ENGINEERING MATHEMATICS-1: (GATE - 2021) - REPORTS

Q. 1

Solution Video | Have any Doubt ?

Consider the following points for a non-singular matrix A of order 4?

I. Rank $(A^{-1}) = 4$
II. Rank $(A^3) = 3$
III. Rank $(2A) = 2$
IV. Rank (adj. (A)) = 4

Which of the above is true?

A. Only I and II

B. Only III and IV

C. I, II and IV

D. Only I and IV — Correct Option

Solution :
(d)
• Rank of inverted matrix remains same as the original matrix.
• So, the statement I and IV are correct statements.

QUESTION ANALYTICS +

Q. 2

? FAQ | Solution Video | Have any Doubt ?

Let $A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & -1 & \lambda \\ 5 & 7 & 1 & \lambda^2 \end{pmatrix}, \lambda \in R.$ Then rank of A is 2 if λ = ______.

A. 1, 3

B. −1, 3 — Correct Option

Solution :
(b)
Here to make the rank 2 of the matrix A, we need to make 3 × 3 determine to be zero.

So, let's take $$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & \lambda \\ 5 & 1 & \lambda^2 \end{pmatrix}$$

If put $\lambda = -1$, then we see $C_2 = C_3$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 5 & 1 & 1 \end{vmatrix} = 0$$

So, $\lambda = -1$, satisfies the condition.
Similarly, $\lambda = 3$ also satisfies and make the rank 2 of the matrix A.

C. 1, −3

D. −1, −3

QUESTION ANALYTICS +

Q. 3

? FAQ | Solution Video | Have any Doubt ?

What is the standard deviation of a uniformly distributed variable between 0 and $\frac{1}{3}$?

A. $\frac{1}{2\sqrt{12}}$

B. $\frac{1}{6\sqrt{3}}$ — Correct Option

Solution :
(b)

• $\text{Variance} = \frac{(b-a)^2}{12}$

$$a = 0, b = \frac{1}{3}$$

So, $$\text{Variance} = \frac{\left(\frac{1}{3}-0\right)^2}{12} = \frac{\frac{1}{9}}{12} = \frac{1}{108}$$

- $$\text{Standard deviation} = \sqrt{\text{Variance}} = \sqrt{\frac{1}{108}} = \frac{1}{6\sqrt{3}}$$

C $\frac{1}{\sqrt{34}}$

D $\frac{1}{9}$

QUESTION ANALYTICS +

Q. 4

? FAQ | Solution Video | Have any Doubt ?

The value of t so that $\begin{bmatrix} 4 \\ -1 \end{bmatrix}$ is an eigen vector of $\begin{bmatrix} 3 & 4 \\ 2 & t \end{bmatrix}$ is ______.

A -1

B 2

C 10 — Correct Option

Solution :
(c)

If we go by options using hit and trial method and take $t = 10$, then the matrix becomes $\begin{bmatrix} 3 & 4 \\ 2 & 10 \end{bmatrix}$.

We know, $AX = \lambda X$

$$\begin{bmatrix} 3 & 4 \\ 2 & 10 \end{bmatrix}\begin{bmatrix} 4 \\ -1 \end{bmatrix} = \lambda\begin{bmatrix} 4 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 12-4 \\ 8-10 \end{bmatrix} = \lambda\begin{bmatrix} 4 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 8 \\ -2 \end{bmatrix} = \lambda\begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

If $\lambda = 2$, then matrix satisfies. Hence $t = 10$ is correct answer.

D 5

QUESTION ANALYTICS +

Q. 5

? FAQ | Solution Video | Have any Doubt ?

The normal distribution N(μ, σ^2) with mean $\mu \in R$ and variance $\sigma^2 > 0$ has probability distribution function:

$$N(Z|\mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\left(\frac{-1}{2\sigma^2}(Z-\mu)^2\right)} \text{ for } Z \in R$$

The difference of median and mode is ______.

A μ

B 0 — Correct Option

Solution :
(b)
Mean = Median = Mode are all same for (μ) for normal distribution.

C −μ

D σ^2

QUESTION ANALYTICS +

Q. 6

Solution Video | Have any Doubt ?

A bag contains 10 defective items and 25 non-defective items. If four are selected at random without replacement, what will be the probability that all four items are defective? (Upto 3 decimal places).

0.004 (0.002 - 0.005) — Correct Option

Solution :
0.004 (0.002 - 0.005)

$$\text{Required Probability} = \frac{10}{35}\times\frac{9}{34}\times\frac{8}{33}\times\frac{7}{32} = \frac{10}{35}\times\frac{9}{34}\times\frac{8}{33}\times\frac{7}{32}$$
$$= \frac{2}{7}\times\frac{3}{34}\times\frac{1}{11}\times\frac{7}{4} = \frac{3}{34\times11\times2}$$

QUESTION ANALYTICS

Q. 7

Solution Video | Have any Doubt ?

Consider the following matrix:

$$A = \begin{bmatrix} 0 & 2 & 0 \\ 0 & 0 & 2 \\ 0 & -6 & -8 \end{bmatrix}$$

Out of all the eigen values, the minimum eigen value is ________.

-6 — Correct Option

Solution :
-6

$$A = \begin{bmatrix} 0 & 2 & 0 \\ 0 & 0 & 2 \\ 0 & -6 & -8 \end{bmatrix}$$

To find eigen values,

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 0-\lambda & 2 & 0 \\ 0 & 0-\lambda & 2 \\ 0 & -6 & -8-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \quad -\lambda(-\lambda(8-\lambda)+12) = 0$$
$$\Rightarrow \quad -\lambda(8\lambda+\lambda^2+12) = 0$$
$$\Rightarrow \quad \lambda(\lambda+2)(\lambda+6) = 0$$
$$\lambda = 0, \lambda = -2, \lambda = -6$$

Since, they are asking the minimum among different eigen values.
So, the correct answer will be –6.

QUESTION ANALYTICS

Q. 8

FAQ | Solution Video | Have any Doubt ?

A three by three matrix B is known to have eigenvalues 0, 1 and 2. This information is enough to find which of these

A. The rank of B — Correct Option

B. The determinant of $B^T B$ — Correct Option

C. The eigenvalues of $B^T B$

D. The eigenvalues of $(B^2 + I)^{-1}$ — Correct Option

YOUR ANSWER - NA	CORRECT ANSWER - a,b,d	STATUS - SKIPPED

Solution :
(a, b, d)

(a) B has 0 as an eigenvalue and is therefore singular (not invertible). Since B is a three by three matrix, this means that its rank can be at most 2. Since B has two distinct non zero eigen values, its rank is exactly 2.

(b) Since B is singular, det(B) = 0. Thus $\det(B^T B) = \det(B^T)\det(B) = 0$.

(c) There is not enough information to find the eigenvalues of $B^T B$.

(d) Eigen values would be 1, $\frac{1}{2}$ and $\frac{1}{5}$.

QUESTION ANALYTICS

Q. 9

FAQ | Solution Video | Have any Doubt ?

Let A be a 3 × 3 matrix with rank 2. Then AX = 0 has

A. The trivial solution X = 0. — Correct Option

B. One independent solution. — Correct Option

C. Two independent solutions.

D. Three independent solutions.

YOUR ANSWER - NA

CORRECT ANSWER - a,b

STATUS - SKIPPED

Solution :

(a, b)
If r is the rank of matrix A and $n \times n$ is the order of matrix then we shall have $(n - r)$ linearly independent non-trivial solutions. Any linear combination of these $(n - r)$ solutions will also be a solution of AX = 0.

QUESTION ANALYTICS

+

Q. 10

? FAQ Solution Video Have any Doubt ?

Which one of the following represents the eigen vectors of matrix $\begin{bmatrix} 4 & 6 \\ 2 & 8 \end{bmatrix}$?

