

# HW#3 Function Approximation Using Fuzzy Logic

NO USING MATLABS Fuzzy Logic TOOLBOX

Need your own programming in either Matlab, Python, R, C, or other

Professor Kelly Cohen

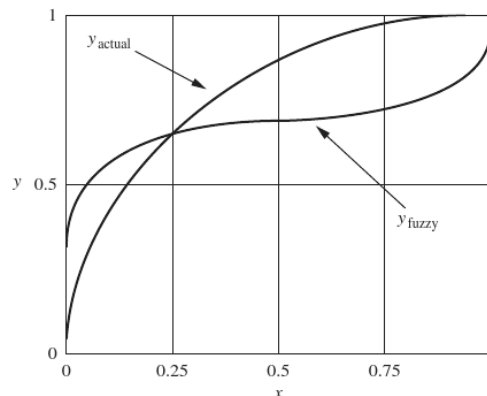
Soft Computing based AI

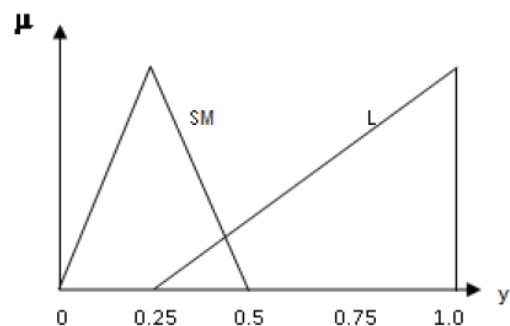
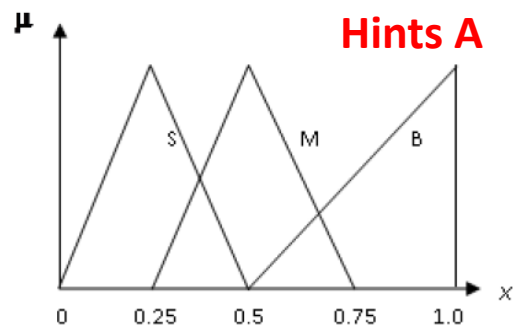
Spring 2019

### Problem 3.1

A video monitor's cathode-ray tube (CRT) has a nonlinear characteristic between the illuminance output and the voltage input. This nonlinear characteristic is  $y = x^{2.2}$ , where  $y$  is the illumination and  $x$  is the voltage. The CCD (charge-coupled device) in a video camera has a linear light-in to voltage-out characteristic. To compensate for the nonlinear characteristic of the monitor, a "gamma correction" circuit is usually employed in a CCD camera. This nonlinear circuit has a transfer function of  $y = x^{\text{gamma}}$ , where the gamma factor is usually 0.45 (i.e.,  $1/2.2$ ) to compensate for the 2.2 gamma characteristic of the monitor. The net result should be a linear response between the light incident on the CCD and the light produced by the monitor. Figure P8.1 shows the nonlinear gamma characteristic of a CCD camera ( $y_{\text{actual}}$ ). Both the input,  $x$ , and the output,  $y$ , have a universe of discourse of  $[0, 1]$ .

Partition the input variable,  $x$ , into three partitions, say small, S, medium, M, and big, B, and partition the output variable,  $y$ , into two partitions, say small, SM, and large, L. Using your own few simple rules for the nonlinear function  $y = x^{0.45}$  and the crisp inputs  $x = 0, 0.25, 0.5, 0.75, 1.0$ , determine whether your results produce a solution roughly similar to  $y_{\text{fuzzy}}$  in Figure P8.1 (which was developed with another fuzzy model (Ross, 1995)). Comment on the form of your solution and why it does or does not conform to the actual result.





