

Chapter 1

Introduction

1.1 Preliminary Remarks

Optimization The desire for optimality (perfection) is inherent in humans. The search for extremes inspires mountaineers, scientists, mathematicians, and the rest of the human race. The development of Calculus of Variation was driven by this noble desire. A mathematical technique of minimization of curves was developed in eighteenth century to solve the problems of the best possible objects: The minimal surface, the shortest distance, or the trajectory of fastest travel.

In twentieth century, control theory emerged to address the extremal problems in science, engineering, and decision-making. These problems specialize the available the degrees of freedom by the so-called controls; these are constrained functions that can be optimally chosen. Optimal design theory addresses space-dependent analog of control problems. Minimax problems address optimization in a conflict situation or in undetermined environment. A special branch of the theory uses minimization principles to create effective algorithms such as finite element method to computing the solution.

Description of fundamental laws of Nature For centuries, scientists tried to prove that the Universe is rational, symmetric, or optimal in another sense. The attempts were made to formulate laws of natural sciences as extreme problems (variational principles) and to use the variational calculus as a scientific instrument to derive and investigate the motion and equilibria in Nature (Fermat, Lagrange, Gauss, Hamilton, Gibbs..). It was observed by Fermat that light "chooses" the trajectory that minimizes the time of travel, many equilibria correspond to the local minimum of the energy, motion of mechanical systems correspond to stationarity of a functional called the action, etc. In turn, the variational principles link together conservation laws and symmetries.

Does the actual trajectory minimize the action? This question motivated great researcher starting from Leibnitz and Fermat to develop variational methods to justify the Nature's "desire" to choose the most economic way to move,