Goodwin-Griffith-genetic-oscillator-model

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22 stycznia 2023

Kontrola genów Model zaproponowany przez Griffitha, w którym X to koncentracja pewnego białka proporcjonalna do aktywności opisywanego genu, a Y to koncentracja odpowiedniego mRNA,

$$\dot{X} = -\alpha X + Y$$

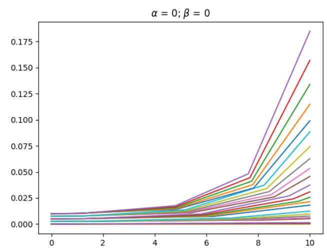
$$\dot{Y} = \frac{x^2}{1 + X^2} - \beta Y$$

1 Pkty stale

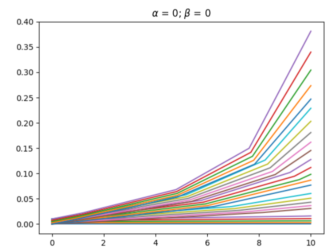
Analize naszego ukladu zaczynamy od analizy pktow stalych. Korzystajac z Wolframa mozemy doknac analizy naszych rownan.

1.1 Rowzionzania Trywialne

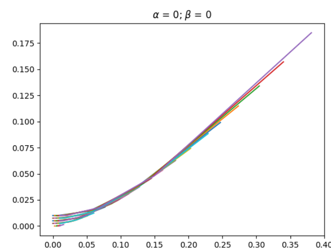
α i $\beta = 0$



Wykresy zachowania X



Wykresy zachowania Y



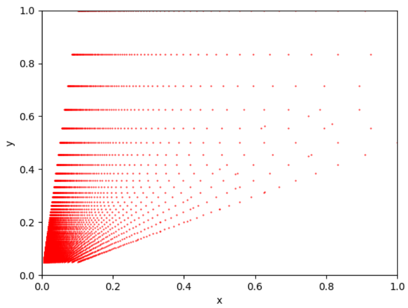
Wykres Fazowy

Mozemy zaobserwowac ze nasze rozwionzania tylko i wylocznie dla x i y = 0 wiec mozemy wysnuac wniosek ze jest to chwiejny pkt stacionarny postaramy sie potem to udowdonic kozystajac z analizy jakobianu.

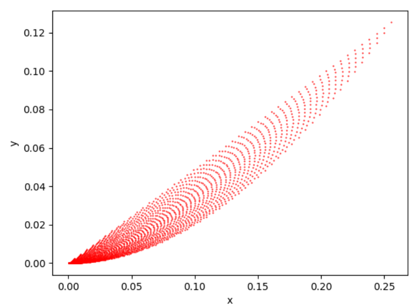
1.2 Rozwionzania nie trywialne

1.2.1 Przypadek pierwszy

$$X = -\frac{\sqrt{1-4\alpha^2\beta^2}-1}{2\beta} \qquad Y = -\frac{\sqrt{1-4\alpha^2\beta^2}-1}{2\beta}$$



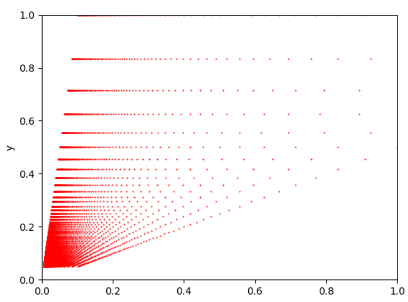
a i b in 0.5 do 10



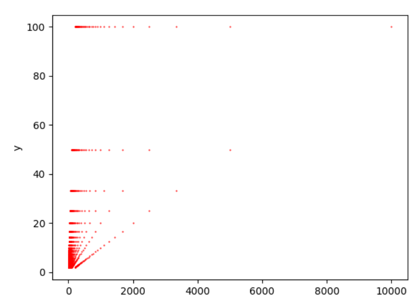
a i b in od 0.01 do 0.5

1.2.2 Przypadek drogi

$$X = \frac{\sqrt{1-4\alpha^2\beta^2}+1}{2\beta} \qquad Y = \frac{\sqrt{1-4\alpha^2\beta^2}+1}{2\beta}$$



a i b in 0.5 do 10



a i b in 0.01 do 0.5

1.3 Jakobian

$$\left\{ \begin{matrix} -\alpha & 1 \\ \frac{2x}{(x^2+1)^2} & \beta \end{matrix} \right\}$$

