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Course: GATE
Computer Science Engineering(CS)

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TOPICWISE : ENGINEERING MATHEMATICS-1 (GATE - 2020) - REPORTS

OVERALL ANALYSIS | COMPARISON REPORT | SOLUTION REPORT

ALL(17) | CORRECT(0) | INCORRECT(0) | SKIPPED(17)

Q. 1 Solution Video | Have any Doubt ?

Identify the correct statement from the following

A. Commutative property not holds for addition of matrices

B. Associative property not holds for addition of matrices

C. Commutative property not holds for multiplication of matrices — Correct Option

Solution :
(c)
Commutative for multiplication of matrices does not hold.

$$AB \neq BA$$

D. None of the above

QUESTION ANALYTICS +

Q. 2 Solution Video | Have any Doubt ?

The density function of repairing a machine is given by $f(x)=\frac{1}{2}e^{\frac{-x}{2}}$, where '$x$' is repair time in hours. The probability that the repair time is more than 2 hours is

A. 0.368 — Correct Option

Solution :
(a)

$$\text{Probability} = \int_2^\infty f(x)dx$$

$$= \int_2^\infty \left[\frac{1}{2}e^{\frac{-x}{2}}\right]dx = \left[-e^{\frac{-x}{2}}\right]_2^\infty = e^{-1} = 0.368$$

B. 0.482

C. 0.518

D. 0.632

QUESTION ANALYTICS +

Q. 3 Solution Video | Have any Doubt ?

The values of a and b for which the following system has no solutions are respectively

$$\begin{aligned} ax + y + 2z &= 0 \\ x + 2y + z &= b \\ 2x + y + az &= 0 \end{aligned}$$

A. $a = -1, b = 0$

B. $a = 2, b = 0$

C. $a = -1, b \neq 0$ — Correct Option

Solution :
(c)
The matrix formed by the coefficients is $\begin{bmatrix} a & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & a \end{bmatrix}$

$$\text{Determinant} = 2a^2 - 2a - 4$$

$$\therefore \quad D = 0 \text{ for } a = 2 \text{ or } a = -1$$

(A) If $D \neq 0$, then the system will have unique solution because the rank of matrix will be 3.

(B) If $a = 2$, the matrix formed by the coefficients is $\begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$

The rank of matrix is 2.
Considering 'z' as side unknown.

The characteristic determinant will be $\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & b \\ 2 & 1 & 0 \end{bmatrix}$

The determinant of this is 0.
The system will have infinite solutions when $a = 2$.

(C) If $a = -1$, the matrix formed by the coefficients is $\begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & -1 \end{bmatrix}$

Its rank is 2.
Considering 'z' as side unknown.

The characteristic matrix is $\begin{bmatrix} -1 & 1 & 0 \\ 1 & 2 & b \\ 2 & 1 & 0 \end{bmatrix}$

The determinant of this matrix is $3b$.
The system will have no solution if $b \neq 0$
$\therefore$ For $a = -1$ and $b \neq 0$, the system will have no solution.

D $a = 2, b \neq 0$

QUESTION ANALYTICS +

Q. 4 Solution Video | Have any Doubt ?

Let X be an exponential random variable with rate parameter λ. Then find variance of X.

A $\frac{1}{\lambda}$

B $\frac{1}{\lambda^2}$ Correct Option

Solution :
(b)
Probability density function:

$$f(x) = \lambda e^{-\lambda x}, x > 0$$

$$E(X) = \int_0^\infty x \cdot f(x) \cdot dx$$

$$= \int_0^\infty x\lambda \cdot e^{-\lambda x} \cdot dx = \frac{1}{\lambda}$$

$$Var(X) = E(X^2) - [E(X)]^2$$

$$E(X^2) = \int_0^\infty x^2 \cdot f(x) \cdot dx$$

$$= \int_0^\infty x^2 \cdot \lambda \cdot e^{-\lambda x} \cdot dx = \frac{2}{\lambda^2}$$

$$\Rightarrow \quad Var(X) = \frac{2}{\lambda^2} - \left(\frac{1}{\lambda}\right)^2 = \frac{1}{\lambda^2}$$

C $\frac{2}{\lambda}$

D $\frac{2}{\lambda^2}$

QUESTION ANALYTICS +

Q. 5 Solution Video | Have any Doubt ?

The rank of matrix, $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ is _____.

A 1

B 2 Correct Option

Solution :
(b)

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$R_2 \leftarrow R_2 - 2(R_1)$ and $R_3 \leftarrow R_3 - R_1$

$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & -1 \\ 0 & -1 & -1 \end{bmatrix}$$

$$R_3 \leftarrow R_3 - R_2$$

$$\Rightarrow \quad A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ Rank of matrix is 2.

C 0

D 3

QUESTION ANALYTICS +

Q. 6 Solution Video | Have any Doubt ?

Consider the following matrix A.

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 5 & x & 6 \end{bmatrix}$$

For which value of x is the matrix A not invertible?

5.5 Correct Option

Solution :
5.5

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 5 & x & 6 \end{bmatrix}$$

$$|A| = |(6 - 2x) - 1\,(0 - 10) + 1\,(0 - 5)| = 11 - 2x$$

If $|A| = 0 \Rightarrow A$ is not invertible

$\therefore \quad 11 - 2x = 0 \Rightarrow x = \frac{11}{2}$

QUESTION ANALYTICS +

Q. 7 Solution Video | Have any Doubt ?

Let A and B be 3 × 3 matrices such that A' = − A and B' = B, then matrix $\lambda AB + 3BA$ is a skewsymmetric matrix for λ equal to _____.

3 Correct Option

Solution :
3

$$(\lambda AB + 3BA)' = \lambda(AB)' + 3(BA)'$$
$$= \lambda B'A' + 3A'B'$$
$$= \lambda(B)(-A) + 3(-A)(B) \quad [\because A' = -A, B' = B]$$
$$= -\lambda BA - 3AB = -(3AB + \lambda BA)$$
$$\Rightarrow \quad \lambda = 3$$

QUESTION ANALYTICS +

Q. 8 Solution Video | Have any Doubt ?

Consider three students in a class, and their marks in exam was 40, 50, 60 and 70. The standard deviation of this data set is _____. (Upto 2 decimal places)

11.18 [11.10 - 11.19] Correct Option

Solution :
11.18 [11.10 - 11.19]

Student	(x_i) Marks	x_i^2
A	40	1600
B	50	2500
C	60	3600
D	70	4900
	220	12600

Here, $n = 4$

$$\text{Standard Deviation (s)} = \sqrt{\frac{n\Sigma x_i^2 - (\Sigma x_i)^2}{n^2}}$$

$$= \sqrt{\frac{4 \times 12600 - (220)^2}{4^2}} = 11.180$$

QUESTION ANALYTICS +

Q. 9

Solution Video | Have any Doubt ?

100 dices are thrown. How many are expected to fall either a 3 or 6 _____. (Upto 2 decimal places)

33.33 (33.30 - 33.39) — Correct Option

Solution :
33.33 (33.30 - 33.39)
The event can be considered as Binomial distribution

$$E(X) = np$$
$$n = 100$$
$$p = p(3)+p(6)=\frac{1}{6}+\frac{1}{6}=\frac{1}{3}$$
$$E(X) = \frac{100}{3}=33.33$$

33.33 fall either a 6 or 3.

QUESTION ANALYTICS +

Q. 10

Solution Video | Have any Doubt ?

Consider the following factorization of a matrix A.

$A = LU$, where $L=\begin{bmatrix}1&0&0\\4&1&0\\7&2&1\end{bmatrix}_{3\times3}$, $U=\begin{bmatrix}a&b&c\\0&d&e\\0&0&f\end{bmatrix}_{3\times3}$ and $A=\begin{bmatrix}a_{11}&a_{12}&a_{13}\\a_{21}&a_{22}&a_{23}\\a_{31}&a_{32}&a_{33}\end{bmatrix}_{3\times3}$

If $i = 1$ then $a_{ij} = j$ otherwise $a_{ij} = 3 + a_{(i-1)j}$. Find the matrix U.

A $\begin{bmatrix}1&2&3\\0&-1&-2\\0&0&0\end{bmatrix}$

B $\begin{bmatrix}1&2&-6\\0&-3&3\\0&0&0\end{bmatrix}$

C $\begin{bmatrix}1&2&3\\0&-3&-6\\0&0&0\end{bmatrix}$ — Correct Option

Solution :
(c)

$$A = LU$$

Given, $L=\begin{bmatrix}1&0&0\\4&1&0\\7&2&1\end{bmatrix}$

$\therefore A=\begin{bmatrix}a_{11}&a_{12}&a_{13}\\a_{21}&a_{22}&a_{23}\\a_{31}&a_{32}&a_{33}\end{bmatrix}$

$a_{ij} = j$, if $i = 1$
$= 3 + a_{(i-1)j}$; otherwise

$\therefore A=\begin{bmatrix}1&2&3\\4&5&6\\7&8&9\end{bmatrix}$

$$A = LU$$

$\Rightarrow \begin{bmatrix}1&2&3\\4&5&6\\7&8&9\end{bmatrix}=\begin{bmatrix}1&0&0\\4&1&0\\7&2&1\end{bmatrix}\begin{bmatrix}a&b&c\\0&d&e\\0&0&f\end{bmatrix}$

(i) $a = 1$, (ii) $b = 2$ (iii) $c = 3$
(iv) $4b + d = 5 \Rightarrow 4 \times 2 + d = 5 \Rightarrow d = -3$
(v) $4c + e = 6 \Rightarrow 4 \times 3 + e = 6 \Rightarrow e = -6$
(vi) $7c + 2e + f = 9 \Rightarrow 7 \times 3 + 2 \times (-6) + f = 9 \Rightarrow f = 0$

$\therefore U=\begin{bmatrix}1&2&3\\0&-3&-6\\0&0&0\end{bmatrix}$

D $\begin{bmatrix}1&2&-6\\0&-1&-6\\0&0&0\end{bmatrix}$

QUESTION ANALYTICS +

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OVERALL ANALYSIS | COMPARISON REPORT | SOLUTION REPORT

ALL(17) | CORRECT(0) | INCORRECT(0) | SKIPPED(17)

Q. 11 Solution Video | Have any Doubt ?

If the proportion of handicapped people in a large population is 0.006, then what is the probability that there will be atmost one handicapped person in a randomly chosen group of 500 people? Use Poisson approximation to compute the probability.

A $3e^{-4}$

B $2e^{-3}$

C $4e^{-3}$ Correct Option

Solution :
(c)

$$P(x = r) = \frac{e^{-\lambda} \cdot \lambda^r}{r!}, \text{ where } \lambda = np$$

$$\lambda = 500 \times 0.006 = 3$$

$$P(x \le 1) = P(x = 0) + P(x = 1)$$

$$= \frac{e^{-\lambda} \cdot \lambda^0}{0!} + \frac{e^{-\lambda} \cdot \lambda^1}{1!}$$

$$= e^{-3} + e^{-3} \cdot 3 = 4e^{-3}$$

D $3e^{-2}$

QUESTION ANALYTICS +

Q. 12 Solution Video | Have any Doubt ?

E[X] = 1 and Var[X] = 2, which one of the following is not correct?

A E[6 X] = 6

B Var[6 X] = 72

C E[1 – X] = 0

D Var[1 – X] = 3 Correct Option

Solution :
(d)

$$E[6X] = 6.E[X] = 6$$

$$Var[6X] = 6^2 Var[X] = 36 \times 2 = 72$$

$$E[1 - X] = 1 + (-1) E[X] = 1 - 1 = 0$$

$$Var[1 - X] = (-1)^2 Var[X] = Var[X] = 2$$

$$\therefore \quad Var[1 - X] \neq 3$$

QUESTION ANALYTICS +

Q. 13 Have any Doubt ?

Which of the following is/are true?
S_1 : Maximum number of distinct eigen values of matrix A less than size of matrix A.
S_2 : Product of eigen values of matrix A is equal to Sum of diagonal elements of A.
S_3 : The characteristic roots (eigen values) of Hermitian matrix are real.

A Only S_2 and S_3

B Only S_1 and S_2

C Only S_3 Correct Option

Solution :
(c)
Maximum number of distinct eigen values = Size of matrix A.
$\therefore$ S_1 is False.
Sum of eigen values = Sum of diagonal elements.
$\therefore$ S_2 is False.

D Only S_1

QUESTION ANALYTICS +

Q. 14 Solution Video | Have any Doubt ?

Find an eigen vector corresponding to largest eigen value of matrix $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$

A $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ Correct Option

Solution :
(a)

$$|\lambda - AI| = (1-\lambda)(\lambda^2 - 2) + (2-\lambda) - \lambda = -\lambda^3 + \lambda^2$$
$$\Rightarrow \quad -\lambda^3 + \lambda^2 = 0$$
$$\Rightarrow \quad -\lambda^2(\lambda - 1) = 0$$
$$\lambda = 0, \lambda = 1$$

The largest eigen value is 1

$$A - I = \begin{bmatrix} 0 & -1 & 1 \\ 1 & -2 & 1 \\ -1 & 1 & 0 \end{bmatrix}_{R_1 \leftrightarrow R_2}$$
$$\Rightarrow \quad \begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ -1 & 1 & 0 \end{bmatrix}_{R_3 \leftarrow R_3 + R_1}$$
$$\Rightarrow \quad \begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ 0 & -1 & 1 \end{bmatrix}_{R_3 \leftarrow R_3 - R_2}$$
$$\Rightarrow \quad \begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}_{R_1 \leftarrow R_1 - 2R_2}$$
$$\Rightarrow \quad \begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$
$$[A - I]\vec{x} = 0$$
$$x_1 - x_3 = 0 \Rightarrow x_1 = x_3,$$
$$-x_2 + x_3 = 0 \Rightarrow x_2 = x_3$$
$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} x_3$$
$$\therefore \quad x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \text{ is an eigen vector.}$$

B $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

C $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$

D $\begin{bmatrix} \frac{1}{2} \\ 1 \\ 0 \end{bmatrix}$

QUESTION ANALYTICS +

Q. 15 Solution Video | Have any Doubt ?

The following system of homogeneous equations:

$$2x + y + 2z = 0$$
$$x + y + 3z = 0$$
$$4x + 3y + bz = 0$$

has non-trivial solution, then the value of 'b' is _____.

5 Correct Option

Solution :
8

For a non-trivial solution of homogeneous system of equations,

$$|A| = 0$$

where $A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 1 & 3 \\ 4 & 3 & b \end{bmatrix}$

$$2\begin{vmatrix} 1 & 3 \\ 3 & b \end{vmatrix} - 1\begin{vmatrix} 1 & 3 \\ 4 & b \end{vmatrix} + 2\begin{vmatrix} 1 & 1 \\ 4 & 3 \end{vmatrix} = 0$$

$\Rightarrow \quad 2\begin{vmatrix} 3 & b \end{vmatrix} - 1\begin{vmatrix} 4 & b \end{vmatrix} + 2\begin{vmatrix} 4 & 3 \end{vmatrix} = 0$

$\Rightarrow \quad b = 8$

QUESTION ANALYTICS +

Q. 16 Solution Video | Have any Doubt ?

Mean and variance of the random variable x having binomial distribution are 4 and 2 respectively then $P(x = 1)$ is _____. (Upto 3 decimal places)

0.03125 [0.031 – 0.032] Correct Option

Solution :
0.03125 [0.031 – 0.032]

Given, $np = 4$

$npq = 2$

$q = \frac{1}{2}, p = \frac{1}{2}, n = 8$

$P(x = 1) = {}^{n}C_1 p^1 q^{n-1}$

$= {}^{8}C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^7 = \frac{8}{2^8} = \frac{1}{2^5} = \frac{1}{32}$

$= 0.03125$

QUESTION ANALYTICS +

Q. 17 Solution Video | Have any Doubt ?

Find the value of x in the matrix A^{-1} for the following matrix A and A^{-1}.

$$A = \begin{pmatrix} 3 & 0 & 2 \\ 0 & 2 & 4 \\ 0 & 0 & 1 \end{pmatrix} \text{ and } A^{-1} = \begin{pmatrix} \frac{1}{3} & 0 & \frac{-2}{3} \\ 0 & \frac{1}{2} & x \\ 0 & 0 & 1 \end{pmatrix}$$

-2 Correct Option

Solution :
-2

$$(A|I) = \left(\begin{array}{ccc|ccc} 3 & 0 & 2 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) R_1 \leftarrow R_1 - 2R_3 \text{ and } R_2 \leftarrow R_2 - 4R_3$$

$$\Rightarrow \left(\begin{array}{ccc|ccc} 3 & 0 & 0 & 1 & 0 & -2 \\ 0 & 2 & 0 & 0 & 1 & -4 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) R_1 \leftarrow R_1/3 \text{ and } R_2 \leftarrow R_2/2$$

$$\Rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{3} & 0 & \frac{-2}{3} \\ 0 & 1 & 0 & 0 & \frac{1}{2} & -2 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) = (I|A^{-1})$$

$$\therefore \quad A^{-1} = \begin{pmatrix} \frac{1}{3} & 0 & \frac{-2}{3} \\ 0 & \frac{1}{2} & -2 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow x = -2$$

QUESTION ANALYTICS +

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Q. 1 Solution Video | Have any Doubt ?

