

# Control Systems

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*Abstract*—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

svn co <https://github.com/gadepall/school/trunk/control/codes>

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8.1. For a unity feedback system shown in Fig 8.1

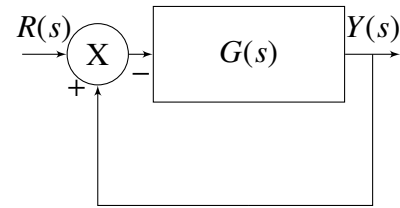


Fig. 8.1

having transfer function

$$G(s) = \frac{K}{(s+3)(s+9)(s+15)} \quad (8.1.1)$$

design the value of gain(K), for a gain margin of 50 dB.

### 8.2. Solution:

Gain Margin:

$$GM = -20 \log |G(j\omega_{pc})| \quad (8.2.1)$$

where,  $\omega_{pc}$  is the phase cross-over frequency, at which

$$\angle G(j\omega_{pc}) = -180^\circ \quad (8.2.2)$$

First substitute,

$$s = j\omega \quad (8.2.3)$$

$$\Rightarrow G(j\omega) = \frac{K}{(-27\omega^2 + 405) + j(-\omega^3 + 207\omega)} \quad (8.2.4)$$

Now the phase will be

$$\angle G(j\omega) = -\tan^{-1}\left(\frac{-\omega^3 + 207\omega}{-27\omega^2 + 405}\right) \quad (8.2.5)$$

Solving for  $\angle G(j\omega) = -180^\circ$  gives

$$\omega_{pc} = 14.3875 \quad (8.2.6)$$

Magnitude :

$$|G(j\omega)| = \frac{K}{\sqrt{(\omega^2 + 9)} \sqrt{(\omega^2 + 81)} \sqrt{(\omega^2 + 225)}} \quad (8.2.7)$$

Substituting value of  $\omega_{pc}$  in (8.2.1) gives

$$K = 16.406 \quad (8.2.8)$$

This can be verified from fig 8.2 The following code generates Fig. 8.2

```
codes/ee18btech11050_1.py
```

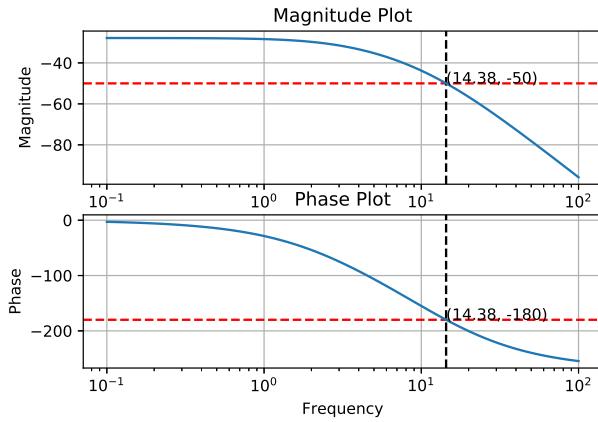


Fig. 8.2

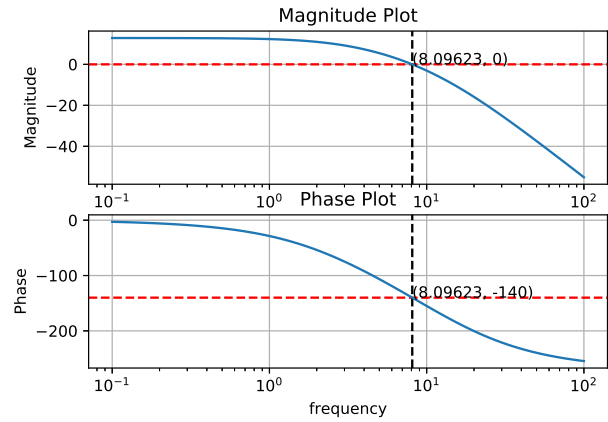


Fig. 8.4

8.3. Design the value gain (K) for a phase margin of  $40^\circ$ .

8.4. **Solution:**

Phase Margin:

