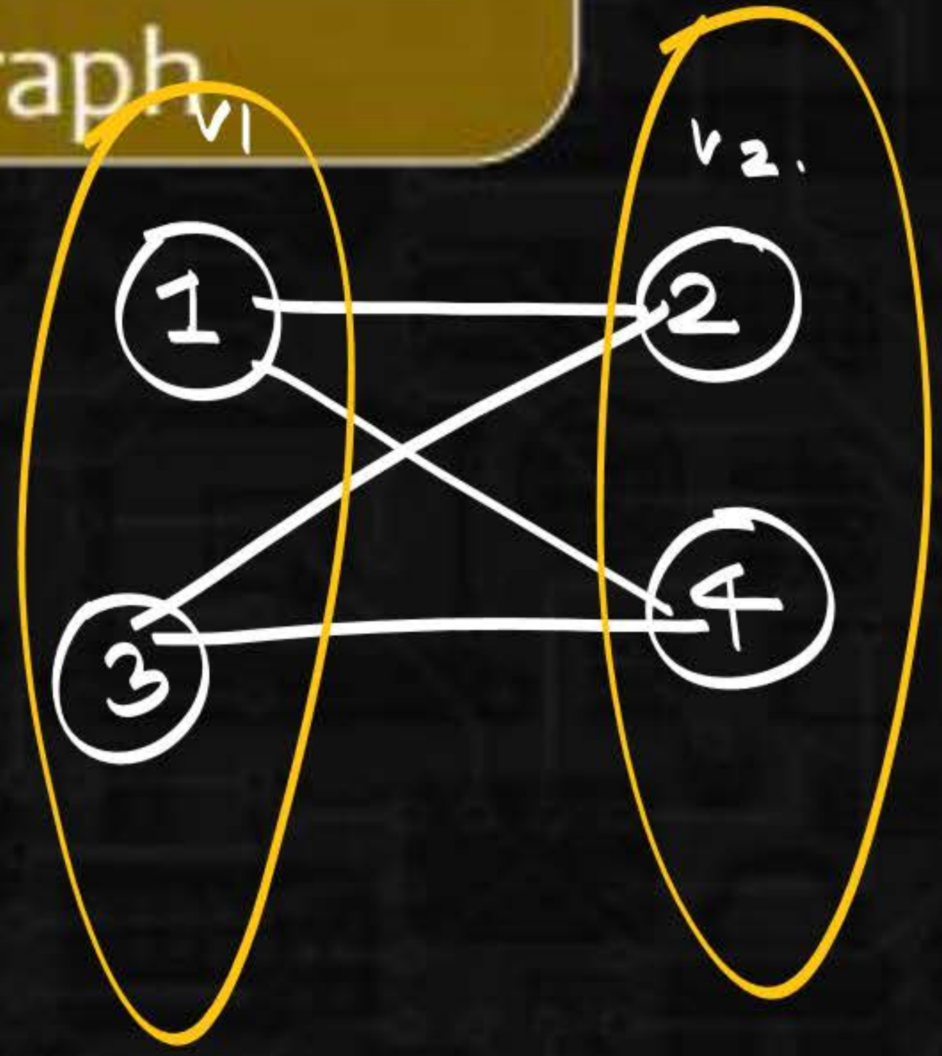
V_1 V_2.

$V \rightarrow$ it creates partition of V into 2 parts V_1, V_2.

$E \rightarrow$ all the edges will be from one set to another set but not in a same set.

