# Number Systems - 1

**Natural Numbers (N):** All the counting numbers are known as Natural Numbers.

**Whole Numbers:** 0, 1, 2, 3, 4, 5 ....

**Integers:** The collection of positive and negative natural numbers including zero.

.... -4, -3, -2, -1, 0, 1, 2, 3, 4 ....

**Rational Numbers:** The numbers which can be expressed in the form of $\frac{p}{q}$, where 'p' and 'q' are integers and $q \neq 0$.

**Equivalent Rational Numbers:** When any rational number is multiplied in both numerator and denominator by the same number, the resultant fraction is known as equivalent rational number.

eg: The equivalent rational numbers of $\frac{3}{7}$ are,

$$\frac{3\times2}{7\times2} = \frac{6}{14}$$

$$\frac{3\times4}{7\times4} = \frac{12}{28}$$

$$\frac{3\times5}{7\times5} = \frac{15}{35}$$

**Co-Primes:** If the HCF of any two numbers is 1, then those numbers are known as Co-Primes.

eg: $\frac{3}{7}, \frac{1}{3}, \frac{5}{7}, \frac{2}{3}, \frac{6}{7}$

## Ex: 1.1

1. Yes! 0 can be represented in the form of $\frac{p}{q}$ where $p, q$ are integers and $q \neq 0$.

eg: $\frac{0}{1}$ (or) $\frac{0}{2}$

2. <u>Sol:</u> $\frac{3\times7}{7}$ $\qquad$ $\frac{4\times7}{7}$

$= \frac{21}{7} \left[\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}, \frac{27}{7}\right] \frac{28}{7}$

3. $\frac{3\times6}{5\times6}$ $\qquad$ $\frac{4\times6}{5\times6}$

$\frac{18}{30} \left[\frac{19}{30}, \frac{20}{30}, \frac{21}{30}, \frac{22}{30}, \frac{23}{30}\right] \frac{24}{30}$

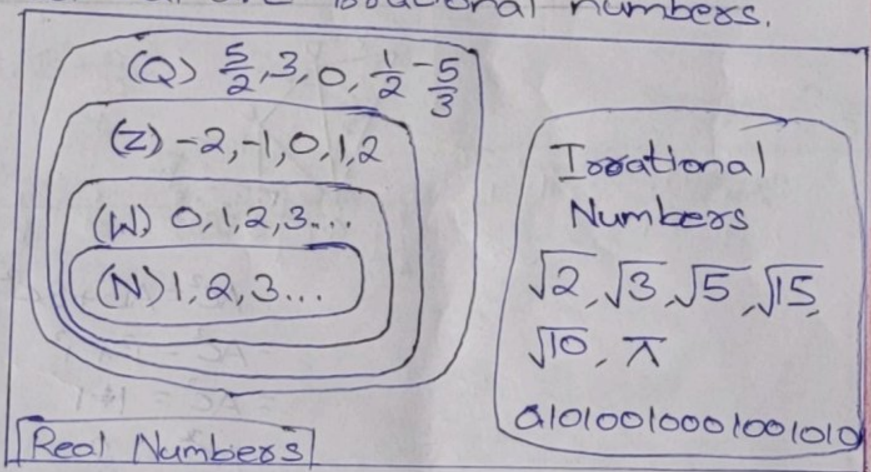
4. i) True
ii) False
iii) False

**Irrational Numbers**: The numbers which cannot be expressed in the form of $p/q$ where $p$ and $q$ are integers and $q \neq 0$.

eg. $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{7}, \pi$

· The decimal expansion of irrational numbers is non-terminating and non-recurring.

**Real Numbers**: Combination (collection) of both rational and irrational numbers.

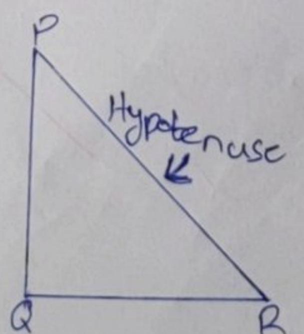



~~Pythagorous~~

**Pythagoras Theorem**: The square of hypotenuse of a right angled triangle is equal to the sum of squares of other two sides.

