Contents

1 Randomness

1.1 Monte carlo

assume table with 50/50 red black

1.1.1 Gamblers fallcaly

If the soccerteam have won/lost 10 times in a row they have loose/win next match.

1.1.2 Regession to the mean - Francis Galton, 1885

If the parents are taller than the mean will the child be shorter than the mean. No bioligy, just analysing the data.
What is says: after na extreme event, you are likley to get an less extreme event. After spinning 10 reds. You are likley to get less than 10 reds on your next 10 spins, but the expected number is still 5.
Now if one is looking at the avrage of the 20 spins it will be closer to the mean, ie. regression to the mean.

1.1.3 Summed up: why are these different?

1. The gambler fallacy sais that we are expected to have

fewer than 5 reds on the 10 spins. \ 2. Regression to the mean sais that we will probably have fewer than 10 reds on the next.

1. Below the mean. \ 2. closer to the mean.

1.2 Sample space

1.2.1 Girl boy paradox

My Friend Nik has two kids, and he told you he has at least one a girl, hence what is the probability of the other kid is female?

First kid	Second Kid	$P()$
Boy	Boy	1/4
Boy	Girl	1/4
Girl	Girl	1/4
Girl	Boy	1/4

You saw my friend Nik was walking on the street with his daughter. And Nik told you he got another child at home, so, what is the probability of the other child was a female?

1.2.2 Monty Hall problem

Still not convinced? Take a 100 doors, would you still switch?

Behind door 1	Behind door 2	Behind door 3	P()
Goat	Goat	Car	1/3
Goat	Car	Goat	1/3
Car	Goat	Goat	1/3

1.3 Random prosesses

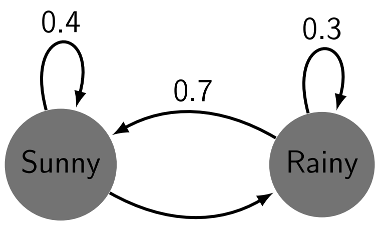
1.3.1 Random walk

1.3.2 makrov chian

The makrov property/ assumption

$$Pr(X_{n+1} = x \mid X_1 = x_1, X_2 = x_2, \ldots, X_n = x_n) = \Pr(X_{n+1} = x \mid X_n = x_n)$$

$$\begin{aligned} p_{ij} &= \mathbf{P}\left(X_{n+1} = j | X_n = i\right) \\ &= \mathbf{P}\left(X_{n+1} = j | X_n = i, X_{n-1}, \ldots, X_0\right) \end{aligned}$$



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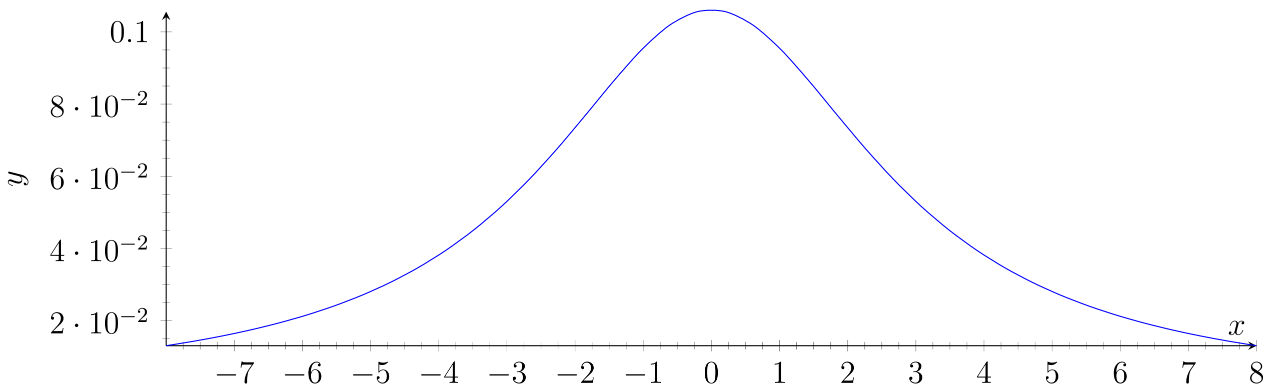
Random walk does not have a transission probility matrix since the state space are not finite. For markov chian with finite state space matrecies we just have an matrix describing all the probabilities.