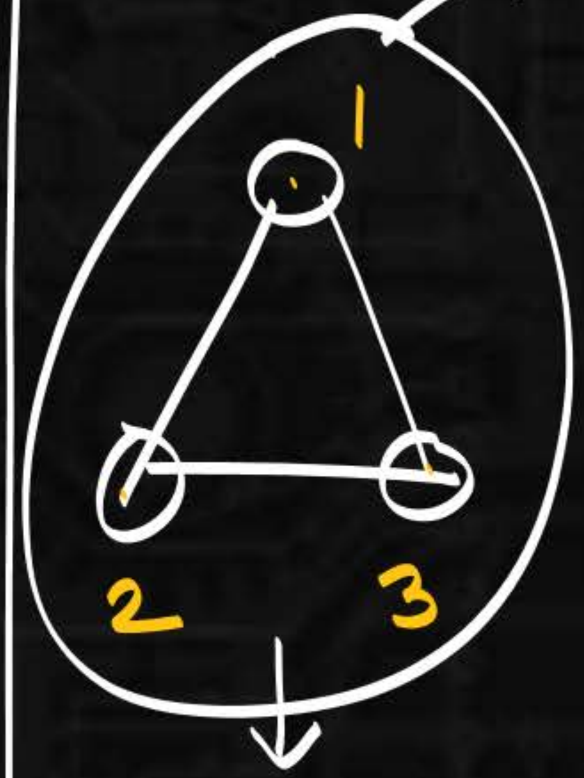
Basics of Graph

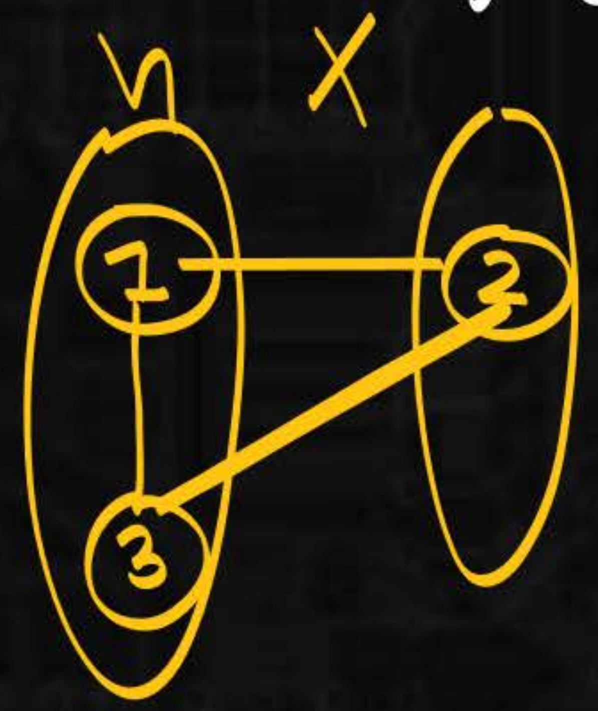




