

Control Systems

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Abstract—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

svn co <https://github.com/gadepall/school/trunk/control/codes>

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1 SIGNAL FLOW GRAPH

1.1 Mason's Gain Formula

1.2 Matrix Formula

1.3 Example

2 BODE PLOT

2.1 Introduction

2.2 Example

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3.1 Damping

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5 STATE-SPACE MODEL

5.1 Controllability and Observability

5.2 Second Order System

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5.5 Example

5.6 Example

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6.2 Example

7 COMPENSATORS

7.1 Phase Lead

7.2 Lag Lead

7.3 Example

8 GAIN MARGIN

8.1 Introduction

8.2 Example

8.3 Example

8.4 Example

8.1. For a unity feedback system shown in Fig 8.1 having transfer function



Fig. 8.1

$$G(s) = \frac{K}{(s+3)(s+9)(s+15)} \quad (8.1.1)$$

design the value of gain(K), for a gain margin of 50dB.

8.2. Solution:

Gain Margin:

$$GM = -20 \log |G(j\omega_{pc})| \quad (8.2.1)$$

where, ω_{pc} is the phase cross-over frequency, at which

$$\angle G(j\omega_{pc}) = -180^\circ \quad (8.2.2)$$

First substitute,

$$s = j\omega \quad (8.2.3)$$

$$\Rightarrow G(j\omega) = \frac{K}{(-27\omega^2 + 405) + j(-\omega^3 + 207\omega)} \quad (8.2.4)$$

Now the phase will be

$$\angle G(j\omega) = -\tan^{-1}\left(\frac{-\omega^3 + 207\omega}{-27\omega^2 + 405}\right) \quad (8.2.5)$$

Solving for $\angle G(j\omega) = -180^\circ$ gives

$$\omega_{pc} = 14.3875 \quad (8.2.6)$$

Magnitude :

$$|G(j\omega)| = \frac{K}{\sqrt{(\omega^2 + 9)} \sqrt{(\omega^2 + 81)} \sqrt{(\omega^2 + 225)}} \quad (8.2.7)$$

Substituting value of ω_{pc} in (8.2.1) gives

$$K = 16.406 \quad (8.2.8)$$

This can be verified from fig 8.2 The following code generates Fig. 8.2

```
codes/ee18btech11050_1.py
```

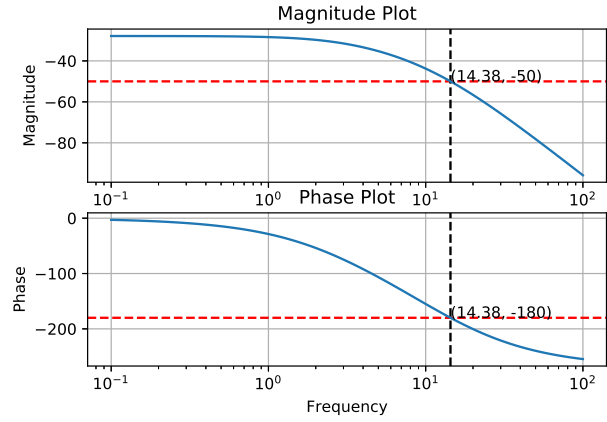


Fig. 8.2

8.3. Design the value gain (K) for a phase margin of 40° .

8.4. Solution:

Phase Margin:

$$PM = 180^\circ + \phi_{gc} \quad (8.4.1)$$

where ϕ_{gc} is the phase angle at the gain cross over frequency ω_{gc} . At gain cross over frequency,

$$|G(j\omega_{gc})| = 1 \quad (8.4.2)$$

$$\Rightarrow -20 \log |G(j\omega_{gc})| = 0 \quad (8.4.3)$$

Given,

$$PM = 40^\circ = 180^\circ + \phi_{gc} \quad (8.4.4)$$

$$\Rightarrow \phi_{gc} = -140^\circ = \angle G(j\omega_{gc}) \quad (8.4.5)$$

From (8.2.5)

