PARTICLE MECHANICS APPLICATION:

THE MATERIAL POINT METHOD

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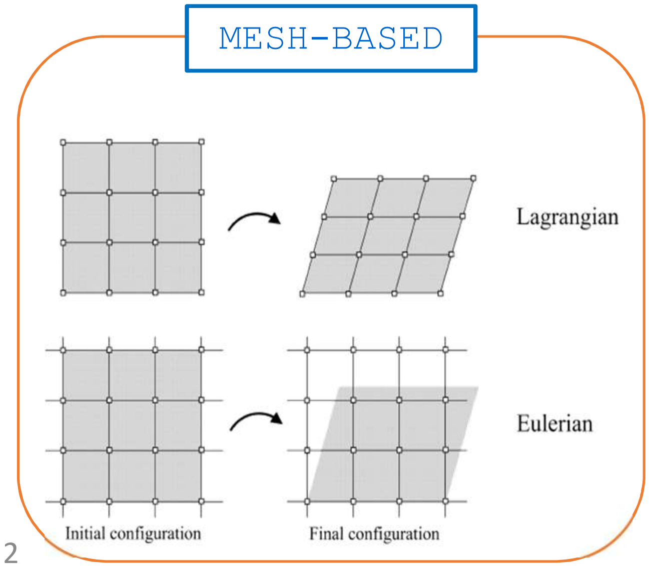
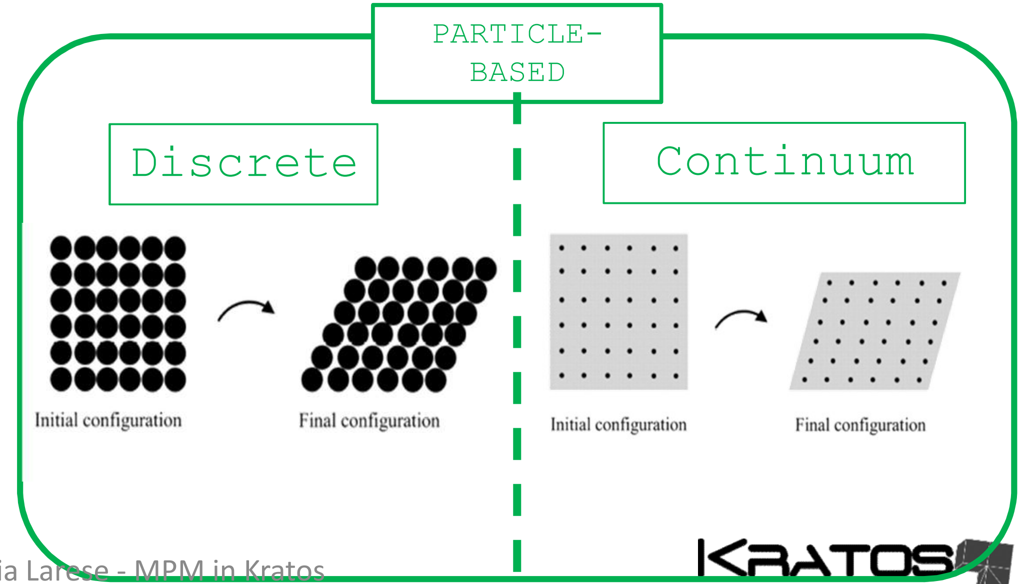
WHY PARTICLE METHODS?

We look for a numerical technique :

- In the continuum mechanics framework
- Able to handle large deformation and displacement (GEOMETRIC NON LINEARITY) without mesh tangling or necessity of remeshing.
- Able to handle history dependent material (MATERIAL NON-LINEARITY).
- Able to handle coupled formulations easily.
- Having good conservation properties.
- Parallelizable and suitable for multi-scale problems.

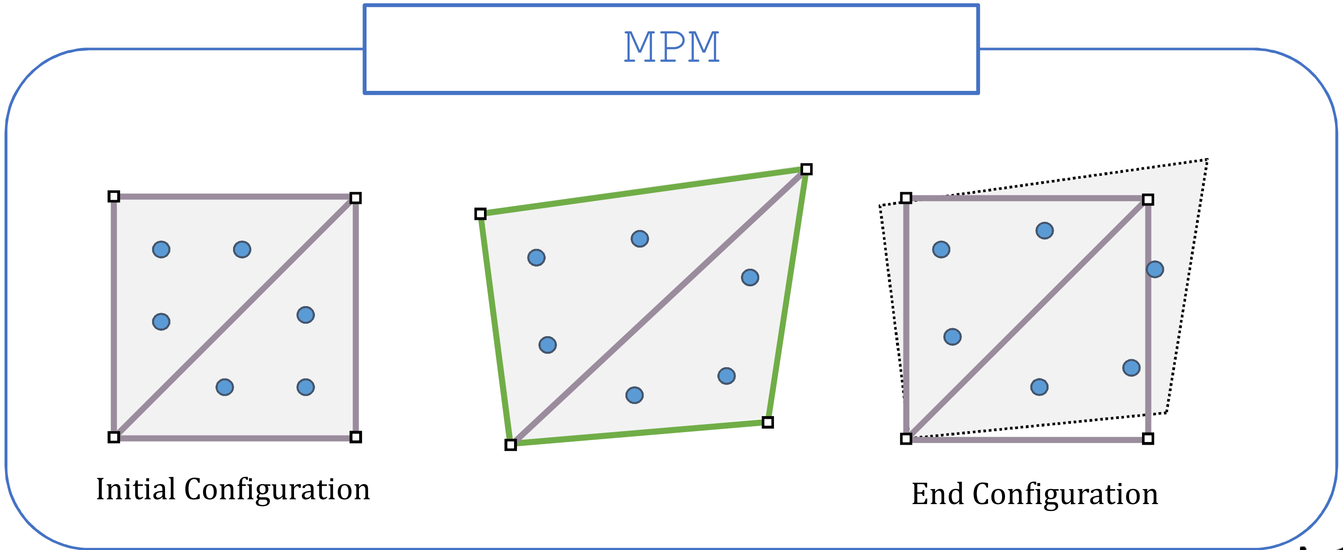




AY Colom. PhD Thesis. 2015

WHY MPM?

- Combines the **strengths of Eulerian and Lagrangian** methods.
- Simple tracking of **geometric** and **material non-linearities**.
- Can simulate **wide range of materials**: solid, fluid, soil, or combination of them.
- Can simulate **static**, **dynamic**, and **transient** problems simultaneously and concurrently.
- **Parallelizable** and suitable for **multi-scale** problems.
- Generally simple, as it is just an **extension of standard FEM**.



IMPLICIT MPM?

- MPM is very popular among the geomechanics community.
- Most of MPM formulations are **explicit.**
- We develop an **IMPLICIT** version of **MPM.**

IMPLICIT vs EXPLICIT FORMULATIONS

× More complicated

× More computationally expensive

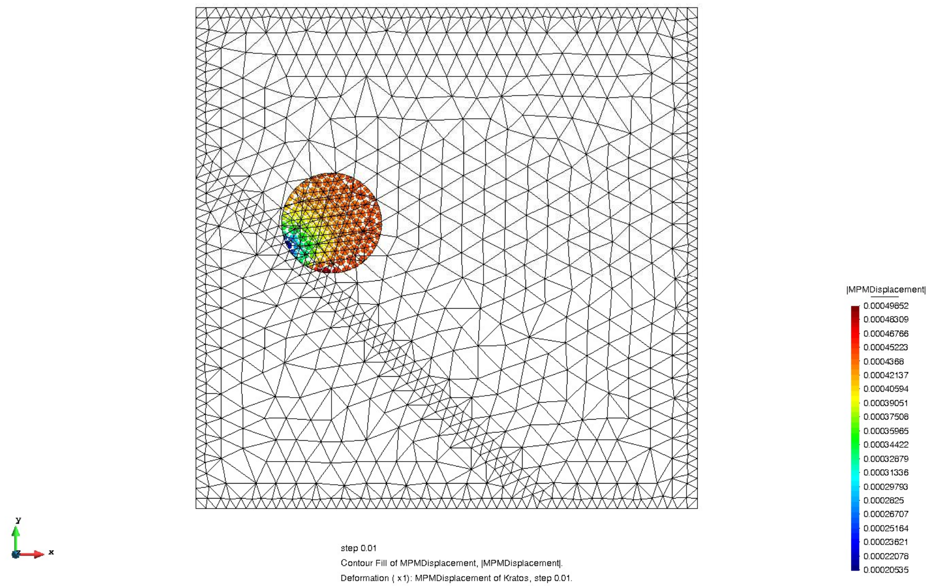
✓ More **accurate**

✓ **Stability** does not depend on the wave propagation speed in the media.

✓ More **robust**

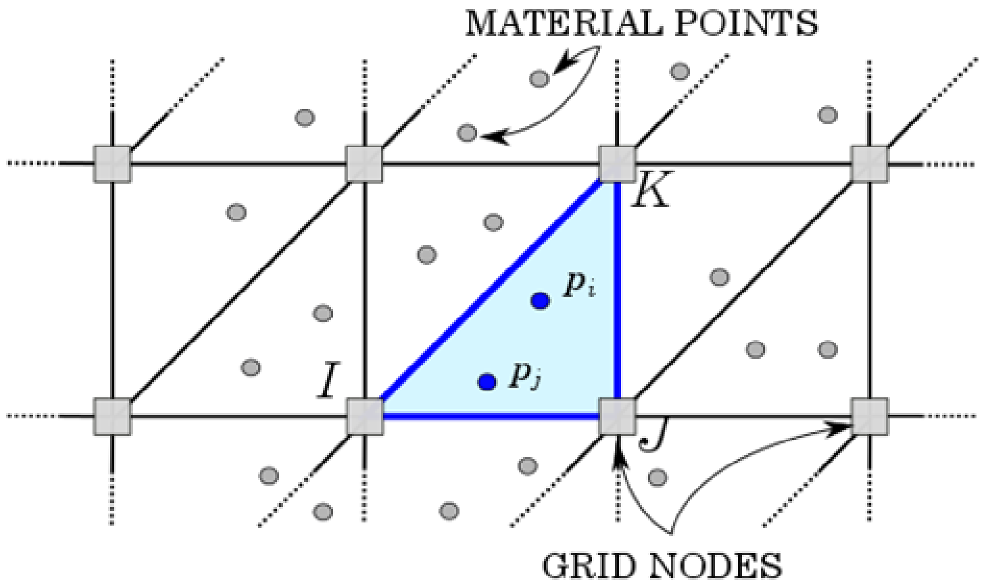
✓ More "FEM like"

✓ Easier for coupling with other numerical methods.



THEORETICAL FRAMEWORKS

IMPLICIT MATERIAL POINT METHOD (MPM)



NOMENCLATURE:

- i, j, k material points (MP)
- I, J, K grid nodes (nodes)

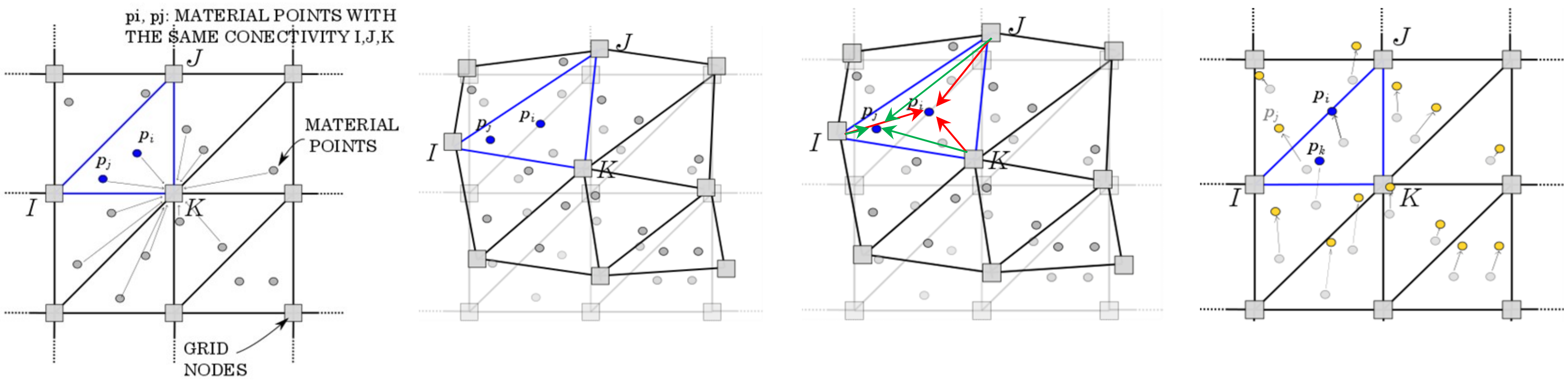
Implicit MPM in KRATOS:

- Each **MATERIAL POINT ELEMENT** is defined by the <u>material point </u>itself and its <u>connectivity.</u>
- At the beginning of each time step a **bin-based search** is performed to identify which background element does the MP lay on.
- **Prediction** and **Correction scheme** is used through a non-linear iteration to obtain problem solutions at each time step.

Solution strategy of MPM

Classical MPM stages are followed at each time step:

1. INITIALIZATION PHASE
2. Updated Lagrangian-FEM CALCULATION PHASE
3. CONVECTIVE PHASE



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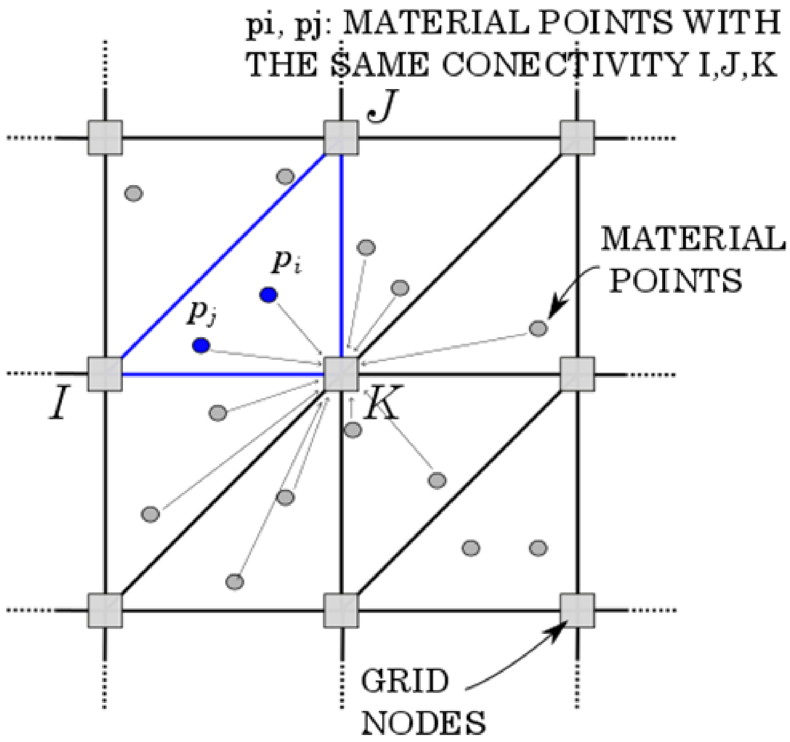
1. Initialization phase



At the beginning of every time step, the **initial conditions** on the background grid's nodes are defined during the initialization phase.

The initialization phase is composed by:

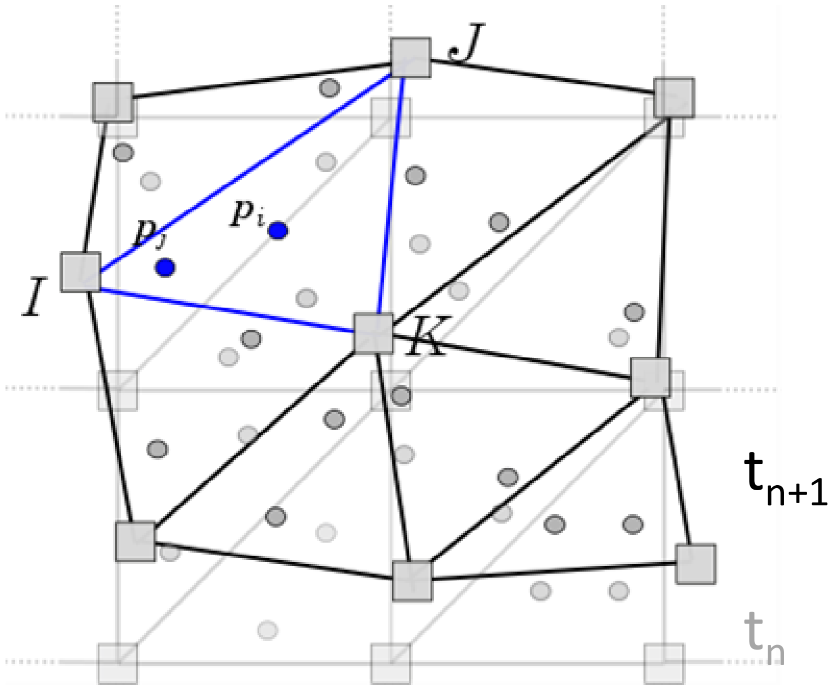
a. <u>Update</u> of material points connectivity

b. <u>Reset</u> of all nodal information

c. <u>Projection</u> of historical information obtained at the previous time step t_n from the material point to the background nodes

d. <u>Prediction</u> of nodal displacement, velocity and acceleration using a Newmark scheme

2. UPDATED LAGRANGIAN PHASE



Ω : initial (un-deformed) configuration

$\varphi(\Omega)$: current configuration

Finite Element procedures:

a. <u>Calculate Elemental System</u>

The **local** left-hand-side (**lhs**) and right-hand side (**rhs**) are evaluated in the current configuration $\varphi(\Omega)$

b. <u>Assemble</u>

The **global LHS** and **RHS** are obtained by assembling the local contributions

c. <u>Solve</u>

The system is iteratively **solved**.