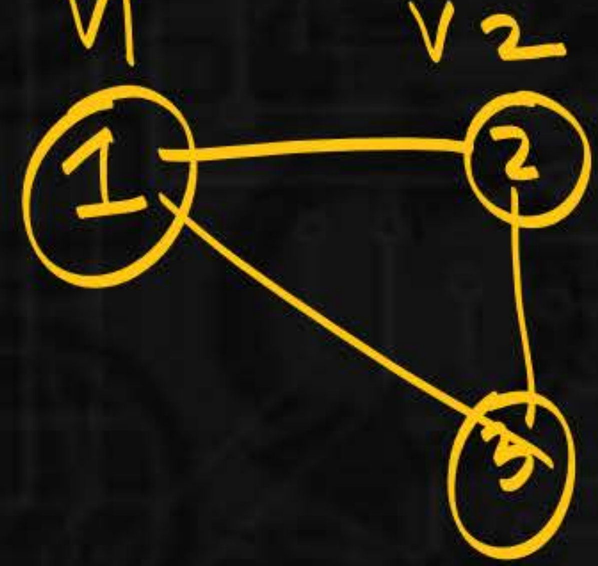
$G = (V, E)$

Odd length cycle

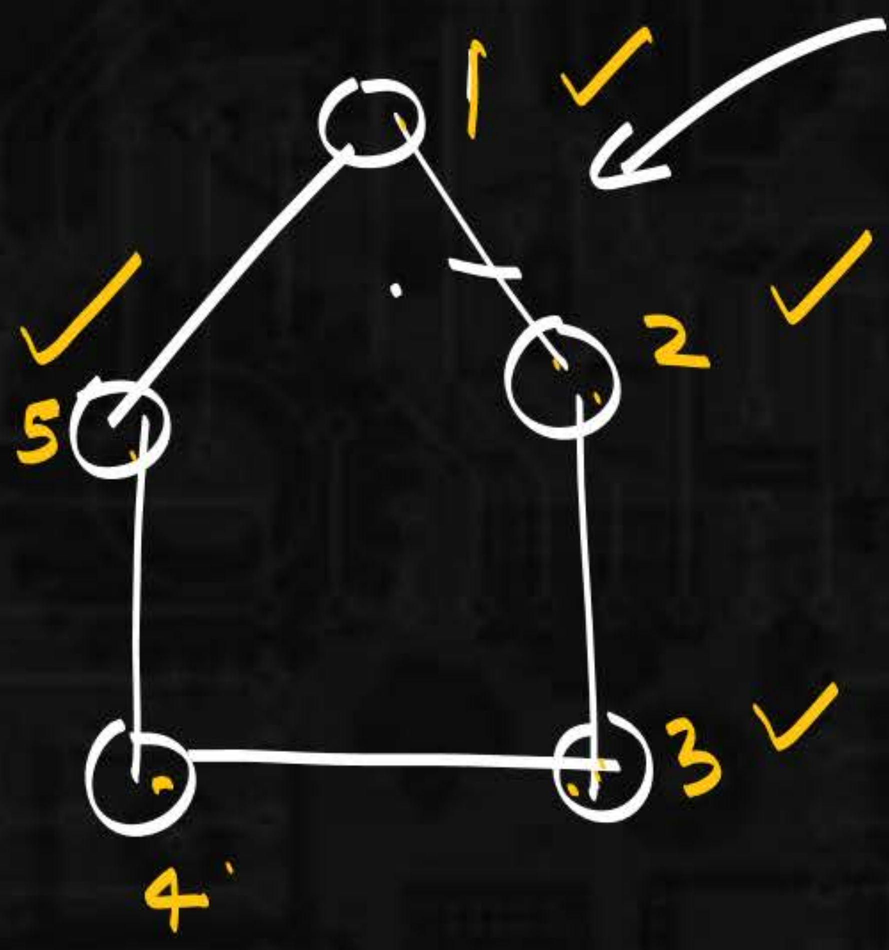




not bipartite Graph.



