Oscillator

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Contents

For the circuit shown in Fig. 1.1, find the loop gain $L(s) = G(s)H(s)$, $L(j\omega)$, the frequency for zero loop phase, and R_2/R_1 for oscillation.

1. Draw the equivalent control system representation for the circuit in Fig. 1.1 as well as the small signal model.
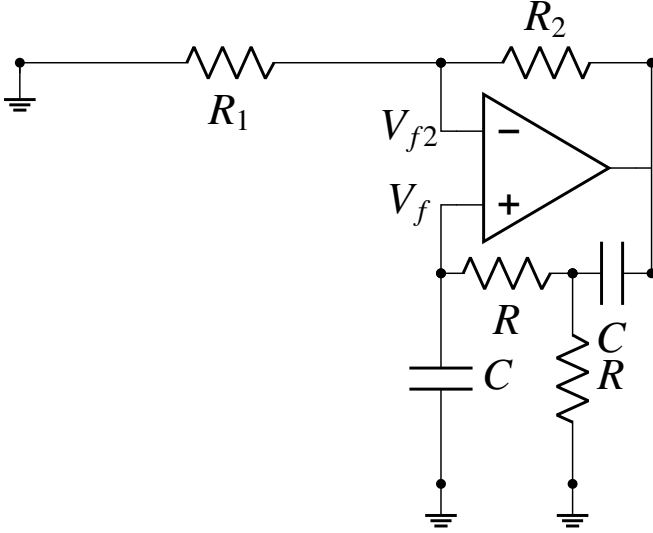   **Solution:** See Figs. 1.2, 1.3 and 1.4

Fig. 1.1

Fig. 1.2: Block diagram

Fig. 1.3: Simplified equivalent block diagram

Solution: See Fig. 1.3. Oscillators do not include input signal.

2. Find the open loop gain G.
   Solution: Let the closed loop gain, open-loop gain of op-amp connected in non-inverting configuration be T_0 and G_0 respectively. From Table **??**

$$T_0 = \frac{G_0(R_1 + R_2)}{(R_1 + R_2) + G_0 R_1} \tag{2.1}$$

$$T_0 = \frac{(R_1 + R_2)}{(R_1 + R_2)/G_0 + R_1} \tag{2.2}$$

Assuming $G_0 \to \infty$

$$T_0 = 1 + \frac{R_2}{R_1} \tag{2.3}$$

The open loop gain of the circuit shown in Fig. 1.1 is equal to the closed loop gain of an op-amp connected in non-inverting configuration.

$$G = T_0 \tag{2.4}$$

$$\implies G = 1 + \frac{R_2}{R_1} \tag{2.5}$$

3. Find the feedback factor H.
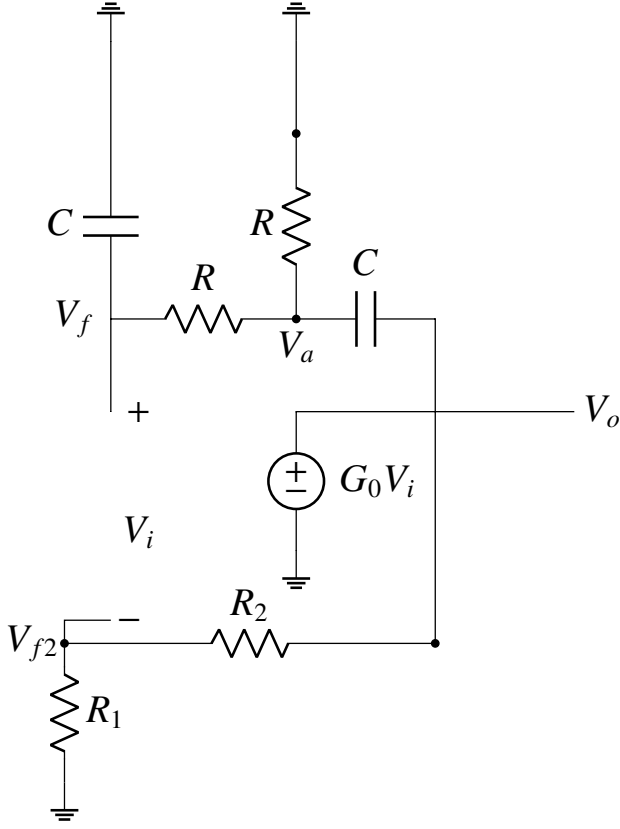   **Solution:** The small signal model is shown in