A $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$

B $\begin{bmatrix} -3 \\ -1 \end{bmatrix}$

C $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$

D $\begin{bmatrix} 2 \\ 2 \end{bmatrix}$

Correct Option

Solution :
(d)
The characteristic equation is $|A - \lambda I| = 0$

i.e., $$\begin{vmatrix} 4-\lambda & 5 \\ 2 & 8-\lambda \end{vmatrix} = 0$$

$$(4 - \lambda)(8 - \lambda) - 10 = 0$$
$$\lambda^2 - 12\lambda + 20 = 0$$
$$(\lambda - 10)(\lambda - 2) = 0$$
$$\lambda = 10, 2$$

Corresponding to $\lambda = 10$, we have

$$[A - \lambda I]X = \begin{bmatrix} -6 & 6 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}$$

which gives $\left.\begin{matrix} -6a+6b=0 \\ 2a-2b=0 \end{matrix}\right\} a = b$

i.e., eigen vector can be the answer and is present in one of the option (d). Similarly $\lambda = 2$ also have eigen vectors i.e. not mentioned in any options.

QUESTION ANALYTICS

+

ENGINEERING MATHEMATICS-1: (GATE - 2021) - REPORTS

Q. 11

Check whether the given system of equation has

$$\begin{aligned} 2x+y+z &= 3 \\ x+2y+z &= 5 \\ 3x+2y+2z &= 6 \end{aligned}$$

A Infinite solution

B No solution

C Unique solution — Correct Option

Solution :
(c)

$$R(A\,|\,B) = \left[\begin{array}{ccc|c} 2 & 1 & 1 & 3 \\ 1 & 2 & 1 & 5 \\ 3 & 2 & 2 & 6 \end{array}\right] \begin{array}{l} R_2 \to 2R_2 - R_1 \\ R_3 \to 2R_3 - 3R_1 \end{array}$$

$$= \left[\begin{array}{ccc|c} 2 & 1 & 1 & 3 \\ 0 & 3 & 1 & 7 \\ 0 & 1 & 1 & 3 \end{array}\right] R_3 \to 3R_3 - R_2$$

$$= \left[\begin{array}{ccc|c} 2 & 1 & 1 & 3 \\ 0 & 3 & 1 & 7 \\ 0 & 0 & 2 & 2 \end{array}\right]$$

$$\text{Rank }(A) = 3 = \text{Rank }(A\,|\,B) = \text{No. of variable}$$

So, the given system of equation has unique solution.

D Inadequate data

QUESTION ANALYTICS +

Q. 12

Twenty coupons are numbered from 1 to 20. 5 coupons are selected at random, one at a time with replacement. The probability that the largest number appearing on every selected coupon is 8 is

A $\left(\frac{2}{5}\right)^5$ — Correct Option

Solution :
(a)

- Probability of selecting a coupon, on which largest printed number can be 8 = $\frac{8}{20} = \frac{2}{5}$.
- Coupons numbered 1 to 8 can be selected out of the 20 available coupons.
- Now coupons are selected one at a time and with replacement, each time probability is

$$p = \frac{8}{20} = \frac{2}{5}$$

Total probability that largest number appearing on selected coupon is 8.

$$= {}^5C_5 P^5 (1-P)^0 = P^5 = \left(\frac{2}{5}\right)^5$$

B $\frac{9}{{}^{20}C_5}$

C $\left(\frac{9}{20}\right)^2$

D $\frac{{}^{20}C_5}{{}^{20}C_{11}}$

QUESTION ANALYTICS +

Q. 13

If a number x is selected from natural numbers 1, 2, 3, 4, ... 20. The probability that x follows $x + \frac{50}{x} > 15$ is ______.

A $\frac{10}{20}$

B $\frac{14}{20}$ Correct Option

Solution :
(b)

$$x + \frac{50}{x} > 15$$

$$\Rightarrow \quad x^2 - 15x + 50 > 0$$

$$x^2 - 5x - 10x + 50 > 0$$

$$(x-5)(x-10) > 0$$

Cases :

(i) $x > 5$ and $x > 10 \Rightarrow x > 10$

(ii) $x < 5$ and $x < 10 \Rightarrow x < 5$

So, $x < 5 = \{1, 2, 3, 4\}$

$x > 10 = \{11, 12, 13, ..., 20\}$

So, total favourable cases : 4 + 10 = 14

$$\text{The required probability} = \frac{14}{20}$$

C $\frac{4}{20}$

D $\frac{15}{20}$

QUESTION ANALYTICS +

Q. 14 Solution Video Have any Doubt ?

The number of novels launched worldwide in a month follows Poission distribution with mean(μ) as 5.2. The probability of launching less than 3 novels in a randomly selected month is ______. (Upto 2 decimal places)

0.10 (0.10 - 0.11) Correct Option

Solution :
0.10 (0.10 - 0.11)

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

$$P(x < 3) = P(x=0) + P(x=1) + P(x=2)$$

$$= \frac{\lambda^0 e^{-\lambda}}{0!} + \frac{\lambda^1 e^{-\lambda}}{1!} + \frac{\lambda^2 e^{-\lambda}}{2!} = \frac{1}{e^\lambda} + \frac{\lambda}{e^\lambda} + \frac{\lambda^2}{2e^\lambda}$$

Given: $\lambda = 5.2$

$$= \frac{2 + 2\lambda + \lambda^2}{2e^\lambda}$$

$$= \frac{2 + 2 \times 5.2 + (5.2)^2}{2 \times e^{5.2}} = 0.108$$

QUESTION ANALYTICS +

Q. 15 ? FAQ Solution Video Have any Doubt ?

If a matrix M has eigen values (4, 5, 10), then the determinant of M^{-1} will be ______.
(Upto 3 decimal places)

0.005 (0.004 - 0.006) Correct Option

Solution :
0.005 (0.004 - 0.006)

Eigen value of M are (4, 5, 10).

Eigen values of M^{-1} are $\left(\frac{1}{4}, \frac{1}{5}, \frac{1}{10}\right)$

$$\text{Determinant of } M^{-1} = \text{Product of eigen values}$$

$$= \frac{1}{4 \times 5 \times 10} = \frac{1}{200} = 0.005$$

QUESTION ANALYTICS +

Q. 16 ? FAQ Solution Video Have any Doubt ?

Which of the following statement is/are true?

A The rank of an $m \times n$ matrix is always less than or equal to m. Correct Option

B For all 3 × 3 matrices A and B, AB = BA.

C If A and B are both invertible matrices of size $m \times m$, then AB is also invertible. Correct Option

D Say you have an $m \times m$ matrix A that is invertible. Then, the columns of A form a basis for R^m. Correct Option

YOUR ANSWER - NA | CORRECT ANSWER - a,c,d | STATUS - SKIPPED

Solution :

(a, c, d)

- Rank of matrix ≤ min (m, n)
  Thus, option (a) is true.
- Matrix multiplication is not commutative thus, AB ≠ BA.
- $(AB)^{-1} = B^{-1} A^{-1}$ and since both B and A are invertible thus AB is also invertible.
- $A_{m \times m}$ which is invertible thus $|A| \neq 0$ i.e. linear combination of column space of A covers the whole R^m. Hence form a basis.

QUESTION ANALYTICS +

Q. 17 FAQ Solution Video Have any Doubt ?

The null space of a matrix $A \in R^{n \times n}$ is trivial, i.e., N(A) = {0}. Which of the following statements are always true?

A There is no solution of $Ax = 0$.

B The linear transformation corresponding to A is one-to-one. Correct Option

C The rank of the matrix A is n. Correct Option

D The determinant of $A = 0$.

YOUR ANSWER - NA | CORRECT ANSWER - b,c | STATUS - SKIPPED

Solution :

(b, c)

The correct answers are:

The linear transformation corresponding to A is one-to-one.

The rank of the matrix A is n.

QUESTION ANALYTICS +

ENGINEERING MATHEMATICS-2: (GATE - 2021) - REPORTS

Q. 1

Solution Video | Have any Doubt ?

