EE2227 Control Systems

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Question 20/GATE EC-2015

Question

A unity negative feedback system has the open loop transfer function

$$G(s) = \frac{K}{s(s+1)(s+3)}$$

The value of the gain K (>0) at which the root locus crosses the imaginary axis is ?

Solution

Given open loop transfer function

$$G(s) = \frac{K}{s(s+1)(s+3)}$$

we have P = 3 poles, at s = 0,-1,-3 and Z = 0 zeroes.

For unity negative feedback, closed loop transfer function is:

$$T(s) = \frac{G(s)H(s)}{1 + G(s)H(s)}$$

Here H(s) = 1

$\Longrightarrow$

$$T(s) = \frac{K}{s(s+1)(s+3) + K}$$

Solution

In the root locus diagram, we can observe the path of the closed loop poles
Poles of closed loop transfer function are the roots of the Characteristic Equation.
Characteristic Equation is:

$$1 + G(s)H(s) = 0$$

$$\Longrightarrow$$

$$s^3 + 4s^2 + 3s + K = 0$$

For the construction of root locus, **If all elements of any row of the Routh array table are zero, then the root locus branch intersects the imaginary axis and vice-versa**

Solution

Routh Array Table: Order	Coefficients	
s^3	1	3
s^2	4	K
s^1	(12-K)/4	0
s^0	K	

For poles to be on imaginary axis, row s^1 should be zero. So,

$$\frac{12 - K}{4} = 0$$

Hence,

$$\mathbf{K = 12}$$

Solution

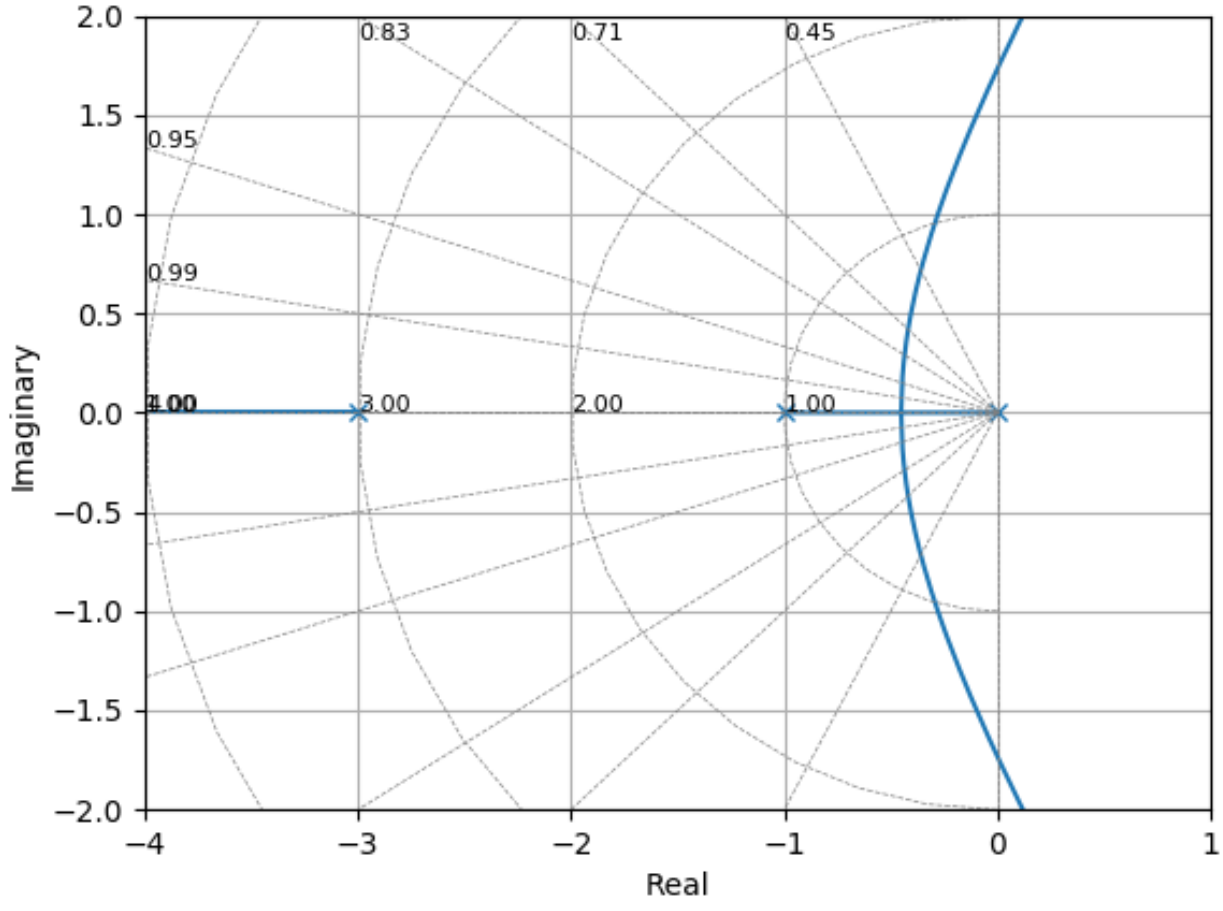

Figure 1: Root Locus Plot