At the point $x = 1$, the function $f(x) = \begin{cases} x^3 - 1; & 1 < x < \infty \\ x - 1; & -\infty < x \le 1 \end{cases}$ is

A Continuous and differentiable

B Continuous and not differentiable — Correct Option

Solution :
(b)

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (x - 1) = 0$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (x^3 - 1) = 0$$

Also $f(1) = 0$

Thus $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$

$\Rightarrow$ f is continuous at $x = 1$

And Lf'(1) = 2, Rf'(1) = 1

$\Rightarrow$ f is not differentiable at $x = 1$

C Discontinuous and differentiable

D Discontinuous and not differentiable

QUESTION ANALYTICS +

Q. 2 Solution Video | Have any Doubt ?

Consider the following function.

$f(x) = \sqrt{36 - 4x^2}$

Find the points at which f has absolute minimum and absolute maximum respectively.

A $x = 0, x = 6$

B $x = 6, x = 0$

C $x = 0, x = 3$

D $x = 3, x = 0$ — Correct Option

Solution :
(d)

$$f(x) = \sqrt{36 - 4x^2}$$

At $x = 0$, $f(x) = 6$

If $x \neq 0 \Rightarrow f(x) < 6$

$\therefore$ f has absolute maximum at $x = 0$

At $x = 3$, $f(x) = 0$

$\therefore$ f has absolute minimum at $x = 3$.

QUESTION ANALYTICS +

Q. 3 Solution Video | Have any Doubt ?

Which of the following represents the LU decomposition of the given matrix. (Using Crout's method)

$A = \begin{bmatrix} 2 & 4 \\ 6 & 3 \end{bmatrix}$

A $L = \begin{bmatrix} 1 & 0 \\ 6 & 1 \end{bmatrix}$ $U = \begin{bmatrix} 2 & 2 \\ 0 & -9 \end{bmatrix}$

B $L = \begin{bmatrix} 1 & 0 \\ 6 & -9 \end{bmatrix}$ $U = \begin{bmatrix} 2 & 2 \\ 0 & 1 \end{bmatrix}$

C $L=\begin{bmatrix}2 & 0\\ 6 & -9\end{bmatrix}\ U=\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}$ Correct Option

Solution :
(c)
Using Crout's method

$$A = \begin{bmatrix}l_{11} & 0\\ l_{21} & l_{22}\end{bmatrix}\begin{bmatrix}1 & u_{12}\\ 0 & 1\end{bmatrix}$$

$$\begin{bmatrix}2 & 4\\ 6 & 3\end{bmatrix} = \begin{bmatrix}l_{11} & 0\\ l_{21} & l_{22}\end{bmatrix}\begin{bmatrix}1 & u_{12}\\ 0 & 1\end{bmatrix}$$

$l_{11} = 2 \qquad l_{11}u_{12} = 4$

$u_{12} = \frac{4}{2} = 2$

$l_{21} = 6 \qquad l_{21}u_{12} + l_{22} = 3$

$6 \times 2 + l_{22} = 3$

$l_{22} = 3 - 12$

$l_{22} = -9$

So, LU decomposition of given matrix is

$$L = \begin{bmatrix}2 & 0\\ 6 & -9\end{bmatrix} \qquad U = \begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}$$

Note: Candidates can use options to solve such questions.

D $L=\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}\ U=\begin{bmatrix}2 & 0\\ 6 & 9\end{bmatrix}$

QUESTION ANALYTICS +

Q. 4 Solution Video Have any Doubt ?

Consider $f(x) = 2x^3 - 15x^2 + 36x + 1$, then which of the following represents the function is increasing in the interval ?

A] 2, 3 [

B] − ∞, 3 [

C] − ∞, 2 [∪] 3, ∞ [Correct Option

Solution :
(c)
Given function, $f(x) = 2x^3 - 15x^2 + 36x + 1$

$f'(x) = 6x^2 - 30x + 36$

$= 6(x - 2)(x - 3)$

Hence, $f'(x) > 0$ when $x < 2$ and $x > 3$. So $f(x)$ is increasing in] − ∞, 2 [∪] 3, ∞ [

D None of these

QUESTION ANALYTICS +

Q. 5 Solution Video Have any Doubt ?

Consider the function

$$f(x) = \begin{cases}x^4, & x^2 < 1\\ x, & x^2 \geq 1\end{cases}$$

Then which of the following is true

A $f(x)$ is continuous but not differentiable at $x = -1$.

B $f(x)$ is neither continuous nor differentiable at $x = -1$. Correct Option

Solution :
(b)

$$f(x) = \begin{cases}x^4 & x^2 < 1 \quad \text{or} \quad -1 < x < 1\\ x & x^2 \geq 1 \quad \text{or} \quad x \geq 1 \text{ or } x \leq -1\end{cases}$$

$f(-1) = x|_{x=-1} = -1$

$f(-1^-) = x|_{x=-1} = -1$

$f(-1^+) = x^4|_{x=-1} = 1$

At $x = -1$

Limit does not exists ⇒ $f(x)$ is not continuous

⇒ $f'(-1)$ does not exist.

C $f(x)$ is continuous and differentiable at $x = -1$.

D $f(x)$ is not continuous but differentiable at $x = -1$.

QUESTION ANALYTICS +

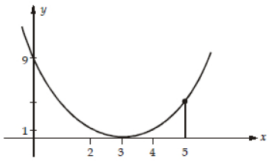
Q. 6 Solution Video | Have any Doubt ?

Consider the function $y = x^2 - 6x + 9$. The maximum value of y obtained when x varies over the interval 2 to 5 will be at _____.

5 — Correct Option

Solution :
5

$$y = x^2 - 6x + 9 = (x - 3)^2$$
$$y(2) = 1$$
$$y(5) = 4$$



$\therefore$ Maximum value of y over the interval 2 to 5 will be at $x = 5$.

QUESTION ANALYTICS +

Q. 7 Solution Video | Have any Doubt ?

The value of $\int_{-2}^{2} |1 - x^4| dx$ is _____.

12 — Correct Option

Solution :
12

$$I = \int_{-2}^{2} |1 - x^4| dx$$

The given function is an even function i.e., $f(x) = f(-x)$

$$\Rightarrow \quad I = 2\int_{0}^{2} |1 - x^4| dx$$
$$= 2\left\{\int_0^1 (1 - x^4) dx + \int_1^2 (x^4 - 1) dx\right\}$$
$$= 2\left\{\left[x - \frac{x^5}{5}\right]_0^1 + \left[\frac{x^5}{5} - x\right]_1^2\right\} = 12$$

QUESTION ANALYTICS +

Q. 8 Solution Video | Have any Doubt ?

Consider the given matrix:

$$\begin{vmatrix} 6 & 3 & 7 \\ 32 & 13 & 37 \\ 10 & 4 & 11 \end{vmatrix}$$

The value of the determinant of the above matrix is _____.

10 — Correct Option

Solution :
10

$$6(13 \times 11 - 4 \times 37) - 3(32 \times 11 - 10 \times 37) + 7(32 \times 4 - 10 \times 13)$$
$$= -30 + 54 - 14 = 10$$

QUESTION ANALYTICS +

Q. 9 Solution Video | Have any Doubt ?

The absolute maximum value of the function $f(x, y) = 2 + 2x + 2y - x^2 - y^2$ is _____.

4 — Correct Option

Solution :
4

$$\frac{\partial f}{\partial} = 2 - 2x \qquad \frac{\partial f}{\partial} = 2 - 2y$$

∂x $\qquad\qquad$ ∂y

$$r = \frac{\partial^2 f}{\partial x^2} = -2 \qquad t = \frac{\partial^2 f}{\partial y^2} = -2, \qquad s = \frac{\partial^2 f}{\partial x \partial y} = 0$$

Finding stationary points,

$$\frac{\partial f}{\partial x} = 2 - 2x = 0$$

$\Rightarrow x = 1$

$$\frac{\partial f}{\partial y} = 2 - 2y = 0$$

$\Rightarrow y = 1$

At the stationary point (1, 1)

$$rt - s^2 = (-2)(-2) - 0 = 4 > 0$$

So, $f(x, y)$ is maxima at (1, 1)

Maximum value of $f(x, y) = 2 + 2 + 2 - 1 - 1 = 4$

QUESTION ANALYTICS +

Q. 10

Solution Video | Have any Doubt ?

If $\lim_{x \to 1} \frac{x^x - x}{x - 1 - \log x} = A$, then A is _____.

