Fundamentals of Discrete Mathematics

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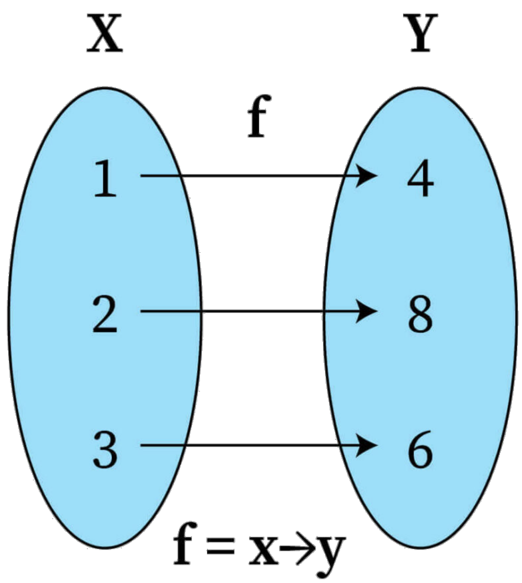

An in depth look into discrete mathematics

Discrete Mathematics
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1 What is Discrete Mathematics

Discrete mathematics is a study that uses quantities that change in steps rather than continuously. Discrete mathematics is essential for writing complicated software because it builds the ability to form effective propositions and identify abstract patterns from data. Discrete math envolopes universal concepts for all computer science languages such as Set Theory, algorithms, and combinations. These skills make mastery in discrete mathematics essential for a successful career in computer science or software development.

Discrete math uses propositional logic which is where propositions are used for all statements and are then tested. A propostition is a declarative statement which means that it must be true or false. Propositions must be a definitive assertion that is completely unambiguous in its truth value.

2 Set Theory

A set is a collection of one or more elements. Elements can be anything, not only numbers. The elements in a set are called its members and they must be distinct. The order of members does not matter. Explicit sets are written with {...} surrounding it ($ex.N = \{1, 2, 3\}$). The symbol $\in$ is used to indicate membership in a set ($ex.N = \{1, 2, 3\}, 1 \in N$).

A finite set is a set that has a limited number of members. This means that finite sets have a real cardinality, Which is denoted by ——. Cardinality is the number of members in a set and must be a non-negative integer ($ex.D = \{Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday\}, |D| = 7$).

A function is a rule that associates each member of the first set with exactly one member of the second set. if f is a function that maps the set X = {1} to the set Y = {2}, then f(x) = 2. The way to write that the function f maps the set X to the set Y is $f : X \to Y$.

3 Pigeonhole Principle

The Pigeonhole principle is a mathematical concept that states:

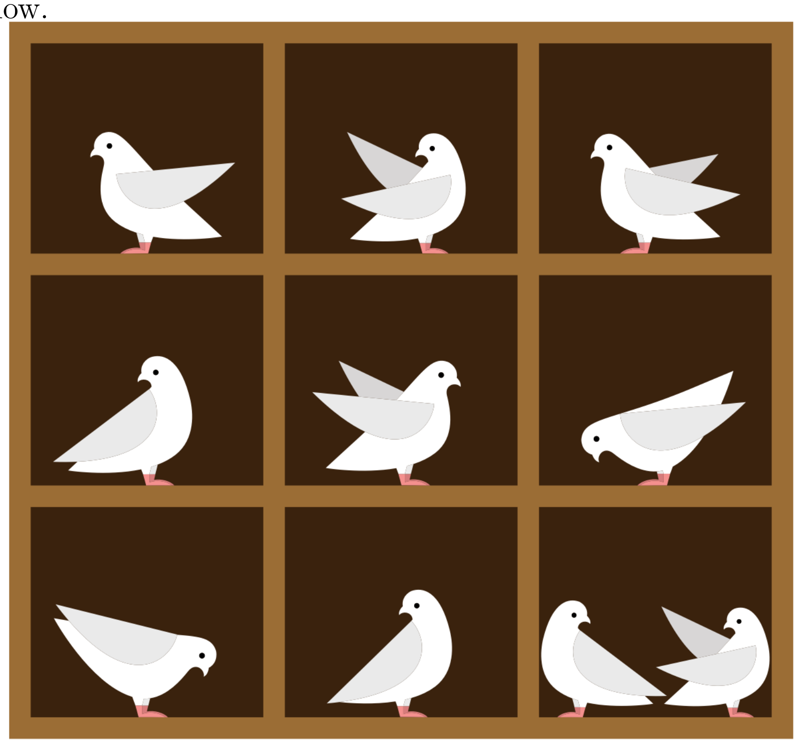
If $f : X \to Y$ and $|X| > |Y|$, then there are elements $x_1, x_2 \in X$, such that $x_1 \neq x_2$ and $f(x_1) = f(x_2)$

This can also be written in completely mathematical notation as well:

$f : X \to Y, |X| > |Y| \Rightarrow \exists\, x_1, x_2 \in X : x_1 \neq x_2 \wedge f(x_1) = f(x_2)$

In laymen's terms it is saying that if the set X is larger than the Set Y, then there must be two members of X that map to the same member of Y.

An example of this, and where the principle get its namesake from, is with pigeons and pigeonholes. If there are 10 pigeons and only 9 pigeonholes, then there must 2 pigeons in 1 of the pigeonholes. This is shown in the picture below.



A proposition can be built from this situation. *If there are 10 pigeons and 9 pigeonholes, then there must be 2 pigeons in the same pigeonhole.* However, this is an ineffective proposition because of how specific it is, so let's try to make it more general. first we can simplify the numbers. this turns the proposition into: *If there are more pigeons than pigeonholes, then there must be atleast 2 pigeons in the same pigeonhole.* This is better but we are still constraining it to pigeons and pigeonholes, so let's turn a group of pigeons into set X, a group of pigeonholes into set Y, and the pigeons going into pigeonholes as a function of mapping $X \to Y$. This turns the proposition into: *If X maps to Y and $|X| > |Y|$, then there exists atleast 2 elements of X mapped to the same element of Y.*. Now, except for a few diferences in notation, this says the exact same thing as the Pigeonhole Principle.

Extended Pigeonhole Principle:

The extended pigeonhole principle is a continuation of the original pigeonhole principle. It is used to find the amount of members of X that map to the same member of Y. It states that for any finite sets X and Y and any positive integer k such that $|X| > k \cdot |Y|$, if $f : X \to Y$, then there are at least k + 1 distinct members $x_1, ..., x_{k+1} \in X$ such that $f(x_1) = ... = f(x_{k+1})$. The original Pigeonhole Principle is the case where k = 1. You can find k by using the formula $\lceil \frac{|X|}{|Y|} \rceil$.