Hand-in 1

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Numerical Algorithms

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1 Excercise 1

1.1

first we define:

$$f(x) = y$$
$$\hat{f}(x) = \hat{y}$$

If we fill in we get:

$$f(x) = 1.557$$
$$\hat{f}(x) = 1.667$$

Now the Forward and the Backward error are defined as follow:

$$\text{Forward error, } \Delta y = \hat{y} - y$$
$$\text{Backward error, } \Delta x = \hat{x} - x$$

$$\text{Forward error, } \Delta y = 1.09 * 10^{-1}$$
$$\text{Backward error, } \Delta x = 3.04 * 10^{-2}$$

for x = 1.4 we get:

$$\text{Forward error, } \Delta y = 4.13 * 10^{1}$$
$$\text{Backward error, } \Delta x = 1.50 * 10^{-1}$$

1.2

Using the definition that:

$$f(\hat{x}) = \hat{y}$$

we have:

$$\begin{aligned}
y &= f(x) \\
x &= x \\
\hat{x} &= \hat{x} \\
\hat{y} &= f(x + \Delta x) \\
\hat{x} &= x + \Delta x
\end{aligned}$$

So the forward error now becomes:

$$\text{Forward error} = f(x + \Delta x) - f(x)$$

Now if we Taylor expand the first term we get:

$$f(x + \Delta x) \approx f(x) + \Delta x * f^{'}(x)$$

where $f^{'}(x)$ is the derivative of $f(x)$ The relative errors then become:

$$\begin{aligned}
\text{Relative Forward error} &= \frac{\hat{y} - y}{y} \\
&\approx \frac{f(x) + \Delta x * f^{'}(x) - f(x)}{f(x)} \\
&\approx \frac{\Delta x * f^{'}(x)}{f(x)} \\
\text{Relative Backward error} &= \frac{\hat{x} - x}{x} \\
&= \frac{\Delta x}{x}
\end{aligned}$$

Thus we can evaluate $f(x)$ by implementing the relative condition number using the relative

Forward error and the relative Backward error:

$$
\begin{aligned}
\text{Relative Condition Error} &= \frac{\text{Relative Forward Error}}{\text{Relative Backward error}} \\
&\approx \frac{\frac{\Delta x * f'(x)}{f(x)}}{\frac{\Delta x}{x}} \\
&\approx \frac{x * f'(x)}{f(x)}
\end{aligned}
$$

For $x = 1$ we get a Relative Condition Error of $2.20 * 10^0$ while for $x = 1.4$ we get a Relative Condition Error of $8.36 * 10^0$, this means that the sensitivity of the function for the change from $x = 1$ to $x = 1.4$ is big.

1.3

In order to evaluate the function around $\hat{f}(x)$ we use the same Relative Condition Error function we got before but replace $f(x)$ with $\hat{f}(x)$ like so:

$$
\text{Relative Condition Error} \approx \frac{x * \hat{f}'(x)}{\hat{f}(x)}
$$

For $x = 1$ we get a Relative Condition Error of $2.60 * 10^0$ while for $x = 1.4$ we get a Relative Condition Error of $9.80 * 10^1$.
Here we can see that the sensitivity of $\hat{f}(x)$ for the change of $x = 1$ to $x = 1.4$ is enormous

2 Exc 2

The comparison can be seen in the figure below:

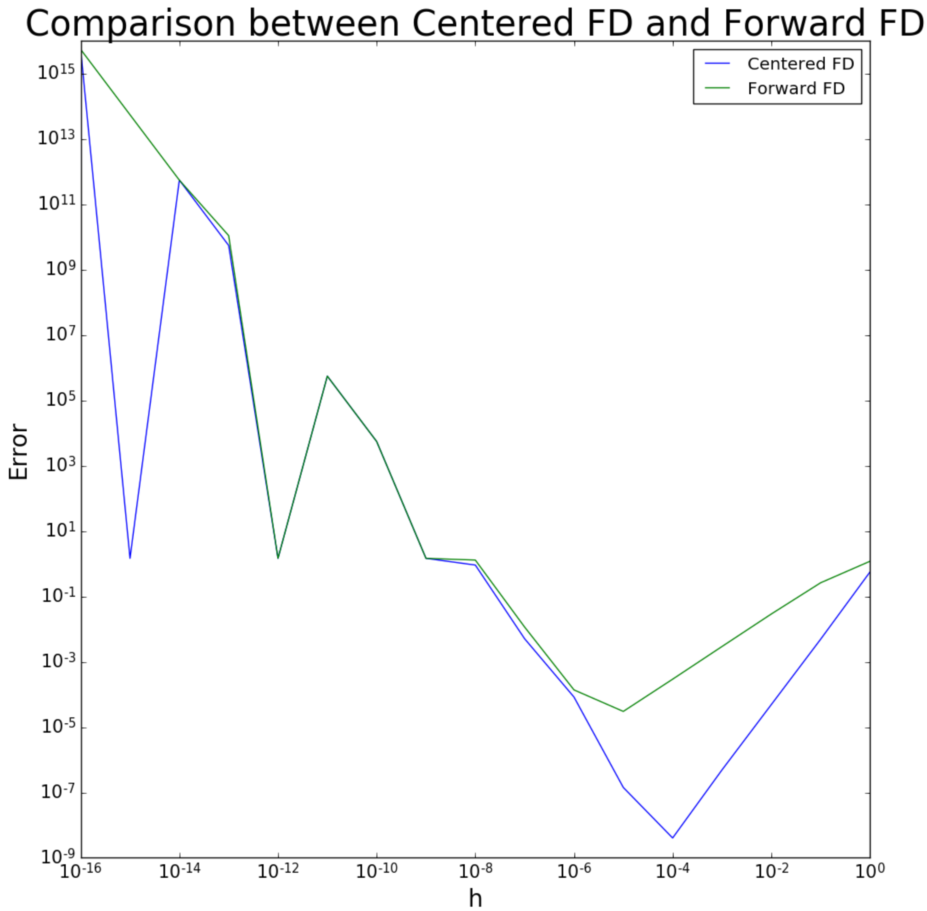

Figure 1: Centered FD plotted with Forward FD against the exact solution, On the y-axis we have the error compared to the exact solution and on the x-axis we have the h step

