第7章 小波和多分辨率处理

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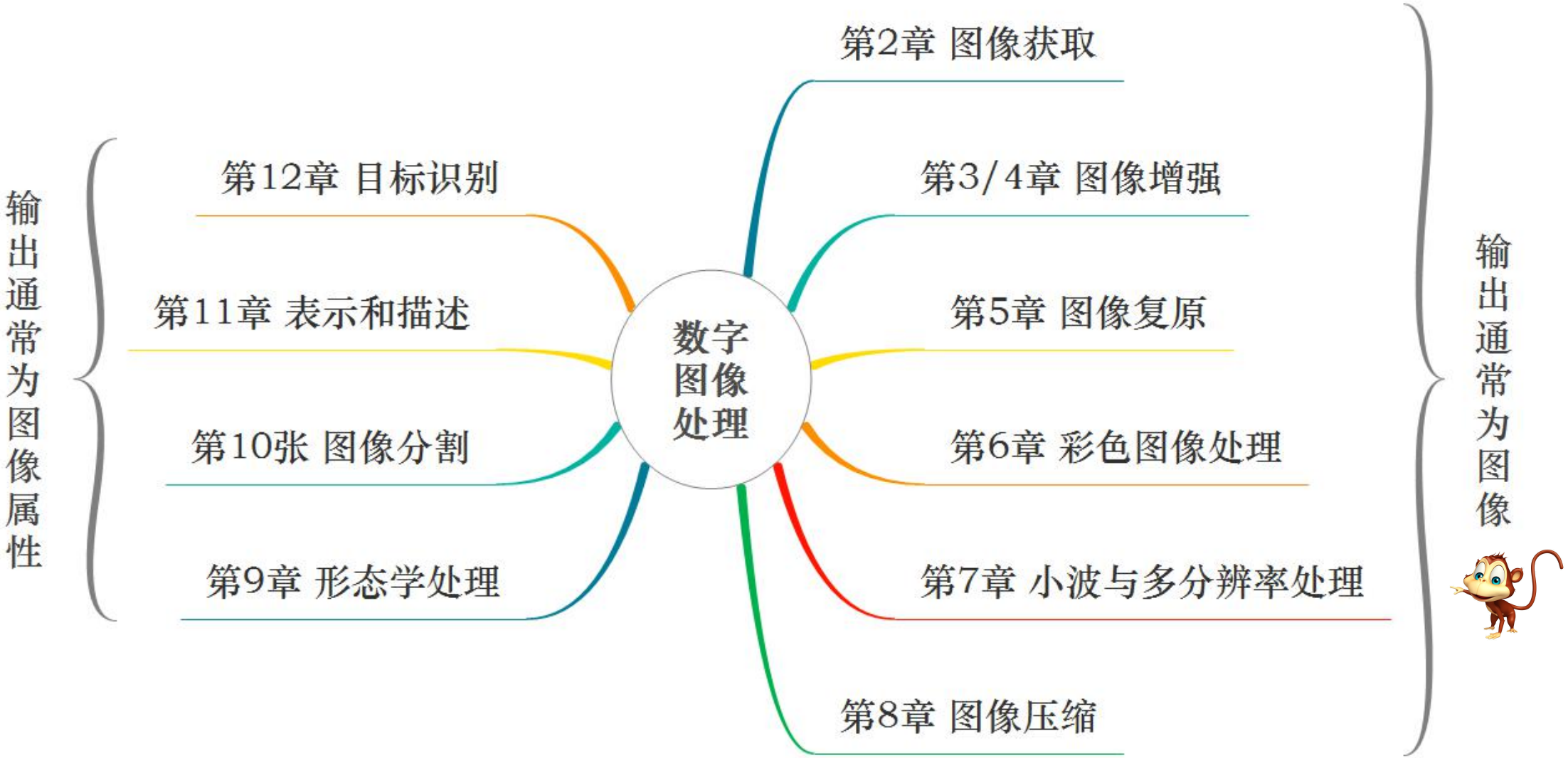
数字图像处理的基本步骤及内容



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- 6.1 彩色基础
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- 6.4 全彩色图像处理
  - 彩色变换
  - 平滑和锐化、彩色噪声
  - 彩色图像分割

6.4 全彩色图像处理

Full color

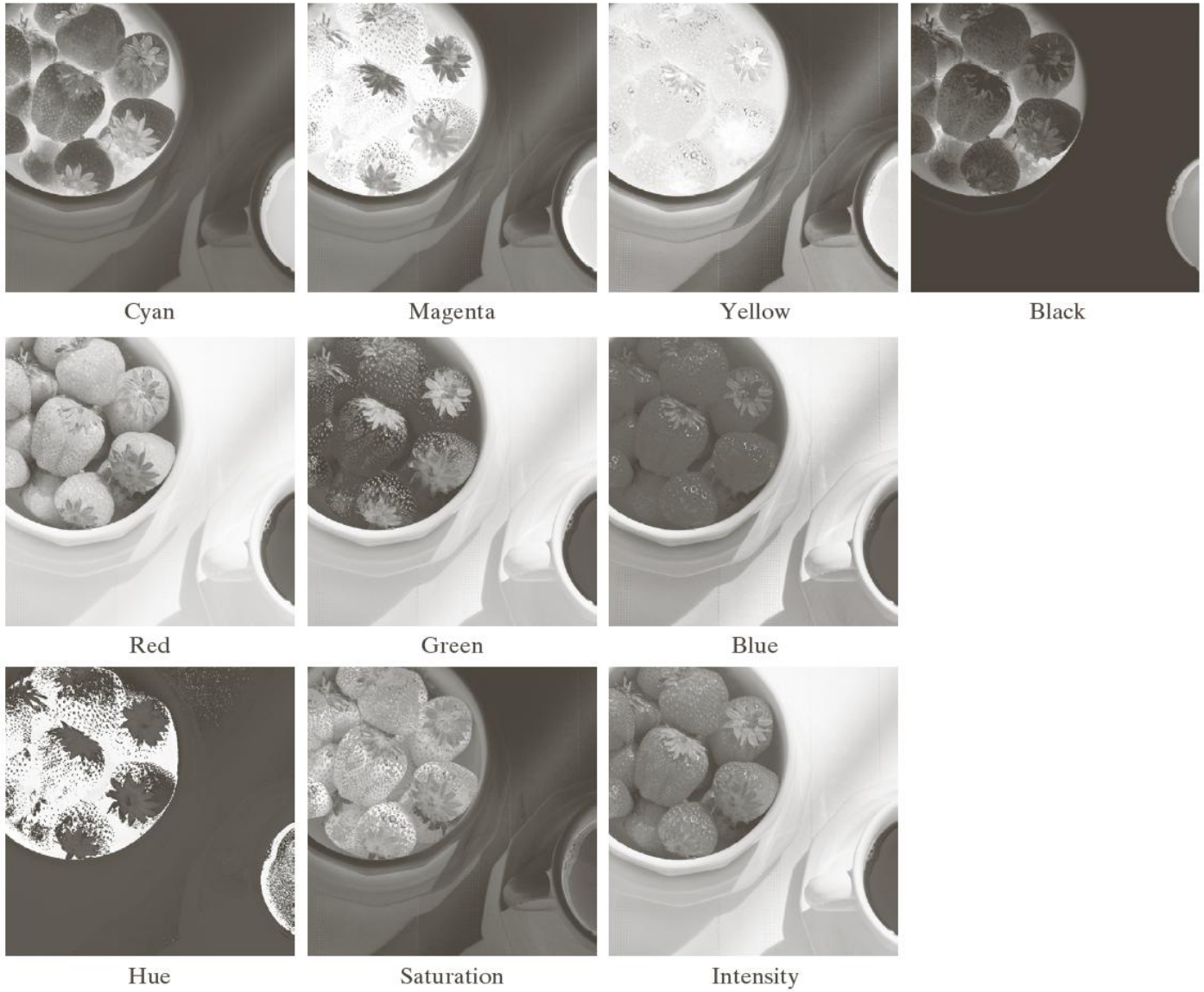


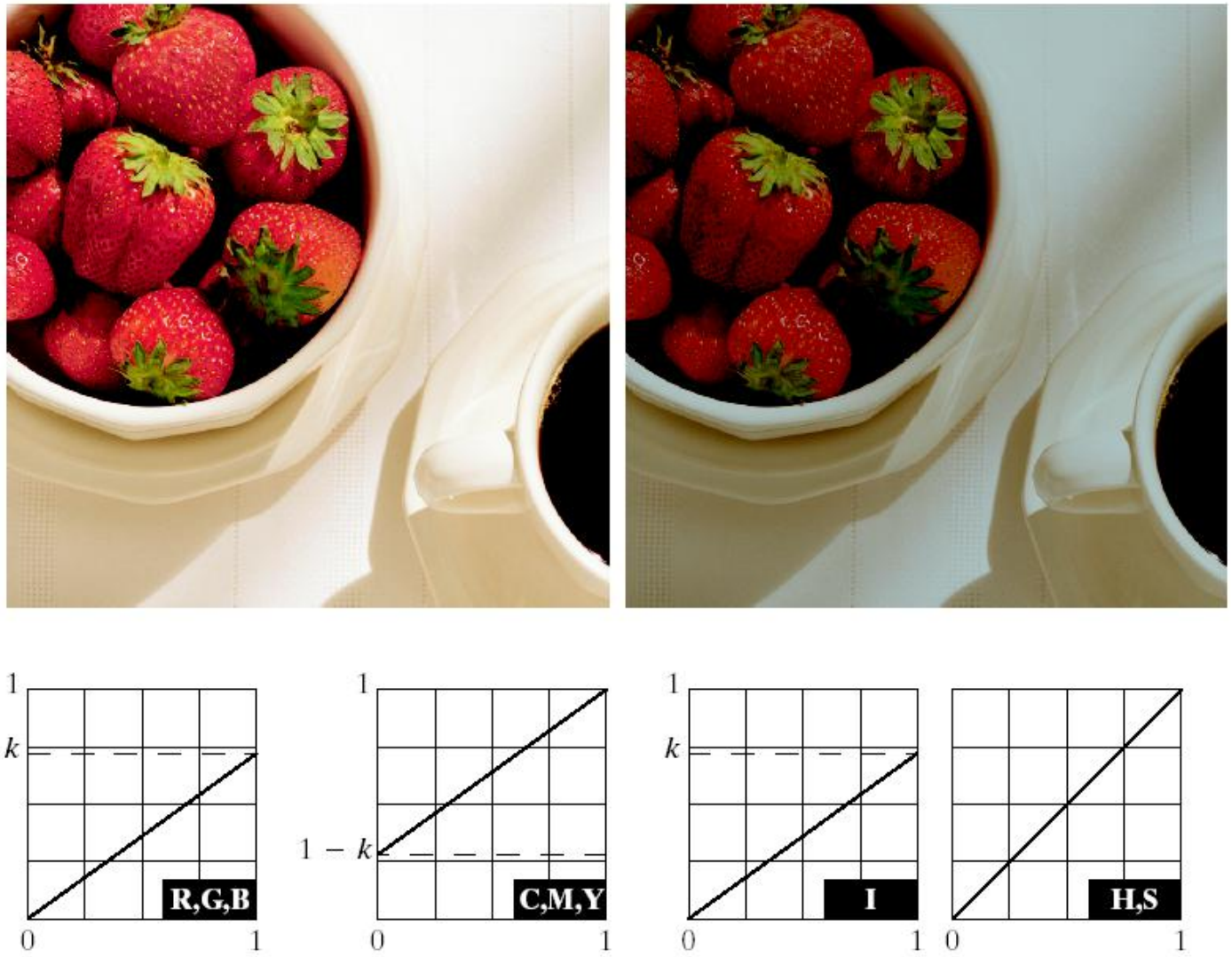
FIGURE 6.30 A full-color image and its various color-space components. Interactive.)

6.4 全彩色图像处理

- 如何将一幅图像调亮？
- 直方图均衡化？
- 彩色图像模糊/锐化
- 边缘检测
- 彩色噪声

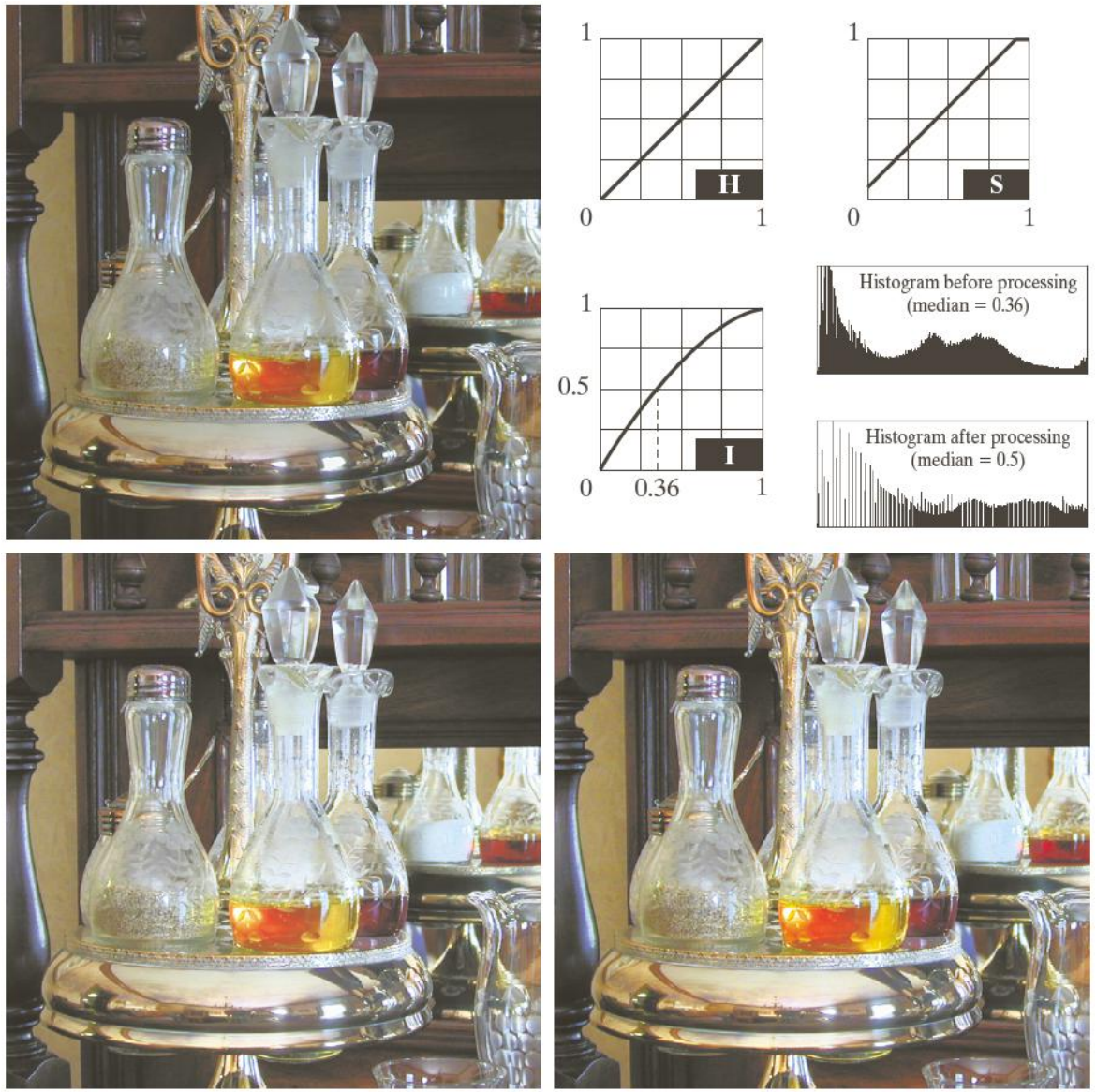
a b
c d e

FIGURE 6.31 Adjusting the intensity of an image using color transformations. (a) Original image. (b) Result of decreasing its intensity by 30% (i.e., letting $k = 0.7$). (c)–(e) The required RGB, CMY, and HSI transformation functions. (Original image courtesy of MedData Interactive.)



6.4 全彩色图像处理

- **色调和彩色校正**
  - 直方图处理
    - 均匀地展开彩色灰度
    - 保持彩色本身（即色调）不变



a b
c d

FIGURE 6.37 Histogram equalization (followed by saturation adjustment) in the HSI color space.

