EC 415: Homework 2

Due by Friday 03/05/2021 6:00PM

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Exercise 3.1

Use specsquare.m to investigate the relationship between the time behavior of the square wave and its spectrum. The Matlab command zoom on is often helpful for viewing details of the plots.

a. Try square waves with different frequencies: f=20, 40, 100, 300 Hz. How do the time plots change? How do the spectra change?

b. Try square waves of different lengths, time=1, 10, 100 seconds. How does the spectrum change in each case?

c. Try different sampling times, T_s=1/100, 1/10000 seconds. How does the spectrum change in each case?

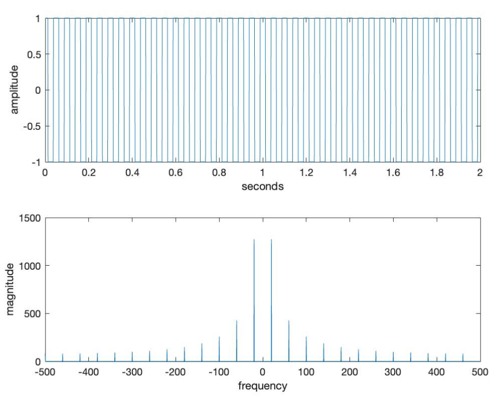
Solution

To obtain all solutions to this exercise, a copy of the provided specsquare.m was used. For each part of the exercise, the relevant variable was changed before re-running the script and generating a plot.

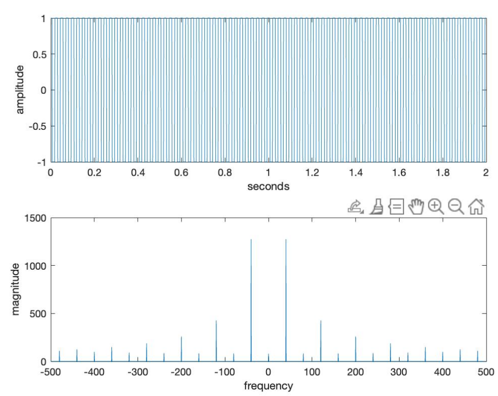
Listing 1: MATLAB code for Exercise 3.1

```
% specsquare.m plot the spectrum of a square wave
f = 10;                    % "frequency" of square wave, 10
time = 2;                  % length of time, 2
Ts = 1/10000;              % time interval between samples, 1/1000
t = Ts:Ts:time;            % create a time vector
x = sign(cos(2*pi*f*t));   % square wave = sign of cos wave
plotspec(x,Ts)             % call plotspec to draw spectrum
```

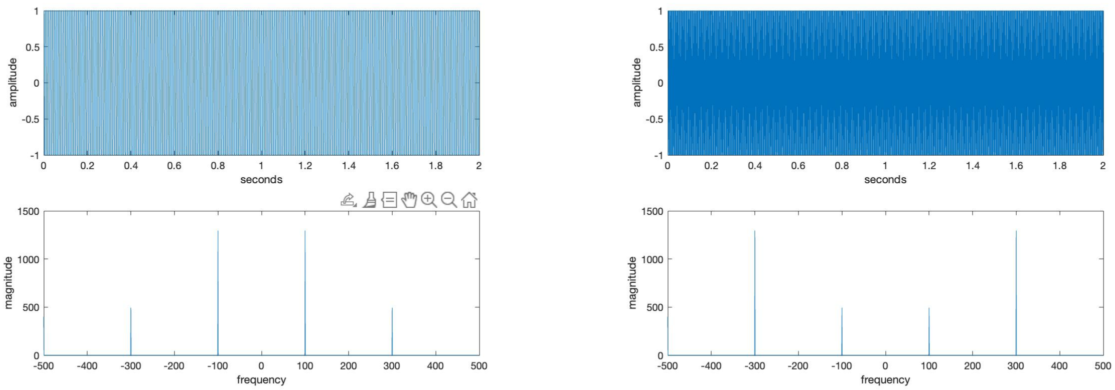
a. As the frequency increases, the density of square waves on the time plot increases. TODO how do the spectra change



