

EC 415: Homework 3

Due by Friday 03/19/2021 6:00PM

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Exercise 3.18

Mimic the code in `speccos.m` with sampling interval $T_s=1/100$ to find the spectrum of a square wave with fundamental $f=10, 20, 30, 33, 43$ Hz. Can you predict where the spikes will occur in each case? Which of the square waves show aliasing?

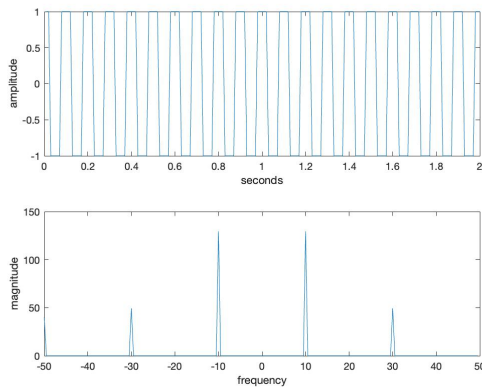
Solution

In each case the spikes occur at $+f$ and $-f$. In this example the sampling frequency is 100Hz, therefore the Nyquist frequency is 50Hz. In the images below, as the signal frequency approaches the Nyquist frequency, more aliasing occurs.

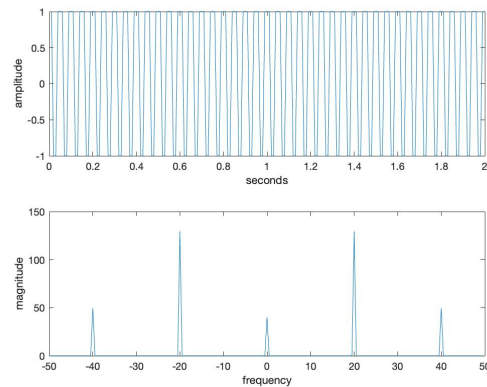
This was confirmed by using the following code and changing the value of f appropriately

Listing 1: MATLAB code for Exercise 3.18

```
f=10; phi=0;           % specify frequency and phase
time=2;                % length of time
Ts=1/100;              % time interval between samples
t=Ts:Ts:time;          % create a time vector
x=sign(cos(2*pi*f*t+phi)); % create cos wave
plotspec(x,Ts)         % draw waveform and spectrum
```



(a) $f=10\text{Hz}$



(b) $f=20\text{Hz}$

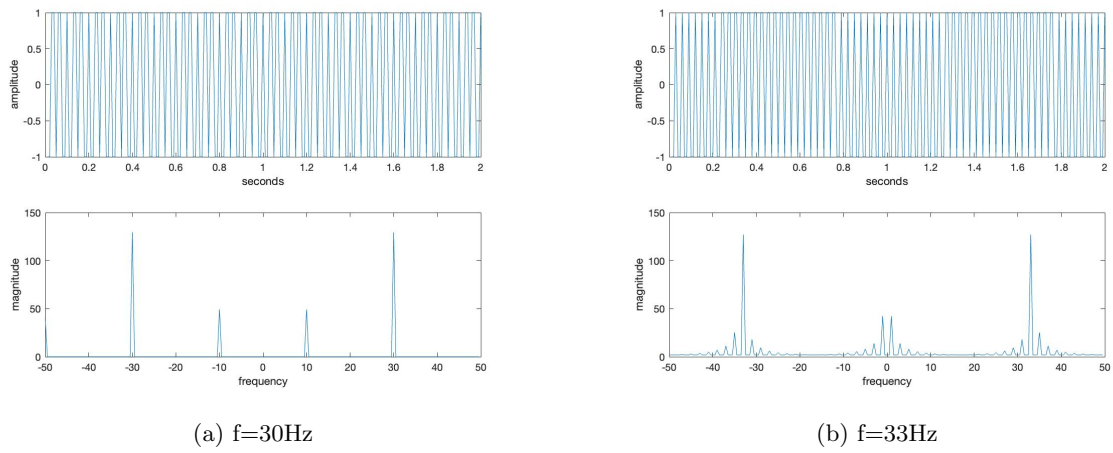
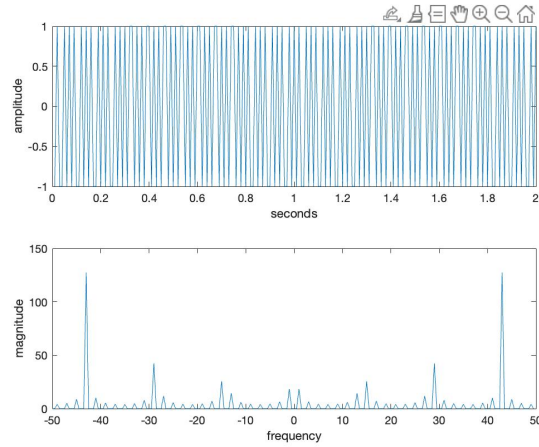


