Engineering Mathematics

Workbook

All Branches

GATE & ESE

Engineering Mathematics

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Sorting HAT Technologies Pvt. Ltd.

353, 12th 'A' cross, off double road, Opp. City Centre

Residency, Indira nagar, 1st stage Bangalore

Bangalore KA 560038 IN.

GATE Syllabus

Electronics & Communication (EC) :

Linear Algebra : Vector space, basis, linear dependence and independence, matrix algebra, eigenvalues and eigenvectors, rank, solution of linear equations- existence and uniqueness. **Calculus :** Mean value theorems, theorems of integral calculus, evaluation of definite and improper integrals, partial derivatives, maxima and minima, multiple integrals, line, surface and volume integrals, Taylor series. **Differential Equations :** First order equations (linear and nonlinear), higher order linear differential equations, Cauchy's and Euler's equations, methods of solution using variation of parameters, complementary function and particular integral, partial differential equations, variable separable method, initial and boundary value problems. **Vector Analysis :** Vectors in plane and space, vector operations, gradient, divergence and curl, Gauss's, Green's and Stokes' theorems. **Complex Analysis :** Analytic functions, Cauchy's integral theorem, Cauchy's integral formula, sequences, series, convergence tests, Taylor and Laurent series, residue theorem. **Probability and Statistics :** Mean, median, mode, standard deviation, combinatorial probability, probability distributions, binomial distribution, Poisson distribution, exponential distribution, normal distribution, joint and conditional probability.

Electrical Engineering (EE) :

Linear Algebra : Matrix Algebra, Systems of linear equations, Eigenvalues, Eigenvectors. **Calculus :** Mean value theorems, Theorems of integral calculus, Evaluation of definite and improper integrals, Partial Derivatives, Maxima and minima, Multiple integrals, Fourier series, Vector identities, Directional derivatives, Line integral, Surface integral, Volume integral, Stokes's theorem, Gauss's theorem, Divergence theorem, Green's theorem. **Differential equations :** First order equations (linear and nonlinear), Higher order linear differential equations with constant coefficients, Method of variation of parameters, Cauchy's equation, Euler's equation, Initial and boundary value problems, Partial Differential Equations, Method of separation of variables. **Complex variables :** Analytic functions, Cauchy's integral theorem, Cauchy's integral formula, Taylor series, Laurent series, Residue theorem, Solution integrals. **Probability and Statistics :** Sampling theorems, Conditional probability, Mean, Median, Mode, Standard Deviation, Random variables, Discrete and Continuous distributions, Poisson distribution, Normal distribution, Binomial distribution, Correlation analysis, Regression analysis.

Mechanical Engineering (ME) :

Linear Algebra : Matrix algebra, systems of linear equations, eigenvalues and eigenvectors.

Calculus : Functions of single variable, limit, continuity and differentiability, mean value theorems, indeterminate forms; evaluation of definite and improper integrals; double and triple integrals; partial derivatives, total derivative, Taylor series (in one and two variables), maxima and minima, Fourier series; gradient, divergence and curl, vector identities, directional derivatives, line, surface and volume integrals, applications of Gauss, Stokes and Green's theorems. **Differential equations :** First order equations (linear and nonlinear); higher order linear differential equations with constant coefficients; Euler-Cauchy equation; initial and boundary value problems; Laplace transforms; solutions of heat, wave and Laplace's equations. **Complex variables :** Analytic functions; Cauchy-Riemann equations; Cauchy's integral theorem and integral formula; Taylor and Laurent series. **Probability and Statistics :** Definitions of probability, sampling theorems, conditional probability; mean, median, mode and standard deviation; random variables, binomial, Poisson and normal distributions. **Numerical Methods :** Numerical solutions of linear and non-linear algebraic equations; integration by trapezoidal and Simpson's rules; single and multi-step methods for differential equations.

Civil Engineering (CE) :

Linear Algebra : Matrix algebra; Systems of linear equations; Eigen values and Eigen vectors. **Calculus :** Functions of single variable; Limit, continuity and differentiability; Mean value theorems, local maxima and minima; Taylor series; Evaluation of definite and indefinite integrals, application of definite integral to obtain area and volume; Partial derivatives; Total derivative; Gradient, Divergence and Curl, Vector identities; Directional derivatives; Line, Surface and Volume integrals. **Ordinary Differential Equation (ODE) :** First order (linear and non-linear) equations; higher order linear equations with constant coefficients; Euler-Cauchy equations; initial and boundary value problems. **Partial Differential Equation (PDE) :** Fourier series; separation of variables; solutions of one- dimensional diffusion equation; first and second order one-dimensional wave equation and two-dimensional Laplace equation. Probability and Statistics : Sampling theorems; Conditional probability; Descriptive statistics – Mean, median, mode and standard deviation; Random Variables – Discrete and Continuous, Poisson and Normal Distribution; Linear regression. **Numerical Methods :** Error analysis. Numerical solutions of linear and non-linear algebraic equations; Newton's and Lagrange polynomials; numerical differentiation; Integration by trapezoidal and Simpson's rule; Single and multi-step methods for first order differential equations.

Computer Science & Information Technology (CS & IT) :

Discrete Mathematics : Propositional and first order logic. Sets, relations, functions, partial orders and lattices. Monoids, Groups. Graphs: connectivity, matching, coloring. Combinatorics: counting, recurrence relations, generating functions.

Linear Algebra : Matrices, determinants, system of linear equations, eigenvalues and eigenvectors, LU decomposition.

Calculus : Limits, continuity and differentiability. Maxima and minima. Mean value theorem. Integration.

Probability and Statistics : Random variables. Uniform, normal, exponential, poisson and binomial distributions. Mean, median, mode and standard deviation. Conditional probability and Bayes theorem.

Instrumentation Engineering (IN) :

Linear Algebra : Matrix algebra, systems of linear equations, consistency and rank, Eigen value and Eigen vectors. **Calculus :** Mean value theorems, theorems of integral calculus, partial derivatives, maxima and minima, multiple integrals, Fourier series, vector identities, line, surface and volume integrals, Stokes, Gauss and Green's theorems.

Differential equations : First order equation (linear and nonlinear), second order linear differential equations with constant coefficients, method of variation of parameters, Cauchy's and Euler's equations, initial and boundary value problems, solution of partial differential equations: variable separable method. **Analysis of complex variables :** Analytic functions, Cauchy's integral theorem and integral formula, Taylor's and Laurent's series, residue theorem, solution of integrals. **Probability and Statistics :** Sampling theorems, conditional probability, mean, median, mode, standard deviation and variance; random variables: discrete and continuous distributions: normal, Poisson and binomial distributions. **Numerical Methods :** Matrix inversion, solutions of non-linear algebraic equations, iterative methods for solving differential equations, numerical integration, regression and correlation analysis.

Chemical Engineering (CH) :

Linear Algebra : Matrix algebra, Systems of linear equations, Eigen values and eigenvectors. **Calculus :** Functions of single variable, Limit, continuity and differentiability, Taylor series, Mean value theorems, Evaluation of definite and improper integrals, Partial derivatives, Total derivative, Maxima and minima, Gradient, Divergence and Curl, Vector identities, Directional derivatives, Line, Surface and Volume integrals, Stokes, Gauss and Green's theorems. **Differential equations :** First order equations (linear and nonlinear), Higher order linear differential equations with constant coefficients, Cauchy's and Euler's equations, Initial and boundary value problems, Laplace transforms, Solutions of one dimensional heat and wave equations and Laplace equation. **Complex variables :** Complex number, polar form of complex number, triangle inequality. **Probability and Statistics :** Definitions of probability and sampling theorems, Conditional probability, Mean, median, mode and standard deviation, Random variables, Poisson, Normal and Binomial distributions, Linear regression analysis. **Numerical Methods :** Numerical solutions of linear and non-linear algebraic equations. Integration by trapezoidal and Simpson's rule. Single and multi-step methods for numerical solution of differential equations.

Production and Industrial Engineering (PI) :

Linear Algebra : Matrix algebra, Systems of linear equations, Eigen values and Eigen vectors. **Calculus :** Functions of single variable, Limit, continuity and differentiability, Mean value theorems, Evaluation of definite and improper integrals, Partial derivatives, Total derivative, Maxima and minima, Gradient, Divergence and Curl, Vector identities, Directional derivatives; Line, Surface and Volume integrals; Stokes, Gauss and Green's theorems. **Differential Equations :** First order equations (linear and nonlinear), Higher order linear differential equations with constant coefficients, Cauchy's and Euler's equations, Initial and boundary value problems, Laplace transforms. **Complex Variables :** Analytic functions, Cauchy's integral theorem, Taylor series.

Probability and Statistics : Definitions of probability and sampling theorems, Conditional probability, Mean, median, mode and standard deviation, Linear regression, Random variables, Poisson, normal, binomial and exponential distributions. **Numerical Methods :** Numerical solutions of linear and nonlinear algebraic equations, Integration by trapezoidal and Simpson's rules, Single and multi-step methods for differential equations.

ESE Syllabus

Electrical Engineering (EE) :

Matrix theory, Eigen values & Eigen vectors, system of linear equations, Numerical methods for solution of non-linear algebraic equations and differential equations, integral calculus, partial derivatives, maxima and minima, Line, Surface and Volume Integrals. Fourier series, linear, non-linear and partial differential equations, initial and boundary value problems, complex variables, Taylor's and Laurent's series, residue theorem, probability and statistics fundamentals, Sampling theorem, random variables, Normal and Poisson distributions, correlation and regression analysis.

Contents

1 Linear Algebra

Objective Questions

1. With reference to the conventional Cartesian (x, y) coordinate system, the vertices of triangle have the following coordinates :

$(x_1, y_1) = (1, 0)$; $(x_2, y_2) = (2, 2)$ and $(x_3, y_3) = (4, 3)$. The area of the triangle is equal to

(A) $\frac{3}{2}$ (B) $\frac{3}{4}$

(C) $\frac{4}{5}$ (D) $\frac{5}{2}$

[GATE-14 (CE, SET-1)]

2. If $[A] = 6$ where $A = \begin{bmatrix} a & b & c \\ x & y & z \\ p & q & r \end{bmatrix}$ then det $[(2A)^{-1}]$ = ________.

3. Let A be $n \times n$ matrix with entries 0 and 1 and n > 1. If there is exactly one non-zero entry in each row and each column of A. Then the determinant value of A, must be ________.

[IISc]

4. Let be a 10×10 matrix in which each row has exactly one entry to 1 the remaining nine entries of the row being 0 which of the following nine entries of the row being 0 which of the following is not possible value for the determinant of the matrix.

[IISc]

(A) 0 (B) -1

(C) 10 (D) 1

5. Number of terms in the expansion of general determinant of order n is

(A) n^2 (B) n!

(C) n (D) $(n+1)^2$

[GATE-1999 (CE)]

6. Which one of the following does NOT equal $\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$? **[GATE-2013 (CS)]**

(A) $\begin{vmatrix} 1 & x(x+1) & x+1 \\ 1 & y(y+1) & y+1 \\ 1 & z(z+1) & z+1 \end{vmatrix}$

(B) $\begin{vmatrix} 1 & x+1 & x^2+1 \\ 1 & y+1 & y^2+1 \\ 1 & z+1 & z^2+1 \end{vmatrix}$

(C) $\begin{vmatrix} 0 & x-y & x^2-y^2 \\ 0 & y-z & y^2-z^2 \\ 1 & z & z^2 \end{vmatrix}$

(D) $\begin{vmatrix} 2 & x+y & x^2+y^2 \\ 2 & y+z & y^2+z^2 \\ 1 & z & z^2 \end{vmatrix}$

7. Consider the matrix

$$J_6 = \begin{vmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{vmatrix}$$

Which is obtained by reserving the order of the column of the identity matrix I_6.

Let $P = I_6 + \alpha J_6$, where α is a non-negative real number. The number of α for which det(P)=0 is ___________.

[GATE-2014 (EC, SET-1)]

8. Let $M^4 = I$ (where I denotes the identity matrix) and $M \neq I$, $M^2 \neq I$ and $M^3 \neq I$. Then, for any natural number k, M^{-1} equals :

(A) M^{4k+1} (B) M^{4k+2}

(C) M^{4k+3} (D) M^{4k}

[GATE-16 (EC-SET-1)]

9. An $n \times n$ array V is defined as follows $v[i, j] = i - j$ for all $i, j \in 1$, $i, j \in n$ then the sum of the elements of the array V is

(A) 0 (B) $n-1$

(C) $n^2 - 3n + 2$ (D) $n(n+1)$

[GATE-2000 (CS)]

10. Let A be a 3×3 matrix, whose elements are $a_{ij} = i^2 - j^2$ then $A^{-1} =$ _____ .

(A) A itself (B) Adj. A

(C) A^T (D) does not exist

11. The number of different $n \times n$ symmetric matrices with each elements being either 0 or 1 is

[GATE-2004 (CS)]

(A) 2^n (B) 2^{n^2}

(C) $2^{\frac{n^2+n}{2}}$ (D) $2^{\frac{n^2-n}{2}}$

12. The Maximum value of the determinant among all 2×2 real symmetric matrices with trace 14 is ______

[GATE-2014, (EC, SET-2)]

13. Let A, B, C, D be $n \times n$ matrices, each with non zero determinant and ABCD = I then $B^{-1} =$

(A) $D^{-1}C^{-1}A^{-1}$ (B) CDA

(C) ABC (D) does not exist

[GATE-2004 (CS)]

14. The number of different matrices that can be formed with elements 0, 1, 2 and 3; each matrix having 4 elements is

(A) 2×4^4 (B) 3×4^4

(C) 4×4^4 (D) 3×2^4

15. If X and Y are two singular matrices such that XY = Y and YX = X then $X^2 + Y^2$ equals

(A) X + Y (B) XY

(C) YX (D) 2(X + Y)

[IISc]

16. If matrix A is $m \times n$ and B is $n \times p$, the number of multiplication operations and addition operations needed to calculate the matrix AB, respectively, are:

(A) mn^2p, mpn (B) mpn, $(n-1)$

(C) mpn, $mp(n-1)$ (D) mn^2p, $(m+p)n$

[GATE-1995]

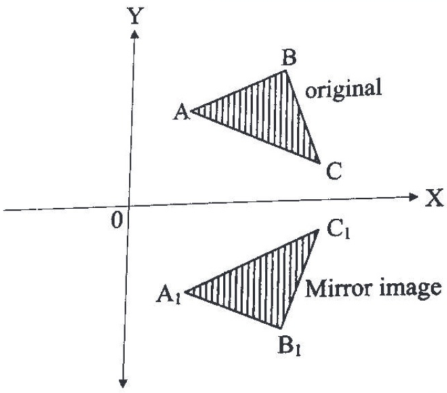
17. What is the minimum number of multiplications involved in computing the matrix product PQR? Matrix P has 4 rows and 2 columns, matrix Q has 2 rows and 4 columns, and matrix R has 4 rows and 1 column _________

[GATE-2013 (CE)]

18. Let A_1, A_2, A_3 and A_4 be four matrices of dimensions 10×5, 5×20, 20×10 and 10×5, respectively. The minimum number of scalar multiplications required to find the product $A_1A_2A_3A_4$ using the basic matrix multiplication method is ____________.

[GATE-16 (CS, SET-2)]

19. The figure shows a shape ABC and its mirror image $A_1B_1C_1$ across the horizontal axis (x-axis). The coordinate transformation matrix that maps ABC to $A_1B_1C_1$ is



[GATE-17 (IN)]

20. For the matrix $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$, $A^TA = t$ and a, b , c are positive real numbers and abc = 1, then $a^3 + b^3 + c^3$ is

21. If $A = \begin{bmatrix} 1 & -2 & -1 \\ 2 & 3 & 1 \\ 0 & 5 & -2 \end{bmatrix}$ and

$\text{Adj}\, A = \begin{bmatrix} -11 & -9 & 1 \\ 4 & -2 & -3 \\ 10 & k & 7 \end{bmatrix}$ then k = _______

(A) –5 (B) 3

(C) –3 (D) 5

22. If matrix $X = \begin{bmatrix} -a^2+a-1 & 1-a \end{bmatrix}$ and $X^2 - X + 1 = 0$ then the inverse of X is

(A) $\begin{bmatrix} 1-a & -1 \\ a^2 & a \end{bmatrix}$

(B) $\begin{bmatrix} 1-a & -1 \\ a^2-a+1 & a \end{bmatrix}$

(C) $\begin{bmatrix} a^2-a+1 & 1 \\ -a^2+a-1 & a-1 \end{bmatrix}$

(D) $\begin{bmatrix} a^2-a+1 & a \\ a & 1-a \end{bmatrix}$

[GATE-2004]

23. A sequence x[n] is specified as

$$\begin{bmatrix} x[n] \\ x[n-1] \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \text{ for } n \geq 2.$$

The initial conditions are x[0] = 1, x[1] = 1 and x[n] = 0 for n < 0. The value of x[12] is ____

[GATE-2016]

24. Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ and $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$ be two matrices such that AB = I. Let $C = A\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and CD = I. Express the elements of D in terms of the elements of B.

[GATE-96 (CS)]

25. Let $S = \{A; A = (a_{ij})_{5\times5}$ where $a_{ij} = 0$ (or) $1 \forall i, j$ and $\sum_i a_{ij} = 1 \forall j$, $\sum_j a_{ij} = 1 \forall i\}$. Then the number of elements in set S is ______

(A) 25 (B) 5^5

(C) 5! (D) 55

26. If $A = (a_{ij})_{m\times n}$ is defined as $a_{ij} = i + j \ \forall \ i, j$

(A) non-singular (B) singular

(C) transpose (D) minor

then the sum of all elements of matrix A = ______.

(A) $\frac{mn}{2}(m+n+1)$ (B) $\frac{mn}{2}(m+n+2)$

(C) $\frac{m}{2}\left(\frac{n(n+1)}{2}\right)$ (D) $\frac{n}{2}\left(\frac{m(m+1)}{2}\right)$

[GATE-1994 (PI)]

27. Let $A=\begin{bmatrix} \frac{1}{9} & -\frac{4}{9} & \frac{8}{9} \\ \frac{8}{9} & \frac{4}{9} & \frac{1}{9} \\ \frac{\alpha}{9} & -\frac{7}{9} & \frac{\beta}{9} \end{bmatrix}$ for some $\alpha, \beta \in R$.

If A is orthogonal then ___________

(A) $\alpha = -4, \beta = -4$ (B) $\alpha = 12, \beta = -5$

(C) $\alpha = 4, \beta = -4$ (D) $\alpha = -4, \beta = -3$

28. Let A be a 3×3 matrix such that $A\begin{bmatrix}2\\-1\\0\end{bmatrix}=\begin{bmatrix}2\\-1\\0\end{bmatrix}$, $A\begin{bmatrix}-1\\2\\-1\end{bmatrix}=\begin{bmatrix}-2\\4\\-2\end{bmatrix}$, $A\begin{bmatrix}0\\-1\\2\end{bmatrix}=\begin{bmatrix}0\\-3\\6\end{bmatrix}$

Suppose $Q=\begin{bmatrix}2 & -1 & 0\\-1 & 2 & -1\\0 & -1 & 2\end{bmatrix}$. Then AQ is ______

(A) $\begin{bmatrix}2 & -1 & 0\\-2 & 4 & -2\\0 & 0 & 6\end{bmatrix}$ (B) $\begin{bmatrix}2 & -1 & 0\\0 & 0 & 6\\-2 & 4 & -2\end{bmatrix}$

(C) $\begin{bmatrix}2 & -2 & 0\\-1 & 4 & -3\\0 & -2 & 6\end{bmatrix}$ (D) $\begin{bmatrix}2 & -2 & 0\\0 & -2 & 6\\-1 & 4 & -3\end{bmatrix}$

Based On Concept

29. If for a matrix, rank equals both the number of rows and number of columns, then the matrix is called

[GATE-1994 (PI)]

30. The rank of the following $(n+1)\times(n+1)$ matrix, where 'a' is a real number is

$$\begin{bmatrix}1 & a & a^2 & \dots & a^n\\1 & a & a^2 & \dots & a^n\\ \vdots & & & & \\1 & a & a^2 & \dots & a^n\end{bmatrix}$$

(A) 1

(B) 2

(C) n

(D) depends on value of a

[GATE-1996 (EE)]

31. The rank of the matrix $\begin{bmatrix}1 & 4 & 8 & 7\\0 & 0 & 3 & 0\\4 & 2 & 3 & 1\\3 & 12 & 24 & 2\end{bmatrix}$

(A) 3 (B) 1

(C) 2 (D) 4

[GATE-1998 (CS)]

32. Let $A=[a_{ij}]$ $1\le i, j\le n$ with $n\ge3$ and $a_{ij}=i\,j$ the rank of A is

(A) 0 (B) 1

(C) n – 1 (D) n

[GATE-2015 (CE, SET-2)]

33. The rank of a 3×3 matrix C (= AB), found by multiplying a non-zero column matrix A of size 3×1 and a non-zero row matrix B of size 1×3, is

(A) 0 (B) 1

(C) 2 (D) 3

[GATE-2001]

34. $x=[x_1x_2....x_n]$ is an n-tuple non-zero vector. The $n\times n$ matrix $V=xx^T$.

38. Let u and v be two vectors in R^2 whose Euclidean norms satisfy $\|u\|=2\|v\|$. What is the value of α such that

(A) has rank zero (B) has rank 1

(C) is orthogonal (D) has rank n

[GATE-2007 (CE)]

35. Two matrices A and B are given below:

$A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$ $B = \begin{bmatrix} p^2 + q^2 & pr + qs \\ pr + qs & r^2 + s^2 \end{bmatrix}$

If the rank of matrix A is N, then the rank of matrix B is

(A) $\frac{N}{2}$ (B) $N - 1$

(C) N (D) 2N

[GATE-2014 (EE, SET-3)]

36. Let $P = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$ and $Q = \begin{bmatrix} -1 & -2 & -1 \\ 6 & 12 & 6 \\ 5 & 10 & 5 \end{bmatrix}$

be two matrices. Then the rank of P + Q is ______.

[GATE-2017 (CS, SET-2)]

37. If the rank of a $P(5 \times 6)$ matrix Q is 4, then which one of the following statements is correct?

(A) Q will have four linearly independent rows and four linearly independent columns

(B) Q will have four linearly independent rows and five linearly independent columns

(C) QQ^T will be invertible

(D) Q^TQ will be invertible

[GATE-2008 (EE)]

What is the value of α such that $w = u + \alpha v$ bisects the angle between u and v?

(A) 2 (B) 1/2

(C) 1 (D) -1/2

[GATE-2017]

39. Let x and y be two vector in a 3-dimensional space and <x, y> denote their dot product. Then the determinant det $\begin{bmatrix} \langle x, x \rangle & \langle x, y \rangle \\ \langle y, x \rangle & \langle y, y \rangle \end{bmatrix} =$ ______.

(A) is zero when x and y are linearly independent

(B) is positive when x and y are linearly independent

(C) is non-zero for all non-zero x and y

(D) is zero only when either x (or) y is zero

[GATE-07 (EE)]

40. $q_1, q_2, q_3, \ldots\ldots q_m$ are n-dimensional vectors with m < n. This set of vectors is linearly dependent.

Q is the matrix with 1, 2 3, m q, q, q, ...q as the columns. The rank of Q is

(A) less than m

(B) m

(C) between m and n

(D) n

[GATE-07 (EE)]

41. Let P be 2×2 real orthogonal matrix and $\bar{x}$ is a real vector $\begin{bmatrix} x_1 & x_2 \end{bmatrix}^T$ with length $\|\bar{x}\| = \left(x_1^2 + x_2^2\right)^{1/2}$. Then which one

44. Let A be an $n \times n$ real matrix such that $A^2 = I$ and Y be an n-dimensional vector. Then the linear system of equations AX = Y has

(A) no solution

of the following statement is correct?

(A) $\|P\bar{x}\| \le \|\bar{x}\|$ where at least one vector satisfies $\|P\bar{x}\| < \|\bar{x}\|$

(B) $\|P\bar{x}\| = \|\bar{x}\|$ for all vectors $\bar{x}$

(C) $\|P\bar{x}\| \ge \|\bar{x}\|$ where atleast one vector satisfies $\|P\bar{x}\| > \|\bar{x}\|$

(D) No relationship can be established between $\|\bar{x}\|$ and $\|P\bar{x}\|$

[GATE-08 (EE)]

42. A set of linear equation is represented by the matrix equation AX = B. The necessary condition for the existence of a solution for this system is

(A) A must be invertible

(B) B must be linearly dependent of columns of A.

(C) B must be linearly independent on the columns of A.

(D) None

[GATE-1998 (EE)]

43. A is 3×4 matrix and AX = B is an inconsistent system of equations. The highest possible rank of A is

(A) 1 (B) 2

(C) 3 (D) 4

[GATE 2005 (ME)]

(A) no solution

(B) unique solution

(C) more than one but infinitely many dependent solutions

(D) infinitely many dependent solutions

[GATE 2007 (IN)]

45. Let A be 3×3 matrix with rank 2. Then AX = 0 has

(A) only the trivial solution X = 0

(B) one independent solution

(C) two independent solutions

(D) three independent solutions

[GATE-2005 (IN)]

46. Let A be an $n \times n$ matrix with rank $r(0 < r < n)$. Then AX = 0 has p independent solutions, where p is

(A) r (B) n

(C) n − r (D) n + r

[GATE-2015 (EE, SET-2)]

47. Let $c_1, \ldots\ldots\ldots, c_n$ be scalars, not all zero, such that $\sum_{i=1}^{n} c_i a_i = 0$ where a_i are column vectors in R^n. Consider the set of linear equations Ax = b. Where $A = [a_i, \ldots\ldots\ldots, a_n]$ and $b = \sum_{i=1}^{n} a_i$. The set of equations has

(A) a unique solution at $x = J_n$ where J_n denotes a n-dimensional vector of all 1

(B) no solution

(C) infinitely many solutions

(D) finitely many solutions

[GATE-2017]

48. Let $v_1 = \begin{vmatrix} 1 \\ 1 \\ 0 \end{vmatrix}$ and $v_2 = \begin{vmatrix} 0 \\ 1 \\ 1 \end{vmatrix}$ let M be the matrix whose columns are v_1, v_2, $2v_1 - v_2$, v_1, $+2v_2$ in that order. Then the

53. The dimension of the null space of the matrix $\begin{bmatrix} 0 & 1 & 1 \\ 1 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix}$ is

(A) 0 (B) 1

(C) 2 (D) 3

number of linearly independent solution of the homogeneous system of linear equations $Mx = 0$ is _______

49. Let $M=\begin{bmatrix}\alpha & 1 & 1\\ 1 & \beta & 1\\ 1 & 1 & \gamma\end{bmatrix}$, $\alpha\beta\gamma = 1$, $\alpha,\beta,\gamma \in R$ and $x=\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} \in R^3$. Then $Mx = 0$ has infinitely many solutions if trace (M) is _______

50. If the system of equations $ax+y+z=0$, $x+by+z=0$ and $x+y+cz=0$ $(a,b,c \neq 1)$ has a non-trivial solution then the value of $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}$ is,

(A) -1 (B) c
(C) 1 (D) none of these

51. The system of equation $x+2ay+az=0$, $x+3by+bz=0$, $x+4cy+cz=0$ has non-zero solution then a, b, c are

(A) in A.P (B) in G.P
(C) in H.P (D) in A.G.P

52. The system of equations $x+2y+3z=\lambda x$, $3x+y+2z=\lambda y$, $2x+3y+3z=\lambda x$ has non-zero solution when $\lambda =$ ___

(A) 2 (B) 4
(C) 6 (D) 8

[GATE-2013 (IN)]

54. Consider the following system of linear equation $x_1+2x_2=b_1$; $2x_1+4x_2=b_2$; $3x_1+7x_2=b_3$; $3x_1+9x_2=b_4$.

Which one of the following conditions ensures that a solution exists for the above system?

(A) $b_2=2b_1$ and $3b_1-6b_3+b_4=0$

(B) $b_3=2b_1$ and $3b_1-6b_3+b_4=0$

(C) $b_3=2b_1$ and $6b_1-3b_3+b_4=0$

(D) $b_2=2b_1$ and $6b_1-3b_3+b_4=0$

[GATE-20 (EC)]

55. The number of purely real elements in a lower triangular representation of the given 3×3 matrix, obtained through the given decomposition is __________.

$$\begin{bmatrix}2 & 3 & 3\\ 3 & 2 & 1\\ 3 & 1 & 7\end{bmatrix}=\begin{bmatrix}a_{11} & 0 & 0\\ a_{12} & a_{22} & 0\\ a_{13} & a_{23} & a_{33}\end{bmatrix}\begin{bmatrix}a_{11} & 0 & 0\\ a_{12} & a_{22} & 0\\ a_{13} & a_{23} & a_{33}\end{bmatrix}^T$$

56. If $v_1, v_2, \ldots\ldots, v_6$ are six vectors in R^4, which one of the following statements is FALSE?

(A) These vectors are not linearly independent

(B) Any four of these vectors form a basis for R^4

(C) It is not necessary that these vectors span R^4

(D) If $\{v_1, v_3, v_5, v_6\}$ spans R^4, then it forms a basis for R^4

57. If the vectors $e_1=(1,0,2)$, $e_2=(0,1,0)$ and $e_3=(-2,0,1)$ form an orthogonal basis of the three dimensional real space R^3, then the vector $u=(4,3,-3)\in R^3$ can be expressed as

(A) 2M

(B) M + 1

(C) M

(D) dependent on the choice of $X_1, X_2, \ldots . X_M$

[GATE-2007, 2 MARKS]

(A) $u = -\frac{2}{5}e_1 - 3e_2 - \frac{11}{5}e_3$

(B) $u = -\frac{2}{5}e_1 - 3e_2 + \frac{11}{5}e_3$

(C) $u = -\frac{2}{5}e_1 + 3e_2 + \frac{11}{5}e_3$

(D) $u = -\frac{2}{5}e_1 + 3e_2 - \frac{11}{5}e_3$

[GATE-16-EC-SET 3]

58. An orthogonal set of vectors having a span that contains P, Q, R is ______, where $P = (-10,-1,3)'$, $Q = (-2,-5,9)'$ & $R = (2,-7,12)'$ are three vectors.

(A) $\begin{bmatrix}-6\\-3\\6\end{bmatrix}\begin{bmatrix}4\\-2\\3\end{bmatrix}$ (B) $\begin{bmatrix}-4\\2\\4\end{bmatrix}\begin{bmatrix}5\\7\\-11\end{bmatrix}\begin{bmatrix}6\\2\\-3\end{bmatrix}$

(C) $\begin{bmatrix}6\\7\\-1\end{bmatrix}\begin{bmatrix}-3\\2\\-2\end{bmatrix}\begin{bmatrix}3\\9\\-4\end{bmatrix}$ (D) $\begin{bmatrix}4\\3\\11\end{bmatrix}\begin{bmatrix}1\\31\\3\end{bmatrix}\begin{bmatrix}5\\3\\4\end{bmatrix}$

[GATE-2006, 2 MARKS]

59. If V_1 and V_2 are 4-dimensional subspaces of a 6-dimensional vector space V, then the smallest possible dimensions of $V_1 \cap V_2$ is ________

[GATE-2014 (CS-SET 3)]

60. It is given that $X_1, X_2, \ldots X_M$ are M non-zero, orthogonal vectors. The dimension of the vector space spanned by the 2M vectors $X_1, X_2, \ldots X_M$, $-X_1, -X_2, \ldots -X_M$ is

61. The vectors (2, 2, 0); (3, 0, 2) & (2, -2, 2) are

(A) linearly independent and a basis of R^3

(B) linearly dependent and a basis of R^3

(C) linearly independent and not a basis of R^3

(D) linearly dependent and not a basis of R^3

62. For what values of 'k' $\{(k,1,1); (0,1,1); (k,0,k)\}$ from a basis of R^3.

63. Consider the set of (column) vector defined by $X = \{\{x \notin R^3 \mid x_1 + x_2 + x_3 = 0$, where $x^T = [x_1, x_2, x_3]^T\}$. Which of the following TRUE?

(A) $\{[1,-1,0]^T, [1,0,-1]^T\}$ is a basis for the subspace X.

(B) $\{[1,-1,0]^T, [1,0,-1]^T\}$ is a linearly independent set, but it does not span X and therefore is not a basis of X

(C) X is not a subspace for R^3

(D) None of the above

[GATE-2007 (CS), 2 MARKS]

64. Let A be an $m \times n$ matrix with rank r, the dimension of the space of solutions of the system of linear equations AX = 0 is

(A) r (B) n − r

(C) m − r (D) min {m, n} − r

[GATE]

65. The dimension of the null space of the

68. The product of Eigen values of the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & \ldots & 0 \\ 1 & \frac{1}{2} & 0 & 0 & \ldots & 0 \\ 1 & \frac{1}{2} & \frac{1}{3} & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots \end{bmatrix}$$ is ______

matrix $\begin{bmatrix} 0 & 1 & 1 \\ 1 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix}$ is

(A) 0 (B) 1

(C) 2 (D) 3

[GATE-2013 (IN), 1 MARK]

66. The Eigen values of $A = \begin{bmatrix} a & 1 & 0 \\ 1 & a & 1 \\ 0 & 1 & a \end{bmatrix}$ are ________.

(A) a, a, a (B) 0, a, 2a

(C) -a, 2a, 2a (D) $a, a+\sqrt{2}, a-\sqrt{2}$

67. The sum of Eigen values of the matrix

$$A = \begin{bmatrix} \frac{1}{1\cdot 2} & 0 & 0 & 0 & \dots & 0 \\ \frac{1}{1\cdot 2} & \frac{1}{2\cdot 3} & 0 & 0 & \dots & 0 \\ \frac{1}{1\cdot 2} & \frac{1}{2\cdot 3} & \frac{1}{3\cdot 4} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ \frac{1}{1\cdot 2} & \frac{1}{2\cdot 3} & \frac{1}{3\cdot 4} & \frac{1}{4\cdot 5} & \dots & \frac{1}{n(n+1)} \end{bmatrix}$$

is ________.