$\Rightarrow AC^2 = AB^2 + BC^2$

eg: Find PR, if $PQ = 2cm$
$QR = 1cm$



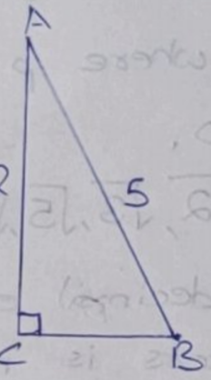

(i) $PR^2 = PQ^2 + QR^2$

$= PR^2 = 2^2 + 1^2$

$= PR^2 = 4 + 1$

$= PR^2 = 5 \Rightarrow PR = \sqrt{5}$

(ii) if $AB = 5cm$
$AC = 2cm$
$BC = ?$




$= 5^2 = 2^2 + BC^2$

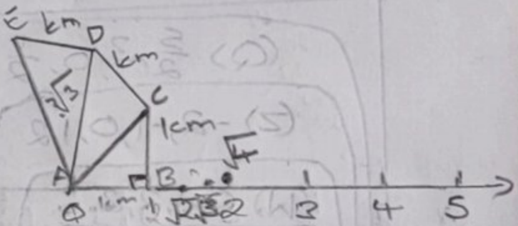
$= 25 = 4 + BC^2$

$= 25 - 4 = BC^2$

$= 21 = BC^2$

$= \sqrt{21} = BC$

## Irrational Numbers on Number Line:




$AC^2 = AB^2 + BC^2$
$= AC^2 = 1^2 + 1^2$
$= AC^2 = 1 + 1$
$= AC^2 = 2$
$\Rightarrow AC = \sqrt{2}$

In $\Delta ACD$

$AD^2 = AC^2 + CD^2$
$= AD^2 = (\sqrt{2})^2 + 1^2$
$= AD^2 = 2 + 1$
$= AD^2 = 3$
$= AD = \sqrt{3}$

In $\Delta ADE$

$AE^2 = AD^2 + DE^2$
$= (\sqrt{3})^2 + 1^2$
$= 3 + 1$
$AE^2 = 4$
$AE = \sqrt{4} = 2$

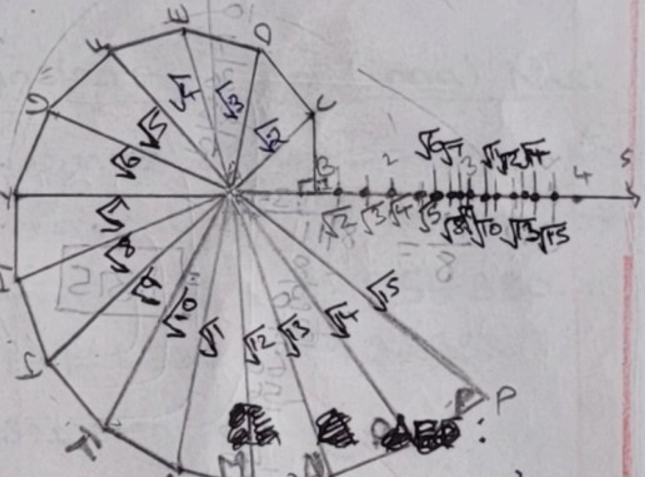
## Ex 1.2 :

1. (i) True
   (ii) False
   (iii) False

2. No, not all are irrational.
   Eg: $\sqrt{49}=7$, $\sqrt{4}=2$, $\sqrt{16}=4$, $\sqrt{81}=9$, $\sqrt{100}=10$
   $\sqrt{121}=11$

3. In $\Delta$ ABC:

$AC^2 = AB^2 + BC^2$
$= AC^2 = 1^2 + 1^2$
$= AC^2 = 1+1$
$= AC^2 = 2$
$= AC = \sqrt{2}$



In $\Delta$ ACD:

$AD^2 = AC^2 + CD^2$
$= AD^2 = (\sqrt{2})^2 + 1^2$
$= AD^2 = 2+1$
$= AD^2 = 3$
$= AD = \sqrt{3}$

In $\Delta$ ADE:

$AE^2 = AD^2 + DE^2$
$= AE^2 = (\sqrt{3})^2 + 1^2$
$= AE^2 = 3+1$
$= AE^2 = 4$
$= AE = \sqrt{4} = 2$

$AF^2 = AE^2 + EF^2$
$= AF^2 = (\sqrt{4})^2 + 1^2$
$= AF^2 = 4+1$
$= AF^2 = 5$
$= AF = \sqrt{5}$

## Real Numbers and Their Decimal Expansions:

(i) $\frac{10}{7} = 7\overline{)10} = \boxed{1.\overline{42857}}$

(ii) $\frac{7}{8} = $ 8 ) 7 (quotient 0.875) $= \boxed{0.875}$

$-0$
70
$-64$
60
$-56$
40
$-40$
0

(iii) $\frac{1}{7} = $

7 ) 1 (quotient 0.3̶4̶2857)
$-0$
10
$-7$
30
$-28$
2

7 ) 1 (quotient $0.\overline{142857}$)
$-0$
10
$-7$
30
$-28$
20
$-14$
60
$-56$
40
$-35$
50
$-49$
1

$= \boxed{0.\overline{142857}}$

(iv) $\frac{11}{8} = $ 8 ) 11 (quotient 1.375) $= \boxed{1.375}$

$-8$
30
$-24$
60
$-56$
40
$-40$
0

(v) $\frac{21}{8}$ 8 ) 21 (quotient 2.625) $=$

$-16$
50
$-48$
20
$-16$
40

$\therefore$ The rational numbers decimal expansions would be either terminating or non-terminating and recurring.

