



Publication Bias in Meta-Analysis

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Cochrane Library

Database of high-quality, systematic reviews in clinical science.

Currently ~ 8,000 reviews, prepared by independent groups.

Reviews are peer-reviewed and prepared after guidelines.



Cochrane Library Dataset

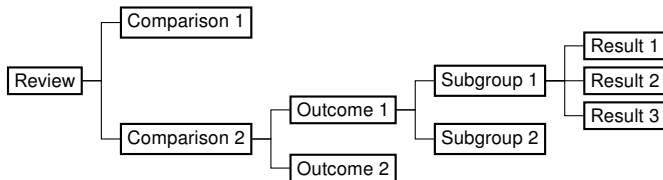
5,016 systematic reviews with studies published until 2018.

52,995 studies.

463,820 study results.



Dataset Structure





Dataset Structure

- Comparison: What is compared, e.g. treatment vs. control
- Outcome: How it is compared
- Subgroup: Subgroup affiliation
- ~~Meta-Analysis Group~~: Results from same comparison, outcome and subgroup



Review Example: binary outcome

Barbiturate efficacy for head injury treatment

Study	Comparison	Outcome	Events	Total	Events_c	Total_c
Bohn 1989	Barbiturate vs no b	Death at the end of	11	41	11	41
Bohn 1989	Barbiturate vs no b	Death or severe dis	18	41	13	41
Eisenberg 1988	Barbiturate vs no b	Uncontrolled ICP du	25	37	30	36
Eisenberg 1988	Barbiturate vs no b	Hypotension during	23	37	18	36
Perez-Barcena 2008	Pentobarbital vs Th	Death at the end of	16	21	9	21
Perez-Barcena 2008	Pentobarbital vs Th	Death or severe dis	17	21	13	21
Perez-Barcena 2008	Pentobarbital vs Th	Uncontrolled ICP du	18	22	11	22
Perez-Barcena 2008	Pentobarbital vs Th	Hypotension during	20	22	21	22
Schwartz 1984	Barbiturate vs Mann	Death at the end of	6	15	7	14
Schwartz 1984	Barbiturate vs Mann	Uncontrolled ICP du	19	28	12	31
Ward 1985	Barbiturate vs no b	Mean ICP during tre	0	27	0	26
Ward 1985	Barbiturate vs no b	Mean arterial press	0	27	0	26
Ward 1985	Barbiturate vs no b	Mean body temperatu	0	27	0	26



Dataset Properties

Missing data:


Missing mean values and mean differences	984
Missing standard deviations and standard errors	1300
Missing sample sizes	12173
Missing study year	44649





Dataset Properties

Review and study properties:



	5% quantile	median	mean	95% quantile
Study number	1	7	12	40
Comparison number	1	2	4	12
Group number	2	19	37	132
Study years	1981	2002	2000	2013
Study sample size	13	78	750	890



Pooling Studies - Meta-Analysis

Results are pooled in a meta-analysis

Multiple results in a meta-analysis group can be pooled:

n	Number of groups	Cumulative sum of groups
1	102344	188079
2	31686	85735
3	16072	54049
4	9628	37977
5	6444	28349
6	4230	21905
7	2961	17675
8	2114	14714
9	1592	12600
10	11008	11008

1)
%



Meta-analysis

Benefits:

- Summary of evidence (e.g. of a treatment)
- More reliable evidence (?)

Assumptions:

- Identical study settings (can be relaxed)
- Random sample of studies



Small Study Effects

“The tendency for the smaller studies to show larger treatment effects” ([Sterne et al., 2001](#))



Small Study Effects

Causes:

- Selective publication of studies with large effects
- Bias in smaller studies
- Systematical differences in study settings
- ...