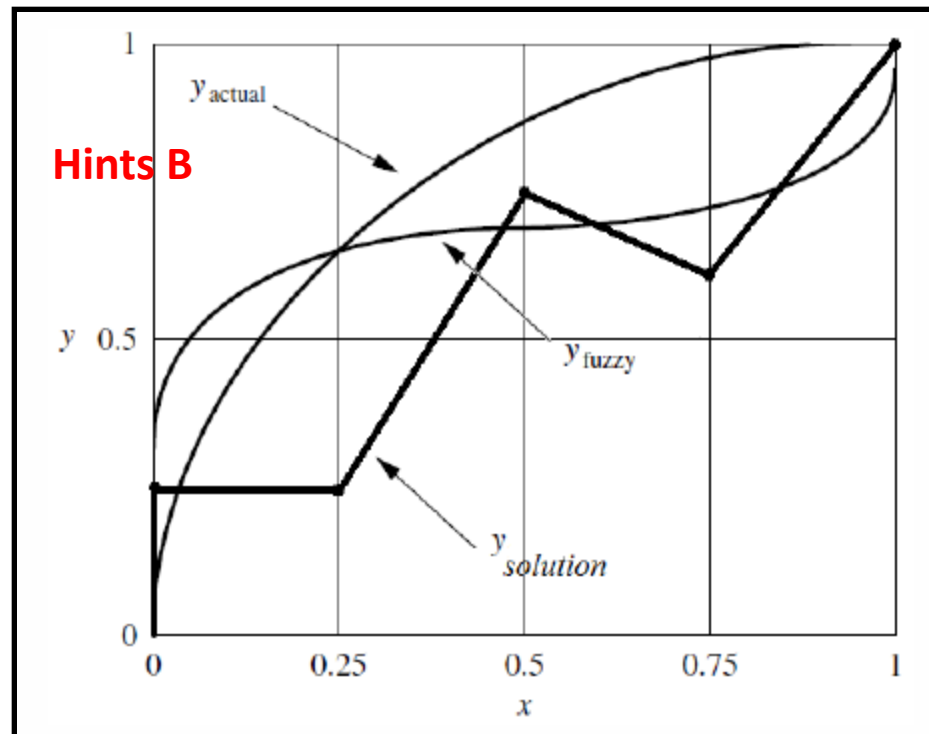
Formulating rules

- 1) IF  $x$  is S THEN  $y$  is SM
- 2) IF  $x$  is M THEN  $y$  is L
- 3) IF  $x$  is L THEN  $y$  is L

crisp inputs

- a)  $x = 0$ , then rule 1 is fired,  $y = 0.25$   
(using center area method)
- b)  $x = 0.25$ , then rule 1 is fired  
 $y = 0.25$
- c)  $x = 0.5$ , then rule 2 is fired,  $y = 0.75$
- d)  $x = 0.75$ , then rule 3 is fired,  
 $y = 0.5(1 + 0.25) = 0.625$
- e)  $x = 1.0$ , then rule 3 is fired and  $y = 1$

The solution can be improved by using a greater number of values and fuzzy classes.

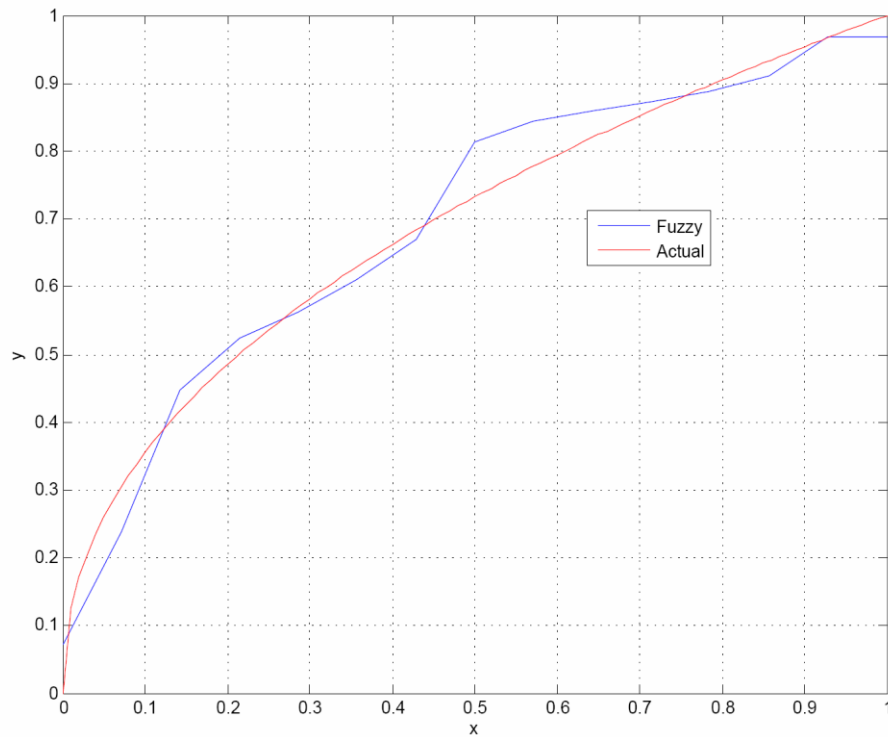


### Class Task for Problem 3.1

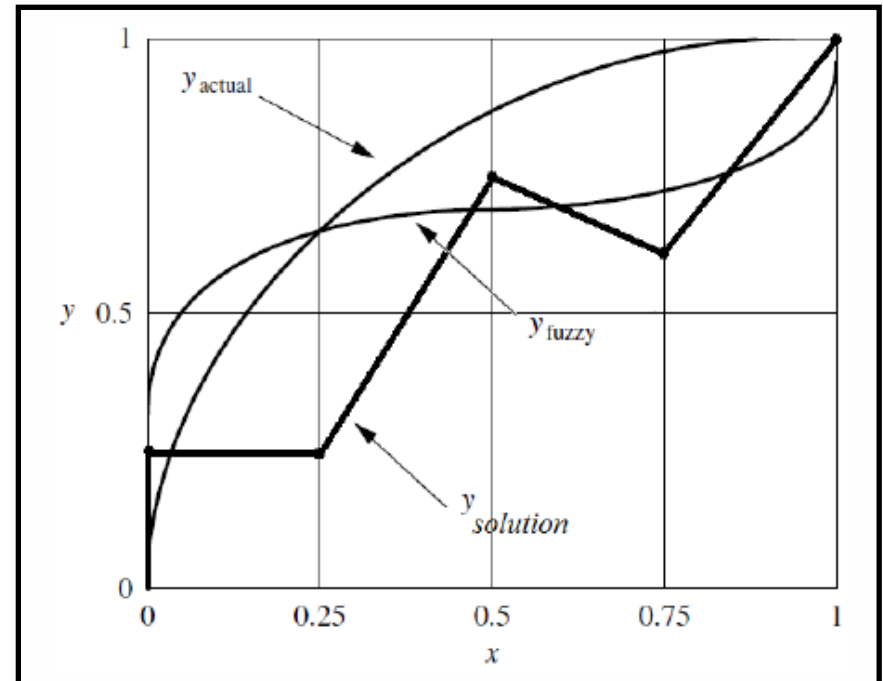
1. Improve the solution by developing a fuzzy system “as close as possible” to the actual solution by:
  - Increasing the number of membership functions for the input ( $x$ ) and the output ( $y$ )
  - Increase the number of rules as well
2. Write a paragraph concerning your observations and lessons learned doing this exercise

# Problem 3.1 (Hint)

Try getting something better than Kelly's Fuzzy on Left



Kelly's Fuzzy



Ross's Solution Manual

# Problem 3.2

Psycho-acoustic research has shown that *white noise* has different effects on people's moods, depending on the average pitch of the tones that make up the noise. Very high and very low pitches make people nervous, whereas midrange noise has a calming effect. The annoyance level of white noise can be approximated by a function of the square of the deviance of the average pitch of the noise from the central pitch of the human hearing range, approximately 10 kHz. As shown in Figure P8.5a, the human annoyance level can be modeled by the nonlinear function  $y = x^2$ , where  $x$  = deviance (kilohertz) from 10 kHz. The range of  $x$  is  $[-10, 10]$ ; outside that range pitches are not audible to humans.

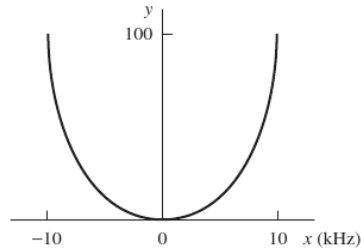


FIGURE P8.5a

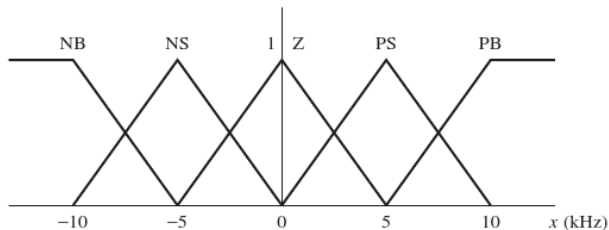


FIGURE P8.5b

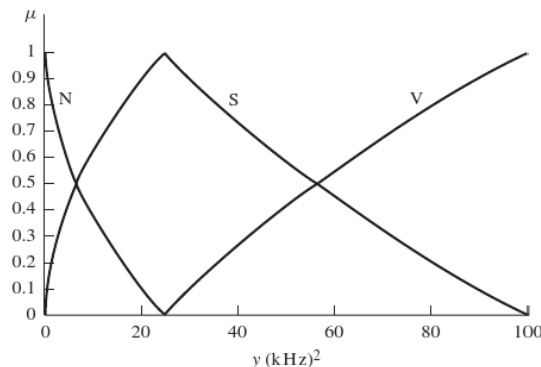


FIGURE P8.5c

The partitions for the input variable,  $x$ , are the five partitions on the range  $[-10, 10]$  kHz, as shown in Figure P8.5b, and the partitions for the output space for  $y = x^2$  are shown in Figure P8.5c. Using the following three simple rules,

1. IF  $x = Z$ , THEN  $y = N$
2. IF  $x = NS$  or  $PS$ , THEN  $y = S$
3. IF  $x = NB$  or  $PB$ , THEN  $y = V$

show how a similar plot of fuzzy results as shown in Figure 8.5d is determined.

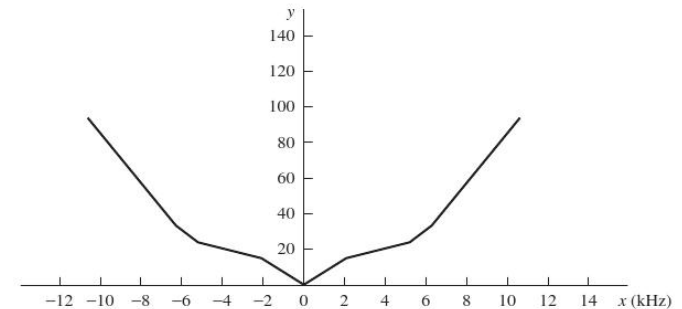


FIGURE P8.5d

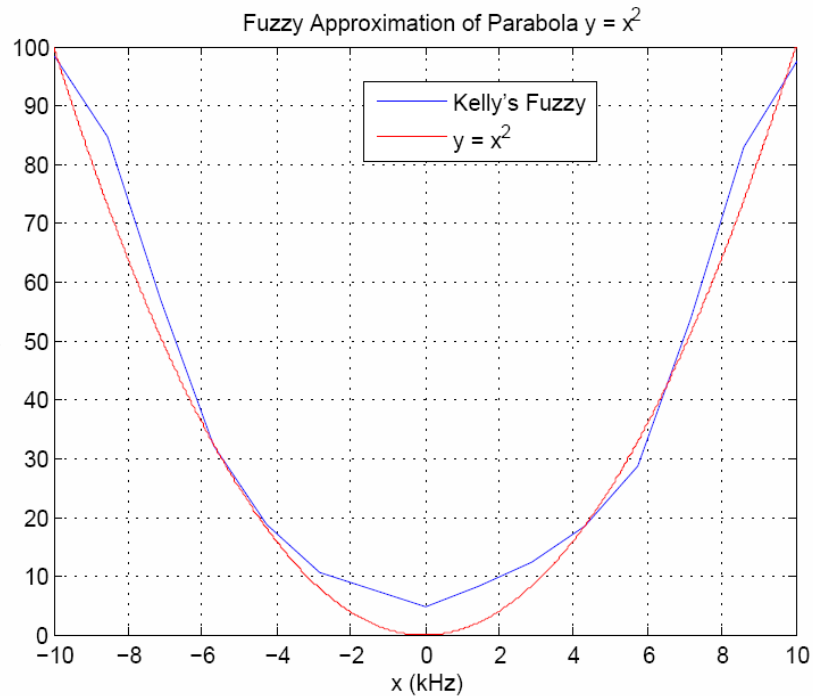
## Class Task for Problem 3.2

1. Develop your very own fuzzy system to obtain a solution "as close as possible" to the actual solution in Figure P8.5a
  - Increasing the number and varying the type of membership functions for the input ( $x$ ) and the output ( $y$ )
  - Increase the number of rules as well
2. Write a paragraph concerning your observations and lessons learned doing this exercise

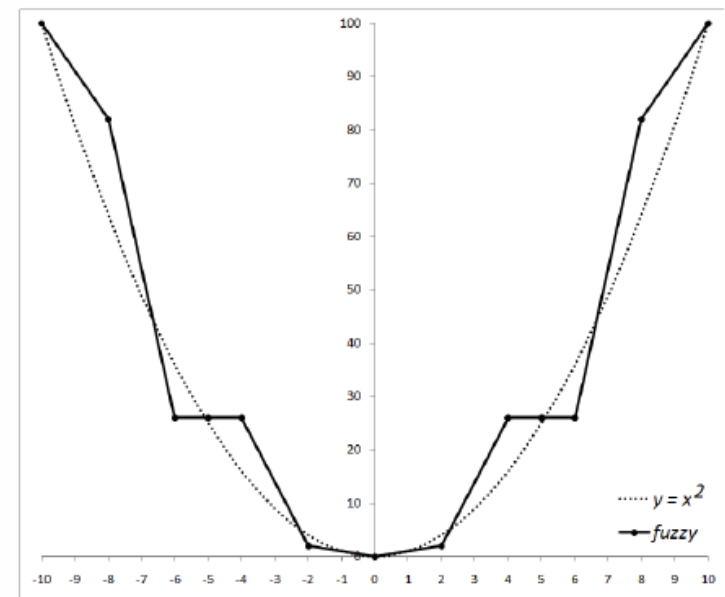
Based on Problem 8.5 in Ross Text Book

## Problem 3.2 (Hint)

Try getting something better than Kelly's Fuzzy on Left



Kelly's Fuzzy



Ross's Solution Manual

# Problem 3.3

In the field of image processing, a *limiter* function is used to enhance an image when background lighting is too high. The limiter function is shown in Figure P8.7a.

(a) Using the following rules, construct three matrix relations using the input (Figure P8.7b) and output (Figure P8.7c) partitions:

Rule 1: IF  $x = Z$ , THEN  $y = S$

Rule 2: IF  $x = PB$ , THEN  $y = PM$

Rule 3: IF  $x = NB$ , THEN  $y = NM$ .