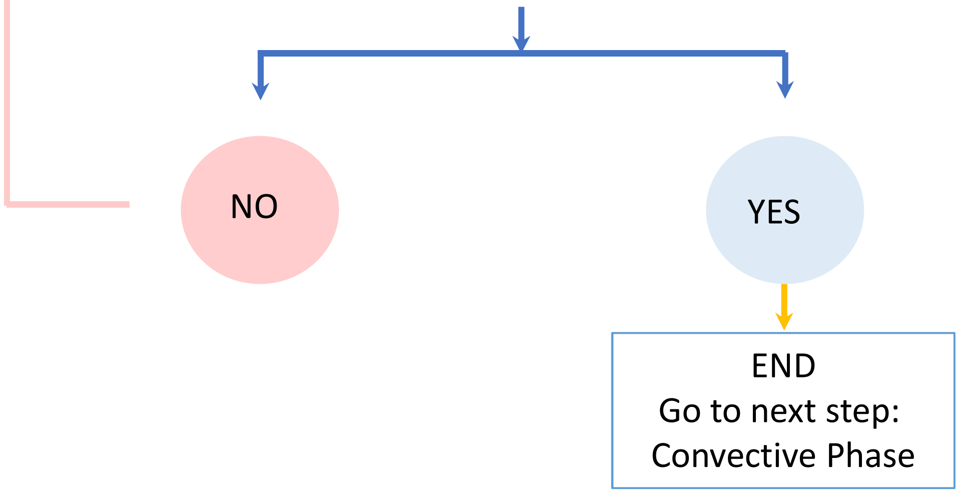
${}^{k+1}\delta \boldsymbol{u}_I^{n+1}$ is calculated

The material points **do not change their local position** within the geometrical element until the solution has reached convergence.

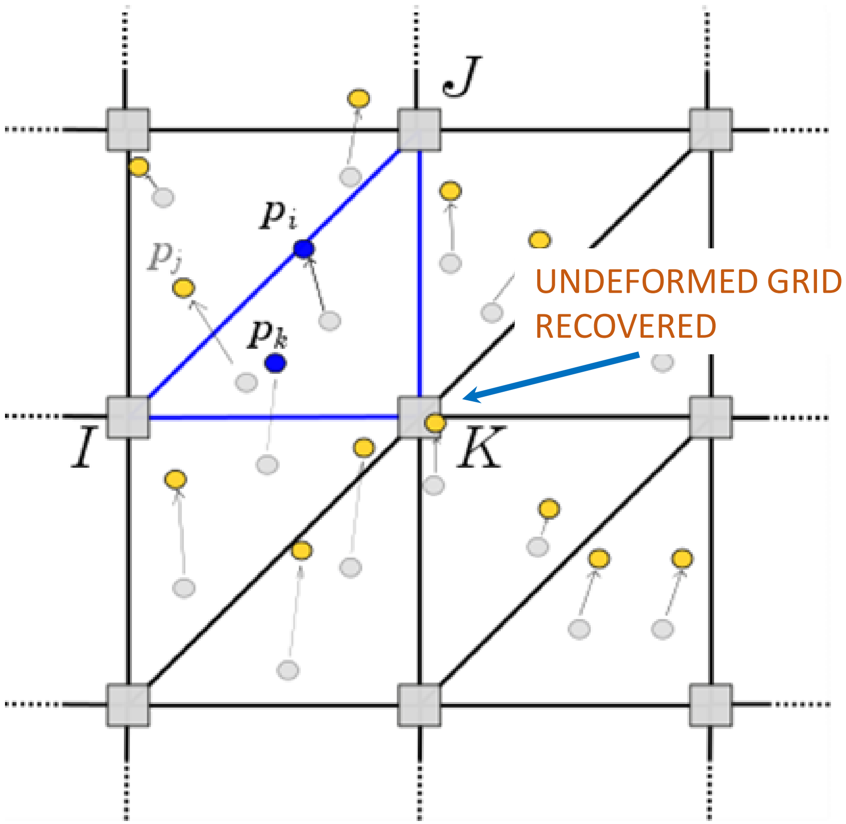
Governing equations: CHECKING CONVERGENCE

PROCEDURE:

a. **CALCULATE ELEMENTAL SYSTEM**
b. **ASSEMBLE**
c. **SOLVE**
   The system is iteratively **solved**
   ${}^{k+1}\delta u_I^{n+1}$ is calculated
d. **NEWMARK UPDATE**
e. **CHECK CONVERGENCE**



3. CONVECTIVE PHASE



The convective phase is composed by:

1. <u>Interpolation</u>
   Nodal information at time t_{n+1} are interpolated back onto the material points.

   MP displacement: $$\Delta \boldsymbol{u}_p^{n+1} = \sum_{n=1}^{n_n} N_I\left(\xi_p, \eta_p\right) \Delta \boldsymbol{u}_I^{n+1}$$

   MP acceleration: $$\boldsymbol{a}_p^{n+1} = \sum_{n=1}^{n_n} N_I\left(\xi_p, \eta_p\right) \boldsymbol{a}_I^{n+1}$$

   MP velocity: $$\boldsymbol{v}_p^{n+1} = \boldsymbol{v}_p^{n} + \frac{1}{2}\Delta t \left(\boldsymbol{a}_p^{n} + \boldsymbol{a}_p^{n+1}\right)$$

2. <u>Update</u>
   MP position is updated

   MP position (update): $$\mathbf{x}_p^{n+1} = \mathbf{x}_p^{n} + \Delta \mathbf{u}_p^{n+1}$$

3. <u>Reset</u>
   The background grid is reset to its original position

KRATOS MULTI-PHYSICS

MIXED FORMULATION

Idea: solve for pressure as an additional DOF.

$$\begin{cases} -\nabla \cdot \sigma = f & \text{in} \quad \varphi(\Omega) \\ \sigma \cdot n = \bar{t} & \text{on} \quad \varphi(\partial\Omega_N) \\ u = \bar{u} & \text{on} \quad \varphi(\partial\Omega_D) \end{cases}$$

$$\begin{cases} -\nabla \cdot (\sigma^{\text{dev}} + p\mathbf{1}) = f & \text{in} \quad \varphi(\Omega) \\ p - (\frac{1}{3}\mathbf{1} : \sigma) = 0 & \text{in} \quad \varphi(\Omega) \\ (\sigma^{\text{dev}} + p\mathbf{1}) \cdot n = \bar{t} & \text{on} \quad \varphi(\partial\Omega_N) \\ u = \bar{u} & \text{on} \quad \varphi(\partial\Omega_D) \end{cases}$$

Weak form
Discretization
Linearization

Stabilization for the treatment of the incompressibility constraint: Polynomial Pressure Projection [Dohrmann & Bochev]

$$\begin{bmatrix} \mathbf{K}^{\tan} & \mathbf{B} \\ \mathbf{B}^* & -\mathbf{M} - \mathbf{M}^{\text{stab}} \end{bmatrix} \begin{bmatrix} \delta u \\ \delta p \end{bmatrix} = - \begin{bmatrix} R_u \\ R_p + R_p^{\text{stab}} \end{bmatrix}$$

$$\begin{bmatrix} {}^m\mathbf{K}^{\tan} & \mathbf{B} \\ \mathbf{B}^* & -\mathbf{M} \end{bmatrix} \begin{bmatrix} \delta u \\ \delta p \end{bmatrix} = - \begin{bmatrix} R_u \\ R_p \end{bmatrix}$$

$$\begin{bmatrix} D_u G_{(u,p)}(w) & D_p G_{(u,p)}(w) \\ D_u G_{(u,p)}(q) & D_p G_{(u,p)}(q) \end{bmatrix}$$

Iaconeta, I., Larese, A., Rossi, R., & Oñate, E. (2018). A stabilized mixed implicit Material Point Method for non-linear incompressible solid mechanics. Computational Mechanics, 1-18.

SOME IMPLEMENTED FEATURES

FEATURES

Available Features:

- Elements:
  - *Updated-Lagrangian* elements (tri, quad, tetra, hexa)
  - *Axis-symmetric* Updated-Lagrangian elements (tri, quad)
  - *Mixed* Updated-Lagrangian elements (tri) (one phase)

- Nodal boundary conditions:
  - Neumann bcs
  - Slip bcs

- Particle boundary conditions:
  - Particle-based Neumann conditions
  - Non-conforming boundary condition.

- Constitutive laws:
  - Solid: linear elastic, hyperelastic Neohookean
  - Soil (one-phase): Mohr Coulomb, Mohr Coulomb with Strain Softening, Modified Cam Clay

FEATURES

Available Features:

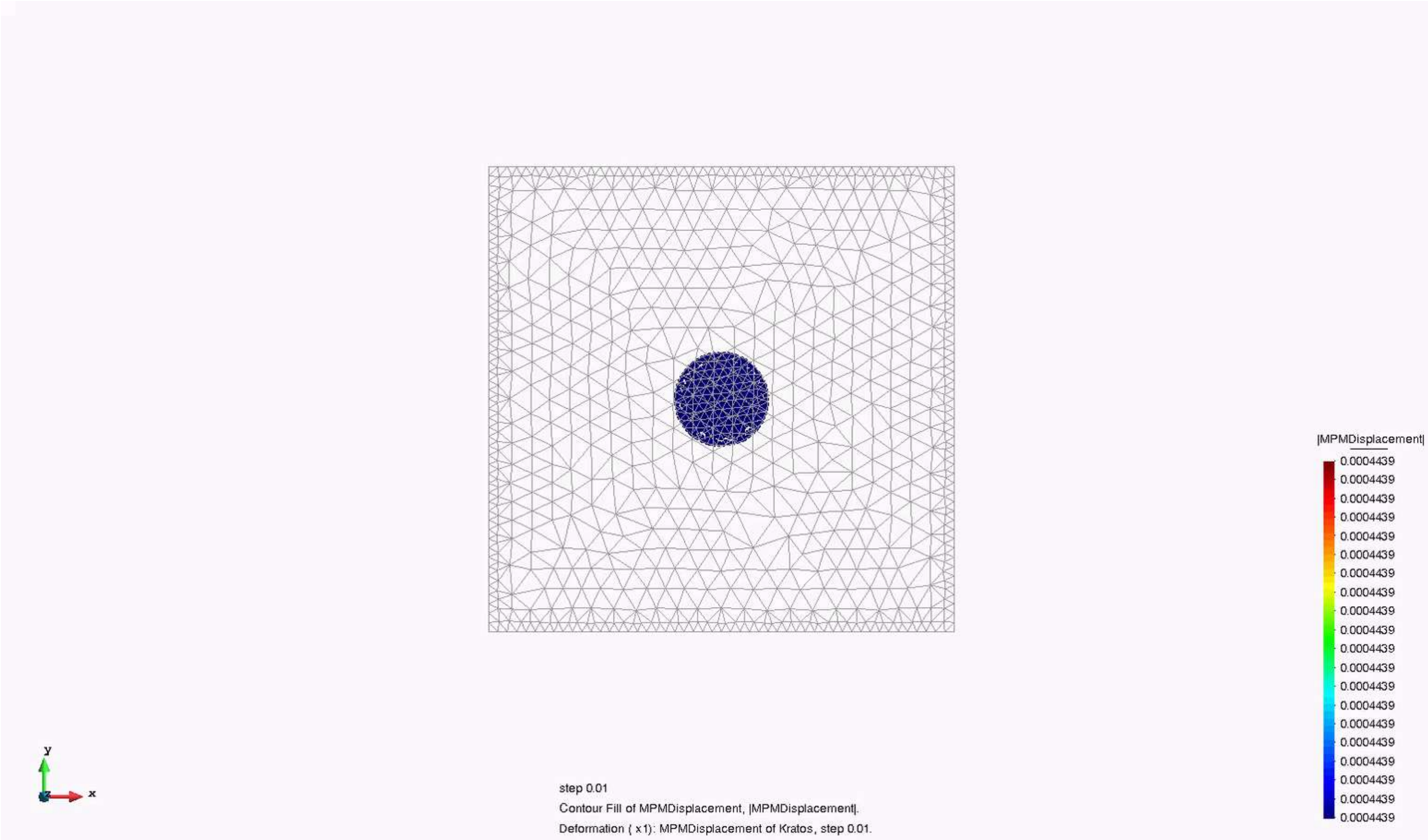
- Other features:
  - particle erase process,
  - particle output processes
- Integrated pre and post processing
- Tests and validation examples

Features Under Development:

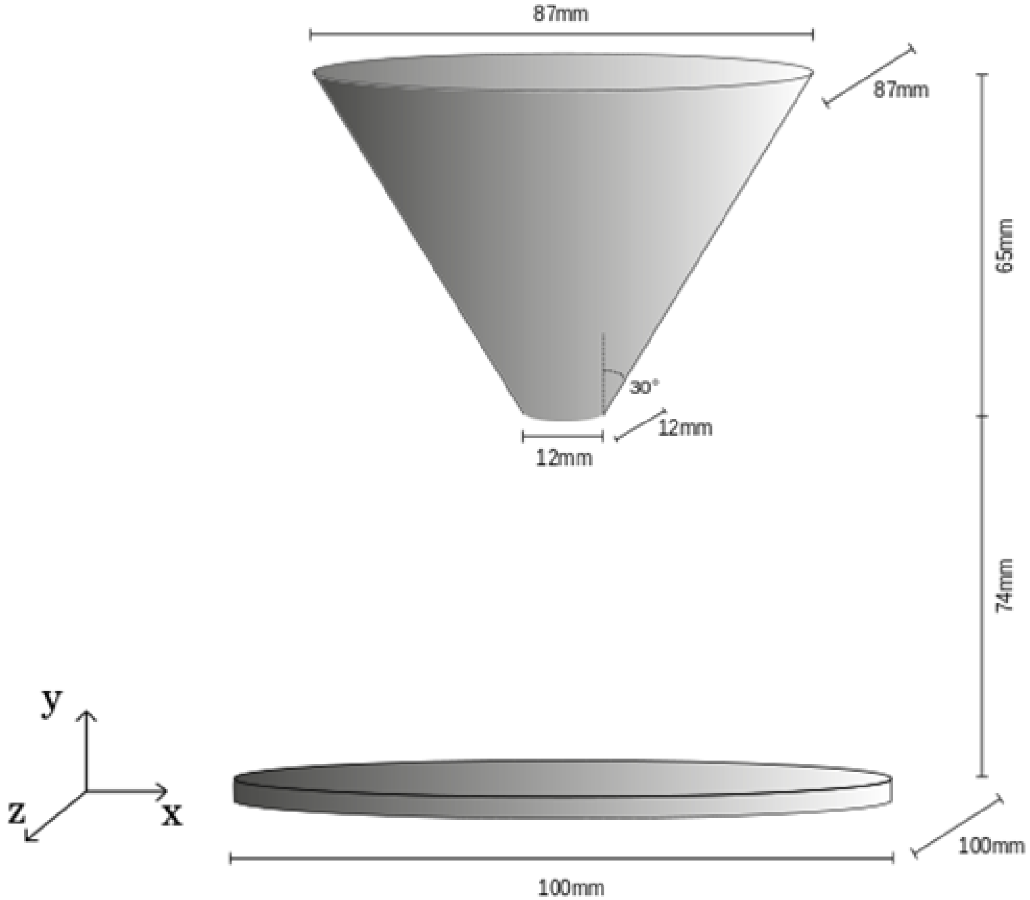
- Constitutive laws:
  - Soil (one-phase): Drucker-Prager, Nor-Sand, Bounding Surface Plasticity, Viscoplastic laws, Hypo-elastic law.
  - Newtonian fluid
- Coupling Strategy with Finite Element Method.
- Using **Isogeometric Analysis** for the definition of the interpolation function

SOME EXAMPLES

Transient Problems



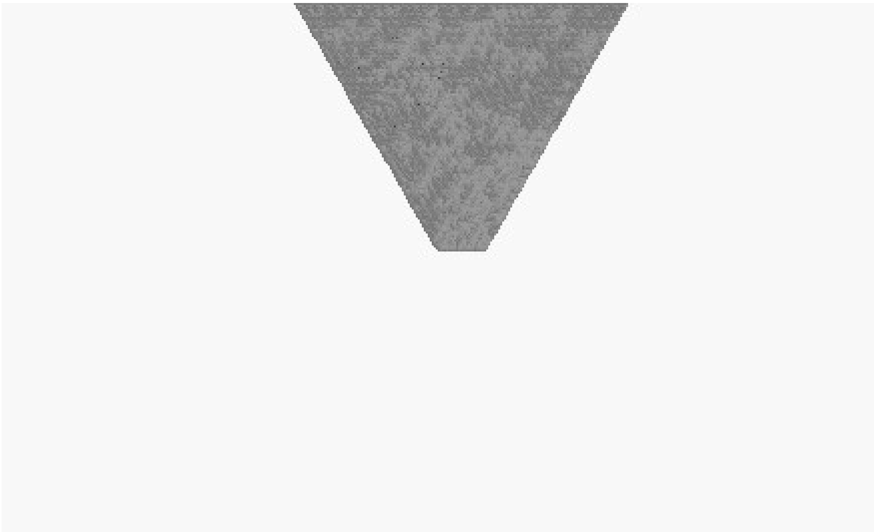
Sugar repose angle test

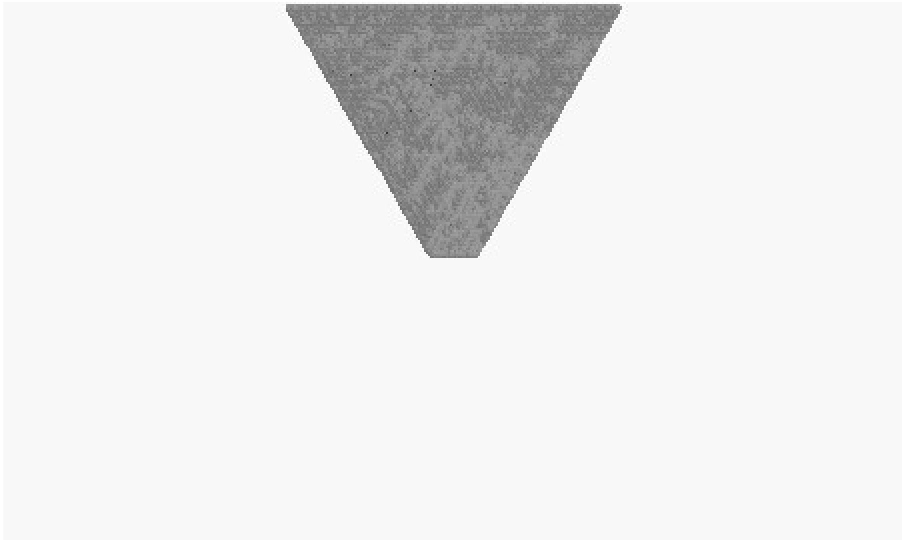


REPOSE ANGLE TEST (granular non cohesive material)