6.4 全彩色图像处理

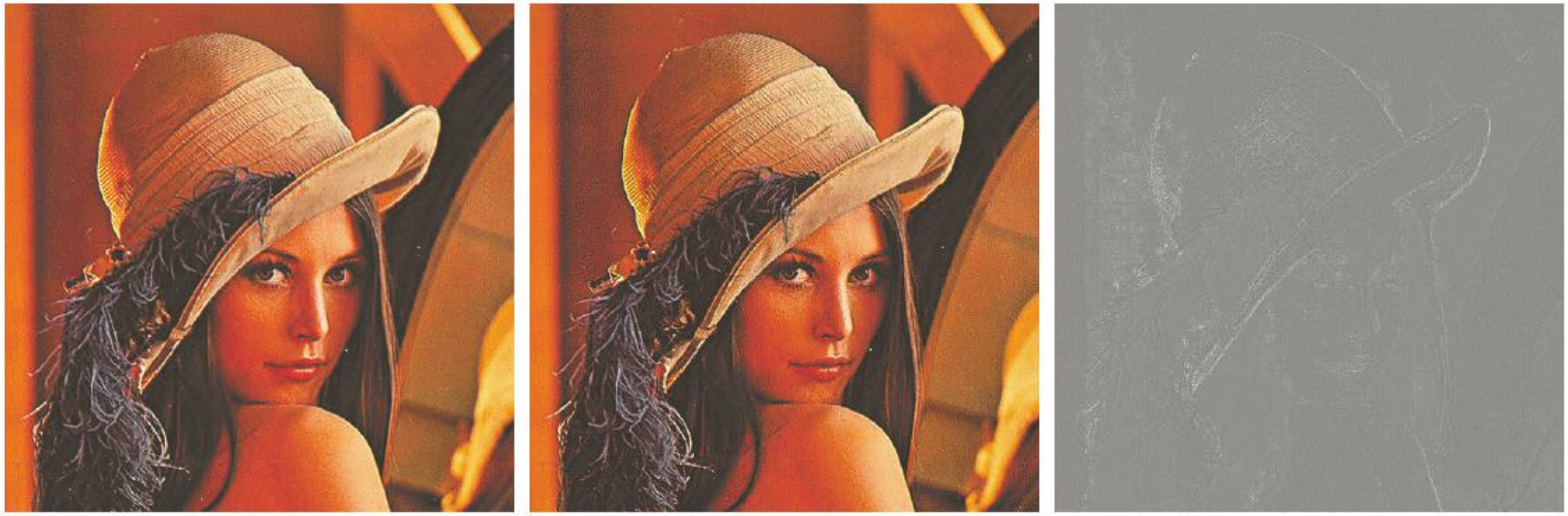
- **平滑和锐化**
  - 彩色图像平滑
    - 邻域平均法

a b c

FIGURE 6.40 Image smoothing with a 5×5 averaging mask. (a) Result of processing each RGB component image. (b) Result of processing the intensity component of the HSI image and converting to RGB. (c) Difference between the two results.

6.4 全彩色图像处理

- **平滑和锐化**
  - 彩色图像锐化
    - 拉普拉斯



a b c

FIGURE 6.41 Image sharpening with the Laplacian. (a) Result of processing each RGB channel. (b) Result of processing the HSI intensity component and converting to RGB. (c) Difference between the two results.

6.4 全彩色图像处理

- 彩色边缘检测

a b
c d

FIGURE 6.46
(a) RGB image.
(b) Gradient computed in RGB color vector space.
(c) Gradients computed on a per-image basis and then added.
(d) Difference between (b) and (c).

a b c

FIGURE 6.47 Component gradient images of the color image in Fig. 6.46. (a) Red component, (b) green component, and (c) blue component. These three images were added and scaled to produce the image in Fig. 6.46(c).

6.4 全彩色图像处理

- 彩色噪声

a b
c d

FIGURE 6.48
(a)–(c) Red, green, and blue component images corrupted by additive Gaussian noise of mean 0 and variance 800. (d) Resulting RGB image. [Compare (d) with Fig. 6.46(a).]

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- 7.1 图像金字塔
- 7.2 小波变换
- 7.3 二维小波变换

7.1 图像金字塔

- 图像局部变化统计特性难以描述

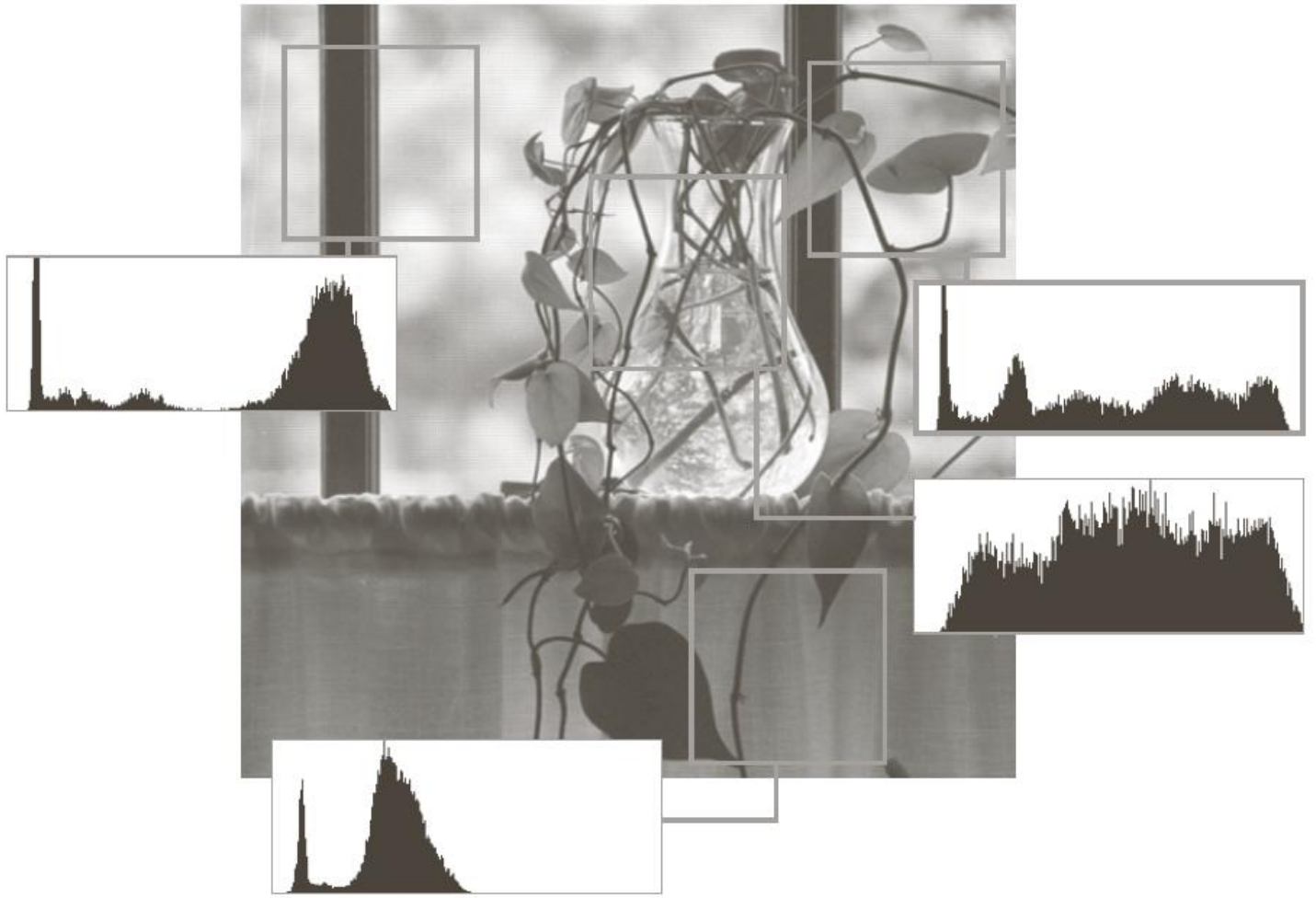
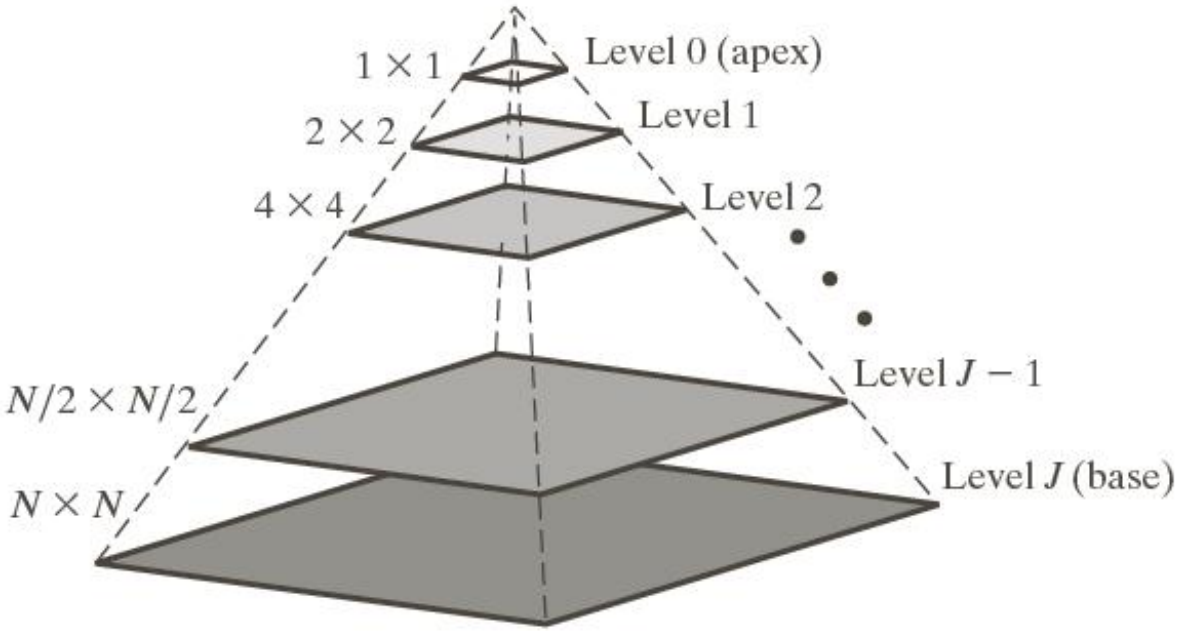


FIGURE 7.1
An image and its local histogram variations.

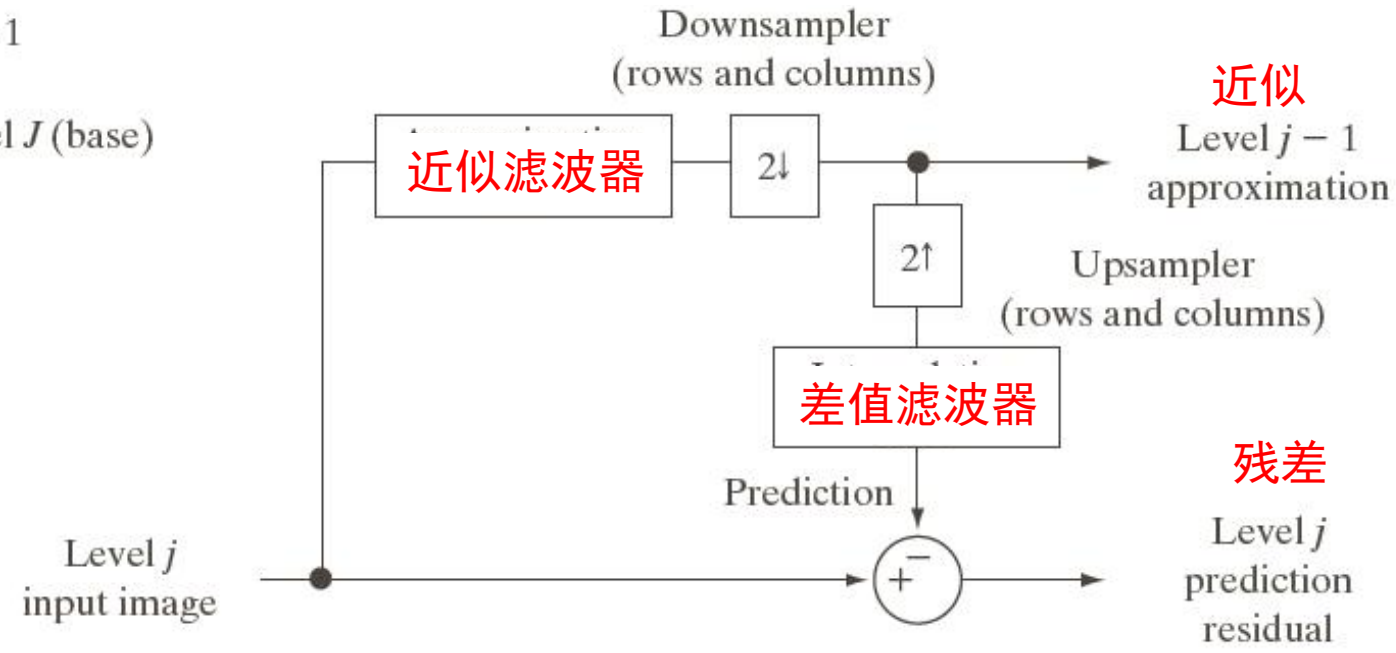
7.1 图像金字塔

- **图像金字塔**



a
b

FIGURE 7.2
(a) An image pyramid. (b) A simple system for creating approximation and prediction residual pyramids.