Fig. 1.4

Fig. 1.4 Applying KCL at node V_f

$$\frac{V_f - 0}{\frac{1}{sC}} + \frac{V_f - V_a}{R} = 0 \qquad (3.1)$$

$$V_f\left(sC + \frac{1}{R}\right) = \frac{V_a}{R} \qquad (3.2)$$

$$V_a = V_f(sRC + 1) \qquad (3.3)$$

Applying KCL at node V_a

$$\frac{V_a - V_f}{R} + \frac{V_a - 0}{R} + \frac{V_a - V_o}{\frac{1}{sC}} = 0 \qquad (3.4)$$

$$V_a\left(\frac{2}{R} + sC\right) = \frac{V_f}{R} + V_o sC \qquad (3.5)$$

Substitute V_a value from equation(3.3)

$$V_f(sRC + 1)\left(\frac{2}{R} + sC\right) = \frac{V_f}{R} + V_o sC \qquad (3.6)$$

$$V_f\left(3 + sRC + \frac{1}{sRC}\right) = V_o \qquad (3.7)$$

The feedback factor H is given by

$$H = \frac{V_f}{V_o} \qquad (3.8)$$

$$\implies H = \frac{1}{\left(3 + sRC + \frac{1}{sRC}\right)} \qquad (3.9)$$

4. Find the loop gain L(s).
   Solution: The transfer function of the equivalent positive feedback circuit in Fig. 1.3 is

$$T = \frac{G}{1 - GH} \qquad (4.1)$$

   Therefore, loop gain is given by

$$L = GH \qquad (4.2)$$

   From equations (2.5) and (3.9)

$$L(s) = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{1}{3 + sRC + \frac{1}{sRC}}\right) \qquad (4.3)$$

$$\implies L(s) = \left(\frac{1 + \frac{R_2}{R_1}}{3 + sRC + \frac{1}{sRC}}\right) \qquad (4.4)$$

5. Find the loop gain in terms of $j\omega$.
   Solution: Substitute $s = j\omega$ in equation (4.4)

$$L(j\omega) = \left(\frac{1 + \frac{R_2}{R_1}}{3 + j\omega RC + \frac{1}{j\omega RC}}\right) \qquad (5.1)$$

$$\implies L(j\omega) = \left(\frac{1 + \frac{R_2}{R_1}}{3 + j\left(\omega RC - \frac{1}{\omega RC}\right)}\right) \qquad (5.2)$$

6. Find the frequency for zero loop phase.
   Solution: The frequency at which loop phase will be zero (i.e. loop gain will be a real number).To obtain the required frequency, equate the imaginary part of the loop gain $L(j\omega)$ to zero.

$$j\left(\omega RC - \frac{1}{\omega RC}\right) = 0 \qquad (6.1)$$

$$\omega^2 = \frac{1}{(RC)^2} \qquad (6.2)$$

$$\implies \omega = \frac{1}{RC} \qquad (6.3)$$

7. Find R_2/R_1 for oscillation.

Solution: For oscillations to start,

- the imaginary part of the loop gain should become zero.
- the loop gain must be at least equal to unity.

From equation (5.2)

$$\left(\frac{1 + \frac{R_2}{R_1}}{3 + j(0)}\right) \geq 1 \qquad (7.1)$$

$$1 + \frac{R_2}{R_1} \geq 3 \qquad (7.2)$$

$$\implies \frac{R_2}{R_1} \geq 2 \qquad (7.3)$$

8. Draw the block diagram and circuit diagram for H.

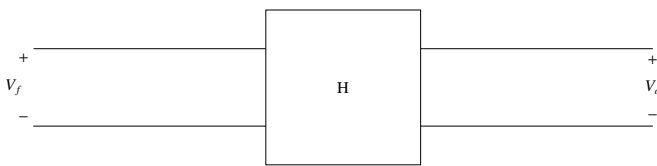   **Solution:** See figs 8.5 and 8.6 .From Fig.
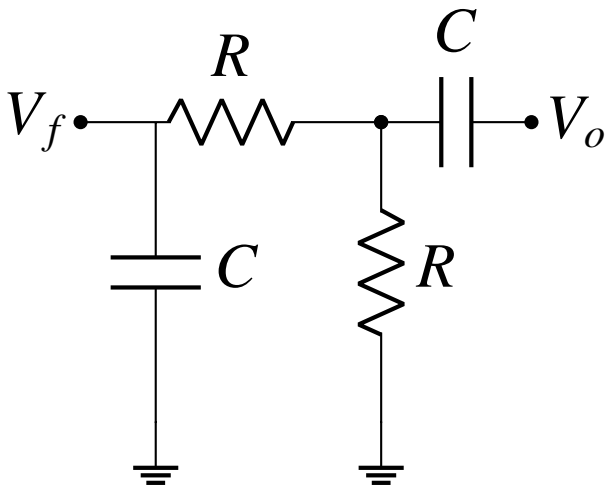