(A) $\frac{1}{n}$ (B) $1-\frac{1}{n+1}$

(C) $\frac{1}{n!}$ (D) $\frac{2}{n(n+1)}$

$\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \dots & \frac{1}{n} \end{bmatrix}$

(A) n^2+n+1 (B) $\frac{n(n+1)}{2}$

(C) $\frac{1}{n!}$ (D) $\frac{1}{n}$

69. A real $n \times n$ matrix $A = \left[a_{ij}\right]$ is defined as $a_{ij} = \begin{cases} i & \forall i = j \\ 0 & \text{otherwise} \end{cases}$

70. The sum of all n Eigen value of matrix A is

(A) $\frac{n(n+1)}{2}$ (B) $\frac{n(n-1)}{2}$

(C) n^2 (D) $\frac{n(n+1)(2n+1)}{6}$

[GATE]

71. Suppose the matrix

$A = \begin{bmatrix} 40 & -29 & -11 \\ -18 & 30 & -12 \\ 26 & 24 & -50 \end{bmatrix}$ has a certain complex number $\lambda \neq 0$ as an Eigen value. Which of the following must also be an Eigen value of A.

(A) $\lambda + 20$ (B) $\lambda - 20$

(C) $20 - \lambda$ (D) $-20 - \lambda$

[CSIR]

72. Consider the 5×5 matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 1 & 2 & 3 & 4 \\ 4 & 5 & 1 & 2 & 3 \\ 3 & 4 & 5 & 1 & 2 \\ 2 & 3 & 4 & 5 & 1 \end{bmatrix}$$

It is given that A has only one real eigen value. Then the real eigen value of A is

(A) – 2.5 (B) 0

(C) n distinct paris of complex conjugate numbers

(D) n pairs of complex conjugate numbers, not necessarily distinct

[GATE-2007 (PI)]

76. The eigen values of a 2×2 matrix X are -2 and -3. The eigen values of matrix $(X+I)^{-1}(X+5I)$ are

(C) 15 (D) 25

[GATE-2017 (EC), 1 MARK]

73. The product of the non-zero eigen values of the matrix $\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$ is

_______ **[GATE-2014 (CS), 2 MARKS]**

74. The linear operation L(x) is defined by the cross product L(x) = b × X, where $b = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T$ and $X = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T$ are three dimensional vectors. The 3×3 matrix M of this operation satisfies $L(x) = M \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ then the eigen values of M are

(A) 0, +1, -1 (B) 1, -1, 1

(C) i, -i, 1 (D) j, -i, 0

[GATE-2007]

75. If A is square symmetric real values matrix of dimensions 2n, then the eigen values of A are

(A) 2n distinct real values

(B) 2n real values not necessarily distinct

(A) -3, -4 (B) -1, -2

(C) -1, -3 (D) -2, -4

[GATE-2009 (IN)]

77. For the matrix A satisfying the equation given below, the eigen values are

$$\begin{bmatrix} A \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 7 & 8 & 9 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

(A) (1, -j, j) (B) (1, 1, 0)

(C) (1, 1, -1) (D) (1, 0, 0)

[GATE-2014 (IN, SET-1)]

78. A real 4×4 matrix A satisfies the equation $A^2 = I$, where I is the 4×4 identity matrix. The positive eigen value of A is _________.

[GATE (EC, SET-1)]

79. Let the eigen values of a 2×2 matrix A be 1, -2 with eigen vectors x_1 and x_2 respectively. Then the eigen values and eigen vectors of the matrix $A^2 - 3A + 4I$ would, respectively, be

(A) $2, 14, x_1, x_2$

(B) $2, 14, x_1 + x_2, x_1 - x_2$

(C) $2, 0, x_1, x_2$

(D) $2, 0, x_1 + x_2, x_1 - x_2$

[GATE-2016 (EE, SET-1)]

80. The value of x for which all the eigen values of the matrix given below are real is $\begin{bmatrix} 10 & 5+j & 4 \\ x & 20 & 2 \\ 4 & 2 & -10 \end{bmatrix}$

(A) 5 + j (B) 5 – j

(C) 1 – 5j (D) 1 + 5j

[GATE-2015 (EC, SET-2)]

81. Consider the matrix $A = \begin{bmatrix} 50 & 70 \\ 70 & 80 \end{bmatrix}$

84. For the matrix $M = \begin{bmatrix} 2 & 3+2i & -4 \\ 3-2i & 5 & 6i \\ -4 & -6i & 3 \end{bmatrix}$

Which of the following are correct

P : M is skew-Hermition and iM is Hermitian

Q : M is Hermitian and iM is skew – Hermition

R : Eigen values of M are real

whose eigen vectors corresponding to eigen values λ_1 and λ_2 are

$$X_1 = \begin{bmatrix} 70 \\ \lambda_1 - 50 \end{bmatrix} \text{ and } X_2 = \begin{bmatrix} \lambda_2 - 80 \\ 70 \end{bmatrix}$$

respectively. The value of $X_1^T X_2$ is

_____

82. Consider the matrix $A = \begin{bmatrix} 0 & 1 & 2 & 0 \\ 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 2 \\ 0 & 0 & 2 & 0 \end{bmatrix}$.

Then _____

(A) A has no real Eigen value

(B) All Eigen values of A are positive and real

(C) All real Eigen values of A are negative

(D) A has both positive and negative real Eigen values.

83. Let $M = \begin{bmatrix} 1 & 1+i & 2i & 9 \\ 1-i & 3 & 4 & 7-i \\ -2i & 4 & 5 & i \\ 9 & 7+i & -i & 7 \end{bmatrix}$ then

(A) M has only real Eigen values

(B) M has only imaginary Eigen values

(C) All Eigen values of M are zero

(D) None of the above

S : Eigen values of iM are real

(A) P and R only (B) Q and R only

(C) P and S only (D) Q and S only

85. If a 3×3 real skew symmetric matrix has an Eigen value 2i, then one of the remaining Eigen value is _________.

(A) $\frac{1}{2!}$ (B) $-\frac{1}{2!}$

(C) 0 (D) 1

86. If $A = \begin{bmatrix} 1 & 0 & 0 \\ i & \frac{-1+i\sqrt{3}}{2} & 0 \\ 0 & 1+2i & \frac{-1-i\sqrt{3}}{2} \end{bmatrix}$ then trace of A^{102} is

(A) 0 (B) 1

(C) 2 (D) 3

87. Let $\alpha = e^{2\pi/5}$ and matrix

$$M = \begin{bmatrix} 1 & \alpha & \alpha^2 & \alpha^3 & \alpha^4 \\ 0 & \alpha & \alpha^2 & \alpha^3 & \alpha^4 \\ 0 & 0 & \alpha^2 & \alpha^3 & \alpha^4 \\ 0 & 0 & 0 & \alpha^3 & \alpha^4 \\ 0 & 0 & 0 & 0 & \alpha^4 \end{bmatrix}$$ then trace of

the matrix $I + M + M^2 =$ ______.

[GATE]

88. The condition for which the eigen values of the matrix $A = \begin{bmatrix} 2 & 1 \\ 1 & k \end{bmatrix}$ are positive, is

(A) $k > \frac{1}{2}$ (B) $k > -2$

(C) $k > 0$ (D) $k < -\frac{1}{2}$

[GATE-2016 (ME), 1 MARK]

89. Let a, b, c are positive real numbers such that $b^2 + c^2 < a < 1$. Consider the

92. Which of the following is true?

I. The matrix $\begin{bmatrix} 2 & 2 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$ is diagonalizable

II. The matrix $\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$ is diagonalizable

(A) Only I

(B) Only II

(C) Both (I) and (II)

3×3 matrix $A = \begin{bmatrix} 1 & b & c \\ b & a & 0 \\ c & 0 & 1 \end{bmatrix}$

(A) All the Eigen values of A are negative real numbers

(B) All the Eigen values of A are positive real numbers

(C) A can have a positive as well as negative Eigen value

(D) Eigen values of A can be complex numbers

90. Let A be a 4×4 matrix with real entries such that -1, 1, 2, -2 are its Eigen values. If $B = A^4 - 5A^2 + 5I$ then which of the following is incorrect.

(A) det (A+B) = 0

(B) trace of A + B is 4

(C) det B = 1

(D) trace of A – B is 0

91. Let A be a 3×3 matrix such that $|A - I| = 0$. If trace of A = 13 and det A = 32 then the sum of squares of the eigen values of A is ________

(A) 82 (B) 13

(C) 169 (D) 81

[GATE]

(D) None of these

93. Let A be $n\times n$ real valued square symmetric matrix of rank 2 with $\sum_{i=1}^{n}\sum_{j=1}^{n} A_{ij}^2 = 50$. Consider the following statements.

(I) One eigen value must be in [-5, 5]

(II) The eigen value with the largest magnitude must be strictly greater than 5.

Which of the above statements about eigen values of A is / are necessarily correct?

(A) Both (I) and (II)

(B) (I) only

(C) (II) only

(D) Neither (I) nor (II)

[GATE-2017 (CS/IT)]

94. Consider a matrix P whose only eigen vectors are the multiples of $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$

Consider the following statements.

(I) P does not have an inverse

(II) P has a repeated eigen value

(III) P cannot be diagonalized

Which one of the following options is correct?

(A) Only I and III are necessarily true

(B) Only II is necessarily true

(C) Only I and II are necessarily true

(D) Only II and III are necessarily true

[GATE-18 (CS/IT)]

95. Let M be a real 4×4 matrix. Consider the following statements:

S1: M has 4 linearly independent eigen vectors

(A) $A - I$ (B) $A + I$

(C) $-I$ (D) 0

[GATE]

99. Let A be an $n\times n$ complex matrix whose characteristic polynomial is $f(t) = t^n + c_{n-1}t^{n-1} + ... + c_1 t + c_0$ then ____

(A) $\det A = c_{n-1}$

(B) $\det A = c_0$

(C) $\det A = (-1)^{n-1} c_{n-1}$

(D) $\det A = (-1)^n c_0$

S2: M has 4 distinct eigen values.

S3: M is non-singular (invertible).

Which one among the following is TRUE?

(A) S1 implies S2 (B) S1 implies S3

(C) S2 implies S1 (D) S3 implies S2

[GATE-2018 (EC)]

96. Number of linearly independent eigen vectors of matrix $A=\begin{bmatrix}2&0&1\\0&2&1\\0&0&3\end{bmatrix}$ is _____

(A) 1

(B) 2

(C) 3

(D) cannot be determined

97. The number of linearly independent eigen vectors of matrix $A=\begin{bmatrix}2&0&1\\0&2&1\\0&0&3\end{bmatrix}$ is ______. **[GATE-2016 (ME), 2 MARKS]**

98. In matrix $A=\begin{bmatrix}a&b\\c&d\end{bmatrix}$ if $a+d=ad-bc=1$ then $A^3=$ _____

[GATE]

100. Let $A=\begin{bmatrix}1&0&-1\\-1&2&0\\0&0&-2\end{bmatrix}$ and $B=A^3-A^2-4A+5I$, where I is the 3×3 identity matrix. The determinant of B is _____ (upto 1 decimal place).

[GATE-2018 (EE), 2 MARKS]

101. The constant term of the characteristic polynomial of the matrix $\begin{bmatrix}1&2&6&5\\-1&3&2&-5\\2&4&12&10\\3&-2&1&-4\end{bmatrix}$ is ______.

(A) 0 (B) 1

(c) trace of A (D) 2

102. Let $P=\begin{bmatrix}0&-2&-3\\-1&1&-1\\a&2&b\end{bmatrix}$ for some $a,b\in R$, suppose 1 and 2 are eigen values of P and $P\begin{bmatrix}1\\0\\-1\end{bmatrix}=\begin{bmatrix}3\\0\\-3\end{bmatrix}$ then $P^4\begin{bmatrix}1\\-1\\0\end{bmatrix}$ is ______.

(A) $\begin{bmatrix}1\\-1\\0\end{bmatrix}$ (B) $\begin{bmatrix}1\\1\\0\end{bmatrix}$

(C) $\begin{bmatrix}16\\16\\0\end{bmatrix}$ (D) $\begin{bmatrix}16\\-16\\0\end{bmatrix}$

[CSIR]

103. Consider a 3×3 real symmetric matrix S such that two of its eigen values are $a\neq0$, $b\neq0$ with respective eigen vectors $\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix},\begin{bmatrix}y_1\\y_2\\y_3\end{bmatrix}$.

107. The minimal polynomial of the matrix $\begin{bmatrix}3&3&0\\3&3&0\\0&0&6\end{bmatrix}$ is

(A) $x(x-1)(x-6)$ (B) $x(x-3)$

(C) $(x-3)(x-6)$ (D) $x(x-6)$

108. If a square matrix of order 100 has exactly 15 distinct eigen values, then the degree of the minimal polynomial is

(A) At least 15 (B) At most 15

(C) Always 15 (D) Exactly 100

[ESE 2017 (EE)]

If $a \neq b$ then $x_1y_1 + x_2y_2 + x_3y_3$ equals

(A) a
(B) b
(C) ab
(D) 0

[GATE-2014 (ME, SET-3)]

104. Let M be a 3×3 real symmetric with eigen values 0, 2, a with respective eigen vectors $u = (4, b, c)^T$, $v = (-1, 2, 0)^T$, $w = (1, 1, 1)^T$ then $a + b - c =$ ______.

105. Let $\alpha, \beta, \gamma, \delta$ be the eigen values of the matrix $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$.

The $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 =$ ______

106. In the LU decomposition of the matrix $\begin{bmatrix} 2 & 2 \\ 4 & 9 \end{bmatrix}$, if the diagonal elements of U are both 1, then the lower diagonal entry l_{22} of L is ________.

[GATE]

109. Let the sets of eigenvalues and eigenvectors of a matrix *B* be $\{\lambda_k \mid 1 \le k \le n\}$ and $\{V_k \mid 1 \le k \le n\}$, respectively. For any invertible matrix *P*, the sets of eigenvalues and eigenvectors of the matrix *A*, where $B = P^{-1}AP$, respectively, are

(A) $\{\lambda_k \det(A) \mid 1 \le k \le n\}$ and $\{PV_k \mid 1 \le k \le n\}$

(B) $\{\lambda_k \mid 1 \le k \le n\}$ and $\{V_k \mid 1 \le k \le n\}$

(C) $\{\lambda_k \mid 1 \le k \le n\}$ and $\{PV_k \mid 1 \le k \le n\}$

(D) $\{\lambda_k \mid 1 \le k \le n\}$ and $\{P^{-1}V_k \Big| \le 1 \le k \le n\}$

[GATE-2023 (EC)]

110. Let *x* be an $n \times 1$ real column vector with length $l = \sqrt{x^T x}$. The trace of matrix $P = xx^T$ is

(A) l^2
(B) $\frac{l^2}{4}$
(C) l
(D) $\frac{l^2}{2}$

[GATE-2023 (EC)]

111. The state equation of a second order system is $\dot{x}(t) = Ax(t)$, $x(0)$ is the initial condition.

Suppose λ_1 and λ_2 are two distinct Eigen values of *A* and V_1 and V_2 are the corresponding Eigen vectors. For constants α_1 and α_2, the solution, $x(t)$, of the state equation is

(A) $\sum_{i=1}^{2} \alpha_i e^{\lambda_i t} V_i$
(B) $\sum_{i=1}^{2} \alpha_i e^{2\lambda_i t} V_i$
(C) $\sum_{i=1}^{2} \alpha_i e^{3\lambda_i t} V_i$
(D) $\sum_{i=1}^{2} \alpha_i e^{4\lambda_i t} V_i$

[GATE-2023 (EC)]

112. Choose solution set *S* corresponding to the systems of two equations

$x - 2y + z = 0$

$x - z = 0$

Note :R denotes the set of real numbers

114. If *M* is an arbitrary real $n \times n$ matrix, then which of the following matrices will have non-negative eigenvalues?

(A) M^2
(B) MM^T
(C) M^TM
(D) $(M^T)^2$

[GATE-2023 (CE-1)]

115. For the matrix $[A] = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix}$, which of the following statements is/are TRUE?

(A) The eigenvalues of $[A]^T$ are same as the eigenvalues of $[A]$

(B) The eigenvalues of $[A]^{-1}$ are the reciprocals of the eigenvalues of $[A]$

(C) The eigenvectors of $[A]^T$ are same as the eigenvectors of $[A]$

(D) The eigenvectors of $[A]^{-1}$

numbers

(A) $S = \left\{ \alpha \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \middle| \alpha \in R \right\}$

(B) $S = \left\{ \alpha \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \middle| \alpha, \beta, \in R \right\}$

(C) $S = \left\{ \alpha \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \middle| \alpha, \beta, \in R \right\}$

(D) $S = \left\{ \alpha \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \middle| \alpha \in R \right\}$

[GATE-2023 (IN)]

113. The rank of the matrix A given below is one. The ratio $\frac{\alpha}{\beta}$ is ________.(rounded off to the nearest integer)

$$A = \begin{bmatrix} 1 & 4 \\ -3 & \alpha \\ \beta & 6 \end{bmatrix}$$

[GATE-2023 (IN)]

(D) The eigenvectors of $[A]^{-1}$ are same as the eigenvectors of $[A]$

[GATE-2023 (CE-1)]

116. For the matrix

$$[A] = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

Which of the following statements is/are TRUE?

(A) $[A]\{x\} = \{b\}$ has a unique solution

(B) $[A]\{x\} = \{b\}$ does not have a unique solution

(C) $[A]$ has three linearly independent eigenvectors

(D) $[A]$ is a positive definite matrix

[GATE-2023 (CE-2)]

117. Two vectors $\begin{bmatrix} 2 & 1 & 0 & 3 \end{bmatrix}^T$ and $\begin{bmatrix} 1 & 0 & 1 & 2 \end{bmatrix}^T$ belong to the null space of a 4 × 4 matrix of rank 2. Which one of the following vectors also belongs to the null space?

(A) $\begin{bmatrix} 1 & 1 & -1 & 1 \end{bmatrix}^T$ (B) $\begin{bmatrix} 2 & 0 & 1 & 2 \end{bmatrix}^T$

(C) $\begin{bmatrix} 0 & -2 & 1 & -1 \end{bmatrix}^T$ (D) $\begin{bmatrix} 3 & 1 & 1 & 2 \end{bmatrix}^T$

[GATE-2023 (CE-2)]

118. Cholesky decomposition is carried out on the following square matrix $[A]$.

$$[A] = \begin{bmatrix} 8 & -5 \\ -5 & a_{22} \end{bmatrix}$$

Let l_{ij} and a_{ij} be the $(i, j)^{th}$ elements of matrices $[L]$ and $[A]$, respectively. If the element l_{22} of the decomposed lower triangular matrix $[L]$ is 1.968, what is the value (rounded off to the nearest integer) of the element a_{22}?

(A) 5 (B) 7

(C) 9 (D) 11

[GATE-2023 (CE-2)]

119. Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \\ 3 & 4 & 1 & 2 \\ 2 & 3 & 4 & 1 \end{bmatrix}$ and

Let $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ and λ_5 be the five eigenvalues of A. Note that these eigenvalues need not be distinct.

The value of $\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 =$ ________.

[GATE-2023 (CSE)]

121. Let $P = \begin{bmatrix} 4 & -2 & 2 \\ 6 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$, $Q = \begin{bmatrix} 3 & -2 & 2 \\ 1 & -4 & 6 \\ 2 & -3 & 5 \end{bmatrix}$. The eigen value of both P & Q are 1, 1 & 2.

(A) Both P&Q are diagonalizable

(B) Both P&Q are not diagonalizable

(C) P is diagonalizable and Q is not.

(D) Q is diagonalizable and P is not.

[GATE-2023 (XE)]

122. Let A be a 3×3 square matrix having eigen values 1, 2 & 3. If $B = A^2 + 2A + I$

$$B = \begin{bmatrix} 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{bmatrix}$$

Let det (A) and det (B) denote the determinants of the matrices A and B, respectively. Which one of the options given below is true ?

(A) $\det(A) = \det(B)$

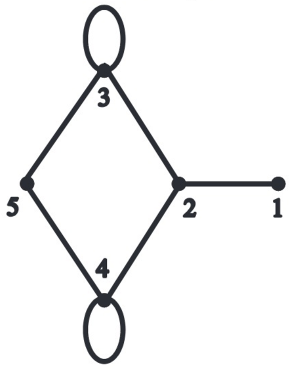
(B) $\det(B) = -\det(A)$

(C) $\det(A) = 0$

(D) $\det(AB) = \det(A) + \det(B)$

[GATE-2023 (CSE)]

120. Let A be the adjacency matrix of the graph with vertices {1, 2, 3, 4, 5}.



where I is the 3×3 identity matrix, then the eigen values of B are

(A) 4, 9, 16 (B) 9, 16, 25

(C) 1, 4, 9 (D) 1, 2, 3

[GATE-2023 (XE)]

❑❑❑

2 Calculus (Part-1)

Objective Questions

1. $\lim\limits_{x\to\infty}\left(e^{\frac{1}{5x}}-1\right)\left(5x+\frac{x}{5}\sin\frac{1}{x}\right) =$ ______

[CSIR]

2. The values of a and b for which is function

$$f(x) = \begin{cases} 2x+1, & \text{if } x \le 1 \\ ax^2+b, & \text{if } 1 < x < 3 \\ 5x+2a & \text{if } x \ge 3 \end{cases}$$

is continuous every where

(A) a = 2, b = 1 (B) a = 1, b = 2

(C) a = 3, b = 2 (D) a = 2, b = 3

(A) – 4 (B) 0

(C) 2 (D) 4

7. Let $f(x) = \begin{cases} \dfrac{\sin x}{x}, & x \ne 0 \\ 1, & x = 0 \end{cases}$

Then

(A) f is not continuous at $x = 0$

(B) f is continuous at $x = 0$ but not differentiable $x = 0$

(C) f is differentiable at $x = 0$ and $f'(0) = 20$

(D) f is differentiable at $x = 0$ and

[IISC]

3. $\lim\limits_{x \to 0} e^x (\cos x) \sin^{\frac{1}{2}} x =$

(A) 1 (B) $e^{-1/2}$

(C) $e^{1/2}$ (D) e

[CSIR]

4. The values of a and b such that

$\lim\limits_{x \to 0} \dfrac{a \sin^2 x + b \log(\cos x)}{x^4} = \dfrac{1}{2}$

(A) -1, -2 (B) 1, 2

(C) -1, 2 (D) 1, -2

[IAS]

5. $\lim\limits_{x \to 0} \dfrac{x(e^x - 1) + 2(\cos x - 1)}{x(1 - \cos x)} =$ ________

[GATE-1993 (ME)]

6. Let $L = \underset{x \to \pi/2}{\mathrm{Lt}} \dfrac{\sin^2 2x}{(x - \pi/2)^2}$. Then L is equal to ________.

(D) f is differentiable at $x = 0$ and $f'(0) = 1$

8. What should be the value of λ such that the function defined below is continuous at

$x = \pi/2 \, ? \, f(x) = \begin{cases} \dfrac{\lambda \cos x}{\frac{\pi}{2} - x} & \text{if } x \neq \pi/2 \\ 1 & \text{if } x = \pi/2 \end{cases}$

(A) zero (B) $2/\pi$

(C) 1 (D) $\pi/2$

[GATE-2011-CE]

9. Which one of the following function is continuous at $x = 3$?

(A) $f(x) = \begin{cases} 2, & \text{if } x = 3 \\ x - 1, & \text{if } x > 3 \\ \dfrac{x+3}{3}, & \text{if } x < 3 \end{cases}$

(B) $f(x) = \begin{cases} x + 3, & \text{if } x \leq 3 \\ x - 4, & \text{if } x > 3 \end{cases}$

(C) $f(x) = \begin{cases} 4, & \text{if } = 3 \\ 8 - x & \text{if } x \neq 3 \end{cases}$

(D) $f(x) = \dfrac{1}{x^3 - 27}$, if $x \neq 3$

[GATE-2013 (CS)]

10. The value of $\lim\limits_{(x,y) \to (0,0)} \dfrac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$ is

(A) 0 (B) $\dfrac{1}{2}$

(C) 1 (D) ∞

[GATE-2015 (PI)]

11. $f(x) = \begin{cases} x^a \sin \dfrac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ is differentiable at $x = 0$ for all a in the inter values

(A) $(-\infty, 1)$ (B) $(-1, \infty)$

14. Let $g(x) = \begin{cases} -x, & x \leq 1 \\ x + 1, & x \geq 1 \end{cases}$ and

$f(x) = \begin{cases} 1 - x, & x \leq 0 \\ x^2, & x > 0 \end{cases}$.

Consider the composition of f and g, i.e., $(f \circ g)(x) = f(g(x))$. The number of discontinuities in $(f \circ g)(x)$ present in the interval $(-\infty, 0)$ is

(A) 0 (B) 1

(C) 2 (D) 4

[GATE-17-EE]

15. A function $f(x)$ is defined as $f(x) = \begin{cases} e^x & , x < 1 \\ \ln x + ax^2 + bx & , x \geq 1 \end{cases}$, where $x \in R$. Which one of the following statement is TRUE?

(A) f(x) is NOT differentiable at x = 1 for

(C) $(1,\infty)$ (D) $(-1,-\infty)$

12. The values of x for which the function $f(x)=\dfrac{x^2-3x-4}{x^2+3x-4}$ is NOT continuous are

(A) 4 and -1 (B) 4 and 1

(C) -4 and 1 (D) -4 and -1

[GATE-16-ME-SET2]

13. At $x=0$, the function is

$$f(x)=\left|\sin\frac{2\pi x}{L}\right|(-\infty<x<\infty, L>0)$$

(A) continuous and differentiable

(B) not continuous and not differentiable

(C) not continuous but differentiable

(D) continuous but not differentiable

[GATE-16-PI-SET1]

any values of a and b.

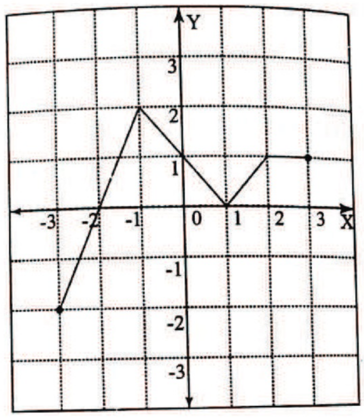
(B) f(x) is differentiable at x = 1 for the unique values of a and b.

(C) f(x) is differentiable at x = 1 for all values of a and b such that a + b = e.

(D) f(x) is differentiable at x = 1 for all values of a and b.

[GATE-17-EE]

16. Which of the following functions is an accurate description of the graph for the range(s) indicated?



(i) $y=2x+4$ for $-3\le x\le -1$

(ii) $y=|x-1|$ for $-1\le x\le 2$

(iii) $y=||x|-1|$ for $-1\le x\le 2$

(iv) $y=1$ for $2\le x\le 3$

(A) i, ii and iii only (B) i, ii and iv only

(C) i and iv only (D) ii and iv only

[GATE-18-CE-SET]

17. Consider the function $f(x)=|x|^3$, where x is real. Then the function $f(x)$ at $x=0$ is

(A) continuous but not differentiable

(B) once differentiable but not twice

(C) twice differentiable but not thrice.

(D) thrice differentiable

[GATE-07 (IN)]

(A) $x=y-|y|$ (B) $x=-(y-|y|)$

(C) $x=y+|y|$ (D) $x=-(y+|y|)$

[GATE-15-CS-SET 3]

20. If $y=|x|$ for $x<0$ and $y=x$ for $x\ge 0$ then

(A) $\dfrac{dy}{dx}$ is discontinuous at $x=0$

(B) y is discontinuous at $x=0$

(C) y is not defined at $x=0$

(D) Both y and $\dfrac{dy}{dx}$ are discontinuous at $x=0$

[GATE-97]

21. A discontinuous real function can be expressed as

(A) Taylor's series and Fourier's series

(B) Taylor's series and not by Fourier's

18. The function $f(x)=|x+1|$ on the interval [-2, 0] is ___________.

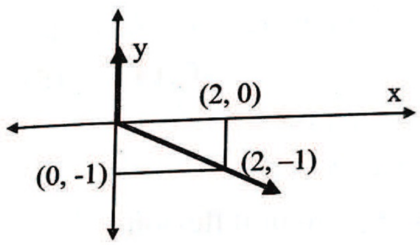
(A) continuous and differentiable

(B) continuous on the interval but not differentiable at all points

(C) Neither continuous nor differentiable

(D) Differentiable but not continuous

19. Choose the most appropriate equation for the function drawn as a thick line, in the plot below.



series

(C) Neither Taylor'sseries nor Fourier's series

(D) not by Taylor's series, but by Fourier's

[GATE-98]

22. The polynomial p(x) = $x^5 + x + 2$ has

(A) all real roots

(B) 3 real and 2 complex roots

(C) 1 real and 4 complex roots

(D) all complex roots

[GATE-2007 (IN)]

23. A polynomial

$$f(x)=a_4x^4+a_3x^3+a_2x^2+a_1x-a_0$$

with all coefficients positive has

(A) no real roots

(B) no negative real root

(C) odd number of real roots

(D) at least one positive and one negative real root

[GATE-2013]

24. The number of roots $e^x+0.5x^2-2=0$ in the range [-5, 5]is

(A) 0 (B) 1

(C) 2 (D) 3

[GATE-2017 PAPER 2 (CS)]

25. Given that 0.8 is one root of the equation, $x^3-0.6x^2-1.84x+1.344=0$. The other roots of this equation will be

(A) 1.1 and -1.4 (B) -1.2 and 1.4

(C) 1.2 and -1.4 (D) -1.1 and 1.4

[ESE 2018 (COMMON PAPER)]

26. A non-zero polynomial f(x) of degree 3 has roots at x = 1, x = 2 and x = 3. Which one of the following must be TRUE?

28. If a continuous function f(x) does not have a root in the interval [a, b], then which one of the following statements is TRUE?

(A) f(a) . f(b) = 0 (B) f(a) . f(b) > 0

(C) f(a) . f(b) < 0 (D) f(a) / f(b) $\leq$ 0

[GATE-2015 (EE-SET1)]

29. By applying Lagranges mean value for the function $f(x)=(1+x)\log(1+x)$ on $[0,1]$ value of $c\in(0,1)$ is

(A) $\frac{4}{e}$ (B) $\frac{1}{e}$

(C) $\frac{4-e}{e}$ (D) $\frac{1-e}{e}$

30. Curve 'C' is defined as $x=a\cos^3\theta$, $y=a\sin^3\theta$ in $[0, \pi/2]$. What will be the point P on curve C where the tangent to the curve is parallel to the chord joining points (a, 0) & (0, a).

(A) (a, a) (B) $\left(\frac{a}{2}, \frac{a}{2}\right)$

(A) f(0)f(4) < 0 (B) f(0)f(4) > 0

(C) f(0) + f(4) < 0 (D) f(0) + f(4) > 0

[GATE-14 (CS-SET 2)]

27. A function f(x) is continuous in interval (0, 2). It is known that f(0) = f(2) = -1 and f(1) = 1. Which one of the following statements must be true?

(A) There exists a 'y' in the interval (0, 1) such that f(y) = f(y + 1)

(B) For every 'y' in the interval (0,.1), f(y) = f(2-y)

(C) the maximum value of the function in the interval (0, 2) is 1

(D) There exists a 'y' in the interval (0, 1) such that f(y) = -f(2 - y).

[GATE-2014 (CS-SET 1)]

(C) $\left(\frac{a}{\sqrt{2}}, \frac{a}{\sqrt{2}}\right)$ (D) $\left(\frac{a}{2\sqrt{2}}, \frac{a}{2\sqrt{2}}\right)$

31. If $f'(x) = \frac{1}{3-x^2}$ and $f(0) = 1$ then the lower bound and upper bound of $f(1)$ estimated by mean value theorem are

(A) 1, 1.2 (B) 1.33, 1.5

(C) 1.5, 1.75 (D) None

32. If $f(0) = 2.0$ and $f'(x) = \frac{1}{5-x^2}$, then the lower and upper bounds of $f(1)$ estimated by the mean value theorem are

(A) 1.9, 2.2

(B) 2.2, 2.25

(C) 2.25, 2.5

(D) None of the above

[GATE-1995]

33. Let the function

$$f(\theta) = \begin{vmatrix} \sin\theta & \cos\theta & \tan\theta \\ \sin\left(\frac{\pi}{6}\right) & \cos\left(\frac{\pi}{6}\right) & \tan\left(\frac{\pi}{6}\right) \\ \sin\left(\frac{\pi}{3}\right) & \cos\left(\frac{\pi}{3}\right) & \tan\left(\frac{\pi}{3}\right) \end{vmatrix}$$

Where $\theta \in \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$ and $f'(\theta)$ denoted the derivative of f with respect to θ. Which of the following statement is/are TRUE?

I. There exists $\theta \in \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$ such that $f'(\theta) = 0$.

II. There exists $\theta \in \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$ such $f'(\theta) \neq 0$.

(A) I only (B) II only

(C) Both I and II (D) Neither I nor II

[GATE-2014 (CS-SET1)]

36. Consider the function $f(x) = x + \ln x$ and f is differentiable on $(1, e)$ and $f(x)$ is continuous on $[1, e]$. Determine the c value using mean value theorem.