From Figure 1 we can easily see that the Centered FD function has an error which approaches zero more often than Forward FD this means that the Centered FD function is generally more accurate.

3 Exc 3

After making the assignment no clear difference between implementation a (iterative method) and b(exponential method) are found (see Table 1.

n	Solutions of the Iterative method (a)	Solutions of the Exponential function (b)
1.0	105.0000	105.0000
4.0	105.0945	105.0945
12.0	105.1162	105.1162
365.0	105.1267	105.1267

Table 1: Values of the solution found for both method a and b

Furthermore, the compounding interest doesn't increase anymore once $\frac{r}{n}$ approaches $1 * 10^{-16}$ This is because my machine uses double precision and when $\frac{r}{n}$ approaches double precision $(1-\frac{r}{n})$ becomes just 1. Making the compounding interest constant. This is shown in Figure 2

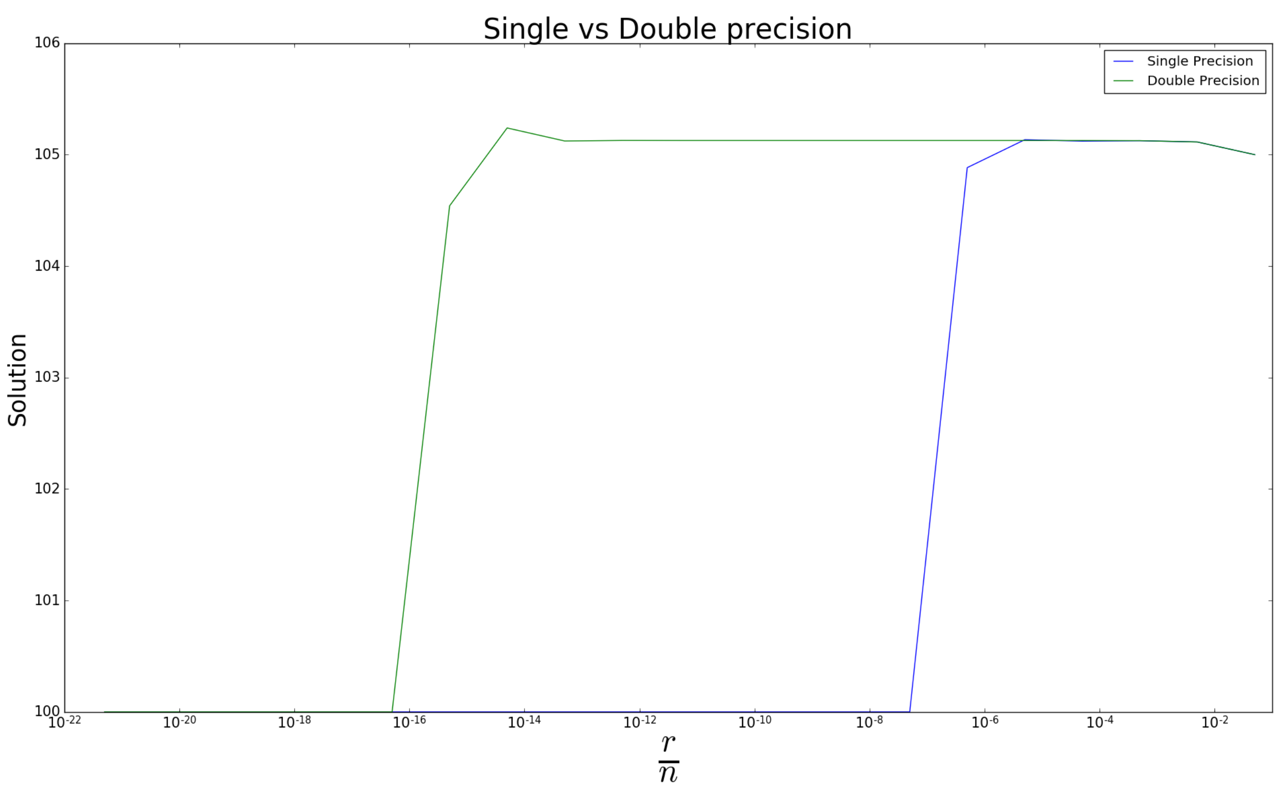

Figure 2: Plot of the function $f = a * \exp(n * \log(1 + \frac{r}{n}))$ for both single and Double precision. On the x-axis we have the $\frac{r}{n}$ part of the function and on the y-axis we have the solution of the formula.

From Figure 2 we can clearly see that when the value of $\frac{r}{n}$ is approximately $1 * 10^{-8}$ for single- and $1 * 10^{-16}$ for double precision we get a solution of 100 instead of ≈ 105 we should get. Which is, as earlier explained, because $(1 - \frac{r}{n})$ becomes 1.