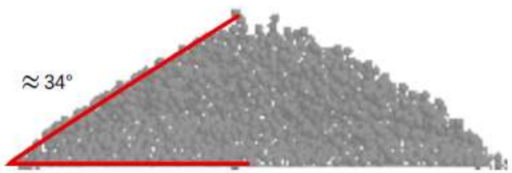
Material 1

Material 2

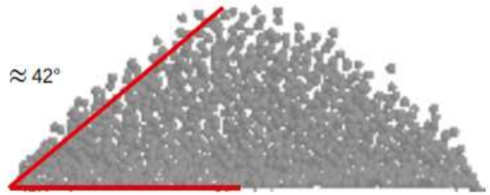




Material 1

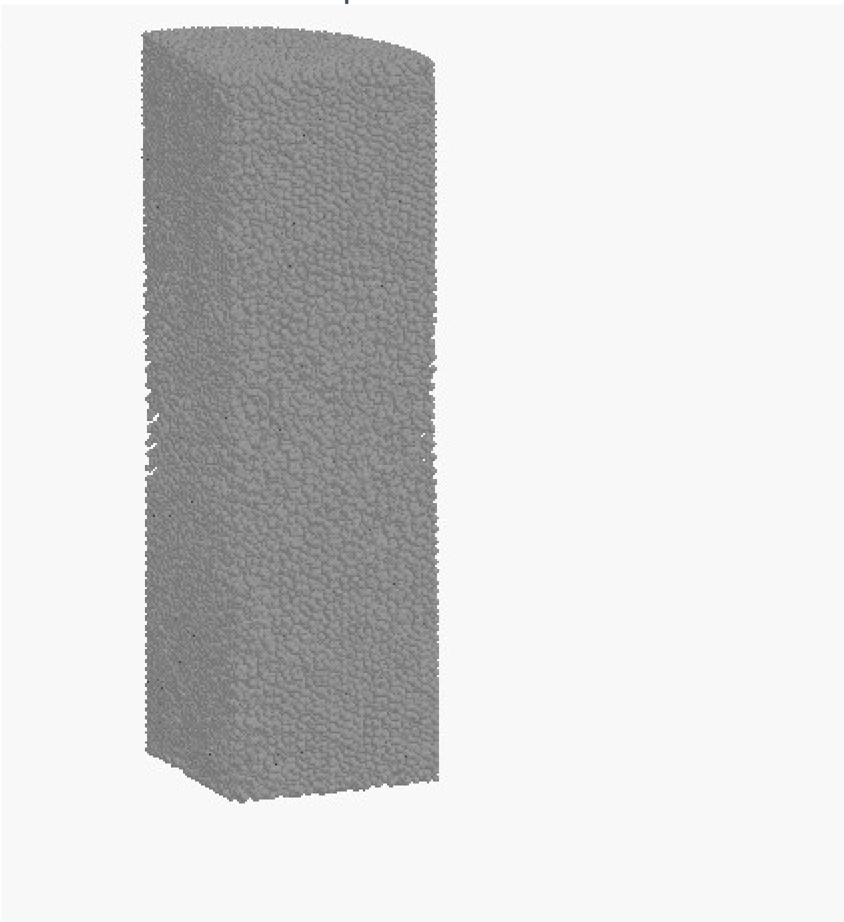


Material 2



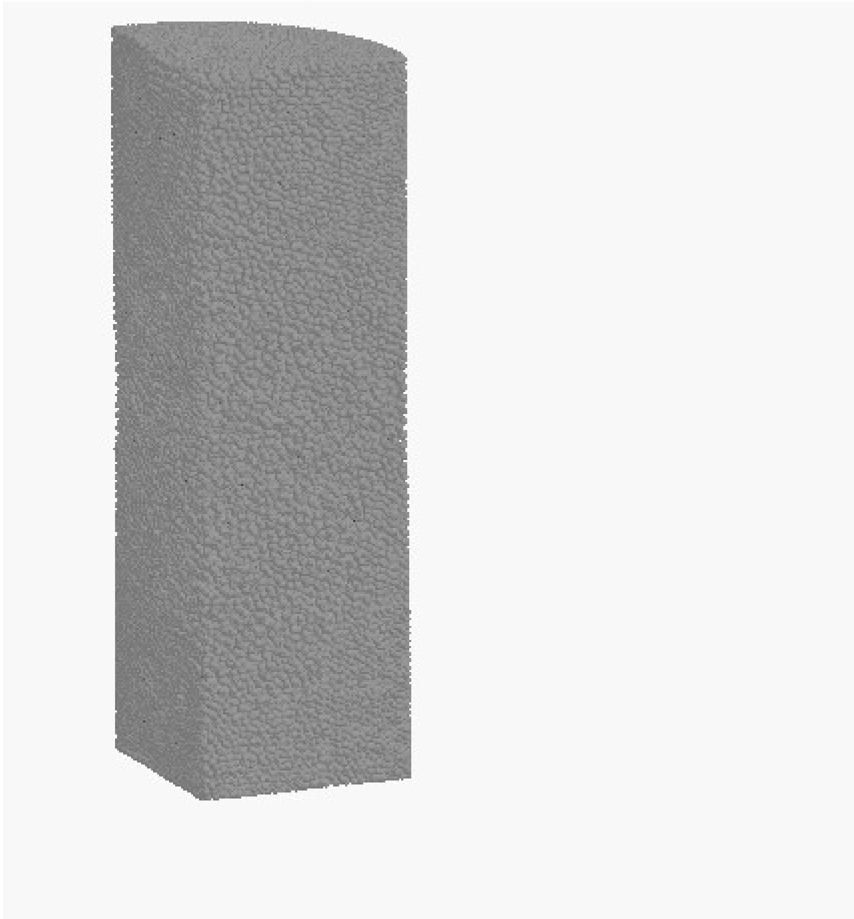
GRANUFALL TEST (granular non cohesive material)

CASE a ϕ = 18 mm



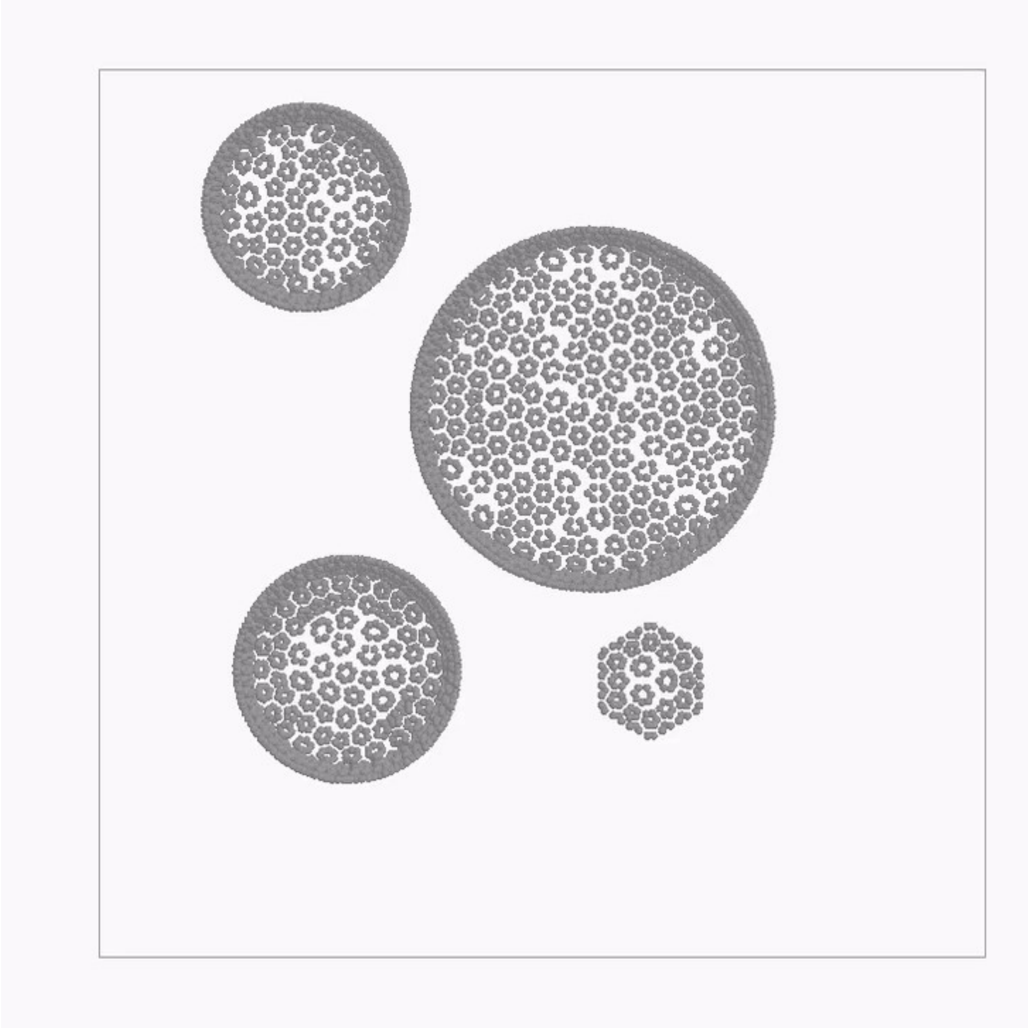
VOLUMETRIC FLOW [mL/s] = 63.07

CASE b ϕ = 8 mm

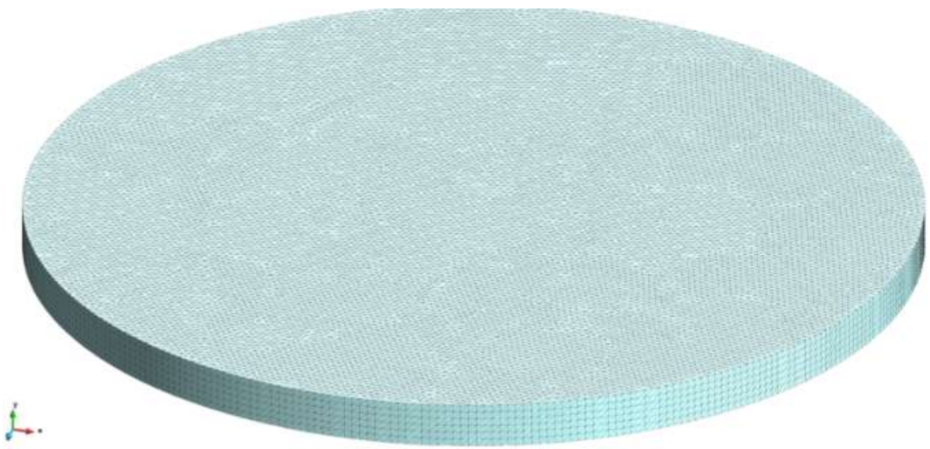


VOLUMETRIC FLOW [mL/s] = 7.86

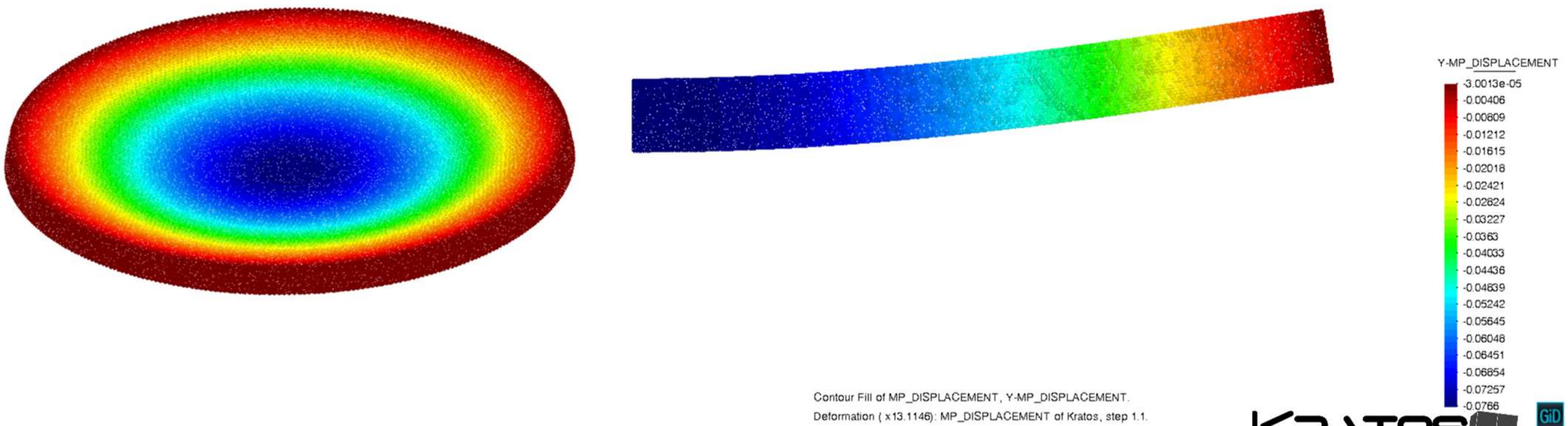
Multiple MATERIALS



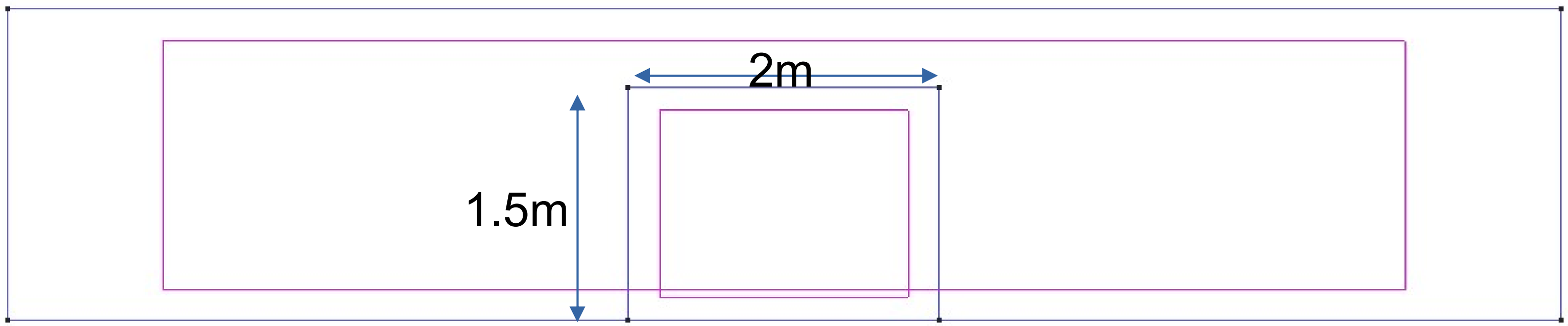
AXIS-SYMMETRIC CIRCULAR PLATE SUBJECTED TO SURFACE LOAD



$E = 10^7$Pa
$\nu = 0.24$
Analytical results:
For Surface pressure: 101.84 Pa
Max deflection: 0.0758 m



PLASTICITY LAWS



Constitutive laws in Particle Mechanics Application are implemented assuming finite deformation and in implicit form. All models have their own independent material parameters, yield criterion, hardening laws, flow rules, and return mapping algorithm.

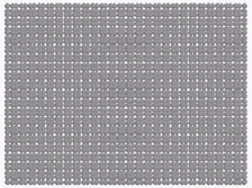
Available models:

1. Mohr Coulomb
2. Mohr Coulomb Strain Softening
3. Modified Cam Clay

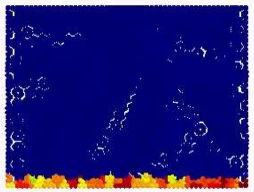
PLASTICITY LAWS – MOHR COULOMB

Independent material parameters:
- Density
- Young modulus
- Poisson Ratio
- Friction angle (ϕ)
- Cohesion(c)
- Dilatancy angle(ψ)*

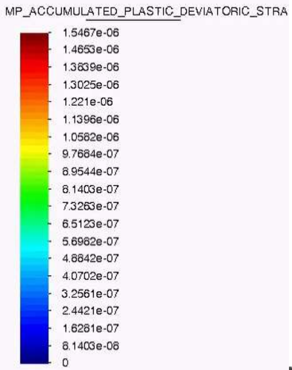
* Since our return mapping algorithm is non-associative.



Quadrilateral Elements



Triangular Elements

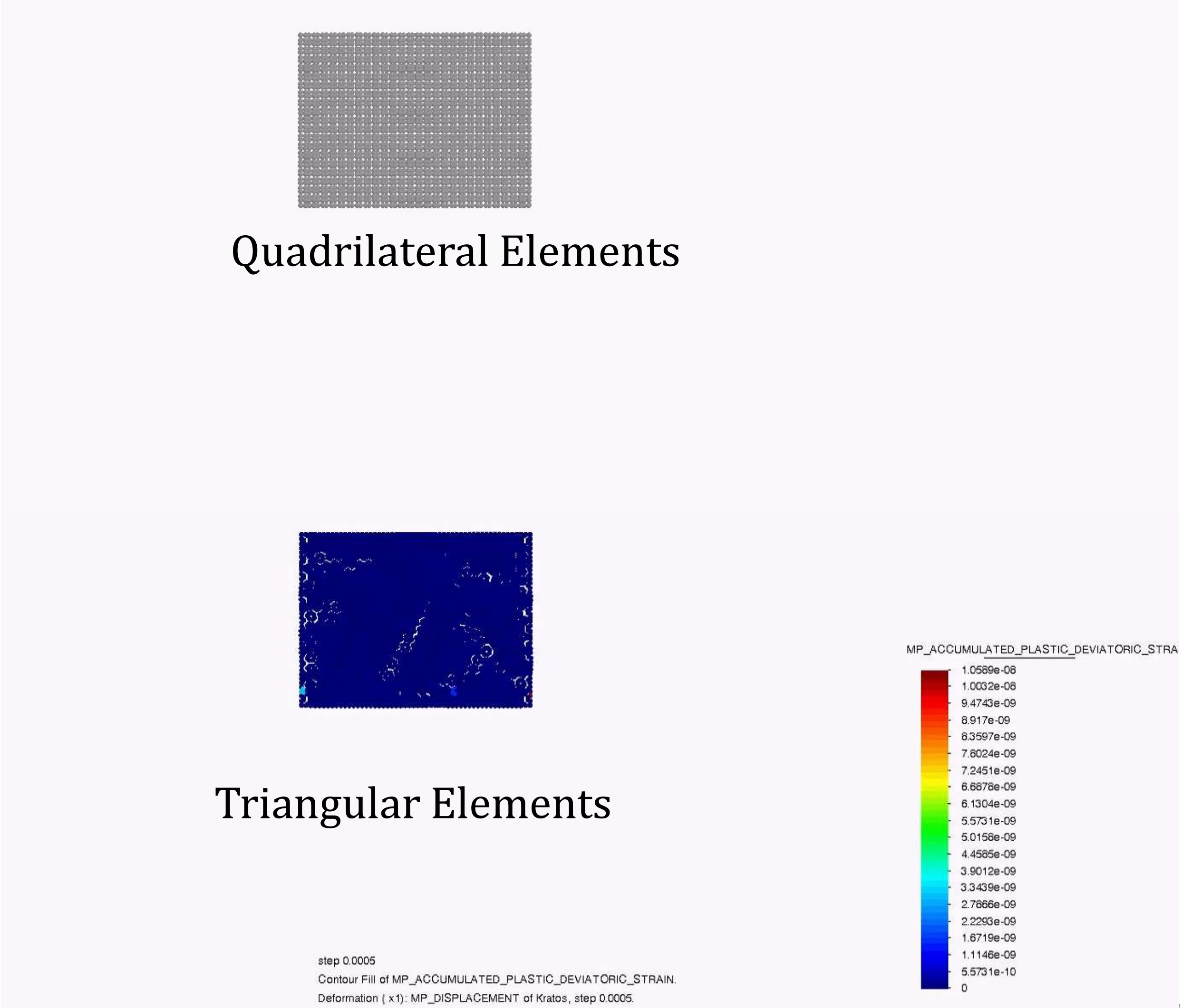


MULTI-PHYSICS

PLASTICITY LAWS – MOHR COULOMB Strain Softening

Independent material parameters:

- Density
- Young modulus
- Poisson Ratio
- Peak Friction angle
- Residual Friction angle
- Peak Cohesion
- Residual Cohesion
- Peak Dilatancy angle
- Residual Dilatancy angle
- Shape Function β

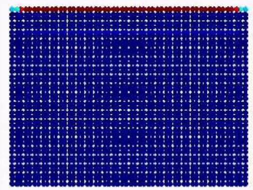


IMPLEMENTED FEATURES

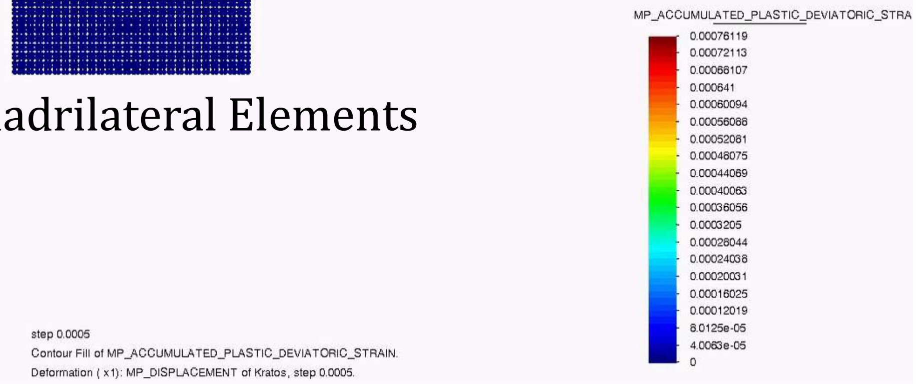
PLASTICITY LAWS – MODIFIED CAM CLAY

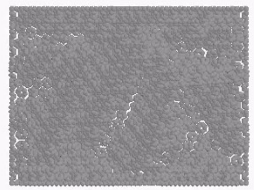
Independent material parameters:

- Density
- Preconsolidation Pressure (p'_c)
- Overconsolidation Ratio (OCR)
- Swelling Slope ($\hat{\kappa}$)
- Normal Consolidation Line Slope ($\hat{\lambda}$)
- Critical State Line Slope (M)
- Initial Shear Modulus (μ_0)
- Volumetric-Deviatoric Coupling constant (α)



Quadrilateral Elements

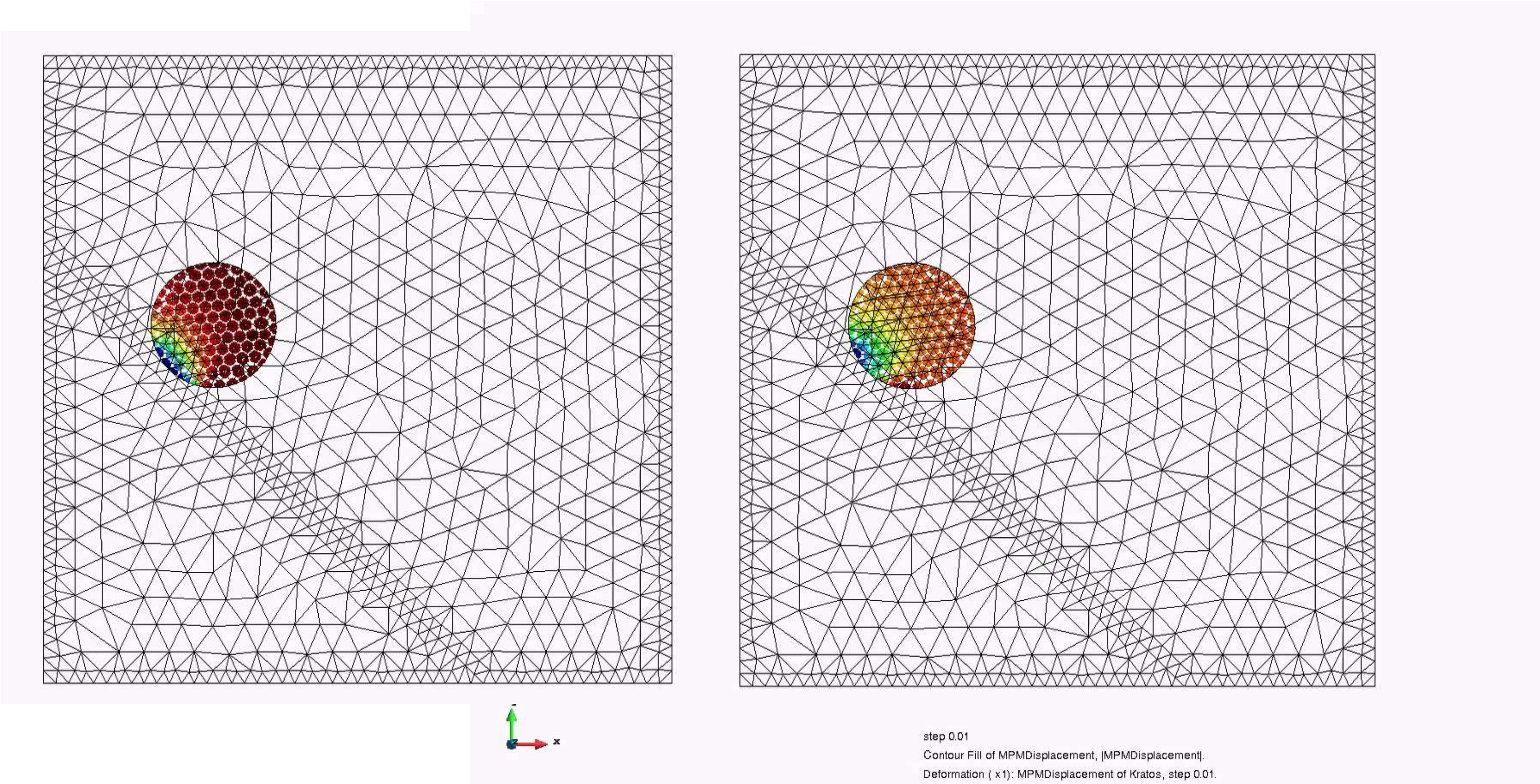




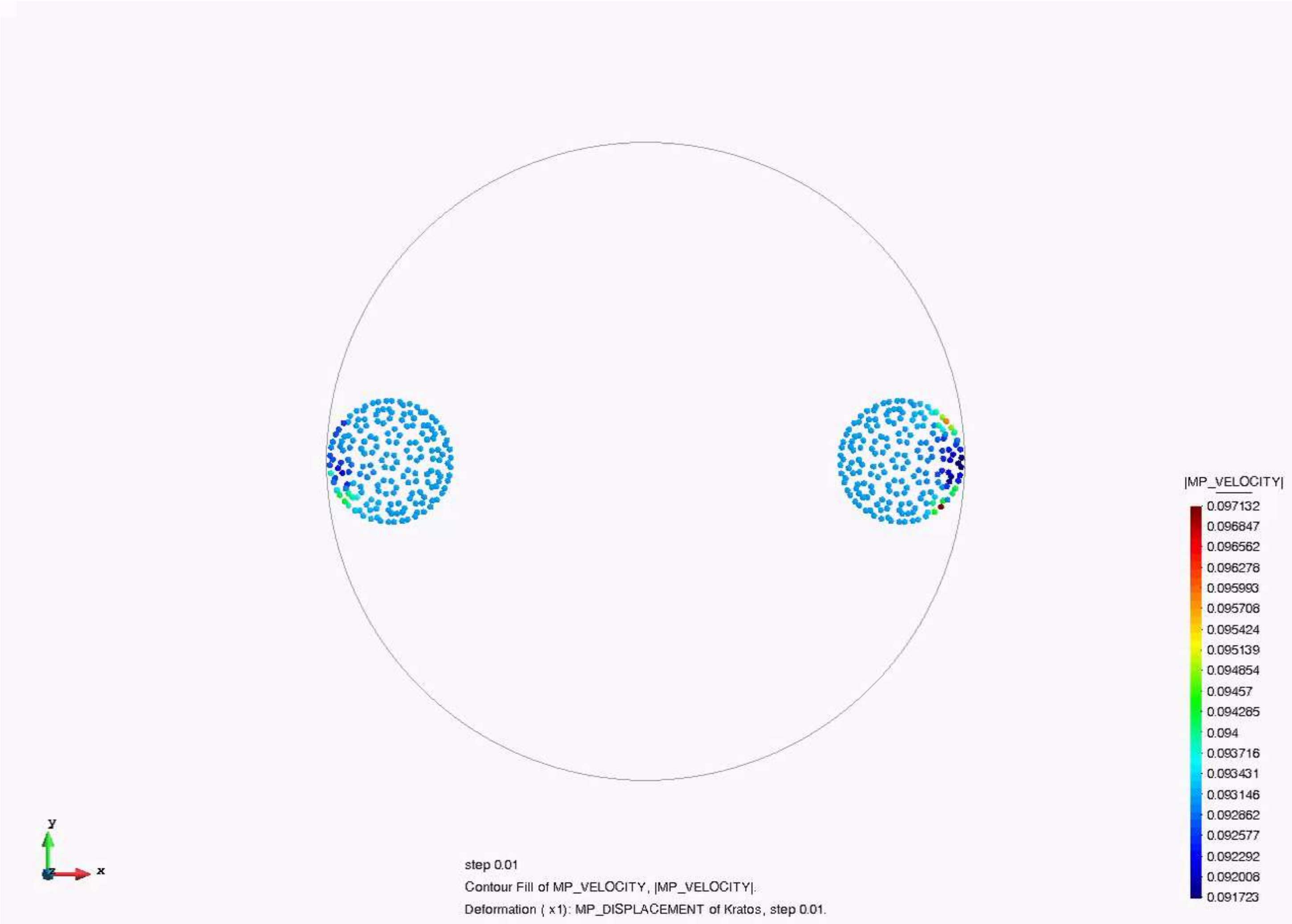
Triangular Elements

KRATOS MULTI-PHYSICS

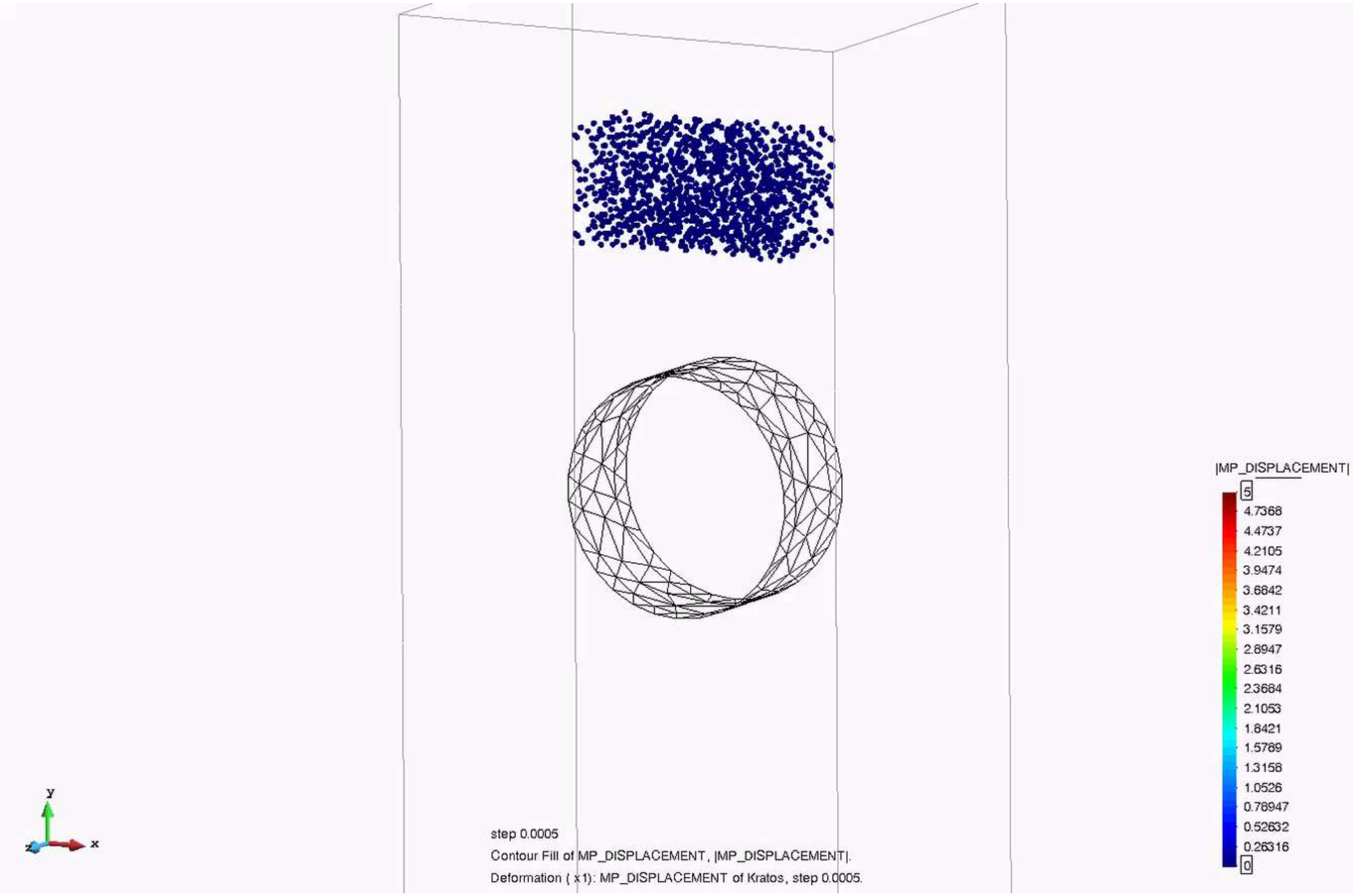
IMPLEMENTED FEATURES
INCLUDING SLIP BOUNDARIES



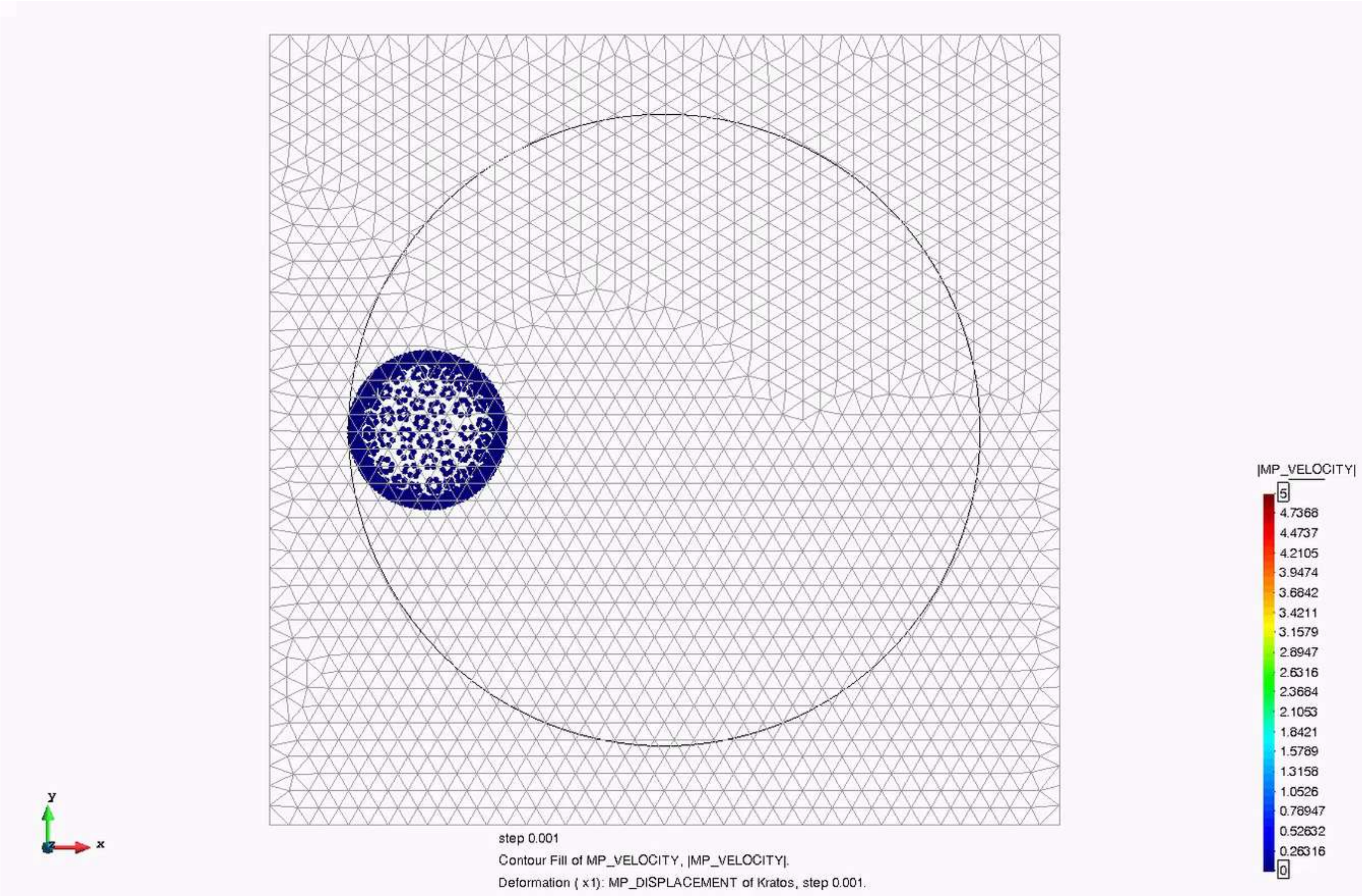
IMPLEMENTED FEATURES
INCLUDING SLIP BOUNDARIES



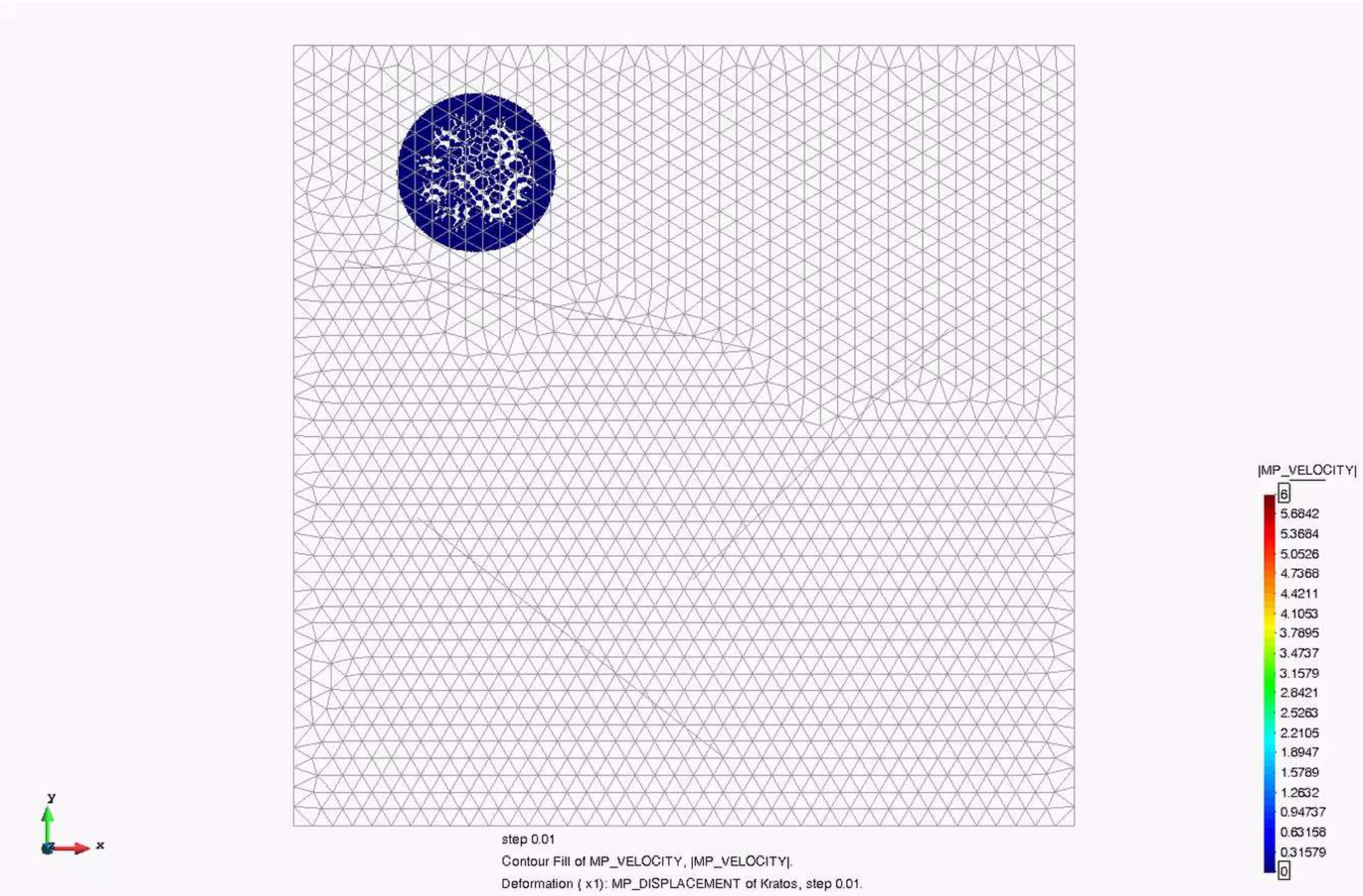
NONCONFORMING BOUNDARIES



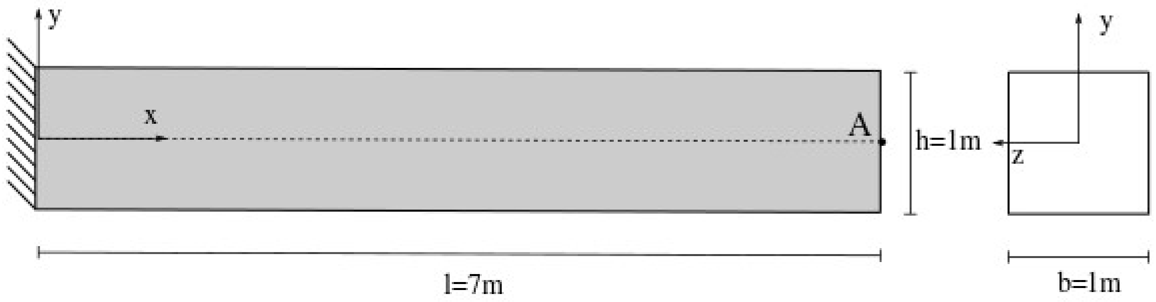
IMPLEMENTED FEATURES
NONCONFORMING NONSLIP BOUNDARIES



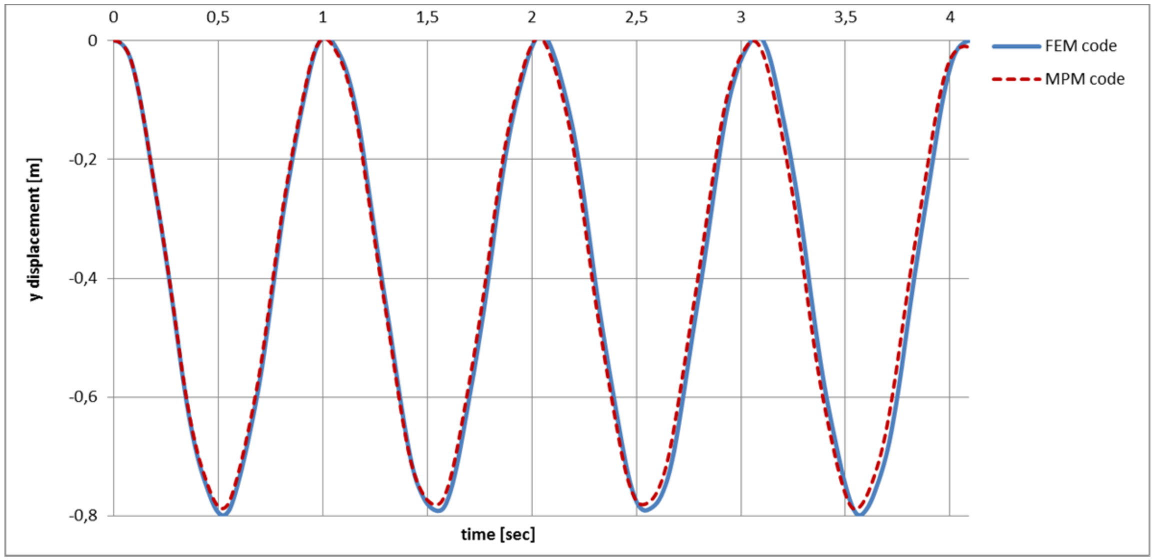
IMPLEMENTED FEATURES
NONCONFORMING SLIP BOUNDARIES



2D CANTILEVER BEAM SUBJECTED TO SELF-WEIGHT



DATA PROBLEM	
Young Modulus[Pa], E	9.0e7 Pa
Poisson's ratio, ν	0.0[-]
Density ρ	1000[Kg/m^3]
Length, L	7[m]
Width, h	1[m]
Depth, b	1[m]
Static analytical solution	-0.39256[m]
Natural frequency, f_{nf}	0.9889[Hz]



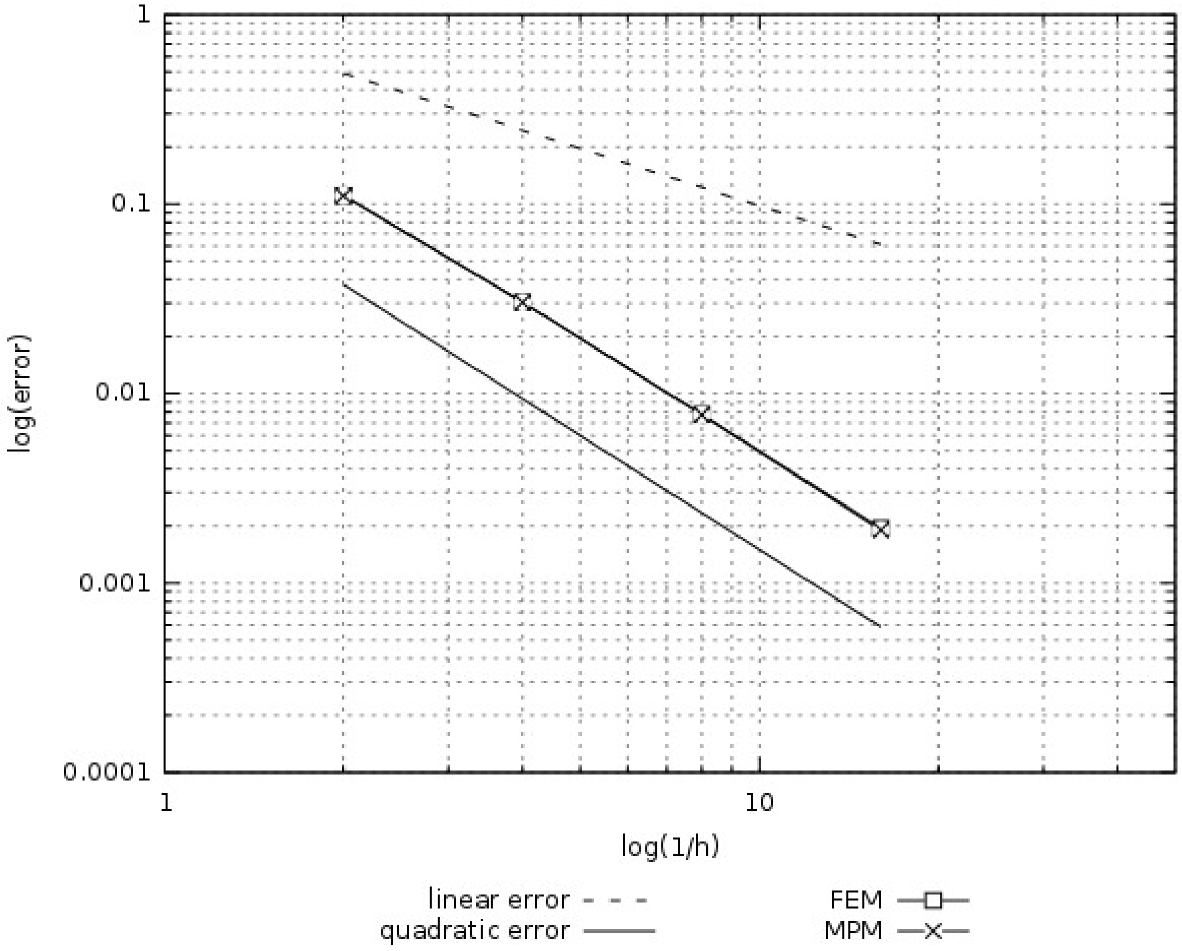
Neo-Hookean
Hyperelastic material

2D CANTILEVER BEAM SUBJECTED TO SELF-WEIGHT

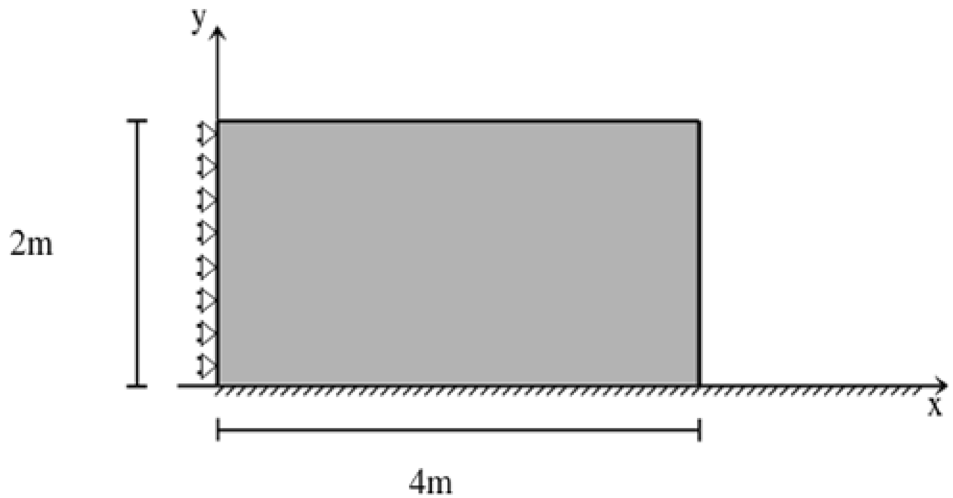
FEM CODE

MPM CODE

2D CANTILEVER BEAM SUBJECTED TO SELF-WEIGHT



COHESIVE SOIL COLUMN COLLAPSE

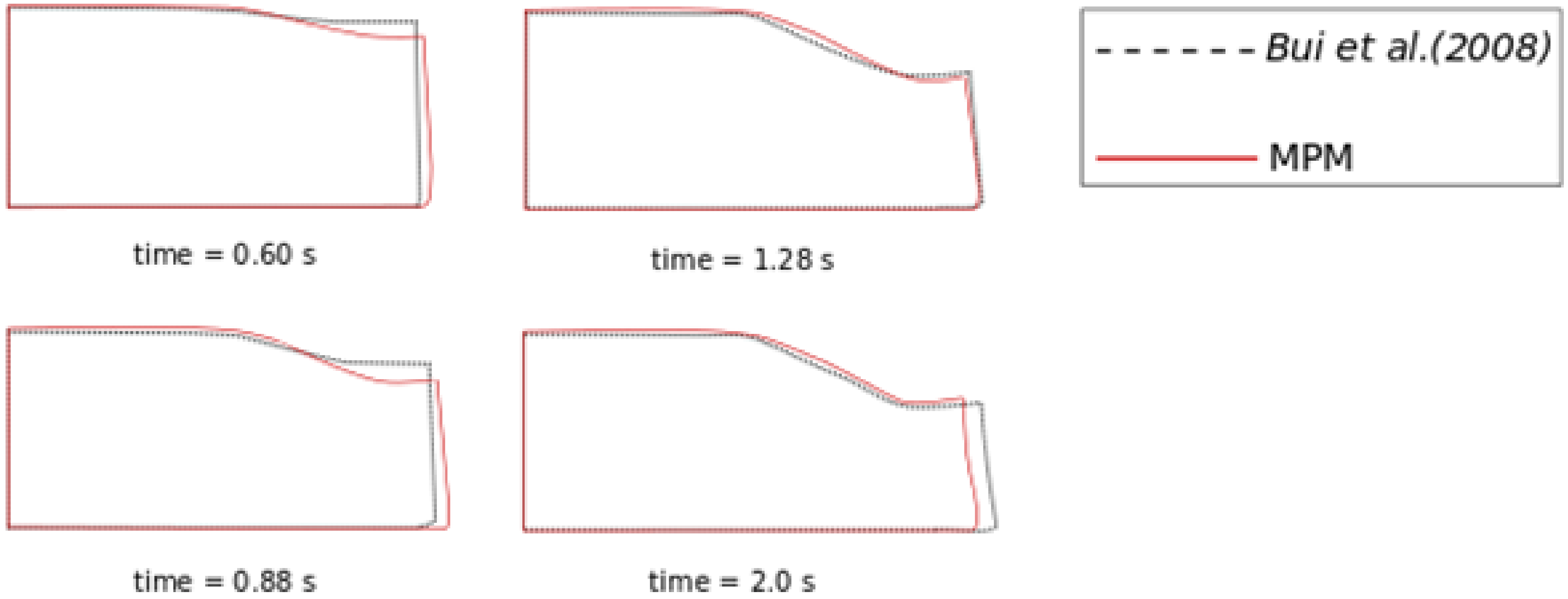


	Material data
Bulk density ρ [kg/mc]	1850
Young Modulus E $[Pa]$	1.8e6
Poisson's ratio ν	0.3
Apparent cohesion c $[kPa]$	5
Friction angle ϕ [°]	25
Dilatancy angle δ [°]	0

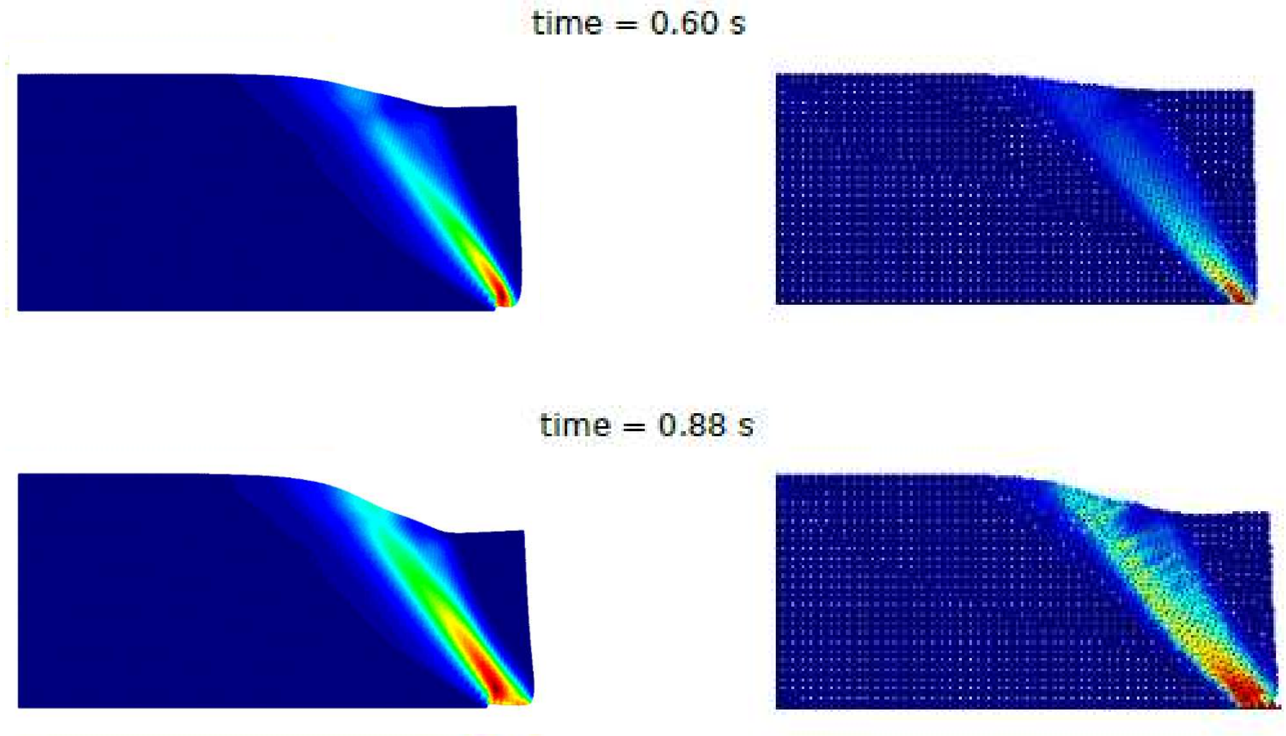
Bui et al.(2008)

