EC 415: Homework 2

Due by Friday 03/05/2021 6:00PM

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Exercise 3.1

Use specsquare.m to investigate the relationship between the time behavior of the square wave and its spectrum. The Matlab command zoom on is often helpful for viewing details of the plots.

a. Try square waves with different frequencies: f=20, 40, 100, 300 Hz. How do the time plots change? How do the spectra change?

b. Try square waves of different lengths, time=1, 10, 100 seconds. How does the spectrum change in each case?

c. Try different sampling times, T_s=1/100, 1/10000 seconds. How does the spectrum change in each case?

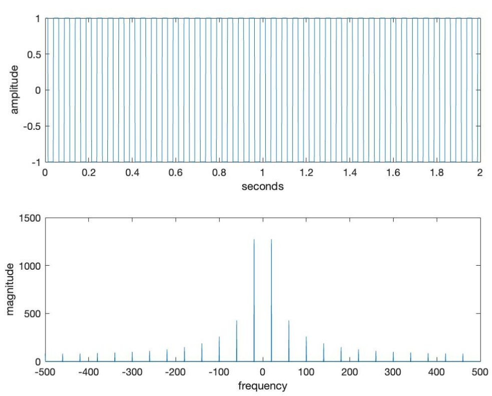
Solution

To obtain all solutions to this exercise, a copy of the provided specsquare.m was used. For each part of the exercise, the relevant variable was changed before re-running the script and generating a plot.

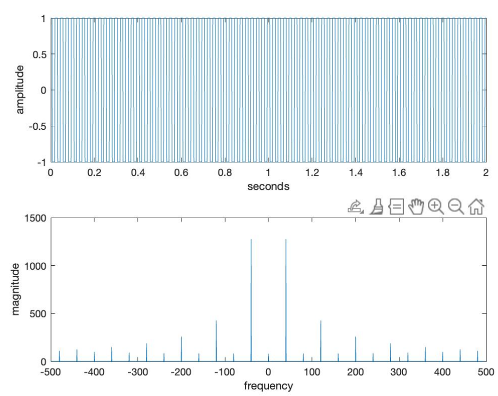
Listing 1: MATLAB code for Exercise 3.1

```
% specsquare.m plot the spectrum of a square wave
f = 10;                   % "frequency" of square wave, 10
time = 2;                 % length of time, 2
Ts = 1/10000;             % time interval between samples, 1/1000
t = Ts:Ts:time;           % create a time vector
x = sign(cos(2*pi*f*t)); % square wave = sign of cos wave
plotspec(x,Ts)            % call plotspec to draw spectrum
```

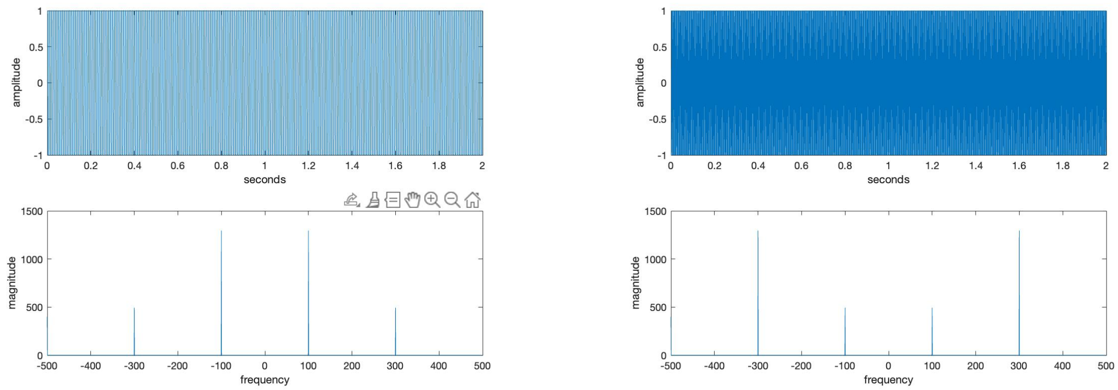
a. As the frequency increases, the density of square waves on the time plot increases. TODO how do the spectra change