The inverse of $\begin{bmatrix} 2 & 2 & 0 \\ -2 & 1 & 1 \\ 3 & 0 & 1 \end{bmatrix}$ is

A $\begin{bmatrix} \frac{1}{12} & \frac{1}{6} & \frac{1}{6} \\ \frac{5}{12} & \frac{1}{6} & -\frac{1}{6} \\ -\frac{1}{4} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

B $\begin{bmatrix} \frac{1}{12} & -\frac{1}{6} & \frac{1}{6} \\ \frac{5}{12} & \frac{1}{6} & -\frac{1}{6} \\ -\frac{1}{4} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

Correct Option

Solution :
(b)

$$A^{-1} = \frac{\text{adj}(A)}{|A|}$$

$$\text{adj}(A) = \begin{bmatrix} 1 & 5 & -3 \\ -2 & 2 & 6 \\ 2 & -2 & 6 \end{bmatrix}^T = \begin{bmatrix} 1 & -2 & 2 \\ 5 & 2 & -2 \\ -3 & 6 & 6 \end{bmatrix}$$

$$|A| = 12$$

$$A^{-1} = \begin{bmatrix} \frac{1}{12} & -\frac{1}{6} & \frac{1}{6} \\ \frac{5}{12} & \frac{1}{6} & -\frac{1}{6} \\ -\frac{1}{4} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

C $\begin{bmatrix} \frac{1}{12} & -\frac{1}{6} & \frac{1}{6} \\ \frac{5}{12} & \frac{1}{6} & -\frac{1}{6} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

D $\begin{bmatrix} \frac{1}{12} & -\frac{1}{6} & \frac{1}{6} \\ \frac{5}{12} & \frac{1}{6} & \frac{1}{6} \\ -\frac{1}{4} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

QUESTION ANALYTICS +

Q. 2

? FAQ | Solution Video | Have any Doubt ?

The maximum and minimum of the function $f(x) = x^3 - 6x^2 + 9x + 1, x \in [0, 5]$, attain at x = ______ respectively.

A 0 and 5

B 5 and 0

Correct Option

Solution :
(b)

$$\begin{aligned} f(x) &= x^3 - 6x^2 + 9x + 1 \\ f'(x) &= 3x^2 - 12x + 9 = 0 \\ x^2 - 4x + 3 &= 0 \\ x^2 - 3x - x + 3 &= 0 \\ x(x-3) - 1(x-3) &= 0 \\ (x-1)(x-3) &= 0 \\ x &= 1, x = 3 \\ f''(x) &= 6x - 12 \end{aligned}$$

We need to check at all the extremum points i.e. 1, 3, 0, 5.

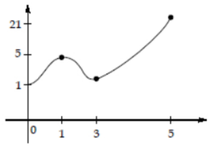
At 1, $f''(x) = -6 < 0$ (maximum)

At 3, $f''(x) = 6 > 0$ (minimum)

Taking into account all points:

$$f(0) = 1$$
$$f(1) = 5$$
$$f(3) = 1$$
$$f(5) = 21$$

Hence roughly graph can be drawn like:



Thus, maximum at 5 and minimum can be at 0 or 3.

C 3 and 0

D −1 and −3

QUESTION ANALYTICS +

Q. 3 ? FAQ Solution Video Have any Doubt ?

Which one of the following functions is strictly bounded?

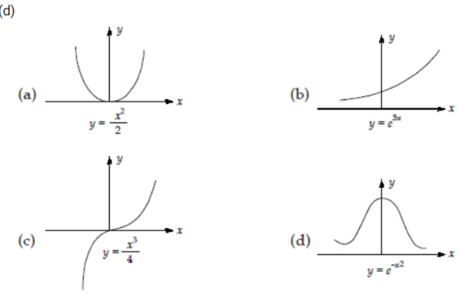
A $\frac{x^2}{2}$

B e^{3x}

C $\frac{x^3}{4}$

D e^{-x^2} Correct Option

Solution :
(d)



$y = e^{-x^2}$ is bounded.

QUESTION ANALYTICS +

Q. 4 ? FAQ Solution Video Have any Doubt ?

At the point $x = 3$, the function:

$$f(x) = \begin{cases} x^3 - 27 & : \quad 3 < x < \infty \\ x - 3 & : \quad -\infty < x \le 3 \end{cases}$$

is

A Discontinuous and not differentiable

B Discontinuous and differentiable

C Continuous and differentiable

D Continuous and not differentiable Correct Option

Solution :
(d)

- $\lim\limits_{x \to 3^-} f(x) = \lim\limits_{x \to 3^-} (x-3) = 0$
- $\lim\limits_{x \to 3^+} f(x) = \lim\limits_{x \to 3^+} (x^3 - 27) = 0$
- At 3, $f(3) = 3 - 3 = 0$

Thus, $\lim\limits_{x \to 3^-} f(x) = \lim\limits_{x \to 3^+} f(x) = f(3)$

$\therefore f$ is continuous at $x = 3$.

$$f'(x) = \begin{cases} 3x^2, & 3 < x < \infty \end{cases}$$

$f(x) = \begin{cases} \ldots \\ 1, & -\infty < x \leq 3 \end{cases}$

$Lf'(3) = 1$ and $Rf'(3) = 27$, f is not differentiable at $x = 3$.

QUESTION ANALYTICS

Q. 5

? FAQ Solution Video Have any Doubt ?

If $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ is skew symmetric, then the values of a and b are

A $a = 0, b = 0$ Correct Option

Solution :
(a)
If A is skew-symmetric,

$$A^T = -A$$

$$\begin{bmatrix} a & b \\ b & a \end{bmatrix} = -\begin{bmatrix} a & b \\ b & a \end{bmatrix}$$

$$a = -a \Rightarrow a = 0$$

$$b = -b \Rightarrow b = 0$$

Therefore, option (a) is correct answer.

B $a = 0, b = 1$

C $a = 1, b = 0$

D $a = 1, b = 1$

QUESTION ANALYTICS

Q. 6

Solution Video Have any Doubt ?

The value of $\lim_{x \to 4} \frac{(2x)^{1/3} - 2}{2x - 8}$ is ________. (Upto 2 decimal places)

0.08 (0.08 - 0.09) Correct Option

Solution :
0.08 (0.08 - 0.09)

$$\lim_{x \to 4} \frac{(2x)^{1/3} - 2}{2x - 8} \qquad \left(\frac{0}{0} \text{ form}\right)$$

So, apply L'Hospital rule

$$= \lim_{x \to 4} \frac{\frac{1}{3}(2x)^{-2/3} \times 2}{2}$$

$$= \lim_{x \to 4} \frac{1}{3}(2x)^{-2/3} = \frac{1}{12} = 0.083$$

QUESTION ANALYTICS

Q. 7

Solution Video Have any Doubt ?

Let A be skew-symmetric matrix of odd order. Then, det(A) = ________.

0 Correct Option

Solution :
0
If A be skew-symmetric matrix of odd order, then determinant is always zero.

QUESTION ANALYTICS

Q. 8

? FAQ Solution Video Have any Doubt ?

Which of the following functions is/are one to one?

A The function f from {a, b, c, d} to {1, 2, 3, 4, 5} with $f(a) = 4$, $f(b) = 5$, $f(c) = 1$, and $f(d) = 3$. Correct Option

B The function $f(x) = x^2$ from the set of integers to the set of integers.

C The function $f(x) = x + 1$ from the set of real numbers to itself. Correct Option

D Suppose that each worker in a group of employees is assigned a job from a set of possible jobs, each to be done by a single worker. Correct Option

YOUR ANSWER - NA | CORRECT ANSWER - a,c,d | STATUS - SKIPPED

Solution :

(a, c, d)

The function f is one-to-one because f takes on different values at the four elements of its domain

The function $f(x) = x^2$ is not one-to-one because, for instance, $f(1) = f(-1) = 1$

The function $f(x) = x + 1$ is a one-to-one function note that if x and y are two different workers, then $f(x)$ not equal to $f(y)$ because the two workers x and y must be assigned different jobs. Thus, the function f that assigns a job to each worker is one-to-one.

QUESTION ANALYTICS +

Q. 9

? FAQ Solution Video Have any Doubt ?

Which of the following function(s) is/are a bijection from R to R?

A $f(x) = 2x + 1$ Correct Option

B $f(x) = x^2 + 1$

C $f(x) = x^3$ Correct Option

D $f(x) = \dfrac{(x^2+1)}{(x^2+2)}$

YOUR ANSWER - NA | CORRECT ANSWER - a,c | STATUS - SKIPPED

Solution :

(a, c)

(a) One way to determine whether a function is a bijection is to try to construct its inverse. This function is a bijection, since its inverse (obtained by solving $y = 2x + 1$ for x) is the function $g(y) = \dfrac{(y-1)}{2}$.

b) This function is not a bijection, since its range is the set of real numbers greater than or equal to 1 (which is sometimes written $[1, \infty)$), not all of R. (It is not injective either.)