(a) f=20Hz



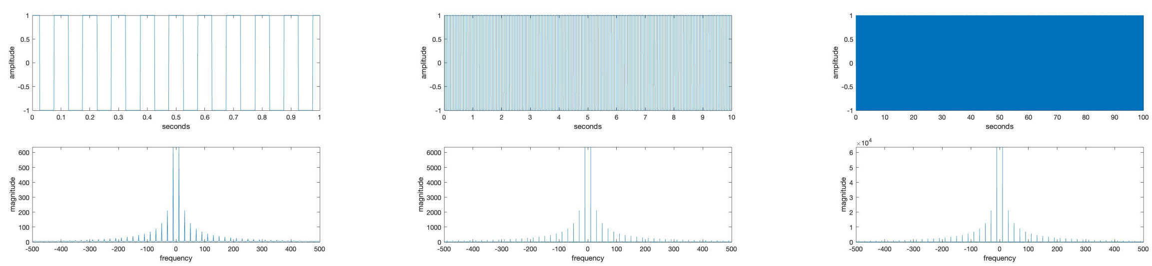
(b) f=40Hz



(a) f=100Hz (b) f=300Hz

Figure 2: square waves of increasing frequency

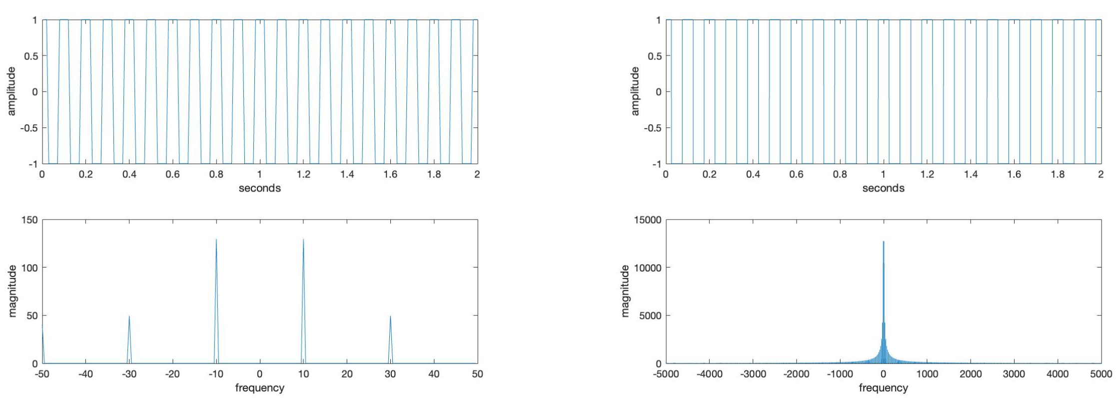
b. As the length of time increases, the magnitude of the spectra increases.



(a) t=1sec (b) t=10sec (c) t=100sec

Figure 3: longer running square waves.

c. As the sampling time decreases, the sampling frequency increases. This means the spectrum plot is a more complete picture with more plotted spectra. However, this does not change the actual shape of the spectrum.



(a) T_s=1/100 (b) T_s=1/10000

Figure 4: square waves of increasing sample rate

Exercise 3.3

Mimic the code in specsquare.m to find the spectrum of:

a. An exponential pulse $s(t) = e^{-t}$ for $0 < t < 10$

b. A scaled exponential pulse $s(t) = 5e^{-t}$ for $0 < t < 10$

c. A Gaussian pulse $s(t) = e^{-t^2}$ for $-2 < t < 2$

d. A Gaussian pulse $s(t) = e^{-t^2}$ for $-20 < t < 20$

e. The sinusoids $s(t) = sin(2\pi ft + \phi)$ for $f = 20, 100, 1000$ with $\phi = 0, \pi/4, \pi/2$ and $0 < t < 10$

Solution

a. Here is the MATLAB code:

Listing 2: MATLAB code for part a

```
% 3.3 PART A
time = 10; % run fo 10 sec
Ts = 1/10000; % ms timescale
t = Ts:Ts:time; % time vector
x = exp(-t); % signal
plotspec(x, Ts) % plot spectrum
```

The resulting plot is shown in the figure below.

