# Reflected power calculation

## **General Description**

Cameras are being increasingly used in robotics and for the observations of doors, traffic, processes and of operators. The advantages compared to traditional sensors are in the ability to have more detailed information about what happens in the scenery and how the objects are reacting. This allows a faster and more adequate decision process of the automatic system in how to react to a certain event.

As an introduction for the designers and the developers of such systems, this application note helps to understand the basic principles of how the optical system needs to be designed and how it works in respect to the illumination power.

This note touches on the most important aspects of the illumination power design for the monochrome and the 3D time-of-flight (TOF) cameras.

# **Topics**

- Rough estimation of an optical design.
- General understanding of the light-power budget of a self-illumin ating camera system.
- Camera functionality, independent of the ambient-light level.
- Camera can operate with modulated light sources e.g. for a 3D TOF or a synchronous signal detection.

# Possible applications

- 2D and 3D TOF cameras
- Object counting, automatic control of doors, machines and safety
- 3D limit and proximity switches, area scanners
- 3D distance measurements, volumetric mapping of objects
- Gesture control of man-machine interfaces
- Control of occupied seats, vehicle guidance

#### 1. Introduction

Increasingly, camera systems (CCD, 3D TOF) are being used for observation and control functions in different application segments. Most of them have their own light sources (IR or visible light) in order to be independent of the existing ambient-light. In these self illuminated cases, the observation field is in relation to the illuminated scenery (defining the usable operating range (see Figure 1)). This self illumination is especially important for 3D TOF systems as they are based on the principle of reading back their own modulated light. This application note describes step by step the general principles to design such an all-in-one camera system.

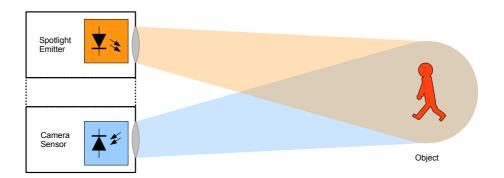


Figure 1: The principle of an all-in-one camera system

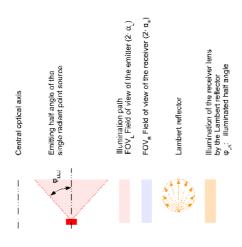
# 2. Optical path of the system

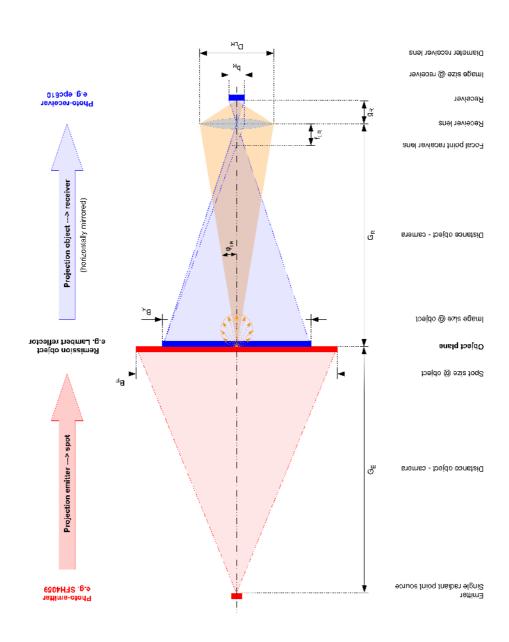
Figure 2 shows the optical schematic of such a system. For easier reading, the observation part is horizontally mirrored.

There are two main parts: the illumination and the observation. The illumination section projects a picture (spot) of the artificial light source onto the scenery. In parallel, a sensor or a camera observes this field and catches the reflected light of the subject therein (refer to Figure 1).

#### 2.1. Basic assumptions

In order to simplify the calculations, we will use a design of a rotationally symmetric optical system aligned to the central optical axis. All the calculations are valid for small, narrow angles e.g.  $-6^{\circ} < \theta < +6^{\circ}$ . This means that the small-angle approximation is valid:  $\sin \theta \approx \theta$  or  $\tan \theta \approx \theta$ . The light source is a single radiant point source with only one emitted wavelength (e.g. 850nm, monochromatic light). Refer for other sources to chapter 4.: Notes.





