Simulation of Light Transmission and Reflection at a Glass Plate Using the 1D-Yee Algorithm

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Contents

1 Introduction

This project investigates how electromagnetic waves interact with dielectric media using the 1D-Yee algorithm. The focus is on understanding transmission and reflection at a glass interface. In the first setup, a thin glass plate is introduced to observe the effect of violating the Courant condition. The second configuration replaces the plate with a semi-infinite medium to calculate the reflection coefficient. In both cases, a wave packet originates from the left, and absorbing boundary conditions ensure reflectionless edges.

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2 Simulation Model and Method

We simulate a 1D system with a thin glass plate (green in Fig. 1), bounded by insulators (grey).

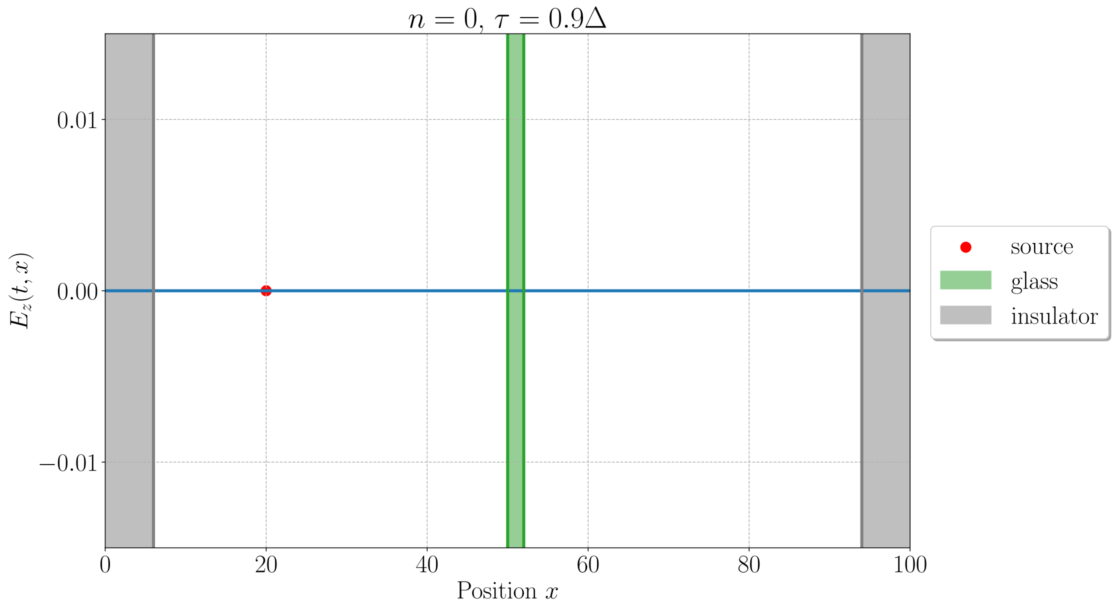

Figure 1: sketch of the overall system. The glass plate is depicted in green in the middle, and at the system's boundaries there are two insulators (grey). The source at x_s is represented by the red point.

A wave packet is generated by a source at x_s (red dot), modeled as

$$J_{source,z}(x,t) = \begin{cases} \sin(\omega t) e^{-((t-30)/10)^2} & \text{if } x = x_s \\ 0 & \text{otherwise} \end{cases} . \tag{1}$$

We impose boundary conditions $E_z(0,t) = E_z(X,t) = 0$, where X is the domain length. The plate has permittivity ϵ, permeability μ, and refractive index n_d, while the surrounding medium is vacuum ($n = 1$).

Assuming a linear, isotropic, nondispersive, and lossless medium with no free charges, Maxwell's equations reduce to:

$$\begin{aligned} \frac{\partial H_y}{\partial t} &= \frac{1}{\mu(x)} \left(\frac{\partial E_z}{\partial x} - \sigma^*(x) H_y \right) \\ \frac{\partial E_z}{\partial t} &= \frac{1}{\epsilon(x)} \left(\frac{\partial H_y}{\partial x} - J_{source,z}(x,t) - \sigma(x) E_z \right) \end{aligned}$$