Small Study Effect Tests

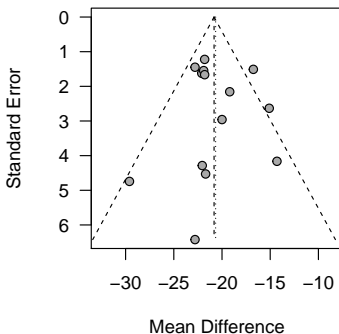
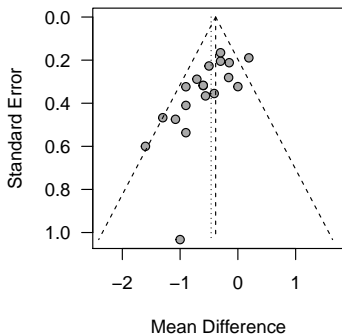
Tests applicable if:

- ~~n large~~
- variation in the estimated variances of effects
(here: $\frac{\sigma_{\max}}{\sigma_{\min}} > 4$)

Adjustments required if variance is dependent on effect size
(e.g. log odds ratios)

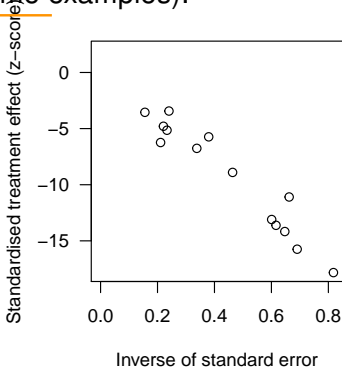
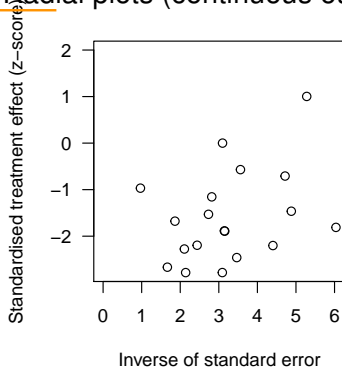
Small Study Effect Tests

Funnel plots (continuous outcome examples):



Regression based Tests

Radial plots (continuous outcome examples):





Regression based Tests

Continuous outcomes tests:

i being the i th study in a meta-analysis:

Let $y_i = \text{effect}_i / \text{se}_i$ and $x_i = 1 / \text{se}_i$

- Egger et al. (1997) :

$$y_i \sim N(\beta_0 + \beta_1 x_i, v_i)$$

- Thompson and Sharp (1999) :

$$y_i \sim N(\beta_0 + \beta_1 x_i, v_i + \tau^2)$$

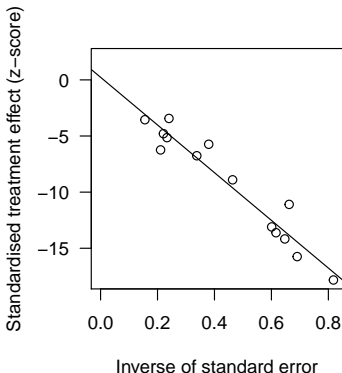
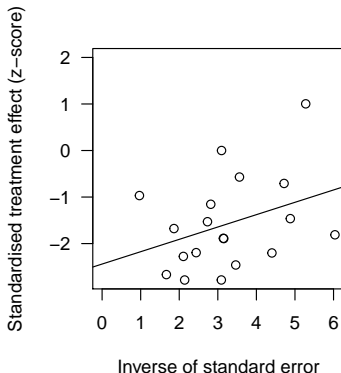


```
## sample estimates:
```

```
##          bias      se.bias      slope
```

```
## -2.4398295  0.5958069  0.2653786
```

Egger's Tests

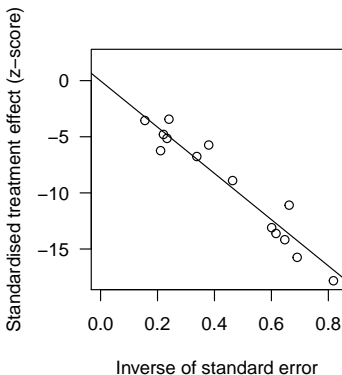
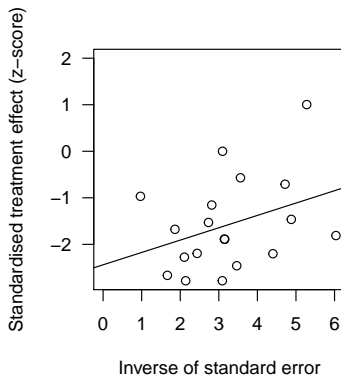


```
##
```



```
## sample estimates:
##          bias      se.bias      slope
## -2.4398295    0.6210298    0.2653786
```