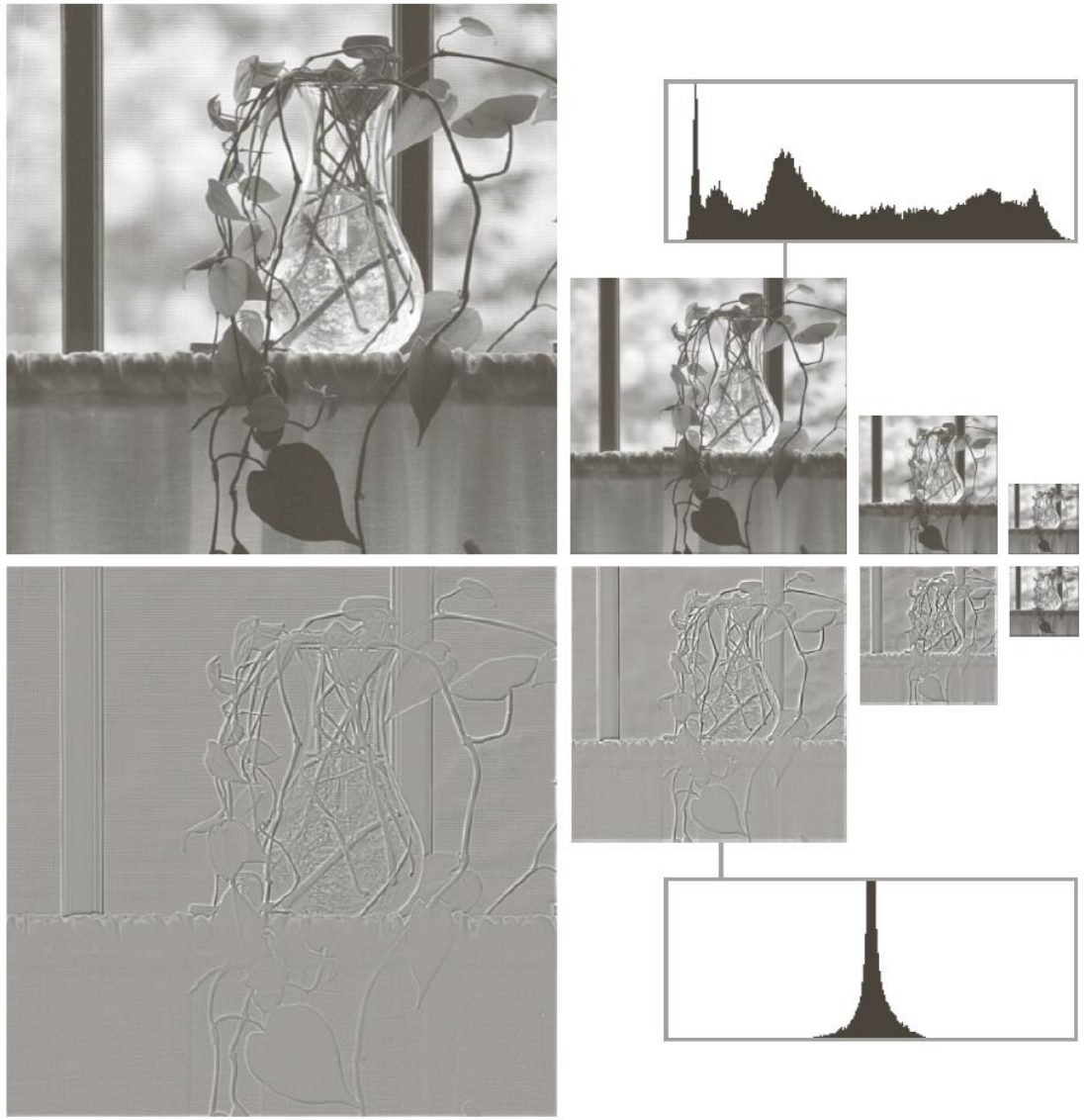


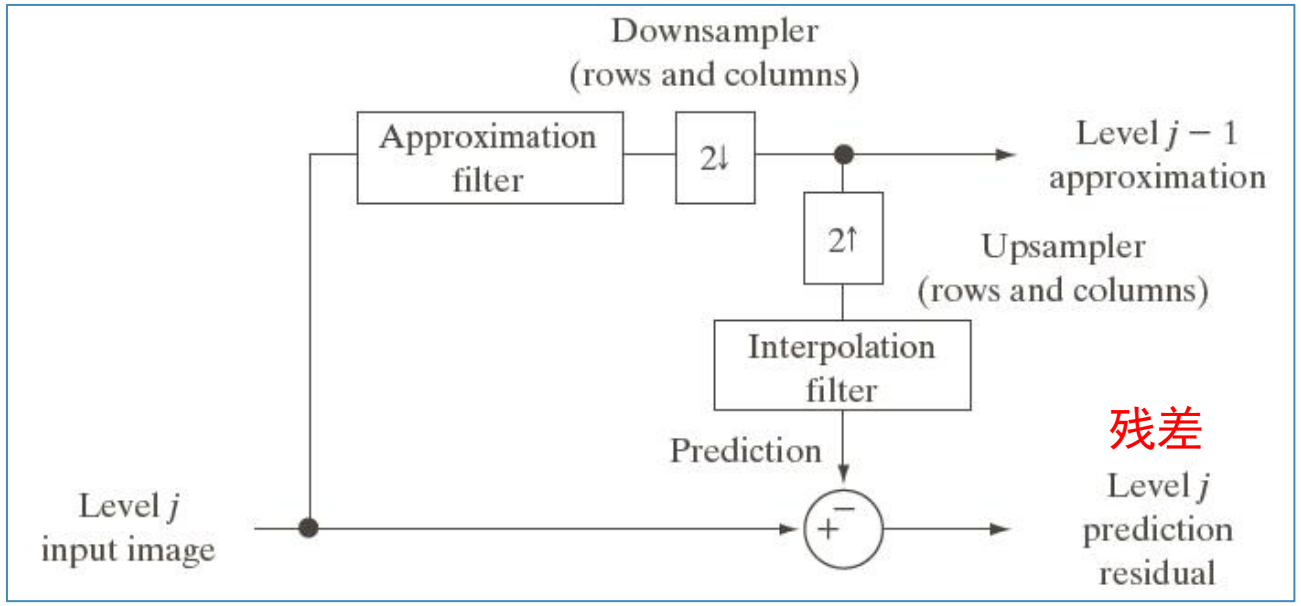
7.1 图像金字塔

- **图像金字塔：近似滤波器**
  - 邻域平均→平均金字塔
  - 低通高斯滤波→高斯金字塔
  - 不过滤→子采样金字塔

a
b

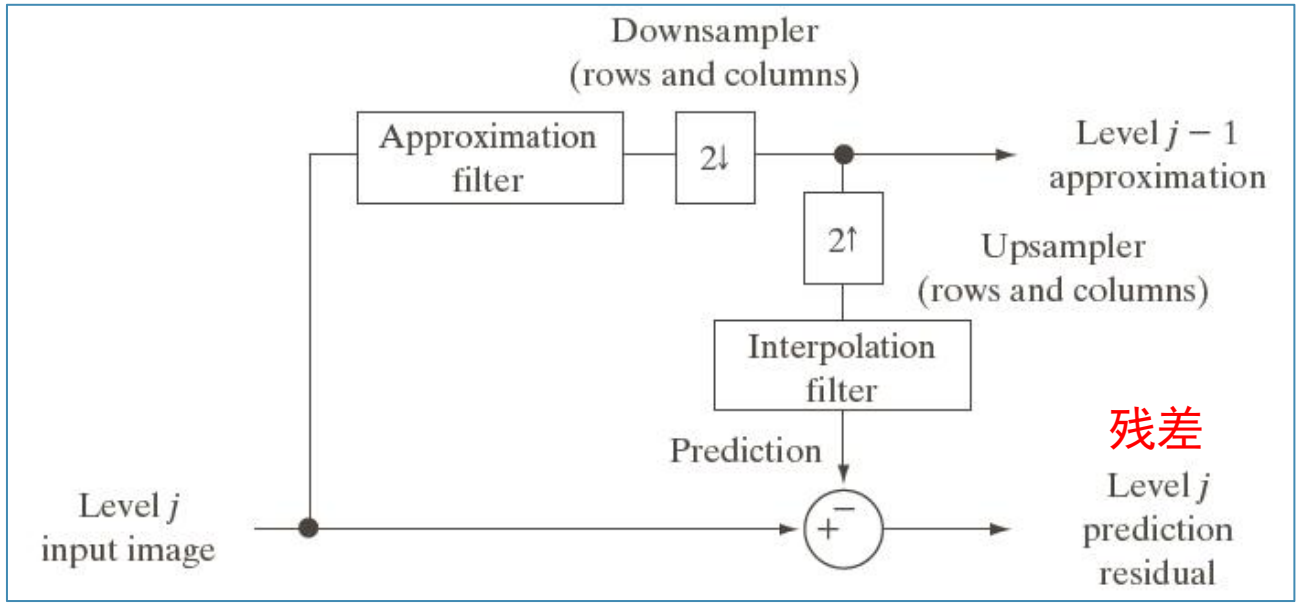
FIGURE 7.3 Two image pyramids and their histograms: (a) an approximation pyramid; (b) a prediction residual pyramid.





7.1 图像金字塔

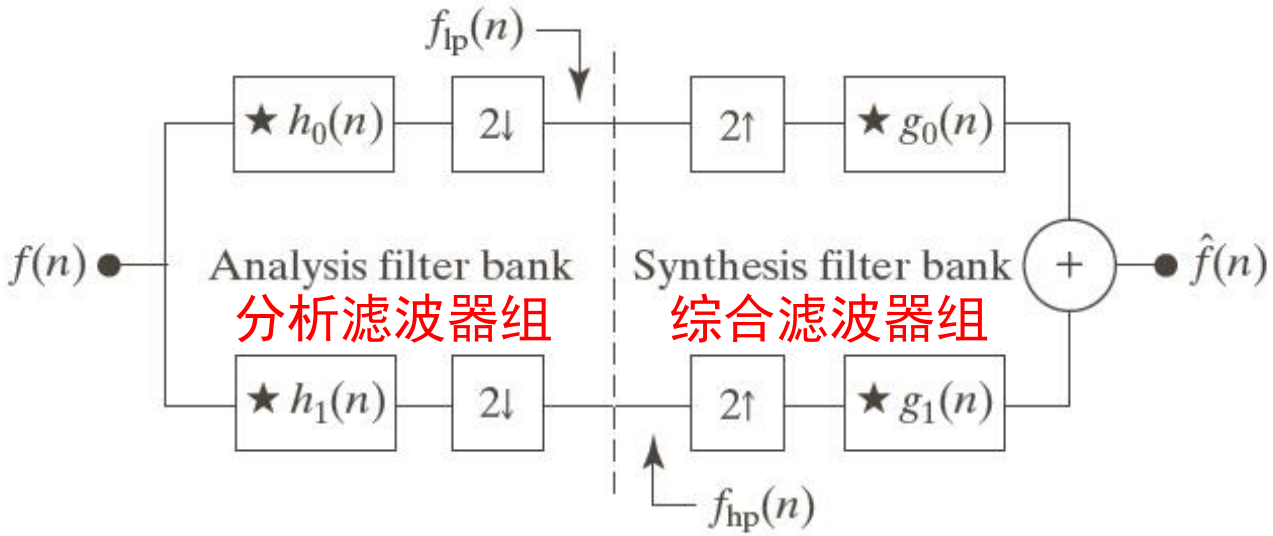
- 练习：为如下图新构建一个全填充的近似金字塔和相应的预测金字塔



11	12	13	14
15	16	17	18
19	20	21	22
23	24	25	26

7.1 图像金字塔

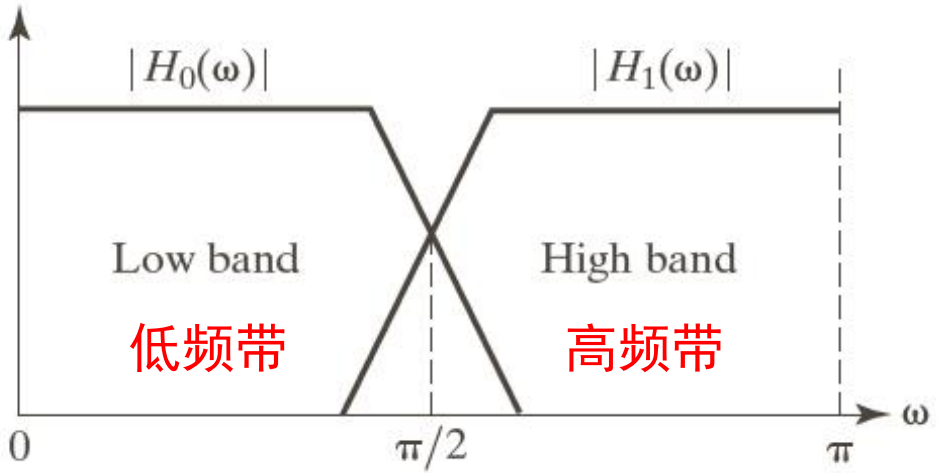
- **子带编码**
  - 子带：一幅图像被分解为一组频带受限的分量



a
b

FIGURE 7.6
(a) A two-band subband coding and decoding system, and (b) its spectrum splitting properties.

  - 右图为一个二带子带编码和解码系统



频谱可分性

7.1 图像金字塔

- **二维子带编码**
  - 近似子带 $a(m,n)$
  - 垂直细节子带 $d^V(m,n)$
  - 水平细节子带 $d^H(m,n)$
  - 对角线细节子带 $d^D(m,n)$

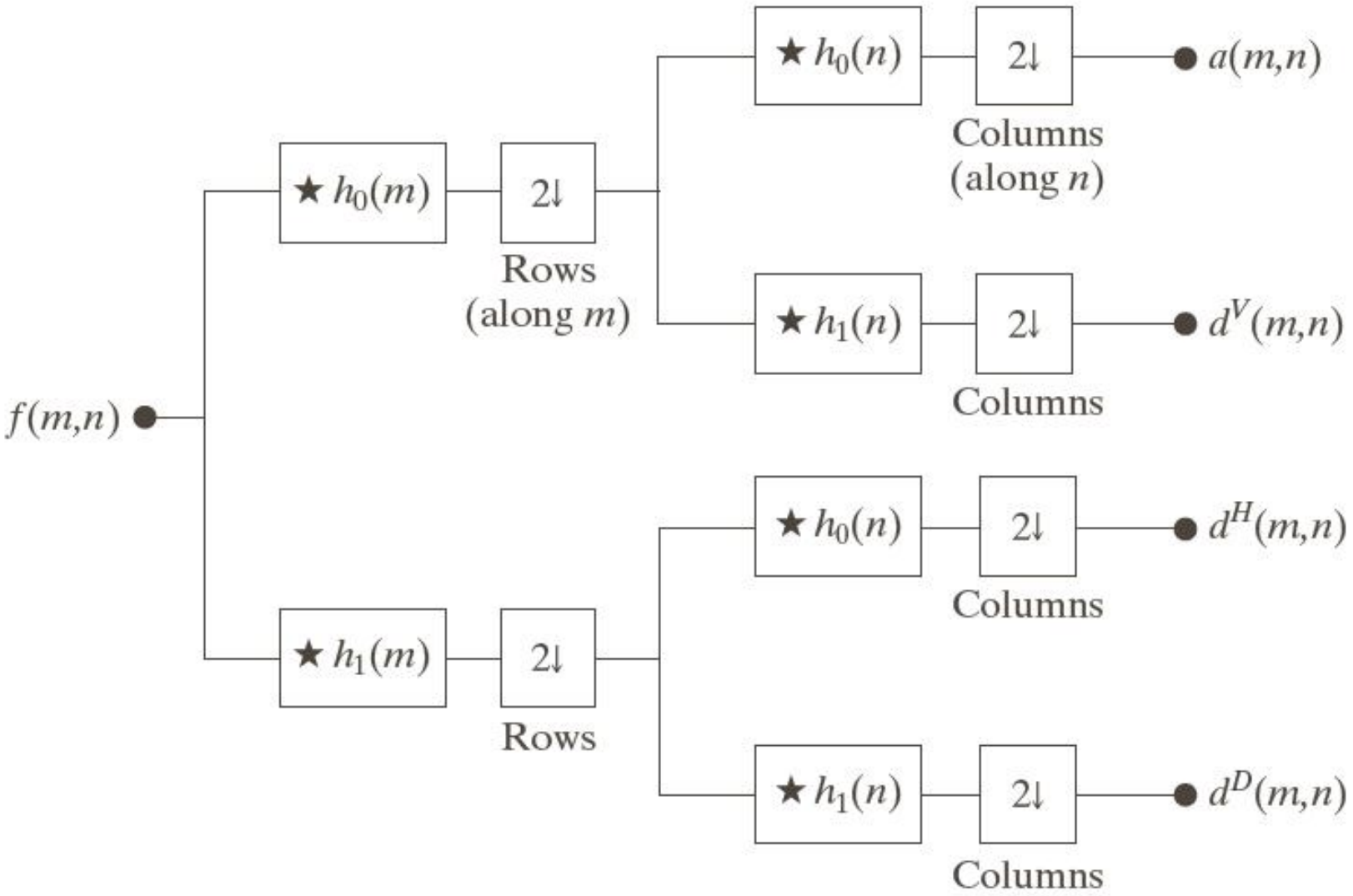
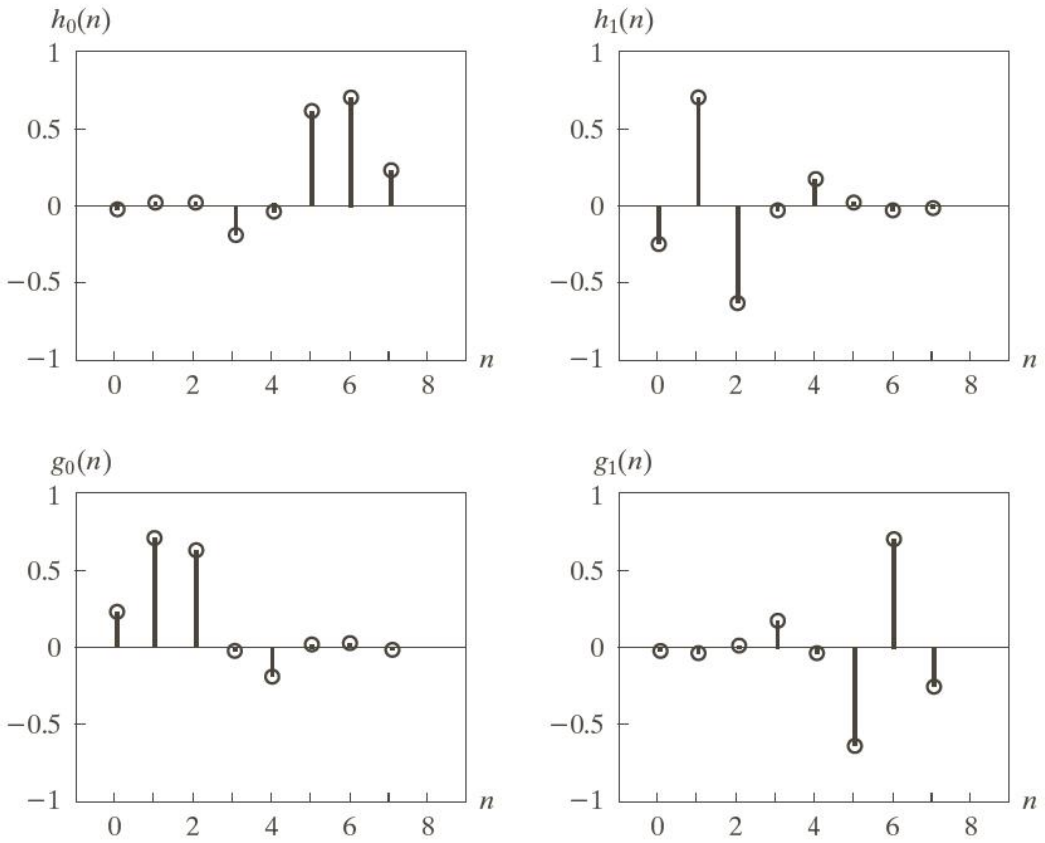


FIGURE 7.7
A two-dimensional, four-band filter bank for subband image coding.

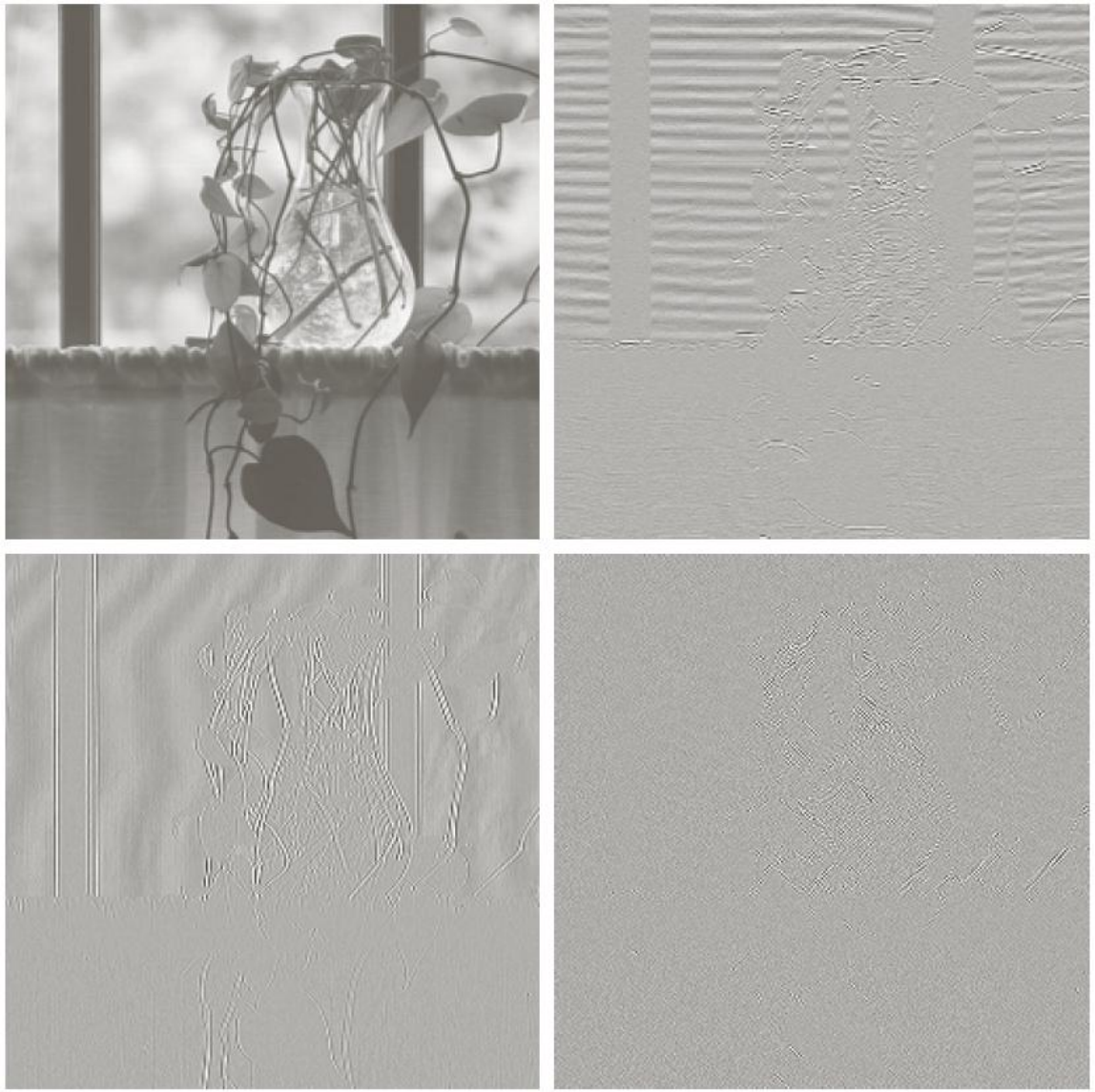
7.1 图像金字塔

- 二维子带编码



a b
c d

FIGURE 7.8 The impulse responses of four 8-tap Daubechies orthonormal filters. See Table 7.1 for the values of $g_0(n)$ for $0 \le n \le 7$.



a b
c d

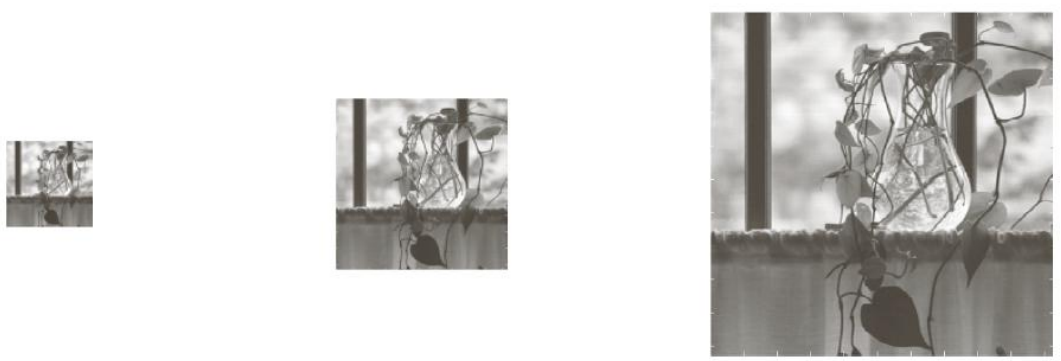
FIGURE 7.9 A four-band split of the vase in Fig. 7.1 using the subband coding system of Fig. 7.7. The four subbands that result are the (a) approximation, (b) horizontal detail, (c) vertical detail, and (d) diagonal detail subbands.

7.1 图像金字塔

- 二维子带编码

a
b c d

FIGURE 7.10
(a) A discrete wavelet transform using Haar $\mathbf{H}_2$ basis functions. Its local histogram variations are also shown. (b)–(d) Several different approximations (64×64, 128×128, and 256×256) that can be obtained from (a).



