



MMG Day
2018

Vicente
Mataix
Ferrández

Introduction

Objectives
Kratos

Theory

Level set
remeshing

Hessian
remeshing

Metric
intersection

Internal
variables

Numerical
contact

Cases

Level set
remeshing

Hessian
remeshing

Internal
variables

Numerical
contact

Conclusions

Problems
Conclusions
Visit us

Implementation of the MMG library in the Kratos Multiphysics (AKA *Kratos*) framework

MMG Day 2018

Vicente Mataix Ferrández¹ Riccardo Rossi¹ Eugenio Oñate Ibañez
de Navarra¹
vmataix@cimne.upc.edu

¹CIMNE. International Center for Numerical Methods in Engineering, Technical University of Catalonia (UPC). Barcelona. Spain

January 17, 2018



Overview

MMG Day
2018

Vicente
Mataix
Ferrández

Introduction

Objectives
Kratos

Theory

Level set
remeshing

Hessian
remeshing

Metric
intersection

Internal
variables

Numerical
contact

Cases

Level set
remeshing

Hessian
remeshing

Internal
variables

Numerical
contact

Conclusions

Problematic
Conclusions

Visit us

1 Introduction

- Objectives
- Kratos

2 Theory

- Level set remeshing
- Hessian remeshing
- Metric intersection
- Internal variables
- Numerical contact

3 Cases

- Level set remeshing
- Hessian remeshing
- Internal variables
- Numerical contact

4 Conclusions

- Problematic
- Conclusions
- Visit us



MMG Day
2018

Vicente
Mataix
Ferrández

Introduction

Objectives

Kratos

Theory

Level set
remeshing

Hessian
remeshing

Metric
intersection

Internal
variables

Numerical
contact

Cases

Level set
remeshing

Hessian
remeshing

Internal
variables

Numerical
contact

Conclusions

Problems
Conclusions

Visit us

Section 1

Introduction



Objectives

MMG Day

2018

Vicente
Mataix
Ferrández

Introduction

Objectives

Kratos

Theory

Level set
remeshing

Hessian
remeshing

Metric
intersection

Internal
variables

Numerical
contact

Cases

Level set
remeshing

Hessian
remeshing

Internal
variables

Numerical
contact

Conclusions

Problems
Conclusions

Visit us

The main aim of this work is to integrate into *MMG API* on the *Kratos* framework





Kratos is a framework for building multi-disciplinary finite element programs.

Features

- **KERNEL:** The kernel and application approach is used to reduce the possible conflicts arising between developers of different fields.
- **OBJECT ORIENTED:** The modular design, hierarchy and abstraction of these approaches fits to the generality, flexibility and re-usability required for the current and future challenges in numerical methods. The main code is developed in *C++* and the *Python* language is used for scripting
- **OPEN SOURCE:** The *BSD (Berkeley Software Distribution)* licence allows to use and distribute the existing code without any restriction, but with the possibility to develop new parts of the code on an open or close basis depending on the developers.
- **FREE:** Because is devoted mainly to developers, researchers and students and, therefore, is the most fruitful way to share knowledge and built a robust numerical methods laboratory adapted to their users' needs. Please, read the license for more detailed information.



Kratos structure classes

MMG Day
2018

Vicente
Mataix
Ferrández

Introduction

Objectives

Kratos

Theory

Level set
remeshing

Hessian
remeshing

Metric
intersection

Internal
variables

Numerical
contact

Cases

Level set
remeshing

Hessian
remeshing

Internal
variables

Numerical
contact

Conclusions

Problems
Conclusions
Visit us

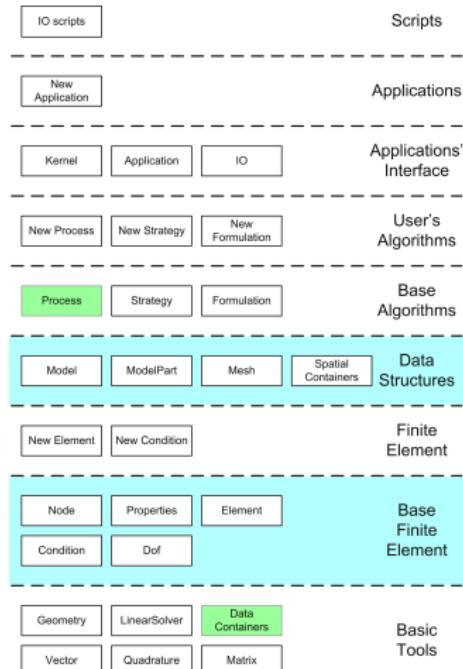


Figure 1: Kratos structure classes

Groups

- **Scripts:** Simple scripted programs created in order to reduce the workload and simplify run problems
- **Applications:** This is the base of the modularity of **Kratos**. Each application can be defined o solve an specific problem and couple them later
- **App interface (core):** Communicate each components and define the framework behaviour
- **Algorithms:** Operations that are used to solve the problem (strategies, time schemes, algorithms, etc...)
- **Data structure:** Contains the information of the problem (geometries, elements, etc...)
- **Finite element:** The base components necessaries to define a **FE** problem (DoF, elements, nodes, etc...)
- **Basic tools:** Algebraic and mathematic components

The groups and classes in **cyan** and **green** will be detailed later for being more related with the **Kratos-MMG** integration



Data structures classes

MMG Day
2018

Vicente
Mataix
Ferrández

Introduction

Objectives
Kratos

Theory

Level set
remeshing

Hessian
remeshing

Metric
intersection

Internal
variables

Numerical
contact

Cases

Level set
remeshing

Hessian
remeshing

Internal
variables

Numerical
contact

Conclusions

Problematic
Conclusions

Visit us

Model

Model stores the whole model to be analyzed. All Nodes, Properties, Elements, Conditions and solution data

ModelPart

ModelPart holds all data related to an arbitrary part of model. It stores all existing components and data like Nodes, Properties, Elements, Conditions and solution data related to a part of model

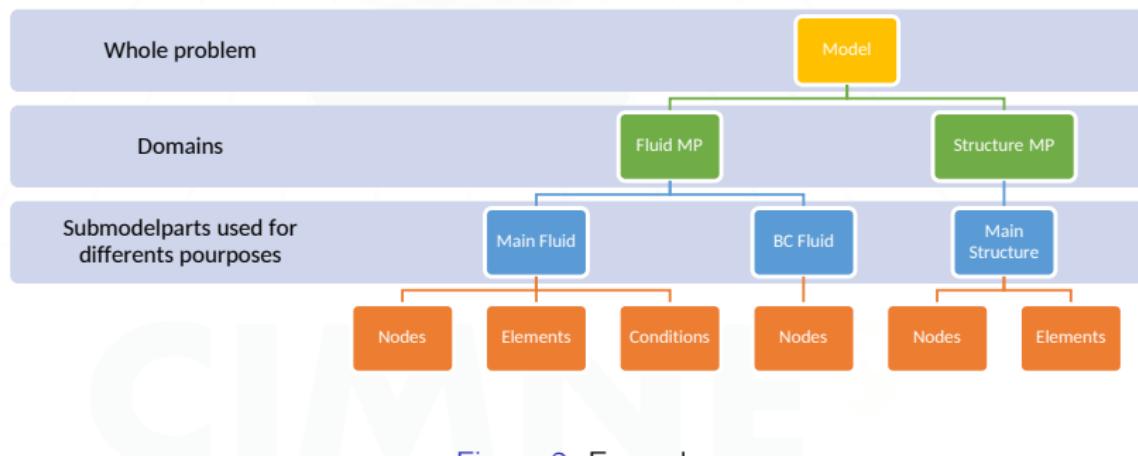


Figure 2: Example



Finite element classes

MMG Day
2018

Vicente
Mataix
Ferrández

Introduction

Objectives
Kratos

Theory
Level set
remeshing
Hessian
remeshing
Metric
intersection
Internal
variables
Numerical
contact

Cases

Level set
remeshing

Hessian
remeshing

Internal
variables

Numerical
contact

Conclusions

Problems
Conclusions
Visit us

Node

Node It is a point with additional facilities. Stores the nodal data, historical nodal data, and list of *DoF*

Condition

Condition encapsulates data and operations necessary for calculating the local contributions of Condition to the global system of equations. *Neumann* conditions are example