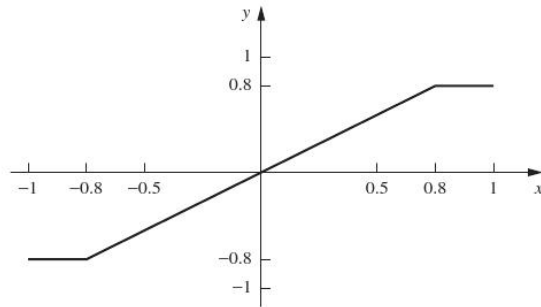


FIGURE P8.7a

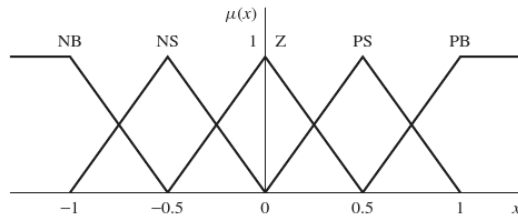


FIGURE P8.7b

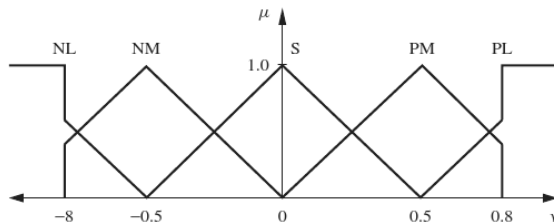


FIGURE P8.7c

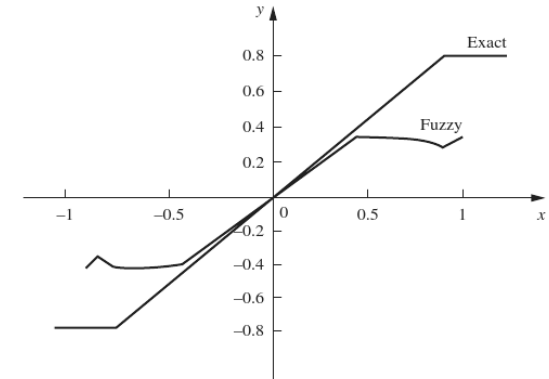


FIGURE P8.7d

(b) For crisp input values  $x = -1, -0.8, -0.6, -0.4, -0.2$ , and  $0$ , use graphical techniques or max-min composition and centroidal defuzzification to determine the associated fuzzy outputs. Because of symmetry, values for  $0 \leq x \leq 1$  are equal to  $|x|$  for  $-1 \leq x \leq 0$ . Verify that these results follow Figure P8.7d.

This problem is posed inappropriately. We need to change the rules as follows:

Rule 1: If  $x$  is  $Z$ , then  $y$  is  $S$

Rule 2: If  $x$  is  $NS$ , then  $y$  is  $NM$

Rule 3: If  $x$  is  $NB$ , then  $y$  is  $NL$

Rule 4: If  $x$  is  $PS$ , then  $y$  is  $PM$

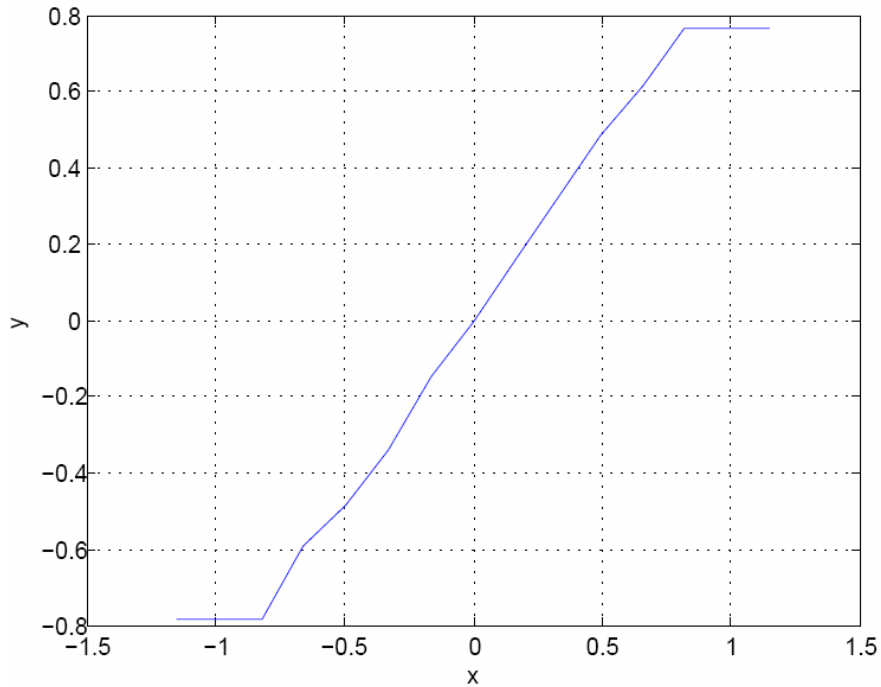
Rule 5: If  $x$  is  $PB$ , then  $y$  is  $PL$

We will use max-min composition method. As the function is symmetric, we need to analyze only one half of it.