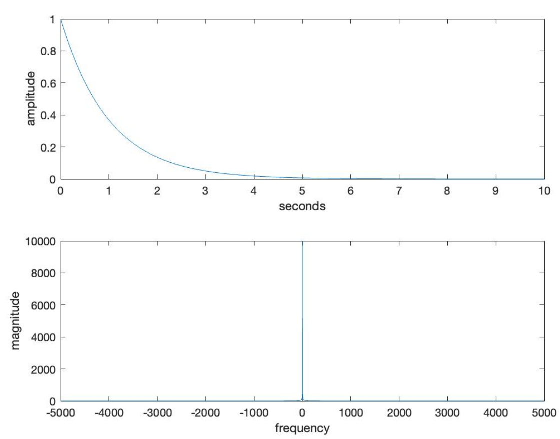


Figure 5: p3.3a d

b. Here is the MATLAB code:

Listing 3: MATLAB code for part b

```
%p3_3b.m
time = 10; % run fo 10 sec
Ts = 1/1000; % ms timescale
t = Ts:Ts:time; % time vector
x = 5*exp(-t); % signal
plotspec(x, Ts) % plot spectrum
```

The resulting plot is shown in the figure below.

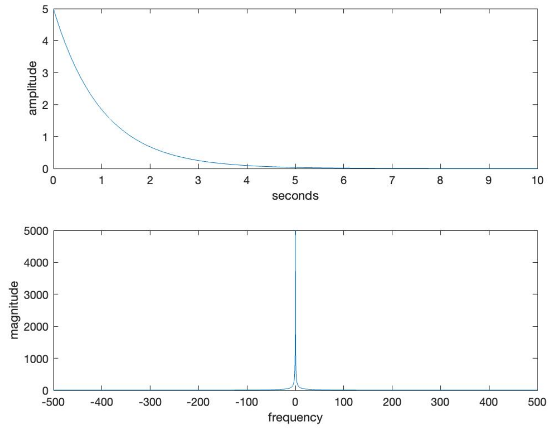


Figure 6: p3.3b

c. Here is the MATLAB code:

Listing 4: MATLAB code for part c

```
%p3_3c.m
time = 2; % run fo 10 sec
Ts = 1/1000; % ms timescale
t = (0-time):Ts:time; % time vector from -time to time
x = 5*exp(-t); % signal
plotspec(x, Ts) % plot spectrum
```

The resulting plot is shown in the figure below.

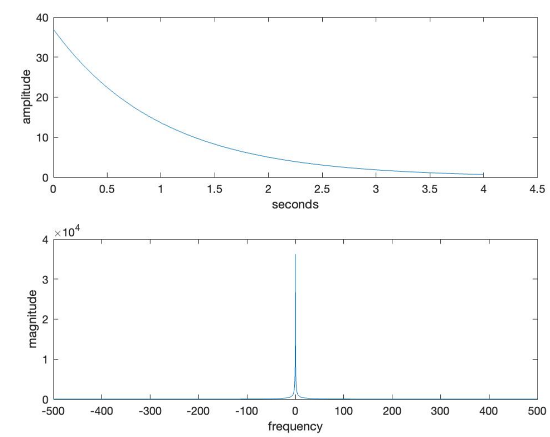


Figure 7: p3.3c

d. Here is the MATLAB code:

Listing 5: MATLAB code for part d

```
%p3_3d.m
time = 20; % run fo 10 sec
Ts = 1/1000; % ms timescale
t = (0-time):Ts:time; % time vector from -time to time
x = 5*exp(-t); % signal
plotspec(x, Ts) % plot spectrum
```

The resulting plot is shown in the figure below.

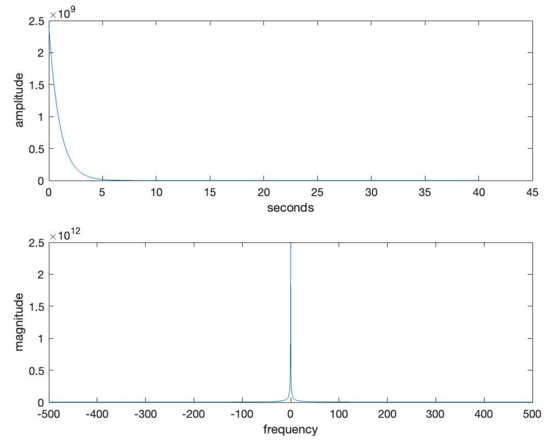


Figure 8: p3.3d

e. Here is the MATLAB code: TODO

Exercise 3.6

Mimic the code in speccos.m to find the spectrum of a cosine wave

a. For different frequencies f=1, 2, 20, 30 Hz

b. for different phases $\phi = 0$, 0.1, $\pi/8$, $\pi/2$ radians

c. For different sampling rates T_s=1/10, 1/1000, 1/100000.