A 0

B 1

C 2 — Correct Option

Solution :

(c)

$$\lim_{x \to 1} \frac{x^x - x}{x - 1 - \log x} \qquad \left(\frac{0}{0} \text{ form}\right)$$

$$= \lim_{x \to 1} \frac{\frac{d}{dx}(x^x) - 1}{1 - 0 - \frac{1}{x}}$$

Let, $y = x^x$

$\log y = x \log x$

$\therefore \quad \frac{1}{y}\frac{dy}{dx} = x \cdot \frac{1}{x} + 1 \cdot \log x$

or $\quad \frac{d}{dx}(x^x) = x^x(1 + \log x)$

$$= \lim_{x \to 1} \frac{x^x(1 + \log x) - 1}{1 - \frac{1}{x}} \quad \left(\frac{0}{0} \text{ form}\right)$$

$$= \lim_{x \to 1} \frac{\frac{d}{dx}(x^x) \cdot (1 + \log x) + x^x\left(\frac{1}{x}\right) - 0}{\frac{1}{x^2}}$$

$$= \lim_{x \to 1} \frac{x^x(1 + \log x)^2 + x^x\left(\frac{1}{x}\right)}{x^{-2}} = \frac{1(1+0)^2 + 1 \cdot 1}{1} = 2$$

D Limit does not exists

QUESTION ANALYTICS +

Kunal Jha
Course: GATE
Computer Science Engineering(CS)

TOPICWISE : ENGINEERING MATHEMATICS-2 (GATE - 2020) - REPORTS

OVERALL ANALYSIS | COMPARISON REPORT | SOLUTION REPORT

ALL(17) | CORRECT(0) | INCORRECT(0) | SKIPPED(17)

Q. 11 Solution Video | Have any Doubt ?

The value of integral $\int_1^3 \frac{|x-2|}{x}dx$ is _____.

A $2\ln\frac{4}{3}$ — Correct Option

Solution :
(a)

$$|x-2| = \begin{cases} -(x-2); & x<2 \\ (x-2); & x>2 \end{cases}$$

$$\int_1^3 \frac{|x-2|}{x}dx = \int_1^2 \frac{-(x-2)}{x}dx + \int_2^3 \frac{x-2}{x}dx$$

$$= \int_1^2 \left(-1+\frac{2}{x}\right)dx + \int_2^3 \left(1-\frac{2}{x}\right)dx$$

$$= [-x]_1^2 + [2\ln x]_1^2 + [x]_2^3 - 2[\ln x]_2^3$$

$$= -(2-1) + 2\ln 2 - 2\ln\frac{3}{2} + (3-2)$$

$$= 2\ln 2 - 2\ln\frac{3}{2}$$

$$= 2\ln\frac{2}{\frac{3}{2}} = 2\ln\frac{4}{3}$$

B $2\ln\frac{3}{4}$

C $\ln\frac{4}{3}$

D $4\ln\frac{2}{3}$

QUESTION ANALYTICS +

Q. 12 Solution Video | Have any Doubt ?

Evaluate $\int \frac{\sin\sqrt{x}}{\sqrt{x}}dx$

A $-\cos\sqrt{x}+c$

B $-2\cos(x)^{3/2}+c$

C $-2\sin\sqrt{x}+c$

D $-2\cos\sqrt{x}+c$ — Correct Option

Solution :
(d)

Let $u = \sqrt{x}$

Then $du = \frac{1}{2\sqrt{x}}dx$

$\therefore dx = du \cdot 2\sqrt{x}$

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}dx = \int \frac{\sin u}{\sqrt{x}} \cdot 2\sqrt{x}\,du = 2\int \sin u\,du$$

$$= -2\cos\sqrt{x}+c$$

QUESTION ANALYTICS +

Q. 13

Solution Video | Have any Doubt ?

Which of the following represents the LU decomposition of the given matrix. (Using Doolittle method)

$$A=\begin{bmatrix}25 & 5 & 1\\ 64 & 8 & 1\\ 144 & 12 & 1\end{bmatrix}$$

A $L=\begin{bmatrix}1 & 0 & 0\\ 2.56 & 1 & 0\\ 5.76 & 3.5 & 1\end{bmatrix} U=\begin{bmatrix}25 & 5 & 1\\ 0 & -4.8 & -1.56\\ 0 & 0 & 0.7\end{bmatrix}$

Correct Option

Solution :
(a)

Using Doolittle method:

$$A = LU$$

$$\begin{bmatrix}25 & 5 & 1\\ 64 & 8 & 1\\ 144 & 12 & 1\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\ l_{21} & 1 & 0\\ l_{31} & l_{32} & 1\end{bmatrix}\begin{bmatrix}u_{11} & u_{12} & u_{13}\\ 0 & u_{22} & u_{23}\\ 0 & 0 & u_{33}\end{bmatrix}$$

$u_{11}=25,\ u_{12}=5,\ u_{13}=1$

$$u_{11}\,l_{21} = 64$$

$$l_{21} = \frac{64}{25}=2.56$$

$$l_{21}u_{12}+u_{22} = 8$$
$$2.56\,u_{12}+u_{22} = 8$$
$$u_{22} = -4.8$$
$$u_{13}\,l_{21}+u_{23} = 1$$
$$u_{23} = -1.56$$
$$l_{31}\,u_{11} = 144$$

$$l_{31} = \frac{144}{25}=5.76$$

$$l_{31}u_{12}+l_{32}u_{22} = 12$$
$$(5.76 * 5)+(u_{22}\,l_{32}) = 12$$
$$l_{32} = 3.5$$
$$l_{31}\,u_{13}+l_{32}\,u_{23}+u_{33} = 1$$
$$u_{33} = 0.7$$

So, LU decomposition is

$$L=\begin{bmatrix}1 & 0 & 0\\ 2.56 & 1 & 0\\ 5.76 & 3.5 & 1\end{bmatrix}\quad U=\begin{bmatrix}25 & 5 & 1\\ 0 & -4.8 & -1.56\\ 0 & 0 & 0.7\end{bmatrix}$$

B $L=\begin{bmatrix}25 & 0 & 0\\ 2.56 & 1 & 0\\ 5.76 & 3.5 & 0.7\end{bmatrix} U=\begin{bmatrix}1 & 5 & 1\\ 0 & -4.8 & -1.56\\ 0 & 0 & 1\end{bmatrix}$

C $L=\begin{bmatrix}1 & 0 & 0\\ 2.56 & 1 & 0\\ 5.76 & 3.5 & 1\end{bmatrix} U=\begin{bmatrix}25 & -5 & 1\\ 0 & 4.8 & 1.56\\ 0 & 0 & -0.7\end{bmatrix}$

D $L=\begin{bmatrix}25 & 0 & 0\\ 2.56 & -4.8 & 0\\ 5.76 & 3.5 & 0.7\end{bmatrix} U=\begin{bmatrix}1 & 5 & 1\\ 0 & 1 & -1.56\\ 0 & 0 & 1\end{bmatrix}$

QUESTION ANALYTICS +

Q. 14

Solution Video | Have any Doubt ?

What is the value of $\int_0^{\pi/2}\log(\tan x)\,dx$?

A $-2\pi\log 2$

B $-\pi\log 2$

C 1

D 0

Correct Option

Solution :
(d)

$$I = \int_0^{\pi/2}\log\left(\frac{\sin x}{\cos x}\right)dx$$

$$= \int_0^{\pi/2}[\log(\sin x)dx-\log(\cos x)dx]$$

$$= \int_0^{\pi/2}\log\sin\left(\frac{\pi}{2}-x\right)dx-\int_0^{\pi/2}\log(\cos x)dx$$

$$I = 0$$

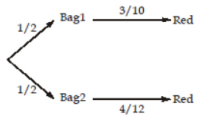
QUESTION ANALYTICS +

Q. 15 Solution Video | Have any Doubt ?

Suppose we have 2 bags. Bag 1 contains 3 red and 7 green balls. Bag 2 contains 4 red and 8 green balls. A person tosses a coin and if it is heads goes to bag 1 and draws a ball. If it is tails, he goes to bag 2 and draws a ball. Given that the ball draw is red, then what is probability that it came from bag 1? _____.
(Upto 2 decimal places)

0.317 [0.31 - 0.32] Correct Option

Solution :
0.317 [0.31 - 0.32]
The tree diagram for above problem, is shown below:



$$P(\text{bag1}\,|\,\text{Red}) = \frac{P(\text{bag1} \cap \text{Red})}{P(\text{Red})}$$

$$= \frac{\frac{1}{2}\times\frac{3}{10}}{\frac{1}{2}\times\frac{3}{10}+\frac{1}{2}\times\frac{1}{3}} = \frac{\frac{3}{20}}{\frac{3}{20}+\frac{1}{6}} = 0.317$$

QUESTION ANALYTICS +

Q. 16 Have any Doubt ?

A product is an assemble of 4 different components. The product can be sequentially assembled in two possible ways. If the 4 components are placed in a box and these are drawn at random from the box, then the probability of getting a correct sequence is _____. (Upto 3 decimal places)

0.083 [0.082 - 0.084] Correct Option

Solution :
0.083 [0.082 - 0.084]
The total number of ways of drawing the 4 components sequentially from the box = 4!.
There are only two possible ways the product can be assembled.