[By computing function $f'(c) = \frac{f(b) - f(a)}{b - a}$]

(A) e (B) $e - 1$

(C) $\frac{e}{e-1}$ (D) $\frac{e-1}{e}$

37. If $Z = e^{ax+by}F(ax - by)$; the value of $b.\frac{\partial Z}{\partial x} + a\frac{\partial Z}{\partial y}$ is

(A) 2Z (B) 2a

(C) 2b (D) 2abZ

[ESE 2018 (EE)]

38. If $z = xy\ln(xy)$, then

(A) $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 0$ (B) $y\frac{\partial z}{\partial x} = x\frac{\partial z}{\partial y}$

34. The value of ε in the mean value theorem of $f(b) - f(a) = (b - a)f'(\varepsilon)$ for the function $f(x) = Ax^2 + Bx + C$ in (a, b) is

(A) $b + a$ (B) $b - a$

(C) $\frac{b+a}{2}$ (D) $\frac{b-a}{2}$

[GATE-94]

35. A function $y = 5x^2 + 10x$ is defined over an open interval $x = (1, 2)$. At least at one point in this interval, $\frac{dy}{dx}$ is exactly

(A) 20 (B) 25

(C) 30 (D) 35

[GATE-2013 (EE)]

(C) $x\frac{\partial z}{\partial x} = y\frac{\partial z}{\partial y}$ (D) $y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = 0$

[GATE-2014]

39. Let $f = y^x$. What is $\frac{\partial^2 f}{\partial x\, \partial y}$ at $x = 2,\ y = 1$?

(A) 0 (B) $\ln 2$

(C) 1 (D) $\frac{1}{\ln 2}$

[GATE-2008]

40. If $u = \begin{vmatrix} x^2 & y^2 & z^2 \\ x & y & z \\ 1 & 1 & 1 \end{vmatrix}$ then $u_x + u_y + u_z =$ ________.

41. If $f(x, y, z) = (x^2 + y^2 + z^2)^{-\frac{1}{2}}$ then $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$ is equal to

(A) 0 (B) 1

(C) 2 (D) $-3(x^2 + y^2 + z^2)^{-\frac{5}{2}}$

[GATE-2000]

42. If $\sin u = \frac{x + 2y + 3z}{x^8 + y^8 + z^8}$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} =$

(A) $\frac{1}{7}\tan u$ (B) $-7\tan u$

(C) $\frac{1}{7}\sec u$ (D) $-\frac{1}{7}\tan u$

43. If $u = \frac{x^3 + y^3}{x - y} + x\sin\left(\frac{x}{y}\right)$ then $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} =$

(A) 0 (B) $2\left(\frac{x^3 + y^3}{x - y}\right)$

45. If $u = \log\left(\frac{x^2 + y^2}{x + y}\right)$, what is the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$?

(A) 0 (B) 1

(C) u (D) e^u

[ESE 2018 (COMMON PAPER)]

46. Let $r = x^2 + y - z$ and $z^3 - xy + yz + y^3 = 1$. Assume that x and y are independent variables. At $(x, y, z) = (2, -1, 1)$, the value (correct to two decimal places) of $\frac{\partial r}{\partial x}$ is _____.

[EC, GATE-2018 : 2 MARKS]

47. If $x^{\sin y} = y^{\sin x}$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{x^2 \cos x \log x - y \sin y}{x^2 \cos x \log x - x \sin x}$

(B) $\frac{y^2 \cos y \log y - x \sin x}{y^2 \cos y \log y - y \sin y}$

(C) $x\sin\dfrac{x}{y}$ (D) u

44. Let $r = x^2 + y - z$ and $z^3 - xy + yz + y^3 = 1$. Assume that x and y are independent variables. At $(x,y,z) = (2,-1,1)$, the value (correct to two decimal places) of $\dfrac{\partial r}{\partial x}$ is ________.

(A) $\dfrac{f}{n}$ (B) $\dfrac{n}{f}$

(C) nf (D) $n\sqrt{f}$

[GATE-2005 (IN)]

(C) $\dfrac{xy\cos x\log y - y\sin y}{xy\cos x\log y - x\sin x}$

(D) $\dfrac{xy\cos x\log y - y\sin y}{xy\cos x\cos y - x\sin x}$

48. Consider two functions: $x = \psi\ln\phi$ and $y = \phi\ln\psi$. Which one of the following is the correction expression for $\dfrac{\partial\psi}{\partial x}$?

(A) $\dfrac{x\ln\phi}{\ln\phi\ln\psi - 1}$ (B) $\dfrac{x\ln\psi}{\ln\phi\ln\psi - 1}$

(C) $\dfrac{\ln\phi}{\ln\phi\ln\psi - 1}$ (D) $\dfrac{\ln\psi}{\ln\phi\ln\psi - 1}$

[GATE-19-CE-SET1]

49. Let $w = f(x,y)$, where x and y are functions of t. Then, according to the chain rule, $\dfrac{dw}{dt}$ is equal to

(A) $\dfrac{dw}{dx}\dfrac{dx}{dt} + \dfrac{dw}{dy}\dfrac{dt}{dt}$

(B) $\dfrac{\partial w}{\partial x}\dfrac{\partial x}{\partial t} + \dfrac{\partial w}{\partial y}\dfrac{\partial t}{\partial t}$

(C) $\dfrac{\partial w}{\partial x}\dfrac{dx}{dt} + \dfrac{\partial w}{\partial y}\dfrac{\partial y}{\partial t}$

(D) $\dfrac{dw}{dx}\dfrac{\partial x}{\partial t} + \dfrac{dw}{dy}\dfrac{\partial y}{\partial t}$

[GATE-17-CE]

50. If $z - e^{xy^2}, x = t\cos t, y = t\sin t$ then $\dfrac{dz}{dt}$ at $t = \dfrac{\pi}{2}$ is

(A) $\dfrac{\pi^3}{8}$ (B) $\dfrac{\pi^3}{4}$

(C) $\dfrac{\pi^3}{2}$ (D) $\dfrac{-\pi^3}{8}$

53. The summation of series

$S = 2 + \dfrac{5}{2} + \dfrac{8}{2^2} + \dfrac{11}{2^3} + ... + \infty$ is

(A) 4.50 (B) 6.0

(C) 6.75 (D) 10.0

[GATE-2004 : 1 MARK]

54. For $|x| \ll 1$, cot h(x) can be approximated as

(A) x (B) x^2

(C) $\dfrac{1}{x}$ (D) $\dfrac{1}{x^2}$

[GATE-2007-EC; 1 MARK]

55. For the function e^{-x}, the linear approximation around $x = 2$ is

(A) $(3-x)e^{-2}$

(B) $1 - x$

(C) $\left[3 + 2\sqrt{2} - (1+\sqrt{2})x\right]e^{-2}$

(D) e^{-2}

[GATE-2007]

56. Which of the following functions would

51. If $z = f(x,y)$ where $x = e^u + e^{-v}, y = e^{-u} - e^v$ then $z_u - z_v =$

(A) $xz_x - yz_y$ (B) $xz_x + yz_y$

(C) $xz_y + yz_x$ (D) $xz_y - yz_x$

52. For $x = \frac{\pi}{6}$, the sum of the series $\sum_1^{\infty}(\cos x)^{2n} = \cos^2 x + \cos^4 x + ...$ is

(A) π (B) 3

(C) ∞ (D) 1

[GATE-1998]

have only odd powers of x in its Taylor series expansion about the point x = 0?

(A) $\sin(x^3)$ (B) $\sin(x^2)$

(C) $\cos(x^3)$ (D) $\cos(x^2)$

[GATE-2008]

57. In the Taylor series expansion of e^x about $x = 2$, the coefficient of $(x-2)^4$ is

(A) $1/4!$ (B) $2^4/4!$

(C) $e^2/4!$ (D) $e^4/4!$

[GATE 2008-ME]

58. The Taylor series expansion of $\frac{\sin x}{x-\pi}$ at $x = \pi$ is given by

(A) $1+\frac{(x-\pi)^2}{3!}+....$

(B) $-1-\frac{(x-\pi)^2}{3!}+....$

(C) $1-\frac{(x-\pi)^2}{3!}+....$

(D) $-1+\frac{(x-\pi)^2}{3!}+....$

[GATE-2009]

59. The Taylor series expansion of $3\sin x + 2\cos x$ is

(A) $2+3x-x^2-\frac{x^3}{2}+....$

(B) $2-3x+x^2-\frac{x^3}{2}+....$

(C) $2-3x-x^2+\frac{x^3}{2}+....$

(D) $2+3x+x^2+\frac{x^3}{2}+....$

(A) $1+x+x^2+x^3$

(B) $1+x+\frac{3}{2}x^2+x^3$

(C) $1+x+\frac{3}{2}x^2+\frac{7}{6}x^3$

(D) $1+x+3x^2+7x^3$

[GATE-17-EC]

62. Taylor series expansion of $f(x)=\int_0^x e^{-\left(\frac{t^2}{2}\right)}dt$ around $x=0$ has the form $f(x)=a_0+a_1x+a_2x^2+....$

The coefficient a_2 (correct to two decimal places) is equal to _________.

[GATE 2018 (EC)]

63. The function $f(x,y)=2x^2+2xy-y^2$ has

(A) only one stationary point at (0, 0)

(B) two stationary points at (0, 0) and $\left(\frac{1}{6},-\frac{1}{3}\right)$

[GATE-2014]

60. The coefficient of x^{12} in

$\left(x^3+x^4+x^5+x^6 ......\right)^3$ is

[GATE-2016-CS-SET 1]

61. Let $f(x)=e^{x+x^2}$ for real x. From among the following. Choose the Taylor series approximation of $f(x)$ around $x=0$, Which includes all, powers of x less than or equal to 3.

(C) two stationary points at (0, 0) and (1, -1)

(D) no stationary point

[GATE-2002]

64. The distance between the origin and the point nearest it on the surface $z^2=1+xy$ is

(A) 1 (B) $\frac{\sqrt{3}}{2}$

(C) $\sqrt{3}$ (D) 2

[GATE-2009]

65. Given a function

$f(x,y)=4x^2+6y^2-8x-4y+8$.

The optimum value of $f(x,y)$ is

(A) a minimum equal to 10/3

(B) a maximum equal to 10/3

(C) a minimum equal to 8/3

(D) a maximum equal to 8/3

[GATE-2010-CE]

66. Find the absolute maxima and minima of the function $f(x,y)=x^2-xy-y^2-6x+2$ on the rectangular plate $0\le x\le 5, -3\le y\le 0$.

[GATE-1995]

67. The absolute maximum value and the absolute minimum value of the function on the unit square $\{(x,y): 0\le x\le 1, 0\le y\le 1\}$, respectively are

(A) 3 and $\frac{3}{2}$ (B) $\frac{3}{2}$ and $\frac{3}{4}$

(C) 3 and $\frac{3}{4}$ (D) 2 and $\frac{3}{4}$

70. The minimum value of the function $f(x)=x^3-3x^2-24x+100$ in the interval [-3, 3] is

(A) 20 (B) 28

(C) 16 (D) 32

71. Number of point of inflections of the function $f(x)=x^4-18x^2+9$

(A) 0 (B) 1

(C) 2 (D) 3

72. Let $f(x)=\int_{\sin x}^{\cos x} e^{-t^2}dt$, $f'=(\pi/4)$ equals

(A) $\sqrt{1/e}$ (B) $-\sqrt{2/e}$

(C) $\sqrt{2/e}$ (D) $-\sqrt{1/e}$

73. Let $f: R\rightarrow R$ be defined as.

$f(t)=\begin{cases}\frac{\tan t}{t}, & t\neq 0\\ 1 & , t=0\end{cases}$. Then the value of

$\lim\limits_{x\to 0}\frac{1}{x^2}\int_{x^2}^{x^3} f(t)dt$

(A) is equal to (-1) (B) is equal to 0

(C) is equal to 1 (D) does not exist

74. Number of inflection points for the curve $y=x+2x^4$ is

(A) 3 (B) 1

(C) 0 (D) $(n+1)^2$

[GATE-1999]

68. The value of a and b for which the function $f(x) = x^3 + ax^2 + bx$ has local minima at $x = 4$ and point of inflection at $x = 1$ are

(A) 3, 24 (B) $-3, -24$

(C) -3, 24 (D) 0, 0

69. For the function $y = 1 - x^4$, the point $x = 0$ is a point of

(A) inflection (B) minima

(C) maxima (D) absolute minima

75. The function $f(x,y) = 2x^4 + y^2 - x^2 - 2y$ has a relative __________.

(A) maxima at $\left(\frac{1}{2}, 1\right)$

(B) minima at $\left(\frac{1}{2}, 1\right)$

(C) maxima at (0, 1)

(D) minima at (0, 1)

[GATE]

76. Let $V_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ and $V_2 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ be two vectors. The value of the coefficient α in the expression $V_1 = \alpha V_2 + e$, which minimizes the length of the error vector *e*, is

(A) $\frac{7}{2}$ (B) $-\frac{2}{7}$

(C) $\frac{2}{7}$ (D) $-\frac{7}{2}$

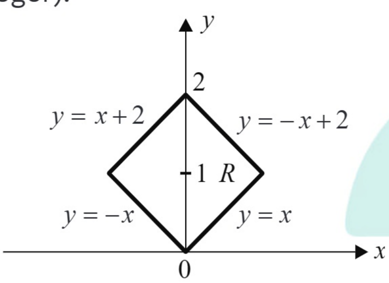
[GATE-2023 (EC)]

77. The value of the line integral $\int_P^Q (z^2 dx + 3y^2 dy + 2xz\, dz)$ along the straight line joining the points *P*(1,1,2) and *Q*(2,3,1) is

(A) 20 (B) 24

(C) 29 (D) -5

78. The value of the integral $\iint_R xy\, dx\, dy$ over the region *R*, given in the figure, is ________ (rounded off to the nearest integer).



[GATE-2023 (EC)]

(A) $\frac{\pi}{3}(2\sqrt{2} - 1)$ (B) $\frac{\pi}{6}(2\sqrt{2} - 1)$

(C) $\pi(2\sqrt{2} - 1)$ (D) $\frac{\pi}{2}(2\sqrt{2} - 1)$

[GATE-2023 (XE)]

❑❑❑

79. What is $\lim_{x\to 0} f(x)$, where $f(x) = x\sin\frac{1}{x}$?

(A) 0 (B) 1

(C) ∞ (D) Limit does not exist

[GATE-2023 (IN)]

80. The surface area of applications of the portion of the paraboloid $z = x^2 + y^2$ that lies between theplanes $z = 0$ & $z = \frac{1}{4}$ is

2 Calculus (Part-2)

Objective Questions

1. For $n \in N$, the value of $\int_0^n \frac{1-(x/n)^n}{n-x} dx =$

(A) 0

(B) $1+\frac{1}{2}+\frac{1}{3}+.........+\frac{1}{n}$

(C) $1+\frac{1}{2}+.....+\frac{1}{n+1}$

(D) $1+\frac{1}{2}+.....+\frac{1}{n+2}$

[CSIR]

2. $\int_0^{\pi/4} \log(1+\tan x)dx$

(A) 0 (B) $\frac{\pi}{4}\log 2$

(C) $\frac{\pi}{8}\log 2$ (D) $\frac{\pi}{2}\log 2$

3. $\int_0^{\pi/2} (a^2\cos^2 x + b^2\sin^2 x)dx =$

(A) 0 (B) $\frac{\pi}{2}(a^2+b^2)$

(C) $\frac{\pi}{4}(a^2+b^2)$ (D) $\frac{\pi}{8}(a^2+b^2)$

[JNU]

[GATE-2018 (EE)]

5. $\int_0^2 |1-x|dx =$ ________

(A) 0 (B) -1

(C) 1 (D) 2

6. The value of $\int_1^e \frac{1}{x^2} e^{(1+\ln x)}dx$ is

(A) 1 (B) $1/e$

(C) e (D) 0

7. The integral $\int_0^{\pi/2} \min(\sin x, \cos x)dx$ equals

(A) $\sqrt{2}-2$ (B) $2-\sqrt{2}$

(C) $2\sqrt{2}$ (D) $2+\sqrt{2}$

8. The value of the integral $\int_0^{\pi/2} \frac{\tan x}{\tan x + \cot x}dx$ is

(A) $\pi/6$ (B) $\pi/2$

(C) 0 (D) $\pi/4$

9. The value of $\int_{-5\pi/2}^{5\pi/2} f(x)dx$, where

4. Let 'f' be a real valued function of a real variable defined as $f(x) = x - [x]$, where $[x]$ denote the largest integer less than or equal to x. The value of $\int_{0.25}^{1.25} f(x)\,dx$ ________is (up to 2 decimal places).

$f(x) = e^{\pi x^2} \cdot \sin^3 x + 4\cos x$, equals _____.

(A) 4 (B) 8

(C) $\frac{5\pi}{2}$ (D) $\frac{-5\pi}{2}$

10. The value of $\int_0^{100\pi} |\sin x|\,dx$.

(A) 100 (B) 100π

(C) 200π (D) 200

11. $\int_0^{\pi} \frac{1}{a^2\cos^2 x + b^2 \sin^2 x}\,dx =$

(A) 0 (B) $\frac{\pi}{ab}$

(C) πab (D) $\frac{\pi}{a^2 + b^2}$

12. The value of the integral $\int_0^{2\pi}\left(\frac{3}{9+\sin^2\theta}\right)d\theta$ is

(A) $\frac{2\pi}{\sqrt{10}}$ (B) $2\sqrt{10}\,\pi$

(C) $\sqrt{10}\,\pi$ (D) 2π

[ESE PRELIMS-2017]

13. The value of the integral $\int_0^{2} \frac{(x-1)^2 \sin(x-1)}{(x-1)^2 + \cos(x-1)}\,dx$ is

(A) 3 (B) 0

(C) -1 (D) -2

[ME, GATE-2014 : 1 MARK]

14. $\int_0^{\pi/2} \log(\sin x)\,dx =$

(A) 0 (B) $\frac{-\pi}{2}\log 2$

(C) $-\pi\log 2$ (D) $\log 2$

(C) $\frac{1}{48}$ (D) $\frac{1}{96}$

17. $\int_0^{\pi} x \sin^6 x \cos^4 x\, dx =$

(A) $\frac{3\pi^2}{512}$ (B) $\frac{5\pi^2}{256}$

(C) $\frac{3\pi^2}{256}\pi ab$ (D) 0

18. The value of $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x}\,dx =$

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$

(C) $\frac{\pi^2}{4}$ (D) $\frac{\pi^2}{8}$

19. Consider the following definite integral

$$I = \int_0^1 \frac{(\sin^{-1} x)^2}{\sqrt{1-x^2}}\,dx.$$

The value of the integral is

(A) $\frac{\pi^3}{24}$ (B) $\frac{\pi^3}{12}$

(C) $\frac{\pi^3}{48}$ (D) $\frac{\pi^3}{64}$

[GATE-17-CE]

20. The value of $\int_0^{\pi/4} x\cos(x^2)\,dx$ correct to three decimal places (assuming that $\pi = 3.14$) is ____.

[GATE-18-CSIT]

15. If $\int_0 |x \sin x| dx = k\pi$, then the values of k is equal to________.

[GATE-2014 (CS-SET 3)]

16. $\int_0^{\pi/2} \sin^5 x \cos^3 x \, dx =$

(A) $\frac{1}{16}$ (B) $\frac{1}{24}$

21. If $f(x) = R\sin\left(\frac{\pi x}{2}\right) + S, f'\left(\frac{1}{2}\right) = \sqrt{2}$ and $\int_0^1 f(x)dx = \frac{2R}{\pi}$, then the constants R and S are respectively.

(A) $\frac{2}{\pi}$ and $\frac{16}{\pi}$ (B) $\frac{2}{\pi}$ and 0

(C) $\frac{4}{\pi}$ and 0 (D) $\frac{4}{\pi}$ and $\frac{16}{\pi}$

[GATE-17-CSIT]

22. The value of the integral $\int_{-\pi}^{\pi/4} \sqrt{\frac{1-\cos 2x}{2}} dx$ is

(A) $-1-\frac{1}{\sqrt{2}}$ (B) $1-\frac{1}{\sqrt{2}}$

(C) $3-\frac{1}{\sqrt{2}}$ (D) $2-\frac{1}{\sqrt{2}}$

23. The value of the integral $\int_0^9 \frac{dy}{\sqrt{y}\sqrt{1+\sqrt{y}}}$ is

(A) 4 (B) $4\left(\sqrt{10}-1\right)$

(C) 8 (D) 12

24. Let $A(t)$ denote the area bounded by the curve $y = e^{|x|}$, the x- axis and the straight lines $x = -t$ and $x = t$, then $\lim_{t \to \infty} A(t)$ is equal to

(A) 2 (B) 1

(C) 1/2 (D) 0

25. $\int_0^{\infty} e^{-y^3} . y^{1/2} dy =$

(A) $\sqrt{\pi}$ (B) $\frac{\sqrt{\pi}}{3}$

(C) $\frac{\sqrt{\pi}}{2}$ (D) 0

26. The value of $\int_0^1 \frac{dt}{\sqrt{-\ln t}}$ is ________

(A) $\frac{3\sqrt{\pi}}{2}$ (B) $\frac{2\sqrt{\pi}}{3}$

(C) $\frac{\sqrt{\pi}}{3}$ (D) $\frac{\sqrt{\pi}}{2}$

28. The value of $\int_0^{\infty} \frac{x^a}{a^x} dx$ is ________

(A) $\frac{\sqrt{a}}{(\ln a)^a}$ (B) $\frac{\sqrt{a+1}}{(\ln a)^{a+1}}$

(C) $\frac{\sqrt{a}}{(\ln a)^{a+1}}$ (D) $\frac{\sqrt{a+1}}{(\ln a)^a}$

29. $\int_0^1 x^6 \sqrt{1-x^2} \, dx =$

(A) $\frac{5\pi}{256}$ (B) $\frac{5\pi}{128}$

(C) $\frac{5\pi}{512}$ (D) $\frac{3\pi}{512}$

30. The value of $\int_0^1 \frac{x^6}{1-x^2} dx =$

(A) $\frac{\pi}{32}$ (B) $\frac{3\pi}{32}$

(C) $\frac{5\pi}{32}$ (D) 0

31. The value of $\int_0^1 x^4 (1-5x)^5 dx$ is ________

(A) $\frac{1}{5015}$ (B) $\frac{1}{15015}$

(C) $\frac{1}{5005}$ (D) $\frac{1}{5001}$

32. The value of $\int_0^{\pi/2} \sqrt{\cot\theta} \, d\theta$ is

(A) $\frac{\sqrt{\pi}}{2}$ (B) $\sqrt{\pi}$

(C) $-\sqrt{\pi}$ (D) $\frac{-\sqrt{\pi}}{2}$

27. The value of $\int_0^{\sigma} e^{-5x} 4\sqrt{x}\, dx$ is ________

32. The value of $\int_0 \sqrt{\cot\theta}\, d\theta$ is ________

(A) $\frac{\pi}{\sqrt{2}}$ (B) $\frac{\pi}{\sqrt{3}}$

(C) $\frac{\pi}{2}$ (D) $\frac{\pi}{4}$

33. The value of $\int_0^{\infty} \frac{x}{1+x^6}\, dx$ is ________

(A) $\frac{\pi}{\sqrt{3}}$ (D) $\frac{\pi}{3\sqrt{3}}$

(C) $\frac{\pi}{\sqrt{2}}$ (D) $\frac{\pi}{2\sqrt{2}}$

34. The value of $\int_0^{\infty} \frac{x^4\left(1+x^5\right)}{\left(1+x\right)^{15}} dx$ is ________

(A) $\frac{1}{5005}$ (B) $\frac{1}{5001}$

(C) $\frac{1}{1001}$ (D) $\frac{1}{10001}$

35. The length of the arc $y = x^{3/2}, z = 0$ from (0, 0, 0) to (4, 8, 0) is ________

(A) $\frac{8}{27}\left(10^{3/2}+1\right)$ (B) $\frac{8}{27}\left(10^{3/2}-2\right)$

(C) $\frac{8}{27}\left(10^{3/2}-1\right)$ (D) $\frac{8}{27}\left(10^{3/2}+2\right)$

36. Let f be increasing, differentiate function. If the curve $y = f\left(x\right)$ passes through (1, 1) and has length $L = \int_1^2 \sqrt{1+\frac{1}{4x^2}}\, dx$ $1 \le x \le 2$, then curve is ________

(A) $y = \ln\left(\sqrt{x}\right) - 1$ (B) $y = 1 - \ln\sqrt{x}$

(C) $y = \ln\left(1+\sqrt{x}\right)$ (D) $y = 1+\ln\left(\sqrt{x}\right)$

37. The length of the arc $x = a\left(t-\sin t\right), y = a\left(1-\cos t\right)$ between $t = 0$ to $t = 2\pi$ is ______

(A) 8a (B) 4a

(C) $4\sqrt{2}a$ (D) $2\sqrt{2}a$

from by $x\left(t\right) = \cos t$, $y\left(t\right) = \sin t$, $z\left(t\right) = \frac{2}{\pi}t$, $0 \le t \le \frac{\pi}{2}$

The length of the curve is ________

[ME, GATE-2015 : 2 MARKS]

40. A parametric curve defined by $x\cos\left(\frac{\pi u}{2}\right), y = \sin\left(\frac{\pi u}{2}\right)$ in the range $0 \le u \le 1$ is rotated about the X-axis by 360^0. Area of the surface generated is

(A) $\frac{\pi}{2}$ (B) π

(C) 2π (D) 4π

[GATE-17-ME]

41. Let $W = f\left(x,y\right)$, where x and y are functions of t. Then, according to the chain rule, $\frac{dw}{dt}$ is equal to

(A) $\frac{dw}{dx}\frac{dx}{dt} + \frac{dw}{dy}\frac{dt}{dt}$

(B) $\frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial x}{\partial t}$

(C) $\frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dx}{dt}$

(D) $\frac{dw}{dx}\frac{\partial x}{\partial t} + \frac{dw}{dy}\frac{\partial x}{\partial t}$

42. The surface area obtained by revolving $y = 2x$ for $x \in \left[0,2\right]$ about y –axis is ____

(A) $2\pi\sqrt{5}$ (B) $4\pi\sqrt{5}$

38. The length of the arc $r = a(1+\cos\theta)$ between $\theta = 0$ to π is _____

39. Consider a spatial curve in three-dimensional space given in parametric

(C) $2\sqrt{5\pi}$ (D) $4\sqrt{5\pi}$

43. The surface area generated by rotations $x = a\cos^3\theta, y = a\sin^3\theta, 0 \le \theta \le \pi$ about y- axis

(A) $\frac{12}{5}\pi a^2$ (B) $\frac{5}{12}\pi a^2$

(C) $\frac{6}{5}\pi a^2$ (D) $\frac{5}{6}\pi a^2$

44. The surface area of the solid generated by revolving line segment $y = x+2$ for $0 \le x \le 1$ about the line $y = 2$ is __________

(A) $\sqrt{2\pi}$ (B) 2π

(C) $2\sqrt{2\pi}$ (D) $\sqrt{2\pi}$

45. If the line $y = mx$, $0 \le x \le 2$ is rotated about the line $y = -1$ then the area of the generated surface is _________

(A) $4\pi(1+m)\sqrt{1+m}$

(B) $4\pi(1+m^2)\sqrt{1+m}$

(C) $4\pi(1+\sqrt{m})\sqrt{1+m^2}$

(D) $4\pi(1+m)\sqrt{1+m^2}$

46. The volume generated by revolving the area bounded by the parabola $y^2 = 8x$, y – axis and the lines $y = -4$ to $y = 4$ about y-axis is

(A) 32π

(B) $\frac{32\pi}{5}$

(C) $\frac{128\pi}{5}$

(D) None of the above

(A) $\frac{128\pi}{5}$ (B) $\frac{5}{128\pi}$

(C) $\frac{127}{5\pi}$ (D) $\frac{32\pi}{5}$

48. The area bounded by $x^2 = 2y$ and $y^2 = 2x$ is _______

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$

(C) $\frac{4}{3}$ (D) 4

49. The area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is

(A) $\frac{2}{3}a^2$ (B) $\frac{14}{3}a^2$

(C) $\frac{16}{3}a^2$ (D) $\frac{17}{3}a^2$

[EE, ESE-2019]

50. The area of the region bounded by the parabola $y = x^2 + 1$ and the straight line $x + y = 3$ is

(A) $\frac{59}{6}$ (B) $\frac{9}{2}$

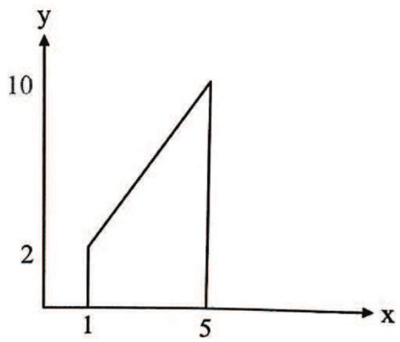
(C) $\frac{10}{3}$ (D) $\frac{7}{6}$

51. Let $I = c\iint_R xy^2\,dx\,dy$, where R is the region shown in the figure and $c = 6\times10^{-4}$. The value of I equals ______. (Give the answer up to two

[EE-1994]

47. The volume generated by revolving the area bounded by $y^2 = 8x$ and the line $x = 2$, about $y-$axis is

decimal places)



52. The area bounded by the curves $x^2 = 4-2y$ and $x^2 = y+4$ is

(A) 16 (B) 24

(C) 30 (D) 36

53. Let D be the region in the first quadrant lying between $x^2 + y^2 = 4$. The value of the integral $\iint_D \sin\left(x^2+y^2\right)dx\,dy$ is

(A) $\frac{\pi}{4}(\cos 1 - \cos 2)$

(B) $\frac{\pi}{4}(\cos 1 - \cos 4)$

(C) $\frac{\pi}{2}(\cos 1 - \cos 2)$

(D) $\frac{\pi}{2}(\cos 1 - \cos 4)$

54. $\int_1^2\int_0^x \frac{1}{\left(x^2+2\right)^{3/2}}dy\,dx$ transforms to $\int_0^a\int_b^c \frac{1}{r^2}dr\,d\theta$ in polar coordinates then a, b, c respective are ______

(A) $\frac{\pi}{4}, \sec\theta$ and $2\sec\theta$

(B) $\frac{\pi}{4}$, $2\sec\theta$, $4\sec\theta$

55. The value of $\iint_D xy\,dx\,dy$ over the region common to the circles $x^2 + y^2 = x$ and $x + y = y$ is ________

(A) $\frac{1}{192}$ (B) $\frac{1}{96}$

(C) $\frac{1}{48}$ (D) $\frac{1}{24}$

56. What is the area common to the circle $= a$ and $r = 2a\cos\theta$?

(A) $0.524\ a^2$ (B) $0.614\ a^2$

(C) $1.047\ a^2$ (D) $1.228\ a^2$

[GATE-2006]

57. A surface $S(x,y) = 2x + 5y - 3$ is integrated once over a path consisting of the points that satisfy $(x+1)^2 + (y-1)^2 = \sqrt{2}$. The integral evaluates to

(A) $17\sqrt{2}$ (B) $\frac{17}{\sqrt{2}}$

(C) $\frac{\sqrt{2}}{17}$ (D) 0

58. The value of $\int_0^\infty\int_0^\infty e^{-\left(x^2+y^2\right)}dx\,dy =$

(A) $\frac{\sqrt{\pi}}{2}$ (B) $\sqrt{\pi}$

(C) π (D) $\frac{\pi}{4}$

59. The evaluate the double integral

(C) $\frac{}{2}$, $2\sec\theta$, $4\sec\theta$

(D) $\frac{\pi}{2}$, $\sec\theta$ and $2\sec\theta$

$\int_0^8 \left(\int_{y/2}^{(y/2)+1} \left(\frac{2x-y}{2} \right) dx \right) dy$, we make the substitution $u = \left(\frac{2x-y}{2} \right)$ and $v = \frac{y}{2}$.

The integral will reduce to

(A) $\int_0^4 \left(\int_0^2 2u\,du \right) dv$ (B) $\int_0^4 \left(\int_0^1 2u\,du \right) dv$

(C) $\int_0^4 \left(\int_0^1 u\,du \right) dv$ (D) $\int_0^4 \left(\int_0^2 u\,du \right) dv$

[GATE-14-EE-SET2]

60. The double integral $\int_1^2 \int_x^{2x} f(x,y) dy\, dx$ under the transformation $x = 4(1-v)$, $y = uv$ is transformed into

(A) $\int_{1/2}^{2/3} \int_{1/(1-v)}^{2(1-v)} f(u-uv, uv) du\, dx$

(B) $\int_{1/2}^{2/3} \int_{1/(1-v)}^{2/(1-v)} f(u-uv, uv) du\, dx$

(C) $\int_{2/3}^{1} \int_{1/(1-v)}^{2/(1-v)} f(u-uv, uv) u\, du\, dx$

(D) $\int_{1/2}^{2/3} \int_{1/(1-v)}^{2/(1-v)} f(u-uv, uv) u\, du\, dx$

61. By change the order of integration $\int_1^2 \int_{x^2}^{2x} f(x,y) dy\, dx$ may be represented as

(A) $\int_0^2 \int_{x^2}^{2x} f(x,y)\, dy\, dx$

(B) $\int_0^2 \int_y^{\sqrt{y}} f(x,y)\, dy\, dx$

(C) $\int_0^4 \int_{y/2}^{\sqrt{y}} f(x,y)\, dy\, dx$

[GATE]

62. Changing the order of the integration in the double integral $I = \int_0^8 \int_{x/4}^2 f(x,y)\, dy\, dx$ leads to $I = \int_r^s \int_p^q f(x,y)\, dx\, dy$. What is q?

