Control Systems

G V V Sharma*

Contents

1 Signal Flow Graph 2
- 1.1 Mason's Gain Formula . . . 2
- 1.2 Matrix Formula 2
- 1.3 Example 2

2 Bode Plot 2
- 2.1 Introduction 2
- 2.2 Example 2
- 2.3 Phase 2

3 Second order System 2
- 3.1 Damping 2
- 3.2 Peak Overshoot 2
- 3.3 Example 2
- 3.4 Settling Time 2

4 Routh Hurwitz Criterion 2
- 4.1 Routh Array 2
- 4.2 Marginal Stability 2
- 4.3 Stability 2
- 4.4 Example 2
- 4.5 Example 2

5 State-Space Model 2
- 5.1 Controllability and Observability 2
- 5.2 Second Order System 2
- 5.3 Example 2
- 5.4 Example 2
- 5.5 Example 2
- 5.6 Example 2
- 5.7 Example 2

6 Nyquist Plot 2
- 6.1 Introduction 2
- 6.2 Example 2

7 Compensators 2
- 7.1 Phase Lead 2
- 7.2 Lag Lead 2
- 7.3 Example 2

8 Gain Margin 2
- 8.1 Introduction 2
- 8.2 Example 2
- 8.3 Example 2
- 8.4 Example 2

9 Phase Margin 4
- 9.1 Intoduction 4
- 9.2 Example 4

10 Oscillator 4
- 10.1 Introduction 4
- 10.2 Example 4

11 Root Locus 4
- 11.1 Introduction 4
- 11.2 Example 4
- 11.3 Example 4

12 Polar Plot 4
- 12.1 Introduction 4

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in.

1 Signal Flow Graph

1.1 Mason's Gain Formula

1.2 Matrix Formula

1.3 Example

2 Bode Plot

2.1 Introduction

2.2 Example

2.3 Phase

3 Second order System

3.1 Damping

3.2 Peak Overshoot

3.3 Example

3.4 Settling Time

4 Routh Hurwitz Criterion

4.1 Routh Array

4.2 Marginal Stability

4.3 Stability

4.4 Example

4.5 Example

5 State-Space Model

5.1 Controllability and Observability

5.2 Second Order System

5.3 Example

5.4 Example

5.5 Example

5.6 Example

5.7 Example

6 Nyquist Plot

6.1 Introduction

6.2 Example

7 Compensators

7.1 Phase Lead

7.2 Lag Lead

7.3 Example

8 Gain Margin

8.1 Introduction

8.2 Example

8.3 Example

8.4 Example

8.1. For a unity feedback system shown in Fig 8.1

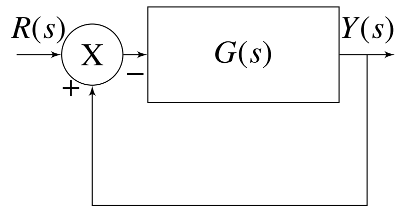

Fig. 8.1

having transfer function

$$G(s) = \frac{K}{(s+3)(s+9)(s+15)} \tag{8.1.1}$$

design the value of gain(K), for a gain margin of 50 dB.

8.2. **Solution:**
Gain Margin:

$$GM = -20\log|G(j\omega_{pc})| \tag{8.2.1}$$

where, ω_{pc} is the phase cross-over frequency. First substitute,

$$s = j\omega \tag{8.2.2}$$

$$\implies G(j\omega) = \frac{K}{(-27\omega^2+405)+j(-\omega^3+207\omega)} \tag{8.2.3}$$

Now the phase will be

$$\angle G(j\omega) = -\tan^{-1}\left(\frac{-\omega^3+207\omega}{-27\omega^2+405}\right) \tag{8.2.4}$$

Solving for $\angle G(j\omega) = -180^\circ$ gives

$$\omega_{pc} = 14.3875 \tag{8.2.5}$$

Magnitude :

$$|G(j\omega)| = \frac{K}{\sqrt{(\omega^2+9)}\sqrt{(\omega^2+81)}\sqrt{(\omega^2+225)}} \tag{8.2.6}$$

Substituting value of ω_{pc} in (8.2.1) gives

$$K = 16.406 \tag{8.2.7}$$

This can be verified from fig 8.2 The following code generates Fig. 8.2

```
codes/ee18btech11050_1.py
```

