

readme

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1 Norms

properties

1. $\|x\| \geq 0$, and $\|x\| = 0 \iff x = 0$
2. $\|\alpha x\| = |\alpha| \|x\|$
3. $\|x + y\| \leq \|x\| + \|y\|$

$$\|A\|_1 = \max_{1 \leq j \leq n} \sum_{i=1}^m |a_{ij}|$$
$$\|A\|_\infty = \max_{1 \leq i \leq m} \sum_{j=1}^n |a_{ij}|$$

2 svd

Example: $A = \begin{bmatrix} 0 & 1 & 1 \\ \sqrt{2} & 2 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

The SVD is defined as

$$A = P \Sigma Q^T$$

Method

Computing:

$$AA^T = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 6 & 2 \\ 2 & 2 & 2 \end{bmatrix}$$

$$\begin{aligned} -\lambda^3 + 10\lambda^2 - 16\lambda &= -\lambda(\lambda^2 - 10\lambda + 16) \\ &= -\lambda(\lambda - 8)(\lambda - 2) \end{aligned} \tag{1}$$

Eigenvals of AA^T are $\lambda = 8, \lambda = 2, \lambda = 0$, thus the singular values are $\sigma_1 = 2\sqrt{2}, \sigma_2 = \sqrt{2}$ (and $\sigma_3 = 0$).

Giving out the matrix

$$\Sigma = 0_{3 \times 3} + \sigma = \begin{bmatrix} 2\sqrt{2} & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Finding the eigenvectors $(A - \lambda I)\mathbf{x} = \mathbf{0}$, we get respectively to the eigenvalue who is described before: $p_1 = \left(\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$, $p_2 = \left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$ and $p_3 = \left(\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right)$ (note: normalised vectors).

Yielding

$$P = [p_1^T p_2^T p_3^T] = \begin{bmatrix} \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$A^T A = \begin{bmatrix} 2 & 2\sqrt{2} & 0 \\ 2\sqrt{2} & 6 & 2 \\ 0 & 2 & 2 \end{bmatrix}$$

With the eigenvalues $\lambda = 8, \lambda = 2, \lambda = 0$ with eigenvectors $q_1 = \left(\frac{1}{\sqrt{6}}, \frac{3}{\sqrt{12}}, \frac{1}{\sqrt{12}}\right)$, $q_2 = \left(\frac{1}{\sqrt{3}}, 0, -\frac{2}{\sqrt{6}}\right)$ and $q_3 = \left(\frac{1}{\sqrt{2}}, -\frac{1}{2}, \frac{1}{2}\right)$. (Actually can also use the formula $p_i = \frac{1}{\sigma_i} A^T p_i$ to get the various q_i).

$$Q = [q_1^T q_2^T q_3^T] = \begin{bmatrix} \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{3}{\sqrt{12}} & 0 & -\frac{1}{2} \\ \frac{1}{\sqrt{12}} & -\frac{2}{\sqrt{6}} & \frac{1}{2} \end{bmatrix}$$