(c) This function is a bijection, since it has an inverse function.

(d) This function is not a bijection. It is easy to see that it is not injective, since x and $-x$ have the same image, for all real numbers x.

QUESTION ANALYTICS +

Q. 10

Solution Video Have any Doubt ?

Let $P = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$ and $D = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$. If $A = PDP^{-1}$, then A^5 is

A $\begin{bmatrix} 8 & 12 \\ 0 & -5 \end{bmatrix}$

B $\begin{bmatrix} 32 & 0 \\ 0 & -32 \end{bmatrix}$

C $\begin{bmatrix} 32 & -192 \\ 0 & -32 \end{bmatrix}$ Correct Option

Solution :

(c)

$$P = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}$$

$$A = PDP^{-1} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -6 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -12 \\ 0 & -2 \end{bmatrix}$$

$$A^2 = A \times A = \begin{bmatrix} 2 & -12 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 2 & -12 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$A^4 = A^2 \times A^2 = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix}$$

$$A^5 = A^4 \times A = \begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix}\begin{bmatrix} 2 & -12 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 32 & -192 \\ 0 & -32 \end{bmatrix}$$

D

$$\begin{bmatrix} 32 & -96 \\ 0 & -32 \end{bmatrix}$$

QUESTION ANALYTICS

ENGINEERING MATHEMATICS-2: (GATE - 2021) - REPORTS

Q. 11 FAQ | Solution Video | Have any Doubt ?

The value of the integral $\int\left[e^x \ln x + \frac{e^x}{x}\right]dx$ will be equal to

(A) $\frac{xe^x}{\ln x} + C$

(B) $e^x \ln x + C$ — Correct Option

Solution :
(b)
We know

$$\int e^x (f(x) + f'(x))dx = e^x f(x) + C$$

Here, $f(x) = \ln x,\ f'(x) = \frac{1}{x}$

$$\int e^x\left(\ln x + \frac{1}{x}\right)dx = e^x \ln x + C$$

So, option (b) is correct answer.

(C) $\frac{e^x}{x} + C$

(D) $\frac{xe^x}{1+\ln x} + C$

QUESTION ANALYTICS +

Q. 12 Solution Video | Have any Doubt ?

Assume A and B are matrix of size $n \times n$, which of the following is true?

(A) If A is idempotent non-singular matrix, then A must be the identity matrix. — Correct Option

Solution :
(a)
(a) A is idempotent. So $A^2 = A$. Since A is non-singular, so it is invertible i.e., A^{-1} exist.

$$I = A^{-1}. A = A^{-1}. A^2 = IA = A$$

So, A must be identity matrix. So, it is true.

(b) $ABA^{-1} = B$

$AB = BA$ since matrix multiplication is not communicative. So, false even if A is invertible.

(c) If coefficient matrix A is invertible for $AX = B$ then $X = A^{-1}B$ which gives unique solution. So it is false.

(d) If B is zero matrix, then also $AB = B$ = zero matrix. So, it is false.

(B) If A is invertible then $ABA^{-1} = B$.

(C) If the coefficient matrix A of the system $AX = B$ is invertible, then the system has infinitely many solution.

(D) If $AB = B$ then B is identity matrix.

QUESTION ANALYTICS +

Q. 13 Solution Video | Have any Doubt ?

$\lim_{x\to 0}\left(\frac{a+x}{a-x}\right)^{1/x} =$ ______.

(A) $e^{2/a}$ — Correct Option

Solution :
(a)

$$\lim\left(\frac{a+x}{a-x}\right)^{1/x} = \lim\left(\frac{1+\frac{x}{a}}{\dots}\right)^{1/x}$$

$$\lim_{x\to 0}\left(a-x\right) \qquad \lim_{x\to 0}\left(1-\frac{x}{a}\right)$$

Since, $\lim_{x\to 0}(1+ax)^{1/x} = e^a$

$$= \frac{e^{1/a}}{e^{-1/a}} = e^{2/a}$$

Therefore, option (a) is correct answer.

B $e^{1/a}$

C e

D e^a

QUESTION ANALYTICS +

Q. 14

Solution Video | Have any Doubt ?

The result of $\int_0^{\pi/3} \sin^3 \theta d\theta$ is ______. (Upto 2 decimal places)

0.20 (0.20 - 0.21) Correct Option

Solution :
0.20 (0.20 - 0.21)

$$I = \int_0^{\pi/3} (1-\cos^2\theta)\sin\theta d\theta$$

Put $\cos\theta = x$

$$-\sin\theta\frac{d\theta}{dx} = 1$$

At $\theta = 0$, $x = 1$

$$\theta = \frac{\pi}{3}, x = \frac{1}{2}$$

$$I = \int_1^{1/2} -(1-t^2)dt = -\left[t-\frac{t^3}{3}\right]_1^{1/2} = -\frac{11}{24}+\frac{2}{3} = \frac{5}{24} = 0.20$$

QUESTION ANALYTICS +

Q. 15

? FAQ | Solution Video | Have any Doubt ?

A man is known to speak truth 2 out of 3 times. He throws a die and reports that number obtained is a four. The probability that the number obtained is actually a four is ______. (Upto 2 decimal places)

0.28 (0.28 - 0.30) Correct Option

Solution :
0.28 (0.28 - 0.30)

- Let X be the event that number four is obtained.
- Let T_1 be the event that 4 occurs.

$$P(T_1) = \frac{1}{6}$$

- Let T_2 be the event that 4 did not occurred.

$$P(T_2) = \frac{5}{6}$$

- Probability that man reports the number obtained is actually a four $P(X|T_1)$.
- Similarly man reports number is 4 but he is lying = $P(X|T_2)$.

Using Baye's theorem, the number obtained is actually a four

$$= P(T_1|X)$$

$$P(T_1 \mid X) = \frac{P(T_1)\cdot P(X|T_1)}{P(T_1)P(X|E_1)+P(T_2)P(X|T_2)}$$

$$= \frac{\frac{1}{6}\times\frac{2}{3}}{\frac{1}{6}\times\frac{2}{3}+\frac{5}{6}\times\frac{1}{3}} = \frac{2}{7} = 0.28$$

QUESTION ANALYTICS +

Q. 16

? FAQ | Solution Video | Have any Doubt ?

If a random variable X and Y are independent and Z is given a third event then which of the following is/are true?

A $P(X) = P(X|Y)$ Correct Option

B $P(X \cap Y) = P(X)\,P(Y)$ Correct Option

(C) $P(X|Z) = P(X|Y, Z)$ Correct Option

(D) $P(X \cap Y|Z) = P(X|Z)\,P(Y|Z)$ Correct Option

YOUR ANSWER - NA | CORRECT ANSWER - a,b,c,d | STATUS - SKIPPED

Solution :

(a, b, c, d)

All the options are true.

For option (c):

$$P(X|Y, Z) = \frac{P(X \cap Y|Z)}{P(Y|Z)} = \frac{P(X|Z) \cap P(Y|Z)}{P(Y|Z)} = P(X|Z)$$

QUESTION ANALYTICS +

Q. 17

? FAQ | Solution Video | Have any Doubt ?

Suppose that E and F are events in a sample space and $p(E) = \frac{1}{3}$, $p(F) = \frac{1}{2}$ and $p(E|F) = \frac{2}{5}$. Then

(A) $p(F|E) = \frac{1}{3}$

(B) $p(F|E) = \frac{3}{5}$ Correct Option

(C) $p(E|F) = \frac{1}{3}$

(D) $p(E|F) = \frac{3}{5}$

YOUR ANSWER - NA | CORRECT ANSWER - b | STATUS - SKIPPED

Solution :

(b)

QUESTION ANALYTICS +

ENGINEERING MATHEMATICS (GATE - 2021) - REPORTS

Q. 1 Solution Video | Have any Doubt ?

The standard deviation of a uniformly distributed random variable between 0 and 1 is

A $\frac{1}{\sqrt{12}}$ Correct Option

Solution :
(a)

$$P(x) = \frac{1}{\beta-\alpha} = \frac{1}{1-0} = 1$$

Mean, $\mu = \Sigma x\, P(x)$

$$\Rightarrow \quad = \int_0^1 x = \left[\frac{x^2}{2}\right]_0^1 = \frac{1}{2}$$

Variance, $\sigma^2 = \Sigma x^2\, P(x) - \mu^2$

$$= \int_0^1 x^2 \frac{1}{4} = \left[\frac{x^3}{3}\right]_0^1 - \frac{1}{4}$$

$$= \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$$

$$\text{Standard deviation} = \sqrt{\text{variance}} = \frac{1}{\sqrt{12}}$$

B $\frac{1}{\sqrt{3}}$

C $\frac{5}{\sqrt{12}}$

D $\frac{7}{\sqrt{12}}$

QUESTION ANALYTICS +

Q. 2 FAQ | Solution Video | Have any Doubt ?

From a pack of regular playing cards, two cards are drawn at random. What is the probability that both cards will be Kings, if first card is NOT replaced?