8.3. Design the value gain (K) for a phase margin of 40°.

8.4. **Solution:**

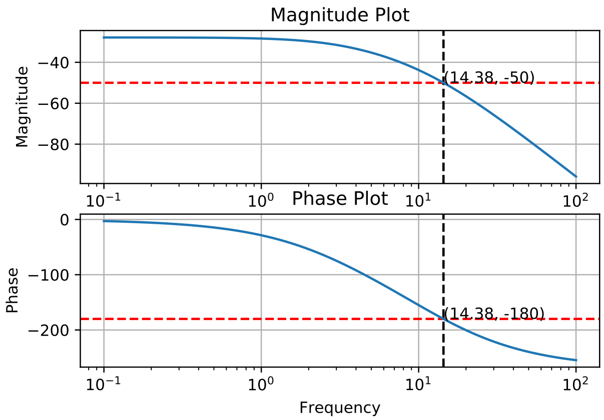

Fig. 8.2

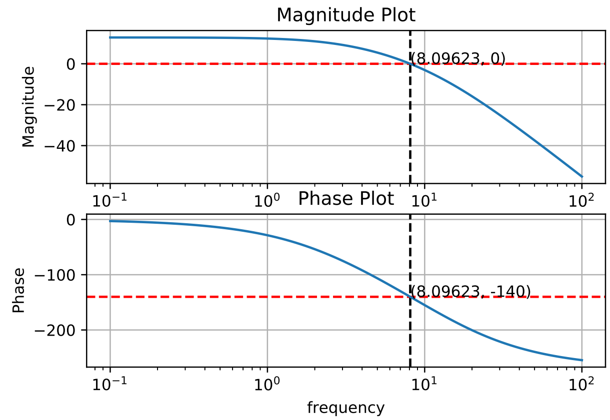

Fig. 8.4

Phase Margin:

$$PM = 180^\circ + \phi_{gc} \tag{8.4.1}$$

where ϕ_{gc} is the phase angle at the gain cross over frequency ω_{gc}.

$$-20\log|G(j\omega_{gc})| = 0 \tag{8.4.2}$$

Given,

$$PM = 40^\circ = 180^\circ + \phi_{gc} \tag{8.4.3}$$

$$\implies \phi_{gc} = -140^\circ = \angle G(j\omega_{gc}) \tag{8.4.4}$$

From (8.2.4)

$$\angle G(j\omega_{gc}) = -\tan^{-1}\left(\frac{-\omega_{gc}^3 + 207\omega_{gc}}{-27\omega_{gc}^2 + 405}\right) \tag{8.4.5}$$

$$\implies \omega_{gc} = 8.09623 \tag{8.4.6}$$

Substituting this value in (8.4.2), we get

$$20\log K = 65.016 \tag{8.4.7}$$

$$\implies K = 1781.56 \tag{8.4.8}$$

This again can be verified from fig 8.4. The following code generates Fig. 8.4

```
codes/ee18btech11050_2.py
```

8.5. Design the value of gain (K) to yield maximum peak overshoot of 20% for a step input.

8.6. **Solution:** Closed loop transfer function:

$$T(s) = \frac{K}{(s+3)(s+6)(s+15)+K} \tag{8.6.1}$$

Output will be:

$$\implies Y(s) = \frac{1}{s}\frac{K}{(s+3)(s+6)(s+15)+K} \tag{8.6.2}$$

Maximum peak overshoot :

$$M_p = \frac{y(t_p) - y(\infty)}{y(\infty)} \tag{8.6.3}$$

which is given as 20%. Here, t_p is the peak time.

$$\implies \frac{y(t_p)}{y(\infty)} = 1.2 \tag{8.6.4}$$

Plotting y(t) for different values of K, we choose the value of K, which gives the above ratio, which is verfied from fig 8.6. Thus, we get

$$t_p = 0.505 \tag{8.6.5}$$

$$\implies K = 928.035 \tag{8.6.6}$$

The following code generates fig 8.6

```
codes/ee18btech11050_3.py
```

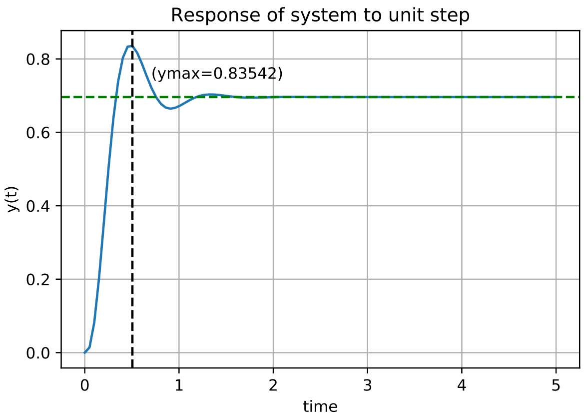

Fig. 8.6

9 Phase Margin

9.1 Intoduction

9.2 Example

10 Oscillator

10.1 Introduction

10.2 Example

11 Root Locus

11.1 Introduction

11.2 Example

11.3 Example

12 Polar Plot

12.1 Introduction