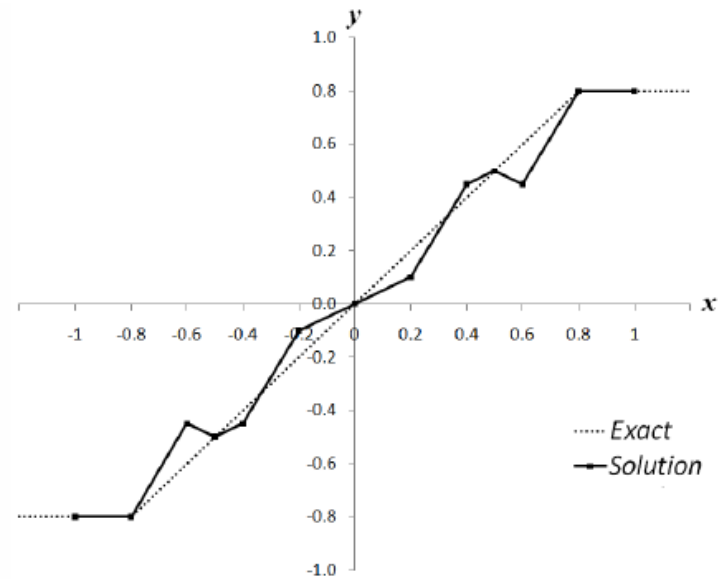
# Problem 3.3 (Hint)

Try getting something better than Kelly's Fuzzy on Left

Problem 8.7



Kelly's Fuzzy



Ross's Solution Manual



# Problem 3.4

When constructing a CMU wall, there is a direct correlation between the CMU block width (W), length (L) and wall strength (S). The following two rules apply:

Rule 1: IF W is small, and L is small, THEN S is small.

Rule 2: IF W is large and L is small, THEN S is medium.

Use symmetric triangles to construct the MFs. For the width W, use a triangle centered on the interval [0, 8] inches for Small, and use a triangle centered on the interval [4, 10] inches for Large. For the length L, use a triangle centered on the interval [0, 16] inches for Small. For the strength S, use a triangle centered on the interval [0, 4] ksi for Small, and use a triangle centered on the interval [1, 5] ksi for Medium.

Conduct a simulation for the inputs  $W = 6$  inches and  $L = 10$  inches.

We use max-product composition in this problem.

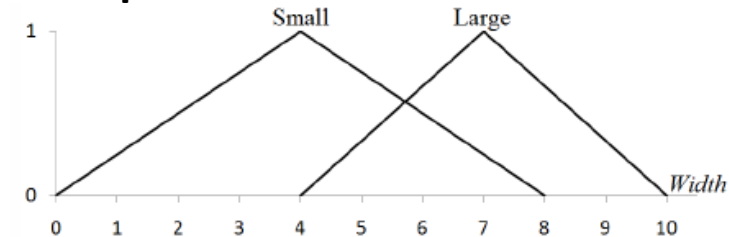
$W = 6$  &  $L = 10$  fires rules 1 & 2

Rule 1:  $W = \text{Small} = 0.5$  &  $L = \text{Small} = 0.75$  then  
 $\text{Strength} = \text{Small} = 0.5 * 0.75 * 0.375$  thus  
 $\text{Strength} = 2$

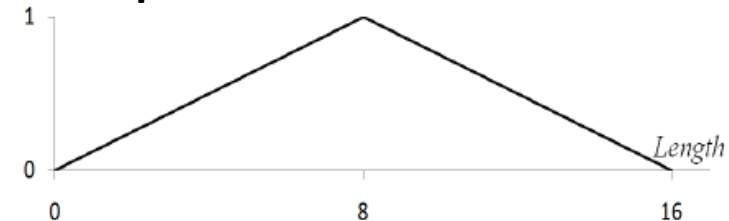
Rule 2:  $W = \text{Large} = 0.67$  &  $L = \text{Small} = 0.75$   
then  $\text{Strength} = \text{Medium} = 0.67 * 0.75 = 0.5025$   
thus  $\text{Strength} = 3$

Therefore  $\text{Strength} = \max(2, 3) = 3$

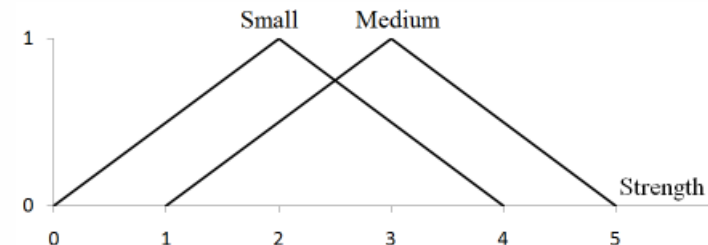
Input 'W'



Input 'L'



Output 'S'



# Problem 3.5

For the nonlinear function,  $y = x^2/2$  develop a fuzzy rule-based system using four simple rules to approximate the output  $y$ . To develop the system, partition the range of the input  $x$ ,  $[-4, 4]$  into five triangular MFs, and partition the output range of  $y$ ,  $[0, 8]$  into three triangular MFs. Use input labels, negative-big (NB), negative-small (NS), zero (Z), positive-small (PS), and positive-big (PB). Use output labels zero (Z), positive-small (PS), and positive-big (PB). Use the following rules for the simulation:

Rule 1: IF  $x$  is Z, THEN  $y$  is Z.

