Control Systems

G V V Sharma*

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***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in.

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8.1. For a unity feedback system shown in Fig 8.1 having transfer function

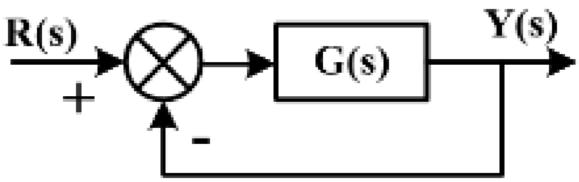

Fig. 8.1

$$G(s) = \frac{K}{(s+3)(s+9)(s+15)} \tag{8.1.1}$$

design the value of gain(K), for a gain margin of 50dB.

8.2. **Solution:**
Gain Margin:

$$GM = -20\log|G(j\omega_{pc})| \tag{8.2.1}$$

where, ω_{pc} is the phase cross-over frequency, at which

$$\angle G(j\omega_{pc}) = -180^\circ \tag{8.2.2}$$

First substitute,

$$s = j\omega \tag{8.2.3}$$

$$\implies G(j\omega) = \frac{K}{(-27\omega^2+405)+j(-\omega^3+207\omega)} \tag{8.2.4}$$

Now the phase will be

$$\angle G(j\omega) = -\tan^{-1}\left(\frac{-\omega^3+207\omega}{-27\omega^2+405}\right) \tag{8.2.5}$$

Solving for $\angle G(j\omega) = -180^\circ$ gives

$$\omega_{pc} = 14.3875 \tag{8.2.6}$$

Magnitude :

$$|G(j\omega)| = \frac{K}{\sqrt{(\omega^2+9)}\sqrt{(\omega^2+81)}\sqrt{(\omega^2+225)}} \tag{8.2.7}$$

Substituting value of ω_{pc} in (8.2.1) gives

$$K = 16.406 \tag{8.2.8}$$

This can be verified from fig 8.2 The following code generates Fig. 8.2

```
codes/ee18btech11050_1.py
```

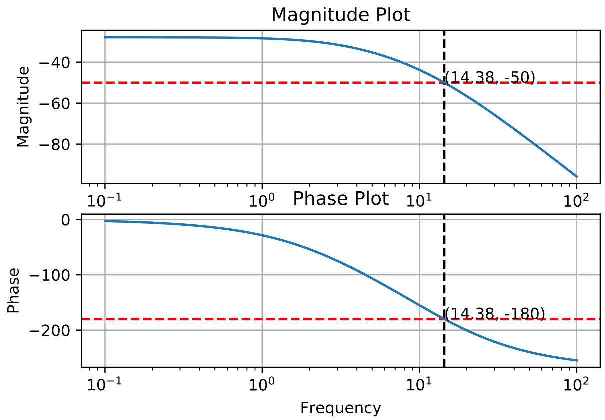

Fig. 8.2

8.3. Design the value gain (K) for a phase margin of 40°.

8.4. **Solution:**
Phase Margin:

$$PM = 180^\circ + \phi_{gc} \tag{8.4.1}$$

where ϕ_{gc} is the phase angle at the gain cross over frequency ω_{gc}. At gain cross over frequency,

$$|G(j\omega_{gc})| = 1 \tag{8.4.2}$$

$$\implies -20\log|G(j\omega_{gc})| = 0 \tag{8.4.3}$$

Given,

$$PM = 40^\circ = 180^\circ + \phi_{gc} \tag{8.4.4}$$

$$\implies \phi_{gc} = -140^\circ = \angle G(j\omega_{gc}) \tag{8.4.5}$$

From (8.2.5)

$$\angle G(j\omega_{gc}) = -\tan^{-1}\left(\frac{-\omega_{gc}^3+207\omega_{gc}}{-27\omega_{gc}^2+405}\right) \tag{8.4.6}$$

$$\implies \omega_{gc} = 8.09623 \tag{8.4.7}$$

Substituting this value in (8.4.3), we get

$$20\log K = 65.016 \tag{8.4.8}$$

$$\implies K = 1781.56 \tag{8.4.9}$$

This again can be verified from fig 8.4. The following code generates Fig. 8.4

```
codes/ee18btech11050_2.py
```

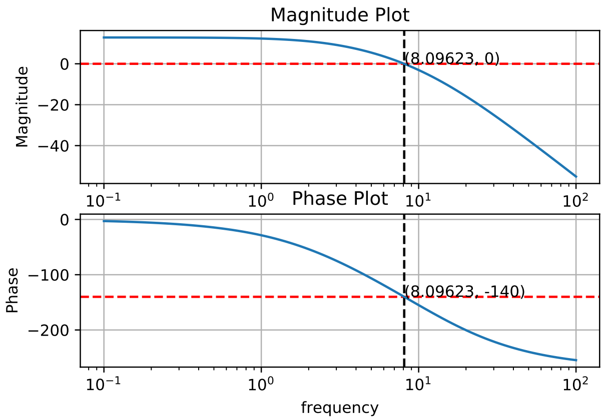

Fig. 8.4

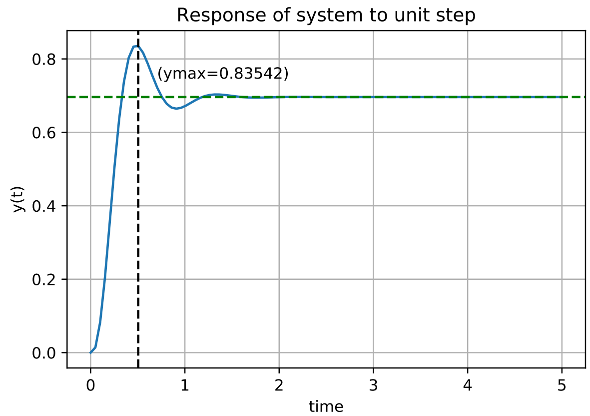

Fig. 8.6

8.5. Design the value of gain (K) to yield maximum peak overshoot of 20% for a step input.

8.6. **Solution:** Closed loop transfer function:

$$T(s) = \frac{G(s)}{1 + G(s)H(s)} \tag{8.6.1}$$

where H(s) = 1

$$\implies T(s) = \frac{K}{(s+3)(s+6)(s+15)+K} \tag{8.6.2}$$

Output will be:

$$\implies Y(s) = \frac{1}{s}\frac{K}{(s+3)(s+6)(s+15)+K} \tag{8.6.3}$$

Maximum peak overshoot :

$$M_p = \frac{y(t_p) - y(\infty)}{y(\infty)} \tag{8.6.4}$$

which is given as 20%. Here, t_p is the peak time. Solving this, we get

$$\implies \frac{y(t_p)}{y(\infty)} = 1.2 \tag{8.6.5}$$

Plotting y(t) for different values of K, we choose the value of K, which gives the above ratio, which is verfied from fig 8.6. Thus, we get

$$t_p = 0.505 \tag{8.6.6}$$

$$\implies K = 928.035 \tag{8.6.7}$$

The following code generates fig 8.6

```
codes/ee18btech11050_3.py
```

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