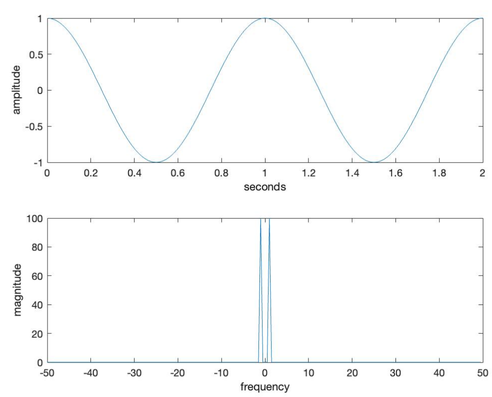
Solution

To obtain all solutions to this exercise, a copy of the provided speccos.m was used. For each part of the exercise, the relevant variable was changed before re-running the script and generating a plot.

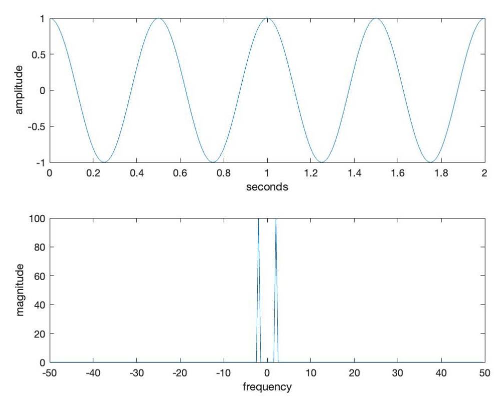
Listing 6: MATLAB code for Exercise 3.6

```
% speccos.m plot the spectrum of a cosine wave
f=10; phi=0;                  % specify frequency and phase
time=2;                       % length of time
Ts=1/100;                   % time interval between samples
t=Ts:Ts:time;                 % create a time vector
x=cos(2*pi*f*t+phi);          % create cos wave
plotspec(x,Ts)                % draw waveform and spectrum
```

