1 Estimate the helicopter angles

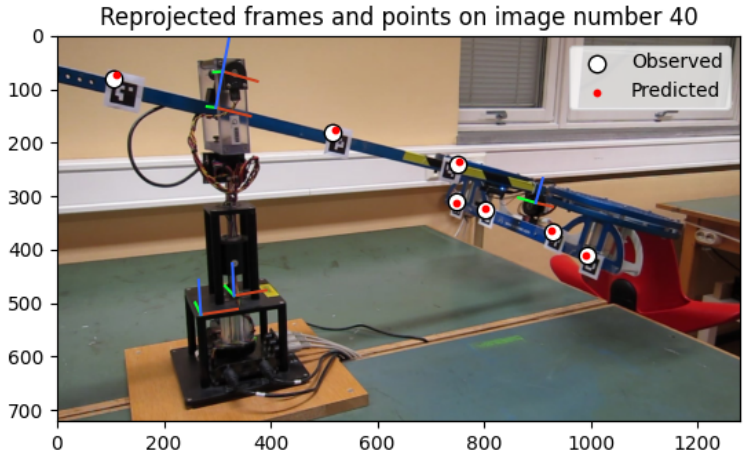
Task 1.1

Task 1.2

Task 1.3

Optimal angles (residuals) on image 0:

5.24245604	4.80020124	1.19885845	-4.99317996	-4.47286417	0	0.04965743
-8.87519959	-5.02385631	-6.06002405	-2.97156827	0.43079638	0	-6.26710301



Reprojection errors at solution:
Marker 1: 10.60 px
Marker 2: 7.26 px
Marker 3: 5.06 px
Marker 4: 2.70 px
Marker 5: 1.85 px
Marker 6: 3.57 px
Marker 7: 1.90 px
Average: 4.71 px
Median: 3.57 px

Task 1.4

$\boldsymbol{p_0}$	steps	Avg.Err.	Med.Err
$\begin{bmatrix}0.0 & 0.0 & 0.0\end{bmatrix}^T$	3	4.71 px	3.38 px
$\begin{bmatrix}0.5 & 0.5 & 0.5\end{bmatrix}^T$	3	4.71 px	3.61 px
$\begin{bmatrix}1.0 & 1.0 & 1.0\end{bmatrix}^T$	4	4.71 px	3.80 px

As can be seen in the above table, the initial value $\boldsymbol{p_0}$ does not change the amount of steps nor the reprojection error significantly.

Task 1.5

We get this warning because we are trying to solve an under-determined system of linear equations what results in having infinite many solutions. The "numpy.linalg.solve()" function cannot handle this.

Task 1.6

For simply extending the dimensions $\boldsymbol{p} \in \mathbb{R}^4$ we will get a Jacobian $\boldsymbol{J} \in \mathbb{R}^{n \times 4}$ and an approximate Hessian $\boldsymbol{J}^T\boldsymbol{J} \in \mathbb{R}^{4 \times 4}$. The modification of the helicopter model might result in not having unique configurations for one specific model state, e.g. ϕ and ψ are nearly indistinguishable for straight upwards pointing helicopter.

Task 1.7

(a) The maximum error is 19.4877 pixels.

(b) This maximum error occurs in image 104. The minimum error occurs in image 152.

(c) Parameter :
- Yaw: max = 0.7999 , image: 187
- Pitch: max = 0.2431, image: 327
- Roll: max = 0.0748, image: 350

The minimum errors are 0 at image 0 because of the offset synchronization we use.

Task 1.8

(a) Because of the way the residue function has been implemented, with the weights for valid entries, the residue of unobserved markers are set to zero. Followingly, the corresponding index of the jacobian matrix J will be zero too - leading to a potentially singular matrix, which prevents solving for a unique solution. The Levenberg-Marquardt method has circumvented this problem by fixing the step length to 1 and adding a term $\mu * I$ to the normal equation. This guarantees that the equation remains solvable and that the estimation algorithm can move past the points with insufficient observational data.

(b) The added term μI adds $\mu 1$ to the diagonal elements of $J^T J$, with μhaving a variable magnitude. The Levenberg-Marquardt method is set up in such a way that the μ term decreases when approaching the minimum, and increases otherwise. This means that the $J^T J$ term will dominate when close to the minimum, leveraging the benefits of Newton-Gauss, while having the '+1' term dominate further from the minimum - where Newton-Gauss struggles. The downside of this method, is the fixed step length of 1. While Levenberg-Marquardt is robust in ensuring that each step of the optimization remains solvable, it suffers from a sort of 'maximum accuracy' - with no guarantees beyond 'minimum + 1'. This must be taken into account when determining the stopping criterion.