Elements

Element encapsulates the elemental formulation in one object and provides an interface for calculating the local matrices and vectors necessary for assembling the global system of equations. It holds its geometry that meanwhile is its array of Nodes. Also stores the elemental data

Properties

Properties encapsulates data shared by different Elements or Conditions. It can store any type of data

DoF

DoF represents a degree of freedom (*DoF*). This class enables the system to work with different set of *DoFs* and also represents the *Dirichlet* condition assigned to each *DoF*



Colours identification

MMG Day
2018

Vicente
Mataix
Ferrández

Introduction

Objectives

Kratos

Theory

Level set
remeshing

Hessian
remeshing

Metric
intersection

Internal
variables

Numerical
contact

Cases

Level set
remeshing

Hessian
remeshing

Internal
variables

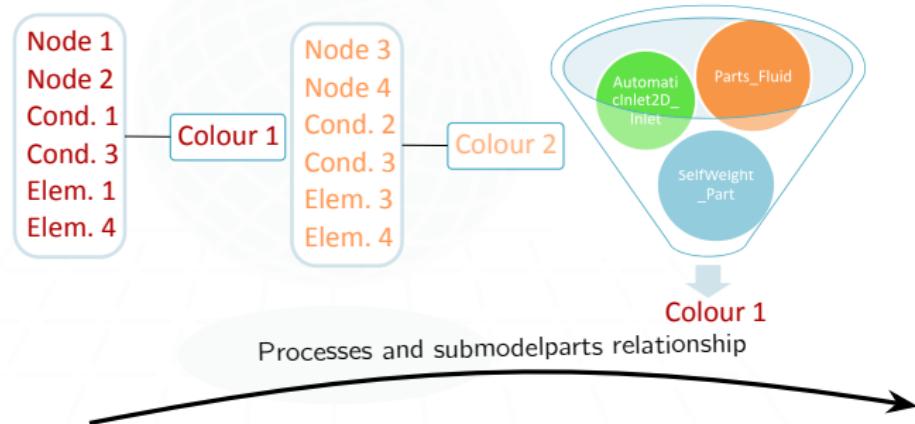
Numerical
contact

Conclusions

Problems
Conclusions

Visit us

In our implementations we use processes to set the BC (both *Neumann* or *Dirichlet*)



```
"python_module" : "apply_inlet_process",
"kratos_module" : "KratosMultiphysics.FluidDynamicsApplication",
"help" : [],
"process_name" : "ApplyInletProcess",
"Parameters" : {
    "model_part_name" : "AutomaticInlet2D_Inlet",
    "variable_name" : "VELOCITY",
    "modulus" : 1.0,
    "direction" : "automatic_inwards_normal",
    "interval" : [0, "End"]
}
```

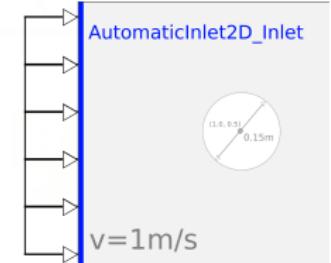


Figure 3: Example of BC in *json* format

Figure 4: Subm. BC



Other classes

MMG Day
2018

Vicente
Mataix
Ferrández

Introduction

Objectives
Kratos

Theory

Level set
remeshing

Hessian
remeshing

Metric
intersection

Internal
variables

Numerical
contact

Cases

Level set
remeshing

Hessian
remeshing

Internal
variables

Numerical
contact

Conclusion
15

Problems
16

Conclusions
17

Visit us

Process

Process is the place for adding new algorithms to *Kratos*. Mapping algorithms, Optimization procedures and many other type of algorithms can be implemented as a new process in *Kratos*.

Data containers

Data containers A data value container is a heterogeneous container with a variable base interface designed to hold the value for any type of variable

Listing 1: Example using *MMG API* in Process

```
template<>
void MmgProcess<2>::InitMesh()
{
    mmgMesh = nullptr;
    mmgSol = nullptr;

    // We init the MMG mesh and sol
    MMG2D_Init_mesh(
        MMG5_ARG_start,
        MMG5_ARG_ppMesh,
        &mmgMesh,
        MMG5_ARG_ppMet,
        &mmgSol,
        MMG5_ARG_end);

    InitVerbosity();
}
```

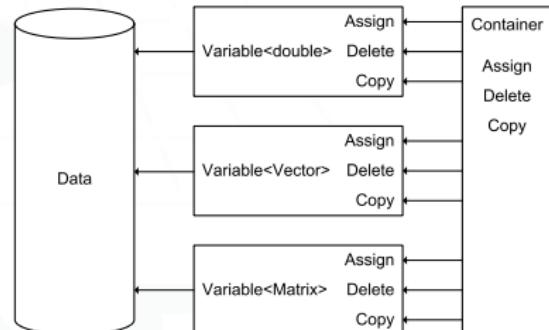


Figure 5: Variable



MMG Day

2018

Vicente

Mataix

Ferrández

Introduction

Objectives

Kratos

Theory

Level set
remeshing

Hessian
remeshing

Metric
intersection

Internal
variables

Numerical
contact

Cases

Level set
remeshing

Hessian
remeshing

Internal
variables

Numerical
contact

Conclusions

Problems
Conclusions

Visit us

Section 2

Theory



Level set remeshing

MMG Day
2018

Vicente
Mataix
Ferrández

Introduction

Objectives

Kratos

Theory

Level set
remeshing

Hessian
remeshing

Metric
intersection

Internal
variables

Numerical
contact

Cases

Level set
remeshing

Hessian
remeshing

Internal
variables

Numerical
contact

Conclusions

Problematic
Conclusions

Visit us

We compute the gradient (1) of a scalar variable f in order to compute an anisotropic metric to remesh, using the procedure from (2)

$$\nabla f = \frac{\partial f}{\partial x} \mathbf{i} + \frac{\partial f}{\partial y} \mathbf{j} + \frac{\partial f}{\partial z} \mathbf{k} \quad (1)$$

Level set metric computation

Calling h the element size and ρ the anisotropic ratio

The scalar value f and ∇f the gradient from that scalar. \mathcal{M} is the metric

We compute the following auxiliar coefficients:

$$\begin{cases} c_0 = \frac{1.0}{h^2} & \text{Isotropic metric} \\ c_1 = \frac{c_0}{2} & \text{Applying anisotropic ratio} \end{cases} \quad (2a)$$

For 2D:

$$\mathcal{M} = \begin{pmatrix} c_0(1 - \nabla f_x^2) + c_1 \nabla f_x^2 & (c_1 - c_0) \nabla f_x \nabla f_y \\ (c_1 - c_0) \nabla f_x \nabla f_y & c_0(1 - \nabla f_y^2) + c_1 \nabla f_y^2 \end{pmatrix} \quad (2b)$$

For 3D:

$$\mathcal{M} = \begin{pmatrix} c_0(1 - \nabla f_x^2) + c_1 \nabla f_x^2 & (c_1 - c_0) \nabla f_x \nabla f_y & (c_1 - c_0) \nabla f_x \nabla f_z \\ (c_1 - c_0) \nabla f_x \nabla f_y & c_0(1 - \nabla f_y^2) + c_1 \nabla f_y^2 & (c_1 - c_0) \nabla f_y \nabla f_z \\ (c_1 - c_0) \nabla f_x \nabla f_z & (c_1 - c_0) \nabla f_y \nabla f_z & c_0(1 - \nabla f_z^2) + c_1 \nabla f_z^2 \end{pmatrix} \quad (2c)$$

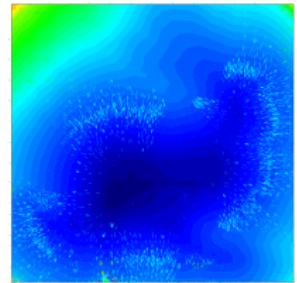


Figure 6: Scalar and its gradient



Hessian remeshing

MMG Day
2018

Vicente
Mataix
Ferrández

Introduction

Objectives

Kratos

Theory

Level set
remeshing

Hessian
remeshing

Metric
intersection

Internal
variables

Numerical
contact

Cases

Level set
remeshing

Hessian
remeshing

Internal
variables

Numerical
contact

Conclusions

Problems
Conclusions

Visit us

Following a similar procedure like in the case of the level set, we can compute the hessian matrix (3) of a scalar variable f

$$H = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \cdots & \frac{\partial^2 f}{\partial x_1 \partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 f}{\partial x_n \partial x_1} & \cdots & \frac{\partial^2 f}{\partial x_n^2} \end{bmatrix} \text{ or, just: } H_{i,j} = \frac{\partial^2 f}{\partial x_i \partial x_j} \quad (3)$$