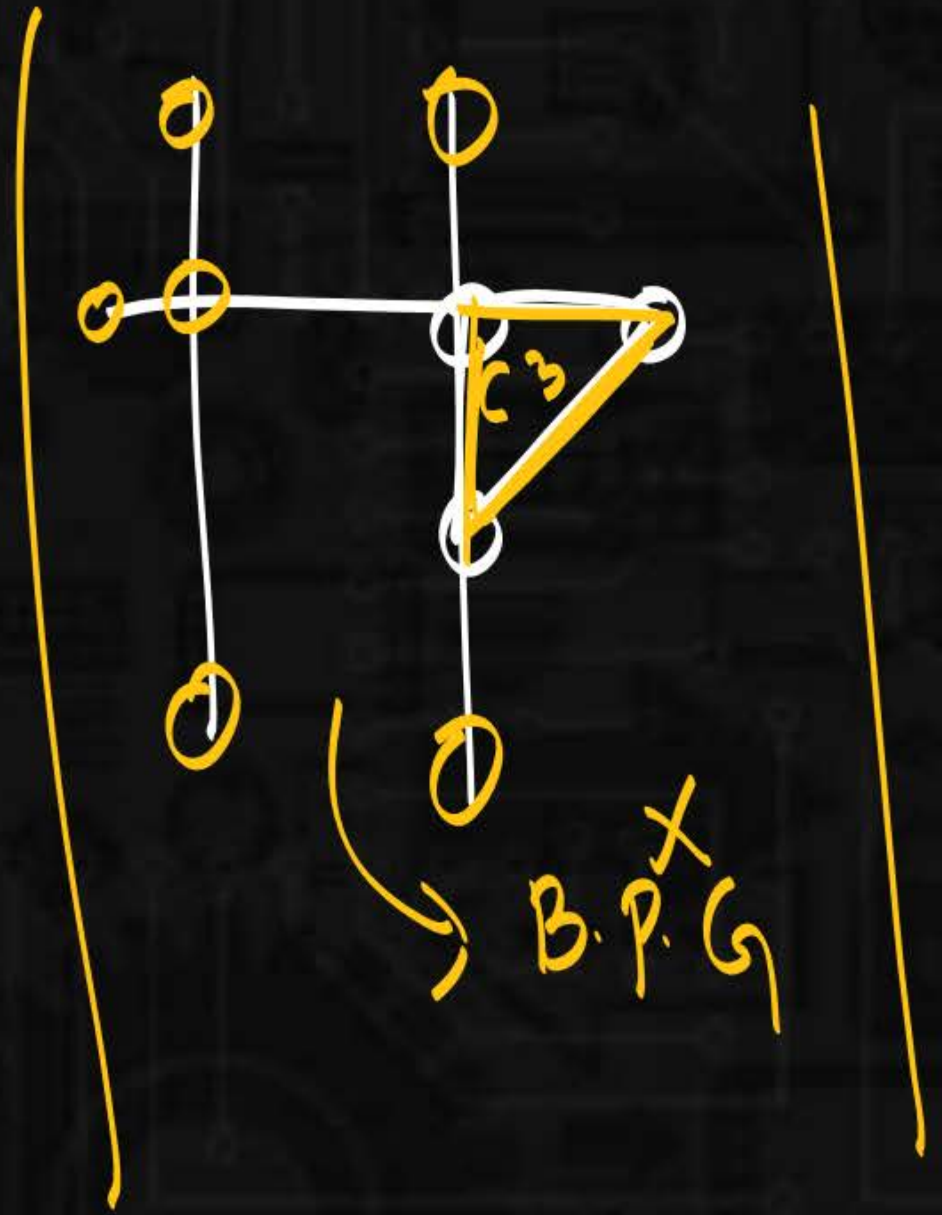
Basics of Graph



we can not adjust 4 → it is not bipartite Graph.

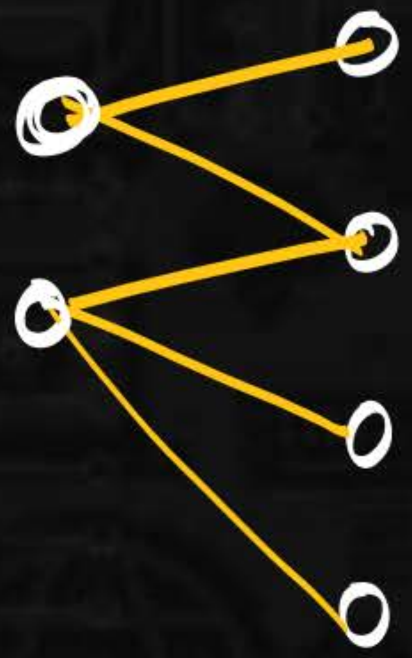
Basics of Graph

Thm: Bipartite Graph does not contains odd length cycle.



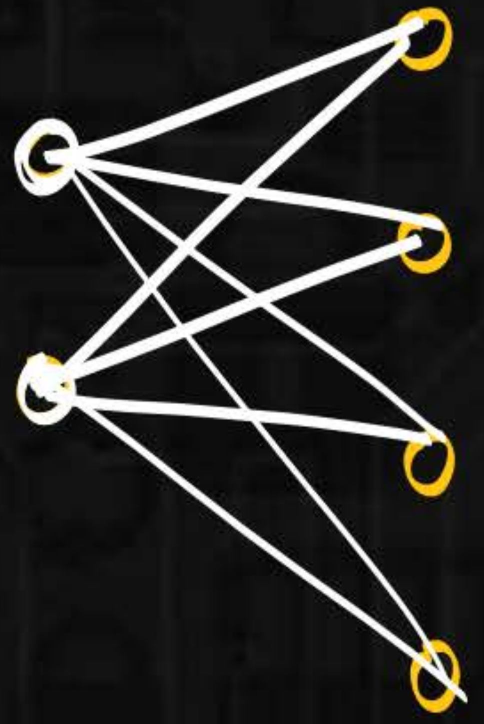
Basics of Graph

Bipartite Graph



Complete bipartite Graph. $\left(K_{m,n} \right)$

$K_{2,4}$.



