# Unsupervised Learning of Visual Features by Contrasting Cluster Assignments

[Caron et al., NeurIPS 2020]

-Summary-

- Current contrastive learning strategy :
  - Compares pairs (positive, true) (negative, true) to pull (positive, true) and push (negative, true).
  - Problem: computing all the pairs on a large dataset (ex: ImageNet) is not practical
  - Alternatives:
    - 1 : compute contrastive loss based on random batch (widely adopted)
    - 2 : clustering-based methods discriminating between groups of images with similar features instead of individual images (Caron et al., 2018)
- This paper tries to improve their previous work (2) by changing clustering-based methods.

- SwAV (**Sw**apping **A**ssignments between multiple **V**iews of the same image) :
  - Goal : To learn a useful prototype  $\{c_1,\dots,c_k\}$  that discriminate feature vectors  $\{z_1,\dots z_B\}$ , which goes along with the concept of CL by using swapping assignments.
  - Method:
  - 1. Image  $x_n$  is transformed into an augmented view  $x_{nt}$  by transformation  $t \sim T$ .
  - 2.  $x_t$  is mapped into <u>normalized</u> feature vector  $z_{nt} = \frac{f_{\theta}(x_{nt})}{\|f_{\theta}(x_{nt})\|_2} \in \mathbb{R}^D$  by model  $f_{\theta}$ .
  - 3. Then, we compute a 'code'  $q_{nt}$  by exploiting 'optimal transport' between  $z_{nt}$  and prototype vectors  $\{c_1, ..., c_K\} \subset \mathbb{R}^D$

Note: prototype matrix  $C = [c_1, ... c_K] \in \mathbb{R}^{D \times K}$ , code matrix  $Q = [q_1, ... q_B] \in \mathbb{R}^{K \times B}$  (where D = dimension of feature  $z_{nt}$ , B = batch size, K = the number of polytopes)

Recall that SwAV clusters multiple samples to the prototypes.

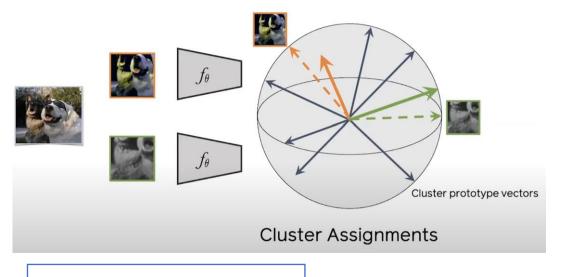


Illustration of clustering on SwAV

Note: why we use optimal transportation to get code  $q_{nt}$ ?

- 1.  $q_{nt}$  should be probability vector (used as loss) (achieved by constraint in optimal transportation)
- 2. Higher coupling value for positive pair / Smaller coupling value for negative pair
- # Above objectives can be achieved by optimal transportation by setting  $-C^TZ$  as the cost matrix.
- But how to exploit optimal transportation to compute the codes  $q_{nt}$ ?
  - Set code matrix Q as the coupling matrix in optimal transportation.
  - Set  $-C^TZ$  as the cost matrix in optimal transportation. (where  $Z=[z_1,...,z_B]\in\mathbb{R}^{D\times B}$ )
  - Now, we want to minimize the coupling  $\times$  cost entirely (= solving optimal transportation)

• Why we choose 
$$-C^TZ = \begin{bmatrix} -c_1^Tz_1 & \cdots & -c_B^Tz_1 \\ \vdots & \ddots & \vdots \\ -c_K^Tz_1 & \cdots & -c_K^Tz_B \end{bmatrix} \in \mathbb{R}^{K \times B}$$
 as the cost matrix?

- This enforces the optimal transportation to assign high coupling value  $(=q_{ij})$  for positive pairs.
  - # Logic : Positive pair -> high similarity (= $c^Tz$ ) -> low cost (= $-c^Tz$ ) -> high coupling value assigned by optimal transportation algorithm
- Also, this method assign low coupling value (=  $q_{ij}$ ) for negative pair in the similar logic above.
- This optimization maximizes the similarity between the features and the prototypes.