Rule 2: IF  $x$  is NB or PB, THEN  $y$  is PB.

Rule 3: IF  $x$  is NS or PS, THEN  $y$  is PS.

Conduct a simulation for the inputs:  $[-3, -1, 0, 1, 3]$ .

$x = -3$  then NB=0.5 fires R2, from NB:

$$y = \frac{5.3 \cdot 0.5 + 7 \cdot 1}{0.5 + 1} = 6.4$$

or NS=0.5 fires R3, from PS:  $y=4$

thus,  $y=6.4$

$x = -1$  then NS=0.5 fires R3, from PS:  $y=4$

or Z=0.5 fires R1, from Z:

$$y = \frac{1 \cdot 1 + 0.5 \cdot 2.7}{1 + 0.5} = 1.6$$

thus,  $y=1.6$

$x = 0$  then Z=1 fires R1, from Z:  $y=1.6$

$x = 1$  then PS=0.5 fires R3, from PS:  $y=4$

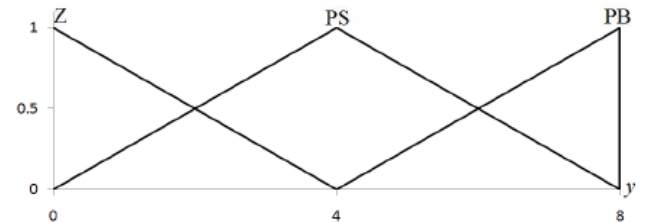
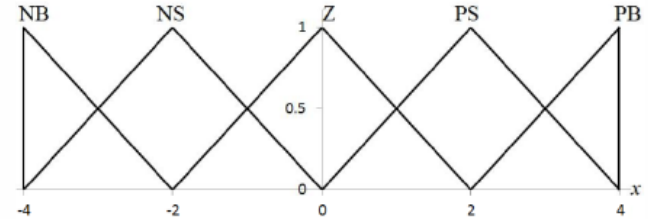
or Z=0.5 fires R1, from Z:  $y=1.6$

thus,  $y=4$

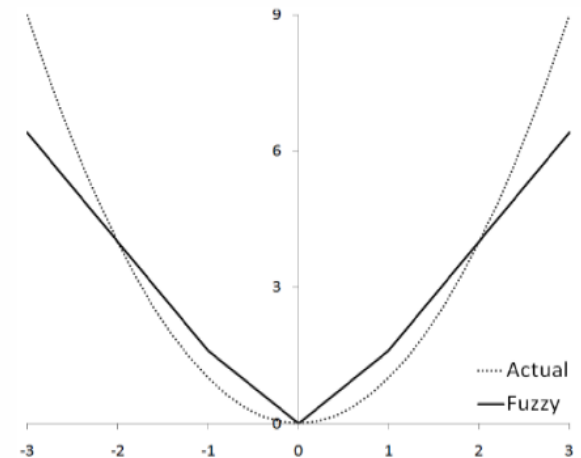
$x = 3$  then PB=0.5 fires R2, from PB:  $y=6.4$

or PS=0.5 fires R3, from PS:  $y=4$

thus,  $y=6.4$



Rule	x	y
1	Z	Z
2	NB or PB	PB
3	NS or PS	PS



## **HW#3 Format for Submission**

- 1. All work detailed neatly on a PowerPoint presentation (include programming files such as m files for all problems)**
- 2. Send Prof. Cohen your PPT + programming files via email (Kelly.Cohen@uc.edu)**
- 3. This assignment is INDEPENDENT and your work/ideas needs to be ORIGINAL!**
- 4. Make sure you include your name on the ppt file and on front page as well as on the programming files**