Mohr-Coulomb plastic law in finite strains [Clausen et al.(2007)]

COHESIVE SOIL COLUMN COLLAPSE



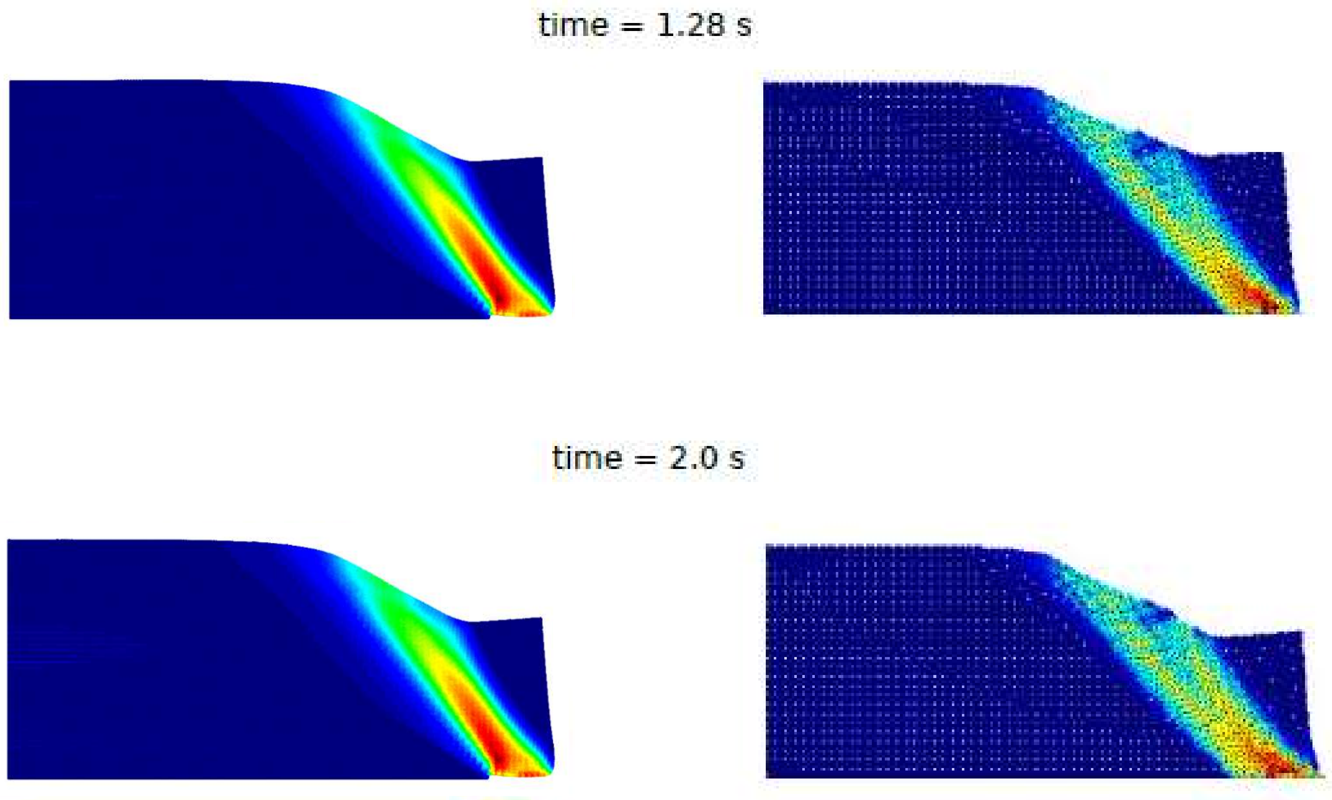
COHESIVE SOIL COLUMN COLLAPSE



MPM

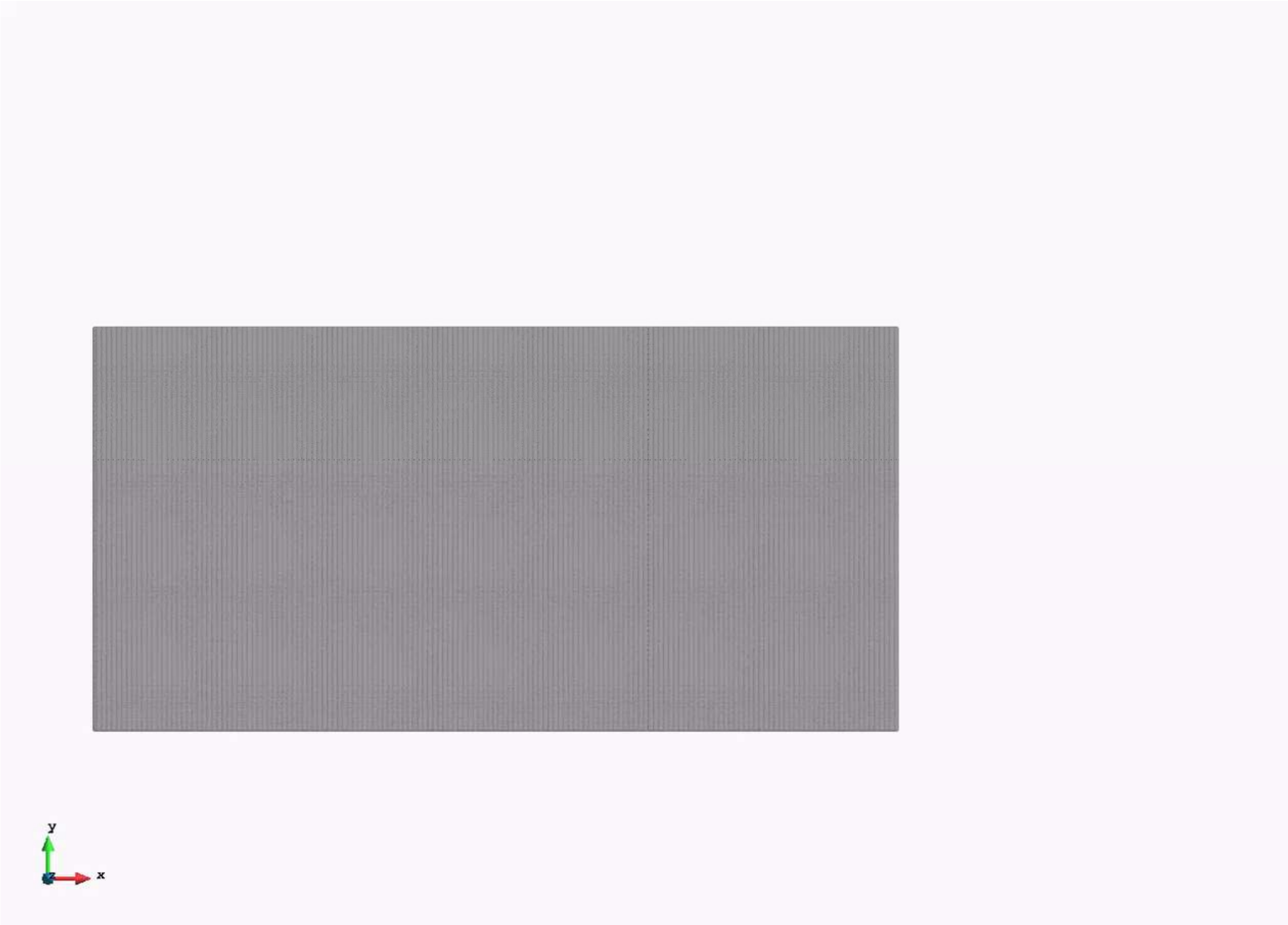
SPH (BUI, 2008)

COHESIVE SOIL COLUMN COLLAPSE

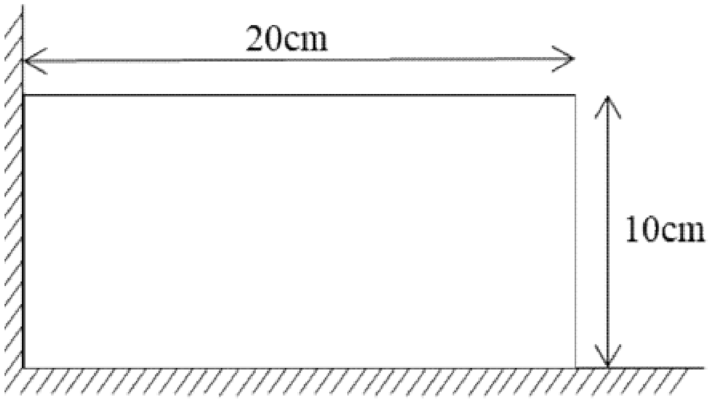


MPM　　　　SPH (BUI, 2008)

COHESIVE SOIL COLUMN COLLAPSE

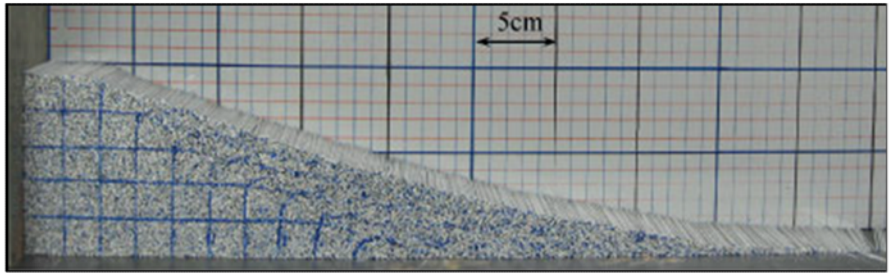


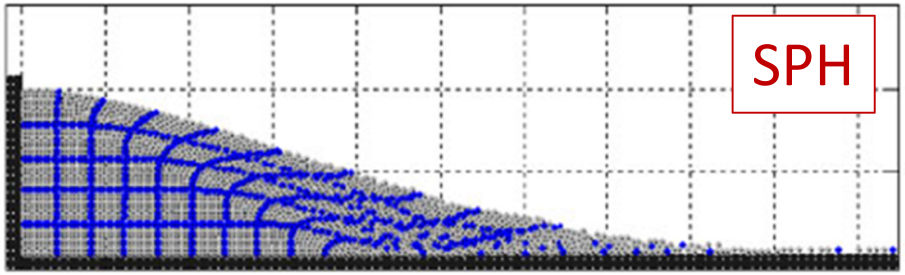
NON-COHESIVE SOIL COLUMN COLLAPSE

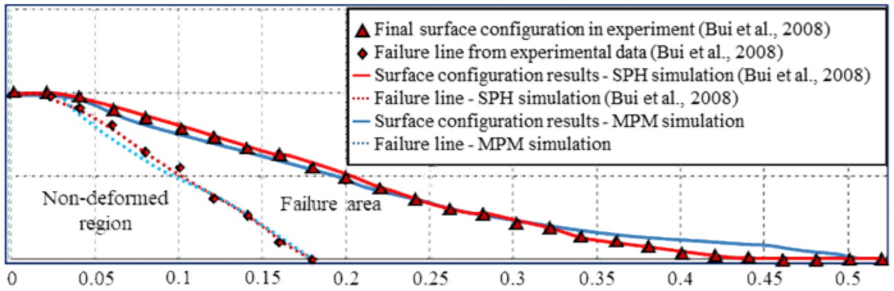


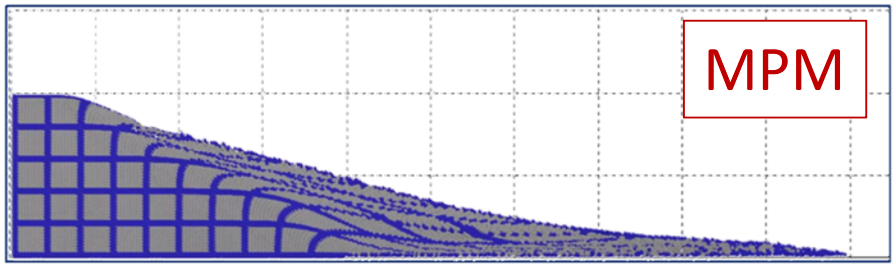
Dimension	Density $[kg/m^3]$	Young's Modulus $[kPa]$	Poisson's ratio
20cm×10cm	2650	840	0.3

Angle of internal friction[°]	Cohesion $[kPa]$	Dilatancy angle[°]
19.8	0.0	0.0

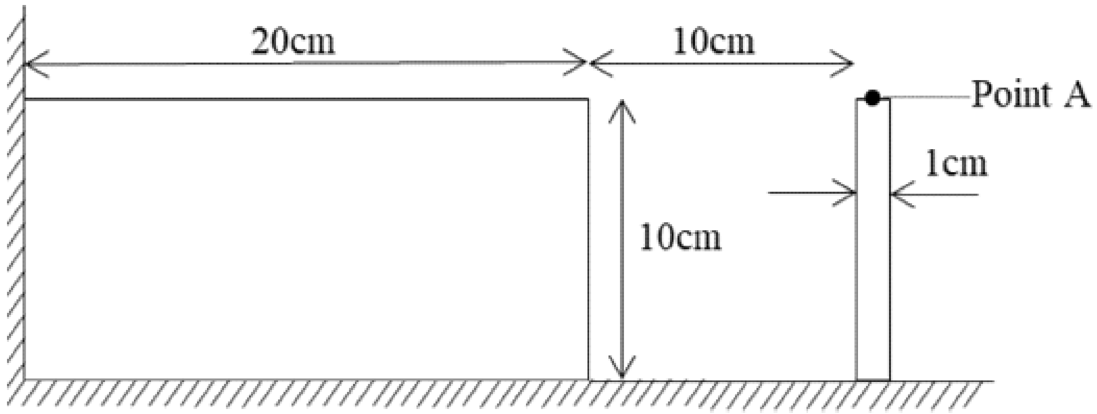


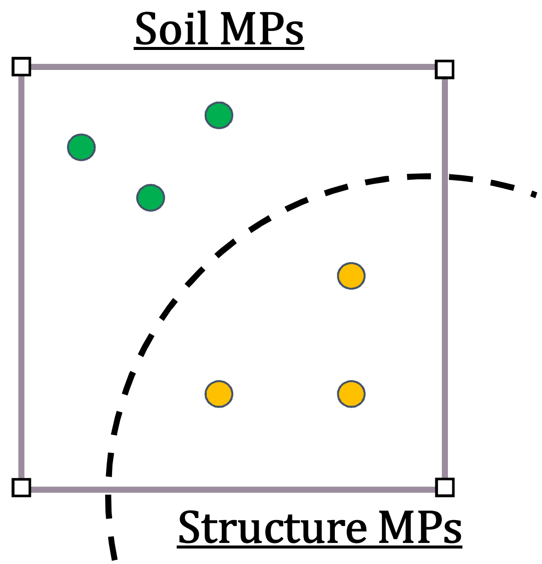






NON-COHESIVE SOIL COLUMN COLLAPSE coupled with Structure



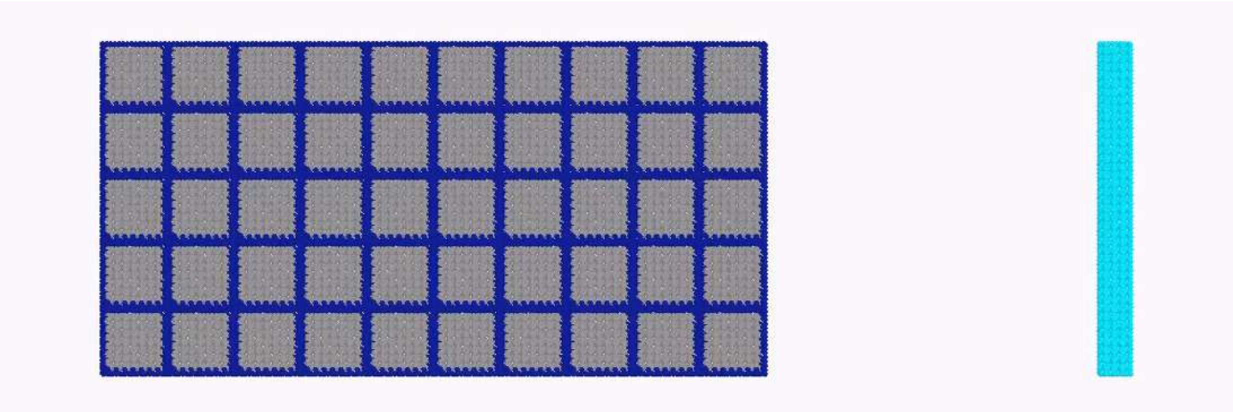


$$\begin{bmatrix} \mathbf{K}_S & \mathbf{K}_{S\Gamma} & 0 \\ \mathbf{K}_{\Gamma S} & \mathbf{K}_{\Gamma\Gamma} & \mathbf{K}_{\Gamma St} \\ 0 & \mathbf{K}_{St\Gamma} & \mathbf{K}_{St} \end{bmatrix}_{IK}^{tan} \begin{Bmatrix} \delta\mathbf{u}_S \\ \delta\mathbf{u}_\Gamma \\ \delta\mathbf{u}_{St} \end{Bmatrix}_K = -\begin{Bmatrix} \mathbf{R}_S \\ \mathbf{R}_\Gamma \\ \mathbf{R}_{St} \end{Bmatrix}_I$$

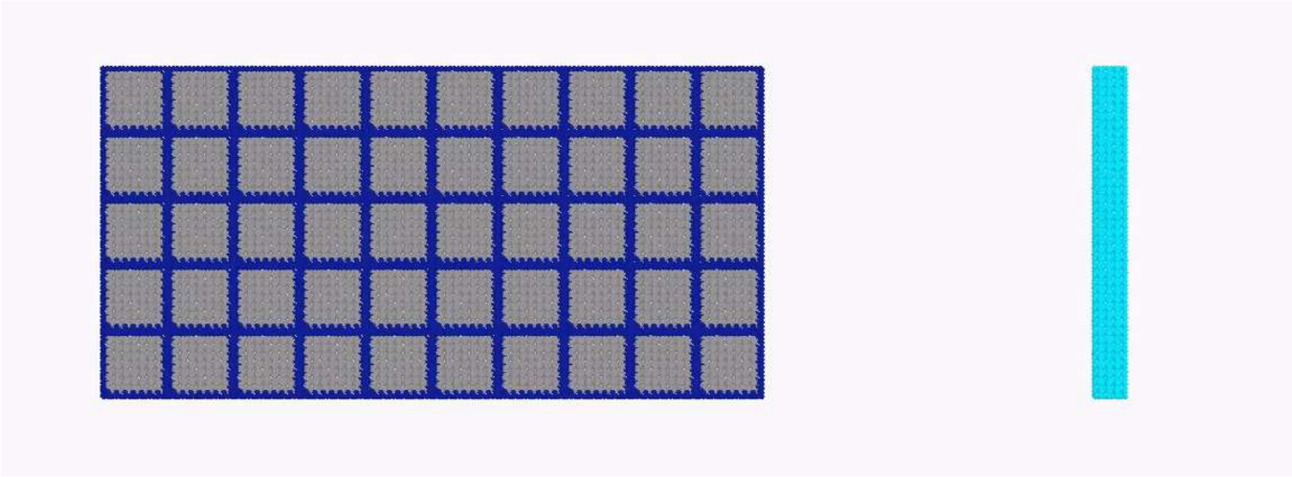
	Material type	Density [kg/m^3]	Young's Modulus [MPa]
a.)	Concrete-like (rigid)	2550	3×10^4
b.)	Rubber-like (flexible)	1100	1

NON-COHESIVE SOIL COLUMN COLLAPSE coupled with Structure

a.)