Figure 2

Figure 3: $f=43\text{Hz}$

Exercise 3.19

Mimic the code in speccos.m with $T_s=1/1000$ to find the spectrum of the output $y(t)$ of a squaring block when the input is $x(t) = \cos(2\pi f_1 t) + \cos(2\pi f_2 t)$ for $f_1=100$ and $f_2=150\text{Hz}$

Solution

Using this code

Listing 2: MATLAB code for Exercise 3.19

```
f1=100; f2=150;           % specify frequency
time=2;                   % length of time
Ts=1/1000;                % time interval between samples
t=Ts:Ts:time;             % create a time vector
x=cos(2*pi*f1*t) + cos(2*pi*f2*t); % create cos wave
y=x.*x;
plotspec(y,Ts)            % draw waveform and spectrum
```

This output was generated

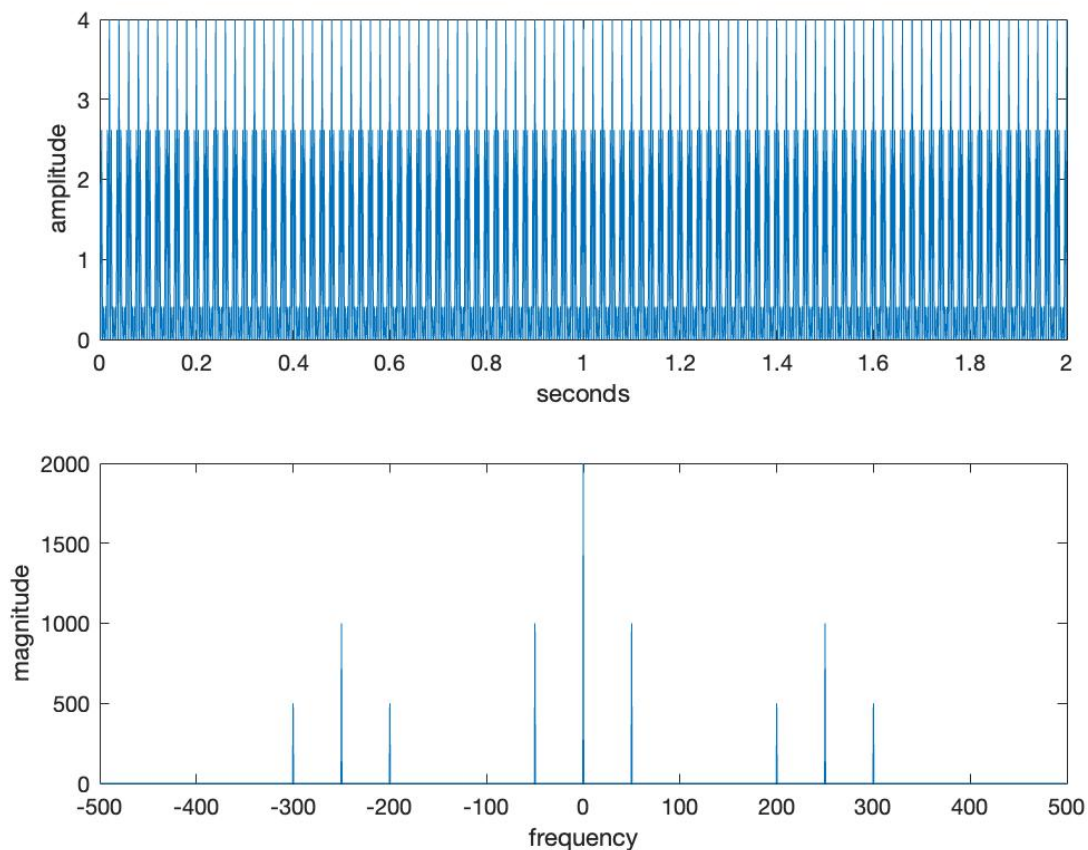


Figure 4: $x(t) = \cos(2\pi f_1 t) + \cos(2\pi f_2 t)$ for $f_1=100$ and $f_2=150\text{Hz}$

Exercise 3.20

TRUE or FALSE: The bandwidth of $x^4(t)$ cannot be greater than that of $x(t)$. Explain.

Solution

FALSE: for any given signal, $Bandwidth(x^n(t)) = n * Bandwidth(x(t))$.

