# Control Systems

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svn co https://github.com/gadepall/school/trunk/ control/codes

#### 1 SIGNAL FLOW GRAPH

- 1.1 Mason's Gain Formula
- 1.2 Matrix Formula

#### 2 Bode Plot

- 2.1 Introduction
- 2.2 Example
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#### 3 SECOND ORDER SYSTEM

- 3.1 Damping
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#### 6 Compensators

- 6.1 Phase Lead
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#### 7 Gain Margin

- 7.1 Introduction
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#### 8 Phase Margin

- 8.1 Intoduction
- 8.2 Example

#### 9 OSCILLATOR

- 9.1 Introduction
- 9.2 Example

#### 10 Root Locus

- 10.1 Introduction
- 10.1. A unity negative feedback system has the

$$G(s) = \frac{K}{s(s+1)(s+3)}$$
 (10.1.1)

The value of the gain K(>0) at which the root locus crosses the imaginary axis is?

#### **Solution:**

### 10.2. Root Locus:

The Root locus is the locus of the roots of the characteristic equation, which are the poles of closed loop transfer function, by varying system gain K from 0 to  $\infty$ .

10.3. The characteristic equation of the closed loop control system is:

$$1 + G(s)H(s) = 0 (10.3.1)$$

The points on the root locus branches must satisfy the angle condition. We can find the value of K for the points on the root locus branches by using magnitude condition.

#### 10.4. Angle Condition:

Given the Characteristic equation, we can write 10.8. Verification: it as:

$$G(s)H(s) = -1 + i0$$
 (10.4.1)

The phase angle of G(s)H(s) is:  $\angle G(s)H(s) =$ 

$$\arctan(\frac{0}{-1}) = (2n+1)\pi$$

The angle condition is the point at which the angle of the transfer function is an odd multiple of 180.

#### 10.5. Magnitude Condition

Magnitude of G(s)H(s) is:

$$|G(s)H(s)| = \sqrt{(-1)^2 + 0^2}$$
 (10.5.1)

$$|G(s)H(s)| = 1$$
 (10.5.2)

The magnitude condition is that the point (which satisfied the angle condition) at which the magnitude of the transfer function is one.

### 10.6. For given transfer function:

H(s) = 1. So, closed loop transfer function will be

$$T(s) = \frac{K}{s(s+1)(s+3) + K}$$
 (10.6.1)

Poles of closed loop transfer function are the roots of the Characteristic Equation. So, characteristic Equation is:

$$s^3 + 4s^2 + 3s + K = 0 ag{10.6.2}$$

#### 10.7. Routh Array Table:

If all elements of any row of the Routh array table are zero, then the root locus branch intersects the imaginary axis

Routh Array Table:

$$\begin{vmatrix} s^{3} \\ s^{2} \\ s^{1} \\ s^{0} \end{vmatrix} \begin{vmatrix} 1 & 3 \\ 4 & K \\ (12 - K)/4 & 0 \\ K \end{vmatrix}$$
 (10.7.1)

For poles to be on imaginary axis, row  $s^1$ should be zero. So,

$$\frac{12 - K}{4} = 0 \tag{10.7.2}$$

Hence, K = 12.

Auxilliary equation:

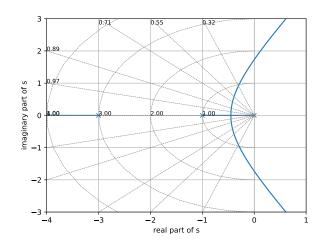
$$4s^2 + K = 0 (10.8.1)$$

$$4s^2 + 12 = 0 \tag{10.8.2}$$

$$\implies$$
 s =  $-j\sqrt{3}$ ,+ $j\sqrt{3}$ 

Thus a pair of poles lie on imaginary axis for K = 12.

#### 10.9. Root Locus plot



Code to plot root locus: