1. Methodology Summary

This study looks at the performance and risk features of four equity portfolio strategies made from NIFTY 50 constituent stocks from **2009 to 2022**.

The main goal is to compare optimized quantitative strategies with simple allocation methods and the market benchmark, NIFTY 50.

1.1 Data Preparation and Cleaning

- **Data Source:** Daily closing prices for 50 NIFTY constituents and the NIFTY 50 index were analyzed from (**January 1, 2009**) **to** (**December 31, 2022**).
- **Filtering:** Following the assignment instructions for missing data, any stock with missing or partial observations was removed. This led to the exclusion of 5 stocks, resulting in a final total of **45 stocks.**
- **Returns:** Daily **simple returns** were computed as:

$$R_t = \frac{P_t}{P_{t-1}} - 1$$

- **Risk-Free Rate:** The **daily annualized risk-free rate** in the factor data was averaged over the whole period, around 2.45%. This average serves as the constant risk-free rate for calculating the Sharpe ratio.

1.2 Portfolio Construction Strategies

Each portfolio was rebuilt for each rolling window using a 6-month formation period. The strategies are summarized below:

- **Global Minimum Variance (GMV):**
  - **Objective:** Minimize portfolio volatility while ignoring expected returns.
  - **Method:** Weights calculated as

$$w = \frac{\Sigma^{-1}\mathbf{1}}{\mathbf{1}^T\Sigma^{-1}\mathbf{1}}$$

  Where Σ is the sample covariance matrix.

- **Mean-Variance (Tangency):**
  - **Objective:** Maximize the portfolio Sharpe Ratio for better risk-adjusted returns.
  - **Method:** Weights calculated as

$$w = \frac{\Sigma^{-1}\mu_{excess}}{\mathbf{1}^T\Sigma^{-1}\mu_{excess}},$$

  Using sample mean excess returns (μ_{excess})

- **Equal-Weighted (EW):**
  - **Objective:** Naive diversification. Which means “we don't know which stock is better, so we will treat them exactly the same” .
  - **Method:** Allocates equal capital $1/45$ to each stock.

- **Active Portfolio:**
  - **Objective:** Select stocks that show statistically significant alpha.
  - **Method:** A CAPM regression ($R_i - R_f = \alpha_i + \beta_i (R_m - R_f)$) was run for each stock. Stocks with a statistically significant alpha (α) at the 95% confidence level ($p < 0.05$) were selected and equally weighted

1.3 Backtesting Framework

- **Rolling Window Design:**
  - **Formation Period:** 6 months, used for estimating mean returns and the covariance matrix.
  - **Holding Period:** 3 months (used for realized, out-of-sample returns)
- **Total Windows:** 54 quarterly windows between July 2009 – December 2022
- **Constraints:** Portfolios were fully invested ($\Sigma Wi = 1$) and allowed short positions (negative weights permitted).

1.4 Risk Management (VaR)

To assess downside risk, a **99% Historical Value at Risk (VaR)** was calculated for each 3-month holding period using **Historical Simulation**.

- **Simulation:** We generated 50,000 bootstrapped return paths from the 6-month formation window.
- **Estimation:** The **1st percentile** of the simulated 3-month return distribution was noted as the **99% VaR.**

2. Performance Analysis

2.1 Cumulative Wealth Growth

The cumulative wealth curve (Figure 1) shows a significant difference between optimized portfolios and benchmark portfolios.

- **Optimized Portfolios (GMV/MV):** These portfolios showed better long-term growth, increasing 1 unit of initial capital to about 12 units by 2022.
- **Benchmark (NIFTY 50):** The index fell behind, increasing to just about 2.5 units.
- **Observation:** The GMV portfolio's steady growth shows the advantage of reducing fluctuations in unstable emerging markets.