Exercise 3.23

Suppose that the output of a nonlinear block is the rectification (absolute value) of the input $y(t) = |x(t)|$. Find the spectrum of the output when the input is $x(t) = \cos(2\pi f_1 t) + \cos(2\pi f_2 t)$ for $f_1 = 100$ and $f_2 = 125$ Hz

Solution

Using this code

Listing 3: MATLAB code for Exercise 3.23

```
f1=100; f2=125;           % specify frequency
time=2;                   % length of time
Ts=1/100;                 % time interval between samples
t=Ts:Ts:time;             % create a time vector
x=cos(2*pi*f1*t) + cos(2*pi*f2*t); % create cos wave
y = abs(x);               % Take abs value
plotspec(y,Ts)            % draw waveform and spectrum
```

This output was generated

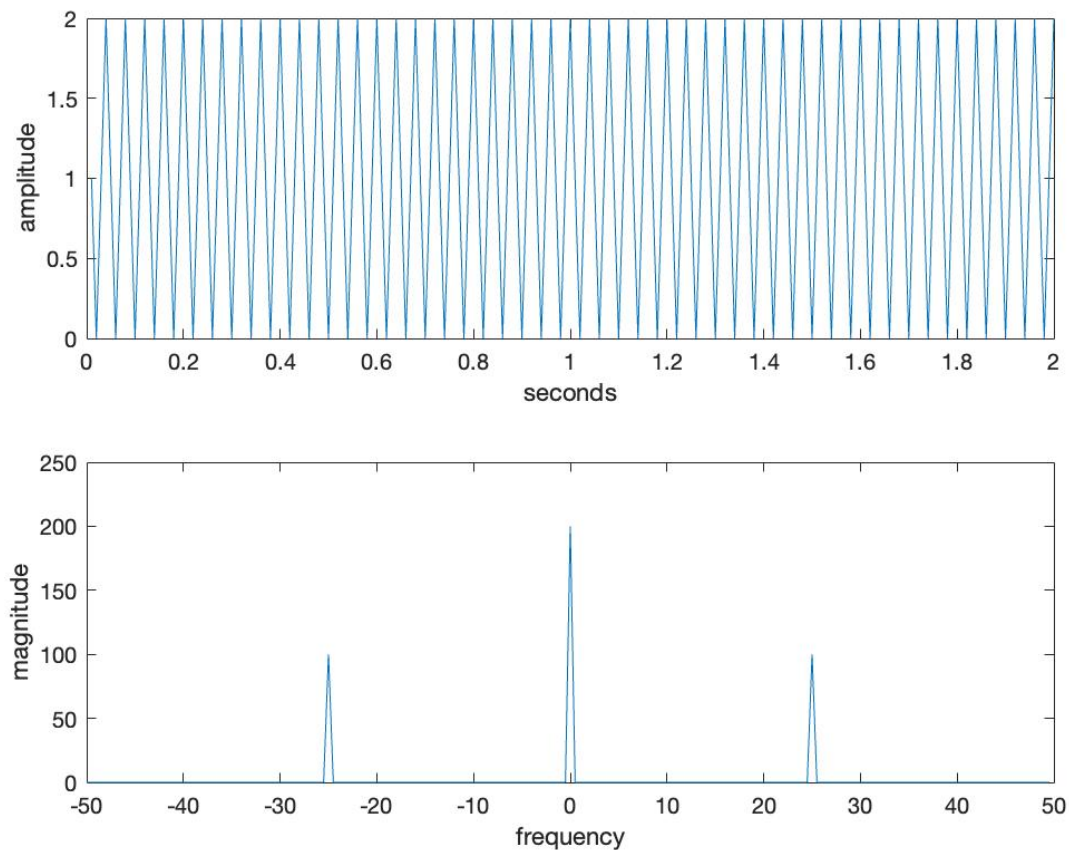


Figure 5: $x(t) = \cos(2\pi f_1 t) + \cos(2\pi f_2 t)$ for $f_1 = 100$ and $f_2 = 125$ Hz

Exercise 3.25

Quantization of an input is another kind of common nonlinearity. The Matlab function `quantalph.m` (available on the website) quantizes a signal to the nearest element of a desired set. Its help file reads

```
% y=quantalph (x , alphabet )
% quantize the input signal x to the alphabet
% using nearest neighbor method
% input x - vector to be quantized
% alphabet vector of discrete values
% that y can assume
% sorted in ascending order
% output y quantized vector
```

Let x be a random vector $x = \text{randn}(1, n)$ of length n . Quantize x to the nearest $[3, 1, 1, 3]$.

- What percentage of the outputs are 1s? 3s?
- Plot the magnitude spectrum of x and the magnitude spectrum of the output.
- Now let $x = 3 * \text{randn}(1, n)$ and answer the same questions.
- Explain what the `randn(1, n)` function does.