#### 2.2. Calculation of the optical receiver parameters

First of all, the object size: the distance from the object to the camera and the size of the sensor will determine the optical properties of camera system.

For the calculations, we will use the following parameters:

 $G_R$  distance from the camera to the object: e.g. 0.60m  $B_R$  image size at the location of the object: e.g. 74 x 74mm

b<sub>R</sub> active area of the sensor (imager): e.g. epc610: 0.32 x 0.32mm (8x8 pixel sensor)

For the first step, we calculate the focal length  $f_{LR}$  of the receiver lens as a function of: the size  $B_R$  of the image at the location of the object, the size  $b_R$  of the imager (active area of the receiver) and the distance  $G_R$  from the camera to the object:

Equation 1:

$$f_{LR} \,=\, \frac{G_R}{1+\frac{B_R}{b_R}}$$

e.g. 
$$f_{LR} = \frac{600}{1 + \frac{74mm}{0.32mm}} = 2.583mm$$

Next is the estimation of the receiver's field of view ( FOV  $_{\text{R}}$  ;the half angle  $\alpha_{\text{R}}$  ) for the image:

Equation 2:

$$\alpha_{\mathsf{R}} \; = \; \mathsf{atan} \left( \! \frac{\mathsf{B}_{\mathsf{R}}}{2 \! \cdot \! \mathsf{G}_{\mathsf{R}}} \! \right)$$

e.g. 
$$\alpha_{R} = atan\left(\frac{74mm}{2\cdot600mm}\right) = 3.528deg$$

The distance  $g_R$  between the imager (receiver) and its lens is:

Equation 3:

$$g_R = G_R \cdot \frac{b_R}{B_R}$$

e.g. 
$$g_R = 600 \text{mm} \cdot \frac{0.32 \text{mm}}{74 \text{mm}} = 2.595 \text{mm}$$

You can also do calculations 1 and 2 for a single pixel of the imager, for the image diagonal to get the free optical diameter for the optics or in case of a rectangular image for the parameters length and width. E.g. for the above examples, the diagonal dimension for the image is:  $B_R = 104$ mm and  $\alpha_R = 4.98$ deg.

### 2.3. Calculation of the optical emitter parameters

For the optical emitter parameters, we will continue from above and use:

 $G_E = G_R$  distance from the camera to the object: e.g. 0.60m (see above)  $\phi_{LED}$  emitting half angle of the single point light source: e.g. IR LED SFH4059: ±10deg

It is common that the manufacturer specifies the light source (e.g. IR LED SFH4059) as a single radiant point light source. The relevant parameter is the emitting half angle  $\phi_{\text{LED}}$  of the emitter (source).

If there is no additional lens in front of the emitter, then the emitter's field of view ( FOV  $_{\text{E}}$ , the half angle  $\alpha_{\text{E}}$ ) for the illumination spot is equal to the half angle  $\phi_{\text{LED}}$  of the emitter (LED). This allows the calculation of the diameter  $B_{\text{E}}$  of the spot size at the location of the object as follows:

$$\mathsf{B}_\mathsf{E} \ = \ \mathsf{2} \cdot \mathsf{G}_\mathsf{E} \cdot \mathsf{tan} \, \alpha_\mathsf{E}$$

e.g. 
$$B_E = 2 \cdot 600 \text{mm} \cdot \text{tan} (10 \text{deg}) = 212 \text{mm}$$

To cover all the mechanical and optical tolerances, the diameter  $B_{\epsilon}$  should be, as a good value, around two times larger than the necessary diameter of the image. This guarantees that the image area is always illuminated by the spot area.

Equation 5:

$$B_E > diameter(B_R)$$

e.g. 
$$B_E = 212mm > diameter(B_R) = 74mm \cdot \sqrt{2} = 104mm$$

These are the basic set of the optical parameters.