$e = 2 \cdot 4$

$|V_1| = m$

$|V_2| = n.$

Total vertices $= m+n$

Total edges $= m \cdot n.$

Basics of Graph

Star Graph:

$K_{1,n-1}$

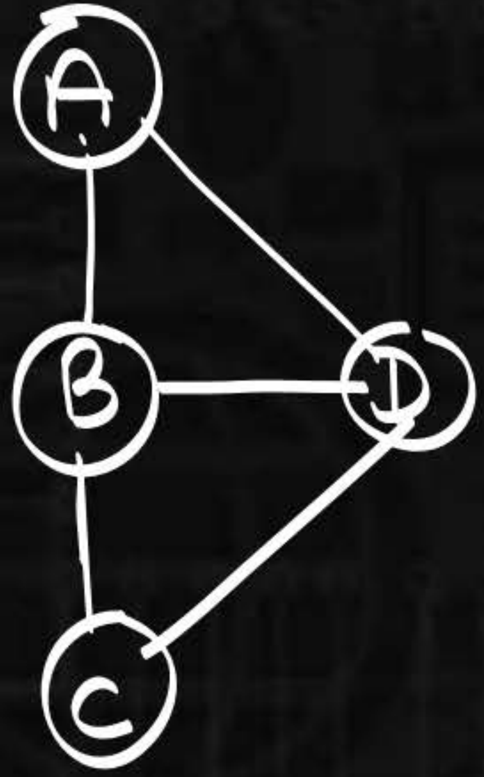
Total vertices $= n$

Total edges $= 1 \times n-1 = n-1$.

Draw a star Graph of 6 vertices.