(a) f=20Hz



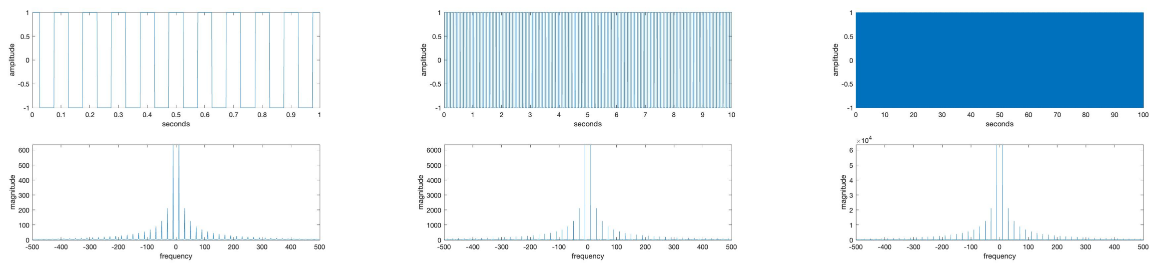
(b) f=40Hz



(a) f=100Hz (b) f=300Hz

Figure 2: square waves of increasing frequency

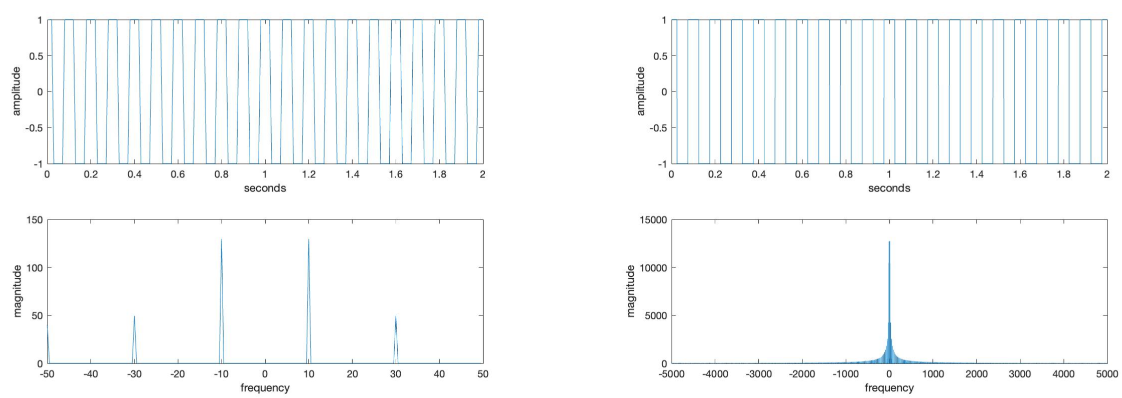
b. As the length of time increases, the magnitude of the spectra increases.



(a) t=1sec (b) t=10sec (c) t=100sec

Figure 3: longer running square waves.

c. As the sampling time decreases, the sampling frequency increases. This means the spectrum plot is a more complete picture with more plotted spectra. However, this does not change the actual shape of the spectrum.



(a) T_s=1/100 (b) T_s=1/10000

Figure 4: square waves of increasing sample rate

Exercise 3.3

Mimic the code in specsquare.m to find the spectrum of:

a. An exponential pulse $s(t) = e^{-t}$ for $0 < t < 10$

b. A scaled exponential pulse $s(t) = 5e^{-t}$ for $0 < t < 10$

c. A Gaussian pulse $s(t) = e^{-t^2}$ for $-2 < t < 2$

d. A Gaussian pulse $s(t) = e^{-t^2}$ for $-20 < t < 20$

e. The sinusoids $s(t) = sin(2\pi ft + \phi)$ for $f = 20, 100, 1000$ with $\phi = 0, \pi/4, \pi/2$ and $0 < t < 10$

Solution

a. Here is the MATLAB code:

Listing 2: MATLAB code for part a

```
% 3.3 PART A
time = 10; % run fo 10 sec
Ts = 1/10000; % ms timescale
t = Ts:Ts:time; % time vector
x = exp(-t); % signal
plotspec(x, Ts) % plot spectrum
```

The resulting plot is shown in the figure below.

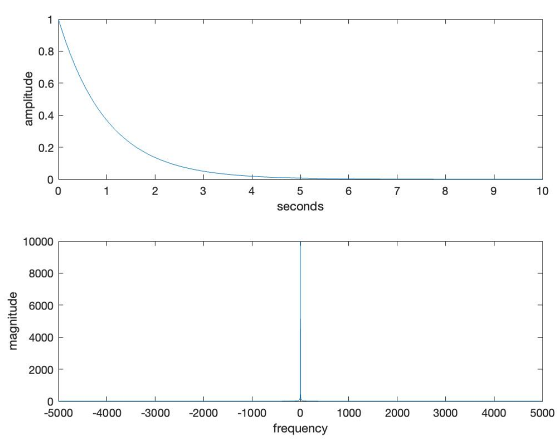


Figure 5: p3.3a d

b. Here is the MATLAB code:

Listing 3: MATLAB code for part b

```
%p3_3b.m
time = 10; % run fo 10 sec
Ts = 1/1000; % ms timescale
t = Ts:Ts:time; % time vector
x = 5*exp(-t); % signal
plotspec(x, Ts) % plot spectrum
```

The resulting plot is shown in the figure below.