$$\angle G(j\omega_{gc}) = -\tan^{-1}\left(\frac{-\omega_{gc}^3 + 207\omega_{gc}}{-27\omega_{gc}^2 + 405}\right) \quad (8.4.6)$$

$$\Rightarrow \omega_{gc} = 8.09623 \quad (8.4.7)$$

Substituting this value in (8.4.3), we get

$$20 \log K = 65.016 \quad (8.4.8)$$

$$\Rightarrow K = 1781.56 \quad (8.4.9)$$

This again can be verified from fig 8.4. The following code generates Fig. 8.4

```
codes/ee18btech11050_2.py
```

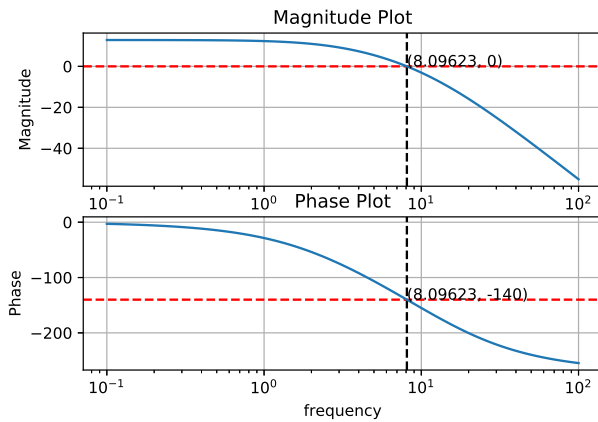


Fig. 8.4

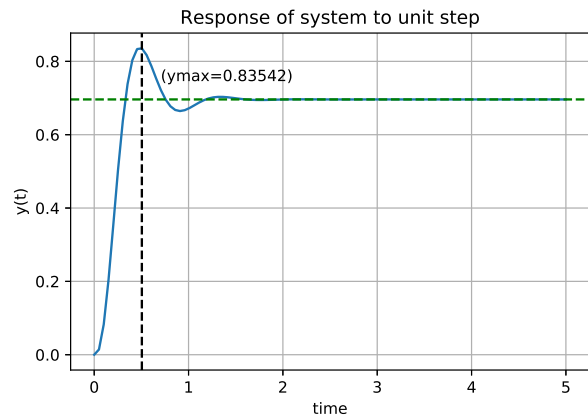


Fig. 8.6

8.5. Design the value of gain (K) to yield maximum peak overshoot of 20% for a step input.

8.6. **Solution:** Closed loop transfer function:

$$T(s) = \frac{G(s)}{1 + G(s)H(s)} \quad (8.6.1)$$

where $H(s) = 1$

$$\Rightarrow T(s) = \frac{K}{(s+3)(s+6)(s+15) + K} \quad (8.6.2)$$

Output will be:

$$\Rightarrow Y(s) = \frac{1}{s} \frac{K}{(s+3)(s+6)(s+15) + K} \quad (8.6.3)$$

Maximum peak overshoot :

$$M_p = \frac{y(t_p) - y(\infty)}{y(\infty)} \quad (8.6.4)$$

which is given as 20%. Here, t_p is the peak time. Solving this, we get

$$\Rightarrow \frac{y(t_p)}{y(\infty)} = 1.2 \quad (8.6.5)$$

Plotting $y(t)$ for different values of K, we choose the value of K, which gives the above ratio, which is verified from fig 8.6. Thus, we get

$$t_p = 0.505 \quad (8.6.6)$$

$$\Rightarrow K = 928.035 \quad (8.6.7)$$

The following code generates fig 8.6

```
codes/ee18btech11050_3.py
```

9 PHASE MARGIN

9.1 Introduction

9.2 Example

10 OSCILLATOR

10.1 Introduction

10.2 Example

11 ROOT LOCUS

11.1 Introduction

11.2 Example

11.3 Example

12 POLAR PLOT

12.1 Introduction