(A) 4y (B) $16y^2$

(C) x (D)8

[GATE-2005]

63. The value of $\int_0^1 \int_0^{x^2} e^{y/x} dy\, dx =$ ________

[JNU]

64. The value of $\int_0^{\pi/2} \int_x^{\pi/2} \frac{\cos y}{y} dy\, dx =$ ________

[CSIR]

65. The value of $\int_0^1 \int_{2y}^2 e^{x^2} dx\, dy =$ ________

(A) e^4 (B) $e^4 - 1$

(C) $\frac{e^4 - 1}{4}$ (D) $\frac{e^4}{4}$

66. Let

$$\int_0^1 \int_y^1 xy \sin(xy)\, dx\, dy = \int_0^1 \int_a^b xy \sin xy\, dx\, dy$$

then

(A) $a = 0, b = x$ (B) $a = 1, b = x$

(C) $a = 0, b = 1$ (D) $a = -1, b = x$

67. $\int_0^\infty \int_{1/4}^\infty x^4 . e^{-x^3 y}\, dx\, dy =$ ________

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$

(C) 1 (D) 1

(D) $\int_{x^2}^{2x}\int_0^2 f(x,y)\,dy\,dx$

(C) $\frac{}{3}$ (D) 1

68. The value of

$\int_0^{\pi}\int_x^{\pi}\int_0^2 \frac{\sin y}{y}\,dz\,dy\,dx =$

(A) -2 (B) 2

(C) -4 (D) 4

[GATE]

69. $\int_0^1\int_y^1 y\sqrt{1+x^3}\,dx\,dy =$

(A) $2\sqrt{2}$ (B) $\frac{2\sqrt{2}-1}{2}$

(C) $\frac{2\sqrt{2}-1}{8}$ (D) $\frac{2\sqrt{2}-1}{9}$

[IISC]

70. The integral $\int_0^1\int_{x^2}^{x} \frac{x}{y} e^{-x^3 y}\,dy\,dx =$ _____

(A) $\frac{e-2}{e}$ (B) $\frac{e-1}{2e}$

(C) $\frac{e-1}{2}$ (D) $\frac{e-1}{2e}$

71. The volume enclosed by the surface $f(x,y) = e^x$ over the triangle bounded by the lines $x = y; x = 0; y = 1$ in the xy plane is _________

72. The integral $\frac{1}{2\pi}\iint_D (x+y+10)\,dx\,dy$ where D denotes the disc: $x^2+y^2 \le 4$, evaluates to __________

[GATE-16-EC-SET 1]

73. The region specified by

$\left\{(\rho,\varphi,Z) : 3 \le \rho \le 5,\ \frac{\pi}{8} \le \phi \le \frac{\pi}{4},\ 3 \le z \le 4.5\right\}$

in cylindrical coordinates has volume of __________

[GATE-16-EC-SET 1]

74. A triangle in the xy-plane is bounded by the straight lines $2x = 3y$, $y = 0$ and $x = 3$. The volume above the triangle and under the plane $x+y+z=6$ is __________

[GATE-16-EC-SET 3]

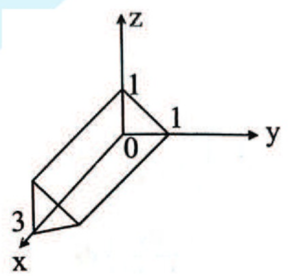
75. A three dimensional region R of finite volume is described by

$x^2+y^2 \le z^3; 0 \le z \le 1.$

Where x, y, z are real. The volume of R (upto two decimal places) is ______

[GATE-17-EC; SESSION 1]

76. For the solid S shown below, the value of $\iiint_S x\,dx\,dy\,dz$ (rounded off to two decimal places) is



[GATE-20-EC]

77. The area of an ellipse represented by an equation $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

(A) πab (B) $\frac{\pi ab}{4}$

(C) $\frac{\pi ab}{2}$ (D) $\frac{4\pi ab}{3}$

[GATE-20-CE-SET1]

78. The volume of

$\iiint \frac{1}{(x^2+y^2+z^2)^{3/2}}\,dx\,dy\,dz$

where R is the region bounded by

$x^2 + y^2 + z^2 = b^2$

(A) $\ln(b/a)$ (B) $4\pi\ln(b/a)$

(C) $2\pi\ln(b/a)$ (D) $8\pi\ln(b/a)$

79. The volume of

$\iiint (x^2 + y^2 + z^2)\,dx\,dy\,dz$

taken over the volume enclosed by $x^2 + y^2 + z^2 = 1$ is __________

(A) $\frac{4\pi}{5}$ (B) $\frac{8\pi}{5}$

(C) $\frac{2\pi}{5}$ (D) $\frac{\pi}{5}$

80. The volume of the solid surrounded by the surface $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} + \left(\frac{z}{c}\right)^{2/3} = 1$

(A) $\frac{4\pi abc}{35}$ (B) $\frac{abc}{35}$

(C) $\frac{2\pi abc}{35}$ (D) $\frac{\pi abc}{35}$

[EE, ESE-2019]

81. For the integral $I = \int_{-1}^{1} \frac{1}{x^2}dx$, which of the following statements is TRUE?

(A) $I = 0$ (B) $I = 2$

(C) $I = -2$ (D) The integral does not converge

[GATE-2023 (CE-1)]

82. For the function $f(x) = e^x |\sin x|; x \in \mathbb{R}$, which of the following statements is/are TRUE?

(A) The function is continuous at all x

(B) The function is differentiable at all x

(C) The function is periodic

(D) The function is bounded

[GATE-2023 (CE-1)]

83. Let $f(x) = x^3 + 15x^2 - 33x - 36$ be a real-valued function. Which of the following statements is/are TRUE?

(A) $f(x)$ does not have a local maximum.

(B) $f(x)$ has a local maximum.

(C) $f(x)$ does not have a local minimum.

(D) $f(x)$ has a local minimum.

[GATE-2023 (CSE)]

84. The value of the definite integral $\int_{-3}^{3}\int_{-2}^{2}\int_{-1}^{1}(4x^2y - z^3)\,dz\,dy\,dz$ is____.

(Rounded off to the nearest integer)

[GATE-2023 (CSE)]

85. Let $f : R^2 \to 12$ be a function defined by

$$f(x,y) = \begin{cases} \frac{xy}{|x| + y}, y \neq -|x|, \text{otherwise} \end{cases}$$

Then which one of following statement is true?

(A) f is not continuous at $(0,0)$

(B) $\frac{\partial f}{\partial x}(0,0) = 1$, and $\frac{\partial f}{\partial y}(0,0) = 0$

(C) $\frac{\partial f}{\partial x}(0,0) = 0$, and $\frac{\partial f}{\partial y}(0,0) = 1$

(D) $\frac{\partial f}{\partial x}(0,0) = 0$, $\frac{\partial f}{\partial y}(0,0) = 0$

[GATE-2023 (XE)]

❑❑❑

3 Vector Calculus

Objective Questions

1. If P, Q and R are three points having coordinates (3, -2, -1), (1, 3, 4), (2, 1, -2) in XYZ space (O, being the origin of the coordinate) then distance of point P from plane OQR is

(A) 3 (B) 7

(C) 5 (D) 9

[GATE-2003]

2. If a vector $\bar{R}(t)$ has a constant magnitude than

(A) $\bar{R}\cdot\frac{d\bar{R}}{dt}=0$ (B) $\bar{R}\times\frac{d\bar{R}}{dt}=0$

(C) $\bar{R}\cdot\bar{R}\frac{d\bar{R}}{dt}=0$ (D) $\bar{R}\times\bar{R}=\frac{d\bar{R}}{dt}$

[GATE-2005 ; (IN)]

3. The area of a triangle formed by the tips of vectors $\bar{a}$, $\bar{b}$ and $\bar{c}$ is

(A) $\frac{1}{2}(\bar{a}-\bar{b})\cdot(\bar{a}-\bar{c})$

(B) $\frac{1}{2}\left|(\bar{a}-\bar{b})\times(\bar{a}-\bar{c})\right|$

(C) $\frac{1}{2}\left|\bar{a}\times\bar{b}\times\bar{c}\right|$

(C) $\frac{1}{2}(\bar{a}\times\bar{b})\cdot c$

[GATE-2007-ME]

4. If $\bar{a}$ and $\bar{b}$ are two arbitrary vectors with magnitudes a and b, respectively, $\left|\bar{a}\times\bar{b}\right|^2$ will be equal to

(A) $a^2b^2-(\bar{a}\cdot\bar{b})^2$ (B) $ab-\bar{a}\cdot\bar{b}$

(C) $a^2b^2+(\bar{a}\cdot\bar{b})^2$ (D) $ab+\bar{a}\,\bar{b}$

[GATE-2011-CE]

5. If A (0, 4, 3), B (0, 0, 0) and C (3, 0, 4) are three points defined in x, y, z coordinate system, then which one of the following vectors is perpendicular to both the vectors $\overline{AB}$ and $\overline{BC}$

(A) $16\hat{i}-9\hat{j}-12\hat{k}$ (B) $16\hat{i}-9\hat{j}+12\hat{k}$

(C) $16\hat{i}-9\hat{j}-12\hat{k}$ (D) $-16\hat{i}-9\hat{j}+12\hat{k}$

[GATE-2011 (PI)]

6. A particle, starting from origin at t = 0s, is traveling along x-axis with velocity

$$V=\frac{\pi}{2}\cos\left(\frac{\pi}{2}t\right)m/s$$

At t = 3s, the difference between the distance covered by the particle and the magnitude of displacement from the origin is _______

[GATE-2014]

7. A particle move along a curve whose parametric equations are: $x=t^3+2t$, $y=-3e^{-2t}$ and z = 2 sin (5t), where x, y and z show variations of the distance covered by the particle (in cm) with time t (in s). The magnitude of the acceleration of the particle $(\text{in cm}/s^2)$ at t = 0 is _________

[GATE-2014 (PI-SET 1)]

8. The smaller angle (in degrees) between the planes $x+y+z=1$ and $2x-y+2z=0$ is _____

[GATE-2017-EC SESSION-II]

9. The derivative of f(x, y) at point (1, 2) in the direction of vector $\hat{i}+\hat{j}$ is $2\sqrt{2}$ and in the direction of the vector $-2\bar{j}$ is -3. Then the derivative of f(x, y) in direction $-\hat{i}-2\hat{j}$ is

(A) $2\sqrt{2}+3/2$ (B) $-7/\sqrt{5}$

(C) $-2\sqrt{2}-3/2$ (D) $1/\sqrt{5}$

[GATE-95]

10. The magnitude of the directional derivative of the function $f(x,y)=x^2+3y^2$ in a direction normal to the circle $x^2+y^2=2$, at the point (1, 1) is

(a) $4\sqrt{2}$ (B) $5\sqrt{2}$

(C) $7\sqrt{2}$ (D) $9\sqrt{2}$

[GATE-2000 (CE)]

11. The maximum value of the directional derivative of the function $\phi=2x^2+3y^2+5z^2$ at a point (1, 1, -1) is

(A) 10 (B) -4

(C) $\sqrt{152}$ (D) 152

[GATE-2002]

12. A scalar field is given by $f=x^{2/3}+y^{2/3}$, where x and y are the Cartesian coordinates. The derivate of 'f' along the line y = x directed way from the origin at the point (8, 8) is

(A) $\frac{\sqrt{2}}{3}$ (B) $\frac{\sqrt{3}}{2}$

(C) $\frac{2}{\sqrt{3}}$ (D) $\frac{3}{\sqrt{2}}$

[GATE-2005 (IN)]

13. A sphere of unit radius is centred at the origin. The unit normal at a point (x, y, z) on the surface of the sphere is the vector.

(A) (x, y, z) (B) $\left(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)$

(C) $\left(\frac{x}{\sqrt{3}},\frac{y}{\sqrt{3}},\frac{z}{\sqrt{3}}\right)$ (D) $\left(\frac{x}{\sqrt{2}},\frac{y}{\sqrt{2}},\frac{z}{\sqrt{2}}\right)$

[GATE-2009 (IN)]

14. The directional derivative of $f(x,y)=\frac{xy}{\sqrt{2}}(x+y)$ at (1, 1) in the direction of the unit vector at an angle of $\frac{\pi}{4}$ with y-axis, is given by ______

[2014-EC-SEC 4]

15. For the function $\phi=ax^2y-y^3$ to represent the velocity potential of an ideal fluid, $\nabla^2\phi$ should be equal to zero. In that case, the value of 'a' has to be

(A) -1 (B) 1

(C) -3 (D) 3

[GATE-99]

16. If $\bar{F}=r^n\bar{r}$ is solenoidal then n = ______

(A) -1 (B) -2

(C) -3 (D) -4

[GATE]

17. The value of $\nabla^2\left(\frac{1}{r}\right) =$ ______ where $\bar{r}$ is the position vector of any point. If $|\bar{F}| = r^n$ and $\nabla \cdot \bar{F} = 0$ then n = ________

[GATE]

18. If $\bar{r} = xa_x + ya_y + za_z$ and $|\bar{r}| = r$, then div $r^2\nabla(\ln r) =$ _____

[GATE-2014 (EC-SET 2)]

19. For a position vector $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$ the norm of the vector can be defined as $|\bar{r}| - \sqrt{x^2 + y^2 + z^2}$. Given a function $\phi = \ln|\bar{r}|$, its gradient $\nabla\phi$ is

(A) $\bar{r}$ (B) $\frac{\bar{r}}{|\bar{r}|}$

(C) $\frac{\bar{r}}{\bar{r} \cdot \bar{r}}$ (D) $\frac{\bar{r}}{|\bar{r}|^3}$

[GATE-2018 (ME-AFTERNOON SESSION)]

20. Divergence of the vector field 21.

$$\bar{v}(x,y,z) = -(x\cos xy + y)\hat{i} + (y\cos xy)\hat{j} + \left[(\sin z^2) + x^2 + y^2\right]\hat{k}$$

(A) $2z\cos z^2$ (B) $\sin xy + 2z\cos z^2$

(C) $x\sin xy - \cos z$ (D) none of these

[GATE-2007 (PI)]

21. For a vector E, which one of the following statement is NOT TRUE?

(A) If $\nabla \cdot E = 0$, E is called solenoidal

(B) If $\nabla \times E = 0$, E is called conservative

(C) If $\nabla \times E = 0$, E is called irrotational

(D) If $\nabla \cdot E = 0$, E is called irrotational

[GATE-013 (IN)]

22. A vector $\bar{P}$ is given by

$$\bar{P} = x^3y\bar{a}_x - x^2y^2\bar{a}_y - x^2yz\bar{a}_z.$$

Which one of the following statement is TRUE?

(A) $\bar{P}$ is solenoidal, but not irrotational

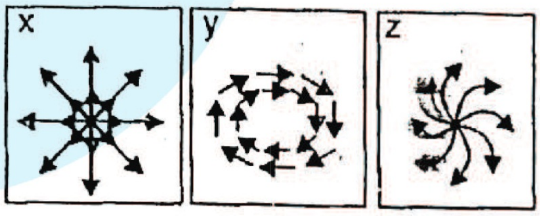
(B) $\bar{P}$ is irrotational, but not solenoidal

(C) $\bar{P}$ is neither solenoidal nor irrotational

(D) $\bar{P}$ is both solenoidal and irrotational

[2015-EC-SET 1]

23. The figures show diagrammatic representations of vector fields, $\bar{X}$, $\bar{Y}$ and $\bar{Z}$ respectively. Which one of the following choices is true?



(A) $\nabla \cdot \bar{X} = 0, \nabla \times \bar{Y} \neq 0, \nabla \times \bar{Z} \neq 0$

(B) $\nabla \cdot \bar{X} \neq 0, \nabla \times \bar{Y} = 0, \nabla \times \bar{Z} \neq 0$

(C) $\nabla \cdot \bar{X} \neq 0, \nabla \times \bar{Y} \neq 0, \nabla \times \bar{Z} \neq 0$

(D) $\nabla \cdot \bar{X} = 0, \nabla \times \bar{Y} = 0, \nabla \times \bar{Z} = 0$

[GATE-2017-EE SESSION-II]

24. Consider the two-dimensional velocity field given by

$$\bar{V} = (5 + a_1x + b_1y)\hat{i} + (4 + a_2x + b_2y)\hat{j}$$

where $a_1b_1a_2$ and b_2 are constants. Which one of the following conditions needs to be satisfied for the flow to be incompressible?

(A) $a_1 + b_1 = 0$ (B) $a_1 + b_2 = 0$

(C) $a_2 + b_2 = 0$ (D) $a_2 + b_2 = 0$

[GATE-2017-ME SESSION-1]

25. $f = \begin{cases} \dfrac{x^3 + y^3}{x^2 - y^2} & x^2 - y^2 \neq 0 \\ 0 & x^2 - y^2 = 0 \end{cases}$. The directional derivative off at (0, 0) in the direction of $\frac{4}{5}\hat{i} + \frac{3}{5}\hat{j}$ is _______

26. Consider points P and Q in xy – plane with P = (1, 0) and Q = (0, 1). The line integral $2\int_P^Q (x\,dx + y\,dy)$ along the semicircle with the line segment PQ as its diameter

(A) is -1

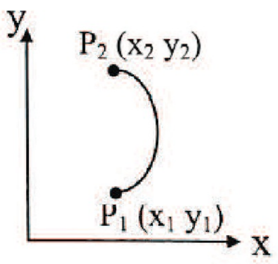
(B) is 0

(C) 1

(D) depends on the direction (clockwise or anti-clockwise) of the semicircle

[GATE-08 (EC)]

27. The line integral $\int_{P_1}^{P_2} (y\,dx + x\,dy)$ from $P_1(x_1, y_1)$ to $P_2(x_2, y_2)$ along the semi-circle P_1P_2 shown in the figure is



(A) $x_2y_2 - x_1y_1$

(B) $\left(y_2^2 - y_1^2\right) + \left(x_2^2 - x_1^2\right)$

(C) $\left(x_2 - x_1\right)\left(y_2 - y_1\right)$

(D) $\left(y_2 - y_1\right)^2 + \left(x_2 - x_1\right)^2$

[GATE-11 (PI)]

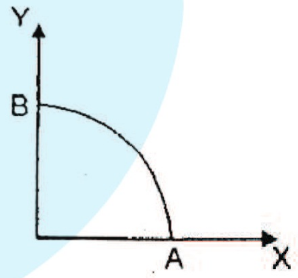
28. $F(x, y) = \left(x^2 + xy\right)a_x + \left(y^2 + xy\right)a_y$. It's line integral over the straight line from (x, y) = (0, 2) to (2, 0) evaluates to

(A) -8 (B) 4

(C) 8 (D) 0

[GATE-2009-EE]

29. A path AB in the form of one quarter of a circle of unit radius is shown in the figure. Integration of $(x + y)^2$ on path AB traversed in a counter clockwise sense is



(A) $\frac{\pi}{2} - 1$ (B) $\frac{\pi}{2} + 1$

(C) $\frac{\pi}{2}$ (D) 1

[GATE-2009]

30. The value of the line integral $\int \left(2xy^2dx + 2x^2ydy + dz\right)$ along a path joining the origin (0, 0, 0) and the point (1, 1, 1)

(A) 0 (B) 2

(C) 4 (D) 6

[GATE-2016-EE-SET 2]

31. A scalar potential ϕ has one following gradient; $\nabla\phi = yz\hat{i} + xz\hat{j} + xyk$.

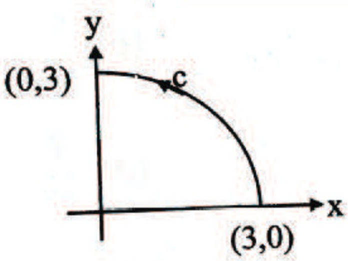
Consider the integral $\int_c \nabla\phi \cdot d\bar{r}$ on the curve $\bar{r} = x\hat{i} + y\hat{j} + zk$. The curve C is parameterized as follows:

$$\begin{cases} x = t \\ y = t^2 & \text{and} \quad 1 \le t \le 3 \\ z = 3t^2 \end{cases}$$

The value of the integral is _________

[GATE-2016]

32. As shown in the figure, C is the arc from the point (3, 0) to the point (0, 3) on the circle $x^2 + y^2 = 9$. The value of the integral $\int_c \left(y^2 + 2yx\right)dx + \left(2xy + x^2\right)dy$ is _________. (up to 2 decimal places).

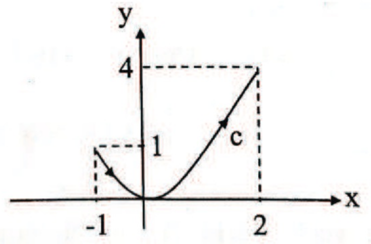


[GATE-18-EE]

33. If $f = 2x^3 + 3y^2 + 4z$, the value of line integral $\int_C (\text{grad} f). \text{Dr}$ evaluated over contour C formed by the segments (-3, -3, 2) $\rightarrow$ (2, -3, 2) $\rightarrow$ (2, 6, 2) $\rightarrow$ (2, 6, -1) is _______.

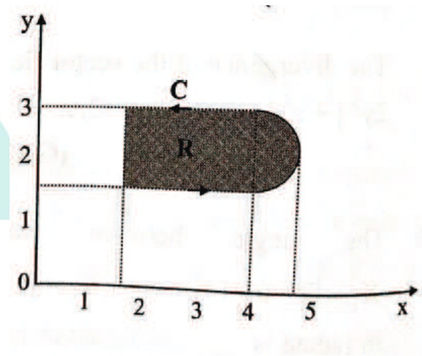
[GATE-19-EE]

34. Let a_x and a_y be unit vectors along x and y directions, respectively. A vector function is given by $F = a_x y - a_y x$. The line integral of the above function $\int_C F \cdot d\ell$ along the curve C, which follows the parabola $y = x^2$ as shown below is __________ (rounded off to 2 decimal places).



[GATE-20-EE]

35. Consider the line integral $\int_C (x\,dy - y\,dx)$ the integral being taken in a counterclockwise direction over the closed curve C that forms the boundary of the region R shown in the figure below. The region R is the area enclosed by the union of a 2×3 rectangle and a semi-circle of radius 1. The line integral evaluates to



(A) $6 + \pi/2$ (B) $8 + \pi$

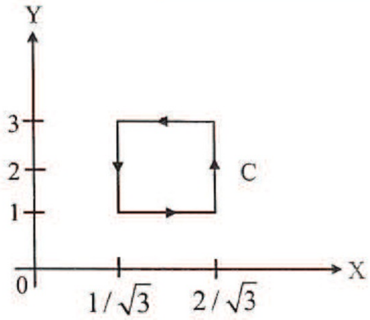
(C) $12 + \pi$ (D) $16 + 2\pi$

[GATE-19-EC]

36. The value of the line integral $\oint_C \bar{F} \cdot \bar{r}\,ds$, where C is a circle of radius $\frac{4}{\sqrt{\pi}}$ units is ______. Here, $\bar{F}(x,y) = y\hat{i} + 2x\hat{j}$ and $\bar{r}$ is the UNIT tangent vector on the curve C at an arc length s from a reference point on the curve. $\hat{i}$ and $\hat{j}$ are the basis vectors in the x-y Cartesian reference. In evaluating the line integral, the curve has to be traversed in the counter-clockwise direction.

[GATE-16-ME-SET 3]

37. If $\vec{A} = xya_x + x^2a_y$ then $\oint \vec{A} \cdot d\bar{r}$ over the path shown in the figure is



(A) 0 (B) $\frac{2}{\sqrt{3}}$

(C) 1 (D) $2\sqrt{3}$

[GATE-10 (EC)]

38. Let $C = \{(x,y) \in R^2, \max\{|x|,|y| = 1\}\}$ the value of line integral

$$\int_C \left(xy^2 + 2y + \sin(e^x)\right)dx + \left(x^2y + \cos(e^y)\right)dy$$

is ________ **[CSIR]**

39. Let $I = \int_C \frac{e^y}{x}dx + \left(e^y \ln x + x\right)dy$, where C is the positive oriented boundary of the region enclosed by $y = 1 + x^2$, y = 2 and $x = \frac{1}{2}$ then the value of I = _______

(A) $\frac{1}{8}$ (B) $\frac{5}{24}$

(C) $\frac{7}{24}$ (D) $\frac{3}{8}$

[GATE]

40. Value of the integral $\oint_C \left(xy\,dy - y^2dx\right)$, where C is the square cut from the first quadrant by the lines x = 1 and y = 1 will be (use Green's theorem to change the line integral into double integral)

(A) $\frac{1}{2}$ (B) 1

(C) $\frac{3}{2}$ (D) $\frac{5}{3}$

[GATE-2005]

41. For a > 0, b > 0 let $\bar{F} = \frac{-y\hat{i} + x\hat{j}}{b^2x^2 + a^2y^2}$ be a planar vector field. Let

$$C = \left\{(x,y) \in R^2 \,/\, x^2 + y^2 = a^2 - b^2\right\}$$

be the circle oriented anticlockwise. Then $\int_C \bar{F} \cdot d\bar{r} =$ _____

(A) $\frac{2\pi}{ab}$ (B) 2π

(C) $2\pi ab$ (D) 0

42. The value of $\int_C \frac{x\,dy - y\,dx}{x^2 + y^2}$ taken in the positive direction over any closed continuous curve C with origin inside it.

43. The line integral of function F = yzi, in the counter clockwise direction, along the circle $x^2 + y^2 = 1$ at z = 1 is

(A) -2π (B) $-\pi$

(C) π (D) 2π

[GATE-2014-EE-SET 1]

44. The value of $\iint_S \text{curl}\bar{v}\cdot n\,ds$ where $\bar{v} = 2y\hat{i} + 3x\hat{j} - z^2\hat{k}$ and S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 9$, n is the positive unit normal vector to s and c is its boundary ____

(A) 3π (B) 9π

(C) 18π (D) 32π

45. The value of $\iint_S (\nabla \times \bar{F})\cdot n\,ds$ where $\bar{F} = z\bar{i} + x\bar{j} + y\bar{k}$ and S is a hemisphere $z = \sqrt{1 - x^2 - y^2}$ of unit radius above xy plane.

(A) π (B) 2π

(C) $\frac{\pi}{2}$ (D) 4π

46. The value of $\int_C \sin z\,dx - \cos x\,dy + \sin y\,dz$ where c is the boundary of the rectangle $0 \le x \le \pi$, $0 \le y \le 2$, $z = 4$.

(A) 1 (B) 2

(C) 3 (D) 4

47. The value of $\int_C \bar{F}\cdot d\bar{r}$ where $\bar{F} = y^2\hat{i} + x^2\hat{j} - (x + 2z)\hat{k}$ where C is the boundary of the triangle with vertices at (0, 0, 0), (a, 0, 0), (a, a, 0).

48. The value of $\int_C y\,dx + z\,dy + x\,dz$ where C is the intersection of $x^2 + y^2 + z^2 = a^2$ and $x + z = a$.

49. The value of $\int_C \left((2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}\right)d\bar{r}$. Where C is the boundary of upper half of the surface of the sphere $x^2 + y^2 + z^2 = 1$ above xy plane is

(A) π (B) $-\pi$

(C) 2π (D) 0

50. Given vector $\bar{u} = \frac{1}{3}\left(-y^3\hat{i} + x^3\hat{j} + z^3\hat{k}\right)$ and n as the unit normal vector to the surface of the hemisphere $(x^2 + y^2 + z^2 = 1; z \ge 0)$, the value of integral $\int (\nabla \times \bar{u})\cdot\bar{n}\,dS$ evaluated on the curved surface of the hemisphere S is

(A) $-\frac{\pi}{2}$ (B) $\frac{\pi}{3}$

(C) $\frac{\pi}{2}$ (D) π

[GATE-19-ME-SET 2]

51. A vector field is defined as

$$\vec{f}(x,y,z) = \frac{x}{(x^2 + y^2 + z^2)^{3/2}}\hat{i} + \frac{y}{(x^2 + y^2 + z^2)^{3/2}}\hat{j} + \frac{z}{(x^2 + y^2 + z^2)^{3/2}}\hat{k}$$

where $\hat{i}$, $\hat{j}$, $\hat{k}$ are unit vectors along the axes of a right-handed rectangular / Cartesian coordinate system. The surface integral $\iint \vec{f}\cdot d\vec{S}$ (where $d\vec{S}$ is an elemental surface area vector) evaluated over the inner and outer surfaces of a spherical shell formed by two concentric spheres with origin as the center, and internal and external radii of 1 and 2, respectively, is

(A) 4π (B) 0

(C) 2π (D) 8π

[GATE-20-ME-SET 1]

52. The value of $\int_S \left(4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}\right) \cdot n\,ds$ where S is bounded by $x^2 + y^2 = 4$, $z = 0$ and $z = 2$ is

(A) 16π (B) 48π

(C) 32π (D) 84π

53. The value of $\int_S \bar{F} \cdot n\,ds$ where S is the surface of the sphere $x^2 + y^2 + z^2 = 4$ where n is the unit normal and $\bar{F} = x\bar{i} + y\bar{j} + z\bar{k}$ is ________

(A) 32π (B) 16π

(C) 8π (D) 64π

54. The flux of the vector filled $\bar{F} = x\bar{i} + y\bar{j} + z\bar{k}$ flowing out through the surface of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. $a > b > c > 0$ is

(A) πabc (B) $3\pi abc$

(C) $2\pi abc$ (D) $4\pi abc$

55. The value of $\int_S \left(\left(x^3 - yz\right)\bar{i} - 2x^2y\bar{j} + z\bar{k}\right) \cdot n\,ds$ where S is the surface of the cube bounded by $x = y = z = 2$ and co-ordinate planes is ________

(A) $\frac{32}{3}$ (B) $\frac{56}{3}$

(C) $\frac{16}{3}$ (D) $\frac{64}{3}$

56. Let S be the sphere $x^2 + y^2 + z^2 = 1$. The value of surface integral $\iint_S \left(x\sin y, \cos^2 x, 2z - z\sin y\right)\left(x, y, z\right) ds$ is ________

(A) $\frac{\pi}{3}$ (B) $\frac{2\pi}{3}$

(C) $\frac{4\pi}{3}$ (D) $\frac{8\pi}{3}$

57. Let S be the unit sphere $x^2 + y^2 + z^2 = 1$. Then the value of surface integral $\iint_S \left[\left(2x^2 + 3x\right) - y^2 + 5z^2\right] ds$ is

(A) 2π (B) 4π

(C) 8π (D) 12π

[CSIR]

58. The surface integral $\iint_S F \cdot n\,ds$ over the surface S of the sphere $x^2 + y^2 + z^2 = 9$, where $F = (x + y)i + (x + z)j + (y + z)k$ and n is the unit outward surface normal, yields _____.

[GATE-17-ME]

59. Let a > 0 and let $S = \left\{(x, y, z) \in R^3 \left| x^2 + y^2 + z^2 = a^2 \right.\right\}$.

Evaluate $\iint_S \left(x^4 + y^4 + z^4\right) ds$.

(A) $\frac{2\pi a^6}{5}$ (B) $\frac{12\pi a^4}{5}$

(C) $\frac{12\pi a^8}{5}$ (D) $\frac{12\pi a^6}{5}$

60. Consider the function $\bar{F} = \frac{1}{r^2}\hat{r}$, where r is distance from the origin and $\hat{r}$ is the unit vector in the radial direction. The divergence of this function over a

sphere of radius1, which includes the

origin is ______.

(A) 0 (B) 2π

(C) 4π (D) $R\pi$

61. Let, $B = \{(x, y, z) \notin R^3 \; \& \; x^2 + y^2 + z^2 \le 4\}$

Let, $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$ be vector valued function defined on B. If $r^2 = x^2 + y^2 + z^2$ then the value of $\iiint_S \nabla \cdot r^2\bar{r} \, dv$ is ______.

(A) 16π (B) 32π

(C) 64π (D) 128π

62. Let ϕ be a scalar field, and $\boldsymbol{u}$ be a vector field. Which of the following identities is true for div($\phi\boldsymbol{u}$)?

(A) div($\phi\boldsymbol{u}$) = ϕdiv($\boldsymbol{u}$) + $\boldsymbol{u}\cdot$ grad(ϕ)

(B) div($\phi\boldsymbol{u}$) = ϕdiv($\boldsymbol{u}$) + $\boldsymbol{u}$ × grad(ϕ)

(C) div($\phi\boldsymbol{u}$) = ϕgrad($\boldsymbol{u}$) + $\boldsymbol{u}\cdot$ grad(ϕ)

(D) div($\phi\boldsymbol{u}$) = ϕgrad($\boldsymbol{u}$) + $\boldsymbol{u}$ × grad(ϕ)

[GATE-2023 (CE-2)]

63. The value of m for which the vector fields $\vec{F} = (4x^m y^2 - 2xy^m)\hat{i} + (2x^4 y - 3x^2 y^2)\hat{j}$ is a conservative vector field is _________.

[GATE-2023 (XE)]

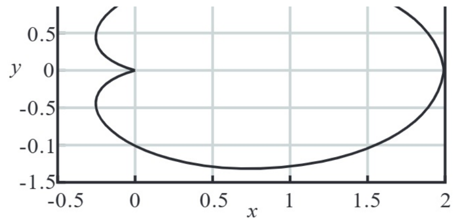
64. For a given vector $W = [1, 2, 3]^T$, the vector normal to the plane defined by $W^T x = 1$ is

(A) $[-2, -2, 2]^T$ (B) $[3, 0, -1]^T$

(C) $[3, 2, 1]^T$ (D) $[1, 2, 3]^T$

[GATE-2023 (EE)]

65. The closed curve shown in the figure is described by $r = 1 + \cos\theta$, where $r = \sqrt{x^2 + y^2}$; $x = r\cos\theta$, $y = r\sin\theta$. The magnitude of the line integral of the vector field $\vec{F} = -y\hat{i} + x\hat{j}$ around the closed curve is _________. (round off to two decimal places)



[GATE-2023 (EE)]

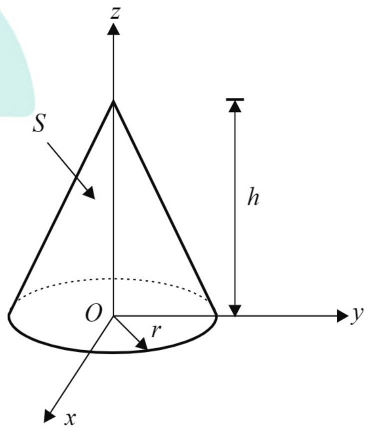
66. A quadratic function of two variables is given as

$f(x_1, x_2) = x_1^2 + 2x_2^2 + 3x_1 + 3x_2 + x_1 x_2 + 1.$

The magnitude of maximum rate of change of the function at the point (1,1) is ______. (Rounded off to the nearest integer)

[GATE-2023 (EE)]

67. A vector field $B(x, y, z) = x\hat{i} + y\hat{i} - 2z\hat{k}$ is defined over a conical region having height h = 2, base radius r = 3 and axis along z, as shown in the figure. The base of the cone lies in the x-y plane and is centered at the origin.If n denotes the unit outward normal to the curved surface S of the cone, the value of the integral $\int_S B.ndS$ equals _________. (Answer in integer)



[GATE-2023 (ME)]

❑❑❑

4 Differential Equations

1. The degree of the differential equation

$$\left[y + x\left(\frac{d^2y}{dx^2}\right)^2\right]^{1/4} = \frac{d^3y}{dx^3} \text{ is.}$$

2. Match each of the items A, B, C with an appropriate item from 1, 2, 3, 4 and 5

(A) $a_1\frac{d^2y}{dx^2}\cdot a_2y\frac{dy}{dx}\cdot a_3y = a_4$

(B) $a_1\frac{d^3y}{dx^3}\cdot a_2y = a_3$

(C) $a_1\frac{d^3y}{dx^2}\cdot a_2x\frac{dy}{dx} + a_3x^2y = 0$

(1) non-linear differential equation

(2) linear differential equation with constant coefficients

(3) linear homogeneous differential equation

(4) non-linear homogeneous differential equation

(5) non-linear first order differential equation

(A) A – 1, B – 2, C – 3

(B) A – 3, B – 4, C – 2

(C) A – 2, B – 4, C – 3

(D) A – 3, B – 1, C – 2

[GATE-1994 (EC)]

3. The differential equation

$$y^{11} + \left(y^3\sin x\right)^5 y^1 + y = \cos x^3 \text{ is}$$

(A) homogeneous

(B) nonlinear

(C) second order linear

(D) non homogeneous with constant coefficients

[GATE-1995]

4. $\frac{d^2y}{dx^2} + \left(x^2 + 4x\right)\frac{dy}{dx} + y = x^8 - 8$.