Hessian metric computation

Once the *Hessian* matrix has been computed we can compute the corresponding anisotropic metric by the following

$$\mathcal{M} = \mathcal{R}^t \tilde{\Lambda}^t \mathcal{R} \text{ where } \tilde{\Lambda} = (\tilde{\lambda}_i) \text{ being } \tilde{\lambda}_i = \min \left(\max \left(\frac{c_d |\lambda_i|}{\epsilon}, \frac{1}{h_{\max}^2} \right), \frac{1}{h_{\min}^2} \right) \quad (4a)$$

Being ϵ the error threshold and c_d a constant ratio of a mesh constant and the interpolation ratio
For an isotropic mesh the metric will be:

$$\mathcal{M}_{iso} = diag(\max(\tilde{\lambda}_i)) = \begin{pmatrix} \max(\tilde{\lambda}_i) & 0 & 0 \\ 0 & \max(\tilde{\lambda}_i) & 0 \\ 0 & 0 & \max(\tilde{\lambda}_i) \end{pmatrix} \quad (4b)$$

For anisotropic mesh will be:

$$\mathcal{M}_{aniso} = \mathcal{R}^t \begin{pmatrix} \max(\min(\tilde{\lambda}_1, \tilde{\lambda}_{\max}), R_{\lambda rel}) & 0 & 0 \\ 0 & \max(\min(\tilde{\lambda}_2, \tilde{\lambda}_{\max}), R_{\lambda rel}) & 0 \\ 0 & 0 & \max(\min(\tilde{\lambda}_3, \tilde{\lambda}_{\max}), R_{\lambda rel}) \end{pmatrix} \mathcal{R} \quad (4c)$$

Being $R_{\lambda rel} = |\tilde{\lambda}_{\max} - \tilde{\lambda}|$ where $R_{\lambda} = (1 - \rho)|\tilde{\lambda}_{\max} - \tilde{\lambda}_{\min}|$



Metric intersection

MMG Day
2018

Vicente
Mataix
Ferrández

Introduction

Objectives
Kratos

Theory

Level set
remeshing

Hessian
remeshing

Metric
intersection

Internal
variables
Numerical
contact

Cases

Level set
remeshing

Hessian
remeshing

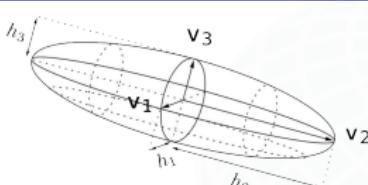
Internal
variables

Numerical
contact

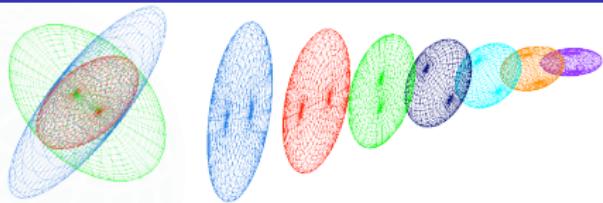
Conclusions

Problems
Conclusions

Visit us



(a) Metric analogy



(b) Interpolation

The metric intersection consists in keeping the most restrictive size constraint in all directions imposed by this set of metrics[2]

Procedure

The simultaneous reduction enables to find a common basis such that \mathcal{M}_1 and \mathcal{M}_2 are congruent to a diagonal matrix, in this basis then \mathcal{N} is introduced

$$\mathcal{N} = \mathcal{M}_1^{-1} \mathcal{M}_2 \text{ considering that can be decomposed in } \lambda_i = e_i^t \mathcal{M}_1 e_i \text{ and } \mu_i = e_i^t \mathcal{M}_2 e_i \quad (5a)$$

Considering $\mathcal{P} = (e_1 e_2 e_3)$ be the matrix the columns of which are the eigenvectors of \mathcal{N} (common basis)

$$\mathcal{M}_1 = \mathcal{P}^{-t} \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} \mathcal{P}^{-1} \text{ and } \mathcal{M}_2 = \mathcal{P}^{-t} \begin{pmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_2 & 0 \\ 0 & 0 & \mu_3 \end{pmatrix} \mathcal{P}^{-1} \quad (5b)$$

Computing the metric intersection as:

$$\mathcal{M}_{1\cap 2} = \mathcal{M}_1 \cap \mathcal{M}_2 = \mathcal{P}^{-t} \begin{pmatrix} \max(\lambda_1, \mu_1) & 0 & 0 \\ 0 & \max(\lambda_2, \mu_2) & 0 \\ 0 & 0 & \max(\lambda_3, \mu_3) \end{pmatrix} \mathcal{P}^{-1} \quad (5c)$$



Internal variables transfer

MMG Day
2018

Vicente
Mataix
Ferrández

Introduction

Objectives

Kratos

Theory

Level set
remeshing

Hessian
remeshing

Metric
intersection

Internal
variables

Numerical
contact

Cases

Level set
remeshing

Hessian
remeshing

Internal
variables

Numerical
contact

Conclusions

Problematic
Conclusions

Visit us

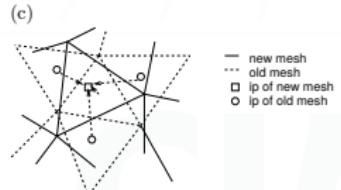
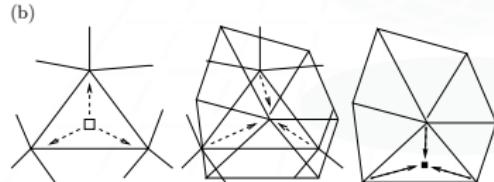
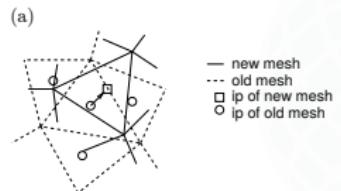


Figure 7: Transfer operators

The Figure 7 shows graphically how each one of the transfer methods work

Techniques

- **CPT:** Closest Point Transfer. (a)
It just takes the value from the closest point
It provides acceptable results at low cost
- **SFT:** Shape Function Projection transfer. (b)
It interpolates the values using the standard FEM shape functions
Leads to an artificial damage diffusion, but preserves the original shape of the damage profile
- **LST:** Least-Square Projection transfer. (c)
It uses an *least-square* transfer across the closest points
It is probably the most accurate technique but computationally more expensive

Some example will be shown following



Numerical contact. Introduction

MMG Day
2018

Vicente
Mataix
Ferrández

Introduction
Objectives
Kratos

Theory
Level set
remeshing
Hessian
remeshing
Metric
intersection

Internal
variables
Numerical
contact

Cases
Level set
remeshing
Hessian
remeshing
Internal
variables
Numerical
contact

Conclusions
Problematic
Conclusions
Visit us

Fundamentals in frictionless contact

The potential corresponding with the frictionless contact is defined in (6a).

$$\mathcal{W}^{CO}(u, \lambda_n) = \int_{\Gamma_C^{(1)}} \lambda_n \cdot g_n dA_0 \quad (6a)$$

We write the gap function as (6b).

$$g_n = \int_{\Gamma_C^{(1)}} n^{(1)} \cdot (u^{(1)} - u^{(2)}) dA_0 \quad (6b)$$

The normal contact conditions can then be represented as **Karush-Kuhn-Tucker (KKT)** (Figure 8) conditions.

$$\begin{cases} g_n \geq 0 \\ p_n \leq 0 \\ p_n g_n = 0 \end{cases} \quad \text{on } \Gamma_C^{(i)} \times [0, T] \quad (6c)$$