## P/Q form of a decimal number:

(i) $0.5 = \dfrac{\cancel{5}^{1}}{\cancel{10}_{2}} = \dfrac{1}{2}$

(ii) $0.75 = \dfrac{\cancel{75}^{3}}{\cancel{100}_{4}} = \dfrac{3}{4}$

(iii) $0.125 = \dfrac{\cancel{125}^{\cancel{5}\,1}}{\cancel{1000}_{\cancel{40}\,8}} = \dfrac{1}{8}$

(iv) $0.1375 = \dfrac{\cancel{1375}^{\cancel{55}\,11}}{\cancel{10000}_{\cancel{400}\,80}} = \dfrac{11}{80}$

(v) $1.125 = \dfrac{\cancel{1125}^{\cancel{45}\,9}}{\cancel{1000}_{\cancel{40}\,8}} = \dfrac{9}{8}$

## Decimal Expansion of Irrational Nos:

- The decimal expansion of irrational numbers will be non-terminating and non-recurring.

eg: $\sqrt{2} = 1.4142135623730954880\ldots$

$\pi = 3.14159265358979323 8\ldots$

[upto 1.23 trillion digits...]

## Ex: 1.3

1. (i) $\dfrac{36}{100} = 0.36$, Terminating

(ii) $\dfrac{1}{11} =$

$$\begin{array}{r|l} & 0.0\overline{9}\ldots \\ 11 & 1 \\ & -0 \\ & \overline{10} \\ & -0 \\ & \overline{100} \\ & -99 \\ & \overline{1} \end{array}$$

$= 0.090909\ldots = 0.\overline{09}$,

- Non-Terminating and Recurring

(iii) $4\dfrac{1}{8}$

$= \dfrac{33}{8}$

$$\begin{array}{r|l} & 4.125\ldots \\ 8 & 33 \\ & -32 \\ & \overline{10} \\ & -8 \\ & \overline{20} \\ & -16 \\ & \overline{40} \\ & -40 \end{array}$$

$=$ ~~4.125~~ $4.125$

~~Non-Terminating and Recurring~~

- Terminating

(vi) $\frac{329}{400} = 400\overline{)3290}$ with quotient $0.8225$

$$\begin{array}{r} 3290 \\ -3200 \\ \hline 900 \\ -800 \\ \hline 1000 \\ -800 \\ \hline 2000 \\ 2000 \\ \hline 0 \end{array}$$

$= 0.8225$

• Terminating

(iv) $\frac{3}{13} = 13\overline{)3}$ with quotient $0.230769$

$$\begin{array}{r} 3 \\ -0 \\ \hline 30 \\ -26 \\ \hline 40 \\ -39 \\ \hline 10 \\ -0 \\ \hline 100 \\ -91 \\ \hline 90 \\ -78 \\ \hline 120 \\ -117 \\ \hline 3 \end{array}$$

$= 0.\overline{230769}$

• Non-Terminating

• Recurring

(v) $\frac{2}{11} = 11\overline{)2}$ with quotient $0.18$

$$\begin{array}{r} 2 \\ -0 \\ \hline 20 \\ -11 \\ \hline 90 \\ -88 \\ \hline 2 \end{array}$$

$= 0.\overline{18}$

• Non-Terminating

• Recurring

2. Given, $\frac{1}{7} = 0.\overline{142857}$

Now $\frac{2}{7} = 2 \times \frac{1}{7}$

$= 2 \times 0.142857$

$= 0.\overline{285714}$

$\frac{3}{7} = 3 \times \frac{1}{7}$

$= 3 \times 0.142857$

$= 0.\overline{428571}$

$\frac{4}{7} = 4 \times \frac{1}{7}$

$= 4 \times 0.142857$

$= 0.\overline{571428}$

$\frac{5}{7} = 5 \times \frac{1}{7}$

$= 5 \times 0.142857$

$= 0.\overline{714285}$

$\frac{6}{7} = 6 \times \frac{1}{7}$

$= 6 \times 0.142857$

$= 0.857142$

3. (i) $0.\overline{6}$

Let $x = 0.\overline{6}$

$x = 0.6666\ldots$ — ①

Multiply with 10 [∵ 1 digit is repeating.]