The above equation is a

(A) partial differential equation

(B) nonlinear differential equation

(C) non-homogeneous differential equation

(D) ordinary differential equation

[GATE-1999]

5. The degree of the differential equation

$\frac{d^2x}{dt^2} + 2x^3 = 0$ is

(A) zero (B) 1

(C) 2 (D) 3

[GATE-2007-CE]

6. The Blasius equation, $\frac{d^3t}{d\eta^3} + \frac{fd^2t}{2d\eta^2} = 0$, is a

(A) second order nonlinear ordinary differential equation

(B) third order nonlinear ordinary differential equation

(C) third order linear ordinary differential equation

(D) mixed order nonlinear ordinary differential equation

[GATE-2010-ME]

7. The order and degree of the differential equation $\frac{d^3y}{dx^3} + 4\sqrt{\left(\frac{dy}{dx}\right)^3 + y^2} = 0$ are

(IV) y_1, y_2 and y_3 are linearly independent on $-1 \le x \le 1$

Which on among the following is correct?

respectively.

(A) 3 and 3 (B) 2 and 3

(C) 3 and 3 (D) 3 and 1

[GATE-2010-CE]

8. The partial differential equation

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \frac{\partial^2 u}{\partial x^2} \text{ is a}$$

(A) linear equation of order 2

(B) non-linear equation of order 1

(C) linear equation of order 1

(D) non-linear equation of order 2

[GATE-2013-ME]

Linearly Dependent & Independent Solutions

9. Choose the CORRECT set of functions, which are linearly dependent.

(A) $\sin x$, $\sin^2 x$ and $\cos^2 x$

(B) cos x, sin x and tan x

(C) $\cos 2x$, $\sin^2 x$ and $\cos^2 x$

(D) cos 2x, sin x and cos x

[GATE-2013 (ME)]

10. Consider the following statements about the linear dependence of the real valued functions $y_1 = 1$, $y_2 = x$ and $y_3 = x^2$, over the field of real numbers.

(I) y_1, y_2 and y_3 are linearly independent on $-1 \le x \le 0$

(II) y_1, y_2 and y_3 are linearly dependent on $0 \le x \le 1$

(III) y_1, y_2 and y_3 are linearly dependent on $0 \le x \le 1$

correct?

(A) Both I and IV are true

(B) Both I and III are true

(C) Both II and IV are true

(D) Both III and IV are true

[GATE-2017 EC SESSION-1]

Formation of Differential Equation

11. The rate of which bacteria multiply is proportional to the instantaneous number present, if the original number doubles in 2 hours then it will be triple in

(A) $2\frac{\log 3}{\log 2}$ (B) $2\frac{\log 2}{\log 3}$

(C) $\frac{\log 3}{\log 2}$ (D) $\frac{\log 2}{\log 3}$

12. A radium decomposes at a rate proportional to the amount of radius present at that time. If 5% grams of original amount disappears after 50 years then the amount will be remain after 100 years is

(A) 95.95% of the original amount

(B) 95% of the original amount

(C) 90% of the original amount

(D) 90.25% of the original amount

12. The orthogonal trajectory of family of straight lines $y = k(x-1)$, $k \notin R$ are given by

(A) $(x-1)^2 + (y-1)^2 = c^2$

(B) $x^2 + y^2 = c^2$

(C) $x^2 + (y-1)^2 = c^2$

(D) $(x-1)^2 + y^2 = c^2$ **[GATE]**

13. The differential equations of the family of circles of radius ‘r’ and whose centre lies on ‘x’ axis.

(A) $r^2\left(1+\left(\frac{dy}{dx}\right)^2\right) = x^2$

(A) $\sin(x+y) - e^x = \text{constant}$

(B) $e^x \tan(x+y) = \text{constant}$

(C) $x\cos(x+y) - e^x + e^x = \text{constant}$

(D) $x\sin(x+y) - e^{x} + e^{x} = \text{constant}$

[GATE]

(B) $y^2\left(1+\left(\frac{dy}{dx}\right)^2\right) = r^2$

(C) $x^2\left(1+\left(\frac{dy}{dx}\right)^2\right) = r^2$

(D) $y^2\left(1-\left(\frac{dy}{dx}\right)^2\right) = r^2$

14. A spherical naphthalene ball exposed to the atmosphere loses volume at a rate proportional to its instantaneous surface area due to evaporation. If the initial diameter of the ball is 2 cm and the diameter reduces to 1 cm after 3 months, the ball completely evaporates in

(A) 6 months (B) 9 months

(C) 12 months (D) infinite time

[GATE-2006]

15. A body originally at 60^0C cools down to 40^0C in 15 min when kept in air at a temperature of 25^0C. What will be the temperature of the body at the end of 30 min?

(A) 35.2^0C (B) 31.5^0C

(C) 28.7^0C (D) 15^0C

[GATE-2007-CE]

Solution of Differential Equations

16. A solution of the first order differential equation

$$y'\cos(x+y) + \frac{\sin(x+y)}{x} = e^x - \cos(x+y)$$

is

17. Consider the following differential equation :

$$x(ydx + xdy)\cos\frac{y}{x} = y(xdy - ydx)\sin\frac{y}{x}$$

Which of the following is the solution of the above equation (c is an arbitrary constant?

(A) $\frac{x}{y}\cos\frac{y}{x} = c$ (B) $\frac{x}{y}\sin\frac{y}{x} = c$

(C) $xy\cos\frac{y}{x} = c$ (D) $xy\sin\frac{y}{x} = c$

[GATE-2015-CE-SET-II]

18. The solution of the differential equation $(x^2 + y^2 + 2x)dx + 2y\,dy = 0$ is

(A) $e^x(x^2 - y^2) = c$ (B) $e^x(x^2 + y^2) = c$

(C) $e^{-x}(x^2 + y^2) = c$ (D) $e^{-x}(x^2 - y^2) = c$

19. The solution of

$(y - xy^2)dx - (x + x^2y)dy = 0$ is

(A) $\ln\left(\frac{x}{y}\right) - \frac{x}{y} = c$ (B) $\ln\left(\frac{x}{y}\right) + \frac{x}{y} = c$

(C) $\ln\left(\frac{x}{y}\right) + xy = c$ (D) $\ln\left(\frac{x}{y}\right) - xy = c$

Exact Differential Equation

20. The differential equation

$$(27x^2 + ky\cos x)dx + (2\sin x - 27y^3)dy = 0$$

is exact for k = ____. **[GATE]**

21. If the integrating factor of

$(x^7y^2 + 3y)dx + (3x^8y - x)dy = 0$

is $x^{\alpha}y^{\beta}$ then $\alpha =$ ____ and $\beta =$ ____.

[GATE]

22. The differential equation

26. The figure shows the plot of y as a function of x. The function shown is the solution of the differential equation (assuming all initial conditions to be zero) is

$\frac{d^2y}{}$ ... $\frac{dy}{}$

$(\alpha xy^3 + y\cos x)dx + (x^2y^2 + \beta\sin x)dy = 0$

is exact for

(A) $\alpha = \frac{3}{2}, \beta = 1$ (B) $\alpha = 1, \beta = \frac{3}{2}$

(C) $\alpha = \frac{2}{3}, \beta = 1$ (D) $\alpha = 1, \beta = \frac{2}{3}$

[GATE]

23. For the differential equation,

$f(x, y)\frac{dy}{dx} + g(x, y) = 0$

to be exact,

(A) $\frac{\partial f}{\partial y} = \frac{\partial g}{\partial x}$ (B) $\frac{\partial f}{\partial x} = \frac{\partial g}{\partial y}$

(C) $f = g$ (D) $\frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 g}{\partial y^2}$

[GATE-1997-CE]

Variable Separable

24. The solution of $\frac{dy}{dx} = y^2$ with initial value $y(0) = 1$ bounded in the interval

(A) $-\infty < x < \infty$ (B) $-\infty < x \le 1$

(C) $x < 1, x > 1$ (D) $-2 \le x \le 2$

[GATE-2007-ME]

25. The solution of the differential equation

$y\sqrt{1-x^2}dy + x\sqrt{1-y^2}dx = 0$ is

(A) $\sqrt{1-x^2} = c$

(B) $\sqrt{1-y^2} = c$

(C) $\sqrt{1-x^2} + \sqrt{1-y^2} = c$

(D) $\sqrt{1+x^2} + \sqrt{1+y^2} = c$

[ESE-2017 (EE)]

(A) $\frac{d^2y}{dx^2} = 1$ (B) $\frac{dy}{dx} = +x$

(C) $\frac{dy}{dx} = -x$ (D) $\frac{dy}{dx} = |x|$

[GATE-2014 (IN-SET 1)]

Homogeneous Differential Equation

27. The solution of $\frac{dy}{dx} = \sin(x+y)$ is

(A) $\tan(x+y) - \sec(x+y) = x + c$

(B) $\sec(x+y) - \tan(x+y) = \frac{x^2}{2} + c$

(C) $\tan(x+y) - \cos(x+y) = x + c$

(D) $\tan(x+y) - \cot(x+y) = x + c$

28. The solution of $\frac{dy}{dx} = \left(\frac{y}{x}\right) + \tan\left(\frac{y}{x}\right)$ is

(A) $\sin\left(\frac{y}{x}\right) = xc$ (B) $\tan\left(\frac{y}{x}\right) = xc$

(C) $\operatorname{cosec}\left(\frac{y}{x}\right) = xc$ (D) $\cot\left(\frac{y}{x}\right) = xc$

29. Solve the differential equation,

$xy^2\frac{dy}{dx} = x^3 + y^3$

[GATE-1994-ME]

30. A curve passes through the point (x = 1, y = 0) and satisfies the differential equation $\frac{dy}{dx} = \frac{x^2 + y^2}{2y} + \frac{y}{x}$. The equation that describes the curve is

(A) $\ln\left(1 + \frac{y^2}{x^2}\right) = x - 1$

(B) $\frac{1}{2}\ln\left(1 + \frac{y^2}{x^2}\right) = x - 1$

(C) $\ln\left(1 + \frac{y}{x}\right) = x - 1$

(D) $\frac{1}{2}\ln\left(1 + \frac{y}{x}\right) = x - 1$

(B) $\frac{dv}{dt} + (1-n)pv = (1+n)q$

(C) $\frac{dv}{dt} + (1+n)pv = (1-n)q$

(D) $\frac{dv}{dt} + (1+n)pv = (1+n)q$

[GATE-2005-CE]

[GATE 2018 (EC)]

Linear Differential Equation of 1st Order

31. The general solution of the differential equation $\frac{dy}{dx}+\tan x\cdot\tan y=\cos x\cdot\sec y$ is

(A) $2\sin y=(x+c-\sin\cos x)\sec x$

(B) $\sin y=(x+c)\cos x$

(C) $\cos y=(x+c)\sin x$

(d) $\sec y=(x+c)\cos x$ **[GATE]**

32. The general solution of $(x^3y^2+xy)\frac{dx}{dy}=1$ is

(A) $\frac{-1}{y}=x^2-2+c\cdot e^{-x^2/2}$

(B) $\frac{1}{y}=x^2+2+c\cdot e^{-x^2/2}$

(C) $\frac{1}{y}=x^2+2+c\cdot e^{x^2/2}$

(D) $\frac{1}{y}=x^2+1+c\cdot e^{-x^2/2}$

33. Transformation to linear form by substituting $v=y^{1-n}$ of the equation $\frac{dy}{dt}+p(t)y=q(t)y^n$; $n>0$ will be

(A) $\frac{dv}{dt}+(1-n)pv=(1-n)q$

34. A system described by a linear, constant coefficient, ordinary, first order differential equation has an exact solution given by y(t) for y > 0, when the forcing function is x(t) and the initial condition is y(0). If one wishes to modify the system so that the solution becomes -2y(t) for t > 0, we need to

(A) change the initial condition to -y(0) and the forcing function to 2x(t)

(B) change the initial condition to 2y(0) and the forcing function to (t)

(C) change the initial condition to $j\sqrt{2}y(0)$ and the forcing function to $j\sqrt{2}x(t)$

(D) change the initial condition to -2y(0) and the forcing function to -2x(t)

[GATE-2013 (EC)]

35. Which ONE of the following is a linear non-homogeneous differential equation, where x and y are the independent and dependent variables respectively?

(A) $\frac{dy}{dx}+xy=e^{-x}$ (B) $\frac{dy}{dx}+xy=0$

(C) $\frac{dy}{dx}+xy=e^{-y}$ (D) $\frac{dy}{dx}+e^{-y}=0$

[GATE-2014-EC-SET 3]

36. The integrating factor for the differential equation $\frac{dp}{dt}+K_2p=K_1L_0e^{-K_1t}$ is

(A) e^{-K_1t} (B) e^{-K_2t}

(C) e^{K_1t} (D) e^{K_2t}

[GATE-2014-CE-SET 2]

37. Consider the differential equation

(A) both b and c are positive

(B) b is positive, and c is negative

(C) b is negative but c is positive

(D) both b and c are negative

41. The differential equation $y''+y=0$ is subjected to the conditions $y(0)=0$, $y(\lambda)=0$. In order that the equation has

$\left(t^2-81\right)\frac{dy}{dt}+5ty=\sin(t)$ with $y(1)=2\pi$.

There exists a unique solution for this differential equation when t belongs to the interval

(A) (-2, 2) (B) (-10, 10)

(C) (-10, 2) (D) (0, 10)

[GATE-2017 EE SESSION-1]

38. The integrating factor of

$(\cos y \sin 2x)dx+\left(\cos^2 y-\cos^2 x\right)dy=0$

is

(A) $\sec^2 y+\sec y\cdot\tan y$

(B) $\tan^2 y+\sec y\cdot\tan y$

(C) $\frac{1}{\sec^2 y+\sec y\cdot\tan y}$

(D) $\frac{1}{\tan^2 y+\sec y\cdot\tan y}$

[GATE]

39. The solution of the differential equation

$\left(x^2+y^2+2x\right)dx+2ydy=0$

(A) $e^x\left(x^2-y^2\right)=c$ (B) $e^x\left(x^2+y^2\right)=c$

(C) $e^{-x}\left(x^2+y^2\right)=c$ (D) $e^{-x}\left(x^2-y^2\right)=c$

40. Consider $\frac{d^2y}{dx^2}+b\frac{dy}{dx}+cy=0$ where b & c are real constants. If $y=x\cdot e^{-5x}$ is a solution then

non-trivial solutions the general value of λ is

[GATE-1993 (ME)]

42. The Solution to the differential equation $f''(x)+4f'(x)+4f(x)=0$ is

(A) $f_1(x)=e^{-2x}$

(B) $f_1(x)=e^{2x}, f_2(x)=xe^{-2x}$

(C) $f_1(x)=e^{-2x}, f_2(x)=xe^{-2x}$

(D) $f_1(x)=e^{2x}, f_2(x)=xe^{-x}$

[GATE-1995-ME]

43. The complete solution of the ordinary differential equation $\frac{d^2y}{dx^2}+p\frac{dy}{dx}+qy=0$ is $y=c_1e^{-x}+c_2e^{-3x}$. Then, p and q are

(A) p = 3, q = 3 (B) p = 3, q = 4

(C) p = 4, q = 3 (D) p = 4, q = 4

[GATE-2005-ME]

44. For the equation

$x''(t)+3x'(t)+2x(t)=5$,

the solution x(t) approaches to the following value as $t\to\infty$

(A) 0 (B) 5/2

(C) 5 (D) 10

[GATE-2005-EE]

45. For the differential equation

$\frac{d^2y}{dx^2}+k^2y=0$

the boundary conditions are

(i) y = 0 for x = 0 and

(ii) y = 0 for x = a

The form of non-zero solutions y (where m varies over all integers) are

(A) $y=\sum_m A_m \sin\frac{m\pi x}{a}$

48. If the characteristic equation of the differential equation

$\frac{d^2y}{dx^2}+2\alpha\frac{dy}{dx}+y=0$

has two equal roots, then the values of α are

(A) ±1 (B) 0, 0

(C) $\pm j$ (D) $\pm1/2$

[GATE-2014-EC-SET 2]

(B) $y = \sum_m A_m \cos \frac{m\pi x}{a}$

(C) $y = \sum_m A_m x^{\frac{m\pi}{a}}$

(D) $y = \sum_m A_m e^{\frac{m\pi x}{a}}$

[GATE-2006-EC]

46. The homogeneous part of the differential equation $\frac{d^2y}{dx^2} + p\frac{dy}{dx} + qy = r$ (p, q, r are constants) has real distinct roots if

(A) $p^2 \cdot 4q > 0$ (B) $p^2 \cdot 4q < 0$

(C) $p^2 \cdot 4q = 0$ (D) $p^2 \cdot 4q = r$

[GATE-2009 (PI)]

47. A function n(x) satisfies the differential equation $\frac{d^2n(x)}{dx^2} - \frac{n(x)}{L^2} = 0$ where L is a constant. The boundary conditions are: n(0) = K and $n(\infty) = 0$. The solution to this equation is

(A) $n(x) = K\exp(x/L)$

(B) $n(x) = K\exp(-x/\sqrt{L})$

(C) $n(x) = K^2\exp(-x/L)$

(D) $n(x) = K\exp(-x/L)$

[GATE-2010-EC]

49. Consider two solution $x(t) = x_1(t)$ and $x(t) = x_2(t)$ of the differential equation $\frac{d^2x(t)}{dt^2} + x(t) = 0,\ t > 0$, such that

$x_1(0) = 1,\ \left.\frac{dx_1(t)}{dt}\right|_{t=0} = 0,\ x_2 = 0$

$\left.\frac{dx_2(t)}{dt}\right|_{t=0} = 1.$

The Wronskian $W(t) = \begin{vmatrix} x_1(t) & x_2(t) \\ \frac{dx_1(t)}{dt} & \frac{dx_2(t)}{dt} \end{vmatrix}$

at $t = \pi/2$ is

(A) 1 (B) -1

(C) 0 (D) $\pi/2$

[GATE-2014-ME-SET 3]

50. The differential equation $\frac{d^2y}{dx^2} + 16y = 0$ for y(x) with the two boundary conditions $\left.\frac{dy}{dx}\right|_{x=0} = 1$ and $\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{2}} = -1$ has

(A) no solution

(B) exactly two solutions

(C) exactly one solution

(D) infinitely many solutions

[GATE 2017]

51. If e^{-x} and xe^{-x} are two independent solutions of $\frac{d^2y}{dx^2} + \lambda\frac{dy}{dx} + y = 0$ then the value of $\lambda =$ _____.

Particular Integral

52. For initial value problem

$y + 2y + (101)y = (104)e^x$, y(0) = 1.1 and y(0) = -0.9. Various solutions are written in the following groups. Match

(A) $y = (y_1 - y_2)\exp(-x/k^2) + y_2$

(B) $y = (y_2 - y_1)\exp(-x/k) + y_1$

(C) $y = (y_1 - y_2)\sinh(x/k) + y_1$

(D) $y = (y_1 - y_2)\exp(-x/k) + y_2$

[GATE-2007-EC]

54. Consider the following second-order differential equation:

$y'' - 4y' + 3y = 2t - 3t^2$.

the type of solution with the correct expression.

Group-1

P. General solution of homogeneous equations

Q. Particular integral

S. Total solution satisfying boundary conditions

Group-II

(1) $0.1e^{x}$

(2) $e^{-x}\left[A\cos 10x + B\sin 10x\right]$

(3) $e^{-x}\cos 10x + 0.1e^{x}$

(A) P – 2, Q – 1, R – 3

(B) P – 1, Q – 3, R – 2

(C) P – 1, Q – 2, R – 3

(D) P – 3, Q – 2, R – 1

[GATE-2006 (IN)]

53. The solution of the differential equation $k^2\frac{d^2y}{dx^2} = y - y_2$ under the boundary conditions

(i) $y = y_1$ at x = 0 and

(ii) $y = y_2$ at $x = \infty$, where k, y_1 and y_2 are constants, is

The particular solution of the differential equation is

(A) $-2-2t-t^2$ (B) $-2t-t^2$

(C) $2t-3t^2$ (D) $-2-2t-3t^2$

[GATE-2017 CE SESSION-II]

55. Consider $y''-y=2e^{x}$ if y(0) = 0, $y'(0)=0$ then y(1) =

(A) e + sin h (1) (B) cos h(1)

(C) sin h(1) (D) cos h(1) + 1

[CSIR]

56. Suppose $y_p(x) = x\cos 2x$ is a particular integral of $y''+\alpha y = -4\sin 2x$, then the constant α is _____

[GATE]

Cauchy Homogeneous Linear Differential Equations

57. The general solution of the differential equation $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$ is:

(A) $Ax + Bx^2$ (A, B are constants)

(B) $Ax + B\log x$ (A, B are constants)

(C) $Ax + Bx^2\log x$ (A, B are constants)

(D) $Ax + Bx\log x$ (A, B are constants)

[GATE-1998]

58. The radial displacement in a rotating disc is governed by the differential equation $\frac{d^2u}{dx^2} + \frac{1}{x}\frac{du}{dx} - \frac{u}{x^2} = 8x$ where u is the displacement and x is the radius. If u = 0 and x = 0, and u = 2 at x = 1, calculate the displacement at $x = \frac{1}{2}$

[GATE-1998]

59. The solution to $x^2y''+xy'-y=0$ is

(A) $y = C_1x^2 + C_2x^{-3}$ (B) $y = C_1 + C_2x^{-2}$

63. The solution of $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$, $u(0,y) = 8e^{-3y}$ is _________

(A) $u(x,y) = 8e^{-12x-3y}$

(B) $u(x,y) = 8e^{-3x-12y}$

(C) $u(x,y) = 8e^{-3y-4x}$

(D) $u(x,y) = 8e^{-3x-3y}$

64. The solution of $\sqrt{p} + \sqrt{q} = 1$ is _____

(A) $z = ax + by + c$

(B) $z = ax + \left(1-\sqrt{a}\right)^2 y + c$

(C) $y = C_1x + \frac{C_2}{x}$ (D) $y = C_1x + C_2x^4$

[GATE-2015 (PI)]

60. The differential equation for which x, x ln x and x^2 are independent solutions is

(A) $x^3y''' + x^2y'' - 3xy' + 3y = 0$

(B) $x^3y''' - 2x^2y'' + 3xy' - 6y = 0$

(C) $x^3y''' - x^2y'' + 2xy' - 2y = 0$

(D) $y''' - y'' + 2y' - 3y = 0$

[CSIR]

61. Consider the differential equation $x^2y'' - 3xy' + 4y = 0$ then the two linearly independent solutions of the differential equations are given by

(a) $x^2,\ x^3$ (B) $x^2,\ x^2 \ln x$

(C) v (D) $x,\ xe^x$

[CSIR]

62. If $x^2\frac{dy}{dx} + 2xy = \frac{2\log x}{x}$ and y(1) = 0 then y(e) = 0

(A) e (B) 1

(C) $\frac{1}{e}$ (D) $\frac{1}{e^2}$

[GATE]

(C) $z = ax + \left(1 - \sqrt{a}\right)y + c$

(D) $z = ax - \sqrt{a}y + c$

65. The solution of $p(1+q) = qz$ is ______

(A) $az - 1 = e^{x+ay+c}$

(B) $z = e^{x+ay+c}$

(C) $z = ax + by + c$

(D) $z = ax + by + f(a,b)$

66. The solution of $p^2 + q^2 = x + y$ is ____

(A) $z = ax + by + c$

(B) $z = \frac{2}{3}(a+x)^{3/2} + \frac{2}{3}(y-a)^{3/2} + b$

(C) $z = \frac{3}{2}(a+x)^{2/3} + \frac{3}{2}(y-a)^{2/3} + b$

(D) $z = (a+x)^{1/2} + (y-a)^{1/2} + b$

67. The general integral of the partial differential equation $y^2p - xyq = x(z - 2y)$ is

(A) $\phi(x^2 + y^2, y^2 - yz) = 0$

(B) $\phi(x^2 - y^2, y^2 + yz) = 0$

(C) $\phi(xy, yz) = 0$

(D) $\phi(x + y, \ln x - z) = 0$

[ESE 2018 (EE)]

68. The solution at x = 1, t = 1 of the partial differential equation, $\frac{\partial^2 u}{\partial x^2} = 25\frac{\partial^2 u}{\partial t^2}$ subject to initial conditions of u(0) = 3x and $\frac{\partial u}{\partial t}(0) = 3$ is ______

(A) 1 (B) 2

(C) 4 (D) 6

[GATE-2018 (CE-MORNING SESSION)]

69. The partial differential equation that can be formed from z = ax + by + ab has the form $\left(\text{with } p = \frac{\partial z}{\partial x} \text{ and } q = \frac{\partial z}{\partial y}\right)$

Classification of Partial Differential Equation

72. Consider the following partial differential equation:

$$3\frac{\partial^2\phi}{\partial x^2} + B\frac{\partial^2\phi}{\partial x\partial y} + 3\frac{\partial^2\phi}{\partial y^2} + 4\phi = 0$$

For this equation to be classified as parabolic, the value of B^2 must be _____

[GATE 2017]

73. The type of partial differential equation $\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + 3\frac{\partial^2 P}{\partial x\partial y} + 2\frac{\partial P}{\partial x} - \frac{\partial P}{\partial y} = 0$ is

(A) elliptic (B) parabolic

(C) hyperbolic (D) none of these

has the form $\left(\text{with } p = \frac{\partial z}{\partial x} \text{ and } q = \frac{\partial z}{\partial y}\right)$

(A) z = px + qy

(B) z = px + pq

(C) z = px + qy + pq

(D) z = py + pq

[GATE-2010-CE]

70. The complete integral of

$(z - px - qy)^3 = pq + 1(p^2 + q)^2$ is

(A) $z = ax + by + \sqrt[3]{pq + 2(p^2 + q)^2}$

(B) $z = ax + by + \sqrt[3]{ab + 2(a^2 + b)^2}$

(C) $z = ax + by + \sqrt[3]{ab} + \sqrt[3]{2(a^2 + b)^2}$

(D) $z = ax + by + c$

[ESE 2017 (COMMON PAPER)]

71. The solution of the following partial differential equation $\frac{\partial^2 u}{\partial x^2} = 9\frac{\partial^2 u}{\partial y^2}$ is

(A) $\sin(3x - y)$ (B) $3x^2 + y^2$

(C) $\sin(3x - 3y)$ (D) $(3y^2 - x^2)$

[ESE 2017 (COMMON PAPER)]

(C) hyperbolic (D) none of these

[GATE-2016-CE-SET 1]

74. The solution of the initial boundary value problem $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ $0 < x <$, $t > 0$ with boundary and initial conditions $\frac{\partial u}{\partial x}(0,t) = 0 = u(\pi,t)$, $t > 0$ and $u(x,0) = f(x)$, $0 < x < \pi$ is ____

(A) $u(x,t) = \sum_{n=0}^{\infty} A_n e^{\left\{-\left(-\frac{2n+1}{2}\right)^2 t\right\}} \cos\left(\frac{2n+1}{2}x\right)$

with $A_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos\left(\frac{2n+1}{2}x\right)dx$

(B) $u(x,t) = \sum_{n=0}^{\infty} A_n e^{\left(-n^2 t \cos n_x\right)}$

with $A_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\, dx$

(C) $u(x,t) = \sum_{n=0}^{\infty} A_n e^{\left\{-\left(\frac{2n+1}{2}\right)^2\right\}} \sin\left\{\left(\frac{2n+1}{2}\right)x\right\}$

with $A_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin\left(\frac{2n+1}{2}x\right)dx$

(D) $u(x,t) = \sum_{n=1}^{\infty} A_n \exp(-n^2 t)\sin(nx)$

with $A_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin(nx)dx$

75. If u = (x, t) is such that $\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}$, $0 \le x \le \pi$, $t \ge 0$, $u(0,t) = u(\pi,t) = 0$

$u(x,0) = 0$; $\frac{\partial u}{\partial t}(x,0) = \sin x$ then $u\left(\frac{\pi}{3}, \frac{\pi}{6}\right)$ is ________

(A) $\frac{3}{4}$ (B) $\frac{3}{8}$

(C) $\frac{\sqrt{3}}{4}$ (D) $\frac{\sqrt{3}}{8}$

76. The number of boundary conditions required to solve the differential

78. Which one of the following is a property of the solutions to the Laplace equation: $\nabla^2 f = 0$?

(A) The solutions have neither maxima nor minima anywhere except at the boundaries.

(B) The solutions are not separable in the coordinates.

(C) The solutions are not continuous.

(D) The solutions are not dependent on the boundary conditions.

[GATE-2016, 2 MARKS]

79. Solution of Laplace's equation having equation $\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}$ is

(A) 2 (B) 0

(C) 4 (D) 1

[GATE-2001 (CE)]

77. The solution of the partial differential equation $\frac{\partial u}{\partial t}=\alpha\frac{\partial^2 u}{\partial x^2}$ is of the form

(A) $C\cos(kt)\left[C_1e^{(\sqrt{k/\alpha})x}+C_2e^{-(\sqrt{k/\alpha})x}\right]$

(B) $Ce^{kt}\left[C_1e^{(\sqrt{k/\alpha})x}+C_2e^{-(\sqrt{k/\alpha})x}\right]$

(c) $Ce^{kt}\left[C_1\cos\left(\sqrt{\frac{k}{\alpha}}\right)x+C_2\sin\left(-\sqrt{\frac{k}{\alpha}}\right)x\right]$

(D) $C\sin(kt)\left[C_1\cos\left(\sqrt{\frac{k}{\alpha}}\right)x+C_2\sin\left(-\sqrt{\frac{k}{\alpha}}\right)x\right]$

[GATE-2016-CE-SET 1]

continuous second-order partial derivatives are called

(A) biharmonic functions

(B) harmonic functions

(C) conjugate harmonic functions

(D) error functions

[GATE-2016; 2 MARKS]

80. The solution $x(t)$, $t\geq 0$, to the differential equation $\ddot{x}=-k\dot{x}$, $k>0$ with initial conditions x(0) = 1 and $\dot{x}(0)=0$ is

(A) $x(t)=2e^{-kt}+2kt-1$

(B) $x(t)=2e^{-kt}-1$

(C) $x(t)=1$

(D) $x(t)=2e^{-kt}-kt-1$

[GATE-2023 (IN)]

81. Consider the second-order linear ordinary differential equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=0, x\geq 1$ with initial condition $y(x=1)=6, \left.\frac{dy}{dx}\right|_{x=1}=2$. The value of y at x = 2 equals __________ (Answer in integer).

[GATE-2023 (ME)]

82. In the differential equation

$$\frac{dy}{dx}+\alpha xy=0,$$

α is a positive constant. If y = 1.0 at x = 0.0, and y = 0.8 at x = 1.0, the value of α is________ (rounded off to three decimal places).

[GATE-2023 (CE-1)]

83. The solution of the differential equation

$$\frac{d^3y}{dx^3}-5.5\frac{d^2y}{dx^2}+9.5\frac{dy}{dx}-5y=0$$

is expressed as $y=C_1e^{2.5x}+C_2e^{\alpha x}+C_3e^{\beta x}$, where C_1, C_2, C_3, α and β are constants, with α and β being distinct and not equal to 2.5. Which of the following options is correct for the values of α

and β?