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- 7.2 小波变换
- 7.3 二维小波变换

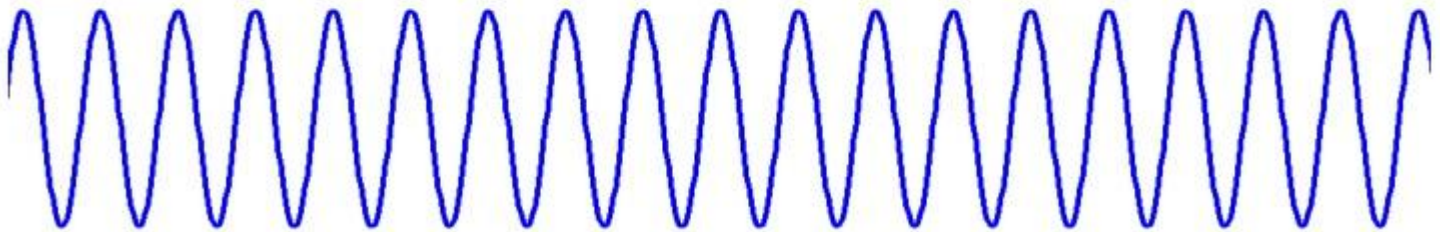
7.2 小波变换

傅里叶变换

$$F(w) = \int_{-\infty}^{\infty} f(t) * e^{-iwt} dt$$

原信号

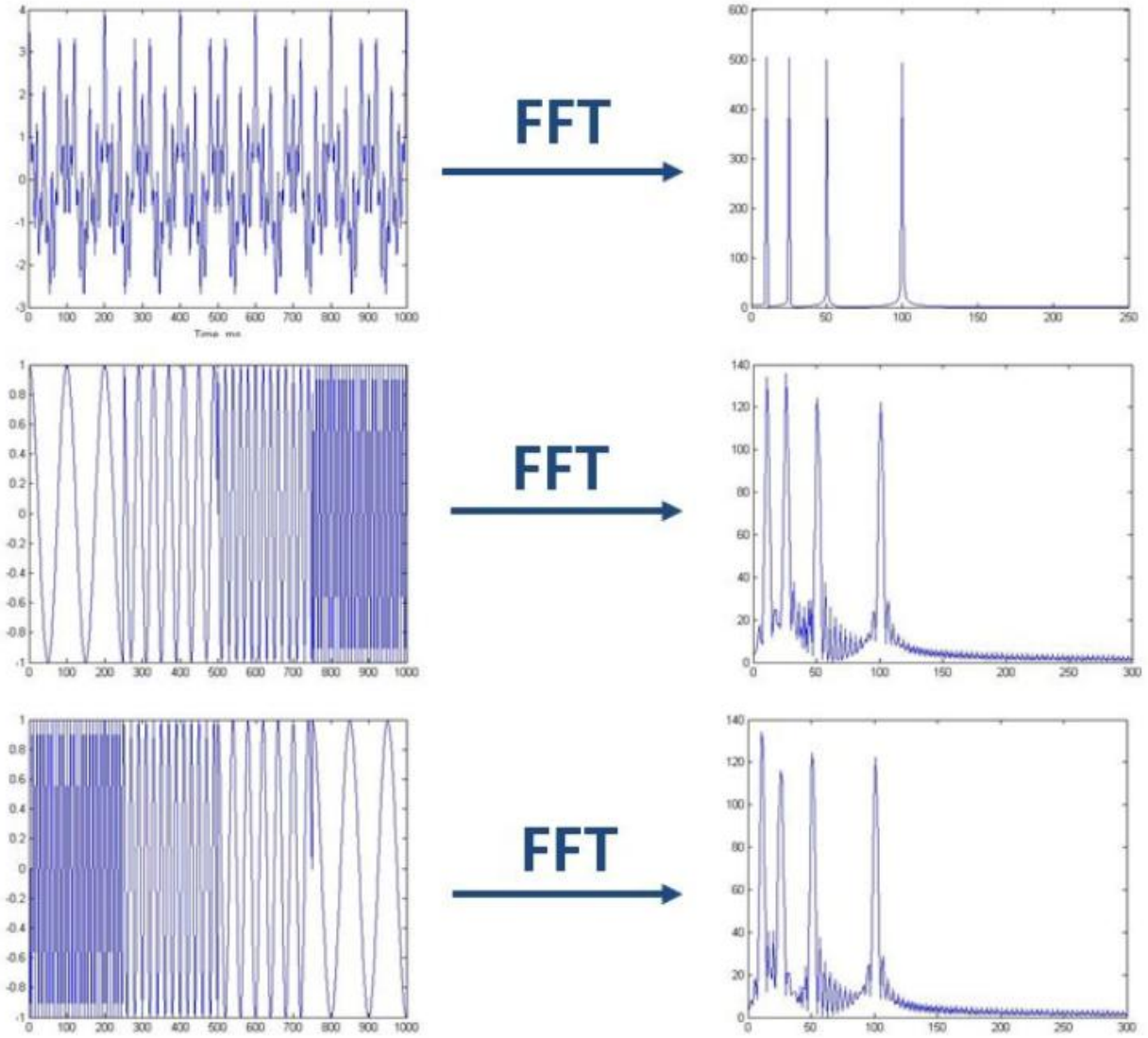
基函数



铺满了整个时域

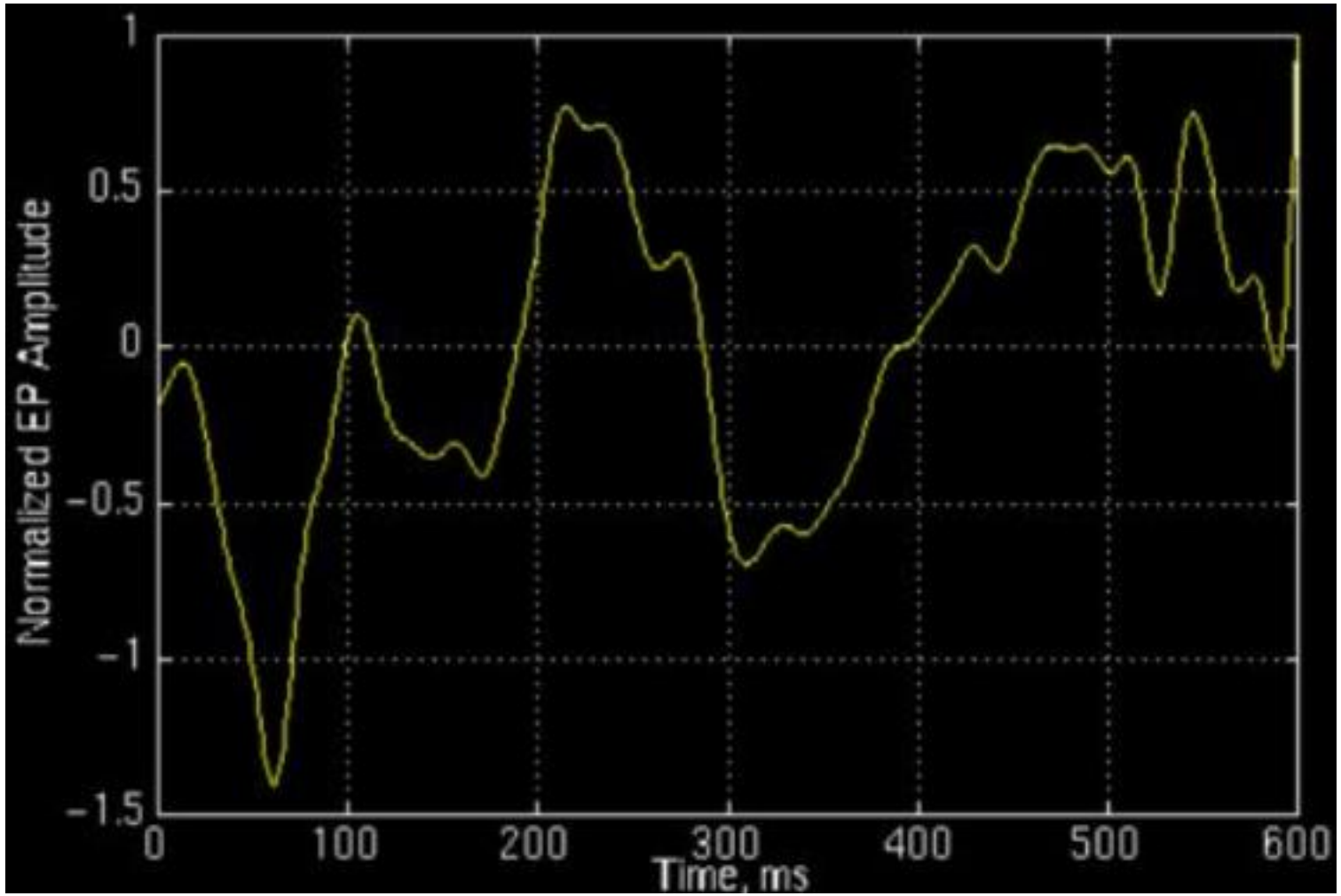
7.2 小波变换

- **非平稳信号**
  - 频率随着时间变化



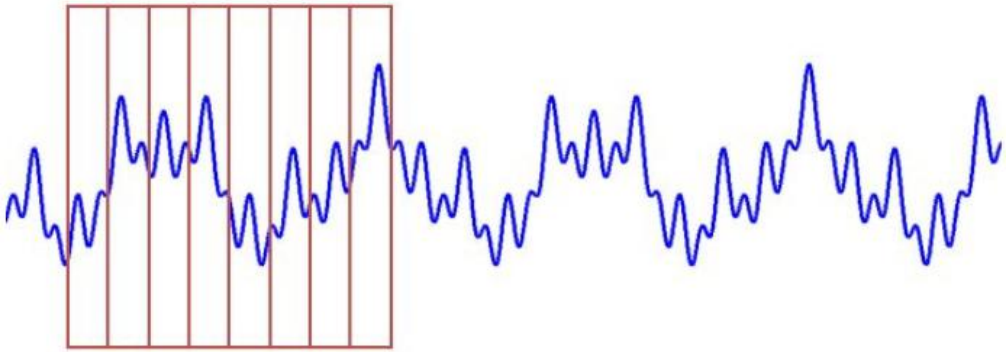
7.2 小波变换

- 平稳信号大多是人为制造出来的，自然界的大量信号几乎都是非平稳的

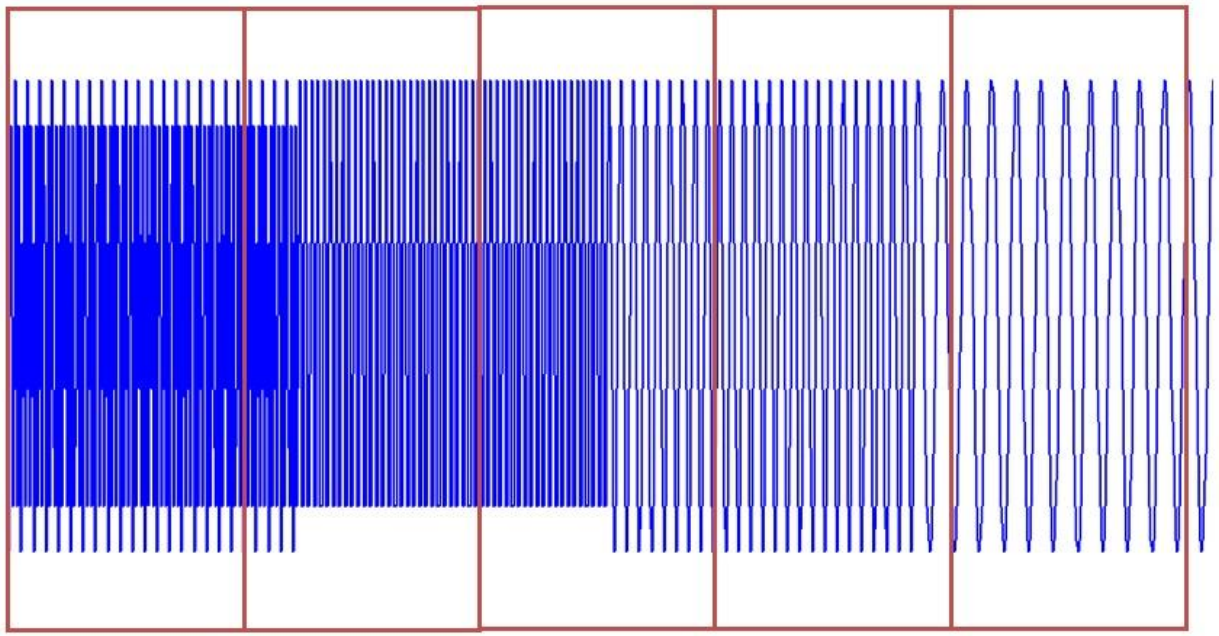


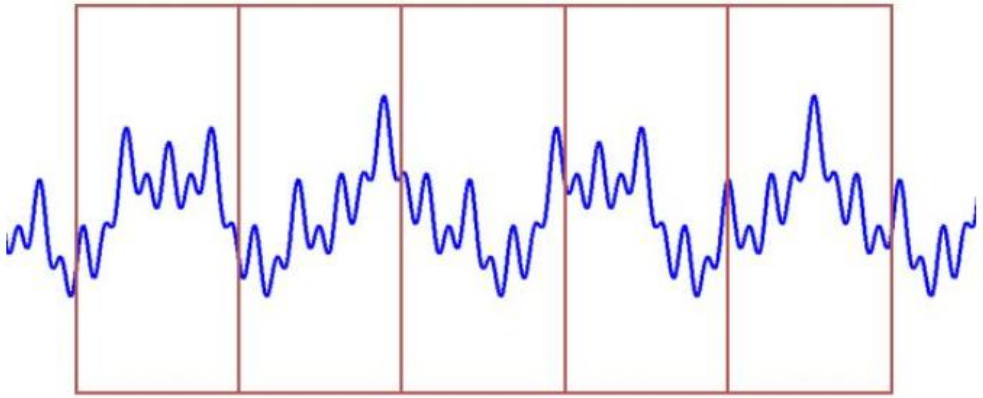
7.2 小波变换

- **短时傅里叶变换**
  - 窗太窄，窗内的信号太短，会导致频率分析不够精准，频率分辨率差。
  - 窗太宽，时域上又不够精细，时间分辨率低。



框太窄 → 频率分辨率差

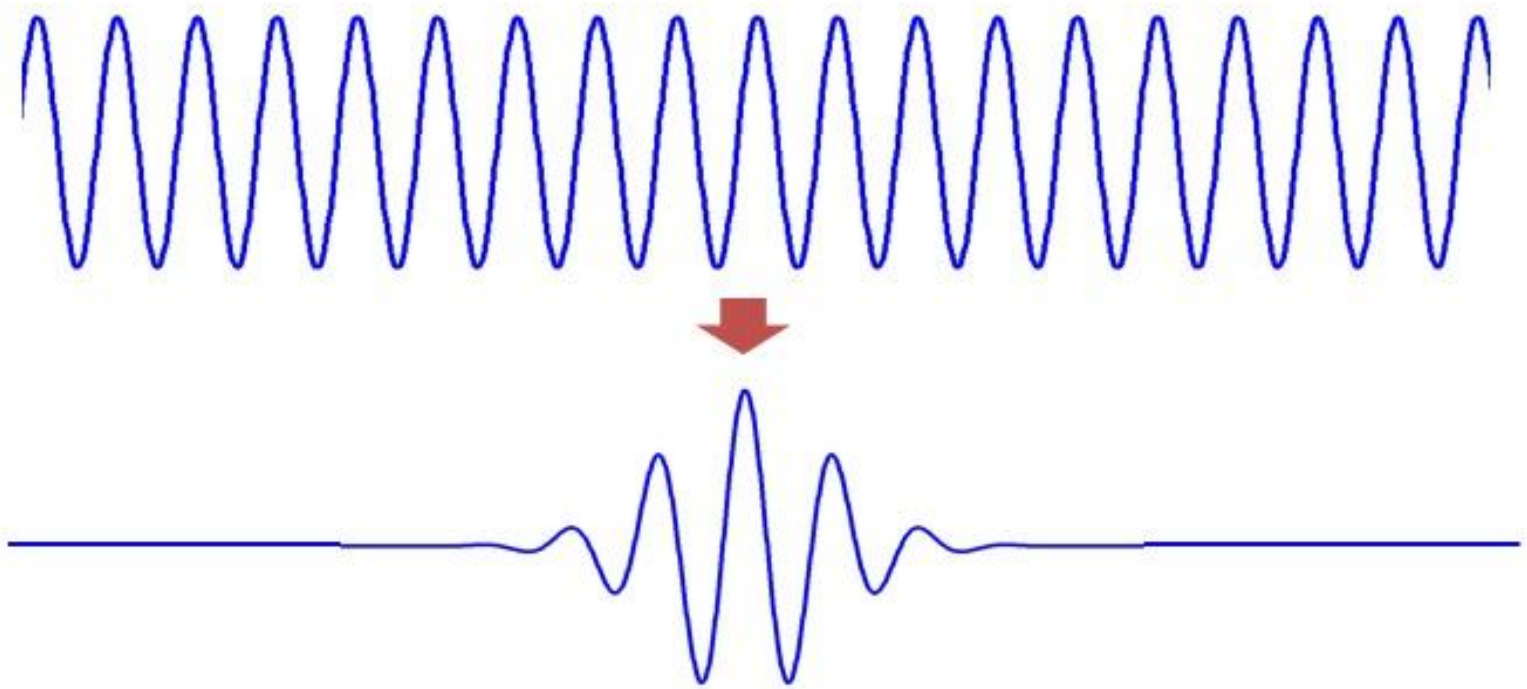




框太宽 → 时间分辨率差

小波变换

$$F(w) = \int_{-\infty}^{\infty} f(t) * e^{-iwt} dt \quad \Rightarrow \quad WT(a,\tau) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} f(t) * \psi(\frac{t-\tau}{a}) dt$$

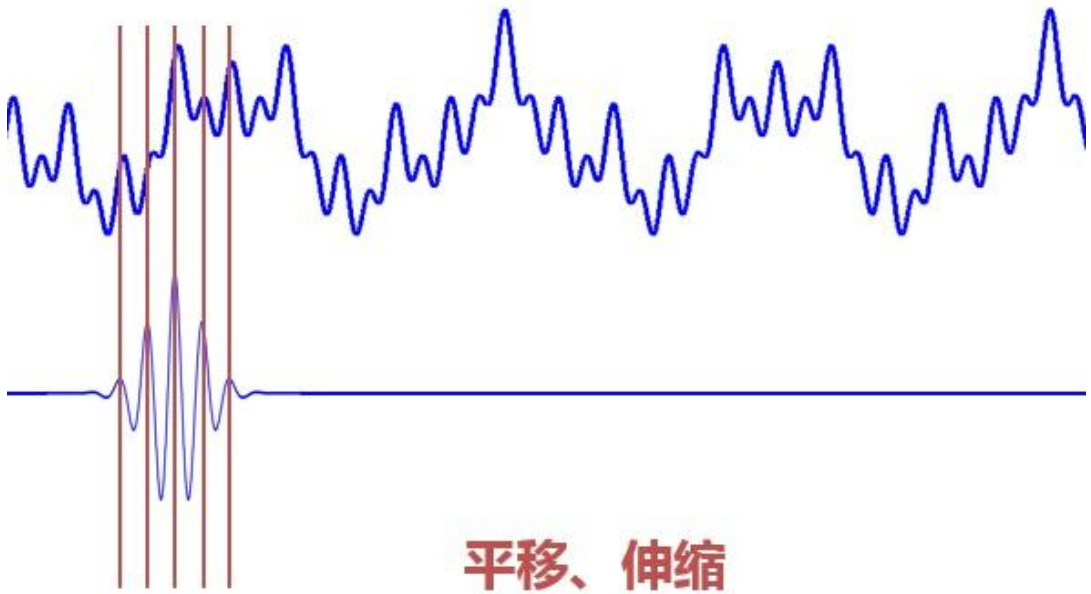


7.2 小波变换

- **小波**
  - 傅里叶变换，变量只有频率ω，

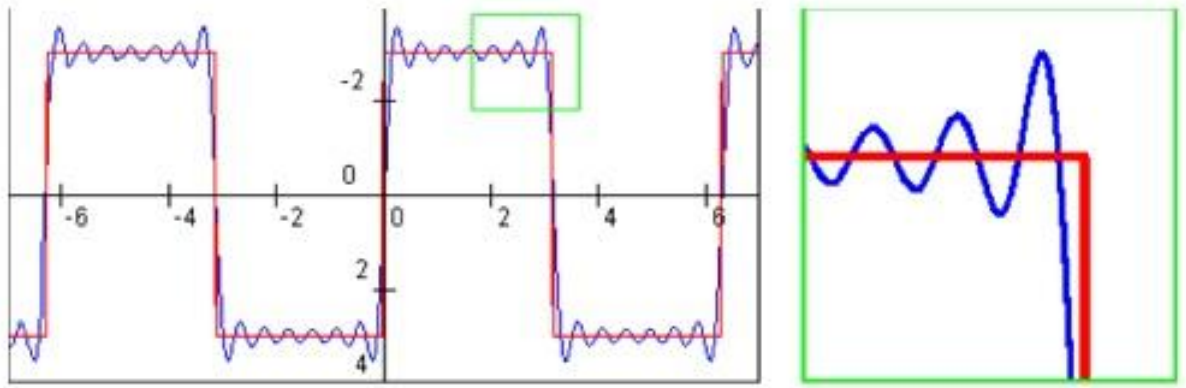
$$WT(a,\tau)=\frac{1}{\sqrt{a}}\int_{-\infty}^{\infty}f(t)*\psi(\frac{t-\tau}{a})dt$$

  - 小波变换有两个变量：
    - 尺度a（scale）和平移量 τ（translation）。
      - 尺度a控制小波函数的伸缩，
      - 平移量 τ控制小波函数的平移。
    - 尺度就对应于频率（反比），平移量 τ就对应于时间。