We use the Yee algorithm with a staggered grid: E_z at $x = l\Delta$ ($L+1$ points), H_y at $x = (l+1/2)\Delta$ (L points). Update equations:

$$\begin{aligned} H_y\Big|_{l+1/2}^{n+1} &= A_{l+1/2} H_y\Big|_{l+1/2}^{n} + B_{l+1/2} \frac{E_z\Big|_{l+1}^{n+1/2} - E_z\Big|_{l}^{n+1/2}}{\Delta} \\ E_z\Big|_{l}^{n+1/2} &= C_l E_z\Big|_{l}^{n-1/2} + D_l \left[\frac{H_y\Big|_{l+1/2}^{n} - H_y\Big|_{l-1/2}^{n}}{\Delta} - \delta_{l,i_s} J_{source}(n\tau) \right] \end{aligned} \tag{2}$$

with coefficients:

$$A_{l+1/2} = \frac{1 - \frac{\sigma^* \tau}{2\mu}}{1 + \frac{\sigma^* \tau}{2\mu}}, \qquad B_{l+1/2} = \frac{\frac{\tau}{\mu}}{1 + \frac{\sigma^* \tau}{2\mu}}$$

$$C_l = \frac{1 - \frac{\sigma \tau}{2\epsilon}}{1 + \frac{\sigma \tau}{2\epsilon}}, \qquad D_l = \frac{\frac{\tau}{\epsilon}}{1 + \frac{\sigma \tau}{2\epsilon}}$$

The implementation in code:

$$\begin{aligned}
\texttt{E[1:-1]} &= \frac{\texttt{D}_\texttt{l}\texttt{[1:-1]*(H[1:]-H[:-1])}}{\Delta} + \texttt{C}_\texttt{l}\texttt{[1:-1]*E[1:-1]} \\
\texttt{E[i}_\texttt{s}\texttt{]} &- = \texttt{D}_\texttt{l}\texttt{[i}_\texttt{s}\texttt{]*J}_\texttt{source}\texttt{(n,}\tau\texttt{)} \\
\texttt{H} &= \frac{\texttt{B}_\texttt{l}\texttt{[:-1]*(E[1:]-E[:-1])}}{\Delta} + \texttt{A}_\texttt{l}\texttt{[:-1]*H}.
\end{aligned}$$

Only current and previous steps are stored to minimize memory use.

Reflection Coefficient

The reflection coefficient is defined as:

$$R := \frac{|E^{\max}_{\text{reflected}}|^2}{|E^{\max}_{\text{incident}}|^2} \tag{3}$$

We identify maxima in the intervals $t_{\text{in}} = [1700\tau, 2000\tau]$ and $t_{\text{ref}} = [4700\tau, 4950\tau]$ using `np.max`, focusing on spatial indices $l \geq 1000$ to avoid transmitted components.

3 Results

In this section, we present the results for two different systems.

3.1 The Thin Glass Plate

The first system was already discussed in the previous section and fig.(1), containing two insulators at the boundaries and a thin glass plate in the center. Mathematically, the insulators are described by the electrical conductivity σ and the magnetic loss σ^*

$$\sigma(x) = \sigma^*(x) = \begin{cases} 1 & \text{if} \quad 0 \leq x \leq 6\lambda \\ 0 & \text{if} \quad 6\lambda < x < L\Delta - 6\lambda \\ 1 & \text{if} \quad L\Delta - 6\lambda \leq x \leq L\Delta. \end{cases} \tag{4}$$

Further, the glass plate is described by the electric permittivity

$$\epsilon(x) = \begin{cases} 1 & \text{if} \quad 0 \leq x < L\Delta/2 \\ n_d^2 & \text{if} \quad L\Delta/2 \leq x < L\Delta/2 + 2\lambda \\ 1 & \text{if} \quad L\Delta/2 + 2\lambda \leq x \leq L\Delta, \end{cases} \tag{5}$$

where n_d is the refractive index of the glass with $n_d = 1.46$.[1] Moreover, the system has a constant magnetic permeability

$$\mu(x) = 1. \tag{6}$$

In fig.(2), the results for the different times (a) $n = 2500$ ($t \approx 45$), (b) $n = 3500$ ($t \approx 63$), (c) $n = 3510$ ($t \approx 63$), (d) $n = 4500$ ($t \approx 81$), and (e) $n = 20000$ ($t \approx 360$), and $\tau = 1.05\Delta = 0.021$ for (f) $n = 500$ ($t \approx 9$) are presented and the parameters of the system are $\lambda = 1$, $\Delta = \lambda/50 = 0.02$, $\tau = 0.9\Delta = 0.018$, $X = L\Delta = 100\lambda = 100$, and $L = 5000$.

