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1  # EXCERCISE 4
2
3  import numpy as np
4  import matplotlib.pyplot as plt
5
6  # Create Hilbert matrix
7  def CreateMatrix(n):
8      Matrix = np.empty((n,n))
9      for i in range(n):
10         for j in range(n):
11             Matrix[i][j] = 1./((i+1)+(j+1)-1)
12     return Matrix
13
14 # Generate vector of ones
15 def GenX(n):
16     x = np.ones((n))
17     return x
18
19 # get solution and xhat for the matrix and x
20 def SolveMatrix(n):
21     x = GenX(n)
22     M = CreateMatrix(n)
23     b = np.dot(M,x)
24     xhat = np.linalg.solve(M,b)
25     return b,xhat,M,x
26
27 # calculate the residual and the relative error and take the inf
28 • norms of them.
29 # Also calculate the norm of the condition number
30 def NormResidual(n):
31     b,xhat,M,x = SolveMatrix(n)
32     residual = b-np.dot(M,xhat)
33     r = np.linalg.norm(residual,ord=np.inf)
34     relerror = (((xhat-x)*100)/x)
35     errorx = np.linalg.norm(relerror,ord =np.inf)
36     cond = np.linalg.cond(M)
37     return r,errorx,cond
38
39 # define lists and amount of arrays of size nxn
40 nsize = 26
41 xlist = []
42 Reslist = []
43 Errlist = []
44 Condloglist = []
45 Condlist = []
46 for i in range(2,nsize):

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46     r,errorx,cond = NormResidual(i)
47     xlist.append(i)
48     Reslist.append(r)
49     Errlist.append(errorx)
50     Condlist.append(cond)
51     # take the log10 of the condition number in order to find the
52     • amount of digits that we lose
53     Condloglist.append(np.log10(cond))
54
55 # plot everything
56 fig, (ax1,ax2,ax3,ax4) = plt.subplots(4,1, sharex=True)
57 plt.suptitle("Excercise 2.6",fontsize = 30)
58 ax1.plot(xlist,Reslist)
59 ax1.tick_params(axis='both', labels=17)
60 ax1.set_ylabel('Residuals', fontsize = 15)
61 ax2.plot(xlist,Errlist)
62 ax2.set_ylabel("Relative Error [%]", fontsize = 15)
63 ax2.tick_params(axis='both', labels=17)
64 ax2.axhline(100,color='r')
65 ax3.plot(xlist,Condlist)
66 ax3.set_ylabel("Condition Number",fontsize = 15)
67 ax3.set_xlabel("N",fontsize = 20)
68 ax3.tick_params(axis='both', labels=17)
69 ax4.set_ylabel("Log10(Condition Number)",fontsize = 15)
70 ax4.set_xlabel("N",fontsize = 20)
71 ax4.tick_params(axis='both', labels=17)
72 ax4.plot(xlist,Condloglist)
73 plt.show()

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