(A) 1 and 2 (B) −1 and −2

(C) 2 and 3 (D) −2 and −3

[GATE-2023 (CE-2)]

❑❑❑

5 Complex Functions

Objective Questions

1. If $|z-1|=2$, then $z\bar{z}-z-\bar{z}=$ ____

2. Given two complex numbers

$z_1=5+\left(5\sqrt{3}\right)i$ and $z_2=\dfrac{2}{\sqrt{3}}+2i$

the argument of $\dfrac{z_1}{z_2}$ in degree is

(A) 0 (B) 30

(C) 60 (D) 90

3. If 1, ω, ω^2 are cube root of unity, then the roots of $(x-1)^3+8=0$ are

(A) $\dfrac{z_1}{z_2}=\dfrac{z_1\overline{z_2}}{|z_2|^2}$

(B) $|z_1+z_2|\le|z_1|+|z_2|$

(C) $|z_1-z_2|\le|z_1|-|z_2|$

(D) $|z_1+z_2|^2+|z_1-z_2|^2=2|z_1|^2+2|z_2|^2$

[GATE-2005-CE]

7. Consider the circle $|z-5-5i|=2$ in the complex number plane (x, y) with $z=x+iy$. The minimum distance from the roots of $(x-1)^3 + 8 = 0$ are

(A) $-1, -1, -1$

(B) $1, \omega, 2\omega$

(C) $-1, 1, 2\omega, 1, 2\omega^2$

(D) $-1, 1-2\omega, 1-2\omega^2$

4. Which of the following is possible value for the imaginary part of $\ln\left(\sqrt{i}\right)$

(A) π (B) $\frac{\pi}{2}$

(C) $\frac{\pi}{4}$ (D) $\frac{\pi}{8}$

[GATE]

5. e^z is a periodic with a period of

(A) 2π (B) $2\pi i$

(C) π (D) $i\pi$

[GATE-1997-CE]

6. Which one of the following is not true for complex number z_1 and z_2?

the origin to the circle is

(A) $5\sqrt{2}-2$ (B) $\sqrt{54}$

(C) $\sqrt{34}$ (D) $5\sqrt{2}$

[GATE-2005 (IN)]

8. Let $z^3 = \bar{z}$, where z is a complex number not equal to zero. Then z is a solution of

(A) $z^2 = 1$ (B) $z^3 = 1$

(C) $z^4 = 1$ (D) $z^9 = 1$

[GATE-2005 (IN)]

9. If a complex number $z = \frac{\sqrt{3}}{2} + i\frac{1}{2}$, then z^4 is

(A) $2\sqrt{2} + 2i$ (B) $\frac{1}{2} + i\frac{\sqrt{3}}{2}$

(C) $\frac{\sqrt{3}}{2} - i\frac{1}{2}$ (D) $\frac{\sqrt{3}}{8} - i\frac{1}{8}$

[GATE-2007 (PI)]

10. The equation sin (z) = 10 has

(A) no real (or) complex solution

(B) exactly two distinct complex solution

(C) a unique solution

(D) an infinite number of complex solutions

[GATE-2008 (ME)]

11. If $z = x + jy$, where x, y are real when the value of $\left|e^{jz}\right|$ is

(A) 1 (B) $e^{\sqrt{x^2-y^2}}$

(C) e^y (D) e^{-y}

[GATE-2009 (IN)]

12. One of the roots of equation $x^3 = j$, where j is the positive square roots of -1 is

(D) $\cos\left(\frac{3\pi}{4}\right) + i\sin\left(-\frac{3\pi}{4}\right)$,

$\cos\left(-\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)$

[GATE-2013-EE]

15. All the values of the multi-valued complex function 1^i, where $i = \sqrt{-1}$, are

(A) purely imaginary

(B) real and non-negative

(C) on the unit circle

(D) equal in real and imaginary parts

[GATE-2014 – EE-SET 2]

16. Given two complex number

$z_1 = 5 + \left(5\sqrt{3}\right)i$ and $z_2 = \frac{2}{\sqrt{3}} + 2i$,

(A) j (B) $\frac{\sqrt{3}}{2}+\frac{j}{2}$

(C) $\frac{\sqrt{3}}{2}-\frac{j}{2}$ (D) $-\frac{\sqrt{3}}{2}-\frac{j}{2}$

[GATE-2009 (IN)]

13. If $x=\sqrt{-1}$, then the value of x^x is

(A) $e^{-\pi/2}$ (B) $e^{\pi/2}$

(C) x (D) 1

[GATE-2012-EC, EE, IN]

14. Square roots of -i, where $i=\sqrt{-1}$, are

(A) i, -1

(B) $\cos\left(-\frac{\pi}{4}\right)+i\sin\left(-\frac{\pi}{4}\right)$,

$\cos\left(\frac{3\pi}{4}\right)+i\sin\left(\frac{3\pi}{4}\right)$

(C) $\cos\left(\frac{\pi}{4}\right)+i\sin\left(\frac{3\pi}{4}\right)$,

$\cos\left(\frac{3\pi}{4}\right)+i\sin\left(\frac{\pi}{4}\right)$

the argument of $\frac{z_1}{z_2}$ in degree is

(A) 0 (B) 30

(C) 60 (D) 90

[GATE-2015-ME-SET 1]

17. Which one of the following options correctly describes the location of the roots of the equation $s^4+s^2+1=0$ on the complex plane?

(A) Four left half plane (LHP) roots

(B) One right half plane (RHP) root, one LHP root and two roots on the imaginary axis

(C) Two RHP roots and two LHP roots

(D) All four roots are on the imaginary axis

[GATE-2017 EC SESSION-1]

18. Let $z=x+jy$, where $j=\sqrt{-1}$. Then $\overline{\cos z}=$

(A) cos z (B) $\cos\bar{z}$

(C) sin z (D) $\sin\bar{z}$

[GATE-2017 (IN)]

Graph of Complex Function

19. The function $f(z)=z^2$ maps first quadrant onto ______

(A) itself

(B) upper half plane

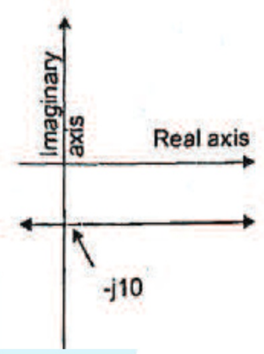
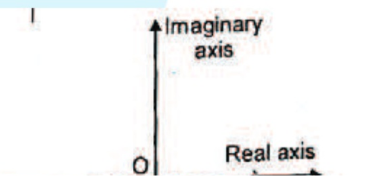
(C) third quadrant

(D) right half plane

20. The bilinear transformation $w=\frac{z-1}{z+1}$

(A) maps the inside of the unit circle in the z-plane to the left half of the w-plane

22. A complex variable z = x + j(0.1) has its real part x varying in the range $-\infty$ to ∞. Which one of the following is the locus (shown in thick lines) of $\frac{1}{z}$ in the complex plane?

(A)	
(B)	

plane

(B) maps the outside the unit circle in the z-plane to the left half of the w-plane

(C) maps the inside of the unit circle in the z-plane to right half of the w-plane

(D) maps the outside the unit circle in the z-plane to the right half of the w-plane

[GATE-2002 (IN)]

21. For the function of a complex variable W = ln Z (where, W = u + jv and Z = x + jy), then u = constant lines get mapped in z-plane as

(A) set of radial straight lines

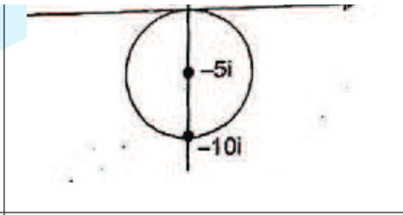
(B) set of concentric circles

(C) set of confocal hyperbolas

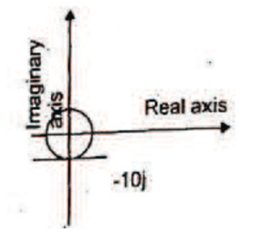
(D) set of confocal ellipses

[GATE-2006-EC]

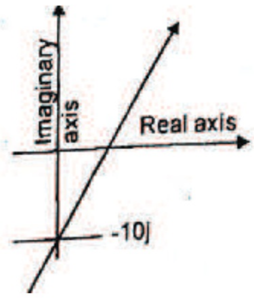
(B)



(C)

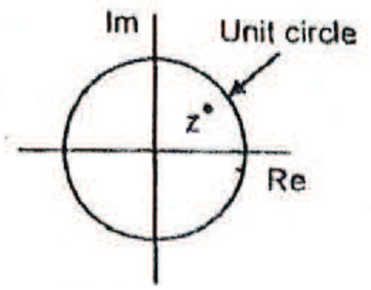


(D)



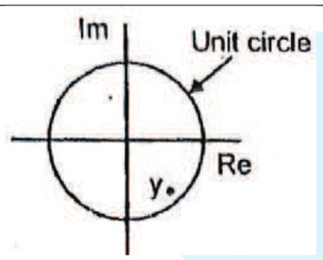
[GATE-2008 (IN)]

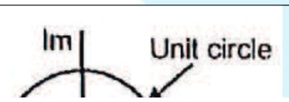
23. A point z has been plotted in the complex plane, as shown in figure below



$\frac{1}{z}$ lies in the curve

(A)





24. Let S be the set of points in the complex plane corresponding to the unit circle. That is $S = [z : |z| = 1]$.

Consider the function f(z) = zz’, where z’ denotes the complex conjugate of z. The f(z) maps S to which one of the following in the complex plane

(A) unit circle

(B) horizontal axis line segment from origin to (1, 0)

(C) the point (1, 0)

(D) the entire horizontal axis

[GATE-EE-SET 1]

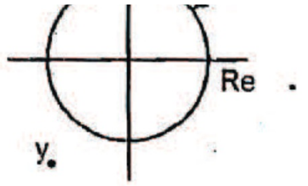
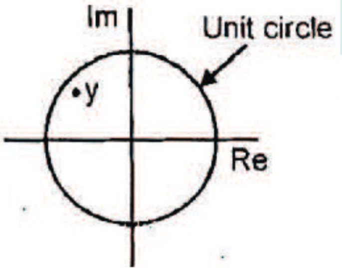
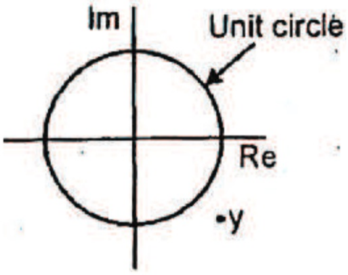
Cauchy-Riemann Equations

25. The value of ‘P’ such that the function

$$f(z) = \frac{1}{2}\log\left(x^2+y^2\right) + i\tan^{-1}\left(\frac{px}{y}\right)$$ is

analytic is ______. **[JNU]**

26. If $f(z) = \left(x^2 + ay^2\right) + ibxy$ is complex analytic function of $z = x + iy$, where

(B)	
(C)	
(D)	

[GATE-2011-EE]

$i = \sqrt{-1}$, then

(A) $a = -1, b = -1$ (B) $a = -1, b = 2$

(C) $a = 1, b = 2$ (D) $a = 2, b = 2$

[GATE 2017]

27. If $\phi(x, y)$ and $\psi(x, y)$ are functions with continuous second derivatives, then $\phi(x, y) + i\psi(x, y)$ can be expressed as an analytic function of $x + iy\left(i = \sqrt{-1}\right)$ when

(A) $\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial x}, \frac{\partial \phi}{\partial y} = \frac{\partial \psi}{\partial y}$

(B) $\frac{\partial \phi}{\partial y} = \frac{\partial \psi}{\partial x}, \frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}$

(C) $\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 1$

(D) $\frac{\partial \phi}{\partial x} + \frac{\partial \psi}{\partial y} = \frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial y} = 0$

[GATE-2007-CE]

28. Consider the complex valued function $f(z) = 2z^3 + b|z|^3$, where z is a complex variable. The value of b for which the function f(z) is analytic is ______.

[GATE-2016-EC-SET 2]

29. What is value of the m for which $2x - x^2 + my^2$ is harmonic?

(A) 1 (B) -1

(C) 2 (D) -2

[ESE 2017 (EE)]

Construction of An Analytic Function

30. The real part of an analytic function f(z) where $z = x + jy$ is given by $e^{-y}\cos(x)$. The imaginary part of f(z) is

(A) $e^{y}\cos(x)$ (B) $e^{-y}\sin(x)$

(C) $-e^{y}\sin(x)$ (D) $-e^{-y}\sin(x)$

32. The value of $\int_c \bar{z}dz$ from z = 0 to z = 4 + 2i along the curve 'c' given by $z = t^2 + it$

(A) $10 - \frac{8i}{3}$ (B) $10i + \frac{8}{3}$

(C) $10 - \frac{8}{3i}$ (D) 0

33. If z is a complex variable, the value of $\int_5^{3i} \frac{dz}{z}$

(A) -0.511 – 1.57i

(B) – 0.511 + 1.57i

(C) 0.511 – 1.57i

(D) 0.511 + 1.57i

[GATE-2014-ME-SET 1]

34. Consider the line integral

$I = \int_c (x^2 + iy^2)dz$,

[GATE-2014-EC-SET 2]

31. If $W = \phi + i\psi$ represents the complex potential for an electric field.

Given : $\psi = x^2 - y^2 + \dfrac{x}{x^2 + y^2}$, then the function ϕ is

(A) $-2x + \dfrac{y}{x^2 + y^2} + C$

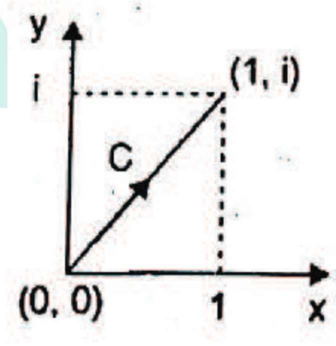
(B) $2xy + \dfrac{x}{x^2 + y^2} + C$

(C) $-2x + \dfrac{x}{x^2 + y^2} + C$

(D) $2xy - \dfrac{y}{x^2 + y^2} + C$

[ESE 2017 (COMMON PAPER)]

where, z = x + iy. The line c is shown in the figure below.



The value of I is

(A) $\dfrac{1}{2}i$ (B) $\dfrac{2}{3}i$

(C) $\dfrac{3}{4}i$ (D) $\dfrac{4}{5}i$

[GATE-2017 EE SESSION-I]

Complex Integration Using Cauchy Integral Theorem

35. Consider likely applicability of Cauchy's integral theorem to evaluate the following integral counter clockwise around the unit circle C, $I = \oint_c \sec z dz$, z being a complex variable. The of I will be

(A) I = 0 singularities set = ϕ

(B) I = 0 singularities set

$= \left\{ \pm \dfrac{2n+1}{2}\pi,\ n = 0, 1, 2.... \right\}$

(C) $I = \dfrac{\pi}{2}$, singularities set

$= \{\pm n\pi;\ n = 0, 1, 2....\}$

(D) None of the above

[GATE-2005-CE]

Complex Integration Using Cauchy Resdue Theorem

38. If $f(z) = c_0 + c_1 z^{-1}$, then $\oint\limits_{\substack{\text{unit}\\\text{circle}}} \dfrac{1 + f(z)}{z} dz$ is given by

(A) $2\pi C_1$ (B) $2\pi(1 + C_0)$

(C) $2\pi j C_1$ (D) $2\pi j(1 + C_0)$

[GATE-2009-EC]

39. If C is a circle of radius r with centre z_0, in the complex z-plane and if n is a non-zero integer, then $\oint_C \dfrac{dz}{(z - z_0)^{nt}}$ equals

(A) $2\pi n j$ (B) 0

(C) $\dfrac{nj}{2\pi}$ (D) $2\pi n$

[GATE-2015-EC-SET 3]

40. The value of $\oint_C \dfrac{dz}{(1 + z^2)}$, where C is the

36. Let $\beta = e^{\frac{i\pi}{10}}$, then residue of $f(z) = \frac{1}{1+z^{10}}$ at $z = \beta$ is

(A) $-\frac{\beta}{10}$ (B) $\frac{\beta}{10}$

(C) $\frac{-\pi i\beta}{5}$ (D) $\frac{\pi i\beta}{5}$

37. The residue of $f(z) = \frac{\sin z}{z^8}$ at z = 0 is

(A) 0 (B) $-\frac{1}{7!}$

(C) $\frac{1}{7!}$ (D) none

[GATE]

contour $\left|z - \frac{i}{2}\right| = 1$ is

(A) $2\pi i$ (B) π

(C) $\tan^{-1} z$ (D) $\pi i \tan^{-1} z$

[GATE-2007-EC]

41. Given $X(z) = \frac{z}{(z-a)^2}$ with $|z| > a$, the residue of $X(z)z^{n-1}$ at $z = a$ for n = 0 will be

(A) a^{n-1} (B) a^n

(C) $n\, a^n$ (D) $n\, a^{n-1}$

[GATE-2008 (EE)]

42. Let z = x + iy be a complex variable. Consider that contour integration is performed along the unit circle in anticlockwise direction. Which one of the following statements is NOT TRUE?

(A) the residue of $\frac{z}{z^2-1}$ at z = 1 is 1/2

(B) $\oint_C z^2 dz = 0$

(C) $\frac{1}{2\pi}\oint_C \frac{1}{z} dz = 1$

(D) $\bar{z}$ (complex conjugate of z) is an analytical function.

[GATE-2015-EC-SET 1]

43. In the following integral, the contour C encloses the points $2\pi j$ and $-2\pi j$.

$$-\frac{1}{2\pi}\oint_C \frac{\sin z}{(z-2\pi j)^3} dz$$

The value of the integral is ______

[GATE-2016]

44. Evaluate $\int \frac{dz}{z \sin z}$, where c is $x^2 + y^2 = 1$

(A) 1 (B) 2

Laurent Expansion

47. The Taylor series expansion of f(z) = sin z about $z = \frac{\pi}{4}$ is

(A) $\frac{1}{\sqrt{2}}\left[1 + \left(z - \frac{\pi}{4}\right) + \frac{\left(z-\frac{\pi}{4}\right)^2}{2!} + \frac{\left(z-\frac{\pi}{4}\right)^3}{3!} + \ldots\right]$

(B) $\frac{1}{\sqrt{2}}\left[1 + \left(z - \frac{\pi}{4}\right) - \frac{\left(z-\frac{\pi}{4}\right)^2}{2!} - \frac{\left(z-\frac{\pi}{4}\right)^3}{3!} + \ldots\right]$

(C) $z - \frac{z^3}{3!} + \frac{z^5}{5!} - \ldots$

(D) none **[GATE]**

48. For the function $\frac{\sin z}{z^3}$ of a complex variable z, the point z = 0 is

(A) a pole of order 3

(B) a pole of order 2

(C) a pole of order 1

(D) not a singularity

[GATE-2007 (IN)]

49. The contour integral $\int_C e^{1/z} dz$ with C as

(C) 0 (D) -1

[ESE 2017 (EE)]

45. The countour C given below is on the complex plane z = x + jy, where $j=\sqrt{-1}$.

The value of the integral $\frac{1}{\pi}\oint_C \frac{dz}{z^2-1}$ is

___________ [GATE 2018 (EC)]

46. What is the residue of the function $\frac{1-e^{2z}}{z^4}$ at its pole?

(A) $\frac{4}{3}$ (B) $-\frac{4}{3}$

(C) $-\frac{2}{3}$ (D) $\frac{2}{3}$

[ESE 2018 (COMMON PAPER)]

the counter clock wise unit circle in the z-plane is equal to

(A) 0 (B) 2π

(C) $2\pi\sqrt{-1}$ (D) α

[GATE-2011 (IN)]

50. In the Laurent expansion of

$$f(z)=\frac{1}{(z-1)(z-2)}$$

valid in the region $1<|z|<2$, the coefficient of $\frac{1}{z^2}$ is

(A) 0 (B) $\frac{1}{2}$

(C) 1 (D) -1

[ESE 2018 (COMMON PAPER)]

51. The coefficient of $\frac{1}{z}$ in the laurent series expansion of $\log\left(\frac{z}{z-1}\right)$ valid in $|z|>1$ is __________. [GATE]

52. In Laurent series expansion of $f(z)=\frac{1}{z-1}-\frac{1}{z-2}$ valid in the region $|z|>2$, the coefficient of $\frac{1}{z^2}$ is

__________. [GATE]

53. The value of the contour integral, $\oint_c\left(\frac{z+2}{z^2+2z+2}\right)dz$, where the contour C is $\left\{z:\left|z+1-\frac{3}{2}j\right|=1\right\}$, taken in the counter clockwise direction, is

(A) $-\pi(1+j)$ (B) $\pi(1+j)$

(C) $\pi(1-j)$ (D) $-\pi(1-j)$

[GATE-2023 (EC)]

54. Let $f(z)=j\left(\frac{1-z}{1+z}\right)$, where z denotes a complex number and *j* denotes $\sqrt{-1}$. The inverse function $f^{-1}(z)$ maps the

differentiable at any other point in the complex plane *C*?

(A) $f(z)=\begin{cases}e^{1/z}, z\neq 0\\ 0, z=0\end{cases}$ for $z\in\tau$

(B) $f(z)=\sin(z), z\in C$

(C) $f(z)=z\bar{z}, z\in C$

(D) $f(z)=e^{-z^2}, z\in C$

[GATE-2023 (XE)]

❑❑❑

real axis to the________.

(A) Unit circle with centre at the origin

(B) Unit circle with centre not at the origin

(C) imaginary axis

(D) real axis

[GATE-2023 (IN)]

55. The value of k that makes the complex-valued function

$f(z) = e^{-kx}(\cos 2y - i\sin 2y)$ analytic,

where $z = x + iy$, is ________. (Answer in integer)

[GATE-2023 (ME)]

56. Which one of the following functions is differentiable at $z = 0$ but not

6 Probability and Statistics

Objective Questions

1. The chance of a student passing an exam is 20%. The chance of a student passing the exam and getting above 90% in it is 5%. Given that a student passes the examination, the probability that the student gets above 90% marks is

(A) $\frac{1}{18}$ (B) $\frac{1}{4}$

(C) $\frac{2}{9}$ (D) $\frac{5}{18}$

[GATE 1992]

2. The probability that a number selected at random between 100 and 999 (both inclusive) will not contain the digit 7 is

(A) $\frac{16}{25}$ (B) $\left(\frac{9}{10}\right)^3$

(C) $\frac{27}{75}$ (D) $\frac{18}{25}$

3. A box contains 5 black balls and 3 red balls. A total of three balls are picked from the box one after another, without replacing them back. The probability of getting two black balls and one red ball

5. Manish has to travel from A to D changing buses at stops B and C enroute. The maximum waiting time at either stop can be 8 minutes each, but any time of waiting up to 8 minutes is equally likely at both places. He can afford up to 13 minutes of total waiting time if he is to arrive at D on time. What is the probability that Manish will arrive late at D?

(A) $\frac{8}{13}$ (B) $\frac{13}{64}$

(C) $\frac{119}{128}$ (D) $\frac{9}{128}$

[GATE-2002-ME]

6. In class of 200 students, 125 students have taken programming language course, 85 students have taken data structures course, 65 students have taken computer organization course, 50 students have taken both programming languages and data structures, 35 students have taken both programming languages and computer organization, 30 students have taken both data is

(A) $\frac{3}{8}$ (B) $\frac{2}{15}$

(C) $\frac{15}{28}$ (D) $\frac{1}{2}$

[GATE-1997]

4. Seven car accidents occurred in a week, what is the probability that they all occurred on the same day?

(A) $\frac{1}{7^7}$ (B) $\frac{1}{7^6}$

(C) $\frac{1}{2^7}$ (D) $\frac{2}{2^7}$

[GATE-2001]

structures and computer organization, 15 students have taken all the three courses. How many students have not taken any of the three courses?

(A) 15 (B) 20

(C) 25 (D) 35

[GATE-2004 (IT)]

7. A bag contains 10 blue marbles, 20 black marbles and 30 red marbles. A marble is drawn from the bag, its colour recorded and it is put back in the bag. This process is repeated 3 times. The probability that no two of the marbles drawn have the same colour is

(A) $\frac{1}{36}$ (B) $\frac{1}{6}$

(C) $\frac{1}{4}$ (D) $\frac{1}{3}$

[GATE-2005 (IT)]

8. In a game, two players X and Y are tossing a coin alternately. Whoever gets a 'head' first, wins the game and the game is terminated. Find the chance that player X will win the game if he starts?

(A) 1/3 (B) 1/2

(C) 2/3 (D) 3/4

9. The box 1 contains chips numbered 3, 6, 9, 12 and 15. The box 2 contains chips numbered 11, 6, 16, 21 and 26. Two chips, one from each box are drawn at random.

The number written on these chips are multiplied. The probability for the product to be an even number is _____

(A) $\frac{6}{25}$ (B) $\frac{2}{5}$

(C) $\frac{3}{5}$ (D) $\frac{19}{25}$

[GATE 2009 (IN)]

10. A and B are friends. They decide to meet between 1PM and 2 PM on a given day. There is a condition that whoever arrives first will not wait for the other

1980 and 1985. What is the smallest N so that the probability of this event exceeds 0.5?

(A) 20 (B) 7

(C) 15 (D) 16

[GATE-2009-EE]

12. What is the probability that divisor of 10^{99} is a multiple of 10^{96}?

(A) 1/625 (B) 4/625

(C) 12/625 (D) 16/625

[GATE-2010 (CS)]

13. The probability that a student knows the correct answer to a multiple choice question is 2/3. If the student does not know the answer, then the student guesses the answer. The probability of the guessed answer being correct is 1/4. Given that the student has answered the question correctly, the conditional probability that the student knows the correct answer is

(A) $\frac{2}{3}$ (B) $\frac{3}{4}$

(C) $\frac{5}{6}$ (D) $\frac{8}{9}$

[GATE-2013-ME]

14. An automobile plant contracted to buy shock absorbers from two suppliers X and Y. X supplies 60% and Y supplies 40% of the shock absorbers. All shock absorbers are subjected to a quality for more than 15 minutes. The probability that they will meet on that day is

(A) $\frac{1}{4}$ (B) $\frac{1}{16}$

(C) $\frac{7}{16}$ (D) $\frac{9}{16}$

11. Assume for simplicity that N people, all born in April (a month of 30 days), are collected in a room. Consider the event of at least two people in the room being born on the same date of the month, even if in different year, e.g.

test. The ones that pass the quality test are considered reliable. Of X's shock absorbers, 96% are reliable. Of Y's shock absorbers, 72% are reliable.

The probability that a randomly chosen shock absorber, which is found to be reliable, is made by Y is

(A) 0.288 (B) 0.334

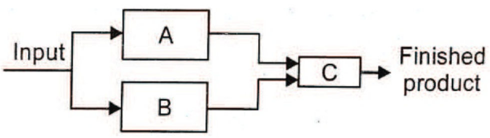
(C) 0.667 (D) 0.720

[GATE-2012]

15. The figure shown the schematic of a production process with machines A, B and C. an input job needs to be

preprocessed either by A or by B before it is fed to C, from which the final finished product comes out. The probabilities of failure of the machines are given as:

$P_A = 0.15, P_B = 0.05$ and $P_C = 0.1$



Assuming, independence of failures of the machines, the probability that a given job is successfully processed (up to the third decimal place) is

[GATE-2014 (IN-SET1)]

16. Parcels from sender S receiver R pass sequentially through two post-offices. Each post-office has a probability $\frac{1}{5}$ of losing an incoming parcel, independently of all other parcels. Given that a parcel is lost, the probability that it was lost by the second post-office is ____

[GATE-2014-EC-SET 4]

17. The security system at an IT office is composed of 10 computers of which exactly four are working. To check whether the system is functional, the officials inspect four of the computers picked at random (without replacement). The system is deemed functional if at least three of the four

(A) $\frac{1}{18}$ (B) $\frac{1}{4}$

(C) $\frac{2}{9}$ (D) $\frac{5}{18}$

[GATE-2015 (ME-SET2)]

19. Candidates were asked to come to an interview with 3 pens each. Black, Blue, green and red were the permitted pen colours that the candidate could bring. The probability that a candidate comes with all 3 pens having the same colour is ____.

[GATE-2016-EE-SET 1]

20. P and Q are considering to apply for a job. The probability that P applies for the job is $\frac{1}{4}$. The probability that P applies for the job given that Q applies for the job is $\frac{1}{2}$, and the probability that Q applies for the job given that P applies for the job is $\frac{1}{3}$. Then the probability that P does not apply for the job given that Q does not apply for the job is

(A) $\frac{4}{5}$ (B) $\frac{5}{6}$

(C) $\frac{7}{8}$ (D) $\frac{11}{12}$

[GATE-2017 PAPER-2 (CS)]

21. A class of twelve children has two more functional if at least three of the four computers inspected are working. Let the probability that the system is deemed functional be denoted by p. Then 100p = _________.

[GATE-2014 (CS-SET2)]

18. The chance of a student passing an exam is 20%. The chance of student passing the exam and getting above 90% marks in it is 5%. Given that a student passes the examination, the probability that the student gets above 90% marks is

21. A class of twelve children has two more boys, then girls. A group of three children are randomly picked from this class to accompany the teacher on a field trip. What is the probability that the group accompanying the teacher contains more girls than boy?

(A) 0 (B) $\frac{325}{864}$

(C) $\frac{525}{864}$ (D) $\frac{5}{12}$

[GATE 2018 (EE)]

22. A person moving through a tuberculosis prone zone has a 50% probability of becoming infected. However, only 30% of infected people developed the disease. What percentage of people moving through a tuberculosis prone zone remains infected but does not show symptoms of disease?

(A) 15 (B) 33

(C) 35 (D) 37

[GATE-2016]

23. A pair of dice is rolled again and again till a total of 5 or 7 is obtained. The chance that a total 5 comes before a total of 7 is

(A) $\frac{2}{5}$ (B) $\frac{3}{7}$

(C) $\frac{3}{13}$ (D) none of these

24. A bag P contains 3 white and 4 black balls and another bag Q contains 4 white and three black balls. A ball is transferred (at random) from bag P to the bag Q and then a ball is transferred from bag Q to the bag P. A ball is then taken out from the bag P. The chance that it is a white ball is

(A) $\frac{31}{56}$ (B) $\frac{25}{49}$

(C) $\frac{25}{56}$ (D) none of these

25. There are two identical locks with two identical keys and the key are among six different ones which a person carries in his pocket. In hurry he drops

probability that two specified person do not sit together is

(A) $\frac{2}{n-1}$ (B) $\frac{n-3}{n-1}$

(C) $\frac{n-2}{n-1}$ (D) $\frac{1}{n-1}$

[GATE]

27. The letters of the word PROBABILITY are arranged in all possible ways. The chance that B's and also two I's occur together is

(A) $\frac{1}{55}$ (B) $\frac{2}{55}$

(C) $\frac{4}{165}$ (D) none of these

28. From 6 positive and 8 negative numbers 4 numbers are drawn at random without replacement and multiplied, the probability that the product is a positive number is

(A) $\frac{505}{1001}$ (B) $\frac{50}{1001}$

(C) $\frac{5}{101}$ (D) $\frac{55}{1001}$

[GATE]

29. If two squares are chosen at random on a chess board, the probability that they have a side in common is

(A) $\frac{1}{9}$ (B) $\frac{2}{7}$

(C) $\frac{1}{18}$ (D) none of these

Random Variable

30. Let X and Y be two independent

one key somewhere. Then the probability that the locks can still opened by drawing one key at random is equal to

(A) $\frac{1}{3}$ (B) $\frac{5}{6}$

(C) $\frac{1}{12}$ (D) $\frac{1}{30}$ **[GATE]**

26. A party of n persons takes their seats at random at a round table, then the

random variables. Which one of the relations between expectation (E), variance (Var) and covariance (Cov) given below is FALSE?

(A) $E(XY) = E(X)\,E(Y)$

(B) $Cov(X, Y) = 0$

(C) $Var(X + Y) = Var(X) + Var(Y)$

(D) $E(X^2Y^2) = (E(X))^2 (E(Y))^2$

[GATE-2007-ME]

31. If the standard deviation of the spot speed of vehicles in a highway is 8.8km/h and the mean speed of the vehicles is 33 km/h, the coefficient of variation in speed is

(A) 0.1517 (B) 0.1867

(C) 0.2666 (D) 0.3646

[GATE-2007-CE]

32. If the difference between the expectation of the square of a random variable $\left[E(X^2)\right]$ and the square of the expectation of the random variable $\left[E(X)\right]^2$ is denoted by R, then

(A) R = 0 (B) R < 0

(C) $R \geq 0$ (D) R > 0

[GATE-2011 (CS)]

33. A simple random sample of 100 observations was taken from a large population. The sample mean & the standard deviation were determined to be 80 to 12 respectively. The standards error of mean is ______

[GATE-2014 (PI-SET1)]

34. Marks obtained by 100 students in an examination are given in the table:

S. No	Marks obtained	Number of students
1	25	20
2	30	20
3	35	40
4	40	20

What would be the mean, median and mode of the marks obtained by the students?

(A) Mean 33; Median 35; Mode 40

[Note answer with one decimal accuracy]

[GATE-2016 (CE-SET 2)]

36. A sample of 15 data is as follows 17, 18, 17, 17, 13, 18, 5, 5, 6, 7, 8, 9 20, 17, 3. The mode of the data is

(A) 4 (B) 13

(C) 17 (D) 20

[GATE 2017 ME SESSION-II]

37. The standard deviation of liner dimensions P and Q are 3μm and 4μm respectively. When assembled, the standard deviation $(\text{in}\,\mu\text{m})$ of the resulting linear dimension (P+Q) is ______.

[GATE-2017 ME SESSION-II]

38. The following sequence of numbers is arranged in increasing order : 1, x, x, x, y, y, 9, 16, 18. Given that the mean and median are equal, and are also equal to twice the mode, the value of y is

(A) 5 (B) 6

(C) 7 (D) 8

[GATE-2017 (CH)]

Discrete Random Variable

39. The mean of squares of first 23 natural number is ______.

40. A random variate has the following distribution:

x	0	1	2	3	4	5	6	7
p(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

The value of k is ______.

41. A six face fair dice is rolled a large

(B) Mean 35; Median 32; Mode 40
(C) Mean 33; Median 35; Mode 35
(D) Mean 35; Median 32; Mode 35

[GATE-2014 (PI-SET 1)]

35. The spot speed (expressed in km/hr) observed at a road section are 66, 62, 45, 79, 32, 51, 56, 60, 53 and 49. The median speed expressed in km/hr is ____.