$\therefore$ The required probability = $\frac{2}{4!} = 0.083$

QUESTION ANALYTICS +

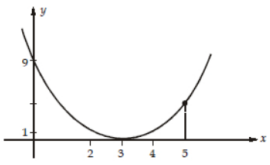
Q. 17 Have any Doubt ?

Consider the function $y = x^2 - 6x + 9$. The maximum value of y obtained when x varies over the interval 2 to 5 will be at _____.

5 Correct Option

Solution :
5

$$y = x^2 - 6x + 9 = (x-3)^2$$
$$y(2) = 1$$
$$y(5) = 4$$



$\therefore$ maximum value of y over the interval 2 to 5 will be at $x = 5$.

QUESTION ANALYTICS +

Kunal Jha
Course: GATE
Computer Science Engineering(CS)

SINGLE SUBJECT : ENGINEERING MATHEMATICS (GATE - 2020) - REPORTS

OVERALL ANALYSIS | COMPARISON REPORT | SOLUTION REPORT

ALL(33) | CORRECT(0) | INCORRECT(0) | SKIPPED(33)

Q. 1 Solution Video | Have any Doubt ?

Consider the function

$$f(x)=\begin{cases}x^4, & x^2<1\\ x, & x^2\geq 1\end{cases}$$

Then which of the following is true

A $f(x)$ is continuous but not differentiable at $x = -1$.

B $f(x)$ is neither continuous nor differentiable at $x = -1$. Correct Option

Solution :
(b)

$$f(x) = \begin{cases}x^4 & x^2<1 \text{ or } -1<x<1\\ x & x^2\geq 1 \text{ or } x\geq 1 \text{ or } x\leq -1\end{cases}$$

$$f(-1) = x|_{x=-1} = -1$$

$$f(-1^-) = x|_{x=-1} = -1$$

$$f(-1^+) = x^4|_{x=-1} = 1$$

At $x = -1$
Limit does not exists $\Rightarrow f(x)$ is not continuous
$\Rightarrow f'(-1)$ does not exist.

C $f(x)$ is continuous and differentiable at $x = -1$.

D $f(x)$ is not continuous but differentiable at $x = -1$.

QUESTION ANALYTICS +

Q. 2 Solution Video | Have any Doubt ?

Consider a matrix $[A] = \begin{bmatrix}a & 5\\ -3 & b\end{bmatrix}$. If the two eigen value are 1, 3. Then the number of values that a can take is

A 1

B 2 Correct Option

Solution :
(b)
Characteristic equation $|A - \lambda I| = 0$

$$\begin{bmatrix}a-\lambda & 5\\ -3 & b-\lambda\end{bmatrix} = \lambda^2 - (a+b)\lambda + ab + 15 = 0$$

also, from eigen values 1, 3
characteristic equation is $(\lambda - 1)(\lambda - 3) = 0$

$$\lambda^2 - 4\lambda + 3 = 0$$

on comparison

$$a + b = 4$$

$$ab + 15 = 3$$

$$\Rightarrow \quad ab = -12$$

$$a - \frac{12}{a} = 4$$

$$\Rightarrow \quad a = 6 \text{ or } -2$$

C 3

D 4

QUESTION ANALYTICS +

Q. 3 Solution Video | Have any Doubt ?

A function $f(x)$ is defined as

$$\begin{cases}\pi\frac{\sin x}{} & x\neq 0\end{cases}$$

$f(x)=\begin{cases} x \\ \frac{22}{7} \end{cases} \quad x=0$

then $f(x)$ as $x = 0$ is

A Neither continuous nor differentiable — Correct Option

Solution :
(a)

LHL $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{\pi \sin x}{x} \to \left[\frac{0}{0}\text{form}\right]$

Applying L Hospital rule,

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \pi \cos x = \pi$ .....(1)

Similarly,

RHL $\lim_{x\to 0^+} f(x) = \pi$ .....(2)

Also,

RHL $f(0) = \frac{22}{7}$ .....(3)

So, from (1), (2) and (3)

LHL = RHL ≠ functional value

$\Rightarrow$ $f(x)$ is not continuous at $x = 0$

Note : $\pi \neq \frac{22}{7}$. It is approximated to $\frac{22}{7}$.

B Continuous but not differentiable

C Continuous and differentiable

D Not defined at $x = 0$

QUESTION ANALYTICS +

Q. 4 Solution Video | Have any Doubt ?

The value of $\lim_{x\to 0} \frac{e^{ax}-e^{-ax}}{\log(1+bx)}$ is

A $\frac{a}{b}$

B $\frac{2a}{b}$ — Correct Option

Solution :
(b)

$$\lim_{x\to 0} \frac{e^{ax}-e^{-ax}}{\log(1+bx)} = \lim_{x\to 0} \frac{(e^{ax}-e^{-ax})\times 2ax\times b}{2ax\times b\times \log(1+bx)}$$

$$= \lim_{x\to 0}\left(\frac{e^{ax}-e^{-ax}}{2ax}\right)\times \lim_{x\to 0}\frac{bx}{\log(1+bx)}\left(\frac{2a}{b}\right)$$

$$= \lim_{x\to 0}\left(\frac{\sinh ax}{ax}\right)\lim_{x\to 0}\frac{bx}{\log(1+bx)}\left(\frac{2a}{b}\right)$$

$$= 1\times 1\times \frac{2a}{b} = \frac{2a}{b}$$

C $\frac{b}{a}$

D Does not exists

QUESTION ANALYTICS +

Q. 5 Solution Video | Have any Doubt ?

The manufacturer knows that the TV's he make contain on an average 1% defective. He packs them in boxes of 100. What is the probability the box picked at random will contain 2 or more faulty TV's _____?

A $\frac{e-1}{e}$

B $\frac{e-2}{e}$ — Correct Option

Solution :
(b)

$\lambda = np = \frac{1}{100}\times 100 = 1$

$P(X > 2) = 1 - (P(X = 0) + P(X = 1))$

$P(X = 0) = \frac{e^{-\lambda}\cdot\lambda^0}{} = e^{-\lambda}$

$$P(X=1) = \frac{e^{-\lambda}\lambda^{1}}{1!} = e^{-\lambda}\cdot\lambda$$

$$P(X>2) = 1-e^{-1}(2) = \frac{1-2}{e} = \frac{e-2}{e}$$

C 0

D None of these

QUESTION ANALYTICS +

Q. 6 Solution Video | Have any Doubt ?

For a given matrix A of 3 × 3 order, find $|Adj(A)|$
Where,

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 5 & 1 \\ 1 & 4 & 3 \end{bmatrix}$$

A 8

B 32

C 64 Correct Option

Solution :
(c)

$$|Adj(A)| = |A|^{n-1} \quad \text{.....(1)}$$

Where n is order of A,
Now,

$$|A| = 1\begin{vmatrix} 5 & 1 \\ 4 & 3 \end{vmatrix} + 0 + 1.\begin{vmatrix} 2 & 1 \\ 5 & 1 \end{vmatrix}$$

$$\Rightarrow |A| = 11 - 3$$

$$\Rightarrow |A| = 8$$

Using (1),

$$|Adj(A)| = 8^{3-1} = 8^2 = 64$$

D 512

QUESTION ANALYTICS +

Q. 7 Solution Video | Have any Doubt ?

If $A_{2\times 3}$, $B_{3\times 4}$ and $C_{4\times 2}$ are multiplied to get ABC then minimum number of multiplications to get ABC is

A 32

B 36

C 40

D 44

QUESTION ANALYTICS +

Q. 8 Solution Video | Have any Doubt ?

Let $P = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$. Which of the following equations does the matrix P satisfy?

A $P^3 + 2P^2 - 15P - 6I = 0$

B $P^3 + 3P^2 + 5P - 6I = 0$

C $P^3 + 3P^2 - 15P - 6I = 0$

D $P^3 - 2P^2 - 5P + 6I = 0$ Correct Option

Solution :
(d)
Every matrix satisfies it's own characteristic equation.
$|P - \lambda I|$ gives the characteristic equation.

$$= \begin{vmatrix} 2-\lambda & -2 & 3 \\ 1 & 1-\lambda & 1 \\ 1 & 3 & -1-\lambda \end{vmatrix} = \lambda^3 - 2\lambda^2 - 5\lambda + 6 = 0$$

$\therefore$ Option (d) is correct.