A $\frac{1}{26}$

B $\frac{1}{52}$

C $\frac{1}{169}$

D $\frac{1}{221}$ Correct Option

Solution :
(d)

$$\frac{{}^4C_1}{{}^{52}C_1} \times \frac{{}^3C_1}{{}^{51}C_1} = \frac{1}{221}$$

QUESTION ANALYTICS +

Q. 3 FAQ | Solution Video | Have any Doubt ?

A function $f(x) = 8x^2 + 7x + 1$ is defined over an open interval $x = (2, 5)$. What is the exact point where $\frac{dy}{dx}$ hold?

A 63 Correct Option

Solution :
(a)

$$f(x) = 8x^2 + 7x + 1$$

Using Lagrange's mean value theorem:

At $x = 2$, $f(x) = 8(2)^2 + 7 \times 2 + 1 = 47$

$x = 5$, $f(x) = 8(5)^2 + 7 \times 5 + 1 = 236$

$$f'(x) = \frac{dy}{dx} = \frac{f(b) - f(a)}{b - a} = \frac{236 - 47}{5 - 2}$$

$$= \frac{189}{3} = 63$$

B 65.33

C 64.88

D 60.81

QUESTION ANALYTICS +

Q. 4

Solution Video Have any Doubt ?

A function $f(x) = 1 - x^2 + x^3$ is defined in the closed interval [−1, 1]. The value of x in the open interval (−1, 1) for which the mean value theorem is satisfied, is

A $-\frac{1}{2}$

B $-\frac{1}{3}$

Correct Option

Solution :
(b)

Since $f(1) \neq f(-1)$, Roll's mean value theorem does not apply.

By Lagrange mean value theorem

$$f'(x) = \frac{f(1) - f(-1)}{1 - (-1)} = \frac{2}{2} = 1$$

$$-2x + 3x^2 = 1$$

$$x = 1, -\frac{1}{3}$$

x lies in $(-1, 1) \Rightarrow$ $x = -\frac{1}{3}$

C $\frac{1}{3}$

D $\frac{1}{2}$

QUESTION ANALYTICS +

Q. 5

? FAQ Solution Video Have any Doubt ?

The $\lim_{x \to 0} \frac{\sin\left(\frac{2}{3}x\right)}{x}$ is

A $\frac{2}{3}$

Correct Option

Solution :
(a)

$$\lim_{x \to 0} \frac{\sin\left(\frac{2}{3}x\right)}{x} = \lim_{\frac{2}{3}x \to 0} \frac{\sin\left(\frac{2}{3}x\right)}{\frac{2}{3}x} \cdot \frac{2}{3} = (1)\left(\frac{2}{3}\right) = \frac{2}{3}$$

B 1

C $\frac{3}{2}$

D ∞

QUESTION ANALYTICS +

Q. 6

? FAQ Solution Video Have any Doubt ?

A 3 x 3 matrix P has eigen values 1, 0.5 and −3. What will be the eigen values of $P^3 + 2P + I$?

A 3 × 3 matrix P has eigen values 1, 0.5 and −3. What will be the eigen values of $P^3 + 2P + I$?

A 4, 12, −16

B 4, 2.125 and −32 Correct Option

Solution :
(b)

Eigen value of P are (1, 0.5, –3).

Eigen values of $P^3 + 2P + I$

$$= [(1)^3 + 2 \times 1 + 1, (0.5)^3 + 2 \times 0.5 + 1, (-3)^3 + 2 \times (-3) + 1]$$
$$= [4, 2.125, -32]$$

C 9, 0.5 and −9

D 9, 2, and −1.6

QUESTION ANALYTICS +

Q. 7

Solution Video | Have any Doubt ?

Consider the following functions:

$$f(x) = \begin{cases} -2x^2, & x \le -2 \\ 5x - 5 & x > -2 \end{cases}$$

Which of the following is true at $x = -2$?

A Neither continuous nor differentiable Correct Option

Solution :
(a)

$$f(-2) = -2 \times 4 = -8$$
$$f(-2^+) = 5(-2) - 5 = -15$$
$$f(-2^-) = -2 \times 4 = -8$$

Here, $f(-2^+) \ne f(-2^-)$

Hence, function is not continuous. So, it also can not be differential.

B Continuous but not differentiable

C Continuous and differentiable

D Differential but not continuous

QUESTION ANALYTICS +

Q. 8

Solution Video | Have any Doubt ?

Consider a function $f(x) = x^4 + 16 - 8x^2$. Which of the following is true about given function?

A $f(x)$ has only 2 stationary points.

B $f(x)$ has 2 maxima and 2 minima points.

C $f(x)$ has only 2 minima points.

D $f(x)$ has 1 maxima and 2 minima points. Correct Option

Solution :
(d)

$$f(x) = x^4 + 16 - 8x^2$$
$$= (x^2 - 4)^2$$
$$f'(x) = 2(x^2 - 4) \times 2x$$
$$= 4x(x^2 - 4) = 0$$
$$x = 0, x = 2, x = -2$$

So, $f(x)$ has 3 stationary points 0, 2 and –2.

Now, $f''(x) = 12x^2 - 16$

$$f''(0) = -16 < 0 \text{ (It is maxima points)}$$
$$f''(2) = 32 > 0 \text{ (It is minima points)}$$
$$f''(-2) = 32 > 0 \text{ (It is minima points)}$$

So, $f(x)$ has 1 maxima and 2 minima points.

QUESTION ANALYTICS +

Q. 9

Solution Video | Have any Doubt ?

The maximum value of n such that the probability of getting no tail in tossing a fair coin n times is greater than 0.2 is _______.

2 Correct Option

Solution :
2

The probability of getting a tail $P = \frac{1}{2}$

$$P(x = 0) = {}^nC_0P^0(1 - P)^n$$

$$= {}^nC_0P^0\left(1-\frac{1}{2}\right)^n > 0.2$$

i.e., $$\left(\frac{1}{2}\right)^n > 0.2$$

$$= 2^n < 5$$

when $n = 1, 2$ then $2^n < 5$

So, maximum value of $n = 2$.

QUESTION ANALYTICS +

Q. 10

Solution Video | Have any Doubt ?

Consider the following probability mass function (p.m.f.) of a random variable *X*.

$$p(X, q) = \begin{cases} q & \text{If } X = 0 \\ 1-q & \text{If } X = 1 \\ 0 & \text{otherwise} \end{cases}$$

If $q = 0.4$, the variance of *X* is ______. (Upto 2 decimal place)

0.24 (0.20 - 0.30) Correct Option

Solution :
0.24 (0.20 - 0.30)

Given : $q = 0.4$

X	0	1
p(X)	0.4	0.6

$$\text{Required value} = V(X) = E(X^2) - [E(X)]^2$$

$$E(X) = \sum_i X_ip_i = 0 \times 0.4 + 1 \times 0.6 = 0.6$$

$$E(X^2) = \sum_i X_i^2p_i = 0^2 \times 0.4 + 1^2 \times 0.6 = 0.6$$

$$\therefore \quad V(X) = E(X^2) - [E(X)]^2 = 0.6 - 0.36 = 0.24$$

QUESTION ANALYTICS +

ENGINEERING MATHEMATICS (GATE - 2021) - REPORTS

Q. 11

Solution Video | Have any Doubt ?

