Control Systems

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***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

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10.1 Introduction

10.1. A unity negative feedback system has the

$$G(s) = \frac{K}{s(s+1)(s+3)} \quad (10.1.1)$$

The value of the gain K (>0) at which the root locus crosses the imaginary axis is ?

Solution:

10.2. Root Locus:
The Root locus is the locus of the roots of the characteristic equation, which are the poles of closed loop transfer function, by varying system gain K from 0 to ∞.

10.3. The characteristic equation of the closed loop control system is:

$$1 + G(s)H(s) = 0 \quad (10.3.1)$$

The points on the root locus branches must satisfy the **angle condition.** We can find the value of K for the points on the root locus branches by using **magnitude condition.**

10.4. Angle Condition:
Given the Characteristic equation, we can write it as:

$$G(s)H(s) = -1 + j0 \quad (10.4.1)$$

The phase angle of G(s)H(s) is: ∠G(s)H(s) =

$$\arctan(\frac{0}{-1}) = (2n+1)\pi$$

The angle condition is the point at which the angle of the transfer function is an odd multiple of 180.

10.5. Magnitude Condition

Magnitude of G(s)H(s) is:

$$|G(s)H(s)| = \sqrt{(-1)^2 + 0^2} \quad (10.5.1)$$

$\implies$

$$|G(s)H(s)| = 1 \quad (10.5.2)$$

The magnitude condition is that the point (which satisfied the angle condition) at which the magnitude of the transfer function is one.

10.6. For given transfer function:
H(s) = 1. So, closed loop transfer function will be

$$T(s) = \frac{K}{s(s+1)(s+3)+K} \quad (10.6.1)$$

Poles of closed loop transfer function are the roots of the Characteristic Equation. So, characteristic Equation is:

$$s^3 + 4s^2 + 3s + K = 0 \quad (10.6.2)$$

10.7. Routh Array Table:
If all elements of any row of the Routh array table are zero, then the root locus branch intersects the imaginary axis
Routh Array Table:

$$\begin{vmatrix} s^3 \\ s^2 \\ s^1 \\ s^0 \end{vmatrix} \begin{vmatrix} 1 & 3 \\ 4 & K \\ (12-K)/4 & 0 \\ K & \end{vmatrix} \quad (10.7.1)$$

For poles to be on imaginary axis, row s^1 should be zero. So,

$$\frac{12-K}{4} = 0 \quad (10.7.2)$$

Hence, K = 12.

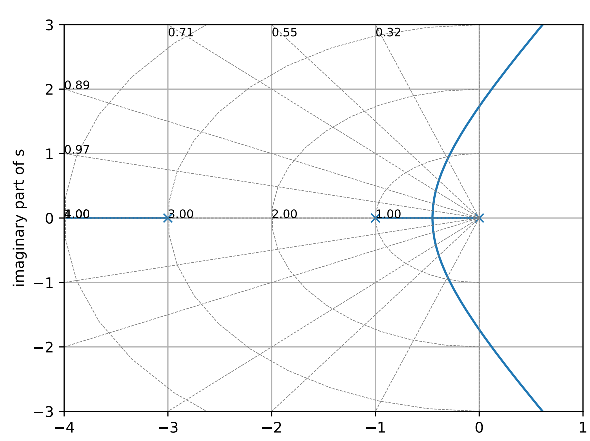
10.8. Verification:
Auxilliary equation:

$$4s^2 + K = 0 \quad (10.8.1)$$

$$4s^2 + 12 = 0 \quad (10.8.2)$$

$\implies$ s = $-j\sqrt{3}, +j\sqrt{3}$
Thus a pair of poles lie on imaginary axis for K = 12.

10.9. Root Locus plot



Code to plot root locus:

```
codes/ee18btech11050.py
```