EE2227-CONTROL SYSTEMS

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QUESTION-41

Consider the system described by the following state space representation

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \mathrm{u(t)}$$

$$\mathrm{y(t)} = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

If u(t) is a unit step input and $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, the value of output y(t) at t = 1 sec(rounded off to three decimal places) is

SOLUTION

From the given ,

Let $A = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$

$X(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$

$\dot{X}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix}$ where $\dot{x}(t) = \frac{d}{dt}(x(t))$

SOLUTION

LAPLACE TRANSFORMS

$L\{\dot{f}(t)\} = sF(s) - f(0)$

$L\{u(t)\} = \frac{1}{s}$

$L\{t\} = \frac{1}{s^2}$

$L\{e^{at}\} = \frac{1}{s-a}$

$L\{e^{-at}\} = \frac{1}{s+a}$

SOLUTION

$\dot{X}(t)$ = Ax(t) + Bu(t) $\qquad --(1)$

Laplace transform on equation (1) results in

sX(s) - x(0) = AX(s) + Bu(s)

(sI - A)X(s) = x(0) +B$\frac{1}{s}$

X(s) = (sI - A)$^{-1}[x(0) + B\frac{1}{s}]$ $\qquad --(2)$

SOLUTION

$$sI - A = s\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} - \begin{bmatrix}0 & 1\\0 & -2\end{bmatrix} = \begin{bmatrix}s & -1\\0 & s+2\end{bmatrix}$$

$$(sI - A)^{-1} = \frac{1}{s(s+2)}\begin{bmatrix}s+2 & 1\\0 & s\end{bmatrix} = \begin{bmatrix}\frac{1}{s} & \frac{1}{s(s+2)}\\0 & \frac{1}{s+2}\end{bmatrix}$$

$$Substituting\ \ (sI - A)^{-1}\ \ in\ \ (2)$$

$$X(s) = \begin{bmatrix}\frac{1}{s} & \frac{1}{s(s+2)}\\0 & \frac{1}{s+2}\end{bmatrix}\left[\begin{bmatrix}1\\0\end{bmatrix} + \frac{1}{s}\begin{bmatrix}0\\1\end{bmatrix}\right] \qquad since, \begin{bmatrix}x_1(0)\\x_2(0)\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix}$$

SOLUTION

$$X(s) = \begin{bmatrix} \frac{1}{s} & \frac{1}{s(s+2)} \\ 0 & \frac{1}{s+2} \end{bmatrix} \begin{bmatrix} 1 \\ \frac{1}{s} \end{bmatrix}$$

$$X(s) = \begin{bmatrix} \frac{1}{s} + \frac{1}{s^2(s+2)} \\ \frac{1}{s(s+2)} \end{bmatrix}$$

On splitting into partial fractions

$$X(s) = \begin{bmatrix} \frac{1}{4(s+2)} + \frac{3}{4s} + \frac{1}{2s^2} \\ \frac{1}{2s} - \frac{1}{2(s+2)} \end{bmatrix}$$

SOLUTION

Inverse laplace transform on X(s) .....

$$\mathrm{L}^{-1}\{X(s)\} = \begin{bmatrix} L^{-1}\{\frac{1}{4(s+2)} + \frac{3}{4s} + \frac{1}{2s^2}\} \\ L^{-1}\{\frac{1}{2s} - \frac{1}{2(s+2)}\} \end{bmatrix}$$

$$x(t) = \begin{bmatrix} \frac{1}{4}e^{-2t} + \frac{3}{4} + \frac{1}{2}t \\ \frac{1}{2} - \frac{1}{2}e^{-2t} \end{bmatrix}$$

SOLUTION

Given that y(t) = C x(t)

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{4}e^{-2t} + \frac{3}{4} + \frac{1}{2}t \\ \frac{1}{2} - \frac{1}{2}e^{-2t} \end{bmatrix}$$

$$y(t) = \frac{1}{4}e^{-2t} + \frac{3}{4} + \frac{1}{2}t$$

$$y(1) = \frac{1}{4}e^{-2} + \frac{3}{4} + \frac{1}{2}(1)$$

y(1) = 1.28383

Rounding off to three decimals......

y(1) = 1.284