$K_{1,5}$

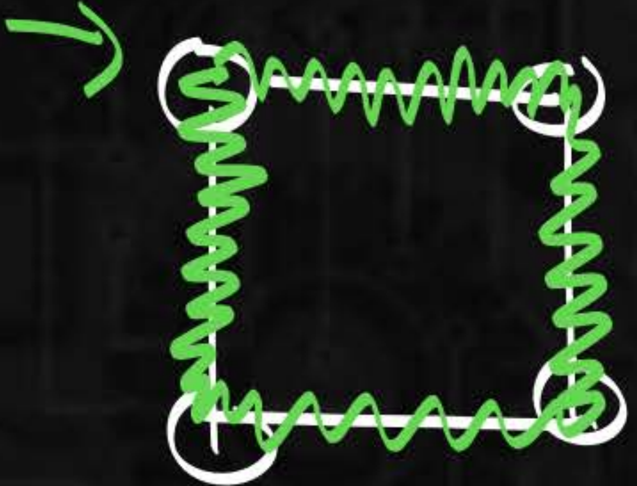
Basics of Graph

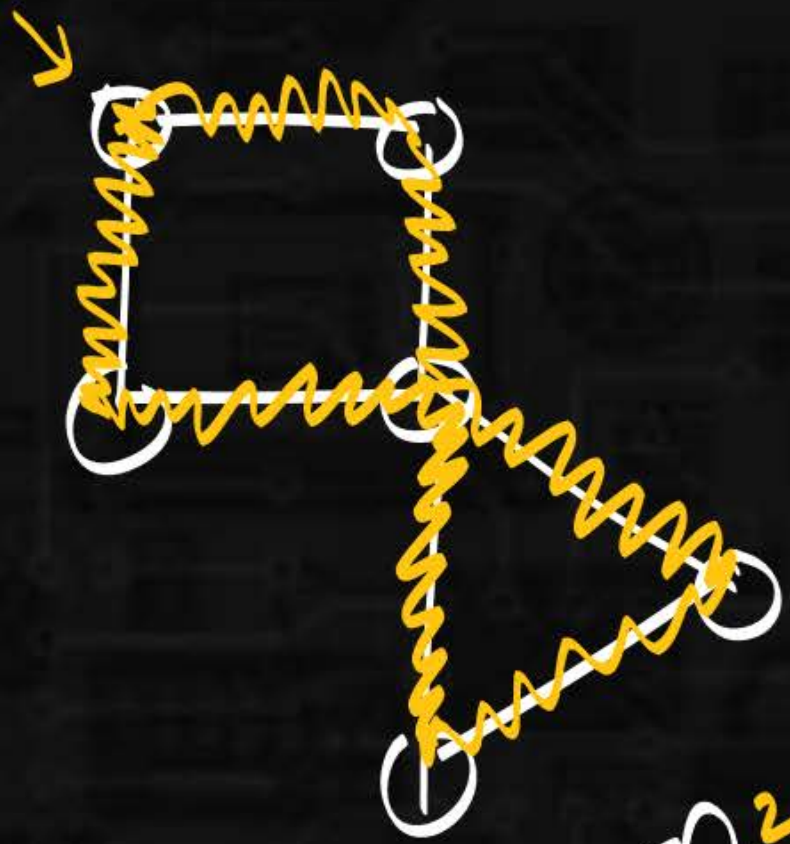


→ closed.
A

cover
all edges
exactly
once.

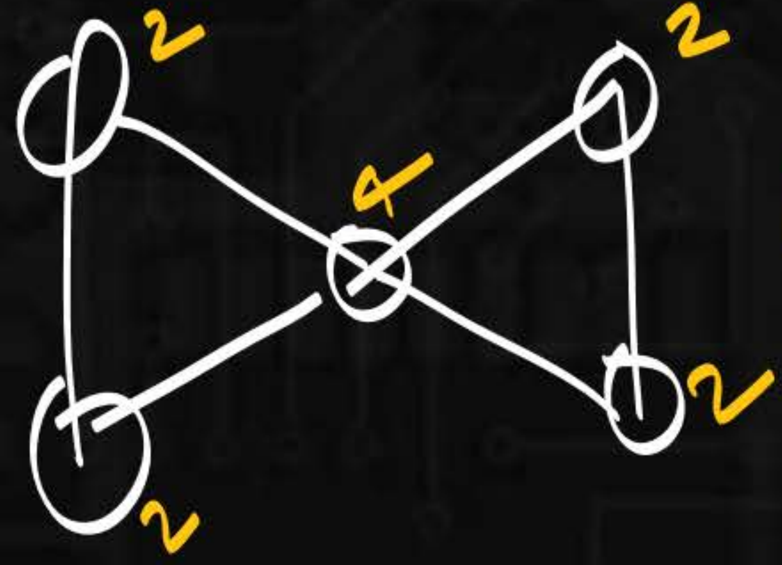
Basics of Graph





Closed
all edges
Euler Graph

Thm:
Graph is Euler Graph iff degrees of all vertices are even.
(connected)