# 3. The power budget

As noted in the introduction, additional illumination is necessary in all cases to be independent of the ambient-light (e.g. to recognize persons or objects in the dark) or to use modulated light for synchronous detection modes (e.g. for 3D time-of-flight applications). This artificial light source must fulfill the aspects of the camera's sensitivity, of the operating distance and of the object's remission (reflectivity).

The light in the scenery is reflected back to the camera. The power calculation for the optical path allows us to estimate the camera's exposure time (or integration time) and/or the number of spotlights needed.

#### 3.1. Irradiance of the scenery

Equation 6 shows the formula to calculate the illumination of the scenery. It is defined by the irradiance ( light-power per area )  $E_{EO}$  [W/m<sup>2</sup>]. At the location of the object, the illumination depends on:

 $I_{\text{E LED}}$  radiant intensity of the emitter : e.g. SFH4059: 100mW/sr @  $i_{\text{NOM}}$  = 70mA

 $i_{NOM}$  current for the nominal radiant intensity  $I_{E LED}$ 

 $\begin{array}{ll} i_{APP} & \text{effective current per emitter in the application:} & \text{e.g. 180mA} \\ n_E & \text{number of emitters used in the application:} & \text{e.g. 2 pieces} \\ A_{OE} & \text{area of the spot size @ object; illuminated by the half angle} & \phi_{\text{LED}} \, \text{of the emitter} \\ \end{array}$ 

v<sub>E</sub> losses caused by the current efficiency of the emitter, by the wavelength efficiency of the emitter

and by the transmission of the emitter cover or lens.

The irradiance at the scenery is:

Equation 6: 
$$\mathsf{E}_{\mathsf{EO}} \ = \ \mathsf{I}_{\mathsf{E}\,\mathsf{LED}} \cdot \frac{\mathsf{i}_{\mathsf{APP}}}{\mathsf{i}_{\mathsf{NOM}}} \cdot \mathsf{n}_{\mathsf{E}} \cdot \mathsf{4}\,\pi \cdot \mathsf{sin}^2 \bigg(\frac{\phi_{\mathsf{LED}}}{2}\bigg) \cdot \frac{\mathsf{v}_{\mathsf{E}}}{\mathsf{A}_{\mathsf{OE}}}$$

$$\text{e.g.} \qquad \qquad \text{E}_{\text{EO}} \ = \ 100 \text{mW/sr} \cdot \frac{180 \text{mA}}{70 \text{mA}} \cdot 2 \cdot 4 \pi \cdot \text{sin}^2 \bigg( \frac{10 \text{deg}}{2} \bigg) \cdot \frac{100 \%}{\pi \cdot \bigg( \frac{0.212 \text{m}}{2} \bigg)^2} \ = \ 1.391 \mu \text{W/mm}^2 \times \frac{100 \text{m}}{2} \times \frac{100 \text{m}}{$$

Note:

For optical designs with additional emitter optics (lens or mirror), only the area  $A_{\text{OE}}$  of the spot size needs to be taken in account. This corresponds to the illumination caused by the half angle  $\phi_{\text{LED}}$  of the emitter.

## 3.2. Irradiance of a pixel

To calculate the irradiance of a single pixel of the imager, we first need to know the half angle  $\phi_{LR}$  of the illumination of the receiver lens. It describes the area where the receiver lens is able to sample the backscattered light from the Lambert reflector (from the object).

Equation 7: 
$$\phi_{LR} \ = \ \text{atan} \left( \frac{D_{LR}}{2 \cdot G_{R}} \right)$$

e.g. 
$$\phi_{LR} \ = \ \text{atan} \left( \frac{1.5 \text{mm}}{2 \cdot 600 \text{mm}} \right) \ = \ 0.0716 \text{deg}$$

The irradiance at the pixel is

Equation 8: 
$$\mathsf{E}_{\mathsf{PIXEL}} \ = \ \mathsf{E}_{\mathsf{EO}} \cdot \rho \cdot 2 \cdot \mathsf{sin}^2 \left( \frac{\phi_{\mathsf{LR}}}{2} \right) \cdot \frac{\mathsf{A}_{\mathsf{OPIXEL}}}{\mathsf{A}_{\mathsf{RPIXEL}}} \cdot \mathsf{v}_{\mathsf{R}}$$

with the parameters:

remission factor of the target (reflectivity of the object): e.g. 90%

 $b_{RPIXEL}$  active area of a pixel: e.g. epc610: 0.04 x 0.04mm (sensor 0.32 x 0.32mm with 8x8 pixel)