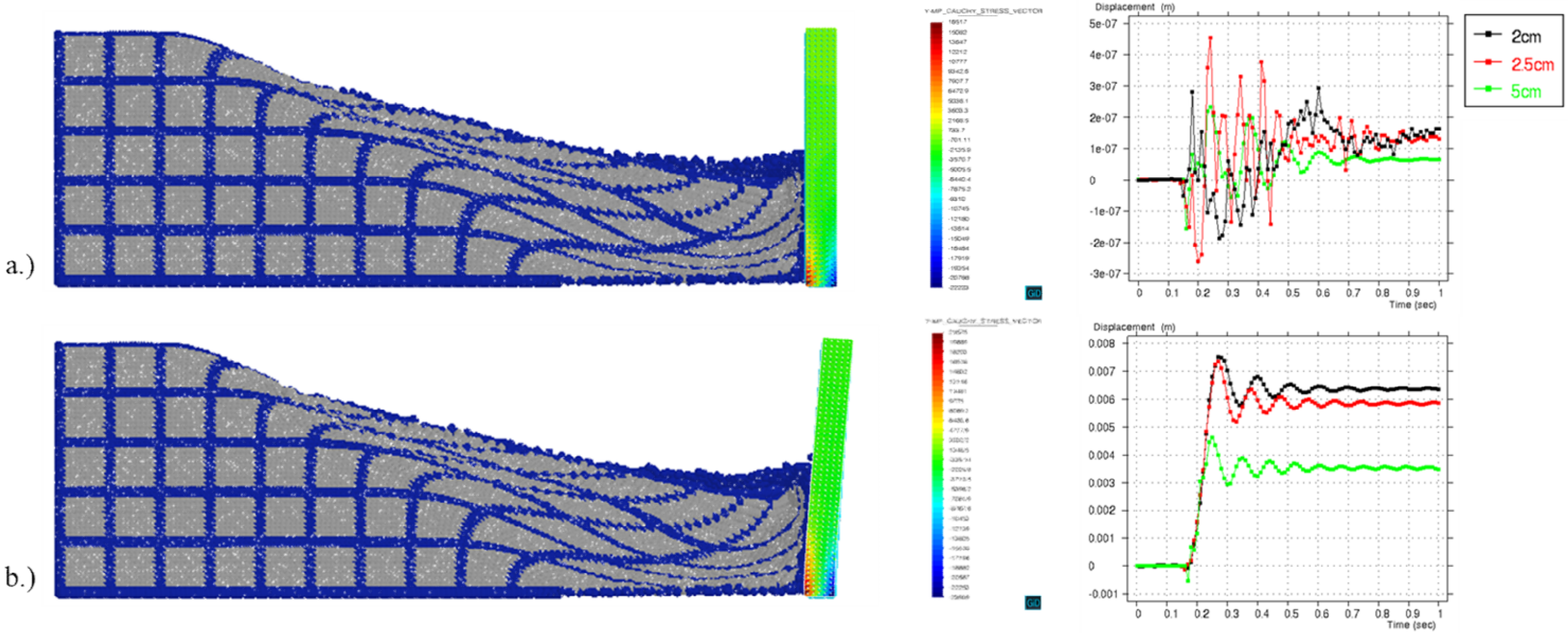
b.)



(a.) with "rigid" structure, (b.) with flexible structure

NUMERICAL EXAMPLES

NON-COHESIVE SOIL COLUMN COLLAPSE coupled with Structure

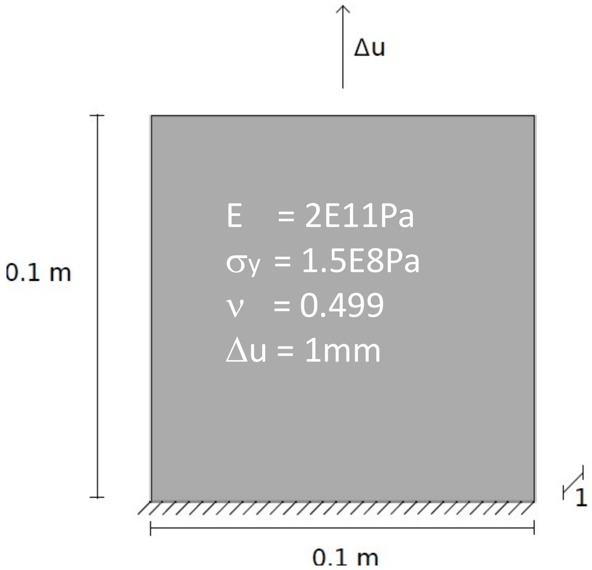


(a.) with "rigid" structure, (b.) with flexible structure

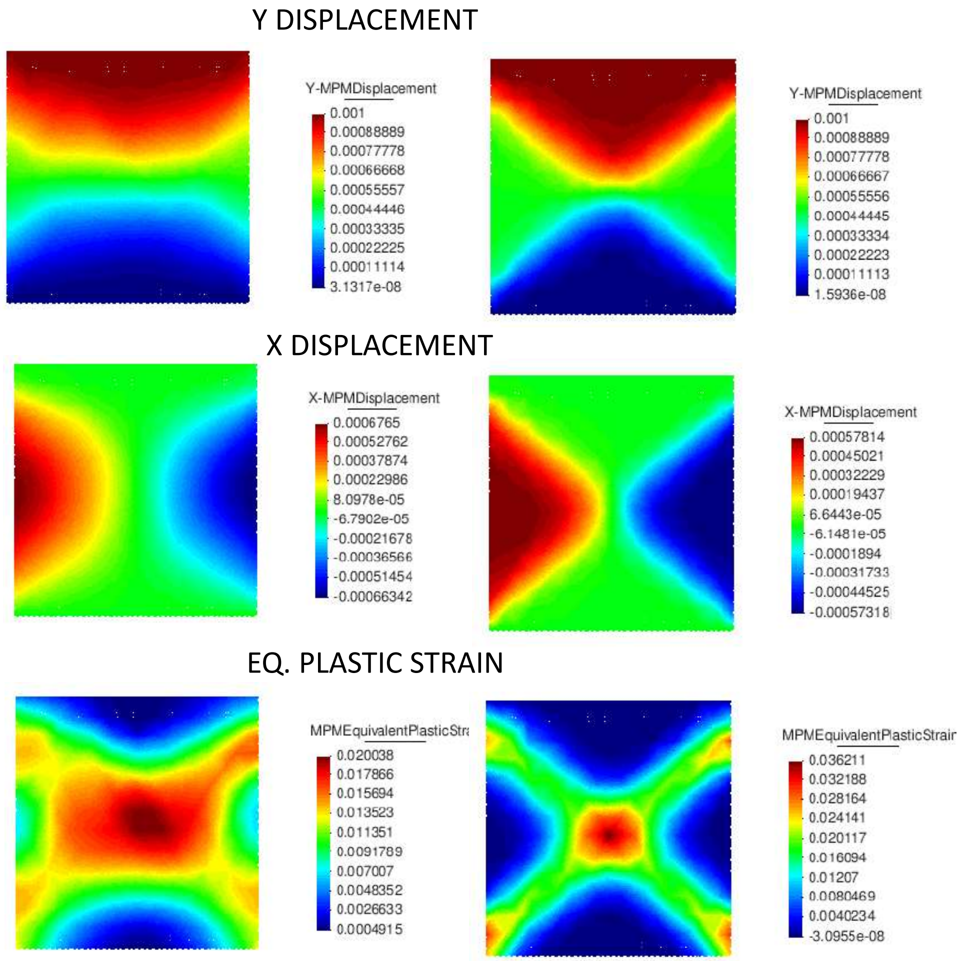
KRATOS MULTI-PHYSICS

MIXED FORMULATION

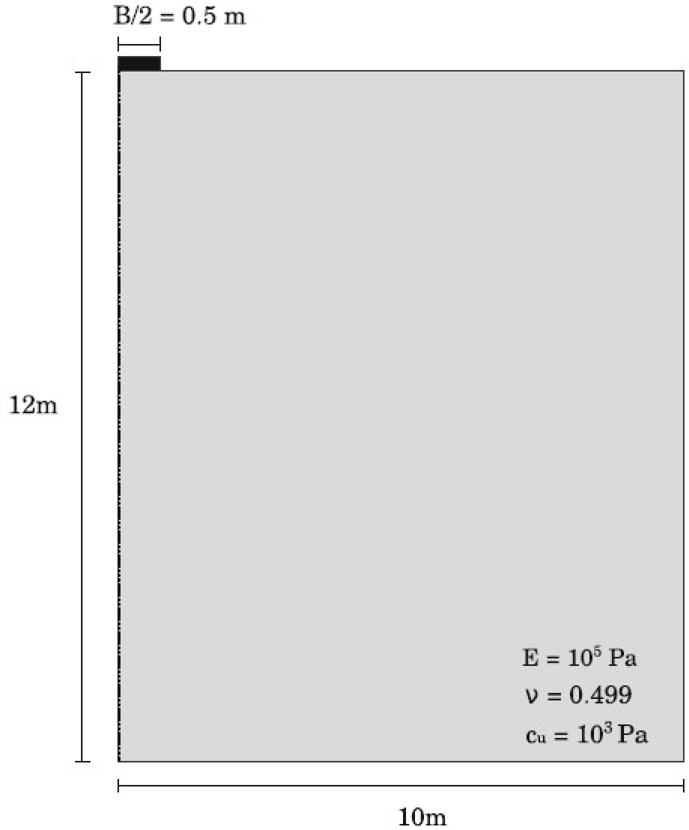
Both IRREDUCIBLE (u) and MIXED FORMULATION (u-p) to deal with incompressibility constraint

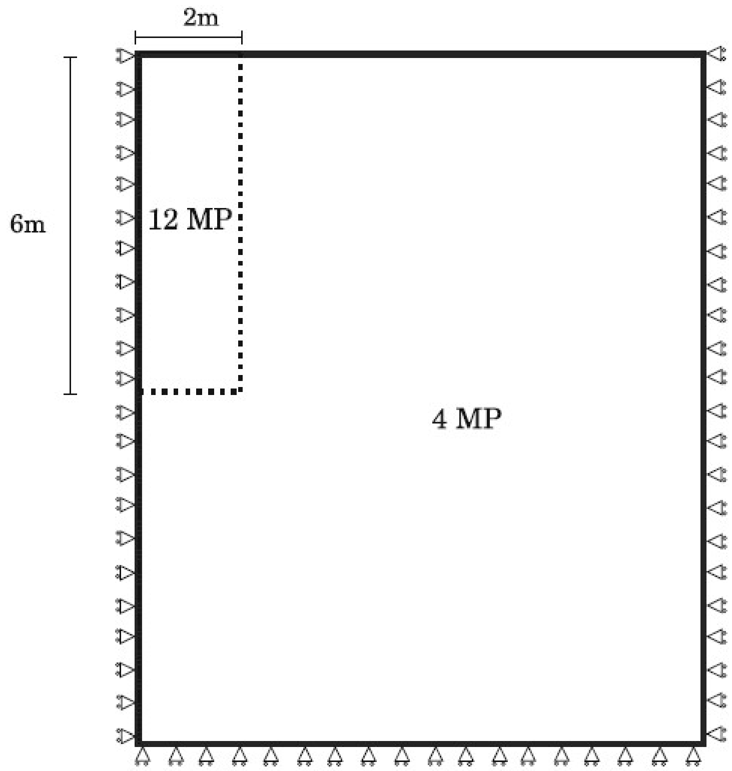


IRREDUCIBLE (u) | MIXED (u-p)



Plain strain rigid footing on undrained soil



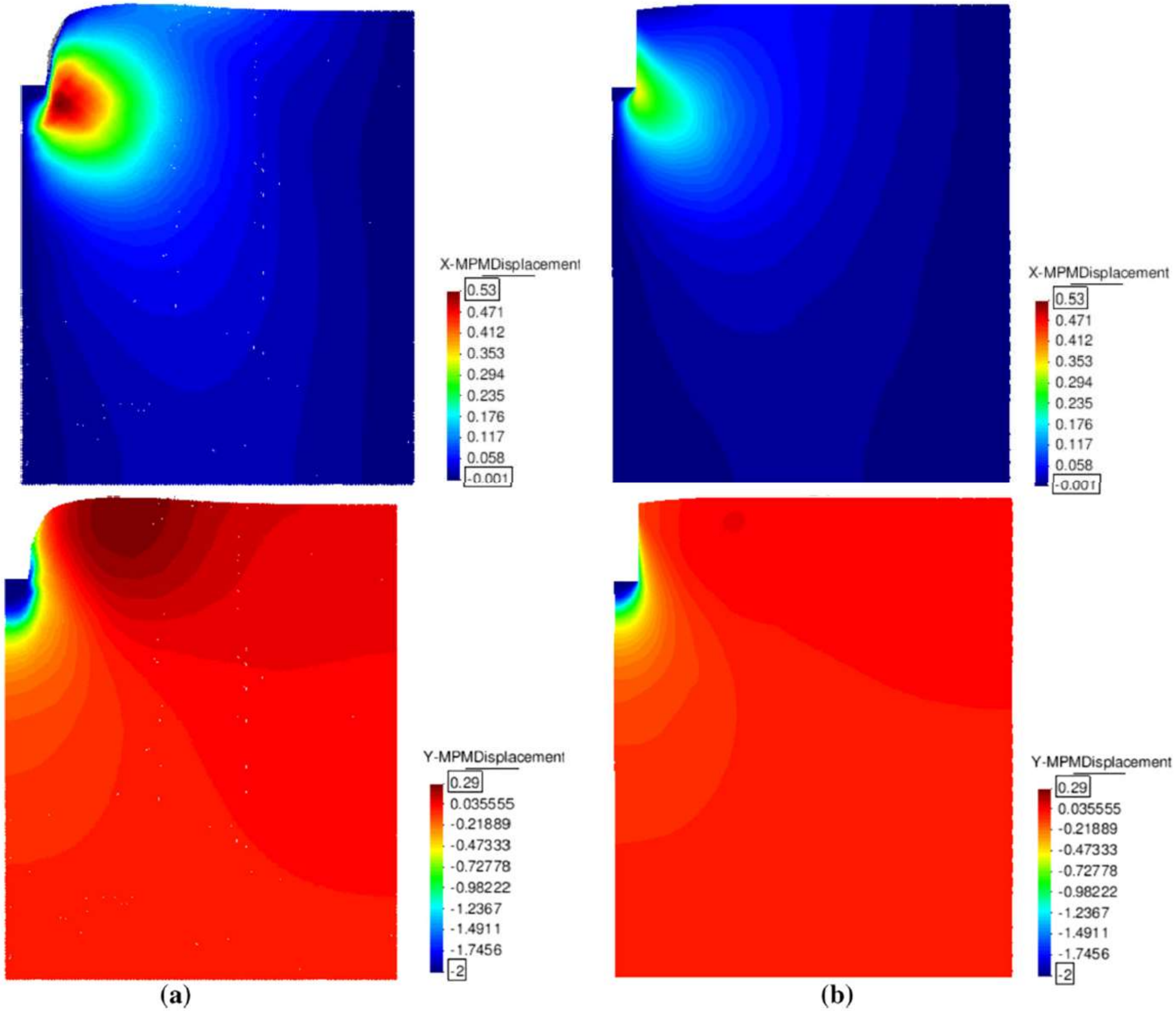


Iaconeta, I., Larese, A., Rossi, R., & Oñate, E. (2018). A stabilized mixed implicit Material Point Method for non-linear incompressible solid mechanics. Computational Mechanics, 1-18.