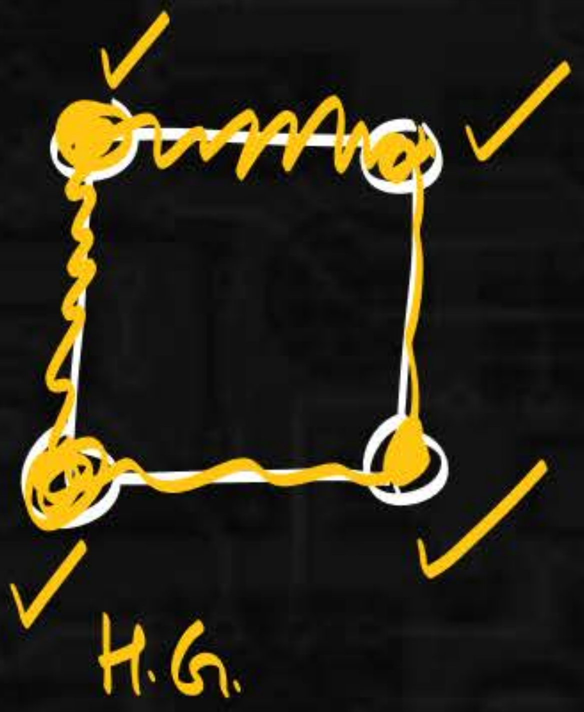


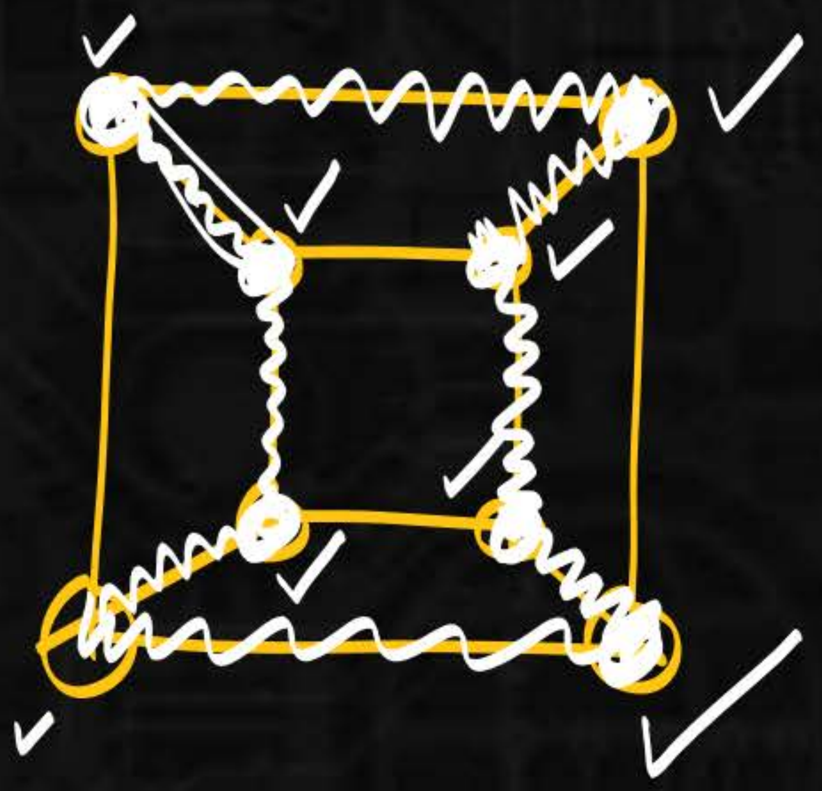
E.G ✓

Basics of Graph

Hamiltonian Graph:

Closed
+
all vertices
must be
covered.