2 PDF

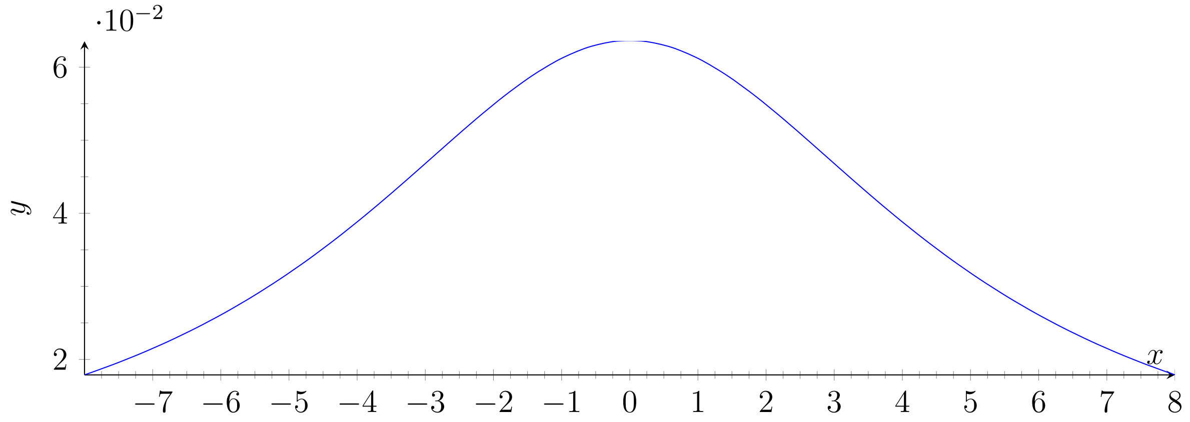
2.1 Normal distribution

$$\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

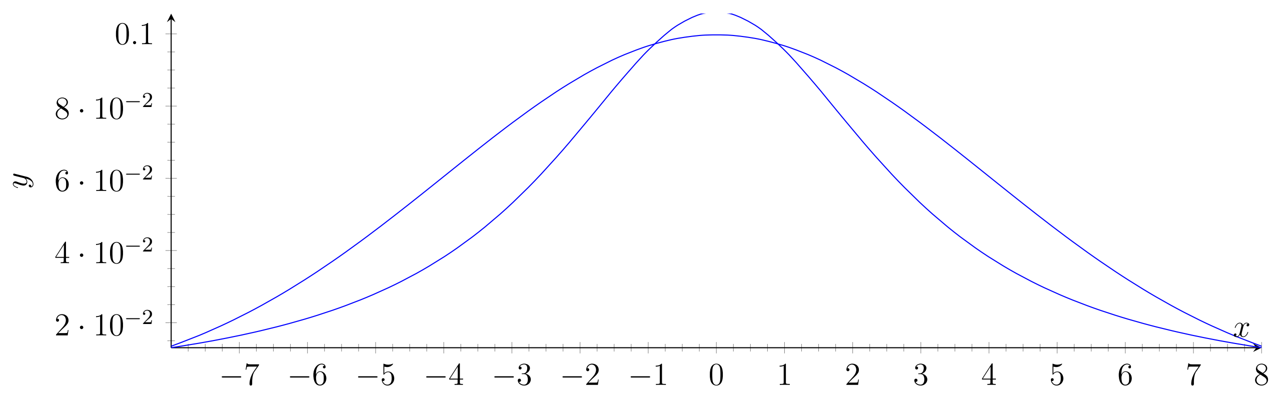


2.2 Cauchy distribution

$$\frac{1}{\pi\gamma\left[1+\left(\frac{x-x_0}{\gamma}\right)^2\right]}$$

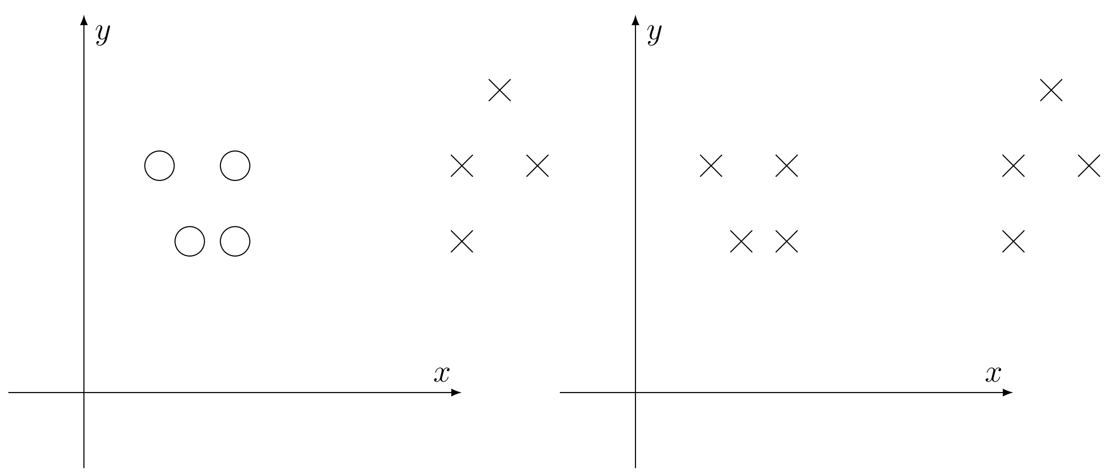


2.3 Together



3 Learning

3.1 supervised vs unsupervised learning



4 Moving average / moving variance

Digital filter

$$\begin{aligned} y(k) &= \frac{1}{N+1}\sum_{n=0}^{n} b_N u(k-n) \\ y(k) &= b_0 u(k) + b_1 u(k-1)\dots b_N u(k-n) \end{aligned} \tag{1}$$

let n be the size of the filter, and y_k be the mesurments

$$\begin{aligned} y_k' &= y_k\frac{1}{n} + y_{k-1}\frac{1}{n} + y_{k-2}\frac{1}{n} + \dots + y_{k-n+1}\frac{1}{n} \\ &= \frac{1}{n}\sum_{i=0}^{n-1} y_{k-i} \end{aligned}$$

4.1 Aanalyse of 6

$$\begin{aligned} y(k) &= \frac{1}{N+1}\sum_{n=0}^{n} u(k-n) \\ y(k) &= \frac{1}{6}\sum_{n=0}^{n} u(k-n) \end{aligned} \tag{2}$$