41. A six-face fair dice is rolled a large number of times. The mean value of the outcomes is ____.

[GATE 2017]

42. An exam paper has 150 multiple choice questions of 1 mark each, with each question having four choices. Each incorrect answer fetches -0.25 marks. Suppose 1000 students choose all their

answer randomly with uniform probability. The sum total of the expected marks obtained by all the students is

(A) 0 (B) 2550
(C) 7525 (D) 9375

[GATE-2004 (CS)]

43. The following data about the flow of liquid was observed in a continuous chemical process plant.

Flow rate	Frequency (litres / sec)
7.5 to 7.7	1
7.7 to 7.9	5
7.9 to 8.1	35
8.1 to 8.3	17
8.3 to 8.5	12
8.5 to 8.7	10

Mean flow rate of the liquid is
(A) 8.00 litres / sec (B) 8.06 litres / sec
(C) 8.16 litres / sec (D) 8.26 litres / sec

[GATE-2004]

44. The random variable X takes on the values 1, 2, (or) 3 with probabilities $\frac{2+5P}{5}$, $\frac{1+3P}{5}$ and $\frac{1.5+2P}{5}$ respectively the value of P and E(X) are respectively.

(A) 0.05, 1.87 (B) 1.90, 5.87
(C) 0.05, 1.10 (D) 0.25, 1.40

[GATE-2007 (PI)]

45. A fair coin is tossed repeatedly till both head and tail appear at least once. The average number of tosses required is ___.

[GATE-2014-EC SET3]

(The answer should be rounded to one decimal place)

[GATE-2014 (CS-SET2)]

47. Let the random variable X represent the number of times a fair coin needs to be tossed till two consecutive heads appear for the first time. The expectation of X is _____.

[GATE-2005]

48. Passengers try repeatedly to get a seat reservation in any train running between two stations until they are successful. If there is 40% chance of getting reservation in any attempt by a passenger then the average number of attempts that passengers need to make to get a seat reserved is ____.

[GATE-2017 EC SESSION-II]

49. A person decides to toss a fair coin repeatedly until he gets a head. He will make at most 3 tosses. Let the random variable Y denote the number of heads. The value of var{Y}, where var{.} denotes tie variance, equals:

(A) $\frac{7}{8}$ (B) $\frac{49}{64}$
(C) $\frac{7}{64}$ (D) $\frac{105}{64}$

[GATE 2017 EE SESSION-II]

Binomial Distribution

50. The probability of India winning a test match against England is 0.5. Assuming independence of the result of various matches. The chance that in a 5 match series, India's second win occurs at 3rd test is _________.

51. A dice has four blank faces and two

46. Each of the nine words in the sentence, "The Quick brownfox jumps over the lazy dog" is written in a separate piece of paper. These nine pieces of paper are kept in a box. One of the piece is drawn at random from the box. The expected length of the word drawn is _____.

faces marked 3. The chance of getting a total of 12 in 5 throws is

(A) $^5C\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)$ (B) $^5C_4\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^4$

(C) $^5C_4\left(\frac{1}{6}\right)^5$ (D) None of these

52. A man takes a step forward with probability 0.4 and backward with probability 0.6. The probability that at the end of eleven steps he is one step away from the starting point is

(A) $\left(\frac{6}{25}\right)^5$ (B) $462\left(\frac{6}{25}\right)^5$

(C) $538\left(\frac{1}{25}\right)^5$ (D) $\left(\frac{1}{25}\right)^5$

[JNU]

53. In a manufacturing plant, the probability of making a defective bolt is 0.1. The mean and standard deviation of defective bolts in a total of 900 bolts are respectively

(A) 90 and 9 (B) 9 and 90

(C) 81 and 9 (D) 9 and 81

[GATE, EE: 2000]

54. An unbiassed coin is tossed an infinite number of times. The probability that the fourth head appears at the tenth toss is

(A) 0.067 (B) 0.073

(C) 0.082 (D) 0.091

[GATE-2014-EC-SET 3]

55. The probability that a screw manufactured by a company is defective is 0.1 the company sells screws in packets containing 5 screws and gives a guarantee of replacement if one or more screws in the packet are found to be defective. The probability that a packet would have to be replaced is _____.

[GATE 2016-ME-SET 2]

Position Distribution

(A) 0.029 (B) 0.034

(C) 0.039 (D) 0.044

[GATE, ME ; 2014 (SET-4)]

57. It is estimated that the average number of events during a year is three. What is the probability of occurrence of not more than two events over a two year duration? Assume that the number of events follow a Poisson distribution

(A) 0.052 (B) 0.062

(C) 0.072 (D) 0.082

[GATE, PI : 2011]

58. A manufacturer knows that the condensers he makes contain on an average 1% defectives. He packs them, in boxes of 100, What is the probability that a box picked up at random will contain 3 or more faulty condensers

(A) $1-\frac{3}{2}e^{-1}$ (B) $1-\frac{5}{2}e^{-1}$

(C) $1-\frac{2}{e}$ (D) $1-\frac{5}{e}$

[GATE]

59. Suppose p is the number of cars per minute passing through a certain road junction between 5 PM and 6 PM, and p has a Poisson distribution with mean 3. What is the probability of observing fewer than 3 cars during any given minute in this interval?

(A) $8/(2e^3)$ (B) $9/(2e^3)$

(C) $17/(2e^3)$ (D) $26/(2e^3)$

[GATE-2013 (CS)]

60. An observer counts 240 veh/h at a

56. The number of accidents occurring in a plant in a month follows Poisson distribution with mean as 5.2. The probability of occurrence of less than 2 accidents in the plant during a randomly selected month is

specific highway location. Assume that the vehicles arrival at the location is Poisson distributed, the probability of having one vehicle arriving over a 30-second time interval is_____.

[GATE-2014 (CE-SET 2)]

61. The second moment of a Poisson-distributed random variable is 2. The mean of the random variable is _______.

[GATE-2016]

62. Vehicles arriving at an intersection from one of the approach roads follow the Poisson distribution. The mean rate of arrival is 900 vehicles per hour. If a gap is defined as the time difference between two successive vehicle arrivals (with vehicles assumed to be points), the probability (up to four decimal places) that the gap is greater than 8 seconds is ____.

[GATE-2017 CE SESSION-I]

63. If a random variable X has a Poisson distribution with mean 5, then the expectation $E\left[(X+2)^2\right]$ equals _____.

[GATE-2017 PAPER-2 (CS)]

Continuous Random Variable

64. A probability density function is of the form $p(x) = Ke^{-\alpha x},\ x \in (-\infty, \infty)$, the value of k is

(A) 0.5 (B) 1

(D) 0.5a (D) a

[GATE-2006]

65. Consider the continuous random variable with probability density function

$f(t) = 1+t$ for $-1 \le t \le 0$ = 1 – t for $0 \le t \le 1$

The standard deviation of the random variable is

(A) 1 (B) 1

66. Let X be a random variable with probability density function

$$f(x) = \begin{cases} 0.2, & \text{for } |x| \le 1 \\ 0.1, & \text{for } 1 < |x| \le 4 \\ 0, & \text{otherwise} \end{cases}$$

The probability P(0.5 < X < 5) is _____.

[GATE-2014-EC-SET 2]

67. The variance of the random variable X with probability density function $f(x) = \frac{1}{2}|x|e^{-|x|}$ is _____.

[GATE-2015 (CS-SET 3)]

68. The probability density function of random variable X is $P_x(x) = e^{-x}$ for $x \ge 0$ and 0 otherwise. The expected value of the function $g_x(x) = e^{3x/4}$ is _______.

[GATE-2015 (IN)]

69. Two random variables X and Y are distributed according to

$$f_{x,y}(x,y) = \begin{cases} (x+y), & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$$

The probability $P(X+Y \le 1)$ is _____.

[GATE-2016-EC-SET 2]

70. Let the probability density function of a random variable, X, be given as:

$$f_x(x) = \frac{3}{2}e^{-3x}u(x) + ae^{4x}u(-x)$$

where, u(x) is the unit step function.

Then the value of 'a' and Prob $\{X \le 0\}$, respectively, are

(A) $2, \frac{1}{2}$ (B) $4, \frac{1}{2}$

(C) $2, \frac{1}{4}$ (D) $4, \frac{1}{4}$

(A) $\frac{1}{\sqrt{3}}$ (B) $\frac{1}{\sqrt{6}}$

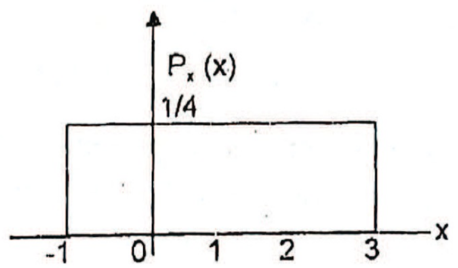
(C) $\frac{1}{3}$ (D) $\frac{1}{6}$

[GATE-2006- ME]

[GATE-2016 EC SET-3]

71. For a random variable x having the PDF shown in the figure given below

The mean and the variance are, respectively



(A) 0.5 and 0.66 (B) 2.0 and 1.33

(C) 1.0 and 0.66 (D) 1.0 and 1.33

[ESE 2017 (EC)]

72. A random variable X has a probability density function

$$f(x) = \begin{cases} kx^n e^{-x}; & x \geq 0 \\ 0; & \text{otherwise} \end{cases}$$

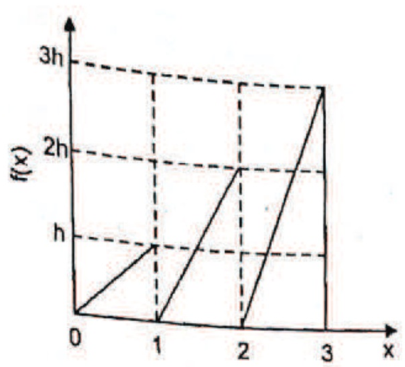
(n is an integer) with mean 3. The values of {k, n} are

(A) $\left\{\frac{1}{2}, 1\right\}$ (B) $\left\{\frac{1}{4}, 2\right\}$

(C) $\left\{\frac{1}{2}, 2\right\}$ (D) {1, 2}

[ESE 2017 (EE)]

73. The graph of a function f(x) is shown in the figure.



For f(x) to be a valid probability density function, the value of h is

(A) 1/3 (B) 2/3

(C) 1 (D) 3

[GATE 2018 (CE-AFTERNOON SESSION)]

within the rectangle with corners at (0, 0), (1, 0), (1, 2) and (0, 2). If p is the length of the position vector of the point, then the expected value of 'p^2' is ________

[GATE 2004]

75. X is a uniformly distributed random variable that takes values between 0 and 1. The value of $E(X^3)$ will be

(A) 0 (B) 1/8

(C) 1/4 (D) 1/2

[GATE-2008-EE]

76. A random variable is uniformly distributed over the interval 2 to 10. Its variance will be

(A) 16/3 (B) 6

(C) 256/9 (D) 36

[GATE-2008 (IN)]

77. The independent random variables X and Y are uniformly distributed in the interval [-1, 1]. The probability that max [X, Y] is less than 1/2 is

(A) 3/4 (B) 9/16

(C) 1/4 (D) 2/3

[GATE-2012-EC, EE, IN]

78. Let X_1, X_2 and X_3 be independent and identically distributed random variables with uniform distribution on {0, 1}. The probability P(X_1 is the largest) is ______.

[GATE-2014 (EC-SET1)]

79. Suppose you break a stick of unit length at a point chosen uniformly at random. Then the expected length of the shorter stick is _____.

[GATE-2016 (CE-SET1)]

80. Let X_1, X_2 and X_3 be independent and identically distributed random variables with the uniform distribution

Uniform Probability Distribution

74. A point is randomly selected with uniform probability in the x-y plane

variables with the uniform distribution on [0, 1]. The probability $P\{X_1 + X_2 \le X_3\}$ is the largest is ____.

[GATE-2016 (CE-SET1)]

81. Assume that in a traffic junction, the cycle of the traffic signal lights is 2 minutes of green (vehicle does not stop) and 3 minutes of red (vehicle stops). Consider that the arrival time of vehicles at the junction is uniformly distributed over 5 minute cycle. The expected waiting time (in minutes) for the vehicle at the junction is _______

[GATE 2017 – EE SESSION-1]

Exponential Distribution

82. Assume that the duration in minutes of a telephone conversation follows the exponential distribution

$f(x) = \frac{1}{5}e^{-x/5}$, $x \ge 0$. The probability that the conversation will exceed five minutes is

(A) $\frac{1}{e}$ (B) $1-\frac{1}{e}$

(C) $\frac{1}{e^2}$ (D) $1-\frac{1}{e^2}$

[GATE-2007 (IN)]

83. If for a single server with Poisson arrival and exponential service time, the arrival rate is 12 per hour. Which one of the following service rates will provide a steady state finite queue length?

(A) 6 per hour (B) 10 per hour

(C) 12 per hour (D) 24 per hour

[GATE 2017 ME SESSION-II]

84. The arrival of customers over fixed time intervals in a bank follow a poisson distribution with an average of 30 customers / hour. The probability that the time between successive customer arrival is between 1 and 3 minutes is ____ (correct to two decimal places).

[GATE-2018 (ME-AFTERNOON SESSION)]

85. Let X_1 and X_2 be two independent exponentially distributed random variables with means 0.5 and 0.25.

(A) exponentially distributed with mean 1/6

(B) exponentially distributed with mean 2

(C) normally distributed with mean ¾

(C) normally distributed with mean 1/6

[GATE 2018 (ME-AFRTERNOON SESSION)]

Normal Distribution

86. A nationalized bank has found that the daily balance available in its savings accounts follows a normal distribution with a mean of Rs. 500 and a standard deviation of Rs. 50. The percentage of savings account holders, who maintain an average daily balance more than Rs. 500 is _____.

87. For a random variable $x(-\infty < x < \infty)$ following normal distribution, the mean is $\mu = 100$ if the probability is $P = \alpha$ for $x \ge 110$. Then the probability of x lying between 90 and 110 i.e., $P(90 \le x \le 110)$ and equal to

(A) $1-2\alpha$ (B) $1-\alpha$

(C) $1-\alpha/2$ (D) 2α

[GATE, PI : 2008]

88. Suppose X is a normal random variable with mean 0 and variance 4. Then the mean of the absolute value of X is

(A) $\frac{1}{\sqrt{2\pi}}$ (B) $\frac{2\sqrt{2}}{\sqrt{\pi}}$

(C) $\frac{2\sqrt{2}}{\pi}$ (D) $\frac{2}{\sqrt{\pi}}$

[GATE 1999]

89. Which one of the following statements is not true?

(A) The measure of skewness is dependent upon the amount of dispersion

(B) In a symmetric distribution, the values of mean, mode and median are the same respectively. Then $Y = \min(X_1, X_2)$ is

(C) In a positively skewed distribution, mean > median > mode

(D) In a negatively skewed distribution, mode > mean > median

[GATE-2005]

90. The value of

$$I = \frac{1}{\sqrt{2\pi}} \int_0^{\infty} \exp\left(\frac{-x^2}{8}\right) dx \text{ is } \_\_\_\_\_\_\_.$$

[GATE 2006]

91. The standard normal probability function can be approximated as

$$F(X_N) = \frac{1}{1 + \exp\left(-1.72555 X_N \mid X_N \mid^{0.12}\right)},$$

where X_N = standard normal deviate. If mean and standard deviation of annual precipitation are 102 cm and 27 cm respectively, the probability that the annual precipitation will be between 90 cm and 102 cm is

(A) 66.7% (B) 50.0%

(C) 33.3% (D) 16.7%

[GATE-2008-CE]

92. Let X be a random variable following ND with mean +1 and variance 4. Let Y be another normal variable with mean -1 and variance unknown. If $P(X \le -1) = P(Y \ge 2)$. The standard deviation of Y is _____.

[GATE-2008]

93. Let U and V be two independent zero mean Gaussian random variables of variances $\frac{1}{4}$ and $\frac{1}{9}$ respectively. The probability $P(3V \ge 2U)$ is

(A) $\frac{4}{9}$ (B) $\frac{1}{2}$

(C) $\frac{2}{3}$ (D) $\frac{5}{9}$

[GATE-2013 (EC)]

94. Let X be a zero mean unit variance Gaussian random variable. E[|X|] is equal to _____.

95. If f(x) is a continuous real valued random variable defined over the interval $(-\infty, +\infty)$ and its occurance is defined by the density function given as

$$f(x) = \frac{1}{\sqrt{2\pi b}} \exp\left\{-\frac{1}{2}\left(\frac{x-a}{b}\right)^2\right\}$$ where a and b are the statistical attributes of the random variable {x}. The value of the integral

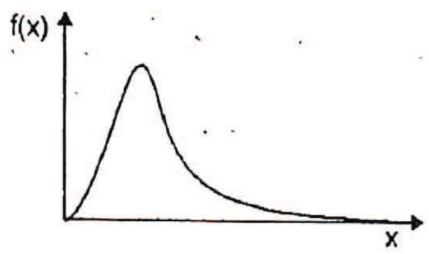
$$\int_{-\infty}^{a} \frac{1}{\sqrt{2\pi b}} \exp\left\{-\frac{1}{2}\left(\frac{x-a}{b}\right)^2\right\} dx \text{ is}$$

(A) 1 (B) 0.5

(C) π (D) $\pi/2$

[GATE-2014-CE-SET 2]

96. A probability distribution with right skew is shown in the figure.



The correct statement for the probability distribution is

(a) Mean is equal to mode

(b) Mean is greater than median but less than mode

(c) Mean is greater than median and mode

(d) Mode is greater than median

[GATE 2018 (CE-AFTERNOON SESSION)]

97. Let X_1, X_2, X_3 and X_4 be independent normal random variables with zero mean and unit variance. The probability that X_4 is the smallest among the four is _______.

[GATE-2018 (EC)]

[GATE-2014-EC-SET 4]

Cummulative Distribution Function

98. Consider a Gaussian distributed random variable with zero mean and standard deviation σ. The value of its cumulative distribution function at the origin will be

(A) 0 (B) 0.5

(C) 1 (D) 10σ

[GATE-2008 (IN)]

99. The cumulative distribution function of a random variable x is the probability that X takes the value

(A) less than or equal to x

(B) equal to x

(C) greater than x

(D) zero **[ESE 2017 (EC)]**

100. The probability density function $F(x) = ae^{-b|x|}$, where x is a random variable whose allowable value range is from $x = -\infty$ to $x = +\infty$. The CDF for this function for $x \geq 0$ is

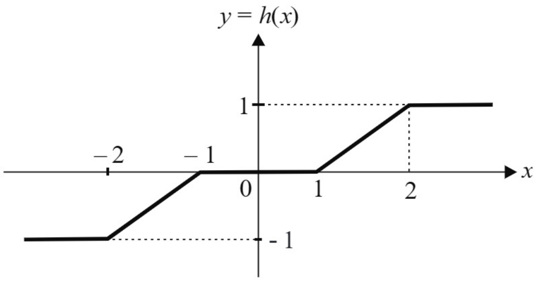
(A) $\frac{a}{b}e^{bx}$ (B) $\frac{a}{b}\left(2 - e^{-bx}\right)$

(C) $-\frac{a}{b}e^{bx}$ (D) $-\frac{a}{b}\left(2 + e^{-bx}\right)$

[ESE 2017 (EC)]

101. A random variable X, distributed normally as N(0, 1), undergoes the transformation $Y = h(X)$, given in the figure. The form of probability density function of Y is

(In the options given below, a, b, care non-zero constants and g(y) is piecewise continuous function)



(A) $a\delta[y-1] + b\delta[y+1] + g[y]$

(B) $a\delta[y+1] + b\delta[y] + c\delta[y-1] + g[y]$

(C) $a\delta[y+2] + b\delta[y] + c\delta[y-2] + g[y]$

(D) $a\delta[y+2] + b\delta[y-2] + g[y]$

[GATE-2023 (EC)]

102. The frequency of occurrence of 8 symbols (a-h) is shown in the table below. A symbol is chosen and it is determined by asking a series of "yes/no" questions which are assumed to be truthfully answered. The average number of questions when asked in the most efficient sequence, to determine the chosen symbol, is _______ (rounded off to two decimal places).

Symbol	a	b	c	d	e	f	g	h
Frequency of occurrence	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{128}$	$\frac{1}{128}$

[GATE-2023 (EC)]

103. One million random numbers are generated from a statistically stationary processwith a Gaussian distribution with mean zero and standard deviation σ_0.

The σ_0 is estimated by randomly drawing out 10,000 numbers of samples (x_n). Theestimates $\hat{\sigma}_1, \hat{\sigma}_2$ are computed in the following two ways.

$$\hat{\sigma}_1^2 = \frac{1}{10000}\sum_{n=1}^{10000} x_n^2, \hat{\sigma}_2^2 = \frac{1}{9999}\sum_{n=1}^{10000} x_n^2$$

Which of the following statements is true?

(A) $E(\hat{\sigma}_2^2) = \sigma_0^2$ (B) $E(\hat{\sigma}_2) = \sigma_0$

(C) $E(\hat{\sigma}_1^2) = \sigma_0^2$ (D) $E(\hat{\sigma}_1) = E(\hat{\sigma}_2)$

[GATE-2023 (EE)]

104. The expected number of trials for first occurrence of a "head" in a biased coin is known to be 4. The probability of first occurrence of a "head" in the second trial is _____________ (Round off to 3 decimal places). **[GATE-2023 (EE)]**

105. X is a discrete random variable which takes values 0, 1 and 2. The probabilities are $P(X=0)=0.25$ and $P(X=1)=0.5$. With $E[\bullet]$ denoting the expectation operator, the value of $E[X]-[X^2]$ is ________(rounded off to one decimal places).

[GATE-2023 (IN)]

106. How many five digit numbers can be formed using the integers 3, 4, 5 and 6 with exactly one digit appearing twice?

[GATE-2023 (IN)]

107. A machines produces a defective components with a probability of 0.015. The number of defective components in a packed box containing 200 components produced by the machine follows a Poisson distribution. The mean and the variance of the distribution are

(A) 3 and 3, respectively

(B) $\sqrt{3}$ and $\sqrt{3}$, respectively

(C) 0.015 and 0.015, respectively

(D) 3 and 9, respectively

[GATE-2023 (ME)]

108. The smallest perimeter that a rectangle with area of 4 square units can have is ________ units. (Answer in integer)

[GATE-2023 (ME)]

109. The probabilities of occurrences of two independent events A and B are 0.5 and 0.8, respectively. What is the probability of occurrence of at least A or B (rounded off to one decimal place)?

[GATE-2023 (CE-1)]

110. A remote village has exactly 1000 vehicles with sequential registration numbers starting from 1000. Out of the total vehicles, 30% are without pollution clearance certificate. Further, even- and odd-numbered vehicles are operated on even- and odd-numbered dates, respectively.

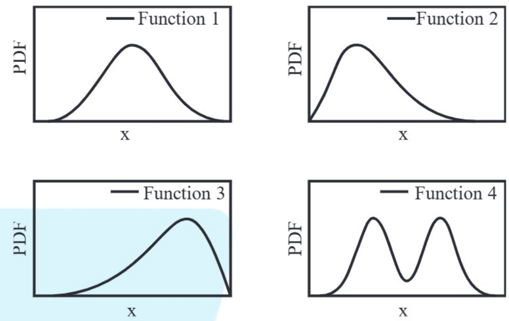
If 100 vehicles are chosen at random on an even-numbered date, the number of vehicles expected without pollution clearance certificate is__________.

(A) 15 (B) 30

(C) 50 (D) 70

[GATE-2023 (CE-2)]

111. Which of the following probability distribution functions (PDFs) has the mean greater than the median?



(A) Function 1 (B) Function 2

(C) Function 3 (D) Function 4

[GATE-2023 (CE-2)]

112. Consider a random experiment where two fair coins are tossed. Let A be the event that denotes HEAD on both the throws, B be the event that denotes HEAD on the first throw, and Cbe the event that denotes HEAD on the second throw.

Which of the following statements is/are TRUE?

(A) A and B are independent

(B) A and C are independent

(C) B and C are independent

(D) Prob(B/C) = Prob(B)

[GATE-2023 (CSE)]

113. The probability of a person telling the truth is 4/6. An unbiased die is thrown by the same person twice & the person reports that the numbers appeared in both the throws are same then the probability that actually the numbers appeared in both the throws are same is _______.

[GATE-2023 (XE)]

❑❑❑

7 Numerical Methods

Objective Questions

Direct Method

1. In the interval $[0, \pi]$ the equation x = cos x has
(A) No solution
(B) exactly one solution
(C) exactly two solution
(D) an infinite number of solutions
[GATE-1995 (CS)]

2. How many distinct values of x satisfy the equation sin(x) = x/2, where x is in radians?
(A) 1 (B) 2
(C) 3 (D) 4 or more
[GATE-2016]

Newton-Raphson Method (TYPE-I) :

3. The following equation needs to be numerically solved using the Newton-Raphson method $x^3 + 4x - 9 = 0$. The Iterative equation for this purpose is (k indicates the iteration level)

(A) $X_{k+1} = \dfrac{2x_k^3 + 9}{3X_k^2 + 4}$

(B) $X_{k+1} = \dfrac{2x_k^2 + 4}{3X_k^2 + 9}$

(C) $X_{k+1} = x_k - 3x_k^2 + 4$

(D) $X_{k+1} = \dfrac{4x_k^2 + 3}{9x_k^2 + 2}$

[GATE-2007-CE]

4. Identify the Newton-Raphson iteration scheme for finding the square root of 2

(A) $x_{n+1} = \dfrac{1}{2}\left(x_n + \dfrac{2}{x_n}\right)$

(B) $x_{n+1} = \dfrac{1}{2}\left(x_n - \dfrac{2}{x_n}\right)$

(C) $x_{n+1} = \dfrac{2}{x_n}$

(D) $x_{n+1} = \sqrt{2 + x_n}$ **[GATE-2007 (IN)]**

5. The recursion relation to solve $x = e^{-x}$ using Newton-Raphson method is

(A) $x_{n+1} = e^{-x_n}$

(B) $x_{n+1} = x_n - e^{-x_n}$

(C) $x_{n+1} = (1 + x_n)\dfrac{e^{-x_n}}{1 + e^{-x_n}}$

(D) $x_{n+1} = \dfrac{x_n^2 - e^{-x_n}(1 + x_n) - 1}{x_n - e^{-x_n}}$

[GATE-2008-EC]

Newton-Raphson Method (TYPE-II) :

6. Consider the series $x_{n+1} = \dfrac{x_n}{2} + \dfrac{9}{8x_n}$, $x_0 = 0.5$ obtained from the Newton-Raphson method. The series converges to
(A) 1.5 (B) $\sqrt{2}$
(C) 1.6 (D) 1.4
[CS, GATE-2007]

7. The Newton-Raphson iteration $x_{n+1} = \dfrac{1}{2}\left(x_n + \dfrac{R}{x_n}\right)$ can be used to compute the
(A) square of R (B) reciprocal of R
(C) square root of R (D) logarithm of R
[GATE-2008 (CS)]

Newton-Raphson Method (TYPE-III) :

8. Given a > 0, we wish to calculate its reciprocal value 1/a by using Newton Raphson method for f(x) = 0, then for a

= 7 and starting with $x_0 = 0.2$, the first two iterations will be

(A) 0,11, 0.1299 (B) 0.12, 0.1392

(C) 0.12, 0.1416 (D) 0.13, 0.1428

[GATE-2005-CE]

9. A numerical solution of the equation $f(x) = x + \sqrt{x} - 3 = 0$ can be obtained using Newton-Raphson method. If the starting value is x = 2 for the iteration, the value of x that is to be used in the next step is

(A) 0.306 (B) 0.739

(C) 1.694 (D) 2.306

[GATE-2011-EC]

10. What is the value of $(1525)^{0}$ to 2 decimal places?

(A) 4.33 (B) 4.36

(C) 4.38 (D) 4.30

[ESE-2018 (COMMON PAPER)]

11. What is the cube root of 1468 to 3 decimal places?

(A) 11.340 (B) 11.353

(C) 11.365 (D) 11.362

[ESE 2018 (COMMON PAPER)]

Newton-Rapson Method(General Question)

12. The Newton-Raphson method is to be used to find the root of the equation and f'(x) is the derivative of f. The method converges

(A) always

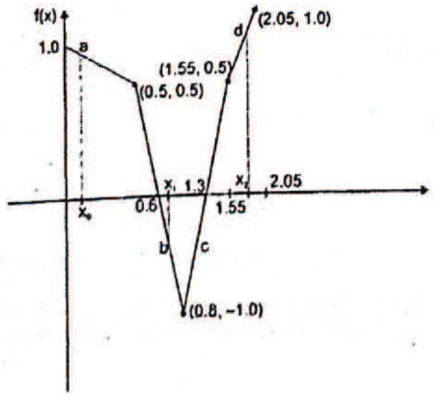
(B) only if f is a polynomial

(C) only if $f(x_0) < 0$

(D) none of the above

[GATE-1999 (CS)]

13. A piecewise linear function f(x) is plotted using thick solid lines in the figure (the plot is drawn to scale).



If we use the Newton-Raphson method to find the roots of f(x) = 0 using x_0, x_1 and x_2 respectively as initial guesses, the roots obtained would be

(A) 1.3, 0.6 and 0.6 respectively

(B) 0.6, 0.6 and 1.3 respectively

(C) 1.3, 1.3 and 0.6 respectively

(D) 1.3, 0.6 and 1.3 respectively

[CS, GATE-2003, 2 MARKS]

14. Solution of the variables x_1 and x_2 for the following equations is to be obtained by employing the Newton-Raphson iterative method

Equation (i) $10x_2 \sin x_1 - 0.8 = 0$

Equation (ii)

$10x_2^2 - 10x_2 \cos x_1 - 0.6 = 0$

Assuming the initial values $x_1 = 0.0$ and $x_2 = 1.0$, the Jacobian matrix is

(A) $\begin{bmatrix} 10 & -0.8 \\ 0 & -0.6 \end{bmatrix}$ (B) $\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & -0.8 \\ 10 & -0.6 \end{bmatrix}$ (D) $\begin{bmatrix} 10 & 0 \\ 10 & -10 \end{bmatrix}$

[GATE-2011 EE]

15. The function $f(x) = e^x - 1$ is to be solved using Newton-Raphson method. If the initial value of x is taken as 1.0, then the absolute error observed at 2nd iteration is ______.

[GATE-2014-EE-SET2]

16. In the Newton-Raphson method, an initial guess of $x_0 = 2$ is made and the sequence $x_0, x_1, x_2, \ldots\ldots$ is obtained for the function $0.75x^3 - 2x^2 - 2x + 4 = 0$
Consider the statements
(i) $x_3 = 0$
(ii) The method converges to a solution in finite number of iterations
Which of the following is TRUE?
(A) only (i)
(B) only (ii)
(C) Both (i) and (ii)
(D) neither (i) nor (ii)
[GATE-14 (CS-SET2)]

17. In Newton Raphson iterative method, the initial guess value (x_{ini}) is considered as zero, while finding the roots of the equation $f(x) = -2 + 6x - 4x^2 + 0.5x^3$. The correction Δx, to be added to X_{ini} in the first iteration is ______.
[GATE-2015-CE-SET II]

18. The quadratic equation $x^2 - 4x + 4 = 0$ is to be solved numerically starting with the initial guess $x_0 = 3$. The Newton-Raphson method is applied once to get a new estimate and then the Secant method is applied once using the initial guess and this new estimate. The estimated value of the root after the application of the Secant method is ______.
[GATE-2015-CE-SET I]

Bisection Method

19. The bisection method is applied to compute a zero of the function $f(x) = x^4 - x^3 - x^2 - 4$ in the interval [1, 9]. The method converges to a solution after _____ interations.
(A) 1 (B) 3
(C) 5 (D) 7
[CS, GATE-2012, 2 MARKS]

20. The real root of the equation 5x-2cosx-1=0 (up to two decimal accuracy) is ________
[GATE-2014-ME-SET 2]

21. Only one of the real roots of $f(x) = x^6 - x - 1$ lies in the interval $1 \le x \le 2$ and bisection method is used to find its value. For achieving an accuracy of 0.001, the required minimum number of iterations is _____
[GATE-2017-EE-SESSION I]

Trapezoidal Rule

22. A 2nd degree polynomial f(x) has values of 1, 4 and 15 at x = 0, 1 and 2, respectively. The integral $\int_0^2 f(x)dx$ is to be estimated by applying the Trapezoidal rule to this data. What is the error (defined as true value-approximate value) in the estimate?
(A) $-\frac{4}{3}$ (B) $-\frac{2}{3}$
(C) 0 (D) $\frac{2}{3}$

23. Using the trapezoidal rule, and dividing the interval of integration into three equal subintervals, the definite integral $\int_{-1}^{+1} |x|\, dx$ is _______
[GATE-2014-ME-SET 1]

Simpson's Rule

24. Simpson's rule for integration gives exact result when f(x) is a polynomial degree
(A) 1 (B) 2
(C) 3 (D) 4
[GATE-1993 (ME)]

25. The estimate of $\int_{0.5}^{1.5} \frac{dx}{x}$ obtained using Simpson's rule with three-point function evaluation exceeds the exact value by

(A) 0.235 (B) 0.068
(C) 0.024 (D) 0.012

[GATE-2012-CE]

26. Find the magnitude of the error (correct to two decimal places) in the estimation of following integral using Simpson's 1/3 Rule. Take the step length as 1.

$$\int_0^4 \left(x^4 + 10\right)dx$$

[GATE-2013-CE]

27. For step-size; $\Delta x = 0.4$ the value of the following integral

$$\int_0^{0.8} \begin{pmatrix} 0.2 + 25x - 200x^2 + 675x^3 \\ -900x^4 + 400x^5 \end{pmatrix} dx$$

using Simpson's 1/3 rule is ____.

[GATE-2015-CE-SET II]

28. Simpson's 1/3 rule is used to integrate the function $f(x) = \frac{3}{5}x^2 + \frac{9}{5}$ between x = 0 and x = 1 using the least number of equal sub intervals. The value of integral is ______.