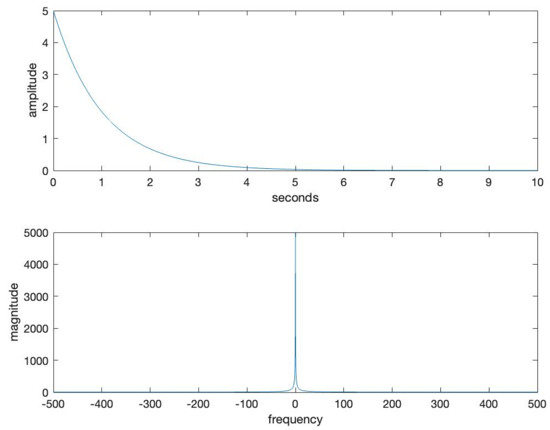


Figure 6: p3.3b

c. Here is the MATLAB code:

Listing 4: MATLAB code for part c

```
%p3_3c.m
time = 2; % run fo 10 sec
Ts = 1/1000; % ms timescale
t = (0-time):Ts:time; % time vector from -time to time
x = 5*exp(-t); % signal
plotspec(x, Ts) % plot spectrum
```

The resulting plot is shown in the figure below.

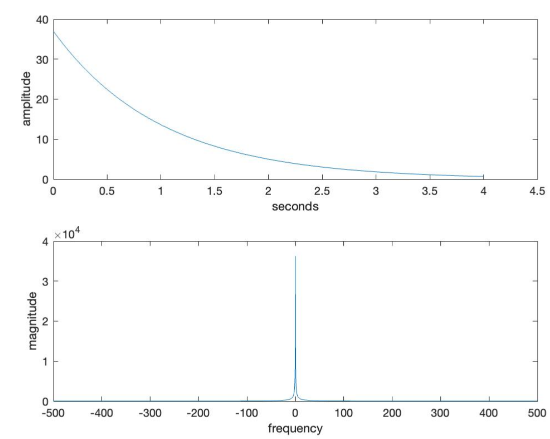


Figure 7: p3.3c

d. Here is the MATLAB code:

Listing 5: MATLAB code for part d

```
%p3_3d.m
time = 20; % run fo 10 sec
Ts = 1/1000; % ms timescale
t = (0-time):Ts:time; % time vector from -time to time
x = 5*exp(-t); % signal
plotspec(x, Ts) % plot spectrum
```

The resulting plot is shown in the figure below.

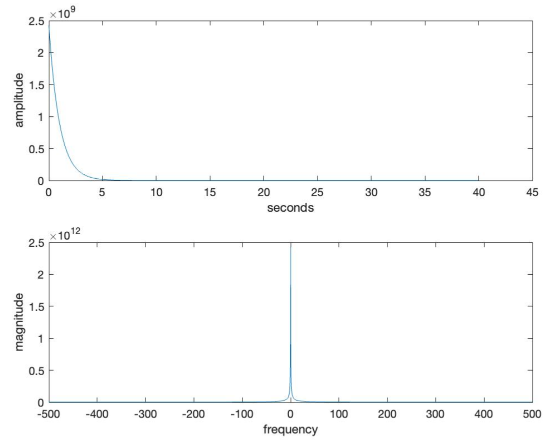


Figure 8: p3.3d

e. Here is the MATLAB code: TODO

Exercise 3.6

Mimic the code in speccos.m to find the spectrum of a cosine wave

a. For different frequencies f=1, 2, 20, 30 Hz, b. for different phases $\phi = 0$, 0.1, $\pi/8$, $\pi/2$ radians

b. For different sampling rates T_s=1/10, 1/1000, 1/100000.

Solution

a. A

b. B

Exercise 3.9

Mimic the code in filternoise.m to create a filter that

a. Passes all frequencies above 500 Hz

b. Passes all frequencies below 3000 Hz

c. Rejects all frequencies between 1500 and 2500 Hz

Solution

a. A

b. B

c. C

Exercise 3.10

Change the sampling rate to T_s=1/20000. Redesign the three filters from Exercise 3.9.

Solution

soln here

Exercise 3.11

Let $x_1(t)$ be a cosine wave of frequency f = 800, $x_2(t)$ be a cosine wave of frequency f = 2000, and $x_3(t)$ be a cosine wave of frequency f = 4500. Let $x(t) = x_1(t) + 0.5 * x_2(t) + 2 * x_3(t)$. Use x(t) as input to each of the three filters in filternoise.m. Plot the spectra, and explain what you see.