Fig. 8.5: Feedback block diagram

Fig. 8.6: Feedback circuit

8.6,the analysis is same as problem 3

$$\frac{V_f}{V_o} = \frac{1}{\left(3 + sRC + \frac{1}{sRC}\right)} \qquad (8.1)$$

$$\implies H = \frac{1}{\left(3 + sRC + \frac{1}{sRC}\right)} \qquad (8.2)$$

9. Find the input and output resistances of the feedback network.

   Solution: To find the input resistance R_{11} short the output node V_o to ground.

   $$R_{11} = Z\|(R + (R\|Z)) \qquad (9.1)$$

   where $Z = \frac{1}{sC}$ is the impedance of the capacitor.

   $$\implies R_{11} = \left(\frac{1}{sC}\|\left(R + R\|\frac{1}{sC}\right)\right) \qquad (9.2)$$

   To find the output resistance R_{22} short the input node V_f to ground.

   $$R_{22} = Z + (R\|R) \qquad (9.3)$$

   $$\implies R_{22} = \frac{1}{sC} + \frac{R}{2} \qquad (9.4)$$

10. Draw the block diagram and circuit diagram for G.

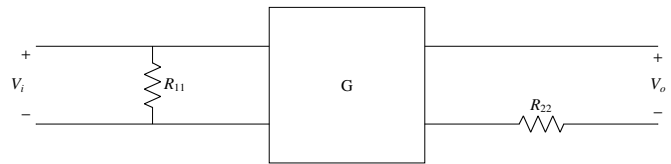   **Solution:** See figs 10.7 and 10.8.From

Fig. 10.7: Open loop block diagram

Fig.10.8

$$V_{f_2} = \left(\frac{R_1}{R_1 + R_2}\right) V_o \qquad (10.1)$$

$$G_1 = \frac{V_{f_2}}{V_o} \qquad (10.2)$$

$$\implies G_1 = \frac{R_1}{R_1 + R_2} \qquad (10.3)$$

From Fig. 1.2 ,G_1 is the negative feedback factor and G_o is the gain of the op-amp.Therefore,equivalent G is given by

$$G = \frac{G_o}{1 + G_o G_1} \qquad (10.4)$$
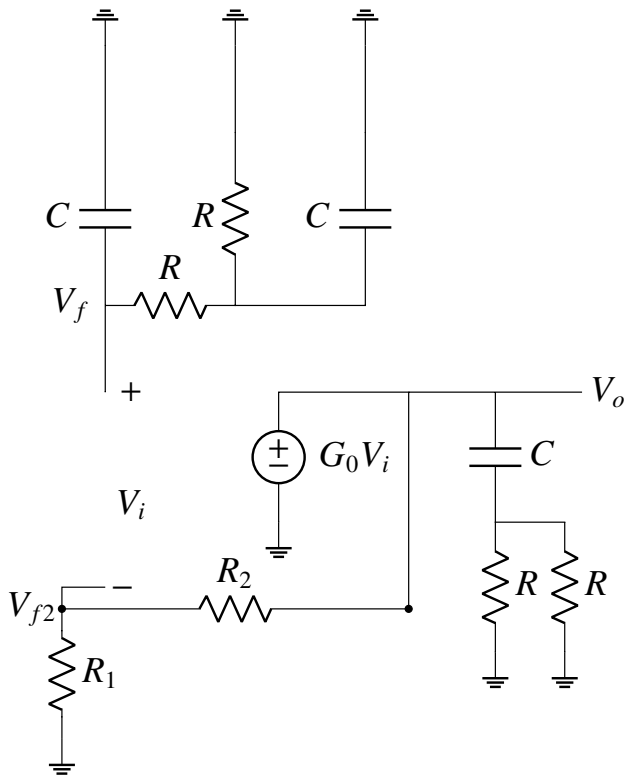
$$G = \frac{1}{\frac{1}{G_o} + G_1} \qquad (10.5)$$

Fig. 10.8: Open loop circuit diagram

We assumed $G_0 \to \infty$.

$$\implies G = \frac{1}{G_1} \qquad (10.6)$$

From equation (10.3).

$$\implies G = \frac{R_1 + R_2}{R_1} = 1 + \frac{R_2}{R_1} \qquad (10.7)$$

Hence verified with equation (2.5).