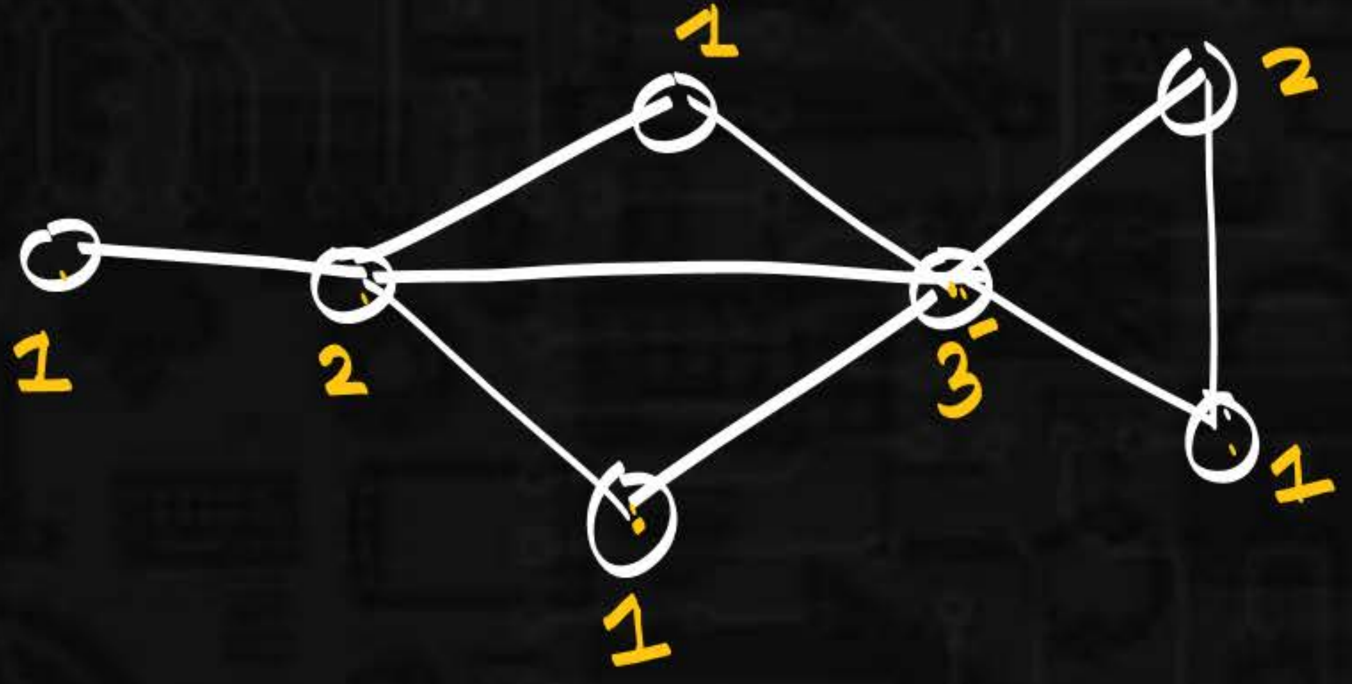
Euler → edge
Hamiltonian → vertex

Basics of Graph

Coloring:

Chromatic no. ($\chi(G)$)

min. no. of colors such that adjacent should not have same color.



$\chi(G) = 3.$

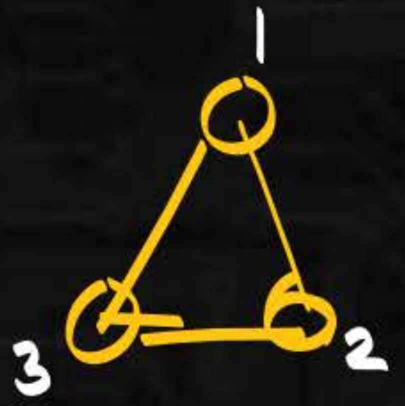
Basics of Graph

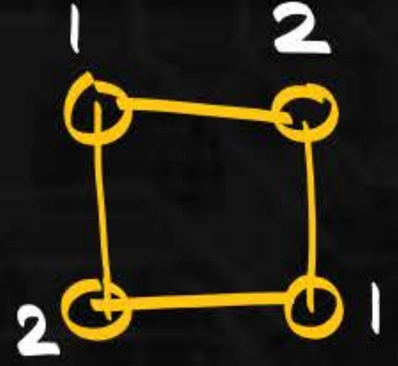
$\chi(C_n) =$

$\chi(W_n) =$

$\chi(K_n) = n$

$\chi(K_{m,n}) = 2$

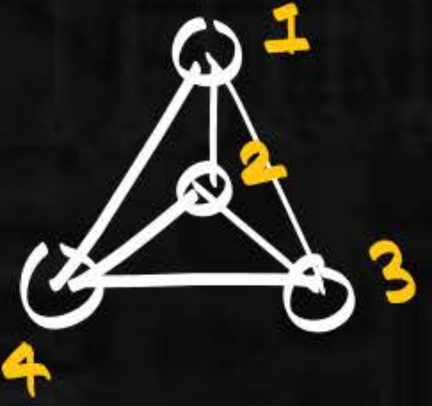


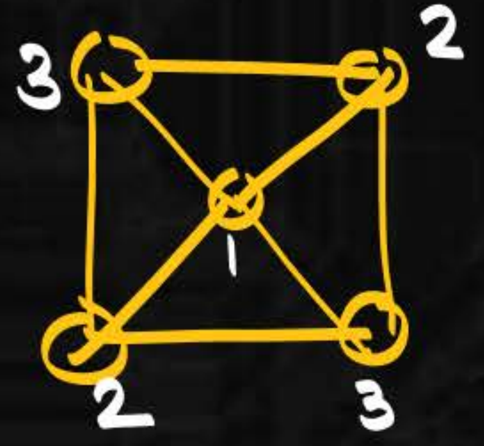


$\chi(C_3) = 3$ $\quad \chi(C_4) = 2$

$$\chi(C_n) = 2 \quad n \to \text{even}$$

$$\chi(C_n) = 3 \quad n \to \text{odd}$$





$\chi(W_4) = 4$ $\quad \chi(W_5) = 3$

$$\chi(W_n) = 4 \quad n \to \text{even}$$

$$\chi(W_n) = 3 \quad n \to \text{odd}$$


THANK YOU GW
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