Figure 8: KKT condition

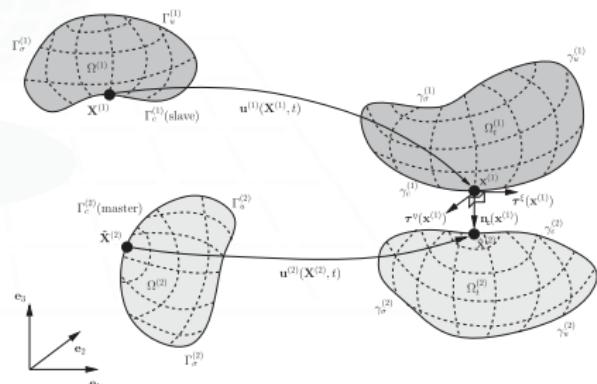


Figure 9: Contact kinematics



Augmented Lagrangian formulation

MMG Day
2018

Vicente
Mataix
Ferrández

Introduction

Objectives
Kratos

Theory

Level set
remeshing

Hessian
remeshing

Metric
intersection

Internal
variables

Numerical
contact

Cases

Level set
remeshing

Hessian
remeshing

Internal
variables

Numerical
contact

Conclusions

Problems
Conclusions
Visit us

ALM Functional

$$\mathcal{W}^{CO}(u, \lambda_n) = \int_{\Gamma_C(1)} k \lambda_n \cdot g_n + \frac{\epsilon}{2} g_n^2 - \frac{1}{2\epsilon} (k \lambda_n + \epsilon g_n)^2 dA_0 \text{ with } \langle x \rangle \begin{cases} x & x \geq 0 \\ 0 & x < 0 \end{cases} \quad (7)$$

Where ϵ is a positive penalty parameter, k is a positive scale factor

$$\mathcal{W}^{CO}(u, \lambda_n) = \int_{\Gamma_C(1)} \begin{cases} k \lambda_n \cdot g_n + \frac{\epsilon}{2} g_n^2 dA_0 & \text{if } k \lambda_n + \epsilon g_n \leq 0 \text{ (Contact zone)} \\ -\frac{k}{2\epsilon} \lambda_n^2 dA_0 & \text{if } k \lambda_n + \epsilon g_n > 0 \text{ (Gap zone)} \end{cases} \quad (8)$$

$$\delta \mathcal{W}^{CO}(u, \lambda_n) = \int_{\Gamma_C(1)} \begin{cases} \hat{\lambda}_n \cdot \delta g_n + k g_n \delta \lambda_n dA_0 & \text{if } \hat{\lambda}_n \leq 0 \text{ (Contact zone)} \\ -\frac{k^2}{\epsilon} \lambda_n \delta \lambda_n dA_0 & \text{if } \hat{\lambda}_n > 0 \text{ (Gap zone)} \end{cases} \quad (9)$$

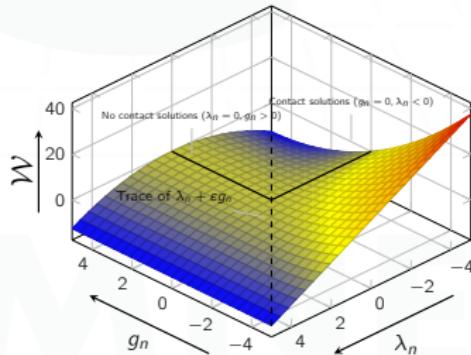


Figure 10: Augmented Lagrangian function for the contact problem



Mortar operators

MMG Day

2018

Vicente
Mataix
Ferrández

Introduction

Objectives

Kratos

Theory

Level set
remeshing

Hessian
remeshing

Metric
intersection

Internal
variables

Numerical
contact

Cases

Level set
remeshing

Hessian
remeshing

Internal
variables

Numerical
contact

Conclusions

Problems
Conclusions

Visit us

Definition

Numerical integration of the mortar coupling terms is exclusively performed on the slave side $\Gamma_{c,h}$ of the interface

$$-\delta\mathcal{W}_{co,h} = \sum_{j=1}^{m(1)} \sum_{k=1}^{n(1)} \lambda_{nj}^T \left(\int_{\Gamma_{c,h}^{(1)}} \phi_j N_k^{(1)} dA_0 \right) \delta d_{nk}^{(1)} - \sum_{j=1}^{m(1)} \sum_{l=1}^{n(2)} \lambda_{nj}^T \left(\int_{\Gamma_{c,h}^{(1)}} \phi_j \left(N_l^{(2)} \circ \chi_h \right) dA_0 \right) \delta d_{nl}^{(2)} \quad (10)$$

$$\mathbf{D}[j, k] = D_{jk} \mathbf{l}_{ndim} = \int_{\Gamma_{c,h}^{(1)}} \phi_j N_k^{(1)} dA_0 \mathbf{l}_{ndim}, j = 1, \dots, m^{(1)}, k = 1, \dots, n^{(1)} = \boxed{\sum_{g=1}^{ngp} w_g \phi_{gj} N_{gk}^{(1)} J_g^{(1)}} \quad (11a)$$

$$\mathbf{M}[j, l] = M_{jl} \mathbf{l}_{ndim} = \int_{\Gamma_{c,h}^{(1)}} \phi_j \left(N_l^{(2)} \circ \chi_h \right) dA_0 \mathbf{l}_{ndim}, j = 1, \dots, m^{(1)}, k = 1, \dots, n^{(2)} = \boxed{\sum_{g=1}^{ngp} w_g \phi_{gj} N_{gk}^{(2)} J_g^{(1)}} \quad (11b)$$

Derivatives

$$\Delta \mathbf{D}[j, k] = \boxed{\sum_{g=1}^{ngp} w_g \Delta \phi_{gj} N_{gk}^{(1)} J_g^{(1)} + \sum_{g=1}^{ngp} w_g \phi_{gj} \Delta N_{gk}^{(1)} J_g^{(1)} + \sum_{g=1}^{ngp} w_g \phi_{gj} N_{gk}^{(1)} \Delta J_g^{(1)}} \quad (12a)$$

$$\Delta \mathbf{M}[j, l] = \boxed{\sum_{g=1}^{ngp} w_g \Delta \phi_{gj} N_{gk}^{(2)} J_g^{(1)} + \sum_{g=1}^{ngp} w_g \phi_{gj} \Delta N_{gk}^{(2)} J_g^{(1)} + \sum_{g=1}^{ngp} w_g \phi_{gj} N_{gk}^{(2)} \Delta J_g^{(1)}} \quad (12b)$$



Semismooth Newton method

MMG Day
2018

Vicente
Mataix
Ferrández

Introduction

Objectives

Kratos

Theory

Level set
remeshing

Hessian
remeshing

Metric
intersection

Internal
variables

Numerical
contact

Cases

Level set
remeshing

Hessian
remeshing

Internal
variables

Numerical
contact

Conclusions

Problems
Conclusions

Visit us

Introduction

Unilateral contact causes one major additional complexity with regard to global solution schemes which divide the set of all discrete constraints into two *a priori* unknown sets of **active** and **inactive** constraints

We re-arrange the **KKT** conditions such that a *Newton–Raphson* type algorithm can be applied

This function in the case of frictionless case basically corresponds with the augmented normal contact pressure

$$\hat{\lambda}_n = k\lambda_n + \varepsilon g_n \begin{cases} \hat{\lambda}_n < 0 & \text{Active} \\ \hat{\lambda}_n \geq 0 & \text{Inactive} \end{cases} \quad (13)$$

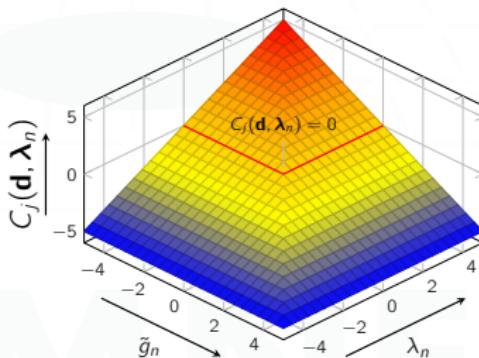


Figure 11: Complementary function of $\hat{\lambda}_n$



Numerical contact remeshing (I)

MMG Day
2018

Vicente
Mataix
Ferrández

Introduction

Objectives
Kratos

Theory
Level set
remeshing
Hessian
remeshing
Metric
intersection

Internal
variables
Numerical
contact

Cases

Level set
remeshing

Hessian
remeshing

Internal
variables
Numerical
contact

Conclusions

Problematic
Conclusions
Visit us

NOTE: Work of *Anna Rehr* from **TUM**

Error estimation[8]