=> $10 \times x = 10 \times 0.66666\ldots$

=> $10x = 6.6666$ — ②

=> Subtracting ① from ②

$$\begin{aligned} 10x &= 6.6666 \\ - \quad x &= 0.6666 \\ \hline 9x &= 6.0 = 6 \end{aligned}$$

$= x = \frac{\cancel{6}^{2}}{\cancel{9}_{3}} = \frac{2}{3}$

$\therefore 0.\overline{6} = \frac{2}{3}$

(ii) $0.4\bar{7}$

Let $x = 0.4\bar{7}$

$x = 0.477777\ldots$ — ①

Multiply with 10 [∵ one digit is repeating]

$\Rightarrow 10 \times x = 10 \times 0.47777\ldots$

$\Rightarrow 10x = 4.7777$ — ②

Subtract ① from ②

$$10x = 4.7777\ldots$$
$$x = 0.4777\ldots$$
$$\overline{9x = 4.3000 = 4.3}$$

$\Rightarrow x = \frac{4.3}{9} = \frac{43}{10 \times 9}$

$\Rightarrow x = \frac{43}{90}$

$\Rightarrow 0.4\bar{7} = \frac{43}{90}$

(iii) $0.\overline{001}$

Let $x = 0.\overline{001}$

$x = 0.001001001\ldots$ — ①

Multiply with 1000 [∵ ~~or~~ 3 digits are repeating]

$= 1000 \times x = 0.001001 \times 1000$

$\Rightarrow 1000x = 1.001$ — ②

Subtract ① from ②

$= \cancel{10x =}$ $1000x = 1.001$

$- \quad x = 0.\overline{001}$

$$\overline{999x = 1}$$

$= x = \frac{1}{999}$

$= 0.\overline{001} = \frac{1}{999}$

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HW Q.5

4. Given, 0.9999

Let $x = 0.9999\ldots$ — ①

Multiply with 10

$= 10 \times x = 10 \times 0.9999\ldots$

$= 10x = 9.9999$ — ②

Subtract ① from ②

$$
\begin{array}{rl}
= & 10x = 9.9999 \\
- & x = 0.9999 \\
\hline
 & 9x = 9.0
\end{array}
$$

$= 9x = 9$

$= x = \frac{\cancel{9}}{\cancel{9}}$

$= x = \frac{1}{1} = 1$ [($p/q$) form]

$\therefore 0.9999 = 1$

5. $\frac{1}{17} = $

$$
\begin{array}{r}
17 \overline{)\,1\,}\; 0.0588235294117647 \\
-0 \\
\hline
10 \\
-0 \\
\hline
100 \\
-85 \\
\hline
150 \\
-136 \\
\hline
140 \\
-136 \\
\hline
40 \\
-34 \\
\hline
60 \\
-51 \\
\hline
90 \\
-85 \\
\hline
50 \\
-34 \\
\hline
160 \\
-153 \\
\hline
70 \\
-68 \\
\hline
20 \\
-17 \\
\hline
30 \\
-17 \\
\hline
130 \\
-119 \\
\hline
11
\end{array}
\qquad
\begin{array}{r}
110 \\
-102 \\
\hline
80 \\
-68 \\
\hline
120 \\
-119 \\
\hline
1
\end{array}
$$

$\Rightarrow 0.0588235294117647$

$\therefore$ 16 digits are the maximum that can be in the ~~decimal expansion~~ repeating block of the decimal expansion of $\frac{1}{17}$

6. The q in the form of p/q to get terminating decimal expansion should be either multiple of 10 or product of 2 and 5.

eg: $\frac{5}{10} = 0.5$

$\frac{8}{100} = 0.08$

$\frac{12}{1000} = 0.012$

$\frac{17}{2\times5} = \frac{17}{10} = 1.7$

7. 1) $\sqrt{2} = 1.414213562\ldots$

2) $\pi = 3.1415926\ldots$

3) $0.001000101101\ldots$

8. ~~5~~

$$
\begin{array}{r}
0.\overline{714285} \\
7\,\overline{)\,5\phantom{0}} \\
-0\phantom{0} \\
\hline
50 \\
-49 \\
\hline
10 \\
-7 \\
\hline
30 \\
-28 \\
\hline
20 \\
-14 \\
\hline
60 \\
-56 \\
\hline
40 \\
-35 \\
\hline
5
\end{array}
\qquad
\begin{array}{r}
0.\overline{81} \\
11\,\overline{)\,9\phantom{0}} \\
-0\phantom{0} \\
\hline
90 \\
-88 \\
\hline
20 \\
-11 \\
\hline
9
\end{array}
$$

$= \frac{5}{7} = 0.\overline{714285}\ldots$

$= \frac{9}{11} = 0.\overline{81}\ldots$

$\therefore$ the 3 irrational nos. are:

(i) 0.753239

(ii) 0.7912345

(iii) 0.80010239

9. (i) $\sqrt{23}$ - irrational

(ii) $\sqrt{225}$ - rational [$\because \sqrt{225} = 15^2$]

(iii) 0.3796 - rational

(iv) 7.478478 - rational

(v) 1.101001000100001... - irrational

Operations on Real Numbers:

① Add $2\sqrt{2} + 5\sqrt{3}$ and $\sqrt{2} - 3\sqrt{3}$

$2\sqrt{2} + 5\sqrt{3} + \sqrt{2} - 3\sqrt{3}$

$= 3\sqrt{2} + 2\sqrt{3}$

② Multiply $6\sqrt{5}$ and $2\sqrt{5}$

$= 6\sqrt{5} \times 2\sqrt{5}$

$= 12 \times (\sqrt{5})^2$

$= 12 \times 5$

$= 60$

Identities Relating to Square Roots:

1. $\sqrt{ab} = \sqrt{a}\sqrt{b}$

2. $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

3. $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b}) = a-b$

4. $(a+\sqrt{b})(a-\sqrt{b}) = a^2-b$

5. $(\sqrt{a}+\sqrt{b})(\sqrt{c}+\sqrt{d}) = \sqrt{ac}+\sqrt{ad}+\sqrt{bc}+\sqrt{bd}$

6. $(\sqrt{a}+\sqrt{b})^2 = a+2\sqrt{ab}+b$

eg①:

$(5+\sqrt{7})(2+\sqrt{5})$

$5(2+\sqrt{5})+\sqrt{7}(2+\sqrt{5})$

$= 5\times2+5\times\sqrt{5}+\sqrt{7}\times2+\sqrt{7}\times\sqrt{5}$

$= 10+5\sqrt{5}+2\sqrt{7}+\sqrt{35}$

②: $(5+\sqrt{5})(5-\sqrt{5})$

$\because (a+b)(a-b) = a^2-b^2$

$= 5^2-(\sqrt{5})^2$

$= 5^2-5$

$= 25-5$

$= 20$

③ $(\sqrt{3}+\sqrt{7})^2$

$= \because (a+b)^2 = a^2+2ab+b^2$

$= (\sqrt{3})^2+2\times3\times7+(\sqrt{7})^2$

$= 3+42+7 = 52$

Ex 1.4
① ②

③ $(\sqrt{3}+\sqrt{7})^2$

$\because (a+b)^2 = a^2+2ab+b^2$

$= (\cancel{\sqrt{}}3)^{\cancel{2}} + 2\times\sqrt{3}\times\sqrt{7} + (\cancel{\sqrt{}}7)^{\cancel{2}}$

$= 3 + 2\sqrt{21} + 7$

$= 10 + 2\sqrt{21}$

④ $(\sqrt{11}-\sqrt{7})(\sqrt{11}+\sqrt{7})$

$\because (a-b)(a+b) = a^2-b^2$

$= (\cancel{\sqrt{}}11)^{\cancel{2}} - (\cancel{\sqrt{}}7)^{\cancel{2}}$

$= \cancel{11-7}$

$= 4$

## Ex : 1.4

1. (i) $2-\sqrt{5}$ = Irrational

(ii) $3+(\sqrt{23})-\sqrt{23}$

$= 3+\sqrt{\cancel{23}}-\sqrt{\cancel{23}}$

$= 3$ = Rational

(iii) $\frac{2\sqrt{7}}{7\sqrt{7}}$

$= \frac{2\times\cancel{\sqrt{7}}}{7\times\cancel{\sqrt{7}}}$

$= \frac{2}{7}$ = Rational

(iv) $\frac{1}{\sqrt{2}}$ = Irrational

(v) $2\pi = 2\times 3.14\ldots = 6.28\ldots$ = Irrational

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2. (i) $(3+\sqrt{3})(2+\sqrt{2})$

$= 3(2+\sqrt{2})+\sqrt{3}(2+\sqrt{2})$

$= 3\times 2+3\times\sqrt{2}+\sqrt{3}\times 2+\sqrt{3}\times\sqrt{2}$

$= 6+3\sqrt{2}+2\sqrt{3}+\sqrt{6}$

(ii) $(3+\sqrt{3})(3-\sqrt{3})$

$[\because (a+b)(a-b) = a^2-b^2]$:

$= 3^2-(\sqrt{3})^2$

$= 3^2-3$

$= 9-3$

$= 6$

(iii) $(\sqrt{5}+\sqrt{2})^2$

$[\because (a+b)^2 = a^2+2ab+b^2]$:

$= (\sqrt{5})^2+2\times\sqrt{5}\times\sqrt{2}+(\sqrt{2})^2$

$= 5+2\times\sqrt{10}+2$

$= 7+2\sqrt{10}$

(iv) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})$

$[\because (a-b)(a+b) = a^2-b^2]$:

$= (\sqrt{5})^2-(\sqrt{2})^2$

$= 5-2$

$= 3$

3. The measurement of circumference of a circle and diametery circle are not absolute. They are only approximate values. Hence it is mathematically proven that $\pi$ is an irrational number.

