Министерство образования и науки Российской Федерации
Федеральное государственное автономное образовательное учреждение высшего образования

САНКТ-ПЕТЕРБУРГСКИЙ НАЦИОНАЛЬНЫЙ ИССЛЕДОВАТЕЛЬСКИЙ УНИВЕРСИТЕТ ИНФОРМАЦИОННЫХ ТЕХНОЛОГИЙ, МЕХАНИКИ И ОПТИКИ
Факультет систем управления и робототехники

Отчет по лабораторной работе №4
«LQR, LQE и LQG»
по дисциплине «Теория автоматического управления»

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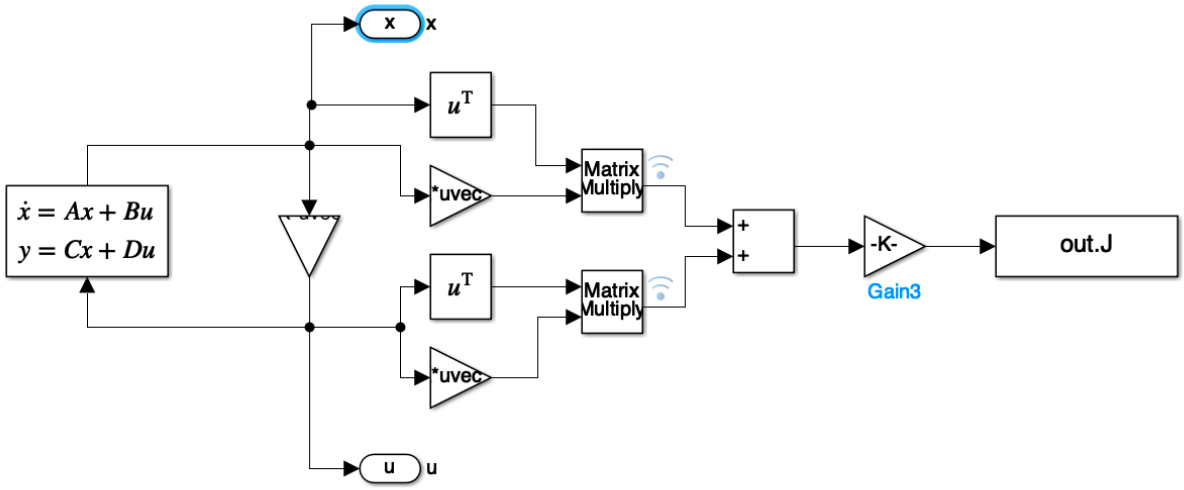
Санкт-Петербург 2022

1. **Цель работы:**
   Синтез и анализ регуляторов LQR, LQE и LQG

2. **Выполнение работы:**
   2.1. Исследование LQR

№5	$A = \begin{bmatrix} -6 & 19 & -13 & 10 \\ 0 & -9 & 6 & 0 \\ 0 & -15 & 9 & 0 \\ -4 & 8 & -7 & 6 \end{bmatrix}$	$B = \begin{bmatrix} -2 & 0 \\ 2 & 0 \\ 4 & 0 \\ 0 & 0 \end{bmatrix}$	$C = \begin{bmatrix} 1 & -3 & 2 & -1 \\ 0 & 6 & -3 & 0 \end{bmatrix}$	$D = \begin{bmatrix} 0 & 0 \\ 0 & 4 \end{bmatrix}$

Схема моделирования:



Программный код:

Init data

```
A = [ -6,  19, -13, 10;
       0,  -9,   6,  0;
       0, -15,   9,  0;
      -4,   8,  -7,  6];


B = [ -2, 0;
       2, 0;
       4, 0;
       0, 0];


x0 = [3; 3; 3; 3];
```

Собственные значение A

```
eig(A)
```

```
ans = 4×1 complex
```

```
  -0.0000 + 2.0000i
  -0.0000 - 2.0000i
  -0.0000 + 3.0000i
  -0.0000 - 3.0000i
```

Управляемость

```
U = [B A*B A*A*B A*A*A*B];
rank(U)
```

```
ans = 4
```

Система управляема

Soft controler

```
Q1 = [1/3, 0, 0, 0;
      0, 1/3, 0, 0;
      0, 0, 1/3, 0;
      0, 0, 0, 1/3];


R1 = [135, 0;
      0, 78];
```

Наблюдаемость

```
V = [Q1; Q1*A; Q1*A*A; Q1*A*A*A];
rank(V)
```

```
ans = 4
```

```
[K1, P1, e1] = lqr(A, B, Q1, R1)
```

```
K1 = 2×4
    0.1469   -0.6539    0.5385   -0.2773
   -0.0000    0.0000   -0.0000    0.0000
P1 = 4×4
   17.2056  -56.8769   41.9986  -26.1308
  -56.8769  237.5090 -169.2624   88.5004
   41.9986 -169.2624  123.8039  -66.6751
  -26.1308   88.5004  -66.6751   44.3887
e1 = 4×1 complex
  -0.1292 + 1.9946i
  -0.1292 - 1.9946i
  -0.1469 + 3.0051i
  -0.1469 - 3.0051i
```

Минимум J

```
min_J1 = x0'*P1*x0
```

```
min_J1 = 414.1329
```

J модели

```
sum(out.J)
```

```
ans = 408.1080
```

Medium controler

```
Q2 = [3, 0, 0, 0;
      0, 3, 0, 0;
      0, 0, 3, 0;
      0, 0, 0, 3];
```

```
R2 = [3, 0;
      0, 3];
```

Наблюдаемость

```
V = [Q2; Q2*A; Q2*A*A; Q2*A*A*A];
rank(V)
```

```
ans = 4
```

Синтез LQR

```
[K2, P2, e2] = lqr(A, B, Q2, R2)
```

```
K2 = 2×4
    3.2598   -7.7697    7.3175   -5.8081
         0         0         0         0
P2 = 4×4
   22.4459  -40.2440   33.7898  -37.2786
  -40.2440   91.7520  -71.8253   75.2596
   33.7898  -71.8253   58.2956  -60.6251
  -37.2786   75.2596  -60.6251   70.3144
e2 = 4×1 complex
  -0.7286 + 2.4083i
  -0.7286 - 2.4083i
  -2.8769 + 2.0019i
  -2.8769 - 2.0019i
```

Минимум J

```
min_J2 = x0'*P2*x0
```

```
min_J2 = 368.6460
```

J модели

```
sum(out.J)
```

```
ans = 371.1802
```

Hard controler

```
Q3 = [12, 0, 0, 0;
      0, 5, 0, 0;
      0, 0, 5, 0;
      0, 0, 0, 12];


R3 = [1/10, 0;
      0, 1/10];
```

Наблюдаемость

```
V = [Q3; Q3*A; Q3*A*A; Q3*A*A*A];
rank(V)
```

```
ans = 4
```

Синтез LQR

```
[K3, P3, e3] = lqr(A, B, Q3, R3)
```

```
K3 = 2×4
   25.7086  -52.1630   49.4870  -54.5244
         0         0         0         0
P3 = 4×4
   46.2026  -76.0677   61.7779  -76.0656
  -76.0677  164.5241 -121.6000  154.4441
```

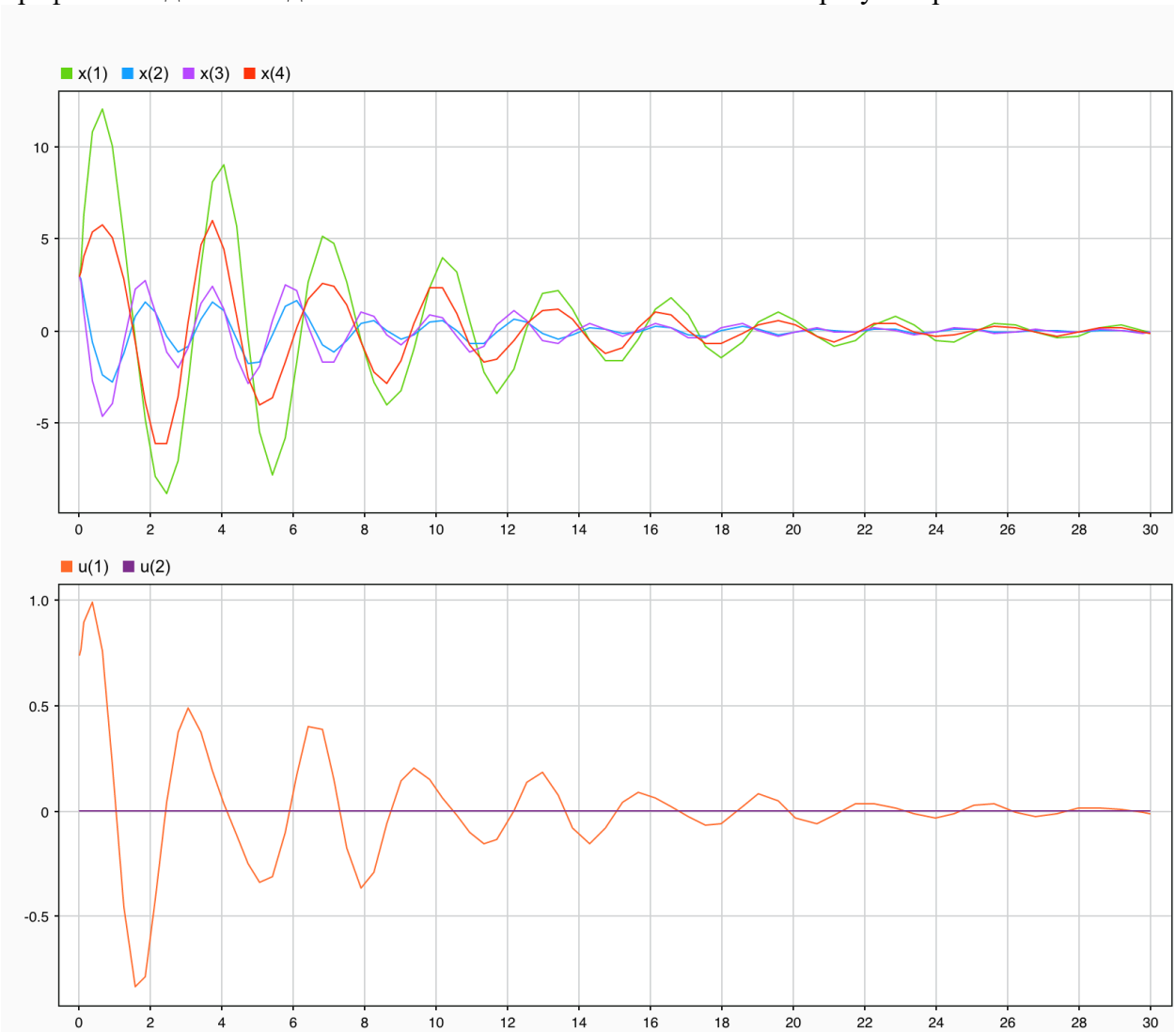
```
   61.7779 -121.6000   92.9261 -116.6180
  -76.0656  154.4441 -116.6180  150.5503
e3 = 4×1 complex
  -0.7959 + 2.6533i
  -0.7959 - 2.6533i
  -2.3887 + 0.0000i
 -38.2244 + 0.0000i
```

```
min_J3 = x0'*P3*x0
min_J3 = 953.5002
```

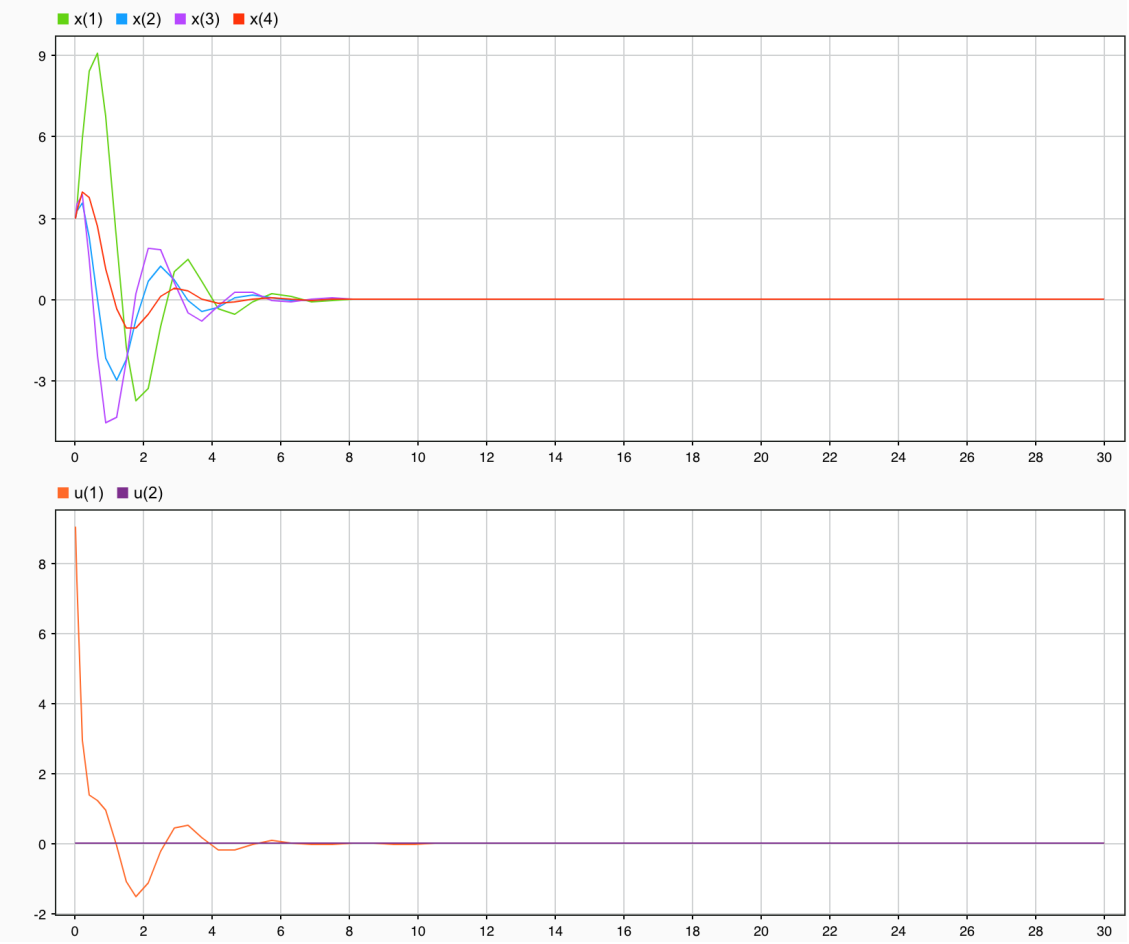
J модели

```
sum(out.J)
ans = 978.8383
```

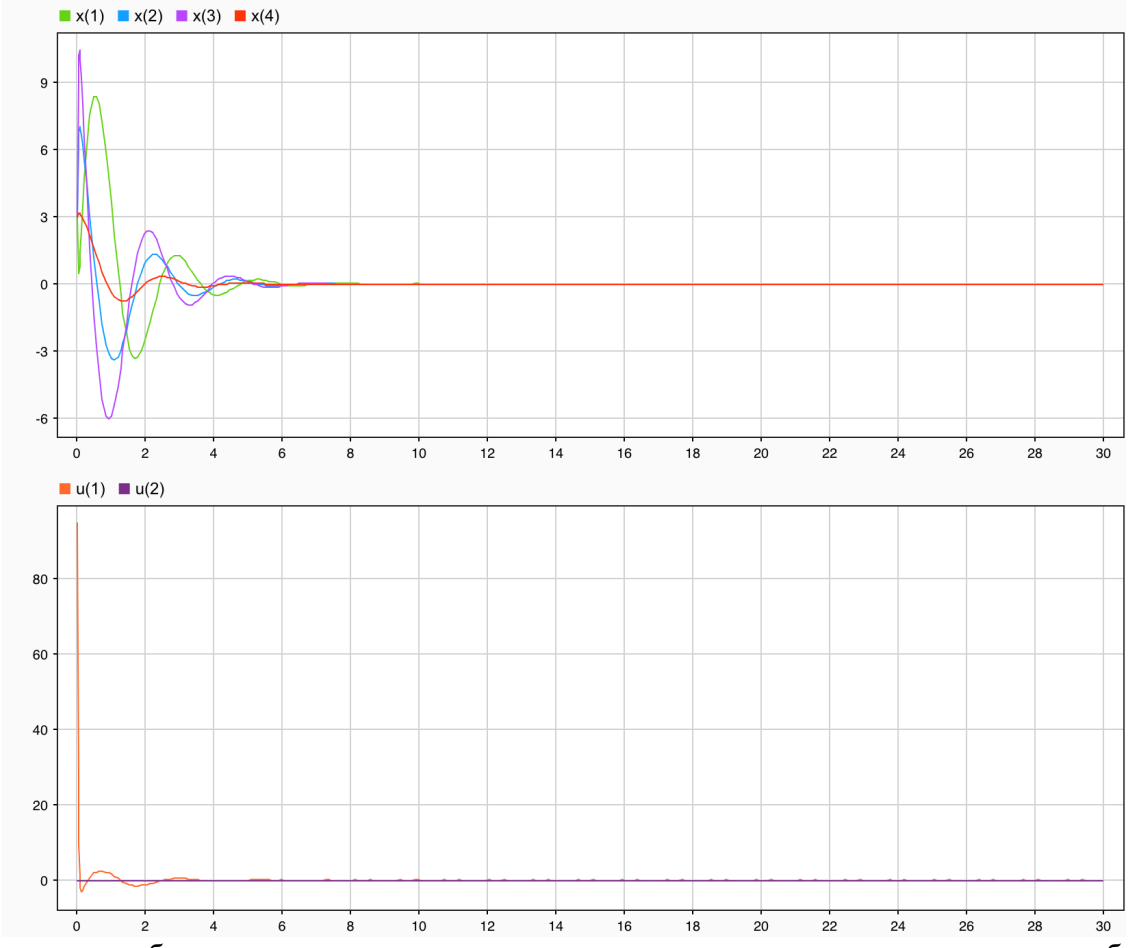
Графики входного воздействия и компонент системы с мягким регулятором



Графики входного воздействия и компонент системы со средним регулятором



Графики входного воздействия и компонент системы со жестким регулятором



В зависимость от выборного регулятора меняется управление: наименьшие у мягкого, наибольшее у жесткого, также изменяется время переходного процесса: наибольшее у мягкого, наименьшие у

жесткого. Соответственно отношение Q к R задает затраты на управление и время переходного процесса.

