Control Systems

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Contents

***Abstract*—This manual is an introduction to control systems in feedback circuits. Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/feedback/codes
```

1 Op-Amp RC Oscillator Circuits

1.1. For the circuit shown in Fig. 1.1.1, find $L(s)$, $L(j\omega)$, the frequency for zero loop phase, and R_2/R_1 for oscillation.

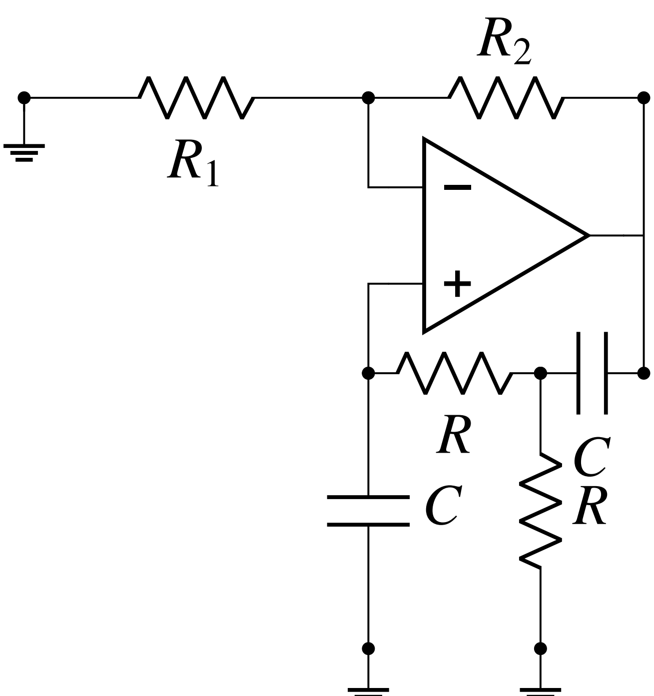

Fig. 1.1.1

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Solution: The equivalent control system representation is shown in Fig. 1.1.2. Oscillators do not include input signal.

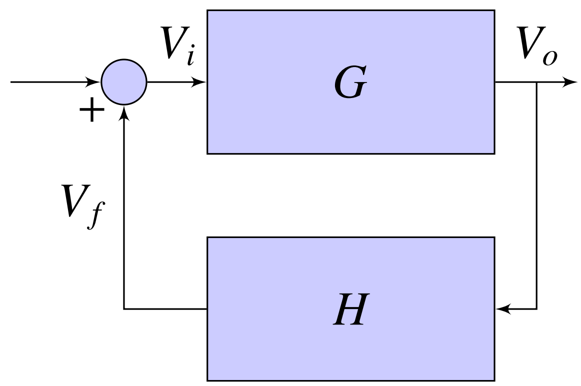

Fig. 1.1.2

1.2. Find the open loop gain G.
Solution: Let the closed loop gain, open-loop gain of op-amp connected in non-inverting configuration be T_1 and G_1 respectively. From Table **??**

$$T_1 = \frac{G_1 (R_1 + R_2)}{(R_1 + R_2) + G_1 R_1} \tag{1.2.1}$$

$$T_1 = \frac{(R_1 + R_2)}{(R_1 + R_2)/G_1 + R_1} \tag{1.2.2}$$

Assuming $G_1 \to \infty$

$$T_1 = 1 + \frac{R_2}{R_1} \tag{1.2.3}$$

The open loop gain of the circuit shown in Fig. 1.1.1 is equal to the closed loop gain of an op-amp connected in non-inverting configuration.

$$G = T_1 \tag{1.2.4}$$

$$\implies G = 1 + \frac{R_2}{R_1} \tag{1.2.5}$$