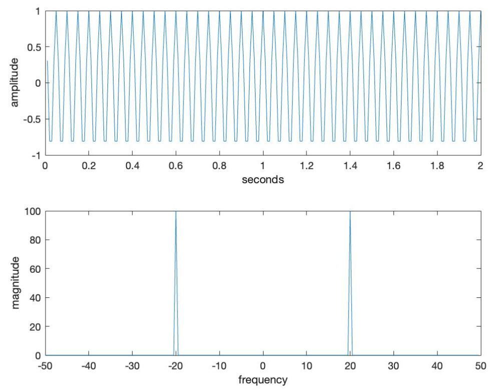
a. A



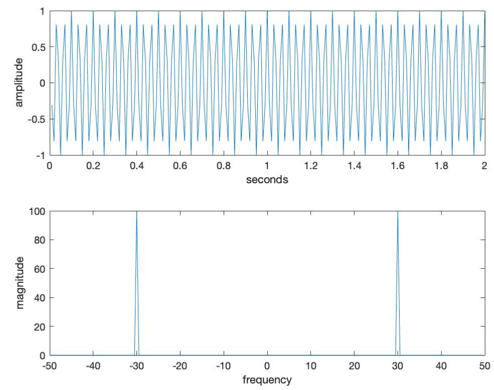
(a) f=1Hz



(b) f=2Hz



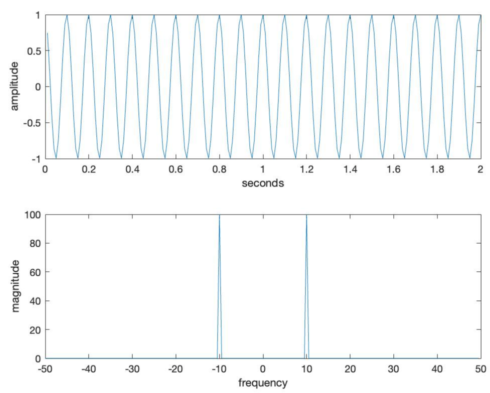
(a) f=20Hz



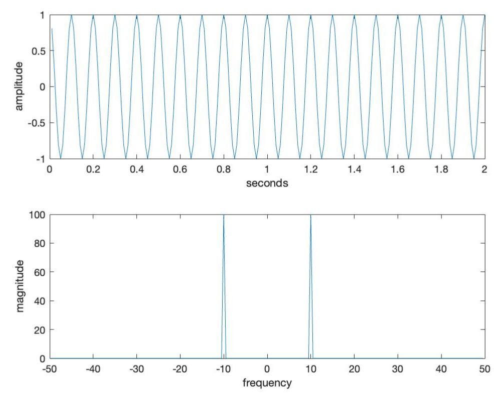
(b) f=30Hz

Figure 10: square waves of increasing frequency

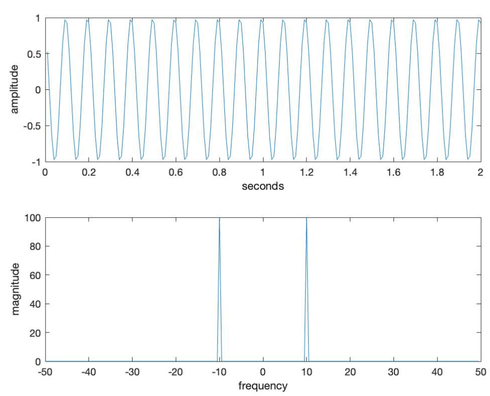
b. B



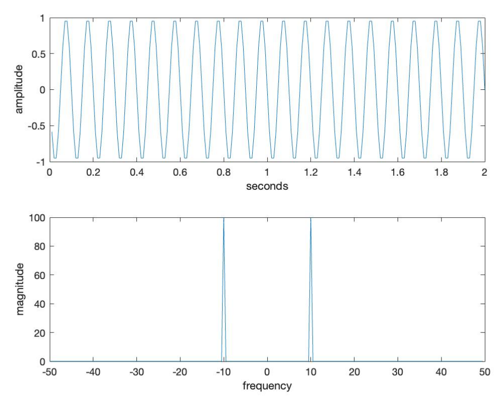
(a) phi=0



(b) phi=2Hz



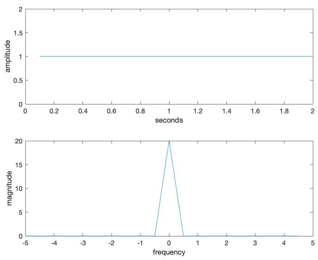
(a) $phi=pi/8$



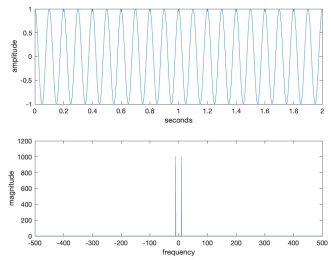
(b) $phi=pi/2$

Figure 12: square waves of increasing frequency

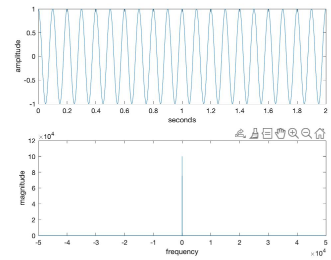
c. C



(a) T_s=1/10



(b) T_s=1/1000



(c) T_s=1/100000

Figure 13: longer running square waves.

Exercise 3.9

Mimic the code in filternoise.m to create a filter that

a. Passes all frequencies above 500 Hz

b. Passes all frequencies below 3000 Hz

c. Rejects all frequencies between 1500 and 2500 Hz

Solution

a. A

Listing 7: MATLAB code for Exercise 3.9a

```
% filternoise.m filter a noisy signal three ways
time=3;                             % length of time
Ts=1/10000;                         % time interval between samples
x=randn(1,time/Ts);                 % generate noise signal
figure(1),plotspec(x,Ts)            % draw spectrum of input

freqs=[0 0.74 0.76 1];
amps=[0 0 1 1];
b=firpm(100,freqs,amps);            % specify the HP filter
yhp=filter(b,1,x);                  % do the filtering
figure(4),plotspec(yhp,Ts)          % plot the output spectrum
```