7.2 小波变换

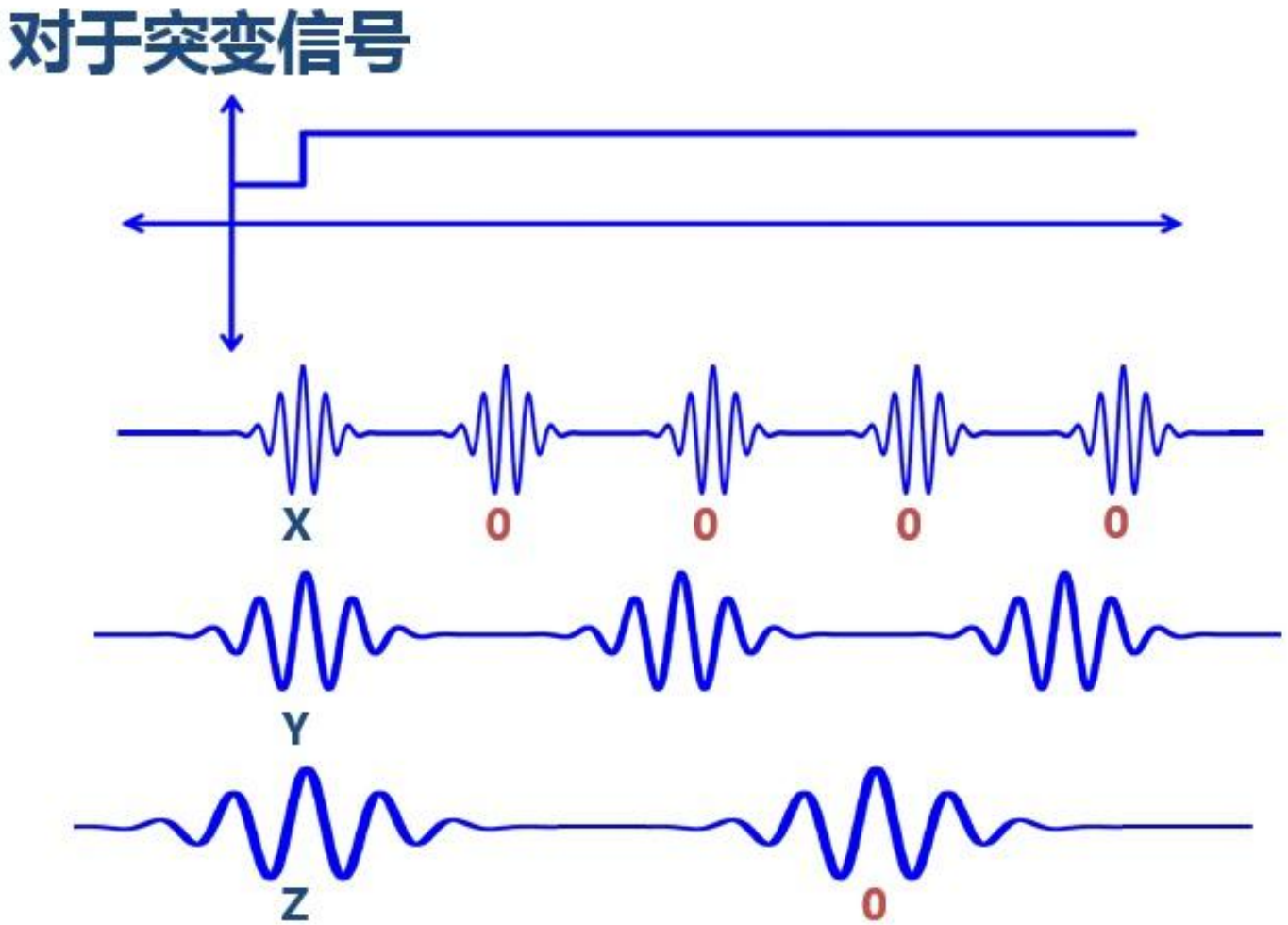
对于突变信号



对于这种变换突然而剧烈的信号，即使只有一小段变换，傅立叶也不得不用大量的三角波去拟合。

——吉布斯效应

7.2 小波变换

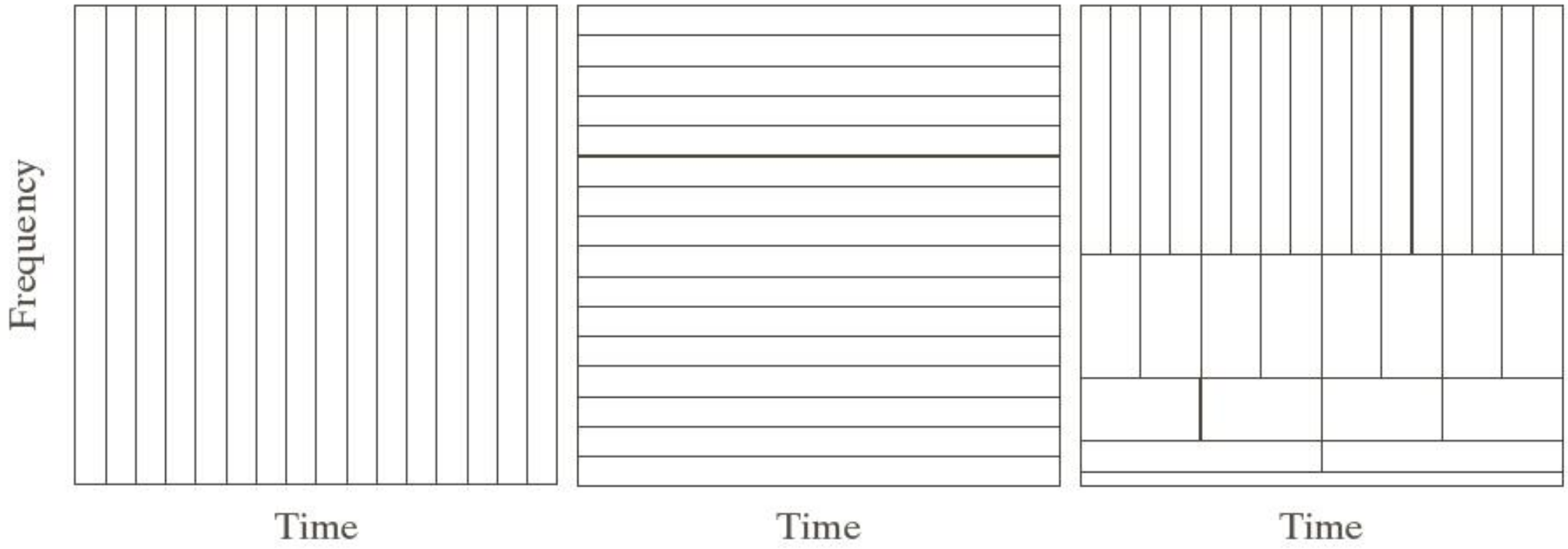


只有小波函数和信号突变处重叠时，系数不为0

7.2 小波变换

- **时间—频率片**

解释?



a b c

FIGURE 7.23 Time-frequency tilings for the basis functions associated with (a) sampled data, (b) the FFT, and (c) the FWT. Note that the horizontal strips of equal height rectangles in (c) represent FWT scales.