2.2. Сравнение LQR с не-LRQ

Программный код:

Init data

```
A = [ -6,  19, -13, 10;
       0,  -9,   6,  0;
       0, -15,   9,  0;
      -4,   8,  -7,  6];


B = [ -2, 0;
       2, 0;
       4, 0;
       0, 0];


x0 = [3; 3; 3; 3];
```

Medium controler

```
Q2 = [3, 0, 0, 0;
      0, 3, 0, 0;
      0, 0, 3, 0;
      0, 0, 0, 3];


R2 = [3, 0;
      0, 3];
```

Наблюдаемость

```
V = [Q2; Q2*A; Q2*A*A; Q2*A*A*A];
rank(V)
```

```
ans = 4
```

Синтез LQR

```
[K2, P2, e2] = lqr(A, B, Q2, R2)
```

```
K2 = 2×4
    3.2598   -7.7697    7.3175   -5.8081
         0         0         0         0
P2 = 4×4
   22.4459  -40.2440   33.7898  -37.2786
  -40.2440   91.7520  -71.8253   75.2596
   33.7898  -71.8253   58.2956  -60.6251
  -37.2786   75.2596  -60.6251   70.3144
e2 = 4×1 complex
  -0.7286 + 2.4083i
  -0.7286 - 2.4083i
  -2.8769 + 2.0019i
  -2.8769 - 2.0019i
```

Минимум J

```
min_J2 = x0'*P2*x0
```

```
min_J2 = 368.6460
```

Modal regulator

```
G = [-1, 1, 0, 0;
       0,-1, 0, 0;
       0, 0,-2, 1;
```

```
       0, 0, 0,-2];


Y = [1, 1, 1, 1;
     1, 1, 1, 1];


P_m
```

```
P_m = 4×4
    0.6000    1.3600    0.0385    0.3964
   -0.4000   -0.6800   -0.1538   -0.3550
   -0.2000   -0.6400    0.1538   -0.1065
    0.6000    1.0000    0.3077    0.4985
```

Синтез регулятора

```
K_m = -Y*inv(P_m)
```

```
K_m = 2×4
   -0.4000    2.0667   -2.7333   -0.8000
   -0.4000    2.0667   -2.7333   -0.8000
```

```
eig(A+B*K_m)
```

```
ans = 4×1 complex
  -2.0000 + 0.0000i
  -2.0000 + 0.0000i
  -1.0000 + 0.0000i
  -1.0000 - 0.0000i
```

LMI regulator

```
a = 1;
Y_lmi
```

```
Y_lmi = 2×4
    4.2473    4.9577  -18.6653  -18.3194
         0         0         0         0
```

```
P_lmi
```

```
P_lmi = 4×4
   16.8932   -6.4725  -13.6403    3.4454
   -6.4725    8.5757    9.0818   -4.0970
  -13.6403    9.0818   20.1762    1.2934
    3.4454   -4.0970    1.2934    7.0601
```

Синтез регулятора

```
K_lmi = Y_lmi*inv(P_lmi)
```

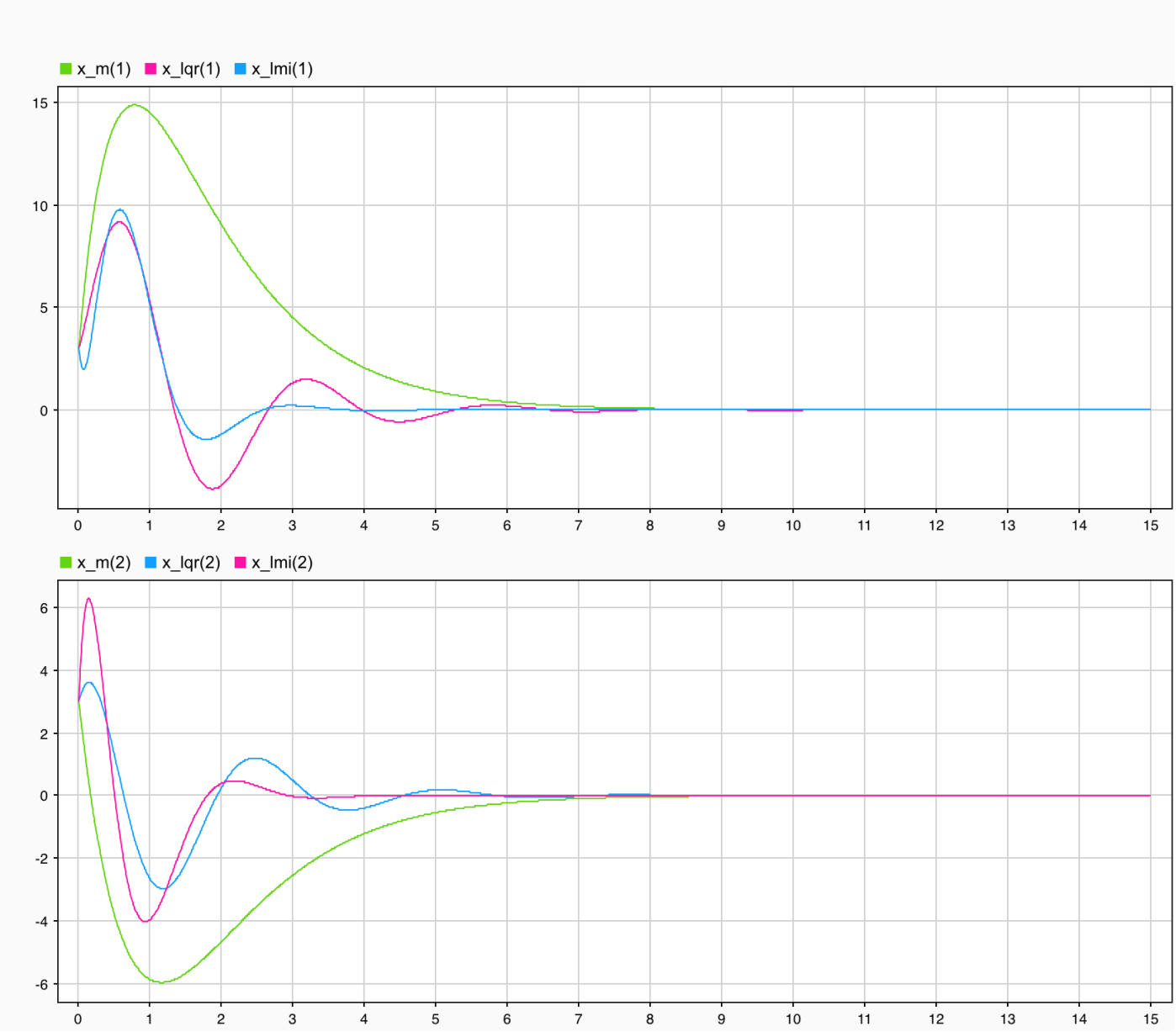
```
K_lmi = 2×4
  -10.7288   21.7753  -19.1847   18.7921
         0         0         0         0
```

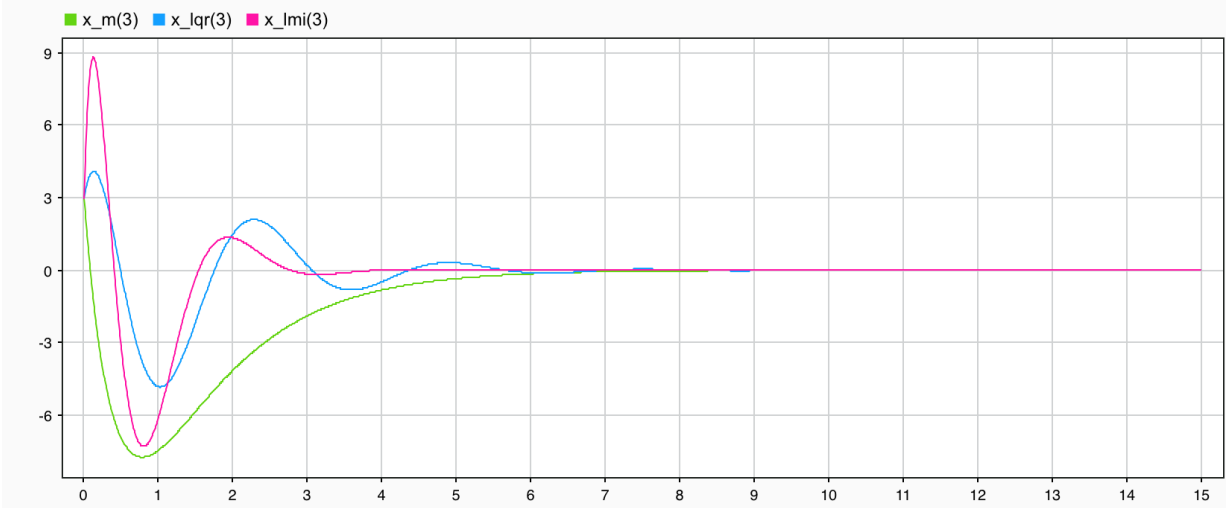
Calculate J

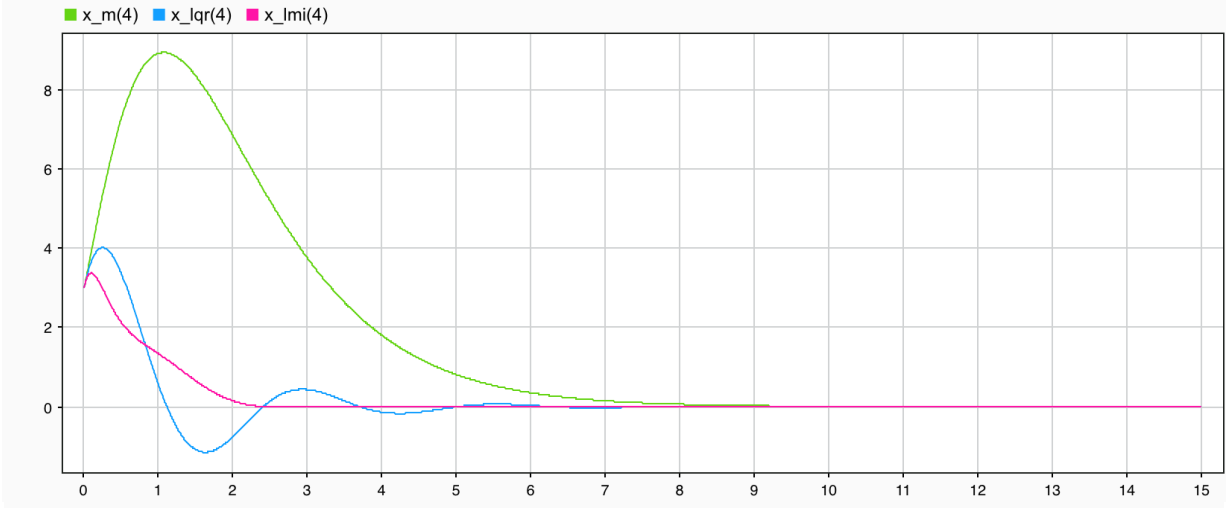
```
J_lqr = zeros(2943, 1);
J_m = zeros(2943, 1);
J_lmi = zeros(2943, 1);
J_lqr(1) = out.J_lqr(1);
J_m(1) = out.J_m(1);
J_lmi(1) = out.J_lmi(1);
for i = 2:2943
    J_lqr(i) = J_lqr(i-1) + out.J_lqr(i);
    J_m(i) = J_m(i-1) + out.J_m(i);
    J_lmi(i) = J_lmi(i-1) + out.J_lmi(i);
end
```

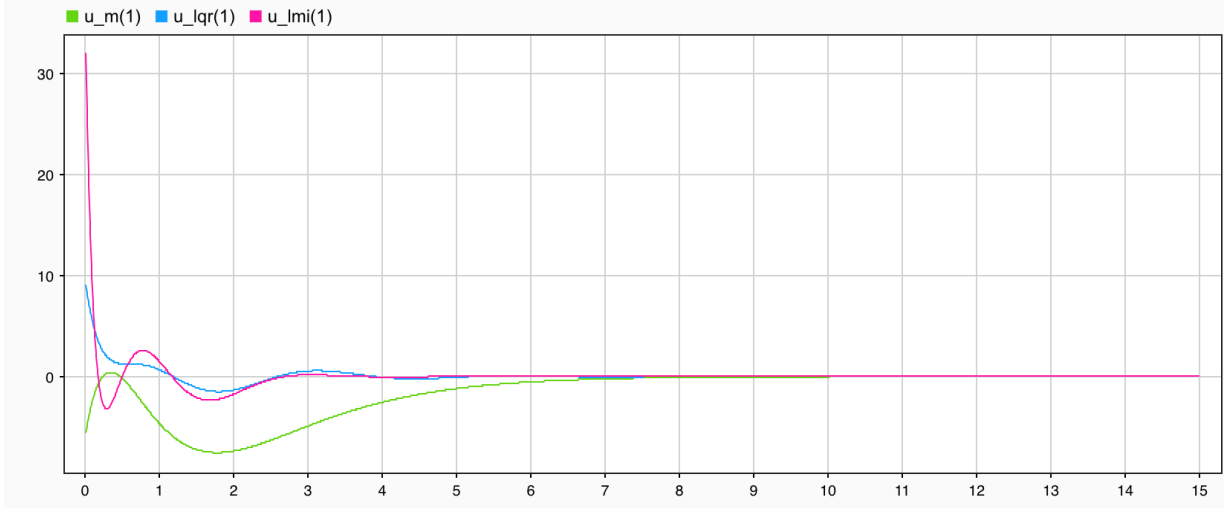
```
cvx_begin sdp
variable P_m(4, 4)
A*P_m-P_m*G == B*Y;
cvx_end

cvx_begin sdp
variable P_lmi(4,4)
variable Y_lmi(2,4)
P_lmi > 0.0001*eye(4);
P_lmi*A' + A*P_lmi + 2*a*P_lmi + (Y_lmi)'*B' + B*Y_lmi <= 0;
cvx_end
```

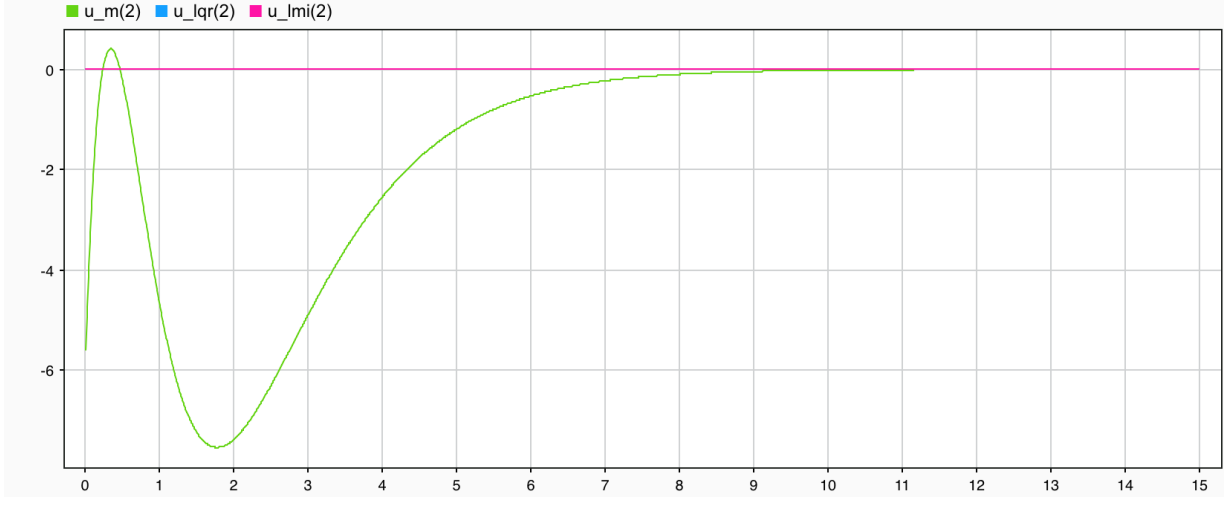
Сравнительные график компонент и входного воздействия:



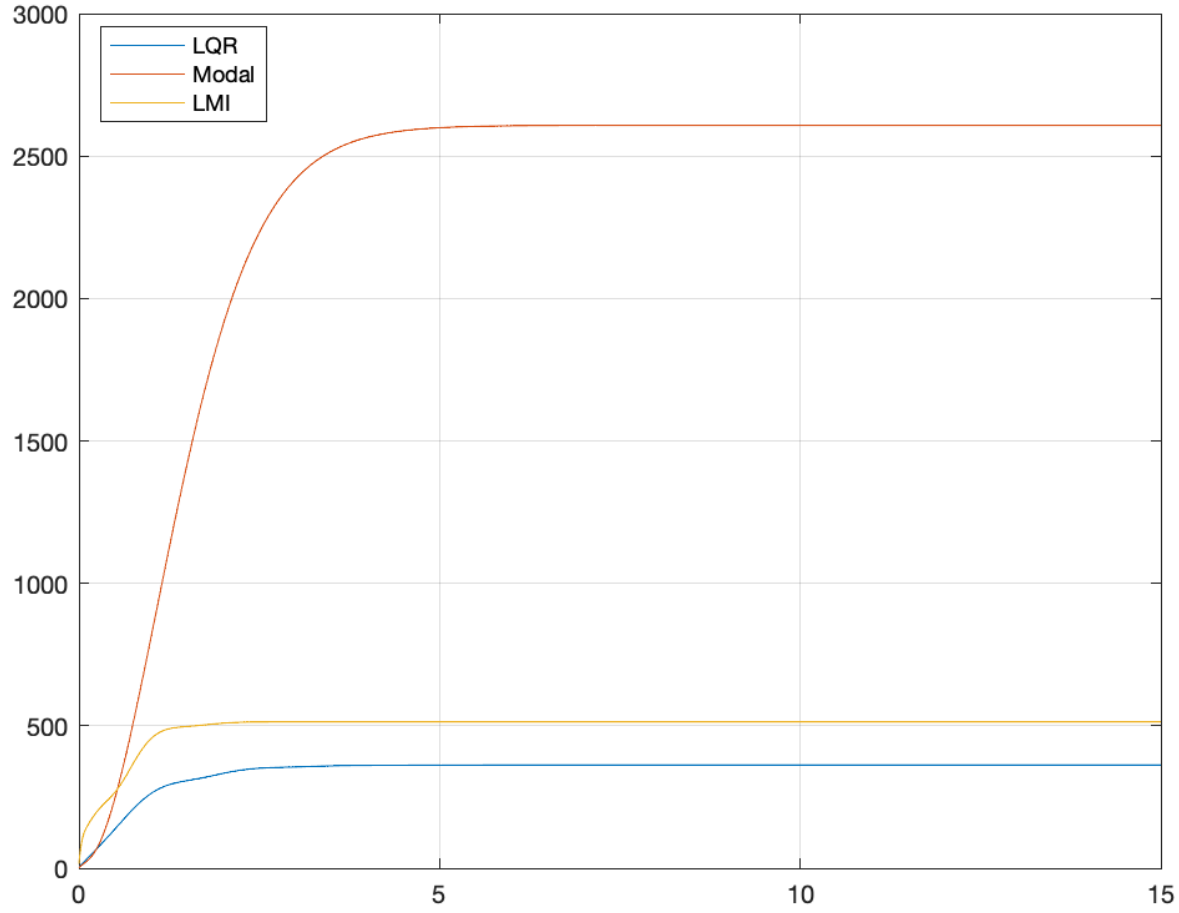

x_m(3)
x_lqr(3)
x_lmi(3)
9
6
3
0
-3
-6
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15


x_m(4)
x_lqr(4)
x_lmi(4)
8
6
4
2
0
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15


u_m(1)
u_lqr(1)
u_lmi(1)
30
20
10
0
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15


u_m(2)
u_lqr(2)
u_lmi(2)
0
-2
-4
-6
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