[1] In the code, both, the σ and ϵ can simply be implemented via `if`-statements

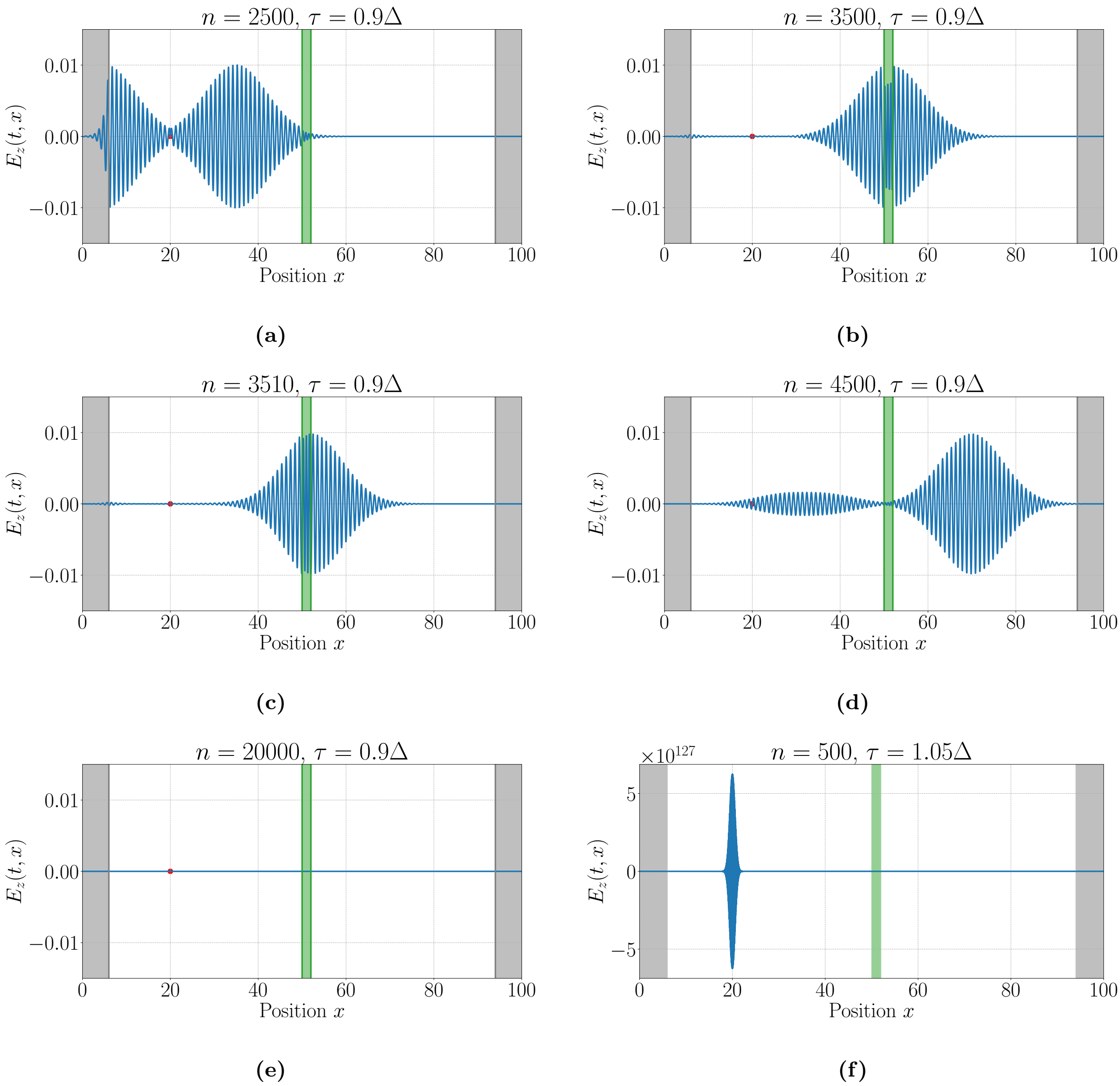

Figure 2: Numerical solutions of the Maxwell equations using the Yee algorithm in one dimension for the system with the thin glass plate. The figures show the z-component of the electric field. The plots contain the glass plate in green and the insulator insulators in grey. The discretizations are $\lambda = 1$, $\Delta = \lambda/50 = 0.02$, $\tau = 0.9\Delta = 0.018$, $X = L\Delta = 100\lambda = 100$, and $L = 5000$. The plots show the solution of the algorithm at different times: (a) $n = 2500$ ($t \approx 45$), (b) $n = 3500$ ($t \approx 63$), (c) $n = 3510$ ($t \approx 63$), (d) $n = 4500$ ($t \approx 81$), and (e) $n = 20000$ ($t \approx 360$), and $\tau = 1.05\Delta = 0.021$ for (f) $n = 500$ ($t \approx 9$).

3.2 The Thick Glass Plate

The second system that we study is very similar. The electrical conductivity σ, the magnetic loss σ^*, and the magnetic permeability μ are the same as before. However, now, we consider a thick glass plate that extends from the middle up to the end of the simulated spatial interval. This is described by the electric permittivity[2]

$$\epsilon(x) = \begin{cases} 1 & \text{if} \quad 0 \leq x < L\Delta/2 \\ n_d^2 & \text{if} \quad L\Delta/2 \leq x < L\Delta, \end{cases} \tag{7}$$

where we have the same refractive index $n_d = 1.46$. This system, at its initial time $t = 0$, can be seen in fig.(3a). The obtained simulation results are presented in fig.(3) for the times t and time discretisations τ.

Furthermore, according to eq.(3) and the described method, we determined the following result for the reflection coefficient R