Let A be matrix $\begin{bmatrix} 2 \\ -4 \\ 7 \end{bmatrix}\begin{bmatrix} 1 & 9 & 5 \end{bmatrix}$. Let x, y and z be the eigen values of A. The product of xyz is equal to ______.

0 — Correct Option

Solution :
0

$$A = \begin{bmatrix} 2 \\ -4 \\ 7 \end{bmatrix}\begin{bmatrix} 1 & 9 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 18 & 10 \\ -4 & -36 & -20 \\ 7 & 63 & 35 \end{bmatrix}$$

$$R_2 = -2 \times R_1$$

Two rows are dependent. Hence, the determinant of matrix A is 0. Since the product of eigen values is determinant of the matrix. Hence, answer is 0.

QUESTION ANALYTICS +

Q. 12

Solution Video | Have any Doubt ?

The value of $\lim_{x \to 0}\left(\frac{1-\cos x}{x^2}\right)$ will be ______. (Upto 2 decimal places)

0.5 (0.50 – 0.50) — Correct Option

Solution :
0.5 (0.50 – 0.50)

$$\lim_{x \to 0}\left(\frac{1-\cos x}{x^2}\right) \qquad \left[\frac{0}{0} \text{ form}\right]$$

Apply L'Hospital rule,

$$= \lim_{x \to 0}\left(\frac{\sin x}{2x}\right) = \frac{1}{2}\lim_{x \to 0}\left(\frac{\sin x}{x}\right)$$

$$= \frac{1}{2} = 0.5$$

QUESTION ANALYTICS +

Q. 13

FAQ | Solution Video | Have any Doubt ?

The probability of getting at least 3 consecutive tails in 4 tosses of a fair coin is equal to ______.
(Upto 2 decimal places)

0.18 (0.18 – 0.20) — Correct Option

Solution :
0.18 (0.18 – 0.20)

Favourable cases = 3 as shown below.

TTTT, TTTH, HTTT

$$\text{Total cases} = 2^4 = 16$$

$$\text{Probability} = \frac{3}{16}$$

QUESTION ANALYTICS +

Q. 14

Solution Video | Have any Doubt ?

The values of x for which the function $f(x) = \frac{x^2 - 3x - 4}{x^2 + 3x - 4}$ is NOT continuous are

A 4

B -4 Correct Option

C -1

D 1 Correct Option

YOUR ANSWER - NA | CORRECT ANSWER - b,d | STATUS - SKIPPED

Solution :

(b, d)

$$f(x) = \frac{x^2 - 3x - 4}{x^2 + 3x - 4} \text{ is not continuous}$$

$$\begin{aligned} \text{when} \quad x^2 + 3x - 4 &= 0 \\ (x+4)(x-1) &= 0 \\ x &= -4, 1 \end{aligned}$$

QUESTION ANALYTICS +

Q. 15

Solution Video | Have any Doubt ?

[*A*] is square matrix which is neither symmetric nor skew-symmetric and $[A]^T$ is its transpose. The sum and difference of these matrices are defined as [*S*] = [*A*] + $[A]^T$ and [*D*] = [*A*] – $[A]^T$ respectively. Which of the following statements is false?

A Both [*S*] and [*D*] are symmetric. Correct Option

B Both [*S*] and [*D*] are skew-symmetric Correct Option

C [*S*] is skew-symmetric and [*D*] is symmetric Correct Option

D [*S*] is symmetric and [*D*] is skew-symmetric.

YOUR ANSWER - NA | CORRECT ANSWER - a,b,c | STATUS - SKIPPED

Solution :

(a, b, c)

$$\begin{aligned} \text{Since} \quad S^t &= (A + A^t)^t = A^t + (A^t)^t \\ &= A^t + A = S \\ \text{i.e.,} \quad S^t &= S \end{aligned}$$

$\therefore$ S is symmetric

$$\begin{aligned} \text{Since} \quad D^t &= (A - A^t)^t = A^t - (A^t)^t = A^t - A = -(A - A^t) = -D \\ \text{i.e.,} \quad D^t &= -D \end{aligned}$$

So, D is skew-symmetric.

QUESTION ANALYTICS +

Q. 16

? FAQ | Solution Video | Have any Doubt ?

Which one of the following statements is true?

A Skewness is a measure of assymetry of the probability distribution of a real valued random variable about its mean. Correct Option

B In a symmetric distribution, the values of mean, mode and median arc the same. Correct Option

C In a positively skewed distribution : mean > median > mode Correct Option

D In a negatively skewed distribution : mode > mean > median.

YOUR ANSWER - NA | CORRECT ANSWER - a,b,c | STATUS - SKIPPED

Solution :

(a, b, c)

Option (a), (b), (c) are true but (d) is not true since in a negatively skewed distribution, mode > median > mean.

QUESTION ANALYTICS +

Q. 17

? FAQ | Solution Video | Have any Doubt ?

Consider the following system of equations:

$$\begin{aligned} 8x + 3y - 2z &= 8 \\ 2x + 3y + 5z &= 9 \\ 2x + 3y + \lambda z &= \mu \end{aligned}$$

The system of equations has no solution for values of λ and μ given by

A $\lambda = 5$ and $\mu \neq 9$ Correct Option

Solution :
(a)
Augmented matrix:

$$[A\,|\,B] = \left[\begin{array}{ccc|c} 8 & 3 & -2 & 8 \\ 2 & 3 & 5 & 9 \\ 2 & 3 & \lambda & \mu \end{array}\right]$$

$$R_3 \leftarrow R_3 - R_2$$
$$R_2 \leftarrow 4R_2 - R_1$$

$$\left[\begin{array}{ccc|c} 8 & 3 & -2 & 8 \\ 0 & 9 & 22 & 28 \\ 0 & 0 & \lambda-5 & \mu-9 \end{array}\right]$$

If $\lambda = 5$ and $\mu \neq 9$, then system has no solution because Rank$[A\ |\ B] \neq$ Rank [4].

B $\lambda = 5$ and $\mu = 9$

C $\lambda \neq 5$ and $\mu = 9$

D $\lambda \neq 5$ and $\mu \neq 9$

QUESTION ANALYTICS +

Q. 18 Solution Video Have any Doubt ?

For the matrix $P = \begin{bmatrix} 3 & -2 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$, one of the eigen values is equal to −2. Then the eigen vector corresponding to this eigen value of P is

A $\begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$ Correct Option

Solution :
(a)
For eigen value $\lambda = -2$

$$(P - \lambda I)X = 0$$

$$\begin{bmatrix} 3-(-2) & -2 & 2 \\ 0 & -2-(-2) & 1 \\ 0 & 0 & 1-(-2) \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 5 & -2 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$5x - 2y + 2z = 0$$

So, only option (a) satisfies.

B $\begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix}$

C $\begin{bmatrix} -1 \\ 3 \\ 5 \end{bmatrix}$

D $\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$

QUESTION ANALYTICS +

Q. 19 Solution Video Have any Doubt ?

Consider the following function:

$$f(x) = \begin{cases} x^2 + c, & \text{if } x > 0 \\ \dfrac{x-c}{3+c}, & \text{if } x \leq 0 \end{cases}$$

Which of the following value of c, for which function is continuous for every x?

A 3

B -4 Correct Option

Solution :
(b)

Function $f(x)$ is continuous for every $x \neq 0$ (since $\dfrac{x-c}{3+c}$ and $x^2 + c$ are polynomials and polynomials are continuous).

$$f(0) = \frac{0-c}{3+c} = \frac{-c}{3+c}$$

$$\lim_{x\to 0^+} \frac{0-c}{3+c} = \frac{-c}{3+c}$$

$$\lim_{x\to 0^+} 0^2 + c = c$$

Since function $f(x)$ is continuous.

$\Rightarrow$ LHL = RHL = $f(0)$

$\Rightarrow$ $\frac{c}{3+c} = c = -\frac{c}{3+c}$

i.e. $-\frac{c}{3+c} = c$

i.e. $c = -4$

C -2

D 2

QUESTION ANALYTICS +

Q. 20

Solution Video | Have any Doubt ?