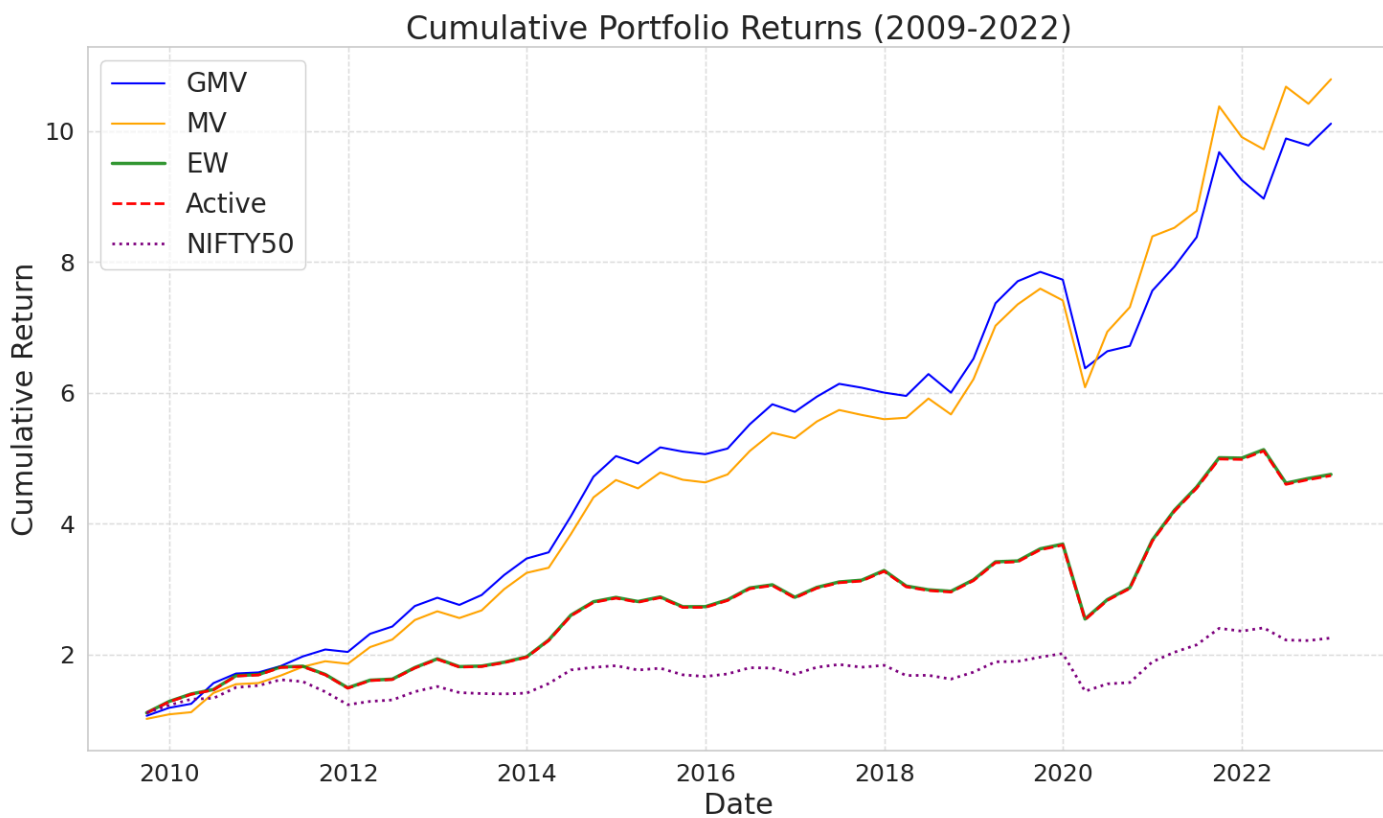


Figure :01

2.2 Performance Metrics

The table below summarizes annualized performance metrics:

Portfolio	Annualized Mean Return	Annualized Std Dev	Sharpe Ratio	Information Ratio
GMV	19.69%	13.56%	1.27	0.89
MV	20.37%	14.27%	1.26	0.90
EW	13.82%	16.73%	0.68	1.49
Active	13.79%	16.71%	0.68	1.48
NIFTY 50	7.46%	15.16%	0.33	-

1. Annualized Mean Return = $(1+\bar{R}_{\text{period}})^N - 1$
2. Annualized Standard Deviation (Volatility) = $\sigma_{\text{period}} \times \sqrt{N}$
3. Sharpe Ratio (Annualized) = $\dfrac{R_{\text{annualized}} - R_f}{\sigma_{\text{annualized}}}$
4. $\text{Information Ratio} = \dfrac{E[R_p - R_b]}{\sigma(R_p - R_b)}$

Annualized Risk-Free Rate : 0.0245

Key Insights:

- **Risk-Adjusted Winner:** The **GMV Portfolio** achieved the **highest Sharpe Ratio (1.27)** : delivering **2.6 times higher return** than NIFTY 50 while maintaining **lower volatility** (13.56% vs. 15.16%).
- **Active vs. Equal-Weighted:** The **Active portfolio's performance** closely mirrors the **Equal-Weighted** strategy, indicating that the alpha-significance filter (p<0.05) was **not selective enough** — likely including most stocks each period, effectively collapsing into EW behavior.

