Phase Shift Oscillator

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1. For the circuit in Fig. 1.1 (ignore the amplitude stabilization circuitry), find the loop gain GH by breaking the circuit at node X.

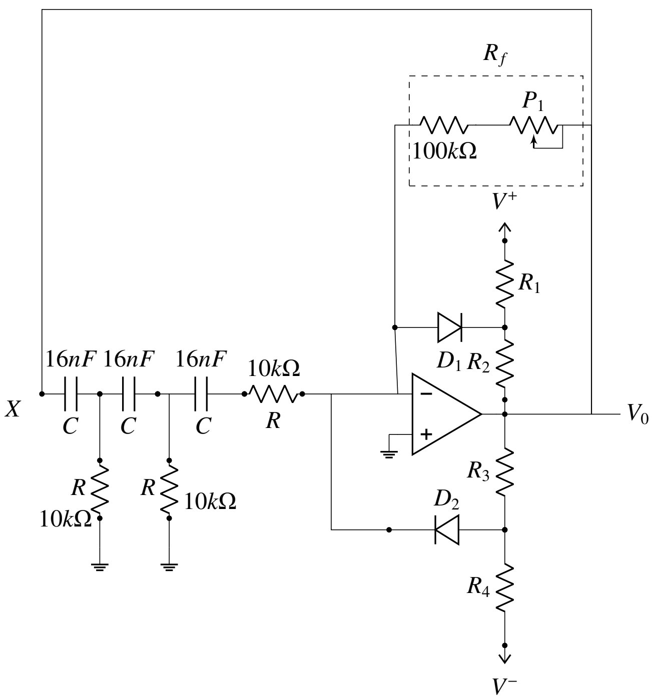

Fig. 1.1

2. **Solution:** The equivalent control system representation of Oscillator circuit is shown in Fig. 2.2. Oscillator circuits do not have input. After removing the amplitude stabilization circuitry, when we break the loop at X, from Fig. 2.3 the value of gain

$$GH = \frac{v_o(j\omega)}{v_x(j\omega)} \tag{2.1}$$

$$v_1 = 0 \tag{2.2}$$

$$\implies v_o = -i_x R_f \tag{2.3}$$

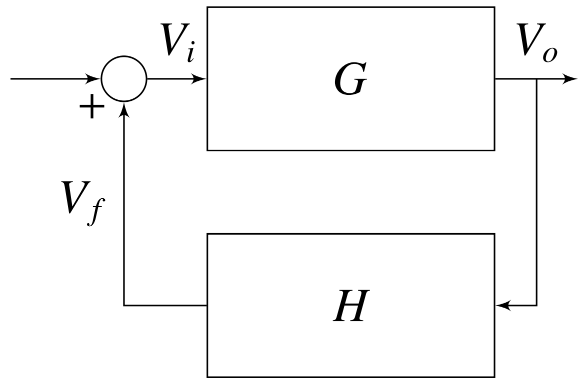

Fig. 2.2

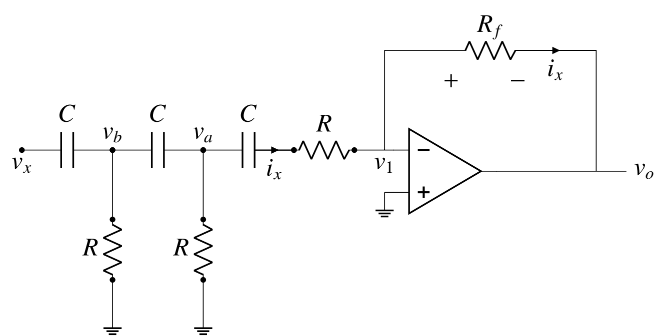

Fig. 2.3

$$v_a = \left(\frac{1 + sRC}{sC}\right) i_x \tag{2.4}$$

$$\implies v_b = \left(R + \frac{3}{sC} + \frac{1}{s^2C^2R}\right) i_x \tag{2.5}$$

$$\implies v_x = \left(R + \frac{6}{sC} + \frac{5}{s^2C^2R} + \frac{1}{s^3C^3R^2}\right) i_x \tag{2.6}$$

$$\implies \frac{v_x}{i_x} = \left(R + \frac{6}{sC} + \frac{5}{s^2C^2R} + \frac{1}{s^3C^3R^2}\right) \tag{2.7}$$

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From (2.3)

$$\frac{v_x}{v_o} = -\frac{R}{R_f}\left(1 + \frac{6}{sCR} + \frac{5}{s^2C^2R^2} + \frac{1}{s^3C^3R^3}\right) \tag{2.8}$$

$$\implies \frac{v_o}{v_x} = -\frac{R_f s^3C^3R^3}{R\left(s^3C^3R^3 + 6s^2C^2R^2 + 5sCR + 1\right)} \tag{2.9}$$

Substituting $s = j\omega$ gives us the transfer function

$$\frac{v_o}{v_x} = GH = \frac{\omega^2C^2R_fR}{(5 - \omega^2C^2R^2) + j(6\omega CR - \frac{1}{\omega CR})} \tag{2.10}$$