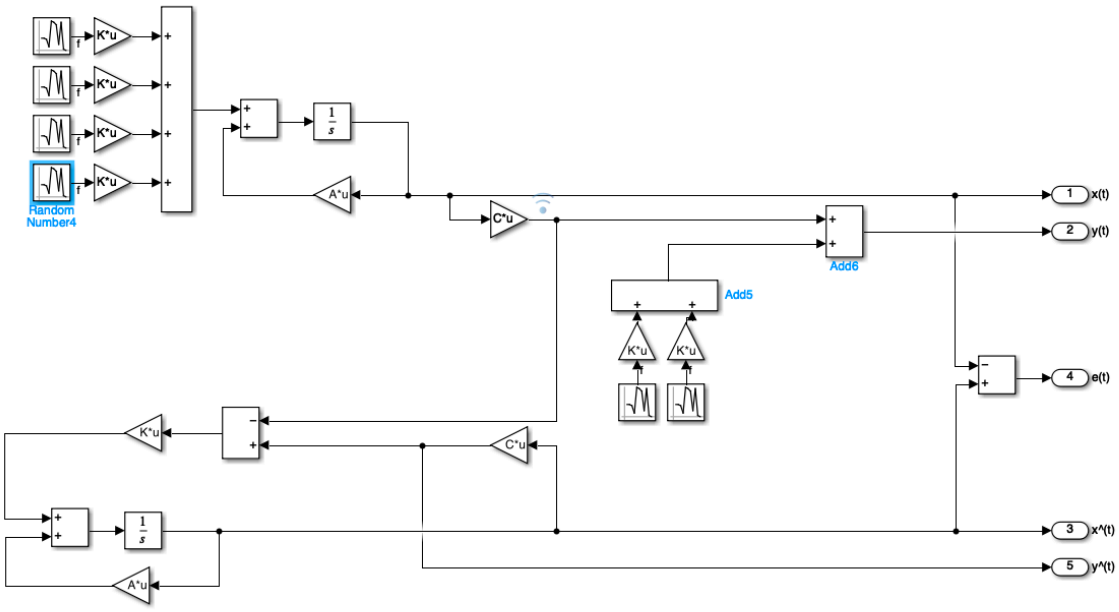
Сравнительный графики J



В LRQ выделяется наибольшем временем переходного процесса, но при этом обладает наименьшим отклонением. Также LQR обладает наименьшим управлением и функционалам качества.

2.3. Фильтр Калмана

Схема моделирования:



Программный код:

Init data

```
A = [ -6,  19, -13, 10;
       0,  -9,   6,  0;
       0, -15,   9,  0;
      -4,   8,  -7,  6];


C = [1, -3,  2, -1;
     0,  6, -3,  0];


x0_1 = [3; 3; 3; 3];
x0_2 = [-3;-3;-3;-3];
```

Собственный значения

```
eig(A)
```

```
ans = 4×1 complex
  -0.0000 + 2.0000i
  -0.0000 - 2.0000i
  -0.0000 + 3.0000i
  -0.0000 - 3.0000i
```

Наблюдаемость

```
V = [A; C*A; C*A*A; C*A*A*A];
rank(V)
```

```
ans = 4
```

Система наблюдаема

Soft estimator

```
Q1 = [1, 0, 0, 0;
      0, 1, 0, 0;
      0, 0, 1, 0;
      0, 0, 0, 1];


R1 = [135, 0;
      0, 135];


G = [1, 0, 0, 0;
     0, 1, 0, 0;
     0, 0, 1, 0;
     0, 0, 0, 1];
```

Управляемость

```
U = [Q1 A*Q1 A*A*Q1 A*A*A*Q1];
rank(U)
```

```
ans = 4
```

Синтез LQE

```
[L1, P1, e1] = lqe(A, G, C, Q1, R1)
```

```
L1 = 4×2
    1.1278   -0.0891
    0.0008    0.2297
    0.0338    0.2366
    0.6387   -0.2086
P1 = 4×4
  397.0404   -2.6188   -1.2271  250.1934
   -2.6188   21.6959   33.0542   -1.7077
```

```
   -1.2271   33.0542   55.4631    5.9694
  250.1934   -1.7077    5.9694  181.0320
e1 = 4×1 complex
  -0.2780 + 2.0119i
  -0.2780 - 2.0119i
  -0.3335 + 2.9864i
  -0.3335 - 2.9864i
```

Medium estimator

```
Q2 = [3, 0, 0, 0;
      0, 3, 0, 0;
      0, 0, 3, 0;
      0, 0, 0, 3];


R2 = [3, 0;
      0, 3];
```

Управляемость

```
U = [Q2 A*Q2 A*A*Q2 A*A*A*Q2];
rank(U)
```

```
ans = 4
```

Синтез LQE

```
[L2, P2, e2] = lqe(A, G, C, Q2, R2)
```

```
L2 = 4×2
   11.5313   -5.4703
   -1.8719    2.7432
   -2.7109    3.2586
    7.2606   -4.0837
P2 = 4×4
  160.2357  -30.1510  -54.8318  106.4313
  -30.1510   10.5012   18.2591  -19.5205
  -54.8318   18.2591   33.2597  -34.9572
  106.4313  -19.5205  -34.9572   73.2964
e2 = 4×1 complex
  -1.0836 + 2.4186i
  -1.0836 - 2.4186i
  -3.2620 + 0.0000i
  -5.7194 + 0.0000i
```

Hard estimator

```
Q3 = [130, 0, 0, 0;
      0, 78, 0, 0;
      0, 0, 78, 0;
      0, 0, 0, 130];


R3 = [1, 0;
      0, 1];
```

Управляемость

```
U = [Q3 A*Q3 A*A*Q3 A*A*A*Q3];
rank(U)
```

```
ans = 4
```

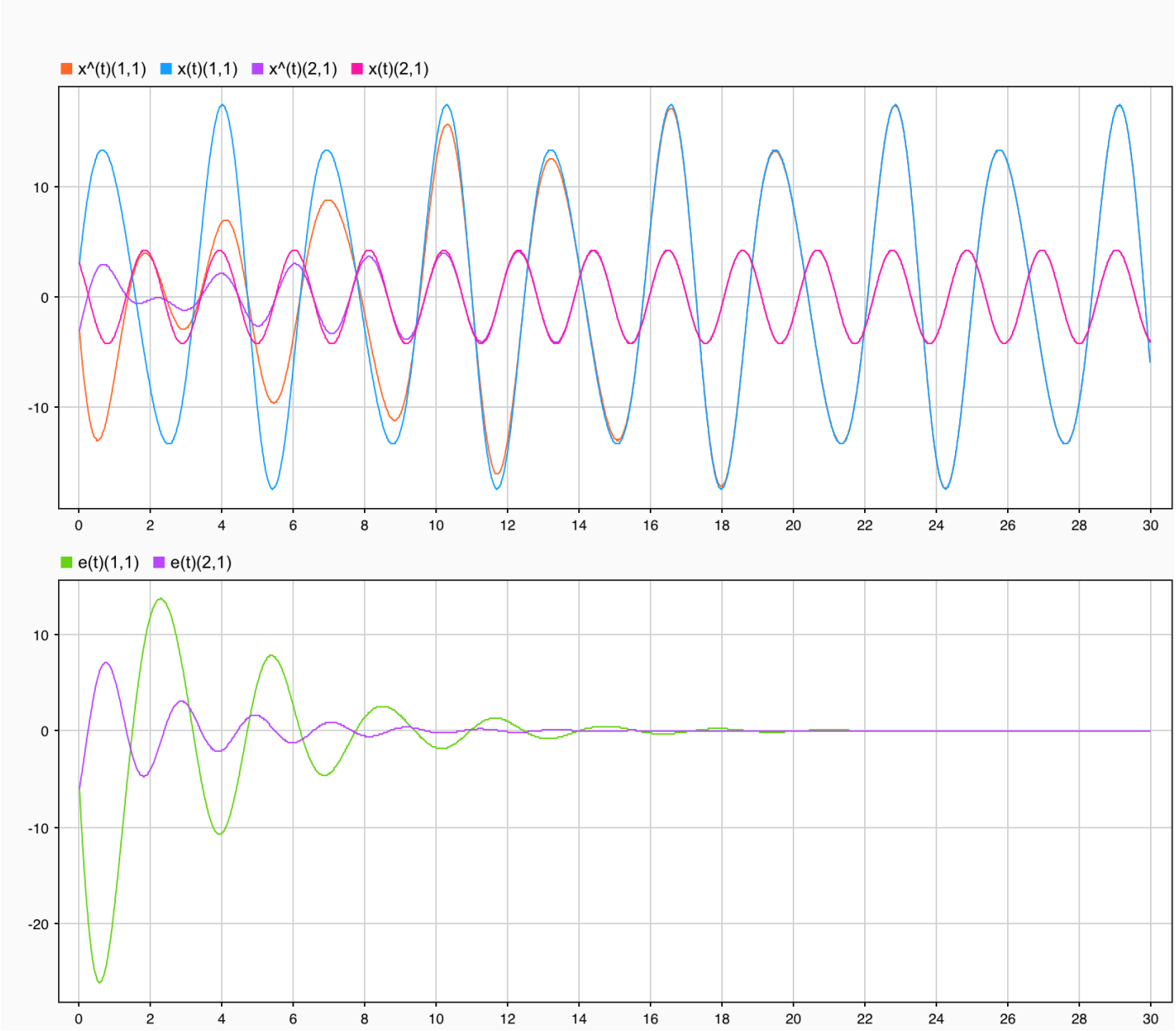
Синтез LQE

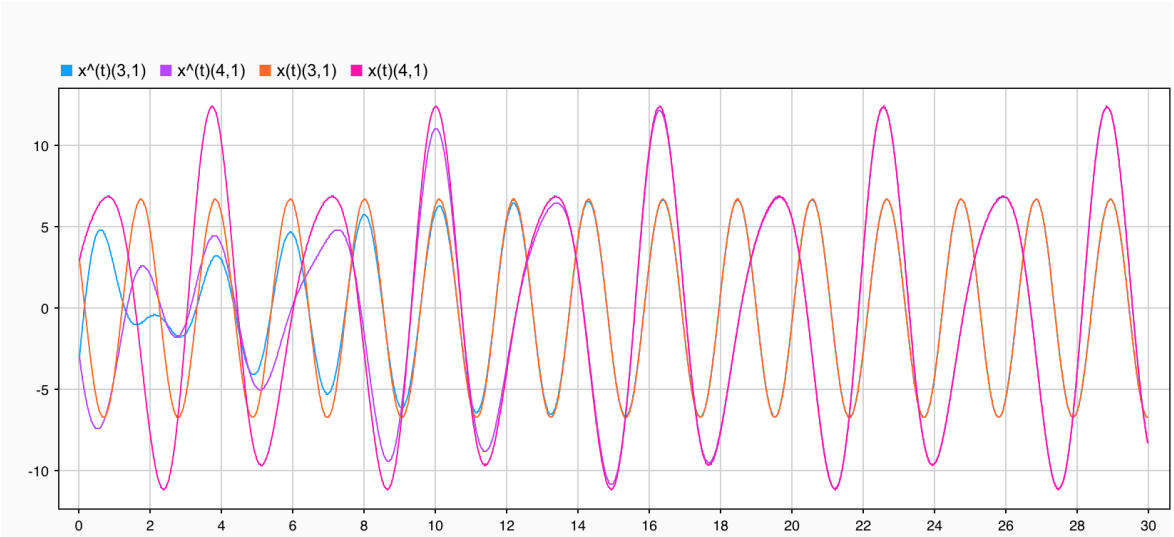
```
[L3, P3, e3] = lqe(A, G, C, Q3, R3)
```

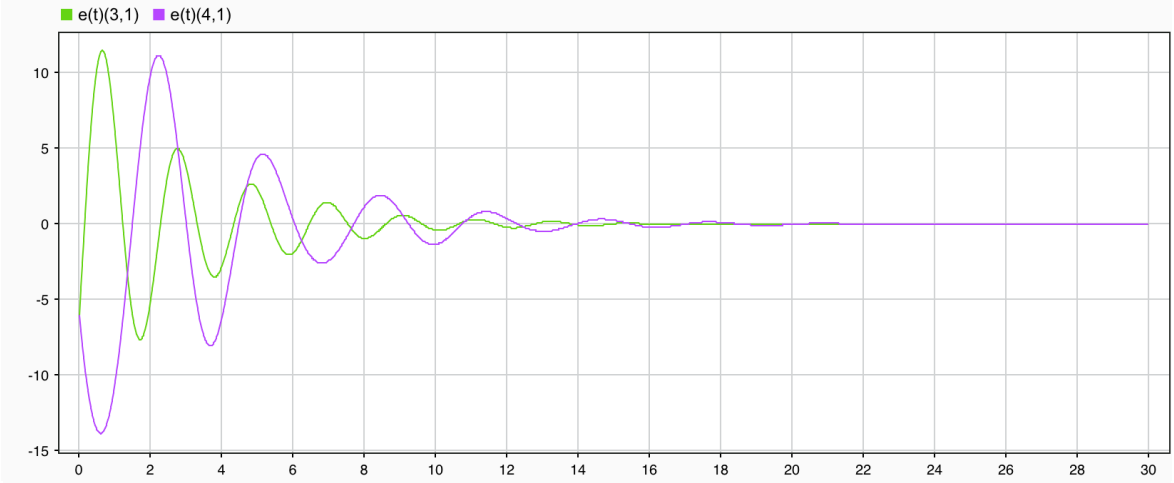
```
L3 = 4×2
  103.0118  -30.2507
  -17.8509   24.7606
```

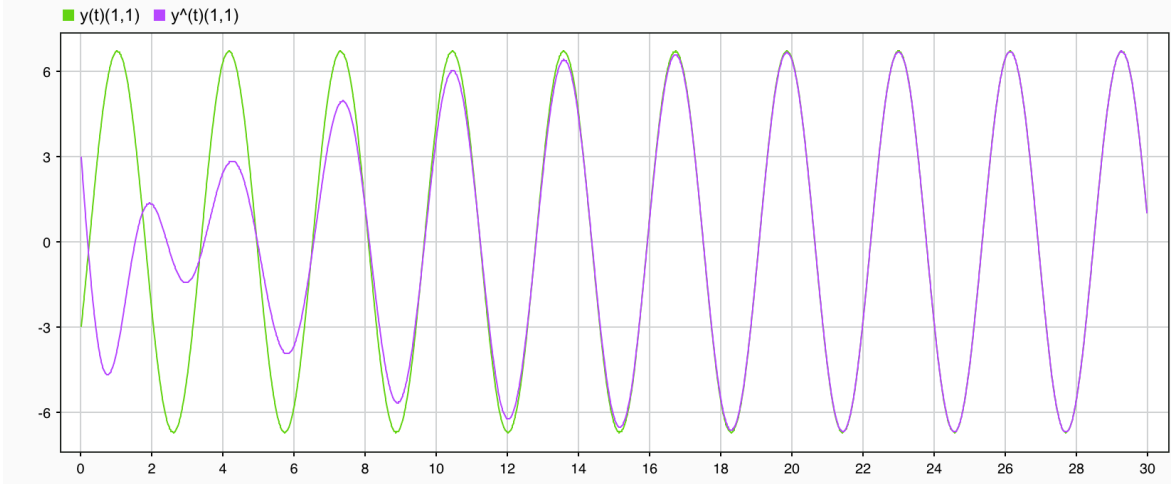
```
  -28.1106   30.9536
   69.5040  -19.8519
P3 = 4×4
10^3 ×
    2.0122   -0.4636   -0.9172    1.4657
   -0.4636    0.1588    0.3093   -0.3035
   -0.9172    0.3093    0.6084   -0.6004
    1.4657   -0.3035   -0.6004    1.1059
e3 = 4×1
   -1.9411
   -3.3085
  -13.8228
  -67.4701
```

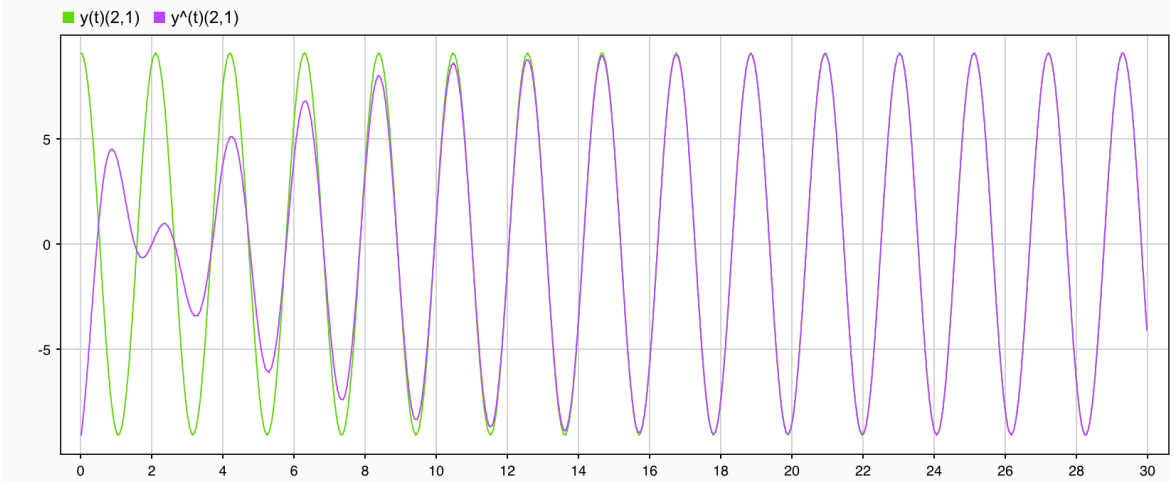
Графики для ξ, $f = 0$
Soft:



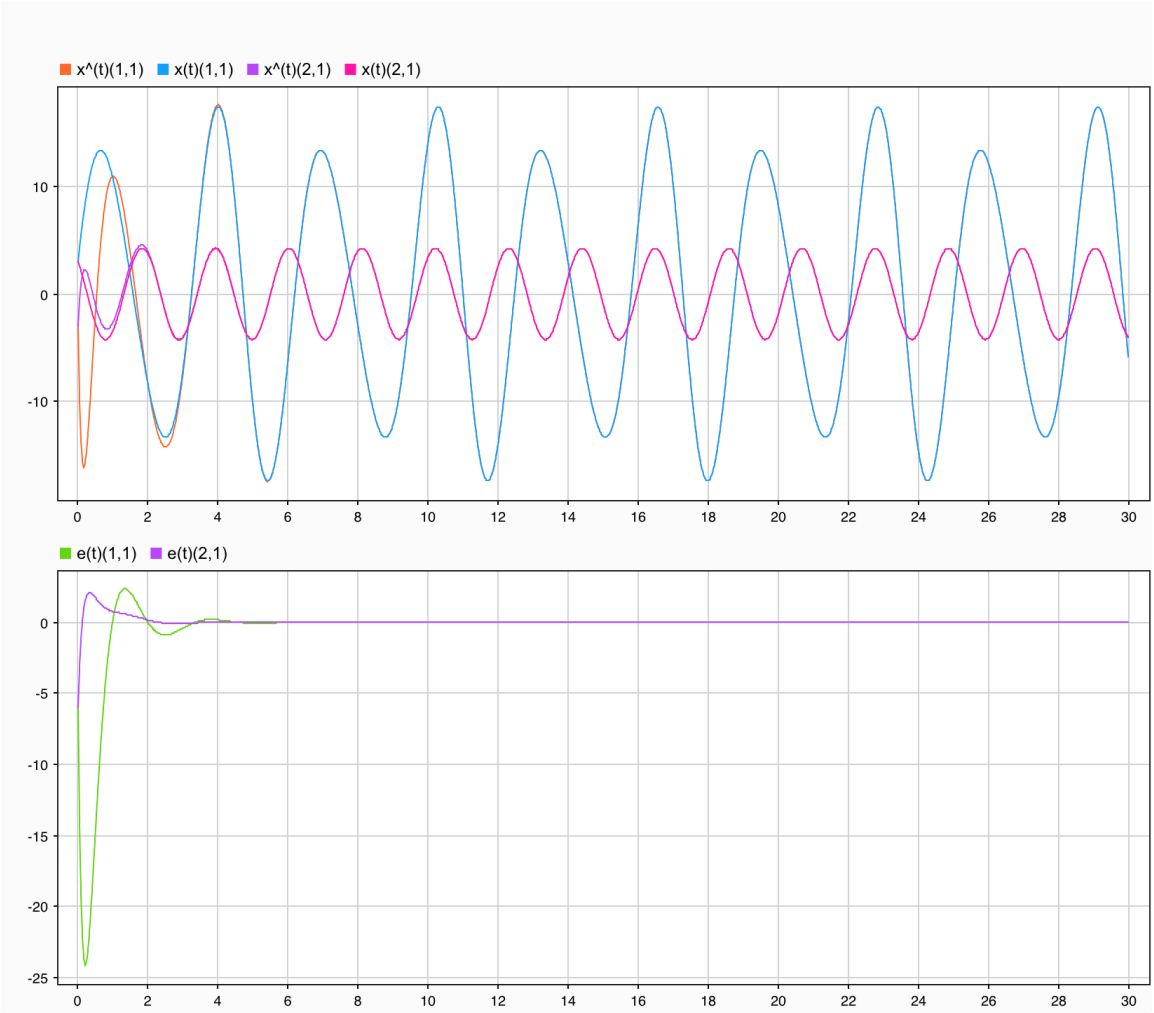

x^(t)(3,1)
x^(t)(4,1)
x(t)(3,1)
x(t)(4,1)
10
5
0
-5
-10
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30

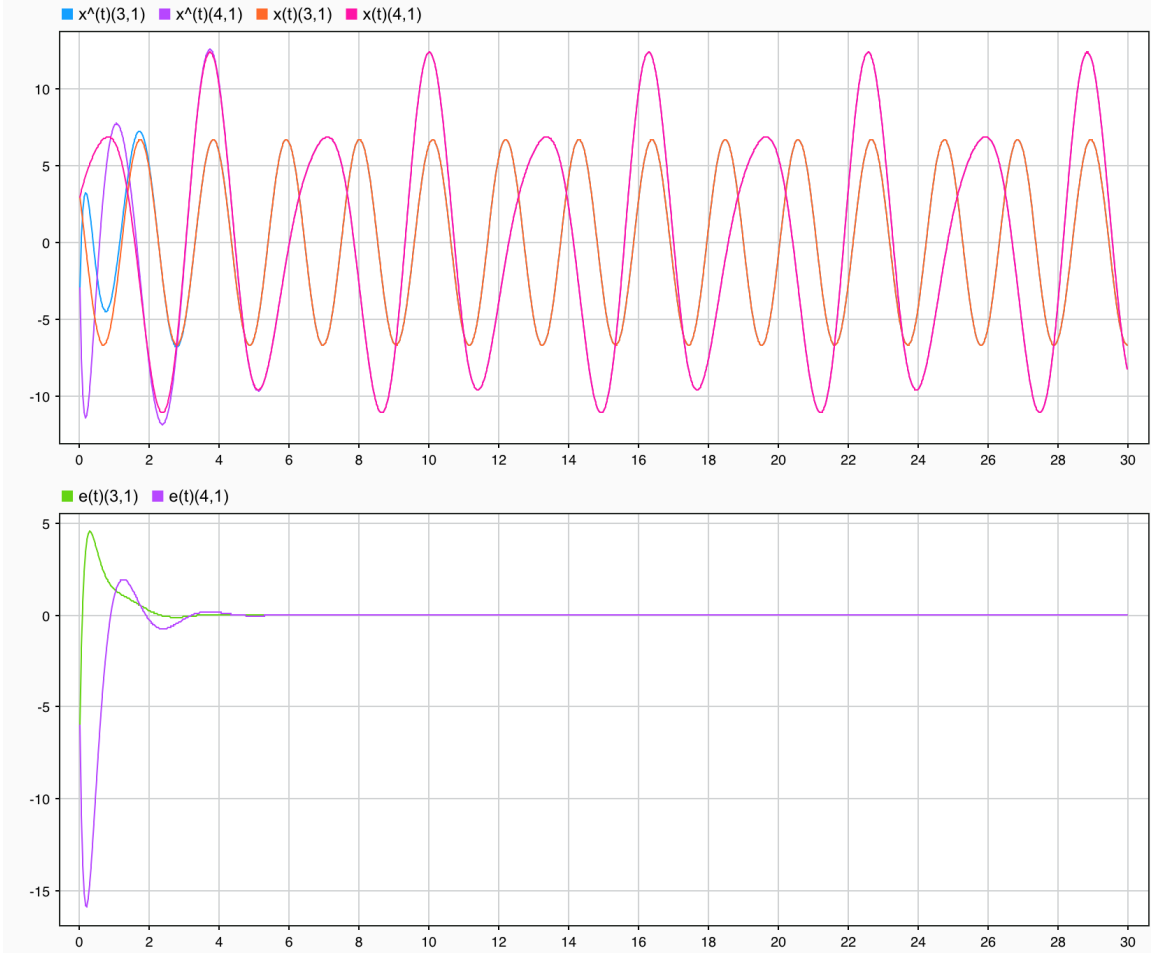

e(t)(3,1)
e(t)(4,1)
10
5
0
-5
-10
-15
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30

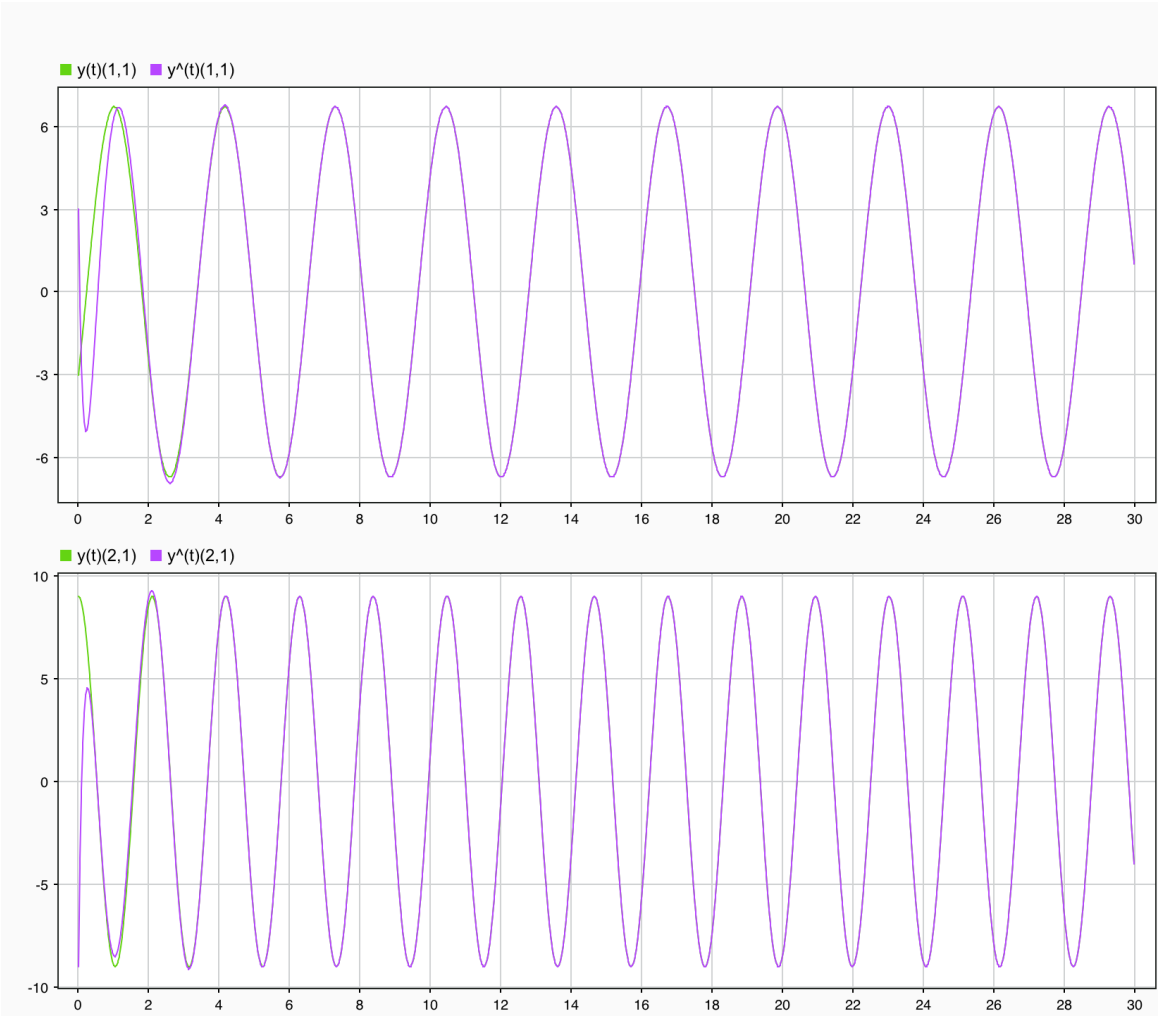

y(t)(1,1)
y^(t)(1,1)
6
3
0
-3
-6
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30


y(t)(2,1)
y^(t)(2,1)
5
0
-5
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30

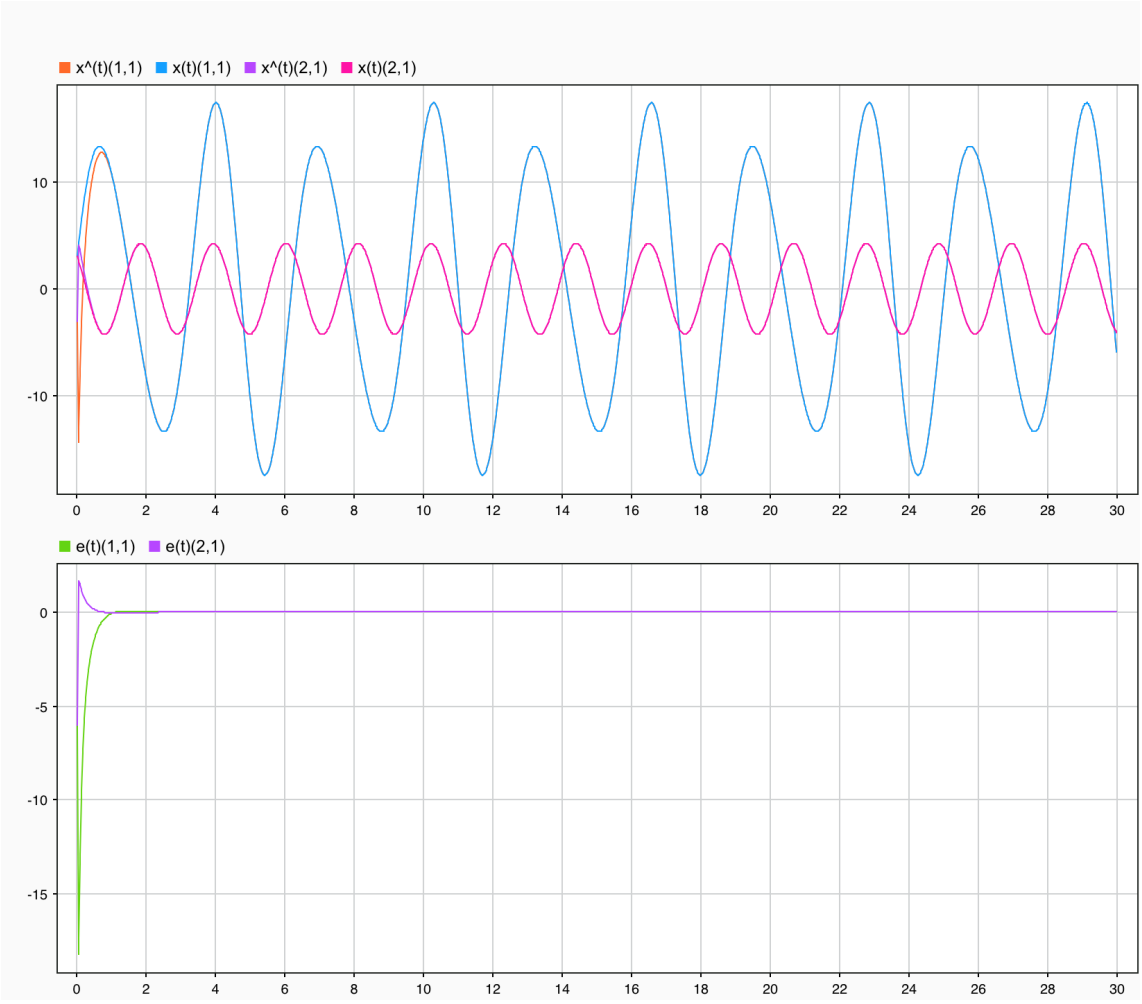
Med:

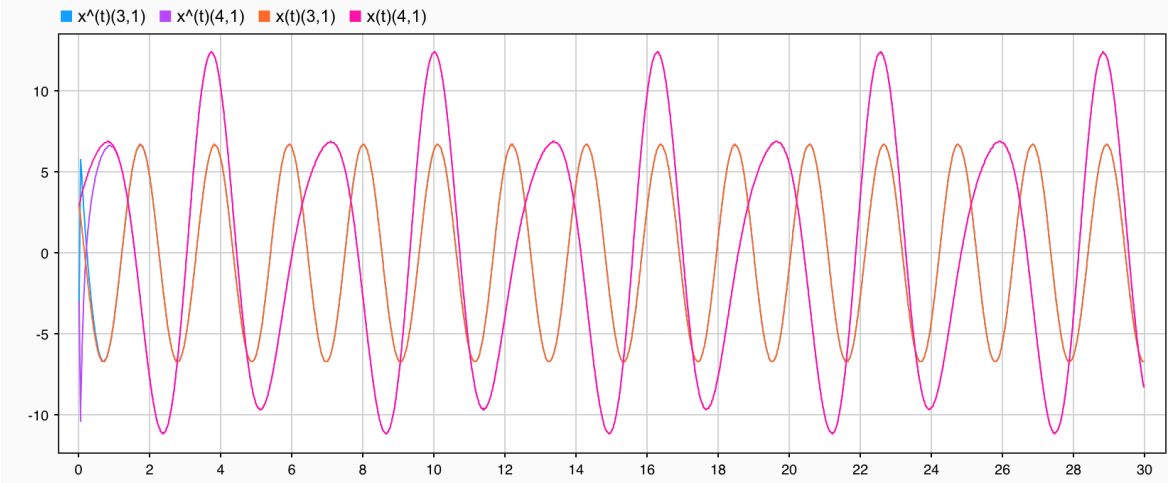


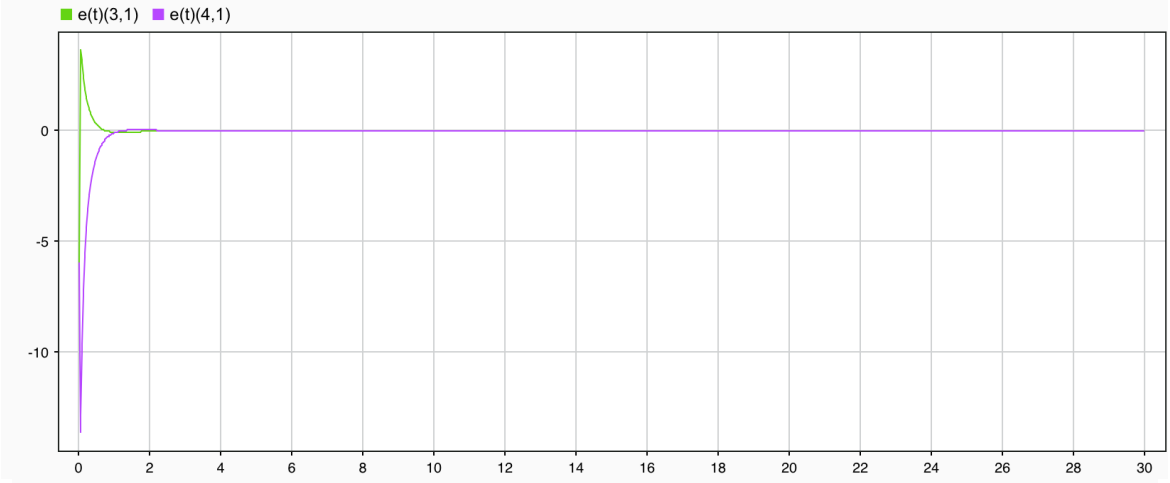


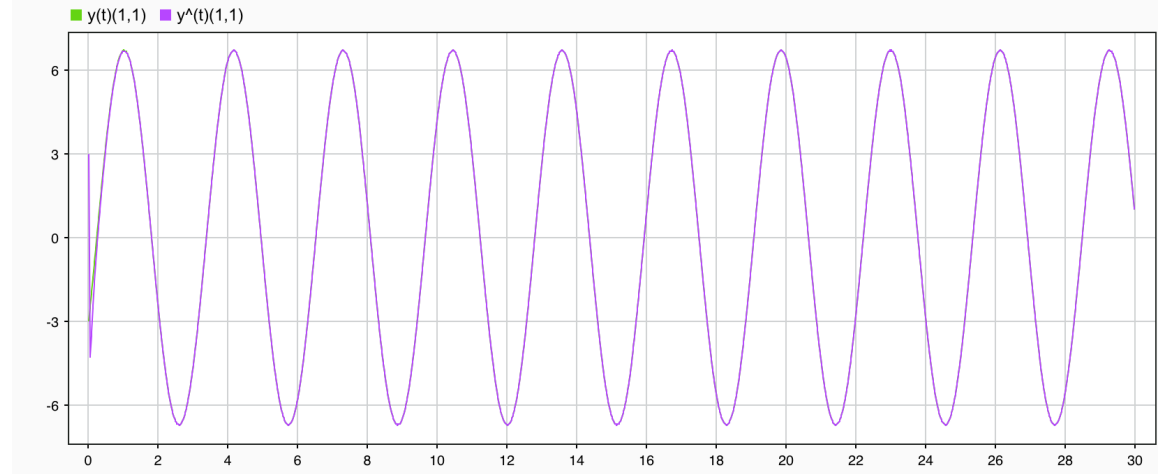


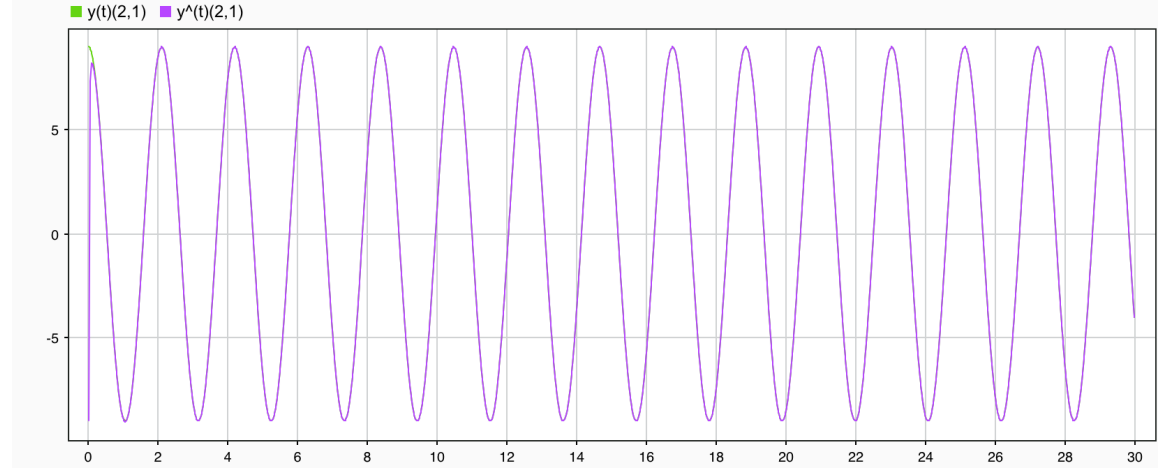
Hard



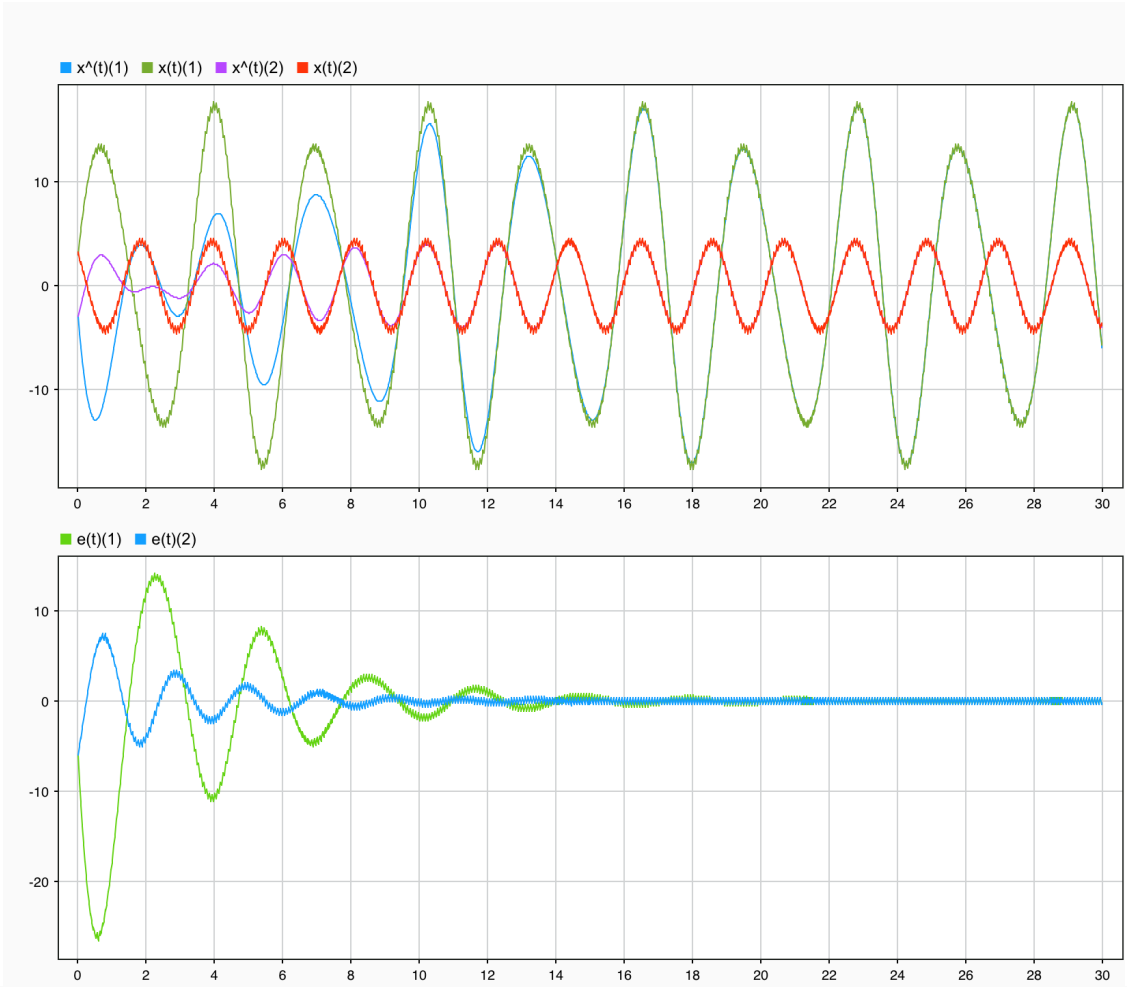

x^(t)(3,1)
x^(t)(4,1)
x(t)(3,1)
x(t)(4,1)
10
5
0
-5
-10
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30

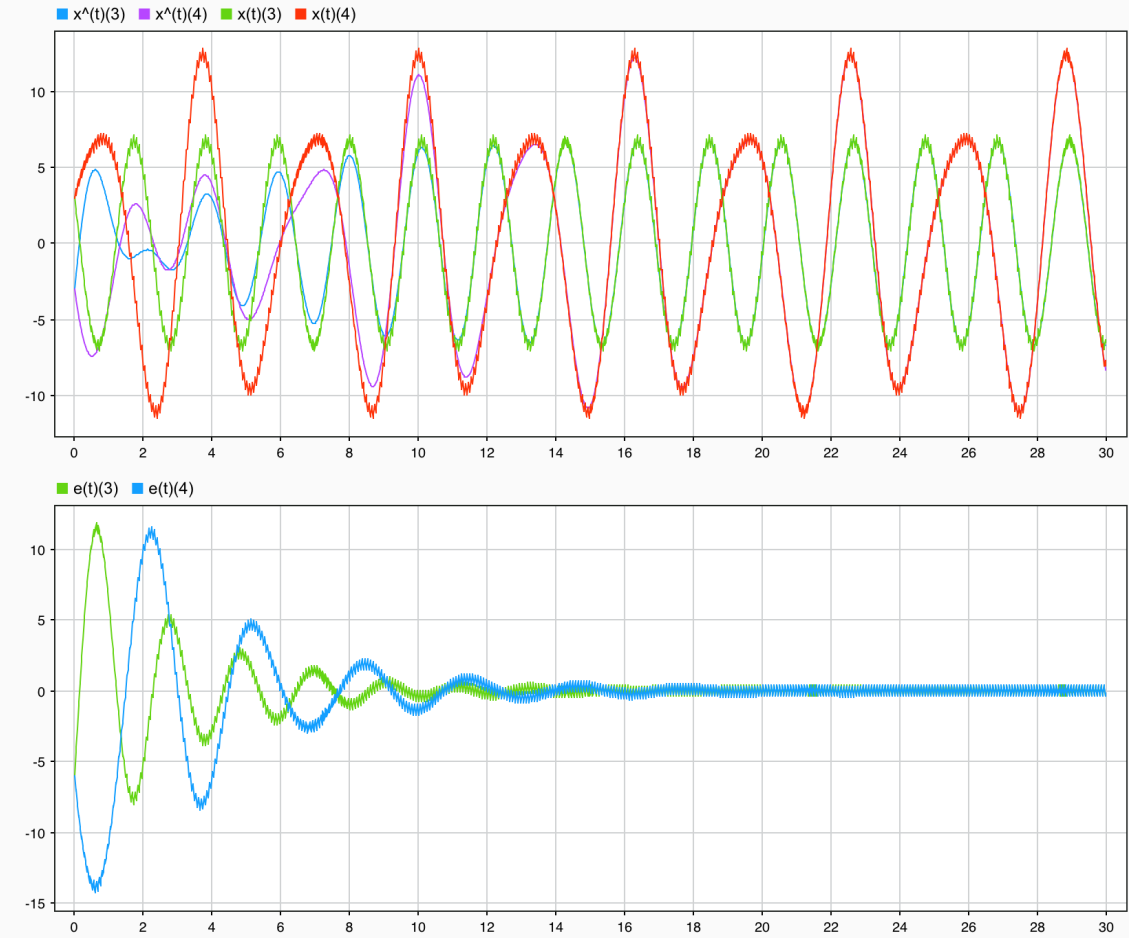

e(t)(3,1)
e(t)(4,1)
0
-5
-10
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30

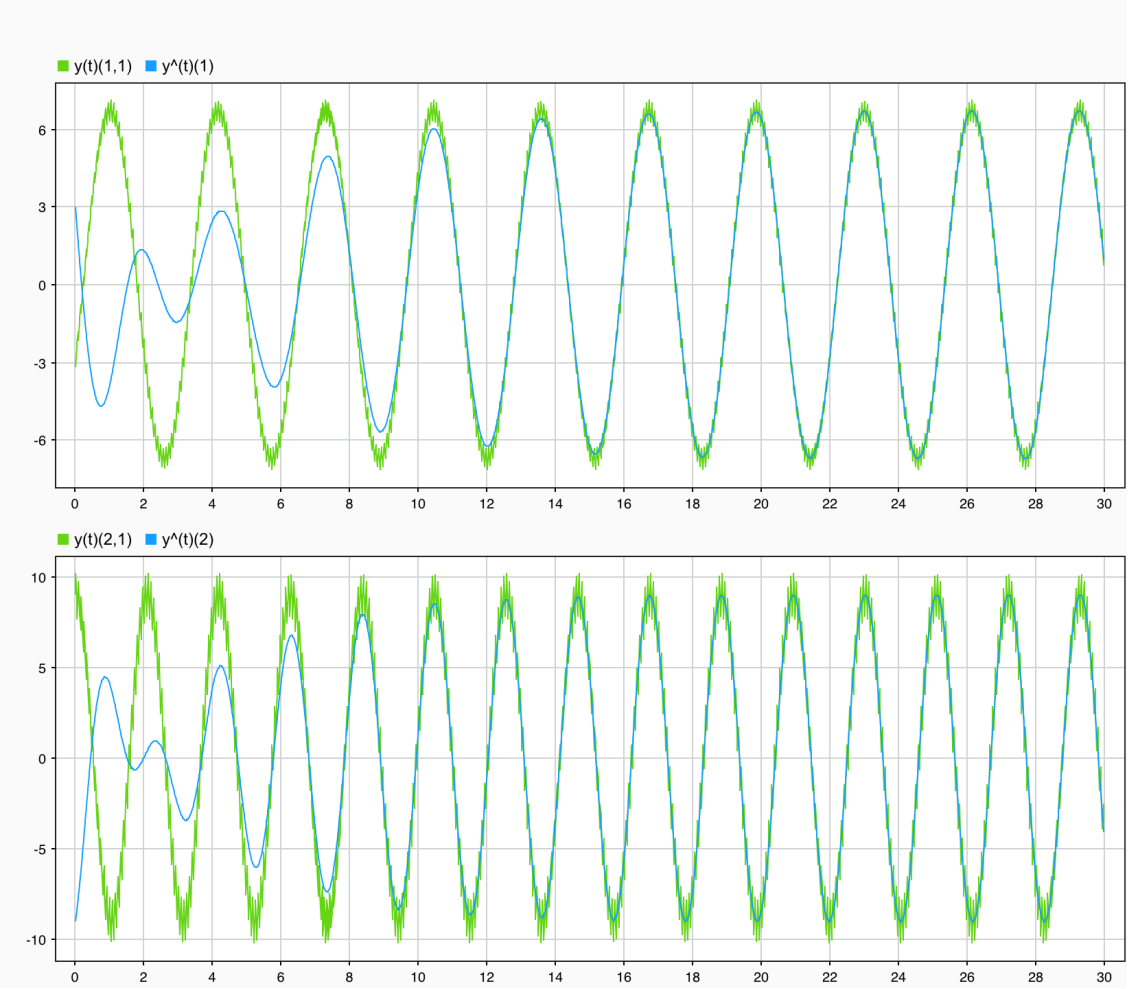

y(t)(1,1)
y^(t)(1,1)
6
3
0
-3
-6
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30


y(t)(2,1)
y^(t)(2,1)
5
0
-5
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30

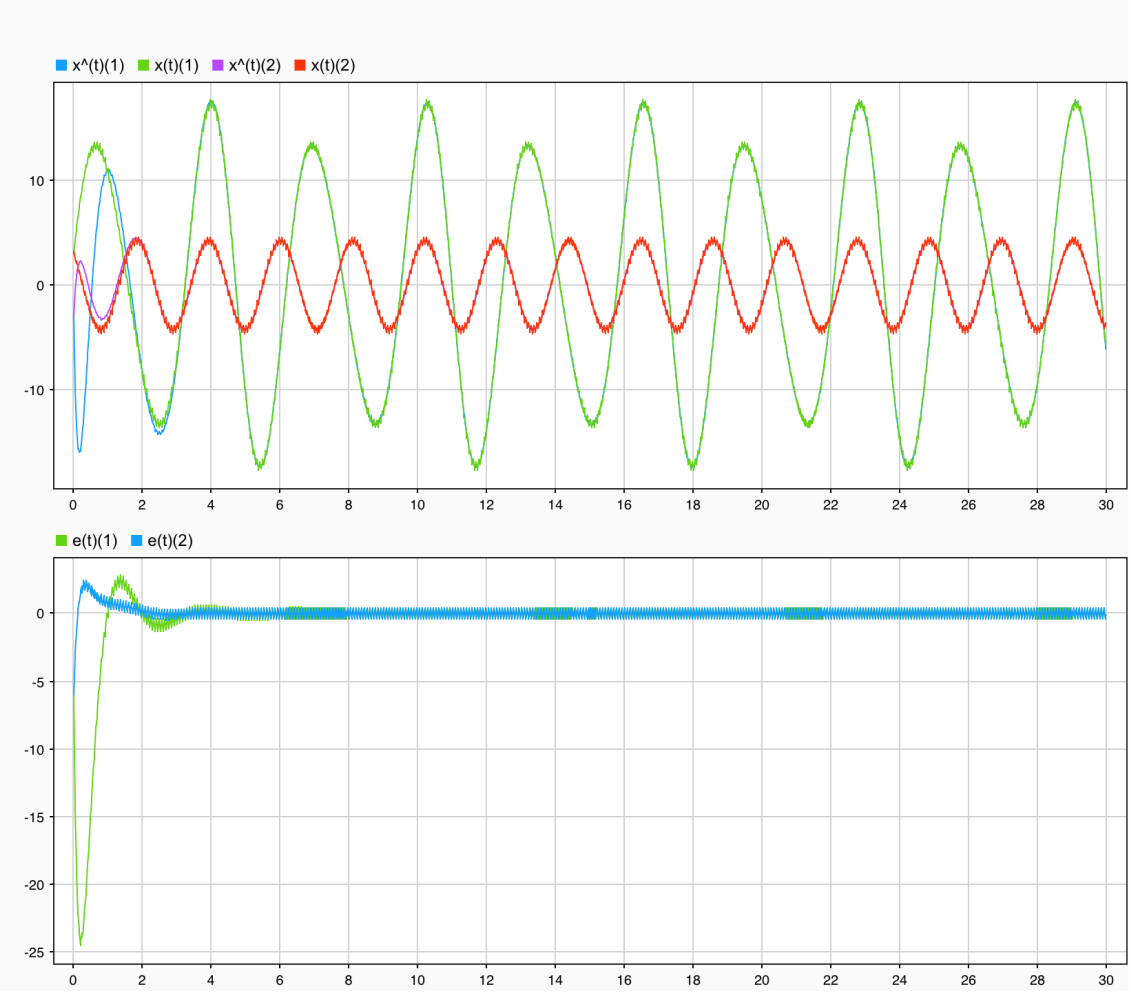
Графики для $\xi = 0$, $f = 30\sin(78x+pi/2)$
Soft:

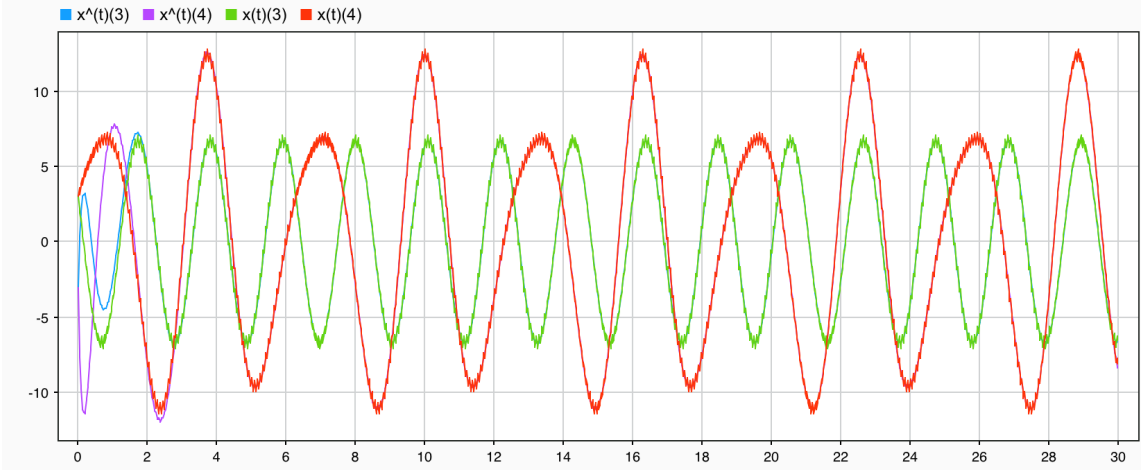


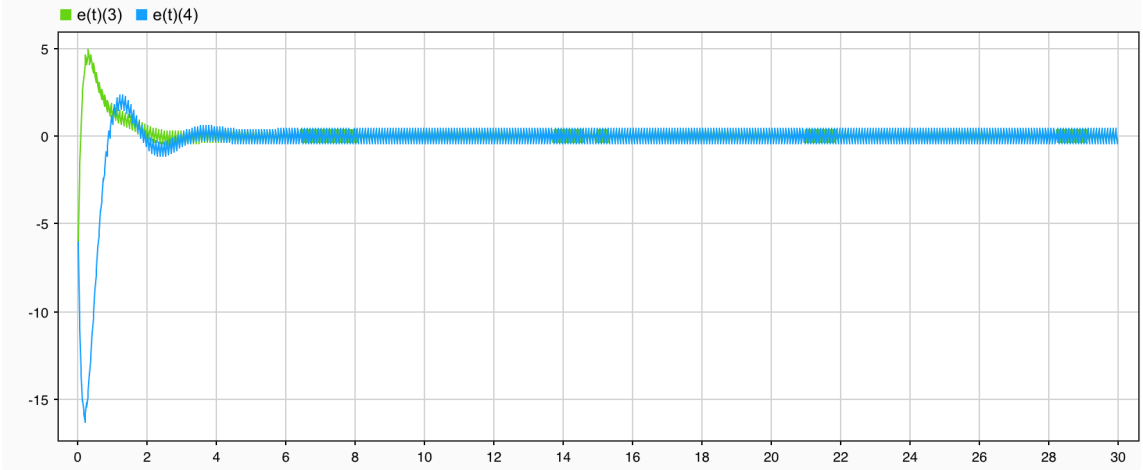


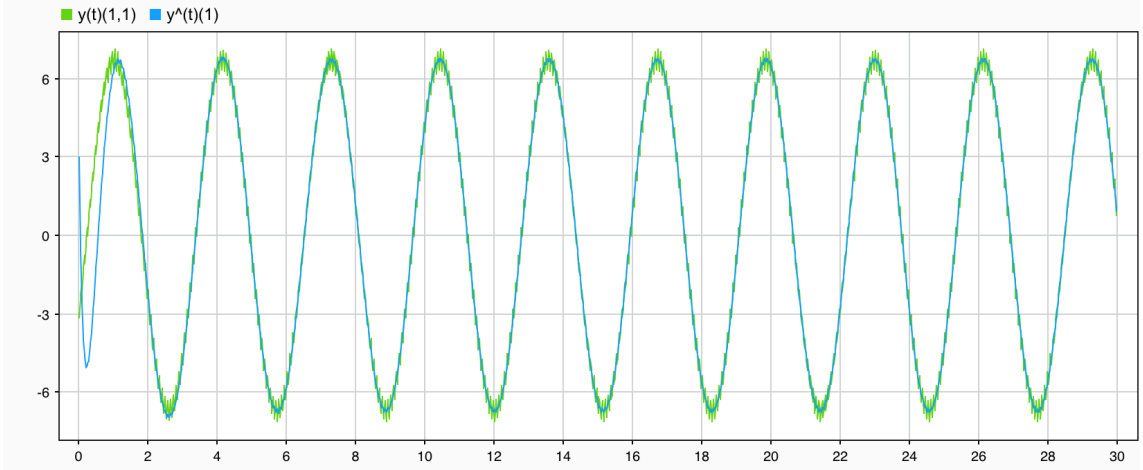


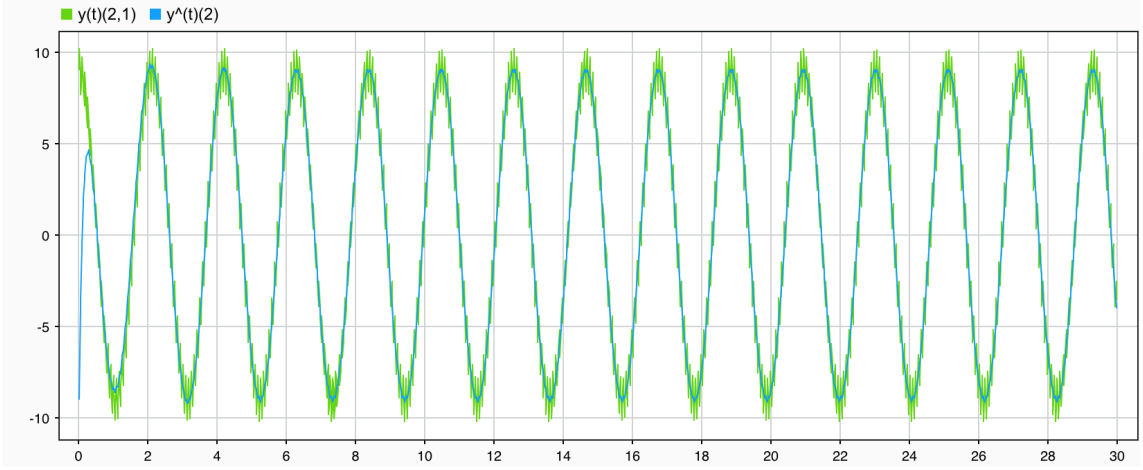
Med:



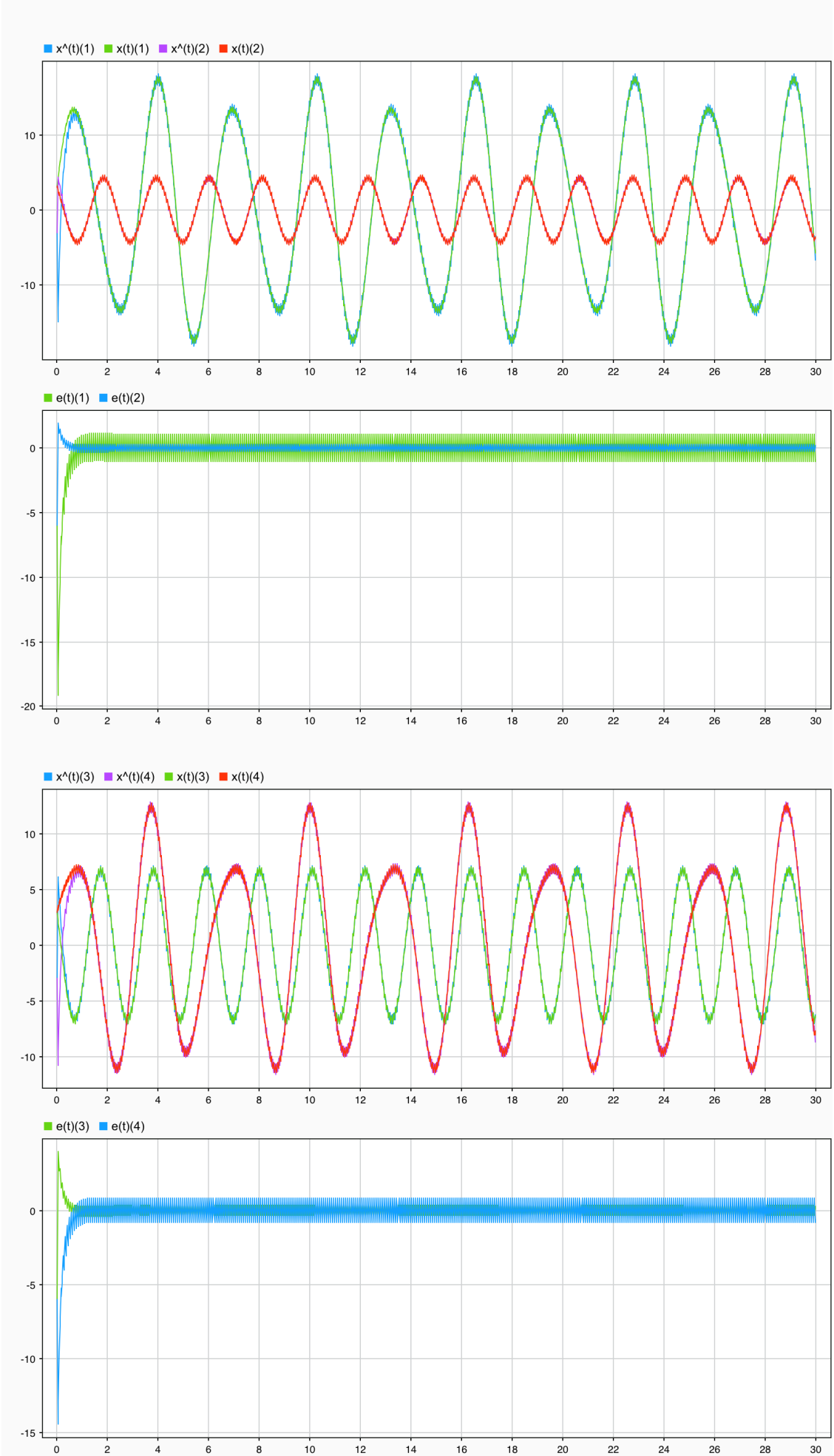

x^(t)(3)
x^(t)(4)
x(t)(3)
x(t)(4)
10
5
0
-5
-10
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30

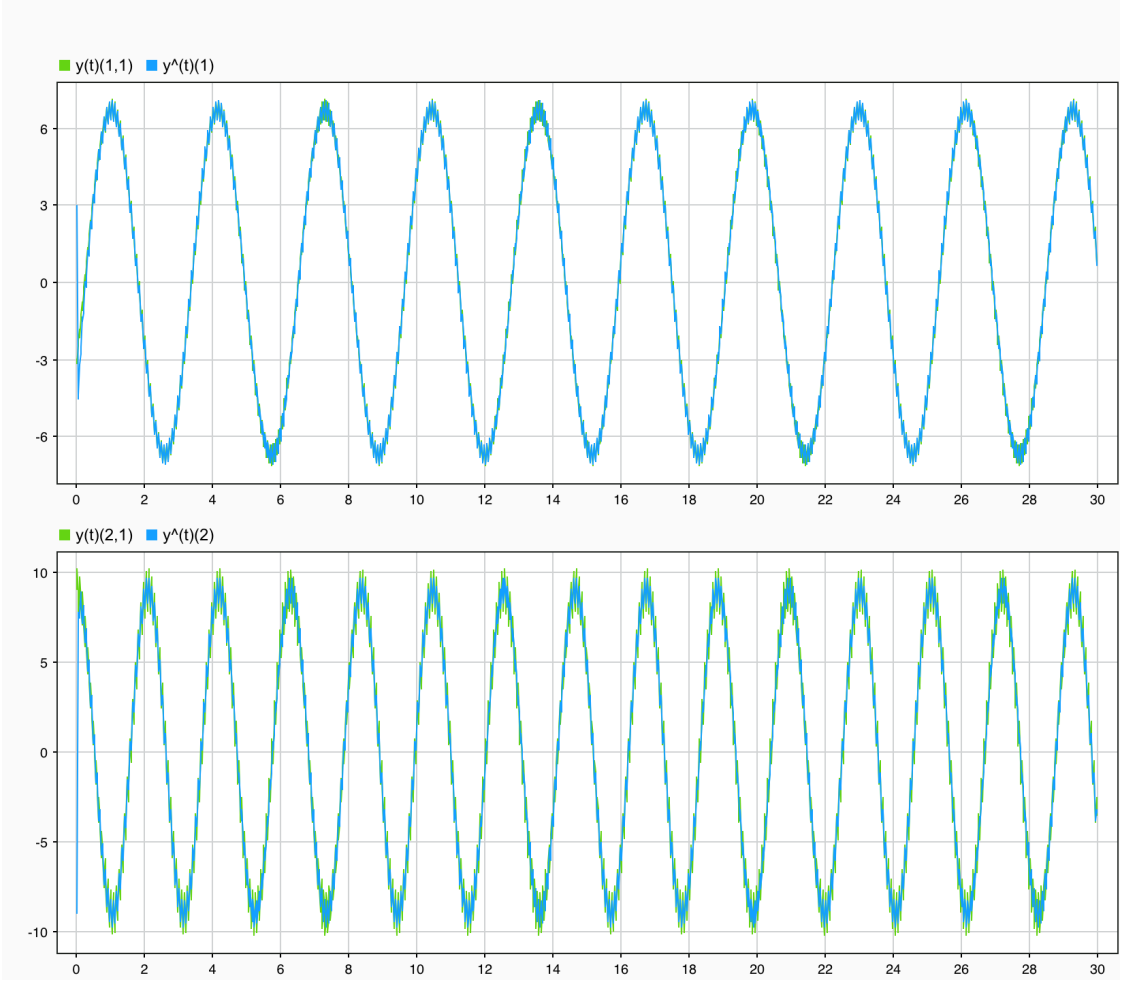

e(t)(3)
e(t)(4)
5
0
-5
-10
-15
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30


y(t)(1,1)
y^(t)(1)
6
3
0
-3
-6
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30


y(t)(2,1)
y^(t)(2)
10
5
0
-5
-10
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30

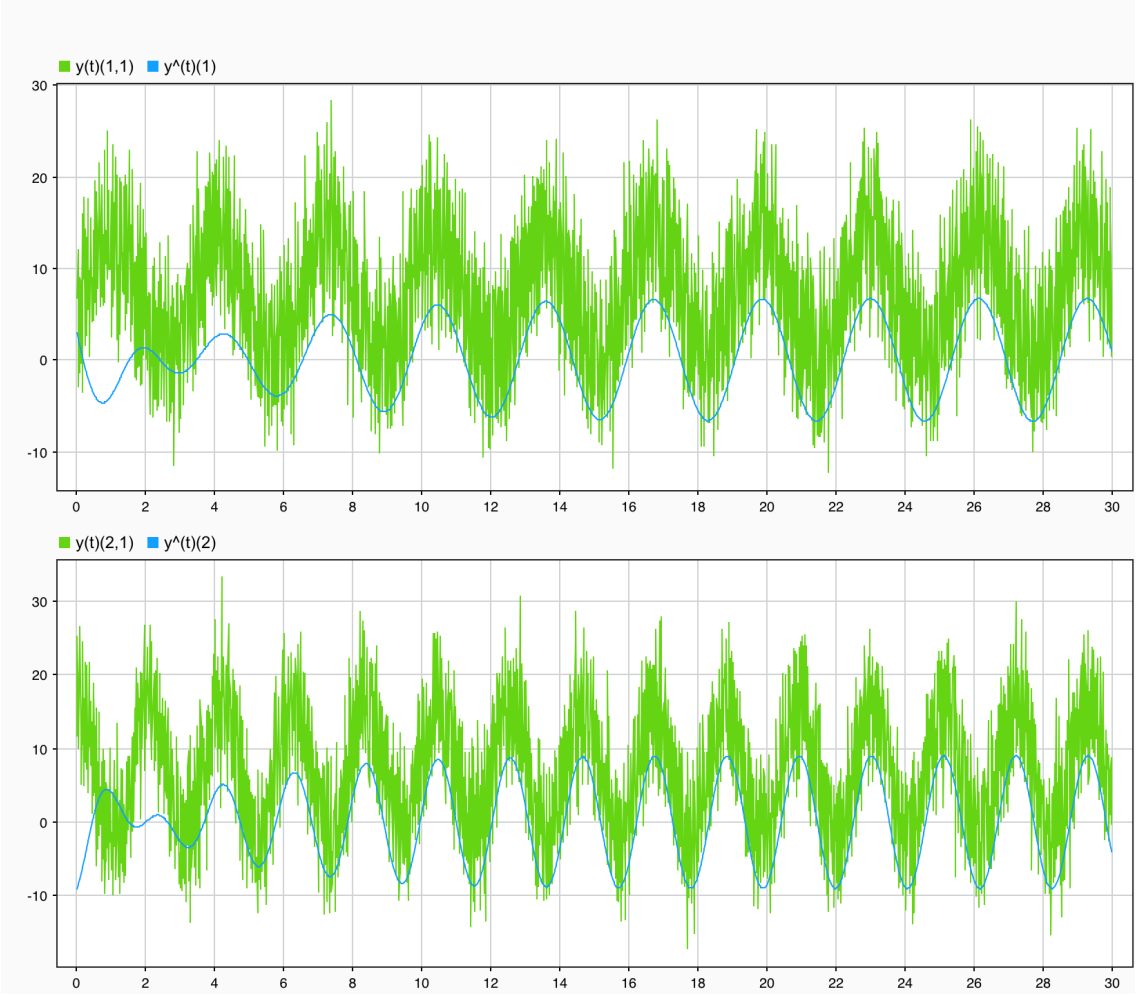
Hard:



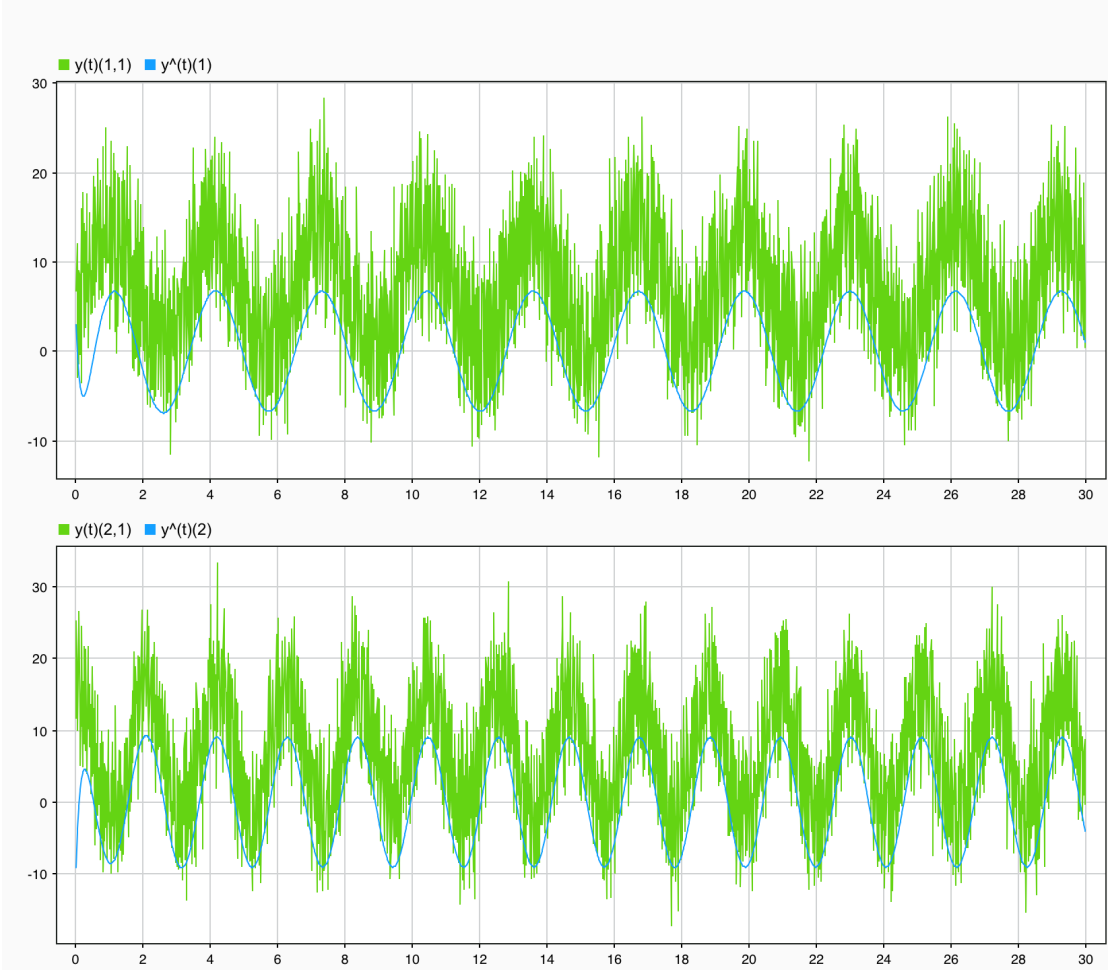


Графики для $\xi = N(\mu = 7, \sigma^2 = 23)$, $f = 0$

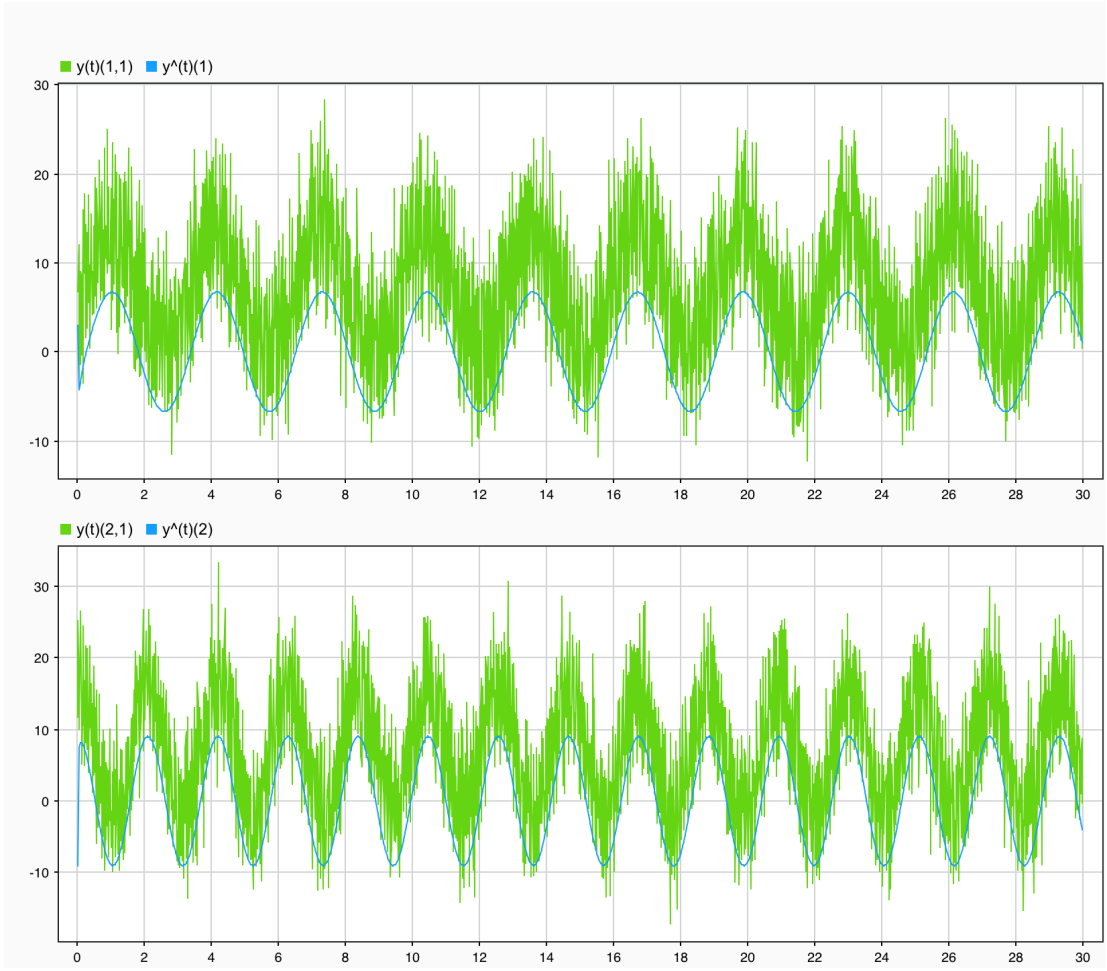
Soft



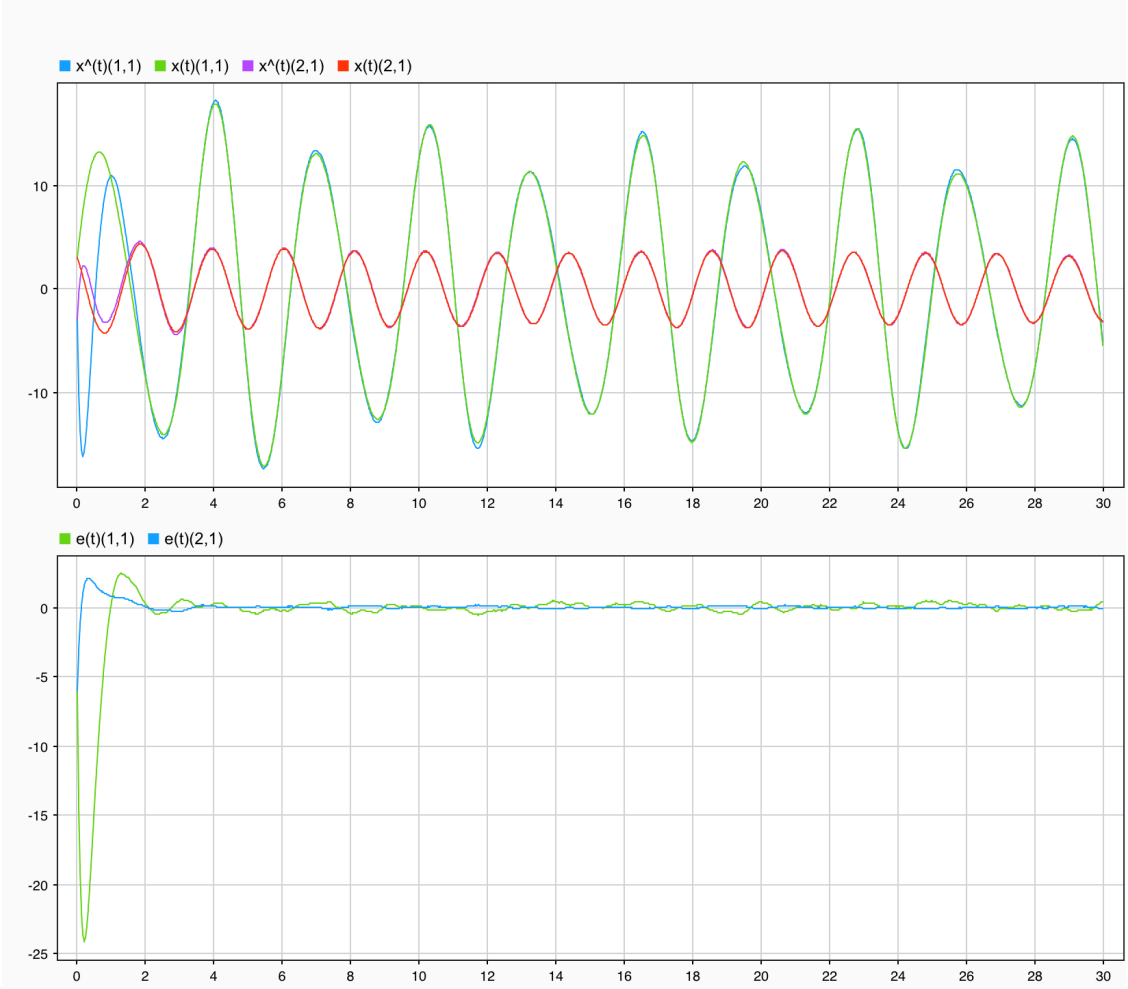
Med

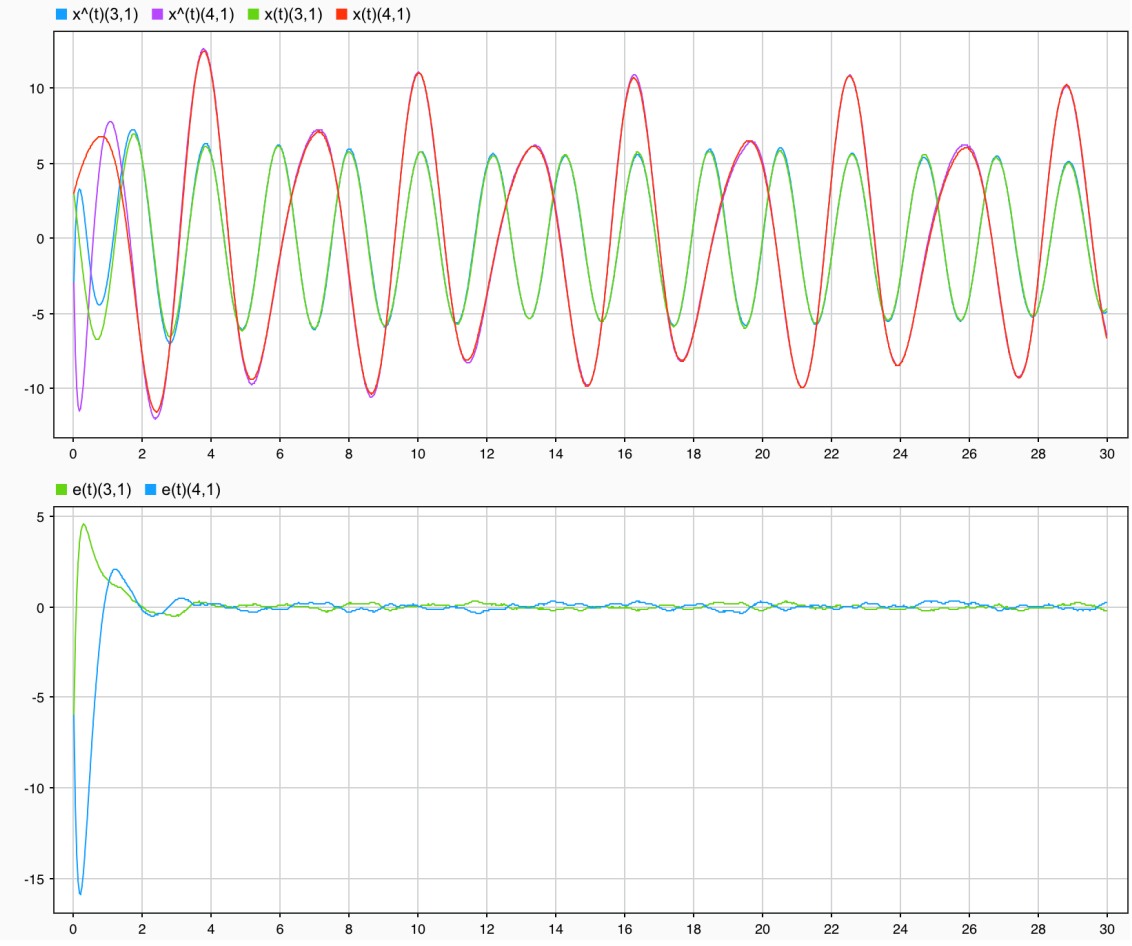


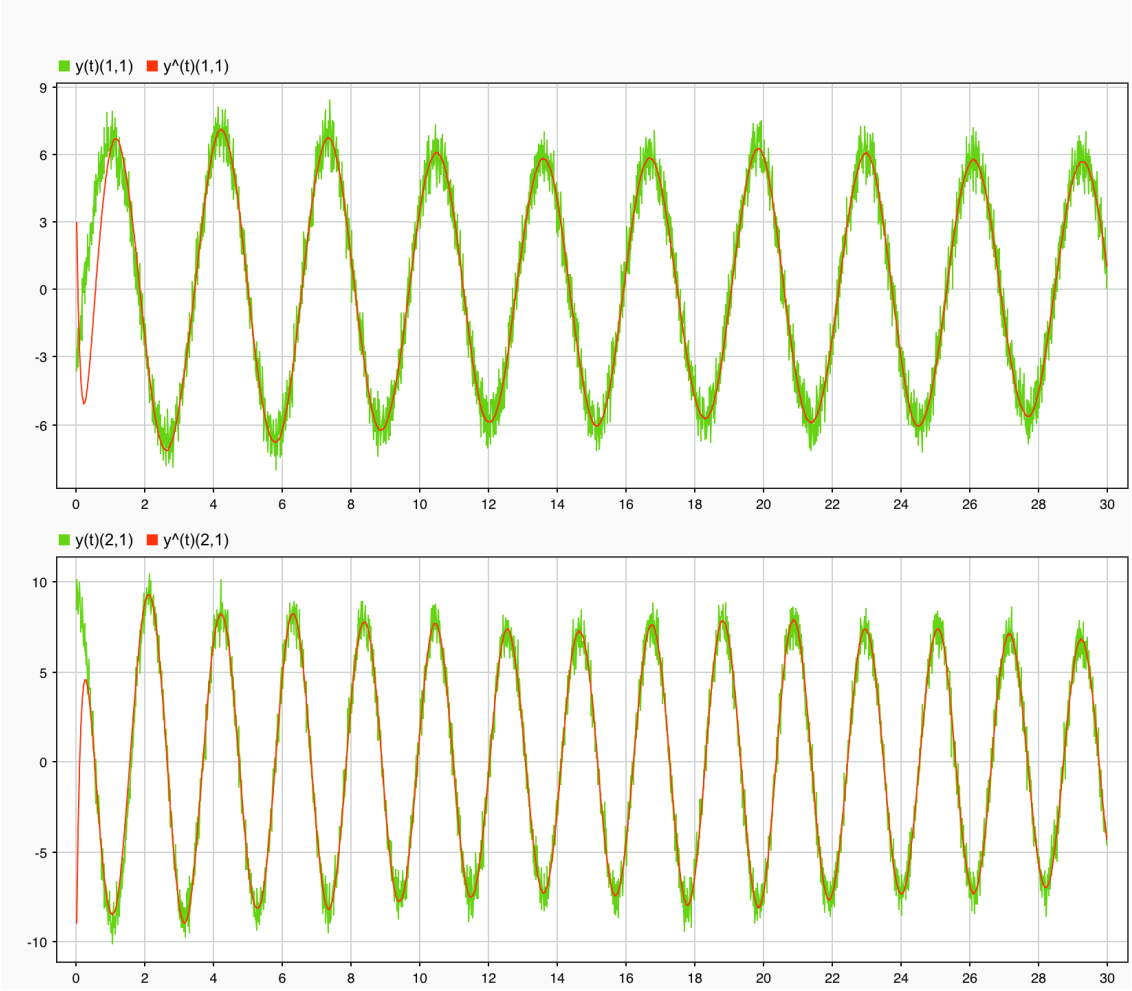
Hard



Возмущения соответствует критерию оптимальности для medium estimator.
$f \sim N(\mu = 0, \sigma^2 = 1/3), \qquad \xi = N(\mu = 0, \sigma^2 = 1/3)$