7.2 小波变换

- **小波变换基础**
- **3个概念**
  - (1) 序列展开

$$f(x)=\sum_k a_k u_k(x)$$

$$a_k=\langle u'_k(x), f(x)\rangle=\int u'^{*}_k(x) f(x)\mathrm{d}x$$

  - (2) 缩放函数

$$u_{j,k}(x)=2^{j/2}u(2^j x-k)$$

$$u(x)=\sum_k h_u(k)\sqrt{2}\,u(2x-k)$$

  - (3) 小波函数

$$v_{j,k}(x)=2^{j/2}v(2^j x-k)$$

$$f(x)=\sum_k a_k v_{j,k}(x)$$

7.2 小波变换

- **小波变换基础**

$$f(x)=\sum_k a_k u_k(x)$$

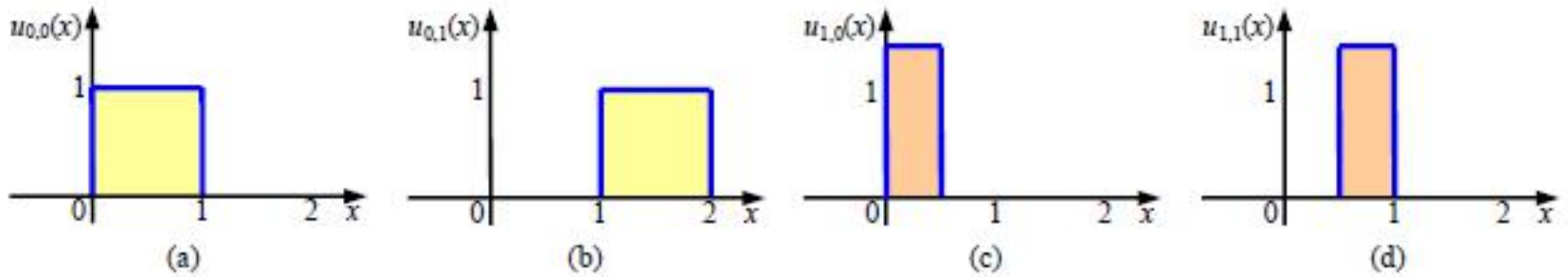


图 8.3.1　U_0和 U_1中的缩放函数

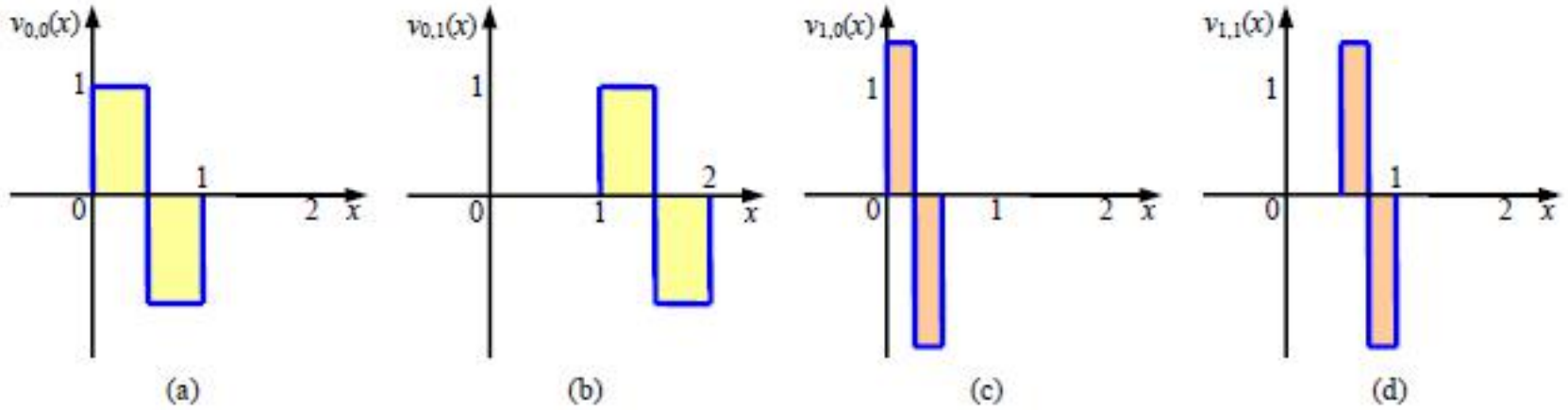


图 8.3.2　V_0和 V_1中的小波函数

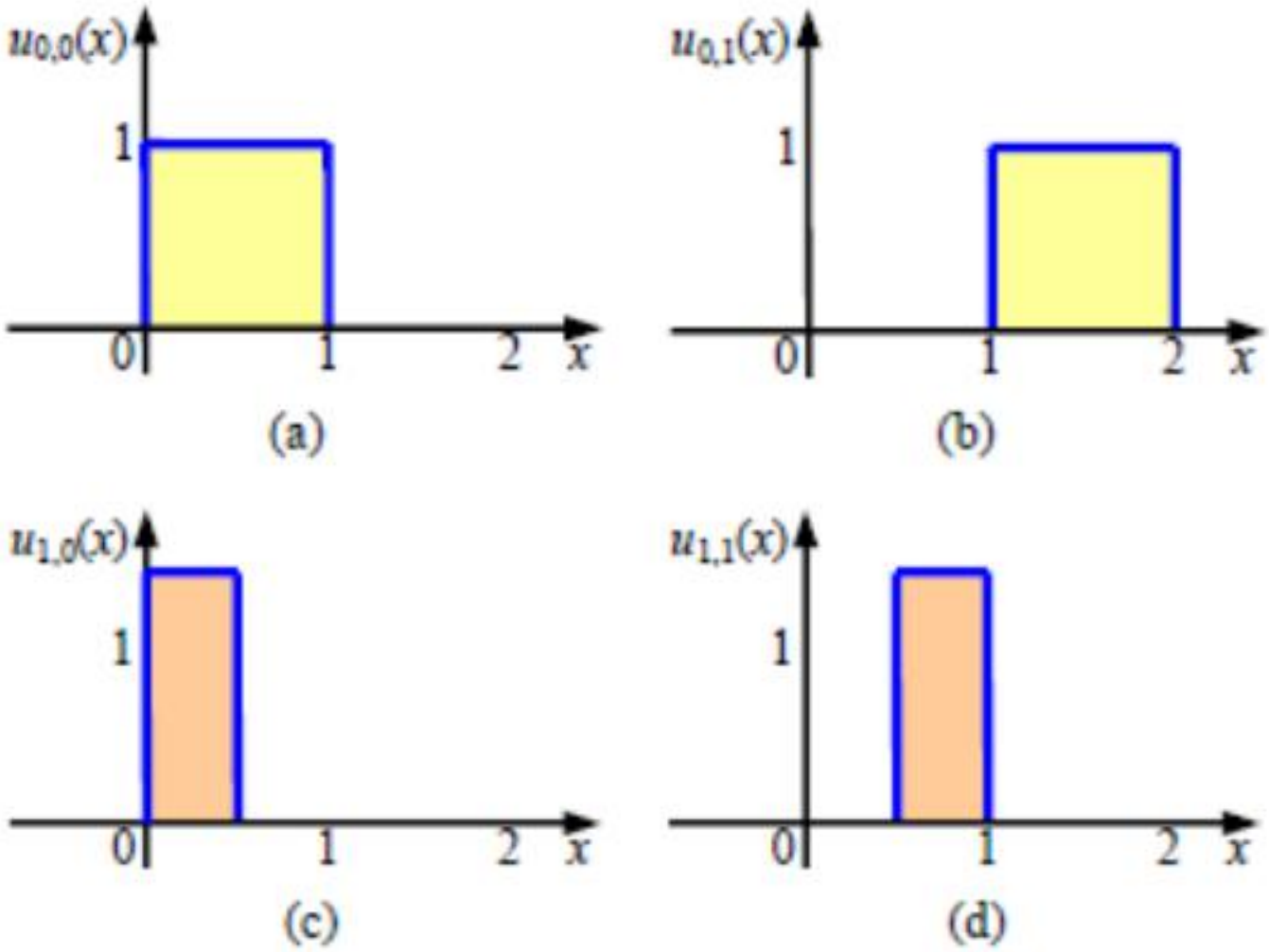
$$u_{j,k}(x)=2^{j/2}u(2^j x-k)$$

$$v_{j,k}(x)=2^{j/2}v(2^j x-k)$$

7.2 小波变换

- **小波变换基础**

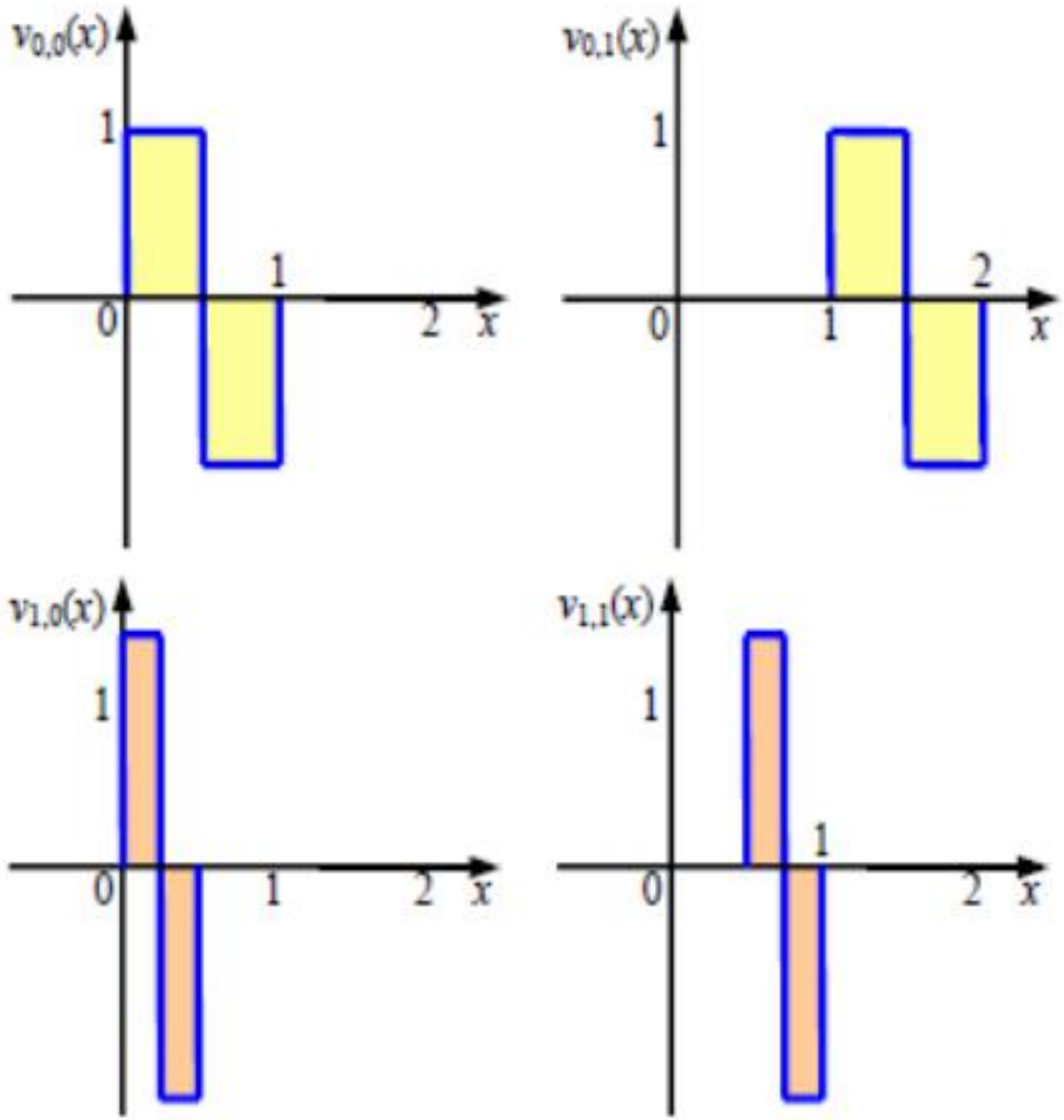
$$u_{j,k}(x) = 2^{j/2} u(2^j x - k)$$



7.2 小波变换

- 小波变换基础

$$v_{j,k}(x) = 2^{j/2} v(2^j x - k)$$



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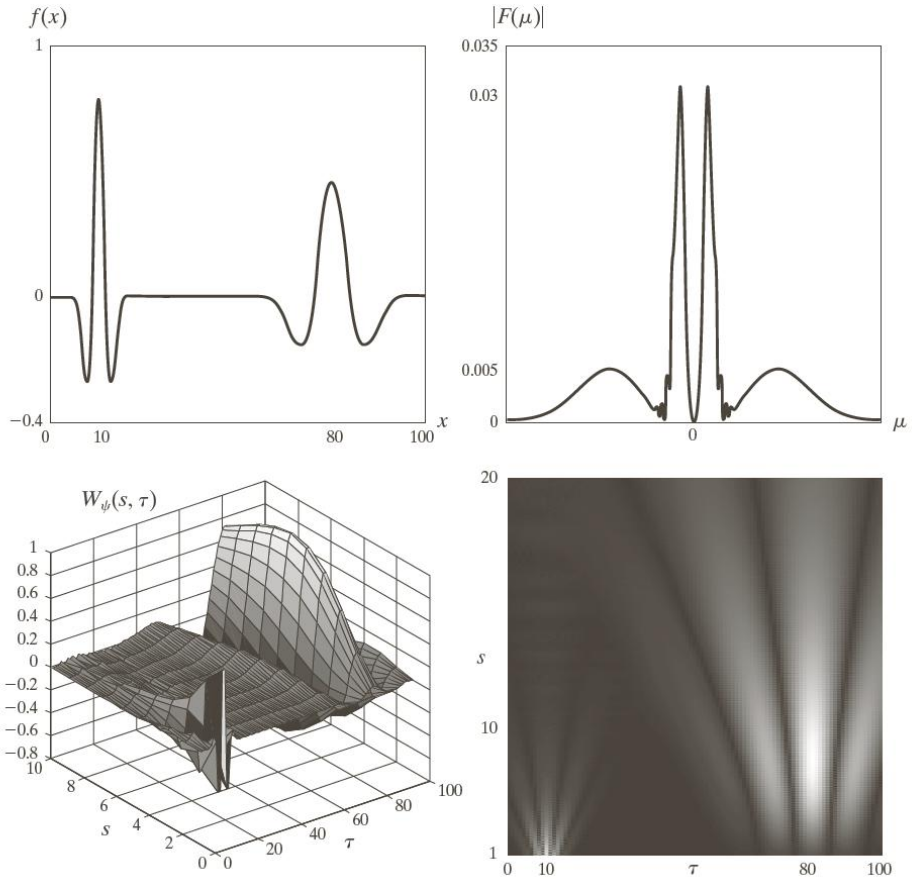
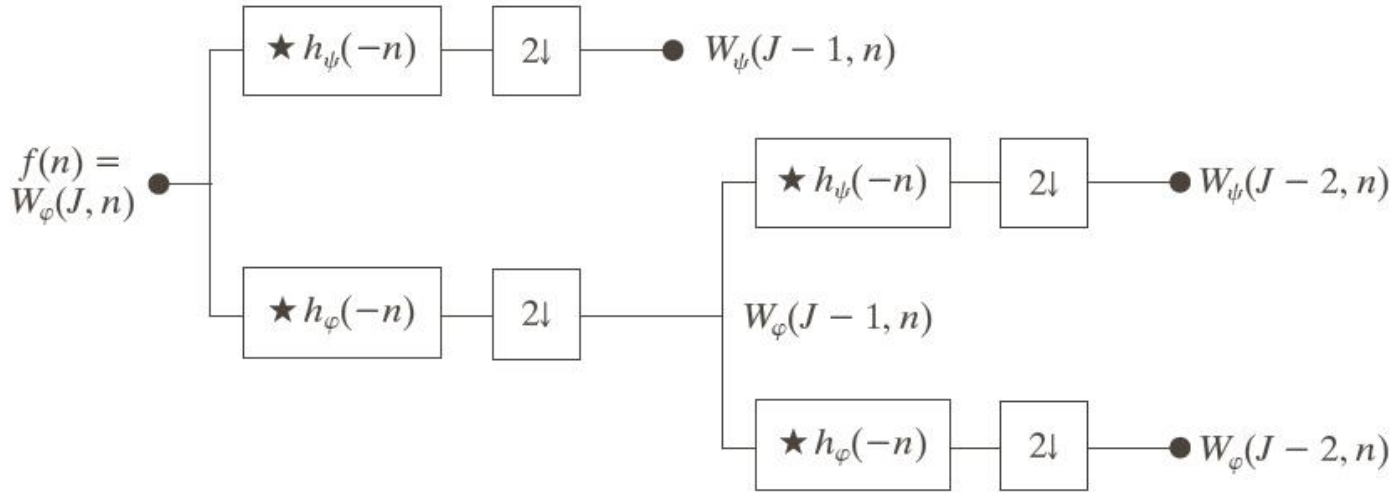
7.3 二维小波变换

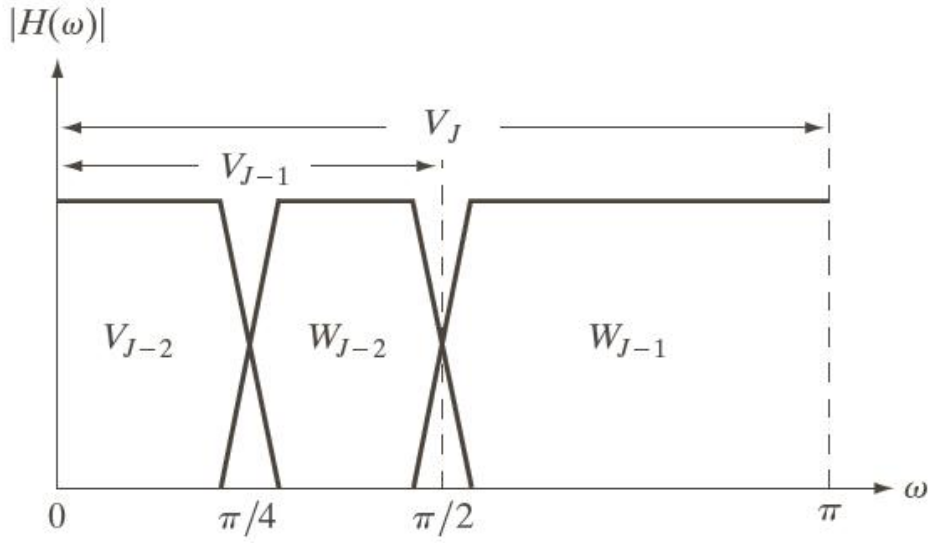
- **一维连续小波变换**
  - 双尺度





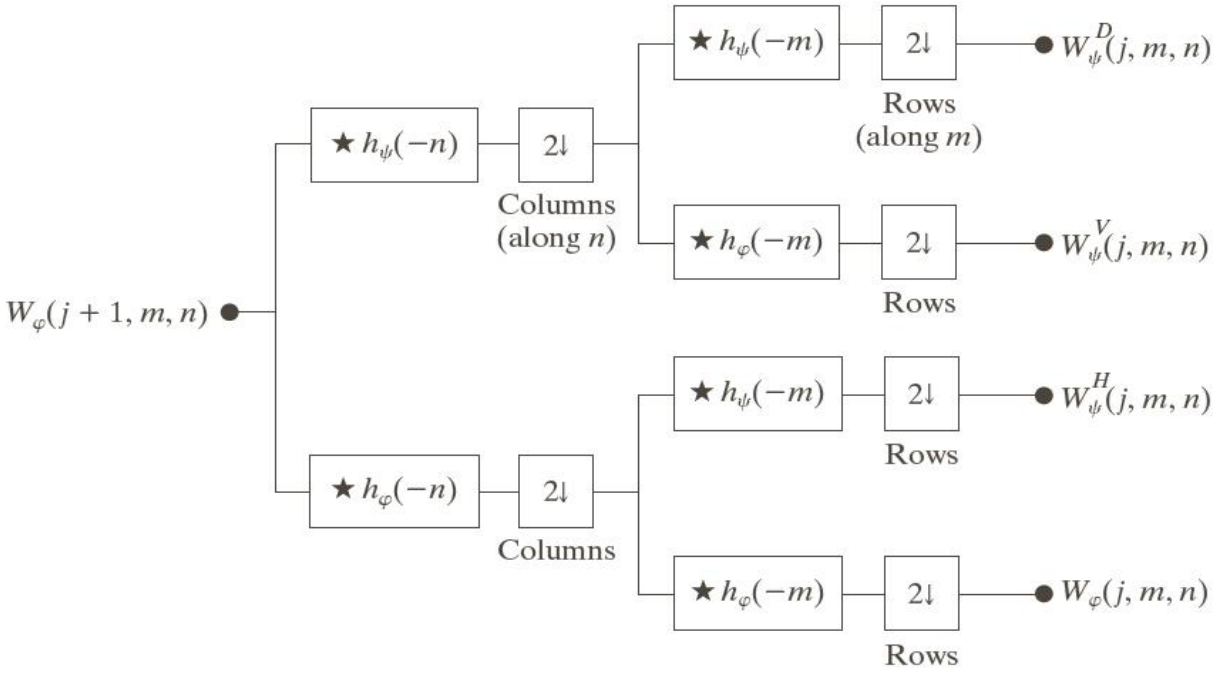
a
b

FIGURE 7.18
(a) A two-stage or two-scale FWT analysis bank and (b) its frequency splitting characteristics.



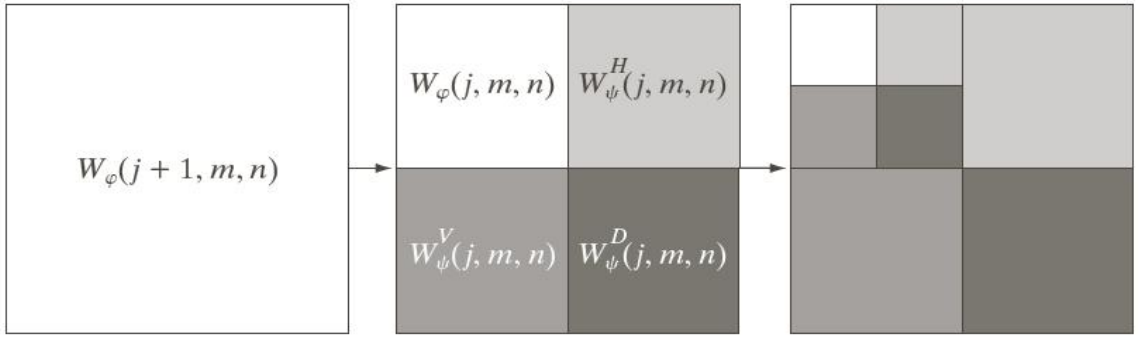
7.3 二维小波变换

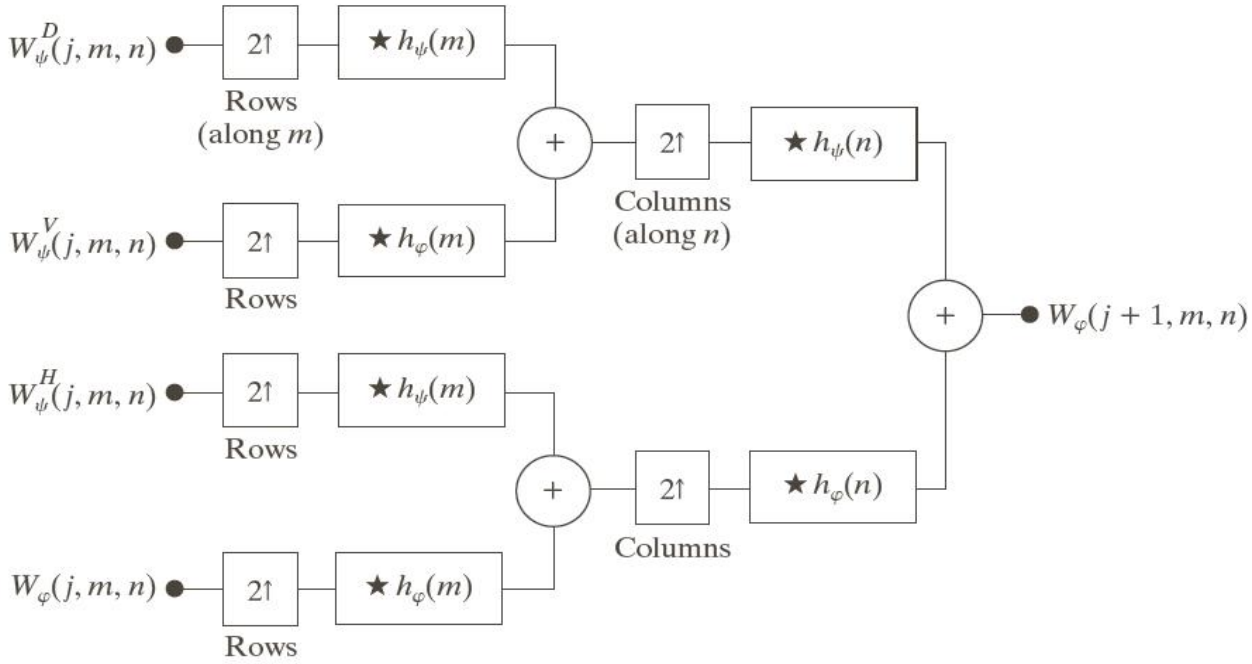
- **图像小波分解**



a
b
c

FIGURE 7.24 The 2-D fast wavelet transform: (a) the analysis filter bank; (b) the resulting decomposition; and (c) the synthesis filter bank.