$$PM = 180^\circ + \phi_{gc} \quad (8.4.1)$$

where  $\phi_{gc}$  is the phase angle at the gain cross over frequency  $\omega_{gc}$ . At gain cross over frequency,

$$|G(j\omega_{gc})| = 1 \quad (8.4.2)$$

$$\Rightarrow -20 \log |G(j\omega_{gc})| = 0 \quad (8.4.3)$$

Given,

$$PM = 40^\circ = 180^\circ + \phi_{gc} \quad (8.4.4)$$

$$\Rightarrow \phi_{gc} = -140^\circ = \angle G(j\omega_{gc}) \quad (8.4.5)$$

From (8.2.5)

$$\angle G(j\omega_{gc}) = -\tan^{-1}\left(\frac{-\omega_{gc}^3 + 207\omega_{gc}}{-27\omega_{gc}^2 + 405}\right) \quad (8.4.6)$$

$$\Rightarrow \omega_{gc} = 8.09623 \quad (8.4.7)$$

Substituting this value in (8.4.3), we get

$$20 \log K = 65.016 \quad (8.4.8)$$

$$\Rightarrow K = 1781.56 \quad (8.4.9)$$

This again can be verified from fig 8.4. The following code generates Fig. 8.4

```
codes/ee18btech11050_2.py
```

8.5. Design the value of gain (K) to yield maximum peak overshoot of 20% for a step input.

8.6. **Solution:** Closed loop transfer function:

$$T(s) = \frac{G(s)}{1 + G(s)H(s)} \quad (8.6.1)$$

where  $H(s) = 1$

$$\Rightarrow T(s) = \frac{K}{(s+3)(s+6)(s+15) + K} \quad (8.6.2)$$

Output will be:

$$\Rightarrow Y(s) = \frac{1}{s} \frac{K}{(s+3)(s+6)(s+15) + K} \quad (8.6.3)$$

Maximum peak overshoot :

$$M_p = \frac{y(t_p) - y(\infty)}{y(\infty)} \quad (8.6.4)$$

which is given as 20%. Here,  $t_p$  is the peak time. Solving this, we get

$$\Rightarrow \frac{y(t_p)}{y(\infty)} = 1.2 \quad (8.6.5)$$

Plotting  $y(t)$  for different values of K, we choose the value of K, which gives the above ratio, which is verified from fig 8.6. Thus, we get

$$t_p = 0.505 \quad (8.6.6)$$

$$\Rightarrow K = 928.035 \quad (8.6.7)$$

The following code generates fig 8.6

```
codes/ee18btech11050_3.py
```

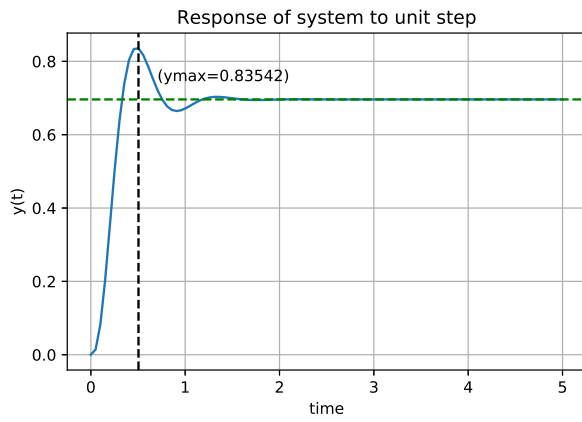


Fig. 8.6

## 9 PHASE MARGIN

### 9.1 Introduction

### 9.2 Example

## 10 OSCILLATOR

### 10.1 Introduction

### 10.2 Example

## 11 ROOT LOCUS

### 11.1 Introduction

### 11.2 Example

### 11.3 Example

## 12 POLAR PLOT

### 12.1 Introduction