QUESTION ANALYTICS +

Q. 9 Solution Video | Have any Doubt ?

The polynomial, $P(x) = x^5 + x + 2$, has

A All real roots

B 3 real and 2 complex roots

C 1 real and 4 complex roots — Correct Option

Solution :
(c)

$$P(x) = x^5 + x + 2$$

It has a real root at $x = -1$

$$\Rightarrow P(x) = (x^4 - x^3 + x^2 - x + 2)(x + 1)$$

Now, $x^4 - x^3 + x^2 + x + 2$ will give other 4 roots
To find roots,

$$\Rightarrow x^4 - x^3 + x^2 - x + 2 = 0$$
$$\Rightarrow x^3(x - 1) + x(x - 1) + 2 = 0$$
$$\Rightarrow x(x^2 + 1)(x - 1) + 2 = 0$$

In the above expression, $x^2 + 1$ is always positive. So, either 'x' or '$x - 1$' should be negative in order to satisfy the equation.
For $x > 1$, both (x) and $(x - 1)$ are positive and,
For $x < 0$, both (x) and $(x - 1)$ are negative
$\therefore$ x should lie within 0 and 1 in order to have real roots.
As $x \in (0, 1)$

$$\Rightarrow |x| < 1$$
$$\Rightarrow |x^2 + 1| < 2, |x| < 1 \text{ and } |x - 1| < 1$$

$\therefore$ The product of these three will be less than 2 and hence, no real value of 'x' can satisfy the equation

$$x^4 - x^3 + x^2 - x + 2 = 0$$

$\therefore$ The equation will have four imaginary roots apart from one real roots.

D None of these

QUESTION ANALYTICS +

Q. 10 Solution Video | Have any Doubt ?

Consider the function $f(x) = x + \ln x$ and f is differentiable on (1, e) and $f(x)$ is continuous on [1, e]. Determine the c value using mean value theorem.
[By computing $f'(c) = \frac{f(b) - f(a)}{b - a}$]

A e

B $e - 1$ — Correct Option

Solution :
(b)

$$f(x) = x + \ln x$$
$$f'(x) = 1 + \frac{1}{x} \quad \ldots (1)$$
$$f'(c) = \frac{f(b) - f(a)}{b - a} = \frac{f(e) - f(1)}{e - 1}$$
$$= \frac{e + \ln e - (1 + \ln 1)}{e - 1} = \frac{e + 1 - 1 + 0}{e - 1} = \frac{e}{e - 1}$$
$$\Rightarrow f'(c) = \frac{e}{e - 1} \text{ [from equation (1)]}$$
$$\Rightarrow 1 + \frac{1}{c} = \frac{e}{e - 1}$$
$$\Rightarrow \frac{1}{c} = \frac{e}{e - 1} - 1$$
$$\Rightarrow \frac{1}{c} = \frac{e - e + 1}{e - 1}$$
$$\Rightarrow c = e - 1$$

C $\frac{e}{e - 1}$

D $\frac{e-1}{e}$

QUESTION ANALYTICS +

Kunal Jha
Course: GATE
Computer Science Engineering(CS)

SINGLE SUBJECT : ENGINEERING MATHEMATICS (GATE - 2020) - REPORTS

Q. 11 Solution Video | Have any Doubt ?

A uniform random variable X is defined such that its probability density function is given by

$$f(x) = \begin{cases} \frac{1}{4} & \text{if } 2 < x < 6 \\ 0 & \text{else} \end{cases}$$

Find variance (X) if mean (X) = 4 _____. (Upto 2 decimal places)

1.33 [1.32 - 1.34] Correct Option

Solution :
1.33 [1.32 - 1.34]
For uniform random variable,

$$f(x) = \begin{cases} \frac{1}{\beta-\alpha} & \alpha < x < \beta \\ 0 & \text{else} \end{cases}$$

$$\text{Variance (X)} = \frac{(\beta-\alpha)^2}{12} = \frac{(6-2)^2}{12} = \frac{4}{3} = 1.33$$

QUESTION ANALYTICS +

Q. 12 Solution Video | Have any Doubt ?

The probability of getting exactly two & consecutive heads in five tosses is _____. (Upto 3 decimal places)

0.125 [0.125 - 0.125] Correct Option

Solution :
0.125 [0.125 - 0.125]
Favourable cases → HHTTT, THHTT, TTHHT, TTTHH → 4
Total cases → 2^5

$$\Rightarrow \quad \text{Probability} = \frac{4}{32} = \frac{1}{8} = 0.125$$

QUESTION ANALYTICS +

Q. 13 Solution Video | Have any Doubt ?

The following system of homogeneous equations

$$2x + y + 2z = 0$$
$$x + y + 3z = 0$$
$$4x + 3y + bz = 0$$

has non-trivial solution, then the value of '*b*' is _____.

8 Correct Option

Solution :
8
For a non-trivial solution of homogeneous system of equations,

$$|A| = 0$$

where

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 1 & 3 \\ 4 & 3 & b \end{bmatrix}$$

$$\Rightarrow \quad 2\begin{vmatrix} 1 & 3 \\ 3 & b \end{vmatrix} - 1\begin{vmatrix} 1 & 3 \\ 4 & b \end{vmatrix} + 2\begin{vmatrix} 1 & 1 \\ 4 & 3 \end{vmatrix} = 0$$

$$\Rightarrow \quad b = 8$$

QUESTION ANALYTICS +

Q. 14 Solution Video | Have any Doubt ?

A product is an assemble of 4 different components. The product can be sequentially assembled in two possible ways. If the 4 components are placed in a box and these are drawn at random from the box, then the probability of getting a correct sequence is _____. (Upto 3 decimal places)

0.083 [0.082 - 0.084] Correct Option

Solution :
0.083 [0.082 - 0.084]
The total number of ways of drawing the 4 components sequentially from the box = 4!
There are only two possible ways the product can be assembled.

$\therefore$ The required probability $= \frac{2}{4!} = 0.083$

QUESTION ANALYTICS +

Q. 15 Solution Video Have any Doubt ?

$\int \frac{\sin(\ln x)}{x} dx = -\frac{1}{a}\cos(\ln x) + c$ then value of a is _____.

1 Correct Option

Solution :
1
Let $\ln x = t$

Then $dt = \frac{1}{x}dx$

Hence, $I = \int \sin t \, dt = -\cos t + c = -\cos(\ln x) + c$

QUESTION ANALYTICS +

Q. 16 Solution Video Have any Doubt ?

The inverse of matrix, $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, is given by $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$. The sum of $a + b + c + d$ is _____.

0 Correct Option

Solution :
0

$|A| = 1 \times 4 - 2 \times 3 = -2$

$A^{-1} = \frac{1}{|A|}\text{adj}(A) = \left(\frac{1}{-2}\right)\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

$\therefore \quad a + b + c + d = \frac{4 - 2 - 3 + 1}{-2} = 0$

QUESTION ANALYTICS +

Q. 17 Solution Video Have any Doubt ?

An exam can be given maximum four times. The probability of selection in first attempt is $\frac{1}{24}$ and it increases by 50% for every next attempt. The probability of selection is_____. Given that individual can give only one successful attempt

A 0.2

B 0.3 Correct Option

Solution :
(b)
Let, P_1, P_2, P_3, P_4 be probability of selection in 1st, 2nd, 3rd and 4th attempt respectively,
Now,

$P_1 = \frac{1}{24}; \quad P_2 = \frac{1}{24}[1 + 0.5]$

$P_2 = \frac{1}{24} \times \frac{3}{2}$

$P_3 = \frac{1}{24} \times \frac{3}{2}[1 + 0.5] = \frac{1}{24} \times \left(\frac{3}{2}\right)^2$

$P_4 = \frac{1}{24} \times \left(\frac{3}{2}\right)^3$

Now let A_i be selection in on attempt and $\overline{A_i}$ be unsuccessful attempt,
So,

$P_{\text{selection}} = A_1 + \overline{A_1}A_2 + \overline{A_1}\,\overline{A_2}A_3 + \overline{A_1}\,\overline{A_2}\,\overline{A_3}A_4$

$= \frac{1}{24} + \frac{23}{24} \times \frac{1}{24} \times \frac{3}{2} + \frac{23}{24}\left(1 - \frac{3}{48}\right) \times \frac{1}{24} \times \left(\frac{3}{2}\right)^2 + \frac{23}{24}\left(1 - \frac{3}{48}\right)\left(1 - \frac{9}{96}\right) \times \frac{1}{24} \times \left[\frac{3}{2}\right]^3 \simeq 0.3$

C 0.4

D 0.5

QUESTION ANALYTICS +

Q. 18 Solution Video | Have any Doubt ?

If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then for every positive integer n, A^n is equal to

A $\begin{bmatrix} 1+2n & 4n \\ n & 1+2n \end{bmatrix}$

B $\begin{bmatrix} 3^n & (-4)^n \\ n & (-1)^n \end{bmatrix}$

C $\begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ Correct Option

Solution :
(c)

$$A^2 = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -8 \\ 2 & -3 \end{bmatrix}$$

D Undefined

QUESTION ANALYTICS +

Q. 19 Solution Video | Have any Doubt ?

If $P(X) = \frac{1}{2}$; $P(Y) = \frac{1}{3}$ and $P\left(\frac{X}{Y}\right) = \frac{1}{3}$ then $P(X \cap Y)$ is

