Fundamentals of Discrete Mathematics

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An in depth look into discrete mathematics

Discrete Mathematics
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Question 12.1

(a) There is somebody who loves everyone.

$$\exists x \forall y Loves(x, y)$$

(b) There is somebody who loves no one.

$$\exists x \forall y \neg Loves(x, y)$$

(c) There is no one person who loves everyone.

$$\neg \exists x \forall y Loves(x, y)$$

Question 12.3

(a) $U = N$, $R = \{\langle x, y\rangle : x < y\}$
The formula is not true.
(b) $U = N$, $R = \{\langle x, x + 1\rangle : x \geq 0\}$
The formula is true.
(c) U is the set of all bit strings, $R = \{\langle x, y\rangle : x$ is lexicographically earlier than $y\}$
The formula is true.
(d) U is the set of all bit strings, $R = \{\langle x, y\rangle : y = x0 or y = x1\}$
The formula is not true.
(e) $U = \mathcal{P}(N)$, $R = \{\langle A, B\rangle : A \subseteq B\}$
The formula is true.

Question 12.5

(a) Any two sets have a union; that is, a set containing all and only the members of the two sets.

$$\forall x \forall y \exists z \forall w (w \in z \Leftrightarrow (w \in x \vee w \in y))$$

(b) Every set has a complement.

$$\forall x \exists y \forall z (z \in y \Leftrightarrow \neg(z \in x))$$

(c) Any member of a subset of a set is a member of that set.

$$\forall x \forall y (\forall z (z \in y \Rightarrow z \in x))$$

(d) There is a set which has no members and is a subset of every set.

$$\exists x \forall y (\forall z (z \notin x) \wedge (x \subseteq y))$$

(e) The power set of any set exists.

$$\forall x \exists y \forall z (z \subseteq x \Rightarrow z \in y)$$

Question 12.7

The formula is:

$$\forall x \forall y (P(x,y) \Leftrightarrow \neg P(y,x)) \wedge \forall x \exists y P(x,y) \wedge \forall x \forall y \forall z (P(x,y) \wedge P(y,z) \Rightarrow P(x,z))$$

This formula is satisfiable but has no finite model.