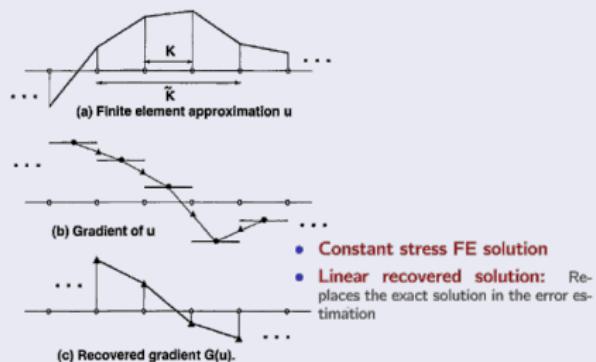
Residual based methods

- **Internal residual (r):** Error in the differential equation
- **Boundary error (R):**
 - **Traction boundaries:** Difference stress and traction
 - **Interelement boundaries:** Stress jumps
 - **Contact boundary:** Difference stress and contact pressure

$$\|e\|_{E,K} \approx \|\hat{e}\|_{E,K} = C[h_K^2 \|r\|_{L_2(K)} + h_K^2 \|R\|_{L_2(\partial K)}]$$

- Sound mathematical error bounds
- Determination of the constant C not trivial
- Quality depends heavily on the chosen constant

Recovery based methods



$$\|e\|_{E,K} \approx \|\hat{e}\|_{E,K} = \left[\int_{\Omega_K} (\sigma^* - \sigma_h)^T D^{-1} (\sigma^* - \sigma_h) \right]^{\frac{1}{2}} d\Omega_K$$

- Robust
- Easy implementation, no unknown constant, easy extensibility
- No sound mathematical error bounds



Numerical contact remeshing (II)

MMG Day
2018

Vicente
Mataix
Ferrández

Introduction

Objectives
Kratos

Theory

Level set
remeshing

Hessian
remeshing

Metric
intersection

Internal
variables

Numerical
contact

Cases

Level set
remeshing

Hessian
remeshing

Internal
variables

Numerical
contact

Conclusions

Problematic
Conclusions

Visit us

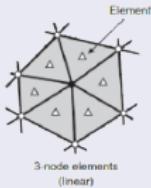
NOTE: Work of *Anna Rehr* from **TUM**

The present work introduces a modified version of the **SPR**[8] method

Modified SPR

Element patch:

All elements that are neighboring one node



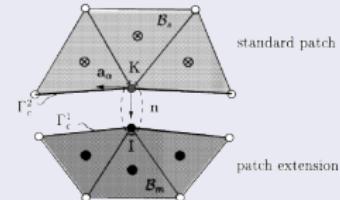
- **Concept:**

- Stresses at the integration points show superior convergence behavior
- Use these points to compute the superior stress field

- **Procedure:**

- Execute a polynomial least square fit with the integration points
- Compute with this polynomial recovered stress at the center node
- Compute the stress field by interpolation with the shape functions

Extension for contact mechanics



- **Existing approach (penalty formulation):**

- Couple patches at the contact boundary
- Enforce stress continuity in the recovery procedure by a penalty formulation

- **Anna Rehr's work:**

- Patch coupling not necessary: contact pressure is known (Lagrange Multiplier)
- Contact BC are regarded in the recovered stress calculation by a penalty formulation which forces the stresses to coincide with the contact pressure



MMG Day
2018

Vicente
Mataix
Ferrández

Introduction

Objectives

Kratos

Theory

Level set
remeshing

Hessian
remeshing

Metric
intersection

Internal
variables

Numerical
contact

Cases

Level set
remeshing

Hessian
remeshing

Internal
variables

Numerical
contact

Conclusions

Problematic
Conclusions

Visit us

Section 3

Cases



Coarse sphere

MMG Day
2018

Vicente
Mataix
Ferrández

Introduction

Objectives
Kratos

Theory

Level set
remeshing

Hessian
remeshing

Metric
intersection

Internal
variables

Numerical
contact

Cases

Level set
remeshing

Hessian
remeshing

Internal
variables

Numerical
contact

Conclusions

Problematic

Conclusions

Visit us

In this problem we remesh using the gradient of the distance function, which is the distance to the plane contained in the sphere center.

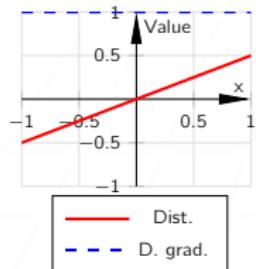


Figure 12: Distance function

The function can be seen in the Figure12

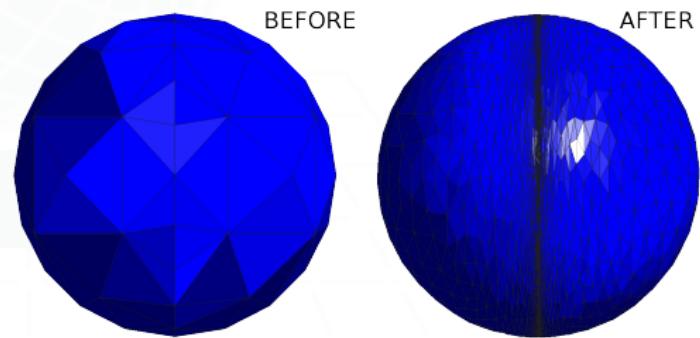


Figure 13: Mesh before and after remeshing



Stanford's bunny

MMG Day
2018

Vicente
Mataix
Ferrández

Introduction

Objectives

Kratos

Theory

Level set
remeshing

Hessian
remeshing

Metric
intersection

Internal
variables

Numerical
contact

Cases

Level set
remeshing

Hessian
remeshing

Internal
variables

Numerical
contact

Conclusions

Problems
Conclusions

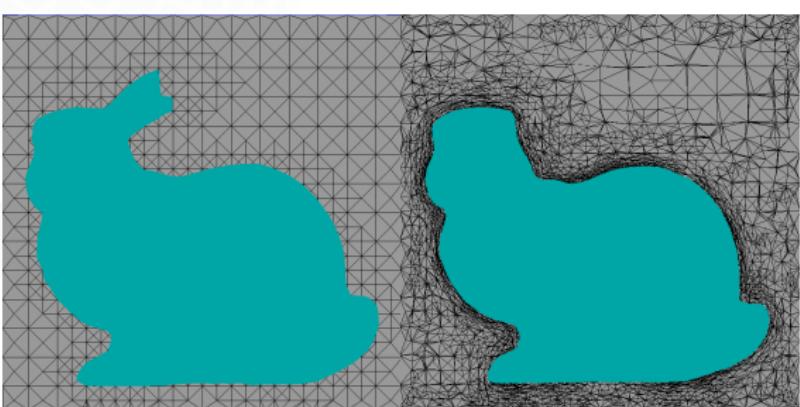
Visit us



Figure 14: Stanford's bunny

Anisotropically remesh the geometry using the distance gradient as error measure .

Previously meshed with an embedded octree mesher (*GID*).



(a) Octree mesh

(b) Anisotropic mesh

Figure 15: Mesh before and after remeshing



Embedded fluid channel 2D

MMG Day
2018

Vicente
Mataix
Ferrández

Introduction

Objectives

Kratos

Theory

Level set
remeshing

Hessian
remeshing

Metric
intersection

Internal
variables

Numerical
contact

Cases

Level set
remeshing

Hessian
remeshing

Internal
variables

Numerical
contact

Conclusions

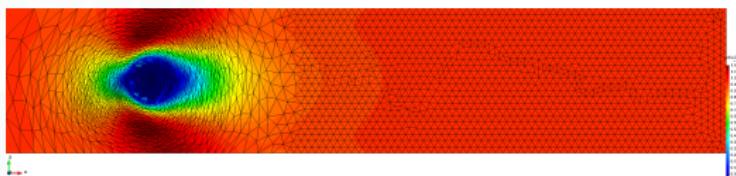
Problematic
Conclusions

Visit us

Adaptative anisotropic remeshing of 2D fluid channel with sphere using as level set the distance function. The problem is solved using an embedded formulation
It consists in a channel 5x1, a sphere of 0.3 diameter and with a velocity of 1 m/s in the inlet and zero pressure in the outlet. The resulting flow has *Reynolds* number of **100**.