Let $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and $B = A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I.$ The matrix B is

A $\begin{bmatrix} 8 & 5 & 5 \\ 0 & 3 & 0 \\ 5 & 5 & 8 \end{bmatrix}$

Correct Option

Solution :

(a)

The characteristic equation of the matrix A is

$$\begin{bmatrix} 2-\lambda & 1 & 1 \\ 0 & 1-\lambda & 0 \\ 1 & 1 & 2-\lambda \end{bmatrix} = 0$$

$$\lambda^3 - 5\lambda^2 + 7\lambda - 3 = 0$$

Using Cayley-Hamilton theorem we have

$$A^3 - 5A^2 + 7A - 3I = 0$$

$$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$$

$$= A^5(A^3 - 5A^2 + 7A - 3I) + A(A^3 - 5A^2 + 7A - 3I) + A^2 + A + I$$

$$= A^2 + A + I$$

$$= \begin{bmatrix} 5 & 4 & 4 \\ 0 & 1 & 0 \\ 4 & 4 & 5 \end{bmatrix} + \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 5 & 5 \\ 0 & 3 & 0 \\ 5 & 5 & 8 \end{bmatrix}$$

B $\begin{bmatrix} 3 & 6 & 9 \\ 2 & 3 & 0 \\ 6 & 6 & 8 \end{bmatrix}$

C $\begin{bmatrix} 3 & 1 & 0 \\ 7 & 5 & 4 \\ 6 & 5 & 2 \end{bmatrix}$

D None of these

QUESTION ANALYTICS +

ENGINEERING MATHEMATICS (GATE - 2021) - REPORTS

Q. 21 Solution Video Have any Doubt ?

The value of the integral given below is

$$\int_{\pi/6}^{\pi/3} \frac{\operatorname{cosec}^2 x}{\cot^2 x} dx$$

A $\frac{2}{\sqrt{3}}$ Correct Option

Solution :
(a)

Take $t = \cot x$

$$\frac{dt}{dx} = -\operatorname{cosec}^2 x$$

$$dt = -\operatorname{cosec}^2 x\, dx$$

$x = \frac{\pi}{6} \Rightarrow$ $t = \sqrt{3}$

$x = \frac{\pi}{3} \Rightarrow$ $t = \frac{1}{\sqrt{3}}$

$$\int_{\sqrt{3}}^{1/\sqrt{3}} \frac{-dt}{t^2} = -\left[\frac{t^{-2+1}}{-2+1}\right]_{\sqrt{3}}^{\frac{1}{\sqrt{3}}} = \left[t^{-1}\right]_{\sqrt{3}}^{1/\sqrt{3}}$$

$$= \left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) = \frac{2}{\sqrt{3}}$$

B $3\sqrt{3}$

C $\frac{3}{5}$

D $3\sqrt{2}$

QUESTION ANALYTICS +

Q. 22 ? FAQ Solution Video Have any Doubt ?

Given matrix $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$, $[AA^T]^{-1}$ is

A $\begin{bmatrix} \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{bmatrix}$ Correct Option

Solution :
(a)

B $\begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{bmatrix}$

C $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

D $\begin{bmatrix} \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{\ } & 0 & 0 \end{bmatrix}$

$$\begin{bmatrix} 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{4} \end{bmatrix}$$

QUESTION ANALYTICS +

Q. 23

Solution Video | Have any Doubt ?

Two matrixes A and B are given below :

$$A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}, B = \begin{bmatrix} p^2+q^2 & pr+qs \\ pr+qs & r^2+s^2 \end{bmatrix}$$

If the rank of matrix A is N, then the rank of matrix B is

A) $\frac{N}{2}$

B) $N-1$

C) N — Correct Option

Solution :
(c)

$$A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$$

$$B = \begin{bmatrix} p^2+q^2 & pr+qs \\ pr+qs & r^2+s^2 \end{bmatrix} = AA^t$$

There are three cases for the rank of A.

Case I : rank $(A) = 0$

$\Rightarrow A$ is null. So, $B = AA^1$ also has to be null and hence rank (B) is also equal to 0. Therefore in this case rank (A) = rank (B).

Case II : rank $(A) = 2$

So, A has to be non-singular, i.e., $|A| \neq 0$. Therefore, $|B| = |A|^2$ is also $\neq 0$. So, rank $(B) = 2$.
Therefore, in this case also rank (A) = rank (B).
Therefore, in all three cases rank (A) = rank (B). So, rank of A is N, then the rank of matrix B is also N.

D) $2N$

QUESTION ANALYTICS +

Q. 24

Solution Video | Have any Doubt ?

For $A = \begin{bmatrix} 1 & \tan x \\ \tan x & 1 \end{bmatrix}$, the determinant of A^TA^{-1} is

A) $\sec^2 x$

B) $\cos 4x$

C) 1 — Correct Option

Solution :
(c)

Long Method :

$$A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$$

$$\text{adj } A = \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}[\text{adj. }(A)]^T = \frac{1}{1+\tan^2 x}\begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$$

$$= \frac{1}{\sec^2 x}\begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$$

Here,

$$A^TA^{-1} = \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}\frac{1}{\sec^2 x}\begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$$

$$= \frac{1}{\sec^2 x}\begin{bmatrix} 1-\tan^2 x & -2\tan x \\ 2\tan x & 1-\tan^2 x \end{bmatrix}$$

$$A^TA^{-1} = \begin{bmatrix} \frac{1-\tan^2 x}{\sec^2 x} & \frac{-2\tan x}{\sec^2 x} \\ \frac{2\tan x}{\sec^2 x} & \frac{1-\tan^2 x}{\sec^2 x} \end{bmatrix}$$

$$|A^TA^{-1}| = \left(\frac{1-\tan^2 x}{\sec^2 x}\right)^2 + \left(\frac{2\tan x}{\sec^2 x}\right)^2$$

$$1+\tan^4 x - 2\tan^2 x + 4\tan^2 x$$

$$= \frac{\cdots}{\sec^4 x} = 1$$

OR

Short Method :

Since
$$|AB| = |A|\,|B|$$
$$|A^TA^{-1}| = |A^T|\,|A^{-1}|$$
$$= |A| \times \frac{1}{|A|} = 1 \qquad \left(\text{Note} : |A^T| = |A| \text{ and } |A^{-1}| = \frac{1}{|A|}\right)$$

D 0

QUESTION ANALYTICS +

Q. 25

Solution Video | Have any Doubt ?

For the function $f(x) = x^2e^{-x}$, the maximum occurs when x is equal to ______.

2 — Correct Option

Solution :
2

$$f(x) = x^2e^{-x}$$
$$f'(x) = x^2(-e^{-x}) + e^x \times 2x = e^{-x}(2x - x^2)$$

Putting $f'(x) = 0$
$$e^{-x}(2x - x^2) = 0$$
$$e^{-x} \times (2 - x) = 0$$

$x = 0$ or $x = 2$ are the stationary points.

Now,
$$f''(x) = e^{-x}(2 - 2x) + (2x - x^2)(-e^{-x})$$
$$= e^{-x}(2 - 2x) + (2x - x^2) = e^{-x}(x^2 - 4x + 2)$$

at $x = 0$, $\quad f''(x) = e^{-0}(0 - 0 + 2) = 2$

Since $f''(x) = 2$ is > 0 at $x = 0$ we have a minima.

Now at $x = 2$
$$f''(2) = e^{-2}(2^2 - 4 \times 2 + 2)$$
$$= e^{-2}(4 - 8 + 2)$$
$$= -2e^{-2} < 0$$

$\therefore$ at $x = 2$ we have a maxima.

QUESTION ANALYTICS +

Q. 26

Solution Video | Have any Doubt ?

Machine *A* produces 50% of the output and machine *B* produces 50%. On an average 15 items in 1000 produced by *A* are defective and 5 item in 300 produced by *B* is defective. An item drawn at random from a day's output is defective. The probability that it was produced by *A* is ______.
(Upto 2 decimal places)

0.48 (0.46 – 0.50) — Correct Option

Solution :
0.48 (0.46 – 0.50)

Output produced by A = 50%
$$P(A) = 0.5$$
$$P(B) = 0.5$$
$$P\left(\frac{D}{A}\right) = \text{Probability that item produced by } A \text{ is defective}$$
$$P\left(\frac{D}{A}\right) = \frac{15}{1000} = 0.015$$
$$P\left(\frac{D}{B}\right) = \frac{5}{300} = 0.016$$
$$P\left(\frac{A}{D}\right) = \text{Probability that defective item produced by } A$$
$$P\left(\frac{A}{D}\right) = \frac{P(A) \times P\left(\frac{D}{A}\right)}{P(A) \times P\left(\frac{D}{A}\right) + P(B) \times P\left(\frac{D}{B}\right)}$$

QUESTION ANALYTICS +

Q. 27

Solution Video | Have any Doubt ?