b. B

Listing 8: MATLAB code for Exercise 3.9b

```
% filternoise.m filter a noisy signal three ways
time=3;                             % length of time
Ts=1/10000;                         % time interval between samples
x=randn(1,time/Ts);                 % generate noise signal
figure(1),plotspec(x,Ts)            % draw spectrum of input

freqs=[0 0.2 0.21 1];
amps=[1 1 0 0];
b=firpm(100,freqs,amps);            % specify the LP filter
ylp=filter(b,1,x);                  % do the filtering
figure(2),plotspec(ylp,Ts)          % plot the output spectrum
```

c. C

Listing 9: MATLAB code for Exercise 3.9c

```
% filternoise.m filter a noisy signal three ways
time=3;                             % length of time
Ts=1/10000;                         % time interval between samples
x=randn(1,time/Ts);                 % generate noise signal
figure(1),plotspec(x,Ts)            % draw spectrum of input

freqs=[0 0.24 0.26 0.5 0.51 1];
```

```
amps=[0 0 1 1 0 0];
b=firpm(100,freqs,amps);          % BP filter
ybp=filter(b,1,x);                % do the filtering
figure(3),plotspec(ybp,Ts)        % plot the output spectrum
```

Exercise 3.10

Change the sampling rate to T_s=1/20000. Redesign the three filters from Exercise 3.9.

Solution

soln here

Exercise 3.11

Let $x_1(t)$ be a cosine wave of frequency f = 800, $x_2(t)$ be a cosine wave of frequency f = 2000, and $x_3(t)$ be a cosine wave of frequency f = 4500. Let $x(t) = x_1(t) + 0.5 * x_2(t) + 2 * x_3(t)$. Use x(t) as input to each of the three filters in filternoise.m. Plot the spectra, and explain what you see.

Solution

Listing 10: MATLAB code for Exercise 3.11

```
% filternoise.m filter a noisy signal three ways
time=3;                         % length of time
Ts=1/10000;                     % time interval between samples
x=randn(1,time/Ts);             % generate noise signal
figure(1),plotspec(x,Ts)        % draw spectrum of input

freqs=[0 0.2 0.21 1];
amps=[1 1 0 0];
b=firpm(100,freqs,amps);        % specify the LP filter
ylp=filter(b,1,x);              % do the filtering
figure(2),plotspec(ylp,Ts)      % plot the output spectrum

freqs=[0 0.24 0.26 0.5 0.51 1];
amps=[0 0 1 1 0 0];
b=firpm(100,freqs,amps);        % BP filter
ybp=filter(b,1,x);              % do the filtering
figure(3),plotspec(ybp,Ts)      % plot the output spectrum

freqs=[0 0.74 0.76 1];
amps=[0 0 1 1];
b=firpm(100,freqs,amps);        % specify the HP filter
yhp=filter(b,1,x);              % do the filtering
figure(4),plotspec(yhp,Ts)      % plot the output spectrum

%Here's how the figure filternoise.eps was actually drawn
N=length(x);                    % length of the signal x
t=Ts*(1:N);                     % define a time vector
ssf=(-N/2:N/2-1)/(Ts*N);        % frequency vector
fx=fftshift(fft(x(1:N)));
figure(5), subplot(4,1,1), plot(ssf,abs(fx))
xlabel('magnitude spectrum at input')
fyl=fftshift(fft(ylp(1:N)));
subplot(4,1,2), plot(ssf,abs(fyl))
xlabel('magnitude spectrum at output of low pass filter')
fybp=fftshift(fft(ybp(1:N)));
subplot(4,1,3), plot(ssf,abs(fybp))
xlabel('magnitude spectrum at output of band pass filter')
fyhp=fftshift(fft(yhp(1:N)));
subplot(4,1,4), plot(ssf,abs(fyhp))
xlabel('magnitude spectrum at output of high pass filter')
```

TODO: change matlab script and get spectra and explain

Exercise 3.26

Mimic the code in modulate.m to find the spectrum of the output y(t) of a modulator block (with modulation frequency $f_c = 1000$ Hz) when

a. The input is $x(t) = cos(2 * \pi * f_1 * t) + cos(2 * \pi * f_2 * t)$ for $f_1 = 100$ and $f_2 = 150$ Hz

b. The input is a square wave with fundamental f = 150 Hz

c. The input is a noise signal with all energy below 300 Hz

Solution

a. A

b. B

c. C