Solution

This code was used to get the following answers

Listing 4: MATLAB code for Exercise 3.25

```
% Set up
Ts=1/1000;
time=1;
n=time/Ts;
alphabet = [3, 1, 1, 3];

% For A and B
x = randn(1, n);
y = quantalph (x , alphabet );
% For C
% x = 3*randn(1, n);
% y = quantalph (x , alphabet );

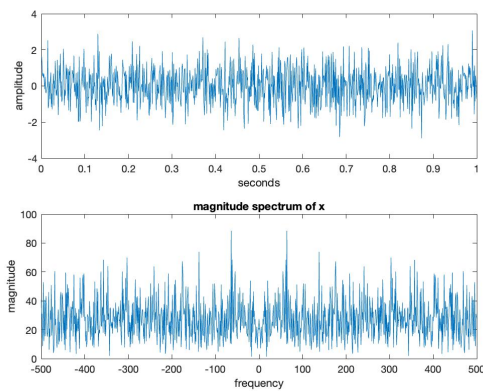
% Get percentages of 1 & 3
num1=0; num3=0;
for i=1:n

    if y(i) == 1
        num1 = num1 + 1;
    elseif y(i) == 3
        fprintf(' trig\n')
        num3 = num3 + 1;
    end
end
fprintf(' 1 appeared %d percent of the time\n', num1/n*100)
fprintf(' 3 appeared %d percent of the time\n', num3/n*100)
```

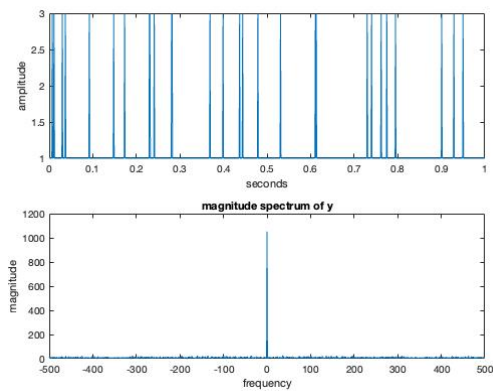
```
% Plot
% plotspec(x,Ts)
% title('magnitude spectrum of x')
% plotspec(y,Ts)
% title('magnitude spectrum of y')
```

a. 98% of the outputs are 1 and the other 2% of outputs are 3

b. Magnitude spectrums plotted below

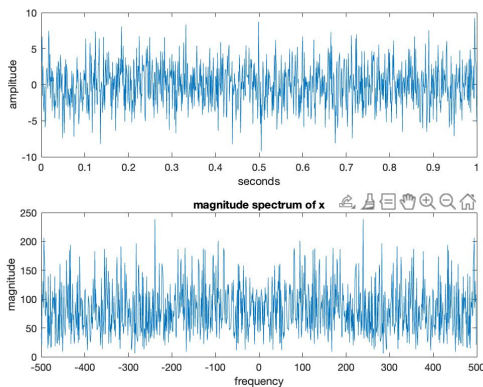


(a) magnitude spectrum of x

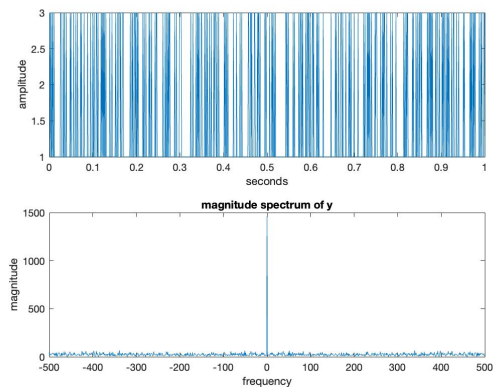


(b) magnitude spectrum of output

c. 71% of the outputs are 1 and the other 29% of outputs are 3



(a) magnitude spectrum of x



(b) magnitude spectrum of output

d. `randn(1,n)` generates a 1-by-n vector of random numbers.

Exercise 4.1

Calculate the Fourier transform of $\delta(t - t_0)$ from the definition. Now calculate it using the time-shift property (A.37). Are they the same?

Hint: they had better be.

Solution

Using the definition of a Fourier transform:

$$F(x(t)) = \int_{-\infty}^{+\infty} x(t) e^{-j2\pi t} dt$$

$$F(\delta(t - t_0)) = \int_{-\infty}^{+\infty} \delta(t - t_0) e^{-j2\pi t} dt$$

$$F(\delta(t - t_0)) = e^{-j2\pi t_0}$$

Using the time-shift property:

$$F(w(t - t_0)) = W(f) e^{-j2\pi f t_0}$$

$$W(\delta(t)) = 1$$

$$F(\delta(t - t_0)) = e^{-j2\pi f t_0}$$

Exercise 4.10

Suppose that a system has an impulse response that is an exponential pulse. Mimic the code in convolex.m to find its output when the input is a white noise (recall specnoise.m on page 42).