$$R = 0.03537 \tag{8}$$

[2] In the code, this can simply be implemented via `if`-statements

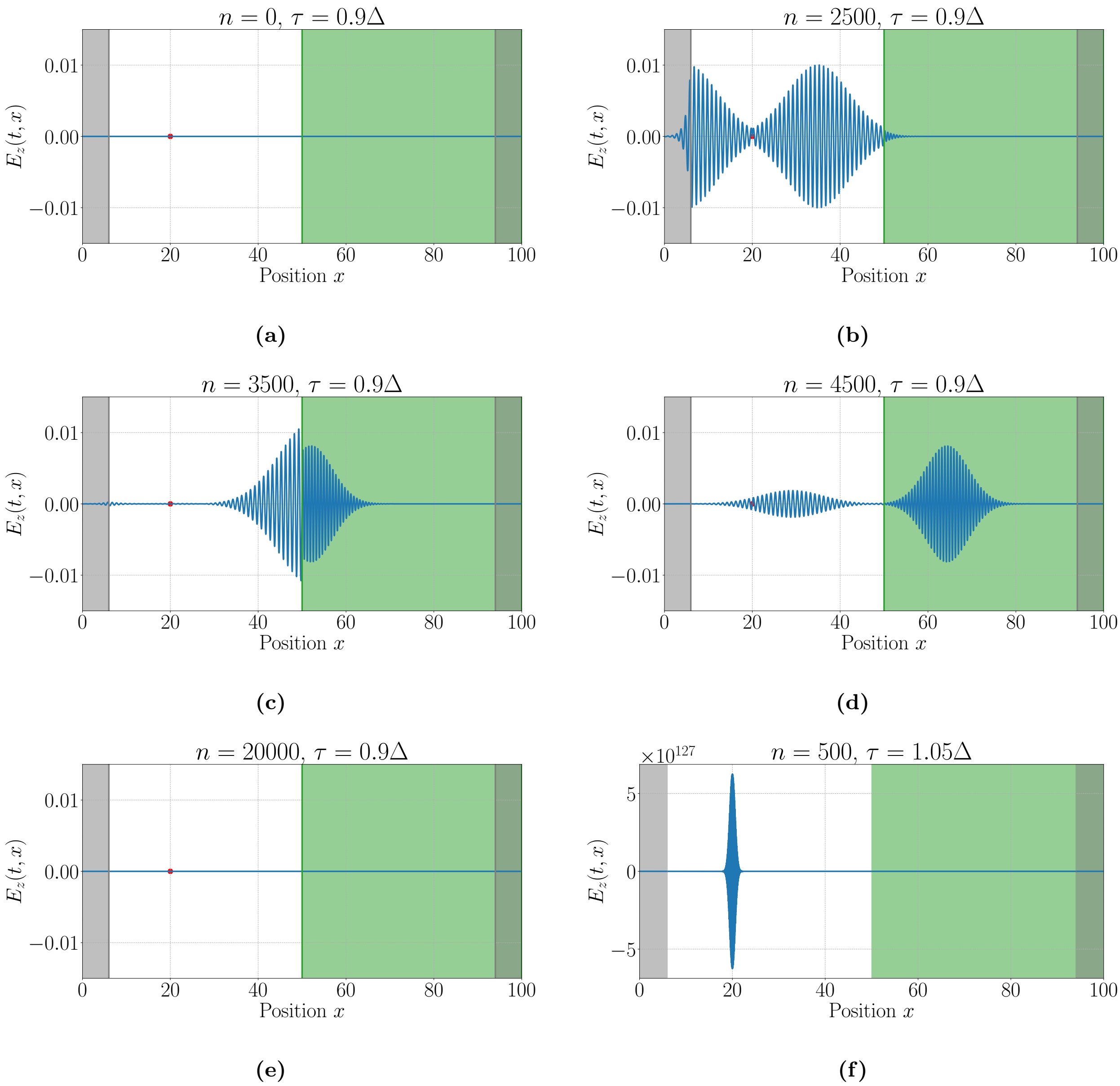

Figure 3: Numerical solutions of the Maxwell equations using the Yee algorithm in one dimension for the system with the thick glass plate. The figures show the z-component of the electric field. The plots contain the glass plate in green and the insulator insulators in grey. The discretizations are $\lambda = 1$, $\Delta = \lambda/50 = 0.02$, $X = L\Delta = 100\lambda = 100$, and $L = 5000$. The plots show the solution of the algorithm at different times and time discretizations: (a) $n = 0$ ($t \approx 0$) (b) $n = 2500$ ($t \approx 45$), (c) $n = 3500$ ($t \approx 63$), (d) $n = 4500$ ($t \approx 81$), and (e) $n = 20000$ ($t \approx 360$), and $\tau = 1.05\Delta = 0.021$ for (f) $n = 500$ ($t \approx 9$).

4 Discussion

First, in fig.(2), we present the results for the system with a thin glass plate at five different times, with the initial value $n = 0$ in fig.(1). We first discuss the results for $\tau = 0.9\Delta$ in figs. ((a)-(e)), followed by the results for $\tau = 1.05\Delta$ in fig. (f). In fig.(2a), we observe that the source at x_s generates two wavepackets in opposite directions. The left-moving wave is mostly absorbed by the insulator (grey regions), while the right-moving wave passes through the glass plate. In fig.(2b), we observe that the amplitude inside the glass is smaller, which results from interference with reflected waves. In fig.