A<sub>OPIXEL</sub> area of a pixel @ the place of the object

ARPIXEL area of a pixel @ the receiver

v<sub>R</sub> losses caused by the wavelength efficiency of the receiver and by the transmission of the receiver lens.

e.g. 
$$E_{\text{PIXEL}} \ = \ 1.391 \mu \text{W/mm}^2 \cdot 90\% \cdot 2 \cdot \sin^2 \! \left( \frac{0.0716 deg}{2} \right) \cdot \frac{(9.25 mm)^2}{(0.04 mm)^2} \cdot 81\% \ = \ 42.34 n \text{W/mm}^2$$

The important relationships from Equation 7 & Equation 8 for the influencing factors to the sensor's irradiance E PIXEL are:

- The irradiance increases with the square of the diameter of the receiver lens (aperture).
  - This shows the importance having a lens with the maximum size in order to get an efficient light-power throughput for the sensor.
- The irradiance decreases with the square of the distance.
- The irradiance is proportional to the remission of the object (reflectivity). The better the remission factor, the better the sensor signal.

We can also express the illumination as the receiving power  $\Phi_{PIXEL}$  @ a pixel of the imager:

Equation 9: 
$$\Phi_{\text{PIXEL}} = E_{\text{PIXEL}} \cdot A_{\text{RPIXEL}} \qquad \text{e.g.} \qquad \Phi_{\text{PIXEL}} = 42.34 \text{nW/mm}^2 \cdot (0.04 \text{mm})^2 = 0.067 \text{nW}$$

The received light power integrated over time gives the energy level which corresponds to the signal in the imager. This means that the sensitivity  $E_{MAX}$  of a sensor [nW/mm²], which is specified in the datasheet, is always related to a specified exposure time (e.g nominal integration time  $t_{NOM}$  [µs]). The necessary integration time  $t_{APP}$  or a given sensor sensitivity is calculated according to the following formula:

Equation 10: 
$$t_{APP} \leqslant t_{NOM} \cdot \frac{E_{MAX}}{E_{PIXEL}}$$
 e.g. 
$$t_{APP} \leqslant 103 \mu s \cdot \frac{160 nW/mm^2}{42.34 nW/mm^2} = 389 \ \mu s$$

E<sub>MAX</sub> maximum irradiance of the imager: e.g. for epc610: 160nW/mm<sup>2</sup> @  $t_{NOM} = 103$ µs

 $t_{NOM}$  integration time for the specified irradiance  $E_{MAX}$ 

The integration time can also be influenced by the illumination. This means that you can use more or fewer emitters to adjust the light level accordingly. Refer to chapter 3.1.: Irradiance of the scenery.

# 4. Notes

epc has published a simple "reflected-power calculator" as an application note AN02.1. This allows the designer to do a rough power estimation for a system following the formulas of this application note.