$$\begin{aligned} y(k) &= \frac{1}{6}[u(k-n) + u(k-1) + u(k-2) + u(k-3) + u(k-4) + u(k-5)] \\ \mathcal{Z}\{y(k)\} &= \frac{1}{6}\left[z + z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5}\right] \\ Y(z) &= \frac{1}{6}\cdot\frac{1 + z^1 + z^2 + z^3 + z^4 + z^5}{z^5} \end{aligned} \tag{3}$$

Frekvens respons (demping)

$$\begin{aligned} Y(jw) &= \frac{1}{6}\left(1 + e^{jwTs}\cdots\right) \\ |Y(w)| &= \frac{1}{N}\left|\frac{\sin\left(\frac{wN}{2}\right)}{\sin\left(\frac{w}{2}\right)}\right| \end{aligned} \tag{4}$$

5 moving avrage continued

$$X_t^{AR} = c + \varepsilon_t \qquad + \sum_{i=1}^{p} \varphi_i X_{t-i}. \tag{5}$$

$$X_t^{MA} = \mu + \varepsilon_t \qquad + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} \tag{6}$$

$$X_t^{ARMA} = c + \varepsilon_t \qquad + \sum_{i=1}^{p} \varphi_i X_{t-i} + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i}. \tag{7}$$

ARIMA

6 Chaos and Bifurcations

6.1 example

$\ddot{x} + 0.1\dot{x} + x^5 = 6\sin t$

$$\begin{aligned} x(0) = 2, \quad \dot{x}(0) = 3 \\ x(0) = 2.01, \quad \dot{x}(0) = 3.01 \end{aligned} \tag{8}$$

from x 0 to 50

7 to be added

winer process markov process