- To prevent trivial solution such as giving all coupling values to smallest cost-valued item, we add entropy term  $H(Q) := -\sum_{i,j} Q_{ij} \log Q_{ij}$ .
- To sum up, our optimization problem is following:

$$\max_{Q \in \mathcal{Q}} \langle Q, C^T Z \rangle_F + \epsilon H(Q) \qquad \text{(A, B)}_{F} = \sum_{i,j} A_{ij} B_{ij} = Tr(A^T B)$$

Note (Frobenius inner product):

$$< A, B>_F = \sum_{i,j} A_{ij} B_{ij} = Tr(A^T B)$$

where 
$$Q = \left\{Q \in \mathbb{R}_+^{K \times B} \middle| Q \cdot 1_B = \frac{1}{k} 1_K$$
,  $Q^T \cdot 1_K = \frac{1}{B} 1_B\right\}$  (finite resource constraint in optimal transport.)

- Luckily, there is a good algorithm to solve this problem (Sinkhorn-Knopp algorithm):
  - Solution :  $Q^* = diag(u) \cdot K \cdot diag(v)$

where 
$$K = \exp(\frac{C^T Z}{\epsilon})$$
,  $u^{(k+1)} = \frac{r}{KV^{(k)}}$ ,  $v^{(k+1)} = \frac{c}{K^T u^{(k+1)}}$  (here,  $r = \frac{1}{k} 1_K$ ,  $c = \frac{1}{B} 1_B$ )

Note: Overall loss adopting  $L(z_t, z_S)$  can be rewritten as follows: (N:# of data)

• (Back to) Method:

$$-\frac{1}{N} \sum_{n=1}^{N} \sum_{s,t \sim \mathcal{T}} \left[ \frac{1}{\tau} \mathbf{z}_{nt}^{\top} \mathbf{C} \mathbf{q}_{ns} + \frac{1}{\tau} \mathbf{z}_{ns}^{\top} \mathbf{C} \mathbf{q}_{nt} - \log \sum_{k=1}^{K} \exp \left( \frac{\mathbf{z}_{nt}^{\top} \mathbf{c}_{k}}{\tau} \right) - \log \sum_{k=1}^{K} \exp \left( \frac{\mathbf{z}_{ns}^{\top} \mathbf{c}_{k}}{\tau} \right) \right]$$

4. Calculate 'estimated similarity vector  $p_{nt}$ ' of  $z_{nt}$  based on the previous projection:

$$p_{nt}^{(k)} = \frac{\exp\left(\frac{1}{\tau}z_{nt}^Tc_k\right)}{\sum_{k'}\exp\left(\frac{1}{\tau}z_{nt}^Tc_{k'}\right)}$$

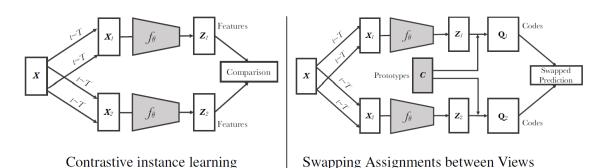
Softmax of columns of  $z_{nt}^T C$  with temperature  $\tau$ 

Why the loss is based on swapped prediction?

Idea:  $z_t$ ,  $z_s$  should be assigned into same prototype.

5. Calculate loss based on **swapped prediction**: (Note:  $z_t$ ,  $z_s$  are two different augmentations of x)

$$L(z_{nt}, z_{ns}) = l(z_{nt}, q_{ns}) + l(z_{ns}, q_{nt}), \quad l(z_{nt}, q_{ns}) = -\sum_{k} q_{ns}^{(k)} \log p_{nt}^{(k)}$$



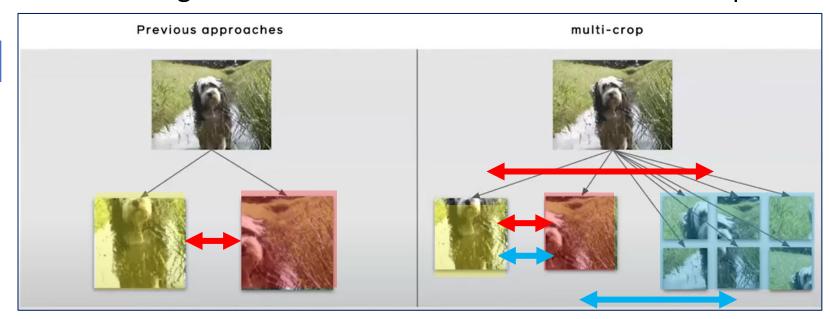
Comparison between CL instance learning vs. SwAV