Main Takeaway for Performance: Portfolio optimization strategies, GMV and MV, worked well. They produced returns that were 2-3 times higher than the NIFTY50 benchmark while taking on less risk. This led to a Sharpe Ratio that was almost four times greater.

2.3 Rolling Returns

The rolling_return.csv file contains a 54x5 matrix of realized 3-month (quarterly) holding period returns for each of the five portfolios.
These returns are the main output of the rolling window backtest method. This process, which avoids look-ahead bias, was done as follows:

1. A **6-month "formation" window** was used to estimate parameters such as mean, covariance, alpha, and beta. It also calculated the optimal portfolio weights.
2. These fixed weights were then applied during the following **3-month "holding" window** to calculate a single out-of-sample realized return.
3. The entire 9-month (6+3) window was then moved forward by 3 months, and the process was repeated from 2009 to 2022.

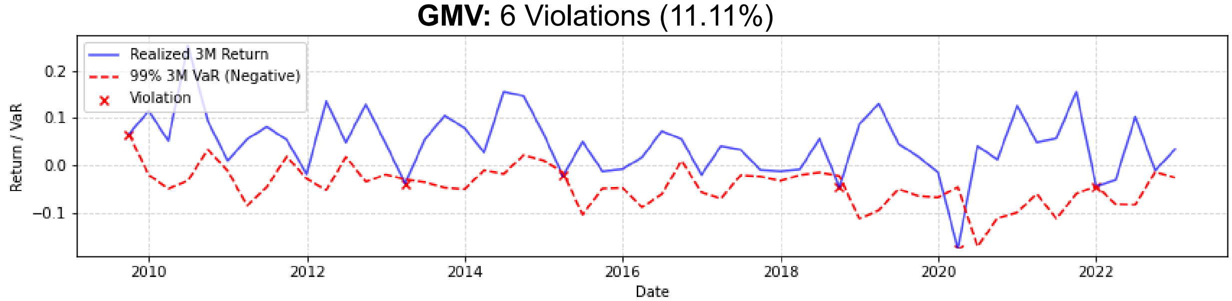
This created a time series of 54 non-overlapping returns. This rolling returns dataset, not the original daily returns, was used as the basis for all performance analysis, including the cumulative return plot, the annualized performance table, and the Information Ratios.

3. VaR Backtest Results

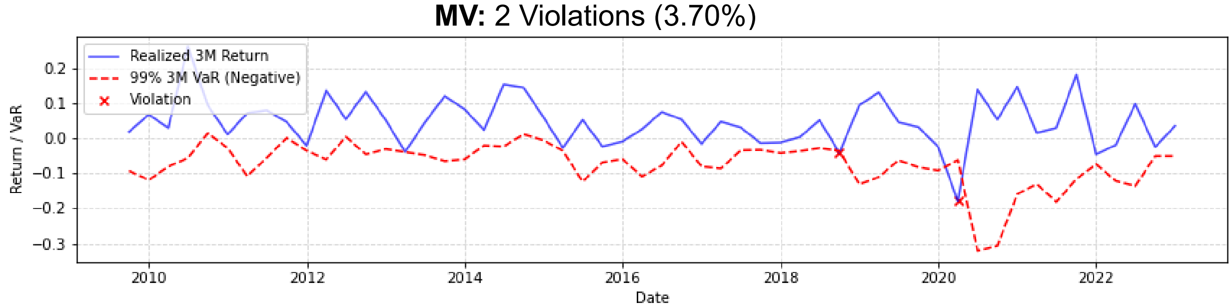
The 99% Historical VaR model was thoroughly tested across 54 windows. A "violation" happens when the actual 3-month loss goes beyond the estimated VaR limit.