[GATE-2015-ME-SET 1]

29. The accuracy of Simpson's rule quadrature for a step size h is

(A) $O(h^2)$ (B) $O(h^3)$
(C) $O(h^4)$ (D) $O(h^5)$

[GATE-2003]

30. Match the correct pairs

Numerical integration scheme	Order of Fitting Polynomial
P. Simpson's 3/8 Rule	1. First
Q. Trapezoidal Rule	2. Second
R. Simpson's 1/3 Rule	3. Third

(A) P – 2, Q – 1, R – 3
(B) P – 3, Q – 2, R - 1
(C) P – 1, Q – 2, R – 3
(D) P – 3, Q – 1, R – 2

[GATE-2013-ME]

31. The integral $\int_{x_1}^{x_2} x^2 dx$ with $x_2 > x_1 > 0$ is evaluated analytically as well as numerically using a single application of the trapezoidal rule. If I is the exact value of the integral obtained analytically and J is the approximate value obtained using the trapezoidal rule, which of the following statements is correct about their relationship?

(A) J > I
(B) J < I
(D) J = I
(D) Insufficient data to determine the relationship

[GATE-2015-CE-SET I]

32. Numerical integration using trapezoidal rule gives the best result for a single variable function, which is

(A) linear (B) parabolic
(C) logarithmic (D) hyperbolic

[GATE-2016-ME-SET II]

33. P(0, 3), Q (0.5, 4) and R (1, 5) are three points on the curve defined by f(x). Numerical integration is carried out using both Trapezoidal rule and Simpson's rule within limits x = 0 and x = 1 for the curve. The difference between the two results will be

(A) 0 (B) 0.25
(C) 0.5 (D) 1

[GATE-2017 ME SESSION-I]

Forward Euler Method

34. Consider the equation $\frac{du}{dt} = 3t^2 + 1$ with u = 0 at t = 0. This is numerically solved by using the forward Euler method with a step size. $\Delta t = 2$. The absolute error in the solution in the end of the first time step is _____.

[GATE-2017]

35. During the numerical solution of a first order differential equation using the Euler (also known as Euler Cauchy) method with step size h the local truncation error is of the order of

(A) h^2 (B) h^3

(C) h^4 (D) h^5

[GATE-2009 (PI)]

36. The differential equation $\frac{dx}{dt} = \frac{1-x}{\tau}$ is discretised using Euler's numerical integration method with a time step $\Delta T > 0$. What is the maximum permissible value of ΔT to ensure stability of the solution of the corresponding discrete time equation?

(A) 1 (B) $\tau / 2$

(C) τ (D) 2τ

[GATE-2011]

37. The ordinary differential equation $\frac{dx}{dt} = -3x + 2$, with x (0) = 1 is to be solved using the forward Euler method. The largest time step that can be used to solve the equation without making the numerical solution unstable is ________.

[GATE-2016-EC-SET 2]

38. Variation of water depth (y) in a gradually varied open channel flow is given by the first order differential equation

$$\frac{dy}{dx} = \frac{1 - e^{-\frac{10}{3}\ln y}}{250 - 45e^{-3\ln y}}$$

Given initial condition y (x = 0) = 0.8 m. The depth (in m, up to three decimal places) of flow of 2 downstream section at x = 1m from one calculation step of Single Step Euler Method is ________.

[GATE-2018 (CE-MORNING SESSION)]

Backward Euler Method

39. The differential equation $\frac{dy}{dx} = 0.25y^2$ is to be solved using the backward (implicit) Euler's method with the boundary condition y = 1 at x = 0 and with a step size of 1. What would be the value of y at x = 1?

(A) 1.33 (B) 1.67

(C) 2.00 (D) 2.33

[GATE-2006]

Modified Euler Method

40. Given the differential equation $y' = x - y$ with initial condition y(0) = 0. The value of y(0, 1) calculated numerically upto the third place of decimal by the 2nd order Runge Kutta method with step size h = 0.1 is

[GATE-1993 (ME)]

41. Consider the first order initial value problem $y' = y + 2x - x^2$, y(0) = 1, $(0 \le x < \infty)$ with exact solution $y(x) = x^2 + e^x$. For x = 0.1, the percentage difference between the exact solution and the solution obtained using a single iteration of the second-order Runge-Kutta method with step-size h = 0.1 is _______.

[GATE-2016-EC-SET 3]

Runge-Kutta Method Of 4th Order

42. Consider an ordinary differential equation $\frac{dx}{dt} = 4t + 4$. If $x = x_0$ at t = 0, the increment in x calculated using Runge-Kutta fourth order multistep method with a step size of $\Delta t = 0.2$ is

(A) 0.22 (B) 0.44

(C) 0.66 (D) 0.88

[GATE-2014-ME-SET 4]

Gauss Seidal And Gauss Jacobi Method

43. Gauss-Seidel method is used to solve the following equations (as per the given order) :

$x_1 + 2x_2 + 3x_3 = 5$, $2x_1 + 3x_2 + x_3 = 1$,

$3x_1 + 2x_2 + x_3 = 3$

Assuming initial guess as $x_1 = x_2 = x_3 = 0$, the value of x_3 after the first iteration is _____.

[GATE-2016]

Forward Difference Operator

44. The values of a function f(x) are tabulated below

x	0	1	2	3
f(x)	1	2	1	10

Using Newton's forward difference formula, the cubic polynomial that can be fitted to the above data, is

(A) $2x^3 + 7x^2 - 6x + 2$

(B) $2x^3 - 7x^2 + 6x - 2$

(C) $x^3 - 7x^2 - 6x + 1$

(D) $2x^3 - 7x^2 + 6x + 1$

[GATE-2004]

45. The following table lists an n^{th} order polynomial $f(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0$ and the forward differences evaluated at equally spaced values of x. The order of the polynomial is

x	f(x)	Δf	$\Delta^2 f$	$\Delta^3 f$
-0.4	1.7648	-0.2965	0.089	-0.03
-0.3	1.4683	-0.2075	0.059	-0.0228
-0.2	1.2608	-0.1485	0.0362	-0.0156
-0.1	1.1123	-0.1123	0.0206	-0.0084
0	1	-0.0917	0.0122	-0.0012
0.1	0.9083	-0.0795	0.011	0.006
0.2	0.8288	0.0685	0.017	0.0132

(A) 1 (B) 2

(C) 3 (D) 4

[GATE-2017 (IN)]

46. In the following differential equation, the numerically obtained value of y(t), at t =1, is ______ (Round off to 2 decimal places).

$$\frac{dy}{dt} = \frac{e^{-\alpha t}}{2 + \alpha t}, \ \alpha = 0.01 \text{ and } y(0) = 0$$

[GATE-2023 (EE)]

47. The initial value problem

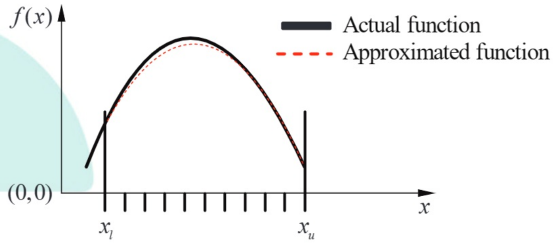
$$\frac{dy}{dt} + 2y = 0, y(0) = 1$$

is solved numerically using the forward Euler's method with a constant and positive time step of Δt.

Let y_n represent the numerical solution obtained after n steps. The condition $|y_{n+1}| \le |y_n|$ is satisfied if and only if Δt does not exceed ______. (Answer in integer)

[GATE-2023 (ME)]

48. A function $f(x)$, that is smooth and convex-shaped between interval (x_l, x_u) is shown in the figure. This function is observed at odd number of regularly spaced points. If the area under the function is computed numerically, then _________.



(A) the numerical value of the area obtained using the trapezoidal rule will be less than the actual

(B) the numerical value of the area obtained using the trapezoidal rule will be more than the actual

(C) the numerical value of the area obtained using the trapezoidal rule will be exactly equal to the actual

(D) with the given details, the numerical value of area cannot be obtained using trapezoidal rule

[GATE-2023 (CE-1)]

49. The differential equation, $\frac{du}{dt} + 2tu^2 = 1$ is solved by employing a backward difference scheme within the finite difference framework. The value of u at the $(n - 1)^{th}$ time-step, for some n, is 1.75. The corresponding time (t) is 3.14 s. Each time step is 0.01 s long. Then, the value of $(u_n - u_{n-1})$ is_______. (round off to three decimal places).

[GATE-2023 (CE-1)]

❑❑❑

8 Laplace Transforms

Laplace Transform Using Main Definition

1. If $f(t)=\begin{cases}\frac{1}{k}, \text{ where } 0<t<k \\ 1 \text{ when } t>k\end{cases}$

then $L(f(t))=$

(A) $\frac{1-e^{-ks}}{ks^2}$ (B) $\frac{1+e^{-ks}}{ks^2}$

(C) $\frac{k-e^{-ks}}{s^2}$ (D) $\frac{1+(k-1)e^{-ks}}{ks}$

2. Laplace transform of $(a+bt)^2$ where 'a' and 'b' are constants is given by:

(A) $(a+bs)^2$

(B) $\frac{1}{(a+bs)^2}$

(C) $\frac{a^2}{s}+\frac{2ab}{s^2}+\frac{2b^2}{s^3}$

(D) $\frac{a^2}{s}+\frac{2ab}{s^2}+\frac{b^2}{s^3}$

3. The Laplace Transform of the following function is

$f(t)=\begin{cases}\sin t \text{ for } 0\le t\le \pi \\ 0 \text{ for } t>\pi\end{cases}$

(A) $\frac{1}{1+s^2}$ for all s > 0

(B) $\frac{1}{1+s^2}$ for all s < π

(C) $\frac{1+e^{-ks}}{1+s^2}$ for all s > 0

(D) $\frac{e^{-ks}}{1+s^2}$ for all s > 0

[GATE-2002]

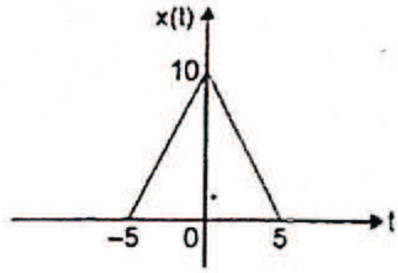
4. The bilateral Laplace transform of a function $f(t)=\begin{cases}1 \text{ if } a\le t\le b \\ 0 \text{ otherwise}\end{cases}$ is

(A) $\frac{a-b}{s}$ (B) $\frac{e^s(a-b)}{s}$

(C) $\frac{e^{-as}-e^{-bs}}{s}$ (D) $\frac{e^{s(a-b)}}{s}$

[GATE-2015-EC-SET 2]

5. If x(t) is as shown in the figure, its Laplace transform is



(A) $\frac{2e^{+5s}+2e^{-5s}}{s^2}$

(B) $\frac{2e^{+5s}-4+2e^{-5s}}{s^2}$

(C) $\frac{2e^{+5s}-2+2e^{-5s}}{s^2}$

(D) $\frac{2e^{+5s}+4-2e^{-5s}}{s^2}$

[ESE-2018 (EC)]

Properties Of Laplace Transform

6. $L(\sin^2 t)=$

(A) $\frac{1}{s(s^2+1)}$ (B) $\frac{2}{s(s^2+4)}$

(C) $\frac{2}{s^2(s^2+4)}$ (D) $\frac{4-s^2}{2s(s^2+4)}$

7. $L\left(\frac{\sin 2t}{t}\right)=$

(A) $\cos^{-1} s$ (B) $\cot^{-1} s$

12. The Laplace transform of $f(t)=2\sqrt{t/\pi}$ is $s^{-3/2}$.

(A) cos⁻¹ s (B) cot⁻¹ s

(C) $\cot^{-1}\frac{s}{2}$ (D) $\tan^{-1}\frac{s}{2}$

8. $L\left(\frac{1-e^t}{t}\right)=$

(A) $\log\left(\frac{s-1}{s}\right)$ (B) $\log\left(\frac{s+\pi}{s}\right)$

(C) $\log\left(\frac{s}{s-1}\right)$ (D) $\log\left(\frac{s-1}{s+1}\right)$

9. The Laplace transform of $e^{\alpha t}\cos\alpha t$ is equal to ...

(A) $\frac{s-\alpha}{(s-\alpha)^2+\alpha^2}$ (B) $\frac{s+\alpha}{(s-\alpha)^2+\alpha^2}$

(C) $\frac{1}{(s-\alpha)^2}$ (D) None

[GATE-1997-EC]

10. If F(s) is the Laplace transform of the function f(t) then Laplace transform of $\int_0^t f(\tau)d\tau$ is

(A) $\frac{1}{s}F(s)$ (B) $\frac{1}{s}F(s)-f(0)$

(C) $sF(s)-f(0)$ (D) $\int F(s)ds$

[GATE-2007 (ME)]

11. The unilateral Laplace transform of f(t) is $\frac{1}{s^2+s+1}$. The unilateral Laplace transform of t f(t) is

(A) $-\frac{s}{(s^2+s+1)^2}$ (B) $-\frac{2s+1}{(s^2+s+1)^2}$

(C) $\frac{s}{(s^2+s+1)^2}$ (D) $\frac{2s+1}{(s^2+s+1)^2}$

[GATE-2012 (EC, EE, IN)]

The Laplace transform of $g(t)=\sqrt{1/\pi t}$ is

(A) $3s^{-5/2}/2$ (B) $s^{-1/2}$

(C) $s^{1/2}$ (D) $s^{3/2}$

[GATE-2014-EE-SET 2]

13. The Laplace transform of e^{i5t} where $i=\sqrt{-1}$ is

(A) $\frac{s-5i}{s^2-25}$ (B) $\frac{s+5i}{s^2+25}$

(C) $\frac{s+5i}{s^2-25}$ (D) $\frac{s-5i}{s^2+25}$

[GATE 2015 – ME-SET 2]

14. F(S) is the Laplace transform of the function $f(t)=2t^2e^{-t}$

F(1) is _______ (correct to two decimal places).

[GATE 2018 (ME-MORNING SESSION)]

Application Of Laplace Transform In Real Integrals

15. The value of the integral

$$\int_{-\infty}^{\infty}12\cos(2\pi t)\frac{\sin(4\pi t)}{4\pi t}dt \text{ is } \_\_\_\_$$

[2015 EC-SET 2]

16. The value of $\int_0^\infty \frac{e^{-t}\sin t}{t}dt=$

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$

(C) $\frac{\pi}{3}$ (D) 0

17. Evaluate $\int_0^\infty \frac{\sin t}{t}dt$

(A) π (B) $\frac{\pi}{2}$

(C) $\frac{\pi}{4}$ (D) $\frac{\pi}{8}$

[GATE-2007]

18. The value of the integral

$2\int_{-\infty}^{\infty}\left(\frac{\sin 2\pi t}{\pi t}\right)dt$ is equal to

(A) 0 (B) 0.5

$\frac{dy}{dx}=7$ at x = 0 using the Laplace transform technique.

[GATE-1997-ME]

24. The function f(t) satisfies the

(C) 1 (D) 2

[GATE-2016 (EE-SET 2)]

Inverse Laplace Transform

19. $L^{-1}\left[\dfrac{1}{(s+1)(s-2)}\right] =$

(A) $\dfrac{e^{2t}}{3}$ (B) $\dfrac{e^{-t}}{3}$

(C) $\dfrac{e^{2t}-e^{-t}}{3}$ (D) $\dfrac{e^{2t}+e^{-t}}{3}$

20. The inverse Laplace transform of $\dfrac{s+9}{s^2+6s+13}$ is

(A) $\cos 2t + 9\sin 2t$

(B) $e^{-3t}\cos 2t - 3e^{-3t}\sin 2t$

(C) $e^{-3t}\sin 2t + 3e^{-3t}\cos 2t$

(D) $e^{-3t}\cos 2t + 3e^{-3t}\sin 2t$

[GATE-1995]

21. The inverse Laplace transform of the function $\dfrac{s+5}{(s+1)(s+3)}$ is ....

(A) $2e^{-t} - e^{-3t}$ (B) $2e^{-t} + e^{-3t}$

(C) $e^{-t} - 2e^{-3t}$ (D) $e^{-t} + 2e^{-3t}$

[GATE-1996-EC]

22. The Laplace transform of a function f(t) is $\dfrac{1}{s^2(s+1)}$. The function f(t) is

(A) $t-1+e^{-t}$ (B) $t+1+e^{-t}$

(C) $-1+e^{-t}$ (D) $2t+e^{t}$

Application Of Laplace Transform In Differential Equation

23. Solve the initial value problem $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 3y = 0$ with y = 3 and

24. The function f(t) satisfies the differential equation $\dfrac{d^2f}{dt^2} + f = 0$ and the auxiliary conditions, f(0) = 0, $\dfrac{df}{dt}(0) = 4$. The Laplace transform of f(t) is given by

(A) $\dfrac{2}{s+1}$ (B) $\dfrac{4}{s+1}$

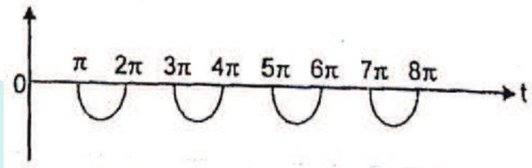
(C) $\dfrac{4}{s^2+1}$ (D) $\dfrac{2}{s^4+1}$

[GATE-2013-ME]

Laplace Transform Of Periodic Functions

25. The Laplace Transform of the periodic function f(t) described by the curve below is

$$f(t) = \begin{cases} \sin t, & \text{if } (2n-1)\pi < t < 2n\pi \; (n = 1, 2, 3, \ldots) \\ 0 & \text{otherwise} \end{cases}$$



[GATE-1993 (ME)]

Initial & Final Value Theorem

26. If $L(f) = F(s) = \dfrac{5s^2+23s+6}{s(s^2+2s+2)}$ then $\lim_{t\to\infty} f(t) =$ _____.

(A) 0, 2 respectively

(B) 2, 0 respectively

(C) 0, 1 respectively

(D) 2/5, 0 respectively

[GATE-1995-EC]

27. If $F(s) = \dfrac{2}{s(1+s)}$ then $\lim_{t\to\infty} f(t) =$ _____ where L(f(t)) = F(s).

(A) 0, 2 respectively

(B) 2, 0 respectively

(C) 0, 1 respectively

(D) 2/5, 0 respectively

[GATE-1995-EC]

28. If L(f(t)) = $\dfrac{2(s+1)}{\ldots}$ then f(0+) and

32. A delayed unit step function is defined as $u(t-a) = \begin{cases} 0, & \text{for } t < a \\ 1, & \text{for } t \ge a \end{cases}$. Its Laplace transform is

(A) $a \cdot e^{-as}$ (B) $\dfrac{e^{-as}}{s}$

28. If $L\{f(t)\} = \frac{[illegible]}{s^2+2s+1}$ then $f(0^+)$ and $f(\infty)$ given by ....

(A) 0, 2 respectively
(B) 2, 0 respectively
(C) 0, 1 respectively
(D) 2/5, 0 respectively

[GATE-1995-EC]

29. If $L\{f(t)\} = \frac{\omega}{s^2+\omega^2}$ then the value of $\lim\limits_{t\to\infty} f(t) =$ ______.

(A) can not be determined
(B) zero
(C) unity
(D) infinite **[GATE-1998-EC]**

30. Given $L^{-1}\left[\frac{3s+1}{s^3+4s^2+(K-3)s}\right]$.

If $\lim\limits_{t\to\infty} f(t) = 1$, then the value of K is

(A) 1 (B) 2
(C) 3 (D) 4

[GATE-2010-EE]

31. If $L\{f(t)\} = \frac{s+2}{s^2+1}$,

$L\{g(t)\} = \frac{s^2+1}{(s+3)(s+2)}$

$h(t) = \int_0^t f(T)g(t-T)dT$ then L{h(t)} is

(A) $\frac{s^2+1}{s+3}$

(B) $\frac{1}{s+3}$

(C) $\frac{s^2+1}{(s+3)(s+2)} + \frac{s+2}{s^2+1}$

(D) None of these

[GATE-2000-EC]

(C) $\frac{e^{as}}{s}$ (D) $\frac{e^{as}}{a}$

33. The integral $\int_{-\infty}^{\infty} \delta\left(t-\frac{\pi}{6}\right) 6\sin(t)\,dt$ evaluates to

(A) 6 (B) 3
(C) 1.5 (D) 0

[GATE-2010 (IN)]

34. Given two continuous time signal $x(t) = e^{-t}$ and $y(t) = e^{-2t}$ which exists for t > 0 then the convolution z(t) = x(t) * y(t) is

(A) $e^{-t} - e^{-2t}$ (B) e^{-2t}
(C) e^{-t} (D) $e^{-t} + e^{-3t}$

[GATE-2011]

35. The solution of the differential equation, for t > 0,
$y''(t) + 2y'(t) + y(t) = 0$ with initial conditions y(0) = 0 and y'(0) = 1, is? (u(t) denotes the unit step function),

(A) $te^{-t}u(t)$ (B) $(e^{-t} - te^{-t})u(t)$
(C) $(-e^{-t} + te^{-t})u(t)$ (D) $e^{-t}u(t)$

[GATE-2016]

36. The Laplace transform of $f(t) = t^n e^{-\alpha t} u(t)$ is

(A) $\frac{(n+1)!}{(s+\alpha)^{n+1}}$ (B) $\frac{n!}{(s+\alpha)^n}$

(C) $\frac{(n-1)!}{(s+\alpha)^{n+1}}$ (D) $\frac{n!}{(s+\alpha)^{n+1}}$

[ESE-2018 (EE)]

37. Which one of the options given is the inverse Laplace transform of $\frac{1}{s^3-s}$?
$u(t)$ denotes the unit-step function.

(A) $\left(-1+\frac{1}{2}e^{-t}+\frac{1}{2}e^{t}\right)u(t)$

(B) $\left(\frac{1}{3}e^{-t}-e^{t}\right)u(t)$

(C) $\left(-1+\frac{1}{2}e^{-(t-1)}+\frac{1}{2}e^{(t-1)}\right)u(t-1)$

(D) $\left(-1-\frac{1}{2}e^{-(t-1)}-\frac{1}{2}e^{(t-1)}\right)u(t-1)$

[GATE-2023 (ME)]

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9 Fourier Series

Objective Questions

1. The Fourier series of the function, $f(x)=0,\ -\pi<x\leq 0$

$= \pi - x,\ 0<x<\pi$

3. Infinite number of discontinuities

Select the correct answer using the codes given below :

in the interval $[-\pi, \pi]$ is

$$f(x) = \frac{\pi}{4} + \frac{2}{\pi}\left[\frac{\cos x}{1^2} + \frac{\cos 3x}{3^2} + \ldots\ldots\right] + \left[\frac{\sin x}{1} + \frac{\sin 2x}{2} + \frac{\sin 3x}{3} + \ldots\ldots\right].$$

The convergence of the above Fourier series at x = 0 gives

(A) $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$

(B) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2} = \frac{\pi^2}{12}$

(C) $\sum_{n=1}^{\infty} \frac{1}{(2n-1)} = \frac{\pi^2}{8}$

(D) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} = \frac{\pi}{4}$

[GATE-2016-CE-SET 2]

2. Let $g : [0, \infty) \rightarrow [0, \infty)$ be a function defined by $g(x) = x - [x]$, where [x] represents the integer part of x. (That is, it is the largest integer which is less than or equal to x). The value of the constant term in the Fourier series expansion of g(x) is _______

[GATE-2014-EE-SET 1]

3. Fourier series of any periodic signal x(t) can be obtained if

1. $\int_0^T |x(t)| dt < \infty$

2. Finite number of discontinuities within finite time interval t

(A) 1, 2 and 3 (B) 1 and 3 only

(C) 1 and 2 only (D) 2 and 3 only

[ESE 2017 (EE)]

4. The Fourier series expansion of the saw-toothed waveform $f(x) = x$ in $(-\pi, \pi)$ of period 2π gives the series,

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots\ldots = ?$$

(A) $\frac{\pi}{2}$ (B) $\frac{\pi^2}{4}$

(C) $\frac{\pi^2}{2}$ (D) $\frac{\pi}{4}$

[ESE 2017 (EE)]

5. The Fourier cosine series for an even function f(x) is given by $f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos(nx)$. The value of the coefficient a_2 for the function $f(x) = \cos^2(x)$ in $[0, \pi]$ is

(A) -0.5 (B) 0.0

(C) 0.5 (D) 1.0

[GATE-2018 (ME-AFTERNOON SESSION)]

6. In the Fourier series expansion of $f(x) = x^2$ in $[-\pi, \pi]$ the sum of absolute values of the Fourier coefficients of f is _____.

(A) $\frac{\pi^2}{6}$ (B) $\frac{\pi^2}{3}$

(C) $\frac{2\pi^2}{3}$ (D) π^2

7. The Fourier series of the periodic function

$$f(x) = |x|, \ -1 < x < 1, \ f(x+2) = f(x)$$

is given by $\frac{1}{2} - \sum_{n=1}^{\infty} 4\frac{\cos(2n-1)\pi x}{(2n-1)^2 \pi^2}$. Using the above, the sum of $1 + \frac{1}{3^2} + \frac{1}{5^2} + \ldots..$ is _______

(A) $\frac{\pi}{4}$ (B) $\frac{3\pi}{8}$

(C) $\frac{\pi^2}{8}$ (D) $\frac{\pi^2}{2}$

8. The values of the fourier coefficient A_0 of the series $\left\{A_0 + \sum_{n=1}^{\infty} A_n \cos nx + B_n \sin nx\right\}$ of a function $f(x) = x^2$ with period 2π defined over an interval $0 \le x \le 2\pi$ is ________.

(A) $\frac{4\pi^2}{3}$ (B) $\frac{2\pi^2}{3}$

(C) $\frac{\pi^2}{3}$ (D) $\frac{\pi^2}{6}$

9. The following function is defined over the interval $[-L, L]$: $f(x) = px^4 + qx^5$

If it is expressed as a Fourier series,

$$f(x) = a_0 + \sum_{n=1}^{\infty}\left\{a_n \sin\left(\frac{\pi x}{L}\right) + b_n \cos\left(\frac{\pi x}{L}\right)\right\}$$

which options amongst the following are true?

(A) $a_n, n- = 1, 2, \ldots, \infty$ depend on p

(B) $a_n, n- = 1, 2, \ldots, \infty$ depend on q

(C) $b_n, n- = 1, 2, \ldots, \infty$ depend on p

(D) $b_n, n- = 1, 2, \ldots, \infty$ depend on q

[GATE-2023 (CE-1)]

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10 Partial Differential Equations

Objective Questions

1. If $u = u(x,t)$ is such that $\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}$, $0 \le x \le \pi$, $t \ge 0$, $u(0,t) = u(\pi,t) = 0$, $u(x,0) = 0$, $\frac{\partial u}{\partial t}(x,0) = \sin x$, then

$u(x,0) = 0$, $\frac{\partial u(x,0)}{\partial t} = \pi$ for $0 \le x \le \pi$?

(A) $u(x,t) = \sum_{n=0}^{\infty} a_n \sin\left(\left(n+\frac{1}{2}\right)t\right)$

$\sin\left(\left(n+\frac{1}{2}\right)x\right)$

$u\left(\frac{\pi}{3},\frac{\pi}{6}\right)$ is

(A) $\frac{3}{4}$ (B) $\frac{3}{8}$

(C) $\frac{\sqrt{3}}{4}$ (D) $\frac{\sqrt{3}}{8}$

2. Let $u(x,t)$ be the solution of the initial value problem $\frac{\partial^2 u}{\partial t^2} = 9\frac{\partial^2 u}{\partial x^2}$, $t > 0$, $-\infty < x < \infty$, $u(x,0) = x+5$, $\frac{\partial u}{\partial t}(x,0) = 0$.

Then $u(2,2)$ is

(A) 7 (B) 13

(C) 14 (D) 26

3. The general solution of $x(z^2 - y^2)\frac{\partial z}{\partial x} + y(x^2 - z^2)\frac{\partial z}{\partial y} = z(y^2 - x^2)$ is

(A) $F(x^2 + y^2 + z^2, xyz) = 0$

(B) $F(x^2 + y^2 - z^2, xyz) = 0$

(C) $F(x^2 - y^2 + z^2, xyz) = 0$

(D) $F(-x^2 + y^2 + z^2, xyz) = 0$

4. Which one of the following is a possible solution to the partial differential equation $\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0$ with boundary conditions $u(0,t) = 0$, $\frac{\partial u(\pi,t)}{\partial x} = 0$, for $t \geq 0$,

(B) $u(x,t) = \sum_{n=0}^{\infty} a_n \cos\left(\left(n+\frac{1}{2}\right)t\right) \sin\left(\left(n+\frac{1}{2}\right)x\right)$

(C) $u(x,t) = \sum_{n=0}^{\infty} a_n \sin\left(\left(n+\frac{1}{2}\right)t\right) \cos\left(\left(n+\frac{1}{2}\right)x\right)$

(D) $u(x,t) = \sum_{n=0}^{\infty} a_n \cos\left(\left(n+\frac{1}{2}\right)t\right) \cos\left(\left(n+\frac{1}{2}\right)x\right)$

5. The solution of the initial boundary value problem

$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $0 < x < \pi$, $t > 0$, with boundary and initial conditions $\frac{\partial u}{\partial x}(0,t) = 0 = u(\pi,t)$, $t > 0$ and $u(x,0) = f(x)$, $0 < x < \pi$, is

(A) $u(x,t) = \sum_{n=0}^{\infty} A_n \exp\left(-\left(\frac{2n+1}{2}\right)^2 t\right) \cos\left(\frac{2n+1}{2}x\right)$,

with $A_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos\left(\frac{2n+1}{2}x\right)dx$

(B) $u(x,t) = \sum_{n=0}^{\infty} A_n \exp(-n^2 t)\cos(nx)$,

with $A_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos(nx)dx$

(C) $u(x,t) = \sum_{n=1}^{\infty} A_n \exp\left(-\left(\frac{2n+1}{2}\right)^2 t\right) \sin\left(\frac{2n+1}{2}x\right)$,

with $A_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin\left(\frac{2n+1}{2}x\right)dx$

7. Which one of the following partial differential equations cannot be reduced to two ordinary differential equations by the method of separation of variables?

(A) $\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = 0$

(B) $\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0$

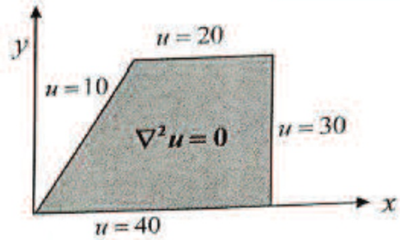
(C) $\frac{\partial^2 u}{xt^2} + \frac{\partial^2 u}{\partial t \partial x} + \frac{\partial u}{\partial x} = 0$

(D) $\frac{\partial^2 u}{} + \frac{\partial^2 u}{} + \frac{\partial^2 u}{} = 0$

(D) $u(x,t)=\sum_{n=1}^{\infty}A_n\exp(-n^2t)\sin(nx)$,

with $A_n=\frac{2}{\pi}\int_0^{\pi}f(x)\sin(nx)dx$

6. For the solution of $\nabla^2u=0$, the domain the boundary conditions are shown below.



Which of the following statements is TRUE?

(A) The solution cannot be obtained using separation of variables because the governing equation is non-separable.

(B) The solution cannot be obtained using separation of variables because all the boundary values are non-zero.

(C) The solution cannot be obtained using separation of variables because not all the boundaries are along constant coordinate lines.

(D) The solution can be obtained by separation of variables.

(D) $\frac{}{xt^2}+\frac{}{\partial t\partial x}+\frac{}{\partial x^2}=0$

8. Which of the following is quasi-linear partial differential equation?

(A) $\frac{\partial^2u}{\partial t^2}+u^2=0$

(B) $\left(\frac{\partial u}{\partial t}\right)^2+\frac{\partial u}{\partial x}=0$

(C) $\left(\frac{\partial u}{\partial t}\right)^2-\left(\frac{\partial u}{\partial x}\right)^2=0$

(D) $\left(\frac{\partial u}{\partial t}\right)^4-\left(\frac{\partial u}{\partial x}\right)^3=0$

9. If u(x,t) = g(t) sin x is the solution of the wave equation $u_{tt}=u_{xx}$, t > 0, $0<x<\pi$, with the initial conditions $u(x,0)=2\sin x, u_t(x,0)=0, 0\le x\le\pi$, and the boundary conditions $u(0,t)=u(\pi,t)=0$, $t\ge 0$, then the value of $g\left(\frac{\pi}{3}\right)$ is ___

10. Let u(x, t) satisfy the initial and boundary value problem $\frac{\partial u}{\partial t}=2\frac{\partial^2u}{\partial x^2}$, $0<x<\pi$, t > 0, $u(0,t)=0\,u(\pi,t)$, t > 0, $u(x,0)=\sin x+2\sin 4x$, $0<x<\pi$. Then the value of $u\left(\frac{\pi}{2},\ln(5)\right)$ is ________.

11. If the transformation $u(x,t)=e^xv(x,t)$ reduces the partial differential equation $\frac{\partial^2u}{\partial x^2}-2\frac{\partial u}{\partial x}-\frac{\partial u}{\partial t}+u=9u=9$ to the equation $\frac{\partial v}{\partial t}-\frac{\partial^2v}{\partial x^2}=9f(x)$, then f(x) equals

(A) $-e^{-x}$ (B) e^{-x}

(C) $-2e^{-x}$ (D) $2e^{-x}$

12. Let $y_1(x)$ and $y_2(x)$ be two linearly independent solutions of the differential equation

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-4y=0,\ x>0.$$

If $y_1(x)=x^2$, then $\lim_{x\to\infty} y_2(x)$ is ____.

13. Let $u(x,t)$ be the solution of the initial boundary value problem.

$$\frac{\partial u}{\partial t}-\frac{\partial^2 u}{dx^2}=0, x\in(0,2), t>0$$

$$u(x,0)=\sin(\pi x), x\in(0,2)$$

$$u(0,t)=4(2,t)=0$$

Then the value of $e^{\pi^2}\left[4\left(\frac{1}{2},1\right)-4\left(\frac{3}{2},1\right)\right]$ is ________.

[GATE-2023 (XE)]

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