Control Systems

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Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

1 State-Space Model

1.1 Example

1.1.1. Consider the system described by the following state space representation

$$\dot{\mathbf{x}} = \begin{pmatrix} 0 & 1 \\ 0 & -2 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \mathbf{u} \tag{1.1.1.1}$$

$$\mathbf{y} = \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} \tag{1.1.1.2}$$

If u(t) is a unit step input and

$$\mathbf{x}(0) = \begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{1.1.1.3}$$

Find the value of output y(t) at t=1 sec(rounded off to three decimals)
Solution: The general state space system is given by

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \tag{1.1.1.4}$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t) \tag{1.1.1.5}$$

with parameters listed in Table 1.1.1.

Variable	**Size**	**Description**
$\mathbf{u}$	$p \times 1$	input(control) vector
$\mathbf{y}$	$q \times 1$	output vector
$\mathbf{x}$	$n \times 1$	state vector
$\mathbf{A}$	$n \times n$	state or system matrix
$\mathbf{B}$	$n \times p$	input matrix
$\mathbf{C}$	$q \times n$	output matrix
$\mathbf{D}$	$q \times p$	feedthrough matrix

TABLE 1.1.1

1.1.2. Find the output function $\mathbf{Y}(s)$ of the system.
Solution: Apply Laplace transform for the equation(1.1.1.4)

$$s\mathbf{I}\mathbf{X}(s) - \mathbf{x}(0) = \mathbf{A}\mathbf{X}(s) + \mathbf{B}\mathbf{U}(s) \tag{1.1.2.1}$$

$$(s\mathbf{I} - \mathbf{A})\mathbf{X}(s) = \mathbf{B}\mathbf{U}(s) + \mathbf{x}(0) \tag{1.1.2.2}$$

$$\mathbf{X}(s) = (s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}\mathbf{U}(s) \tag{1.1.2.3}$$

$$+ (s\mathbf{I} - \mathbf{A})^{-1}\mathbf{x}(0) \tag{1.1.2.4}$$

Now apply Laplace transform for the equation(1.1.1.5)

$$\mathbf{Y}(s) = \mathbf{C}\mathbf{X}(s) + \mathbf{D}\mathbf{U}(s) \tag{1.1.2.5}$$

Substitute $\mathbf{X}(s)$ from equation(1.1.2.4)

$$\mathbf{Y}(s) = (\mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D})\mathbf{U}(s) + \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{x}(0) \tag{1.1.2.6}$$

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1.1.3. Given

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 0 & -2 \end{pmatrix} \tag{1.1.3.1}$$

$$\mathbf{B} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{1.1.3.2}$$

$$\mathbf{C} = \begin{pmatrix} 1 & 0 \end{pmatrix} \tag{1.1.3.3}$$

$$\mathbf{D} = \begin{pmatrix} 0 & 0 \end{pmatrix} \tag{1.1.3.4}$$

$$\mathbf{x}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{1.1.3.5}$$

Substituting the above in equation(1.1.2.6)

$$Y(s) = \frac{s^2 + 2s + 1}{s^3 + 2s^2} \tag{1.1.3.6}$$

Splitting into partial fractions ,

$$Y(s) = \frac{1}{4(s+2)} + \frac{3}{4s} + \frac{1}{2s^2} \tag{1.1.3.7}$$

Applying inverse laplace transform on Y(s),

$$y(t) = (\frac{1}{4}e^{-2t} + \frac{3}{4} + \frac{1}{2}t)u(t) \tag{1.1.3.8}$$

y(t) at t=1 sec is y(1)=1.284(rounded off to three decimals)
The following code verifies the answer.

```
codes/ee18btech11047.py
```