Thompson and Sharp's Tests



```
##
```



Regression based Tests

Adjustments for binary outcomes:

- **Peters et al. (2006)** : x_i = inverse of total sample size, variance_i as weight.
- **Harbord et al. (2006)** : $x : i$ = score of the log-likelihood of a proportion, variance_i as weight.
- **?** : Use arcsine variance stabilizing transformation for variances and effects, do e.g. Egger's test.



Rank based tests

Begg (1988) Let y_i the standardized effect size of a study i , v_i it's variance and n the number of studies

u the number of pairs (y_i, v_i) ranked in the same order, l the number of pairs in the opposite order

$$Z = (u - l) / \sqrt{n(n-1)(2n+5)/18}$$

can be used as a test statistic (based on Kendall's Tau)



Rank based tests

Schwarzer et al. (2007) Let e_t the number of events in the treatment group

Given constant log odds ratio, E_t follows a hypergeometric distribution.

$\mathbb{E}(E_t)$ and the variance can be estimated and used as in Begg (1988)



Test Results

Application of tests if:

- $n \geq 10$
- at least one statistically significant effect in a study
- $\frac{\sigma_{\max}}{\sigma_{\min}} > 4$
- $I^2 < 0.5$

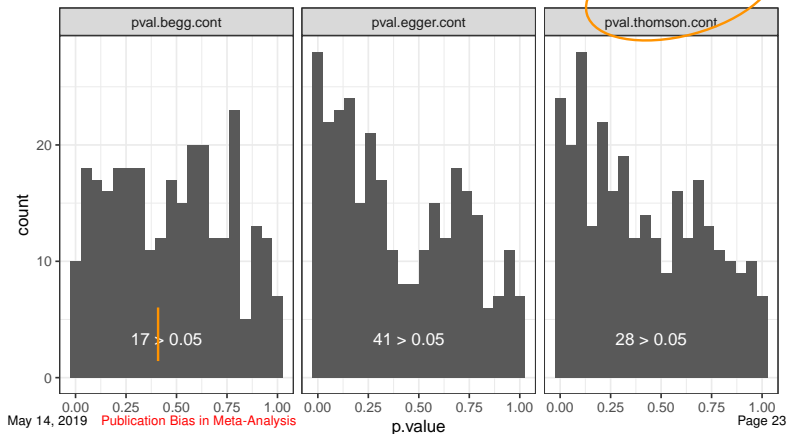
100%

From 5338 with $n \geq 10$, 1602 remain.



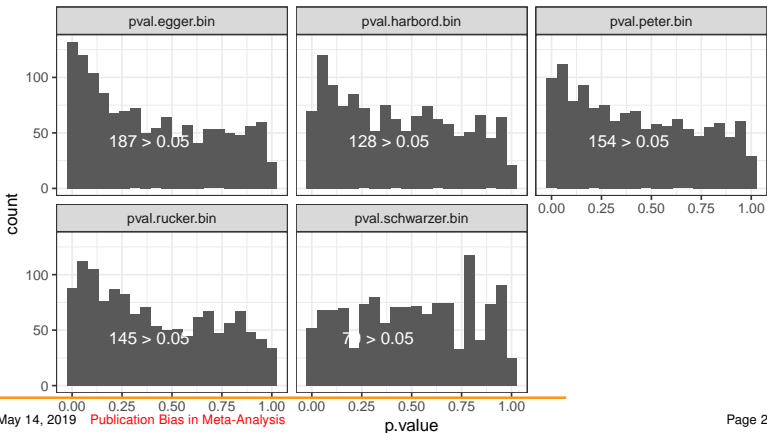
Continuous Outcome Test Results

p -values distribution:



Binary Outcome Test Results

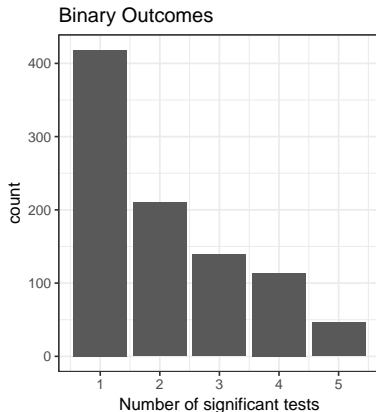
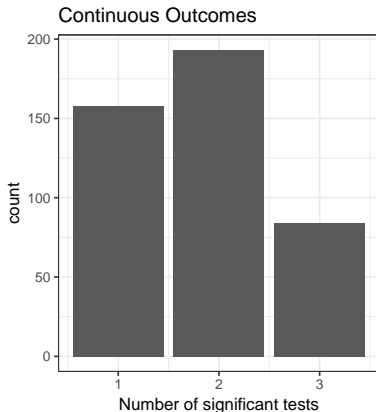
p -values distribution:





Agreement in significance

Number of significant test results per meta-analysis:





Small Study Effect Adjustment

Three methods:

- Regression
- Copas selection model
- Trim-and-fill



Adjustment by regression

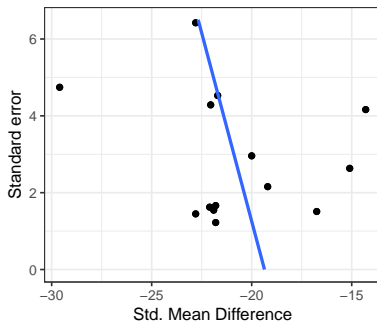
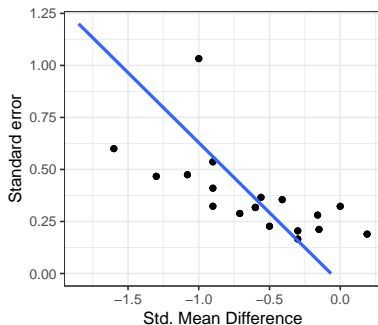
Similar to the tests, but with unnormalized effect y_i :

$$y_i = \beta_0 + \beta_1 x_i \quad (1)$$

β_0 corresponds to y_i with $x_i = 0$

Adjustment by regression

Linear regression method:





Shrinkage Regression

Extended random effects model:

$$y_i = \beta_0 + \beta_1(\sqrt{v_i + \tau^2}) + \epsilon_i(\sqrt{s_i + \tau^2}), \epsilon_i \stackrel{\text{iid}}{\sim} N(0, 1)$$

$$\mathbb{E}(y_i) \rightarrow \beta_0 + \beta_1\tau \text{ if } \sqrt{v_i} \rightarrow 0$$

β_0, β_1 and τ can be estimated e.g. by ML and REML.

Shrinkage Regression

shrinking the within study variance:

$$y_i = \beta_0^* + \beta_1^*(\sqrt{v_i/M + \tau^2}) + \epsilon_i(\sqrt{s_i/M + \tau^2})$$

Letting $M \rightarrow \infty$ and substituting for all parameters and the observed residual

$$y_{\infty,i} = \beta_0^* + \sqrt{\frac{\tau^2}{v_i + \tau^2}}(y_i - \beta_0^*) \quad (2)$$



Shrinkage Regression

Three different treatment effect estimates:

$$y_{\infty,i} = \beta_0^* + \sqrt{\frac{\tau^2}{v_i + \tau^2}}(y_i - \beta_0^*) \quad (3)$$



References

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- Egger, M., Smith, G. D., Schneider, M., and Minder, C. (1997). Bias in meta-analysis detected by a simple, graphical test. *BMJ*, 315(7109):629–634.
- Harbord, R. M., Egger, M., and Sterne, J. A. C. (2006). A modified test for small-study effects in meta-analyses of controlled trials with binary endpoints. *Statistics in Medicine*, 25(20):3443–3457.
- Peters, J., Sutton, A., R Jones, D., Abrams, K., and Rushton, L. (2006). Comparison of two methods to detect publication bias in meta-analysis. *JAMA : the journal of the American Medical Association*.