1.3. Find the feedback factor H.
Solution: The small signal model is shown in Fig. 1.3 Applying KCL at node V_f

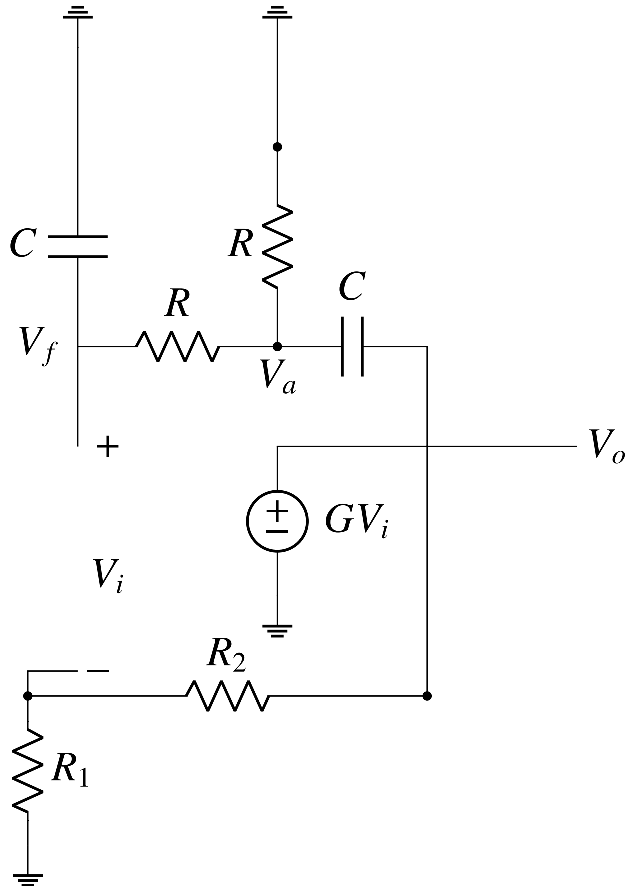

Fig. 1.3

$$\frac{V_f - 0}{\frac{1}{sC}} + \frac{V_f - V_a}{R} = 0 \tag{1.3.1}$$

$$V_f\left(sC + \frac{1}{R}\right) = \frac{V_a}{R} \tag{1.3.2}$$

$$V_a = V_f\,(sRC + 1) \tag{1.3.3}$$

Applying KCL at node V_a

$$\frac{V_a - V_f}{R} + \frac{V_a - 0}{R} + \frac{V_a - V_o}{\frac{1}{sC}} = 0 \tag{1.3.4}$$

$$V_a\left(\frac{2}{R} + sC\right) = \frac{V_f}{R} + V_o sC \tag{1.3.5}$$

Substitute V_a value from equation(1.3.3)

$$V_f(sRC + 1)\left(\frac{2}{R} + sC\right) = \frac{V_f}{R} + V_o sC \tag{1.3.6}$$

$$V_f\left(3 + sRC + \frac{1}{sRC}\right) = V_o \tag{1.3.7}$$

The feedback factor H is given by

$$H = \frac{V_f}{V_o} \tag{1.3.8}$$

$$\implies H = \frac{1}{\left(3 + sRC + \frac{1}{sRC}\right)} \tag{1.3.9}$$

1.4. Find the loop gain L(s).
Solution: The transfer function of the equivalent positive feedback circuit in Fig. 1.1.2 is

$$T = \frac{G}{1 - GH} \tag{1.4.1}$$

Therefore, loop gain is given by

$$L = GH \tag{1.4.2}$$

From equations (1.2.5) and (1.3.9)

$$L(s) = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{1}{3 + sRC + \frac{1}{sRC}}\right) \tag{1.4.3}$$

$$\implies L(s) = \left(\frac{1 + \frac{R_2}{R_1}}{3 + sRC + \frac{1}{sRC}}\right) \tag{1.4.4}$$

1.5. Find the loop gain in terms of $j\omega$.
Solution: Substitute $s = j\omega$ in equation (1.4.4)

$$L(j\omega) = \left(\frac{1 + \frac{R_2}{R_1}}{3 + j\omega RC + \frac{1}{j\omega RC}}\right) \tag{1.5.1}$$

$$\implies L(j\omega) = \left(\frac{1 + \frac{R_2}{R_1}}{3 + j\left(\omega RC - \frac{1}{\omega RC}\right)}\right) \tag{1.5.2}$$

1.6. Find the frequency for zero loop phase.
Solution: The frequency at which loop phase will be zero (i.e. loop gain will be a real number).To obtain the required frequency, equate the imaginary part of the loop gain $L(j\omega)$ to zero.

$$j\left(\omega RC - \frac{1}{\omega RC}\right) = 0 \tag{1.6.1}$$

$$\omega^2 = \frac{1}{(RC)^2} \tag{1.6.2}$$

$$\implies \omega = \frac{1}{RC} \tag{1.6.3}$$

1.7. Find R_2/R_1 for oscillation.

Solution: For oscillations to start,

- the imaginary part of the loop gain should become zero.
- the loop gain must be made greater than unity.

From equation (1.5.2)

$$\left(\frac{1+\frac{R_2}{R_1}}{3+j(0)}\right) > 1 \tag{1.7.1}$$

$$1+\frac{R_2}{R_1} > 3 \tag{1.7.2}$$

$$\implies \frac{R_2}{R_1} > 2 \tag{1.7.3}$$