5. (i) $\frac{1}{\sqrt{7}}$

Rationalise:

$= \frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$

$= \frac{\sqrt{7}}{\sqrt{49}} = \frac{\sqrt{7}}{7}$

(ii) $\frac{1}{\sqrt{7}-\sqrt{6}}$

Rationalise:

$\frac{1}{\sqrt{7}-\sqrt{6}} \times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}} = \frac{\sqrt{7}+\sqrt{6}}{(\sqrt{7})^2-(\sqrt{6})^2}$ $[\because (a-b)(a+b) = a^2-b^2]$

$\frac{\sqrt{7}+\sqrt{6}}{7-6}$

$= \frac{\sqrt{7}+\sqrt{6}}{1} = \sqrt{7}+\sqrt{6}$

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}}$

Rationalise:

$= \frac{1}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}} = \frac{\sqrt{5}-\sqrt{2}}{5-2} = \frac{\sqrt{5}-\sqrt{2}}{3}$

$= \frac{\sqrt{5}-\sqrt{2}}{(\sqrt{5})^2-(\sqrt{2})^2}$

(iv) $\frac{1}{\sqrt{7}-2}$

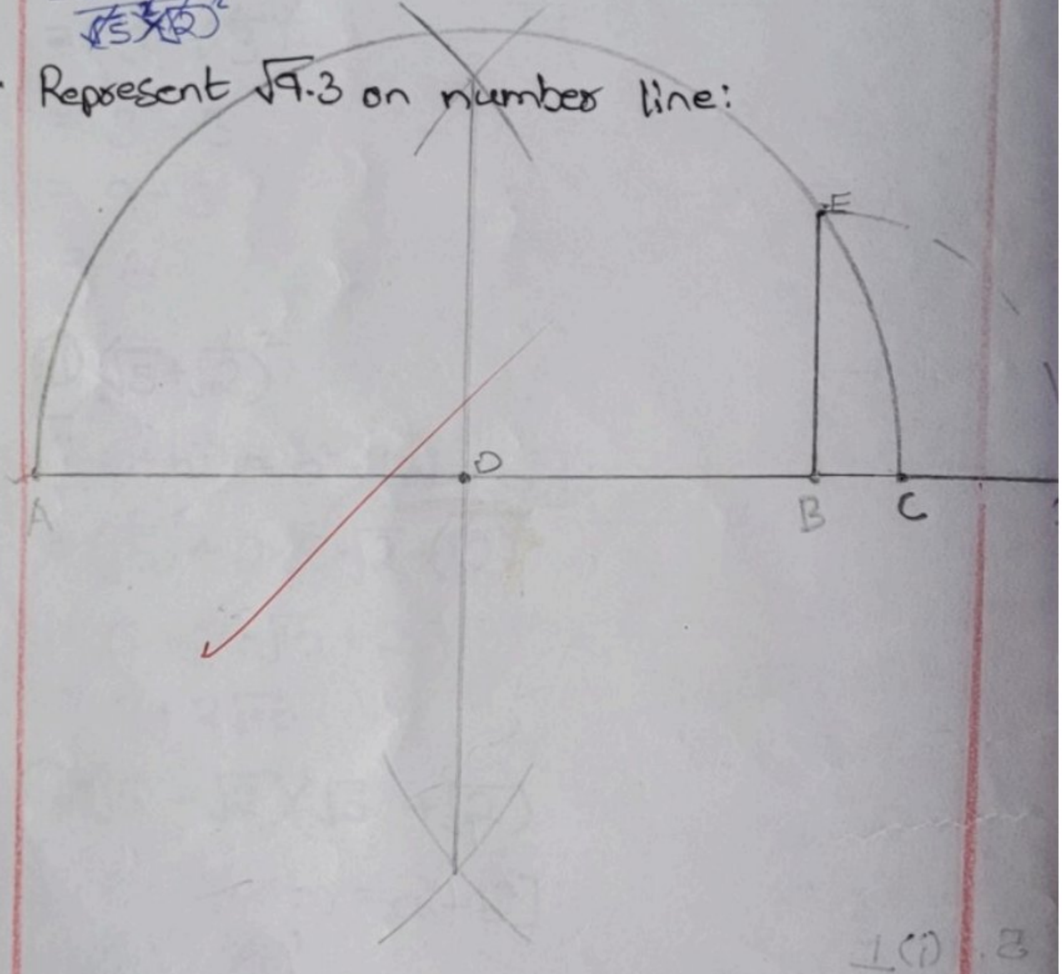
$= \frac{1\times\sqrt{7}+2}{\sqrt{7}-2\times\sqrt{7}+2}$