Solution

soln here

Exercise 3.26

Mimic the code in modulate.m to find the spectrum of the output y(t) of a modulator block (with modulation frequency $f_c = 1000$ Hz) when

a. The input is $x(t) = cos(2 * \pi * f_1 * t) + cos(2 * \pi * f_2 * t)$ for $f_1 = 100$ and $f_2 = 150$ Hz

b. The input is a square wave with fundamental f = 150 Hz

c. The input is a noise signal with all energy below 300 Hz

Solution

a. A

b. B

c. C

Problem 1

What is Wi-Fi? What frequencies does it operate on? List out various 802.11 protocols and their respective frequencies and data rates. Finally, specify three factors that contribute to the weakening of Wi-Fi signals as they propagate in space.

Solution

Wi-Fi is technology for radio wireless local area networking of devices based on the IEEE 802.11 standards. *Wi-Fi* is a trademark of the Wi-Fi Alliance, which restricts the use of the term *Wi-Fi Certified* to products that successfully complete interoperability certification testing.[1]

Wi-Fi operates on the following frequencies:

- 2.4 GHz
- 5 GHz
- 60 GHz

The table below lists out different Wi-Fi protocols, the frequency on which they operate and data rates associated with them.[2]

Designation	Spectrum	single-MIMO PHY
802.11a	5 GHz	54 Mbps
802.11b	2.4 GHz	11 Mbps
802.11g	2.4 GHz	54 Mbps
802.11n	2.4 GHz / 5 GHz	144 Mbps / 300 Mbps
802.11ac	5 GHz	433 Mbps
802.11ad	60 GHz	5 Gbps
802.11ax (draft)	2.4 GHz / 5 GHz	500 Mbps
802.11ay (draft)	60 GHz	40 Gbps

The three concepts that explain weakening of the Wi-Fi signal strength as it propagates in space are:

1. Path loss
2. Shadowing
3. Multipath fading

Problem 2

Find the Fourier Transform of the rectangular pulse:

$$\Pi(t) = \begin{cases} 1, & -T/2 \leq t \leq T/2 \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

[1] "What is Wi-Fi (IEEE 802.11x)? A Webopedia Definition", `https://www.webopedia.com/TERM/W/Wi_Fi.html`

[2] "Downloading the newest Wi-Fi protocols: 802.11ax and 802.11ay explained", `https://arstechnica.com/gadgets/2018/11/802-eleventy-which-802-11ax-and-802-11ay-explained/`

Solution

The Fourier Transform of the rectangular pulse is calculated in the following manner:

$$\begin{aligned} W(f) = \int_{t=-\infty}^{\infty} \Pi(t) e^{-j2\pi f t}\, dt = \int_{t=-T/2}^{T/2} (1) e^{-j2\pi f t}\, dt = \left. \frac{e^{-j2\pi f t}}{-j2\pi f} \right|_{t=-T/2}^{T/2} \\ = \frac{e^{-j\pi f T} - e^{j\pi f T}}{-j2\pi f} = T \frac{\sin(\pi f T)}{\pi f T} \equiv T \mathrm{sinc}(fT) \end{aligned} \quad (2)$$

Problem 3

Write a Matlab code to upconvert an input sine signal (frequency f_i) to a higher frequency (f_c) of a carrier so that it can more easily propagate over long distances and plot the result. Be sure to also plot the input and the carrier signal.

Solution

Here is the Matlab code:

Listing 6: Sample code from Matlab

```
% modulate.m: change the frequency of the input
time=.5; Ts=1/10000;              % time and sampling interval
t=Ts:Ts:time;                     % define a 'time' vector
fc=1000; cmod=cos(2*pi*fc*t);     % create cos of freq fc
fi=100; x=cos(2*pi*fi*t);         % input is cos of freq fi
y=cmod.*x;                        % multiply input by cmod
figure(1), plotspec(cmod,Ts)      % find spectra and plot
figure(2), plotspec(x,Ts)
figure(3), plotspec(y,Ts)

%Here's how the figure was actually drawn
N=length(x);                                % length of the signal x
t=Ts*(1:N);                                 % define a time vector
ssf=(-N/2:N/2-1)/(Ts*N);                    % frequency vector
fx=fftshift(fft(x(1:N)));
figure(4), subplot(3,1,1), plot(ssf,abs(fx))
xlabel('magnitude spectrum at input')
fcmod=fftshift(fft(cmod(1:N)));
subplot(3,1,2), plot(ssf,abs(fcmod))
xlabel('magnitude spectrum of the oscillator')
fy=fftshift(fft(y(1:N)));
subplot(3,1,3), plot(ssf,abs(fy))
xlabel('magnitude spectrum at output')
```

The resulting plots are shown in the figures below.