A $\frac{1}{6}$

B $\frac{1}{9}$ Correct Option

Solution :
(b)

Since, $P\left(\frac{X}{Y}\right) = \frac{P(X \cap Y)}{P(Y)}$

$\Rightarrow$ $P(X \cap Y) = P\left(\frac{X}{Y}\right) \cdot P(Y) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

C $\frac{1}{12}$

D $\frac{1}{18}$

QUESTION ANALYTICS +

Q. 20 Have any Doubt ?

Determinant of matrix.

$$D = \begin{vmatrix} 2 & 3 & 4 & 5 \\ 1 & 4 & 4 & 5 \\ 1 & 3 & 5 & 5 \\ 1 & 3 & 4 & 6 \end{vmatrix}$$ is

A 20

B 14 Correct Option

Solution :
(b)

$$C_1 \to C_1 + C_2 + C_3 + C_4$$

$$\begin{vmatrix} 14 & 3 & 4 & 5 \\ 14 & 4 & 4 & 5 \end{vmatrix}$$

$\Rightarrow \quad D = \begin{vmatrix} 14 & 3 & 5 & 5 \\ 14 & 3 & 4 & 6 \end{vmatrix}$

$R_2 - R_1, R_3 - R_1, R_4 - R_1$

$$D = \begin{vmatrix} 14 & 3 & 4 & 5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$$\Rightarrow \quad D = 14 \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 14$$

C 10

D 8

QUESTION ANALYTICS +

Kunal Jha
Course: GATE
Computer Science Engineering(CS)

SINGLE SUBJECT : ENGINEERING MATHEMATICS (GATE - 2020) - REPORTS

OVERALL ANALYSIS | COMPARISON REPORT | SOLUTION REPORT

ALL(33) | CORRECT(0) | INCORRECT(0) | SKIPPED(33)

Q. 21 Solution Video | Have any Doubt ?

Number of tigers in a reserve is normally distributed with mean and variance respectively as 1200 and 9×10^4. The probability of finding more than 1800 tiger is approx.

A 0.0125

B 0.025 — Correct Option

Solution :
(b)

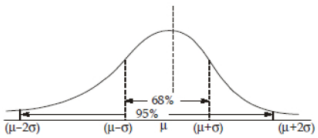
Given,

Mean, $\mu = 1200$

Variance, $\sigma^2 = 9 \times 10^4$

$\Rightarrow$ Standard deviation, $\sigma = \sqrt{9 \times 10^4} = 300$

Using Standard normal curve,



Probability of finding figures between

$(\mu - 2\sigma)$ and $(\mu + 2\sigma) = 0.95$

$\mu - 2\sigma = 1200 - 2 \times 300 = 600$

$\mu + 2\sigma = 1200 + 2 \times 300 = 1800$

i.e. $P(600 \le X \le 1800) = 0.95$

$\Rightarrow P(X \le 600) + P(X \ge 1800) = 0.05$

Since normal curve is symmetric w.r.t. mean value,

So, $P(X \le 600) = P(X \ge 1800)$

$\Rightarrow 2P(X \ge 1800) = 0.05$

$\Rightarrow P(X \ge 1800) = 0.025$

C 0.05

D None of these

QUESTION ANALYTICS +

Q. 22 Solution Video | Have any Doubt ?

A function $f(x)$ is defined as

$$f(x) = \begin{cases} 3x+4 & x<0 \\ -4 & x=0 \\ 4-6x & x>0 \end{cases}$$

then at $x = 0$ for $f(x)$

A Limit does not exists

B Limit exists but discontinuous — Correct Option

Solution :
(b)

For limit to exist, LHL = RHL

Now,

RHL $\lim_{x \to 0^+} f(x) = 4 - 6x = 4$

LHL $\lim_{x \to 0^-} f(x) = 3x + 4 = 4$

$\Rightarrow$ Limit exists

Now for continuity,

LHL = RHL = Functional value

At $x = 0$

$f(x) = -4 \neq$ RHL

$\Rightarrow$ Discontinuous

C Limit exists and is continuous

D Is not defined

QUESTION ANALYTICS +

Q. 23 Solution Video | Have any Doubt ?

If P, Q, R and S are non singular matrix of order three such that PQRS = I then R^{-1} is

A PQS

B QPS

C SPQ Correct Option

Solution :
(c)

Given, $PQRS = I$

$\Rightarrow PQRSS^{-1} = I \cdot S^{-1}$

$\Rightarrow PQR = S^{-1}$

$\Rightarrow PQRR^{-1} = S^{-1}R^{-1}$

$\Rightarrow PQ = S^{-1}R^{-1}$

$\Rightarrow SPQ = S \cdot S^{-1}R^{-1} = R^{-1}$

$\Rightarrow R^{-1} = SPQ$

D PSQ

QUESTION ANALYTICS +

Q. 24 Solution Video | Have any Doubt ?

Matrix A is expressed as $A = B + C$ where B and C are respectively symmetric and skew symmetric matrix then matrix B is

Given, $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 4 & 1 \\ 3 & 1 & 5 \end{bmatrix}$

A $\begin{bmatrix} 1 & 5/2 & 5/2 \\ 5 & 8 & 2 \\ 5 & 2 & 10 \end{bmatrix}$

B $\begin{bmatrix} 1 & 5/2 & 5/2 \\ 5/2 & 4 & 1 \\ 5/2 & 1 & 5 \end{bmatrix}$ Correct Option

Solution :
(b)

$$A = \frac{A+A^T}{2} + \frac{A-A^T}{2}$$

Where, $B = \frac{A+A^T}{2}$ is symmetric matrix

$C = \frac{A-A^T}{2}$ is skew symmetric matrix

Now, $A^T = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 1 \\ 2 & 1 & 5 \end{bmatrix}$

$$\Rightarrow B = \frac{A+A^T}{2} = \frac{1}{2}\begin{bmatrix} 2 & 5 & 5 \\ 5 & 8 & 2 \\ 5 & 2 & 10 \end{bmatrix} = \begin{bmatrix} 1 & 5/2 & 5/2 \\ 5/2 & 4 & 1 \\ 5/2 & 1 & 5 \end{bmatrix}$$

C $\begin{bmatrix} 2 & 5 & 5 \\ 5 & 8 & 2 \\ 5 & 2 & 10 \end{bmatrix}$

D $\begin{bmatrix} 1 & 5/2 & 5/2 \\ 5/2 & 4 & 1 \\ 5/2 & 1 & 10 \end{bmatrix}$

QUESTION ANALYTICS +

Q. 25 Solution Video | Have any Doubt ?

Which of the following matrices is not invertible?

A $A_1 = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$

B $A_2 = \begin{bmatrix} 1 & 0 & 4 \\ -1 & 1 & -1 \\ -1 & 0 & 3 \end{bmatrix}$

C $A_3 = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 5 & 2 \\ 4 & 7 & 4 \end{bmatrix}$ Correct Option

Solution :
(c)
The matrix which has 0 determinant will not be invertible.
Determinant of A_1, $|A_1| = 3 \times 2 - 4 \times 1 = 2$
Determinant of A_2, $|A_2| = 1[-3 - 0] + 0 + 4\,[0 + 1] = 1$
Determinant of A_3, $|A_3| = 1\,(20 - 14) - 3(8 - 8) + 1\,(14 - 20) = 0$
Determinant of A_4, $|A_4| = 2(0 - 2) - 3\,(6 - 3) + 1\,(3 - 0) = -4 - 9 + 3 = -10$

D $A_4 = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 0 & 1 \\ 3 & 1 & 2 \end{bmatrix}$

QUESTION ANALYTICS +

Q. 26 Solution Video | Have any Doubt ?

Consider A = $[a_{ij}]$, $1 \le i, j \le n$ where $n \ge 3$ and $a_{ij} = i * j$. Then which of the following represents the rank of A?

A 0

B 1 Correct Option

Solution :
(b)
Consider $n = 3$

Then $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$

and $|A| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{vmatrix}$ $R_3 \leftarrow 3R_1 - R_3$, $R_2 \leftarrow 2R_1 - R_2$

$= \begin{vmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}$

This if $n = 3$ then Rank $(A) = 1$.

C $n - 1$

D n

QUESTION ANALYTICS +

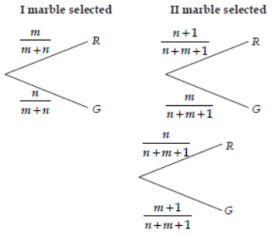
Q. 27 Solution Video | Have any Doubt ?

A Jar-1 contains m red and n green marbles Jar-2 contains n red and m green marbles. It a marble is taken from 1st Jar and put into 2nd Jar, and then a marble is randomly picked from 2nd Jar, what is the probability that the marble selected is red?