(2c), we see that at $n = 3510$, the amplitude inside the glass matches outside, confirming the previous observation. At $n = 4500$, three wavepackets are visible: one transmitted wave and two reflected waves from the glass sides. At $n = 20000$, all waves are absorbed at the boundaries. These results match our physical expectations, validating the choice of $\tau = 0.9\Delta$.

For $\tau = 1.05\Delta$, shown in fig.(2f), the amplitude diverges, indicating numerical instability when the Courant condition is violated, as seen with an amplitude of 10^{127} at $n = 500$.

Next, we present results for the system with a thick glass plate in fig.(3). The behavior at $\tau = 0.9\Delta$ is similar to the thin glass case. At $n = 3500$, the wave transmitted into the glass has a smaller amplitude, while interference causes a larger amplitude outside the glass. The reflected and transmitted waves are clearer in fig.(3d) at $n = 4500$, and all waves are absorbed at $n = 20000$.

For $\tau = 1.05\Delta$, the results are similar to the thin glass case, with an amplitude of order 10^{127}, confirming numerical instability.

Finally, the reflection coefficient R was calculated and compared to the theoretical value. We find:

$$|R_{theory} - R| = 4 \cdot 10^{-4}.$$

This small difference, within numerical uncertainties, confirms the accuracy of our simulation, and we conclude that the Yee-algorithm provides reasonable physical results.