Solution

Using this code

Listing 5: MATLAB code for Exercise 4.10

```
% Create Noise signal
time=1;                                % length of time
Ts=1/10000;                            % time interval between samples
t=Ts:Ts:time;                          % create time vector
x=randn(1,time/Ts);                   % Ts points of noise for time seconds
h=exp(-t);                             % define impulse response
y=conv(h,x);                           % Do convolution
subplot(3,1,1), plot(t,x)             % Plot
subplot(3,1,2), plot(t,h)
subplot(3,1,3), plot(t,y(1:length(t)))
% actual commands used to draw figure:
subplot(3,1,1), plot(t,x)
ylabel('input')
subplot(3,1,2), plot(t,h)
ylabel('impulse_response')
subplot(3,1,3), plot(t,y(1:length(t)))
ylabel('output')
xlabel('time_in_seconds')
```

This output was generated

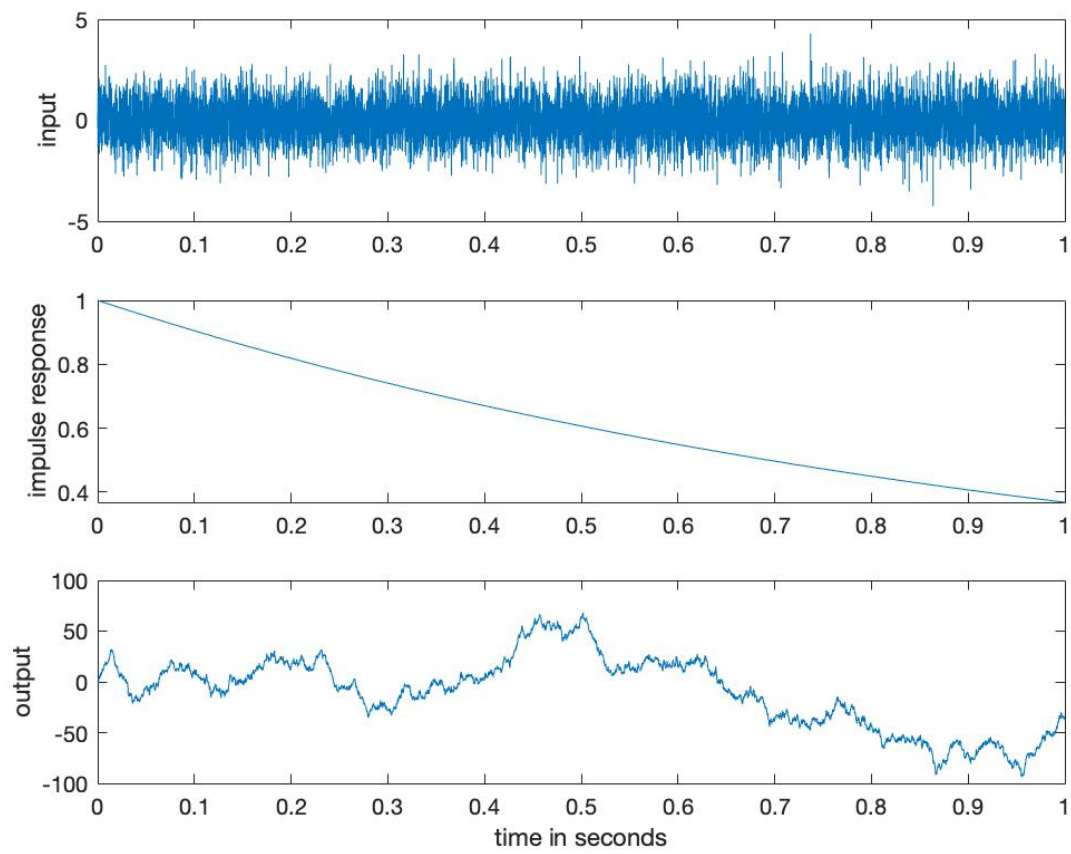


Figure 8: output of exponential pulse response to noisy input

Exercise 4.17

Suppose a system has an impulse response that is a sinc function. Using `freqresp.m`, find the frequency response of the system. What kind of filter does this represent?

Hint: center the sinc in time; for instance, use $h = \text{sinc}(10 * (t - \text{time}/2))$.

Solution

Using this code

Listing 6: MATLAB code for Exercise 4.17

```
Ts=1/100; time=10;           % sampling interval and total time
t=0:Ts:time;                 % create time vector
h=sinc(10 * (t - time/2));    % define impulse response
plotspec(h,Ts)                % find and plot frequency response
```