7.3 二维小波变换

- 图像小波分解

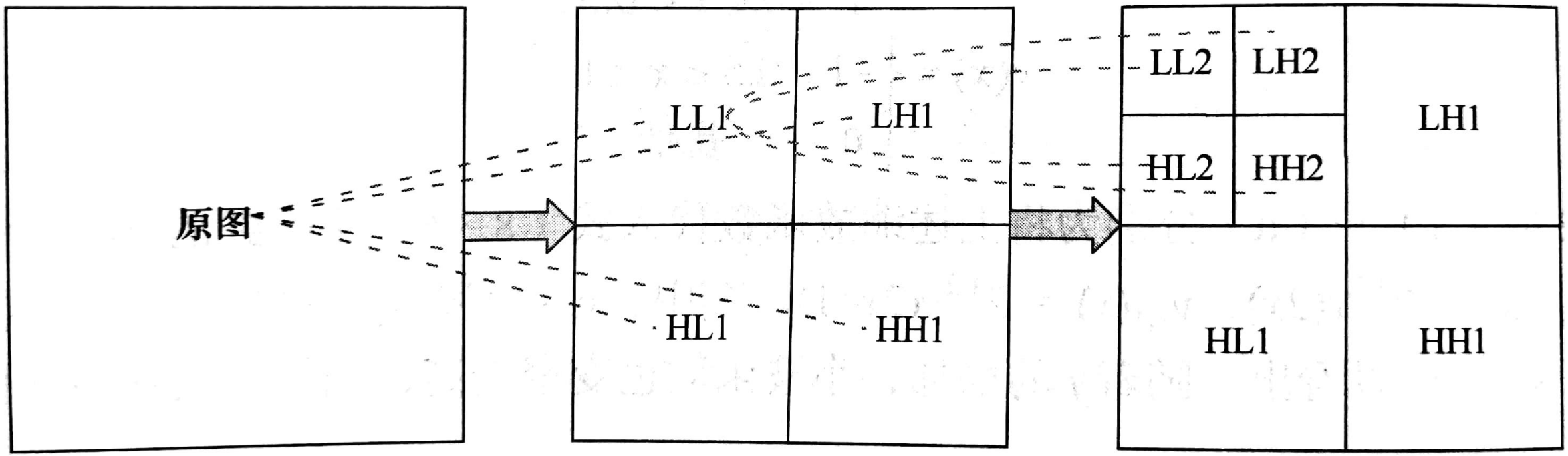
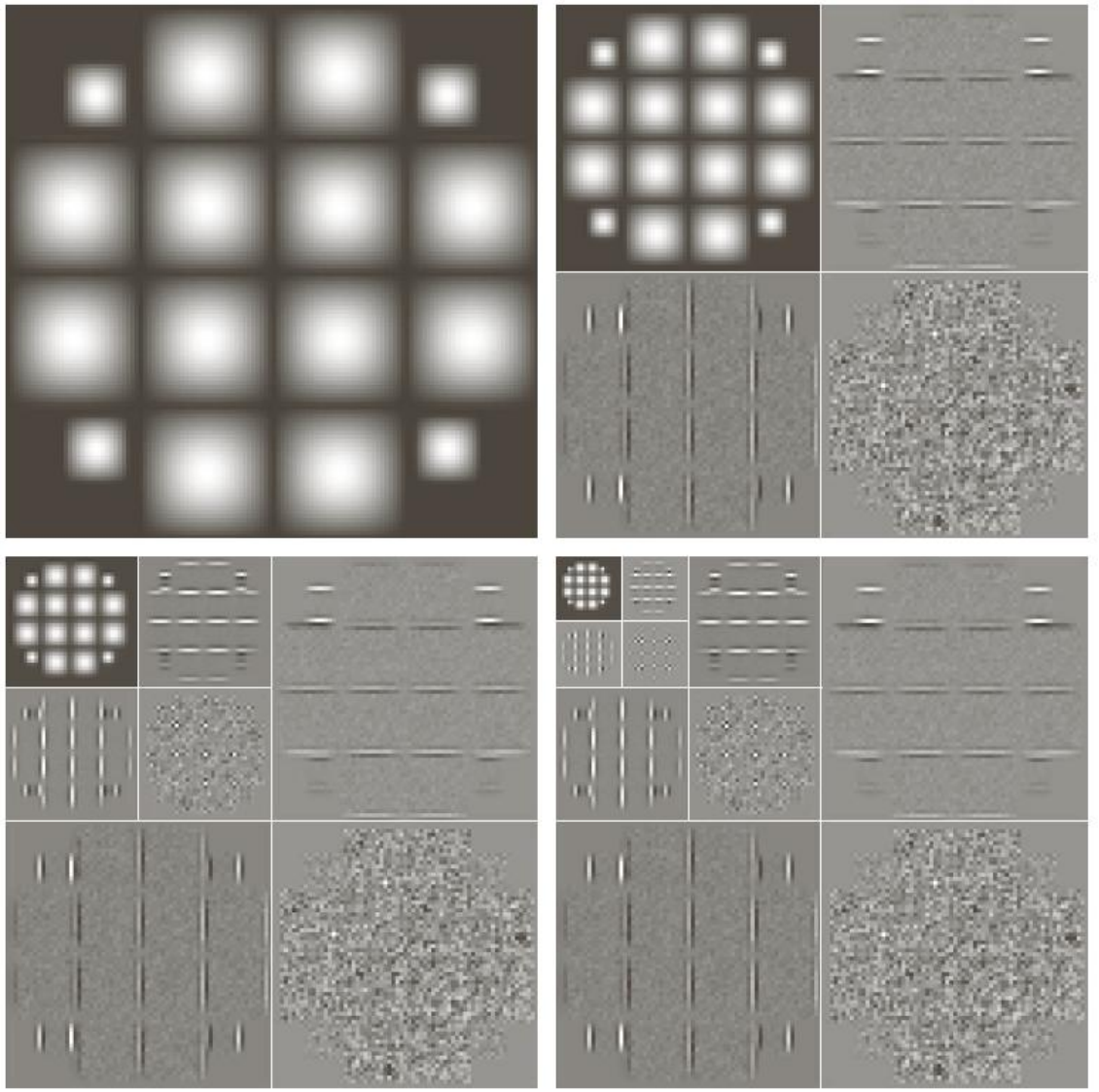


图 8.3.4　2-D 图像的二级小波分解示意图

8.3 小波变换编码

- **利用小波包进行分解**
  - Wavelet toolbox



a b
c d

FIGURE 7.25
Computing a 2-D three-scale FWT: (a) the original image; (b) a one-scale FWT; (c) a two-scale FWT; and (d) a three-scale FWT.

8.3 小波变换编码

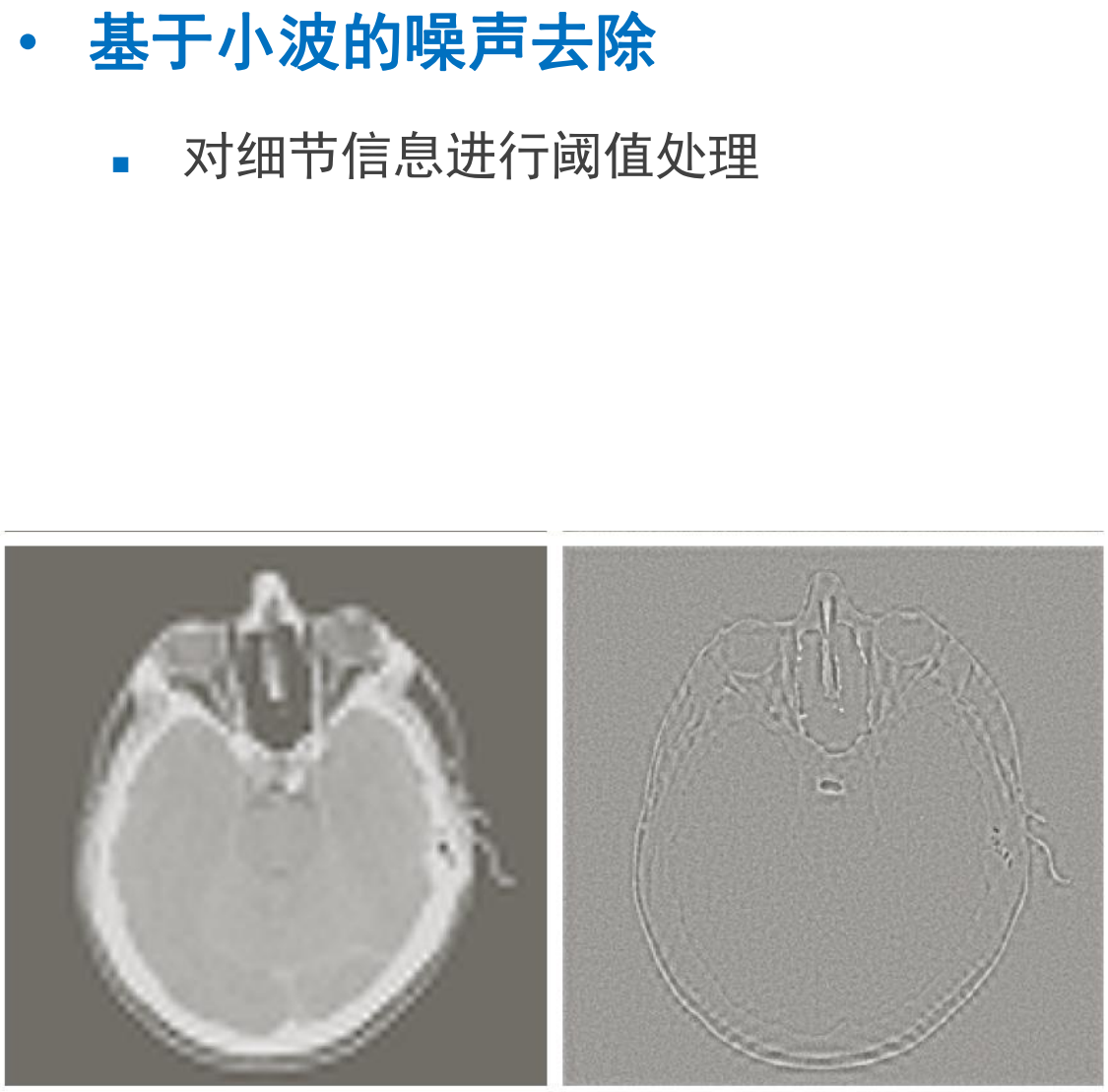
- **利用小波包进行分解**
  - Wavelet toolbox
  - 多种小波基

a b
c d

FIGURE 8.46 Three-scale wavelet transforms of Fig. 8.9(a) with respect to (a) Haar wavelets, (b) Daubechies wavelets, (c) symlets, and (d) Cohen-Daubechies Feauveau biorthogonal wavelets.

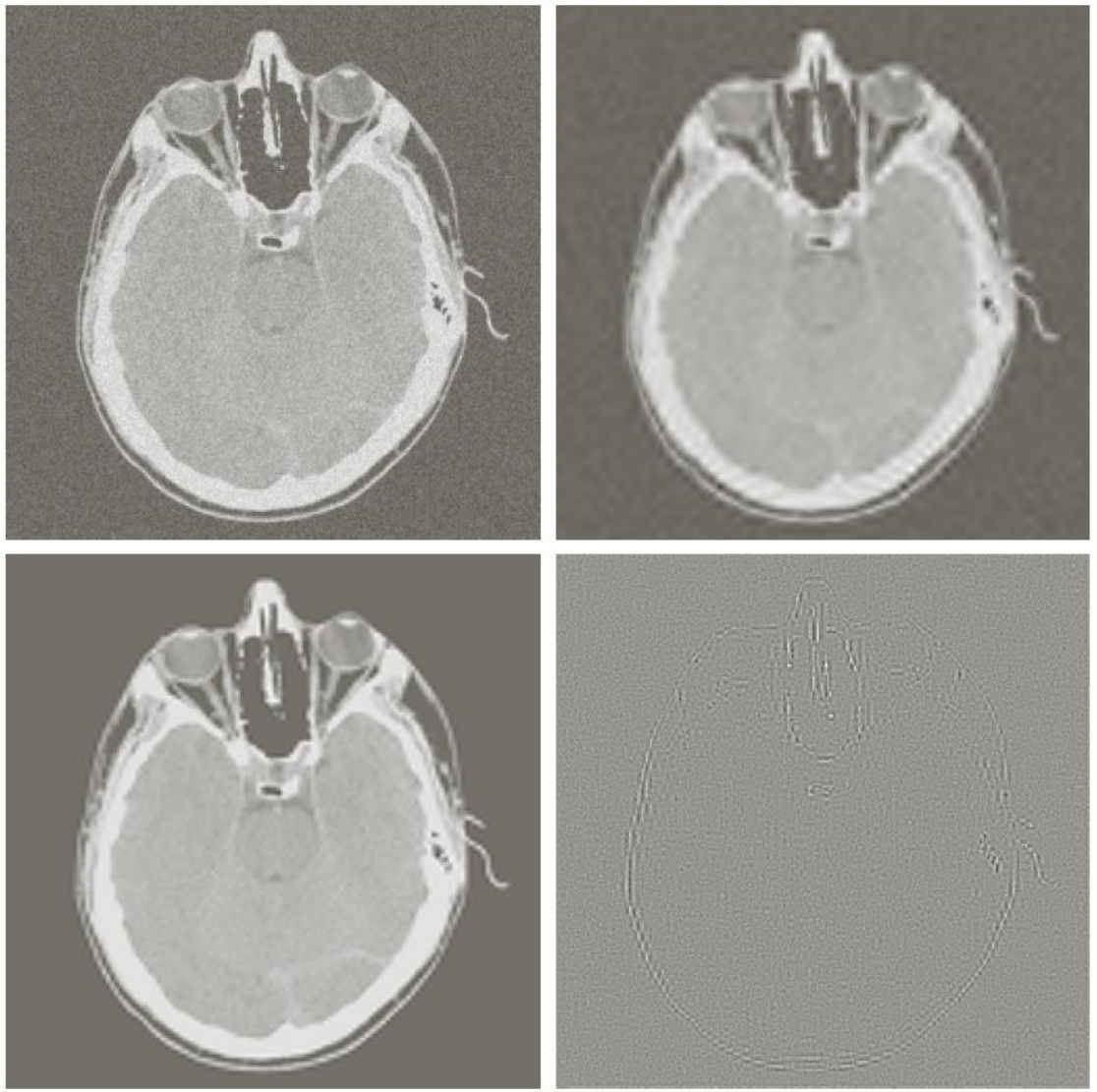
8.3 小波变换编码

- **基于小波的噪声去除**
  - 对细节信息进行阈值处理



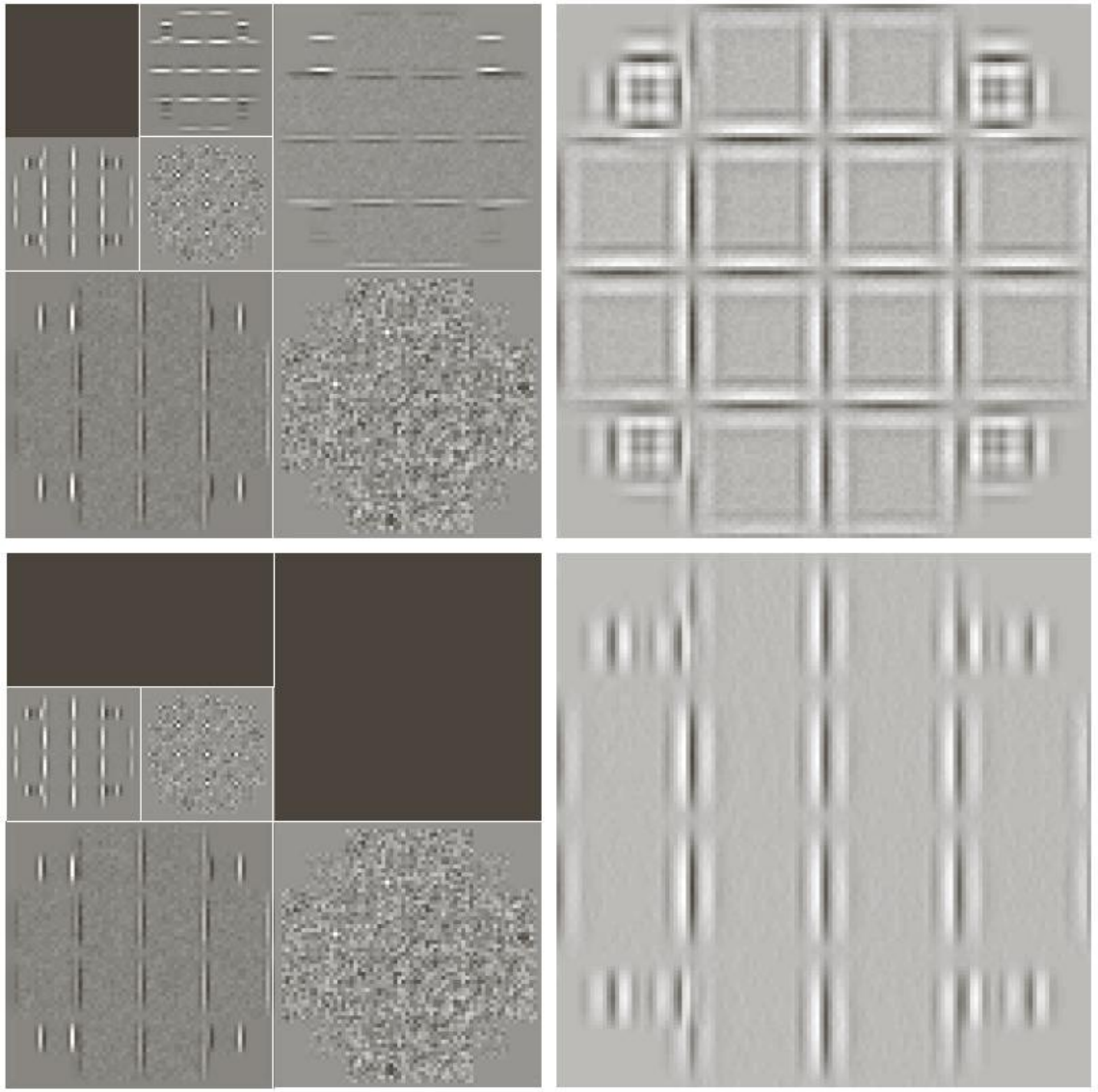
a b
c d
e f

FIGURE 7.28 Modifying a DWT for noise removal: (a) a noisy CT of a human head; (b), (c) and (e) various reconstructions after thresholding the detail coefficients; (d) and (f) the information removed during the reconstruction of (c) and (e). (Original image courtesy Vanderbilt University Medical Center.)



8.3 小波变换编码

- **基于小波的边缘检测**
  - 只利用高频进行重建



a b
c d

FIGURE 7.27 Modifying a DWT for edge detection: (a) and (c) two-scale decompositions with selected coefficients deleted; (b) and (d) the corresponding reconstructions.

Thank You !

Q & A