ADDENDUM TO SIMULATION OF FRISBEE FLIGHT

M. Hubbard and S. A. Hummel
Department of Mechanical and Aeronautical Engineering
University of California, Davis
CA 95616 USA

Since publication in the 5th Conference on Mathematics and Computers in Sport held at the University of Technology, Sydney, New South Wales, Australia, June 2000, several changes have been made, as well as corrections to errors present in the original paper. The following serves to provide a list of the nomenclature used and a reference to the changes.

Nomenclature

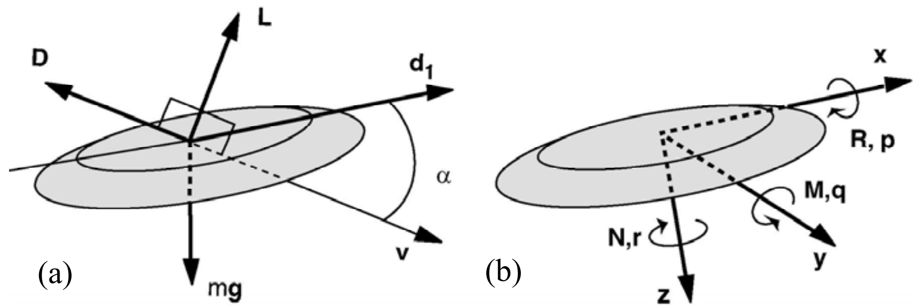
m	frisbee mass
A	frisbee planform area
d	frisbee diameter
ρ	air density
I_d	diametrial inertia
I_a	axial inertia
$\boldsymbol{\omega}$	total angular velocity of disc
$\boldsymbol{v}$	velocity of the center of mass
ϕ, θ, γ	123 Euler angle rotations
$\mathbf{b}_1$, $\mathbf{b}_2$, $\mathbf{b}_3$	aeroballistic axes
$\mathbf{d}_1$, $\mathbf{d}_2$, $\mathbf{d}_3$	body fixed axes
x, y, z	roll, pitch, yaw axes
p, q, r	roll, pitch, yaw angular velocities (rad s^{-1})
u, v, w	linear velocity components
$\mathbf{F}$	sum of aerodynamic forces
$\mathbf{M}$	sum of aerodynamic moments
L	lift force
D	drag force
$C_l = C_{lo} + C_{l\alpha}\,\alpha$	lift coefficient
$C_d = C_{do} + C_{d\alpha}\,(\alpha-\alpha_{eq})^2$	drag coefficient
C_{lo}	lift coefficient at $\alpha = 0$
$C_{l\alpha}$	lift coefficient dependent on α
C_{do}	drag coefficient at $\alpha = \alpha_{eq}$
$C_{d\alpha}$	drag coefficient dependent on α
$\alpha_{eq} = - C_{lo} / C_{l\alpha}$	alpha at zero lift and minimum drag
M_1, M_2, M_3	rolling, pitching, and yawing moments
R, M, N	equivalent to M_1, M_2, M_3 (used in figure shown below)
$C_{M\,o}$	pitching moment coefficient at α=0
$C_{M\,\alpha}$	pitching moment coefficient dependent on alpha
$C_{M\,q}$	pitching moment damping coefficient
$C_{R\,r}$	roll moment coefficient due to spin r
$C_{R\,p}$	roll moment damping coefficient
$C_{N\,r}$	spin moment damping coefficient

Equation 3
Gravity term was missing, correct equation is

$$\mathbf{F} - mg\mathbf{k} = m\,(d\mathbf{v}/dt + \boldsymbol{\omega}_{\mathbf{D}} \times \boldsymbol{v}) \qquad (3)$$

Figure 4:
The y axis is in meters, not millimeters

Second paragraph, last sentence of Section 4 Simulation results
Because the duration of the flight was so small compared to the characteristic time of spin decay (about 7 sec), the Frisbee lost only about 10 % of its initial spin rate of -8.91 rad/sec.
Characteristic time of spin decay is closer to 60 secs, not 7 secs.



Force and Moment equations

Lift force: $L = C_l\,\rho A v^2/2 = (C_{Lo} + C_{L\alpha}\alpha)\,\rho A v^2/2$

Drag force: $D = C_d\,\rho A v^2/2 = (C_{Do} + C_{D\alpha}(\alpha - \alpha_{eq})^2)\,\rho A v^2/2$

Rolling moment: $M_1 = R = (C_{Rr} r + C_{Rp} p)\,\rho d A v^2/2$

Pitching moment: $M_2 = M = (C_{Mo} + C_{M\alpha}\alpha + C_{Mq} q)\,\rho d A v^2/2$

Spin-down moment: $M_3 = N = C_{Nr}\, r\, \rho d A v^2/2$

The aerodynamic forces and moments are parameterized by the (*now*) ten coefficients. These equations differ from those previously reported as follows:

Drag Force:
$\alpha - \alpha_{eq}$ replaces α

Rolling Moment:
C_{Rr} replaces $C_{R\gamma'}$
C_{Rp} replaces $C_{M\alpha'}$

Pitching Moment:
C_{Mo} term added
C_{Mq} replaces $C_{M\alpha'}$

The current estimated values of the ten coefficients are, as reported in the paper *Identification of Frisbee aerodynamic coefficients using flight data* by Hummel and Hubbard (2002):

Source	C_{Lo}	$C_{L\alpha}$	C_{Do}	$C_{D\alpha}$	C_{Mo}	$C_{M\alpha}$	C_{Mq}	C_{Rr}	C_{Rp}	C_{Nr}
Four Flights	0.188	2.37	0.15	1.24	-0.06	0.38	0.0008	0.0004	-0.013	-2.8E-5
P & C	0.2	2.96	0.08	2.60	-0.02	0.13	---	---	---	---

Equations of Motion

The equations are correct as follows:

$$u' = F_{b1} / m - (w\ \theta' + v\ \phi' \sin\theta)$$

$$v' = F_{b2} / m + (w\ \phi' \cos\theta - u\ \phi' \sin\theta)$$

$$w' = F_{b3} / m + (u\ \theta' - v\ \phi' \cos\theta\ )$$

$$\phi'' = (M_{b1} + 2\ I_d\ \phi'\theta' \sin\theta - I_a\ \theta'\ \phi' \sin\theta + \gamma') \cos\theta) / I_d \cos\theta$$

$$\theta'' = (M_{b2} - I_d\ \phi'^{\,2} \cos\theta \sin\theta\ + I_a\ \phi' \cos\theta\, \phi' \sin\theta\ + \gamma')\ \ ) / I_d$$

$$\gamma''\ = (M_{b3} - I_a\ (\phi'' \sin\theta\ + \phi'\ \theta' \cos\theta\ ) / I_a$$