(Note: Gray color implies trainable parameters)

#### Introduction (Multi-crop)

- It is widely known that comparing more views during CL training improves the resulting model. (Misra et al., 2019)
- However, most CL methods compare 'one' pair of transformations per image for saving comp. cost.
  - To exploit the advantage of multi view training, they propose to use V low resolution crops that cover only small parts of the image in addition to two standard resolution crops.

*Illustration of multi-crop concept* 



#### Introduction (Multi-crop)

- Detailed description of multi-crop :
  - Recall the loss based on swapped prediction appeared in SwAV.

$$L(z_{t_1}, z_{t_2}) = l(z_{t_1}, q_{t_2}) + l(z_{t_2}, q_{t_1})$$

which adopts only two standard resolution crops  $z_{t_1}$ ,  $z_{t_2}$ .

• We now sample V additional low resolution crops  $z_{t_3}$ , ...  $z_{t_{V+2}}$  and generalize the loss :

$$L(z_{t_1}, z_{t_2}, \dots z_{t_{V+2}}) = \sum_{i \in \{1,2\}} \sum_{v=1}^{V+2} 1_{v \neq i} l(z_{t_v}, q_{t_i})$$

Note: Using low resolution images ensures only a small increase in computational cost.

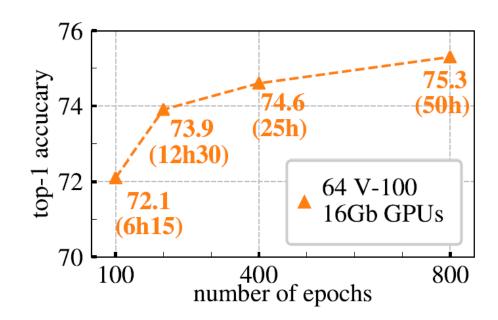
#### **Experiments**

Multi-crop (x)

Multi-crop (o)

• To verify downstream performance, they used linear evaluation protocol.

	Top-1		Δ
Method	2x224	2x160+4x96	•
Supervised	76.5	76.0	-0.5
Contrastive-inst	tance app 68.2	proaches 70.6	+2.4
Clustering-base	ed approa	ches	
SeLa-v2	67.2	71.8	+4.6
DeepCluster-v2	70.2	74.3	+4.1
SwÁV	70.1	74.1	+4.0



Left: Comparision between clustering-based and contrastive instance methods and impact of multi-crop. (400 /200 epochs for unsupervised / supervised)

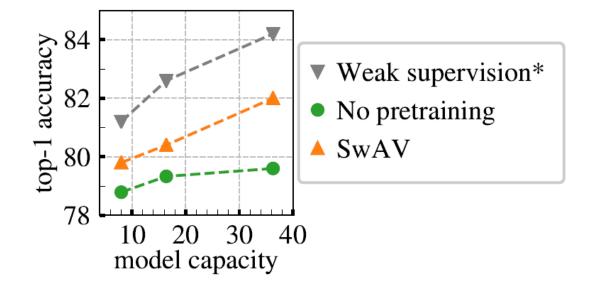
Right: SwAV (with multi-crop) downstream performance as a function of epochs

Note: DeepCluster-v2 is not online which makes it impractical for extremely large datasets (= requires much longer training compared to SwAV)

#### **Experiments**

• Further experiments demonstrate the SwAV's higher performance on representation learning.

Method	Frozen	Finetuned
Random	15.0	76.5
MoCo	-	77.3*
SimCLR	60.4	77.2
SwAV	66.5	<b>77.8</b>



Left: Top-1 acc on ImageNet for pretrained models on uncurated 1B Instagram images (linear evaluation on frozen features or finetuned features)

Right: Performance of finetuned models as we increase the capacity of ResNext

Note (weak supervision): pretrained on a curated 1B Instragram images filtered with 1.5k hashtags similar to ImageNet classes.