Plain strain rigid footing on undrained soil

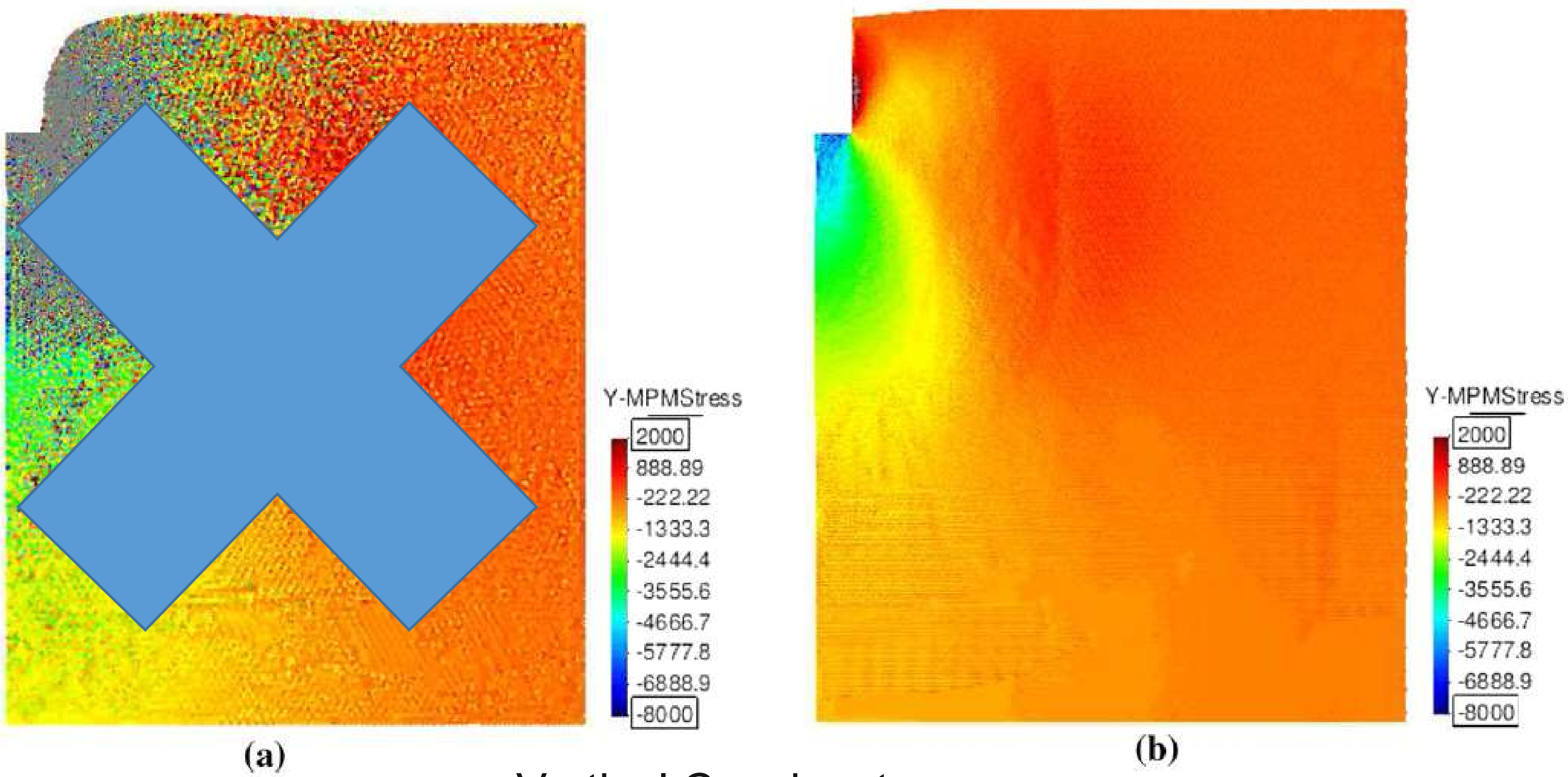
Horizontal displacement

Vertical displacement



(a.) irreducible formulation,
(b.) mixed formulation

Plain strain rigid footing on undrained soil

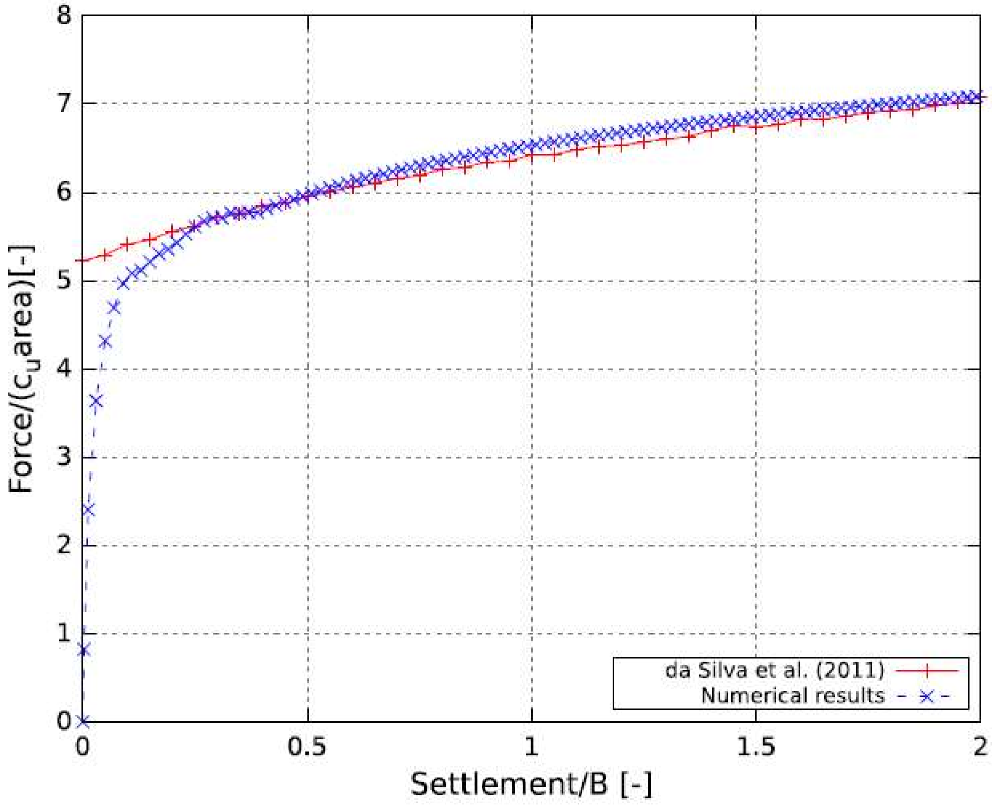


Vertical Cauchy stress

(a.) irreducible formulation, (b.) mixed formulation

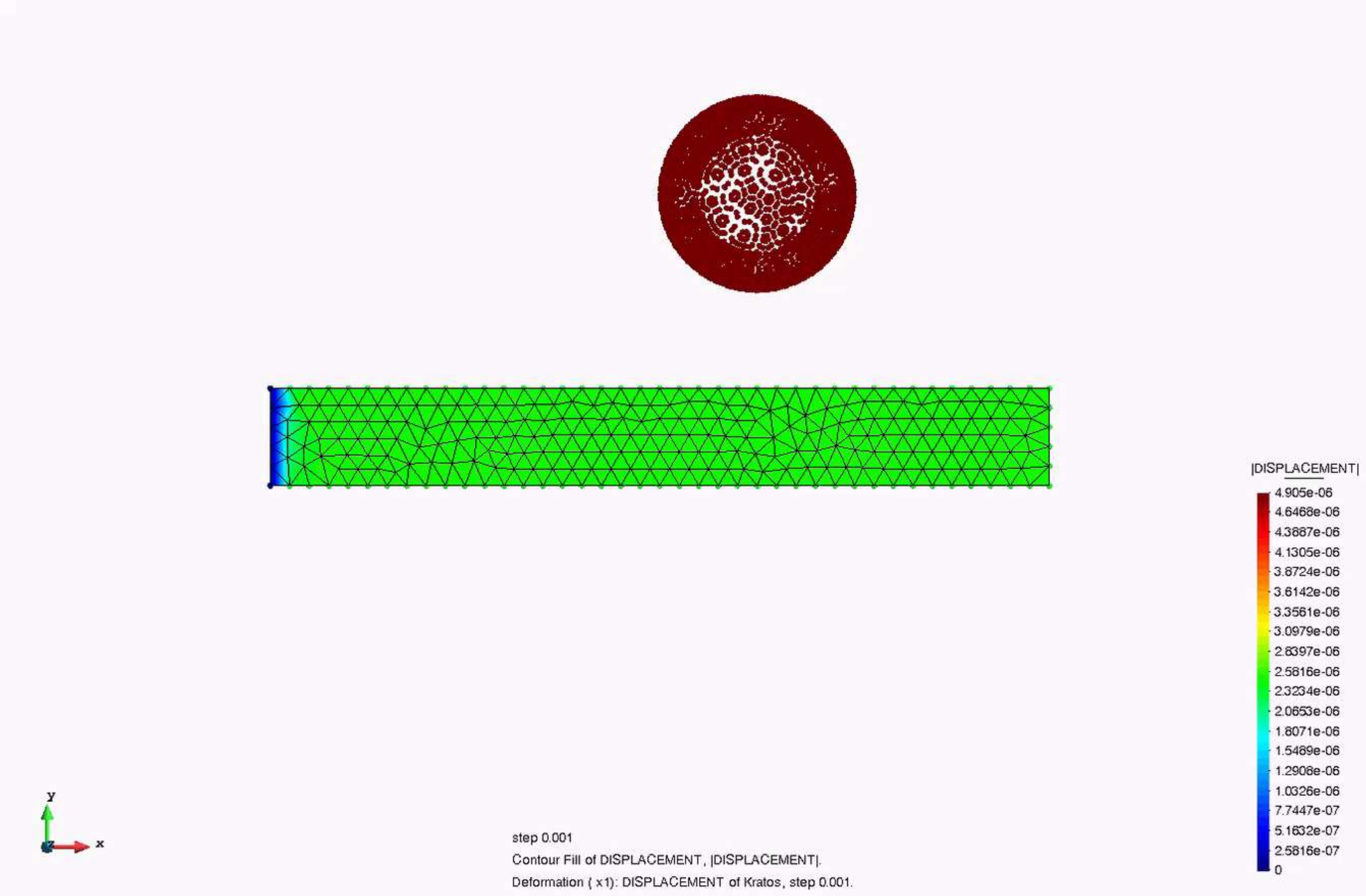
Plain strain rigid footing on undrained soil

Normalized load-displacement curve

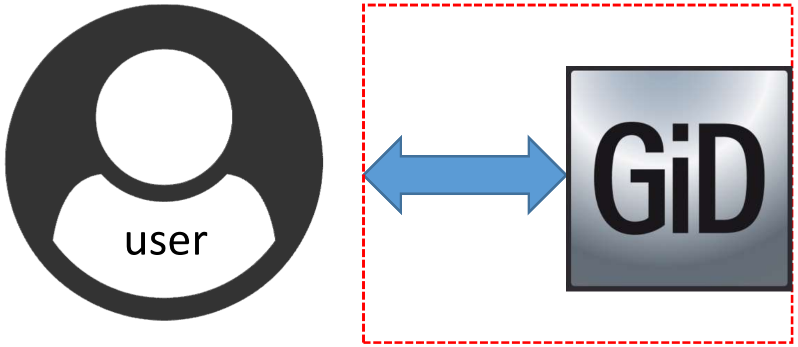


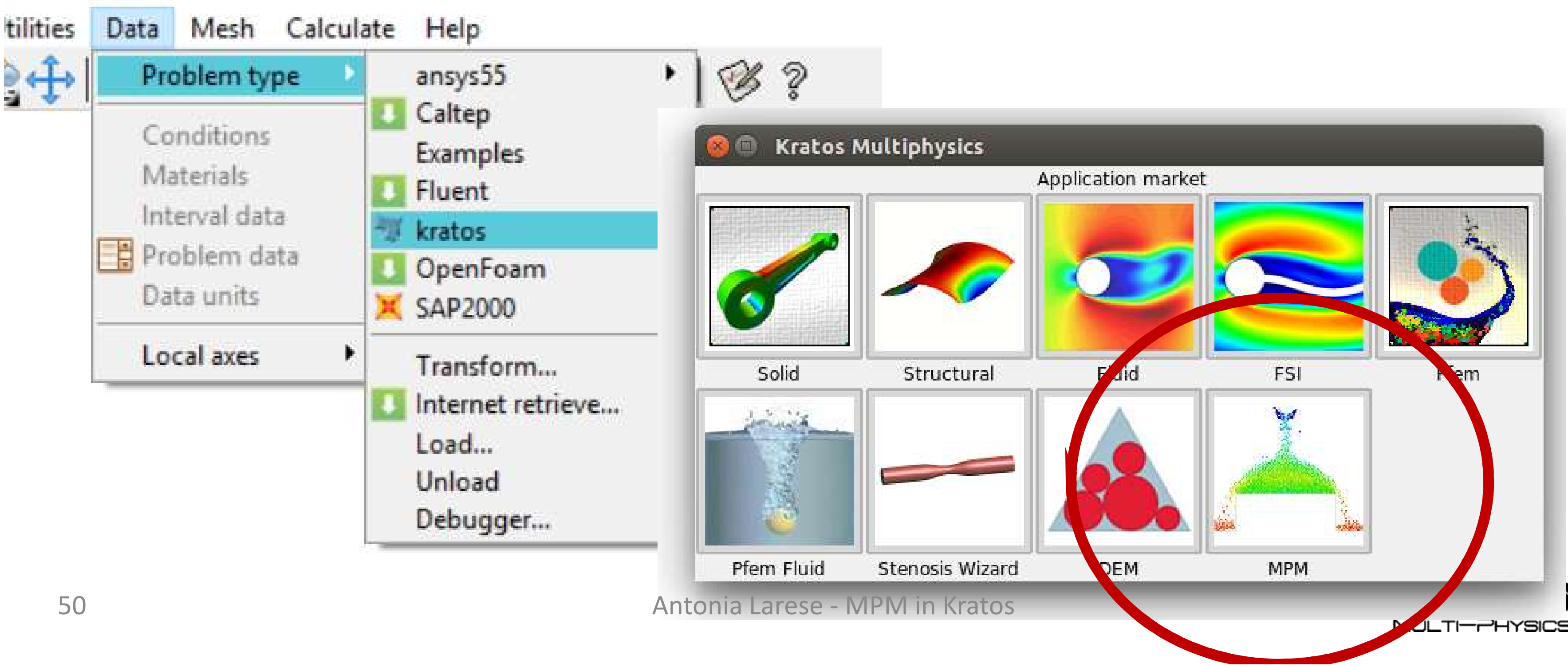
KRATOS MULTI-PHYSICS

SOIL-STRUCTURE INTERACTION: CURRENT DEVELOPMENT

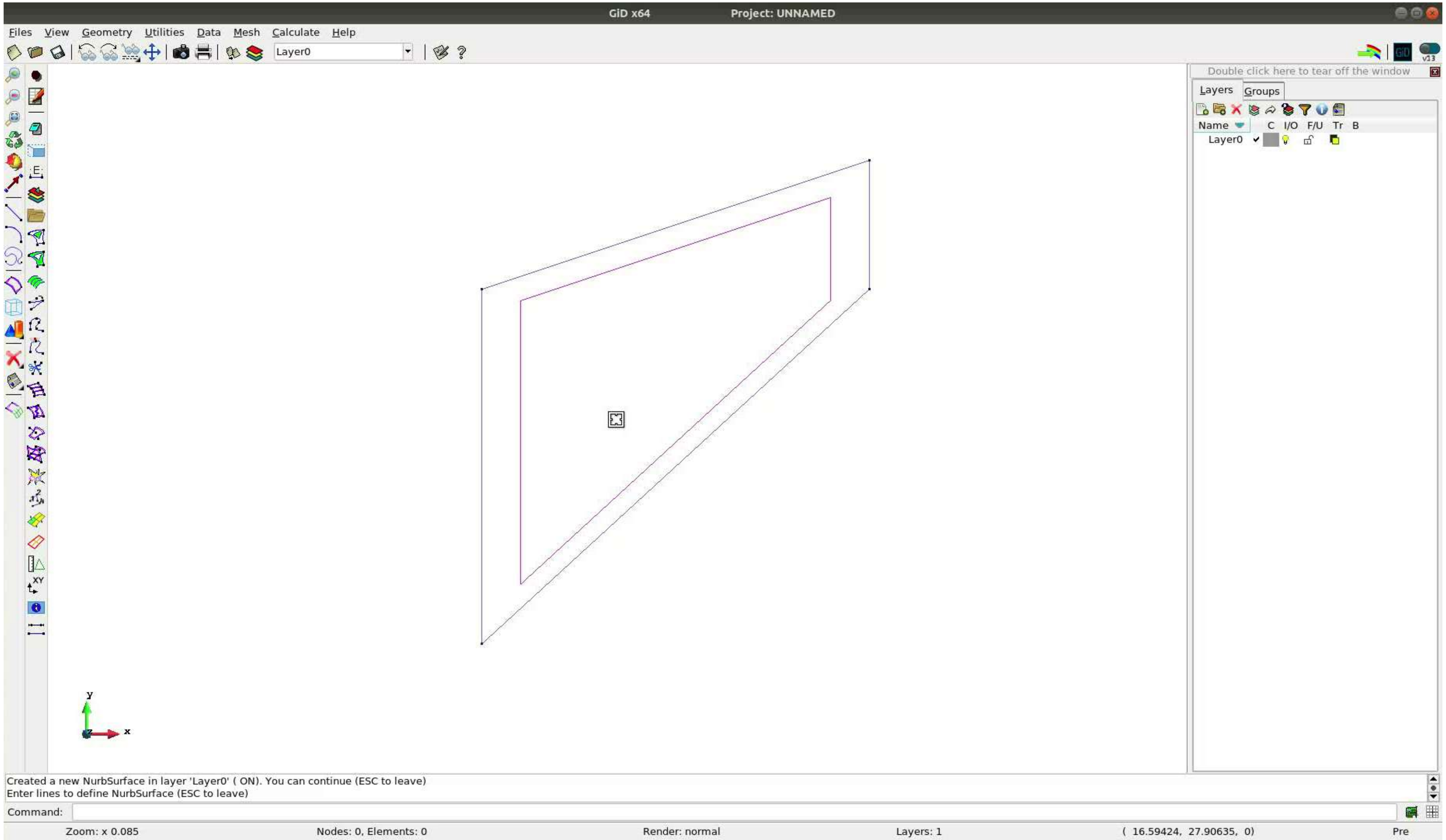


GiD INTERFACE

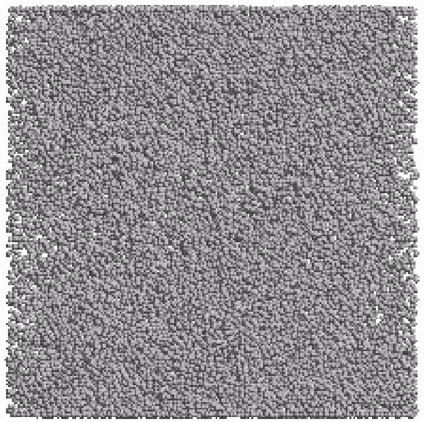




HOW TO USE MPM in KRATOS: DEMO



THANK YOU FOR YOUR ATTENTION!



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