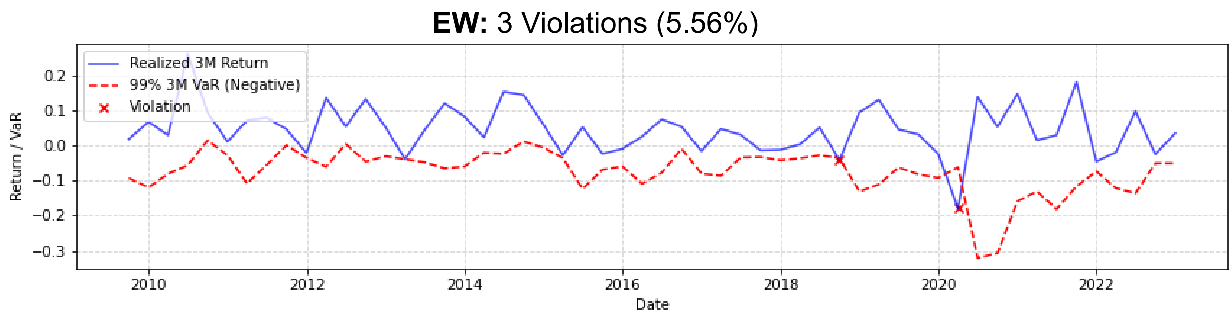
Expected Violations: With a 99% confidence level, we anticipate a violation rate of 1%, or about 0.5 violations over **54 windows.**

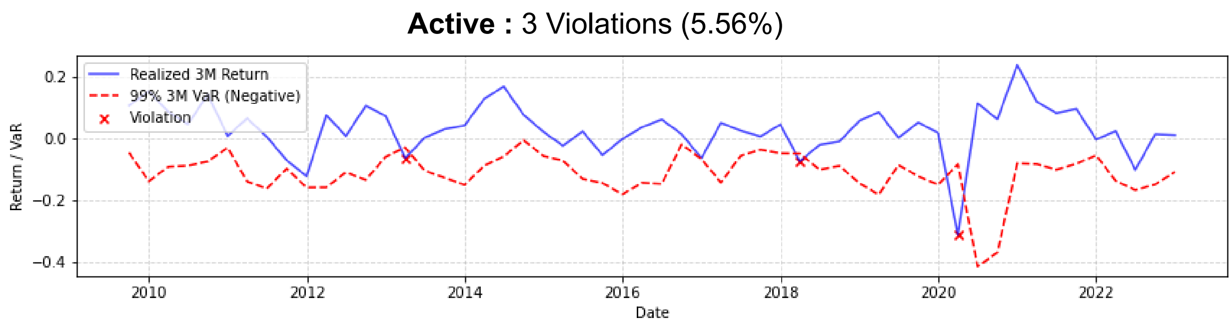
GMV: 6 Violations (11.11%)



MV: 2 Violations (3.70%)







Interpretations:

1. **Underestimation of Tail Risk:**
   The historical simulation method **consistently underestimated downside risk.** It produced violation rates that were much higher than the 1% target.
2. **Regime Sensitivity:**
   The 6-month lookback only captures recent calm periods. Therefore, sudden spikes in volatility, such as during **Covid 19 in year 2020**, lead to failures in the **VaR model**.
   GMV Paradox:
   Even though the GMV strategy is considered a “low-risk” portfolio, it showed the **highest violation rate at 9.26%.** This illustrates the **“fat tail” phenomenon**, where stable portfolios can unexpectedly endure large losses.

4. Key Takeaways

1. **Mathematical Optimization Adds Value**

   Both **GMV** and **Tangency (MV)** portfolios outperformed the market benchmark. This shows the benefit of systematic portfolio optimization compared to naïve or market-cap weighting.

2. **Historical Alpha ≠ Predictive Power**
   The **Active strategy** did not provide better performance than the Equal-Weighted strategy. This suggests that **short-term historical alpha** does not reliably predict future outperformance in Indian equities.

3. **Historical VaR Is Lagged and Reactive**
   The Historical Simulation VaR model reacts too slowly to changes in market conditions. This leads to a consistent underestimation of risk. More responsive models like **EWMA (Exponentially Weighted Moving Average)** could improve adaptability.

5. Conclusion

The evidence shows that **quantitative optimization** frameworks like GMV and MV regularly outperform the NIFTY benchmark in both **absolute** terms and **adjusted for risk**.

However, basic historical simulation risk models like VaR may not suffice in rapidly changing markets.

Future developments could include **machine learning-based alpha signals** to enhance return prediction and risk management.