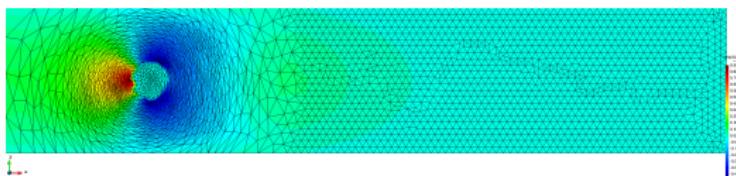
(a) Problem



(a) Velocity



(b) Initial mesh



(b) Pressure



(c) Remeshed

Figure 16: Setup



Lamborghini

MMG Day
2018

Vicente
Mataix
Ferrández

Introduction

Objectives

Kratos

Theory

Level set
remeshing

Hessian
remeshing

Metric
intersection

Internal
variables

Numerical
contact

Cases

Level set
remeshing

Hessian
remeshing

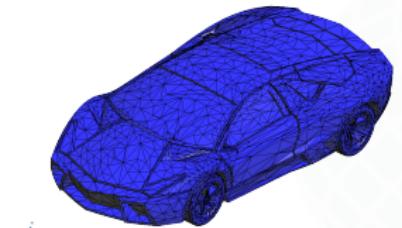
Internal
variables

Numerical
contact

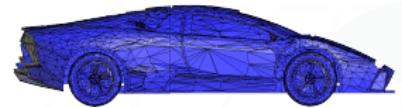
Conclusions

Problems
Conclusions

Visit us



(c) View 1



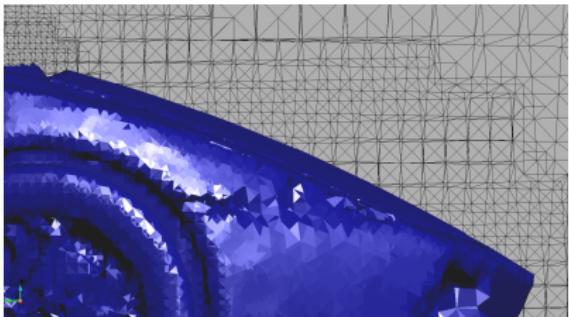
(d) View 2

Figure 17: Lamborghini

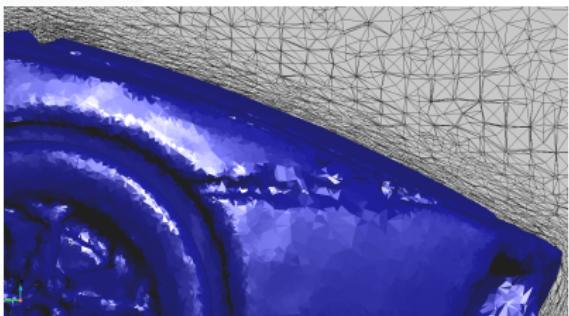
In this test case we want to remesh anisotropically the geometry of *Lamborghini*, more complex than the previous bunny.

Anisotropically remesh the geometry using the distance gradient as error measure.

Previously meshed with an embedded octree mesher (*GID*).



(a) Octree mesh



(b) Anisotropic mesh

Figure 18: Mesh before and after remeshing



Hessian 2D

MMG Day
2018

Vicente
Mataix
Ferrández

Introduction

Objectives

Kratos

Theory

Level set
remeshing

Hessian
remeshing

Metric
intersection

Internal
variables

Numerical
contact

Cases

Level set
remeshing

Hessian
remeshing

Internal
variables

Numerical
contact

Conclusions

Problems
Conclusions

Visit us

The problem corresponds with the example proposed in reference[2]

The objective is to remesh the structured 1x1 mesh with the error function from Figure 19 and equation (14)

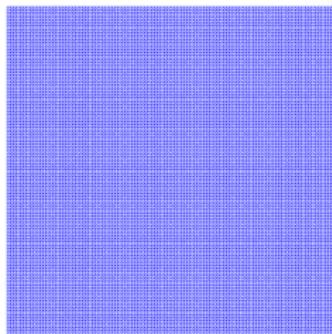
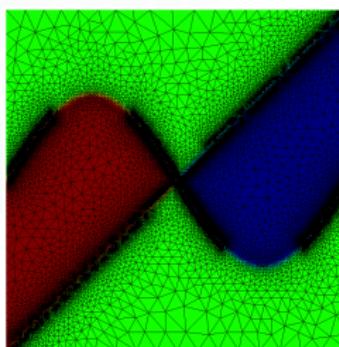
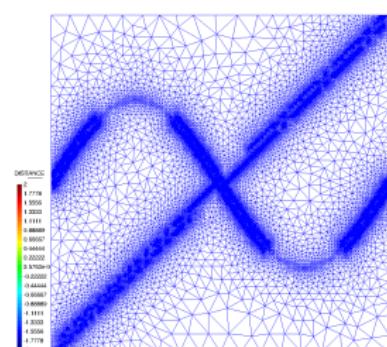


Figure 19: Initial mesh



(a) Error estimation



(b) New mesh

The χ shaped function:

$$f(x, y) = \tanh(-100(y - 0.5 - 0.25 \sin(2\pi x))) + \tanh(100(y - x)) \quad (14)$$

Figure 20: Solution



Hessian 3D

MMG Day
2018

Vicente
Mataix
Ferrández

Introduction

Objectives

Kratos

Theory

Level set
remeshing

Hessian
remeshing

Metric
intersection

Internal
variables

Numerical
contact

Cases

Level set
remeshing

Hessian
remeshing

Internal
variables

Numerical
contact

Conclusions

Problems
Conclusions

Visit us

An extension of the previous problem to 3D in several remeshing iterations

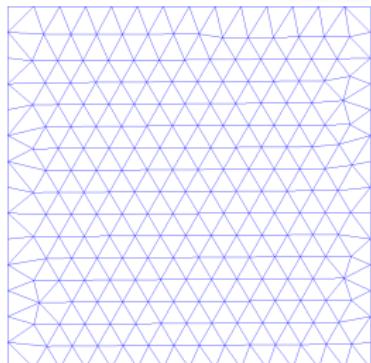
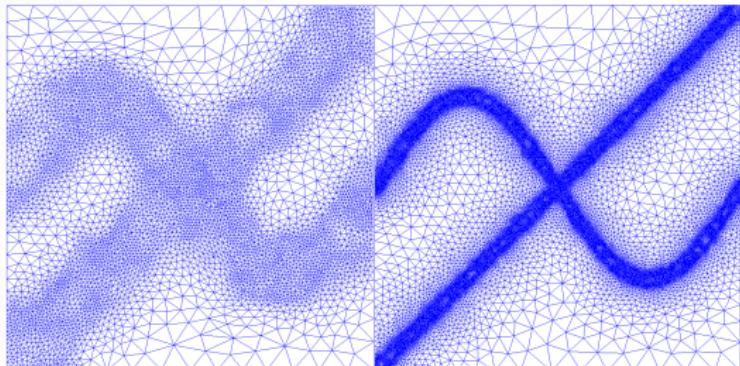
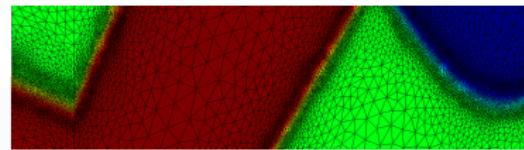


Figure 21: Initial mesh



(a) Iteration 1

(b) Iteration 2



(c) Error estimation

Figure 22: Solution



Beam 2D

MMG Day
2018

Vicente
Mataix
Ferrández

Introduction

Objectives
Kratos

Theory
Level set
remeshing
Hessian
remeshing
Metric
intersection

Internal
variables
Numerical
contact

Cases

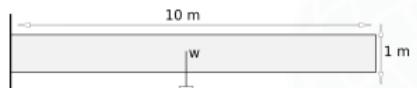
Level set
remeshing
Hessian
remeshing

Internal
variables
Numerical
contact

Conclusions

Problematic
Conclusions
Visit us

The simulation considers 100 time steps of 0.01s. The problem will be remeshed each ten steps considering the Hessian of the displacement



(a) Problem



(b) Initial mesh



(c) Mesh 1



(d) Mesh 2



(e) Mesh 3

Figure 23: Setup

CIMNE

KRATOS

Adaptative remeshing of a 2D beam using the displacement Hessian as metric (MMG lib.)