$= \frac{\sqrt{7}+2}{(\sqrt{7})^2-2^2}$

$= \frac{\sqrt{7}+2}{7-4}$

$= \frac{\sqrt{7}+2}{3}$

4. Represent $\sqrt{9.3}$ on number line:




Steps:

1. Draw a line segment $\overline{AB}$ of 9.3cm
2. Extend the line segment $\overline{AB}$ by 1cm and ~~nam~~ name as 'c'.
3. Draw the perpendicular bisectors from points A and B.
4. Draw a semicircle with radius $\overline{AD}$

Laws of Exponents :-

(i) $a^m \times a^n = a^{m+n}$

(ii) $\frac{a^m}{a^n} = a^{m-n}$

(iii) $(a^m)^n = a^{mn}$

(iv) $a^m \times b^m = (ab)^m$

(v) $\frac{1}{a^n} = a^{-n}$

(vi) $a^0 = 1$

(vii) $a^{-n} = \frac{1}{a^n}$

(viii) $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Ex : 1.5

1. Find

(i) $64^{\frac{1}{2}}$

$(8^2)^{\frac{1}{2}}$

$= 8^{\cancel{2} \times \frac{1}{\cancel{2}}}$

$= 8$

(ii) $32^{\frac{1}{5}}$

$= (2^5)^{\frac{1}{5}}$

$= 2^{\cancel{5} \times \frac{1}{\cancel{5}}}$

$= 2$

| | |
|---|---|
| 2 | 32 |
| 2 | 16 |
| 2 | 8 |
| 2 | 4 |
| 2 | 2 |
| | 1 |

(iii) ~~125~~ $125^{\frac{1}{3}}$

$= (5^3)^{\frac{1}{3}}$

$= 5^{\cancel{3} \times \frac{1}{\cancel{3}}}$

$= 5$

| | |
|---|---|
| 2 | 32 |
| 2 | 16 |
| 2 | 8 |
| 2 | 4 |
| 2 | 2 |
| | 1 |

2. Find:

(i) $9^{3/2}$

$= (3^2)^{3/2}$

$= 3^{\cancel{2}\times\frac{3}{\cancel{2}}}$

$= 3^3 = 3\times3\times3$

$= 27$

(ii) $32^{2/5}$

$= (2^5)^{2/5}$

$= 2^{\cancel{5}\times\frac{2}{\cancel{5}}}$

$= 2^2 = 2\times2$

$= 4$

(iii) $\cancel{32^{2/5}}$ $16^{3/4}$

$= (2^4)^{3/4}$

$= 2^{\cancel{4}\times\frac{3}{\cancel{4}}}$

$= 2^3$

$= 8$

| | |
|---|---|
| 2 | 16 |
| 2 | 8 |
| 2 | 4 |
| 2 | 2 |
| | 1 |

(iv) $\cancel{125^{-\frac{1}{3}}}$ $125^{-\frac{1}{3}}$

$= \cancel{\frac{1}{125}^{-\frac{1}{3}}}$

$= \cancel{\frac{1}{5^3}^{\frac{1}{3}}}$

$= \cancel{\frac{1}{1}}$

$= \frac{1}{(125)^{\frac{1}{3}}}$

$= \frac{1}{(5^3)^{\frac{1}{3}}}$

$= \frac{1}{5^{\cancel{3}\times\frac{1}{\cancel{3}}}}$

$= \frac{1}{5}$

3. Simplify:

(i) $2^{\frac{2}{3}} \times 2^{\frac{1}{5}}$

$= 2^{\frac{2}{3}+\frac{1}{5}}$

$= 2^{\frac{10+3}{15}}$

$= 2^{\frac{13}{15}}$

(ii) $\left(\frac{1}{3^3}\right)^7$

$= \frac{1^7}{(3^3)^7}$

$= \frac{1}{3^{21}}$

(iii) $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$

$= 11^{\frac{1}{2}-\frac{1}{4}}$

$= 11^{\frac{4-2}{8}}$

$= 11^{\frac{\cancel{2}^1}{\cancel{8}_4}}$

$= 11^{\frac{1}{4}}$

(iv) $7^{\frac{1}{2}} \times 8^{\frac{1}{2}}$

$= (7\times 8)^{\frac{1}{2}}$

$= 56^{\frac{1}{2}}$

Extra Questions:

1. Find the value of a and b if $\frac{\sqrt{3}-1}{\sqrt{3}+1} = a+b\sqrt{3}$

Given, $\frac{\sqrt{3}-1}{\sqrt{3}+1} = a+b\sqrt{3}$

Rationalising Factor $= \sqrt{3}-1$

$= \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$

$= \frac{(\sqrt{3}-1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)}$

$= \frac{(\sqrt{3})^2 - 2\times\sqrt{3}\times 1 + 1^2}{(\sqrt{3})^2-(1)^2}$

$= \frac{3-2\sqrt{3}+1}{3-1}$

$= \frac{4-2\sqrt{3}}{2}$

$= \frac{\cancel{2}(2-\sqrt{3})}{\cancel{2}}$

$= 2-\sqrt{3}$

$= 2+(-1)\sqrt{3} = a+b\sqrt{3}$

On comparing:

$a = 2$

$b = -1$

HW: (i) $\frac{3+\sqrt{2}}{3-\sqrt{2}} = a+b\sqrt{2}$

(ii) $\cancel{5+\sqrt{2}}$ $\frac{5+2\sqrt{3}}{7+4\sqrt{3}} = a+b\sqrt{3}$

(iii) $\frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = a-b\sqrt{6}$

Given,

(i) $\frac{3+\sqrt{2}}{3-\sqrt{2}} = a+b\sqrt{2}$

Rationalising Factor $= 3+\sqrt{2}$

$= \frac{3+\sqrt{2} \times 3+\sqrt{2}}{3-\sqrt{2} \times 3+\sqrt{2}}$

$= \frac{(3+\sqrt{2})^2}{(3-\sqrt{2})(3+\sqrt{2})}$

$= \cancel{3+\sqrt{2}}$ $\frac{3^2+2\times3\times\sqrt{2}+(\sqrt{2})^2}{3^2-(\sqrt{2})^2}$

$=$

$= \frac{9+6\sqrt{2}+2}{9-2}$

$= \frac{11+6\sqrt{2}}{7}$

$= \frac{11}{7} + \frac{6\sqrt{2}}{7}$

$= a+b\sqrt{2}$

On Comparing:

$a = \frac{11}{7}$, $b = \frac{6}{7}$

(ii) Given, $\frac{5+2\sqrt{3}}{7+4\sqrt{3}} = a+b\sqrt{3}$

$=$ Rationalising Factor $= 7-4\sqrt{3}$

$= \frac{(5+2\sqrt{3})\times 7-4\sqrt{3}}{7+4\sqrt{3}\times 7-4\sqrt{3}}$

$= \frac{5(7-4\sqrt{3})+2\sqrt{3}(7-4\sqrt{3})}{7^2-(4\sqrt{3})^2}$

$= \frac{35-20\sqrt{3}+14\sqrt{3}-8\sqrt{3}\sqrt{3}}{49-16\sqrt{3}\sqrt{3}}$

$= \frac{35-20\sqrt{3}+14\sqrt{3}-24}{49-48}$

$= \frac{11-6\sqrt{3}}{1}$

$= 11-6\sqrt{3}$

$= 11+(-6)\sqrt{3}$

$a+b\sqrt{3}$

On Comparing

$a=11$

$b=-6$

(iii) Given, $\frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = a-b\sqrt{6}$

$=$ Rationalising Factor $= 3\sqrt{2}+2\sqrt{3}$

$= \frac{\sqrt{2}+\sqrt{3}\times 3\sqrt{2}+2\sqrt{3}}{(3\sqrt{2}-2\sqrt{3})(3\sqrt{2}+2\sqrt{3})}$

$= \frac{\sqrt{2}(3\sqrt{2}+2\sqrt{3})+\sqrt{3}(3\sqrt{2}+2\sqrt{3})}{(3\sqrt{2})^2-(2\sqrt{3})^2}$

$$= \frac{3\sqrt{2}\sqrt{2} + 2\sqrt{6} + 3\sqrt{6} + 2\sqrt{3}\sqrt{3}}{(3\sqrt{2})^2 - (2\sqrt{3})^2}$$

$$= \frac{6 + 2\sqrt{6} + 3\sqrt{6} + 6}{3^2 \times (\sqrt{2})^2 - 2^2 \times (\sqrt{3})^2}$$

$$= \frac{6 + 2\sqrt{6} + 3\sqrt{6} + 6}{9 \times 2 - 4 \times 3}$$

$$\cancel{= 12 + 2\sqrt{6} + 3\sqrt{6}}$$

$$= \frac{12 + 5\sqrt{6}}{18 - 12}$$

$$= \frac{12 + 5\sqrt{6}}{6}$$

$$= \frac{\overset{2}{\cancel{12}}}{\cancel{6}} + \frac{5\sqrt{6}}{6}$$

$$= 2 - \left(-\frac{5\sqrt{6}}{6}\right)$$

$$a - b\sqrt{6}$$

On Comparing:

$a = 2$

$b = -\frac{5}{6}$

Nawnh
16/4/24