$A = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$ and $B = \begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$ are two square matrix. If $(\theta - \Phi) = \frac{\pi}{2}$, then rank AB is ______.

0 — Correct Option

Solution :
0

$$AB = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}\begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta\cos\phi\cos(\theta-\phi) & \cos\theta\sin\phi\cos(\theta-\phi) \\ \sin\theta\cos\phi\cos(\theta-\phi) & \sin\theta\sin\phi\cos(\theta-\phi) \end{bmatrix}$$

As given, $\theta - \phi = \frac{\pi}{2}$

$$AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Then rank of AB is 0.

QUESTION ANALYTICS +

Q. 28

? FAQ ▶ Solution Video Have any Doubt ?

Evaluate : $\lim_{x\to 0^+} \log_{\sin x} \sin 2x$

1

Correct Option

Solution :
1

$$\lim_{x\to 0^+} \log_{\sin x} \sin 2x = \lim_{x\to 0^+} \frac{\log \sin 2x}{\log \sin x} \qquad \left(\frac{-\infty}{-\infty} \text{ form}\right)$$

$$= \lim_{x\to 0^+} \frac{\frac{2\cos 2x}{\sin 2x}}{\frac{1}{\sin x}\cdot \cos x} \qquad \text{(L-Hospital's rule)}$$

$$= \lim_{x\to 0} \frac{\left(\frac{2x}{\sin 2x}\right)\cos 2x}{\left(\frac{x}{\sin x}\right)\cos x}$$

$$= \lim_{x\to 0} \frac{\cos 2x}{\cos x} = 1$$

QUESTION ANALYTICS +

Q. 29

? FAQ ▶ Solution Video Have any Doubt ?

Consider there are 4 true coins and 2 false coin with head on both sides. A coin is chosen at random and tossed 4 times. If head is occurs at every toss then the probability that false coin is selected for toss is ______. (Upto 2 decimal places)

0.88 (0.86 – 0.90)

Correct Option

Solution :
0.88 (0.86 – 0.90)

$$P(\text{choosing true coin}) = \frac{4}{6} = \frac{2}{3}$$

$$P(\text{choosing false coin}) = \frac{2}{6} = \frac{1}{3}$$

Probability for obtaining head

$$= (\text{Probability with true coin} + \text{Probability with false coin})$$

$$= \frac{2}{3}\times\left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)+\frac{1}{3}\times(1\times1\times1\times1)$$

$$= \frac{2}{3}\times\frac{1}{16}+\frac{1}{3}=\frac{1+8}{24}=\frac{9}{24}$$

So, P(choosing false coin/head on 4 tosses)

$$= \frac{\frac{1}{3}\times 1}{\frac{9}{24}} = \frac{24}{3\times 9} = 0.88$$

QUESTION ANALYTICS +

Q. 30

? FAQ ▶ Solution Video Have any Doubt ?

An examination consists of two papers, Paper-I and Paper-II. The probability of failing in Paper-I is 0.3 and that in Paper-II is 0.2. Given that a student has failed in Paper-II,the probability of a student the failing in Paper -I is 0.6. The probability of a student failing in both the papers is ______. (Upto 2 decimal places)

0.12 (0.12 - 0.13)

Correct Option

Solution :
0.12 (0.12 - 0.13)

Let,

A denote the event of failing in Paper-I and B denote the event of failing in Paper-II.

Given, $P(A) = 0.3,\ P(B) = 0.2$

$$P\left(\frac{A}{B}\right) = 0.6$$

Probability of failing in both

$$P(A\cap B) = P(B)\times P\left(\frac{A}{B}\right)$$

$$= 0.2 \times 0.6 = 0.12$$

QUESTION ANALYTICS

+

ENGINEERING MATHEMATICS (GATE - 2021) - REPORTS

Q. 31

FAQ Solution Video Have any Doubt ?

If P and Q are two random events, then which of the following is false?

A. Independence of P and Q implies that probability $(P \cap Q) = 0$ — Correct Option

B. Probability $(P \cup Q) \geq$ Probability (P) + Probability (Q) — Correct Option

C. If P and Q are mutually exclusive, then they must be independent. — Correct Option

D. Probability $(P \cap Q) \leq$ Probability (P)

YOUR ANSWER - NA | CORRECT ANSWER - a,b,c | STATUS - SKIPPED

Solution :

(a, b, c)

Option (a) is false since if P and Q are independent

$$\text{pr}(P \cap Q) = \text{pr}(P) \times \text{pr}(Q)$$

which need not be zero.

Option (b) is false since

$$\text{pr}(P \cup Q) = \text{pr}(P) + \text{pr}(Q) - \text{pr}(P \cap Q)$$

$$\therefore \quad \text{pr}(P \cup Q) \leq \text{pr}(P) + \text{pr}(Q)$$

Option (c) is false since independence and mutually exclusive are unrelated properties.

Option (d) is true, since

$$P \cap Q \subseteq P$$

$$\Rightarrow \quad n(P \cap Q) \leq n(P)$$

Dividing both sides by $n(S)$, we get

$$\frac{n(P \cap Q)}{n(S)} \leq \frac{n(P)}{n(S)}$$

$$\Rightarrow \quad \text{pr}(P \cap Q) \leq \text{pr}(P)$$

QUESTION ANALYTICS +

Q. 32

FAQ Solution Video Have any Doubt ?

Below are the given different scenario of question. Which of the following are correct?

A. The probability of throwing six perfect dices and getting six different faces is $^{6!}/_{6^6}$ — Correct Option

B. The probability of throwing six perfect dices and getting six different faces is $\frac{6!}{6^2}$.

C. The probability of getting atleast three consecutive heads in four tosses of a fair coin is $\frac{3}{16}$. — Correct Option

D. The probability of throw consecutive heads four tosses of a fair coin is $\frac{5}{16}$.

YOUR ANSWER - NA | CORRECT ANSWER - a,c | STATUS - SKIPPED

Solution :

(a, c)

- Option (a) is correct.
- Option (c) is correct. Lets look at how.

  P(getting 3 consecutive heads) = $P_{HHHT} + P_{THHH} + P_{HHHH}$

  $$= \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{3}{16}$$

QUESTION ANALYTICS +

Q. 33

FAQ Solution Video Have any Doubt ?

Let $A_{n \times n}$ be an invertible matrix with real entries whose row sums are all equal to C. Which of the following statements are correct?

A. Every row in the matrix of $2A$ sums to $2C$. — Correct Option

B. Every row in the matrix A^2 sums to C^2 — Correct Option

C Every row in the matrix A^{-1} sums to C^{-1}. Correct Option

D None of these

YOUR ANSWER - NA | CORRECT ANSWER - a,b,c | STATUS - SKIPPED

Solution :

(a, b, c)

All (a), (b) and (c) are true.

Lets take a matrix, $\begin{bmatrix} x & y \\ y & x \end{bmatrix}$

The sum of each row $= x + y$

Now, $$2A = 2\times\begin{bmatrix} x & y \\ y & x \end{bmatrix} = \begin{bmatrix} 2x & 2y \\ 2y & 2x \end{bmatrix} = 2(x+y)$$

$$= \text{Sum of each row} = 2 = C$$

Similarly, $$A^2 = \begin{bmatrix} x^2+y^2 & xy+yx \\ xy+yx & x^2+y^2 \end{bmatrix}$$

$$= \text{Sum of each row} = (x^2 + y^2 + 2xy) = (x + y)^2 = C^2$$

Similarly, $$A = \frac{1}{x^2-y^2}\begin{bmatrix} x & -y \\ -y & x \end{bmatrix}$$

$$= \frac{(x-y)}{(x^2-y^2)} = \frac{(x-y)}{(x+y)(x-y)} = \frac{1}{(x+y)} = \frac{1}{C}$$

QUESTION ANALYTICS +