AN02.1 Reflected power calculation	flected power calculation Input Data				Calculated Data			
Emitter	SFH-4059-R							
Emitting half-angle (I <sub>ELED</sub> = 50%)	$\Phi_{\text{LED}}$	10	[± deg]					
Minimum radiant intensity @ i <sub>NOM</sub>	min I <sub>E LED</sub>	33	[mW/sr]		Nominal radiant intensity @ i <sub>NOM</sub>	nom I <sub>E LED</sub>	100	[mW/sr]
Maximum radiant intensity @ i <sub>NOM</sub>	max I <sub>E LED</sub>	167	[mW/sr]					
Current for nominal radiant intensity	I <sub>NOM</sub>	70	[mA]					
Effective current per emitter @ application	i <sub>APP</sub>	180	[mA]					
Number of emitters @ application	n <sub>E</sub>	2	[pcs]					
Efficiency caused by total losses emitter	ν <sub>E</sub>	100	[%]					
Object / Target		White wall						
Distance camera lens to object	G <sub>R</sub>	0.60	[m]		Image size: length @ object	B <sub>RL</sub>	0.074	[m]
Field of view emitter (diameter; half angle)	$\alpha_{E}$	10	[± deg]		Image size: width @ object	B <sub>RW</sub>	0.074	[m]
Field of view receiver (diameter; half angle)	$\alpha_R$	5	[± deg]		Pixel size (length and width) @ object	B <sub>PIXEL</sub>	9.28	[mm]
					Spot diameter emitter @ object	B <sub>ED</sub>	0.212	[m]
					Spot diameter receiver @ object	B <sub>RD</sub>	0.105	[m]
					Ratio spot emitter / spot receiver	R <sub>R/E</sub>	2.015	[#]
Remission (reflectivity) of object	ρ	90	[%]		Irradiance @ object	E <sub>EO</sub>	1.396	[µW/mm
					Illuminated half angle receiver lens	$\phi_{LR}$	0.072	[± deg]
Receiver		epc610						
Number of pixels: length	n <sub>PL</sub>	8	[pcs]		Focal length receiver lens	f <sub>LR</sub>	2.575	[mm]
Number of pixels: width	n <sub>PW</sub>	8	[pcs]		Distance lens – receiver	f <sub>LR</sub>	2.586	[mm]
Pixel size (length and width) @ receiver	b <sub>PIXEL</sub>	40	[µm]					
Diameter receiver lens	D <sub>LR</sub>		[mm]					
Efficiency caused by total losses receiver	ν <sub>E</sub>	81	[%]					
Maximum irradiance @ t <sub>NOM</sub>	E <sub>R</sub>	160	[nW/mm²]		Irradiance @ receiver	E <sub>EO</sub>	42.793	[nW/mm
					Received light power @ receiver	Φ <sub>PIXEL</sub>	0.068	[nW]
Integration time for maximum irradiance	t <sub>NOM</sub>	103.00	fusl		Max. necessary integ. time @ application	t <sub>APP</sub>	385.11	fus1

For detailed description refer to the application note AN02 Reflected power calculation

Figure 3: Spreadsheet of the "reflected-power calculator"

Until now, all the considerations are based on the assumption of a subject near the optical axis and having small angles of view. In reality, the camera does not focus on a spot in most of the situations. It is focused onto a scenery with a wider angle of view ( (FOV)  $\alpha_E$  and  $\alpha_R$ ). For other points inside the observation fields, additional influencing effects have to be taken into consideration:

For the given radiation characteristics of the emitter, the radiant intensity drops down to 50% of its peak value at the half angle  $\phi_E$  (e.g. ±10deg). This leads to an inhomogeneous irradiation of an extended object.

The following parameters in the application are a function of the wavelength and of the bandwidth

- the relative spectral emission of the emitter.
- the relative spectral transmission of the lenses and of the used optical filters.
- the relative spectral sensitivity of the sensor.

Light passing beside the optical axis is additionally attenuated by Lambert's cos-law.

For more detailed designs, refer to either the specialized literature, to the scientific publications or use an optical simulation tool (e.g. the ZEMAX program).

#### 4.1. Cross-talk in the optical system path

Highly sensitive camera systems can easily be influenced by foreign or cross-talking light. This requires a careful design that does not under estimate such effects as reflections(??) inside the housing, over faceplates, etc. Design features like light traps, shutters, non-transparent materials for the used wavelengths or light hermetic closures help to overcome such phenomena.

### 5. Conclusion

All-in-one cameras for automation and for robotics have to be designed to the point of their specific need. This results in a "light-powerful" system with an optimized parameter setting (operating range, frame rate, exposure time, etc.) to achieve either a short reaction time, a long distance detection, a wide angle of view or 3D imaging. Most of these cameras need their own spotlight to be fully independent of the ambient illumination or to enable at all the principle functionality (e.g. of a 3D TOF camera). For the 3D TOF camera, this is a core requirement for having good accuracy over the operating range.

If you need more information, please go to www.espros.ch or contact us at info@espros.ch.

### 6. References

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