Vicente Mataix Ferrández (CIMNE)



Fluid channel 2D

MMG Day
2018

Vicente
Mataix
Ferrández

Introduction

Objectives

Kratos

Theory

Level set
remeshing

Hessian
remeshing

Metric
intersection

Internal
variables

Numerical
contact

Cases

Level set
remeshing

Hessian
remeshing

Internal
variables

Numerical
contact

Conclusions

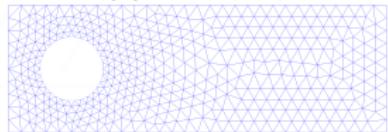
Problematic
Conclusions

Visit us

Adaptative remeshing of 2D fluid channel with sphere using Hessian of velocity as metric measure. It consists in a channel 3x1, a sphere of 0.5 diameter and with a velocity of 1 m/s in the inlet and zero pressure in the outlet.



(a) Problem



(b) Initial mesh

Figure 24: Setup

CIMNE **KRATOS** MULTIPHYSICS

Adaptative remeshing of 2D fluid channel with sphere using Hessian of velocity as metric (MMG lib.)

Vicente Mataix Ferrández (CIMNE)



Fluid channel 3D

MMG Day
2018

Vicente
Mataix
Ferrández

Introduction

Objectives
Kratos

Theory

Level set
remeshing

Hessian
remeshing

Metric
intersection

Internal
variables

Numerical
contact

Cases

Level set
remeshing

Hessian
remeshing

Internal
variables

Numerical
contact

Conclusions

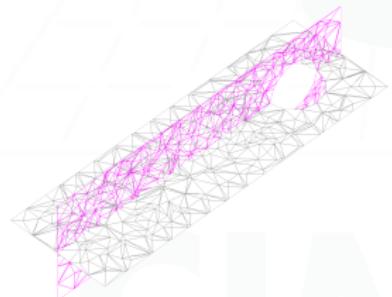
Problematic
Conclusions

Visit us

Adaptive remeshing of 3D fluid channel with sphere using Hessian of velocity as metric measure. It consists in a channel 3x1x1, a sphere of 0.5 diameter and with a velocity of 1 m/s in the inlet and zero pressure in the outlet.

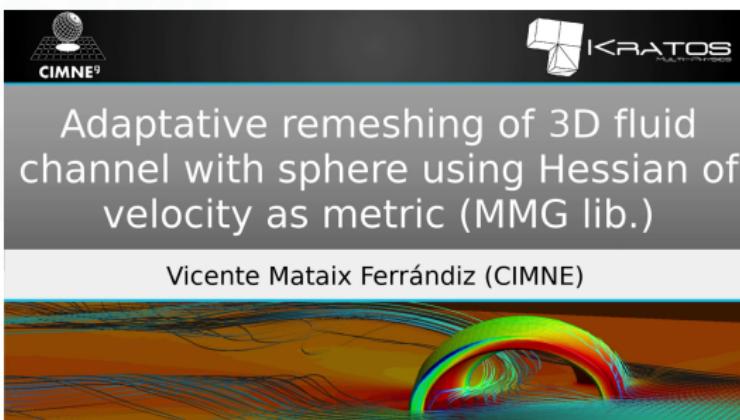


(a) Problem



(b) Initial mesh

Figure 25: Setup





Beam 2D

MMG Day

2018

Vicente

Mataix

Ferrández

Introduction

Objectives

Kratos

Theory

Level set
remeshing

Hessian
remeshing

Metric
intersection

Internal
variables

Numerical
contact

Cases

Level set
remeshing

Hessian
remeshing

Internal
variables

Numerical
contact

Conclusions

Problematic

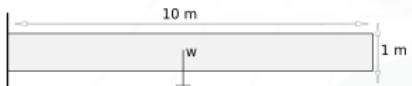
Conclusions

Visit us

The simulation considers 10 time steps of 0.01s

The problem will be remeshed each ten steps considering the Hessian of the displacement

A J2-plasticity law has been considered



(a) Problem

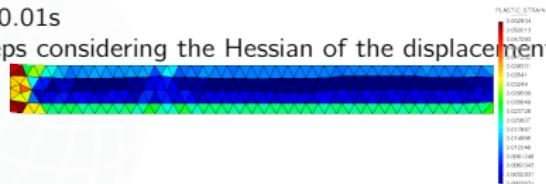


(b) Initial mesh

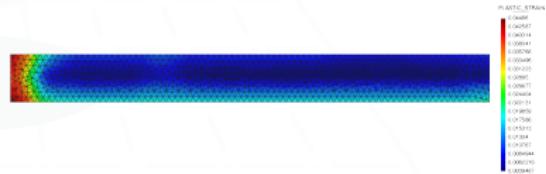


(c) After remeshing

Figure 26: Setup



(a) Initial mesh



(b) LST



(c) CPT



Patch test

MMG Day
2018

Vicente
Mataix
Ferrández

Introduction

Objectives

Kratos

Theory

Level set
remeshing

Hessian
remeshing

Metric
intersection

Internal
variables

Numerical
contact

Cases

Level set
remeshing

Hessian
remeshing

Internal
variables

Numerical
contact

Conclusions

Problems
Conclusions

Visit us

NOTE: Work of *Anna Rehr* from **TUM**

The patch test is the most basic test to pass to verify a contact formulation. It has been solved in 2D using the modification of the **SPR**.

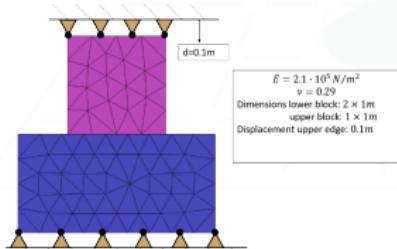


Figure 27: Setup

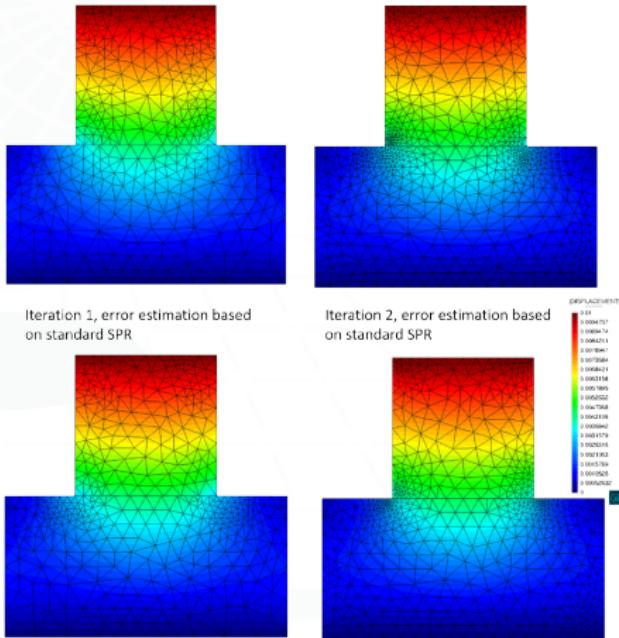


Figure 28: Solution



Hertz problem

MMG Day
2018

Vicente
Mataix
Ferrández

Introduction

Objectives

Kratos

Theory

Level set
remeshing

Hessian
remeshing

Metric
intersection

Internal
variables

Numerical
contact

Cases

Level set
remeshing

Hessian
remeshing

Internal
variables

Numerical
contact

Conclusions

Problematic
Conclusions

Visit us

NOTE: Work of *Anna Rehr* from **TUM**

The **Hertz** test is a very used
benchmark for contact mechanics.
It has been solved both in 2D and 3D
using the modification of the **SPR**