Potecialne pkty zerujace rownania:
$-\alpha X + Y = 0 \Rightarrow Y = \alpha X$
$\frac{x^2}{1+X^2} - \beta Y = 0 \Rightarrow \frac{x^2}{1+X^2} = \beta Y$
$\frac{x^2}{1+x^2} = \beta\alpha x$
rozwionzania:
trywialne x= 0
nie trywialne dla α i $\beta \neq 0$
$x = \frac{1-\sqrt{1-4\alpha^2\beta^2}}{2\alpha\beta}$
$x = \frac{\sqrt{1-4\alpha^2\beta^2}-1}{2\alpha\beta}$

1.3.1 Rozwionzania dla $x = \frac{1-\sqrt{1-4\alpha^2\beta^2}}{2\alpha\beta}$

$$\lambda_1 =$$

$(16\ \alpha^3\beta^2 + 16\alpha^2\beta^3 + 8\alpha\sqrt{1-4\alpha^2\beta^2} + 8\beta\sqrt{1-4\alpha^2\beta^2}$

-$\sqrt{(-16\alpha^3\beta^2 - 16\alpha^2\beta^3 - 8\alpha\sqrt{1-4\alpha^2\beta^2} - 8\beta\sqrt{1-4\alpha^2\beta^2} + 8\alpha + 8\beta)^2}$

$\overline{-4(1024\alpha^5\beta^5 - 768\alpha^3\beta^3 - 128\alpha\beta\sqrt{1-4\alpha^2\beta^2)} - 256\alpha^5\beta^5\sqrt{1-4\alpha^2\beta^2}}$

$\overline{+512\alpha^3\beta^3\sqrt{1-4\alpha^2\beta^2} + 128\alpha\beta))} - 8\alpha - 8\beta$

/(2 (2 - 2 $\sqrt{1-4\alpha^2\beta^2})^2$)

$$\lambda_2 =$$

$(16\ \alpha^3\beta^2 + 16\alpha^2\beta^3 + 8\alpha\sqrt{1-4\alpha^2\beta^2} + 8\beta\sqrt{1-4\alpha^2\beta^2}$

+ $\sqrt{(-16\alpha^3\beta^2 - 16\alpha^2\beta^3 - 8\alpha\sqrt{1-4\alpha^2\beta^2} - 8\beta\sqrt{1-4\alpha^2\beta^2} + 8\alpha + 8\beta)^2}$

$\overline{-4(1024\alpha^5\beta^5 - 768\alpha^3\beta^3 - 128\alpha\beta\sqrt{1-4\alpha^2\beta^2} - 256\alpha^5\beta^5\sqrt{1-4\alpha^2\beta^2} + 512\alpha^3\beta^3\sqrt{1-4\alpha^2\beta^2} + 128\alpha\beta)}$

- 8 $\alpha - 8\beta)$

/(2 (2 - 2 $\sqrt{1-4\alpha^2\beta^2})^2$)

1.3.2 Rozwionzania dla $x = \frac{\sqrt{1-4\alpha^2\beta^2}-1}{2\alpha\beta}$

$$\lambda_1 =$$

$(16\ \alpha^3\beta^2 + 16\alpha^2\beta^3 + 8\alpha\sqrt{1-4\alpha^2\beta^2} + 8\beta\sqrt{1-4\alpha^2\beta^2}$

- $\sqrt{(-16\alpha^3\beta^2 - 16\alpha^2\beta^3 - 8\alpha\sqrt{1-4\alpha^2\beta^2} - 8\beta\sqrt{1-4\alpha^2\beta^2} + 8\alpha + 8\beta)^2}$

$\overline{-4(-512\alpha^5\beta^5 - 256\alpha^3\beta^3 - 128\alpha\beta\sqrt{1-4\alpha^2\beta^2} + 256\alpha^5\beta^5\sqrt{1-4\alpha^2\beta^2} + 128\alpha\beta)}$

- $8\ \alpha - 8\beta)/(2(2-2\sqrt{1-4\alpha^2\beta^2})^2)$

$$\lambda_2 =$$

$(16\ \alpha^3\beta^2 + 16\alpha^2\beta^3 + 8\alpha\sqrt{1-4\alpha^2\beta^2} + 8\beta\sqrt{1-4\alpha^2\beta^2}$

$+\ \sqrt{(-16\alpha^3\beta^2 - 16\alpha^2\beta^3 - 8\alpha\sqrt{1-4\alpha^2\beta^2} - 8\beta\sqrt{1-4\alpha^2\beta^2} + 8\alpha + 8\beta)^2}$

$\overline{-4(-512\alpha^5\beta^5 - 256\alpha^3\beta^3 - 128\alpha\beta\sqrt{1-4\alpha^2\beta^2} + 256\alpha^5\beta^5\sqrt{1-4\alpha^2\beta^2} + 128\alpha\beta)}$

- $8\ \alpha - 8\beta)/(2(2-2\sqrt{1-4\alpha^2\beta^2})^2)$