This output was generated

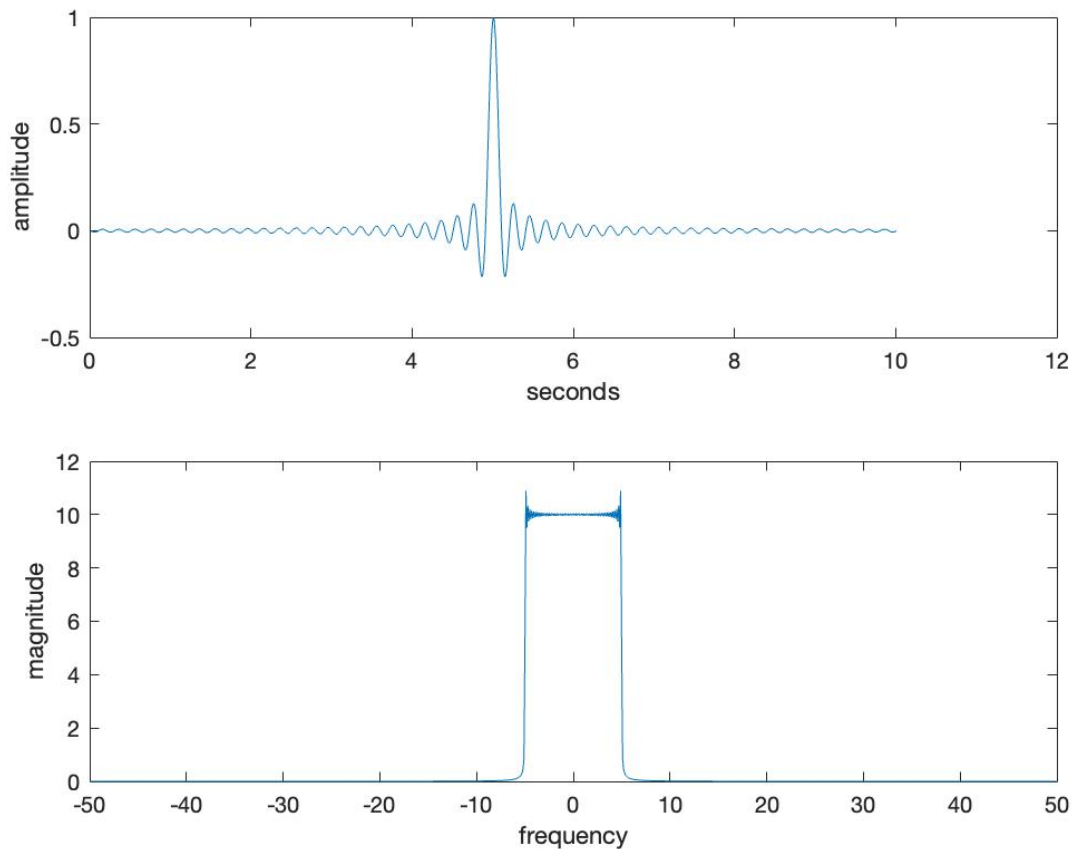


Figure 9: frequency response of system with sinc impulse response

This figure represents a low pass filter.

Exercise 4.18

Suppose a system has an impulse response that is a sin function. Using `freqresp.m`, find the frequency response of the system. What kind of filter does this represent? Can you predict the relationship between the frequency of the sine wave and the location of the peaks in the spectrum?

Hint: try $h = \sin(25 * t)$.

Solution

Using this code

Listing 7: MATLAB code for Exercise 4.18

```
Ts=1/100; time=10;           % sampling interval and total time
t=0:Ts:time;                 % create time vector
h=sin(25*t);                 % define impulse response
plotspec(h,Ts)               % find and plot frequency response
```