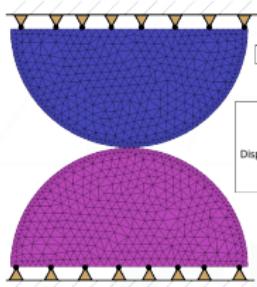
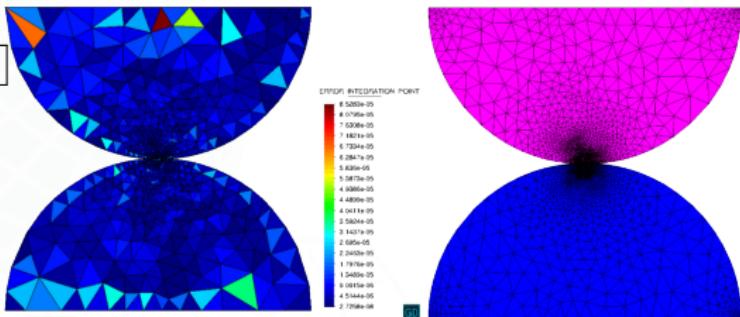
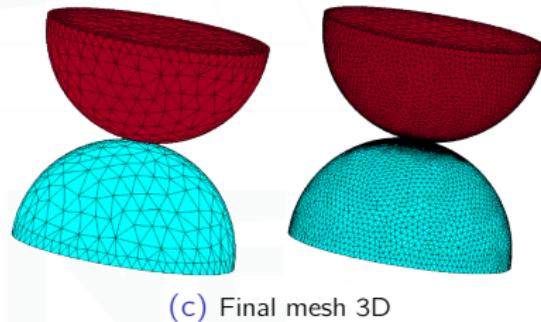


Figure 29: Setup



(a) Error

(b) Final mesh 2D



(c) Final mesh 3D



MMG Day
2018

Vicente
Mataix
Ferrández

Introduction

Objectives

Kratos

Theory

Level set
remeshing

Hessian
remeshing

Metric
intersection

Internal
variables

Numerical
contact

Cases

Level set
remeshing

Hessian
remeshing

Internal
variables

Numerical
contact

Conclusions

Problems
Conclusions
Visit us

Section 4

Conclusions



Problematic on implementation

MMG Day
2018

Vicente
Mataix
Ferrández

Introduction

Objectives

Kratos

Theory

Level set
remeshing

Hessian
remeshing

Metric
intersection

Internal
variables

Numerical
contact

Cases

Level set
remeshing

Hessian
remeshing

Internal
variables

Numerical
contact

Conclusions

Problematic

Conclusions

Visit us

During the integration of [MMG](#) we have some problems, like the following:

Problems

- **Some bugs in the creation of new geometries:** Ghost geometries appeared some times
- **Submodelparts integration:** The nodes are problematic (can belong to several geometries at the same time)
- **Some processors give problems:** Some flags of optimization are removed in order get things to work
- **Tensor notation:** Notation are always problematic



Thanks for the fast reply in the [MMG](#) forum!!



Conclusions and future work

MMG Day
2018

Vicente
Mataix
Ferrández

Introduction

Objectives
Kratos

Theory

Level set
remeshing

Hessian
remeshing

Metric
intersection

Internal
variables

Numerical
contact

Cases

Level set
remeshing

Hessian
remeshing

Internal
variables

Numerical
contact

Conclusions

Problematic

Conclusions

Visit us

Conclusions

- *MMG*: We have implemented into *Kratos* the **MMG API**
- *Metrics*: We have implemented several metric measures and utilities
- *Internal values*: We have implemented several utilities in order to interpolate internal values
- *Contact*: We have implemented an extended version of **SPR** in order to be able to compute contact remeshing

Future works

- *New problems*: Use the process for new problems not computed yet (for example *FSI*)
- *Contact*: Improve the contact remeshing implementation
- *Extend*: Extend the *Kratos/MMG* integration



Visit us at GitHub

MMG Day
2018

Vicente
Mataix
Ferrández

Introduction

Objectives
Kratos

Theory

Level set
remeshing

Hessian
remeshing

Metric
intersection

Internal
variables

Numerical
contact

Cases

Level set
remeshing

Hessian
remeshing

Internal
variables

Numerical
contact

Conclusions

Problems
Conclusions

Visit us

Kratos Multiphysics / Kratos

Code Issues Pull requests Projects Wiki Insights

Kratos Multiphysics (A.K.A Kratos) is a framework for building parallel multi-disciplinary simulation software. Modularity, extensibility and HPC are the main objectives. Kratos has BSD license and is written in C++ with extensive Python interface. <http://www.cimne.com/kratos/>

kratos fem dem parallel-computing openmp numerical-methods c-plus-plus python multi-platform bsd-license multiphysics mpl

26,907 commits 146 branches 13 releases 50 contributors

Branch: master New pull request Create new file Upload files Find file Clone or download

roigcarlo Merge pull request #1288 from KratosMultiphysics/dem/fix-test-name ... Latest commit 7fd48f2 11 hours ago

Commit	Message	Time
applications	Fixing test name	14 hours ago
benchmarking	Added unittests to the nightly run	2 years ago
cmake_build	getting from master some files that should not be touched	8 months ago
cmake_modules	adding the cotire file as downloaded from github	2 months ago
documents	adding ams packages to doxygen latex preamble	6 years ago
embedded_python	adding cotire command (conditionally)	a month ago
external_libraries	fix for old boost versions	21 days ago

Figure 30: <https://github.com/KratosMultiphysics/Kratos>



Many thanks

MMG Day
2018

Vicente
Mataix
Ferrández

Introduction

Objectives

Kratos

Theory

Level set
remeshing

Hessian
remeshing

Metric
intersection

Internal
variables

Numerical
contact

Cases

Level set
remeshing

Hessian
remeshing

Internal
variables

Numerical
contact

Conclusions

Problems
Conclusions

Visit us



(a) International Center
for Numerical Methods
in Engineering



(b) Chair of Structural Analysis Tech-
nical University of Munich

AIRBUS

(a) Airbus Defence and Space (b)
Stress Methods Optimisation Technology
Department

SIEMENS

(b) Siemens AG Corporate

ONERA
THE FRENCH AEROSPACE LAB
(c) ONERA, The French
Aerospace Lab Applied
Aerodynamics Department

Figure 32: Known Users

And many thanks to our community!!



References

MMG Day
2018

Vicente
Mataix
Ferrández

Introduction

Objectives

Kratos

Theory

Level set
remeshing

Hessian
remeshing

Metric
intersection

Internal
variables

Numerical
contact

Cases

Level set
remeshing

Hessian
remeshing

Internal
variables

Numerical
contact

Conclusions

Problematic
Conclusions

Visit us



P. Dadvand, R. Rossi, E. Oñate: An Object-oriented Environment for Developing Finite Element Codes for Multi-disciplinary Applications. Computational Methods in Engineering. 2010



F. Alauzet: Metric-Based Anisotropic Mesh Adaptation. Course material, CEA-EDF-INRIA Schools. Numerical Analysis Summer School. 2007



P. Tremblay: 2-D, 3-D and 4-D Anisotropic Mesh Adaptation for the Time-Continuous Space-Time Finite Element Method with Applications to the Incompressible Navier-Stokes Equations. PhD thesis Ottawa-Carleton Institute for Mechanical and Aerospace Engineering, Department of Mechanical Engineering, University of Ottawa. 2007



G. Turk, M. Levoy: The Stanford 3D Scanning Repository.



P.J. Frey, F. Alauzet: Anisotropic mesh adaptation for CFD computations. Comput. Methods Appl. Mech. 2004



P.J. Frey, F. Alauzet: Anisotropic mesh adaptation for transient flows simulations



M. Bellet: Adaptive mesh technique for thermal-metallurgical numerical simulation of arc welding processes, International Journal for Numerical Methods in Engineering, 2008.



Zienkiewicz, O. C. and Zhu, J.Z. and Taylor, Robert L.: The Finite Element Method: its Basis and Fundamentals, Butterworth-Heinemann, 2013.



P.J. Frey, F. Alauzet: Estimateur d'erreur géométrique et métriques anisotropes pour l'adaptation de maillage. Partie I : aspects théoriques, HAL Id: inria-00071827, 2006





MMG Day
2018

Vicente
Mataix
Ferrández

Introduction

Objectives

Kratos

Theory

Level set
remeshing

Hessian
remeshing

Metric
intersection

Internal
variables

Numerical
contact

Cases

Level set
remeshing

Hessian
remeshing

Internal
variables

Numerical
contact

Conclusions

Problems
Conclusions

Visit us

Thank you very much for your attention