3. Find frequency of oscillation f_0.
4. **Solution:** For system to oscillate at a frequency ω_0,

$$L(j\omega_0) = G(j\omega_0)H(j\omega_0) = 1 \tag{4.1}$$

$$\implies \angle(G(j\omega_0)H(j\omega_0)) = 0 \tag{4.2}$$

$$\implies 6\omega_0CR = \frac{1}{\omega_0CR} \tag{4.3}$$

$$\implies \omega_0 = \frac{1}{\sqrt{6}CR} \tag{4.4}$$

$$\implies \omega_0 = 2551.55rad/sec \tag{4.5}$$

$$\implies f_0 = 406.1Hz \tag{4.6}$$

5. Find R_f for oscillation to begin.
6. **Solution:** From (4.1)

$$Re(G(j\omega_0)H(j\omega_0)) = 1 \tag{6.1}$$

$$\implies \frac{R_f\omega_0^2C^2R}{5 - \omega_0^2C^2R^2} = 1 \tag{6.2}$$

$$\implies R_f = 29R \tag{6.3}$$

$$\implies R_f = 290k\Omega \tag{6.4}$$

Thus, for the oscillations to begin,

$$R_f \geq 290k\Omega \tag{6.5}$$

7. Find feedback gain H.
8. **Solution:** From feedback circuit as shown in fig 8.4, feedback gain

$$H(j\omega) = \frac{v_1(j\omega)}{v_o(j\omega)} \tag{8.1}$$

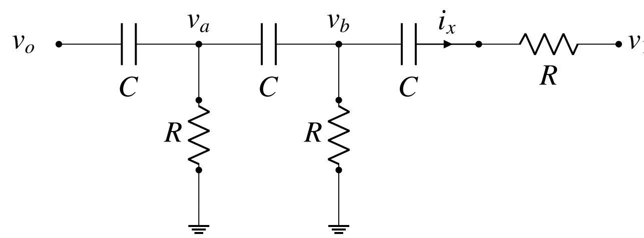

Fig. 8.4

$$v_a = \left(\frac{sCR}{sCR + 1}\right)v_o \tag{8.2}$$

$$\implies v_b = \left(\frac{s^2C^2R^2}{(sCR + 1)^2}\right)v_o \tag{8.3}$$

$$v_1 = v_b - i_x\left(R + \frac{1}{sC}\right) \tag{8.4}$$

From (2.3),

$$v_1 = v_b + \frac{v_o}{R_f}\left(R + \frac{1}{sC}\right) \tag{8.5}$$

$$v_1 = \left(\frac{R}{R_f} + \frac{1}{sCR_f} + \frac{s^2C^2R^2}{(sCR + 1)^2}\right)v_o \tag{8.6}$$

$$\implies \frac{v_1}{v_o} = \frac{R}{R_f} + \frac{1}{sCR_f} + \frac{s^2C^2R^2}{(sCR + 1)^2} \tag{8.7}$$

$$\implies H(j\omega) = \frac{R}{R_f} + \frac{1}{j\omega CR_f} - \frac{\omega^2C^2R^2}{1 - \omega^2C^2R^2 + j(2\omega CR)} \tag{8.8}$$

9. Tabulate your results.
10. **Solution:** See table 10

Parameter	Value
ω_0	2551.55 rad/s
f_0	406.1 Hz
R_f	$290k\Omega$

TABLE 10: calculated parameters

11. Verify results using Spice Simulation.

12. **Solution:** Following readme provides instructions for simulation in spice

codes/ee18btech11050/spice/README.md

The following netlist simulates the given circuit in 1.1

codes/ee18btech11050/spice/ ee18btech11050_sim.net

The following code plots the oscillator output from spice simulation, which is shown in fig 12.5

codes/ee18btech11050/spice/ ee18btech11050_sim.py

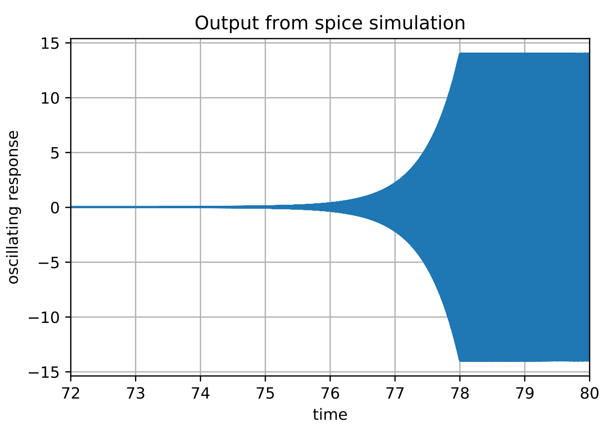

Fig. 12.5

The following code plots a part of spice output generated above, where a sinusoidal output can be clearly observed shown in fig 12.6

codes/ee18btech11050/spice/ ee18btech11050_sim2.py

From fig 12.6, time period is calculated from one cycle:

$$T = 78.91131 - 78.908846 = 0.002464 sec \tag{12.1}$$

$$\implies f = 405.844 Hz \tag{12.2}$$

Hence frequency is verified through spice simulation.

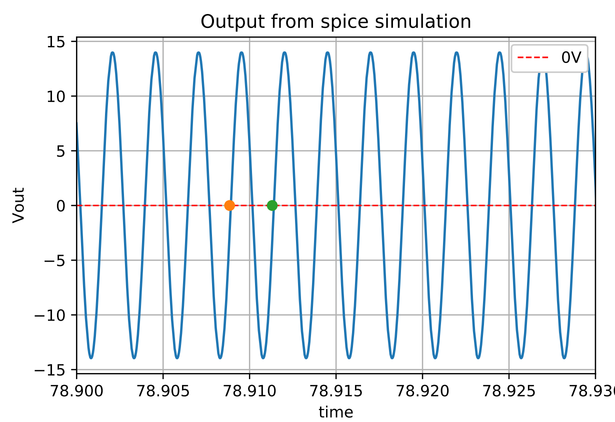

Fig. 12.6