This output was generated

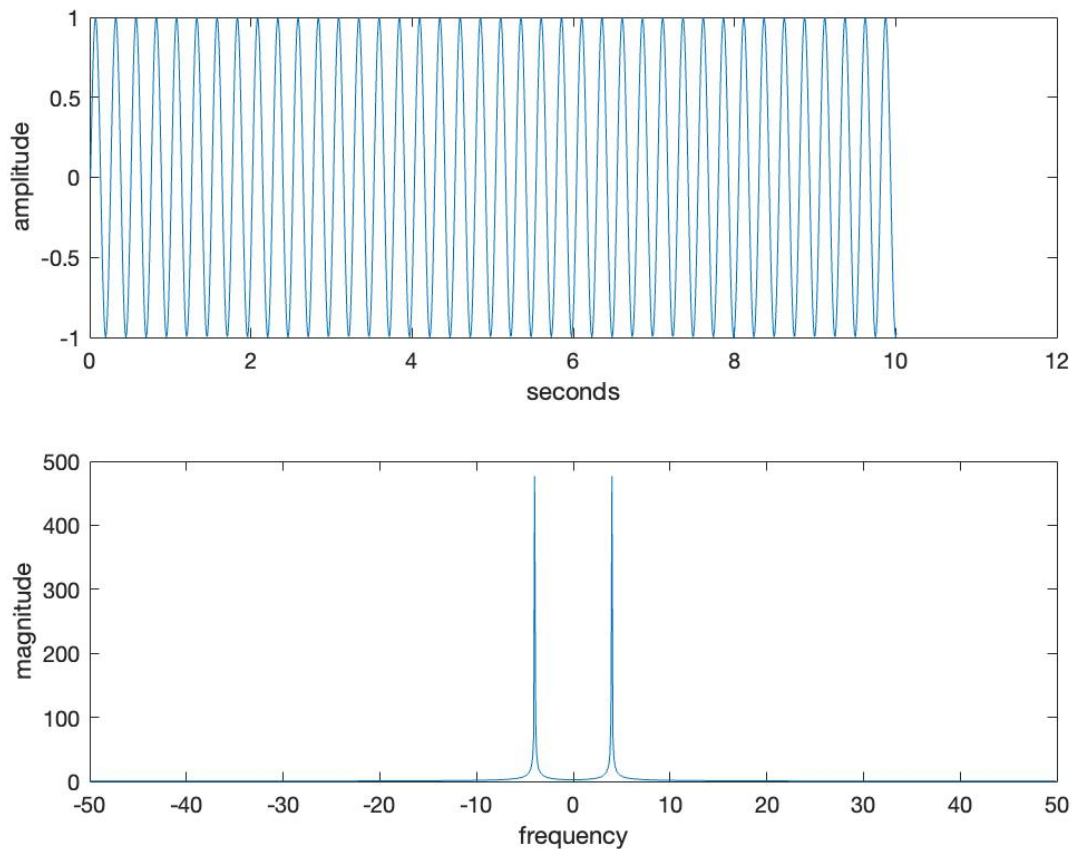


Figure 10: frequency response of system with sinc impulse response

This figure represents a narrow band pass filter. For this image the spectrum peaks are located at the frequency of the sine wave.

$25 = 2 * \pi * 3.979$ so $f = 4Hz$ which corresponds to the spectrum peaks in the image above.

Plotting by hand

Plot by hand the magnitude and argument of the spectrum of:

a. $\cos(40\pi t) + \sin(20\pi t)$.

b. $\cos(20\pi t) + \sin(20\pi t)$.