11. Find the amplitude and frequency for some arbitrary R,C values given in Table 11.
    Solution: From equation (2.5)

Parameter	Value
R	250Ω
C	$1mF$
R_2	$2k\Omega$
R_1	$1k\Omega$

TABLE 11

$$G = 1 + \frac{R_2}{R_1} = 3 \qquad (11.1)$$

From equation (3.9)

$$H = \frac{1}{3 + 0.25s + \frac{1}{0.25s}} \qquad (11.2)$$

From equation (4.1)

$$T = \frac{3\left(0.0625s^2 + 0.75s + 1\right)}{0.0625s^2 + 1} \qquad (11.3)$$

The following code plots the oscillating response of the system.
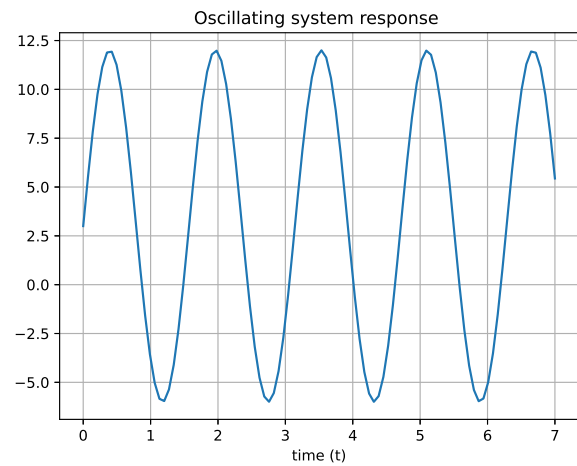
codes/ee18btech11047/ee18btech11047.py

Fig. 11

Amplitude: From Fig. 11 V(peak-peak) is

$$V_{p-p} = 11.929 - (-5.957) = 17.886 \qquad (11.4)$$

$$V_{max} = \frac{V_{p-p}}{2} = 8.943 \qquad (11.5)$$

Frequency: From equation (6.3)

$$\omega = \frac{1}{RC} = 4rad/sec \qquad (11.6)$$

$$f = \frac{\omega}{2\pi} = 0.636Hz \qquad (11.7)$$

12. Verify the amplitude and frequency using spice simulation.
    Solution: The following readme file provides necessary instructions to simulate the circuit in spice.

codes/ee18btech11047/spice/README.md

The following netlist simulates the given circuit.

> codes/ee18btech11047/spice/ee18btech11047.
>   net

The following code plots the output from the oscillator spice simulation which is shown in Fig. 12.10.

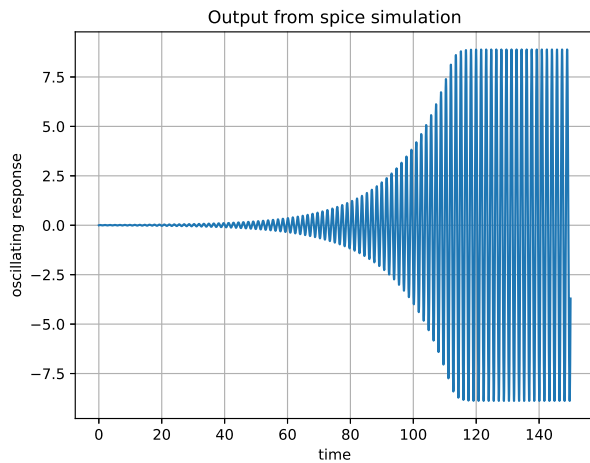> codes/ee18btech11047/spice/
>   ee18btech11047_spice.py

Fig. 12.10

The following code plots a part of the spice output from which we can observe a clear sinusoidal output shown in Fig. 12.11.

> codes/ee18btech11047/spice/
>   ee18btech11047_spice2.py
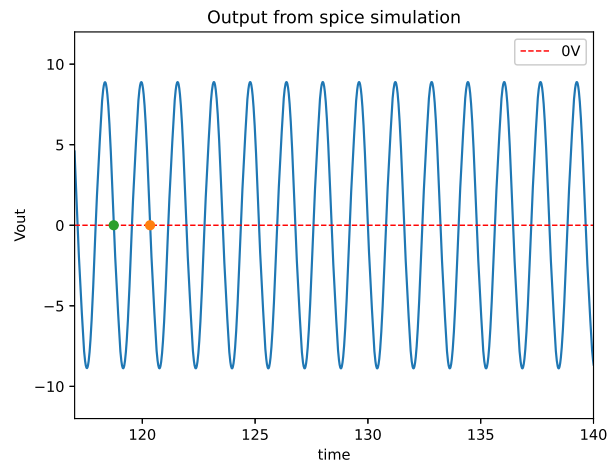

Fig. 12.11

Amplitude: From Fig. 12.11 V(peak-peak) is

$$V_{p-p} = 8.89 - (-8.89) = 17.78 \qquad (12.1)$$

$$V_{max} = \frac{V_{p-p}}{2} = 8.89 \qquad (12.2)$$

Frequency: From Fig. 12.11 time period is calculated by any two end points of one cycle,

$$T = 120.344 - (-118.734) = 1.61 \, sec \qquad (12.3)$$

$$f = \frac{1}{T} = 0.621 Hz \qquad (12.4)$$

Hence, the ampitude and frequency are verified through the spice simulation.