A $\frac{n}{m+n}$

B $\frac{m(n+1)+n^2}{(m+n)(m+n+1)}$ Correct Option

Solution :
(b)
The tree diagram for problem is



$$p(R) = \frac{m}{m+n} \times \frac{n \times 1}{n+m+1} + \frac{n}{m+n} \times \frac{n}{n+m+1}$$

$$= \frac{m(n+1)+n^2}{(m+n)(m+n+1)}$$

C $\frac{2mn}{(m+n)^2}$

D None of these

QUESTION ANALYTICS +

Q. 28 Solution Video Have any Doubt ?

There are 3 fair coins and 1 false coin with tails on both sides. A coin is chosen at random and tossed 4 times. What is the probability that the false coin has been chosen for tossing if 'tails' occurs in all 4 times?

A 0.44

B 0.54

C 0.84 Correct Option

Solution :
(c)

$$\text{Required probability} = \frac{\text{Favorable outcomes}}{\text{Total possible outcomes}}$$

$$\text{Favorable outcomes} = \text{A false coin is chosen and flipped every time}$$

$$\text{Probability of selecting a false coin} = \frac{1}{4}$$

Probability of getting a tail on every flip of false coin = 1.

$$\therefore \quad \text{Favorable outcomes} = \frac{1}{4}\times 1 = \frac{1}{4}$$

$$\text{Total possible outcomes} = \text{Favourable outcomes} + \text{Unfavourable outcomes}$$

$$\text{Unfavourable outcomes} = \text{A fair coin is chosen and flipped every time to get tail}$$

$$\text{Probability of selecting a fair coin} = \frac{3}{4}$$

Probability of flipping a fair coin 4 times and getting

$$\text{tails every time} = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$$

$$\therefore \quad \text{Unfavourable outcomes} = \frac{3}{4}\times\frac{1}{16} = \frac{3}{64}$$

$$\text{Total possible outcomes} = \frac{1}{4}+\frac{3}{64} = \frac{19}{64}$$

$$\therefore \quad \text{Required probability} = \frac{\frac{1}{4}}{\frac{19}{64}} = \frac{16}{19} = 0.84$$

D 0.48

QUESTION ANALYTICS +

Q. 29 Solution Video Have any Doubt ?

Let X be a continuous random variable with following probability density function.

$f(x) = 2x, x \in [0,1]$

$= 0, x \notin [0,1]$

Find the variance of X _____. (Upto 3 decimal places)

0.055 [0.053 - 0.056] Correct Option

Solution :
0.055 [0.053 - 0.056]

$$\text{Variance of X} = \text{Var[X]}$$

$$\text{Expectation} = \text{E[X]}$$

$$= \int_{-\infty}^{\infty} x\cdot f(x)\cdot dx = \int_0^1 x\cdot(2x)\cdot dx = 2\cdot\int_0^1 x^2\cdot dx = 2\left[\frac{x^3}{3}\right]_0^1 = \frac{2}{3}$$

$$\text{E}[X^2] = \int_{-\infty}^{\infty} x^2\cdot f(x)\cdot dx = \int_0^1 x^2\cdot(2x)\cdot dx = 2\cdot\int_0^1 x^3\cdot dx = 2\cdot\left[\frac{x^4}{4}\right]_0^1 = \frac{1}{2}$$

$$\therefore \quad \text{Var[X]} = \text{E}[X^2] - \text{E}[X]^2$$

$$= \frac{1}{2} - \left(\frac{2}{3}\right)^2 = \frac{1}{2} - \frac{4}{9} = \frac{1}{18}$$

QUESTION ANALYTICS +

Q. 30 Solution Video Have any Doubt ?

A 2 × 2 matrix is defined as $A = \begin{bmatrix} p & 2 \\ -1 & 0 \end{bmatrix}$. The matrix A satisfies the equation $A + 3I + qA^{-1} = 0$. The value of $\frac{p}{q}$ is _____. (Upto 1 decimal place)

-1.5 [-1.4 - -1.6] Correct Option

Solution :
-1.5 [-1.4 - -1.6]

$$A = \begin{bmatrix} p & 2 \\ -1 & 0 \end{bmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} p-\lambda & 2 \\ -1 & 0-\lambda \end{vmatrix} = 0$$

$$(p - \lambda)(-\lambda) + 2 = 0$$
$$\lambda^2 - p\lambda + 2 = 0$$

Since this is characteristic equation

$$A^2 - pA + 2I = 0$$

Multiplying by A^{-1}, we get

$$A - pI + 2A^{-1} = 0$$
$$A + 3I + qA^{-1} = 0 \quad \text{(given)}$$
$$p = -3$$
$$q = 2$$
$$\frac{p}{q} = -1.5$$

QUESTION ANALYTICS +

Kunal Jha
Course: GATE
Computer Science Engineering(CS)

SINGLE SUBJECT : ENGINEERING MATHEMATICS (GATE - 2020) - REPORTS

Q. 31

Solution Video | Have any Doubt ?

You purchase a certain product. The manual states that the life-time T of the product, defined as the amount of time (in years) the product works properly until it breaks down, satisfies

$P(T \geq t) = e^{-t/5}$, for all $t \geq 0$

You purchase the product and use it for two years without any problems. The probability that it breaks down in the third year is _____. (Upto 3 decimal places).

0.181 [0.180 - 0.185] — Correct Option

Solution :

0.181 [0.180 - 0.185]

Let be the event that a purchased product breaks down in the third year. Also, let B be the event that a purchased product does not break down in the first two years.

We are interested in $P(A|B)$.

We have $P(B) = P(T \geq 2) = e^{-2/5}$

We also have $P(A) = P(2 \leq T \leq 3) = P(T \geq 2) - P(T \geq 3) = e^{-2/5} - e^{-3/5}$

Finally, since $A \subset B$, we have $A \cap B = A$.

Therefore, $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)} = \frac{e^{-2/5} - e^{-3/5}}{e^{-2/5}} = 0.1813$

QUESTION ANALYTICS +

Q. 32

Solution Video | Have any Doubt ?

A random variable X has probability distribution shown below:

x:	-2	0	6	9
$P(X=x)$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{12}$	$\frac{5}{12}$

Then $E[(2X+3)^2]$ is _____. (Upto 2 decimal places)

205.67 [205.65 - 205.69] — Correct Option

Solution :

205.67 [205.65 - 205.69]

$E[(2X+3)^2] = 4E(X^2) + 9 + 12E(X)$ ....(1)

Now, $E(X) = -2 \times \frac{1}{6} + 0 \times \frac{1}{3} + 6 \times \frac{1}{12} + 9 \times \frac{5}{12}$

$= \frac{-1}{3} + \frac{1}{2} + \frac{15}{4} = \frac{47}{12}$ ....(2)

$E(X^2) = (-2)^2 \times \frac{1}{6} + 0^2 \times \frac{1}{3} + 6^2 \times \frac{1}{12} + 9^2 \times \frac{5}{12}$

$= \frac{4}{6} + \frac{36}{12} + \frac{405}{12} = \frac{449}{12}$ ....(3)

Using (1), (2) and (3)

$E[(2X+3)^2] = 4 \times \frac{449}{12} + 9 + 12 \times \frac{47}{12} = 205.667 = 205.67$

QUESTION ANALYTICS +

Q. 33

Solution Video | Have any Doubt ?

A 2 × 2 matrix M has eigen values 2 and 3 with eigen vectors $\begin{bmatrix}2\\1\end{bmatrix}$ and $\begin{bmatrix}1\\2\end{bmatrix}$ respectively. The sum of all elements of Matrix M is _____.

5 — Correct Option

Solution :

5

Let matrix, $M = \begin{bmatrix}p & q\\ r & s\end{bmatrix}$

Now for eigen value $\lambda_1 = 2$ and eigen vector $X_1 = \begin{bmatrix}2\\1\end{bmatrix}$,

$MX_1 = \lambda_1 X_1$

$\Rightarrow \begin{bmatrix}p & q\\ r & s\end{bmatrix}\begin{bmatrix}2\\1\end{bmatrix} = 2\begin{bmatrix}2\\1\end{bmatrix}$

$\Rightarrow \begin{bmatrix}2p+q\\ 2r+s\end{bmatrix} = \begin{bmatrix}4\\2\end{bmatrix}$ .....(1)

Also for eigen value λ_2 with eigen vector X_2,

$$MX_2 = \lambda_2 X_2$$

From (1) and (2) we have,

$$2p + q = 4 \; ; \; 2r + s = 2$$

$$p + 2q = 3 \; ; \; r + 2s = 6$$

$$\Rightarrow \quad 3p + 3q = 7$$

and $\quad 3r + 3s = 8$

$$\Rightarrow \quad 3(p + q + r + s) = 15$$

$$\Rightarrow \quad p + q + r + s = 5$$

QUESTION ANALYTICS +