Solution

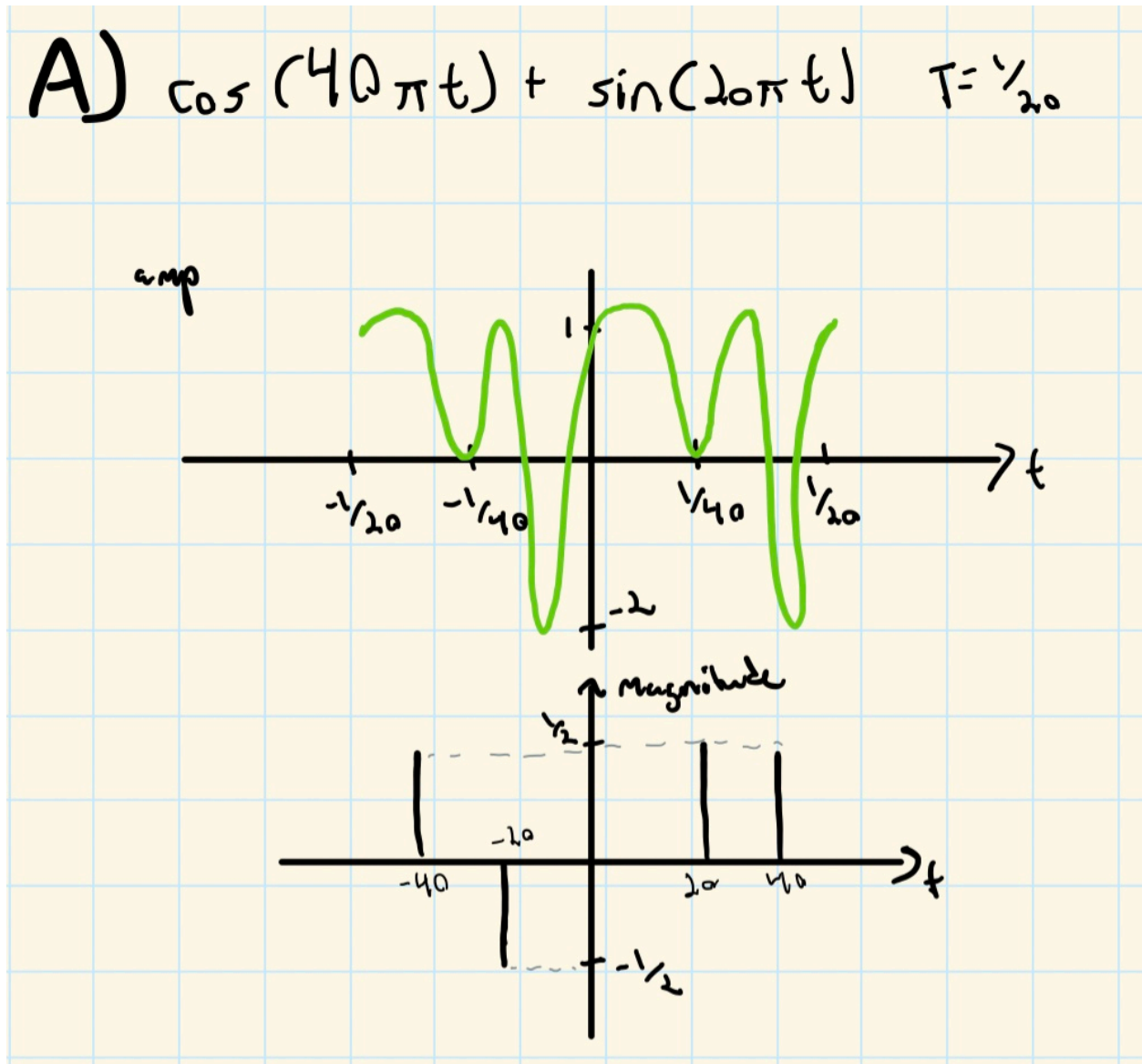
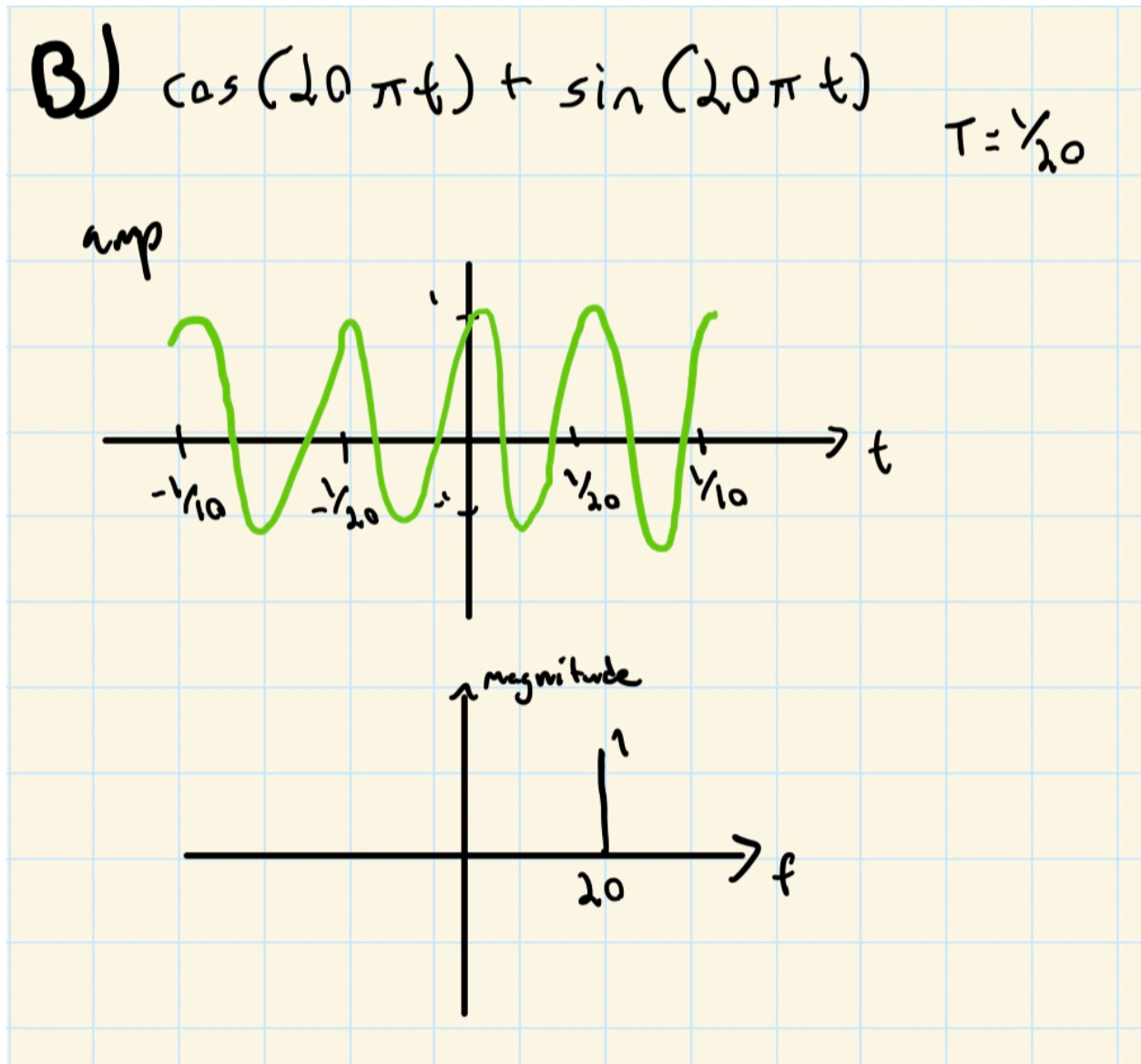


Figure 11: spectrum plot of $\cos(40\pi t) + \sin(20\pi t)$

Figure 12: spectrum plot of $\cos(20\pi t) + \sin(20\pi t)$