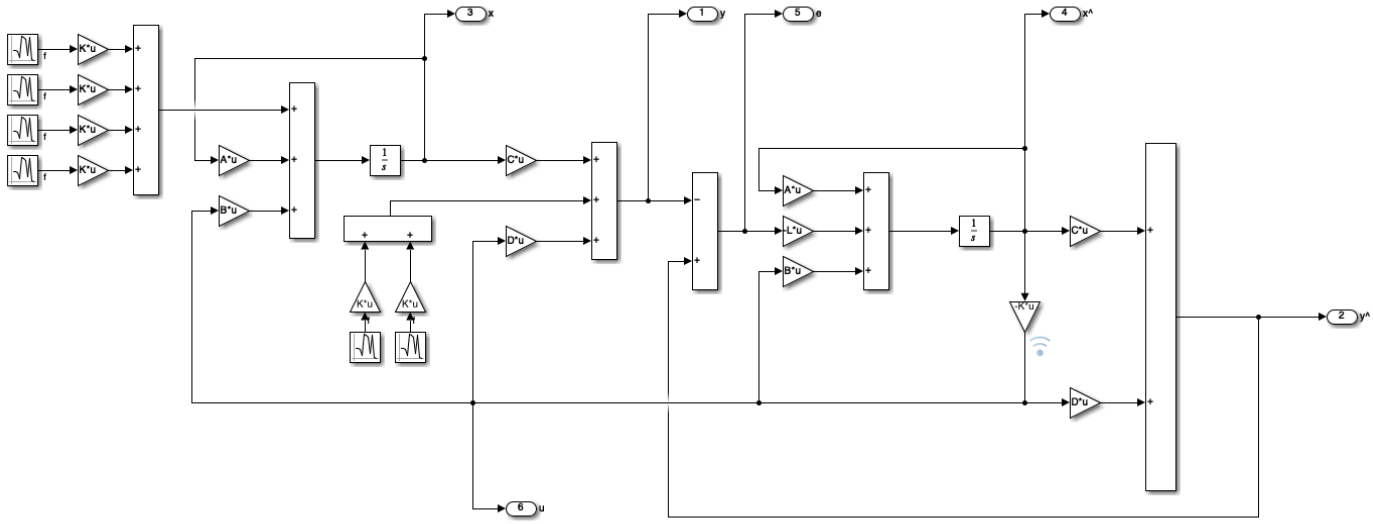






2.4. Синтез LGQ

Схема моделирования:



Программный код:

Init data

```
A = [ -6,  19, -13, 10;
       0,  -9,   6,  0;
       0, -15,   9,  0;
      -4,   8,  -7,  6];
```

```
B = [ -2, 0;
       2, 0;
       4, 0;
       0, 0];


C = [1, -3,  2, -1;
     0,  6, -3,  0];


D = [0, 0;
     0, 4];


G = [1, 0, 0, 0;
     0, 1, 0, 0;
     0, 0, 1, 0;
     0, 0, 0, 1];


x0_1 = [3; 3; 3; 3];
x0_2 = [-3;-3;-3;-3];
```

LQR

```
Q1 = [77, 0, 0, 0;
      0, 77, 0, 0;
      0, 0, 77, 0;
      0, 0, 0, 77];


R1 = [77, 0;
      0, 77];
```

Наблюдаемость

```
V = [Q1; Q1*A; Q1*A*A; Q1*A*A*A];
rank(V)
```

```
ans = 4
```

Синтез LQR

```
[K, P_lqr, e_lqr] = lqr(A, B, Q1, R1)
```

```
K = 2×4
    3.2598   -7.7697    7.3175   -5.8081
   -0.0000    0.0000   -0.0000    0.0000
P_lqr = 4×4
10^3 ×
    0.5761   -1.0329    0.8673   -0.9568
   -1.0329    2.3550   -1.8435    1.9317
    0.8673   -1.8435    1.4963   -1.5560
   -0.9568    1.9317   -1.5560    1.8047
e_lqr = 4×1 complex
  -0.7286 + 2.4083i
  -0.7286 - 2.4083i
  -2.8769 + 2.0019i
  -2.8769 - 2.0019i
```

Min J

```
min_J = x0_1'*P_lqr*x0_1
```

```
min_J = 9.4619e+03
```

LQE

```
Q2 = [7, 0, 0, 0;
      0, 23, 0, 0;
      0, 0, 24, 0;
      0, 0, 0, 35];


R2 = [2, 0;
      0, 77];
```

Управляемость

```
U = [Q2 A*Q2 A*A*Q2 A*A*A*Q2];
rank(U)
```

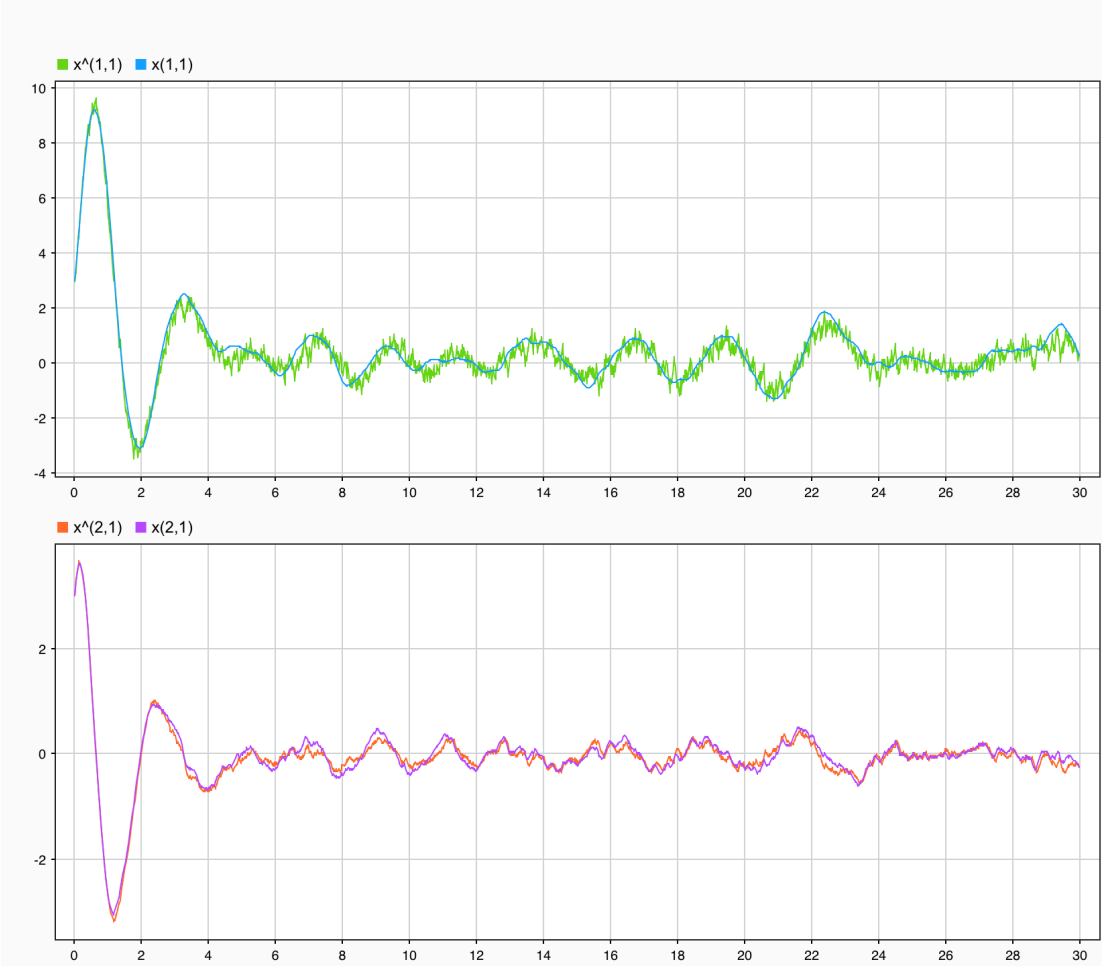
```
ans = 4
```

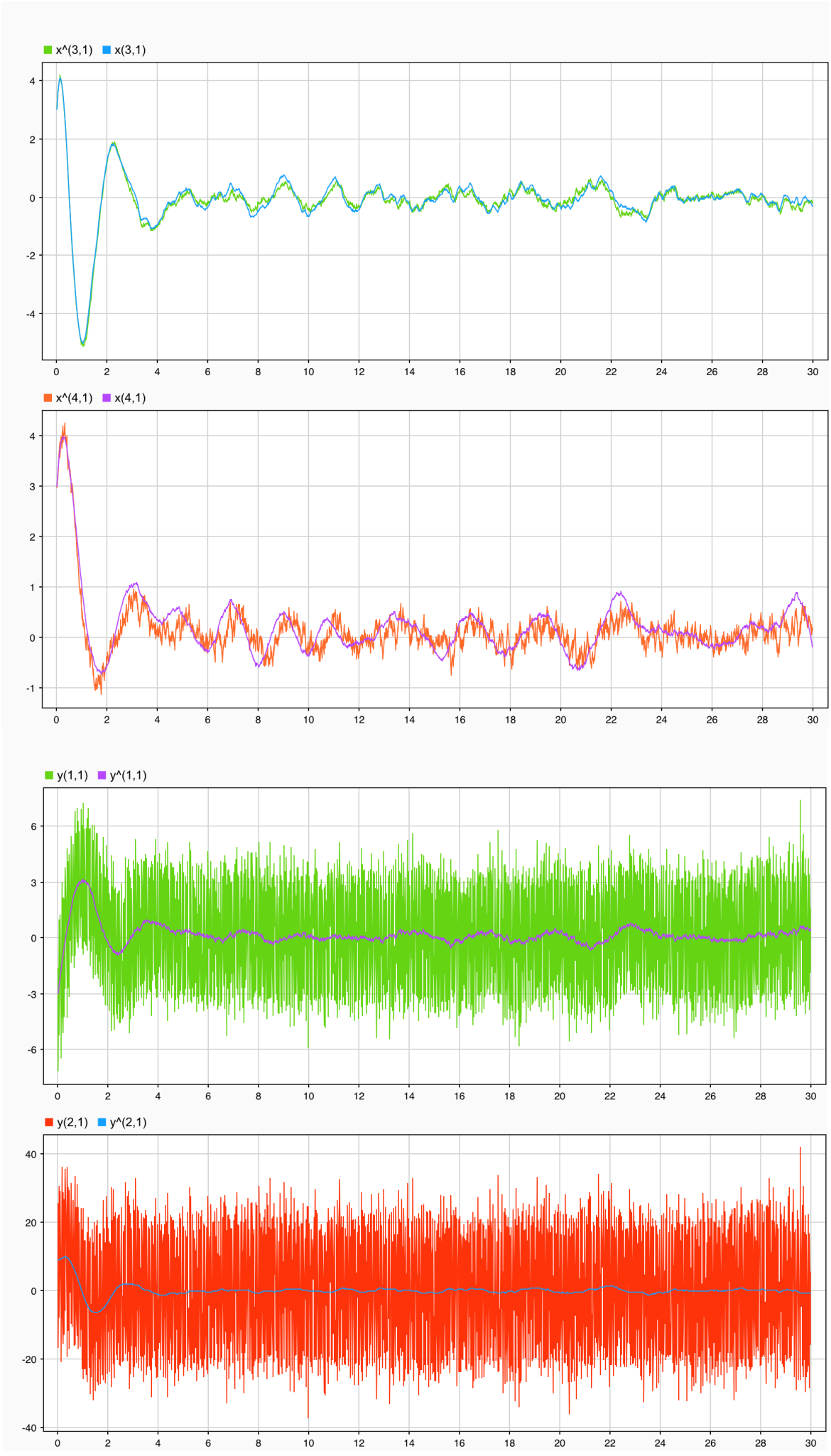
Синтез LQE

```
[L, P_lqe, e_lqe] = lqe(A, G, C, Q2, R2)
```

```
L = 4×2
   42.0726    0.2571
  -10.5354    0.8091
  -15.4519    0.8003
   27.4906   -0.1317
P_lqe = 4×4
  786.9353 -163.6463 -333.8921  525.9446
 -163.6463   83.1001  145.4332 -101.0095
 -333.8921  145.4332  270.3245 -198.6390
  525.9446 -101.0095 -198.6390  376.7138
e_lqe = 4×1 complex
  -0.9058 + 2.9413i
  -0.9058 - 2.9413i
  -3.0188 + 0.0000i
 -12.9076 + 0.0000i
```

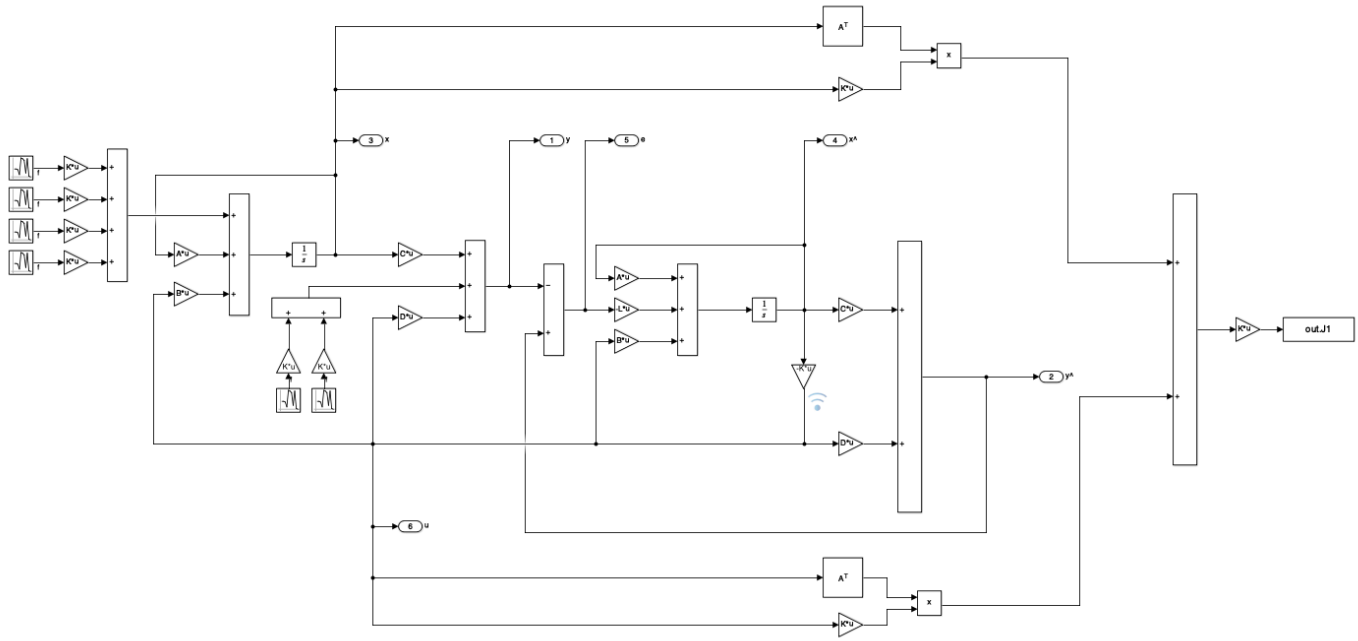
Сравнительные графики:




x^(3,1)
x(3,1)
x^(4,1)
x(4,1)
y(1,1)
y^(1,1)
y(2,1)
y^(2,1)

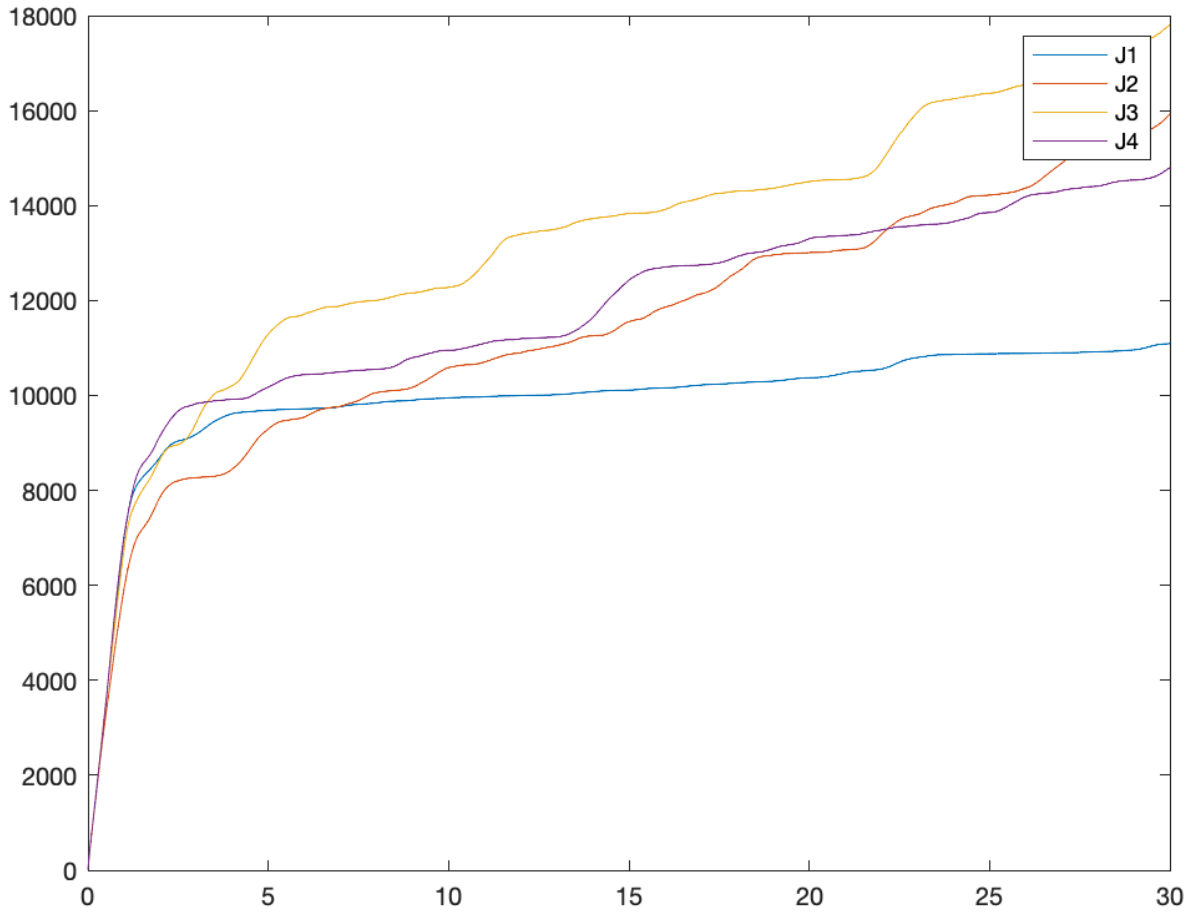
2.5. Маленькое исследование LQG

Схема моделирования:



J1: Seed = 1 для всех J2: Seed = i J3: Seed = i*35 / i*77
J4: Seed = {7; 77; 777; 777} / {35; 3535}

Графики J:



3. Вывод

В ходе лабораторной работы был произведен анализ LQR, LQE, LQG с использованием различных параметров. Так же проведено сравнение LQR с другими регуляторами и рассмотрение различных seed для LQG.